

# Computer algebra independent integration tests

1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.2Quadratic/1.1.2.2(cx)^m(a+bx^2)^p

Nasser M. Abbasi

December 3, 2018

Compiled on December 3, 2018 at 9:52am

## Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>detailed summary tables of results</b>	<b>10</b>
<b>3</b>	<b>Listing of integrals</b>	<b>186</b>
<b>4</b>	<b>Listing of Grading functions</b>	<b>2992</b>

# 1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

## 1.1 Listing of CAS systems tested

The following systems were tested at this time.

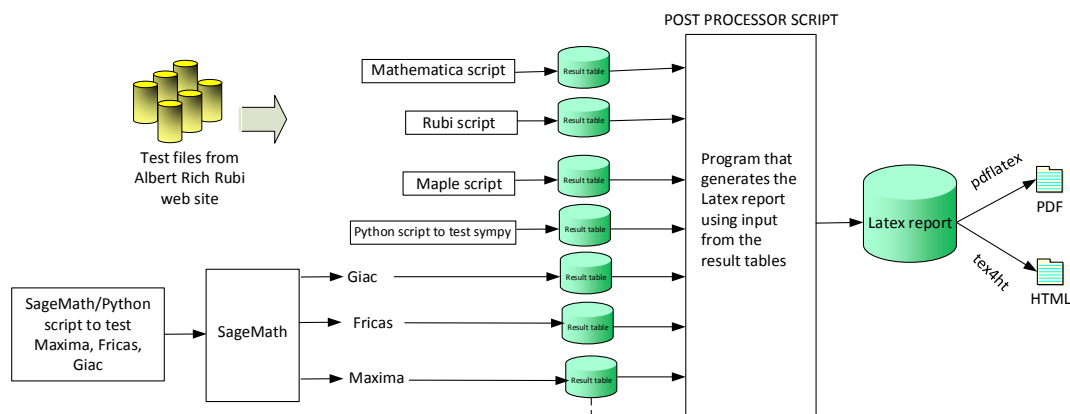
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

## 1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is comma delimited. It contains 12 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

**High level overview of the CAS independent integration test build system**

Nasser M. Abbasi  
June 22, 2018

## 1.3 Timing

The command `AbsoluteTiming[ ]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int', int(expr, x)), output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the `integrate` command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima `integrate` was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express>

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

    except Exception as ee:
        leafCount =1
```

When these cas systems implement a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.



System	solved	Failed
Rubi	% 100. ( 1071 )	% 0. ( 0 )
Rubi in Sympy	% 87.96 ( 942 )	% 12.04 ( 129 )
Mathematica	% 100. ( 1071 )	% 0. ( 0 )
Maple	% 70.49 ( 755 )	% 29.51 ( 316 )
Maxima	% 39.5 ( 423 )	% 60.5 ( 648 )
Fricas	% 62.93 ( 674 )	% 37.07 ( 397 )
Sympy	% 85.81 ( 919 )	% 14.19 ( 152 )
Giac	% 59.76 ( 640 )	% 40.24 ( 431 )

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

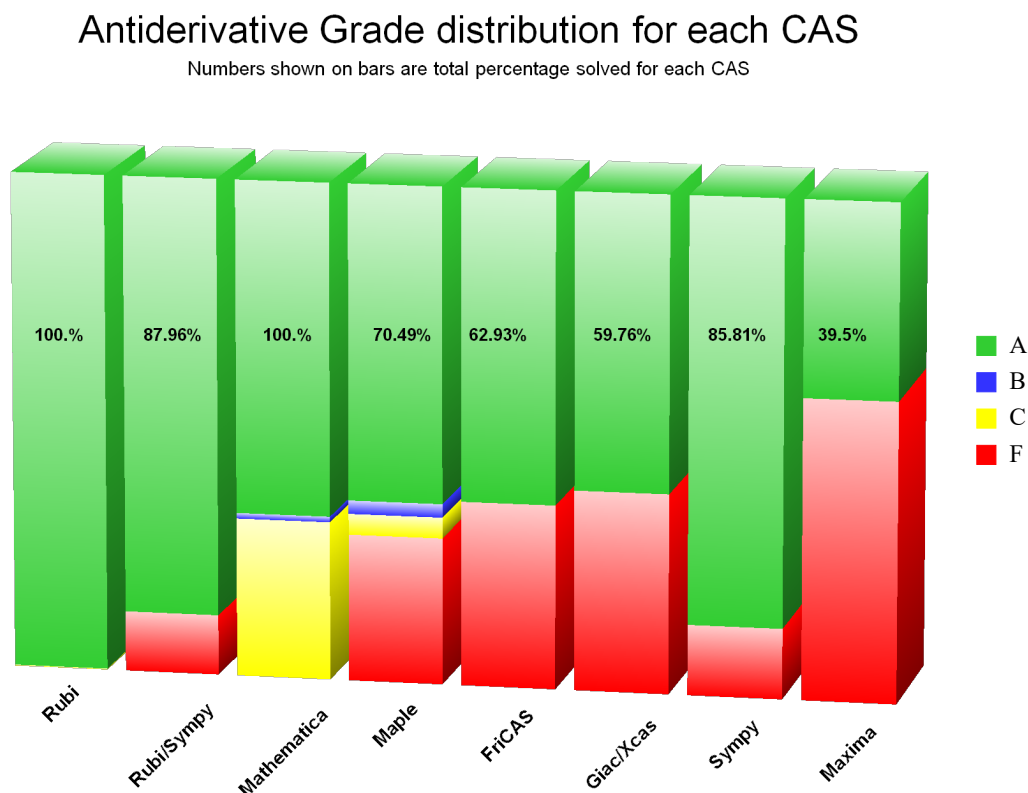
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ul style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ul>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

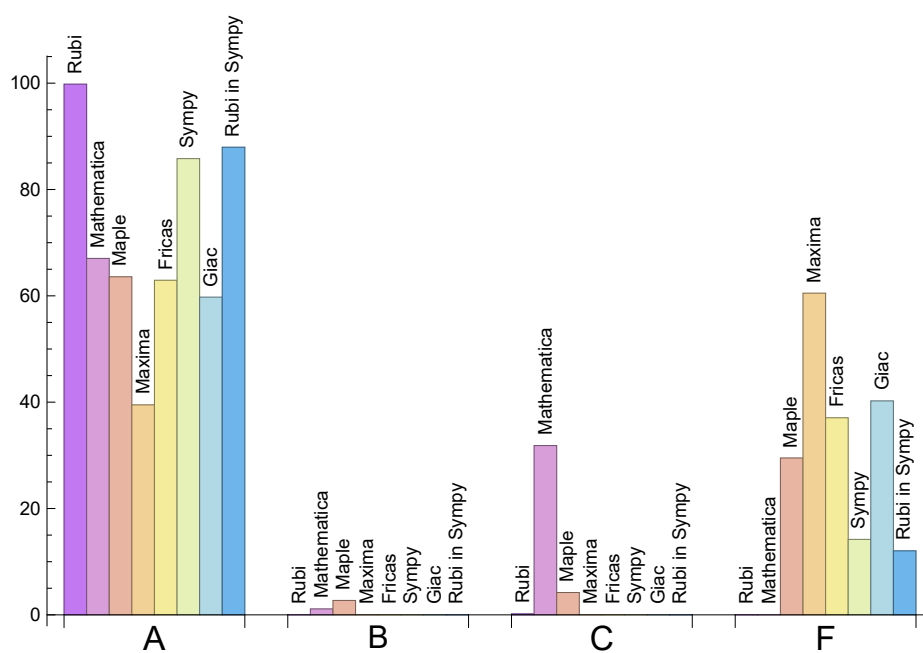
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.81	0.	0.19	0.
Rubi in Sympy	87.96	0.	0.	12.04
Mathematica	67.04	1.12	31.84	0.
Maple	63.59	2.71	4.2	29.51
Maxima	39.5	0.	0.	60.5
Fricas	62.93	0.	0.	37.07
Sympy	85.81	0.	0.	14.19
Giac	59.76	0.	0.	40.24

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



## 1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.19	113.	1.02	78.	1.
Rubi in Sympy	16.76	94.79	0.89	65.	0.89
Mathematica	0.06	69.01	0.86	64.	0.86
Maple	0.01	69.55	0.91	49.	0.84
Maxima	1.39	68.98	1.29	51.	1.16
Fricas	0.24	83.46	1.36	54.	1.2
Sympy	13.98	127.28	1.77	46.	0.94
Giac	0.22	107.47	1.58	77.	1.19

## 1.8 list of integrals that has no closed form antiderivative

}

## 1.9 list of integrals not solved by each system

**Not solved by Rubi** {}

**Not solved by Rubi in Sympy** {4, 5, 6, 7, 15, 18, 19, 21, 22, 34, 35, 36, 45, 48, 59, 60, 61, 62, 63, 74, 79, 92, 93, 94, 95, 96, 97, 98, 99, 113, 121, 124, 125, 126, 127, 128, 129, 130, 131, 132, 145, 146, 147, 148, 149, 150, 151, 152, 153, 168, 169, 170, 171, 172, 181, 182, 183, 192, 193, 194, 206, 207, 208, 209, 210, 225, 799, 800, 801, 802, 803, 804, 813, 815, 816, 817, 818, 819, 820, 821, 843, 844, 845, 846, 847, 848, 849, 858, 863, 864, 865, 866, 867, 868, 869, 898, 899, 900, 901, 902, 903, 904, 949, 950, 951, 952, 953, 954, 991, 992, 993, 994, 995, 996, 997, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1032, 1033, 1034, 1035, 1036, 1037, 1038}

**Not solved by Mathematica** {}

**Not solved by Maple** {344, 345, 346, 347, 348, 349, 350, 351, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 662, 664, 669, 670, 671, 672, 673, 674, 675, 676, 681, 682, 683, 684, 685, 686, 687, 688, 693, 694, 695, 696, 697, 698, 699, 700, 706, 707, 708, 709, 710, 711, 712, 713, 718, 719, 720, 721, 722, 723, 724, 725, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 884, 885, 886, 888, 889, 890, 912, 913, 914, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 932, 933, 934, 935, 936, 937, 938, 939, 944, 945, 949, 950, 951, 952, 953, 954, 955, 956, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 975, 976, 977, 978, 979, 980, 981, 985, 986, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1062, 1064, 1066, 1067, 1068, 1069, 1070, 1071}

**Not solved by Maxima** {125, 127, 129, 131, 133, 135, 137, 139, 141, 143, 146, 148, 150, 152, 154, 156, 158, 160, 162, 164, 166, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 226, 228, 230, 233, 235, 237, 240, 242, 244, 247, 249, 251, 257, 258, 261, 262, 263, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 485, 486, 487, 488, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 502, 503, 504, 505, 506, 507, 508, 513, 514, 515, 516, 517, 518, 519, 528, 529, 530, 531, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 669, 670, 671, 672, 673, 674, 675, 676, 681, 682, 683, 684, 685, 686, 687, 688, 693, 694, 695, 696, 697, 698, 699, 700, 706, 707, 708, 709,

710, 711, 712, 713, 718, 719, 720, 721, 722, 723, 724, 725, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 932, 933, 934, 935, 936, 937, 938, 939, 944, 945, 949, 950, 951, 952, 953, 954, 955, 956, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 975, 976, 977, 978, 979, 980, 981, 985, 986, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1062, 1064, 1066, 1067, 1068, 1069, 1070, 1071}

**Not solved by Fracas** {344, 345, 346, 347, 348, 349, 350, 351, 352, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 672, 673, 674, 675, 676, 684, 685, 686, 687, 688, 696, 697, 698, 699, 700, 709, 710, 711, 712, 713, 721, 722, 723, 724, 725, 733, 734, 735, 736, 737, 738, 739, 740, 741, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 932, 933, 934, 935, 936, 937, 938, 939, 949, 950, 951, 952, 953, 954, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 975, 976, 977, 978, 979, 980, 981, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1062, 1064, 1066, 1067, 1068, 1069, 1070, 1071}

**Not solved by Sympy** {205, 206, 207, 208, 222, 223, 224, 288, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 329, 335, 336, 350, 351, 409, 410, 411, 589, 590, 596, 597, 598, 604, 605, 606, 607, 613, 620, 621, 627, 628, 629, 634, 635, 636, 648, 663, 738, 739, 742, 743, 744, 745, 746, 749, 750, 754, 755, 758, 759, 760, 761, 762, 763, 765, 766, 767, 768, 771, 772, 773, 776, 777, 778, 779, 782, 783, 919, 923, 924, 925, 928, 929, 930, 931, 935, 936, 937, 940, 941, 942, 943, 947, 948, 949, 953, 954, 958, 959, 963, 964, 967, 968, 969, 973, 974, 977, 978, 979, 983, 984, 985, 988, 989, 990, 991, 992, 993, 996, 997, 998, 1004, 1005, 1006, 1007, 1008, 1009, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1069, 1070, 1071}

**Not solved by Giac** {344, 345, 346, 347, 348, 349, 350, 351, 352, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 672, 673, 674, 675, 676, 684, 685, 686, 687, 688, 696, 697, 698, 699, 700, 709, 710, 711, 712, 713, 721, 722, 723, 724, 725, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 932, 933, 934, 935, 936, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1043, 1044, 1045, 1046, 1047,

1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071}

## 1.10 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Rubi in Sympy** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

## 1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {1019, 1020, 1032}

**Mathematica** {}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Rubi in Sympy** Verification phase not implemented yet.

## 2 detailed summary tables of results

### 2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	1	12	18	12
normalized size	1	1.	1.	0.82	1.06	0.06	0.71	1.06	0.71
time (sec)	N/A	0.017	0.002	0.001	1.336	0.181	0.068	0.213	3.181

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	1	12	18	12
normalized size	1	1.	1.	0.82	1.06	0.06	0.71	1.06	0.71
time (sec)	N/A	0.015	0.002	0.002	1.343	0.181	0.065	0.212	3.17

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	1	12	18	12
normalized size	1	1.	1.	0.82	1.06	0.06	0.71	1.06	0.71
time (sec)	N/A	0.015	0.002	0.002	1.35	0.182	0.064	0.211	3.342

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	19	1	12	18	0
normalized size	1	1.	1.	0.82	1.12	0.06	0.71	1.06	0.
time (sec)	N/A	0.015	0.001	0.	1.342	0.184	0.067	0.214	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	1	8	14	0
normalized size	1	1.	1.	0.92	1.17	0.08	0.67	1.17	0.
time (sec)	N/A	0.01	0.	0.001	1.342	0.182	0.065	0.215	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	19	15	10	19	0
normalized size	1	1.	1.	0.92	1.46	1.15	0.77	1.46	0.
time (sec)	N/A	0.013	0.002	0.003	1.352	0.204	0.133	0.214	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	18	5	14	0
normalized size	1	1.	1.	1.1	1.4	1.8	0.5	1.4	0.
time (sec)	N/A	0.013	0.001	0.005	1.345	0.201	0.952	0.216	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	19	23	10	27	10
normalized size	1	1.	1.	0.92	1.46	1.77	0.77	2.08	0.77
time (sec)	N/A	0.014	0.004	0.007	1.342	0.209	1.045	0.217	2.935

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	14	18	10
normalized size	1	1.	1.	0.93	1.2	1.2	0.93	1.2	0.67
time (sec)	N/A	0.015	0.003	0.006	1.345	0.217	1.101	0.212	3.051

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	18	14	18	14
normalized size	1	1.	1.	0.82	1.06	1.06	0.82	1.06	0.82
time (sec)	N/A	0.016	0.004	0.008	1.378	0.226	1.097	0.211	3.116

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	20	20	15	20	14
normalized size	1	1.	1.	0.82	1.18	1.18	0.88	1.18	0.82
time (sec)	N/A	0.015	0.004	0.007	1.337	0.204	1.145	0.206	3.152

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	20	20	15	20	14
normalized size	1	1.	1.	0.82	1.18	1.18	0.88	1.18	0.82
time (sec)	N/A	0.015	0.004	0.007	1.339	0.201	1.144	0.208	3.147

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	1	24	32	24
normalized size	1	1.	1.	0.83	1.07	0.03	0.8	1.07	0.8
time (sec)	N/A	0.061	0.002	0.001	1.347	0.196	0.095	0.208	7.765

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	1	26	32	26
normalized size	1	1.	1.	0.83	1.07	0.03	0.87	1.07	0.87
time (sec)	N/A	0.036	0.001	0.	1.318	0.191	0.089	0.207	5.65

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	1	24	32	0
normalized size	1	1.	1.	0.83	1.07	0.03	0.8	1.07	0.
time (sec)	N/A	0.054	0.001	0.	1.329	0.189	0.085	0.208	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	1	26	32	26
normalized size	1	1.	1.	0.83	1.07	0.03	0.87	1.07	0.87
time (sec)	N/A	0.035	0.001	0.001	1.348	0.183	0.089	0.207	5.772

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	25	19	1	24	19	10
normalized size	1	1.	1.	1.56	1.19	0.06	1.5	1.19	0.62
time (sec)	N/A	0.012	0.004	0.	1.327	0.194	0.085	0.208	2.165

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	1	22	28	0
normalized size	1	1.	1.	0.88	1.12	0.04	0.88	1.12	0.
time (sec)	N/A	0.022	0.001	0.	1.332	0.188	0.078	0.212	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	32	28	20	32	0
normalized size	1	1.	1.	0.96	1.39	1.22	0.87	1.39	0.
time (sec)	N/A	0.038	0.002	0.003	1.324	0.208	1.02	0.222	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	30	34	19	30	19
normalized size	1	1.	1.	0.96	1.25	1.42	0.79	1.25	0.79
time (sec)	N/A	0.03	0.001	0.004	1.333	0.195	1.024	0.212	5.505



Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	32	36	24	43	0
normalized size	1	1.	1.	0.89	1.19	1.33	0.89	1.59	0.
time (sec)	N/A	0.044	0.002	0.007	1.322	0.203	1.119	0.213	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	30	35	20	30	0
normalized size	1	1.	1.	0.96	1.3	1.52	0.87	1.3	0.
time (sec)	N/A	0.03	0.001	0.007	1.332	0.193	1.132	0.208	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	35	38	22	46	24
normalized size	1	1.	1.	0.96	1.46	1.58	0.92	1.92	1.
time (sec)	N/A	0.041	0.002	0.007	1.342	0.203	1.28	0.211	7.168

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	35	35	27	35	24
normalized size	1	1.	1.	0.89	1.25	1.25	0.96	1.25	0.86
time (sec)	N/A	0.032	0.001	0.007	1.341	0.197	1.284	0.209	5.733

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	30	25	32	32	26	32	15
normalized size	1	1.	1.58	1.32	1.68	1.68	1.37	1.68	0.79
time (sec)	N/A	0.018	0.002	0.007	1.333	0.196	1.312	0.208	3.176

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	35	27	35	27
normalized size	1	1.	1.	0.83	1.17	1.17	0.9	1.17	0.9
time (sec)	N/A	0.032	0.001	0.008	1.33	0.211	1.36	0.209	5.751

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	35	27	35	26
normalized size	1	1.	1.	0.83	1.17	1.17	0.9	1.17	0.87
time (sec)	N/A	0.042	0.002	0.008	1.322	0.204	1.45	0.207	7.328

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	35	27	35	27
normalized size	1	1.	1.	0.83	1.17	1.17	0.9	1.17	0.9
time (sec)	N/A	0.031	0.001	0.007	1.469	0.196	1.487	0.206	5.813

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	1	37	47	37
normalized size	1	1.	1.	0.84	1.09	0.02	0.86	1.09	0.86
time (sec)	N/A	0.082	0.004	0.001	1.346	0.19	0.101	0.206	10.874

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	1	37	47	37
normalized size	1	1.	1.	0.84	1.09	0.02	0.86	1.09	0.86
time (sec)	N/A	0.078	0.003	0.	1.342	0.183	0.105	0.205	10.432

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	1	39	47	39
normalized size	1	1.	1.	0.84	1.09	0.02	0.91	1.09	0.91
time (sec)	N/A	0.075	0.003	0.002	1.35	0.18	0.099	0.207	10.056

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	43	36	47	1	37	47	27
normalized size	1	1.	1.26	1.06	1.38	0.03	1.09	1.38	0.79
time (sec)	N/A	0.085	0.003	0.	1.352	0.183	0.1	0.207	8.962

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	36	19	1	37	19	10
normalized size	1	1.	1.	2.25	1.19	0.06	2.31	1.19	0.62
time (sec)	N/A	0.012	0.004	0.001	1.35	0.18	0.098	0.208	2.243

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	49	45	37	49	0
normalized size	1	1.	1.	0.87	1.26	1.15	0.95	1.26	0.
time (sec)	N/A	0.052	0.007	0.003	1.346	0.202	1.057	0.209	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	49	51	37	62	0
normalized size	1	1.	1.	0.88	1.22	1.27	0.92	1.55	0.
time (sec)	N/A	0.058	0.012	0.009	1.347	0.204	1.188	0.209	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	50	53	36	62	0
normalized size	1	1.	1.	0.88	1.25	1.32	0.9	1.55	0.
time (sec)	N/A	0.055	0.008	0.01	1.346	0.2	1.353	0.208	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	53	53	36	63	41
normalized size	1	1.	1.	0.87	1.36	1.36	0.92	1.62	1.05
time (sec)	N/A	0.053	0.008	0.009	1.346	0.202	1.494	0.208	9.023

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	43	36	47	47	37	47	15
normalized size	1	1.	2.26	1.89	2.47	2.47	1.95	2.47	0.79
time (sec)	N/A	0.018	0.011	0.007	1.345	0.195	1.584	0.211	3.25

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	43	36	50	50	39	50	39
normalized size	1	1.	1.08	0.9	1.25	1.25	0.98	1.25	0.98
time (sec)	N/A	0.056	0.007	0.009	1.348	0.194	1.707	0.207	9.324

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	50	50	39	50	41
normalized size	1	1.	1.	0.84	1.16	1.16	0.91	1.16	0.95
time (sec)	N/A	0.057	0.007	0.008	1.35	0.192	1.818	0.207	9.389

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	50	50	39	50	39
normalized size	1	1.	1.	0.84	1.16	1.16	0.91	1.16	0.91
time (sec)	N/A	0.056	0.012	0.007	1.345	0.194	1.9	0.213	9.218

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	1	37	47	37
normalized size	1	1.	1.	0.84	1.09	0.02	0.86	1.09	0.86
time (sec)	N/A	0.046	0.003	0.002	1.344	0.18	0.102	0.208	7.499

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	1	37	47	37
normalized size	1	1.	1.	0.84	1.09	0.02	0.86	1.09	0.86
time (sec)	N/A	0.046	0.003	0.001	1.34	0.18	0.098	0.207	7.468

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	1	39	47	39
normalized size	1	1.	1.	0.84	1.09	0.02	0.91	1.09	0.91
time (sec)	N/A	0.045	0.003	0.001	1.348	0.184	0.101	0.207	7.666

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	42	1	32	42	0
normalized size	1	1.	1.	0.91	1.2	0.03	0.91	1.2	0.
time (sec)	N/A	0.031	0.002	0.002	1.35	0.197	0.098	0.207	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	43	49	29	43	29
normalized size	1	1.	1.	0.97	1.26	1.44	0.85	1.26	0.85
time (sec)	N/A	0.041	0.006	0.005	1.353	0.204	1.036	0.214	7.083

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	46	49	34	46	32
normalized size	1	1.	1.	0.92	1.24	1.32	0.92	1.24	0.86
time (sec)	N/A	0.04	0.007	0.009	1.343	0.206	1.202	0.208	7.13

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	45	50	32	45	0
normalized size	1	1.	1.	0.97	1.32	1.47	0.94	1.32	0.
time (sec)	N/A	0.04	0.009	0.008	1.354	0.21	1.409	0.209	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	36	50	50	39	50	34
normalized size	1	1.	1.	0.92	1.28	1.28	1.	1.28	0.87
time (sec)	N/A	0.042	0.007	0.007	1.346	0.221	1.542	0.207	7.573

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	50	50	39	50	41
normalized size	1	1.	1.	0.84	1.16	1.16	0.91	1.16	0.95
time (sec)	N/A	0.041	0.007	0.008	1.347	0.209	1.722	0.207	7.585

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	50	50	39	50	39
normalized size	1	1.	1.	0.84	1.16	1.16	0.91	1.16	0.91
time (sec)	N/A	0.041	0.012	0.008	1.346	0.218	1.737	0.206	7.724

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	1	65	77	65
normalized size	1	1.	1.	0.84	1.12	0.01	0.94	1.12	0.94
time (sec)	N/A	0.125	0.005	0.003	1.342	0.212	0.127	0.205	17.129

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	1	65	77	65
normalized size	1	1.	1.	0.84	1.12	0.01	0.94	1.12	0.94
time (sec)	N/A	0.116	0.004	0.002	1.349	0.19	0.123	0.207	16.593

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	1	66	77	66
normalized size	1	1.	1.	0.84	1.12	0.01	0.96	1.12	0.96
time (sec)	N/A	0.114	0.004	0.003	1.352	0.188	0.124	0.205	16.223

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	69	58	77	1	65	77	65
normalized size	1	1.	0.96	0.81	1.07	0.01	0.9	1.07	0.9
time (sec)	N/A	0.21	0.004	0.002	1.363	0.189	0.126	0.207	15.631

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	66	57	76	1	63	76	44
normalized size	1	1.	1.25	1.08	1.43	0.02	1.19	1.43	0.83
time (sec)	N/A	0.161	0.004	0.002	1.368	0.188	0.122	0.207	14.435

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	66	57	76	1	63	76	27
normalized size	1	1.	1.94	1.68	2.24	0.03	1.85	2.24	0.79
time (sec)	N/A	0.098	0.003	0.002	1.344	0.195	0.12	0.207	10.575

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	58	19	1	65	19	10
normalized size	1	1.	1.	3.62	1.19	0.06	4.06	1.19	0.62
time (sec)	N/A	0.013	0.004	0.003	1.331	0.192	0.117	0.206	2.192

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	56	78	74	65	78	0
normalized size	1	1.	1.	0.86	1.2	1.14	1.	1.2	0.
time (sec)	N/A	0.082	0.007	0.005	1.338	0.224	1.147	0.208	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	57	78	82	63	92	0
normalized size	1	1.	1.	0.89	1.22	1.28	0.98	1.44	0.
time (sec)	N/A	0.093	0.008	0.009	1.332	0.217	1.256	0.213	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	57	80	82	61	95	0
normalized size	1	1.	1.	0.89	1.25	1.28	0.95	1.48	0.
time (sec)	N/A	0.09	0.012	0.009	1.333	0.221	1.468	0.21	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	57	82	82	63	97	0
normalized size	1	1.	1.	0.89	1.28	1.28	0.98	1.52	0.
time (sec)	N/A	0.09	0.008	0.009	1.325	0.222	1.673	0.211	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	57	82	82	61	95	0
normalized size	1	1.	1.	0.89	1.28	1.28	0.95	1.48	0.
time (sec)	N/A	0.086	0.009	0.011	1.328	0.226	1.986	0.212	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	56	82	82	60	93	68
normalized size	1	1.	1.	0.86	1.26	1.26	0.92	1.43	1.05
time (sec)	N/A	0.081	0.008	0.01	1.336	0.227	2.229	0.209	14.202

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	69	58	77	77	61	77	15
normalized size	1	1.	3.63	3.05	4.05	4.05	3.21	4.05	0.79
time (sec)	N/A	0.019	0.008	0.008	1.331	0.201	2.316	0.209	3.224

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	67	58	80	80	63	80	32
normalized size	1	1.	1.68	1.45	2.	2.	1.58	2.	0.8
time (sec)	N/A	0.058	0.011	0.01	1.332	0.207	2.487	0.207	6.484

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	67	58	80	80	63	80	65
normalized size	1	1.	1.08	0.94	1.29	1.29	1.02	1.29	1.05
time (sec)	N/A	0.086	0.008	0.009	1.338	0.192	2.638	0.216	14.77

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	80	80	63	80	66
normalized size	1	1.	1.	0.84	1.16	1.16	0.91	1.16	0.96
time (sec)	N/A	0.086	0.007	0.009	1.352	0.192	2.835	0.208	14.634

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	80	80	63	80	68
normalized size	1	1.	1.	0.84	1.16	1.16	0.91	1.16	0.99
time (sec)	N/A	0.087	0.007	0.009	1.339	0.197	2.96	0.209	14.883

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	1	66	77	66
normalized size	1	1.	1.	0.84	1.12	0.01	0.96	1.12	0.96
time (sec)	N/A	0.075	0.004	0.002	1.344	0.176	0.12	0.209	11.895

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	1	65	77	65
normalized size	1	1.	1.	0.84	1.12	0.01	0.94	1.12	0.94
time (sec)	N/A	0.071	0.004	0.002	1.347	0.175	0.127	0.208	11.919

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	1	66	77	66
normalized size	1	1.	1.	0.84	1.12	0.01	0.96	1.12	0.96
time (sec)	N/A	0.071	0.004	0.002	1.345	0.175	0.122	0.21	11.983

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	76	1	63	76	63
normalized size	1	1.	1.	0.86	1.15	0.02	0.95	1.15	0.95
time (sec)	N/A	0.069	0.004	0.002	1.337	0.176	0.112	0.208	12.337

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	55	73	1	61	73	0
normalized size	1	1.	1.	0.89	1.18	0.02	0.98	1.18	0.
time (sec)	N/A	0.052	0.002	0.002	1.368	0.181	0.117	0.209	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	56	74	80	58	74	58
normalized size	1	1.	1.	0.92	1.21	1.31	0.95	1.21	0.95
time (sec)	N/A	0.063	0.007	0.006	1.35	0.194	1.189	0.207	11.444

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	55	74	80	58	74	56
normalized size	1	1.	1.	0.92	1.23	1.33	0.97	1.23	0.93
time (sec)	N/A	0.062	0.007	0.009	1.347	0.193	1.314	0.207	11.457



Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	78	80	61	78	60
normalized size	1	1.	1.	0.89	1.24	1.27	0.97	1.24	0.95
time (sec)	N/A	0.064	0.008	0.008	1.355	0.195	1.564	0.207	11.552

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	56	78	80	60	78	56
normalized size	1	1.	1.	0.92	1.28	1.31	0.98	1.28	0.92
time (sec)	N/A	0.064	0.007	0.008	1.354	0.194	1.757	0.206	11.671

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	55	77	80	58	77	0
normalized size	1	1.	1.	0.92	1.28	1.33	0.97	1.28	0.
time (sec)	N/A	0.063	0.01	0.009	1.346	0.194	2.043	0.207	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	58	80	80	63	80	63
normalized size	1	1.	1.	0.89	1.23	1.23	0.97	1.23	0.97
time (sec)	N/A	0.064	0.008	0.008	1.341	0.195	2.232	0.207	12.106

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	58	80	80	63	80	65
normalized size	1	1.	1.	0.87	1.19	1.19	0.94	1.19	0.97
time (sec)	N/A	0.065	0.007	0.008	1.343	0.195	2.36	0.208	12.133

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	80	80	63	80	68
normalized size	1	1.	1.	0.84	1.16	1.16	0.91	1.16	0.99
time (sec)	N/A	0.066	0.008	0.008	1.345	0.192	2.512	0.206	12.441

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	80	80	63	80	66
normalized size	1	1.	1.	0.84	1.16	1.16	0.91	1.16	0.96
time (sec)	N/A	0.066	0.008	0.008	1.346	0.194	2.637	0.211	12.611

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	80	80	63	80	68
normalized size	1	1.	1.	0.84	1.16	1.16	0.91	1.16	0.99
time (sec)	N/A	0.067	0.011	0.008	1.348	0.192	2.806	0.207	12.612

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	108	91	122	1	105	122	105
normalized size	1	1.	0.84	0.71	0.95	0.01	0.81	0.95	0.81
time (sec)	N/A	0.47	0.005	0.002	1.344	0.177	0.164	0.21	26.653

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	108	91	122	1	107	122	100
normalized size	1	1.	0.98	0.83	1.11	0.01	0.97	1.11	0.91
time (sec)	N/A	0.392	0.005	0.002	1.351	0.176	0.164	0.209	29.797

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	106	91	122	1	104	122	80
normalized size	1	1.	1.16	1.	1.34	0.01	1.14	1.34	0.88
time (sec)	N/A	0.324	0.004	0.002	1.348	0.177	0.16	0.208	25.498

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	106	91	122	1	105	122	65
normalized size	1	1.	1.47	1.26	1.69	0.01	1.46	1.69	0.9
time (sec)	N/A	0.263	0.005	0.002	1.345	0.178	0.165	0.208	21.826

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	103	90	120	1	102	120	44
normalized size	1	1.	1.94	1.7	2.26	0.02	1.92	2.26	0.83
time (sec)	N/A	0.205	0.005	0.001	1.349	0.174	0.157	0.206	17.483

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	106	91	122	1	105	122	27
normalized size	1	1.	3.12	2.68	3.59	0.03	3.09	3.59	0.79
time (sec)	N/A	0.118	0.004	0.003	1.347	0.176	0.154	0.208	13.608

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	91	19	1	99	19	10
normalized size	1	1.	1.	5.69	1.19	0.06	6.19	1.19	0.62
time (sec)	N/A	0.014	0.004	0.001	1.344	0.177	0.153	0.207	2.179

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	123	119	102	123	0
normalized size	1	1.	1.	0.89	1.23	1.19	1.02	1.23	0.
time (sec)	N/A	0.132	0.008	0.004	1.349	0.203	1.334	0.209	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	90	123	127	100	136	0
normalized size	1	1.	1.	0.91	1.24	1.28	1.01	1.37	0.
time (sec)	N/A	0.154	0.009	0.01	1.343	0.202	1.438	0.209	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	90	124	127	102	139	0
normalized size	1	1.	1.	0.89	1.23	1.26	1.01	1.38	0.
time (sec)	N/A	0.15	0.009	0.01	1.345	0.202	1.63	0.208	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	89	123	127	95	138	0
normalized size	1	1.	1.	0.95	1.31	1.35	1.01	1.47	0.
time (sec)	N/A	0.148	0.008	0.01	1.353	0.201	1.881	0.21	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	90	127	127	99	142	0
normalized size	1	1.	1.	0.93	1.31	1.31	1.02	1.46	0.
time (sec)	N/A	0.145	0.008	0.01	1.342	0.201	2.172	0.21	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	90	127	127	97	142	0
normalized size	1	1.	1.	0.95	1.34	1.34	1.02	1.49	0.
time (sec)	N/A	0.144	0.008	0.011	1.344	0.202	2.55	0.208	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	90	127	127	97	142	0
normalized size	1	1.	1.	0.89	1.26	1.26	0.96	1.41	0.
time (sec)	N/A	0.143	0.009	0.013	1.335	0.202	2.859	0.213	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	90	127	127	97	139	0
normalized size	1	1.	1.	0.91	1.28	1.28	0.98	1.4	0.
time (sec)	N/A	0.138	0.009	0.012	1.347	0.2	3.319	0.208	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	127	127	95	138	105
normalized size	1	1.	1.	0.89	1.27	1.27	0.95	1.38	1.05
time (sec)	N/A	0.136	0.009	0.011	1.351	0.199	3.712	0.209	23.825

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	100	91	122	122	97	122	15
normalized size	1	1.	5.26	4.79	6.42	6.42	5.11	6.42	0.79
time (sec)	N/A	0.02	0.008	0.01	1.414	0.195	3.96	0.209	3.269

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	106	91	124	124	99	124	32
normalized size	1	1.	2.65	2.28	3.1	3.1	2.48	3.1	0.8
time (sec)	N/A	0.06	0.008	0.009	1.35	0.196	4.201	0.21	6.568

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	104	91	124	124	99	124	53
normalized size	1	1.	1.68	1.47	2.	2.	1.6	2.	0.85
time (sec)	N/A	0.087	0.009	0.008	1.344	0.196	4.41	0.21	9.479

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	106	91	124	124	99	124	73
normalized size	1	1.	1.26	1.08	1.48	1.48	1.18	1.48	0.87
time (sec)	N/A	0.12	0.008	0.01	1.344	0.195	4.711	0.213	13.166

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	124	124	99	124	105
normalized size	1	1.	1.	0.86	1.17	1.17	0.93	1.17	0.99
time (sec)	N/A	0.154	0.008	0.01	1.361	0.197	4.951	0.21	24.336

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	124	124	99	124	109
normalized size	1	1.	1.	0.84	1.15	1.15	0.92	1.15	1.01
time (sec)	N/A	0.14	0.008	0.009	1.343	0.197	5.324	0.213	24.574

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	124	124	99	124	107
normalized size	1	1.	1.	0.84	1.15	1.15	0.92	1.15	0.99
time (sec)	N/A	0.136	0.008	0.009	1.327	0.193	5.583	0.208	24.419

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	124	124	99	124	105
normalized size	1	1.	1.	0.86	1.17	1.17	0.93	1.17	0.99
time (sec)	N/A	0.137	0.008	0.01	1.345	0.194	5.85	0.209	24.474

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	122	1	107	122	107
normalized size	1	1.	1.	0.84	1.13	0.01	0.99	1.13	0.99
time (sec)	N/A	0.122	0.005	0.002	1.344	0.18	0.164	0.207	19.363

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	122	1	107	122	107
normalized size	1	1.	1.	0.84	1.13	0.01	0.99	1.13	0.99
time (sec)	N/A	0.114	0.004	0.001	1.344	0.18	0.157	0.209	19.562

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	122	1	107	122	107
normalized size	1	1.	1.	0.84	1.13	0.01	0.99	1.13	0.99
time (sec)	N/A	0.114	0.005	0.002	1.351	0.188	0.16	0.208	19.983

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	122	1	105	122	105
normalized size	1	1.	1.	0.86	1.15	0.01	0.99	1.15	0.99
time (sec)	N/A	0.111	0.005	0.003	1.347	0.183	0.161	0.208	19.693

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	88	117	1	102	117	0
normalized size	1	1.	1.	0.87	1.16	0.01	1.01	1.16	0.
time (sec)	N/A	0.092	0.002	0.002	1.343	0.18	0.145	0.205	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	119	124	99	119	99
normalized size	1	1.	1.	0.89	1.19	1.24	0.99	1.19	0.99
time (sec)	N/A	0.102	0.018	0.006	1.358	0.196	1.32	0.207	19.081

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	89	120	124	99	120	97
normalized size	1	1.	1.	0.91	1.22	1.27	1.01	1.22	0.99
time (sec)	N/A	0.103	0.015	0.008	1.347	0.206	1.467	0.207	19.501

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	123	124	100	123	99
normalized size	1	1.	1.	0.89	1.23	1.24	1.	1.23	0.99
time (sec)	N/A	0.103	0.017	0.009	1.342	0.198	1.684	0.209	19.267

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	102	89	123	124	100	123	100
normalized size	1	1.	1.	0.87	1.21	1.22	0.98	1.21	0.98
time (sec)	N/A	0.105	0.011	0.009	1.356	0.197	1.901	0.209	19.476

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	102	89	123	124	99	123	100
normalized size	1	1.	1.	0.87	1.21	1.22	0.97	1.21	0.98
time (sec)	N/A	0.105	0.022	0.009	1.33	0.195	2.229	0.211	19.801

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	123	124	97	123	99
normalized size	1	1.	1.	0.89	1.23	1.24	0.97	1.23	0.99
time (sec)	N/A	0.105	0.015	0.009	1.323	0.198	2.533	0.219	19.695

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	89	123	124	95	123	97
normalized size	1	1.	1.	0.91	1.26	1.27	0.97	1.26	0.99
time (sec)	N/A	0.106	0.017	0.009	1.511	0.2	2.89	0.207	19.735

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	88	122	124	94	122	0
normalized size	1	1.	1.	0.89	1.23	1.25	0.95	1.23	0.
time (sec)	N/A	0.107	0.01	0.01	1.347	0.2	3.313	0.214	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	91	124	124	99	124	104
normalized size	1	1.	1.	0.88	1.19	1.19	0.95	1.19	1.
time (sec)	N/A	0.108	0.017	0.008	1.353	0.197	3.628	0.227	20.204

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	124	124	99	124	107
normalized size	1	1.	1.	0.86	1.17	1.17	0.93	1.17	1.01
time (sec)	N/A	0.11	0.016	0.009	1.352	0.198	3.962	0.218	20.234

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	68	92	90	68	93	0
normalized size	1	1.	1.	0.86	1.16	1.14	0.86	1.18	0.
time (sec)	N/A	0.135	0.01	0.005	1.347	0.201	1.331	0.214	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	71	0	1	119	104	0
normalized size	1	1.	1.	0.88	0.	0.01	1.47	1.28	0.
time (sec)	N/A	0.092	0.055	0.009	0.	0.212	1.387	0.212	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	77	76	56	78	0
normalized size	1	1.	1.	0.86	1.17	1.15	0.85	1.18	0.
time (sec)	N/A	0.104	0.011	0.004	1.346	0.201	1.253	0.21	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	60	0	1	107	88	0
normalized size	1	1.	1.	0.88	0.	0.01	1.57	1.29	0.
time (sec)	N/A	0.077	0.045	0.004	0.	0.206	1.314	0.211	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	46	62	61	44	63	0
normalized size	1	1.	1.	0.87	1.17	1.15	0.83	1.19	0.
time (sec)	N/A	0.085	0.009	0.003	1.344	0.198	1.281	0.209	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	49	0	1	95	74	0
normalized size	1	1.	1.	0.89	0.	0.02	1.73	1.35	0.
time (sec)	N/A	0.067	0.043	0.004	0.	0.206	1.305	0.223	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	46	45	32	47	0
normalized size	1	1.	1.	0.88	1.15	1.12	0.8	1.18	0.
time (sec)	N/A	0.068	0.009	0.003	1.343	0.199	1.192	0.244	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	0	1	80	54	0
normalized size	1	1.	1.	0.9	0.	0.02	1.9	1.29	0.
time (sec)	N/A	0.057	0.034	0.004	0.	0.206	1.253	0.223	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	31	30	20	32	0
normalized size	1	1.	1.	0.89	1.15	1.11	0.74	1.19	0.
time (sec)	N/A	0.05	0.008	0.003	1.346	0.199	1.165	0.224	0.



Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	0	1	56	35	26
normalized size	1	1.	1.	0.87	0.	0.03	1.81	1.13	0.84
time (sec)	N/A	0.04	0.016	0.004	0.	0.205	1.168	0.221	6.997

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	10	19	10
normalized size	1	1.	1.	0.93	1.2	1.2	0.67	1.27	0.67
time (sec)	N/A	0.01	0.003	0.002	1.351	0.195	0.227	0.217	2.252

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	0	1	53	20	22
normalized size	1	1.	1.	0.67	0.	0.04	2.21	0.83	0.92
time (sec)	N/A	0.018	0.008	0.002	0.	0.204	0.287	0.208	2.467

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	31	24	15	32	19
normalized size	1	1.	1.	0.95	1.41	1.09	0.68	1.45	0.86
time (sec)	N/A	0.036	0.009	0.006	1.356	0.201	0.496	0.211	6.29

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	0	1	65	39	29
normalized size	1	1.	1.	0.88	0.	0.03	1.91	1.15	0.85
time (sec)	N/A	0.037	0.024	0.006	0.	0.207	1.298	0.216	6.612

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	45	45	31	58	34
normalized size	1	1.	1.	0.91	1.29	1.29	0.89	1.66	0.97
time (sec)	N/A	0.059	0.012	0.009	1.347	0.207	1.589	0.211	9.186

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	39	0	1	87	54	37
normalized size	1	1.	1.	0.91	0.	0.02	2.02	1.26	0.86
time (sec)	N/A	0.054	0.039	0.008	0.	0.209	1.471	0.211	10.5

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	63	61	42	77	48
normalized size	1	1.	1.	0.9	1.29	1.24	0.86	1.57	0.98
time (sec)	N/A	0.074	0.013	0.01	1.354	0.205	1.788	0.211	11.946

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	52	0	1	100	70	49
normalized size	1	1.	1.	0.9	0.	0.02	1.72	1.21	0.84
time (sec)	N/A	0.075	0.046	0.009	0.	0.208	1.748	0.208	14.989

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	78	78	56	95	60
normalized size	1	1.	1.	0.89	1.24	1.24	0.89	1.51	0.95
time (sec)	N/A	0.089	0.013	0.01	1.353	0.208	2.187	0.209	13.952

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	61	0	1	112	84	61
normalized size	1	1.	1.	0.88	0.	0.01	1.62	1.22	0.88
time (sec)	N/A	0.096	0.056	0.01	0.	0.215	2.258	0.208	20.211

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	66	93	93	68	109	71
normalized size	1	1.	1.	0.88	1.24	1.24	0.91	1.45	0.95
time (sec)	N/A	0.103	0.011	0.01	1.344	0.203	2.916	0.212	16.461

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	83	85	119	140	88	139	0
normalized size	1	1.	0.88	0.9	1.27	1.49	0.94	1.48	0.
time (sec)	N/A	0.181	0.057	0.014	1.346	0.198	1.744	0.212	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	93	90	0	1	151	128	0
normalized size	1	1.	0.89	0.86	0.	0.01	1.44	1.22	0.
time (sec)	N/A	0.12	0.132	0.013	0.	0.208	1.831	0.208	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	72	74	104	126	80	124	0
normalized size	1	1.	0.87	0.89	1.25	1.52	0.96	1.49	0.
time (sec)	N/A	0.152	0.038	0.014	1.351	0.2	1.704	0.211	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	82	78	0	1	134	113	0
normalized size	1	1.	0.89	0.85	0.	0.01	1.46	1.23	0.
time (sec)	N/A	0.102	0.097	0.023	0.	0.209	1.765	0.209	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	60	63	88	109	66	108	0
normalized size	1	1.	0.86	0.9	1.26	1.56	0.94	1.54	0.
time (sec)	N/A	0.129	0.039	0.028	1.337	0.201	1.623	0.212	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	68	0	1	124	99	0
normalized size	1	1.	0.9	0.86	0.	0.01	1.57	1.25	0.
time (sec)	N/A	0.091	0.088	0.012	0.	0.207	1.72	0.214	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	52	73	95	53	90	0
normalized size	1	1.	0.86	0.91	1.28	1.67	0.93	1.58	0.
time (sec)	N/A	0.106	0.029	0.014	1.34	0.201	1.589	0.214	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	57	0	1	107	82	0
normalized size	1	1.	0.91	0.86	0.	0.02	1.62	1.24	0.
time (sec)	N/A	0.076	0.077	0.011	0.	0.207	1.665	0.21	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	41	58	76	39	66	0
normalized size	1	1.	0.86	0.93	1.32	1.73	0.89	1.5	0.
time (sec)	N/A	0.081	0.033	0.015	1.357	0.197	1.516	0.213	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	51	43	0	1	83	57	48
normalized size	1	1.	0.93	0.78	0.	0.02	1.51	1.04	0.87
time (sec)	N/A	0.059	0.059	0.01	0.	0.205	1.544	0.207	11.168

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	30	43	47	29	65	26
normalized size	1	1.	0.82	0.91	1.3	1.42	0.88	1.97	0.79
time (sec)	N/A	0.062	0.022	0.012	1.334	0.201	1.355	0.216	8.468

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	0	1	78	47	36
normalized size	1	1.	1.	0.8	0.	0.02	1.73	1.04	0.8
time (sec)	N/A	0.04	0.037	0.009	0.	0.21	1.363	0.207	6.348

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	20	15	19	12
normalized size	1	1.	1.	0.94	1.19	1.25	0.94	1.19	0.75
time (sec)	N/A	0.011	0.004	0.001	1.377	0.198	1.222	0.216	2.289

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	0	1	78	47	36
normalized size	1	1.	1.	0.8	0.	0.02	1.73	1.04	0.8
time (sec)	N/A	0.032	0.045	0.004	0.	0.21	1.42	0.206	4.013

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	50	63	34	63	34
normalized size	1	1.	0.87	0.92	1.32	1.66	0.89	1.66	0.89
time (sec)	N/A	0.065	0.027	0.016	1.552	0.213	1.669	0.211	9.594

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	54	46	0	1	90	63	48
normalized size	1	1.	0.95	0.81	0.	0.02	1.58	1.11	0.84
time (sec)	N/A	0.055	0.067	0.013	0.	0.214	1.698	0.208	10.215

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	46	70	99	49	69	46
normalized size	1	1.	0.84	0.94	1.43	2.02	1.	1.41	0.94
time (sec)	N/A	0.087	0.072	0.018	1.336	0.213	2.047	0.221	12.293

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	67	59	0	1	114	80	61
normalized size	1	1.	0.99	0.87	0.	0.01	1.68	1.18	0.9
time (sec)	N/A	0.073	0.077	0.016	0.	0.212	2.08	0.206	14.29

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	57	61	95	122	68	116	66
normalized size	1	1.	0.86	0.92	1.44	1.85	1.03	1.76	1.
time (sec)	N/A	0.113	0.132	0.02	1.343	0.211	2.563	0.209	15.889

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	80	70	0	1	126	95	75
normalized size	1	1.	0.99	0.86	0.	0.01	1.56	1.17	0.93
time (sec)	N/A	0.101	0.082	0.017	0.	0.21	2.805	0.211	22.124

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	68	73	107	134	78	134	78
normalized size	1	1.	0.85	0.91	1.34	1.68	0.98	1.68	0.98
time (sec)	N/A	0.13	0.114	0.02	1.349	0.205	3.628	0.208	19.069

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	91	81	0	1	138	109	88
normalized size	1	1.	0.97	0.86	0.	0.01	1.47	1.16	0.94
time (sec)	N/A	0.126	0.089	0.017	0.	0.211	4.21	0.21	27.622

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	79	84	124	151	94	149	94
normalized size	1	1.	0.85	0.9	1.33	1.62	1.01	1.6	1.01
time (sec)	N/A	0.15	0.133	0.021	1.347	0.205	5.716	0.211	22.376

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	97	101	150	185	117	154	0
normalized size	1	1.	0.85	0.89	1.32	1.62	1.03	1.35	0.
time (sec)	N/A	0.231	0.055	0.016	1.346	0.199	2.409	0.212	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	85	91	134	169	104	138	0
normalized size	1	1.	0.85	0.91	1.34	1.69	1.04	1.38	0.
time (sec)	N/A	0.197	0.054	0.016	1.359	0.199	2.339	0.207	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	75	80	120	155	90	124	0
normalized size	1	1.	0.86	0.92	1.38	1.78	1.03	1.43	0.
time (sec)	N/A	0.166	0.044	0.015	1.359	0.207	2.248	0.213	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	69	104	139	78	108	0
normalized size	1	1.	0.85	0.93	1.41	1.88	1.05	1.46	0.
time (sec)	N/A	0.14	0.044	0.014	1.346	0.215	2.142	0.214	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	48	58	89	123	66	84	0
normalized size	1	1.	0.74	0.89	1.37	1.89	1.02	1.29	0.
time (sec)	N/A	0.115	0.111	0.013	1.348	0.212	2.064	0.214	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	46	74	93	53	57	41
normalized size	1	1.	0.8	0.94	1.51	1.9	1.08	1.16	0.84
time (sec)	N/A	0.094	0.028	0.013	1.351	0.226	1.826	0.222	12.774

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	49	49	36	30	14
normalized size	1	1.	1.26	1.63	2.58	2.58	1.89	1.58	0.74
time (sec)	N/A	0.02	0.012	0.01	1.352	0.207	1.653	0.21	3.439

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	35	27	19	14
normalized size	1	1.	1.	0.94	1.19	2.19	1.69	1.19	0.88
time (sec)	N/A	0.011	0.004	0.002	1.343	0.223	1.529	0.211	2.224

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	49	81	122	56	80	49
normalized size	1	1.	0.8	0.91	1.5	2.26	1.04	1.48	0.91
time (sec)	N/A	0.093	0.059	0.015	1.351	0.232	2.188	0.219	12.309

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	59	62	104	161	78	111	66
normalized size	1	1.	0.88	0.93	1.55	2.4	1.16	1.66	0.99
time (sec)	N/A	0.117	0.097	0.02	1.333	0.219	3.013	0.215	16.186

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	74	79	124	181	90	108	85
normalized size	1	1.	0.86	0.92	1.44	2.1	1.05	1.26	0.99
time (sec)	N/A	0.146	0.091	0.02	1.348	0.208	4.374	0.217	20.455

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	85	90	139	196	104	149	95
normalized size	1	1.	0.89	0.95	1.46	2.06	1.09	1.57	1.
time (sec)	N/A	0.167	0.14	0.019	1.358	0.219	7.61	0.211	22.77

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	96	101	154	211	116	161	112
normalized size	1	1.	0.86	0.9	1.38	1.88	1.04	1.44	1.
time (sec)	N/A	0.194	0.134	0.022	1.354	0.212	14.512	0.211	27.067

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	99	99	0	1	160	130	0
normalized size	1	1.	0.89	0.89	0.	0.01	1.44	1.17	0.
time (sec)	N/A	0.143	0.146	0.013	0.	0.213	2.405	0.239	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	88	88	0	1	144	113	0
normalized size	1	1.	0.9	0.9	0.	0.01	1.47	1.15	0.
time (sec)	N/A	0.12	0.097	0.014	0.	0.21	2.292	0.232	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	77	0	1	131	99	0
normalized size	1	1.	0.91	0.91	0.	0.01	1.54	1.16	0.
time (sec)	N/A	0.103	0.086	0.014	0.	0.213	2.28	0.226	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	66	63	0	1	107	73	66
normalized size	1	1.	0.89	0.85	0.	0.01	1.45	0.99	0.89
time (sec)	N/A	0.081	0.091	0.014	0.	0.21	2.107	0.211	15.024

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	47	0	1	109	61	56
normalized size	1	1.	0.86	0.73	0.	0.02	1.7	0.95	0.88
time (sec)	N/A	0.063	0.077	0.012	0.	0.211	1.849	0.211	10.316

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	49	0	1	110	68	51
normalized size	1	1.	0.89	0.75	0.	0.02	1.69	1.05	0.78
time (sec)	N/A	0.058	0.052	0.011	0.	0.225	1.789	0.214	8.408

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	51	0	1	105	61	54
normalized size	1	1.	0.89	0.82	0.	0.02	1.69	0.98	0.87
time (sec)	N/A	0.046	0.077	0.006	0.	0.215	1.846	0.21	5.702

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	68	66	0	1	114	77	65
normalized size	1	1.	0.89	0.87	0.	0.01	1.5	1.01	0.86
time (sec)	N/A	0.077	0.077	0.016	0.	0.226	2.414	0.207	14.17



Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	79	79	0	1	138	96	80
normalized size	1	1.	0.91	0.91	0.	0.01	1.59	1.1	0.92
time (sec)	N/A	0.101	0.088	0.019	0.	0.222	3.337	0.223	18.972

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	90	89	0	1	150	108	94
normalized size	1	1.	0.9	0.89	0.	0.01	1.5	1.08	0.94
time (sec)	N/A	0.124	0.102	0.017	0.	0.222	5.476	0.223	24.46

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	101	101	0	1	162	126	107
normalized size	1	1.	0.89	0.89	0.	0.01	1.43	1.12	0.95
time (sec)	N/A	0.153	0.1	0.019	0.	0.221	10.188	0.222	30.336

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	169	199	327	467	258	227	0
normalized size	1	1.	0.78	0.92	1.51	2.16	1.19	1.05	0.
time (sec)	N/A	0.535	0.088	0.027	1.412	0.209	37.967	0.217	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	158	188	312	452	245	212	0
normalized size	1	1.	0.77	0.92	1.52	2.2	1.2	1.03	0.
time (sec)	N/A	0.469	0.066	0.027	1.389	0.211	37.713	0.221	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	145	177	297	435	231	188	0
normalized size	1	1.	0.77	0.94	1.58	2.31	1.23	1.	0.
time (sec)	N/A	0.409	0.067	0.025	1.398	0.217	37.146	0.212	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	116	166	282	405	219	161	170
normalized size	1	1.	0.65	0.93	1.58	2.26	1.22	0.9	0.95
time (sec)	N/A	0.379	0.064	0.017	1.364	0.209	36.144	0.212	51.868

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	101	150	257	257	202	134	14
normalized size	1	1.	5.32	7.89	13.53	13.53	10.63	7.05	0.74
time (sec)	N/A	0.02	0.037	0.014	1.37	0.204	35.838	0.212	3.291

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	90	133	242	242	190	119	31
normalized size	1	1.	2.31	3.41	6.21	6.21	4.87	3.05	0.79
time (sec)	N/A	0.057	0.03	0.015	1.361	0.204	35.133	0.211	7.044

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	79	116	227	227	178	104	48
normalized size	1	1.	1.36	2.	3.91	3.91	3.07	1.79	0.83
time (sec)	N/A	0.081	0.032	0.014	1.346	0.204	34.836	0.209	9.862

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	68	99	212	212	167	89	65
normalized size	1	1.	0.88	1.29	2.75	2.75	2.17	1.16	0.84
time (sec)	N/A	0.108	0.035	0.013	1.378	0.204	34.329	0.209	13.175

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	57	82	197	197	155	74	82
normalized size	1	1.	0.63	0.9	2.16	2.16	1.7	0.81	0.9
time (sec)	N/A	0.164	0.027	0.011	1.367	0.207	33.929	0.212	23.714

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	65	182	182	143	59	66
normalized size	1	1.	0.64	0.9	2.53	2.53	1.99	0.82	0.92
time (sec)	N/A	0.131	0.02	0.01	1.354	0.2	33.932	0.21	19.341

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	48	167	167	131	45	46
normalized size	1	1.	0.66	0.91	3.15	3.15	2.47	0.85	0.87
time (sec)	N/A	0.101	0.024	0.012	1.386	0.205	33.518	0.225	14.764

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	31	153	153	119	30	29
normalized size	1	1.	0.71	0.91	4.5	4.5	3.5	0.88	0.85
time (sec)	N/A	0.066	0.014	0.01	1.348	0.202	33.287	0.224	11.017

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	139	110	19	14
normalized size	1	1.	1.	0.94	1.19	8.69	6.88	1.19	0.88
time (sec)	N/A	0.011	0.005	0.	1.33	0.195	33.142	0.213	2.233

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	120	147	289	537	0	184	156
normalized size	1	1.	0.72	0.89	1.74	3.23	0.	1.11	0.94
time (sec)	N/A	0.279	0.155	0.026	1.378	0.221	0.	0.214	56.325

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	136	167	312	576	0	215	0
normalized size	1	1.	0.74	0.91	1.7	3.13	0.	1.17	0.
time (sec)	N/A	0.411	0.242	0.029	1.396	0.241	0.	0.217	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	151	198	332	597	0	235	0
normalized size	1	1.	0.7	0.91	1.53	2.75	0.	1.08	0.
time (sec)	N/A	0.485	0.186	0.029	1.376	0.231	0.	0.214	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	162	209	347	612	0	252	0
normalized size	1	1.	0.72	0.92	1.54	2.71	0.	1.12	0.
time (sec)	N/A	0.526	0.248	0.03	1.394	0.235	0.	0.213	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	166	228	0	1	314	219	0
normalized size	1	1.	0.72	0.99	0.	0.	1.36	0.95	0.
time (sec)	N/A	0.434	0.166	0.027	0.	0.215	37.617	0.211	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	155	217	0	1	298	203	0
normalized size	1	1.	0.71	1.	0.	0.	1.37	0.93	0.
time (sec)	N/A	0.373	0.156	0.028	0.	0.218	37.16	0.216	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	144	203	0	1	274	177	197
normalized size	1	1.	0.7	0.98	0.	0.	1.32	0.86	0.95
time (sec)	N/A	0.333	0.142	0.027	0.	0.214	36.868	0.209	57.422

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	134	124	0	1	275	165	187
normalized size	1	1.	0.68	0.63	0.	0.01	1.4	0.84	0.95
time (sec)	N/A	0.31	0.145	0.023	0.	0.256	35.963	0.21	50.625

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	138	124	0	1	289	173	187
normalized size	1	1.	0.7	0.63	0.	0.01	1.46	0.87	0.94
time (sec)	N/A	0.314	0.14	0.021	0.	0.216	35.585	0.209	50.695

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	138	122	0	1	291	173	189
normalized size	1	1.	0.69	0.61	0.	0.01	1.46	0.87	0.95
time (sec)	N/A	0.311	0.122	0.021	0.	0.217	35.211	0.21	49.746

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	138	122	0	1	291	173	190
normalized size	1	1.	0.69	0.61	0.	0.	1.46	0.86	0.95
time (sec)	N/A	0.305	0.142	0.02	0.	0.213	34.988	0.24	48.506

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	138	122	0	1	291	173	185
normalized size	1	1.	0.69	0.61	0.	0.	1.45	0.86	0.92
time (sec)	N/A	0.295	0.134	0.021	0.	0.221	34.438	0.238	47.043

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	138	122	0	1	291	173	190
normalized size	1	1.	0.68	0.6	0.	0.	1.44	0.86	0.94
time (sec)	N/A	0.287	0.134	0.02	0.	0.219	34.109	0.214	44.891

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	138	122	0	1	291	173	196
normalized size	1	1.	0.68	0.6	0.	0.	1.43	0.85	0.97
time (sec)	N/A	0.274	0.134	0.02	0.	0.226	34.207	0.211	41.999

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	138	122	0	1	291	173	194
normalized size	1	1.	0.68	0.6	0.	0.	1.43	0.85	0.95
time (sec)	N/A	0.266	0.131	0.019	0.	0.218	34.025	0.209	38.814

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	138	124	0	1	286	173	184
normalized size	1	1.	0.67	0.6	0.	0.	1.4	0.84	0.9
time (sec)	N/A	0.248	0.122	0.019	0.	0.219	33.848	0.211	35.152

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	131	156	0	1	272	165	173
normalized size	1	1.	0.72	0.86	0.	0.01	1.5	0.91	0.96
time (sec)	N/A	0.225	0.194	0.006	0.	0.238	34.06	0.226	30.719

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	147	206	0	1	0	181	184
normalized size	1	1.	0.7	0.99	0.	0.	0.	0.87	0.88
time (sec)	N/A	0.318	0.185	0.027	0.	0.244	0.	0.226	54.208

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	157	219	0	1	0	200	211
normalized size	1	1.	0.71	1.	0.	0.	0.	0.91	0.96
time (sec)	N/A	0.354	0.169	0.03	0.	0.251	0.	0.219	61.41

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	169	230	0	1	0	215	224
normalized size	1	1.	0.73	0.99	0.	0.	0.	0.92	0.96
time (sec)	N/A	0.396	0.179	0.031	0.	0.239	0.	0.218	71.325

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	26	34	31	22	35	0
normalized size	1	1.	1.	0.93	1.21	1.11	0.79	1.25	0.
time (sec)	N/A	0.059	0.009	0.004	1.34	0.226	1.252	0.209	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	0	1	49	39	26
normalized size	1	1.	1.	0.87	0.	0.03	1.58	1.26	0.84
time (sec)	N/A	0.04	0.018	0.003	0.	0.264	1.247	0.209	7.392

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	20	20	12	22	12
normalized size	1	1.	1.	1.	1.25	1.25	0.75	1.38	0.75
time (sec)	N/A	0.011	0.004	0.001	1.349	0.214	0.274	0.213	2.525

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	0	1	46	24	22
normalized size	1	1.	1.	0.67	0.	0.04	1.92	1.	0.92
time (sec)	N/A	0.022	0.008	0.002	0.	0.224	0.331	0.207	2.745

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	23	34	27	15	35	19
normalized size	1	1.	1.	1.	1.48	1.17	0.65	1.52	0.83
time (sec)	N/A	0.037	0.011	0.006	1.355	0.211	0.53	0.209	6.482

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	0	1	58	42	27
normalized size	1	1.	1.	0.88	0.	0.03	1.76	1.27	0.82
time (sec)	N/A	0.038	0.021	0.006	0.	0.212	1.321	0.21	7.145

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	33	47	45	31	58	34
normalized size	1	1.	1.	0.94	1.34	1.29	0.89	1.66	0.97
time (sec)	N/A	0.061	0.014	0.008	1.351	0.206	1.655	0.212	9.485

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	29	34	47	57	29	72	26
normalized size	1	1.	0.83	0.97	1.34	1.63	0.83	2.06	0.74
time (sec)	N/A	0.067	0.022	0.012	1.346	0.201	1.372	0.212	9.151

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	38	0	1	71	53	36
normalized size	1	1.	1.02	0.83	0.	0.02	1.54	1.15	0.78
time (sec)	N/A	0.042	0.053	0.01	0.	0.215	1.389	0.21	6.731

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	22	22	15	22	10
normalized size	1	1.	1.	1.	1.29	1.29	0.88	1.29	0.59
time (sec)	N/A	0.012	0.004	0.002	1.346	0.202	1.246	0.208	2.451

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	38	0	1	71	53	36
normalized size	1	1.	1.02	0.83	0.	0.02	1.54	1.15	0.78
time (sec)	N/A	0.035	0.033	0.007	0.	0.213	1.529	0.211	4.092

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	35	39	55	72	34	69	34
normalized size	1	1.	0.88	0.98	1.38	1.8	0.85	1.72	0.85
time (sec)	N/A	0.068	0.033	0.017	1.345	0.211	1.712	0.219	9.788

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	47	0	1	83	68	48
normalized size	1	1.	0.97	0.81	0.	0.02	1.43	1.17	0.83
time (sec)	N/A	0.057	0.065	0.013	0.	0.219	1.818	0.222	11.08

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	44	51	77	108	49	76	46
normalized size	1	1.	0.85	0.98	1.48	2.08	0.94	1.46	0.88
time (sec)	N/A	0.092	0.059	0.018	1.4	0.239	2.176	0.207	13.015

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	25	35	51	51	34	35	14
normalized size	1	1.	1.25	1.75	2.55	2.55	1.7	1.75	0.7
time (sec)	N/A	0.02	0.016	0.01	1.373	0.251	1.755	0.209	3.57

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	52	0	1	104	72	51
normalized size	1	1.	0.84	0.78	0.	0.01	1.55	1.07	0.76
time (sec)	N/A	0.062	0.056	0.011	0.	0.232	1.855	0.207	8.844

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	22	35	26	22	12
normalized size	1	1.	1.	1.	1.29	2.06	1.53	1.29	0.71
time (sec)	N/A	0.012	0.005	0.	1.359	0.213	1.625	0.208	2.464

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	61	0	1	99	66	54
normalized size	1	1.	0.88	0.95	0.	0.02	1.55	1.03	0.84
time (sec)	N/A	0.051	0.074	0.006	0.	0.227	1.872	0.207	6.049

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	45	55	84	124	56	85	49
normalized size	1	1.	0.79	0.96	1.47	2.18	0.98	1.49	0.86
time (sec)	N/A	0.092	0.062	0.016	1.331	0.224	2.286	0.213	12.726

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	69	56	0	1	107	82	65
normalized size	1	1.	0.88	0.72	0.	0.01	1.37	1.05	0.83
time (sec)	N/A	0.084	0.085	0.015	0.	0.23	2.526	0.213	15.382



Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	60	68	107	163	78	113	66
normalized size	1	1.	0.87	0.99	1.55	2.36	1.13	1.64	0.96
time (sec)	N/A	0.121	0.104	0.018	1.352	0.221	3.069	0.21	17.742

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	25	35	81	81	58	53	29
normalized size	1	1.	0.69	0.97	2.25	2.25	1.61	1.47	0.81
time (sec)	N/A	0.077	0.017	0.011	1.343	0.213	2.824	0.212	10.201

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	81	72	0	1	160	104	90
normalized size	1	1.	0.74	0.66	0.	0.01	1.47	0.95	0.83
time (sec)	N/A	0.108	0.107	0.012	0.	0.227	3.08	0.215	16.567

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	22	65	49	22	12
normalized size	1	1.	1.	1.	1.29	3.82	2.88	1.29	0.71
time (sec)	N/A	0.011	0.008	0.001	1.346	0.222	2.7	0.212	2.686

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	79	107	0	1	146	96	88
normalized size	1	1.	0.79	1.07	0.	0.01	1.46	0.96	0.88
time (sec)	N/A	0.09	0.085	0.005	0.	0.225	3.133	0.21	11.493

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	67	87	143	243	104	115	80
normalized size	1	1.	0.74	0.96	1.57	2.67	1.14	1.26	0.88
time (sec)	N/A	0.143	0.062	0.02	1.363	0.235	5.499	0.216	18.984

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	92	78	0	1	155	112	99
normalized size	1	1.	0.78	0.66	0.	0.01	1.31	0.95	0.84
time (sec)	N/A	0.137	0.105	0.018	0.	0.234	7.749	0.223	26.683

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	83	102	166	282	126	143	100
normalized size	1	1.	0.78	0.96	1.57	2.66	1.19	1.35	0.94
time (sec)	N/A	0.196	0.144	0.022	1.361	0.234	11.546	0.213	28.796

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	23	18	12	24	15
normalized size	1	1.	1.	0.93	1.53	1.2	0.8	1.6	1.
time (sec)	N/A	0.029	0.007	0.008	1.339	0.225	0.314	0.209	5.123

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	23	20	12	24	15
normalized size	1	1.	1.	0.89	1.28	1.11	0.67	1.33	0.83
time (sec)	N/A	0.031	0.008	0.006	1.335	0.228	0.309	0.219	5.165

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	32	38	22	43	26
normalized size	1	1.	1.	0.88	1.23	1.46	0.85	1.65	1.
time (sec)	N/A	0.045	0.01	0.008	1.456	0.225	1.353	0.212	6.565

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	32	38	22	43	26
normalized size	1	1.	1.	0.85	1.19	1.41	0.81	1.59	0.96
time (sec)	N/A	0.048	0.007	0.007	1.346	0.226	0.548	0.212	6.7

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	20	0	1	83	31	27
normalized size	1	1.	0.93	0.67	0.	0.03	2.77	1.03	0.9
time (sec)	N/A	0.058	0.019	0.006	0.	0.228	0.4	0.208	3.58

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	44	33	0	1	87	45	31
normalized size	1	1.	1.19	0.89	0.	0.03	2.35	1.22	0.84
time (sec)	N/A	0.069	0.029	0.008	0.	0.233	0.601	0.21	5.587

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	25	25	38	54	22	49	26
normalized size	1	1.	0.89	0.89	1.36	1.93	0.79	1.75	0.93
time (sec)	N/A	0.048	0.023	0.015	1.353	0.227	1.397	0.214	7.104

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	26	25	38	54	22	49	26
normalized size	1	1.	0.87	0.83	1.27	1.8	0.73	1.63	0.87
time (sec)	N/A	0.052	0.024	0.015	1.354	0.222	1.373	0.209	6.949

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	0	1	60	50	29
normalized size	1	1.	1.06	1.	0.	0.03	1.76	1.47	0.85
time (sec)	N/A	0.056	0.028	0.008	0.	0.228	0.69	0.207	4.215

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	0	1	66	49	29
normalized size	1	1.	1.06	1.	0.	0.03	1.94	1.44	0.85
time (sec)	N/A	0.031	0.018	0.005	0.	0.223	0.621	0.209	4.856

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	38	0	1	104	78	39
normalized size	1	1.	1.	0.76	0.	0.02	2.08	1.56	0.78
time (sec)	N/A	0.104	0.037	0.009	0.	0.232	1.032	0.211	7.621

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	18	24	19	18	19
normalized size	1	1.	1.	0.76	0.86	1.14	0.9	0.86	0.9
time (sec)	N/A	0.014	0.008	0.004	1.349	0.218	19.895	0.205	2.922

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	18	24	19	18	19
normalized size	1	1.	1.	0.76	0.86	1.14	0.9	0.86	0.9
time (sec)	N/A	0.014	0.007	0.004	1.35	0.225	7.705	0.207	2.909

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	18	24	19	18	19
normalized size	1	1.	1.	0.76	0.86	1.14	0.9	0.86	0.9
time (sec)	N/A	0.014	0.007	0.004	1.344	0.219	2.708	0.207	3.05

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	18	22	19	18	19
normalized size	1	1.	1.	0.76	0.86	1.05	0.9	0.86	0.9
time (sec)	N/A	0.014	0.008	0.003	1.343	0.229	1.481	0.206	2.98

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	18	19	17	18	17
normalized size	1	1.	1.	0.79	0.95	1.	0.89	0.95	0.89
time (sec)	N/A	0.014	0.008	0.005	1.338	0.219	0.598	0.208	2.94

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	18	19	17	18	17
normalized size	1	1.	1.	0.84	0.95	1.	0.89	0.95	0.89
time (sec)	N/A	0.014	0.009	0.005	1.344	0.25	1.556	0.21	2.948

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	18	20	17	18	17
normalized size	1	1.	1.	0.74	0.95	1.05	0.89	0.95	0.89
time (sec)	N/A	0.014	0.01	0.004	1.346	0.252	2.155	0.211	3.038

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	18	18	19	18	19
normalized size	1	1.	1.	0.74	0.95	0.95	1.	0.95	1.
time (sec)	N/A	0.014	0.008	0.005	1.346	0.251	4.706	0.205	2.926

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	39	34	32	34
normalized size	1	1.	0.83	0.75	0.89	1.08	0.94	0.89	0.94
time (sec)	N/A	0.032	0.013	0.007	1.343	0.251	37.092	0.206	5.115

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	39	34	32	34
normalized size	1	1.	0.83	0.75	0.89	1.08	0.94	0.89	0.94
time (sec)	N/A	0.031	0.012	0.007	1.354	0.249	20.084	0.208	5.201

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	39	34	32	34
normalized size	1	1.	0.83	0.75	0.89	1.08	0.94	0.89	0.94
time (sec)	N/A	0.03	0.012	0.006	1.348	0.209	7.932	0.208	5.207

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	36	34	32	34
normalized size	1	1.	0.83	0.75	0.89	1.	0.94	0.89	0.94
time (sec)	N/A	0.029	0.012	0.007	1.357	0.21	2.45	0.208	5.369

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	27	32	35	32	32	32
normalized size	1	1.	0.88	0.79	0.94	1.03	0.94	0.94	0.94
time (sec)	N/A	0.029	0.012	0.006	1.351	0.217	2.241	0.206	5.173

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	27	32	35	32	32	32
normalized size	1	1.	0.88	0.79	0.94	1.03	0.94	0.94	0.94
time (sec)	N/A	0.03	0.014	0.006	1.346	0.222	3.504	0.206	5.186

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	27	32	35	32	32	32
normalized size	1	1.	0.88	0.79	0.94	1.03	0.94	0.94	0.94
time (sec)	N/A	0.031	0.013	0.007	1.35	0.214	4.094	0.206	5.143

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	27	34	35	32	34	32
normalized size	1	1.	0.88	0.79	1.	1.03	0.94	1.	0.94
time (sec)	N/A	0.031	0.014	0.007	1.35	0.212	6.117	0.209	5.174

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	47	54	49	47	49
normalized size	1	1.	0.8	0.75	0.92	1.06	0.96	0.92	0.96
time (sec)	N/A	0.043	0.014	0.008	1.348	0.212	70.793	0.207	6.355

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	47	54	49	47	49
normalized size	1	1.	0.8	0.75	0.92	1.06	0.96	0.92	0.96
time (sec)	N/A	0.041	0.014	0.007	1.345	0.214	37.317	0.207	6.262

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	47	54	49	47	49
normalized size	1	1.	0.8	0.75	0.92	1.06	0.96	0.92	0.96
time (sec)	N/A	0.043	0.014	0.007	1.375	0.211	20.296	0.208	6.47

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	47	51	49	47	49
normalized size	1	1.	0.8	0.75	0.92	1.	0.96	0.92	0.96
time (sec)	N/A	0.042	0.013	0.008	1.334	0.215	4.631	0.224	6.385

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	38	47	50	48	47	48
normalized size	1	1.	0.84	0.78	0.96	1.02	0.98	0.96	0.98
time (sec)	N/A	0.04	0.013	0.006	1.348	0.209	6.446	0.218	6.33

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	38	47	50	46	47	46
normalized size	1	1.	0.87	0.81	1.	1.06	0.98	1.	0.98
time (sec)	N/A	0.045	0.015	0.007	1.334	0.213	8.354	0.212	6.341

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	38	47	50	48	47	48
normalized size	1	1.	0.84	0.78	0.96	1.02	0.98	0.96	0.98
time (sec)	N/A	0.041	0.016	0.008	1.326	0.209	9.913	0.222	6.323

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	38	49	50	46	49	46
normalized size	1	1.	0.87	0.81	1.04	1.06	0.98	1.04	0.98
time (sec)	N/A	0.041	0.015	0.007	1.351	0.213	15.239	0.208	6.302

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	203	152	0	207	0	265	204
normalized size	1	1.	0.94	0.71	0.	0.96	0.	1.23	0.95
time (sec)	N/A	0.481	0.12	0.017	0.	0.235	0.	0.216	69.715

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	190	143	0	200	180	240	192
normalized size	1	1.	0.93	0.7	0.	0.98	0.88	1.18	0.94
time (sec)	N/A	0.41	0.054	0.01	0.	0.232	126.62	0.226	65.062

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	189	140	0	144	177	240	190
normalized size	1	1.	0.94	0.69	0.	0.71	0.88	1.19	0.94
time (sec)	N/A	0.375	0.057	0.01	0.	0.233	40.185	0.216	62.75

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	146	132	0	159	170	246	182
normalized size	1	1.	0.76	0.69	0.	0.83	0.89	1.28	0.95
time (sec)	N/A	0.339	0.034	0.008	0.	0.23	18.083	0.22	57.369

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	146	132	0	147	170	246	182
normalized size	1	1.	0.76	0.69	0.	0.77	0.89	1.28	0.95
time (sec)	N/A	0.327	0.037	0.008	0.	0.226	32.733	0.212	56.081

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	189	140	0	180	180	257	190
normalized size	1	1.	0.94	0.69	0.	0.89	0.89	1.27	0.94
time (sec)	N/A	0.381	0.11	0.01	0.	0.23	66.437	0.215	63.851

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	190	143	0	197	184	240	192
normalized size	1	1.	0.93	0.7	0.	0.97	0.9	1.18	0.94
time (sec)	N/A	0.374	0.131	0.012	0.	0.231	160.845	0.224	62.935

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	203	152	0	230	0	270	204
normalized size	1	1.	0.94	0.71	0.	1.07	0.	1.26	0.95
time (sec)	N/A	0.436	0.142	0.015	0.	0.271	0.	0.221	71.127

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	212	158	0	239	0	265	216
normalized size	1	1.	0.92	0.69	0.	1.04	0.	1.15	0.94
time (sec)	N/A	0.425	0.239	0.018	0.	0.289	0.	0.219	70.197

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	199	149	0	234	0	269	204
normalized size	1	1.	0.91	0.68	0.	1.07	0.	1.23	0.94
time (sec)	N/A	0.381	0.248	0.016	0.	0.257	0.	0.226	62.81

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	198	158	0	225	0	269	197
normalized size	1	1.	0.91	0.72	0.	1.03	0.	1.23	0.9
time (sec)	N/A	0.366	0.291	0.016	0.	0.252	0.	0.226	61.947

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	198	158	0	235	0	269	197
normalized size	1	1.	0.91	0.72	0.	1.08	0.	1.23	0.9
time (sec)	N/A	0.383	0.275	0.013	0.	0.252	0.	0.217	64.535

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	199	149	0	221	0	269	204
normalized size	1	1.	0.91	0.68	0.	1.01	0.	1.23	0.94
time (sec)	N/A	0.366	0.28	0.012	0.	0.252	0.	0.219	63.329



Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	212	158	0	259	0	284	218
normalized size	1	1.	0.92	0.69	0.	1.13	0.	1.23	0.95
time (sec)	N/A	0.425	0.305	0.02	0.	0.259	0.	0.224	69.163

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	212	161	0	282	0	265	218
normalized size	1	1.	0.92	0.7	0.	1.23	0.	1.15	0.95
time (sec)	N/A	0.421	0.337	0.02	0.	0.254	0.	0.217	68.191

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	227	172	0	325	0	297	231
normalized size	1	1.	0.93	0.71	0.	1.34	0.	1.22	0.95
time (sec)	N/A	0.485	0.503	0.021	0.	0.258	0.	0.217	78.624

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	221	170	0	319	0	282	224
normalized size	1	1.	0.92	0.71	0.	1.33	0.	1.18	0.94
time (sec)	N/A	0.431	0.199	0.021	0.	0.256	0.	0.22	71.488

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	223	169	0	335	0	286	224
normalized size	1	1.	0.92	0.7	0.	1.38	0.	1.18	0.93
time (sec)	N/A	0.434	0.214	0.021	0.	0.259	0.	0.223	72.412

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	223	169	0	323	0	285	223
normalized size	1	1.	0.92	0.7	0.	1.33	0.	1.18	0.92
time (sec)	N/A	0.421	0.238	0.02	0.	0.258	0.	0.222	71.042

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	220	175	0	321	0	282	224
normalized size	1	1.	0.92	0.73	0.	1.34	0.	1.18	0.94
time (sec)	N/A	0.43	0.219	0.011	0.	0.255	0.	0.22	71.266

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	220	166	0	305	0	282	224
normalized size	1	1.	0.92	0.69	0.	1.28	0.	1.18	0.94
time (sec)	N/A	0.417	0.184	0.01	0.	0.254	0.	0.215	70.268

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	234	178	0	333	0	297	238
normalized size	1	1.	0.93	0.71	0.	1.33	0.	1.18	0.95
time (sec)	N/A	0.479	0.208	0.024	0.	0.264	0.	0.217	79.031

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	234	181	0	356	0	281	238
normalized size	1	1.	0.93	0.72	0.	1.42	0.	1.12	0.95
time (sec)	N/A	0.478	0.236	0.025	0.	0.261	0.	0.214	76.16

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	251	192	0	400	0	313	252
normalized size	1	1.	0.95	0.73	0.	1.52	0.	1.19	0.95
time (sec)	N/A	0.55	0.243	0.027	0.	0.26	0.	0.217	85.171

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	73	66	0	149	128	262	53
normalized size	1	1.	1.26	1.14	0.	2.57	2.21	4.52	0.91
time (sec)	N/A	0.09	0.032	0.012	0.	0.255	17.581	0.219	17.688

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	107	72	113	153	105	113	99
normalized size	1	1.	0.99	0.67	1.05	1.42	0.97	1.05	0.92
time (sec)	N/A	0.145	0.056	0.011	1.495	0.254	22.126	0.21	19.607

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	100	67	107	143	99	107	92
normalized size	1	1.	0.99	0.66	1.06	1.42	0.98	1.06	0.91
time (sec)	N/A	0.137	0.034	0.009	1.496	0.247	7.76	0.215	19.478

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	67	107	143	97	107	90
normalized size	1	1.	1.	0.68	1.08	1.44	0.98	1.08	0.91
time (sec)	N/A	0.13	0.034	0.008	1.506	0.252	3.349	0.209	18.002

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	76	62	100	139	90	100	83
normalized size	1	1.	0.83	0.67	1.09	1.51	0.98	1.09	0.9
time (sec)	N/A	0.128	0.027	0.007	1.487	0.252	2.001	0.215	17.334

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	76	62	100	139	90	100	83
normalized size	1	1.	0.83	0.67	1.09	1.51	0.98	1.09	0.9
time (sec)	N/A	0.123	0.022	0.007	1.489	0.25	2.853	0.209	16.279

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	67	107	157	97	107	90
normalized size	1	1.	1.	0.68	1.08	1.59	0.98	1.08	0.91
time (sec)	N/A	0.133	0.055	0.01	1.516	0.248	6.219	0.211	18.96

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	100	67	107	169	99	107	92
normalized size	1	1.	0.99	0.66	1.06	1.67	0.98	1.06	0.91
time (sec)	N/A	0.128	0.066	0.01	1.502	0.251	12.337	0.209	17.981

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	107	72	116	178	105	116	99
normalized size	1	1.	0.99	0.67	1.07	1.65	0.97	1.07	0.92
time (sec)	N/A	0.14	0.076	0.012	1.528	0.252	32.925	0.213	20.925

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	114	79	123	194	277	123	110
normalized size	1	1.	0.93	0.65	1.01	1.59	2.27	1.01	0.9
time (sec)	N/A	0.146	0.133	0.015	1.497	0.25	62.21	0.21	20.191

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	107	74	116	185	264	116	102
normalized size	1	1.	0.95	0.65	1.03	1.64	2.34	1.03	0.9
time (sec)	N/A	0.147	0.141	0.014	1.507	0.251	35.134	0.215	19.548

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	106	74	116	184	257	116	95
normalized size	1	1.	0.94	0.65	1.03	1.63	2.27	1.03	0.84
time (sec)	N/A	0.141	0.115	0.013	1.514	0.249	20.452	0.228	18.461

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	106	74	116	184	257	116	95
normalized size	1	1.	0.94	0.65	1.03	1.63	2.27	1.03	0.84
time (sec)	N/A	0.145	0.099	0.011	1.505	0.248	12.139	0.214	19.346

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	107	74	116	185	264	116	102
normalized size	1	1.	0.95	0.65	1.03	1.64	2.34	1.03	0.9
time (sec)	N/A	0.141	0.112	0.009	1.508	0.248	16.557	0.209	18.39

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	114	79	124	194	366	124	112
normalized size	1	1.	0.93	0.65	1.02	1.59	3.	1.02	0.92
time (sec)	N/A	0.155	0.15	0.016	1.524	0.246	28.7	0.21	21.662

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	114	79	124	208	366	123	112
normalized size	1	1.	0.93	0.65	1.02	1.7	3.	1.01	0.92
time (sec)	N/A	0.149	0.161	0.015	1.516	0.249	62.766	0.224	20.095

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	121	84	131	215	384	132	121
normalized size	1	1.	0.92	0.64	1.	1.64	2.93	1.01	0.92
time (sec)	N/A	0.163	0.191	0.02	1.511	0.248	164.549	0.21	24.579

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	121	82	134	228	0	127	117
normalized size	1	1.	0.94	0.64	1.04	1.77	0.	0.98	0.91
time (sec)	N/A	0.161	0.09	0.015	1.501	0.25	0.	0.21	20.659

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	121	82	134	231	481	127	117
normalized size	1	1.	0.94	0.64	1.04	1.79	3.73	0.98	0.91
time (sec)	N/A	0.167	0.087	0.014	1.501	0.247	131.675	0.211	21.498

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	121	82	131	225	481	124	116
normalized size	1	1.	0.94	0.64	1.02	1.74	3.73	0.96	0.9
time (sec)	N/A	0.16	0.08	0.015	1.49	0.245	83.86	0.211	20.575

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	121	86	134	231	481	127	117
normalized size	1	1.	0.94	0.67	1.04	1.79	3.73	0.98	0.91
time (sec)	N/A	0.169	0.083	0.01	1.498	0.25	46.909	0.211	21.578

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	121	86	134	228	481	127	117
normalized size	1	1.	0.94	0.67	1.04	1.77	3.73	0.98	0.91
time (sec)	N/A	0.163	0.083	0.01	1.56	0.258	64.726	0.211	20.487

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	128	87	138	235	653	134	128
normalized size	1	1.	0.93	0.63	1.	1.7	4.73	0.97	0.93
time (sec)	N/A	0.18	0.106	0.018	1.493	0.255	111.671	0.212	23.351

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	128	87	138	248	0	134	128
normalized size	1	1.	0.93	0.63	1.	1.8	0.	0.97	0.93
time (sec)	N/A	0.17	0.105	0.016	1.503	0.257	0.	0.212	22.274

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	135	92	144	255	0	143	136
normalized size	1	1.	0.92	0.63	0.98	1.73	0.	0.97	0.93
time (sec)	N/A	0.186	0.113	0.02	1.502	0.256	0.	0.211	24.932

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	35	24	31	31	26	32	12
normalized size	1	1.	2.33	1.6	2.07	2.07	1.73	2.13	0.8
time (sec)	N/A	0.028	0.01	0.012	1.479	0.245	0.913	0.212	5.736

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	100	97	69	92	184	94	92	87
normalized size	1	1.37	1.33	0.95	1.26	2.52	1.29	1.26	1.19
time (sec)	N/A	0.599	0.039	0.102	1.517	0.262	10.996	0.209	158.3

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	87	432	0	495	1999	826	87
normalized size	1	1.	0.9	4.45	0.	5.1	20.61	8.52	0.9
time (sec)	N/A	0.114	0.056	0.01	0.	0.25	13.456	0.218	17.008

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	291	0	339	1221	560	70
normalized size	1	1.	0.9	3.68	0.	4.29	15.46	7.09	0.89
time (sec)	N/A	0.086	0.042	0.009	0.	0.249	8.151	0.215	13.801

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	55	178	0	212	683	346	53
normalized size	1	1.	0.9	2.92	0.	3.48	11.2	5.67	0.87
time (sec)	N/A	0.063	0.041	0.008	0.	0.246	4.915	0.212	10.647

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	39	93	0	115	306	182	36
normalized size	1	1.	0.91	2.16	0.	2.67	7.12	4.23	0.84
time (sec)	N/A	0.046	0.029	0.007	0.	0.248	2.599	0.211	7.779

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	35	0	45	94	69	19
normalized size	1	1.	0.92	1.4	0.	1.8	3.76	2.76	0.76
time (sec)	N/A	0.023	0.024	0.003	0.	0.245	1.107	0.206	4.061

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	88	0	29
normalized size	1	1.	1.05	0.	0.	0.	2.26	0.	0.74
time (sec)	N/A	0.032	0.027	0.043	0.	0.	4.488	0.	4.987

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	374	0	31
normalized size	1	1.	1.05	0.	0.	0.	9.59	0.	0.79
time (sec)	N/A	0.03	0.03	0.053	0.	0.	27.619	0.	4.453

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	1556	0	31
normalized size	1	1.	1.05	0.	0.	0.	39.9	0.	0.79
time (sec)	N/A	0.03	0.032	0.072	0.	0.	96.678	0.	4.451

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	45	0	0	0	92	0	29
normalized size	1	1.	1.02	0.	0.	0.	2.09	0.	0.66
time (sec)	N/A	0.042	0.038	0.044	0.	0.	19.487	0.	5.188

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	95	0	32
normalized size	1	1.	0.95	0.	0.	0.	2.16	0.	0.73
time (sec)	N/A	0.038	0.02	0.043	0.	0.	4.532	0.	5.149

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	58	0	0	0	39	0	24
normalized size	1	1.	1.53	0.	0.	0.	1.03	0.	0.63
time (sec)	N/A	0.035	0.05	0.043	0.	0.	67.745	0.	4.996

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	59	0	0	0	0	0	34
normalized size	1	1.	1.26	0.	0.	0.	0.	0.	0.72
time (sec)	N/A	0.049	0.076	0.044	0.	0.	0.	0.	5.224

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	80	0	0	0	0	0	29
normalized size	1	1.	1.78	0.	0.	0.	0.	0.	0.64
time (sec)	N/A	0.048	0.096	0.043	0.	0.	0.	0.	5.282

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	92	0	0	343	0	27
normalized size	1	1.	1.06	2.56	0.	0.	9.53	0.	0.75
time (sec)	N/A	0.03	0.03	0.097	0.	0.	28.161	0.	4.499

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	61	47	0	77	110	77	71
normalized size	1	1.	0.76	0.59	0.	0.96	1.38	0.96	0.89
time (sec)	N/A	0.122	0.029	0.008	0.	0.233	3.775	0.207	15.672

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	50	36	0	62	87	58	51
normalized size	1	1.	0.85	0.61	0.	1.05	1.47	0.98	0.86
time (sec)	N/A	0.095	0.022	0.008	0.	0.228	1.88	0.208	11.843

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	25	0	46	63	39	31
normalized size	1	1.	1.	0.66	0.	1.21	1.66	1.03	0.82
time (sec)	N/A	0.066	0.018	0.007	0.	0.227	0.85	0.206	7.902

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	19	39	19	12
normalized size	1	1.	1.	0.83	1.06	1.06	2.17	1.06	0.67
time (sec)	N/A	0.011	0.006	0.003	1.349	0.229	0.4	0.208	2.151



Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	47	39	0	1	56	45	31
normalized size	1	1.	1.27	1.05	0.	0.03	1.51	1.22	0.84
time (sec)	N/A	0.074	0.03	0.006	0.	0.246	4.84	0.21	7.521

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	58	63	0	1	42	58	41
normalized size	1	1.	1.23	1.34	0.	0.02	0.89	1.23	0.87
time (sec)	N/A	0.079	0.046	0.006	0.	0.237	6.454	0.209	7.788

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	78	85	0	1	92	84	60
normalized size	1	1.	1.1	1.2	0.	0.01	1.3	1.18	0.85
time (sec)	N/A	0.114	0.059	0.009	0.	0.246	12.128	0.214	10.562

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	91	105	0	1	117	108	82
normalized size	1	1.	0.96	1.11	0.	0.01	1.23	1.14	0.86
time (sec)	N/A	0.153	0.074	0.011	0.	0.257	19.38	0.211	14.218

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	77	77	0	1	117	86	82
normalized size	1	1.	0.82	0.82	0.	0.01	1.24	0.91	0.87
time (sec)	N/A	0.111	0.059	0.009	0.	0.25	18.385	0.212	12.984

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	64	57	0	1	92	68	60
normalized size	1	1.	0.91	0.81	0.	0.01	1.31	0.97	0.86
time (sec)	N/A	0.073	0.038	0.008	0.	0.247	11.534	0.212	9.289

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	49	36	0	1	41	50	39
normalized size	1	1.	1.07	0.78	0.	0.02	0.89	1.09	0.85
time (sec)	N/A	0.032	0.027	0.004	0.	0.246	6.305	0.214	3.127

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	45	54	0	1	56	77	34
normalized size	1	1.	1.07	1.29	0.	0.02	1.33	1.83	0.81
time (sec)	N/A	0.039	0.022	0.006	0.	0.248	4.874	0.215	5.208

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	0	23	42	80	17
normalized size	1	1.	1.	0.86	0.	1.1	2.	3.81	0.81
time (sec)	N/A	0.022	0.015	0.006	0.	0.234	2.086	0.212	3.177

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	28	0	51	68	151	37
normalized size	1	1.	0.93	0.64	0.	1.16	1.55	3.43	0.84
time (sec)	N/A	0.045	0.019	0.006	0.	0.24	2.919	0.213	5.188

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	53	39	0	66	359	186	61
normalized size	1	1.	0.78	0.57	0.	0.97	5.28	2.74	0.9
time (sec)	N/A	0.071	0.024	0.007	0.	0.245	4.498	0.214	8.109

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	64	50	0	81	575	224	85
normalized size	1	1.	0.7	0.54	0.	0.88	6.25	2.43	0.92
time (sec)	N/A	0.101	0.031	0.007	0.	0.265	6.614	0.212	11.626

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	0	92	133	181	70
normalized size	1	1.	0.62	0.59	0.	1.15	1.66	2.26	0.88
time (sec)	N/A	0.118	0.045	0.009	0.	0.237	11.784	0.21	15.637

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	0	77	109	143	51
normalized size	1	1.	0.66	0.61	0.	1.31	1.85	2.42	0.86
time (sec)	N/A	0.094	0.036	0.006	0.	0.238	6.345	0.207	11.77

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	0	61	85	105	31
normalized size	1	1.	0.74	0.66	0.	1.61	2.24	2.76	0.82
time (sec)	N/A	0.066	0.032	0.008	0.	0.234	3.448	0.213	7.914

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	43	61	19	12
normalized size	1	1.	1.	0.83	1.06	2.39	3.39	1.06	0.67
time (sec)	N/A	0.011	0.006	0.004	1.34	0.241	1.626	0.208	2.229

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	62	52	0	1	78	65	44
normalized size	1	1.	1.15	0.96	0.	0.02	1.44	1.2	0.81
time (sec)	N/A	0.099	0.06	0.007	0.	0.248	6.935	0.209	9.449

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	75	0	1	88	77	56
normalized size	1	1.	1.03	1.19	0.	0.02	1.4	1.22	0.89
time (sec)	N/A	0.102	0.075	0.006	0.	0.252	8.53	0.211	9.715

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	73	102	0	1	71	82	63
normalized size	1	1.	1.07	1.5	0.	0.01	1.04	1.21	0.93
time (sec)	N/A	0.107	0.092	0.009	0.	0.251	10.854	0.211	10.381

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	89	122	0	1	119	108	78
normalized size	1	1.	0.97	1.33	0.	0.01	1.29	1.17	0.85
time (sec)	N/A	0.143	0.08	0.012	0.	0.257	18.622	0.21	13.801

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	102	142	0	1	148	127	104
normalized size	1	1.	0.88	1.22	0.	0.01	1.28	1.09	0.9
time (sec)	N/A	0.183	0.129	0.019	0.	0.272	28.53	0.211	17.935

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	88	95	0	1	148	103	104
normalized size	1	1.	0.77	0.83	0.	0.01	1.29	0.9	0.9
time (sec)	N/A	0.136	0.073	0.01	0.	0.265	26.197	0.215	16.926

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	75	75	0	1	119	85	78
normalized size	1	1.	0.82	0.82	0.	0.01	1.31	0.93	0.86
time (sec)	N/A	0.102	0.056	0.008	0.	0.256	17.126	0.211	12.882

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	62	51	0	1	70	66	60
normalized size	1	1.	0.95	0.78	0.	0.02	1.08	1.02	0.92
time (sec)	N/A	0.045	0.059	0.003	0.	0.244	9.9	0.212	4.359

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	58	69	0	1	88	99	56
normalized size	1	1.	0.92	1.1	0.	0.02	1.4	1.57	0.89
time (sec)	N/A	0.053	0.045	0.006	0.	0.247	8.381	0.217	6.099

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	55	92	0	1	78	154	51
normalized size	1	1.	0.9	1.51	0.	0.02	1.28	2.52	0.84
time (sec)	N/A	0.063	0.048	0.007	0.	0.247	7.384	0.217	7.931

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	0	47	68	116	17
normalized size	1	1.	1.	0.86	0.	2.24	3.24	5.52	0.81
time (sec)	N/A	0.021	0.026	0.006	0.	0.234	3.771	0.213	3.203

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	0	66	94	224	37
normalized size	1	1.	0.7	0.64	0.	1.5	2.14	5.09	0.84
time (sec)	N/A	0.044	0.034	0.006	0.	0.242	5.133	0.217	5.169

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	0	81	420	259	61
normalized size	1	1.	0.62	0.57	0.	1.19	6.18	3.81	0.9
time (sec)	N/A	0.07	0.044	0.006	0.	0.261	7.907	0.216	8.053

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	50	0	96	648	297	85
normalized size	1	1.	0.58	0.54	0.	1.04	7.04	3.23	0.92
time (sec)	N/A	0.1	0.047	0.008	0.	0.303	12.247	0.217	11.619

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	0	107	158	301	70
normalized size	1	1.	0.62	0.59	0.	1.34	1.98	3.76	0.88
time (sec)	N/A	0.12	0.053	0.008	0.	0.238	29.368	0.21	15.798

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	0	92	133	243	51
normalized size	1	1.	0.66	0.61	0.	1.56	2.25	4.12	0.86
time (sec)	N/A	0.092	0.041	0.007	0.	0.235	18.533	0.208	11.774

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	0	76	109	186	31
normalized size	1	1.	0.74	0.66	0.	2.	2.87	4.89	0.82
time (sec)	N/A	0.065	0.037	0.007	0.	0.237	11.427	0.209	7.993

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	58	85	95	12
normalized size	1	1.	1.	0.83	1.06	3.22	4.72	5.28	0.67
time (sec)	N/A	0.012	0.008	0.004	1.34	0.243	6.003	0.207	2.139

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	66	0	1	105	84	60
normalized size	1	1.	1.	0.92	0.	0.01	1.46	1.17	0.83
time (sec)	N/A	0.126	0.075	0.007	0.	0.248	11.806	0.211	11.95

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	76	88	0	1	112	99	73
normalized size	1	1.	0.95	1.1	0.	0.01	1.4	1.24	0.91
time (sec)	N/A	0.131	0.109	0.008	0.	0.25	11.419	0.211	11.932

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	82	116	0	1	117	103	78
normalized size	1	1.	0.95	1.35	0.	0.01	1.36	1.2	0.91
time (sec)	N/A	0.133	0.118	0.008	0.	0.258	13.198	0.213	12.907

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	84	139	0	1	99	101	83
normalized size	1	1.	0.94	1.56	0.	0.01	1.11	1.13	0.93
time (sec)	N/A	0.141	0.114	0.014	0.	0.26	16.6	0.21	13.261

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	102	159	0	1	150	127	104
normalized size	1	1.	0.9	1.41	0.	0.01	1.33	1.12	0.92
time (sec)	N/A	0.182	0.13	0.023	0.	0.275	27.987	0.213	17.222

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	112	179	0	1	175	146	122
normalized size	1	1.	0.82	1.31	0.	0.01	1.28	1.07	0.89
time (sec)	N/A	0.225	0.114	0.046	0.	0.302	42.726	0.21	22.065

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	98	113	0	1	175	123	122
normalized size	1	1.	0.72	0.83	0.	0.01	1.29	0.9	0.9
time (sec)	N/A	0.174	0.081	0.01	0.	0.307	36.591	0.214	21.392

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	88	93	0	1	150	104	104
normalized size	1	1.	0.79	0.83	0.	0.01	1.34	0.93	0.93
time (sec)	N/A	0.132	0.063	0.007	0.	0.269	24.586	0.212	16.939

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	66	0	1	97	85	78
normalized size	1	1.	0.85	0.79	0.	0.01	1.15	1.01	0.93
time (sec)	N/A	0.061	0.077	0.004	0.	0.257	14.444	0.212	5.755

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	73	85	0	1	117	117	76
normalized size	1	1.	0.88	1.02	0.	0.01	1.41	1.41	0.92
time (sec)	N/A	0.073	0.06	0.007	0.	0.256	13.09	0.215	7.742

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	73	110	0	1	112	178	78
normalized size	1	1.	0.85	1.28	0.	0.01	1.3	2.07	0.91
time (sec)	N/A	0.079	0.068	0.007	0.	0.25	11.439	0.22	9.217

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	68	130	0	1	105	227	70
normalized size	1	1.	0.83	1.59	0.	0.01	1.28	2.77	0.85
time (sec)	N/A	0.089	0.056	0.012	0.	0.247	12.919	0.226	10.948

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	0	62	95	153	17
normalized size	1	1.	1.	0.86	0.	2.95	4.52	7.29	0.81
time (sec)	N/A	0.022	0.033	0.006	0.	0.237	6.458	0.216	3.194

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	0	81	121	297	37
normalized size	1	1.	0.7	0.64	0.	1.84	2.75	6.75	0.84
time (sec)	N/A	0.045	0.042	0.007	0.	0.255	10.213	0.216	5.205

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	0	96	481	332	61
normalized size	1	1.	0.62	0.57	0.	1.41	7.07	4.88	0.9
time (sec)	N/A	0.071	0.051	0.007	0.	0.313	15.482	0.215	8.042

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	50	0	111	721	370	85
normalized size	1	1.	0.58	0.54	0.	1.21	7.84	4.02	0.92
time (sec)	N/A	0.103	0.055	0.007	0.	0.388	22.692	0.216	11.749

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	64	61	0	126	1012	405	109
normalized size	1	1.	0.55	0.53	0.	1.09	8.72	3.49	0.94
time (sec)	N/A	0.135	0.064	0.009	0.	0.549	32.149	0.213	15.819

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	75	72	0	140	1346	443	133
normalized size	1	1.	0.54	0.51	0.	1.	9.61	3.16	0.95
time (sec)	N/A	0.172	0.069	0.009	0.	0.767	39.243	0.215	20.912

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	94	91	0	196	0	1	150
normalized size	1	1.	0.58	0.57	0.	1.22	0.	0.01	0.93
time (sec)	N/A	0.232	0.089	0.012	0.	0.259	0.	0.224	32.236

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	83	80	0	181	0	1	129
normalized size	1	1.	0.59	0.57	0.	1.29	0.	0.01	0.92
time (sec)	N/A	0.203	0.067	0.01	0.	0.254	0.	0.225	27.578

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	72	69	0	166	0	1	112
normalized size	1	1.	0.59	0.57	0.	1.36	0.	0.01	0.92
time (sec)	N/A	0.175	0.075	0.009	0.	0.252	0.	0.218	24.009

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	61	58	0	151	230	1	92
normalized size	1	1.	0.6	0.57	0.	1.5	2.28	0.01	0.91
time (sec)	N/A	0.147	0.059	0.007	0.	0.245	155.645	0.212	19.508



Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	0	136	204	595	70
normalized size	1	1.	0.62	0.59	0.	1.7	2.55	7.44	0.88
time (sec)	N/A	0.122	0.062	0.009	0.	0.24	116.136	0.224	15.991

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	0	122	180	500	51
normalized size	1	1.	0.66	0.61	0.	2.07	3.05	8.47	0.86
time (sec)	N/A	0.095	0.051	0.007	0.	0.237	85.228	0.218	11.89

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	0	105	156	406	31
normalized size	1	1.	0.74	0.66	0.	2.76	4.11	10.68	0.82
time (sec)	N/A	0.067	0.05	0.006	0.	0.24	62.155	0.21	8.079

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	88	133	267	12
normalized size	1	1.	1.	0.83	1.06	4.89	7.39	14.83	0.67
time (sec)	N/A	0.011	0.014	0.005	1.324	0.255	43.375	0.209	2.175

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	94	94	0	1	160	122	90
normalized size	1	1.	0.87	0.87	0.	0.01	1.48	1.13	0.83
time (sec)	N/A	0.191	0.111	0.007	0.	0.264	36.801	0.21	17.796

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	104	118	0	1	167	136	110
normalized size	1	1.	0.88	1.	0.	0.01	1.42	1.15	0.93
time (sec)	N/A	0.193	0.143	0.008	0.	0.26	37.89	0.212	18.014

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	105	148	0	1	175	143	117
normalized size	1	1.	0.83	1.17	0.	0.01	1.39	1.13	0.93
time (sec)	N/A	0.204	0.154	0.01	0.	0.257	36.197	0.212	19.146

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	105	168	0	1	175	143	117
normalized size	1	1.	0.83	1.33	0.	0.01	1.39	1.13	0.93
time (sec)	N/A	0.204	0.155	0.018	0.	0.251	33.441	0.213	19.117

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	108	190	0	1	173	140	119
normalized size	1	1.	0.84	1.48	0.	0.01	1.35	1.09	0.93
time (sec)	N/A	0.206	0.15	0.031	0.	0.253	35.725	0.211	19.741

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	106	213	0	1	153	139	124
normalized size	1	1.	0.81	1.63	0.	0.01	1.17	1.06	0.95
time (sec)	N/A	0.214	0.174	0.069	0.	0.313	45.248	0.225	20.144

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	123	233	0	1	204	165	144
normalized size	1	1.	0.79	1.5	0.	0.01	1.32	1.06	0.93
time (sec)	N/A	0.264	0.142	0.17	0.	0.362	64.968	0.214	24.957

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	134	253	0	1	231	184	168
normalized size	1	1.	0.75	1.41	0.	0.01	1.29	1.03	0.94
time (sec)	N/A	0.313	0.201	0.437	0.	0.493	92.317	0.213	31.688

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	131	169	0	1	258	180	192
normalized size	1	1.	0.65	0.84	0.	0.	1.28	0.89	0.95
time (sec)	N/A	0.318	0.126	0.016	0.	0.667	100.404	0.214	38.337

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	120	149	0	1	231	161	168
normalized size	1	1.	0.67	0.84	0.	0.01	1.3	0.9	0.94
time (sec)	N/A	0.256	0.098	0.012	0.	0.49	74.344	0.213	31.013

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	109	129	0	1	204	142	144
normalized size	1	1.	0.71	0.84	0.	0.01	1.32	0.92	0.94
time (sec)	N/A	0.204	0.092	0.009	0.	0.381	54.286	0.217	25.76

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	93	96	0	1	151	123	116
normalized size	1	1.	0.76	0.79	0.	0.01	1.24	1.01	0.95
time (sec)	N/A	0.102	0.088	0.005	0.	0.3	35.861	0.219	9.683

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	96	117	0	1	173	155	117
normalized size	1	1.	0.78	0.95	0.	0.01	1.41	1.26	0.95
time (sec)	N/A	0.114	0.073	0.007	0.	0.269	35.074	0.213	11.625

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	95	146	0	1	175	216	121
normalized size	1	1.	0.74	1.14	0.	0.01	1.37	1.69	0.95
time (sec)	N/A	0.127	0.103	0.008	0.	0.271	33.113	0.213	13.766

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	96	166	0	1	175	270	121
normalized size	1	1.	0.74	1.29	0.	0.01	1.36	2.09	0.94
time (sec)	N/A	0.137	0.103	0.013	0.	0.275	35.28	0.23	15.458

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	94	186	0	1	167	324	117
normalized size	1	1.	0.75	1.48	0.	0.01	1.33	2.57	0.93
time (sec)	N/A	0.144	0.085	0.022	0.	0.27	37.649	0.216	16.496

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	90	206	0	1	160	373	107
normalized size	1	1.	0.73	1.66	0.	0.01	1.29	3.01	0.86
time (sec)	N/A	0.15	0.084	0.047	0.	0.272	42.965	0.22	18.227

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	0	92	150	225	17
normalized size	1	1.	1.	0.86	0.	4.38	7.14	10.71	0.81
time (sec)	N/A	0.023	0.048	0.005	0.	0.294	28.865	0.216	3.154

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	0	111	175	443	37
normalized size	1	1.	0.7	0.64	0.	2.52	3.98	10.07	0.84
time (sec)	N/A	0.045	0.056	0.006	0.	0.371	34.672	0.218	5.163

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	0	126	604	478	61
normalized size	1	1.	0.62	0.57	0.	1.85	8.88	7.03	0.9
time (sec)	N/A	0.072	0.067	0.007	0.	0.546	62.157	0.22	8.131

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	50	0	140	867	516	85
normalized size	1	1.	0.58	0.54	0.	1.52	9.42	5.61	0.92
time (sec)	N/A	0.102	0.069	0.008	0.	0.76	83.081	0.217	11.561

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	64	61	0	155	1182	551	109
normalized size	1	1.	0.55	0.53	0.	1.34	10.19	4.75	0.94
time (sec)	N/A	0.135	0.082	0.009	0.	1.084	111.166	0.222	15.89

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	75	72	0	170	1540	589	133
normalized size	1	1.	0.54	0.51	0.	1.21	11.	4.21	0.95
time (sec)	N/A	0.174	0.083	0.01	0.	1.568	114.539	0.22	20.625

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	86	83	0	185	1950	624	156
normalized size	1	1.	0.52	0.51	0.	1.13	11.89	3.8	0.95
time (sec)	N/A	0.214	0.094	0.011	0.	2.353	146.819	0.222	26.506

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	24	54	180	61	46	37
normalized size	1	1.	0.59	0.52	1.17	3.91	1.33	1.	0.8
time (sec)	N/A	0.059	0.016	0.006	1.502	0.222	5.942	0.204	6.625

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	39	46	61	236	75	58	56
normalized size	1	1.	0.62	0.73	0.97	3.75	1.19	0.92	0.89
time (sec)	N/A	0.059	0.023	0.008	1.495	0.227	15.306	0.206	7.556

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	19	35	139	44	31	24
normalized size	1	1.	0.71	0.61	1.13	4.48	1.42	1.	0.77
time (sec)	N/A	0.043	0.009	0.005	1.497	0.226	1.769	0.203	5.538

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	36	32	42	182	54	49	39
normalized size	1	1.	0.8	0.71	0.93	4.04	1.2	1.09	0.87
time (sec)	N/A	0.039	0.026	0.007	1.495	0.226	8.953	0.206	5.877

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	99	27	15	10
normalized size	1	1.	1.	0.8	1.	6.6	1.8	1.	0.67
time (sec)	N/A	0.009	0.003	0.004	1.342	0.225	0.46	0.204	1.931

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	26	120	22	39	22
normalized size	1	1.	1.	0.74	0.96	4.44	0.81	1.44	0.81
time (sec)	N/A	0.014	0.011	0.003	1.496	0.225	0.486	0.204	1.274

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	25	26	138	39	51	24
normalized size	1	1.	1.07	0.83	0.87	4.6	1.3	1.7	0.8
time (sec)	N/A	0.048	0.013	0.005	1.499	0.231	4.275	0.207	5.658

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	34	28	78	19	54	19
normalized size	1	1.	1.	1.36	1.12	3.12	0.76	2.16	0.76
time (sec)	N/A	0.022	0.014	0.006	1.491	0.232	0.662	0.205	3.298

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	43	41	47	189	24	58	32
normalized size	1	1.	1.1	1.05	1.21	4.85	0.62	1.49	0.82
time (sec)	N/A	0.05	0.022	0.006	1.499	0.231	5.925	0.207	5.885

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	92	34	57	15
normalized size	1	1.	1.	0.83	1.06	5.11	1.89	3.17	0.83
time (sec)	N/A	0.016	0.009	0.005	1.497	0.224	3.676	0.206	2.946

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	55	66	269	63	74	46
normalized size	1	1.	0.84	0.96	1.16	4.72	1.11	1.3	0.81
time (sec)	N/A	0.066	0.041	0.008	1.498	0.23	12.424	0.206	6.685

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	34	54	142	61	70	37
normalized size	1	1.	0.59	0.74	1.17	3.09	1.33	1.52	0.8
time (sec)	N/A	0.058	0.02	0.007	1.496	0.227	5.957	0.202	6.91

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	39	46	61	227	167	45	56
normalized size	1	1.	0.62	0.73	0.97	3.6	2.65	0.71	0.89
time (sec)	N/A	0.059	0.023	0.009	1.498	0.236	15.327	0.207	7.721

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	29	35	115	44	43	24
normalized size	1	1.	0.71	0.94	1.13	3.71	1.42	1.39	0.77
time (sec)	N/A	0.043	0.012	0.005	1.494	0.228	1.79	0.201	5.55

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	36	32	42	178	124	35	39
normalized size	1	1.	0.8	0.71	0.93	3.96	2.76	0.78	0.87
time (sec)	N/A	0.04	0.028	0.008	1.505	0.231	9.02	0.208	5.933

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	15	88	27	15	12
normalized size	1	1.	1.	1.47	1.	5.87	1.8	1.	0.8
time (sec)	N/A	0.009	0.004	0.004	1.315	0.23	0.467	0.213	1.953

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	26	119	22	26	22
normalized size	1	1.	1.	0.74	0.96	4.41	0.81	0.96	0.81
time (sec)	N/A	0.014	0.011	0.004	1.524	0.228	0.49	0.217	1.296

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	25	47	68	76	54	24
normalized size	1	1.	1.07	0.83	1.57	2.27	2.53	1.8	0.8
time (sec)	N/A	0.051	0.013	0.006	1.48	0.225	4.521	0.217	5.666

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	34	28	95	20	53	20
normalized size	1	1.	1.	1.36	1.12	3.8	0.8	2.12	0.8
time (sec)	N/A	0.022	0.015	0.006	1.503	0.228	0.67	0.207	3.287

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	43	41	69	131	99	61	31
normalized size	1	1.	1.1	1.05	1.77	3.36	2.54	1.56	0.79
time (sec)	N/A	0.052	0.022	0.006	1.49	0.233	6.334	0.208	5.752

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	19	101	78	99	15
normalized size	1	1.	1.	1.39	1.06	5.61	4.33	5.5	0.83
time (sec)	N/A	0.016	0.013	0.005	1.549	0.23	3.731	0.208	2.942

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	55	88	193	141	77	46
normalized size	1	1.	0.84	0.96	1.54	3.39	2.47	1.35	0.81
time (sec)	N/A	0.067	0.042	0.008	1.51	0.227	12.689	0.206	6.619

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	34	54	180	61	46	37
normalized size	1	1.	0.59	0.74	1.17	3.91	1.33	1.	0.8
time (sec)	N/A	0.055	0.015	0.006	1.492	0.227	5.954	0.203	6.643

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	49	61	77	236	167	59	65
normalized size	1	1.	0.68	0.85	1.07	3.28	2.32	0.82	0.9
time (sec)	N/A	0.065	0.021	0.009	1.48	0.228	15.56	0.206	8.204

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	29	35	139	44	31	24
normalized size	1	1.	0.71	0.94	1.13	4.48	1.42	1.	0.77
time (sec)	N/A	0.042	0.009	0.007	1.487	0.223	1.779	0.202	5.504

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	46	47	58	182	124	50	48
normalized size	1	1.	0.85	0.87	1.07	3.37	2.3	0.93	0.89
time (sec)	N/A	0.047	0.023	0.007	1.51	0.223	9.215	0.205	6.149

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	15	99	27	15	10
normalized size	1	1.	1.	1.47	1.	6.6	1.8	1.	0.67
time (sec)	N/A	0.009	0.003	0.005	1.346	0.221	0.469	0.202	1.916

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	35	42	120	22	41	31
normalized size	1	1.	1.03	0.97	1.17	3.33	0.61	1.14	0.86
time (sec)	N/A	0.02	0.011	0.003	1.494	0.228	0.492	0.206	1.509



Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	25	26	96	82	32	24
normalized size	1	1.	0.93	0.83	0.87	3.2	2.73	1.07	0.8
time (sec)	N/A	0.049	0.013	0.007	1.476	0.231	4.6	0.204	5.704

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	35	48	45	78	19	59	27
normalized size	1	1.	1.03	1.41	1.32	2.29	0.56	1.74	0.79
time (sec)	N/A	0.028	0.014	0.004	1.487	0.228	0.682	0.208	3.505

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	41	47	134	99	39	31
normalized size	1	1.	0.95	1.05	1.21	3.44	2.54	1.	0.79
time (sec)	N/A	0.05	0.016	0.006	1.487	0.231	6.073	0.205	5.809

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	19	92	78	57	14
normalized size	1	1.	1.	1.39	1.06	5.11	4.33	3.17	0.78
time (sec)	N/A	0.016	0.008	0.005	1.476	0.222	3.693	0.206	2.96

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	55	66	196	141	55	46
normalized size	1	1.	0.81	0.96	1.16	3.44	2.47	0.96	0.81
time (sec)	N/A	0.067	0.031	0.007	1.494	0.226	12.517	0.204	6.709

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	24	54	38	68	50	44
normalized size	1	1.	0.59	0.52	1.17	0.83	1.48	1.09	0.96
time (sec)	N/A	0.058	0.022	0.005	1.485	0.218	6.021	0.203	6.702

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	48	55	61	97	83	47	71
normalized size	1	1.	0.67	0.76	0.85	1.35	1.15	0.65	0.99
time (sec)	N/A	0.067	0.03	0.012	1.506	0.235	15.39	0.207	7.787

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	19	35	31	49	34	27
normalized size	1	1.	0.71	0.61	1.13	1.	1.58	1.1	0.87
time (sec)	N/A	0.044	0.014	0.005	1.499	0.234	1.838	0.202	5.506

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	41	42	90	61	38	53
normalized size	1	1.	0.8	0.76	0.78	1.67	1.13	0.7	0.98
time (sec)	N/A	0.047	0.032	0.007	1.504	0.242	9.064	0.209	6.127

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	24	31	16	14
normalized size	1	1.	1.	0.8	1.	1.6	2.07	1.07	0.93
time (sec)	N/A	0.009	0.003	0.003	1.347	0.23	0.512	0.202	1.97

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	29	26	80	34	28	34
normalized size	1	1.	1.	0.81	0.72	2.22	0.94	0.78	0.94
time (sec)	N/A	0.02	0.015	0.005	1.493	0.23	1.475	0.203	1.494

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	25	47	70	44	36	27
normalized size	1	1.	0.93	0.83	1.57	2.33	1.47	1.2	0.9
time (sec)	N/A	0.05	0.015	0.007	1.502	0.231	4.313	0.203	5.562

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	35	43	28	86	32	66	32
normalized size	1	1.	1.03	1.26	0.82	2.53	0.94	1.94	0.94
time (sec)	N/A	0.029	0.025	0.005	1.504	0.23	1.633	0.208	3.465

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	41	69	88	27	42	36
normalized size	1	1.	0.95	1.05	1.77	2.26	0.69	1.08	0.92
time (sec)	N/A	0.051	0.02	0.006	1.499	0.236	6.026	0.205	5.754

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	19	37	120	15
normalized size	1	1.	1.	0.83	1.06	1.06	2.06	6.67	0.83
time (sec)	N/A	0.016	0.01	0.006	1.498	0.225	3.644	0.21	2.945

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	44	55	88	97	68	61	51
normalized size	1	1.	0.77	0.96	1.54	1.7	1.19	1.07	0.89
time (sec)	N/A	0.067	0.035	0.007	1.507	0.239	12.532	0.204	6.62

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	39	36	0	47	68	58	49
normalized size	1	1.	0.7	0.64	0.	0.84	1.21	1.04	0.88
time (sec)	N/A	0.093	0.027	0.008	0.	0.235	2.677	0.203	11.762

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	67	59	0	1	95	73	66
normalized size	1	1.	0.92	0.81	0.	0.01	1.3	1.	0.9
time (sec)	N/A	0.073	0.055	0.007	0.	0.247	12.854	0.208	9.348

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	25	0	31	44	36	29
normalized size	1	1.	0.75	0.69	0.	0.86	1.22	1.	0.81
time (sec)	N/A	0.067	0.02	0.006	0.	0.227	1.749	0.201	7.927

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	52	39	0	1	42	54	41
normalized size	1	1.	1.06	0.8	0.	0.02	0.86	1.1	0.84
time (sec)	N/A	0.045	0.029	0.008	0.	0.235	7.304	0.21	5.836

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	20	18	10
normalized size	1	1.	1.	0.93	1.2	1.2	1.33	1.2	0.67
time (sec)	N/A	0.012	0.004	0.004	1.354	0.228	1.456	0.202	2.18

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	0	1	17	31	22
normalized size	1	1.	1.	0.84	0.	0.04	0.68	1.24	0.88
time (sec)	N/A	0.018	0.011	0.004	0.	0.246	3.589	0.209	2.428

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	31	29	0	1	19	30	22
normalized size	1	1.	1.24	1.16	0.	0.04	0.76	1.2	0.88
time (sec)	N/A	0.053	0.028	0.006	0.	0.232	3.669	0.204	5.794

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	0	23	19	41	14
normalized size	1	1.	1.	0.95	0.	1.21	1.	2.16	0.74
time (sec)	N/A	0.023	0.016	0.005	0.	0.222	1.863	0.208	3.22

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	64	48	0	1	42	65	41
normalized size	1	1.	1.28	0.96	0.	0.02	0.84	1.3	0.82
time (sec)	N/A	0.08	0.038	0.009	0.	0.241	7.673	0.203	7.801

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	29	26	0	36	46	74	36
normalized size	1	1.	0.66	0.59	0.	0.82	1.05	1.68	0.82
time (sec)	N/A	0.045	0.021	0.005	0.	0.223	2.578	0.21	5.309

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	78	68	0	1	97	89	66
normalized size	1	1.	1.05	0.92	0.	0.01	1.31	1.2	0.89
time (sec)	N/A	0.113	0.065	0.01	0.	0.242	13.636	0.206	10.761

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	36	0	62	68	55	48
normalized size	1	1.	0.69	0.65	0.	1.13	1.24	1.	0.87
time (sec)	N/A	0.091	0.028	0.006	0.	0.227	3.017	0.22	11.68

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	61	57	0	1	71	69	61
normalized size	1	1.	0.9	0.84	0.	0.01	1.04	1.01	0.9
time (sec)	N/A	0.072	0.094	0.009	0.	0.246	10.62	0.217	9.429

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	24	23	0	46	41	34	27
normalized size	1	1.	0.75	0.72	0.	1.44	1.28	1.06	0.84
time (sec)	N/A	0.064	0.021	0.007	0.	0.228	2.012	0.209	7.856

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	37	0	1	37	53	36
normalized size	1	1.	1.07	0.86	0.	0.02	0.86	1.23	0.84
time (sec)	N/A	0.045	0.036	0.007	0.	0.242	5.577	0.217	5.797

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	32	24	19	14
normalized size	1	1.	1.	0.94	1.19	2.	1.5	1.19	0.88
time (sec)	N/A	0.011	0.005	0.003	1.325	0.226	1.911	0.207	2.173

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	31	17	19	12
normalized size	1	1.	1.	0.94	1.19	1.94	1.06	1.19	0.75
time (sec)	N/A	0.01	0.012	0.005	1.322	0.218	1.781	0.213	1.265

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	48	43	0	1	184	53	34
normalized size	1	1.	1.17	1.05	0.	0.02	4.49	1.29	0.83
time (sec)	N/A	0.077	0.054	0.007	0.	0.238	5.838	0.21	7.814

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	26	0	47	46	68	34
normalized size	1	1.	0.71	0.68	0.	1.24	1.21	1.79	0.89
time (sec)	N/A	0.034	0.025	0.006	0.	0.228	2.7	0.215	4.016

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	67	63	0	1	73	89	63
normalized size	1	1.	0.99	0.93	0.	0.01	1.07	1.31	0.93
time (sec)	N/A	0.109	0.151	0.007	0.	0.24	11.295	0.216	10.664

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	42	37	0	68	233	143	60
normalized size	1	1.	0.64	0.56	0.	1.03	3.53	2.17	0.91
time (sec)	N/A	0.059	0.032	0.007	0.	0.229	4.233	0.223	6.986

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	73	75	0	1	367	88	83
normalized size	1	1.	0.8	0.82	0.	0.01	4.03	0.97	0.91
time (sec)	N/A	0.099	0.149	0.013	0.	0.255	16.64	0.236	13.618

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	39	36	0	78	138	58	48
normalized size	1	1.	0.72	0.67	0.	1.44	2.56	1.07	0.89
time (sec)	N/A	0.092	0.031	0.006	0.	0.236	3.665	0.212	11.552

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	58	54	0	1	303	69	54
normalized size	1	1.	0.91	0.84	0.	0.02	4.73	1.08	0.84
time (sec)	N/A	0.069	0.121	0.008	0.	0.248	10.011	0.222	9.339

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	45	63	92	32	31
normalized size	1	1.	0.78	0.69	1.25	1.75	2.56	0.89	0.86
time (sec)	N/A	0.065	0.024	0.008	1.322	0.232	3.42	0.215	7.921

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	46	50	44	23	15
normalized size	1	1.	1.	0.86	2.19	2.38	2.1	1.1	0.71
time (sec)	N/A	0.024	0.022	0.007	1.362	0.23	2.423	0.213	3.494

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	47	46	19	15
normalized size	1	1.	1.	0.83	1.06	2.61	2.56	1.06	0.83
time (sec)	N/A	0.011	0.006	0.005	1.324	0.234	3.251	0.219	2.156

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	26	42	63	95	36	32
normalized size	1	1.	0.74	0.67	1.08	1.62	2.44	0.92	0.82
time (sec)	N/A	0.021	0.02	0.005	1.348	0.238	2.726	0.218	2.03

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	63	57	0	1	740	68	51
normalized size	1	1.	1.07	0.97	0.	0.02	12.54	1.15	0.86
time (sec)	N/A	0.104	0.223	0.008	0.	0.253	9.838	0.213	10.571

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	42	39	0	80	165	86	56
normalized size	1	1.	0.7	0.65	0.	1.33	2.75	1.43	0.93
time (sec)	N/A	0.048	0.034	0.008	0.	0.24	4.825	0.214	5.408

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	80	78	0	1	864	100	85
normalized size	1	1.	0.87	0.85	0.	0.01	9.39	1.09	0.92
time (sec)	N/A	0.143	0.184	0.008	0.	0.255	18.055	0.231	14.1

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	53	48	0	97	354	163	80
normalized size	1	1.	0.62	0.56	0.	1.13	4.12	1.9	0.93
time (sec)	N/A	0.078	0.04	0.007	0.	0.249	7.003	0.228	8.95

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	95	111	0	1	3181	123	122
normalized size	1	1.	0.73	0.85	0.	0.01	24.28	0.94	0.93
time (sec)	N/A	0.172	0.2	0.048	0.	0.324	45.556	0.227	22.875

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	61	58	0	138	454	96	87
normalized size	1	1.	0.65	0.62	0.	1.47	4.83	1.02	0.93
time (sec)	N/A	0.145	0.041	0.008	0.	0.257	20.249	0.216	19.103

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	80	88	0	1	2980	105	92
normalized size	1	1.	0.75	0.83	0.	0.01	28.11	0.99	0.87
time (sec)	N/A	0.134	0.133	0.024	0.	0.271	31.445	0.224	17.692

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	50	47	99	123	364	74	68
normalized size	1	1.	0.67	0.63	1.32	1.64	4.85	0.99	0.91
time (sec)	N/A	0.122	0.041	0.008	1.352	0.254	19.916	0.214	15.712

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	139	80	95	23	15
normalized size	1	1.	1.	0.86	6.62	3.81	4.52	1.1	0.71
time (sec)	N/A	0.024	0.031	0.007	1.359	0.249	9.703	0.229	3.233

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	72	108	272	55	53
normalized size	1	1.	0.66	0.61	1.22	1.83	4.61	0.93	0.9
time (sec)	N/A	0.094	0.034	0.007	1.354	0.254	19.606	0.216	11.688

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	115	96	199	39	37
normalized size	1	1.	0.7	0.64	2.61	2.18	4.52	0.89	0.84
time (sec)	N/A	0.049	0.033	0.008	1.348	0.252	10.076	0.221	5.537

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	45	93	180	32	32
normalized size	1	1.	0.74	0.66	1.18	2.45	4.74	0.84	0.84
time (sec)	N/A	0.065	0.027	0.008	1.352	0.251	19.267	0.219	8.103



Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	42	39	95	111	517	58	61
normalized size	1	1.	0.62	0.57	1.4	1.63	7.6	0.85	0.9
time (sec)	N/A	0.075	0.034	0.007	1.35	0.256	10.918	0.219	8.834

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	77	90	19	15
normalized size	1	1.	1.	0.83	1.06	4.28	5.	1.06	0.83
time (sec)	N/A	0.011	0.008	0.005	1.345	0.253	18.901	0.211	2.163

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	51	48	82	123	1265	74	70
normalized size	1	1.	0.66	0.62	1.06	1.6	16.43	0.96	0.91
time (sec)	N/A	0.049	0.03	0.005	1.34	0.259	12.03	0.216	5.047

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	86	85	0	1	5250	109	85
normalized size	1	1.	0.91	0.89	0.	0.01	55.26	1.15	0.89
time (sec)	N/A	0.166	0.26	0.009	0.	0.275	30.807	0.213	17.449

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	64	61	0	139	400	122	97
normalized size	1	1.	0.64	0.61	0.	1.39	4.	1.22	0.97
time (sec)	N/A	0.081	0.048	0.008	0.	0.292	19.876	0.215	9.282

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	102	108	0	1	5540	142	124
normalized size	1	1.	0.77	0.82	0.	0.01	41.97	1.08	0.94
time (sec)	N/A	0.215	0.223	0.009	0.	0.291	50.597	0.215	22.178

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	75	72	0	157	668	198	126
normalized size	1	1.	0.57	0.55	0.	1.19	5.06	1.5	0.95
time (sec)	N/A	0.12	0.052	0.009	0.	0.337	28.719	0.221	14.29

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	24	54	139	44	46	37
normalized size	1	1.	0.59	0.52	1.17	3.02	0.96	1.	0.8
time (sec)	N/A	0.056	0.014	0.004	1.497	0.223	3.424	0.21	6.736

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	36	34	45	176	39	49	39
normalized size	1	1.	0.8	0.76	1.	3.91	0.87	1.09	0.87
time (sec)	N/A	0.039	0.03	0.007	1.482	0.233	1.977	0.212	5.333

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	19	35	99	27	31	24
normalized size	1	1.	0.71	0.61	1.13	3.19	0.87	1.	0.77
time (sec)	N/A	0.042	0.009	0.005	1.503	0.233	0.966	0.209	5.622

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	26	120	22	39	22
normalized size	1	1.	1.	0.74	0.96	4.44	0.81	1.44	0.81
time (sec)	N/A	0.023	0.014	0.006	1.504	0.233	0.533	0.214	3.631

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	51	10	15	10
normalized size	1	1.	1.	0.8	1.	3.4	0.67	1.	0.67
time (sec)	N/A	0.009	0.002	0.004	1.346	0.232	0.307	0.217	1.937

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	22	7	22	7
normalized size	1	1.	1.	0.7	0.8	2.2	0.7	2.2	0.7
time (sec)	N/A	0.007	0.006	0.004	1.501	0.23	0.322	0.213	1.07

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	25	15	12	47	8	39	15
normalized size	1	1.	1.25	0.75	0.6	2.35	0.4	1.95	0.75
time (sec)	N/A	0.033	0.009	0.005	1.485	0.231	3.56	0.211	4.532

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	27	15	31	14
normalized size	1	1.	1.	0.83	1.06	1.5	0.83	1.72	0.78
time (sec)	N/A	0.017	0.009	0.004	1.494	0.228	2.497	0.212	3.032

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	43	30	32	189	44	58	31
normalized size	1	1.	1.1	0.77	0.82	4.85	1.13	1.49	0.79
time (sec)	N/A	0.051	0.026	0.006	1.494	0.235	7.924	0.211	5.474

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	27	22	39	74	32	57	29
normalized size	1	1.	0.73	0.59	1.05	2.	0.86	1.54	0.78
time (sec)	N/A	0.032	0.012	0.004	1.507	0.223	5.757	0.216	4.637

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	44	51	269	63	74	46
normalized size	1	1.	0.84	0.77	0.89	4.72	1.11	1.3	0.81
time (sec)	N/A	0.07	0.04	0.007	1.5	0.231	16.894	0.215	6.56

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	34	54	99	46	58	37
normalized size	1	1.	0.59	0.74	1.17	2.15	1.	1.26	0.8
time (sec)	N/A	0.057	0.015	0.005	1.5	0.224	3.445	0.209	6.795

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	36	34	45	178	39	35	39
normalized size	1	1.	0.8	0.76	1.	3.96	0.87	0.78	0.87
time (sec)	N/A	0.041	0.031	0.01	1.539	0.23	2.	0.216	5.481

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	29	35	70	29	31	24
normalized size	1	1.	0.71	0.94	1.13	2.26	0.94	1.	0.77
time (sec)	N/A	0.044	0.009	0.006	1.488	0.225	0.98	0.208	5.634

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	26	119	22	26	22
normalized size	1	1.	1.	0.74	0.96	4.41	0.81	0.96	0.81
time (sec)	N/A	0.023	0.015	0.007	1.489	0.239	0.537	0.213	3.702

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	15	23	12	15	12
normalized size	1	1.	1.	1.47	1.	1.53	0.8	1.	0.8
time (sec)	N/A	0.009	0.003	0.004	1.351	0.237	0.324	0.207	2.003

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	26	7	8	7
normalized size	1	1.	1.	0.7	0.8	2.6	0.7	0.8	0.7
time (sec)	N/A	0.007	0.006	0.003	1.483	0.238	0.315	0.216	1.126

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	25	15	34	24	27	42	15
normalized size	1	1.	1.25	0.75	1.7	1.2	1.35	2.1	0.75
time (sec)	N/A	0.036	0.01	0.006	1.484	0.239	3.773	0.215	4.597

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	19	50	37	45	14
normalized size	1	1.	1.	1.39	1.06	2.78	2.06	2.5	0.78
time (sec)	N/A	0.017	0.011	0.005	1.479	0.234	2.612	0.219	2.996

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	43	30	54	131	100	61	32
normalized size	1	1.	1.1	0.77	1.38	3.36	2.56	1.56	0.82
time (sec)	N/A	0.053	0.027	0.007	1.491	0.236	8.132	0.235	5.48

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	27	32	39	101	82	99	31
normalized size	1	1.	0.73	0.86	1.05	2.73	2.22	2.68	0.84
time (sec)	N/A	0.034	0.013	0.005	1.485	0.232	5.831	0.231	4.511

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	44	73	186	138	77	48
normalized size	1	1.	0.84	0.77	1.28	3.26	2.42	1.35	0.84
time (sec)	N/A	0.072	0.042	0.007	1.506	0.235	17.181	0.216	6.622

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	34	54	139	44	46	37
normalized size	1	1.	0.59	0.74	1.17	3.02	0.96	1.	0.8
time (sec)	N/A	0.057	0.015	0.004	1.499	0.237	3.463	0.214	6.792

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	46	49	61	176	39	50	48
normalized size	1	1.	0.85	0.91	1.13	3.26	0.72	0.93	0.89
time (sec)	N/A	0.048	0.028	0.009	1.501	0.222	1.936	0.215	5.665

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	29	35	99	27	31	24
normalized size	1	1.	0.71	0.94	1.13	3.19	0.87	1.	0.77
time (sec)	N/A	0.043	0.008	0.004	1.49	0.216	0.961	0.209	5.605

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	35	42	120	22	41	31
normalized size	1	1.	1.03	0.97	1.17	3.33	0.61	1.14	0.86
time (sec)	N/A	0.029	0.015	0.008	1.481	0.218	0.543	0.213	3.794

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	15	51	10	15	10
normalized size	1	1.	1.	1.47	1.	3.4	0.67	1.	0.67
time (sec)	N/A	0.009	0.003	0.004	1.347	0.214	0.311	0.215	1.965

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	43	22	24	22	7	23	15
normalized size	1	1.	2.26	1.16	1.26	1.16	0.37	1.21	0.79
time (sec)	N/A	0.012	0.005	0.003	1.498	0.212	0.314	0.215	1.219

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	15	12	24	27	19	14
normalized size	1	1.	0.9	0.75	0.6	1.2	1.35	0.95	0.7
time (sec)	N/A	0.034	0.009	0.007	1.482	0.229	3.673	0.207	4.48

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	19	27	39	31	12
normalized size	1	1.	1.	1.39	1.06	1.5	2.17	1.72	0.67
time (sec)	N/A	0.016	0.009	0.005	1.484	0.219	2.597	0.22	3.041

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	30	32	134	100	39	31
normalized size	1	1.	0.95	0.77	0.82	3.44	2.56	1.	0.79
time (sec)	N/A	0.051	0.023	0.005	1.593	0.219	8.168	0.216	5.469

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	27	32	39	74	70	57	29
normalized size	1	1.	0.73	0.86	1.05	2.	1.89	1.54	0.78
time (sec)	N/A	0.032	0.012	0.005	1.506	0.223	5.812	0.221	4.505

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	44	51	196	138	55	46
normalized size	1	1.	0.81	0.77	0.89	3.44	2.42	0.96	0.81
time (sec)	N/A	0.069	0.031	0.008	1.492	0.223	16.847	0.218	6.531

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	27	24	54	31	49	50	44
normalized size	1	1.	0.59	0.52	1.17	0.67	1.07	1.09	0.96
time (sec)	N/A	0.059	0.014	0.004	1.486	0.216	3.548	0.22	6.768

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	43	45	90	53	38	53
normalized size	1	1.	0.8	0.8	0.83	1.67	0.98	0.7	0.98
time (sec)	N/A	0.048	0.038	0.009	1.504	0.226	3.005	0.226	5.582

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	19	35	24	31	34	27
normalized size	1	1.	0.71	0.61	1.13	0.77	1.	1.1	0.87
time (sec)	N/A	0.044	0.011	0.004	1.485	0.21	1.025	0.219	5.652

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	29	26	80	36	28	36
normalized size	1	1.	1.	0.81	0.72	2.22	1.	0.78	1.
time (sec)	N/A	0.03	0.019	0.009	1.486	0.222	1.503	0.228	3.877

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	15	14	16	14
normalized size	1	1.	1.	0.8	1.	1.	0.93	1.07	0.93
time (sec)	N/A	0.009	0.002	0.004	1.341	0.227	0.361	0.221	1.987

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	8	63	17	11	17
normalized size	1	1.	1.	0.84	0.42	3.32	0.89	0.58	0.89
time (sec)	N/A	0.012	0.006	0.003	1.499	0.255	1.296	0.223	1.22

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	27	15	34	58	8	20	15
normalized size	1	1.	1.35	0.75	1.7	2.9	0.4	1.	0.75
time (sec)	N/A	0.035	0.01	0.005	1.491	0.234	3.622	0.217	4.544

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	19	15	55	14
normalized size	1	1.	1.	0.83	1.06	1.06	0.83	3.06	0.78
time (sec)	N/A	0.017	0.01	0.004	1.49	0.224	2.579	0.229	3.066

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	30	54	88	46	42	34
normalized size	1	1.	0.95	0.77	1.38	2.26	1.18	1.08	0.87
time (sec)	N/A	0.053	0.03	0.007	1.493	0.241	8.034	0.221	5.419

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	27	22	39	28	36	120	32
normalized size	1	1.	0.73	0.59	1.05	0.76	0.97	3.24	0.86
time (sec)	N/A	0.033	0.015	0.005	1.491	0.226	5.87	0.229	4.536

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	44	44	73	97	65	61	51
normalized size	1	1.	0.77	0.77	1.28	1.7	1.14	1.07	0.89
time (sec)	N/A	0.071	0.038	0.007	1.504	0.256	17.012	0.219	6.445

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	21	0	1	14	30	14
normalized size	1	1.	1.	1.24	0.	0.06	0.82	1.76	0.82
time (sec)	N/A	0.011	0.012	0.004	0.	0.25	3.351	0.222	1.867

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	21	0	1	39	36	14
normalized size	1	1.	1.	1.24	0.	0.06	2.29	2.12	0.82
time (sec)	N/A	0.011	0.013	0.004	0.	0.244	3.52	0.218	1.978

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	0	1	39	31	22
normalized size	1	1.	1.	0.84	0.	0.04	1.56	1.24	0.88
time (sec)	N/A	0.018	0.012	0.004	0.	0.253	3.56	0.217	2.334

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	0	1	17	38	24
normalized size	1	1.	1.	0.81	0.	0.04	0.65	1.46	0.92
time (sec)	N/A	0.017	0.012	0.004	0.	0.252	3.369	0.217	2.078

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	21	0	1	17	30	17
normalized size	1	1.	1.	1.11	0.	0.05	0.89	1.58	0.89
time (sec)	N/A	0.019	0.014	0.007	0.	0.232	3.423	0.225	2.037



Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	21	0	1	46	38	17
normalized size	1	1.	1.	1.11	0.	0.05	2.42	2.	0.89
time (sec)	N/A	0.014	0.013	0.008	0.	0.225	3.605	0.223	2.235

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	0	1	46	34	22
normalized size	1	1.	1.	0.85	0.	0.04	1.7	1.26	0.81
time (sec)	N/A	0.019	0.012	0.005	0.	0.227	3.654	0.228	2.644

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	0	1	20	41	24
normalized size	1	1.	1.	0.82	0.	0.04	0.71	1.46	0.86
time (sec)	N/A	0.019	0.011	0.006	0.	0.252	3.479	0.214	2.346

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	0	1	17	31	22
normalized size	1	1.	1.	0.84	0.	0.04	0.68	1.24	0.88
time (sec)	N/A	0.018	0.01	0.	0.	0.23	3.587	0.217	2.42

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	0	1	46	38	22
normalized size	1	1.	1.	0.81	0.	0.04	1.77	1.46	0.85
time (sec)	N/A	0.018	0.01	0.004	0.	0.234	3.685	0.218	2.329

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	0	1	46	34	22
normalized size	1	1.	1.	0.85	0.	0.04	1.7	1.26	0.81
time (sec)	N/A	0.019	0.012	0.003	0.	0.228	3.788	0.22	2.625

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	0	1	20	41	24
normalized size	1	1.	1.	0.82	0.	0.04	0.71	1.46	0.86
time (sec)	N/A	0.019	0.011	0.004	0.	0.227	3.593	0.215	2.28

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	11	31	19	12	12
normalized size	1	1.	1.	0.94	0.69	1.94	1.19	0.75	0.75
time (sec)	N/A	0.01	0.006	0.005	1.504	0.222	3.519	0.223	2.227

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	155	152	0	0	0	0	170
normalized size	1	1.	0.84	0.83	0.	0.	0.	0.	0.92
time (sec)	N/A	0.326	0.26	0.089	0.	0.	0.	0.	30.263

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	191	221	0	0	0	0	277
normalized size	1	1.	0.63	0.73	0.	0.	0.	0.	0.92
time (sec)	N/A	0.601	0.342	0.032	0.	0.	0.	0.	57.923

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	142	138	0	0	46	0	139
normalized size	1	1.	0.93	0.9	0.	0.	0.3	0.	0.91
time (sec)	N/A	0.249	0.149	0.032	0.	0.	21.252	0.	22.866

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	174	205	0	0	46	0	246
normalized size	1	1.	0.65	0.76	0.	0.	0.17	0.	0.91
time (sec)	N/A	0.505	0.209	0.03	0.	0.	3.346	0.	47.458

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	103	119	0	0	46	0	114
normalized size	1	1.	0.82	0.94	0.	0.	0.37	0.	0.9
time (sec)	N/A	0.2	0.239	0.03	0.	0.	2.885	0.	17.357

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	174	194	0	0	49	0	241
normalized size	1	1.	0.66	0.74	0.	0.	0.19	0.	0.92
time (sec)	N/A	0.509	0.208	0.042	0.	0.	5.14	0.	48.083

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	106	120	0	0	49	0	114
normalized size	1	1.	0.84	0.95	0.	0.	0.39	0.	0.9
time (sec)	N/A	0.2	0.28	0.034	0.	0.	24.159	0.	18.049

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	196	219	0	0	0	0	277
normalized size	1	1.	0.65	0.72	0.	0.	0.	0.	0.91
time (sec)	N/A	0.6	0.289	0.041	0.	0.	0.	0.	59.843

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	166	163	0	0	0	0	196
normalized size	1	1.	0.78	0.77	0.	0.	0.	0.	0.92
time (sec)	N/A	0.367	0.204	0.034	0.	0.	0.	0.	37.125

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	202	232	0	0	0	0	303
normalized size	1	1.	0.61	0.71	0.	0.	0.	0.	0.92
time (sec)	N/A	0.67	0.31	0.033	0.	0.	0.	0.	68.993

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	153	150	0	0	46	0	165
normalized size	1	1.	0.85	0.83	0.	0.	0.25	0.	0.91
time (sec)	N/A	0.305	0.165	0.017	0.	0.	67.974	0.	28.582

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	188	218	0	0	46	0	272
normalized size	1	1.	0.63	0.73	0.	0.	0.15	0.	0.92
time (sec)	N/A	0.58	0.266	0.017	0.	0.	21.482	0.	55.575

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	141	134	0	0	46	0	138
normalized size	1	1.	0.93	0.88	0.	0.	0.3	0.	0.91
time (sec)	N/A	0.245	0.118	0.018	0.	0.	12.113	0.	22.305

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	190	208	0	0	49	0	274
normalized size	1	1.	0.64	0.7	0.	0.	0.17	0.	0.93
time (sec)	N/A	0.575	0.367	0.023	0.	0.	12.581	0.	56.639

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	130	125	0	0	49	0	139
normalized size	1	1.	0.86	0.82	0.	0.	0.32	0.	0.91
time (sec)	N/A	0.25	0.152	0.02	0.	0.	53.088	0.	23.301

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	193	216	0	0	0	0	274
normalized size	1	1.	0.65	0.73	0.	0.	0.	0.	0.92
time (sec)	N/A	0.58	0.327	0.023	0.	0.	0.	0.	57.743

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	121	135	0	0	0	0	139
normalized size	1	1.	0.8	0.89	0.	0.	0.	0.	0.91
time (sec)	N/A	0.257	0.242	0.04	0.	0.	0.	0.	23.691

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	213	234	0	0	0	0	304
normalized size	1	1.	0.64	0.71	0.	0.	0.	0.	0.92
time (sec)	N/A	0.686	0.424	0.046	0.	0.	0.	0.	70.921

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	112	237	0	0	0	0	199
normalized size	1	1.	0.88	1.85	0.	0.	0.	0.	1.55
time (sec)	N/A	0.194	0.188	0.059	0.	0.	0.	0.	54.686

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	86	133	0	0	53	0	124
normalized size	1	1.	0.74	1.14	0.	0.	0.45	0.	1.06
time (sec)	N/A	0.196	0.177	0.041	0.	0.	21.237	0.	15.358

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	106	229	0	0	53	0	173
normalized size	1	1.	1.07	2.31	0.	0.	0.54	0.	1.75
time (sec)	N/A	0.137	0.096	0.036	0.	0.	3.205	0.	50.228

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	80	124	0	0	53	0	97
normalized size	1	1.	0.85	1.32	0.	0.	0.56	0.	1.03
time (sec)	N/A	0.133	0.068	0.039	0.	0.	2.742	0.	12.021

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	83	225	0	0	56	0	168
normalized size	1	1.	0.85	2.3	0.	0.	0.57	0.	1.71
time (sec)	N/A	0.141	0.13	0.049	0.	0.	5.072	0.	50.749

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	78	129	0	0	49	0	100
normalized size	1	1.	0.81	1.34	0.	0.	0.51	0.	1.04
time (sec)	N/A	0.132	0.078	0.041	0.	0.	24.087	0.	12.103

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	144	141	0	0	0	0	144
normalized size	1	1.	0.92	0.9	0.	0.	0.	0.	0.92
time (sec)	N/A	0.26	0.171	0.034	0.	0.	0.	0.	24.06

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	177	210	0	0	44	0	252
normalized size	1	1.	0.65	0.77	0.	0.	0.16	0.	0.92
time (sec)	N/A	0.514	0.23	0.034	0.	0.	140.009	0.	48.862

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	106	125	0	0	44	0	114
normalized size	1	1.	0.83	0.98	0.	0.	0.35	0.	0.9
time (sec)	N/A	0.201	0.211	0.015	0.	0.	10.994	0.	17.58

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	111	132	0	0	44	0	216
normalized size	1	1.	0.47	0.56	0.	0.	0.19	0.	0.92
time (sec)	N/A	0.434	0.094	0.015	0.	0.	2.605	0.	39.297

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	90	104	0	0	44	0	88
normalized size	1	1.	0.93	1.07	0.	0.	0.45	0.	0.91
time (sec)	N/A	0.157	0.05	0.022	0.	0.	3.459	0.	12.766

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	176	196	0	0	48	0	243
normalized size	1	1.	0.66	0.73	0.	0.	0.18	0.	0.91
time (sec)	N/A	0.513	0.2	0.023	0.	0.	7.785	0.	49.893

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	109	123	0	0	48	0	116
normalized size	1	1.	0.84	0.95	0.	0.	0.37	0.	0.9
time (sec)	N/A	0.207	0.274	0.023	0.	0.	53.127	0.	18.071

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	198	219	0	0	0	0	282
normalized size	1	1.	0.65	0.72	0.	0.	0.	0.	0.92
time (sec)	N/A	0.595	0.264	0.025	0.	0.	0.	0.	60.603

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	131	128	0	0	0	0	139
normalized size	1	1.	0.86	0.84	0.	0.	0.	0.	0.91
time (sec)	N/A	0.252	0.127	0.048	0.	0.	0.	0.	24.549

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	168	197	0	0	44	0	243
normalized size	1	1.	0.63	0.74	0.	0.	0.17	0.	0.91
time (sec)	N/A	0.51	0.149	0.045	0.	0.	170.819	0.	49.237

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	115	115	0	0	44	0	110
normalized size	1	1.	0.92	0.92	0.	0.	0.35	0.	0.88
time (sec)	N/A	0.2	0.096	0.017	0.	0.	13.053	0.	18.345

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	166	197	0	0	44	0	236
normalized size	1	1.	0.62	0.74	0.	0.	0.17	0.	0.89
time (sec)	N/A	0.5	0.138	0.018	0.	0.	5.245	0.	47.983

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	117	114	0	0	44	0	110
normalized size	1	1.	0.93	0.9	0.	0.	0.35	0.	0.87
time (sec)	N/A	0.202	0.088	0.025	0.	0.	8.89	0.	17.827

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	180	197	0	0	48	0	272
normalized size	1	1.	0.61	0.67	0.	0.	0.16	0.	0.92
time (sec)	N/A	0.596	0.22	0.027	0.	0.	28.136	0.	59.157

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	130	124	0	0	0	0	139
normalized size	1	1.	0.84	0.81	0.	0.	0.	0.	0.9
time (sec)	N/A	0.258	0.158	0.027	0.	0.	0.	0.	23.693

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	197	219	0	0	0	0	306
normalized size	1	1.	0.6	0.66	0.	0.	0.	0.	0.92
time (sec)	N/A	0.68	0.236	0.027	0.	0.	0.	0.	71.013

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	117	219	0	0	0	0	141
normalized size	1	1.	0.75	1.41	0.	0.	0.	0.	0.91
time (sec)	N/A	0.254	0.472	0.051	0.	0.	0.	0.	25.218

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	195	385	0	0	44	0	269
normalized size	1	1.	0.64	1.27	0.	0.	0.14	0.	0.88
time (sec)	N/A	0.594	0.312	0.049	0.	0.	170.984	0.	59.273

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	137	218	0	0	44	0	136
normalized size	1	1.	0.88	1.4	0.	0.	0.28	0.	0.87
time (sec)	N/A	0.252	0.19	0.019	0.	0.	55.096	0.	24.471

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	194	382	0	0	44	0	267
normalized size	1	1.	0.64	1.26	0.	0.	0.15	0.	0.88
time (sec)	N/A	0.583	0.432	0.02	0.	0.	25.412	0.	57.175

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	115	216	0	0	44	0	139
normalized size	1	1.	0.73	1.38	0.	0.	0.28	0.	0.89
time (sec)	N/A	0.258	0.333	0.028	0.	0.	54.998	0.	23.579

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	208	384	0	0	0	0	304
normalized size	1	1.	0.62	1.15	0.	0.	0.	0.	0.91
time (sec)	N/A	0.671	0.422	0.03	0.	0.	0.	0.	69.199

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	127	227	0	0	0	0	167
normalized size	1	1.	0.69	1.23	0.	0.	0.	0.	0.9
time (sec)	N/A	0.312	0.459	0.028	0.	0.	0.	0.	30.162

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	222	410	0	0	0	0	333
normalized size	1	1.	0.61	1.13	0.	0.	0.	0.	0.92
time (sec)	N/A	0.769	0.406	0.03	0.	0.	0.	0.	82.228



Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	112	235	0	0	51	0	170
normalized size	1	1.	1.05	2.2	0.	0.	0.48	0.	1.59
time (sec)	N/A	0.149	0.135	0.041	0.	0.	139.309	0.	48.472

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	89	131	0	0	51	0	95
normalized size	1	1.	1.01	1.49	0.	0.	0.58	0.	1.08
time (sec)	N/A	0.144	0.096	0.038	0.	0.	10.951	0.	11.406

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	77	165	0	0	51	0	138
normalized size	1	1.	1.15	2.46	0.	0.	0.76	0.	2.06
time (sec)	N/A	0.102	0.097	0.02	0.	0.	2.548	0.	44.394

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	64	117	0	0	51	0	68
normalized size	1	1.	1.02	1.86	0.	0.	0.81	0.	1.08
time (sec)	N/A	0.092	0.095	0.027	0.	0.	3.419	0.	8.285

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	110	228	0	0	54	0	172
normalized size	1	1.	1.03	2.13	0.	0.	0.5	0.	1.61
time (sec)	N/A	0.143	0.109	0.029	0.	0.	7.69	0.	48.614

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	84	132	0	0	54	0	99
normalized size	1	1.	0.86	1.35	0.	0.	0.55	0.	1.01
time (sec)	N/A	0.135	0.099	0.025	0.	0.	53.206	0.	11.712

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	108	229	0	0	51	0	173
normalized size	1	1.	0.98	2.08	0.	0.	0.46	0.	1.57
time (sec)	N/A	0.146	0.105	0.054	0.	0.	170.871	0.	52.096

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	91	126	0	0	51	0	97
normalized size	1	1.	0.97	1.34	0.	0.	0.54	0.	1.03
time (sec)	N/A	0.14	0.119	0.046	0.	0.	12.828	0.	12.782

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	107	227	0	0	51	0	170
normalized size	1	1.	1.06	2.25	0.	0.	0.5	0.	1.68
time (sec)	N/A	0.141	0.08	0.023	0.	0.	5.129	0.	51.445

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	92	122	0	0	51	0	97
normalized size	1	1.	0.96	1.27	0.	0.	0.53	0.	1.01
time (sec)	N/A	0.134	0.102	0.034	0.	0.	8.698	0.	12.531

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	112	228	0	0	54	0	199
normalized size	1	1.	0.8	1.63	0.	0.	0.39	0.	1.42
time (sec)	N/A	0.189	0.104	0.031	0.	0.	28.23	0.	56.322

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	95	133	0	0	0	0	129
normalized size	1	1.	0.72	1.01	0.	0.	0.	0.	0.98
time (sec)	N/A	0.179	0.145	0.029	0.	0.	0.	0.	16.75

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	65	66	0	0	36	0	20
normalized size	1	1.	3.1	3.14	0.	0.	1.71	0.	0.95
time (sec)	N/A	0.033	0.126	0.073	0.	0.	2.302	0.	5.4

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	68	73	0	0	32	0	60
normalized size	1	1.	1.01	1.09	0.	0.	0.48	0.	0.9
time (sec)	N/A	0.092	0.099	0.061	0.	0.	2.136	0.	5.409

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	64	109	0	0	0	54	0	54
normalized size	1	1.28	2.18	0.	0.	0.	1.08	0.	1.08
time (sec)	N/A	0.07	0.118	0.028	0.	0.	22.491	0.	7.949

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	63	63	0	0	0	54	0	53
normalized size	1	1.26	1.26	0.	0.	0.	1.08	0.	1.06
time (sec)	N/A	0.064	0.028	0.026	0.	0.	3.688	0.	7.858

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	63	63	0	0	0	53	0	53
normalized size	1	1.26	1.26	0.	0.	0.	1.06	0.	1.06
time (sec)	N/A	0.064	0.045	0.028	0.	0.	2.932	0.	8.338

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	66	66	0	0	0	53	0	54
normalized size	1	1.38	1.38	0.	0.	0.	1.1	0.	1.12
time (sec)	N/A	0.07	0.05	0.028	0.	0.	5.656	0.	8.35

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	66	66	0	0	0	53	0	54
normalized size	1	1.32	1.32	0.	0.	0.	1.06	0.	1.08
time (sec)	N/A	0.068	0.049	0.027	0.	0.	26.206	0.	8.406

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	63	96	0	0	0	54	0	53
normalized size	1	1.26	1.92	0.	0.	0.	1.08	0.	1.06
time (sec)	N/A	0.069	0.073	0.03	0.	0.	18.09	0.	8.585

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	63	63	0	0	0	48	0	49
normalized size	1	1.26	1.26	0.	0.	0.	0.96	0.	0.98
time (sec)	N/A	0.069	0.049	0.029	0.	0.	8.285	0.	8.466

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	63	63	0	0	0	53	0	53
normalized size	1	1.26	1.26	0.	0.	0.	1.06	0.	1.06
time (sec)	N/A	0.064	0.015	0.	0.	0.	2.925	0.	8.337

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	57	105	0	0	0	41	0	44
normalized size	1	1.24	2.28	0.	0.	0.	0.89	0.	0.96
time (sec)	N/A	0.066	0.104	0.032	0.	0.	21.627	0.	8.13

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	66	110	0	0	0	53	0	54
normalized size	1	1.29	2.16	0.	0.	0.	1.04	0.	1.06
time (sec)	N/A	0.072	0.131	0.03	0.	0.	157.304	0.	8.682

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	97	16	22	22	202	0	14
normalized size	1	1.	5.71	0.94	1.29	1.29	11.88	0.	0.82
time (sec)	N/A	0.021	0.153	0.015	1.498	0.237	68.002	0.	8.338

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	C	C	F	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	127	97	0	22	24	105	0	100
normalized size	1	7.47	5.71	0.	1.29	1.41	6.18	0.	5.88
time (sec)	N/A	0.189	0.047	0.041	1.493	0.27	49.126	0.	19.372

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	131	14	18	22	0	0	12
normalized size	1	1.	8.73	0.93	1.2	1.47	0.	0.	0.8
time (sec)	N/A	0.02	0.182	0.01	1.531	0.227	0.	0.	7.76

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	C	C	F	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	123	131	0	18	35	94	0	97
normalized size	1	8.2	8.73	0.	1.2	2.33	6.27	0.	6.47
time (sec)	N/A	0.18	0.055	0.038	1.497	0.227	53.004	0.	17.851

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	61	47	86	77	1795	77	75
normalized size	1	1.	0.76	0.59	1.08	0.96	22.44	0.96	0.94
time (sec)	N/A	0.125	0.028	0.008	1.35	0.205	9.76	0.218	15.354

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	50	36	63	62	700	58	54
normalized size	1	1.	0.85	0.61	1.07	1.05	11.86	0.98	0.92
time (sec)	N/A	0.096	0.024	0.008	1.357	0.203	6.421	0.217	11.543

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	25	41	46	223	39	34
normalized size	1	1.	1.	0.66	1.08	1.21	5.87	1.03	0.89
time (sec)	N/A	0.068	0.019	0.007	1.396	0.203	4.051	0.219	7.815

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	19	42	19	14
normalized size	1	1.	1.	0.83	1.06	1.06	2.33	1.06	0.78
time (sec)	N/A	0.012	0.008	0.004	1.338	0.203	0.498	0.221	2.157

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	61	0	0	131	46	132	94
normalized size	1	1.	0.6	0.	0.	1.3	0.46	1.31	0.93
time (sec)	N/A	0.189	0.046	0.046	0.	0.214	3.847	0.574	10.447

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	67	0	0	193	42	143	99
normalized size	1	1.	0.63	0.	0.	1.8	0.39	1.34	0.93
time (sec)	N/A	0.173	0.045	0.046	0.	0.217	4.401	0.581	10.819

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	82	0	0	232	42	167	122
normalized size	1	1.	0.61	0.	0.	1.72	0.31	1.24	0.9
time (sec)	N/A	0.231	0.053	0.045	0.	0.221	5.387	0.601	15.649

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	90	0	0	0	29	0	264
normalized size	1	1.	0.29	0.	0.	0.	0.09	0.	0.84
time (sec)	N/A	0.631	0.061	0.04	0.	0.	2.748	0.	21.027

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	78	0	0	0	29	0	240
normalized size	1	1.	0.27	0.	0.	0.	0.1	0.	0.83
time (sec)	N/A	0.415	0.055	0.036	0.	0.	2.379	0.	15.775

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	63	0	0	0	26	0	216
normalized size	1	1.	0.24	0.	0.	0.	0.1	0.	0.81
time (sec)	N/A	0.302	0.034	0.043	0.	0.	2.202	0.	8.118

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	68	0	0	0	29	0	211
normalized size	1	1.	0.26	0.	0.	0.	0.11	0.	0.81
time (sec)	N/A	0.312	0.039	0.038	0.	0.	2.398	0.	9.825

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	88	0	0	0	34	0	235
normalized size	1	1.	0.3	0.	0.	0.	0.12	0.	0.81
time (sec)	N/A	0.402	0.045	0.041	0.	0.	2.845	0.	15.052

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	61	47	86	77	1795	77	75
normalized size	1	1.	0.76	0.59	1.08	0.96	22.44	0.96	0.94
time (sec)	N/A	0.124	0.03	0.008	1.347	0.211	11.024	0.218	15.48

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	50	36	63	62	700	58	54
normalized size	1	1.	0.85	0.61	1.07	1.05	11.86	0.98	0.92
time (sec)	N/A	0.096	0.024	0.007	1.339	0.21	7.029	0.214	11.608

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	39	25	41	47	66	39	34
normalized size	1	1.	1.03	0.66	1.08	1.24	1.74	1.03	0.89
time (sec)	N/A	0.067	0.021	0.007	1.34	0.209	2.224	0.214	7.819

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	19	42	19	14
normalized size	1	1.	1.	0.83	1.06	1.06	2.33	1.06	0.78
time (sec)	N/A	0.011	0.008	0.005	1.33	0.208	1.066	0.214	2.147

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	61	0	0	162	46	132	94
normalized size	1	1.	0.6	0.	0.	1.6	0.46	1.31	0.93
time (sec)	N/A	0.176	0.047	0.033	0.	0.221	3.919	0.596	10.265

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	67	0	0	173	42	144	99
normalized size	1	1.	0.64	0.	0.	1.66	0.4	1.38	0.95
time (sec)	N/A	0.164	0.042	0.045	0.	0.22	4.567	0.585	10.686

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	83	0	0	219	42	170	122
normalized size	1	1.	0.61	0.	0.	1.62	0.31	1.26	0.9
time (sec)	N/A	0.219	0.046	0.043	0.	0.22	5.768	0.612	15.482

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	601	601	90	0	0	0	29	0	502
normalized size	1	1.	0.15	0.	0.	0.	0.05	0.	0.84
time (sec)	N/A	1.031	0.058	0.039	0.	0.	3.29	0.	46.865

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	577	577	79	0	0	0	29	0	476
normalized size	1	1.	0.14	0.	0.	0.	0.05	0.	0.82
time (sec)	N/A	0.884	0.053	0.035	0.	0.	2.691	0.	37.826

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	550	63	0	0	0	26	0	450
normalized size	1	1.	0.11	0.	0.	0.	0.05	0.	0.82
time (sec)	N/A	0.732	0.041	0.042	0.	0.	2.274	0.	26.8

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	538	538	68	0	0	0	29	0	439
normalized size	1	1.	0.13	0.	0.	0.	0.05	0.	0.82
time (sec)	N/A	0.725	0.042	0.039	0.	0.	2.582	0.	28.162

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	575	575	88	0	0	0	34	0	471
normalized size	1	1.	0.15	0.	0.	0.	0.06	0.	0.82
time (sec)	N/A	0.876	0.049	0.039	0.	0.	3.087	0.	36.867

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	86	92	136	181	75
normalized size	1	1.	0.62	0.59	1.08	1.15	1.7	2.26	0.94
time (sec)	N/A	0.132	0.045	0.009	1.354	0.206	20.865	0.221	15.582

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	63	77	112	143	54
normalized size	1	1.	0.66	0.61	1.07	1.31	1.9	2.42	0.92
time (sec)	N/A	0.102	0.034	0.007	1.346	0.208	12.517	0.215	11.693

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	41	61	88	105	34
normalized size	1	1.	0.74	0.66	1.08	1.61	2.32	2.76	0.89
time (sec)	N/A	0.07	0.035	0.008	1.357	0.207	7.493	0.216	7.846

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	43	65	19	14
normalized size	1	1.	1.	0.83	1.06	2.39	3.61	1.06	0.78
time (sec)	N/A	0.012	0.01	0.003	1.34	0.208	3.887	0.213	2.152



Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	76	0	0	143	49	149	109
normalized size	1	1.	0.65	0.	0.	1.22	0.42	1.27	0.93
time (sec)	N/A	0.225	0.051	0.033	0.	0.216	5.392	0.597	13.478

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	73	0	0	185	46	161	112
normalized size	1	1.	0.63	0.	0.	1.59	0.4	1.39	0.97
time (sec)	N/A	0.206	0.065	0.052	0.	0.218	6.154	0.586	13.585

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	80	0	0	216	42	167	121
normalized size	1	1.	0.61	0.	0.	1.64	0.32	1.27	0.92
time (sec)	N/A	0.223	0.05	0.045	0.	0.217	6.965	0.593	15.045

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	100	0	0	0	29	0	284
normalized size	1	1.	0.3	0.	0.	0.	0.09	0.	0.85
time (sec)	N/A	0.595	0.07	0.037	0.	0.	5.998	0.	26.884

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	90	0	0	0	29	0	260
normalized size	1	1.	0.29	0.	0.	0.	0.09	0.	0.84
time (sec)	N/A	0.493	0.071	0.036	0.	0.	4.448	0.	21.011

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	76	0	0	0	26	0	235
normalized size	1	1.	0.27	0.	0.	0.	0.09	0.	0.82
time (sec)	N/A	0.38	0.043	0.043	0.	0.	3.596	0.	10.951

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	78	0	0	0	29	0	228
normalized size	1	1.	0.28	0.	0.	0.	0.1	0.	0.81
time (sec)	N/A	0.372	0.086	0.038	0.	0.	4.188	0.	12.635

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	80	0	0	0	34	0	233
normalized size	1	1.	0.28	0.	0.	0.	0.12	0.	0.82
time (sec)	N/A	0.393	0.045	0.041	0.	0.	4.297	0.	14.243

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	16	12	32	56	12	10
normalized size	1	1.	1.	1.23	0.92	2.46	4.31	0.92	0.77
time (sec)	N/A	0.008	0.008	0.003	1.337	0.206	10.478	0.211	1.634

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	86	62	1690	77	75
normalized size	1	1.	0.62	0.59	1.08	0.78	21.12	0.96	0.94
time (sec)	N/A	0.126	0.032	0.008	1.345	0.211	9.203	0.214	15.485

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	63	47	631	58	54
normalized size	1	1.	0.66	0.61	1.07	0.8	10.69	0.98	0.92
time (sec)	N/A	0.098	0.026	0.006	1.345	0.213	5.84	0.215	11.546

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	25	41	32	178	39	34
normalized size	1	1.	0.74	0.66	1.08	0.84	4.68	1.03	0.89
time (sec)	N/A	0.069	0.022	0.007	1.342	0.214	3.829	0.213	7.899

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	19	24	19	14
normalized size	1	1.	1.	0.83	1.06	1.06	1.33	1.06	0.78
time (sec)	N/A	0.012	0.005	0.004	1.345	0.212	1.545	0.21	2.143

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	48	0	0	117	41	117	80
normalized size	1	1.	0.56	0.	0.	1.36	0.48	1.36	0.93
time (sec)	N/A	0.144	0.038	0.027	0.	0.224	3.642	0.605	7.833

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	69	0	0	197	41	149	100
normalized size	1	1.	0.63	0.	0.	1.79	0.37	1.35	0.91
time (sec)	N/A	0.173	0.048	0.038	0.	0.224	4.374	0.591	10.988

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	82	0	0	196	41	171	126
normalized size	1	1.	0.59	0.	0.	1.42	0.3	1.24	0.91
time (sec)	N/A	0.228	0.054	0.042	0.	0.223	5.558	0.599	15.961

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	580	580	79	0	0	0	27	0	481
normalized size	1	1.	0.14	0.	0.	0.	0.05	0.	0.83
time (sec)	N/A	0.875	0.059	0.036	0.	0.	2.384	0.	38.019

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	556	556	62	0	0	0	27	0	456
normalized size	1	1.	0.11	0.	0.	0.	0.05	0.	0.82
time (sec)	N/A	0.755	0.048	0.036	0.	0.	2.22	0.	29.962

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	529	47	0	0	0	24	0	427
normalized size	1	1.	0.09	0.	0.	0.	0.05	0.	0.81
time (sec)	N/A	0.619	0.023	0.037	0.	0.	2.147	0.	21.423

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	546	546	69	0	0	0	27	0	439
normalized size	1	1.	0.13	0.	0.	0.	0.05	0.	0.8
time (sec)	N/A	0.733	0.046	0.036	0.	0.	2.389	0.	29.221

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	578	578	83	0	0	0	32	0	476
normalized size	1	1.	0.14	0.	0.	0.	0.06	0.	0.82
time (sec)	N/A	0.895	0.051	0.038	0.	0.	2.795	0.	37.096

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	86	62	1690	77	75
normalized size	1	1.	0.62	0.59	1.08	0.78	21.12	0.96	0.94
time (sec)	N/A	0.128	0.03	0.009	1.342	0.214	9.442	0.217	15.384

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	36	63	47	631	58	54
normalized size	1	1.	0.66	0.61	1.07	0.8	10.69	0.98	0.92
time (sec)	N/A	0.101	0.024	0.008	1.331	0.216	6.046	0.213	11.454

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	25	41	31	178	36	34
normalized size	1	1.	0.71	0.66	1.08	0.82	4.68	0.95	0.89
time (sec)	N/A	0.069	0.02	0.007	1.322	0.216	3.918	0.215	7.802

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	19	24	19	14
normalized size	1	1.	1.	0.83	1.06	1.06	1.33	1.06	0.78
time (sec)	N/A	0.012	0.005	0.005	1.325	0.214	1.658	0.217	2.165

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	48	0	0	132	41	117	80
normalized size	1	1.	0.56	0.	0.	1.53	0.48	1.36	0.93
time (sec)	N/A	0.148	0.035	0.029	0.	0.225	3.842	0.567	8.066

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	69	0	0	212	41	147	100
normalized size	1	1.	0.64	0.	0.	1.98	0.38	1.37	0.93
time (sec)	N/A	0.171	0.051	0.04	0.	0.23	4.905	0.562	11.225

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	83	0	0	220	41	171	133
normalized size	1	1.	0.6	0.	0.	1.59	0.3	1.24	0.96
time (sec)	N/A	0.226	0.052	0.043	0.	0.227	6.208	0.567	16.224

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	79	0	0	0	27	0	243
normalized size	1	1.	0.27	0.	0.	0.	0.09	0.	0.83
time (sec)	N/A	0.42	0.057	0.036	0.	0.	2.538	0.	15.546

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	62	0	0	0	27	0	219
normalized size	1	1.	0.23	0.	0.	0.	0.1	0.	0.81
time (sec)	N/A	0.339	0.052	0.035	0.	0.	2.307	0.	10.695

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	47	0	0	0	24	0	197
normalized size	1	1.	0.19	0.	0.	0.	0.1	0.	0.8
time (sec)	N/A	0.236	0.027	0.038	0.	0.	2.188	0.	5.754

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	70	0	0	0	27	0	211
normalized size	1	1.	0.26	0.	0.	0.	0.1	0.	0.8
time (sec)	N/A	0.336	0.052	0.038	0.	0.	2.648	0.	10.401

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	83	0	0	0	32	0	240
normalized size	1	1.	0.28	0.	0.	0.	0.11	0.	0.82
time (sec)	N/A	0.405	0.048	0.039	0.	0.	3.437	0.	14.964

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	86	62	1584	77	75
normalized size	1	1.	0.62	0.59	1.08	0.78	19.8	0.96	0.94
time (sec)	N/A	0.128	0.035	0.007	1.343	0.212	9.919	0.213	15.235

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	38	36	63	46	561	55	54
normalized size	1	1.	0.64	0.61	1.07	0.78	9.51	0.93	0.92
time (sec)	N/A	0.1	0.029	0.008	1.337	0.21	6.193	0.215	11.462

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	41	31	46	36	34
normalized size	1	1.	0.71	0.63	1.08	0.82	1.21	0.95	0.89
time (sec)	N/A	0.071	0.024	0.007	1.476	0.215	2.463	0.213	7.83

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	19	26	19	15
normalized size	1	1.	1.	0.83	1.06	1.06	1.44	1.06	0.83
time (sec)	N/A	0.012	0.005	0.004	1.357	0.213	2.332	0.212	2.139

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	55	0	0	173	41	136	95
normalized size	1	1.	0.53	0.	0.	1.66	0.39	1.31	0.91
time (sec)	N/A	0.191	0.048	0.04	0.	0.226	4.186	0.575	11.028

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	70	0	0	246	41	171	121
normalized size	1	1.	0.56	0.	0.	1.97	0.33	1.37	0.97
time (sec)	N/A	0.223	0.052	0.069	0.	0.23	5.638	0.576	14.872

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	83	0	0	247	41	190	155
normalized size	1	1.	0.52	0.	0.	1.55	0.26	1.19	0.97
time (sec)	N/A	0.284	0.059	0.077	0.	0.229	7.342	0.652	20.879

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	577	577	65	0	0	0	27	0	478
normalized size	1	1.	0.11	0.	0.	0.	0.05	0.	0.83
time (sec)	N/A	0.873	0.062	0.064	0.	0.	2.582	0.	38.324

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	55	0	0	0	27	0	454
normalized size	1	1.	0.1	0.	0.	0.	0.05	0.	0.82
time (sec)	N/A	0.748	0.049	0.034	0.	0.	2.494	0.	29.838

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	58	0	0	0	24	0	449
normalized size	1	1.	0.11	0.	0.	0.	0.04	0.	0.81
time (sec)	N/A	0.757	0.038	0.042	0.	0.	2.44	0.	27.719

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	571	571	70	0	0	0	27	0	469
normalized size	1	1.	0.12	0.	0.	0.	0.05	0.	0.82
time (sec)	N/A	0.869	0.052	0.062	0.	0.	3.349	0.	36.922

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	599	599	83	0	0	0	32	0	498
normalized size	1	1.	0.14	0.	0.	0.	0.05	0.	0.83
time (sec)	N/A	1.022	0.056	0.066	0.	0.	4.124	0.	45.697

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	98	0	0	0	0	0	260
normalized size	1	1.	0.36	0.	0.	0.	0.	0.	0.95
time (sec)	N/A	0.872	0.069	0.044	0.	0.	0.	0.	75.24

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	83	0	0	0	0	0	224
normalized size	1	1.	0.34	0.	0.	0.	0.	0.	0.92
time (sec)	N/A	0.679	0.058	0.035	0.	0.	0.	0.	65.573

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	68	0	0	0	46	0	196
normalized size	1	1.	0.32	0.	0.	0.	0.22	0.	0.93
time (sec)	N/A	0.59	0.05	0.033	0.	0.	6.718	0.	56.962

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	72	0	0	0	49	0	192
normalized size	1	1.	0.35	0.	0.	0.	0.24	0.	0.92
time (sec)	N/A	0.596	0.051	0.036	0.	0.	26.413	0.	57.659

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	47	34	0	0	24
normalized size	1	1.	0.93	0.75	1.68	1.21	0.	0.	0.86
time (sec)	N/A	0.028	0.032	0.006	1.36	0.234	0.	0.	3.557

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	51	31	51	62	0	0	48
normalized size	1	1.	0.89	0.54	0.89	1.09	0.	0.	0.84
time (sec)	N/A	0.058	0.042	0.006	1.396	0.229	0.	0.	6.721

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	63	42	74	77	0	0	73
normalized size	1	1.	0.74	0.49	0.87	0.91	0.	0.	0.86
time (sec)	N/A	0.09	0.049	0.009	1.402	0.231	0.	0.	10.677

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	74	53	86	92	0	0	99
normalized size	1	1.	0.65	0.47	0.76	0.81	0.	0.	0.88
time (sec)	N/A	0.126	0.055	0.008	1.369	0.233	0.	0.	15.415

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	451	98	0	0	0	0	0	437
normalized size	1	1.	0.22	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	1.885	0.064	0.04	0.	0.	0.	0.	45.397

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	85	0	0	0	46	0	403
normalized size	1	1.	0.2	0.	0.	0.	0.11	0.	0.96
time (sec)	N/A	1.538	0.057	0.034	0.	0.	149.214	0.	35.618

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	63	0	0	0	46	0	372
normalized size	1	1.	0.17	0.	0.	0.	0.12	0.	0.98
time (sec)	N/A	1.384	0.043	0.031	0.	0.	5.416	0.	27.732



Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	69	0	0	0	0	0	376
normalized size	1	1.	0.18	0.	0.	0.	0.	0.	0.96
time (sec)	N/A	1.436	0.054	0.037	0.	0.	0.	0.	28.327

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	93	0	0	0	0	0	410
normalized size	1	1.	0.22	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	1.552	0.078	0.041	0.	0.	0.	0.	37.13

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	69	0	0	0	46	0	49
normalized size	1	1.	1.19	0.	0.	0.	0.79	0.	0.84
time (sec)	N/A	0.066	0.052	0.039	0.	0.	11.713	0.	7.495

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	65	0	0	0	46	0	49
normalized size	1	1.	1.12	0.	0.	0.	0.79	0.	0.84
time (sec)	N/A	0.065	0.044	0.035	0.	0.	3.654	0.	7.428

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	72	0	0	0	49	0	51
normalized size	1	1.	1.29	0.	0.	0.	0.88	0.	0.91
time (sec)	N/A	0.064	0.051	0.034	0.	0.	12.922	0.	7.471

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	109	0	0	0	0	0	284
normalized size	1	1.	0.36	0.	0.	0.	0.	0.	0.94
time (sec)	N/A	0.89	0.081	0.034	0.	0.	0.	0.	84.402

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	96	0	0	0	0	0	252
normalized size	1	1.	0.35	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	0.745	0.069	0.033	0.	0.	0.	0.	73.268

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	83	0	0	0	46	0	223
normalized size	1	1.	0.34	0.	0.	0.	0.19	0.	0.92
time (sec)	N/A	0.655	0.066	0.032	0.	0.	150.617	0.	63.639

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	83	0	0	0	49	0	224
normalized size	1	1.	0.36	0.	0.	0.	0.21	0.	0.96
time (sec)	N/A	0.658	0.059	0.039	0.	0.	171.363	0.	63.887

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	83	0	0	0	0	0	218
normalized size	1	1.	0.35	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	0.667	0.068	0.045	0.	0.	0.	0.	64.649

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	31	21	27	58	0	0	24
normalized size	1	1.	1.11	0.75	0.96	2.07	0.	0.	0.86
time (sec)	N/A	0.028	0.044	0.007	1.391	0.223	0.	0.	3.612

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	41	31	51	77	0	0	48
normalized size	1	1.	0.72	0.54	0.89	1.35	0.	0.	0.84
time (sec)	N/A	0.058	0.059	0.006	1.413	0.223	0.	0.	6.755

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	52	42	74	92	0	0	73
normalized size	1	1.	0.61	0.49	0.87	1.08	0.	0.	0.86
time (sec)	N/A	0.089	0.064	0.007	1.392	0.225	0.	0.	10.747

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	109	0	0	0	0	0	462
normalized size	1	1.	0.23	0.	0.	0.	0.	0.	0.96
time (sec)	N/A	1.834	0.078	0.035	0.	0.	0.	0.	54.574

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	96	0	0	0	0	0	430
normalized size	1	1.	0.21	0.	0.	0.	0.	0.	0.96
time (sec)	N/A	1.654	0.063	0.031	0.	0.	0.	0.	43.976

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	83	0	0	0	46	0	401
normalized size	1	1.	0.2	0.	0.	0.	0.11	0.	0.97
time (sec)	N/A	1.493	0.054	0.033	0.	0.	107.832	0.	34.705

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	84	0	0	0	0	0	405
normalized size	1	1.	0.2	0.	0.	0.	0.	0.	0.98
time (sec)	N/A	1.513	0.059	0.039	0.	0.	0.	0.	35.465

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	90	0	0	0	0	0	405
normalized size	1	1.	0.21	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	1.524	0.091	0.043	0.	0.	0.	0.	36.074

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	450	104	0	0	0	0	0	437
normalized size	1	1.	0.23	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	1.676	0.091	0.05	0.	0.	0.	0.	46.797

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	83	0	0	0	0	0	51
normalized size	1	1.	1.41	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.068	0.059	0.033	0.	0.	0.	0.	7.542

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	81	0	0	0	46	0	51
normalized size	1	1.	1.37	0.	0.	0.	0.78	0.	0.86
time (sec)	N/A	0.068	0.059	0.032	0.	0.	68.454	0.	7.564

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	86	0	0	0	49	0	53
normalized size	1	1.	1.51	0.	0.	0.	0.86	0.	0.93
time (sec)	N/A	0.068	0.081	0.039	0.	0.	107.735	0.	7.634

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	98	0	0	0	0	0	267
normalized size	1	1.	0.35	0.	0.	0.	0.	0.	0.96
time (sec)	N/A	0.834	0.073	0.039	0.	0.	0.	0.	77.809

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	87	0	0	0	0	0	236
normalized size	1	1.	0.35	0.	0.	0.	0.	0.	0.96
time (sec)	N/A	0.677	0.067	0.036	0.	0.	0.	0.	67.472

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	69	0	0	0	0	0	197
normalized size	1	1.	0.33	0.	0.	0.	0.	0.	0.94
time (sec)	N/A	0.589	0.057	0.033	0.	0.	0.	0.	58.195

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	57	0	0	0	44	0	170
normalized size	1	1.	0.31	0.	0.	0.	0.24	0.	0.93
time (sec)	N/A	0.53	0.032	0.036	0.	0.	6.908	0.	51.228

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	47	34	36	0	24
normalized size	1	1.	0.93	0.75	1.68	1.21	1.29	0.	0.86
time (sec)	N/A	0.029	0.019	0.006	1.366	0.229	54.73	0.	3.547

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	34	29	51	47	0	0	48
normalized size	1	1.	0.6	0.51	0.89	0.82	0.	0.	0.84
time (sec)	N/A	0.058	0.037	0.007	1.393	0.229	0.	0.	6.654

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	52	42	74	62	0	0	73
normalized size	1	1.	0.61	0.49	0.87	0.73	0.	0.	0.86
time (sec)	N/A	0.089	0.049	0.007	1.403	0.229	0.	0.	10.707

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	63	53	86	77	0	0	99
normalized size	1	1.	0.56	0.47	0.76	0.68	0.	0.	0.88
time (sec)	N/A	0.123	0.056	0.01	1.363	0.234	0.	0.	15.391

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	87	0	0	0	0	0	411
normalized size	1	1.	0.21	0.	0.	0.	0.	0.	0.98
time (sec)	N/A	1.557	0.062	0.035	0.	0.	0.	0.	36.779

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	66	0	0	0	44	0	376
normalized size	1	1.	0.17	0.	0.	0.	0.11	0.	0.97
time (sec)	N/A	1.416	0.052	0.028	0.	0.	69.584	0.	28.081

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	55	0	0	0	31	0	352
normalized size	1	1.	0.15	0.	0.	0.	0.09	0.	0.97
time (sec)	N/A	1.279	0.032	0.039	0.	0.	8.976	0.	21.425

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	72	0	0	0	0	0	382
normalized size	1	1.	0.18	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	1.406	0.058	0.039	0.	0.	0.	0.	28.471

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	93	0	0	0	0	0	413
normalized size	1	1.	0.22	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	1.545	0.084	0.043	0.	0.	0.	0.	37.254

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	0	0	0	44	0	49
normalized size	1	1.	0.98	0.	0.	0.	0.76	0.	0.84
time (sec)	N/A	0.066	0.035	0.036	0.	0.	7.765	0.	7.765

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	0	0	0	46	0	49
normalized size	1	1.	0.98	0.	0.	0.	0.79	0.	0.84
time (sec)	N/A	0.067	0.032	0.042	0.	0.	5.691	0.	7.835

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	74	0	0	0	48	0	51
normalized size	1	1.	1.32	0.	0.	0.	0.86	0.	0.91
time (sec)	N/A	0.067	0.063	0.035	0.	0.	26.731	0.	7.809

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	89	0	0	0	29	0	110
normalized size	1	1.	0.74	0.	0.	0.	0.24	0.	0.91
time (sec)	N/A	0.149	0.064	0.049	0.	0.	3.017	0.	16.15

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	76	0	0	0	29	0	87
normalized size	1	1.	0.78	0.	0.	0.	0.3	0.	0.9
time (sec)	N/A	0.102	0.052	0.037	0.	0.	2.547	0.	11.983

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	62	0	0	0	26	0	66
normalized size	1	1.	0.83	0.	0.	0.	0.35	0.	0.88
time (sec)	N/A	0.052	0.033	0.047	0.	0.	2.32	0.	5.665

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	68	0	0	0	29	0	60
normalized size	1	1.	0.94	0.	0.	0.	0.4	0.	0.83
time (sec)	N/A	0.063	0.04	0.037	0.	0.	2.566	0.	7.642

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	85	0	0	0	34	0	83
normalized size	1	1.	0.86	0.	0.	0.	0.34	0.	0.84
time (sec)	N/A	0.095	0.045	0.041	0.	0.	3.142	0.	11.572

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	94	0	0	0	34	0	104
normalized size	1	1.	0.76	0.	0.	0.	0.28	0.	0.85
time (sec)	N/A	0.137	0.053	0.043	0.	0.	4.013	0.	15.772

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	89	0	0	0	31	0	110
normalized size	1	1.	0.71	0.	0.	0.	0.25	0.	0.87
time (sec)	N/A	0.146	0.088	0.043	0.	0.	3.058	0.	18.938

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	79	0	0	0	31	0	87
normalized size	1	1.	0.78	0.	0.	0.	0.31	0.	0.86
time (sec)	N/A	0.103	0.071	0.027	0.	0.	2.545	0.	14.465

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	63	0	0	0	27	0	66
normalized size	1	1.	0.81	0.	0.	0.	0.35	0.	0.85
time (sec)	N/A	0.056	0.04	0.038	0.	0.	2.377	0.	7.526

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	0	0	0	31	0	61
normalized size	1	1.	0.92	0.	0.	0.	0.41	0.	0.8
time (sec)	N/A	0.068	0.043	0.031	0.	0.	2.631	0.	9.628

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	84	0	0	0	36	0	82
normalized size	1	1.	0.82	0.	0.	0.	0.35	0.	0.8
time (sec)	N/A	0.111	0.051	0.032	0.	0.	3.135	0.	14.085

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	95	0	0	0	36	0	104
normalized size	1	1.	0.74	0.	0.	0.	0.28	0.	0.81
time (sec)	N/A	0.141	0.052	0.034	0.	0.	4.046	0.	18.57

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	89	0	0	0	29	0	0
normalized size	1	1.	0.62	0.	0.	0.	0.2	0.	0.
time (sec)	N/A	0.162	0.079	0.041	0.	0.	4.949	0.	0.

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	78	0	0	0	29	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.24	0.	0.
time (sec)	N/A	0.115	0.058	0.036	0.	0.	3.711	0.	0.

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	63	0	0	0	26	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.28	0.	0.
time (sec)	N/A	0.065	0.036	0.042	0.	0.	2.959	0.	0.

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	68	0	0	0	29	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.33	0.	0.
time (sec)	N/A	0.08	0.04	0.039	0.	0.	3.532	0.	0.

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	88	0	0	0	34	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.28	0.	0.
time (sec)	N/A	0.115	0.047	0.04	0.	0.	4.037	0.	0.

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	94	0	0	0	34	0	0
normalized size	1	1.	0.65	0.	0.	0.	0.23	0.	0.
time (sec)	N/A	0.154	0.05	0.044	0.	0.	5.126	0.	0.



Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	89	0	0	0	31	0	110
normalized size	1	1.	0.71	0.	0.	0.	0.25	0.	0.87
time (sec)	N/A	0.147	0.076	0.043	0.	0.	5.087	0.	18.993

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	80	0	0	0	31	0	87
normalized size	1	1.	0.79	0.	0.	0.	0.31	0.	0.86
time (sec)	N/A	0.105	0.065	0.036	0.	0.	3.753	0.	14.633

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	64	0	0	0	27	0	66
normalized size	1	1.	0.82	0.	0.	0.	0.35	0.	0.85
time (sec)	N/A	0.057	0.04	0.045	0.	0.	3.034	0.	7.516

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	0	0	0	31	0	63
normalized size	1	1.	0.92	0.	0.	0.	0.41	0.	0.83
time (sec)	N/A	0.067	0.041	0.04	0.	0.	3.551	0.	9.755

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	84	0	0	0	36	0	82
normalized size	1	1.	0.82	0.	0.	0.	0.35	0.	0.8
time (sec)	N/A	0.1	0.048	0.049	0.	0.	4.049	0.	13.827

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	95	0	0	0	36	0	107
normalized size	1	1.	0.74	0.	0.	0.	0.28	0.	0.84
time (sec)	N/A	0.142	0.055	0.046	0.	0.	5.216	0.	18.325

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	76	0	0	0	26	0	83
normalized size	1	1.	0.83	0.	0.	0.	0.28	0.	0.9
time (sec)	N/A	0.072	0.049	0.046	0.	0.	5.243	0.	7.28

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	77	0	0	0	27	0	83
normalized size	1	1.	0.8	0.	0.	0.	0.28	0.	0.86
time (sec)	N/A	0.074	0.049	0.039	0.	0.	5.306	0.	9.132

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	76	0	0	0	26	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.23	0.	0.
time (sec)	N/A	0.086	0.048	0.044	0.	0.	10.254	0.	0.

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	77	0	0	0	27	0	83
normalized size	1	1.	0.8	0.	0.	0.	0.28	0.	0.86
time (sec)	N/A	0.075	0.05	0.045	0.	0.	10.288	0.	9.241

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	90	0	0	0	27	0	0
normalized size	1	1.	0.62	0.	0.	0.	0.18	0.	0.
time (sec)	N/A	0.166	0.076	0.036	0.	0.	3.091	0.	0.

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	79	0	0	0	27	0	0
normalized size	1	1.	0.65	0.	0.	0.	0.22	0.	0.
time (sec)	N/A	0.121	0.056	0.04	0.	0.	2.574	0.	0.

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	62	0	0	0	27	0	0
normalized size	1	1.	0.63	0.	0.	0.	0.28	0.	0.
time (sec)	N/A	0.079	0.046	0.034	0.	0.	2.377	0.	0.

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	47	0	0	0	24	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.34	0.	0.
time (sec)	N/A	0.047	0.024	0.038	0.	0.	2.281	0.	0.

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	69	0	0	0	27	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.29	0.	0.
time (sec)	N/A	0.078	0.046	0.041	0.	0.	2.674	0.	0.

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	83	0	0	0	32	0	0
normalized size	1	1.	0.67	0.	0.	0.	0.26	0.	0.
time (sec)	N/A	0.116	0.05	0.038	0.	0.	3.215	0.	0.

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	94	0	0	0	32	0	0
normalized size	1	1.	0.64	0.	0.	0.	0.22	0.	0.
time (sec)	N/A	0.159	0.06	0.039	0.	0.	4.104	0.	0.

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	89	0	0	0	29	0	114
normalized size	1	1.	0.69	0.	0.	0.	0.22	0.	0.88
time (sec)	N/A	0.147	0.093	0.036	0.	0.	3.284	0.	19.24

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	79	0	0	0	29	0	90
normalized size	1	1.	0.76	0.	0.	0.	0.28	0.	0.87
time (sec)	N/A	0.107	0.061	0.036	0.	0.	2.881	0.	14.572

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	64	0	0	0	29	0	68
normalized size	1	1.	0.79	0.	0.	0.	0.36	0.	0.84
time (sec)	N/A	0.073	0.055	0.035	0.	0.	2.488	0.	10.641

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	48	0	0	0	26	0	49
normalized size	1	1.	0.83	0.	0.	0.	0.45	0.	0.84
time (sec)	N/A	0.037	0.029	0.039	0.	0.	2.398	0.	6.151

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	0	0	0	29	0	63
normalized size	1	1.	0.9	0.	0.	0.	0.37	0.	0.8
time (sec)	N/A	0.069	0.049	0.039	0.	0.	2.757	0.	10.058

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	84	0	0	0	34	0	87
normalized size	1	1.	0.79	0.	0.	0.	0.32	0.	0.82
time (sec)	N/A	0.105	0.058	0.038	0.	0.	3.338	0.	13.96

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	95	0	0	0	34	0	114
normalized size	1	1.	0.73	0.	0.	0.	0.26	0.	0.87
time (sec)	N/A	0.144	0.068	0.041	0.	0.	4.225	0.	18.696

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	90	0	0	0	27	0	114
normalized size	1	1.	0.73	0.	0.	0.	0.22	0.	0.92
time (sec)	N/A	0.139	0.076	0.037	0.	0.	3.115	0.	16.391

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	78	0	0	0	27	0	90
normalized size	1	1.	0.78	0.	0.	0.	0.27	0.	0.9
time (sec)	N/A	0.1	0.062	0.037	0.	0.	2.633	0.	12.055

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	62	0	0	0	27	0	68
normalized size	1	1.	0.79	0.	0.	0.	0.35	0.	0.87
time (sec)	N/A	0.067	0.045	0.035	0.	0.	2.448	0.	8.354

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	47	0	0	0	24	0	49
normalized size	1	1.	0.84	0.	0.	0.	0.43	0.	0.88
time (sec)	N/A	0.034	0.024	0.04	0.	0.	2.358	0.	4.315

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	0	0	0	27	0	63
normalized size	1	1.	0.92	0.	0.	0.	0.36	0.	0.83
time (sec)	N/A	0.066	0.045	0.037	0.	0.	3.128	0.	7.939

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	83	0	0	0	32	0	88
normalized size	1	1.	0.81	0.	0.	0.	0.31	0.	0.86
time (sec)	N/A	0.097	0.053	0.04	0.	0.	3.899	0.	11.579

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	94	0	0	0	32	0	112
normalized size	1	1.	0.75	0.	0.	0.	0.25	0.	0.89
time (sec)	N/A	0.134	0.063	0.04	0.	0.	5.073	0.	15.974

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	91	0	0	0	29	0	114
normalized size	1	1.	0.71	0.	0.	0.	0.22	0.	0.88
time (sec)	N/A	0.144	0.082	0.028	0.	0.	3.192	0.	19.101

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	77	0	0	0	29	0	90
normalized size	1	1.	0.74	0.	0.	0.	0.28	0.	0.87
time (sec)	N/A	0.106	0.063	0.028	0.	0.	2.691	0.	14.503

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	64	0	0	0	29	0	68
normalized size	1	1.	0.79	0.	0.	0.	0.36	0.	0.84
time (sec)	N/A	0.07	0.051	0.028	0.	0.	2.456	0.	10.61

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	48	0	0	0	26	0	49
normalized size	1	1.	0.83	0.	0.	0.	0.45	0.	0.84
time (sec)	N/A	0.036	0.025	0.041	0.	0.	2.361	0.	6.041

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	70	0	0	0	29	0	61
normalized size	1	1.	0.9	0.	0.	0.	0.37	0.	0.78
time (sec)	N/A	0.068	0.047	0.029	0.	0.	3.106	0.	10.192

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	84	0	0	0	34	0	88
normalized size	1	1.	0.79	0.	0.	0.	0.32	0.	0.83
time (sec)	N/A	0.101	0.057	0.03	0.	0.	3.949	0.	14.159

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	95	0	0	0	34	0	112
normalized size	1	1.	0.73	0.	0.	0.	0.26	0.	0.85
time (sec)	N/A	0.14	0.062	0.031	0.	0.	5.197	0.	18.728

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	78	0	0	0	27	0	0
normalized size	1	1.	0.63	0.	0.	0.	0.22	0.	0.
time (sec)	N/A	0.144	0.074	0.065	0.	0.	3.261	0.	0.

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	64	0	0	0	27	0	0
normalized size	1	1.	0.64	0.	0.	0.	0.27	0.	0.
time (sec)	N/A	0.1	0.054	0.059	0.	0.	2.89	0.	0.

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	53	0	0	0	27	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.36	0.	0.
time (sec)	N/A	0.067	0.048	0.036	0.	0.	2.827	0.	0.

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	24	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.43	0.	0.
time (sec)	N/A	0.035	0.035	0.043	0.	0.	2.777	0.	0.

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	71	0	0	0	27	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.36	0.	0.
time (sec)	N/A	0.068	0.049	0.062	0.	0.	4.09	0.	0.

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	83	0	0	0	32	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.31	0.	0.
time (sec)	N/A	0.099	0.059	0.066	0.	0.	5.275	0.	0.

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	94	0	0	0	32	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.25	0.	0.
time (sec)	N/A	0.137	0.065	0.07	0.	0.	7.08	0.	0.

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	78	0	0	0	29	0	109
normalized size	1	1.	0.63	0.	0.	0.	0.23	0.	0.88
time (sec)	N/A	0.145	0.076	0.074	0.	0.	3.284	0.	19.458

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	65	0	0	0	29	0	87
normalized size	1	1.	0.64	0.	0.	0.	0.29	0.	0.86
time (sec)	N/A	0.105	0.056	0.068	0.	0.	2.999	0.	15.039

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	54	0	0	0	29	0	65
normalized size	1	1.	0.7	0.	0.	0.	0.38	0.	0.84
time (sec)	N/A	0.071	0.052	0.037	0.	0.	2.91	0.	10.481

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	54	0	0	0	26	0	65
normalized size	1	1.	0.7	0.	0.	0.	0.34	0.	0.84
time (sec)	N/A	0.057	0.042	0.044	0.	0.	2.883	0.	7.924

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	71	0	0	0	29	0	82
normalized size	1	1.	0.72	0.	0.	0.	0.29	0.	0.83
time (sec)	N/A	0.101	0.051	0.07	0.	0.	4.001	0.	14.423

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	84	0	0	0	34	0	109
normalized size	1	1.	0.67	0.	0.	0.	0.27	0.	0.87
time (sec)	N/A	0.141	0.059	0.073	0.	0.	5.246	0.	18.767

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	95	0	0	0	34	0	133
normalized size	1	1.	0.63	0.	0.	0.	0.23	0.	0.88
time (sec)	N/A	0.181	0.07	0.085	0.	0.	7.198	0.	24.288

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	55	0	0	0	24	0	68
normalized size	1	1.	0.71	0.	0.	0.	0.31	0.	0.87
time (sec)	N/A	0.053	0.048	0.044	0.	0.	4.041	0.	5.977

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	72	0	0	0	24	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.31	0.	0.
time (sec)	N/A	0.056	0.089	0.048	0.	0.	7.246	0.	0.

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	75	0	0	0	24	0	87
normalized size	1	1.	0.77	0.	0.	0.	0.25	0.	0.9
time (sec)	N/A	0.074	0.081	0.046	0.	0.	15.386	0.	8.116

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	56	0	0	0	26	0	68
normalized size	1	1.	0.69	0.	0.	0.	0.32	0.	0.84
time (sec)	N/A	0.059	0.048	0.052	0.	0.	4.046	0.	7.798



Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	74	0	0	0	26	0	87
normalized size	1	1.	0.73	0.	0.	0.	0.26	0.	0.86
time (sec)	N/A	0.079	0.093	0.046	0.	0.	7.434	0.	10.13

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	77	0	0	0	26	0	87
normalized size	1	1.	0.76	0.	0.	0.	0.26	0.	0.86
time (sec)	N/A	0.077	0.095	0.051	0.	0.	15.59	0.	9.911

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	54	43	0	0	27	0	0
normalized size	1	1.	0.55	0.43	0.	0.	0.27	0.	0.
time (sec)	N/A	0.094	0.05	0.053	0.	0.	2.831	0.	0.

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	49	38	0	0	27	0	0
normalized size	1	1.	0.6	0.47	0.	0.	0.33	0.	0.
time (sec)	N/A	0.067	0.038	0.03	0.	0.	2.324	0.	0.

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	41	31	0	0	27	0	0
normalized size	1	1.	0.65	0.49	0.	0.	0.43	0.	0.
time (sec)	N/A	0.045	0.025	0.028	0.	0.	2.109	0.	0.

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	24	18	0	0	26	0	0
normalized size	1	1.	0.56	0.42	0.	0.	0.6	0.	0.
time (sec)	N/A	0.025	0.015	0.013	0.	0.	2.038	0.	0.

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	46	33	0	0	29	0	0
normalized size	1	1.	0.73	0.52	0.	0.	0.46	0.	0.
time (sec)	N/A	0.045	0.032	0.032	0.	0.	2.323	0.	0.

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	55	45	0	0	32	0	0
normalized size	1	1.	0.66	0.54	0.	0.	0.39	0.	0.
time (sec)	N/A	0.065	0.043	0.032	0.	0.	2.867	0.	0.

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	62	50	0	0	32	0	0
normalized size	1	1.	0.61	0.5	0.	0.	0.32	0.	0.
time (sec)	N/A	0.088	0.033	0.033	0.	0.	3.703	0.	0.

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	55	50	0	0	29	0	75
normalized size	1	1.	0.66	0.6	0.	0.	0.35	0.	0.9
time (sec)	N/A	0.078	0.061	0.056	0.	0.	2.87	0.	7.016

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	50	45	0	0	29	0	58
normalized size	1	1.	0.77	0.69	0.	0.	0.45	0.	0.89
time (sec)	N/A	0.057	0.05	0.039	0.	0.	2.397	0.	5.244

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	38	0	0	29	0	41
normalized size	1	1.	0.87	0.81	0.	0.	0.62	0.	0.87
time (sec)	N/A	0.036	0.024	0.039	0.	0.	2.129	0.	3.641

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	24	18	0	0	27	0	26
normalized size	1	1.	0.86	0.64	0.	0.	0.96	0.	0.93
time (sec)	N/A	0.015	0.015	0.024	0.	0.	2.071	0.	1.164

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	46	40	0	0	31	0	39
normalized size	1	1.	0.98	0.85	0.	0.	0.66	0.	0.83
time (sec)	N/A	0.034	0.028	0.041	0.	0.	2.386	0.	3.442

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	55	45	0	0	34	0	58
normalized size	1	1.	0.82	0.67	0.	0.	0.51	0.	0.87
time (sec)	N/A	0.054	0.041	0.042	0.	0.	2.942	0.	5.04

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	62	50	0	0	34	0	75
normalized size	1	1.	0.73	0.59	0.	0.	0.4	0.	0.88
time (sec)	N/A	0.075	0.047	0.045	0.	0.	3.662	0.	6.777

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	54	43	0	0	27	0	75
normalized size	1	1.	0.65	0.52	0.	0.	0.33	0.	0.9
time (sec)	N/A	0.077	0.043	0.046	0.	0.	2.727	0.	6.894

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	49	38	0	0	27	0	58
normalized size	1	1.	0.75	0.58	0.	0.	0.42	0.	0.89
time (sec)	N/A	0.054	0.036	0.03	0.	0.	2.333	0.	5.155

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	31	0	0	27	0	41
normalized size	1	1.	0.87	0.66	0.	0.	0.57	0.	0.87
time (sec)	N/A	0.033	0.021	0.029	0.	0.	2.121	0.	3.589

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	18	0	0	26	0	24
normalized size	1	1.	0.89	0.67	0.	0.	0.96	0.	0.89
time (sec)	N/A	0.014	0.012	0.014	0.	0.	2.008	0.	1.076

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	46	33	0	0	29	0	39
normalized size	1	1.	0.94	0.67	0.	0.	0.59	0.	0.8
time (sec)	N/A	0.033	0.026	0.033	0.	0.	2.753	0.	3.377

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	55	45	0	0	32	0	56
normalized size	1	1.	0.82	0.67	0.	0.	0.48	0.	0.84
time (sec)	N/A	0.053	0.026	0.033	0.	0.	3.36	0.	5.358

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	58	50	0	0	32	0	73
normalized size	1	1.	0.68	0.59	0.	0.	0.38	0.	0.86
time (sec)	N/A	0.073	0.049	0.034	0.	0.	4.305	0.	6.678

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	55	0	0	0	29	0	75
normalized size	1	1.	0.66	0.	0.	0.	0.35	0.	0.9
time (sec)	N/A	0.077	0.053	0.033	0.	0.	2.82	0.	7.007

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	50	0	0	0	29	0	58
normalized size	1	1.	0.77	0.	0.	0.	0.45	0.	0.89
time (sec)	N/A	0.057	0.04	0.022	0.	0.	2.351	0.	5.267

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	0	0	0	29	0	41
normalized size	1	1.	0.87	0.	0.	0.	0.62	0.	0.87
time (sec)	N/A	0.035	0.021	0.02	0.	0.	2.196	0.	3.645

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	18	0	0	27	0	24
normalized size	1	1.	0.89	0.67	0.	0.	1.	0.	0.89
time (sec)	N/A	0.016	0.012	0.028	0.	0.	2.143	0.	1.173

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	46	0	0	0	31	0	37
normalized size	1	1.	0.94	0.	0.	0.	0.63	0.	0.76
time (sec)	N/A	0.035	0.029	0.024	0.	0.	2.727	0.	3.449

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	55	0	0	0	34	0	56
normalized size	1	1.	0.82	0.	0.	0.	0.51	0.	0.84
time (sec)	N/A	0.055	0.03	0.025	0.	0.	3.439	0.	5.12

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	58	0	0	0	34	0	73
normalized size	1	1.	0.68	0.	0.	0.	0.4	0.	0.86
time (sec)	N/A	0.075	0.048	0.026	0.	0.	4.389	0.	6.823

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	68	65	0	0	31	0	92
normalized size	1	1.	0.26	0.25	0.	0.	0.12	0.	0.36
time (sec)	N/A	0.337	0.043	0.06	0.	0.	2.862	0.	7.811

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	63	60	0	0	31	0	75
normalized size	1	1.	0.26	0.25	0.	0.	0.13	0.	0.31
time (sec)	N/A	0.283	0.036	0.051	0.	0.	2.46	0.	5.852

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	57	53	0	0	31	0	58
normalized size	1	1.	0.26	0.24	0.	0.	0.14	0.	0.26
time (sec)	N/A	0.241	0.031	0.052	0.	0.	2.199	0.	4.172

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	41	40	0	0	29	0	42
normalized size	1	1.	0.21	0.2	0.	0.	0.15	0.	0.21
time (sec)	N/A	0.206	0.019	0.04	0.	0.	2.145	0.	1.555

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	63	55	0	0	31	0	54
normalized size	1	1.	0.29	0.25	0.	0.	0.14	0.	0.24
time (sec)	N/A	0.242	0.026	0.057	0.	0.	2.495	0.	3.934

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	71	67	0	0	34	0	73
normalized size	1	1.	0.29	0.28	0.	0.	0.14	0.	0.3
time (sec)	N/A	0.283	0.035	0.062	0.	0.	2.934	0.	5.715

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	76	72	0	0	34	0	90
normalized size	1	1.	0.29	0.28	0.	0.	0.13	0.	0.35
time (sec)	N/A	0.327	0.037	0.053	0.	0.	3.776	0.	7.429

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	68	53	0	0	34	0	0
normalized size	1	1.	0.26	0.2	0.	0.	0.13	0.	0.
time (sec)	N/A	0.344	0.043	0.033	0.	0.	2.808	0.	0.

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	63	48	0	0	34	0	0
normalized size	1	1.	0.26	0.2	0.	0.	0.14	0.	0.
time (sec)	N/A	0.29	0.037	0.028	0.	0.	2.343	0.	0.

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	58	41	0	0	34	0	0
normalized size	1	1.	0.26	0.18	0.	0.	0.15	0.	0.
time (sec)	N/A	0.247	0.033	0.029	0.	0.	2.187	0.	0.

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	41	21	0	0	32	0	0
normalized size	1	1.	0.2	0.1	0.	0.	0.16	0.	0.
time (sec)	N/A	0.207	0.019	0.014	0.	0.	2.016	0.	0.

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	63	43	0	0	36	0	0
normalized size	1	1.	0.28	0.19	0.	0.	0.16	0.	0.
time (sec)	N/A	0.248	0.029	0.031	0.	0.	2.292	0.	0.

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	71	48	0	0	39	0	0
normalized size	1	1.	0.29	0.2	0.	0.	0.16	0.	0.
time (sec)	N/A	0.292	0.036	0.033	0.	0.	2.772	0.	0.

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	76	53	0	0	39	0	0
normalized size	1	1.	0.29	0.2	0.	0.	0.15	0.	0.
time (sec)	N/A	0.338	0.042	0.034	0.	0.	3.623	0.	0.

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	68	65	0	0	31	0	92
normalized size	1	1.	0.49	0.47	0.	0.	0.22	0.	0.67
time (sec)	N/A	0.158	0.042	0.06	0.	0.	2.729	0.	7.696

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	63	60	0	0	31	0	75
normalized size	1	1.	0.52	0.5	0.	0.	0.26	0.	0.62
time (sec)	N/A	0.126	0.035	0.051	0.	0.	2.301	0.	5.827

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	57	53	0	0	31	0	58
normalized size	1	1.	0.56	0.52	0.	0.	0.3	0.	0.57
time (sec)	N/A	0.103	0.03	0.051	0.	0.	2.116	0.	4.143

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	41	40	0	0	29	0	42
normalized size	1	1.	0.5	0.49	0.	0.	0.35	0.	0.51
time (sec)	N/A	0.069	0.018	0.036	0.	0.	2.042	0.	1.54

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	63	55	0	0	29	0	54
normalized size	1	1.	0.61	0.53	0.	0.	0.28	0.	0.52
time (sec)	N/A	0.098	0.024	0.056	0.	0.	2.686	0.	3.889

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	68	67	0	0	34	0	73
normalized size	1	1.	0.56	0.55	0.	0.	0.28	0.	0.6
time (sec)	N/A	0.126	0.029	0.059	0.	0.	3.339	0.	5.642

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	73	72	0	0	34	0	90
normalized size	1	1.	0.52	0.51	0.	0.	0.24	0.	0.64
time (sec)	N/A	0.155	0.03	0.056	0.	0.	4.363	0.	7.449

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	68	0	0	0	36	0	99
normalized size	1	1.	0.49	0.	0.	0.	0.26	0.	0.71
time (sec)	N/A	0.162	0.039	0.032	0.	0.	2.718	0.	7.523

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	63	0	0	0	36	0	80
normalized size	1	1.	0.52	0.	0.	0.	0.3	0.	0.66
time (sec)	N/A	0.133	0.035	0.021	0.	0.	2.269	0.	5.724

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	58	0	0	0	36	0	63
normalized size	1	1.	0.56	0.	0.	0.	0.35	0.	0.61
time (sec)	N/A	0.104	0.029	0.02	0.	0.	2.107	0.	4.038

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	41	21	0	0	34	0	44
normalized size	1	1.	0.49	0.25	0.	0.	0.4	0.	0.52
time (sec)	N/A	0.076	0.018	0.014	0.	0.	1.995	0.	1.478

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	63	0	0	0	34	0	58
normalized size	1	1.	0.6	0.	0.	0.	0.32	0.	0.55
time (sec)	N/A	0.103	0.026	0.025	0.	0.	2.672	0.	3.829



Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	68	0	0	0	37	0	78
normalized size	1	1.	0.55	0.	0.	0.	0.3	0.	0.63
time (sec)	N/A	0.13	0.029	0.024	0.	0.	3.367	0.	5.548

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	76	0	0	0	37	0	97
normalized size	1	1.	0.54	0.	0.	0.	0.26	0.	0.69
time (sec)	N/A	0.16	0.034	0.026	0.	0.	4.293	0.	7.344

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	98	0	0	0	0	0	133
normalized size	1	1.	0.64	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.312	0.068	0.049	0.	0.	0.	0.	33.262

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	83	0	0	0	46	0	100
normalized size	1	1.	0.7	0.	0.	0.	0.39	0.	0.85
time (sec)	N/A	0.242	0.054	0.039	0.	0.	22.987	0.	26.628

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	62	0	0	0	46	0	76
normalized size	1	1.	0.7	0.	0.	0.	0.52	0.	0.85
time (sec)	N/A	0.207	0.041	0.037	0.	0.	5.126	0.	23.029

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	69	0	0	0	32	0	85
normalized size	1	1.	0.73	0.	0.	0.	0.34	0.	0.9
time (sec)	N/A	0.207	0.053	0.042	0.	0.	56.321	0.	23.508

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	92	0	0	0	0	0	110
normalized size	1	1.	0.75	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.257	0.068	0.047	0.	0.	0.	0.	28.732

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	103	0	0	0	0	0	141
normalized size	1	1.	0.67	0.	0.	0.	0.	0.	0.92
time (sec)	N/A	0.302	0.079	0.052	0.	0.	0.	0.	35.711

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	83	0	0	0	0	570	134
normalized size	1	1.	0.56	0.	0.	0.	0.	3.88	0.91
time (sec)	N/A	0.257	0.061	0.043	0.	0.	0.	0.262	29.452

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	68	0	0	0	46	498	104
normalized size	1	1.	0.59	0.	0.	0.	0.4	4.29	0.9
time (sec)	N/A	0.195	0.049	0.036	0.	0.	9.161	0.261	23.293

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	72	0	0	0	49	452	97
normalized size	1	1.	0.67	0.	0.	0.	0.46	4.22	0.91
time (sec)	N/A	0.195	0.053	0.043	0.	0.	14.79	0.255	24.021

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	27	34	0	58	24
normalized size	1	1.	0.93	0.75	0.96	1.21	0.	2.07	0.86
time (sec)	N/A	0.027	0.031	0.007	1.386	0.268	0.	0.234	3.612

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	51	31	51	62	0	144	48
normalized size	1	1.	0.89	0.54	0.89	1.09	0.	2.53	0.84
time (sec)	N/A	0.057	0.037	0.007	1.388	0.257	0.	0.229	6.696

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	63	42	74	77	0	244	73
normalized size	1	1.	0.74	0.49	0.87	0.91	0.	2.87	0.86
time (sec)	N/A	0.088	0.041	0.009	1.387	0.257	0.	0.234	10.637

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	74	53	97	92	0	359	99
normalized size	1	1.	0.65	0.47	0.86	0.81	0.	3.18	0.88
time (sec)	N/A	0.126	0.048	0.009	1.386	0.239	0.	0.236	15.317

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	84	0	0	0	48	0	100
normalized size	1	1.	0.69	0.	0.	0.	0.39	0.	0.82
time (sec)	N/A	0.257	0.065	0.056	0.	0.	24.151	0.	36.171

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	66	0	0	0	39	0	76
normalized size	1	1.	0.72	0.	0.	0.	0.42	0.	0.83
time (sec)	N/A	0.216	0.047	0.047	0.	0.	5.779	0.	32.314

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	70	0	0	0	37	0	83
normalized size	1	1.	0.72	0.	0.	0.	0.38	0.	0.86
time (sec)	N/A	0.216	0.056	0.053	0.	0.	58.337	0.	32.622

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	93	0	0	0	0	0	110
normalized size	1	1.	0.73	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.262	0.084	0.057	0.	0.	0.	0.	38.47

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	105	0	0	0	0	0	141
normalized size	1	1.	0.66	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.315	0.105	0.057	0.	0.	0.	0.	45.58

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	84	0	0	0	0	533	311
normalized size	1	1.	0.24	0.	0.	0.	0.	1.55	0.91
time (sec)	N/A	0.846	0.067	0.053	0.	0.	0.	0.263	76.971

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	69	0	0	0	48	459	274
normalized size	1	1.	0.22	0.	0.	0.	0.16	1.5	0.89
time (sec)	N/A	0.65	0.066	0.046	0.	0.	9.444	0.263	66.279

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	72	0	0	0	51	429	267
normalized size	1	1.	0.24	0.	0.	0.	0.17	1.45	0.9
time (sec)	N/A	0.615	0.065	0.05	0.	0.	14.904	0.254	66.39

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	22	28	47	0	61	24
normalized size	1	1.	0.93	0.76	0.97	1.62	0.	2.1	0.83
time (sec)	N/A	0.028	0.034	0.006	1.402	0.223	0.	0.229	3.848

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	52	32	54	62	0	149	48
normalized size	1	1.	0.88	0.54	0.92	1.05	0.	2.53	0.81
time (sec)	N/A	0.057	0.035	0.008	1.406	0.224	0.	0.232	7.263

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	64	43	78	78	0	251	73
normalized size	1	1.	0.73	0.49	0.89	0.89	0.	2.85	0.83
time (sec)	N/A	0.091	0.041	0.007	1.396	0.219	0.	0.236	11.507

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	75	54	103	93	0	367	99
normalized size	1	1.	0.64	0.46	0.88	0.79	0.	3.14	0.85
time (sec)	N/A	0.131	0.045	0.009	1.405	0.217	0.	0.236	16.489

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	69	0	0	382	44	0	105
normalized size	1	1.	0.59	0.	0.	3.26	0.38	0.	0.9
time (sec)	N/A	0.16	0.061	0.055	0.	0.241	13.915	0.	20.88

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	55	0	0	304	44	0	76
normalized size	1	1.	0.66	0.	0.	3.66	0.53	0.	0.92
time (sec)	N/A	0.119	0.029	0.044	0.	0.237	6.589	0.	15.833

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	27	34	36	0	24
normalized size	1	1.	0.93	0.75	0.96	1.21	1.29	0.	0.86
time (sec)	N/A	0.028	0.02	0.008	1.396	0.215	57.558	0.	3.591

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	41	31	51	61	0	0	48
normalized size	1	1.	0.72	0.54	0.89	1.07	0.	0.	0.84
time (sec)	N/A	0.057	0.034	0.007	1.399	0.215	0.	0.	6.772

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	52	42	74	77	0	0	73
normalized size	1	1.	0.61	0.49	0.87	0.91	0.	0.	0.86
time (sec)	N/A	0.087	0.044	0.008	1.403	0.214	0.	0.	10.777

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	87	0	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.204	0.077	0.06	0.	0.	0.	0.	0.

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	69	0	0	0	44	0	0
normalized size	1	1.	0.55	0.	0.	0.	0.35	0.	0.
time (sec)	N/A	0.154	0.063	0.049	0.	0.	150.86	0.	0.

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	57	0	0	0	44	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.53	0.	0.
time (sec)	N/A	0.107	0.032	0.035	0.	0.	3.469	0.	0.

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	75	0	0	0	31	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.34	0.	0.
time (sec)	N/A	0.114	0.058	0.053	0.	0.	14.202	0.	0.

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	88	0	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.159	0.071	0.058	0.	0.	0.	0.	0.

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	103	0	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.21	0.093	0.061	0.	0.	0.	0.	0.

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	71	0	0	417	46	0	275
normalized size	1	1.	0.23	0.	0.	1.35	0.15	0.	0.89
time (sec)	N/A	0.639	0.067	0.054	0.	0.241	13.76	0.	66.837

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	56	0	0	338	46	0	246
normalized size	1	1.	0.21	0.	0.	1.24	0.17	0.	0.9
time (sec)	N/A	0.502	0.038	0.049	0.	0.238	6.613	0.	57.895

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	22	28	47	90	0	24
normalized size	1	1.	0.93	0.76	0.97	1.62	3.1	0.	0.83
time (sec)	N/A	0.029	0.025	0.007	1.398	0.212	55.691	0.	3.889

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	42	32	54	63	0	0	48
normalized size	1	1.	0.71	0.54	0.92	1.07	0.	0.	0.81
time (sec)	N/A	0.06	0.038	0.007	1.396	0.213	0.	0.	7.375

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	53	43	78	78	0	0	73
normalized size	1	1.	0.6	0.49	0.89	0.89	0.	0.	0.83
time (sec)	N/A	0.094	0.052	0.008	1.408	0.214	0.	0.	11.599

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	71	0	0	0	46	0	109
normalized size	1	1.	0.55	0.	0.	0.	0.36	0.	0.85
time (sec)	N/A	0.168	0.066	0.057	0.	0.	150.516	0.	21.066

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	58	0	0	0	46	0	75
normalized size	1	1.	0.64	0.	0.	0.	0.51	0.	0.83
time (sec)	N/A	0.121	0.037	0.039	0.	0.	3.565	0.	15.293

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	76	0	0	0	32	0	61
normalized size	1	1.	1.12	0.	0.	0.	0.47	0.	0.9
time (sec)	N/A	0.081	0.056	0.06	0.	0.	13.779	0.	11.384

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	89	0	0	0	0	0	87
normalized size	1	1.	0.89	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.122	0.083	0.074	0.	0.	0.	0.	16.195

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	104	0	0	0	0	0	116
normalized size	1	1.	0.8	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.172	0.099	0.074	0.	0.	0.	0.	22.214

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	66	0	0	0	44	0	75
normalized size	1	1.	0.77	0.	0.	0.	0.51	0.	0.87
time (sec)	N/A	0.203	0.056	0.039	0.	0.	13.538	0.	23.057

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	0	31	0	61
normalized size	1	1.	0.83	0.	0.	0.	0.47	0.	0.92
time (sec)	N/A	0.166	0.028	0.046	0.	0.	9.172	0.	19.515

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	72	0	0	0	0	0	85
normalized size	1	1.	0.74	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.203	0.055	0.046	0.	0.	0.	0.	23.633

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	92	0	0	0	0	0	114
normalized size	1	1.	0.73	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.244	0.082	0.047	0.	0.	0.	0.	29.217

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	104	0	0	0	0	0	144
normalized size	1	1.	0.66	0.	0.	0.	0.	0.	0.92
time (sec)	N/A	0.299	0.09	0.049	0.	0.	0.	0.	36.468

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	69	0	0	0	44	0	109
normalized size	1	1.	0.59	0.	0.	0.	0.38	0.	0.93
time (sec)	N/A	0.198	0.058	0.038	0.	0.	176.672	0.	24.213

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	57	0	0	0	44	0	76
normalized size	1	1.	0.68	0.	0.	0.	0.52	0.	0.9
time (sec)	N/A	0.158	0.031	0.037	0.	0.	6.826	0.	19.254

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	21	27	34	36	0	22
normalized size	1	1.	0.92	0.81	1.04	1.31	1.38	0.	0.85
time (sec)	N/A	0.028	0.019	0.007	1.403	0.234	28.685	0.	3.585



Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	34	29	51	47	0	0	46
normalized size	1	1.	0.62	0.53	0.93	0.85	0.	0.	0.84
time (sec)	N/A	0.057	0.034	0.007	1.416	0.228	0.	0.	6.749

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	52	42	74	62	0	0	71
normalized size	1	1.	0.63	0.51	0.89	0.75	0.	0.	0.86
time (sec)	N/A	0.087	0.041	0.007	1.392	0.231	0.	0.	10.824

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	68	0	0	0	46	0	76
normalized size	1	1.	0.75	0.	0.	0.	0.51	0.	0.84
time (sec)	N/A	0.209	0.06	0.045	0.	0.	13.446	0.	32.434

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	56	0	0	0	32	0	61
normalized size	1	1.	0.82	0.	0.	0.	0.47	0.	0.9
time (sec)	N/A	0.178	0.032	0.058	0.	0.	9.161	0.	28.379

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	76	0	0	0	0	0	87
normalized size	1	1.	0.76	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.22	0.065	0.054	0.	0.	0.	0.	33.231

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	94	0	0	0	0	0	116
normalized size	1	1.	0.72	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.261	0.098	0.059	0.	0.	0.	0.	39.261

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	105	0	0	0	0	0	146
normalized size	1	1.	0.65	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.315	0.108	0.06	0.	0.	0.	0.	46.842

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	71	0	0	0	46	0	282
normalized size	1	1.	0.23	0.	0.	0.	0.15	0.	0.92
time (sec)	N/A	0.67	0.063	0.05	0.	0.	175.969	0.	68.048

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	58	0	0	0	46	0	246
normalized size	1	1.	0.21	0.	0.	0.	0.17	0.	0.9
time (sec)	N/A	0.547	0.033	0.047	0.	0.	6.878	0.	59.852

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	22	28	35	90	0	22
normalized size	1	1.	0.93	0.81	1.04	1.3	3.33	0.	0.81
time (sec)	N/A	0.03	0.019	0.006	1.392	0.229	28.494	0.	3.931

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	30	53	46	0	0	46
normalized size	1	1.	0.61	0.53	0.93	0.81	0.	0.	0.81
time (sec)	N/A	0.06	0.035	0.007	1.402	0.226	0.	0.	7.675

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	53	43	78	63	0	0	71
normalized size	1	1.	0.62	0.5	0.91	0.73	0.	0.	0.83
time (sec)	N/A	0.094	0.043	0.007	1.406	0.228	0.	0.	11.712

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	73	0	0	486	0	0	138
normalized size	1	1.	0.5	0.	0.	3.33	0.	0.	0.95
time (sec)	N/A	0.21	0.069	0.087	0.	0.261	0.	0.	28.723

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F(-2)	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	60	0	0	400	44	0	99
normalized size	1	1.	0.56	0.	0.	3.74	0.41	0.	0.93
time (sec)	N/A	0.155	0.053	0.04	0.	0.233	60.142	0.	22.375

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	21	27	42	34	0	20
normalized size	1	1.	0.92	0.81	1.04	1.62	1.31	0.	0.77
time (sec)	N/A	0.028	0.018	0.006	1.399	0.209	28.2	0.	3.704

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	34	29	0	45	0	0	46
normalized size	1	1.	0.62	0.53	0.	0.82	0.	0.	0.84
time (sec)	N/A	0.057	0.036	0.008	0.	0.21	0.	0.	6.764

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	52	42	0	62	0	0	71
normalized size	1	1.	0.63	0.51	0.	0.75	0.	0.	0.86
time (sec)	N/A	0.089	0.049	0.008	0.	0.211	0.	0.	10.845

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	63	53	0	77	0	0	95
normalized size	1	1.	0.58	0.49	0.	0.71	0.	0.	0.87
time (sec)	N/A	0.126	0.062	0.008	0.	0.209	0.	0.	15.72

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	87	0	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.213	0.079	0.087	0.	0.	0.	0.	0.

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	73	0	0	0	0	0	0
normalized size	1	1.	0.59	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.156	0.069	0.077	0.	0.	0.	0.	0.

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	60	0	0	0	0	0	0
normalized size	1	1.	0.67	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.058	0.047	0.	0.	0.	0.	0.

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	44	0	0
normalized size	1	1.	1.	0.	0.	0.	0.7	0.	0.
time (sec)	N/A	0.074	0.05	0.039	0.	0.	14.186	0.	0.

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	76	0	0	0	48	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.52	0.	0.
time (sec)	N/A	0.119	0.066	0.082	0.	0.	132.303	0.	0.

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	86	0	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.161	0.082	0.086	0.	0.	0.	0.	0.

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	105	0	0	0	0	0	0
normalized size	1	1.	0.67	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.216	0.122	0.092	0.	0.	0.	0.	0.

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	69	0	0	0	0	0	49
normalized size	1	1.	1.19	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.067	0.058	0.058	0.	0.	0.	0.	8.07

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	0	0	0	44	0	49
normalized size	1	1.	0.98	0.	0.	0.	0.76	0.	0.84
time (sec)	N/A	0.066	0.038	0.042	0.	0.	40.091	0.	8.052

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	0	0	0	44	0	49
normalized size	1	1.	0.98	0.	0.	0.	0.76	0.	0.84
time (sec)	N/A	0.067	0.036	0.042	0.	0.	4.936	0.	8.081

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	0	0	0	44	0	49
normalized size	1	1.	0.98	0.	0.	0.	0.76	0.	0.84
time (sec)	N/A	0.067	0.038	0.044	0.	0.	5.307	0.	8.088

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	44	0	48
normalized size	1	1.	0.98	0.	0.	0.	0.79	0.	0.86
time (sec)	N/A	0.066	0.029	0.045	0.	0.	13.728	0.	8.092

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	75	0	0	0	48	0	51
normalized size	1	1.	1.34	0.	0.	0.	0.86	0.	0.91
time (sec)	N/A	0.066	0.091	0.053	0.	0.	57.257	0.	8.082

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	62	0	0	0	0	0	51
normalized size	1	1.	1.02	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.067	0.059	0.055	0.	0.	0.	0.	8.149

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	62	0	0	0	0	0	51
normalized size	1	1.	1.02	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.067	0.051	0.053	0.	0.	0.	0.	8.124

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	62	0	0	0	0	0	51
normalized size	1	1.	1.02	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.066	0.049	0.05	0.	0.	0.	0.	8.193

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	62	0	0	0	0	0	51
normalized size	1	1.	1.02	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.066	0.047	0.059	0.	0.	0.	0.	8.312

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	63	0	0	0	0	0	49
normalized size	1	1.	1.07	0.	0.	0.	0.	0.	0.83
time (sec)	N/A	0.067	0.045	0.057	0.	0.	0.	0.	8.321

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	74	0	0	0	0	0	53
normalized size	1	1.	1.25	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.066	0.061	0.078	0.	0.	0.	0.	8.346

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	101	0	0	0	29	0	320
normalized size	1	1.	0.29	0.	0.	0.	0.08	0.	0.93
time (sec)	N/A	0.89	0.079	0.049	0.	0.	5.172	0.	29.885

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	90	0	0	0	29	0	296
normalized size	1	1.	0.28	0.	0.	0.	0.09	0.	0.92
time (sec)	N/A	0.668	0.054	0.038	0.	0.	3.815	0.	23.458

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	76	0	0	0	29	0	272
normalized size	1	1.	0.26	0.	0.	0.	0.1	0.	0.92
time (sec)	N/A	0.561	0.058	0.037	0.	0.	3.029	0.	17.831

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	62	0	0	0	26	0	246
normalized size	1	1.	0.23	0.	0.	0.	0.1	0.	0.9
time (sec)	N/A	0.47	0.033	0.043	0.	0.	2.688	0.	10.011

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	68	0	0	0	29	0	240
normalized size	1	1.	0.26	0.	0.	0.	0.11	0.	0.9
time (sec)	N/A	0.46	0.041	0.041	0.	0.	3.189	0.	11.861

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	85	0	0	0	34	0	267
normalized size	1	1.	0.29	0.	0.	0.	0.11	0.	0.9
time (sec)	N/A	0.558	0.05	0.041	0.	0.	4.052	0.	17.24

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	94	0	0	0	34	0	292
normalized size	1	1.	0.29	0.	0.	0.	0.11	0.	0.9
time (sec)	N/A	0.646	0.06	0.045	0.	0.	5.261	0.	22.899

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	108	0	0	0	34	0	316
normalized size	1	1.	0.31	0.	0.	0.	0.1	0.	0.91
time (sec)	N/A	0.756	0.064	0.046	0.	0.	7.457	0.	29.883

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	659	659	90	0	0	0	27	0	0
normalized size	1	1.	0.14	0.	0.	0.	0.04	0.	0.
time (sec)	N/A	1.545	0.082	0.04	0.	0.	3.978	0.	0.

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	635	635	79	0	0	0	27	0	0
normalized size	1	1.	0.12	0.	0.	0.	0.04	0.	0.
time (sec)	N/A	1.365	0.061	0.036	0.	0.	3.153	0.	0.

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	611	611	62	0	0	0	27	0	0
normalized size	1	1.	0.1	0.	0.	0.	0.04	0.	0.
time (sec)	N/A	1.233	0.049	0.035	0.	0.	2.656	0.	0.

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	577	577	47	0	0	0	24	0	0
normalized size	1	1.	0.08	0.	0.	0.	0.04	0.	0.
time (sec)	N/A	1.062	0.025	0.038	0.	0.	2.578	0.	0.

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	586	586	70	0	0	0	27	0	0
normalized size	1	1.	0.12	0.	0.	0.	0.05	0.	0.
time (sec)	N/A	1.179	0.049	0.036	0.	0.	3.126	0.	0.

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	633	633	83	0	0	0	32	0	0
normalized size	1	1.	0.13	0.	0.	0.	0.05	0.	0.
time (sec)	N/A	1.348	0.055	0.038	0.	0.	4.064	0.	0.

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	661	661	94	0	0	0	32	0	0
normalized size	1	1.	0.14	0.	0.	0.	0.05	0.	0.
time (sec)	N/A	1.503	0.061	0.041	0.	0.	5.342	0.	0.

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	89	0	0	0	27	0	299
normalized size	1	1.	0.27	0.	0.	0.	0.08	0.	0.92
time (sec)	N/A	0.687	0.066	0.038	0.	0.	3.167	0.	23.563

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	79	0	0	0	27	0	275
normalized size	1	1.	0.26	0.	0.	0.	0.09	0.	0.92
time (sec)	N/A	0.575	0.055	0.036	0.	0.	2.768	0.	18.152

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	62	0	0	0	27	0	252
normalized size	1	1.	0.22	0.	0.	0.	0.1	0.	0.91
time (sec)	N/A	0.487	0.046	0.037	0.	0.	2.755	0.	12.503

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	47	0	0	0	24	0	228
normalized size	1	1.	0.19	0.	0.	0.	0.1	0.	0.9
time (sec)	N/A	0.389	0.022	0.039	0.	0.	2.759	0.	7.238



Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	70	0	0	0	27	0	246
normalized size	1	1.	0.26	0.	0.	0.	0.1	0.	0.9
time (sec)	N/A	0.481	0.051	0.038	0.	0.	3.981	0.	12.118

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	83	0	0	0	32	0	272
normalized size	1	1.	0.28	0.	0.	0.	0.11	0.	0.91
time (sec)	N/A	0.563	0.047	0.04	0.	0.	5.081	0.	16.897

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	94	0	0	0	32	0	298
normalized size	1	1.	0.29	0.	0.	0.	0.1	0.	0.91
time (sec)	N/A	0.654	0.055	0.042	0.	0.	7.059	0.	22.595

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	654	654	79	0	0	0	27	0	0
normalized size	1	1.	0.12	0.	0.	0.	0.04	0.	0.
time (sec)	N/A	1.517	0.071	0.064	0.	0.	3.497	0.	0.

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	630	630	64	0	0	0	27	0	0
normalized size	1	1.	0.1	0.	0.	0.	0.04	0.	0.
time (sec)	N/A	1.376	0.055	0.062	0.	0.	3.532	0.	0.

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	583	583	53	0	0	0	27	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.05	0.	0.
time (sec)	N/A	1.065	0.049	0.036	0.	0.	3.498	0.	0.

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	555	555	55	0	0	0	24	0	0
normalized size	1	1.	0.1	0.	0.	0.	0.04	0.	0.
time (sec)	N/A	0.821	0.037	0.047	0.	0.	3.339	0.	0.

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	614	614	71	0	0	0	27	0	0
normalized size	1	1.	0.12	0.	0.	0.	0.04	0.	0.
time (sec)	N/A	1.175	0.054	0.083	0.	0.	5.177	0.	0.

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	652	652	83	0	0	0	32	0	0
normalized size	1	1.	0.13	0.	0.	0.	0.05	0.	0.
time (sec)	N/A	1.341	0.058	0.075	0.	0.	7.129	0.	0.

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	680	97	0	0	0	32	0	0
normalized size	1	1.	0.14	0.	0.	0.	0.05	0.	0.
time (sec)	N/A	1.479	0.067	0.075	0.	0.	10.206	0.	0.

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	93	132	143	200	2023	597	85
normalized size	1	1.	0.93	1.32	1.43	2.	20.23	5.97	0.85
time (sec)	N/A	0.139	0.065	0.009	1.366	0.217	34.314	0.238	24.73

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	80	99	132	981	336	58
normalized size	1	1.	0.89	1.11	1.38	1.83	13.62	4.67	0.81
time (sec)	N/A	0.093	0.044	0.008	1.366	0.219	15.867	0.248	17.688

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	42	63	78	364	138	37
normalized size	1	1.	0.83	0.88	1.31	1.62	7.58	2.88	0.77
time (sec)	N/A	0.063	0.029	0.007	1.363	0.219	5.834	0.241	11.053

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	22	0	34	97	28	15
normalized size	1	1.	0.96	0.96	0.	1.48	4.22	1.22	0.65
time (sec)	N/A	0.016	0.006	0.003	0.	0.216	2.133	0.228	2.663

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	51	0	0	0	39	0	31
normalized size	1	1.	1.24	0.	0.	0.	0.95	0.	0.76
time (sec)	N/A	0.05	0.025	0.036	0.	0.	11.04	0.	6.066

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	58	0	0	0	42	0	32
normalized size	1	1.	1.38	0.	0.	0.	1.	0.	0.76
time (sec)	N/A	0.052	0.028	0.048	0.	0.	24.746	0.	6.548

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	49	49	0	0	0	26	0	37
normalized size	1	1.22	1.22	0.	0.	0.	0.65	0.	0.92
time (sec)	N/A	0.041	0.029	0.076	0.	0.	59.01	0.	7.948

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	49	49	0	0	0	26	0	37
normalized size	1	1.22	1.22	0.	0.	0.	0.65	0.	0.92
time (sec)	N/A	0.04	0.022	0.051	0.	0.	29.703	0.	7.887

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	49	49	0	0	0	26	0	37
normalized size	1	1.22	1.22	0.	0.	0.	0.65	0.	0.92
time (sec)	N/A	0.041	0.019	0.045	0.	0.	15.692	0.	8.286

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	44	44	0	0	0	22	0	34
normalized size	1	1.26	1.26	0.	0.	0.	0.63	0.	0.97
time (sec)	N/A	0.025	0.014	0.033	0.	0.	8.786	0.	4.75

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	47	47	0	0	0	26	0	37
normalized size	1	1.24	1.24	0.	0.	0.	0.68	0.	0.97
time (sec)	N/A	0.038	0.017	0.045	0.	0.	16.744	0.	7.842

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0	41
normalized size	1	1.21	1.21	0.	0.	0.	0.	0.	0.98
time (sec)	N/A	0.038	0.025	0.026	0.	0.	0.	0.	8.046

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0	41
normalized size	1	1.21	1.21	0.	0.	0.	0.	0.	0.98
time (sec)	N/A	0.039	0.023	0.027	0.	0.	0.	0.	7.733

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0	41
normalized size	1	1.21	1.21	0.	0.	0.	0.	0.	0.98
time (sec)	N/A	0.037	0.023	0.026	0.	0.	0.	0.	7.837

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0	41
normalized size	1	1.21	1.21	0.	0.	0.	0.	0.	0.98
time (sec)	N/A	0.036	0.021	0.027	0.	0.	0.	0.	7.672

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	49	49	0	0	0	0	0	39
normalized size	1	1.22	1.22	0.	0.	0.	0.	0.	0.98
time (sec)	N/A	0.036	0.021	0.028	0.	0.	0.	0.	7.931

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	49	49	0	0	0	0	0	42
normalized size	1	1.22	1.22	0.	0.	0.	0.	0.	1.05
time (sec)	N/A	0.037	0.025	0.027	0.	0.	0.	0.	7.705

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0	44
normalized size	1	1.21	1.21	0.	0.	0.	0.	0.	1.05
time (sec)	N/A	0.037	0.028	0.027	0.	0.	0.	0.	7.493

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	51	51	0	0	0	0	0	46
normalized size	1	1.21	1.21	0.	0.	0.	0.	0.	1.1
time (sec)	N/A	0.037	0.031	0.029	0.	0.	0.	0.	8.63

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	61	63	0	0	0	51	0	48
normalized size	1	1.15	1.19	0.	0.	0.	0.96	0.	0.91
time (sec)	N/A	0.046	0.049	0.08	0.	0.	111.17	0.	9.219

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	0	0	0	54	0	51
normalized size	1	1.	0.97	0.	0.	0.	0.82	0.	0.77
time (sec)	N/A	0.051	0.038	0.099	0.	0.	112.115	0.	9.445

Problem 1060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	70	66	0	0	0	0	0	54
normalized size	1	1.32	1.25	0.	0.	0.	0.	0.	1.02
time (sec)	N/A	0.053	0.054	0.083	0.	0.	0.	0.	8.673

Problem 1061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	71	81	113	143	0	0	90
normalized size	1	1.	0.68	0.77	1.08	1.36	0.	0.	0.86
time (sec)	N/A	0.133	0.092	0.01	1.343	0.228	0.	0.	17.547

Problem 1062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	70	66	0	0	0	0	0	54
normalized size	1	1.32	1.25	0.	0.	0.	0.	0.	1.02
time (sec)	N/A	0.054	0.057	0.084	0.	0.	0.	0.	9.217

Problem 1063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	46	45	80	90	0	0	56
normalized size	1	1.	0.69	0.67	1.19	1.34	0.	0.	0.84
time (sec)	N/A	0.059	0.063	0.006	1.371	0.228	0.	0.	8.777

Problem 1064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	70	66	0	0	0	0	0	54
normalized size	1	1.32	1.25	0.	0.	0.	0.	0.	1.02
time (sec)	N/A	0.054	0.055	0.085	0.	0.	0.	0.	8.581

Problem 1065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	29	50	46	0	0	26
normalized size	1	1.	1.	0.97	1.67	1.53	0.	0.	0.87
time (sec)	N/A	0.021	0.045	0.004	1.367	0.228	0.	0.	4.039

Problem 1066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	70	66	0	0	0	0	0	53
normalized size	1	1.32	1.25	0.	0.	0.	0.	0.	1.
time (sec)	N/A	0.053	0.054	0.083	0.	0.	0.	0.	9.204

Problem 1067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	56	56	0	0	0	0	0	42
normalized size	1	1.3	1.3	0.	0.	0.	0.	0.	0.98
time (sec)	N/A	0.045	0.036	0.074	0.	0.	0.	0.	8.499

Problem 1068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	69	65	0	0	0	24	0	48
normalized size	1	1.33	1.25	0.	0.	0.	0.46	0.	0.92
time (sec)	N/A	0.048	0.048	0.082	0.	0.	91.214	0.	9.07

Problem 1069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	64	62	0	0	0	0	0	44
normalized size	1	1.31	1.27	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.052	0.047	0.083	0.	0.	0.	0.	8.931

Problem 1070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	69	65	0	0	0	0	0	48
normalized size	1	1.33	1.25	0.	0.	0.	0.	0.	0.92
time (sec)	N/A	0.052	0.053	0.083	0.	0.	0.	0.	8.887

Problem 1071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	64	62	0	0	0	0	0	44
normalized size	1	1.31	1.27	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.053	0.05	0.083	0.	0.	0.	0.	8.656

## 2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [321] had the largest ratio of [ 0.6923 ]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	11	0.091
2	A	2	1	1.	11	0.091
3	A	2	1	1.	11	0.091
4	A	2	1	1.	9	0.111
5	A	1	0	1.	7	0.
6	A	2	1	1.	11	0.091
7	A	2	1	1.	11	0.091
8	A	2	1	1.	11	0.091
9	A	2	1	1.	11	0.091
10	A	2	1	1.	11	0.091
11	A	2	1	1.	11	0.091
12	A	2	1	1.	11	0.091
13	A	3	2	1.	13	0.154
14	A	2	1	1.	13	0.077
15	A	3	2	1.	13	0.154
16	A	2	1	1.	13	0.077
17	A	1	1	1.	11	0.091
18	A	2	1	1.	9	0.111
19	A	3	2	1.	13	0.154
20	A	2	1	1.	13	0.077
21	A	3	2	1.	13	0.154
22	A	2	1	1.	13	0.077
23	A	3	2	1.	13	0.154
24	A	2	1	1.	13	0.077
25	A	1	1	1.	13	0.077
26	A	2	1	1.	13	0.077
27	A	3	2	1.	13	0.154
28	A	2	1	1.	13	0.077
29	A	3	2	1.	13	0.154
30	A	3	2	1.	13	0.154
31	A	3	2	1.	13	0.154
32	A	3	2	1.	13	0.154

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
33	A	1	1	1.	11	0.091
34	A	3	2	1.	13	0.154
35	A	3	2	1.	13	0.154
36	A	3	2	1.	13	0.154
37	A	3	2	1.	13	0.154
38	A	1	1	1.	13	0.077
39	A	3	3	1.	13	0.231
40	A	3	2	1.	13	0.154
41	A	3	2	1.	13	0.154
42	A	2	1	1.	13	0.077
43	A	2	1	1.	13	0.077
44	A	2	1	1.	13	0.077
45	A	2	1	1.	9	0.111
46	A	2	1	1.	13	0.077
47	A	2	1	1.	13	0.077
48	A	2	1	1.	13	0.077
49	A	2	1	1.	13	0.077
50	A	2	1	1.	13	0.077
51	A	2	1	1.	13	0.077
52	A	3	2	1.	13	0.154
53	A	3	2	1.	13	0.154
54	A	3	2	1.	13	0.154
55	A	3	2	1.	13	0.154
56	A	3	2	1.	13	0.154
57	A	3	2	1.	13	0.154
58	A	1	1	1.	11	0.091
59	A	3	2	1.	13	0.154
60	A	3	2	1.	13	0.154
61	A	3	2	1.	13	0.154
62	A	3	2	1.	13	0.154
63	A	3	2	1.	13	0.154
64	A	3	2	1.	13	0.154
65	A	1	1	1.	13	0.077
66	A	3	3	1.	13	0.231
67	A	4	3	1.	13	0.231
68	A	3	2	1.	13	0.154
69	A	3	2	1.	13	0.154
70	A	2	1	1.	13	0.077
71	A	2	1	1.	13	0.077
72	A	2	1	1.	13	0.077
73	A	2	1	1.	13	0.077
74	A	2	1	1.	9	0.111
75	A	2	1	1.	13	0.077
76	A	2	1	1.	13	0.077
77	A	2	1	1.	13	0.077
78	A	2	1	1.	13	0.077
79	A	2	1	1.	13	0.077
80	A	2	1	1.	13	0.077
81	A	2	1	1.	13	0.077

Continued on next page



Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
82	A	2	1	1.	13	0.077
83	A	2	1	1.	13	0.077
84	A	2	1	1.	13	0.077
85	A	3	2	1.	13	0.154
86	A	3	2	1.	13	0.154
87	A	3	2	1.	13	0.154
88	A	3	2	1.	13	0.154
89	A	3	2	1.	13	0.154
90	A	3	2	1.	13	0.154
91	A	1	1	1.	11	0.091
92	A	3	2	1.	13	0.154
93	A	3	2	1.	13	0.154
94	A	3	2	1.	13	0.154
95	A	3	2	1.	13	0.154
96	A	3	2	1.	13	0.154
97	A	3	2	1.	13	0.154
98	A	3	2	1.	13	0.154
99	A	3	2	1.	13	0.154
100	A	3	2	1.	13	0.154
101	A	1	1	1.	13	0.077
102	A	3	3	1.	13	0.231
103	A	4	3	1.	13	0.231
104	A	5	3	1.	13	0.231
105	A	6	3	1.	13	0.231
106	A	3	2	1.	13	0.154
107	A	3	2	1.	13	0.154
108	A	3	2	1.	13	0.154
109	A	2	1	1.	13	0.077
110	A	2	1	1.	13	0.077
111	A	2	1	1.	13	0.077
112	A	2	1	1.	13	0.077
113	A	2	1	1.	9	0.111
114	A	2	1	1.	13	0.077
115	A	2	1	1.	13	0.077
116	A	2	1	1.	13	0.077
117	A	2	1	1.	13	0.077
118	A	2	1	1.	13	0.077
119	A	2	1	1.	13	0.077
120	A	2	1	1.	13	0.077
121	A	2	1	1.	13	0.077
122	A	2	1	1.	13	0.077
123	A	2	1	1.	13	0.077
124	A	3	2	1.	13	0.154
125	A	3	2	1.	13	0.154
126	A	3	2	1.	13	0.154
127	A	3	2	1.	13	0.154
128	A	3	2	1.	13	0.154
129	A	3	2	1.	13	0.154
130	A	3	2	1.	13	0.154

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	3	2	1.	13	0.154
132	A	3	2	1.	13	0.154
133	A	2	2	1.	13	0.154
134	A	1	1	1.	11	0.091
135	A	1	1	1.	9	0.111
136	A	4	4	1.	13	0.308
137	A	2	2	1.	13	0.154
138	A	3	2	1.	13	0.154
139	A	3	2	1.	13	0.154
140	A	3	2	1.	13	0.154
141	A	4	2	1.	13	0.154
142	A	3	2	1.	13	0.154
143	A	5	2	1.	13	0.154
144	A	3	2	1.	13	0.154
145	A	3	2	1.	13	0.154
146	A	4	3	1.	13	0.231
147	A	3	2	1.	13	0.154
148	A	4	3	1.	13	0.231
149	A	3	2	1.	13	0.154
150	A	4	3	1.	13	0.231
151	A	3	2	1.	13	0.154
152	A	4	3	1.	13	0.231
153	A	3	2	1.	13	0.154
154	A	3	3	1.	13	0.231
155	A	3	2	1.	13	0.154
156	A	2	2	1.	13	0.154
157	A	1	1	1.	11	0.091
158	A	2	2	1.	9	0.222
159	A	3	2	1.	13	0.154
160	A	3	3	1.	13	0.231
161	A	3	2	1.	13	0.154
162	A	4	3	1.	13	0.231
163	A	3	2	1.	13	0.154
164	A	5	3	1.	13	0.231
165	A	3	2	1.	13	0.154
166	A	6	3	1.	13	0.231
167	A	3	2	1.	13	0.154
168	A	3	2	1.	13	0.154
169	A	3	2	1.	13	0.154
170	A	3	2	1.	13	0.154
171	A	3	2	1.	13	0.154
172	A	3	2	1.	13	0.154
173	A	3	2	1.	13	0.154
174	A	1	1	1.	13	0.077
175	A	1	1	1.	11	0.091
176	A	3	2	1.	13	0.154
177	A	3	2	1.	13	0.154
178	A	3	2	1.	13	0.154
179	A	3	2	1.	13	0.154

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	3	2	1.	13	0.154
181	A	5	3	1.	13	0.231
182	A	5	3	1.	13	0.231
183	A	5	3	1.	13	0.231
184	A	4	3	1.	13	0.231
185	A	3	2	1.	13	0.154
186	A	3	3	1.	13	0.231
187	A	3	2	1.	9	0.222
188	A	4	3	1.	13	0.231
189	A	5	3	1.	13	0.231
190	A	6	3	1.	13	0.231
191	A	7	3	1.	13	0.231
192	A	3	2	1.	13	0.154
193	A	3	2	1.	13	0.154
194	A	3	2	1.	13	0.154
195	A	3	2	1.	13	0.154
196	A	1	1	1.	13	0.077
197	A	3	3	1.	13	0.231
198	A	4	3	1.	13	0.231
199	A	5	3	1.	13	0.231
200	A	3	2	1.	13	0.154
201	A	3	2	1.	13	0.154
202	A	3	2	1.	13	0.154
203	A	3	2	1.	13	0.154
204	A	1	1	1.	11	0.091
205	A	3	2	1.	13	0.154
206	A	3	2	1.	13	0.154
207	A	3	2	1.	13	0.154
208	A	3	2	1.	13	0.154
209	A	12	3	1.	13	0.231
210	A	12	3	1.	13	0.231
211	A	11	3	1.	13	0.231
212	A	10	2	1.	13	0.154
213	A	10	3	1.	13	0.231
214	A	10	3	1.	13	0.231
215	A	10	3	1.	13	0.231
216	A	10	3	1.	13	0.231
217	A	10	3	1.	13	0.231
218	A	10	3	1.	13	0.231
219	A	10	3	1.	13	0.231
220	A	10	3	1.	13	0.231
221	A	10	2	1.	9	0.222
222	A	11	3	1.	13	0.231
223	A	12	3	1.	13	0.231
224	A	13	3	1.	13	0.231
225	A	3	2	1.	14	0.143
226	A	2	2	1.	14	0.143
227	A	1	1	1.	12	0.083
228	A	1	1	1.	10	0.1

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
229	A	4	4	1.	14	0.286
230	A	2	2	1.	14	0.143
231	A	3	2	1.	14	0.143
232	A	3	2	1.	14	0.143
233	A	2	2	1.	14	0.143
234	A	1	1	1.	12	0.083
235	A	2	2	1.	10	0.2
236	A	3	2	1.	14	0.143
237	A	3	3	1.	14	0.214
238	A	3	2	1.	14	0.143
239	A	1	1	1.	14	0.071
240	A	3	3	1.	14	0.214
241	A	1	1	1.	12	0.083
242	A	3	2	1.	10	0.2
243	A	3	2	1.	14	0.143
244	A	4	3	1.	14	0.214
245	A	3	2	1.	14	0.143
246	A	3	2	1.	14	0.143
247	A	5	3	1.	14	0.214
248	A	1	1	1.	12	0.083
249	A	5	2	1.	10	0.2
250	A	3	2	1.	14	0.143
251	A	6	3	1.	14	0.214
252	A	3	2	1.	14	0.143
253	A	4	4	1.	13	0.308
254	A	4	4	1.	13	0.308
255	A	3	2	1.	13	0.154
256	A	3	2	1.	13	0.154
257	A	1	1	1.	10	0.1
258	A	1	1	1.	18	0.056
259	A	3	2	1.	13	0.154
260	A	3	2	1.	13	0.154
261	A	1	1	1.	14	0.071
262	A	1	1	1.	15	0.067
263	A	1	1	1.	20	0.05
264	A	2	1	1.	13	0.077
265	A	2	1	1.	13	0.077
266	A	2	1	1.	13	0.077
267	A	2	1	1.	13	0.077
268	A	2	1	1.	13	0.077
269	A	2	1	1.	13	0.077
270	A	2	1	1.	13	0.077
271	A	2	1	1.	13	0.077
272	A	2	1	1.	15	0.067
273	A	2	1	1.	15	0.067
274	A	2	1	1.	15	0.067
275	A	2	1	1.	15	0.067
276	A	2	1	1.	15	0.067
277	A	2	1	1.	15	0.067

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	2	1	1.	15	0.067
279	A	2	1	1.	15	0.067
280	A	2	1	1.	15	0.067
281	A	2	1	1.	15	0.067
282	A	2	1	1.	15	0.067
283	A	2	1	1.	15	0.067
284	A	2	1	1.	15	0.067
285	A	2	1	1.	15	0.067
286	A	2	1	1.	15	0.067
287	A	2	1	1.	15	0.067
288	A	12	8	1.	15	0.533
289	A	11	8	1.	15	0.533
290	A	11	8	1.	15	0.533
291	A	10	7	1.	15	0.467
292	A	10	7	1.	15	0.467
293	A	11	8	1.	15	0.533
294	A	11	8	1.	15	0.533
295	A	12	8	1.	15	0.533
296	A	12	9	1.	15	0.6
297	A	11	8	1.	15	0.533
298	A	11	8	1.	15	0.533
299	A	11	8	1.	15	0.533
300	A	11	8	1.	15	0.533
301	A	12	9	1.	15	0.6
302	A	12	9	1.	15	0.6
303	A	13	9	1.	15	0.6
304	A	12	8	1.	15	0.533
305	A	12	9	1.	15	0.6
306	A	12	9	1.	15	0.6
307	A	12	8	1.	15	0.533
308	A	12	8	1.	15	0.533
309	A	13	9	1.	15	0.6
310	A	13	9	1.	15	0.6
311	A	14	9	1.	15	0.6
312	A	4	4	1.	16	0.25
313	A	12	8	1.	13	0.615
314	A	11	8	1.	13	0.615
315	A	11	8	1.	13	0.615
316	A	10	7	1.	13	0.538
317	A	10	7	1.	13	0.538
318	A	11	8	1.	13	0.615
319	A	11	8	1.	13	0.615
320	A	12	8	1.	13	0.615
321	A	12	9	1.	13	0.692
322	A	11	8	1.	13	0.615
323	A	11	8	1.	13	0.615
324	A	11	8	1.	13	0.615
325	A	11	8	1.	13	0.615
326	A	12	9	1.	13	0.692

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
327	A	12	9	1.	13	0.692
328	A	13	9	1.	13	0.692
329	A	12	8	1.	13	0.615
330	A	12	9	1.	13	0.692
331	A	12	9	1.	13	0.692
332	A	12	8	1.	13	0.615
333	A	12	8	1.	13	0.615
334	A	13	9	1.	13	0.692
335	A	13	9	1.	13	0.692
336	A	14	9	1.	13	0.692
337	A	4	4	1.	15	0.267
338	A	11	7	1.37	13	0.538
339	A	2	1	1.	13	0.077
340	A	2	1	1.	13	0.077
341	A	2	1	1.	13	0.077
342	A	2	1	1.	13	0.077
343	A	2	1	1.	11	0.091
344	A	1	1	1.	13	0.077
345	A	1	1	1.	13	0.077
346	A	1	1	1.	13	0.077
347	A	1	1	1.	17	0.059
348	A	1	1	1.	15	0.067
349	A	1	1	1.	17	0.059
350	A	1	1	1.	17	0.059
351	A	1	1	1.	17	0.059
352	A	1	1	1.	16	0.062
353	A	3	2	1.	15	0.133
354	A	3	2	1.	15	0.133
355	A	3	2	1.	15	0.133
356	A	1	1	1.	13	0.077
357	A	4	4	1.	15	0.267
358	A	4	4	1.	15	0.267
359	A	5	5	1.	15	0.333
360	A	6	5	1.	15	0.333
361	A	5	4	1.	15	0.267
362	A	4	4	1.	15	0.267
363	A	3	3	1.	11	0.273
364	A	3	3	1.	15	0.2
365	A	1	1	1.	15	0.067
366	A	2	2	1.	15	0.133
367	A	3	2	1.	15	0.133
368	A	4	2	1.	15	0.133
369	A	3	2	1.	15	0.133
370	A	3	2	1.	15	0.133
371	A	3	2	1.	15	0.133
372	A	1	1	1.	13	0.077
373	A	5	4	1.	15	0.267
374	A	5	5	1.	15	0.333
375	A	5	4	1.	15	0.267

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	6	5	1.	15	0.333
377	A	7	5	1.	15	0.333
378	A	6	4	1.	15	0.267
379	A	5	4	1.	15	0.267
380	A	4	3	1.	11	0.273
381	A	4	4	1.	15	0.267
382	A	4	3	1.	15	0.2
383	A	1	1	1.	15	0.067
384	A	2	2	1.	15	0.133
385	A	3	2	1.	15	0.133
386	A	4	2	1.	15	0.133
387	A	3	2	1.	15	0.133
388	A	3	2	1.	15	0.133
389	A	3	2	1.	15	0.133
390	A	1	1	1.	13	0.077
391	A	6	4	1.	15	0.267
392	A	6	5	1.	15	0.333
393	A	6	5	1.	15	0.333
394	A	6	4	1.	15	0.267
395	A	7	5	1.	15	0.333
396	A	8	5	1.	15	0.333
397	A	7	4	1.	15	0.267
398	A	6	4	1.	15	0.267
399	A	5	3	1.	11	0.273
400	A	5	4	1.	15	0.267
401	A	5	4	1.	15	0.267
402	A	5	3	1.	15	0.2
403	A	1	1	1.	15	0.067
404	A	2	2	1.	15	0.133
405	A	3	2	1.	15	0.133
406	A	4	2	1.	15	0.133
407	A	5	2	1.	15	0.133
408	A	6	2	1.	15	0.133
409	A	3	2	1.	15	0.133
410	A	3	2	1.	15	0.133
411	A	3	2	1.	15	0.133
412	A	3	2	1.	15	0.133
413	A	3	2	1.	15	0.133
414	A	3	2	1.	15	0.133
415	A	3	2	1.	15	0.133
416	A	1	1	1.	13	0.077
417	A	8	4	1.	15	0.267
418	A	8	5	1.	15	0.333
419	A	8	5	1.	15	0.333
420	A	8	5	1.	15	0.333
421	A	8	5	1.	15	0.333
422	A	8	4	1.	15	0.267
423	A	9	5	1.	15	0.333
424	A	10	5	1.	15	0.333

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
425	A	10	4	1.	15	0.267
426	A	9	4	1.	15	0.267
427	A	8	4	1.	15	0.267
428	A	7	3	1.	11	0.273
429	A	7	4	1.	15	0.267
430	A	7	4	1.	15	0.267
431	A	7	4	1.	15	0.267
432	A	7	4	1.	15	0.267
433	A	7	3	1.	15	0.2
434	A	1	1	1.	15	0.067
435	A	2	2	1.	15	0.133
436	A	3	2	1.	15	0.133
437	A	4	2	1.	15	0.133
438	A	5	2	1.	15	0.133
439	A	6	2	1.	15	0.133
440	A	7	2	1.	15	0.133
441	A	3	2	1.	15	0.133
442	A	4	3	1.	15	0.2
443	A	3	2	1.	15	0.133
444	A	3	3	1.	15	0.2
445	A	1	1	1.	13	0.077
446	A	2	2	1.	11	0.182
447	A	4	4	1.	15	0.267
448	A	2	2	1.	15	0.133
449	A	4	4	1.	15	0.267
450	A	1	1	1.	15	0.067
451	A	5	5	1.	15	0.333
452	A	3	2	1.	15	0.133
453	A	4	3	1.	15	0.2
454	A	3	2	1.	15	0.133
455	A	3	3	1.	15	0.2
456	A	1	1	1.	13	0.077
457	A	2	2	1.	11	0.182
458	A	4	4	1.	15	0.267
459	A	2	2	1.	15	0.133
460	A	4	4	1.	15	0.267
461	A	1	1	1.	15	0.067
462	A	5	5	1.	15	0.333
463	A	3	2	1.	15	0.133
464	A	5	4	1.	15	0.267
465	A	3	2	1.	15	0.133
466	A	4	4	1.	15	0.267
467	A	1	1	1.	13	0.077
468	A	3	3	1.	11	0.273
469	A	4	4	1.	15	0.267
470	A	3	3	1.	15	0.2
471	A	4	4	1.	15	0.267
472	A	1	1	1.	15	0.067
473	A	5	5	1.	15	0.333

Continued on next page



Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
474	A	3	2	1.	15	0.133
475	A	5	4	1.	15	0.267
476	A	3	2	1.	15	0.133
477	A	4	4	1.	15	0.267
478	A	1	1	1.	13	0.077
479	A	3	3	1.	11	0.273
480	A	4	4	1.	15	0.267
481	A	3	3	1.	15	0.2
482	A	4	4	1.	15	0.267
483	A	1	1	1.	15	0.067
484	A	5	5	1.	15	0.333
485	A	3	2	1.	15	0.133
486	A	4	3	1.	15	0.2
487	A	3	2	1.	15	0.133
488	A	3	3	1.	15	0.2
489	A	1	1	1.	13	0.077
490	A	2	2	1.	11	0.182
491	A	3	3	1.	15	0.2
492	A	1	1	1.	15	0.067
493	A	4	4	1.	15	0.267
494	A	2	2	1.	15	0.133
495	A	5	4	1.	15	0.267
496	A	3	2	1.	15	0.133
497	A	4	4	1.	15	0.267
498	A	3	2	1.	15	0.133
499	A	3	3	1.	15	0.2
500	A	1	1	1.	13	0.077
501	A	1	1	1.	11	0.091
502	A	4	4	1.	15	0.267
503	A	2	2	1.	15	0.133
504	A	5	4	1.	15	0.267
505	A	3	2	1.	15	0.133
506	A	5	4	1.	15	0.267
507	A	3	2	1.	15	0.133
508	A	4	3	1.	15	0.2
509	A	3	2	1.	15	0.133
510	A	1	1	1.	15	0.067
511	A	1	1	1.	13	0.077
512	A	2	2	1.	11	0.182
513	A	5	4	1.	15	0.267
514	A	3	3	1.	15	0.2
515	A	6	4	1.	15	0.267
516	A	4	3	1.	15	0.2
517	A	7	4	1.	15	0.267
518	A	3	2	1.	15	0.133
519	A	6	3	1.	15	0.2
520	A	3	2	1.	15	0.133
521	A	1	1	1.	15	0.067
522	A	3	2	1.	15	0.133

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
523	A	2	2	1.	15	0.133
524	A	3	2	1.	15	0.133
525	A	3	2	1.	15	0.133
526	A	1	1	1.	13	0.077
527	A	4	2	1.	11	0.182
528	A	7	4	1.	15	0.267
529	A	5	3	1.	15	0.2
530	A	8	4	1.	15	0.267
531	A	6	3	1.	15	0.2
532	A	3	2	1.	15	0.133
533	A	3	2	1.	15	0.133
534	A	3	2	1.	15	0.133
535	A	2	2	1.	15	0.133
536	A	1	1	1.	13	0.077
537	A	1	1	1.	11	0.091
538	A	3	3	1.	15	0.2
539	A	1	1	1.	15	0.067
540	A	4	4	1.	15	0.267
541	A	2	2	1.	15	0.133
542	A	5	4	1.	15	0.267
543	A	3	2	1.	15	0.133
544	A	3	2	1.	15	0.133
545	A	3	2	1.	15	0.133
546	A	2	2	1.	15	0.133
547	A	1	1	1.	13	0.077
548	A	1	1	1.	11	0.091
549	A	3	3	1.	15	0.2
550	A	1	1	1.	15	0.067
551	A	4	4	1.	15	0.267
552	A	2	2	1.	15	0.133
553	A	5	4	1.	15	0.267
554	A	3	2	1.	15	0.133
555	A	4	3	1.	15	0.2
556	A	3	2	1.	15	0.133
557	A	3	3	1.	15	0.2
558	A	1	1	1.	13	0.077
559	A	2	2	1.	11	0.182
560	A	3	3	1.	15	0.2
561	A	1	1	1.	15	0.067
562	A	4	4	1.	15	0.267
563	A	2	2	1.	15	0.133
564	A	5	4	1.	15	0.267
565	A	3	2	1.	15	0.133
566	A	4	3	1.	15	0.2
567	A	3	2	1.	15	0.133
568	A	3	3	1.	15	0.2
569	A	1	1	1.	13	0.077
570	A	2	2	1.	11	0.182
571	A	3	3	1.	15	0.2

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
572	A	1	1	1.	15	0.067
573	A	4	4	1.	15	0.267
574	A	2	2	1.	15	0.133
575	A	5	4	1.	15	0.267
576	A	1	1	1.	11	0.091
577	A	1	1	1.	12	0.083
578	A	2	2	1.	11	0.182
579	A	2	2	1.	12	0.167
580	A	1	1	1.	11	0.091
581	A	1	1	1.	12	0.083
582	A	2	2	1.	13	0.154
583	A	2	2	1.	14	0.143
584	A	2	2	1.	11	0.182
585	A	2	2	1.	12	0.167
586	A	2	2	1.	13	0.154
587	A	2	2	1.	14	0.143
588	A	2	2	1.	13	0.154
589	A	5	4	1.	19	0.21
590	A	6	6	1.	19	0.316
591	A	4	4	1.	19	0.21
592	A	5	5	1.	19	0.263
593	A	3	3	1.	19	0.158
594	A	5	5	1.	19	0.263
595	A	3	3	1.	19	0.158
596	A	6	6	1.	19	0.316
597	A	6	4	1.	19	0.21
598	A	7	6	1.	19	0.316
599	A	5	4	1.	19	0.21
600	A	6	5	1.	19	0.263
601	A	4	3	1.	19	0.158
602	A	6	6	1.	19	0.316
603	A	4	4	1.	19	0.21
604	A	6	5	1.	19	0.263
605	A	4	3	1.	19	0.158
606	A	7	6	1.	19	0.316
607	A	6	6	1.	22	0.273
608	A	5	5	1.	22	0.227
609	A	5	5	1.	22	0.227
610	A	4	4	1.	22	0.182
611	A	5	5	1.	22	0.227
612	A	4	4	1.	22	0.182
613	A	4	3	1.	19	0.158
614	A	5	5	1.	19	0.263
615	A	3	3	1.	19	0.158
616	A	4	4	1.	19	0.21
617	A	2	2	1.	19	0.105
618	A	5	5	1.	19	0.263
619	A	3	3	1.	19	0.158
620	A	6	5	1.	19	0.263

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	4	4	1.	19	0.21
622	A	5	5	1.	19	0.263
623	A	3	3	1.	19	0.158
624	A	5	5	1.	19	0.263
625	A	3	3	1.	19	0.158
626	A	6	6	1.	19	0.316
627	A	4	4	1.	19	0.21
628	A	7	6	1.	19	0.316
629	A	4	3	1.	19	0.158
630	A	6	6	1.	19	0.316
631	A	4	4	1.	19	0.21
632	A	6	5	1.	19	0.263
633	A	4	3	1.	19	0.158
634	A	7	6	1.	19	0.316
635	A	5	4	1.	19	0.21
636	A	8	6	1.	19	0.316
637	A	5	5	1.	22	0.227
638	A	4	4	1.	22	0.182
639	A	4	4	1.	22	0.182
640	A	3	3	1.	22	0.136
641	A	5	5	1.	22	0.227
642	A	4	4	1.	22	0.182
643	A	5	5	1.	22	0.227
644	A	4	4	1.	22	0.182
645	A	5	5	1.	22	0.227
646	A	4	4	1.	22	0.182
647	A	6	6	1.	22	0.273
648	A	5	5	1.	22	0.227
649	A	2	2	1.	20	0.1
650	A	2	2	1.	17	0.118
651	A	2	2	1.28	15	0.133
652	A	2	2	1.26	15	0.133
653	A	2	2	1.26	15	0.133
654	A	2	2	1.38	15	0.133
655	A	2	2	1.32	15	0.133
656	A	2	2	1.26	17	0.118
657	A	2	2	1.26	17	0.118
658	A	2	2	1.26	15	0.133
659	A	2	2	1.24	17	0.118
660	A	2	2	1.29	17	0.118
661	A	1	1	1.	31	0.032
662	C	5	2	7.47	43	0.047
663	A	1	1	1.	29	0.034
664	C	5	2	8.2	38	0.053
665	A	3	2	1.	15	0.133
666	A	3	2	1.	15	0.133
667	A	3	2	1.	15	0.133
668	A	1	1	1.	13	0.077
669	A	6	6	1.	15	0.4

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
670	A	6	6	1.	15	0.4
671	A	7	7	1.	15	0.467
672	A	5	4	1.	15	0.267
673	A	4	4	1.	15	0.267
674	A	3	3	1.	11	0.273
675	A	3	3	1.	15	0.2
676	A	4	4	1.	15	0.267
677	A	3	2	1.	15	0.133
678	A	3	2	1.	15	0.133
679	A	3	2	1.	15	0.133
680	A	1	1	1.	13	0.077
681	A	6	6	1.	15	0.4
682	A	6	6	1.	15	0.4
683	A	7	7	1.	15	0.467
684	A	7	6	1.	15	0.4
685	A	6	6	1.	15	0.4
686	A	5	5	1.	11	0.454
687	A	5	5	1.	15	0.333
688	A	6	6	1.	15	0.4
689	A	3	2	1.	15	0.133
690	A	3	2	1.	15	0.133
691	A	3	2	1.	15	0.133
692	A	1	1	1.	13	0.077
693	A	7	6	1.	15	0.4
694	A	7	7	1.	15	0.467
695	A	7	6	1.	15	0.4
696	A	6	4	1.	15	0.267
697	A	5	4	1.	15	0.267
698	A	4	3	1.	11	0.273
699	A	4	4	1.	15	0.267
700	A	4	3	1.	15	0.2
701	A	1	1	1.	11	0.091
702	A	3	2	1.	15	0.133
703	A	3	2	1.	15	0.133
704	A	3	2	1.	15	0.133
705	A	1	1	1.	13	0.077
706	A	5	5	1.	15	0.333
707	A	6	6	1.	15	0.4
708	A	7	6	1.	15	0.4
709	A	6	5	1.	15	0.333
710	A	5	5	1.	15	0.333
711	A	4	4	1.	11	0.364
712	A	5	5	1.	15	0.333
713	A	6	5	1.	15	0.333
714	A	3	2	1.	15	0.133
715	A	3	2	1.	15	0.133
716	A	3	2	1.	15	0.133
717	A	1	1	1.	13	0.077
718	A	5	5	1.	15	0.333

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
719	A	6	6	1.	15	0.4
720	A	7	6	1.	15	0.4
721	A	4	3	1.	15	0.2
722	A	3	3	1.	15	0.2
723	A	2	2	1.	11	0.182
724	A	3	3	1.	15	0.2
725	A	4	3	1.	15	0.2
726	A	3	2	1.	15	0.133
727	A	3	2	1.	15	0.133
728	A	3	2	1.	15	0.133
729	A	1	1	1.	13	0.077
730	A	6	6	1.	15	0.4
731	A	7	6	1.	15	0.4
732	A	8	6	1.	15	0.4
733	A	6	6	1.	15	0.4
734	A	5	5	1.	15	0.333
735	A	5	5	1.	11	0.454
736	A	6	6	1.	15	0.4
737	A	7	6	1.	15	0.4
738	A	12	11	1.	19	0.579
739	A	11	11	1.	19	0.579
740	A	10	10	1.	19	0.526
741	A	10	10	1.	19	0.526
742	A	1	1	1.	19	0.053
743	A	2	2	1.	19	0.105
744	A	3	2	1.	19	0.105
745	A	4	2	1.	19	0.105
746	A	6	5	1.	19	0.263
747	A	5	5	1.	19	0.263
748	A	4	4	1.	19	0.21
749	A	4	4	1.	19	0.21
750	A	5	5	1.	19	0.263
751	A	2	2	1.	19	0.105
752	A	2	2	1.	19	0.105
753	A	2	2	1.	19	0.105
754	A	13	11	1.	19	0.579
755	A	12	11	1.	19	0.579
756	A	11	10	1.	19	0.526
757	A	11	11	1.	19	0.579
758	A	11	10	1.	19	0.526
759	A	1	1	1.	19	0.053
760	A	2	2	1.	19	0.105
761	A	3	2	1.	19	0.105
762	A	7	5	1.	19	0.263
763	A	6	5	1.	19	0.263
764	A	5	4	1.	19	0.21
765	A	5	5	1.	19	0.263
766	A	5	4	1.	19	0.21
767	A	6	5	1.	19	0.263

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
768	A	2	2	1.	19	0.105
769	A	2	2	1.	19	0.105
770	A	2	2	1.	19	0.105
771	A	12	10	1.	19	0.526
772	A	11	10	1.	19	0.526
773	A	10	10	1.	19	0.526
774	A	9	9	1.	19	0.474
775	A	1	1	1.	19	0.053
776	A	2	2	1.	19	0.105
777	A	3	2	1.	19	0.105
778	A	4	2	1.	19	0.105
779	A	5	4	1.	19	0.21
780	A	4	4	1.	19	0.21
781	A	3	3	1.	19	0.158
782	A	4	4	1.	19	0.21
783	A	5	4	1.	19	0.21
784	A	2	2	1.	19	0.105
785	A	2	2	1.	19	0.105
786	A	2	2	1.	19	0.105
787	A	5	4	1.	15	0.267
788	A	4	4	1.	15	0.267
789	A	3	3	1.	11	0.273
790	A	3	3	1.	15	0.2
791	A	4	4	1.	15	0.267
792	A	5	4	1.	15	0.267
793	A	5	4	1.	16	0.25
794	A	4	4	1.	16	0.25
795	A	3	3	1.	12	0.25
796	A	3	3	1.	16	0.188
797	A	4	4	1.	16	0.25
798	A	5	4	1.	16	0.25
799	A	6	5	1.	15	0.333
800	A	5	5	1.	15	0.333
801	A	4	4	1.	11	0.364
802	A	4	4	1.	15	0.267
803	A	5	5	1.	15	0.333
804	A	6	5	1.	15	0.333
805	A	5	4	1.	16	0.25
806	A	4	4	1.	16	0.25
807	A	3	3	1.	12	0.25
808	A	3	3	1.	16	0.188
809	A	4	4	1.	16	0.25
810	A	5	4	1.	16	0.25
811	A	4	3	1.	11	0.273
812	A	4	3	1.	12	0.25
813	A	5	4	1.	11	0.364
814	A	4	3	1.	12	0.25
815	A	6	4	1.	15	0.267
816	A	5	4	1.	15	0.267

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
817	A	4	4	1.	15	0.267
818	A	3	3	1.	11	0.273
819	A	4	4	1.	15	0.267
820	A	5	4	1.	15	0.267
821	A	6	4	1.	15	0.267
822	A	5	3	1.	16	0.188
823	A	4	3	1.	16	0.188
824	A	3	3	1.	16	0.188
825	A	2	2	1.	12	0.167
826	A	3	3	1.	16	0.188
827	A	4	3	1.	16	0.188
828	A	5	3	1.	16	0.188
829	A	5	3	1.	15	0.2
830	A	4	3	1.	15	0.2
831	A	3	3	1.	15	0.2
832	A	2	2	1.	11	0.182
833	A	3	3	1.	15	0.2
834	A	4	3	1.	15	0.2
835	A	5	3	1.	15	0.2
836	A	5	3	1.	16	0.188
837	A	4	3	1.	16	0.188
838	A	3	3	1.	16	0.188
839	A	2	2	1.	12	0.167
840	A	3	3	1.	16	0.188
841	A	4	3	1.	16	0.188
842	A	5	3	1.	16	0.188
843	A	5	3	1.	15	0.2
844	A	4	3	1.	15	0.2
845	A	3	3	1.	15	0.2
846	A	2	2	1.	11	0.182
847	A	3	3	1.	15	0.2
848	A	4	3	1.	15	0.2
849	A	5	3	1.	15	0.2
850	A	5	4	1.	16	0.25
851	A	4	4	1.	16	0.25
852	A	3	3	1.	16	0.188
853	A	3	3	1.	12	0.25
854	A	4	4	1.	16	0.25
855	A	5	4	1.	16	0.25
856	A	6	4	1.	16	0.25
857	A	3	3	1.	11	0.273
858	A	3	3	1.	11	0.273
859	A	4	3	1.	11	0.273
860	A	3	3	1.	12	0.25
861	A	4	3	1.	12	0.25
862	A	4	3	1.	12	0.25
863	A	5	3	1.	15	0.2
864	A	4	3	1.	15	0.2
865	A	3	3	1.	15	0.2

Continued on next page



Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
866	A	2	2	1.	11	0.182
867	A	3	3	1.	15	0.2
868	A	4	3	1.	15	0.2
869	A	5	3	1.	15	0.2
870	A	4	2	1.	15	0.133
871	A	3	2	1.	15	0.133
872	A	2	2	1.	15	0.133
873	A	1	1	1.	11	0.091
874	A	2	2	1.	15	0.133
875	A	3	2	1.	15	0.133
876	A	4	2	1.	15	0.133
877	A	4	2	1.	15	0.133
878	A	3	2	1.	15	0.133
879	A	2	2	1.	15	0.133
880	A	1	1	1.	11	0.091
881	A	2	2	1.	15	0.133
882	A	3	2	1.	15	0.133
883	A	4	2	1.	15	0.133
884	A	4	2	1.	15	0.133
885	A	3	2	1.	15	0.133
886	A	2	2	1.	15	0.133
887	A	1	1	1.	11	0.091
888	A	2	2	1.	15	0.133
889	A	3	2	1.	15	0.133
890	A	4	2	1.	15	0.133
891	A	7	5	1.	15	0.333
892	A	6	5	1.	15	0.333
893	A	5	5	1.	15	0.333
894	A	4	4	1.	11	0.364
895	A	5	5	1.	15	0.333
896	A	6	5	1.	15	0.333
897	A	7	5	1.	15	0.333
898	A	7	5	1.	15	0.333
899	A	6	5	1.	15	0.333
900	A	5	5	1.	15	0.333
901	A	4	4	1.	11	0.364
902	A	5	5	1.	15	0.333
903	A	6	5	1.	15	0.333
904	A	7	5	1.	15	0.333
905	A	5	3	1.	15	0.2
906	A	4	3	1.	15	0.2
907	A	3	3	1.	15	0.2
908	A	2	2	1.	11	0.182
909	A	3	3	1.	15	0.2
910	A	4	3	1.	15	0.2
911	A	5	3	1.	15	0.2
912	A	5	3	1.	15	0.2
913	A	4	3	1.	15	0.2
914	A	3	3	1.	15	0.2

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
915	A	2	2	1.	11	0.182
916	A	3	3	1.	15	0.2
917	A	4	3	1.	15	0.2
918	A	5	3	1.	15	0.2
919	A	8	7	1.	19	0.368
920	A	7	7	1.	19	0.368
921	A	6	6	1.	19	0.316
922	A	6	6	1.	19	0.316
923	A	7	7	1.	19	0.368
924	A	8	7	1.	19	0.368
925	A	7	7	1.	19	0.368
926	A	6	6	1.	19	0.316
927	A	6	6	1.	19	0.316
928	A	1	1	1.	19	0.053
929	A	2	2	1.	19	0.105
930	A	3	2	1.	19	0.105
931	A	4	2	1.	19	0.105
932	A	7	7	1.	20	0.35
933	A	6	6	1.	20	0.3
934	A	6	6	1.	20	0.3
935	A	7	7	1.	20	0.35
936	A	8	7	1.	20	0.35
937	A	13	10	1.	20	0.5
938	A	12	9	1.	20	0.45
939	A	12	9	1.	20	0.45
940	A	1	1	1.	20	0.05
941	A	2	2	1.	20	0.1
942	A	3	2	1.	20	0.1
943	A	4	2	1.	20	0.1
944	A	6	6	1.	19	0.316
945	A	5	5	1.	19	0.263
946	A	1	1	1.	19	0.053
947	A	2	2	1.	19	0.105
948	A	3	2	1.	19	0.105
949	A	6	5	1.	19	0.263
950	A	5	5	1.	19	0.263
951	A	4	4	1.	19	0.21
952	A	4	4	1.	19	0.21
953	A	5	5	1.	19	0.263
954	A	6	5	1.	19	0.263
955	A	12	9	1.	20	0.45
956	A	11	8	1.	20	0.4
957	A	1	1	1.	20	0.05
958	A	2	2	1.	20	0.1
959	A	3	2	1.	20	0.1
960	A	5	5	1.	20	0.25
961	A	4	4	1.	20	0.2
962	A	3	3	1.	20	0.15
963	A	4	4	1.	20	0.2

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
964	A	5	4	1.	20	0.2
965	A	6	6	1.	19	0.316
966	A	5	5	1.	19	0.263
967	A	6	6	1.	19	0.316
968	A	7	6	1.	19	0.316
969	A	8	6	1.	19	0.316
970	A	6	6	1.	19	0.316
971	A	5	5	1.	19	0.263
972	A	1	1	1.	19	0.053
973	A	2	2	1.	19	0.105
974	A	3	2	1.	19	0.105
975	A	6	6	1.	20	0.3
976	A	5	5	1.	20	0.25
977	A	6	6	1.	20	0.3
978	A	7	6	1.	20	0.3
979	A	8	6	1.	20	0.3
980	A	12	9	1.	20	0.45
981	A	11	8	1.	20	0.4
982	A	1	1	1.	20	0.05
983	A	2	2	1.	20	0.1
984	A	3	2	1.	20	0.1
985	A	7	7	1.	19	0.368
986	A	6	6	1.	19	0.316
987	A	1	1	1.	19	0.053
988	A	2	2	1.	19	0.105
989	A	3	2	1.	19	0.105
990	A	4	2	1.	19	0.105
991	A	6	4	1.	19	0.21
992	A	5	4	1.	19	0.21
993	A	4	4	1.	19	0.21
994	A	3	3	1.	19	0.158
995	A	4	4	1.	19	0.21
996	A	5	4	1.	19	0.21
997	A	6	4	1.	19	0.21
998	A	2	2	1.	19	0.105
999	A	2	2	1.	19	0.105
1000	A	2	2	1.	19	0.105
1001	A	2	2	1.	19	0.105
1002	A	2	2	1.	19	0.105
1003	A	2	2	1.	19	0.105
1004	A	2	2	1.	19	0.105
1005	A	2	2	1.	19	0.105
1006	A	2	2	1.	19	0.105
1007	A	2	2	1.	19	0.105
1008	A	2	2	1.	19	0.105
1009	A	2	2	1.	19	0.105
1010	A	7	5	1.	15	0.333
1011	A	6	5	1.	15	0.333
1012	A	5	5	1.	15	0.333

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1013	A	4	4	1.	11	0.364
1014	A	4	4	1.	15	0.267
1015	A	5	5	1.	15	0.333
1016	A	6	5	1.	15	0.333
1017	A	7	5	1.	15	0.333
1018	A	9	7	1.	15	0.467
1019	A	8	7	1.	15	0.467
1020	A	7	7	1.	15	0.467
1021	A	6	6	1.	11	0.546
1022	A	7	7	1.	15	0.467
1023	A	8	7	1.	15	0.467
1024	A	9	7	1.	15	0.467
1025	A	6	4	1.	15	0.267
1026	A	5	4	1.	15	0.267
1027	A	4	4	1.	15	0.267
1028	A	3	3	1.	11	0.273
1029	A	4	4	1.	15	0.267
1030	A	5	4	1.	15	0.267
1031	A	6	4	1.	15	0.267
1032	A	9	8	1.	15	0.533
1033	A	8	8	1.	15	0.533
1034	A	7	7	1.	15	0.467
1035	A	5	5	1.	11	0.454
1036	A	8	8	1.	15	0.533
1037	A	9	8	1.	15	0.533
1038	A	10	8	1.	15	0.533
1039	A	3	2	1.	13	0.154
1040	A	3	2	1.	13	0.154
1041	A	3	2	1.	13	0.154
1042	A	1	1	1.	11	0.091
1043	A	2	2	1.	13	0.154
1044	A	2	2	1.	13	0.154
1045	A	2	2	1.22	13	0.154
1046	A	2	2	1.22	13	0.154
1047	A	2	2	1.22	13	0.154
1048	A	2	2	1.26	9	0.222
1049	A	2	2	1.24	13	0.154
1050	A	2	2	1.21	15	0.133
1051	A	2	2	1.21	15	0.133
1052	A	2	2	1.21	15	0.133
1053	A	2	2	1.21	15	0.133
1054	A	2	2	1.22	15	0.133
1055	A	2	2	1.22	15	0.133
1056	A	2	2	1.21	15	0.133
1057	A	2	2	1.21	15	0.133
1058	A	2	2	1.15	13	0.154
1059	A	2	2	1.	15	0.133
1060	A	2	2	1.32	17	0.118
1061	A	3	2	1.	17	0.118

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1062	A	2	2	1.32	17	0.118
1063	A	2	2	1.	17	0.118
1064	A	2	2	1.32	17	0.118
1065	A	1	1	1.	17	0.059
1066	A	2	2	1.32	17	0.118
1067	A	2	2	1.3	17	0.118
1068	A	2	2	1.33	15	0.133
1069	A	2	2	1.31	17	0.118
1070	A	2	2	1.33	17	0.118
1071	A	2	2	1.31	17	0.118

### 3 Listing of integrals

#### 3.1 $\int x^4 (a + bx^2) dx$

**Optimal.** Leaf size=17

$$\frac{ax^5}{5} + \frac{bx^7}{7}$$

[Out] (a\*x^5)/5 + (b\*x^7)/7

---

**Rubi [A]** time = 0.0173312, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{ax^5}{5} + \frac{bx^7}{7}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a + b\*x^2), x]

[Out] (a\*x^5)/5 + (b\*x^7)/7

---

**Rubi in Sympy [A]** time = 3.18053, size = 12, normalized size = 0.71

$$\frac{ax^5}{5} + \frac{bx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(b\*x\*\*2+a), x)

[Out] a\*x\*\*5/5 + b\*x\*\*7/7

---

**Mathematica [A]** time = 0.00226804, size = 17, normalized size = 1.

$$\frac{ax^5}{5} + \frac{bx^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a + b\*x^2), x]

[Out] (a\*x^5)/5 + (b\*x^7)/7

---

**Maple [A]** time = 0.001, size = 14, normalized size = 0.8

$$\frac{ax^5}{5} + \frac{bx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(b\*x^2+a), x)

[Out]  $1/5*a*x^5+1/7*b*x^7$

---

**Maxima [A]** time = 1.33607, size = 18, normalized size = 1.06

$$\frac{1}{7}bx^7 + \frac{1}{5}ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x^4,x, algorithm="maxima")`

[Out]  $1/7*b*x^7 + 1/5*a*x^5$

---

**Fricas [A]** time = 0.180676, size = 1, normalized size = 0.06

$$\frac{1}{7}x^7b + \frac{1}{5}x^5a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x^4,x, algorithm="fricas")`

[Out]  $1/7*x^7*b + 1/5*x^5*a$

---

**Sympy [A]** time = 0.067774, size = 12, normalized size = 0.71

$$\frac{ax^5}{5} + \frac{bx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(b*x**2+a),x)`

[Out]  $a*x**5/5 + b*x**7/7$

---

**GIAC/XCAS [A]** time = 0.213043, size = 18, normalized size = 1.06

$$\frac{1}{7}bx^7 + \frac{1}{5}ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x^4,x, algorithm="giac")`

[Out]  $1/7*b*x^7 + 1/5*a*x^5$

### 3.2 $\int x^3 (a + bx^2) dx$

**Optimal.** Leaf size=17

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

[Out]  $(a*x^4)/4 + (b*x^6)/6$

**Rubi [A]** time = 0.0150114, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*x^2), x]

[Out]  $(a*x^4)/4 + (b*x^6)/6$

**Rubi in Sympy [A]** time = 3.17039, size = 12, normalized size = 0.71

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(b\*x\*\*2+a), x)

[Out]  $a*x**4/4 + b*x**6/6$

**Mathematica [A]** time = 0.00168727, size = 17, normalized size = 1.

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*x^2), x]

[Out]  $(a*x^4)/4 + (b*x^6)/6$

**Maple [A]** time = 0.002, size = 14, normalized size = 0.8

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(b\*x^2+a), x)

[Out]  $1/4*a*x^4+1/6*b*x^6$



---

**Maxima [A]** time = 1.34305, size = 18, normalized size = 1.06

$$\frac{1}{6}bx^6 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x^3,x, algorithm="maxima")`

[Out] `1/6*b*x^6 + 1/4*a*x^4`

---

**Fricas [A]** time = 0.181383, size = 1, normalized size = 0.06

$$\frac{1}{6}x^6b + \frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x^3,x, algorithm="fricas")`

[Out] `1/6*x^6*b + 1/4*x^4*a`

---

**Sympy [A]** time = 0.064849, size = 12, normalized size = 0.71

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**2+a), x)`

[Out] `a*x**4/4 + b*x**6/6`

---

**GIAC/XCAS [A]** time = 0.211888, size = 18, normalized size = 1.06

$$\frac{1}{6}bx^6 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x^3,x, algorithm="giac")`

[Out] `1/6*b*x^6 + 1/4*a*x^4`

### 3.3 $\int x^2 (a + bx^2) dx$

**Optimal.** Leaf size=17

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

[Out]  $(a*x^3)/3 + (b*x^5)/5$

---

**Rubi [A]** time = 0.0153141, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Antiderivative was successfully verified.

[In] `Int[x^2*(a + b*x^2), x]`

[Out]  $(a*x^3)/3 + (b*x^5)/5$

---

**Rubi in Sympy [A]** time = 3.34218, size = 12, normalized size = 0.71

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2*(b*x**2+a), x)`

[Out]  $a*x**3/3 + b*x**5/5$

---

**Mathematica [A]** time = 0.00159928, size = 17, normalized size = 1.

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Antiderivative was successfully verified.

[In] `Integrate[x^2*(a + b*x^2), x]`

[Out]  $(a*x^3)/3 + (b*x^5)/5$

---

**Maple [A]** time = 0.002, size = 14, normalized size = 0.8

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^2+a), x)`

[Out]  $1/3*a*x^3+1/5*b*x^5$

---

**Maxima [A]** time = 1.34983, size = 18, normalized size = 1.06

$$\frac{1}{5}bx^5 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x^2,x, algorithm="maxima")`

[Out] `1/5*b*x^5 + 1/3*a*x^3`

---

**Fricas [A]** time = 0.181964, size = 1, normalized size = 0.06

$$\frac{1}{5}x^5b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x^2,x, algorithm="fricas")`

[Out] `1/5*x^5*b + 1/3*x^3*a`

---

**Sympy [A]** time = 0.063534, size = 12, normalized size = 0.71

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**2+a), x)`

[Out] `a*x**3/3 + b*x**5/5`

---

**GIAC/XCAS [A]** time = 0.211488, size = 18, normalized size = 1.06

$$\frac{1}{5}bx^5 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x^2,x, algorithm="giac")`

[Out] `1/5*b*x^5 + 1/3*a*x^3`

### 3.4 $\int x (a + bx^2) dx$

**Optimal.** Leaf size=17

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

[Out]  $(a*x^2)/2 + (b*x^4)/4$

**Rubi [A]** time = 0.0153499, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*x^2), x]

[Out]  $(a*x^2)/2 + (b*x^4)/4$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$a \int x dx + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(b\*x\*\*2+a), x)

[Out]  $a*Integral(x, x) + b*x**4/4$

**Mathematica [A]** time = 0.00147288, size = 17, normalized size = 1.

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*x^2), x]

[Out]  $(a*x^2)/2 + (b*x^4)/4$

**Maple [A]** time = 0., size = 14, normalized size = 0.8

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(b\*x^2+a), x)

[Out]  $1/2*a*x^2+1/4*b*x^4$

---

**Maxima [A]** time = 1.34211, size = 19, normalized size = 1.12

$$\frac{(bx^2 + a)^2}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x,x, algorithm="maxima")

[Out] 1/4\*(b\*x^2 + a)^2/b

---

**Fricas [A]** time = 0.183601, size = 1, normalized size = 0.06

$$\frac{1}{4}x^4b + \frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x,x, algorithm="fricas")

[Out] 1/4\*x^4\*b + 1/2\*x^2\*a

---

**Sympy [A]** time = 0.067092, size = 12, normalized size = 0.71

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x\*\*2+a),x)

[Out] a\*x\*\*2/2 + b\*x\*\*4/4

---

**GIAC/XCAS [A]** time = 0.214222, size = 18, normalized size = 1.06

$$\frac{1}{4}bx^4 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x,x, algorithm="giac")

[Out] 1/4\*b\*x^4 + 1/2\*a\*x^2

### 3.5 $\int (a + bx^2) dx$

**Optimal.** Leaf size=12

$$ax + \frac{bx^3}{3}$$

[Out]  $a*x + (b*x^3)/3$

**Rubi [A]** time = 0.00960205, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0$ .

$$ax + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[a + b\*x^2, x]

[Out]  $a*x + (b*x^3)/3$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bx^3}{3} + \int a dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(b\*x\*\*2+a, x)

[Out]  $b*x**3/3 + \text{Integral}(a, x)$

**Mathematica [A]** time = 0.0000703963, size = 12, normalized size = 1.

$$ax + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*x^2, x]

[Out]  $a*x + (b*x^3)/3$

**Maple [A]** time = 0.001, size = 11, normalized size = 0.9

$$ax + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(b\*x^2+a, x)

[Out]  $a*x+1/3*b*x^3$

---

**Maxima [A]** time = 1.3417, size = 14, normalized size = 1.17

$$\frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(b*x^2 + a,x, algorithm="maxima")`

[Out] `1/3*b*x^3 + a*x`

---

**Fricas [A]** time = 0.182388, size = 1, normalized size = 0.08

$$\frac{1}{3}x^3b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(b*x^2 + a,x, algorithm="fricas")`

[Out] `1/3*x^3*b + x*a`

---

**Sympy [A]** time = 0.064596, size = 8, normalized size = 0.67

$$ax + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(b*x**2+a,x)`

[Out] `a*x + b*x**3/3`

---

**GIAC/XCAS [A]** time = 0.215295, size = 14, normalized size = 1.17

$$\frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(b*x^2 + a,x, algorithm="giac")`

[Out] `1/3*b*x^3 + a*x`

### 3.6 $\int \frac{a+bx^2}{x} dx$

**Optimal.** Leaf size=13

$$a \log(x) + \frac{bx^2}{2}$$

[Out]  $(b \cdot x^2)/2 + a \cdot \text{Log}[x]$

**Rubi [A]** time = 0.0128179, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$a \log(x) + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/x, x]

[Out]  $(b \cdot x^2)/2 + a \cdot \text{Log}[x]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$a \log(x) + b \int x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/x, x)

[Out]  $a \cdot \log(x) + b \cdot \text{Integral}(x, x)$

**Mathematica [A]** time = 0.00174487, size = 13, normalized size = 1.

$$a \log(x) + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/x, x]

[Out]  $(b \cdot x^2)/2 + a \cdot \text{Log}[x]$

**Maple [A]** time = 0.003, size = 12, normalized size = 0.9

$$\frac{bx^2}{2} + a \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/x, x)

[Out]  $1/2 \cdot b \cdot x^2 + a \cdot \ln(x)$



---

**Maxima [A]** time = 1.35155, size = 19, normalized size = 1.46

$$\frac{1}{2}bx^2 + \frac{1}{2}a \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x,x, algorithm="maxima")

[Out] 1/2\*b\*x^2 + 1/2\*a\*log(x^2)

---

**Fricas [A]** time = 0.204498, size = 15, normalized size = 1.15

$$\frac{1}{2}bx^2 + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x,x, algorithm="fricas")

[Out] 1/2\*b\*x^2 + a\*log(x)

---

**Sympy [A]** time = 0.133044, size = 10, normalized size = 0.77

$$a \log(x) + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/x,x)

[Out] a\*log(x) + b\*x\*\*2/2

---

**GIAC/XCAS [A]** time = 0.213848, size = 19, normalized size = 1.46

$$\frac{1}{2}bx^2 + \frac{1}{2}a \ln(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x,x, algorithm="giac")

[Out] 1/2\*b\*x^2 + 1/2\*a\*ln(x^2)

$$3.7 \quad \int \frac{a+bx^2}{x^2} dx$$

**Optimal.** Leaf size=10

$$bx - \frac{a}{x}$$

[Out]  $-(a/x) + b*x$

**Rubi [A]** time = 0.0129526, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$bx - \frac{a}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/x^2, x]

[Out]  $-(a/x) + b*x$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a}{x} + \int b dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/x\*\*2, x)

[Out]  $-a/x + \text{Integral}(b, x)$

**Mathematica [A]** time = 0.00145976, size = 10, normalized size = 1.

$$bx - \frac{a}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/x^2, x]

[Out]  $-(a/x) + b*x$

**Maple [A]** time = 0.005, size = 11, normalized size = 1.1

$$-\frac{a}{x} + bx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/x^2, x)

[Out]  $-a/x+b*x$

**Maxima [A]** time = 1.34505, size = 14, normalized size = 1.4

$$bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/x^2,x, algorithm="maxima")`

[Out] `b*x - a/x`

---

**Fricas [A]** time = 0.20076, size = 18, normalized size = 1.8

$$\frac{bx^2 - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/x^2,x, algorithm="fricas")`

[Out] `(b*x^2 - a)/x`

---

**Sympy [A]** time = 0.952039, size = 5, normalized size = 0.5

$$-\frac{a}{x} + bx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/x**2,x)`

[Out] `-a/x + b*x`

---

**GIAC/XCAS [A]** time = 0.216468, size = 14, normalized size = 1.4

$$bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/x^2,x, algorithm="giac")`

[Out] `b*x - a/x`

### 3.8 $\int \frac{a+bx^2}{x^3} dx$

**Optimal.** Leaf size=13

$$b \log(x) - \frac{a}{2x^2}$$

[Out]  $-a/(2*x^2) + b*\text{Log}[x]$

---

**Rubi [A]** time = 0.0142568, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$b \log(x) - \frac{a}{2x^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)/x^3, x]$

[Out]  $-a/(2*x^2) + b*\text{Log}[x]$

---

**Rubi in Sympy [A]** time = 2.93464, size = 10, normalized size = 0.77

$$-\frac{a}{2x^2} + b \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x**2+a)/x**3, x)$

[Out]  $-a/(2*x**2) + b*\log(x)$

---

**Mathematica [A]** time = 0.00367596, size = 13, normalized size = 1.

$$b \log(x) - \frac{a}{2x^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b*x^2)/x^3, x]$

[Out]  $-a/(2*x^2) + b*\text{Log}[x]$

---

**Maple [A]** time = 0.007, size = 12, normalized size = 0.9

$$-\frac{a}{2x^2} + b \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((b*x^2+a)/x^3, x)$

[Out]  $-1/2*a/x^2+b*\ln(x)$

---

**Maxima [A]** time = 1.34249, size = 19, normalized size = 1.46

$$\frac{1}{2} b \log(x^2) - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^3,x, algorithm="maxima")

[Out] 1/2\*b\*log(x^2) - 1/2\*a/x^2

---

**Fricas [A]** time = 0.208853, size = 23, normalized size = 1.77

$$\frac{2bx^2 \log(x) - a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^3,x, algorithm="fricas")

[Out] 1/2\*(2\*b\*x^2\*log(x) - a)/x^2

---

**Sympy [A]** time = 1.04539, size = 10, normalized size = 0.77

$$-\frac{a}{2x^2} + b \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/x\*\*3,x)

[Out] -a/(2\*x\*\*2) + b\*log(x)

---

**GIAC/XCAS [A]** time = 0.216802, size = 27, normalized size = 2.08

$$\frac{1}{2} b \ln(x^2) - \frac{bx^2 + a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^3,x, algorithm="giac")

[Out] 1/2\*b\*ln(x^2) - 1/2\*(b\*x^2 + a)/x^2

$$3.9 \quad \int \frac{a+bx^2}{x^4} dx$$

**Optimal.** Leaf size=15

$$-\frac{a}{3x^3} - \frac{b}{x}$$

[Out]  $-a/(3*x^3) - b/x$

**Rubi [A]** time = 0.0152293, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{a}{3x^3} - \frac{b}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/x^4, x]

[Out]  $-a/(3*x^3) - b/x$

**Rubi in Sympy [A]** time = 3.05136, size = 10, normalized size = 0.67

$$-\frac{a}{3x^3} - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/x\*\*4, x)

[Out]  $-a/(3*x**3) - b/x$

**Mathematica [A]** time = 0.0033547, size = 15, normalized size = 1.

$$-\frac{a}{3x^3} - \frac{b}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/x^4, x]

[Out]  $-a/(3*x^3) - b/x$

**Maple [A]** time = 0.006, size = 14, normalized size = 0.9

$$-\frac{a}{3x^3} - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/x^4, x)

[Out]  $-1/3*a/x^3-b/x$

---

**Maxima [A]** time = 1.34456, size = 18, normalized size = 1.2

$$-\frac{3bx^2 + a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^4,x, algorithm="maxima")

[Out] -1/3\*(3\*b\*x^2 + a)/x^3

---

**Fricas [A]** time = 0.217099, size = 18, normalized size = 1.2

$$-\frac{3bx^2 + a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^4,x, algorithm="fricas")

[Out] -1/3\*(3\*b\*x^2 + a)/x^3

---

**Sympy [A]** time = 1.10062, size = 14, normalized size = 0.93

$$-\frac{a + 3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/x\*\*4,x)

[Out] -(a + 3\*b\*x\*\*2)/(3\*x\*\*3)

---

**GIAC/XCAS [A]** time = 0.211536, size = 18, normalized size = 1.2

$$-\frac{3bx^2 + a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^4,x, algorithm="giac")

[Out] -1/3\*(3\*b\*x^2 + a)/x^3

$$3.10 \quad \int \frac{a+bx^2}{x^5} dx$$

**Optimal.** Leaf size=17

$$-\frac{a}{4x^4} - \frac{b}{2x^2}$$

[Out]  $-a/(4*x^4) - b/(2*x^2)$

**Rubi [A]** time = 0.015626, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{a}{4x^4} - \frac{b}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/x^5, x]

[Out]  $-a/(4*x^4) - b/(2*x^2)$

**Rubi in Sympy [A]** time = 3.1161, size = 14, normalized size = 0.82

$$-\frac{a}{4x^4} - \frac{b}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/x\*\*5, x)

[Out]  $-a/(4*x**4) - b/(2*x**2)$

**Mathematica [A]** time = 0.00351693, size = 17, normalized size = 1.

$$-\frac{a}{4x^4} - \frac{b}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/x^5, x]

[Out]  $-a/(4*x^4) - b/(2*x^2)$

**Maple [A]** time = 0.008, size = 14, normalized size = 0.8

$$-\frac{a}{4x^4} - \frac{b}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/x^5, x)

[Out]  $-1/4*a/x^4 - 1/2*b/x^2$



---

**Maxima [A]** time = 1.37844, size = 18, normalized size = 1.06

$$-\frac{2bx^2 + a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^5,x, algorithm="maxima")

[Out] -1/4\*(2\*b\*x^2 + a)/x^4

---

**Fricas [A]** time = 0.225828, size = 18, normalized size = 1.06

$$-\frac{2bx^2 + a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^5,x, algorithm="fricas")

[Out] -1/4\*(2\*b\*x^2 + a)/x^4

---

**Sympy [A]** time = 1.0974, size = 14, normalized size = 0.82

$$-\frac{a + 2bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/x\*\*5,x)

[Out] -(a + 2\*b\*x\*\*2)/(4\*x\*\*4)

---

**GIAC/XCAS [A]** time = 0.210825, size = 18, normalized size = 1.06

$$-\frac{2bx^2 + a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^5,x, algorithm="giac")

[Out] -1/4\*(2\*b\*x^2 + a)/x^4

$$3.11 \quad \int \frac{a+bx^2}{x^6} dx$$

**Optimal.** Leaf size=17

$$-\frac{a}{5x^5} - \frac{b}{3x^3}$$

[Out]  $-a/(5*x^5) - b/(3*x^3)$

**Rubi [A]** time = 0.0152197, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{a}{5x^5} - \frac{b}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/x^6, x]

[Out]  $-a/(5*x^5) - b/(3*x^3)$

**Rubi in Sympy [A]** time = 3.15166, size = 14, normalized size = 0.82

$$-\frac{a}{5x^5} - \frac{b}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/x\*\*6, x)

[Out]  $-a/(5*x**5) - b/(3*x**3)$

**Mathematica [A]** time = 0.00355981, size = 17, normalized size = 1.

$$-\frac{a}{5x^5} - \frac{b}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/x^6, x]

[Out]  $-a/(5*x^5) - b/(3*x^3)$

**Maple [A]** time = 0.007, size = 14, normalized size = 0.8

$$-\frac{a}{5x^5} - \frac{b}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/x^6, x)

[Out]  $-1/5*a/x^5-1/3*b/x^3$

---

**Maxima [A]** time = 1.33722, size = 20, normalized size = 1.18

$$-\frac{5bx^2 + 3a}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^6,x, algorithm="maxima")

[Out] -1/15\*(5\*b\*x^2 + 3\*a)/x^5

---

**Fricas [A]** time = 0.203801, size = 20, normalized size = 1.18

$$-\frac{5bx^2 + 3a}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^6,x, algorithm="fricas")

[Out] -1/15\*(5\*b\*x^2 + 3\*a)/x^5

---

**Sympy [A]** time = 1.14459, size = 15, normalized size = 0.88

$$-\frac{3a + 5bx^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/x\*\*6,x)

[Out] -(3\*a + 5\*b\*x\*\*2)/(15\*x\*\*5)

---

**GIAC/XCAS [A]** time = 0.206106, size = 20, normalized size = 1.18

$$-\frac{5bx^2 + 3a}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^6,x, algorithm="giac")

[Out] -1/15\*(5\*b\*x^2 + 3\*a)/x^5

$$3.12 \quad \int \frac{a+bx^2}{x^7} dx$$

**Optimal.** Leaf size=17

$$-\frac{a}{6x^6} - \frac{b}{4x^4}$$

[Out]  $-a/(6*x^6) - b/(4*x^4)$

**Rubi [A]** time = 0.0153288, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{a}{6x^6} - \frac{b}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/x^7, x]

[Out]  $-a/(6*x^6) - b/(4*x^4)$

**Rubi in Sympy [A]** time = 3.14676, size = 14, normalized size = 0.82

$$-\frac{a}{6x^6} - \frac{b}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)/x\*\*7, x)

[Out]  $-a/(6*x**6) - b/(4*x**4)$

**Mathematica [A]** time = 0.003699, size = 17, normalized size = 1.

$$-\frac{a}{6x^6} - \frac{b}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/x^7, x]

[Out]  $-a/(6*x^6) - b/(4*x^4)$

**Maple [A]** time = 0.007, size = 14, normalized size = 0.8

$$-\frac{a}{6x^6} - \frac{b}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/x^7, x)

[Out]  $-1/6*a/x^6 - 1/4*b/x^4$

---

**Maxima [A]** time = 1.33949, size = 20, normalized size = 1.18

$$-\frac{3bx^2 + 2a}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^7,x, algorithm="maxima")

[Out] -1/12\*(3\*b\*x^2 + 2\*a)/x^6

---

**Fricas [A]** time = 0.200904, size = 20, normalized size = 1.18

$$-\frac{3bx^2 + 2a}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^7,x, algorithm="fricas")

[Out] -1/12\*(3\*b\*x^2 + 2\*a)/x^6

---

**Sympy [A]** time = 1.14391, size = 15, normalized size = 0.88

$$-\frac{2a + 3bx^2}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/x\*\*7,x)

[Out] -(2\*a + 3\*b\*x\*\*2)/(12\*x\*\*6)

---

**GIAC/XCAS [A]** time = 0.207989, size = 20, normalized size = 1.18

$$-\frac{3bx^2 + 2a}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^7,x, algorithm="giac")

[Out] -1/12\*(3\*b\*x^2 + 2\*a)/x^6

### 3.13 $\int x^5 (a + bx^2)^2 dx$

**Optimal.** Leaf size=30

$$\frac{a^2x^6}{6} + \frac{1}{4}abx^8 + \frac{b^2x^{10}}{10}$$

[Out]  $(a^2x^6)/6 + (a*b*x^8)/4 + (b^2*x^{10})/10$

**Rubi [A]** time = 0.061023, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^2x^6}{6} + \frac{1}{4}abx^8 + \frac{b^2x^{10}}{10}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(a + b\*x^2)^2,x]

[Out]  $(a^2x^6)/6 + (a*b*x^8)/4 + (b^2*x^{10})/10$

**Rubi in Sympy [A]** time = 7.76487, size = 24, normalized size = 0.8

$$\frac{a^2x^6}{6} + \frac{abx^8}{4} + \frac{b^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(b\*x\*\*2+a)\*\*2,x)

[Out]  $a**2*x**6/6 + a*b*x**8/4 + b**2*x**10/10$

**Mathematica [A]** time = 0.00178551, size = 30, normalized size = 1.

$$\frac{a^2x^6}{6} + \frac{1}{4}abx^8 + \frac{b^2x^{10}}{10}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*x^2)^2,x]

[Out]  $(a^2x^6)/6 + (a*b*x^8)/4 + (b^2*x^{10})/10$

**Maple [A]** time = 0.001, size = 25, normalized size = 0.8

$$\frac{a^2x^6}{6} + \frac{abx^8}{4} + \frac{b^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(b\*x^2+a)^2,x)

[Out]  $1/6*a^2*x^6+1/4*a*b*x^8+1/10*b^2*x^{10}$

---

**Maxima [A]** time = 1.34741, size = 32, normalized size = 1.07

$$\frac{1}{10} b^2 x^{10} + \frac{1}{4} abx^8 + \frac{1}{6} a^2 x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2\*x^5,x, algorithm="maxima")

[Out] 1/10\*b^2\*x^10 + 1/4\*a\*b\*x^8 + 1/6\*a^2\*x^6

---

**Fricas [A]** time = 0.196389, size = 1, normalized size = 0.03

$$\frac{1}{10} x^{10} b^2 + \frac{1}{4} x^8 b a + \frac{1}{6} x^6 a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2\*x^5,x, algorithm="fricas")

[Out] 1/10\*x^10\*b^2 + 1/4\*x^8\*b\*a + 1/6\*x^6\*a^2

---

**Sympy [A]** time = 0.094627, size = 24, normalized size = 0.8

$$\frac{a^2 x^6}{6} + \frac{abx^8}{4} + \frac{b^2 x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(b\*x\*\*2+a)\*\*2,x)

[Out] a\*\*2\*x\*\*6/6 + a\*b\*x\*\*8/4 + b\*\*2\*x\*\*10/10

---

**GIAC/XCAS [A]** time = 0.207913, size = 32, normalized size = 1.07

$$\frac{1}{10} b^2 x^{10} + \frac{1}{4} abx^8 + \frac{1}{6} a^2 x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2\*x^5,x, algorithm="giac")

[Out] 1/10\*b^2\*x^10 + 1/4\*a\*b\*x^8 + 1/6\*a^2\*x^6

### 3.14 $\int x^4 (a + bx^2)^2 dx$

**Optimal.** Leaf size=30

$$\frac{a^2x^5}{5} + \frac{2}{7}abx^7 + \frac{b^2x^9}{9}$$

[Out]  $(a^2x^5)/5 + (2abx^7)/7 + (b^2x^9)/9$

**Rubi [A]** time = 0.0361616, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{a^2x^5}{5} + \frac{2}{7}abx^7 + \frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a + b\*x^2)^2,x]

[Out]  $(a^2x^5)/5 + (2abx^7)/7 + (b^2x^9)/9$

**Rubi in Sympy [A]** time = 5.65006, size = 26, normalized size = 0.87

$$\frac{a^2x^5}{5} + \frac{2abx^7}{7} + \frac{b^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(b\*x\*\*2+a)\*\*2,x)

[Out]  $a**2*x**5/5 + 2*a*b*x**7/7 + b**2*x**9/9$

**Mathematica [A]** time = 0.00137657, size = 30, normalized size = 1.

$$\frac{a^2x^5}{5} + \frac{2}{7}abx^7 + \frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a + b\*x^2)^2,x]

[Out]  $(a^2x^5)/5 + (2abx^7)/7 + (b^2x^9)/9$

**Maple [A]** time = 0., size = 25, normalized size = 0.8

$$\frac{a^2x^5}{5} + \frac{2abx^7}{7} + \frac{b^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(b\*x^2+a)^2,x)

[Out]  $1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9$



---

**Maxima [A]** time = 1.31823, size = 32, normalized size = 1.07

$$\frac{1}{9}b^2x^9 + \frac{2}{7}abx^7 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2*x^4,x, algorithm="maxima")`

[Out] `1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5`

---

**Fricas [A]** time = 0.191311, size = 1, normalized size = 0.03

$$\frac{1}{9}x^9b^2 + \frac{2}{7}x^7ba + \frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2*x^4,x, algorithm="fricas")`

[Out] `1/9*x^9*b^2 + 2/7*x^7*b*a + 1/5*x^5*a^2`

---

**Sympy [A]** time = 0.088753, size = 26, normalized size = 0.87

$$\frac{a^2x^5}{5} + \frac{2abx^7}{7} + \frac{b^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(b*x**2+a)**2,x)`

[Out] `a**2*x**5/5 + 2*a*b*x**7/7 + b**2*x**9/9`

---

**GIAC/XCAS [A]** time = 0.207002, size = 32, normalized size = 1.07

$$\frac{1}{9}b^2x^9 + \frac{2}{7}abx^7 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2*x^4,x, algorithm="giac")`

[Out] `1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5`

### 3.15 $\int x^3 (a + bx^2)^2 dx$

**Optimal.** Leaf size=30

$$\frac{a^2x^4}{4} + \frac{1}{3}abx^6 + \frac{b^2x^8}{8}$$

[Out]  $(a^2x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8$

**Rubi [A]** time = 0.0539066, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^2x^4}{4} + \frac{1}{3}abx^6 + \frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*x^2)^2, x]

[Out]  $(a^2x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^2 \int^{x^2} x dx}{2} + \frac{abx^6}{3} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(b\*x\*\*2+a)\*\*2, x)

[Out]  $a**2*Integral(x, (x, x**2))/2 + a*b*x**6/3 + b**2*x**8/8$

**Mathematica [A]** time = 0.00134393, size = 30, normalized size = 1.

$$\frac{a^2x^4}{4} + \frac{1}{3}abx^6 + \frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*x^2)^2, x]

[Out]  $(a^2x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8$

**Maple [A]** time = 0., size = 25, normalized size = 0.8

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(b\*x^2+a)^2, x)

[Out]  $1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8$

---

**Maxima [A]** time = 1.32923, size = 32, normalized size = 1.07

$$\frac{1}{8}b^2x^8 + \frac{1}{3}abx^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2*x^3,x, algorithm="maxima")`

[Out] `1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4`

---

**Fricas [A]** time = 0.18926, size = 1, normalized size = 0.03

$$\frac{1}{8}x^8b^2 + \frac{1}{3}x^6ba + \frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2*x^3,x, algorithm="fricas")`

[Out] `1/8*x^8*b^2 + 1/3*x^6*b*a + 1/4*x^4*a^2`

---

**Sympy [A]** time = 0.084781, size = 24, normalized size = 0.8

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**2+a)**2,x)`

[Out] `a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8`

---

**GIAC/XCAS [A]** time = 0.208328, size = 32, normalized size = 1.07

$$\frac{1}{8}b^2x^8 + \frac{1}{3}abx^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2*x^3,x, algorithm="giac")`

[Out] `1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4`

### 3.16 $\int x^2 (a + bx^2)^2 dx$

**Optimal.** Leaf size=30

$$\frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

[Out]  $(a^2x^3)/3 + (2abx^5)/5 + (b^2x^7)/7$

**Rubi [A]** time = 0.0348519, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*x^2)^2,x]

[Out]  $(a^2x^3)/3 + (2abx^5)/5 + (b^2x^7)/7$

**Rubi in Sympy [A]** time = 5.77185, size = 26, normalized size = 0.87

$$\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(b\*x\*\*2+a)\*\*2,x)

[Out]  $a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7$

**Mathematica [A]** time = 0.00120314, size = 30, normalized size = 1.

$$\frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*x^2)^2,x]

[Out]  $(a^2x^3)/3 + (2abx^5)/5 + (b^2x^7)/7$

**Maple [A]** time = 0.001, size = 25, normalized size = 0.8

$$\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(b\*x^2+a)^2,x)

[Out]  $1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7$

---

**Maxima [A]** time = 1.34789, size = 32, normalized size = 1.07

$$\frac{1}{7}b^2x^7 + \frac{2}{5}abx^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2*x^2,x, algorithm="maxima")`

[Out] `1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3`

---

**Fricas [A]** time = 0.183359, size = 1, normalized size = 0.03

$$\frac{1}{7}x^7b^2 + \frac{2}{5}x^5ba + \frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2*x^2,x, algorithm="fricas")`

[Out] `1/7*x^7*b^2 + 2/5*x^5*b*a + 1/3*x^3*a^2`

---

**Sympy [A]** time = 0.08922, size = 26, normalized size = 0.87

$$\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**2+a)**2,x)`

[Out] `a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7`

---

**GIAC/XCAS [A]** time = 0.207309, size = 32, normalized size = 1.07

$$\frac{1}{7}b^2x^7 + \frac{2}{5}abx^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2*x^2,x, algorithm="giac")`

[Out] `1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3`

### 3.17 $\int x (a + bx^2)^2 dx$

**Optimal.** Leaf size=16

$$\frac{(a + bx^2)^3}{6b}$$

[Out] (a + b\*x^2)^3/(6\*b)

**Rubi [A]** time = 0.0122781, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{(a + bx^2)^3}{6b}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*x^2)^2, x]

[Out] (a + b\*x^2)^3/(6\*b)

**Rubi in Sympy [A]** time = 2.16477, size = 10, normalized size = 0.62

$$\frac{(a + bx^2)^3}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(b\*x\*\*2+a)\*\*2, x)

[Out] (a + b\*x\*\*2)\*\*3/(6\*b)

**Mathematica [A]** time = 0.0037182, size = 16, normalized size = 1.

$$\frac{(a + bx^2)^3}{6b}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*x^2)^2, x]

[Out] (a + b\*x^2)^3/(6\*b)

**Maple [A]** time = 0., size = 25, normalized size = 1.6

$$\frac{b^2x^6}{6} + \frac{abx^4}{2} + \frac{a^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(b\*x^2+a)^2, x)

[Out] 1/6\*b^2\*x^6+1/2\*a\*b\*x^4+1/2\*a^2\*x^2

---

**Maxima [A]** time = 1.32701, size = 19, normalized size = 1.19

$$\frac{(bx^2 + a)^3}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2\*x,x, algorithm="maxima")

[Out] 1/6\*(b\*x^2 + a)^3/b

---

**Fricas [A]** time = 0.193589, size = 1, normalized size = 0.06

$$\frac{1}{6}x^6b^2 + \frac{1}{2}x^4ba + \frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2\*x,x, algorithm="fricas")

[Out] 1/6\*x^6\*b^2 + 1/2\*x^4\*b\*a + 1/2\*x^2\*a^2

---

**Sympy [A]** time = 0.085306, size = 24, normalized size = 1.5

$$\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x\*\*2+a)\*\*2,x)

[Out] a\*\*2\*x\*\*2/2 + a\*b\*x\*\*4/2 + b\*\*2\*x\*\*6/6

---

**GIAC/XCAS [A]** time = 0.207605, size = 19, normalized size = 1.19

$$\frac{(bx^2 + a)^3}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2\*x,x, algorithm="giac")

[Out] 1/6\*(b\*x^2 + a)^3/b

### 3.18 $\int (a + bx^2)^2 dx$

**Optimal.** Leaf size=25

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

[Out]  $a^2x + (2abx^3)/3 + (b^2x^5)/5$

**Rubi [A]** time = 0.0220631, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^2, x]

[Out]  $a^2x + (2abx^3)/3 + (b^2x^5)/5$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2abx^3}{3} + \frac{b^2x^5}{5} + \int a^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*2, x)

[Out]  $2abx^3/3 + b^2x^5/5 + \text{Integral}(a^2, x)$

**Mathematica [A]** time = 0.00142008, size = 25, normalized size = 1.

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^2, x]

[Out]  $a^2x + (2abx^3)/3 + (b^2x^5)/5$

**Maple [A]** time = 0., size = 22, normalized size = 0.9

$$a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^2, x)

[Out]  $a^2x + 2/3abx^3 + 1/5b^2x^5$



---

**Maxima [A]** time = 1.3319, size = 28, normalized size = 1.12

$$\frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2,x, algorithm="maxima")

[Out] 1/5\*b^2\*x^5 + 2/3\*a\*b\*x^3 + a^2\*x

---

**Fricas [A]** time = 0.187661, size = 1, normalized size = 0.04

$$\frac{1}{5}x^5b^2 + \frac{2}{3}x^3ba + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2,x, algorithm="fricas")

[Out] 1/5\*x^5\*b^2 + 2/3\*x^3\*b\*a + x\*a^2

---

**Sympy [A]** time = 0.077821, size = 22, normalized size = 0.88

$$a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*2,x)

[Out] a\*\*2\*x + 2\*a\*b\*x\*\*3/3 + b\*\*2\*x\*\*5/5

---

**GIAC/XCAS [A]** time = 0.211813, size = 28, normalized size = 1.12

$$\frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2,x, algorithm="giac")

[Out] 1/5\*b^2\*x^5 + 2/3\*a\*b\*x^3 + a^2\*x

$$3.19 \quad \int \frac{(a+bx^2)^2}{x} dx$$

**Optimal.** Leaf size=23

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

[Out]  $a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]$

**Rubi [A]** time = 0.0382137, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

[In]  $Int[(a + b*x^2)^2/x, x]$

[Out]  $a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^2 \log(x^2)}{2} + abx^2 + \frac{b^2 \int x dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $rubi\_integrate((b*x**2+a)**2/x, x)$

[Out]  $a**2*log(x**2)/2 + a*b*x**2 + b**2*Integral(x, (x, x**2))/2$

**Mathematica [A]** time = 0.00161879, size = 23, normalized size = 1.

$$a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

[In]  $Integrate[(a + b*x^2)^2/x, x]$

[Out]  $a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]$

**Maple [A]** time = 0.003, size = 22, normalized size = 1.

$$abx^2 + \frac{b^2x^4}{4} + a^2 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $int((b*x^2+a)^2/x, x)$

[Out]  $a*b*x^2+1/4*b^2*x^4+a^2*ln(x)$

---

**Maxima [A]** time = 1.32409, size = 32, normalized size = 1.39

$$\frac{1}{4} b^2 x^4 + abx^2 + \frac{1}{2} a^2 \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x,x, algorithm="maxima")

[Out] 1/4\*b^2\*x^4 + a\*b\*x^2 + 1/2\*a^2\*log(x^2)

---

**Fricas [A]** time = 0.208355, size = 28, normalized size = 1.22

$$\frac{1}{4} b^2 x^4 + abx^2 + a^2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x,x, algorithm="fricas")

[Out] 1/4\*b^2\*x^4 + a\*b\*x^2 + a^2\*log(x)

---

**Sympy [A]** time = 1.02012, size = 20, normalized size = 0.87

$$a^2 \log(x) + abx^2 + \frac{b^2 x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*2/x,x)

[Out] a\*\*2\*log(x) + a\*b\*x\*\*2 + b\*\*2\*x\*\*4/4

---

**GIAC/XCAS [A]** time = 0.222284, size = 32, normalized size = 1.39

$$\frac{1}{4} b^2 x^4 + abx^2 + \frac{1}{2} a^2 \ln(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x,x, algorithm="giac")

[Out] 1/4\*b^2\*x^4 + a\*b\*x^2 + 1/2\*a^2\*ln(x^2)

$$3.20 \quad \int \frac{(a+bx^2)^2}{x^2} dx$$

**Optimal.** Leaf size=24

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

[Out]  $-(a^2/x) + 2*a*b*x + (b^2*x^3)/3$

**Rubi [A]** time = 0.0303651, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*x^2)^2/x^2, x]`

[Out]  $-(a^2/x) + 2*a*b*x + (b^2*x^3)/3$

**Rubi in Sympy [A]** time = 5.50499, size = 19, normalized size = 0.79

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**2/x**2, x)`

[Out]  $-a**2/x + 2*a*b*x + b**2*x**3/3$

**Mathematica [A]** time = 0.00132313, size = 24, normalized size = 1.

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^2/x^2, x]`

[Out]  $-(a^2/x) + 2*a*b*x + (b^2*x^3)/3$

**Maple [A]** time = 0.004, size = 23, normalized size = 1.

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^2/x^2, x)`

[Out]  $-a^2/x+2*a*b*x+1/3*b^2*x^3$

---

**Maxima [A]** time = 1.3329, size = 30, normalized size = 1.25

$$\frac{1}{3} b^2 x^3 + 2 abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2/x^2,x, algorithm="maxima")`

[Out] `1/3*b^2*x^3 + 2*a*b*x - a^2/x`

---

**Fricas [A]** time = 0.195456, size = 34, normalized size = 1.42

$$\frac{b^2 x^4 + 6 abx^2 - 3 a^2}{3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2/x^2,x, algorithm="fricas")`

[Out] `1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x`

---

**Sympy [A]** time = 1.02389, size = 19, normalized size = 0.79

$$-\frac{a^2}{x} + 2abx + \frac{b^2 x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**2/x**2,x)`

[Out] `-a**2/x + 2*a*b*x + b**2*x**3/3`

---

**GIAC/XCAS [A]** time = 0.211509, size = 30, normalized size = 1.25

$$\frac{1}{3} b^2 x^3 + 2 abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2/x^2,x, algorithm="giac")`

[Out] `1/3*b^2*x^3 + 2*a*b*x - a^2/x`

$$3.21 \quad \int \frac{(a+bx^2)^2}{x^3} dx$$

**Optimal.** Leaf size=27

$$-\frac{a^2}{2x^2} + 2ab \log(x) + \frac{b^2x^2}{2}$$

[Out]  $-a^2/(2*x^2) + (b^2*x^2)/2 + 2*a*b*Log[x]$

**Rubi [A]** time = 0.0437337, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^2}{2x^2} + 2ab \log(x) + \frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^2/x^3, x]

[Out]  $-a^2/(2*x^2) + (b^2*x^2)/2 + 2*a*b*Log[x]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^2}{2x^2} + ab \log(x^2) + \frac{\int^{x^2} b^2 dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*2/x\*\*3, x)

[Out]  $-a**2/(2*x**2) + a*b*log(x**2) + Integral(b**2, (x, x**2))/2$

**Mathematica [A]** time = 0.00156888, size = 27, normalized size = 1.

$$-\frac{a^2}{2x^2} + 2ab \log(x) + \frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^2/x^3, x]

[Out]  $-a^2/(2*x^2) + (b^2*x^2)/2 + 2*a*b*Log[x]$

**Maple [A]** time = 0.007, size = 24, normalized size = 0.9

$$-\frac{a^2}{2x^2} + \frac{b^2x^2}{2} + 2ab \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^2/x^3, x)

[Out]  $-1/2*a^2/x^2+1/2*b^2*x^2+2*a*b*ln(x)$

---

**Maxima [A]** time = 1.32193, size = 32, normalized size = 1.19

$$\frac{1}{2} b^2 x^2 + ab \log(x^2) - \frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^3,x, algorithm="maxima")

[Out] 1/2\*b^2\*x^2 + a\*b\*log(x^2) - 1/2\*a^2/x^2

---

**Fricas [A]** time = 0.203374, size = 36, normalized size = 1.33

$$\frac{b^2 x^4 + 4 abx^2 \log(x) - a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^3,x, algorithm="fricas")

[Out] 1/2\*(b^2\*x^4 + 4\*a\*b\*x^2\*log(x) - a^2)/x^2

---

**Sympy [A]** time = 1.11932, size = 24, normalized size = 0.89

$$-\frac{a^2}{2x^2} + 2ab \log(x) + \frac{b^2 x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*2/x\*\*3,x)

[Out] -a\*\*2/(2\*x\*\*2) + 2\*a\*b\*log(x) + b\*\*2\*x\*\*2/2

---

**GIAC/XCAS [A]** time = 0.213064, size = 43, normalized size = 1.59

$$\frac{1}{2} b^2 x^2 + ab \ln(x^2) - \frac{2 abx^2 + a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^3,x, algorithm="giac")

[Out] 1/2\*b^2\*x^2 + a\*b\*ln(x^2) - 1/2\*(2\*a\*b\*x^2 + a^2)/x^2

$$3.22 \quad \int \frac{(a+bx^2)^2}{x^4} dx$$

**Optimal.** Leaf size=23

$$-\frac{a^2}{3x^3} - \frac{2ab}{x} + b^2x$$

[Out]  $-a^2/(3*x^3) - (2*a*b)/x + b^2*x$

**Rubi [A]** time = 0.0304979, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^2}{3x^3} - \frac{2ab}{x} + b^2x$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^2/x^4, x]

[Out]  $-a^2/(3*x^3) - (2*a*b)/x + b^2*x$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^2}{3x^3} - \frac{2ab}{x} + \int b^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*2/x\*\*4, x)

[Out]  $-a**2/(3*x**3) - 2*a*b/x + \text{Integral}(b**2, x)$

**Mathematica [A]** time = 0.00149176, size = 23, normalized size = 1.

$$-\frac{a^2}{3x^3} - \frac{2ab}{x} + b^2x$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^2/x^4, x]

[Out]  $-a^2/(3*x^3) - (2*a*b)/x + b^2*x$

**Maple [A]** time = 0.007, size = 22, normalized size = 1.

$$-\frac{a^2}{3x^3} - 2\frac{ab}{x} + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^2/x^4, x)

[Out]  $-1/3*a^2/x^3 - 2*a*b/x + b^2*x$



---

**Maxima [A]** time = 1.33237, size = 30, normalized size = 1.3

$$b^2x - \frac{6abx^2 + a^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^4,x, algorithm="maxima")

[Out] b^2\*x - 1/3\*(6\*a\*b\*x^2 + a^2)/x^3

---

**Fricas [A]** time = 0.193146, size = 35, normalized size = 1.52

$$\frac{3b^2x^4 - 6abx^2 - a^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^4,x, algorithm="fricas")

[Out] 1/3\*(3\*b^2\*x^4 - 6\*a\*b\*x^2 - a^2)/x^3

---

**Sympy [A]** time = 1.13156, size = 20, normalized size = 0.87

$$b^2x - \frac{a^2 + 6abx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*2/x\*\*4,x)

[Out] b\*\*2\*x - (a\*\*2 + 6\*a\*b\*x\*\*2)/(3\*x\*\*3)

---

**GIAC/XCAS [A]** time = 0.207674, size = 30, normalized size = 1.3

$$b^2x - \frac{6abx^2 + a^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^4,x, algorithm="giac")

[Out] b^2\*x - 1/3\*(6\*a\*b\*x^2 + a^2)/x^3

$$3.23 \quad \int \frac{(a+bx^2)^2}{x^5} dx$$

**Optimal.** Leaf size=24

$$-\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \log(x)$$

[Out]  $-a^2/(4*x^4) - (a*b)/x^2 + b^2*Log[x]$

**Rubi [A]** time = 0.0409143, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^2/x^5, x]

[Out]  $-a^2/(4*x^4) - (a*b)/x^2 + b^2*Log[x]$

**Rubi in Sympy [A]** time = 7.16817, size = 24, normalized size = 1.

$$-\frac{a^2}{4x^4} - \frac{ab}{x^2} + \frac{b^2 \log(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*2/x\*\*5, x)

[Out]  $-a**2/(4*x**4) - a*b/x**2 + b**2*log(x**2)/2$

**Mathematica [A]** time = 0.00150968, size = 24, normalized size = 1.

$$-\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^2/x^5, x]

[Out]  $-a^2/(4*x^4) - (a*b)/x^2 + b^2*Log[x]$

**Maple [A]** time = 0.007, size = 23, normalized size = 1.

$$-\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^2/x^5, x)

[Out]  $-1/4*a^2/x^4 - a*b/x^2 + b^2*ln(x)$

---

**Maxima [A]** time = 1.34151, size = 35, normalized size = 1.46

$$\frac{1}{2} b^2 \log(x^2) - \frac{4 abx^2 + a^2}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^5,x, algorithm="maxima")

[Out] 1/2\*b^2\*log(x^2) - 1/4\*(4\*a\*b\*x^2 + a^2)/x^4

---

**Fricas [A]** time = 0.202645, size = 38, normalized size = 1.58

$$\frac{4 b^2 x^4 \log(x) - 4 abx^2 - a^2}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^5,x, algorithm="fricas")

[Out] 1/4\*(4\*b^2\*x^4\*log(x) - 4\*a\*b\*x^2 - a^2)/x^4

---

**Sympy [A]** time = 1.27951, size = 22, normalized size = 0.92

$$b^2 \log(x) - \frac{a^2 + 4 abx^2}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*2/x\*\*5,x)

[Out] b\*\*2\*log(x) - (a\*\*2 + 4\*a\*b\*x\*\*2)/(4\*x\*\*4)

---

**GIAC/XCAS [A]** time = 0.210985, size = 46, normalized size = 1.92

$$\frac{1}{2} b^2 \ln(x^2) - \frac{3 b^2 x^4 + 4 abx^2 + a^2}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^5,x, algorithm="giac")

[Out] 1/2\*b^2\*ln(x^2) - 1/4\*(3\*b^2\*x^4 + 4\*a\*b\*x^2 + a^2)/x^4

$$3.24 \quad \int \frac{(a+bx^2)^2}{x^6} dx$$

**Optimal.** Leaf size=28

$$-\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

[Out]  $-a^2/(5*x^5) - (2*a*b)/(3*x^3) - b^2/x$

**Rubi [A]** time = 0.0320402, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^2/x^6, x]

[Out]  $-a^2/(5*x^5) - (2*a*b)/(3*x^3) - b^2/x$

**Rubi in Sympy [A]** time = 5.73289, size = 24, normalized size = 0.86

$$-\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*2/x\*\*6, x)

[Out]  $-a**2/(5*x**5) - 2*a*b/(3*x**3) - b**2/x$

**Mathematica [A]** time = 0.00145336, size = 28, normalized size = 1.

$$-\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^2/x^6, x]

[Out]  $-a^2/(5*x^5) - (2*a*b)/(3*x^3) - b^2/x$

**Maple [A]** time = 0.007, size = 25, normalized size = 0.9

$$-\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^2/x^6, x)

[Out]  $-1/5*a^2/x^5 - 2/3*a*b/x^3 - b^2/x$

---

**Maxima [A]** time = 1.34055, size = 35, normalized size = 1.25

$$-\frac{15b^2x^4 + 10abx^2 + 3a^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^6,x, algorithm="maxima")

[Out] -1/15\*(15\*b^2\*x^4 + 10\*a\*b\*x^2 + 3\*a^2)/x^5

---

**Fricas [A]** time = 0.197064, size = 35, normalized size = 1.25

$$-\frac{15b^2x^4 + 10abx^2 + 3a^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^6,x, algorithm="fricas")

[Out] -1/15\*(15\*b^2\*x^4 + 10\*a\*b\*x^2 + 3\*a^2)/x^5

---

**Sympy [A]** time = 1.28358, size = 27, normalized size = 0.96

$$-\frac{3a^2 + 10abx^2 + 15b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*2/x\*\*6,x)

[Out] -(3\*a\*\*2 + 10\*a\*b\*x\*\*2 + 15\*b\*\*2\*x\*\*4)/(15\*x\*\*5)

---

**GIAC/XCAS [A]** time = 0.208559, size = 35, normalized size = 1.25

$$-\frac{15b^2x^4 + 10abx^2 + 3a^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^6,x, algorithm="giac")

[Out] -1/15\*(15\*b^2\*x^4 + 10\*a\*b\*x^2 + 3\*a^2)/x^5

$$3.25 \quad \int \frac{(a+bx^2)^2}{x^7} dx$$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^3}{6ax^6}$$

[Out]  $-(a + b*x^2)^3/(6*a*x^6)$

Rubi [A] time = 0.017752, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{(a+bx^2)^3}{6ax^6}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*x^2)^2/x^7, x]`

[Out]  $-(a + b*x^2)^3/(6*a*x^6)$

Rubi in Sympy [A] time = 3.17593, size = 15, normalized size = 0.79

$$-\frac{(a+bx^2)^3}{6ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**2/x**7, x)`

[Out]  $-(a + b*x**2)**3/(6*a*x**6)$

Mathematica [A] time = 0.00161879, size = 30, normalized size = 1.58

$$-\frac{a^2}{6x^6} - \frac{ab}{2x^4} - \frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^2/x^7, x]`

[Out]  $-a^2/(6*x^6) - (a*b)/(2*x^4) - b^2/(2*x^2)$

Maple [A] time = 0.007, size = 25, normalized size = 1.3

$$-\frac{a^2}{6x^6} - \frac{ab}{2x^4} - \frac{b^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^2/x^7, x)`

[Out]  $-1/6*a^2/x^6-1/2*a*b/x^4-1/2*b^2/x^2$

---

**Maxima [A]** time = 1.33326, size = 32, normalized size = 1.68

$$\frac{3b^2x^4 + 3abx^2 + a^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^7,x, algorithm="maxima")

[Out] -1/6\*(3\*b^2\*x^4 + 3\*a\*b\*x^2 + a^2)/x^6

---

**Fricas [A]** time = 0.19566, size = 32, normalized size = 1.68

$$\frac{3b^2x^4 + 3abx^2 + a^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^7,x, algorithm="fricas")

[Out] -1/6\*(3\*b^2\*x^4 + 3\*a\*b\*x^2 + a^2)/x^6

---

**Sympy [A]** time = 1.31154, size = 26, normalized size = 1.37

$$\frac{a^2 + 3abx^2 + 3b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*2/x\*\*7,x)

[Out] -(a\*\*2 + 3\*a\*b\*x\*\*2 + 3\*b\*\*2\*x\*\*4)/(6\*x\*\*6)

---

**GIAC/XCAS [A]** time = 0.207993, size = 32, normalized size = 1.68

$$\frac{3b^2x^4 + 3abx^2 + a^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^7,x, algorithm="giac")

[Out] -1/6\*(3\*b^2\*x^4 + 3\*a\*b\*x^2 + a^2)/x^6

$$3.26 \quad \int \frac{(a+bx^2)^2}{x^8} dx$$

**Optimal.** Leaf size=30

$$-\frac{a^2}{7x^7} - \frac{2ab}{5x^5} - \frac{b^2}{3x^3}$$

[Out]  $-a^2/(7*x^7) - (2*a*b)/(5*x^5) - b^2/(3*x^3)$

**Rubi [A]** time = 0.0319263, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^2}{7x^7} - \frac{2ab}{5x^5} - \frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^2/x^8, x]

[Out]  $-a^2/(7*x^7) - (2*a*b)/(5*x^5) - b^2/(3*x^3)$

**Rubi in Sympy [A]** time = 5.75073, size = 27, normalized size = 0.9

$$-\frac{a^2}{7x^7} - \frac{2ab}{5x^5} - \frac{b^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*2/x\*\*8, x)

[Out]  $-a**2/(7*x**7) - 2*a*b/(5*x**5) - b**2/(3*x**3)$

**Mathematica [A]** time = 0.00143672, size = 30, normalized size = 1.

$$-\frac{a^2}{7x^7} - \frac{2ab}{5x^5} - \frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^2/x^8, x]

[Out]  $-a^2/(7*x^7) - (2*a*b)/(5*x^5) - b^2/(3*x^3)$

**Maple [A]** time = 0.008, size = 25, normalized size = 0.8

$$-\frac{a^2}{7x^7} - \frac{2ab}{5x^5} - \frac{b^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^2/x^8, x)

[Out]  $-1/7*a^2/x^7 - 2/5*a*b/x^5 - 1/3*b^2/x^3$



---

**Maxima [A]** time = 1.33012, size = 35, normalized size = 1.17

$$\frac{35 b^2 x^4 + 42 a b x^2 + 15 a^2}{105 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^8,x, algorithm="maxima")

[Out] -1/105\*(35\*b^2\*x^4 + 42\*a\*b\*x^2 + 15\*a^2)/x^7

---

**Fricas [A]** time = 0.210573, size = 35, normalized size = 1.17

$$\frac{35 b^2 x^4 + 42 a b x^2 + 15 a^2}{105 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^8,x, algorithm="fricas")

[Out] -1/105\*(35\*b^2\*x^4 + 42\*a\*b\*x^2 + 15\*a^2)/x^7

---

**Sympy [A]** time = 1.36027, size = 27, normalized size = 0.9

$$\frac{15 a^2 + 42 a b x^2 + 35 b^2 x^4}{105 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*2/x\*\*8,x)

[Out] -(15\*a\*\*2 + 42\*a\*b\*x\*\*2 + 35\*b\*\*2\*x\*\*4)/(105\*x\*\*7)

---

**GIAC/XCAS [A]** time = 0.208513, size = 35, normalized size = 1.17

$$\frac{35 b^2 x^4 + 42 a b x^2 + 15 a^2}{105 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^8,x, algorithm="giac")

[Out] -1/105\*(35\*b^2\*x^4 + 42\*a\*b\*x^2 + 15\*a^2)/x^7

$$3.27 \quad \int \frac{(a+bx^2)^2}{x^9} dx$$

**Optimal.** Leaf size=30

$$-\frac{a^2}{8x^8} - \frac{ab}{3x^6} - \frac{b^2}{4x^4}$$

[Out]  $-a^2/(8*x^8) - (a*b)/(3*x^6) - b^2/(4*x^4)$

**Rubi [A]** time = 0.0424477, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^2}{8x^8} - \frac{ab}{3x^6} - \frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^2/x^9, x]

[Out]  $-a^2/(8*x^8) - (a*b)/(3*x^6) - b^2/(4*x^4)$

**Rubi in Sympy [A]** time = 7.32775, size = 26, normalized size = 0.87

$$-\frac{a^2}{8x^8} - \frac{ab}{3x^6} - \frac{b^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*2/x\*\*9, x)

[Out]  $-a**2/(8*x**8) - a*b/(3*x**6) - b**2/(4*x**4)$

**Mathematica [A]** time = 0.00176919, size = 30, normalized size = 1.

$$-\frac{a^2}{8x^8} - \frac{ab}{3x^6} - \frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^2/x^9, x]

[Out]  $-a^2/(8*x^8) - (a*b)/(3*x^6) - b^2/(4*x^4)$

**Maple [A]** time = 0.008, size = 25, normalized size = 0.8

$$-\frac{a^2}{8x^8} - \frac{ab}{3x^6} - \frac{b^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^2/x^9, x)

[Out]  $-1/8*a^2/x^8 - 1/3*a*b/x^6 - 1/4*b^2/x^4$

---

**Maxima [A]** time = 1.32214, size = 35, normalized size = 1.17

$$-\frac{6b^2x^4 + 8abx^2 + 3a^2}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^9,x, algorithm="maxima")

[Out] -1/24\*(6\*b^2\*x^4 + 8\*a\*b\*x^2 + 3\*a^2)/x^8

---

**Fricas [A]** time = 0.203892, size = 35, normalized size = 1.17

$$-\frac{6b^2x^4 + 8abx^2 + 3a^2}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^9,x, algorithm="fricas")

[Out] -1/24\*(6\*b^2\*x^4 + 8\*a\*b\*x^2 + 3\*a^2)/x^8

---

**Sympy [A]** time = 1.44967, size = 27, normalized size = 0.9

$$-\frac{3a^2 + 8abx^2 + 6b^2x^4}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*2/x\*\*9,x)

[Out] -(3\*a\*\*2 + 8\*a\*b\*x\*\*2 + 6\*b\*\*2\*x\*\*4)/(24\*x\*\*8)

---

**GIAC/XCAS [A]** time = 0.207467, size = 35, normalized size = 1.17

$$-\frac{6b^2x^4 + 8abx^2 + 3a^2}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^9,x, algorithm="giac")

[Out] -1/24\*(6\*b^2\*x^4 + 8\*a\*b\*x^2 + 3\*a^2)/x^8

$$3.28 \quad \int \frac{(a+bx^2)^2}{x^{10}} dx$$

**Optimal.** Leaf size=30

$$-\frac{a^2}{9x^9} - \frac{2ab}{7x^7} - \frac{b^2}{5x^5}$$

[Out]  $-a^2/(9*x^9) - (2*a*b)/(7*x^7) - b^2/(5*x^5)$

**Rubi [A]** time = 0.0312406, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^2}{9x^9} - \frac{2ab}{7x^7} - \frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^2/x^10, x]

[Out]  $-a^2/(9*x^9) - (2*a*b)/(7*x^7) - b^2/(5*x^5)$

**Rubi in Sympy [A]** time = 5.8128, size = 27, normalized size = 0.9

$$-\frac{a^2}{9x^9} - \frac{2ab}{7x^7} - \frac{b^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*2/x\*\*10, x)

[Out]  $-a**2/(9*x**9) - 2*a*b/(7*x**7) - b**2/(5*x**5)$

**Mathematica [A]** time = 0.00140569, size = 30, normalized size = 1.

$$-\frac{a^2}{9x^9} - \frac{2ab}{7x^7} - \frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^2/x^10, x]

[Out]  $-a^2/(9*x^9) - (2*a*b)/(7*x^7) - b^2/(5*x^5)$

**Maple [A]** time = 0.007, size = 25, normalized size = 0.8

$$-\frac{a^2}{9x^9} - \frac{2ab}{7x^7} - \frac{b^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^2/x^10, x)

[Out]  $-1/9*a^2/x^9 - 2/7*a*b/x^7 - 1/5*b^2/x^5$

---

**Maxima [A]** time = 1.46874, size = 35, normalized size = 1.17

$$\frac{63 b^2 x^4 + 90 a b x^2 + 35 a^2}{315 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^10,x, algorithm="maxima")

[Out] -1/315\*(63\*b^2\*x^4 + 90\*a\*b\*x^2 + 35\*a^2)/x^9

---

**Fricas [A]** time = 0.196477, size = 35, normalized size = 1.17

$$\frac{63 b^2 x^4 + 90 a b x^2 + 35 a^2}{315 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^10,x, algorithm="fricas")

[Out] -1/315\*(63\*b^2\*x^4 + 90\*a\*b\*x^2 + 35\*a^2)/x^9

---

**Sympy [A]** time = 1.48724, size = 27, normalized size = 0.9

$$\frac{35 a^2 + 90 a b x^2 + 63 b^2 x^4}{315 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*2/x\*\*10,x)

[Out] -(35\*a\*\*2 + 90\*a\*b\*x\*\*2 + 63\*b\*\*2\*x\*\*4)/(315\*x\*\*9)

---

**GIAC/XCAS [A]** time = 0.205813, size = 35, normalized size = 1.17

$$\frac{63 b^2 x^4 + 90 a b x^2 + 35 a^2}{315 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^10,x, algorithm="giac")

[Out] -1/315\*(63\*b^2\*x^4 + 90\*a\*b\*x^2 + 35\*a^2)/x^9

### 3.29 $\int x^9 (a + bx^2)^3 dx$

**Optimal.** Leaf size=43

$$\frac{a^3 x^{10}}{10} + \frac{1}{4} a^2 b x^{12} + \frac{3}{14} a b^2 x^{14} + \frac{b^3 x^{16}}{16}$$

[Out]  $(a^3 x^{10})/10 + (a^2 b x^{12})/4 + (3 a b^2 x^{14})/14 + (b^3 x^{16})/16$

**Rubi [A]** time = 0.0824747, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^3 x^{10}}{10} + \frac{1}{4} a^2 b x^{12} + \frac{3}{14} a b^2 x^{14} + \frac{b^3 x^{16}}{16}$$

Antiderivative was successfully verified.

[In] Int[x^9\*(a + b\*x^2)^3,x]

[Out]  $(a^3 x^{10})/10 + (a^2 b x^{12})/4 + (3 a b^2 x^{14})/14 + (b^3 x^{16})/16$

**Rubi in Sympy [A]** time = 10.874, size = 37, normalized size = 0.86

$$\frac{a^3 x^{10}}{10} + \frac{a^2 b x^{12}}{4} + \frac{3 a b^2 x^{14}}{14} + \frac{b^3 x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*9\*(b\*x\*\*2+a)\*\*3,x)

[Out]  $a**3*x**10/10 + a**2*b*x**12/4 + 3*a*b**2*x**14/14 + b**3*x**16/16$

**Mathematica [A]** time = 0.00356781, size = 43, normalized size = 1.

$$\frac{a^3 x^{10}}{10} + \frac{1}{4} a^2 b x^{12} + \frac{3}{14} a b^2 x^{14} + \frac{b^3 x^{16}}{16}$$

Antiderivative was successfully verified.

[In] Integrate[x^9\*(a + b\*x^2)^3,x]

[Out]  $(a^3 x^{10})/10 + (a^2 b x^{12})/4 + (3 a b^2 x^{14})/14 + (b^3 x^{16})/16$

**Maple [A]** time = 0.001, size = 36, normalized size = 0.8

$$\frac{a^3 x^{10}}{10} + \frac{a^2 b x^{12}}{4} + \frac{3 a b^2 x^{14}}{14} + \frac{b^3 x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9\*(b\*x^2+a)^3,x)

[Out]  $1/10*a^3*x^{10}+1/4*a^2*b*x^{12}+3/14*a*b^2*x^{14}+1/16*b^3*x^{16}$

**Maxima [A]** time = 1.34573, size = 47, normalized size = 1.09

$$\frac{1}{16}b^3x^{16} + \frac{3}{14}ab^2x^{14} + \frac{1}{4}a^2bx^{12} + \frac{1}{10}a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3*x^9,x, algorithm="maxima")`

[Out]  $1/16*b^3*x^{16} + 3/14*a*b^2*x^{14} + 1/4*a^2*b*x^{12} + 1/10*a^3*x^{10}$

**Fricas [A]** time = 0.189893, size = 1, normalized size = 0.02

$$\frac{1}{16}x^{16}b^3 + \frac{3}{14}x^{14}b^2a + \frac{1}{4}x^{12}ba^2 + \frac{1}{10}x^{10}a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3*x^9,x, algorithm="fricas")`

[Out]  $1/16*x^{16}*b^3 + 3/14*x^{14}*b^2*a + 1/4*x^{12}*b*a^2 + 1/10*x^{10}*a^3$

**Sympy [A]** time = 0.101148, size = 37, normalized size = 0.86

$$\frac{a^3x^{10}}{10} + \frac{a^2bx^{12}}{4} + \frac{3ab^2x^{14}}{14} + \frac{b^3x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9*(b*x**2+a)**3,x)`

[Out]  $a^{**3}*x^{**10}/10 + a^{**2}*b*x^{**12}/4 + 3*a*b^{**2}*x^{**14}/14 + b^{**3}*x^{**16}/16$

**GIAC/XCAS [A]** time = 0.206275, size = 47, normalized size = 1.09

$$\frac{1}{16}b^3x^{16} + \frac{3}{14}ab^2x^{14} + \frac{1}{4}a^2bx^{12} + \frac{1}{10}a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3*x^9,x, algorithm="giac")`

[Out]  $1/16*b^3*x^{16} + 3/14*a*b^2*x^{14} + 1/4*a^2*b*x^{12} + 1/10*a^3*x^{10}$

### 3.30 $\int x^7 (a + bx^2)^3 dx$

**Optimal.** Leaf size=43

$$\frac{a^3x^8}{8} + \frac{3}{10}a^2bx^{10} + \frac{1}{4}ab^2x^{12} + \frac{b^3x^{14}}{14}$$

[Out]  $(a^3x^8)/8 + (3a^2bx^{10})/10 + (ab^2x^{12})/4 + (b^3x^{14})/14$

---

**Rubi [A]** time = 0.0782016, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^3x^8}{8} + \frac{3}{10}a^2bx^{10} + \frac{1}{4}ab^2x^{12} + \frac{b^3x^{14}}{14}$$

Antiderivative was successfully verified.

[In] Int[x^7\*(a + b\*x^2)^3,x]

[Out]  $(a^3x^8)/8 + (3a^2bx^{10})/10 + (ab^2x^{12})/4 + (b^3x^{14})/14$

---

**Rubi in Sympy [A]** time = 10.4321, size = 37, normalized size = 0.86

$$\frac{a^3x^8}{8} + \frac{3a^2bx^{10}}{10} + \frac{ab^2x^{12}}{4} + \frac{b^3x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*7\*(b\*x\*\*2+a)\*\*3,x)

[Out]  $a**3*x**8/8 + 3*a**2*b*x**10/10 + a*b**2*x**12/4 + b**3*x**14/14$

---

**Mathematica [A]** time = 0.00327151, size = 43, normalized size = 1.

$$\frac{a^3x^8}{8} + \frac{3}{10}a^2bx^{10} + \frac{1}{4}ab^2x^{12} + \frac{b^3x^{14}}{14}$$

Antiderivative was successfully verified.

[In] Integrate[x^7\*(a + b\*x^2)^3,x]

[Out]  $(a^3x^8)/8 + (3a^2bx^{10})/10 + (ab^2x^{12})/4 + (b^3x^{14})/14$

---

**Maple [A]** time = 0., size = 36, normalized size = 0.8

$$\frac{a^3x^8}{8} + \frac{3a^2bx^{10}}{10} + \frac{ab^2x^{12}}{4} + \frac{b^3x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7\*(b\*x^2+a)^3,x)

[Out]  $1/8*a^3*x^8+3/10*a^2*b*x^10+1/4*a*b^2*x^12+1/14*b^3*x^14$



---

**Maxima [A]** time = 1.34176, size = 47, normalized size = 1.09

$$\frac{1}{14} b^3 x^{14} + \frac{1}{4} ab^2 x^{12} + \frac{3}{10} a^2 b x^{10} + \frac{1}{8} a^3 x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3\*x^7,x, algorithm="maxima")

[Out] 1/14\*b^3\*x^14 + 1/4\*a\*b^2\*x^12 + 3/10\*a^2\*b\*x^10 + 1/8\*a^3\*x^8

---

**Fricas [A]** time = 0.182858, size = 1, normalized size = 0.02

$$\frac{1}{14} x^{14} b^3 + \frac{1}{4} x^{12} b^2 a + \frac{3}{10} x^{10} b a^2 + \frac{1}{8} x^8 a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3\*x^7,x, algorithm="fricas")

[Out] 1/14\*x^14\*b^3 + 1/4\*x^12\*b^2\*a + 3/10\*x^10\*b\*a^2 + 1/8\*x^8\*a^3

---

**Sympy [A]** time = 0.105183, size = 37, normalized size = 0.86

$$\frac{a^3 x^8}{8} + \frac{3 a^2 b x^{10}}{10} + \frac{a b^2 x^{12}}{4} + \frac{b^3 x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*7\*(b\*x\*\*2+a)\*\*3,x)

[Out] a\*\*3\*x\*\*8/8 + 3\*a\*\*2\*b\*x\*\*10/10 + a\*b\*\*2\*x\*\*12/4 + b\*\*3\*x\*\*14/14

---

**GIAC/XCAS [A]** time = 0.205491, size = 47, normalized size = 1.09

$$\frac{1}{14} b^3 x^{14} + \frac{1}{4} ab^2 x^{12} + \frac{3}{10} a^2 b x^{10} + \frac{1}{8} a^3 x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3\*x^7,x, algorithm="giac")

[Out] 1/14\*b^3\*x^14 + 1/4\*a\*b^2\*x^12 + 3/10\*a^2\*b\*x^10 + 1/8\*a^3\*x^8

### 3.31 $\int x^5 (a + bx^2)^3 dx$

**Optimal.** Leaf size=43

$$\frac{a^3x^6}{6} + \frac{3}{8}a^2bx^8 + \frac{3}{10}ab^2x^{10} + \frac{b^3x^{12}}{12}$$

[Out]  $(a^3x^6)/6 + (3a^2bx^8)/8 + (3ab^2x^{10})/10 + (b^3x^{12})/12$

**Rubi [A]** time = 0.0754728, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^3x^6}{6} + \frac{3}{8}a^2bx^8 + \frac{3}{10}ab^2x^{10} + \frac{b^3x^{12}}{12}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(a + b\*x^2)^3,x]

[Out]  $(a^3x^6)/6 + (3a^2bx^8)/8 + (3ab^2x^{10})/10 + (b^3x^{12})/12$

**Rubi in Sympy [A]** time = 10.0558, size = 39, normalized size = 0.91

$$\frac{a^3x^6}{6} + \frac{3a^2bx^8}{8} + \frac{3ab^2x^{10}}{10} + \frac{b^3x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(b\*x\*\*2+a)\*\*3,x)

[Out]  $a**3*x**6/6 + 3*a**2*b*x**8/8 + 3*a*b**2*x**10/10 + b**3*x**12/12$

**Mathematica [A]** time = 0.00302224, size = 43, normalized size = 1.

$$\frac{a^3x^6}{6} + \frac{3}{8}a^2bx^8 + \frac{3}{10}ab^2x^{10} + \frac{b^3x^{12}}{12}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*x^2)^3,x]

[Out]  $(a^3x^6)/6 + (3a^2bx^8)/8 + (3ab^2x^{10})/10 + (b^3x^{12})/12$

**Maple [A]** time = 0.002, size = 36, normalized size = 0.8

$$\frac{a^3x^6}{6} + \frac{3a^2bx^8}{8} + \frac{3ab^2x^{10}}{10} + \frac{b^3x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(b\*x^2+a)^3,x)

[Out]  $1/6*a^3*x^6+3/8*a^2*b*x^8+3/10*a*b^2*x^10+1/12*b^3*x^12$

---

**Maxima [A]** time = 1.35017, size = 47, normalized size = 1.09

$$\frac{1}{12} b^3 x^{12} + \frac{3}{10} a b^2 x^{10} + \frac{3}{8} a^2 b x^8 + \frac{1}{6} a^3 x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3*x^5,x, algorithm="maxima")`

[Out] `1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6`

---

**Fricas [A]** time = 0.179869, size = 1, normalized size = 0.02

$$\frac{1}{12} x^{12} b^3 + \frac{3}{10} x^{10} b^2 a + \frac{3}{8} x^8 b a^2 + \frac{1}{6} x^6 a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3*x^5,x, algorithm="fricas")`

[Out] `1/12*x^12*b^3 + 3/10*x^10*b^2*a + 3/8*x^8*b*a^2 + 1/6*x^6*a^3`

---

**Sympy [A]** time = 0.098994, size = 39, normalized size = 0.91

$$\frac{a^3 x^6}{6} + \frac{3 a^2 b x^8}{8} + \frac{3 a b^2 x^{10}}{10} + \frac{b^3 x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(b*x**2+a)**3,x)`

[Out] `a**3*x**6/6 + 3*a**2*b*x**8/8 + 3*a*b**2*x**10/10 + b**3*x**12/12`

---

**GIAC/XCAS [A]** time = 0.206851, size = 47, normalized size = 1.09

$$\frac{1}{12} b^3 x^{12} + \frac{3}{10} a b^2 x^{10} + \frac{3}{8} a^2 b x^8 + \frac{1}{6} a^3 x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3*x^5,x, algorithm="giac")`

[Out] `1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6`

### 3.32 $\int x^3 (a + bx^2)^3 dx$

**Optimal.** Leaf size=34

$$\frac{(a + bx^2)^5}{10b^2} - \frac{a(a + bx^2)^4}{8b^2}$$

[Out]  $-(a*(a + b*x^2)^4)/(8*b^2) + (a + b*x^2)^5/(10*b^2)$

**Rubi [A]** time = 0.0848432, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{(a + bx^2)^5}{10b^2} - \frac{a(a + bx^2)^4}{8b^2}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*x^2)^3, x]

[Out]  $-(a*(a + b*x^2)^4)/(8*b^2) + (a + b*x^2)^5/(10*b^2)$

**Rubi in Sympy [A]** time = 8.96201, size = 27, normalized size = 0.79

$$-\frac{a(a + bx^2)^4}{8b^2} + \frac{(a + bx^2)^5}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(b\*x\*\*2+a)\*\*3, x)

[Out]  $-a*(a + b*x**2)**4/(8*b**2) + (a + b*x**2)**5/(10*b**2)$

**Mathematica [A]** time = 0.00322607, size = 43, normalized size = 1.26

$$\frac{a^3x^4}{4} + \frac{1}{2}a^2bx^6 + \frac{3}{8}ab^2x^8 + \frac{b^3x^{10}}{10}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*x^2)^3, x]

[Out]  $(a^3*x^4)/4 + (a^2*b*x^6)/2 + (3*a*b^2*x^8)/8 + (b^3*x^10)/10$

**Maple [A]** time = 0., size = 36, normalized size = 1.1

$$\frac{b^3x^{10}}{10} + \frac{3ab^2x^8}{8} + \frac{a^2bx^6}{2} + \frac{a^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(b\*x^2+a)^3, x)

[Out]  $1/10*b^3*x^10+3/8*a*b^2*x^8+1/2*a^2*b*x^6+1/4*a^3*x^4$

---

**Maxima [A]** time = 1.35244, size = 47, normalized size = 1.38

$$\frac{1}{10} b^3 x^{10} + \frac{3}{8} a b^2 x^8 + \frac{1}{2} a^2 b x^6 + \frac{1}{4} a^3 x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3\*x^3,x, algorithm="maxima")

[Out] 1/10\*b^3\*x^10 + 3/8\*a\*b^2\*x^8 + 1/2\*a^2\*b\*x^6 + 1/4\*a^3\*x^4

---

**Fricas [A]** time = 0.182997, size = 1, normalized size = 0.03

$$\frac{1}{10} x^{10} b^3 + \frac{3}{8} x^8 b^2 a + \frac{1}{2} x^6 b a^2 + \frac{1}{4} x^4 a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3\*x^3,x, algorithm="fricas")

[Out] 1/10\*x^10\*b^3 + 3/8\*x^8\*b^2\*a + 1/2\*x^6\*b\*a^2 + 1/4\*x^4\*a^3

---

**Sympy [A]** time = 0.099702, size = 37, normalized size = 1.09

$$\frac{a^3 x^4}{4} + \frac{a^2 b x^6}{2} + \frac{3 a b^2 x^8}{8} + \frac{b^3 x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(b\*x\*\*2+a)\*\*3,x)

[Out] a\*\*3\*x\*\*4/4 + a\*\*2\*b\*x\*\*6/2 + 3\*a\*b\*\*2\*x\*\*8/8 + b\*\*3\*x\*\*10/10

---

**GIAC/XCAS [A]** time = 0.206832, size = 47, normalized size = 1.38

$$\frac{1}{10} b^3 x^{10} + \frac{3}{8} a b^2 x^8 + \frac{1}{2} a^2 b x^6 + \frac{1}{4} a^3 x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3\*x^3,x, algorithm="giac")

[Out] 1/10\*b^3\*x^10 + 3/8\*a\*b^2\*x^8 + 1/2\*a^2\*b\*x^6 + 1/4\*a^3\*x^4

### 3.33 $\int x (a + bx^2)^3 dx$

**Optimal.** Leaf size=16

$$\frac{(a + bx^2)^4}{8b}$$

[Out] (a + b\*x^2)^4/(8\*b)

**Rubi [A]** time = 0.012089, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{(a + bx^2)^4}{8b}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*x^2)^3, x]

[Out] (a + b\*x^2)^4/(8\*b)

**Rubi in Sympy [A]** time = 2.24312, size = 10, normalized size = 0.62

$$\frac{(a + bx^2)^4}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(b\*x\*\*2+a)\*\*3, x)

[Out] (a + b\*x\*\*2)\*\*4/(8\*b)

**Mathematica [A]** time = 0.00356205, size = 16, normalized size = 1.

$$\frac{(a + bx^2)^4}{8b}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*x^2)^3, x]

[Out] (a + b\*x^2)^4/(8\*b)

**Maple [B]** time = 0.001, size = 36, normalized size = 2.3

$$\frac{b^3x^8}{8} + \frac{ab^2x^6}{2} + \frac{3a^2bx^4}{4} + \frac{a^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(b\*x^2+a)^3, x)

[Out] 1/8\*b^3\*x^8+1/2\*a\*b^2\*x^6+3/4\*a^2\*b\*x^4+1/2\*a^3\*x^2

---

**Maxima [A]** time = 1.3496, size = 19, normalized size = 1.19

$$\frac{(bx^2 + a)^4}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3\*x,x, algorithm="maxima")

[Out] 1/8\*(b\*x^2 + a)^4/b

---

**Fricas [A]** time = 0.179606, size = 1, normalized size = 0.06

$$\frac{1}{8}x^8b^3 + \frac{1}{2}x^6b^2a + \frac{3}{4}x^4ba^2 + \frac{1}{2}x^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3\*x,x, algorithm="fricas")

[Out] 1/8\*x^8\*b^3 + 1/2\*x^6\*b^2\*a + 3/4\*x^4\*b\*a^2 + 1/2\*x^2\*a^3

---

**Sympy [A]** time = 0.098184, size = 37, normalized size = 2.31

$$\frac{a^3x^2}{2} + \frac{3a^2bx^4}{4} + \frac{ab^2x^6}{2} + \frac{b^3x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x\*\*2+a)\*\*3,x)

[Out] a\*\*3\*x\*\*2/2 + 3\*a\*\*2\*b\*x\*\*4/4 + a\*b\*\*2\*x\*\*6/2 + b\*\*3\*x\*\*8/8

---

**GIAC/XCAS [A]** time = 0.207988, size = 19, normalized size = 1.19

$$\frac{(bx^2 + a)^4}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3\*x,x, algorithm="giac")

[Out] 1/8\*(b\*x^2 + a)^4/b

$$3.34 \quad \int \frac{(a+bx^2)^3}{x} dx$$

**Optimal.** Leaf size=39

$$a^3 \log(x) + \frac{3}{2}a^2bx^2 + \frac{3}{4}ab^2x^4 + \frac{b^3x^6}{6}$$

[Out]  $(3*a^2*b*x^2)/2 + (3*a*b^2*x^4)/4 + (b^3*x^6)/6 + a^3*\text{Log}[x]$

**Rubi [A]** time = 0.0519704, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$a^3 \log(x) + \frac{3}{2}a^2bx^2 + \frac{3}{4}ab^2x^4 + \frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^3/x, x]

[Out]  $(3*a^2*b*x^2)/2 + (3*a*b^2*x^4)/4 + (b^3*x^6)/6 + a^3*\text{Log}[x]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^3 \log(x^2)}{2} + \frac{3a^2bx^2}{2} + \frac{3ab^2 \int^{x^2} x dx}{2} + \frac{b^3x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*3/x, x)

[Out]  $a**3*\log(x**2)/2 + 3*a**2*b*x**2/2 + 3*a*b**2*Integral(x, (x, x**2))/2 + b**3*x**6/6$

**Mathematica [A]** time = 0.00654973, size = 39, normalized size = 1.

$$a^3 \log(x) + \frac{3}{2}a^2bx^2 + \frac{3}{4}ab^2x^4 + \frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^3/x, x]

[Out]  $(3*a^2*b*x^2)/2 + (3*a*b^2*x^4)/4 + (b^3*x^6)/6 + a^3*\text{Log}[x]$

**Maple [A]** time = 0.003, size = 34, normalized size = 0.9

$$\frac{3a^2bx^2}{2} + \frac{3ab^2x^4}{4} + \frac{b^3x^6}{6} + a^3 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^3/x, x)



[Out]  $3/2*a^2*b*x^2+3/4*a*b^2*x^4+1/6*b^3*x^6+a^3*\ln(x)$

**Maxima [A]** time = 1.34637, size = 49, normalized size = 1.26

$$\frac{1}{6}b^3x^6 + \frac{3}{4}ab^2x^4 + \frac{3}{2}a^2bx^2 + \frac{1}{2}a^3\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x,x, algorithm="maxima")`

[Out]  $1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + 1/2*a^3*\log(x^2)$

**Fricas [A]** time = 0.20157, size = 45, normalized size = 1.15

$$\frac{1}{6}b^3x^6 + \frac{3}{4}ab^2x^4 + \frac{3}{2}a^2bx^2 + a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x,x, algorithm="fricas")`

[Out]  $1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + a^3*\log(x)$

**Sympy [A]** time = 1.05655, size = 37, normalized size = 0.95

$$a^3\log(x) + \frac{3a^2bx^2}{2} + \frac{3ab^2x^4}{4} + \frac{b^3x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**3/x,x)`

[Out]  $a**3*\log(x) + 3*a**2*b*x**2/2 + 3*a*b**2*x**4/4 + b**3*x**6/6$

**GIAC/XCAS [A]** time = 0.209217, size = 49, normalized size = 1.26

$$\frac{1}{6}b^3x^6 + \frac{3}{4}ab^2x^4 + \frac{3}{2}a^2bx^2 + \frac{1}{2}a^3\ln(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x,x, algorithm="giac")`

[Out]  $1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + 1/2*a^3*\ln(x^2)$

$$3.35 \quad \int \frac{(a+bx^2)^3}{x^3} dx$$

**Optimal.** Leaf size=40

$$-\frac{a^3}{2x^2} + 3a^2b \log(x) + \frac{3}{2}ab^2x^2 + \frac{b^3x^4}{4}$$

[Out]  $-a^3/(2*x^2) + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*\text{Log}[x]$

**Rubi [A]** time = 0.0576987, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^3}{2x^2} + 3a^2b \log(x) + \frac{3}{2}ab^2x^2 + \frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^3/x^3, x]

[Out]  $-a^3/(2*x^2) + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*\text{Log}[x]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^3}{2x^2} + \frac{3a^2b \log(x^2)}{2} + \frac{3ab^2x^2}{2} + \frac{b^3 \int^{x^2} x dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*3/x\*\*3, x)

[Out]  $-a**3/(2*x**2) + 3*a**2*b*\log(x**2)/2 + 3*a*b**2*x**2/2 + b**3*\text{Integral}(x, (x, x**2))/2$

**Mathematica [A]** time = 0.0120422, size = 40, normalized size = 1.

$$-\frac{a^3}{2x^2} + 3a^2b \log(x) + \frac{3}{2}ab^2x^2 + \frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^3/x^3, x]

[Out]  $-a^3/(2*x^2) + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*\text{Log}[x]$

**Maple [A]** time = 0.009, size = 35, normalized size = 0.9

$$-\frac{a^3}{2x^2} + \frac{3ab^2x^2}{2} + \frac{b^3x^4}{4} + 3a^2b \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^3/x^3, x)

[Out]  $-1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*\ln(x)$

**Maxima [A]** time = 1.34689, size = 49, normalized size = 1.22

$$\frac{1}{4}b^3x^4 + \frac{3}{2}ab^2x^2 + \frac{3}{2}a^2b\log(x^2) - \frac{a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^3,x, algorithm="maxima")`

[Out]  $1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*b*\log(x^2) - 1/2*a^3/x^2$

**Fricas [A]** time = 0.204462, size = 51, normalized size = 1.27

$$\frac{b^3x^6 + 6ab^2x^4 + 12a^2bx^2\log(x) - 2a^3}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^3,x, algorithm="fricas")`

[Out]  $1/4*(b^3*x^6 + 6*a*b^2*x^4 + 12*a^2*b*x^2*\log(x) - 2*a^3)/x^2$

**Sympy [A]** time = 1.18794, size = 37, normalized size = 0.92

$$-\frac{a^3}{2x^2} + 3a^2b\log(x) + \frac{3ab^2x^2}{2} + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**3/x**3,x)`

[Out]  $-a**3/(2*x**2) + 3*a**2*b*\log(x) + 3*a*b**2*x**2/2 + b**3*x**4/4$

**GIAC/XCAS [A]** time = 0.209, size = 62, normalized size = 1.55

$$\frac{1}{4}b^3x^4 + \frac{3}{2}ab^2x^2 + \frac{3}{2}a^2b\ln(x^2) - \frac{3a^2bx^2 + a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^3,x, algorithm="giac")`

[Out]  $1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*b*\ln(x^2) - 1/2*(3*a^2*b*x^2 + a^3)/x^2$

$$3.36 \quad \int \frac{(a+bx^2)^3}{x^5} dx$$

**Optimal.** Leaf size=40

$$-\frac{a^3}{4x^4} - \frac{3a^2b}{2x^2} + 3ab^2 \log(x) + \frac{b^3x^2}{2}$$

[Out]  $-a^3/(4*x^4) - (3*a^2*b)/(2*x^2) + (b^3*x^2)/2 + 3*a*b^2*Log[x]$

**Rubi [A]** time = 0.0546777, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^3}{4x^4} - \frac{3a^2b}{2x^2} + 3ab^2 \log(x) + \frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^3/x^5, x]

[Out]  $-a^3/(4*x^4) - (3*a^2*b)/(2*x^2) + (b^3*x^2)/2 + 3*a*b^2*Log[x]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^3}{4x^4} - \frac{3a^2b}{2x^2} + \frac{3ab^2 \log(x^2)}{2} + \frac{\int^{x^2} b^3 dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*3/x\*\*5, x)

[Out]  $-a**3/(4*x**4) - 3*a**2*b/(2*x**2) + 3*a*b**2*log(x**2)/2 + \text{Integral}(b**3, (x, x**2))/2$

**Mathematica [A]** time = 0.00762455, size = 40, normalized size = 1.

$$-\frac{a^3}{4x^4} - \frac{3a^2b}{2x^2} + 3ab^2 \log(x) + \frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^3/x^5, x]

[Out]  $-a^3/(4*x^4) - (3*a^2*b)/(2*x^2) + (b^3*x^2)/2 + 3*a*b^2*Log[x]$

**Maple [A]** time = 0.01, size = 35, normalized size = 0.9

$$-\frac{a^3}{4x^4} - \frac{3a^2b}{2x^2} + \frac{b^3x^2}{2} + 3ab^2 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^3/x^5, x)

[Out]  $-1/4*a^3/x^4-3/2*a^2*b/x^2+1/2*b^3*x^2+3*a*b^2*\ln(x)$

**Maxima [A]** time = 1.3458, size = 50, normalized size = 1.25

$$\frac{1}{2}b^3x^2 + \frac{3}{2}ab^2\log(x^2) - \frac{6a^2bx^2 + a^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^5,x, algorithm="maxima")`

[Out]  $1/2*b^3*x^2 + 3/2*a*b^2*\log(x^2) - 1/4*(6*a^2*b*x^2 + a^3)/x^4$

**Fricas [A]** time = 0.200147, size = 53, normalized size = 1.32

$$\frac{2b^3x^6 + 12ab^2x^4\log(x) - 6a^2bx^2 - a^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^5,x, algorithm="fricas")`

[Out]  $1/4*(2*b^3*x^6 + 12*a*b^2*x^4*\log(x) - 6*a^2*b*x^2 - a^3)/x^4$

**Sympy [A]** time = 1.353, size = 36, normalized size = 0.9

$$3ab^2\log(x) + \frac{b^3x^2}{2} - \frac{a^3 + 6a^2bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**3/x**5,x)`

[Out]  $3*a*b**2*\log(x) + b**3*x**2/2 - (a**3 + 6*a**2*b*x**2)/(4*x**4)$

**GIAC/XCAS [A]** time = 0.207896, size = 62, normalized size = 1.55

$$\frac{1}{2}b^3x^2 + \frac{3}{2}ab^2\ln(x^2) - \frac{9ab^2x^4 + 6a^2bx^2 + a^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^5,x, algorithm="giac")`

[Out]  $1/2*b^3*x^2 + 3/2*a*b^2*\ln(x^2) - 1/4*(9*a*b^2*x^4 + 6*a^2*b*x^2 + a^3)/x^4$

$$3.37 \quad \int \frac{(a+bx^2)^3}{x^7} dx$$

**Optimal.** Leaf size=39

$$-\frac{a^3}{6x^6} - \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} + b^3 \log(x)$$

[Out]  $-a^3/(6*x^6) - (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) + b^3*Log[x]$

**Rubi [A]** time = 0.0528071, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^3}{6x^6} - \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} + b^3 \log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^3/x^7, x]

[Out]  $-a^3/(6*x^6) - (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) + b^3*Log[x]$

**Rubi in Sympy [A]** time = 9.02265, size = 41, normalized size = 1.05

$$-\frac{a^3}{6x^6} - \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} + \frac{b^3 \log(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*3/x\*\*7, x)

[Out]  $-a**3/(6*x**6) - 3*a**2*b/(4*x**4) - 3*a*b**2/(2*x**2) + b**3*log(x**2)/2$

**Mathematica [A]** time = 0.00772855, size = 39, normalized size = 1.

$$-\frac{a^3}{6x^6} - \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} + b^3 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^3/x^7, x]

[Out]  $-a^3/(6*x^6) - (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) + b^3*Log[x]$

**Maple [A]** time = 0.009, size = 34, normalized size = 0.9

$$-\frac{a^3}{6x^6} - \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} + b^3 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^3/x^7, x)

[Out]  $-1/6*a^3/x^6-3/4*a^2*b/x^4-3/2*a*b^2/x^2+b^3*\ln(x)$

**Maxima [A]** time = 1.346, size = 53, normalized size = 1.36

$$\frac{1}{2} b^3 \log(x^2) - \frac{18 ab^2 x^4 + 9 a^2 b x^2 + 2 a^3}{12 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^7,x, algorithm="maxima")`

[Out]  $1/2*b^3*\log(x^2) - 1/12*(18*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)/x^6$

**Fricas [A]** time = 0.202058, size = 53, normalized size = 1.36

$$\frac{12 b^3 x^6 \log(x) - 18 ab^2 x^4 - 9 a^2 b x^2 - 2 a^3}{12 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^7,x, algorithm="fricas")`

[Out]  $1/12*(12*b^3*x^6*\log(x) - 18*a*b^2*x^4 - 9*a^2*b*x^2 - 2*a^3)/x^6$

**Sympy [A]** time = 1.49431, size = 36, normalized size = 0.92

$$b^3 \log(x) - \frac{2a^3 + 9a^2bx^2 + 18ab^2x^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**3/x**7,x)`

[Out]  $b**3*\log(x) - (2*a**3 + 9*a**2*b*x**2 + 18*a*b**2*x**4)/(12*x**6)$

**GIAC/XCAS [A]** time = 0.207988, size = 63, normalized size = 1.62

$$\frac{1}{2} b^3 \ln(x^2) - \frac{11 b^3 x^6 + 18 ab^2 x^4 + 9 a^2 b x^2 + 2 a^3}{12 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^7,x, algorithm="giac")`

[Out]  $1/2*b^3*\ln(x^2) - 1/12*(11*b^3*x^6 + 18*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)/x^6$

$$3.38 \quad \int \frac{(a+bx^2)^3}{x^9} dx$$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^4}{8ax^8}$$

[Out]  $-(a + b*x^2)^4/(8*a*x^8)$

Rubi [A] time = 0.0183981, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{(a+bx^2)^4}{8ax^8}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^3/x^9, x]

[Out]  $-(a + b*x^2)^4/(8*a*x^8)$

Rubi in Sympy [A] time = 3.24988, size = 15, normalized size = 0.79

$$-\frac{(a+bx^2)^4}{8ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*3/x\*\*9, x)

[Out]  $-(a + b*x**2)**4/(8*a*x**8)$

Mathematica [B] time = 0.0113988, size = 43, normalized size = 2.26

$$-\frac{a^3}{8x^8} - \frac{a^2b}{2x^6} - \frac{3ab^2}{4x^4} - \frac{b^3}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^3/x^9, x]

[Out]  $-a^3/(8*x^8) - (a^2*b)/(2*x^6) - (3*a*b^2)/(4*x^4) - b^3/(2*x^2)$

Maple [B] time = 0.007, size = 36, normalized size = 1.9

$$-\frac{a^2b}{2x^6} - \frac{3ab^2}{4x^4} - \frac{a^3}{8x^8} - \frac{b^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^3/x^9, x)

[Out]  $-1/2*a^2*b/x^6 - 3/4*a*b^2/x^4 - 1/8*a^3/x^8 - 1/2*b^3/x^2$



---

**Maxima [A]** time = 1.3449, size = 47, normalized size = 2.47

$$\frac{4b^3x^6 + 6ab^2x^4 + 4a^2bx^2 + a^3}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3/x^9,x, algorithm="maxima")

[Out] -1/8\*(4\*b^3\*x^6 + 6\*a\*b^2\*x^4 + 4\*a^2\*b\*x^2 + a^3)/x^8

---

**Fricas [A]** time = 0.194857, size = 47, normalized size = 2.47

$$\frac{4b^3x^6 + 6ab^2x^4 + 4a^2bx^2 + a^3}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3/x^9,x, algorithm="fricas")

[Out] -1/8\*(4\*b^3\*x^6 + 6\*a\*b^2\*x^4 + 4\*a^2\*b\*x^2 + a^3)/x^8

---

**Sympy [A]** time = 1.58443, size = 37, normalized size = 1.95

$$\frac{a^3 + 4a^2bx^2 + 6ab^2x^4 + 4b^3x^6}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*3/x\*\*9,x)

[Out] -(a\*\*3 + 4\*a\*\*2\*b\*x\*\*2 + 6\*a\*b\*\*2\*x\*\*4 + 4\*b\*\*3\*x\*\*6)/(8\*x\*\*8)

---

**GIAC/XCAS [A]** time = 0.211486, size = 47, normalized size = 2.47

$$\frac{4b^3x^6 + 6ab^2x^4 + 4a^2bx^2 + a^3}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3/x^9,x, algorithm="giac")

[Out] -1/8\*(4\*b^3\*x^6 + 6\*a\*b^2\*x^4 + 4\*a^2\*b\*x^2 + a^3)/x^8

$$3.39 \quad \int \frac{(a+bx^2)^3}{x^{11}} dx$$

**Optimal.** Leaf size=40

$$\frac{b(a+bx^2)^4}{40a^2x^8} - \frac{(a+bx^2)^4}{10ax^{10}}$$

[Out]  $-(a + b*x^2)^4/(10*a*x^{10}) + (b*(a + b*x^2)^4)/(40*a^2*x^8)$

**Rubi [A]** time = 0.0563301, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{b(a+bx^2)^4}{40a^2x^8} - \frac{(a+bx^2)^4}{10ax^{10}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^3/x^11, x]

[Out]  $-(a + b*x^2)^4/(10*a*x^{10}) + (b*(a + b*x^2)^4)/(40*a^2*x^8)$

**Rubi in Sympy [A]** time = 9.3241, size = 39, normalized size = 0.98

$$-\frac{a^3}{10x^{10}} - \frac{3a^2b}{8x^8} - \frac{ab^2}{2x^6} - \frac{b^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*3/x\*\*11, x)

[Out]  $-a**3/(10*x**10) - 3*a**2*b/(8*x**8) - a*b**2/(2*x**6) - b**3/(4*x**4)$

**Mathematica [A]** time = 0.00714586, size = 43, normalized size = 1.08

$$-\frac{a^3}{10x^{10}} - \frac{3a^2b}{8x^8} - \frac{ab^2}{2x^6} - \frac{b^3}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^3/x^11, x]

[Out]  $-a^3/(10*x^10) - (3*a^2*b)/(8*x^8) - (a*b^2)/(2*x^6) - b^3/(4*x^4)$

**Maple [A]** time = 0.009, size = 36, normalized size = 0.9

$$-\frac{a^3}{10x^{10}} - \frac{ab^2}{2x^6} - \frac{b^3}{4x^4} - \frac{3a^2b}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^3/x^11, x)

[Out]  $-1/10*a^3/x^{10}-1/2*a*b^2/x^6-1/4*b^3/x^4-3/8*a^2*b/x^8$

**Maxima [A]** time = 1.34767, size = 50, normalized size = 1.25

$$-\frac{10b^3x^6 + 20ab^2x^4 + 15a^2bx^2 + 4a^3}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^11,x, algorithm="maxima")`

[Out]  $-1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^{10}$

**Fricas [A]** time = 0.193905, size = 50, normalized size = 1.25

$$-\frac{10b^3x^6 + 20ab^2x^4 + 15a^2bx^2 + 4a^3}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^11,x, algorithm="fricas")`

[Out]  $-1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^{10}$

**Sympy [A]** time = 1.70705, size = 39, normalized size = 0.98

$$-\frac{4a^3 + 15a^2bx^2 + 20ab^2x^4 + 10b^3x^6}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**3/x**11,x)`

[Out]  $-(4*a**3 + 15*a**2*b*x**2 + 20*a*b**2*x**4 + 10*b**3*x**6)/(40*x**10)$

**GIAC/XCAS [A]** time = 0.20664, size = 50, normalized size = 1.25

$$-\frac{10b^3x^6 + 20ab^2x^4 + 15a^2bx^2 + 4a^3}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^11,x, algorithm="giac")`

[Out]  $-1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^{10}$

$$3.40 \quad \int \frac{(a+bx^2)^3}{x^{13}} dx$$

**Optimal.** Leaf size=43

$$-\frac{a^3}{12x^{12}} - \frac{3a^2b}{10x^{10}} - \frac{3ab^2}{8x^8} - \frac{b^3}{6x^6}$$

[Out]  $-a^3/(12*x^{12}) - (3*a^2*b)/(10*x^{10}) - (3*a*b^2)/(8*x^8) - b^3/(6*x^6)$

**Rubi [A]** time = 0.0567016, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^3}{12x^{12}} - \frac{3a^2b}{10x^{10}} - \frac{3ab^2}{8x^8} - \frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^3/x^13, x]

[Out]  $-a^3/(12*x^{12}) - (3*a^2*b)/(10*x^{10}) - (3*a*b^2)/(8*x^8) - b^3/(6*x^6)$

**Rubi in Sympy [A]** time = 9.38866, size = 41, normalized size = 0.95

$$-\frac{a^3}{12x^{12}} - \frac{3a^2b}{10x^{10}} - \frac{3ab^2}{8x^8} - \frac{b^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*3/x\*\*13, x)

[Out]  $-a**3/(12*x**12) - 3*a**2*b/(10*x**10) - 3*a*b**2/(8*x**8) - b**3/(6*x**6)$

**Mathematica [A]** time = 0.00743864, size = 43, normalized size = 1.

$$-\frac{a^3}{12x^{12}} - \frac{3a^2b}{10x^{10}} - \frac{3ab^2}{8x^8} - \frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^3/x^13, x]

[Out]  $-a^3/(12*x^{12}) - (3*a^2*b)/(10*x^{10}) - (3*a*b^2)/(8*x^8) - b^3/(6*x^6)$

**Maple [A]** time = 0.008, size = 36, normalized size = 0.8

$$-\frac{a^3}{12x^{12}} - \frac{3a^2b}{10x^{10}} - \frac{3ab^2}{8x^8} - \frac{b^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^3/x^13, x)

[Out]  $-1/12*a^3/x^{12}-3/10*a^2*b/x^{10}-3/8*a*b^2/x^8-1/6*b^3/x^6$

**Maxima [A]** time = 1.35018, size = 50, normalized size = 1.16

$$\frac{20b^3x^6 + 45ab^2x^4 + 36a^2bx^2 + 10a^3}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^13,x, algorithm="maxima")`

[Out]  $-1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^{12}$

**Fricas [A]** time = 0.192495, size = 50, normalized size = 1.16

$$\frac{20b^3x^6 + 45ab^2x^4 + 36a^2bx^2 + 10a^3}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^13,x, algorithm="fricas")`

[Out]  $-1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^{12}$

**Sympy [A]** time = 1.81764, size = 39, normalized size = 0.91

$$-\frac{10a^3 + 36a^2bx^2 + 45ab^2x^4 + 20b^3x^6}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**3/x**13,x)`

[Out]  $-(10*a**3 + 36*a**2*b*x**2 + 45*a*b**2*x**4 + 20*b**3*x**6)/(120*x**12)$

**GIAC/XCAS [A]** time = 0.207445, size = 50, normalized size = 1.16

$$\frac{20b^3x^6 + 45ab^2x^4 + 36a^2bx^2 + 10a^3}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^13,x, algorithm="giac")`

[Out]  $-1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^{12}$

$$3.41 \quad \int \frac{(a+bx^2)^3}{x^{15}} dx$$

**Optimal.** Leaf size=43

$$-\frac{a^3}{14x^{14}} - \frac{a^2b}{4x^{12}} - \frac{3ab^2}{10x^{10}} - \frac{b^3}{8x^8}$$

[Out]  $-a^3/(14*x^{14}) - (a^2*b)/(4*x^{12}) - (3*a*b^2)/(10*x^{10}) - b^3/(8*x^8)$

**Rubi [A]** time = 0.0555315, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^3}{14x^{14}} - \frac{a^2b}{4x^{12}} - \frac{3ab^2}{10x^{10}} - \frac{b^3}{8x^8}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^3/x^15, x]

[Out]  $-a^3/(14*x^{14}) - (a^2*b)/(4*x^{12}) - (3*a*b^2)/(10*x^{10}) - b^3/(8*x^8)$

**Rubi in Sympy [A]** time = 9.21817, size = 39, normalized size = 0.91

$$-\frac{a^3}{14x^{14}} - \frac{a^2b}{4x^{12}} - \frac{3ab^2}{10x^{10}} - \frac{b^3}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*3/x\*\*15, x)

[Out]  $-a**3/(14*x**14) - a**2*b/(4*x**12) - 3*a*b**2/(10*x**10) - b**3/(8*x**8)$

**Mathematica [A]** time = 0.0120506, size = 43, normalized size = 1.

$$-\frac{a^3}{14x^{14}} - \frac{a^2b}{4x^{12}} - \frac{3ab^2}{10x^{10}} - \frac{b^3}{8x^8}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^3/x^15, x]

[Out]  $-a^3/(14*x^{14}) - (a^2*b)/(4*x^{12}) - (3*a*b^2)/(10*x^{10}) - b^3/(8*x^8)$

**Maple [A]** time = 0.007, size = 36, normalized size = 0.8

$$-\frac{a^3}{14x^{14}} - \frac{a^2b}{4x^{12}} - \frac{3ab^2}{10x^{10}} - \frac{b^3}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^3/x^15, x)

[Out]  $-1/14*a^3/x^{14}-1/4*a^2*b/x^{12}-3/10*a*b^2/x^{10}-1/8*b^3/x^8$

**Maxima [A]** time = 1.34464, size = 50, normalized size = 1.16

$$\frac{35 b^3 x^6 + 84 a b^2 x^4 + 70 a^2 b x^2 + 20 a^3}{280 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^15,x, algorithm="maxima")`

[Out]  $-1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^{14}$

**Fricas [A]** time = 0.193665, size = 50, normalized size = 1.16

$$\frac{35 b^3 x^6 + 84 a b^2 x^4 + 70 a^2 b x^2 + 20 a^3}{280 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^15,x, algorithm="fricas")`

[Out]  $-1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^{14}$

**Sympy [A]** time = 1.90044, size = 39, normalized size = 0.91

$$-\frac{20a^3 + 70a^2bx^2 + 84ab^2x^4 + 35b^3x^6}{280x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**3/x**15,x)`

[Out]  $-(20*a**3 + 70*a**2*b*x**2 + 84*a*b**2*x**4 + 35*b**3*x**6)/(280*x**14)$

**GIAC/XCAS [A]** time = 0.21349, size = 50, normalized size = 1.16

$$\frac{35 b^3 x^6 + 84 a b^2 x^4 + 70 a^2 b x^2 + 20 a^3}{280 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^15,x, algorithm="giac")`

[Out]  $-1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^{14}$

### 3.42 $\int x^6 (a + bx^2)^3 dx$

**Optimal.** Leaf size=43

$$\frac{a^3x^7}{7} + \frac{1}{3}a^2bx^9 + \frac{3}{11}ab^2x^{11} + \frac{b^3x^{13}}{13}$$

[Out]  $(a^3x^7)/7 + (a^2bx^9)/3 + (3ab^2x^{11})/11 + (b^3x^{13})/13$

**Rubi [A]** time = 0.046116, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{a^3x^7}{7} + \frac{1}{3}a^2bx^9 + \frac{3}{11}ab^2x^{11} + \frac{b^3x^{13}}{13}$$

Antiderivative was successfully verified.

[In] Int[x^6\*(a + b\*x^2)^3,x]

[Out]  $(a^3x^7)/7 + (a^2bx^9)/3 + (3ab^2x^{11})/11 + (b^3x^{13})/13$

**Rubi in Sympy [A]** time = 7.4988, size = 37, normalized size = 0.86

$$\frac{a^3x^7}{7} + \frac{a^2bx^9}{3} + \frac{3ab^2x^{11}}{11} + \frac{b^3x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6\*(b\*x\*\*2+a)\*\*3,x)

[Out]  $a**3*x**7/7 + a**2*b*x**9/3 + 3*a*b**2*x**11/11 + b**3*x**13/13$

**Mathematica [A]** time = 0.00321871, size = 43, normalized size = 1.

$$\frac{a^3x^7}{7} + \frac{1}{3}a^2bx^9 + \frac{3}{11}ab^2x^{11} + \frac{b^3x^{13}}{13}$$

Antiderivative was successfully verified.

[In] Integrate[x^6\*(a + b\*x^2)^3,x]

[Out]  $(a^3x^7)/7 + (a^2bx^9)/3 + (3ab^2x^{11})/11 + (b^3x^{13})/13$

**Maple [A]** time = 0.002, size = 36, normalized size = 0.8

$$\frac{a^3x^7}{7} + \frac{a^2bx^9}{3} + \frac{3ab^2x^{11}}{11} + \frac{b^3x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6\*(b\*x^2+a)^3,x)

[Out]  $1/7*a^3*x^7+1/3*a^2*b*x^9+3/11*a*b^2*x^11+1/13*b^3*x^13$



---

**Maxima [A]** time = 1.34434, size = 47, normalized size = 1.09

$$\frac{1}{13} b^3 x^{13} + \frac{3}{11} a b^2 x^{11} + \frac{1}{3} a^2 b x^9 + \frac{1}{7} a^3 x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3\*x^6,x, algorithm="maxima")

[Out] 1/13\*b^3\*x^13 + 3/11\*a\*b^2\*x^11 + 1/3\*a^2\*b\*x^9 + 1/7\*a^3\*x^7

---

**Fricas [A]** time = 0.179628, size = 1, normalized size = 0.02

$$\frac{1}{13} x^{13} b^3 + \frac{3}{11} x^{11} b^2 a + \frac{1}{3} x^9 b a^2 + \frac{1}{7} x^7 a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3\*x^6,x, algorithm="fricas")

[Out] 1/13\*x^13\*b^3 + 3/11\*x^11\*b^2\*a + 1/3\*x^9\*b\*a^2 + 1/7\*x^7\*a^3

---

**Sympy [A]** time = 0.101922, size = 37, normalized size = 0.86

$$\frac{a^3 x^7}{7} + \frac{a^2 b x^9}{3} + \frac{3 a b^2 x^{11}}{11} + \frac{b^3 x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*6\*(b\*x\*\*2+a)\*\*3,x)

[Out] a\*\*3\*x\*\*7/7 + a\*\*2\*b\*x\*\*9/3 + 3\*a\*b\*\*2\*x\*\*11/11 + b\*\*3\*x\*\*13/13

---

**GIAC/XCAS [A]** time = 0.207793, size = 47, normalized size = 1.09

$$\frac{1}{13} b^3 x^{13} + \frac{3}{11} a b^2 x^{11} + \frac{1}{3} a^2 b x^9 + \frac{1}{7} a^3 x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3\*x^6,x, algorithm="giac")

[Out] 1/13\*b^3\*x^13 + 3/11\*a\*b^2\*x^11 + 1/3\*a^2\*b\*x^9 + 1/7\*a^3\*x^7

### 3.43 $\int x^4 (a + bx^2)^3 dx$

**Optimal.** Leaf size=43

$$\frac{a^3x^5}{5} + \frac{3}{7}a^2bx^7 + \frac{1}{3}ab^2x^9 + \frac{b^3x^{11}}{11}$$

[Out] (a^3\*x^5)/5 + (3\*a^2\*b\*x^7)/7 + (a\*b^2\*x^9)/3 + (b^3\*x^11)/11

**Rubi [A]** time = 0.0456453, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{a^3x^5}{5} + \frac{3}{7}a^2bx^7 + \frac{1}{3}ab^2x^9 + \frac{b^3x^{11}}{11}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a + b\*x^2)^3,x]

[Out] (a^3\*x^5)/5 + (3\*a^2\*b\*x^7)/7 + (a\*b^2\*x^9)/3 + (b^3\*x^11)/11

**Rubi in Sympy [A]** time = 7.46795, size = 37, normalized size = 0.86

$$\frac{a^3x^5}{5} + \frac{3a^2bx^7}{7} + \frac{ab^2x^9}{3} + \frac{b^3x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(b\*x\*\*2+a)\*\*3,x)

[Out] a\*\*3\*x\*\*5/5 + 3\*a\*\*2\*b\*x\*\*7/7 + a\*b\*\*2\*x\*\*9/3 + b\*\*3\*x\*\*11/11

**Mathematica [A]** time = 0.00292432, size = 43, normalized size = 1.

$$\frac{a^3x^5}{5} + \frac{3}{7}a^2bx^7 + \frac{1}{3}ab^2x^9 + \frac{b^3x^{11}}{11}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a + b\*x^2)^3,x]

[Out] (a^3\*x^5)/5 + (3\*a^2\*b\*x^7)/7 + (a\*b^2\*x^9)/3 + (b^3\*x^11)/11

**Maple [A]** time = 0.001, size = 36, normalized size = 0.8

$$\frac{a^3x^5}{5} + \frac{3a^2bx^7}{7} + \frac{ab^2x^9}{3} + \frac{b^3x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(b\*x^2+a)^3,x)

[Out] 1/5\*a^3\*x^5+3/7\*a^2\*b\*x^7+1/3\*a\*b^2\*x^9+1/11\*b^3\*x^11

---

**Maxima [A]** time = 1.34029, size = 47, normalized size = 1.09

$$\frac{1}{11} b^3 x^{11} + \frac{1}{3} a b^2 x^9 + \frac{3}{7} a^2 b x^7 + \frac{1}{5} a^3 x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3\*x^4,x, algorithm="maxima")

[Out] 1/11\*b^3\*x^11 + 1/3\*a\*b^2\*x^9 + 3/7\*a^2\*b\*x^7 + 1/5\*a^3\*x^5

---

**Fricas [A]** time = 0.180417, size = 1, normalized size = 0.02

$$\frac{1}{11} x^{11} b^3 + \frac{1}{3} x^9 b^2 a + \frac{3}{7} x^7 b a^2 + \frac{1}{5} x^5 a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3\*x^4,x, algorithm="fricas")

[Out] 1/11\*x^11\*b^3 + 1/3\*x^9\*b^2\*a + 3/7\*x^7\*b\*a^2 + 1/5\*x^5\*a^3

---

**Sympy [A]** time = 0.097826, size = 37, normalized size = 0.86

$$\frac{a^3 x^5}{5} + \frac{3 a^2 b x^7}{7} + \frac{a b^2 x^9}{3} + \frac{b^3 x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(b\*x\*\*2+a)\*\*3,x)

[Out] a\*\*3\*x\*\*5/5 + 3\*a\*\*2\*b\*x\*\*7/7 + a\*b\*\*2\*x\*\*9/3 + b\*\*3\*x\*\*11/11

---

**GIAC/XCAS [A]** time = 0.207033, size = 47, normalized size = 1.09

$$\frac{1}{11} b^3 x^{11} + \frac{1}{3} a b^2 x^9 + \frac{3}{7} a^2 b x^7 + \frac{1}{5} a^3 x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3\*x^4,x, algorithm="giac")

[Out] 1/11\*b^3\*x^11 + 1/3\*a\*b^2\*x^9 + 3/7\*a^2\*b\*x^7 + 1/5\*a^3\*x^5

### 3.44 $\int x^2 (a + bx^2)^3 dx$

**Optimal.** Leaf size=43

$$\frac{a^3x^3}{3} + \frac{3}{5}a^2bx^5 + \frac{3}{7}ab^2x^7 + \frac{b^3x^9}{9}$$

[Out]  $(a^3x^3)/3 + (3a^2bx^5)/5 + (3ab^2x^7)/7 + (b^3x^9)/9$

**Rubi [A]** time = 0.0447275, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{a^3x^3}{3} + \frac{3}{5}a^2bx^5 + \frac{3}{7}ab^2x^7 + \frac{b^3x^9}{9}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*x^2)^3,x]

[Out]  $(a^3x^3)/3 + (3a^2bx^5)/5 + (3ab^2x^7)/7 + (b^3x^9)/9$

**Rubi in Sympy [A]** time = 7.66599, size = 39, normalized size = 0.91

$$\frac{a^3x^3}{3} + \frac{3a^2bx^5}{5} + \frac{3ab^2x^7}{7} + \frac{b^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(b\*x\*\*2+a)\*\*3,x)

[Out]  $a**3*x**3/3 + 3*a**2*b*x**5/5 + 3*a*b**2*x**7/7 + b**3*x**9/9$

**Mathematica [A]** time = 0.00279761, size = 43, normalized size = 1.

$$\frac{a^3x^3}{3} + \frac{3}{5}a^2bx^5 + \frac{3}{7}ab^2x^7 + \frac{b^3x^9}{9}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*x^2)^3,x]

[Out]  $(a^3x^3)/3 + (3a^2bx^5)/5 + (3ab^2x^7)/7 + (b^3x^9)/9$

**Maple [A]** time = 0.001, size = 36, normalized size = 0.8

$$\frac{a^3x^3}{3} + \frac{3a^2bx^5}{5} + \frac{3ab^2x^7}{7} + \frac{b^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(b\*x^2+a)^3,x)

[Out]  $1/3*a^3*x^3+3/5*a^2*b*x^5+3/7*a*b^2*x^7+1/9*b^3*x^9$

---

**Maxima [A]** time = 1.34831, size = 47, normalized size = 1.09

$$\frac{1}{9}b^3x^9 + \frac{3}{7}ab^2x^7 + \frac{3}{5}a^2bx^5 + \frac{1}{3}a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3\*x^2,x, algorithm="maxima")

[Out] 1/9\*b^3\*x^9 + 3/7\*a\*b^2\*x^7 + 3/5\*a^2\*b\*x^5 + 1/3\*a^3\*x^3

---

**Fricas [A]** time = 0.183528, size = 1, normalized size = 0.02

$$\frac{1}{9}x^9b^3 + \frac{3}{7}x^7b^2a + \frac{3}{5}x^5ba^2 + \frac{1}{3}x^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3\*x^2,x, algorithm="fricas")

[Out] 1/9\*x^9\*b^3 + 3/7\*x^7\*b^2\*a + 3/5\*x^5\*b\*a^2 + 1/3\*x^3\*a^3

---

**Sympy [A]** time = 0.100597, size = 39, normalized size = 0.91

$$\frac{a^3x^3}{3} + \frac{3a^2bx^5}{5} + \frac{3ab^2x^7}{7} + \frac{b^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(b\*x\*\*2+a)\*\*3,x)

[Out] a\*\*3\*x\*\*3/3 + 3\*a\*\*2\*b\*x\*\*5/5 + 3\*a\*b\*\*2\*x\*\*7/7 + b\*\*3\*x\*\*9/9

---

**GIAC/XCAS [A]** time = 0.207111, size = 47, normalized size = 1.09

$$\frac{1}{9}b^3x^9 + \frac{3}{7}ab^2x^7 + \frac{3}{5}a^2bx^5 + \frac{1}{3}a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3\*x^2,x, algorithm="giac")

[Out] 1/9\*b^3\*x^9 + 3/7\*a\*b^2\*x^7 + 3/5\*a^2\*b\*x^5 + 1/3\*a^3\*x^3

### 3.45 $\int (a + bx^2)^3 dx$

**Optimal.** Leaf size=35

$$a^3x + a^2bx^3 + \frac{3}{5}ab^2x^5 + \frac{b^3x^7}{7}$$

[Out]  $a^3x + a^2bx^3 + (3ab^2x^5)/5 + (b^3x^7)/7$

**Rubi [A]** time = 0.0311187, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$a^3x + a^2bx^3 + \frac{3}{5}ab^2x^5 + \frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^3, x]

[Out]  $a^3x + a^2bx^3 + (3ab^2x^5)/5 + (b^3x^7)/7$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$a^2bx^3 + \frac{3ab^2x^5}{5} + \frac{b^3x^7}{7} + \int a^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*3, x)

[Out]  $a**2*b*x**3 + 3*a*b**2*x**5/5 + b**3*x**7/7 + \text{Integral}(a**3, x)$

**Mathematica [A]** time = 0.00159608, size = 35, normalized size = 1.

$$a^3x + a^2bx^3 + \frac{3}{5}ab^2x^5 + \frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^3, x]

[Out]  $a^3x + a^2bx^3 + (3ab^2x^5)/5 + (b^3x^7)/7$

**Maple [A]** time = 0.002, size = 32, normalized size = 0.9

$$a^3x + a^2bx^3 + \frac{3ab^2x^5}{5} + \frac{b^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^3, x)

[Out]  $a^3x+a^2*b*x^3+3/5*a*b^2*x^5+1/7*b^3*x^7$

---

**Maxima [A]** time = 1.34973, size = 42, normalized size = 1.2

$$\frac{1}{7}b^3x^7 + \frac{3}{5}ab^2x^5 + a^2bx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3,x, algorithm="maxima")

[Out] 1/7\*b^3\*x^7 + 3/5\*a\*b^2\*x^5 + a^2\*b\*x^3 + a^3\*x

---

**Fricas [A]** time = 0.197482, size = 1, normalized size = 0.03

$$\frac{1}{7}x^7b^3 + \frac{3}{5}x^5b^2a + x^3ba^2 + xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3,x, algorithm="fricas")

[Out] 1/7\*x^7\*b^3 + 3/5\*x^5\*b^2\*a + x^3\*b\*a^2 + x\*a^3

---

**Sympy [A]** time = 0.098213, size = 32, normalized size = 0.91

$$a^3x + a^2bx^3 + \frac{3ab^2x^5}{5} + \frac{b^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*3,x)

[Out] a\*\*3\*x + a\*\*2\*b\*x\*\*3 + 3\*a\*b\*\*2\*x\*\*5/5 + b\*\*3\*x\*\*7/7

---

**GIAC/XCAS [A]** time = 0.206649, size = 42, normalized size = 1.2

$$\frac{1}{7}b^3x^7 + \frac{3}{5}ab^2x^5 + a^2bx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3,x, algorithm="giac")

[Out] 1/7\*b^3\*x^7 + 3/5\*a\*b^2\*x^5 + a^2\*b\*x^3 + a^3\*x

$$3.46 \quad \int \frac{(a+bx^2)^3}{x^2} dx$$

Optimal. Leaf size=34

$$-\frac{a^3}{x} + 3a^2bx + ab^2x^3 + \frac{b^3x^5}{5}$$

[Out]  $-(a^3/x) + 3*a^2*b*x + a*b^2*x^3 + (b^3*x^5)/5$

Rubi [A] time = 0.0406878, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^3}{x} + 3a^2bx + ab^2x^3 + \frac{b^3x^5}{5}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^3/x^2, x]

[Out]  $-(a^3/x) + 3*a^2*b*x + a*b^2*x^3 + (b^3*x^5)/5$

Rubi in Sympy [A] time = 7.08272, size = 29, normalized size = 0.85

$$-\frac{a^3}{x} + 3a^2bx + ab^2x^3 + \frac{b^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*3/x\*\*2, x)

[Out]  $-a**3/x + 3*a**2*b*x + a*b**2*x**3 + b**3*x**5/5$

Mathematica [A] time = 0.00647518, size = 34, normalized size = 1.

$$-\frac{a^3}{x} + 3a^2bx + ab^2x^3 + \frac{b^3x^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^3/x^2, x]

[Out]  $-(a^3/x) + 3*a^2*b*x + a*b^2*x^3 + (b^3*x^5)/5$

Maple [A] time = 0.005, size = 33, normalized size = 1.

$$-\frac{a^3}{x} + 3a^2bx + ab^2x^3 + \frac{b^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^3/x^2, x)

[Out]  $-a^3/x+3*a^2*b*x+a*b^2*x^3+1/5*b^3*x^5$



---

**Maxima [A]** time = 1.35274, size = 43, normalized size = 1.26

$$\frac{1}{5} b^3 x^5 + ab^2 x^3 + 3 a^2 b x - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^2,x, algorithm="maxima")`

[Out] `1/5*b^3*x^5 + a*b^2*x^3 + 3*a^2*b*x - a^3/x`

---

**Fricas [A]** time = 0.204492, size = 49, normalized size = 1.44

$$\frac{b^3 x^6 + 5 ab^2 x^4 + 15 a^2 b x^2 - 5 a^3}{5 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^2,x, algorithm="fricas")`

[Out] `1/5*(b^3*x^6 + 5*a*b^2*x^4 + 15*a^2*b*x^2 - 5*a^3)/x`

---

**Sympy [A]** time = 1.03607, size = 29, normalized size = 0.85

$$-\frac{a^3}{x} + 3a^2 b x + ab^2 x^3 + \frac{b^3 x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**3/x**2,x)`

[Out] `-a**3/x + 3*a**2*b*x + a*b**2*x**3 + b**3*x**5/5`

---

**GIAC/XCAS [A]** time = 0.21427, size = 43, normalized size = 1.26

$$\frac{1}{5} b^3 x^5 + ab^2 x^3 + 3 a^2 b x - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^2,x, algorithm="giac")`

[Out] `1/5*b^3*x^5 + a*b^2*x^3 + 3*a^2*b*x - a^3/x`

$$3.47 \quad \int \frac{(a+bx^2)^3}{x^4} dx$$

**Optimal.** Leaf size=37

$$-\frac{a^3}{3x^3} - \frac{3a^2b}{x} + 3ab^2x + \frac{b^3x^3}{3}$$

[Out]  $-a^3/(3*x^3) - (3*a^2*b)/x + 3*a*b^2*x + (b^3*x^3)/3$

**Rubi [A]** time = 0.0404206, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^3}{3x^3} - \frac{3a^2b}{x} + 3ab^2x + \frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^3/x^4, x]

[Out]  $-a^3/(3*x^3) - (3*a^2*b)/x + 3*a*b^2*x + (b^3*x^3)/3$

**Rubi in Sympy [A]** time = 7.12991, size = 32, normalized size = 0.86

$$-\frac{a^3}{3x^3} - \frac{3a^2b}{x} + 3ab^2x + \frac{b^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*3/x\*\*4, x)

[Out]  $-a**3/(3*x**3) - 3*a**2*b/x + 3*a*b**2*x + b**3*x**3/3$

**Mathematica [A]** time = 0.00699355, size = 37, normalized size = 1.

$$-\frac{a^3}{3x^3} - \frac{3a^2b}{x} + 3ab^2x + \frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^3/x^4, x]

[Out]  $-a^3/(3*x^3) - (3*a^2*b)/x + 3*a*b^2*x + (b^3*x^3)/3$

**Maple [A]** time = 0.009, size = 34, normalized size = 0.9

$$-\frac{a^3}{3x^3} - 3\frac{a^2b}{x} + 3ab^2x + \frac{b^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^3/x^4, x)

[Out]  $-1/3*a^3/x^3-3*a^2*b/x+3*a*b^2*x+1/3*b^3*x^3$

---

**Maxima [A]** time = 1.34259, size = 46, normalized size = 1.24

$$\frac{1}{3} b^3 x^3 + 3 a b^2 x - \frac{9 a^2 b x^2 + a^3}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3/x^4,x, algorithm="maxima")

[Out] 1/3\*b^3\*x^3 + 3\*a\*b^2\*x - 1/3\*(9\*a^2\*b\*x^2 + a^3)/x^3

---

**Fricas [A]** time = 0.205746, size = 49, normalized size = 1.32

$$\frac{b^3 x^6 + 9 a b^2 x^4 - 9 a^2 b x^2 - a^3}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3/x^4,x, algorithm="fricas")

[Out] 1/3\*(b^3\*x^6 + 9\*a\*b^2\*x^4 - 9\*a^2\*b\*x^2 - a^3)/x^3

---

**Sympy [A]** time = 1.20199, size = 34, normalized size = 0.92

$$3 a b^2 x + \frac{b^3 x^3}{3} - \frac{a^3 + 9 a^2 b x^2}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*3/x\*\*4,x)

[Out] 3\*a\*b\*\*2\*x + b\*\*3\*x\*\*3/3 - (a\*\*3 + 9\*a\*\*2\*b\*x\*\*2)/(3\*x\*\*3)

---

**GIAC/XCAS [A]** time = 0.20782, size = 46, normalized size = 1.24

$$\frac{1}{3} b^3 x^3 + 3 a b^2 x - \frac{9 a^2 b x^2 + a^3}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3/x^4,x, algorithm="giac")

[Out] 1/3\*b^3\*x^3 + 3\*a\*b^2\*x - 1/3\*(9\*a^2\*b\*x^2 + a^3)/x^3

$$3.48 \quad \int \frac{(a+bx^2)^3}{x^6} dx$$

Optimal. Leaf size=34

$$-\frac{a^3}{5x^5} - \frac{a^2b}{x^3} - \frac{3ab^2}{x} + b^3x$$

[Out]  $-a^3/(5*x^5) - (a^2*b)/x^3 - (3*a*b^2)/x + b^3*x$

Rubi [A] time = 0.0400347, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^3}{5x^5} - \frac{a^2b}{x^3} - \frac{3ab^2}{x} + b^3x$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^3/x^6, x]

[Out]  $-a^3/(5*x^5) - (a^2*b)/x^3 - (3*a*b^2)/x + b^3*x$

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3}{5x^5} - \frac{a^2b}{x^3} - \frac{3ab^2}{x} + \int b^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*3/x\*\*6, x)

[Out]  $-a**3/(5*x**5) - a**2*b/x**3 - 3*a*b**2/x + \text{Integral}(b**3, x)$

Mathematica [A] time = 0.00920143, size = 34, normalized size = 1.

$$-\frac{a^3}{5x^5} - \frac{a^2b}{x^3} - \frac{3ab^2}{x} + b^3x$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^3/x^6, x]

[Out]  $-a^3/(5*x^5) - (a^2*b)/x^3 - (3*a*b^2)/x + b^3*x$

Maple [A] time = 0.008, size = 33, normalized size = 1.

$$-\frac{a^3}{5x^5} - \frac{a^2b}{x^3} - 3\frac{ab^2}{x} + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^3/x^6, x)

[Out]  $-1/5*a^3/x^5 - a^2*b/x^3 - 3*a*b^2/x + b^3*x$

---

**Maxima [A]** time = 1.35443, size = 45, normalized size = 1.32

$$b^3x - \frac{15ab^2x^4 + 5a^2bx^2 + a^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3/x^6,x, algorithm="maxima")

[Out] b^3\*x - 1/5\*(15\*a\*b^2\*x^4 + 5\*a^2\*b\*x^2 + a^3)/x^5

---

**Fricas [A]** time = 0.209635, size = 50, normalized size = 1.47

$$\frac{5b^3x^6 - 15ab^2x^4 - 5a^2bx^2 - a^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3/x^6,x, algorithm="fricas")

[Out] 1/5\*(5\*b^3\*x^6 - 15\*a\*b^2\*x^4 - 5\*a^2\*b\*x^2 - a^3)/x^5

---

**Sympy [A]** time = 1.40911, size = 32, normalized size = 0.94

$$b^3x - \frac{a^3 + 5a^2bx^2 + 15ab^2x^4}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*3/x\*\*6,x)

[Out] b\*\*3\*x - (a\*\*3 + 5\*a\*\*2\*b\*x\*\*2 + 15\*a\*b\*\*2\*x\*\*4)/(5\*x\*\*5)

---

**GIAC/XCAS [A]** time = 0.208819, size = 45, normalized size = 1.32

$$b^3x - \frac{15ab^2x^4 + 5a^2bx^2 + a^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3/x^6,x, algorithm="giac")

[Out] b^3\*x - 1/5\*(15\*a\*b^2\*x^4 + 5\*a^2\*b\*x^2 + a^3)/x^5

$$3.49 \quad \int \frac{(a+bx^2)^3}{x^8} dx$$

**Optimal.** Leaf size=39

$$-\frac{a^3}{7x^7} - \frac{3a^2b}{5x^5} - \frac{ab^2}{x^3} - \frac{b^3}{x}$$

[Out]  $-a^3/(7*x^7) - (3*a^2*b)/(5*x^5) - (a*b^2)/x^3 - b^3/x$

**Rubi [A]** time = 0.0418237, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^3}{7x^7} - \frac{3a^2b}{5x^5} - \frac{ab^2}{x^3} - \frac{b^3}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^3/x^8, x]

[Out]  $-a^3/(7*x^7) - (3*a^2*b)/(5*x^5) - (a*b^2)/x^3 - b^3/x$

**Rubi in Sympy [A]** time = 7.57277, size = 34, normalized size = 0.87

$$-\frac{a^3}{7x^7} - \frac{3a^2b}{5x^5} - \frac{ab^2}{x^3} - \frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*3/x\*\*8, x)

[Out]  $-a**3/(7*x**7) - 3*a**2*b/(5*x**5) - a*b**2/x**3 - b**3/x$

**Mathematica [A]** time = 0.00694587, size = 39, normalized size = 1.

$$-\frac{a^3}{7x^7} - \frac{3a^2b}{5x^5} - \frac{ab^2}{x^3} - \frac{b^3}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^3/x^8, x]

[Out]  $-a^3/(7*x^7) - (3*a^2*b)/(5*x^5) - (a*b^2)/x^3 - b^3/x$

**Maple [A]** time = 0.007, size = 36, normalized size = 0.9

$$-\frac{a^3}{7x^7} - \frac{3a^2b}{5x^5} - \frac{ab^2}{x^3} - \frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^3/x^8, x)

[Out]  $-1/7*a^3/x^7-3/5*a^2*b/x^5-a*b^2/x^3-b^3/x$

---

**Maxima [A]** time = 1.34556, size = 50, normalized size = 1.28

$$\frac{35 b^3 x^6 + 35 a b^2 x^4 + 21 a^2 b x^2 + 5 a^3}{35 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3/x^8,x, algorithm="maxima")

[Out] -1/35\*(35\*b^3\*x^6 + 35\*a\*b^2\*x^4 + 21\*a^2\*b\*x^2 + 5\*a^3)/x^7

---

**Fricas [A]** time = 0.220698, size = 50, normalized size = 1.28

$$\frac{35 b^3 x^6 + 35 a b^2 x^4 + 21 a^2 b x^2 + 5 a^3}{35 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3/x^8,x, algorithm="fricas")

[Out] -1/35\*(35\*b^3\*x^6 + 35\*a\*b^2\*x^4 + 21\*a^2\*b\*x^2 + 5\*a^3)/x^7

---

**Sympy [A]** time = 1.54212, size = 39, normalized size = 1.

$$\frac{5 a^3 + 21 a^2 b x^2 + 35 a b^2 x^4 + 35 b^3 x^6}{35 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*3/x\*\*8,x)

[Out] -(5\*a\*\*3 + 21\*a\*\*2\*b\*x\*\*2 + 35\*a\*b\*\*2\*x\*\*4 + 35\*b\*\*3\*x\*\*6)/(35\*x\*\*7)

---

**GIAC/XCAS [A]** time = 0.206679, size = 50, normalized size = 1.28

$$\frac{35 b^3 x^6 + 35 a b^2 x^4 + 21 a^2 b x^2 + 5 a^3}{35 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3/x^8,x, algorithm="giac")

[Out] -1/35\*(35\*b^3\*x^6 + 35\*a\*b^2\*x^4 + 21\*a^2\*b\*x^2 + 5\*a^3)/x^7

$$3.50 \quad \int \frac{(a+bx^2)^3}{x^{10}} dx$$

Optimal. Leaf size=43

$$-\frac{a^3}{9x^9} - \frac{3a^2b}{7x^7} - \frac{3ab^2}{5x^5} - \frac{b^3}{3x^3}$$

[Out] -a^3/(9\*x^9) - (3\*a^2\*b)/(7\*x^7) - (3\*a\*b^2)/(5\*x^5) - b^3/(3\*x^3)

**Rubi [A]** time = 0.0411805, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^3}{9x^9} - \frac{3a^2b}{7x^7} - \frac{3ab^2}{5x^5} - \frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^3/x^10, x]

[Out] -a^3/(9\*x^9) - (3\*a^2\*b)/(7\*x^7) - (3\*a\*b^2)/(5\*x^5) - b^3/(3\*x^3)

**Rubi in Sympy [A]** time = 7.58532, size = 41, normalized size = 0.95

$$-\frac{a^3}{9x^9} - \frac{3a^2b}{7x^7} - \frac{3ab^2}{5x^5} - \frac{b^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*3/x\*\*10, x)

[Out] -a\*\*3/(9\*x\*\*9) - 3\*a\*\*2\*b/(7\*x\*\*7) - 3\*a\*b\*\*2/(5\*x\*\*5) - b\*\*3/(3\*x\*\*3)

**Mathematica [A]** time = 0.00715706, size = 43, normalized size = 1.

$$-\frac{a^3}{9x^9} - \frac{3a^2b}{7x^7} - \frac{3ab^2}{5x^5} - \frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^3/x^10, x]

[Out] -a^3/(9\*x^9) - (3\*a^2\*b)/(7\*x^7) - (3\*a\*b^2)/(5\*x^5) - b^3/(3\*x^3)

**Maple [A]** time = 0.008, size = 36, normalized size = 0.8

$$-\frac{a^3}{9x^9} - \frac{3a^2b}{7x^7} - \frac{3ab^2}{5x^5} - \frac{b^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^3/x^10, x)



[Out]  $-1/9*a^3/x^9-3/7*a^2*b/x^7-3/5*a*b^2/x^5-1/3*b^3/x^3$

**Maxima [A]** time = 1.34708, size = 50, normalized size = 1.16

$$-\frac{105 b^3 x^6 + 189 a b^2 x^4 + 135 a^2 b x^2 + 35 a^3}{315 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^10,x, algorithm="maxima")`

[Out]  $-1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9$

**Fricas [A]** time = 0.209159, size = 50, normalized size = 1.16

$$-\frac{105 b^3 x^6 + 189 a b^2 x^4 + 135 a^2 b x^2 + 35 a^3}{315 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^10,x, algorithm="fricas")`

[Out]  $-1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9$

**Sympy [A]** time = 1.72183, size = 39, normalized size = 0.91

$$-\frac{35 a^3 + 135 a^2 b x^2 + 189 a b^2 x^4 + 105 b^3 x^6}{315 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**3/x**10,x)`

[Out]  $-(35*a**3 + 135*a**2*b*x**2 + 189*a*b**2*x**4 + 105*b**3*x**6)/(315*x**9)$

**GIAC/XCAS [A]** time = 0.207046, size = 50, normalized size = 1.16

$$-\frac{105 b^3 x^6 + 189 a b^2 x^4 + 135 a^2 b x^2 + 35 a^3}{315 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^10,x, algorithm="giac")`

[Out]  $-1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9$

$$3.51 \quad \int \frac{(a+bx^2)^3}{x^{12}} dx$$

**Optimal.** Leaf size=43

$$-\frac{a^3}{11x^{11}} - \frac{a^2b}{3x^9} - \frac{3ab^2}{7x^7} - \frac{b^3}{5x^5}$$

[Out] -a^3/(11\*x^11) - (a^2\*b)/(3\*x^9) - (3\*a\*b^2)/(7\*x^7) - b^3/(5\*x^5)

**Rubi [A]** time = 0.0412004, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^3}{11x^{11}} - \frac{a^2b}{3x^9} - \frac{3ab^2}{7x^7} - \frac{b^3}{5x^5}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^3/x^12, x]

[Out] -a^3/(11\*x^11) - (a^2\*b)/(3\*x^9) - (3\*a\*b^2)/(7\*x^7) - b^3/(5\*x^5)

**Rubi in Sympy [A]** time = 7.72359, size = 39, normalized size = 0.91

$$-\frac{a^3}{11x^{11}} - \frac{a^2b}{3x^9} - \frac{3ab^2}{7x^7} - \frac{b^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*3/x\*\*12, x)

[Out] -a\*\*3/(11\*x\*\*11) - a\*\*2\*b/(3\*x\*\*9) - 3\*a\*b\*\*2/(7\*x\*\*7) - b\*\*3/(5\*x\*\*5)

**Mathematica [A]** time = 0.011793, size = 43, normalized size = 1.

$$-\frac{a^3}{11x^{11}} - \frac{a^2b}{3x^9} - \frac{3ab^2}{7x^7} - \frac{b^3}{5x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^3/x^12, x]

[Out] -a^3/(11\*x^11) - (a^2\*b)/(3\*x^9) - (3\*a\*b^2)/(7\*x^7) - b^3/(5\*x^5)

**Maple [A]** time = 0.008, size = 36, normalized size = 0.8

$$-\frac{a^3}{11x^{11}} - \frac{a^2b}{3x^9} - \frac{3ab^2}{7x^7} - \frac{b^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^3/x^12, x)

[Out]  $-1/11*a^3/x^{11}-1/3*a^2*b/x^9-3/7*a*b^2/x^7-1/5*b^3/x^5$

**Maxima [A]** time = 1.34607, size = 50, normalized size = 1.16

$$\frac{231 b^3 x^6 + 495 a b^2 x^4 + 385 a^2 b x^2 + 105 a^3}{1155 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^12,x, algorithm="maxima")`

[Out]  $-1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^{11}$

**Fricas [A]** time = 0.217779, size = 50, normalized size = 1.16

$$\frac{231 b^3 x^6 + 495 a b^2 x^4 + 385 a^2 b x^2 + 105 a^3}{1155 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^12,x, algorithm="fricas")`

[Out]  $-1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^{11}$

**Sympy [A]** time = 1.73748, size = 39, normalized size = 0.91

$$\frac{105 a^3 + 385 a^2 b x^2 + 495 a b^2 x^4 + 231 b^3 x^6}{1155 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**3/x**12,x)`

[Out]  $-(105*a**3 + 385*a**2*b*x**2 + 495*a*b**2*x**4 + 231*b**3*x**6)/(1155*x**11)$

**GIAC/XCAS [A]** time = 0.206268, size = 50, normalized size = 1.16

$$\frac{231 b^3 x^6 + 495 a b^2 x^4 + 385 a^2 b x^2 + 105 a^3}{1155 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^12,x, algorithm="giac")`

[Out]  $-1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^{11}$

### 3.52 $\int x^{13} (a + bx^2)^5 dx$

**Optimal.** Leaf size=69

$$\frac{a^5 x^{14}}{14} + \frac{5}{16} a^4 b x^{16} + \frac{5}{9} a^3 b^2 x^{18} + \frac{1}{2} a^2 b^3 x^{20} + \frac{5}{22} a b^4 x^{22} + \frac{b^5 x^{24}}{24}$$

[Out]  $(a^5 x^{14})/14 + (5 a^4 b x^{16})/16 + (5 a^3 b^2 x^{18})/9 + (a^2 b^3 x^{20})/2 + (5 a b^4 x^{22})/22 + (b^5 x^{24})/24$

**Rubi [A]** time = 0.124759, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^5 x^{14}}{14} + \frac{5}{16} a^4 b x^{16} + \frac{5}{9} a^3 b^2 x^{18} + \frac{1}{2} a^2 b^3 x^{20} + \frac{5}{22} a b^4 x^{22} + \frac{b^5 x^{24}}{24}$$

Antiderivative was successfully verified.

[In] Int[x^13\*(a + b\*x^2)^5,x]

[Out]  $(a^5 x^{14})/14 + (5 a^4 b x^{16})/16 + (5 a^3 b^2 x^{18})/9 + (a^2 b^3 x^{20})/2 + (5 a b^4 x^{22})/22 + (b^5 x^{24})/24$

**Rubi in Sympy [A]** time = 17.129, size = 65, normalized size = 0.94

$$\frac{a^5 x^{14}}{14} + \frac{5 a^4 b x^{16}}{16} + \frac{5 a^3 b^2 x^{18}}{9} + \frac{a^2 b^3 x^{20}}{2} + \frac{5 a b^4 x^{22}}{22} + \frac{b^5 x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*13\*(b\*x\*\*2+a)\*\*5,x)

[Out]  $a**5*x**14/14 + 5*a**4*b*x**16/16 + 5*a**3*b**2*x**18/9 + a**2*b**3*x**20/2 + 5*a*b**4*x**22/22 + b**5*x**24/24$

**Mathematica [A]** time = 0.00500101, size = 69, normalized size = 1.

$$\frac{a^5 x^{14}}{14} + \frac{5}{16} a^4 b x^{16} + \frac{5}{9} a^3 b^2 x^{18} + \frac{1}{2} a^2 b^3 x^{20} + \frac{5}{22} a b^4 x^{22} + \frac{b^5 x^{24}}{24}$$

Antiderivative was successfully verified.

[In] Integrate[x^13\*(a + b\*x^2)^5,x]

[Out]  $(a^5 x^{14})/14 + (5 a^4 b x^{16})/16 + (5 a^3 b^2 x^{18})/9 + (a^2 b^3 x^{20})/2 + (5 a b^4 x^{22})/22 + (b^5 x^{24})/24$

**Maple [A]** time = 0.003, size = 58, normalized size = 0.8

$$\frac{a^5 x^{14}}{14} + \frac{5 a^4 b x^{16}}{16} + \frac{5 a^3 b^2 x^{18}}{9} + \frac{a^2 b^3 x^{20}}{2} + \frac{5 a b^4 x^{22}}{22} + \frac{b^5 x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^13*(b*x^2+a)^5,x)`

[Out]  $\frac{1}{14}a^5x^{14} + \frac{5}{16}a^4b^4x^{16} + \frac{5}{9}a^3b^2x^{18} + \frac{1}{2}a^2b^3x^{20} + \frac{5}{22}ab^4x^{22} + \frac{1}{24}b^5x^{24}$

**Maxima [A]** time = 1.34181, size = 77, normalized size = 1.12

$$\frac{1}{24}b^5x^{24} + \frac{5}{22}ab^4x^{22} + \frac{1}{2}a^2b^3x^{20} + \frac{5}{9}a^3b^2x^{18} + \frac{5}{16}a^4bx^{16} + \frac{1}{14}a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^13,x, algorithm="maxima")`

[Out]  $\frac{1}{24}b^5x^{24} + \frac{5}{22}a^4b^4x^{22} + \frac{1}{2}a^2b^3x^{20} + \frac{5}{9}a^3b^2x^{18} + \frac{5}{16}a^4b^4x^{16} + \frac{1}{14}a^5x^{14}$

**Fricas [A]** time = 0.211817, size = 1, normalized size = 0.01

$$\frac{1}{24}x^{24}b^5 + \frac{5}{22}x^{22}b^4a + \frac{1}{2}x^{20}b^3a^2 + \frac{5}{9}x^{18}b^2a^3 + \frac{5}{16}x^{16}ba^4 + \frac{1}{14}x^{14}a^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^13,x, algorithm="fricas")`

[Out]  $\frac{1}{24}x^{24}b^5 + \frac{5}{22}x^{22}b^4a + \frac{1}{2}x^{20}b^3a^2 + \frac{5}{9}x^{18}b^2a^3 + \frac{5}{16}x^{16}ba^4 + \frac{1}{14}x^{14}a^5$

**Sympy [A]** time = 0.127357, size = 65, normalized size = 0.94

$$\frac{a^5x^{14}}{14} + \frac{5a^4bx^{16}}{16} + \frac{5a^3b^2x^{18}}{9} + \frac{a^2b^3x^{20}}{2} + \frac{5ab^4x^{22}}{22} + \frac{b^5x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**13*(b*x**2+a)**5,x)`

[Out]  $a^5x^{14}/14 + 5a^4b^4x^{16}/16 + 5a^3b^2x^{18}/9 + a^2b^3x^{20}/2 + 5a^4b^4x^{22}/22 + b^5x^{24}/24$

**GIAC/XCAS [A]** time = 0.205095, size = 77, normalized size = 1.12

$$\frac{1}{24}b^5x^{24} + \frac{5}{22}ab^4x^{22} + \frac{1}{2}a^2b^3x^{20} + \frac{5}{9}a^3b^2x^{18} + \frac{5}{16}a^4bx^{16} + \frac{1}{14}a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^13,x, algorithm="giac")`

[Out]  $\frac{1}{24}b^5x^{24} + \frac{5}{22}a^4b^4x^{22} + \frac{1}{2}a^2b^3x^{20} + \frac{5}{9}a^3b^2x^{18} + \frac{5}{16}a^4b^4x^{16} + \frac{1}{14}a^5x^{14}$

### 3.53 $\int x^{11} (a + bx^2)^5 dx$

**Optimal.** Leaf size=69

$$\frac{a^5 x^{12}}{12} + \frac{5}{14} a^4 b x^{14} + \frac{5}{8} a^3 b^2 x^{16} + \frac{5}{9} a^2 b^3 x^{18} + \frac{1}{4} a b^4 x^{20} + \frac{b^5 x^{22}}{22}$$

[Out] (a^5\*x^12)/12 + (5\*a^4\*b\*x^14)/14 + (5\*a^3\*b^2\*x^16)/8 + (5\*a^2\*b^3\*x^18)/9 + (a\*b^4\*x^20)/4 + (b^5\*x^22)/22

**Rubi [A]** time = 0.116089, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^5 x^{12}}{12} + \frac{5}{14} a^4 b x^{14} + \frac{5}{8} a^3 b^2 x^{16} + \frac{5}{9} a^2 b^3 x^{18} + \frac{1}{4} a b^4 x^{20} + \frac{b^5 x^{22}}{22}$$

Antiderivative was successfully verified.

[In] Int[x^11\*(a + b\*x^2)^5,x]

[Out] (a^5\*x^12)/12 + (5\*a^4\*b\*x^14)/14 + (5\*a^3\*b^2\*x^16)/8 + (5\*a^2\*b^3\*x^18)/9 + (a\*b^4\*x^20)/4 + (b^5\*x^22)/22

**Rubi in Sympy [A]** time = 16.5927, size = 65, normalized size = 0.94

$$\frac{a^5 x^{12}}{12} + \frac{5a^4 b x^{14}}{14} + \frac{5a^3 b^2 x^{16}}{8} + \frac{5a^2 b^3 x^{18}}{9} + \frac{a b^4 x^{20}}{4} + \frac{b^5 x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*11\*(b\*x\*\*2+a)\*\*5,x)

[Out] a\*\*5\*x\*\*12/12 + 5\*a\*\*4\*b\*x\*\*14/14 + 5\*a\*\*3\*b\*\*2\*x\*\*16/8 + 5\*a\*\*2\*b\*\*3\*x\*\*18/9 + a\*b\*\*4\*x\*\*20/4 + b\*\*5\*x\*\*22/22

**Mathematica [A]** time = 0.00399403, size = 69, normalized size = 1.

$$\frac{a^5 x^{12}}{12} + \frac{5}{14} a^4 b x^{14} + \frac{5}{8} a^3 b^2 x^{16} + \frac{5}{9} a^2 b^3 x^{18} + \frac{1}{4} a b^4 x^{20} + \frac{b^5 x^{22}}{22}$$

Antiderivative was successfully verified.

[In] Integrate[x^11\*(a + b\*x^2)^5,x]

[Out] (a^5\*x^12)/12 + (5\*a^4\*b\*x^14)/14 + (5\*a^3\*b^2\*x^16)/8 + (5\*a^2\*b^3\*x^18)/9 + (a\*b^4\*x^20)/4 + (b^5\*x^22)/22

**Maple [A]** time = 0.002, size = 58, normalized size = 0.8

$$\frac{a^5 x^{12}}{12} + \frac{5a^4 b x^{14}}{14} + \frac{5a^3 b^2 x^{16}}{8} + \frac{5a^2 b^3 x^{18}}{9} + \frac{a b^4 x^{20}}{4} + \frac{b^5 x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11*(b*x^2+a)^5,x)`

[Out]  $1/12*a^5*x^{12}+5/14*a^4*b*x^{14}+5/8*a^3*b^2*x^{16}+5/9*a^2*b^3*x^{18}+1/4*a*b^4*x^{20}+1/22*b^5*x^{22}$

**Maxima [A]** time = 1.34871, size = 77, normalized size = 1.12

$$\frac{1}{22}b^5x^{22} + \frac{1}{4}ab^4x^{20} + \frac{5}{9}a^2b^3x^{18} + \frac{5}{8}a^3b^2x^{16} + \frac{5}{14}a^4bx^{14} + \frac{1}{12}a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^11,x, algorithm="maxima")`

[Out]  $1/22*b^5*x^{22} + 1/4*a*b^4*x^{20} + 5/9*a^2*b^3*x^{18} + 5/8*a^3*b^2*x^{16} + 5/14*a^4*b*x^{14} + 1/12*a^5*x^{12}$

**Fricas [A]** time = 0.189793, size = 1, normalized size = 0.01

$$\frac{1}{22}x^{22}b^5 + \frac{1}{4}x^{20}b^4a + \frac{5}{9}x^{18}b^3a^2 + \frac{5}{8}x^{16}b^2a^3 + \frac{5}{14}x^{14}ba^4 + \frac{1}{12}x^{12}a^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^11,x, algorithm="fricas")`

[Out]  $1/22*x^{22}*b^5 + 1/4*x^{20}*b^4*a + 5/9*x^{18}*b^3*a^2 + 5/8*x^{16}*b^2*a^3 + 5/14*x^{14}*b*a^4 + 1/12*x^{12}*a^5$

**Sympy [A]** time = 0.123252, size = 65, normalized size = 0.94

$$\frac{a^5x^{12}}{12} + \frac{5a^4bx^{14}}{14} + \frac{5a^3b^2x^{16}}{8} + \frac{5a^2b^3x^{18}}{9} + \frac{ab^4x^{20}}{4} + \frac{b^5x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**11*(b*x**2+a)**5,x)`

[Out]  $a**5*x**12/12 + 5*a**4*b*x**14/14 + 5*a**3*b**2*x**16/8 + 5*a**2*b**3*x**18/9 + a*b**4*x**20/4 + b**5*x**22/22$

**GIAC/XCAS [A]** time = 0.20709, size = 77, normalized size = 1.12

$$\frac{1}{22}b^5x^{22} + \frac{1}{4}ab^4x^{20} + \frac{5}{9}a^2b^3x^{18} + \frac{5}{8}a^3b^2x^{16} + \frac{5}{14}a^4bx^{14} + \frac{1}{12}a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^11,x, algorithm="giac")`

[Out]  $1/22*b^5*x^{22} + 1/4*a*b^4*x^{20} + 5/9*a^2*b^3*x^{18} + 5/8*a^3*b^2*x^{16} + 5/14*a^4*b*x^{14} + 1/12*a^5*x^{12}$

### 3.54 $\int x^9 (a + bx^2)^5 dx$

**Optimal.** Leaf size=69

$$\frac{a^5 x^{10}}{10} + \frac{5}{12} a^4 b x^{12} + \frac{5}{7} a^3 b^2 x^{14} + \frac{5}{8} a^2 b^3 x^{16} + \frac{5}{18} a b^4 x^{18} + \frac{b^5 x^{20}}{20}$$

[Out]  $(a^5 x^{10})/10 + (5 a^4 b x^{12})/12 + (5 a^3 b^2 x^{14})/7 + (5 a^2 b^3 x^{16})/8 + (5 a b^4 x^{18})/18 + (b^5 x^{20})/20$

**Rubi [A]** time = 0.114488, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^5 x^{10}}{10} + \frac{5}{12} a^4 b x^{12} + \frac{5}{7} a^3 b^2 x^{14} + \frac{5}{8} a^2 b^3 x^{16} + \frac{5}{18} a b^4 x^{18} + \frac{b^5 x^{20}}{20}$$

Antiderivative was successfully verified.

[In] Int[x^9\*(a + b\*x^2)^5,x]

[Out]  $(a^5 x^{10})/10 + (5 a^4 b x^{12})/12 + (5 a^3 b^2 x^{14})/7 + (5 a^2 b^3 x^{16})/8 + (5 a b^4 x^{18})/18 + (b^5 x^{20})/20$

**Rubi in Sympy [A]** time = 16.2229, size = 66, normalized size = 0.96

$$\frac{a^5 x^{10}}{10} + \frac{5 a^4 b x^{12}}{12} + \frac{5 a^3 b^2 x^{14}}{7} + \frac{5 a^2 b^3 x^{16}}{8} + \frac{5 a b^4 x^{18}}{18} + \frac{b^5 x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*9\*(b\*x\*\*2+a)\*\*5,x)

[Out]  $a**5*x**10/10 + 5*a**4*b*x**12/12 + 5*a**3*b**2*x**14/7 + 5*a**2*b**3*x**16/8 + 5*a*b**4*x**18/18 + b**5*x**20/20$

**Mathematica [A]** time = 0.00401707, size = 69, normalized size = 1.

$$\frac{a^5 x^{10}}{10} + \frac{5}{12} a^4 b x^{12} + \frac{5}{7} a^3 b^2 x^{14} + \frac{5}{8} a^2 b^3 x^{16} + \frac{5}{18} a b^4 x^{18} + \frac{b^5 x^{20}}{20}$$

Antiderivative was successfully verified.

[In] Integrate[x^9\*(a + b\*x^2)^5,x]

[Out]  $(a^5 x^{10})/10 + (5 a^4 b x^{12})/12 + (5 a^3 b^2 x^{14})/7 + (5 a^2 b^3 x^{16})/8 + (5 a b^4 x^{18})/18 + (b^5 x^{20})/20$

**Maple [A]** time = 0.003, size = 58, normalized size = 0.8

$$\frac{a^5 x^{10}}{10} + \frac{5 a^4 b x^{12}}{12} + \frac{5 a^3 b^2 x^{14}}{7} + \frac{5 a^2 b^3 x^{16}}{8} + \frac{5 a b^4 x^{18}}{18} + \frac{b^5 x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(x^9*(b*x^2+a)^5,x)`

[Out]  $\frac{1}{10}a^5x^{10} + \frac{5}{12}a^4b^4x^{12} + \frac{5}{7}a^3b^2x^{14} + \frac{5}{8}a^2b^3x^{16} + \frac{5}{18}ab^4x^{18} + \frac{1}{20}b^5x^{20}$

**Maxima [A]** time = 1.35211, size = 77, normalized size = 1.12

$$\frac{1}{20}b^5x^{20} + \frac{5}{18}ab^4x^{18} + \frac{5}{8}a^2b^3x^{16} + \frac{5}{7}a^3b^2x^{14} + \frac{5}{12}a^4bx^{12} + \frac{1}{10}a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^9,x, algorithm="maxima")`

[Out]  $\frac{1}{20}b^5x^{20} + \frac{5}{18}a^4b^4x^{18} + \frac{5}{8}a^2b^3x^{16} + \frac{5}{7}a^3b^2x^{14} + \frac{5}{12}a^4b^4x^{18} + \frac{1}{10}a^5x^{10}$

**Fricas [A]** time = 0.188498, size = 1, normalized size = 0.01

$$\frac{1}{20}x^{20}b^5 + \frac{5}{18}x^{18}b^4a + \frac{5}{8}x^{16}b^3a^2 + \frac{5}{7}x^{14}b^2a^3 + \frac{5}{12}x^{12}ba^4 + \frac{1}{10}x^{10}a^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^9,x, algorithm="fricas")`

[Out]  $\frac{1}{20}x^{20}b^5 + \frac{5}{18}x^{18}b^4a + \frac{5}{8}x^{16}b^3a^2 + \frac{5}{7}x^{14}b^2a^3 + \frac{5}{12}x^{12}b^4a + \frac{1}{10}x^{10}a^5$

**Sympy [A]** time = 0.124101, size = 66, normalized size = 0.96

$$\frac{a^5x^{10}}{10} + \frac{5a^4bx^{12}}{12} + \frac{5a^3b^2x^{14}}{7} + \frac{5a^2b^3x^{16}}{8} + \frac{5ab^4x^{18}}{18} + \frac{b^5x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9*(b*x**2+a)**5,x)`

[Out]  $a^5x^{10}/10 + 5a^4b^4x^{12}/12 + 5a^3b^2x^{14}/7 + 5a^2b^3x^{16}/8 + 5a^4b^4x^{18}/18 + b^5x^{20}/20$

**GIAC/XCAS [A]** time = 0.205086, size = 77, normalized size = 1.12

$$\frac{1}{20}b^5x^{20} + \frac{5}{18}ab^4x^{18} + \frac{5}{8}a^2b^3x^{16} + \frac{5}{7}a^3b^2x^{14} + \frac{5}{12}a^4bx^{12} + \frac{1}{10}a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^9,x, algorithm="giac")`

[Out]  $\frac{1}{20}b^5x^{20} + \frac{5}{18}a^4b^4x^{18} + \frac{5}{8}a^2b^3x^{16} + \frac{5}{7}a^3b^2x^{14} + \frac{5}{12}a^4b^4x^{18} + \frac{1}{10}a^5x^{10}$

### 3.55 $\int x^7 (a + bx^2)^5 dx$

**Optimal.** Leaf size=72

$$-\frac{a^3 (a + bx^2)^6}{12b^4} + \frac{3a^2 (a + bx^2)^7}{14b^4} + \frac{(a + bx^2)^9}{18b^4} - \frac{3a (a + bx^2)^8}{16b^4}$$

[Out]  $-(a^3*(a + b*x^2)^6)/(12*b^4) + (3*a^2*(a + b*x^2)^7)/(14*b^4) - (3*a*(a + b*x^2)^8)/(16*b^4) + (a + b*x^2)^9/(18*b^4)$

**Rubi [A]** time = 0.20955, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^3 (a + bx^2)^6}{12b^4} + \frac{3a^2 (a + bx^2)^7}{14b^4} + \frac{(a + bx^2)^9}{18b^4} - \frac{3a (a + bx^2)^8}{16b^4}$$

Antiderivative was successfully verified.

[In] Int[x^7\*(a + b\*x^2)^5, x]

[Out]  $-(a^3*(a + b*x^2)^6)/(12*b^4) + (3*a^2*(a + b*x^2)^7)/(14*b^4) - (3*a*(a + b*x^2)^8)/(16*b^4) + (a + b*x^2)^9/(18*b^4)$

**Rubi in Sympy [A]** time = 15.6313, size = 65, normalized size = 0.9

$$\frac{a^5 x^8}{8} + \frac{a^4 b x^{10}}{2} + \frac{5 a^3 b^2 x^{12}}{6} + \frac{5 a^2 b^3 x^{14}}{7} + \frac{5 a b^4 x^{16}}{16} + \frac{b^5 x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*7\*(b\*x\*\*2+a)\*\*5, x)

[Out]  $a**5*x**8/8 + a**4*b*x**10/2 + 5*a**3*b**2*x**12/6 + 5*a**2*b**3*x**14/7 + 5*a*b**4*x**16/16 + b**5*x**18/18$

**Mathematica [A]** time = 0.00363661, size = 69, normalized size = 0.96

$$\frac{a^5 x^8}{8} + \frac{1}{2} a^4 b x^{10} + \frac{5}{6} a^3 b^2 x^{12} + \frac{5}{7} a^2 b^3 x^{14} + \frac{5}{16} a b^4 x^{16} + \frac{b^5 x^{18}}{18}$$

Antiderivative was successfully verified.

[In] Integrate[x^7\*(a + b\*x^2)^5, x]

[Out]  $(a^5*x^8)/8 + (a^4*b*x^10)/2 + (5*a^3*b^2*x^12)/6 + (5*a^2*b^3*x^14)/7 + (5*a*b^4*x^16)/16 + (b^5*x^18)/18$

**Maple [A]** time = 0.002, size = 58, normalized size = 0.8

$$\frac{b^5 x^{18}}{18} + \frac{5 a b^4 x^{16}}{16} + \frac{5 a^2 b^3 x^{14}}{7} + \frac{5 a^3 b^2 x^{12}}{6} + \frac{a^4 b x^{10}}{2} + \frac{a^5 x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(b*x^2+a)^5,x)`

[Out]  $1/18*b^5*x^{18}+5/16*a*b^4*x^{16}+5/7*a^2*b^3*x^{14}+5/6*a^3*b^2*x^{12}+1/2*a^4*b*x^{10}+1/8*a^5*x^8$

**Maxima [A]** time = 1.36291, size = 77, normalized size = 1.07

$$\frac{1}{18}b^5x^{18} + \frac{5}{16}ab^4x^{16} + \frac{5}{7}a^2b^3x^{14} + \frac{5}{6}a^3b^2x^{12} + \frac{1}{2}a^4bx^{10} + \frac{1}{8}a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^7,x, algorithm="maxima")`

[Out]  $1/18*b^5*x^{18} + 5/16*a*b^4*x^{16} + 5/7*a^2*b^3*x^{14} + 5/6*a^3*b^2*x^{12} + 1/2*a^4*b*x^{10} + 1/8*a^5*x^8$

**Fricas [A]** time = 0.189048, size = 1, normalized size = 0.01

$$\frac{1}{18}x^{18}b^5 + \frac{5}{16}x^{16}b^4a + \frac{5}{7}x^{14}b^3a^2 + \frac{5}{6}x^{12}b^2a^3 + \frac{1}{2}x^{10}ba^4 + \frac{1}{8}x^8a^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^7,x, algorithm="fricas")`

[Out]  $1/18*x^{18}*b^5 + 5/16*x^{16}*b^4*a + 5/7*x^{14}*b^3*a^2 + 5/6*x^{12}*b^2*a^3 + 1/2*x^{10}*b*a^4 + 1/8*x^8*a^5$

**Sympy [A]** time = 0.125711, size = 65, normalized size = 0.9

$$\frac{a^5x^8}{8} + \frac{a^4bx^{10}}{2} + \frac{5a^3b^2x^{12}}{6} + \frac{5a^2b^3x^{14}}{7} + \frac{5ab^4x^{16}}{16} + \frac{b^5x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(b*x**2+a)**5,x)`

[Out]  $a**5*x**8/8 + a**4*b*x**10/2 + 5*a**3*b**2*x**12/6 + 5*a**2*b**3*x**14/7 + 5*a*b**4*x**16/16 + b**5*x**18/18$

**GIAC/XCAS [A]** time = 0.207079, size = 77, normalized size = 1.07

$$\frac{1}{18}b^5x^{18} + \frac{5}{16}ab^4x^{16} + \frac{5}{7}a^2b^3x^{14} + \frac{5}{6}a^3b^2x^{12} + \frac{1}{2}a^4bx^{10} + \frac{1}{8}a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^7,x, algorithm="giac")`

[Out]  $1/18*b^5*x^{18} + 5/16*a*b^4*x^{16} + 5/7*a^2*b^3*x^{14} + 5/6*a^3*b^2*x^{12} + 1/2*a^4*b*x^{10} + 1/8*a^5*x^8$

### 3.56 $\int x^5 (a + bx^2)^5 dx$

**Optimal.** Leaf size=53

$$\frac{a^2 (a + bx^2)^6}{12b^3} + \frac{(a + bx^2)^8}{16b^3} - \frac{a (a + bx^2)^7}{7b^3}$$

[Out]  $(a^2*(a + b*x^2)^6)/(12*b^3) - (a*(a + b*x^2)^7)/(7*b^3) + (a + b*x^2)^8/(16*b^3)$

**Rubi [A]** time = 0.16092, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^2 (a + bx^2)^6}{12b^3} + \frac{(a + bx^2)^8}{16b^3} - \frac{a (a + bx^2)^7}{7b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(a + b\*x^2)^5,x]

[Out]  $(a^2*(a + b*x^2)^6)/(12*b^3) - (a*(a + b*x^2)^7)/(7*b^3) + (a + b*x^2)^8/(16*b^3)$

**Rubi in Sympy [A]** time = 14.4351, size = 44, normalized size = 0.83

$$\frac{a^2 (a + bx^2)^6}{12b^3} - \frac{a (a + bx^2)^7}{7b^3} + \frac{(a + bx^2)^8}{16b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(b\*x\*\*2+a)\*\*5,x)

[Out]  $a**2*(a + b*x**2)**6/(12*b**3) - a*(a + b*x**2)**7/(7*b**3) + (a + b*x**2)**8/(16*b**3)$

**Mathematica [A]** time = 0.00368588, size = 66, normalized size = 1.25

$$\frac{a^5 x^6}{6} + \frac{5}{8} a^4 b x^8 + a^3 b^2 x^{10} + \frac{5}{6} a^2 b^3 x^{12} + \frac{5}{14} a b^4 x^{14} + \frac{b^5 x^{16}}{16}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*x^2)^5,x]

[Out]  $(a^5*x^6)/6 + (5*a^4*b*x^8)/8 + a^3*b^2*x^10 + (5*a^2*b^3*x^12)/6 + (5*a*b^4*x^14)/14 + (b^5*x^16)/16$

**Maple [A]** time = 0.002, size = 57, normalized size = 1.1

$$\frac{b^5 x^{16}}{16} + \frac{5 a b^4 x^{14}}{14} + \frac{5 a^2 b^3 x^{12}}{6} + a^3 b^2 x^{10} + \frac{5 a^4 b x^8}{8} + \frac{a^5 x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(b*x^2+a)^5,x)`

[Out]  $\frac{1}{16}b^5x^{16} + \frac{5}{14}a^4b^4x^{14} + \frac{5}{6}a^2b^3x^{12} + a^3b^2x^{10} + \frac{5}{8}a^4bx^8 + \frac{1}{6}a^5x^6$

**Maxima [A]** time = 1.36766, size = 76, normalized size = 1.43

$$\frac{1}{16}b^5x^{16} + \frac{5}{14}ab^4x^{14} + \frac{5}{6}a^2b^3x^{12} + a^3b^2x^{10} + \frac{5}{8}a^4bx^8 + \frac{1}{6}a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^5,x, algorithm="maxima")`

[Out]  $\frac{1}{16}b^5x^{16} + \frac{5}{14}a^4b^4x^{14} + \frac{5}{6}a^2b^3x^{12} + a^3b^2x^{10} + \frac{5}{8}a^4bx^8 + \frac{1}{6}a^5x^6$

**Fricas [A]** time = 0.187868, size = 1, normalized size = 0.02

$$\frac{1}{16}x^{16}b^5 + \frac{5}{14}x^{14}b^4a + \frac{5}{6}x^{12}b^3a^2 + x^{10}b^2a^3 + \frac{5}{8}x^8ba^4 + \frac{1}{6}x^6a^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^5,x, algorithm="fricas")`

[Out]  $\frac{1}{16}x^{16}b^5 + \frac{5}{14}x^{14}b^4a + \frac{5}{6}x^{12}b^3a^2 + x^{10}b^2a^3 + \frac{5}{8}x^8b^4a + \frac{1}{6}x^6a^5$

**Sympy [A]** time = 0.122172, size = 63, normalized size = 1.19

$$\frac{a^5x^6}{6} + \frac{5a^4bx^8}{8} + a^3b^2x^{10} + \frac{5a^2b^3x^{12}}{6} + \frac{5ab^4x^{14}}{14} + \frac{b^5x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(b*x**2+a)**5,x)`

[Out]  $a^5x^6/6 + 5a^4bx^8/8 + a^3b^2x^{10} + 5a^2b^3x^{12}/6 + 5ab^4x^{14}/14 + b^5x^{16}/16$

**GIAC/XCAS [A]** time = 0.207026, size = 76, normalized size = 1.43

$$\frac{1}{16}b^5x^{16} + \frac{5}{14}ab^4x^{14} + \frac{5}{6}a^2b^3x^{12} + a^3b^2x^{10} + \frac{5}{8}a^4bx^8 + \frac{1}{6}a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^5,x, algorithm="giac")`

[Out]  $\frac{1}{16}b^5x^{16} + \frac{5}{14}a^4b^4x^{14} + \frac{5}{6}a^2b^3x^{12} + a^3b^2x^{10} + \frac{5}{8}a^4bx^8 + \frac{1}{6}a^5x^6$

### 3.57 $\int x^3 (a + bx^2)^5 dx$

**Optimal.** Leaf size=34

$$\frac{(a + bx^2)^7}{14b^2} - \frac{a(a + bx^2)^6}{12b^2}$$

[Out]  $-(a*(a + b*x^2)^6)/(12*b^2) + (a + b*x^2)^7/(14*b^2)$

**Rubi [A]** time = 0.0983513, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{(a + bx^2)^7}{14b^2} - \frac{a(a + bx^2)^6}{12b^2}$$

Antiderivative was successfully verified.

[In] `Int[x^3*(a + b*x^2)^5, x]`

[Out]  $-(a*(a + b*x^2)^6)/(12*b^2) + (a + b*x^2)^7/(14*b^2)$

**Rubi in Sympy [A]** time = 10.5751, size = 27, normalized size = 0.79

$$-\frac{a(a + bx^2)^6}{12b^2} + \frac{(a + bx^2)^7}{14b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3*(b*x**2+a)**5, x)`

[Out]  $-a*(a + b*x**2)**6/(12*b**2) + (a + b*x**2)**7/(14*b**2)$

**Mathematica [A]** time = 0.00341422, size = 66, normalized size = 1.94

$$\frac{a^5x^4}{4} + \frac{5}{6}a^4bx^6 + \frac{5}{4}a^3b^2x^8 + a^2b^3x^{10} + \frac{5}{12}ab^4x^{12} + \frac{b^5x^{14}}{14}$$

Antiderivative was successfully verified.

[In] `Integrate[x^3*(a + b*x^2)^5, x]`

[Out]  $(a^5*x^4)/4 + (5*a^4*b*x^6)/6 + (5*a^3*b^2*x^8)/4 + a^2*b^3*x^{10} + (5*a*b^4*x^{12})/12 + (b^5*x^{14})/14$

**Maple [A]** time = 0.002, size = 57, normalized size = 1.7

$$\frac{b^5x^{14}}{14} + \frac{5ab^4x^{12}}{12} + a^2b^3x^{10} + \frac{5a^3b^2x^8}{4} + \frac{5a^4bx^6}{6} + \frac{a^5x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(b*x^2+a)^5, x)`

[Out]  $1/14*b^5*x^{14}+5/12*a*b^4*x^{12}+a^2*b^3*x^{10}+5/4*a^3*b^2*x^8+5/6*a^4*b*x^6+1/4*a^5*x^4$

**Maxima [A]** time = 1.3438, size = 76, normalized size = 2.24

$$\frac{1}{14}b^5x^{14} + \frac{5}{12}ab^4x^{12} + a^2b^3x^{10} + \frac{5}{4}a^3b^2x^8 + \frac{5}{6}a^4bx^6 + \frac{1}{4}a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^3,x, algorithm="maxima")`

[Out]  $1/14*b^5*x^{14} + 5/12*a*b^4*x^{12} + a^2*b^3*x^{10} + 5/4*a^3*b^2*x^8 + 5/6*a^4*b*x^6 + 1/4*a^5*x^4$

**Fricas [A]** time = 0.194538, size = 1, normalized size = 0.03

$$\frac{1}{14}x^{14}b^5 + \frac{5}{12}x^{12}b^4a + x^{10}b^3a^2 + \frac{5}{4}x^8b^2a^3 + \frac{5}{6}x^6ba^4 + \frac{1}{4}x^4a^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^3,x, algorithm="fricas")`

[Out]  $1/14*x^{14}*b^5 + 5/12*x^{12}*b^4*a + x^{10}*b^3*a^2 + 5/4*x^8*b^2*a^3 + 5/6*x^6*b*a^4 + 1/4*x^4*a^5$

**Sympy [A]** time = 0.119794, size = 63, normalized size = 1.85

$$\frac{a^5x^4}{4} + \frac{5a^4bx^6}{6} + \frac{5a^3b^2x^8}{4} + a^2b^3x^{10} + \frac{5ab^4x^{12}}{12} + \frac{b^5x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**2+a)**5,x)`

[Out]  $a**5*x**4/4 + 5*a**4*b*x**6/6 + 5*a**3*b**2*x**8/4 + a**2*b**3*x**10 + 5*a*b**4*x**12/12 + b**5*x**14/14$

**GIAC/XCAS [A]** time = 0.207204, size = 76, normalized size = 2.24

$$\frac{1}{14}b^5x^{14} + \frac{5}{12}ab^4x^{12} + a^2b^3x^{10} + \frac{5}{4}a^3b^2x^8 + \frac{5}{6}a^4bx^6 + \frac{1}{4}a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^3,x, algorithm="giac")`

[Out]  $1/14*b^5*x^{14} + 5/12*a*b^4*x^{12} + a^2*b^3*x^{10} + 5/4*a^3*b^2*x^8 + 5/6*a^4*b*x^6 + 1/4*a^5*x^4$

$$3.58 \quad \int x (a + bx^2)^5 dx$$

**Optimal.** Leaf size=16

$$\frac{(a + bx^2)^6}{12b}$$

[Out]  $(a + b*x^2)^6/(12*b)$

**Rubi [A]** time = 0.0127081, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{(a + bx^2)^6}{12b}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*x^2)^5, x]

[Out]  $(a + b*x^2)^6/(12*b)$

**Rubi in Sympy [A]** time = 2.19247, size = 10, normalized size = 0.62

$$\frac{(a + bx^2)^6}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(b\*x\*\*2+a)\*\*5, x)

[Out]  $(a + b*x**2)**6/(12*b)$

**Mathematica [A]** time = 0.00421162, size = 16, normalized size = 1.

$$\frac{(a + bx^2)^6}{12b}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*x^2)^5, x]

[Out]  $(a + b*x^2)^6/(12*b)$

**Maple [B]** time = 0.003, size = 58, normalized size = 3.6

$$\frac{b^5 x^{12}}{12} + \frac{ab^4 x^{10}}{2} + \frac{5a^2 b^3 x^8}{4} + \frac{5a^3 b^2 x^6}{3} + \frac{5a^4 b x^4}{4} + \frac{a^5 x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(b\*x^2+a)^5, x)

[Out]  $1/12*b^5*x^12+1/2*a*b^4*x^10+5/4*a^2*b^3*x^8+5/3*a^3*b^2*x^6+5/4*a^4*b*x^4+1/2*a^5*x^2$



---

**Maxima [A]** time = 1.33089, size = 19, normalized size = 1.19

$$\frac{(bx^2 + a)^6}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^5\*x,x, algorithm="maxima")

[Out] 1/12\*(b\*x^2 + a)^6/b

---

**Fricas [A]** time = 0.19162, size = 1, normalized size = 0.06

$$\frac{1}{12}x^{12}b^5 + \frac{1}{2}x^{10}b^4a + \frac{5}{4}x^8b^3a^2 + \frac{5}{3}x^6b^2a^3 + \frac{5}{4}x^4ba^4 + \frac{1}{2}x^2a^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^5\*x,x, algorithm="fricas")

[Out] 1/12\*x^12\*b^5 + 1/2\*x^10\*b^4\*a + 5/4\*x^8\*b^3\*a^2 + 5/3\*x^6\*b^2\*a^3 + 5/4\*x^4\*b\*a^4 + 1/2\*x^2\*a^5

---

**Sympy [A]** time = 0.116997, size = 65, normalized size = 4.06

$$\frac{a^5x^2}{2} + \frac{5a^4bx^4}{4} + \frac{5a^3b^2x^6}{3} + \frac{5a^2b^3x^8}{4} + \frac{ab^4x^{10}}{2} + \frac{b^5x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x\*\*2+a)\*\*5,x)

[Out] a\*\*5\*x\*\*2/2 + 5\*a\*\*4\*b\*x\*\*4/4 + 5\*a\*\*3\*b\*\*2\*x\*\*6/3 + 5\*a\*\*2\*b\*\*3\*x\*\*8/4 + a\*b\*\*4\*x\*\*10/2 + b\*\*5\*x\*\*12/12

---

**GIAC/XCAS [A]** time = 0.205511, size = 19, normalized size = 1.19

$$\frac{(bx^2 + a)^6}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^5\*x,x, algorithm="giac")

[Out] 1/12\*(b\*x^2 + a)^6/b

$$3.59 \quad \int \frac{(a+bx^2)^5}{x} dx$$

**Optimal.** Leaf size=65

$$a^5 \log(x) + \frac{5}{2}a^4bx^2 + \frac{5}{2}a^3b^2x^4 + \frac{5}{3}a^2b^3x^6 + \frac{5}{8}ab^4x^8 + \frac{b^5x^{10}}{10}$$

[Out]  $(5*a^4*b*x^2)/2 + (5*a^3*b^2*x^4)/2 + (5*a^2*b^3*x^6)/3 + (5*a*b^4*x^8)/8 + (b^5*x^{10})/10 + a^5*\text{Log}[x]$

**Rubi [A]** time = 0.0819982, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$a^5 \log(x) + \frac{5}{2}a^4bx^2 + \frac{5}{2}a^3b^2x^4 + \frac{5}{3}a^2b^3x^6 + \frac{5}{8}ab^4x^8 + \frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x, x]

[Out]  $(5*a^4*b*x^2)/2 + (5*a^3*b^2*x^4)/2 + (5*a^2*b^3*x^6)/3 + (5*a*b^4*x^8)/8 + (b^5*x^{10})/10 + a^5*\text{Log}[x]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^5 \log(x^2)}{2} + \frac{5a^4bx^2}{2} + 5a^3b^2 \int^{x^2} x dx + \frac{5a^2b^3x^6}{3} + \frac{5ab^4x^8}{8} + \frac{b^5x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x, x)

[Out]  $a**5*\log(x**2)/2 + 5*a**4*b*x**2/2 + 5*a**3*b**2*\text{Integral}(x, (x, x**2)) + 5*a**2*b**3*x**6/3 + 5*a*b**4*x**8/8 + b**5*x**10/10$

**Mathematica [A]** time = 0.00721082, size = 65, normalized size = 1.

$$a^5 \log(x) + \frac{5}{2}a^4bx^2 + \frac{5}{2}a^3b^2x^4 + \frac{5}{3}a^2b^3x^6 + \frac{5}{8}ab^4x^8 + \frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x, x]

[Out]  $(5*a^4*b*x^2)/2 + (5*a^3*b^2*x^4)/2 + (5*a^2*b^3*x^6)/3 + (5*a*b^4*x^8)/8 + (b^5*x^{10})/10 + a^5*\text{Log}[x]$

**Maple [A]** time = 0.005, size = 56, normalized size = 0.9

$$\frac{5a^4bx^2}{2} + \frac{5a^3b^2x^4}{2} + \frac{5a^2b^3x^6}{3} + \frac{5ab^4x^8}{8} + \frac{b^5x^{10}}{10} + a^5 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^5/x,x)`

[Out]  $5/2*a^4*b*x^2+5/2*a^3*b^2*x^4+5/3*a^2*b^3*x^6+5/8*a*b^4*x^8+1/10*b^5*x^{10}+a^5*\ln(x)$

**Maxima [A]** time = 1.33764, size = 78, normalized size = 1.2

$$\frac{1}{10}b^5x^{10} + \frac{5}{8}ab^4x^8 + \frac{5}{3}a^2b^3x^6 + \frac{5}{2}a^3b^2x^4 + \frac{5}{2}a^4bx^2 + \frac{1}{2}a^5\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x,x, algorithm="maxima")`

[Out]  $1/10*b^5*x^{10} + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4*b*x^2 + 1/2*a^5*\log(x^2)$

**Fricas [A]** time = 0.223893, size = 74, normalized size = 1.14

$$\frac{1}{10}b^5x^{10} + \frac{5}{8}ab^4x^8 + \frac{5}{3}a^2b^3x^6 + \frac{5}{2}a^3b^2x^4 + \frac{5}{2}a^4bx^2 + a^5\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x,x, algorithm="fricas")`

[Out]  $1/10*b^5*x^{10} + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4*b*x^2 + a^5*\log(x)$

**Sympy [A]** time = 1.14663, size = 65, normalized size = 1.

$$a^5\log(x) + \frac{5a^4bx^2}{2} + \frac{5a^3b^2x^4}{2} + \frac{5a^2b^3x^6}{3} + \frac{5ab^4x^8}{8} + \frac{b^5x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x,x)`

[Out]  $a^{**5}*\log(x) + 5*a^{**4}*b*x^{**2}/2 + 5*a^{**3}*b^{**2}*x^{**4}/2 + 5*a^{**2}*b^{**3}*x^{**6}/3 + 5*a*b^{**4}*x^{**8}/8 + b^{**5}*x^{**10}/10$

**GIAC/XCAS [A]** time = 0.207529, size = 78, normalized size = 1.2

$$\frac{1}{10}b^5x^{10} + \frac{5}{8}ab^4x^8 + \frac{5}{3}a^2b^3x^6 + \frac{5}{2}a^3b^2x^4 + \frac{5}{2}a^4bx^2 + \frac{1}{2}a^5\ln(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x,x, algorithm="giac")`

[Out]  $1/10*b^5*x^{10} + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^4*b*x^2 + 1/2*a^5*\ln(x^2)$

$$3.60 \quad \int \frac{(a+bx^2)^5}{x^3} dx$$

**Optimal.** Leaf size=64

$$-\frac{a^5}{2x^2} + 5a^4b \log(x) + 5a^3b^2x^2 + \frac{5}{2}a^2b^3x^4 + \frac{5}{6}ab^4x^6 + \frac{b^5x^8}{8}$$

[Out]  $-a^5/(2*x^2) + 5*a^3*b^2*x^2 + (5*a^2*b^3*x^4)/2 + (5*a*b^4*x^6)/6 + (b^5*x^8)/8 + 5*a^4*b*Log[x]$

**Rubi [A]** time = 0.0925029, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^5}{2x^2} + 5a^4b \log(x) + 5a^3b^2x^2 + \frac{5}{2}a^2b^3x^4 + \frac{5}{6}ab^4x^6 + \frac{b^5x^8}{8}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x^3, x]

[Out]  $-a^5/(2*x^2) + 5*a^3*b^2*x^2 + (5*a^2*b^3*x^4)/2 + (5*a*b^4*x^6)/6 + (b^5*x^8)/8 + 5*a^4*b*Log[x]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^5}{2x^2} + \frac{5a^4b \log(x^2)}{2} + 5a^3b^2x^2 + 5a^2b^3 \int^{x^2} x dx + \frac{5ab^4x^6}{6} + \frac{b^5x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x\*\*3, x)

[Out]  $-a**5/(2*x**2) + 5*a**4*b*log(x**2)/2 + 5*a**3*b**2*x**2 + 5*a**2*b**3*Integral(x, (x, x**2)) + 5*a*b**4*x**6/6 + b**5*x**8/8$

**Mathematica [A]** time = 0.00822996, size = 64, normalized size = 1.

$$-\frac{a^5}{2x^2} + 5a^4b \log(x) + 5a^3b^2x^2 + \frac{5}{2}a^2b^3x^4 + \frac{5}{6}ab^4x^6 + \frac{b^5x^8}{8}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x^3, x]

[Out]  $-a^5/(2*x^2) + 5*a^3*b^2*x^2 + (5*a^2*b^3*x^4)/2 + (5*a*b^4*x^6)/6 + (b^5*x^8)/8 + 5*a^4*b*Log[x]$

**Maple [A]** time = 0.009, size = 57, normalized size = 0.9

$$-\frac{a^5}{2x^2} + 5a^3b^2x^2 + \frac{5a^2b^3x^4}{2} + \frac{5ab^4x^6}{6} + \frac{b^5x^8}{8} + 5a^4b \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^5/x^3,x)`

[Out]  $-1/2*a^5/x^2+5*a^3*b^2*x^2+5/2*a^2*b^3*x^4+5/6*a*b^4*x^6+1/8*b^5*x^8+5*a^4*b*\ln(x)$

**Maxima [A]** time = 1.33178, size = 78, normalized size = 1.22

$$\frac{1}{8}b^5x^8 + \frac{5}{6}ab^4x^6 + \frac{5}{2}a^2b^3x^4 + 5a^3b^2x^2 + \frac{5}{2}a^4b\log(x^2) - \frac{a^5}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^3,x, algorithm="maxima")`

[Out]  $1/8*b^5*x^8 + 5/6*a*b^4*x^6 + 5/2*a^2*b^3*x^4 + 5*a^3*b^2*x^2 + 5/2*a^4*b*\log(x^2) - 1/2*a^5/x^2$

**Fricas [A]** time = 0.217206, size = 82, normalized size = 1.28

$$\frac{3b^5x^{10} + 20ab^4x^8 + 60a^2b^3x^6 + 120a^3b^2x^4 + 120a^4bx^2\log(x) - 12a^5}{24x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^3,x, algorithm="fricas")`

[Out]  $1/24*(3*b^5*x^{10} + 20*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4 + 120*a^4*b*x^2*\log(x) - 12*a^5)/x^2$

**Sympy [A]** time = 1.25594, size = 63, normalized size = 0.98

$$-\frac{a^5}{2x^2} + 5a^4b\log(x) + 5a^3b^2x^2 + \frac{5a^2b^3x^4}{2} + \frac{5ab^4x^6}{6} + \frac{b^5x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x**3,x)`

[Out]  $-a**5/(2*x**2) + 5*a**4*b*\log(x) + 5*a**3*b**2*x**2 + 5*a**2*b**3*x**4/2 + 5*a*b**4*x**6/6 + b**5*x**8/8$

**GIAC/XCAS [A]** time = 0.213439, size = 92, normalized size = 1.44

$$\frac{1}{8}b^5x^8 + \frac{5}{6}ab^4x^6 + \frac{5}{2}a^2b^3x^4 + 5a^3b^2x^2 + \frac{5}{2}a^4b\ln(x^2) - \frac{5a^4bx^2 + a^5}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^3,x, algorithm="giac")`

[Out]  $1/8*b^5*x^8 + 5/6*a*b^4*x^6 + 5/2*a^2*b^3*x^4 + 5*a^3*b^2*x^2 + 5/2*a^4*b*\ln(x^2) - 1/2*(5*a^4*b*x^2 + a^5)/x^2$

$$3.61 \quad \int \frac{(a+bx^2)^5}{x^5} dx$$

**Optimal.** Leaf size=64

$$-\frac{a^5}{4x^4} - \frac{5a^4b}{2x^2} + 10a^3b^2 \log(x) + 5a^2b^3x^2 + \frac{5}{4}ab^4x^4 + \frac{b^5x^6}{6}$$

[Out]  $-a^5/(4*x^4) - (5*a^4*b)/(2*x^2) + 5*a^2*b^3*x^2 + (5*a*b^4*x^4)/4 + (b^5*x^6)/6 + 10*a^3*b^2*Log[x]$

**Rubi [A]** time = 0.090487, antiderivative size = 64, normalized size of antiderivative = 1., number of rules used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^5}{4x^4} - \frac{5a^4b}{2x^2} + 10a^3b^2 \log(x) + 5a^2b^3x^2 + \frac{5}{4}ab^4x^4 + \frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x^5, x]

[Out]  $-a^5/(4*x^4) - (5*a^4*b)/(2*x^2) + 5*a^2*b^3*x^2 + (5*a*b^4*x^4)/4 + (b^5*x^6)/6 + 10*a^3*b^2*Log[x]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^5}{4x^4} - \frac{5a^4b}{2x^2} + 5a^3b^2 \log(x^2) + 5a^2b^3x^2 + \frac{5ab^4 \int^{x^2} x dx}{2} + \frac{b^5x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x\*\*5, x)

[Out]  $-a**5/(4*x**4) - 5*a**4*b/(2*x**2) + 5*a**3*b**2*log(x**2) + 5*a**2*b**3*x**2 + 5*a*b**4*Integral(x, (x, x**2))/2 + b**5*x**6/6$

**Mathematica [A]** time = 0.0118605, size = 64, normalized size = 1.

$$-\frac{a^5}{4x^4} - \frac{5a^4b}{2x^2} + 10a^3b^2 \log(x) + 5a^2b^3x^2 + \frac{5}{4}ab^4x^4 + \frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x^5, x]

[Out]  $-a^5/(4*x^4) - (5*a^4*b)/(2*x^2) + 5*a^2*b^3*x^2 + (5*a*b^4*x^4)/4 + (b^5*x^6)/6 + 10*a^3*b^2*Log[x]$

**Maple [A]** time = 0.009, size = 57, normalized size = 0.9

$$-\frac{a^5}{4x^4} - \frac{5a^4b}{2x^2} + 5a^2b^3x^2 + \frac{5ab^4x^4}{4} + \frac{b^5x^6}{6} + 10a^3b^2 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^5/x^5,x)`

[Out]  $-1/4*a^5/x^4-5/2*a^4*b/x^2+5*a^2*b^3*x^2+5/4*a*b^4*x^4+1/6*b^5*x^6+10*a^3*b^2*\ln(x)$

**Maxima [A]** time = 1.33321, size = 80, normalized size = 1.25

$$\frac{1}{6}b^5x^6 + \frac{5}{4}ab^4x^4 + 5a^2b^3x^2 + 5a^3b^2\log(x^2) - \frac{10a^4bx^2 + a^5}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^5,x, algorithm="maxima")`

[Out]  $1/6*b^5*x^6 + 5/4*a*b^4*x^4 + 5*a^2*b^3*x^2 + 5*a^3*b^2*\log(x^2) - 1/4*(10*a^4*b*x^2 + a^5)/x^4$

**Fricas [A]** time = 0.22133, size = 82, normalized size = 1.28

$$\frac{2b^5x^{10} + 15ab^4x^8 + 60a^2b^3x^6 + 120a^3b^2x^4\log(x) - 30a^4bx^2 - 3a^5}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^5,x, algorithm="fricas")`

[Out]  $1/12*(2*b^5*x^{10} + 15*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4*\log(x) - 30*a^4*b*x^2 - 3*a^5)/x^4$

**Sympy [A]** time = 1.46812, size = 61, normalized size = 0.95

$$10a^3b^2\log(x) + 5a^2b^3x^2 + \frac{5ab^4x^4}{4} + \frac{b^5x^6}{6} - \frac{a^5 + 10a^4bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x**5,x)`

[Out]  $10*a**3*b**2*\log(x) + 5*a**2*b**3*x**2 + 5*a*b**4*x**4/4 + b**5*x**6/6 - (a**5 + 10*a**4*b*x**2)/(4*x**4)$

**GIAC/XCAS [A]** time = 0.209507, size = 95, normalized size = 1.48

$$\frac{1}{6}b^5x^6 + \frac{5}{4}ab^4x^4 + 5a^2b^3x^2 + 5a^3b^2\ln(x^2) - \frac{30a^3b^2x^4 + 10a^4bx^2 + a^5}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^5,x, algorithm="giac")`

[Out]  $1/6*b^5*x^6 + 5/4*a*b^4*x^4 + 5*a^2*b^3*x^2 + 5*a^3*b^2*\ln(x^2) - 1/4*(30*a^3*b^2*x^4 + 10*a^4*b*x^2 + a^5)/x^4$

$$3.62 \quad \int \frac{(a+bx^2)^5}{x^7} dx$$

**Optimal.** Leaf size=64

$$-\frac{a^5}{6x^6} - \frac{5a^4b}{4x^4} - \frac{5a^3b^2}{x^2} + 10a^2b^3 \log(x) + \frac{5}{2}ab^4x^2 + \frac{b^5x^4}{4}$$

[Out]  $-a^5/(6*x^6) - (5*a^4*b)/(4*x^4) - (5*a^3*b^2)/x^2 + (5*a*b^4*x^2)/2 + (b^5*x^4)/4 + 10*a^2*b^3*Log[x]$

**Rubi [A]** time = 0.0899219, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^5}{6x^6} - \frac{5a^4b}{4x^4} - \frac{5a^3b^2}{x^2} + 10a^2b^3 \log(x) + \frac{5}{2}ab^4x^2 + \frac{b^5x^4}{4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x^7, x]

[Out]  $-a^5/(6*x^6) - (5*a^4*b)/(4*x^4) - (5*a^3*b^2)/x^2 + (5*a*b^4*x^2)/2 + (b^5*x^4)/4 + 10*a^2*b^3*Log[x]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^5}{6x^6} - \frac{5a^4b}{4x^4} - \frac{5a^3b^2}{x^2} + 5a^2b^3 \log(x^2) + \frac{5ab^4x^2}{2} + \frac{b^5 \int^{x^2} x dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x\*\*7, x)

[Out]  $-a**5/(6*x**6) - 5*a**4*b/(4*x**4) - 5*a**3*b**2/x**2 + 5*a**2*b**3*log(x**2) + 5*a*b**4*x**2/2 + b**5*Integral(x, (x, x**2))/2$

**Mathematica [A]** time = 0.00848019, size = 64, normalized size = 1.

$$-\frac{a^5}{6x^6} - \frac{5a^4b}{4x^4} - \frac{5a^3b^2}{x^2} + 10a^2b^3 \log(x) + \frac{5}{2}ab^4x^2 + \frac{b^5x^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x^7, x]

[Out]  $-a^5/(6*x^6) - (5*a^4*b)/(4*x^4) - (5*a^3*b^2)/x^2 + (5*a*b^4*x^2)/2 + (b^5*x^4)/4 + 10*a^2*b^3*Log[x]$

**Maple [A]** time = 0.009, size = 57, normalized size = 0.9

$$-\frac{a^5}{6x^6} - \frac{5a^4b}{4x^4} - 5\frac{a^3b^2}{x^2} + \frac{5ab^4x^2}{2} + \frac{b^5x^4}{4} + 10a^2b^3 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((b*x^2+a)^5/x^7,x)`

[Out]  $-1/6*a^5/x^6-5/4*a^4*b/x^4-5*a^3*b^2/x^2+5/2*a*b^4*x^2+1/4*b^5*x^4+10*a^2*b^3*\ln(x)$

**Maxima [A]** time = 1.32544, size = 82, normalized size = 1.28

$$\frac{1}{4}b^5x^4 + \frac{5}{2}ab^4x^2 + 5a^2b^3\log(x^2) - \frac{60a^3b^2x^4 + 15a^4bx^2 + 2a^5}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^7,x, algorithm="maxima")`

[Out]  $1/4*b^5*x^4 + 5/2*a*b^4*x^2 + 5*a^2*b^3*\log(x^2) - 1/12*(60*a^3*b^2*x^4 + 15*a^4*b*x^2 + 2*a^5)/x^6$

**Fricas [A]** time = 0.221668, size = 82, normalized size = 1.28

$$\frac{3b^5x^{10} + 30ab^4x^8 + 120a^2b^3x^6\log(x) - 60a^3b^2x^4 - 15a^4bx^2 - 2a^5}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^7,x, algorithm="fricas")`

[Out]  $1/12*(3*b^5*x^{10} + 30*a*b^4*x^8 + 120*a^2*b^3*x^6*\log(x) - 60*a^3*b^2*x^4 - 15*a^4*b*x^2 - 2*a^5)/x^6$

**Sympy [A]** time = 1.67289, size = 63, normalized size = 0.98

$$10a^2b^3\log(x) + \frac{5ab^4x^2}{2} + \frac{b^5x^4}{4} - \frac{2a^5 + 15a^4bx^2 + 60a^3b^2x^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x**7,x)`

[Out]  $10*a**2*b**3*\log(x) + 5*a*b**4*x**2/2 + b**5*x**4/4 - (2*a**5 + 15*a**4*b*x**2 + 60*a**3*b**2*x**4)/(12*x**6)$

**GIAC/XCAS [A]** time = 0.211455, size = 97, normalized size = 1.52

$$\frac{1}{4}b^5x^4 + \frac{5}{2}ab^4x^2 + 5a^2b^3\ln(x^2) - \frac{110a^2b^3x^6 + 60a^3b^2x^4 + 15a^4bx^2 + 2a^5}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^7,x, algorithm="giac")`

[Out]  $1/4*b^5*x^4 + 5/2*a*b^4*x^2 + 5*a^2*b^3*\ln(x^2) - 1/12*(110*a^2*b^3*x^6 + 60*a^3*b^2*x^4 + 15*a^4*b*x^2 + 2*a^5)/x^6$

$$3.63 \quad \int \frac{(a+bx^2)^5}{x^9} dx$$

**Optimal.** Leaf size=64

$$-\frac{a^5}{8x^8} - \frac{5a^4b}{6x^6} - \frac{5a^3b^2}{2x^4} - \frac{5a^2b^3}{x^2} + 5ab^4 \log(x) + \frac{b^5x^2}{2}$$

[Out]  $-a^5/(8*x^8) - (5*a^4*b)/(6*x^6) - (5*a^3*b^2)/(2*x^4) - (5*a^2*b^3)/x^2 + (b^5*x^2)/2 + 5*a*b^4*Log[x]$

**Rubi [A]** time = 0.0863397, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^5}{8x^8} - \frac{5a^4b}{6x^6} - \frac{5a^3b^2}{2x^4} - \frac{5a^2b^3}{x^2} + 5ab^4 \log(x) + \frac{b^5x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x^9, x]

[Out]  $-a^5/(8*x^8) - (5*a^4*b)/(6*x^6) - (5*a^3*b^2)/(2*x^4) - (5*a^2*b^3)/x^2 + (b^5*x^2)/2 + 5*a*b^4*Log[x]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^5}{8x^8} - \frac{5a^4b}{6x^6} - \frac{5a^3b^2}{2x^4} - \frac{5a^2b^3}{x^2} + \frac{5ab^4 \log(x^2)}{2} + \frac{\int^{x^2} b^5 dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x\*\*9, x)

[Out]  $-a**5/(8*x**8) - 5*a**4*b/(6*x**6) - 5*a**3*b**2/(2*x**4) - 5*a**2*b**3/x**2 + 5*a*b**4*log(x**2)/2 + Integral(b**5, (x, x**2))/2$

**Mathematica [A]** time = 0.00855411, size = 64, normalized size = 1.

$$-\frac{a^5}{8x^8} - \frac{5a^4b}{6x^6} - \frac{5a^3b^2}{2x^4} - \frac{5a^2b^3}{x^2} + 5ab^4 \log(x) + \frac{b^5x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x^9, x]

[Out]  $-a^5/(8*x^8) - (5*a^4*b)/(6*x^6) - (5*a^3*b^2)/(2*x^4) - (5*a^2*b^3)/x^2 + (b^5*x^2)/2 + 5*a*b^4*Log[x]$

**Maple [A]** time = 0.011, size = 57, normalized size = 0.9

$$-\frac{a^5}{8x^8} - \frac{5a^4b}{6x^6} - \frac{5a^3b^2}{2x^4} - 5\frac{a^2b^3}{x^2} + \frac{b^5x^2}{2} + 5ab^4 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^5/x^9,x)`

[Out]  $-1/8*a^5/x^8-5/6*a^4*b/x^6-5/2*a^3*b^2/x^4-5*a^2*b^3/x^2+1/2*b^5*x^2+5*a*b^4*\ln(x)$

**Maxima [A]** time = 1.32768, size = 82, normalized size = 1.28

$$\frac{1}{2}b^5x^2 + \frac{5}{2}ab^4\log(x^2) - \frac{120a^2b^3x^6 + 60a^3b^2x^4 + 20a^4bx^2 + 3a^5}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^9,x, algorithm="maxima")`

[Out]  $1/2*b^5*x^2 + 5/2*a*b^4*\log(x^2) - 1/24*(120*a^2*b^3*x^6 + 60*a^3*b^2*x^4 + 20*a^4*b*x^2 + 3*a^5)/x^8$

**Fricas [A]** time = 0.225919, size = 82, normalized size = 1.28

$$\frac{12b^5x^{10} + 120ab^4x^8\log(x) - 120a^2b^3x^6 - 60a^3b^2x^4 - 20a^4bx^2 - 3a^5}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^9,x, algorithm="fricas")`

[Out]  $1/24*(12*b^5*x^{10} + 120*a*b^4*x^8*\log(x) - 120*a^2*b^3*x^6 - 60*a^3*b^2*x^4 - 20*a^4*b*x^2 - 3*a^5)/x^8$

**Sympy [A]** time = 1.98619, size = 61, normalized size = 0.95

$$5ab^4\log(x) + \frac{b^5x^2}{2} - \frac{3a^5 + 20a^4bx^2 + 60a^3b^2x^4 + 120a^2b^3x^6}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x**9,x)`

[Out]  $5*a*b^4*\log(x) + b^5*x^2/2 - (3*a^5 + 20*a^4*b*x^2 + 60*a^3*b^2*x^4 + 120*a^2*b^3*x^6)/(24*x^8)$

**GIAC/XCAS [A]** time = 0.211784, size = 95, normalized size = 1.48

$$\frac{1}{2}b^5x^2 + \frac{5}{2}ab^4\ln(x^2) - \frac{125ab^4x^8 + 120a^2b^3x^6 + 60a^3b^2x^4 + 20a^4bx^2 + 3a^5}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^9,x, algorithm="giac")`

[Out]  $1/2*b^5*x^2 + 5/2*a*b^4*\ln(x^2) - 1/24*(125*a*b^4*x^8 + 120*a^2*b^3*x^6 + 60*a^3*b^2*x^4 + 20*a^4*b*x^2 + 3*a^5)/x^8$

$$3.64 \quad \int \frac{(a+bx^2)^5}{x^{11}} dx$$

**Optimal.** Leaf size=65

$$-\frac{a^5}{10x^{10}} - \frac{5a^4b}{8x^8} - \frac{5a^3b^2}{3x^6} - \frac{5a^2b^3}{2x^4} - \frac{5ab^4}{2x^2} + b^5 \log(x)$$

[Out]  $-a^5/(10*x^{10}) - (5*a^4*b)/(8*x^8) - (5*a^3*b^2)/(3*x^6) - (5*a^2*b^3)/(2*x^4) - (5*a*b^4)/(2*x^2) + b^5*Log[x]$

**Rubi [A]** time = 0.0809067, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^5}{10x^{10}} - \frac{5a^4b}{8x^8} - \frac{5a^3b^2}{3x^6} - \frac{5a^2b^3}{2x^4} - \frac{5ab^4}{2x^2} + b^5 \log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x^11, x]

[Out]  $-a^5/(10*x^{10}) - (5*a^4*b)/(8*x^8) - (5*a^3*b^2)/(3*x^6) - (5*a^2*b^3)/(2*x^4) - (5*a*b^4)/(2*x^2) + b^5*Log[x]$

**Rubi in Sympy [A]** time = 14.2022, size = 68, normalized size = 1.05

$$-\frac{a^5}{10x^{10}} - \frac{5a^4b}{8x^8} - \frac{5a^3b^2}{3x^6} - \frac{5a^2b^3}{2x^4} - \frac{5ab^4}{2x^2} + \frac{b^5 \log(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x\*\*11, x)

[Out]  $-a**5/(10*x**10) - 5*a**4*b/(8*x**8) - 5*a**3*b**2/(3*x**6) - 5*a**2*b**3/(2*x**4) - 5*a*b**4/(2*x**2) + b**5*log(x**2)/2$

**Mathematica [A]** time = 0.008365, size = 65, normalized size = 1.

$$-\frac{a^5}{10x^{10}} - \frac{5a^4b}{8x^8} - \frac{5a^3b^2}{3x^6} - \frac{5a^2b^3}{2x^4} - \frac{5ab^4}{2x^2} + b^5 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x^11, x]

[Out]  $-a^5/(10*x^{10}) - (5*a^4*b)/(8*x^8) - (5*a^3*b^2)/(3*x^6) - (5*a^2*b^3)/(2*x^4) - (5*a*b^4)/(2*x^2) + b^5*Log[x]$

**Maple [A]** time = 0.01, size = 56, normalized size = 0.9

$$-\frac{a^5}{10x^{10}} - \frac{5a^4b}{8x^8} - \frac{5a^3b^2}{3x^6} - \frac{5a^2b^3}{2x^4} - \frac{5ab^4}{2x^2} + b^5 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^5/x^11,x)`

[Out]  $-1/10*a^5/x^{10}-5/8*a^4*b/x^8-5/3*a^3*b^2/x^6-5/2*a^2*b^3/x^4-5/2*a*b^4/x^2+b^5*\ln(x)$

**Maxima [A]** time = 1.33571, size = 82, normalized size = 1.26

$$\frac{1}{2} b^5 \log(x^2) - \frac{300 ab^4 x^8 + 300 a^2 b^3 x^6 + 200 a^3 b^2 x^4 + 75 a^4 b x^2 + 12 a^5}{120 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^11,x, algorithm="maxima")`

[Out]  $1/2*b^5*\log(x^2) - 1/120*(300*a*b^4*x^8 + 300*a^2*b^3*x^6 + 200*a^3*b^2*x^4 + 75*a^4*b*x^2 + 12*a^5)/x^{10}$

**Fricas [A]** time = 0.226869, size = 82, normalized size = 1.26

$$\frac{120 b^5 x^{10} \log(x) - 300 ab^4 x^8 - 300 a^2 b^3 x^6 - 200 a^3 b^2 x^4 - 75 a^4 b x^2 - 12 a^5}{120 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^11,x, algorithm="fricas")`

[Out]  $1/120*(120*b^5*x^{10}*\log(x) - 300*a*b^4*x^8 - 300*a^2*b^3*x^6 - 200*a^3*b^2*x^4 - 75*a^4*b*x^2 - 12*a^5)/x^{10}$

**Sympy [A]** time = 2.22891, size = 60, normalized size = 0.92

$$b^5 \log(x) - \frac{12a^5 + 75a^4bx^2 + 200a^3b^2x^4 + 300a^2b^3x^6 + 300ab^4x^8}{120x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x**11,x)`

[Out]  $b^{**5}*\log(x) - (12*a^{**5} + 75*a^{**4}*b*x^{**2} + 200*a^{**3}*b^{**2}*x^{**4} + 300*a^{**2}*b^{**3}*x^{**6} + 300*a*b^{**4}*x^{**8})/(120*x^{**10})$

**GIAC/XCAS [A]** time = 0.209483, size = 93, normalized size = 1.43

$$\frac{1}{2} b^5 \ln(x^2) - \frac{137 b^5 x^{10} + 300 ab^4 x^8 + 300 a^2 b^3 x^6 + 200 a^3 b^2 x^4 + 75 a^4 b x^2 + 12 a^5}{120 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^11,x, algorithm="giac")`

[Out]  $1/2*b^5*\ln(x^2) - 1/120*(137*b^5*x^{10} + 300*a*b^4*x^8 + 300*a^2*b^3*x^6 + 200*a^3*b^2*x^4 + 75*a^4*b*x^2 + 12*a^5)/x^{10}$

$$3.65 \quad \int \frac{(a+bx^2)^5}{x^{13}} dx$$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^6}{12ax^{12}}$$

[Out]  $-(a + b*x^2)^6/(12*a*x^{12})$

**Rubi [A]** time = 0.0185312, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{(a+bx^2)^6}{12ax^{12}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x^13, x]

[Out]  $-(a + b*x^2)^6/(12*a*x^{12})$

**Rubi in Sympy [A]** time = 3.22412, size = 15, normalized size = 0.79

$$-\frac{(a+bx^2)^6}{12ax^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x\*\*13, x)

[Out]  $-(a + b*x**2)**6/(12*a*x**12)$

**Mathematica [B]** time = 0.00761848, size = 69, normalized size = 3.63

$$-\frac{a^5}{12x^{12}} - \frac{a^4b}{2x^{10}} - \frac{5a^3b^2}{4x^8} - \frac{5a^2b^3}{3x^6} - \frac{5ab^4}{4x^4} - \frac{b^5}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x^13, x]

[Out]  $-a^5/(12*x^{12}) - (a^4*b)/(2*x^{10}) - (5*a^3*b^2)/(4*x^8) - (5*a^2*b^3)/(3*x^6) - (5*a*b^4)/(4*x^4) - b^5/(2*x^2)$

**Maple [B]** time = 0.008, size = 58, normalized size = 3.1

$$-\frac{5a^2b^3}{3x^6} - \frac{a^5}{12x^{12}} - \frac{5ab^4}{4x^4} - \frac{5a^3b^2}{4x^8} - \frac{b^5}{2x^2} - \frac{a^4b}{2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^5/x^13, x)

[Out]  $-5/3*a^2*b^3/x^6-1/12*a^5/x^{12}-5/4*a*b^4/x^4-5/4*a^3*b^2/x^8-1/2*b^5/x^2-1/2*a^4*b/x^{10}$

**Maxima [A]** time = 1.33073, size = 77, normalized size = 4.05

$$\frac{6b^5x^{10} + 15ab^4x^8 + 20a^2b^3x^6 + 15a^3b^2x^4 + 6a^4bx^2 + a^5}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^13,x, algorithm="maxima")`

[Out]  $-1/12*(6*b^5*x^{10} + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4*b*x^2 + a^5)/x^{12}$

**Fricas [A]** time = 0.201099, size = 77, normalized size = 4.05

$$\frac{6b^5x^{10} + 15ab^4x^8 + 20a^2b^3x^6 + 15a^3b^2x^4 + 6a^4bx^2 + a^5}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^13,x, algorithm="fricas")`

[Out]  $-1/12*(6*b^5*x^{10} + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4*b*x^2 + a^5)/x^{12}$

**Sympy [A]** time = 2.31615, size = 61, normalized size = 3.21

$$\frac{a^5 + 6a^4bx^2 + 15a^3b^2x^4 + 20a^2b^3x^6 + 15ab^4x^8 + 6b^5x^{10}}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x**13,x)`

[Out]  $-(a**5 + 6*a**4*b*x**2 + 15*a**3*b**2*x**4 + 20*a**2*b**3*x**6 + 15*a*b**4*x**8 + 6*b**5*x**10)/(12*x**12)$

**GIAC/XCAS [A]** time = 0.208877, size = 77, normalized size = 4.05

$$\frac{6b^5x^{10} + 15ab^4x^8 + 20a^2b^3x^6 + 15a^3b^2x^4 + 6a^4bx^2 + a^5}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^13,x, algorithm="giac")`

[Out]  $-1/12*(6*b^5*x^{10} + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4*b*x^2 + a^5)/x^{12}$

$$3.66 \quad \int \frac{(a+bx^2)^5}{x^{15}} dx$$

**Optimal.** Leaf size=40

$$\frac{b(a+bx^2)^6}{84a^2x^{12}} - \frac{(a+bx^2)^6}{14ax^{14}}$$

[Out]  $-(a + b*x^2)^6/(14*a*x^{14}) + (b*(a + b*x^2)^6)/(84*a^2*x^{12})$

**Rubi [A]** time = 0.0581051, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{b(a+bx^2)^6}{84a^2x^{12}} - \frac{(a+bx^2)^6}{14ax^{14}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x^15, x]

[Out]  $-(a + b*x^2)^6/(14*a*x^{14}) + (b*(a + b*x^2)^6)/(84*a^2*x^{12})$

**Rubi in Sympy [A]** time = 6.4845, size = 32, normalized size = 0.8

$$-\frac{(a+bx^2)^6}{14ax^{14}} + \frac{b(a+bx^2)^6}{84a^2x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x\*\*15, x)

[Out]  $-(a + b*x**2)**6/(14*a*x**14) + b*(a + b*x**2)**6/(84*a**2*x**12)$

**Mathematica [A]** time = 0.0110855, size = 67, normalized size = 1.68

$$-\frac{a^5}{14x^{14}} - \frac{5a^4b}{12x^{12}} - \frac{a^3b^2}{x^{10}} - \frac{5a^2b^3}{4x^8} - \frac{5ab^4}{6x^6} - \frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x^15, x]

[Out]  $-a^5/(14*x^{14}) - (5*a^4*b)/(12*x^{12}) - (a^3*b^2)/x^{10} - (5*a^2*b^3)/(4*x^8) - (5*a*b^4)/(6*x^6) - b^5/(4*x^4)$

**Maple [A]** time = 0.01, size = 58, normalized size = 1.5

$$-\frac{5a^4b}{12x^{12}} - \frac{5ab^4}{6x^6} - \frac{b^5}{4x^4} - \frac{5a^2b^3}{4x^8} - \frac{a^5}{14x^{14}} - \frac{a^3b^2}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^5/x^15, x)



[Out]  $-5/12*a^4*b/x^{12}-5/6*a*b^4/x^6-1/4*b^5/x^4-5/4*a^2*b^3/x^8-1/14*a^5/x^{14}-a^3*b^2/x^{10}$

**Maxima [A]** time = 1.33184, size = 80, normalized size = 2.

$$\frac{21b^5x^{10} + 70ab^4x^8 + 105a^2b^3x^6 + 84a^3b^2x^4 + 35a^4bx^2 + 6a^5}{84x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^15,x, algorithm="maxima")`

[Out]  $-1/84*(21*b^5*x^{10} + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*a^4*b*x^2 + 6*a^5)/x^{14}$

**Fricas [A]** time = 0.207042, size = 80, normalized size = 2.

$$\frac{21b^5x^{10} + 70ab^4x^8 + 105a^2b^3x^6 + 84a^3b^2x^4 + 35a^4bx^2 + 6a^5}{84x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^15,x, algorithm="fricas")`

[Out]  $-1/84*(21*b^5*x^{10} + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*a^4*b*x^2 + 6*a^5)/x^{14}$

**Sympy [A]** time = 2.48686, size = 63, normalized size = 1.58

$$\frac{6a^5 + 35a^4bx^2 + 84a^3b^2x^4 + 105a^2b^3x^6 + 70ab^4x^8 + 21b^5x^{10}}{84x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x**15,x)`

[Out]  $-(6*a**5 + 35*a**4*b*x**2 + 84*a**3*b**2*x**4 + 105*a**2*b**3*x**6 + 70*a*b**4*x**8 + 21*b**5*x**10)/(84*x**14)$

**GIAC/XCAS [A]** time = 0.206607, size = 80, normalized size = 2.

$$\frac{21b^5x^{10} + 70ab^4x^8 + 105a^2b^3x^6 + 84a^3b^2x^4 + 35a^4bx^2 + 6a^5}{84x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^15,x, algorithm="giac")`

[Out]  $-1/84*(21*b^5*x^{10} + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*a^4*b*x^2 + 6*a^5)/x^{14}$

$$3.67 \quad \int \frac{(a+bx^2)^5}{x^{17}} dx$$

**Optimal.** Leaf size=62

$$-\frac{b^2(a+bx^2)^6}{336a^3x^{12}} + \frac{b(a+bx^2)^6}{56a^2x^{14}} - \frac{(a+bx^2)^6}{16ax^{16}}$$

[Out]  $-(a + b*x^2)^6/(16*a*x^{16}) + (b*(a + b*x^2)^6)/(56*a^2*x^{14}) - (b^2*(a + b*x^2)^6)/(336*a^3*x^{12})$

**Rubi [A]** time = 0.0860031, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{b^2(a+bx^2)^6}{336a^3x^{12}} + \frac{b(a+bx^2)^6}{56a^2x^{14}} - \frac{(a+bx^2)^6}{16ax^{16}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x^17, x]

[Out]  $-(a + b*x^2)^6/(16*a*x^{16}) + (b*(a + b*x^2)^6)/(56*a^2*x^{14}) - (b^2*(a + b*x^2)^6)/(336*a^3*x^{12})$

**Rubi in Sympy [A]** time = 14.7702, size = 65, normalized size = 1.05

$$-\frac{a^5}{16x^{16}} - \frac{5a^4b}{14x^{14}} - \frac{5a^3b^2}{6x^{12}} - \frac{a^2b^3}{x^{10}} - \frac{5ab^4}{8x^8} - \frac{b^5}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x\*\*17, x)

[Out]  $-a**5/(16*x**16) - 5*a**4*b/(14*x**14) - 5*a**3*b**2/(6*x**12) - a**2*b**3/x**10 - 5*a*b**4/(8*x**8) - b**5/(6*x**6)$

**Mathematica [A]** time = 0.00756152, size = 67, normalized size = 1.08

$$-\frac{a^5}{16x^{16}} - \frac{5a^4b}{14x^{14}} - \frac{5a^3b^2}{6x^{12}} - \frac{a^2b^3}{x^{10}} - \frac{5ab^4}{8x^8} - \frac{b^5}{6x^6}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x^17, x]

[Out]  $-a^5/(16*x^{16}) - (5*a^4*b)/(14*x^{14}) - (5*a^3*b^2)/(6*x^{12}) - (a^2*b^3)/x^{10} - (5*a*b^4)/(8*x^8) - b^5/(6*x^6)$

**Maple [A]** time = 0.009, size = 58, normalized size = 0.9

$$-\frac{b^5}{6x^6} - \frac{5a^4b}{14x^{14}} - \frac{5ab^4}{8x^8} - \frac{a^2b^3}{x^{10}} - \frac{a^5}{16x^{16}} - \frac{5a^3b^2}{6x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^5/x^17,x)`

[Out]  $-1/6*b^5/x^6-5/14*a^4*b/x^{14}-5/8*a*b^4/x^8-a^2*b^3/x^{10}-1/16*a^5/x^{16}-5/6*a^3*b^2/x^{12}$

**Maxima [A]** time = 1.33772, size = 80, normalized size = 1.29

$$\frac{56 b^5 x^{10} + 210 a b^4 x^8 + 336 a^2 b^3 x^6 + 280 a^3 b^2 x^4 + 120 a^4 b x^2 + 21 a^5}{336 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^17,x, algorithm="maxima")`

[Out]  $-1/336*(56*b^5*x^{10} + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 + 120*a^4*b*x^2 + 21*a^5)/x^{16}$

**Fricas [A]** time = 0.191977, size = 80, normalized size = 1.29

$$\frac{56 b^5 x^{10} + 210 a b^4 x^8 + 336 a^2 b^3 x^6 + 280 a^3 b^2 x^4 + 120 a^4 b x^2 + 21 a^5}{336 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^17,x, algorithm="fricas")`

[Out]  $-1/336*(56*b^5*x^{10} + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 + 120*a^4*b*x^2 + 21*a^5)/x^{16}$

**Sympy [A]** time = 2.63818, size = 63, normalized size = 1.02

$$\frac{21a^5 + 120a^4bx^2 + 280a^3b^2x^4 + 336a^2b^3x^6 + 210ab^4x^8 + 56b^5x^{10}}{336x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x**17,x)`

[Out]  $-(21*a**5 + 120*a**4*b*x**2 + 280*a**3*b**2*x**4 + 336*a**2*b**3*x**6 + 210*a*b**4*x**8 + 56*b**5*x**10)/(336*x**16)$

**GIAC/XCAS [A]** time = 0.216411, size = 80, normalized size = 1.29

$$\frac{56 b^5 x^{10} + 210 a b^4 x^8 + 336 a^2 b^3 x^6 + 280 a^3 b^2 x^4 + 120 a^4 b x^2 + 21 a^5}{336 x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^17,x, algorithm="giac")`

[Out]  $-1/336*(56*b^5*x^{10} + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 + 120*a^4*b*x^2 + 21*a^5)/x^{16}$

$$3.68 \quad \int \frac{(a+bx^2)^5}{x^{19}} dx$$

**Optimal.** Leaf size=69

$$-\frac{a^5}{18x^{18}} - \frac{5a^4b}{16x^{16}} - \frac{5a^3b^2}{7x^{14}} - \frac{5a^2b^3}{6x^{12}} - \frac{ab^4}{2x^{10}} - \frac{b^5}{8x^8}$$

[Out]  $-a^5/(18*x^{18}) - (5*a^4*b)/(16*x^{16}) - (5*a^3*b^2)/(7*x^{14}) - (5*a^2*b^3)/(6*x^{12}) - (a*b^4)/(2*x^{10}) - b^5/(8*x^8)$

**Rubi [A]** time = 0.0864066, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^5}{18x^{18}} - \frac{5a^4b}{16x^{16}} - \frac{5a^3b^2}{7x^{14}} - \frac{5a^2b^3}{6x^{12}} - \frac{ab^4}{2x^{10}} - \frac{b^5}{8x^8}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x^19, x]

[Out]  $-a^5/(18*x^{18}) - (5*a^4*b)/(16*x^{16}) - (5*a^3*b^2)/(7*x^{14}) - (5*a^2*b^3)/(6*x^{12}) - (a*b^4)/(2*x^{10}) - b^5/(8*x^8)$

**Rubi in Sympy [A]** time = 14.6341, size = 66, normalized size = 0.96

$$-\frac{a^5}{18x^{18}} - \frac{5a^4b}{16x^{16}} - \frac{5a^3b^2}{7x^{14}} - \frac{5a^2b^3}{6x^{12}} - \frac{ab^4}{2x^{10}} - \frac{b^5}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x\*\*19, x)

[Out]  $-a**5/(18*x**18) - 5*a**4*b/(16*x**16) - 5*a**3*b**2/(7*x**14) - 5*a**2*b**3/(6*x**12) - a*b**4/(2*x**10) - b**5/(8*x**8)$

**Mathematica [A]** time = 0.00738297, size = 69, normalized size = 1.

$$-\frac{a^5}{18x^{18}} - \frac{5a^4b}{16x^{16}} - \frac{5a^3b^2}{7x^{14}} - \frac{5a^2b^3}{6x^{12}} - \frac{ab^4}{2x^{10}} - \frac{b^5}{8x^8}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x^19, x]

[Out]  $-a^5/(18*x^{18}) - (5*a^4*b)/(16*x^{16}) - (5*a^3*b^2)/(7*x^{14}) - (5*a^2*b^3)/(6*x^{12}) - (a*b^4)/(2*x^{10}) - b^5/(8*x^8)$

**Maple [A]** time = 0.009, size = 58, normalized size = 0.8

$$-\frac{a^5}{18x^{18}} - \frac{5a^4b}{16x^{16}} - \frac{5a^3b^2}{7x^{14}} - \frac{5a^2b^3}{6x^{12}} - \frac{ab^4}{2x^{10}} - \frac{b^5}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^5/x^19,x)`

[Out]  $-1/18*a^5/x^{18}-5/16*a^4*b/x^{16}-5/7*a^3*b^2/x^{14}-5/6*a^2*b^3/x^{12}-1/2*a*b^4/x^{10}-1/8*b^5/x^8$

**Maxima [A]** time = 1.35205, size = 80, normalized size = 1.16

$$-\frac{126b^5x^{10} + 504ab^4x^8 + 840a^2b^3x^6 + 720a^3b^2x^4 + 315a^4bx^2 + 56a^5}{1008x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^19,x, algorithm="maxima")`

[Out]  $-1/1008*(126*b^5*x^{10} + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4 + 315*a^4*b*x^2 + 56*a^5)/x^{18}$

**Fricas [A]** time = 0.192223, size = 80, normalized size = 1.16

$$-\frac{126b^5x^{10} + 504ab^4x^8 + 840a^2b^3x^6 + 720a^3b^2x^4 + 315a^4bx^2 + 56a^5}{1008x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^19,x, algorithm="fricas")`

[Out]  $-1/1008*(126*b^5*x^{10} + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4 + 315*a^4*b*x^2 + 56*a^5)/x^{18}$

**Sympy [A]** time = 2.83529, size = 63, normalized size = 0.91

$$-\frac{56a^5 + 315a^4bx^2 + 720a^3b^2x^4 + 840a^2b^3x^6 + 504ab^4x^8 + 126b^5x^{10}}{1008x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x**19,x)`

[Out]  $-(56*a**5 + 315*a**4*b*x**2 + 720*a**3*b**2*x**4 + 840*a**2*b**3*x**6 + 504*a*b**4*x**8 + 126*b**5*x**10)/(1008*x**18)$

**GIAC/XCAS [A]** time = 0.207796, size = 80, normalized size = 1.16

$$-\frac{126b^5x^{10} + 504ab^4x^8 + 840a^2b^3x^6 + 720a^3b^2x^4 + 315a^4bx^2 + 56a^5}{1008x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^19,x, algorithm="giac")`

[Out]  $-1/1008*(126*b^5*x^{10} + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4 + 315*a^4*b*x^2 + 56*a^5)/x^{18}$

$$3.69 \quad \int \frac{(a+bx^2)^5}{x^{21}} dx$$

**Optimal.** Leaf size=69

$$-\frac{a^5}{20x^{20}} - \frac{5a^4b}{18x^{18}} - \frac{5a^3b^2}{8x^{16}} - \frac{5a^2b^3}{7x^{14}} - \frac{5ab^4}{12x^{12}} - \frac{b^5}{10x^{10}}$$

[Out]  $-a^5/(20*x^{20}) - (5*a^4*b)/(18*x^{18}) - (5*a^3*b^2)/(8*x^{16}) - (5*a^2*b^3)/(7*x^{14}) - (5*a*b^4)/(12*x^{12}) - b^5/(10*x^{10})$

**Rubi [A]** time = 0.0870523, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^5}{20x^{20}} - \frac{5a^4b}{18x^{18}} - \frac{5a^3b^2}{8x^{16}} - \frac{5a^2b^3}{7x^{14}} - \frac{5ab^4}{12x^{12}} - \frac{b^5}{10x^{10}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x^21, x]

[Out]  $-a^5/(20*x^{20}) - (5*a^4*b)/(18*x^{18}) - (5*a^3*b^2)/(8*x^{16}) - (5*a^2*b^3)/(7*x^{14}) - (5*a*b^4)/(12*x^{12}) - b^5/(10*x^{10})$

**Rubi in Sympy [A]** time = 14.8829, size = 68, normalized size = 0.99

$$-\frac{a^5}{20x^{20}} - \frac{5a^4b}{18x^{18}} - \frac{5a^3b^2}{8x^{16}} - \frac{5a^2b^3}{7x^{14}} - \frac{5ab^4}{12x^{12}} - \frac{b^5}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x\*\*21, x)

[Out]  $-a**5/(20*x**20) - 5*a**4*b/(18*x**18) - 5*a**3*b**2/(8*x**16) - 5*a**2*b**3/(7*x**14) - 5*a*b**4/(12*x**12) - b**5/(10*x**10)$

**Mathematica [A]** time = 0.0074764, size = 69, normalized size = 1.

$$-\frac{a^5}{20x^{20}} - \frac{5a^4b}{18x^{18}} - \frac{5a^3b^2}{8x^{16}} - \frac{5a^2b^3}{7x^{14}} - \frac{5ab^4}{12x^{12}} - \frac{b^5}{10x^{10}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x^21, x]

[Out]  $-a^5/(20*x^{20}) - (5*a^4*b)/(18*x^{18}) - (5*a^3*b^2)/(8*x^{16}) - (5*a^2*b^3)/(7*x^{14}) - (5*a*b^4)/(12*x^{12}) - b^5/(10*x^{10})$

**Maple [A]** time = 0.009, size = 58, normalized size = 0.8

$$-\frac{a^5}{20x^{20}} - \frac{5a^4b}{18x^{18}} - \frac{5a^3b^2}{8x^{16}} - \frac{5a^2b^3}{7x^{14}} - \frac{5ab^4}{12x^{12}} - \frac{b^5}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^5/x^21,x)`

[Out]  $-1/20*a^5/x^{20}-5/18*a^4*b/x^{18}-5/8*a^3*b^2/x^{16}-5/7*a^2*b^3/x^{14}-5/12*a*b^4/x^{12}-1/10*b^5/x^{10}$

**Maxima [A]** time = 1.33943, size = 80, normalized size = 1.16

$$\frac{252 b^5 x^{10} + 1050 a b^4 x^8 + 1800 a^2 b^3 x^6 + 1575 a^3 b^2 x^4 + 700 a^4 b x^2 + 126 a^5}{2520 x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^21,x, algorithm="maxima")`

[Out]  $-1/2520*(252*b^5*x^{10} + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x^4 + 700*a^4*b*x^2 + 126*a^5)/x^{20}$

**Fricas [A]** time = 0.196832, size = 80, normalized size = 1.16

$$\frac{252 b^5 x^{10} + 1050 a b^4 x^8 + 1800 a^2 b^3 x^6 + 1575 a^3 b^2 x^4 + 700 a^4 b x^2 + 126 a^5}{2520 x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^21,x, algorithm="fricas")`

[Out]  $-1/2520*(252*b^5*x^{10} + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x^4 + 700*a^4*b*x^2 + 126*a^5)/x^{20}$

**Sympy [A]** time = 2.95956, size = 63, normalized size = 0.91

$$\frac{126 a^5 + 700 a^4 b x^2 + 1575 a^3 b^2 x^4 + 1800 a^2 b^3 x^6 + 1050 a b^4 x^8 + 252 b^5 x^{10}}{2520 x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x**21,x)`

[Out]  $-(126*a**5 + 700*a**4*b*x**2 + 1575*a**3*b**2*x**4 + 1800*a**2*b**3*x**6 + 1050*a*b**4*x**8 + 252*b**5*x**10)/(2520*x**20)$

**GIAC/XCAS [A]** time = 0.209346, size = 80, normalized size = 1.16

$$\frac{252 b^5 x^{10} + 1050 a b^4 x^8 + 1800 a^2 b^3 x^6 + 1575 a^3 b^2 x^4 + 700 a^4 b x^2 + 126 a^5}{2520 x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^21,x, algorithm="giac")`

[Out]  $-1/2520*(252*b^5*x^{10} + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x^4 + 700*a^4*b*x^2 + 126*a^5)/x^{20}$

### 3.70 $\int x^8 (a + bx^2)^5 dx$

**Optimal.** Leaf size=69

$$\frac{a^5 x^9}{9} + \frac{5}{11} a^4 b x^{11} + \frac{10}{13} a^3 b^2 x^{13} + \frac{2}{3} a^2 b^3 x^{15} + \frac{5}{17} a b^4 x^{17} + \frac{b^5 x^{19}}{19}$$

[Out]  $(a^5 x^9)/9 + (5 a^4 b x^{11})/11 + (10 a^3 b^2 x^{13})/13 + (2 a^2 b^3 x^{15})/3 + (5 a b^4 x^{17})/17 + (b^5 x^{19})/19$

**Rubi [A]** time = 0.0746888, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{a^5 x^9}{9} + \frac{5}{11} a^4 b x^{11} + \frac{10}{13} a^3 b^2 x^{13} + \frac{2}{3} a^2 b^3 x^{15} + \frac{5}{17} a b^4 x^{17} + \frac{b^5 x^{19}}{19}$$

Antiderivative was successfully verified.

[In] Int[x^8\*(a + b\*x^2)^5,x]

[Out]  $(a^5 x^9)/9 + (5 a^4 b x^{11})/11 + (10 a^3 b^2 x^{13})/13 + (2 a^2 b^3 x^{15})/3 + (5 a b^4 x^{17})/17 + (b^5 x^{19})/19$

**Rubi in Sympy [A]** time = 11.8948, size = 66, normalized size = 0.96

$$\frac{a^5 x^9}{9} + \frac{5 a^4 b x^{11}}{11} + \frac{10 a^3 b^2 x^{13}}{13} + \frac{2 a^2 b^3 x^{15}}{3} + \frac{5 a b^4 x^{17}}{17} + \frac{b^5 x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*8\*(b\*x\*\*2+a)\*\*5,x)

[Out]  $a**5*x**9/9 + 5*a**4*b*x**11/11 + 10*a**3*b**2*x**13/13 + 2*a**2*b**3*x**15/3 + 5*a*b**4*x**17/17 + b**5*x**19/19$

**Mathematica [A]** time = 0.00388875, size = 69, normalized size = 1.

$$\frac{a^5 x^9}{9} + \frac{5}{11} a^4 b x^{11} + \frac{10}{13} a^3 b^2 x^{13} + \frac{2}{3} a^2 b^3 x^{15} + \frac{5}{17} a b^4 x^{17} + \frac{b^5 x^{19}}{19}$$

Antiderivative was successfully verified.

[In] Integrate[x^8\*(a + b\*x^2)^5,x]

[Out]  $(a^5 x^9)/9 + (5 a^4 b x^{11})/11 + (10 a^3 b^2 x^{13})/13 + (2 a^2 b^3 x^{15})/3 + (5 a b^4 x^{17})/17 + (b^5 x^{19})/19$

**Maple [A]** time = 0.002, size = 58, normalized size = 0.8

$$\frac{a^5 x^9}{9} + \frac{5 a^4 b x^{11}}{11} + \frac{10 a^3 b^2 x^{13}}{13} + \frac{2 a^2 b^3 x^{15}}{3} + \frac{5 a b^4 x^{17}}{17} + \frac{b^5 x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(x^8*(b*x^2+a)^5,x)`

[Out]  $\frac{1}{9}a^5x^9 + \frac{5}{11}a^4b^4x^{11} + \frac{10}{13}a^3b^2x^{13} + \frac{2}{3}a^2b^3x^{15} + \frac{5}{17}ab^4x^{17} + \frac{1}{19}b^5x^{19}$

**Maxima [A]** time = 1.34354, size = 77, normalized size = 1.12

$$\frac{1}{19}b^5x^{19} + \frac{5}{17}ab^4x^{17} + \frac{2}{3}a^2b^3x^{15} + \frac{10}{13}a^3b^2x^{13} + \frac{5}{11}a^4bx^{11} + \frac{1}{9}a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^8,x, algorithm="maxima")`

[Out]  $\frac{1}{19}b^5x^{19} + \frac{5}{17}a^4b^4x^{17} + \frac{2}{3}a^2b^3x^{15} + \frac{10}{13}a^3b^2x^{13} + \frac{5}{11}a^4bx^{11} + \frac{1}{9}a^5x^9$

**Fricas [A]** time = 0.175905, size = 1, normalized size = 0.01

$$\frac{1}{19}x^{19}b^5 + \frac{5}{17}x^{17}b^4a + \frac{2}{3}x^{15}b^3a^2 + \frac{10}{13}x^{13}b^2a^3 + \frac{5}{11}x^{11}ba^4 + \frac{1}{9}x^9a^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^8,x, algorithm="fricas")`

[Out]  $\frac{1}{19}x^{19}b^5 + \frac{5}{17}x^{17}b^4a + \frac{2}{3}x^{15}b^3a^2 + \frac{10}{13}x^{13}b^2a^3 + \frac{5}{11}x^{11}ba^4 + \frac{1}{9}x^9a^5$

**Sympy [A]** time = 0.119994, size = 66, normalized size = 0.96

$$\frac{a^5x^9}{9} + \frac{5a^4bx^{11}}{11} + \frac{10a^3b^2x^{13}}{13} + \frac{2a^2b^3x^{15}}{3} + \frac{5ab^4x^{17}}{17} + \frac{b^5x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*(b*x**2+a)**5,x)`

[Out]  $a^5x^9/9 + 5a^4bx^{11}/11 + 10a^3b^2x^{13}/13 + 2a^2b^3x^{15}/3 + 5ab^4x^{17}/17 + b^5x^{19}/19$

**GIAC/XCAS [A]** time = 0.209475, size = 77, normalized size = 1.12

$$\frac{1}{19}b^5x^{19} + \frac{5}{17}ab^4x^{17} + \frac{2}{3}a^2b^3x^{15} + \frac{10}{13}a^3b^2x^{13} + \frac{5}{11}a^4bx^{11} + \frac{1}{9}a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^8,x, algorithm="giac")`

[Out]  $\frac{1}{19}b^5x^{19} + \frac{5}{17}a^4b^4x^{17} + \frac{2}{3}a^2b^3x^{15} + \frac{10}{13}a^3b^2x^{13} + \frac{5}{11}a^4bx^{11} + \frac{1}{9}a^5x^9$

### 3.71 $\int x^6 (a + bx^2)^5 dx$

**Optimal.** Leaf size=69

$$\frac{a^5 x^7}{7} + \frac{5}{9} a^4 b x^9 + \frac{10}{11} a^3 b^2 x^{11} + \frac{10}{13} a^2 b^3 x^{13} + \frac{1}{3} a b^4 x^{15} + \frac{b^5 x^{17}}{17}$$

[Out]  $(a^5 x^7)/7 + (5 a^4 b x^9)/9 + (10 a^3 b^2 x^{11})/11 + (10 a^2 b^3 x^{13})/13 + (a b^4 x^{15})/3 + (b^5 x^{17})/17$

**Rubi [A]** time = 0.0709719, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{a^5 x^7}{7} + \frac{5}{9} a^4 b x^9 + \frac{10}{11} a^3 b^2 x^{11} + \frac{10}{13} a^2 b^3 x^{13} + \frac{1}{3} a b^4 x^{15} + \frac{b^5 x^{17}}{17}$$

Antiderivative was successfully verified.

[In] Int[x^6\*(a + b\*x^2)^5,x]

[Out]  $(a^5 x^7)/7 + (5 a^4 b x^9)/9 + (10 a^3 b^2 x^{11})/11 + (10 a^2 b^3 x^{13})/13 + (a b^4 x^{15})/3 + (b^5 x^{17})/17$

**Rubi in Sympy [A]** time = 11.9187, size = 65, normalized size = 0.94

$$\frac{a^5 x^7}{7} + \frac{5 a^4 b x^9}{9} + \frac{10 a^3 b^2 x^{11}}{11} + \frac{10 a^2 b^3 x^{13}}{13} + \frac{a b^4 x^{15}}{3} + \frac{b^5 x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6\*(b\*x\*\*2+a)\*\*5,x)

[Out]  $a**5*x**7/7 + 5*a**4*b*x**9/9 + 10*a**3*b**2*x**11/11 + 10*a**2*b**3*x**13/13 + a*b**4*x**15/3 + b**5*x**17/17$

**Mathematica [A]** time = 0.00366061, size = 69, normalized size = 1.

$$\frac{a^5 x^7}{7} + \frac{5}{9} a^4 b x^9 + \frac{10}{11} a^3 b^2 x^{11} + \frac{10}{13} a^2 b^3 x^{13} + \frac{1}{3} a b^4 x^{15} + \frac{b^5 x^{17}}{17}$$

Antiderivative was successfully verified.

[In] Integrate[x^6\*(a + b\*x^2)^5,x]

[Out]  $(a^5 x^7)/7 + (5 a^4 b x^9)/9 + (10 a^3 b^2 x^{11})/11 + (10 a^2 b^3 x^{13})/13 + (a b^4 x^{15})/3 + (b^5 x^{17})/17$

**Maple [A]** time = 0.002, size = 58, normalized size = 0.8

$$\frac{a^5 x^7}{7} + \frac{5 a^4 b x^9}{9} + \frac{10 a^3 b^2 x^{11}}{11} + \frac{10 a^2 b^3 x^{13}}{13} + \frac{a b^4 x^{15}}{3} + \frac{b^5 x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*(b*x^2+a)^5,x)`

[Out]  $\frac{1}{7}a^5x^7 + \frac{5}{9}a^4b^2x^9 + \frac{10}{11}a^3b^2x^{11} + \frac{10}{13}a^2b^3x^{13} + \frac{5}{9}a^4bx^9 + \frac{1}{7}a^5x^7$

**Maxima [A]** time = 1.34668, size = 77, normalized size = 1.12

$$\frac{1}{17}b^5x^{17} + \frac{1}{3}ab^4x^{15} + \frac{10}{13}a^2b^3x^{13} + \frac{10}{11}a^3b^2x^{11} + \frac{5}{9}a^4bx^9 + \frac{1}{7}a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^6,x, algorithm="maxima")`

[Out]  $\frac{1}{17}b^5x^{17} + \frac{1}{3}a^4b^4x^{15} + \frac{10}{13}a^2b^3x^{13} + \frac{10}{11}a^3b^2x^{11} + \frac{5}{9}a^4bx^9 + \frac{1}{7}a^5x^7$

**Fricas [A]** time = 0.174745, size = 1, normalized size = 0.01

$$\frac{1}{17}x^{17}b^5 + \frac{1}{3}x^{15}b^4a + \frac{10}{13}x^{13}b^3a^2 + \frac{10}{11}x^{11}b^2a^3 + \frac{5}{9}x^9ba^4 + \frac{1}{7}x^7a^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^6,x, algorithm="fricas")`

[Out]  $\frac{1}{17}x^{17}b^5 + \frac{1}{3}x^{15}b^4a + \frac{10}{13}x^{13}b^3a^2 + \frac{10}{11}x^{11}b^2a^3 + \frac{5}{9}x^9b^4a + \frac{1}{7}x^7a^5$

**Sympy [A]** time = 0.127308, size = 65, normalized size = 0.94

$$\frac{a^5x^7}{7} + \frac{5a^4bx^9}{9} + \frac{10a^3b^2x^{11}}{11} + \frac{10a^2b^3x^{13}}{13} + \frac{ab^4x^{15}}{3} + \frac{b^5x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6*(b*x**2+a)**5,x)`

[Out]  $a^5x^{7/7} + \frac{5}{9}a^4b^2x^{9/9} + \frac{10}{11}a^3b^2x^{11/11} + \frac{10}{13}a^2b^3x^{13/13} + \frac{ab^4x^{15}}{3} + \frac{b^5x^{17}}{17}$

**GIAC/XCAS [A]** time = 0.20759, size = 77, normalized size = 1.12

$$\frac{1}{17}b^5x^{17} + \frac{1}{3}ab^4x^{15} + \frac{10}{13}a^2b^3x^{13} + \frac{10}{11}a^3b^2x^{11} + \frac{5}{9}a^4bx^9 + \frac{1}{7}a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^6,x, algorithm="giac")`

[Out]  $\frac{1}{17}b^5x^{17} + \frac{1}{3}a^4b^4x^{15} + \frac{10}{13}a^2b^3x^{13} + \frac{10}{11}a^3b^2x^{11} + \frac{5}{9}a^4bx^9 + \frac{1}{7}a^5x^7$

### 3.72 $\int x^4 (a + bx^2)^5 dx$

**Optimal.** Leaf size=69

$$\frac{a^5 x^5}{5} + \frac{5a^4 b x^7}{7} + \frac{10a^3 b^2 x^9}{9} + \frac{10a^2 b^3 x^{11}}{11} + \frac{5ab^4 x^{13}}{13} + \frac{b^5 x^{15}}{15}$$

[Out]  $(a^5 x^5)/5 + (5 a^4 b x^7)/7 + (10 a^3 b^2 x^9)/9 + (10 a^2 b^3 x^{11})/11 + (5 a b^4 x^{13})/13 + (b^5 x^{15})/15$

**Rubi [A]** time = 0.0708871, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{a^5 x^5}{5} + \frac{5a^4 b x^7}{7} + \frac{10a^3 b^2 x^9}{9} + \frac{10a^2 b^3 x^{11}}{11} + \frac{5ab^4 x^{13}}{13} + \frac{b^5 x^{15}}{15}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a + b\*x^2)^5,x]

[Out]  $(a^5 x^5)/5 + (5 a^4 b x^7)/7 + (10 a^3 b^2 x^9)/9 + (10 a^2 b^3 x^{11})/11 + (5 a b^4 x^{13})/13 + (b^5 x^{15})/15$

**Rubi in Sympy [A]** time = 11.9829, size = 66, normalized size = 0.96

$$\frac{a^5 x^5}{5} + \frac{5a^4 b x^7}{7} + \frac{10a^3 b^2 x^9}{9} + \frac{10a^2 b^3 x^{11}}{11} + \frac{5ab^4 x^{13}}{13} + \frac{b^5 x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(b\*x\*\*2+a)\*\*5,x)

[Out]  $a**5*x**5/5 + 5*a**4*b*x**7/7 + 10*a**3*b**2*x**9/9 + 10*a**2*b**3*x**11/11 + 5*a*b**4*x**13/13 + b**5*x**15/15$

**Mathematica [A]** time = 0.00382316, size = 69, normalized size = 1.

$$\frac{a^5 x^5}{5} + \frac{5a^4 b x^7}{7} + \frac{10a^3 b^2 x^9}{9} + \frac{10a^2 b^3 x^{11}}{11} + \frac{5ab^4 x^{13}}{13} + \frac{b^5 x^{15}}{15}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a + b\*x^2)^5,x]

[Out]  $(a^5 x^5)/5 + (5 a^4 b x^7)/7 + (10 a^3 b^2 x^9)/9 + (10 a^2 b^3 x^{11})/11 + (5 a b^4 x^{13})/13 + (b^5 x^{15})/15$

**Maple [A]** time = 0.002, size = 58, normalized size = 0.8

$$\frac{a^5 x^5}{5} + \frac{5a^4 b x^7}{7} + \frac{10a^3 b^2 x^9}{9} + \frac{10a^2 b^3 x^{11}}{11} + \frac{5ab^4 x^{13}}{13} + \frac{b^5 x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(b*x^2+a)^5,x)`

[Out]  $\frac{1}{5}a^5x^5 + \frac{5}{7}a^4bx^7 + \frac{10}{9}a^3b^2x^9 + \frac{10}{11}a^2b^3x^{11} + \frac{5}{13}ab^4x^{13} + \frac{1}{15}b^5x^{15}$

**Maxima [A]** time = 1.34451, size = 77, normalized size = 1.12

$$\frac{1}{15}b^5x^{15} + \frac{5}{13}ab^4x^{13} + \frac{10}{11}a^2b^3x^{11} + \frac{10}{9}a^3b^2x^9 + \frac{5}{7}a^4bx^7 + \frac{1}{5}a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^4,x, algorithm="maxima")`

[Out]  $\frac{1}{15}b^5x^{15} + \frac{5}{13}a^4bx^{13} + \frac{10}{11}a^2b^3x^{11} + \frac{10}{9}a^3b^2x^9 + \frac{5}{7}a^4bx^7 + \frac{1}{5}a^5x^5$

**Fricas [A]** time = 0.175377, size = 1, normalized size = 0.01

$$\frac{1}{15}x^{15}b^5 + \frac{5}{13}x^{13}b^4a + \frac{10}{11}x^{11}b^3a^2 + \frac{10}{9}x^9b^2a^3 + \frac{5}{7}x^7ba^4 + \frac{1}{5}x^5a^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^4,x, algorithm="fricas")`

[Out]  $\frac{1}{15}x^{15}b^5 + \frac{5}{13}x^{13}b^4a + \frac{10}{11}x^{11}b^3a^2 + \frac{10}{9}x^9b^2a^3 + \frac{5}{7}x^7ba^4 + \frac{1}{5}x^5a^5$

**Sympy [A]** time = 0.122299, size = 66, normalized size = 0.96

$$\frac{a^5x^5}{5} + \frac{5a^4bx^7}{7} + \frac{10a^3b^2x^9}{9} + \frac{10a^2b^3x^{11}}{11} + \frac{5ab^4x^{13}}{13} + \frac{b^5x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(b*x**2+a)**5,x)`

[Out]  $a^5x^5/5 + 5a^4bx^7/7 + 10a^3b^2x^9/9 + 10a^2b^3x^{11}/11 + 5ab^4x^{13}/13 + b^5x^{15}/15$

**GIAC/XCAS [A]** time = 0.20979, size = 77, normalized size = 1.12

$$\frac{1}{15}b^5x^{15} + \frac{5}{13}ab^4x^{13} + \frac{10}{11}a^2b^3x^{11} + \frac{10}{9}a^3b^2x^9 + \frac{5}{7}a^4bx^7 + \frac{1}{5}a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^4,x, algorithm="giac")`

[Out]  $\frac{1}{15}b^5x^{15} + \frac{5}{13}a^4bx^{13} + \frac{10}{11}a^2b^3x^{11} + \frac{10}{9}a^3b^2x^9 + \frac{5}{7}a^4bx^7 + \frac{1}{5}a^5x^5$

### 3.73 $\int x^2 (a + bx^2)^5 dx$

**Optimal.** Leaf size=66

$$\frac{a^5 x^3}{3} + a^4 b x^5 + \frac{10}{7} a^3 b^2 x^7 + \frac{10}{9} a^2 b^3 x^9 + \frac{5}{11} a b^4 x^{11} + \frac{b^5 x^{13}}{13}$$

[Out]  $(a^5 x^3)/3 + a^4 b x^5 + (10 a^3 b^2 x^7)/7 + (10 a^2 b^3 x^9)/9 + (5 a b^4 x^{11})/11 + (b^5 x^{13})/13$

**Rubi [A]** time = 0.0685256, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{a^5 x^3}{3} + a^4 b x^5 + \frac{10}{7} a^3 b^2 x^7 + \frac{10}{9} a^2 b^3 x^9 + \frac{5}{11} a b^4 x^{11} + \frac{b^5 x^{13}}{13}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*x^2)^5, x]

[Out]  $(a^5 x^3)/3 + a^4 b x^5 + (10 a^3 b^2 x^7)/7 + (10 a^2 b^3 x^9)/9 + (5 a b^4 x^{11})/11 + (b^5 x^{13})/13$

**Rubi in Sympy [A]** time = 12.337, size = 63, normalized size = 0.95

$$\frac{a^5 x^3}{3} + a^4 b x^5 + \frac{10 a^3 b^2 x^7}{7} + \frac{10 a^2 b^3 x^9}{9} + \frac{5 a b^4 x^{11}}{11} + \frac{b^5 x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(b\*x\*\*2+a)\*\*5, x)

[Out]  $a**5*x**3/3 + a**4*b*x**5 + 10*a**3*b**2*x**7/7 + 10*a**2*b**3*x**9/9 + 5*a*b**4*x**11/11 + b**5*x**13/13$

**Mathematica [A]** time = 0.00358285, size = 66, normalized size = 1.

$$\frac{a^5 x^3}{3} + a^4 b x^5 + \frac{10}{7} a^3 b^2 x^7 + \frac{10}{9} a^2 b^3 x^9 + \frac{5}{11} a b^4 x^{11} + \frac{b^5 x^{13}}{13}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*x^2)^5, x]

[Out]  $(a^5 x^3)/3 + a^4 b x^5 + (10 a^3 b^2 x^7)/7 + (10 a^2 b^3 x^9)/9 + (5 a b^4 x^{11})/11 + (b^5 x^{13})/13$

**Maple [A]** time = 0.002, size = 57, normalized size = 0.9

$$\frac{a^5 x^3}{3} + a^4 b x^5 + \frac{10 a^3 b^2 x^7}{7} + \frac{10 a^2 b^3 x^9}{9} + \frac{5 a b^4 x^{11}}{11} + \frac{b^5 x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^2+a)^5,x)`

[Out]  $\frac{1}{3}a^5x^3+a^4b^5x^5+10/7a^3b^2x^7+10/9a^2b^3x^9+5/11a^4b^4x^{11}+1/13b^5x^{13}$

**Maxima [A]** time = 1.3372, size = 76, normalized size = 1.15

$$\frac{1}{13}b^5x^{13} + \frac{5}{11}ab^4x^{11} + \frac{10}{9}a^2b^3x^9 + \frac{10}{7}a^3b^2x^7 + a^4bx^5 + \frac{1}{3}a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^2,x, algorithm="maxima")`

[Out]  $\frac{1}{13}b^5x^{13} + \frac{5}{11}a^4b^4x^{11} + \frac{10}{9}a^2b^3x^9 + \frac{10}{7}a^3b^2x^7 + a^4b^4x^{11} + \frac{1}{3}a^5x^3$

**Fricas [A]** time = 0.175852, size = 1, normalized size = 0.02

$$\frac{1}{13}x^{13}b^5 + \frac{5}{11}x^{11}b^4a + \frac{10}{9}x^9b^3a^2 + \frac{10}{7}x^7b^2a^3 + x^5ba^4 + \frac{1}{3}x^3a^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^2,x, algorithm="fricas")`

[Out]  $\frac{1}{13}x^{13}b^5 + \frac{5}{11}x^{11}b^4a + \frac{10}{9}x^9b^3a^2 + \frac{10}{7}x^7b^2a^3 + x^5ba^4 + \frac{1}{3}x^3a^5$

**Sympy [A]** time = 0.112364, size = 63, normalized size = 0.95

$$\frac{a^5x^3}{3} + a^4bx^5 + \frac{10a^3b^2x^7}{7} + \frac{10a^2b^3x^9}{9} + \frac{5ab^4x^{11}}{11} + \frac{b^5x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**2+a)**5,x)`

[Out]  $a^5x^3/3 + a^4b^5x^5 + 10a^3b^2x^7/7 + 10a^2b^3x^9/9 + 5a^4b^4x^{11}/11 + b^5x^{13}/13$

**GIAC/XCAS [A]** time = 0.207603, size = 76, normalized size = 1.15

$$\frac{1}{13}b^5x^{13} + \frac{5}{11}ab^4x^{11} + \frac{10}{9}a^2b^3x^9 + \frac{10}{7}a^3b^2x^7 + a^4bx^5 + \frac{1}{3}a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^2,x, algorithm="giac")`

[Out]  $\frac{1}{13}b^5x^{13} + \frac{5}{11}a^4b^4x^{11} + \frac{10}{9}a^2b^3x^9 + \frac{10}{7}a^3b^2x^7 + a^4b^4x^{11} + \frac{1}{3}a^5x^3$

### 3.74 $\int (a + bx^2)^5 dx$

**Optimal.** Leaf size=62

$$a^5x + \frac{5}{3}a^4bx^3 + 2a^3b^2x^5 + \frac{10}{7}a^2b^3x^7 + \frac{5}{9}ab^4x^9 + \frac{b^5x^{11}}{11}$$

[Out]  $a^5x + (5*a^4*b*x^3)/3 + 2*a^3*b^2*x^5 + (10*a^2*b^3*x^7)/7 + (5*a*b^4*x^9)/9 + (b^5*x^{11})/11$

**Rubi [A]** time = 0.0522279, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$a^5x + \frac{5}{3}a^4bx^3 + 2a^3b^2x^5 + \frac{10}{7}a^2b^3x^7 + \frac{5}{9}ab^4x^9 + \frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5, x]

[Out]  $a^5x + (5*a^4*b*x^3)/3 + 2*a^3*b^2*x^5 + (10*a^2*b^3*x^7)/7 + (5*a*b^4*x^9)/9 + (b^5*x^{11})/11$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{5a^4bx^3}{3} + 2a^3b^2x^5 + \frac{10a^2b^3x^7}{7} + \frac{5ab^4x^9}{9} + \frac{b^5x^{11}}{11} + \int a^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5, x)

[Out]  $5*a**4*b*x**3/3 + 2*a**3*b**2*x**5 + 10*a**2*b**3*x**7/7 + 5*a*b**4*x**9/9 + b**5*x**11/11 + \text{Integral}(a**5, x)$

**Mathematica [A]** time = 0.00173559, size = 62, normalized size = 1.

$$a^5x + \frac{5}{3}a^4bx^3 + 2a^3b^2x^5 + \frac{10}{7}a^2b^3x^7 + \frac{5}{9}ab^4x^9 + \frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5, x]

[Out]  $a^5x + (5*a^4*b*x^3)/3 + 2*a^3*b^2*x^5 + (10*a^2*b^3*x^7)/7 + (5*a*b^4*x^9)/9 + (b^5*x^{11})/11$

**Maple [A]** time = 0.002, size = 55, normalized size = 0.9

$$a^5x + \frac{5a^4bx^3}{3} + 2a^3b^2x^5 + \frac{10a^2b^3x^7}{7} + \frac{5ab^4x^9}{9} + \frac{b^5x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((b*x^2+a)^5,x)`

[Out]  $a^5x + \frac{5}{3}a^4bx^3 + 2a^3b^2x^5 + \frac{10}{7}a^2b^3x^7 + \frac{5}{9}a^4bx^3 + a^5x + \frac{1}{11}b^5x^{11}$

**Maxima [A]** time = 1.36761, size = 73, normalized size = 1.18

$$\frac{1}{11}b^5x^{11} + \frac{5}{9}ab^4x^9 + \frac{10}{7}a^2b^3x^7 + 2a^3b^2x^5 + \frac{5}{3}a^4bx^3 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5,x, algorithm="maxima")`

[Out]  $\frac{1}{11}b^5x^{11} + \frac{5}{9}a^4bx^9 + \frac{10}{7}a^2b^3x^7 + 2a^3b^2x^5 + \frac{5}{3}a^4bx^3 + a^5x$

**Fricas [A]** time = 0.180537, size = 1, normalized size = 0.02

$$\frac{1}{11}x^{11}b^5 + \frac{5}{9}x^9b^4a + \frac{10}{7}x^7b^3a^2 + 2x^5b^2a^3 + \frac{5}{3}x^3ba^4 + xa^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5,x, algorithm="fricas")`

[Out]  $\frac{1}{11}x^{11}b^5 + \frac{5}{9}x^9b^4a + \frac{10}{7}x^7b^3a^2 + 2x^5b^2a^3 + \frac{5}{3}x^3b^4a + xa^5$

**Sympy [A]** time = 0.116605, size = 61, normalized size = 0.98

$$a^5x + \frac{5a^4bx^3}{3} + 2a^3b^2x^5 + \frac{10a^2b^3x^7}{7} + \frac{5ab^4x^9}{9} + \frac{b^5x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5,x)`

[Out]  $a^5x + \frac{5a^4bx^3}{3} + 2a^3b^2x^5 + \frac{10a^2b^3x^7}{7} + \frac{5ab^4x^9}{9} + \frac{b^5x^{11}}{11}$

**GIAC/XCAS [A]** time = 0.209262, size = 73, normalized size = 1.18

$$\frac{1}{11}b^5x^{11} + \frac{5}{9}ab^4x^9 + \frac{10}{7}a^2b^3x^7 + 2a^3b^2x^5 + \frac{5}{3}a^4bx^3 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5,x, algorithm="giac")`

[Out]  $\frac{1}{11}b^5x^{11} + \frac{5}{9}a^4bx^9 + \frac{10}{7}a^2b^3x^7 + 2a^3b^2x^5 + \frac{5}{3}a^4bx^3 + a^5x$

$$3.75 \quad \int \frac{(a+bx^2)^5}{x^2} dx$$

**Optimal.** Leaf size=61

$$-\frac{a^5}{x} + 5a^4bx + \frac{10}{3}a^3b^2x^3 + 2a^2b^3x^5 + \frac{5}{7}ab^4x^7 + \frac{b^5x^9}{9}$$

[Out]  $-(a^5/x) + 5*a^4*b*x + (10*a^3*b^2*x^3)/3 + 2*a^2*b^3*x^5 + (5*a*b^4*x^7)/7 + (b^5*x^9)/9$

**Rubi [A]** time = 0.0634389, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^5}{x} + 5a^4bx + \frac{10}{3}a^3b^2x^3 + 2a^2b^3x^5 + \frac{5}{7}ab^4x^7 + \frac{b^5x^9}{9}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x^2, x]

[Out]  $-(a^5/x) + 5*a^4*b*x + (10*a^3*b^2*x^3)/3 + 2*a^2*b^3*x^5 + (5*a*b^4*x^7)/7 + (b^5*x^9)/9$

**Rubi in Sympy [A]** time = 11.4444, size = 58, normalized size = 0.95

$$-\frac{a^5}{x} + 5a^4bx + \frac{10a^3b^2x^3}{3} + 2a^2b^3x^5 + \frac{5ab^4x^7}{7} + \frac{b^5x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x\*\*2, x)

[Out]  $-a**5/x + 5*a**4*b*x + 10*a**3*b**2*x**3/3 + 2*a**2*b**3*x**5 + 5*a*b**4*x**7/7 + b**5*x**9/9$

**Mathematica [A]** time = 0.00701115, size = 61, normalized size = 1.

$$-\frac{a^5}{x} + 5a^4bx + \frac{10}{3}a^3b^2x^3 + 2a^2b^3x^5 + \frac{5}{7}ab^4x^7 + \frac{b^5x^9}{9}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x^2, x]

[Out]  $-(a^5/x) + 5*a^4*b*x + (10*a^3*b^2*x^3)/3 + 2*a^2*b^3*x^5 + (5*a*b^4*x^7)/7 + (b^5*x^9)/9$

**Maple [A]** time = 0.006, size = 56, normalized size = 0.9

$$-\frac{a^5}{x} + 5a^4bx + \frac{10a^3b^2x^3}{3} + 2a^2b^3x^5 + \frac{5ab^4x^7}{7} + \frac{b^5x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^5/x^2,x)`

[Out]  $-a^5/x+5*a^4*b*x+10/3*a^3*b^2*x^3+2*a^2*b^3*x^5+5/7*a*b^4*x^7+1/9*b^5*x^9$

**Maxima [A]** time = 1.34964, size = 74, normalized size = 1.21

$$\frac{1}{9}b^5x^9 + \frac{5}{7}ab^4x^7 + 2a^2b^3x^5 + \frac{10}{3}a^3b^2x^3 + 5a^4bx - \frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^2,x, algorithm="maxima")`

[Out]  $1/9*b^5*x^9 + 5/7*a*b^4*x^7 + 2*a^2*b^3*x^5 + 10/3*a^3*b^2*x^3 + 5*a^4*b*x - a^5/x$

**Fricas [A]** time = 0.194079, size = 80, normalized size = 1.31

$$\frac{7b^5x^{10} + 45ab^4x^8 + 126a^2b^3x^6 + 210a^3b^2x^4 + 315a^4bx^2 - 63a^5}{63x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^2,x, algorithm="fricas")`

[Out]  $1/63*(7*b^5*x^{10} + 45*a*b^4*x^8 + 126*a^2*b^3*x^6 + 210*a^3*b^2*x^4 + 315*a^4*b*x^2 - 63*a^5)/x$

**Sympy [A]** time = 1.18877, size = 58, normalized size = 0.95

$$-\frac{a^5}{x} + 5a^4bx + \frac{10a^3b^2x^3}{3} + 2a^2b^3x^5 + \frac{5ab^4x^7}{7} + \frac{b^5x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x**2,x)`

[Out]  $-a**5/x + 5*a**4*b*x + 10*a**3*b**2*x**3/3 + 2*a**2*b**3*x**5 + 5*a*b**4*x**7/7 + b**5*x**9/9$

**GIAC/XCAS [A]** time = 0.206537, size = 74, normalized size = 1.21

$$\frac{1}{9}b^5x^9 + \frac{5}{7}ab^4x^7 + 2a^2b^3x^5 + \frac{10}{3}a^3b^2x^3 + 5a^4bx - \frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^2,x, algorithm="giac")`

[Out]  $1/9*b^5*x^9 + 5/7*a*b^4*x^7 + 2*a^2*b^3*x^5 + 10/3*a^3*b^2*x^3 + 5*a^4*b*x - a^5/x$

$$3.76 \quad \int \frac{(a+bx^2)^5}{x^4} dx$$

**Optimal.** Leaf size=60

$$-\frac{a^5}{3x^3} - \frac{5a^4b}{x} + 10a^3b^2x + \frac{10}{3}a^2b^3x^3 + ab^4x^5 + \frac{b^5x^7}{7}$$

[Out]  $-a^5/(3*x^3) - (5*a^4*b)/x + 10*a^3*b^2*x + (10*a^2*b^3*x^3)/3 + a*b^4*x^5 + (b^5*x^7)/7$

**Rubi [A]** time = 0.0624357, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^5}{3x^3} - \frac{5a^4b}{x} + 10a^3b^2x + \frac{10}{3}a^2b^3x^3 + ab^4x^5 + \frac{b^5x^7}{7}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x^4, x]

[Out]  $-a^5/(3*x^3) - (5*a^4*b)/x + 10*a^3*b^2*x + (10*a^2*b^3*x^3)/3 + a*b^4*x^5 + (b^5*x^7)/7$

**Rubi in Sympy [A]** time = 11.4573, size = 56, normalized size = 0.93

$$-\frac{a^5}{3x^3} - \frac{5a^4b}{x} + 10a^3b^2x + \frac{10a^2b^3x^3}{3} + ab^4x^5 + \frac{b^5x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x\*\*4, x)

[Out]  $-a**5/(3*x**3) - 5*a**4*b/x + 10*a**3*b**2*x + 10*a**2*b**3*x**3/3 + a*b**4*x**5 + b**5*x**7/7$

**Mathematica [A]** time = 0.00719834, size = 60, normalized size = 1.

$$-\frac{a^5}{3x^3} - \frac{5a^4b}{x} + 10a^3b^2x + \frac{10}{3}a^2b^3x^3 + ab^4x^5 + \frac{b^5x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x^4, x]

[Out]  $-a^5/(3*x^3) - (5*a^4*b)/x + 10*a^3*b^2*x + (10*a^2*b^3*x^3)/3 + a*b^4*x^5 + (b^5*x^7)/7$

**Maple [A]** time = 0.009, size = 55, normalized size = 0.9

$$-\frac{a^5}{3x^3} - 5\frac{a^4b}{x} + 10a^3b^2x + \frac{10a^2b^3x^3}{3} + ab^4x^5 + \frac{b^5x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^5/x^4,x)`

[Out]  $-1/3*a^5/x^3-5*a^4*b/x+10*a^3*b^2*x+10/3*a^2*b^3*x^3+a*b^4*x^5+1/7*b^5*x^7$

**Maxima [A]** time = 1.3472, size = 74, normalized size = 1.23

$$\frac{1}{7}b^5x^7 + ab^4x^5 + \frac{10}{3}a^2b^3x^3 + 10a^3b^2x - \frac{15a^4bx^2 + a^5}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^4,x, algorithm="maxima")`

[Out]  $1/7*b^5*x^7 + a*b^4*x^5 + 10/3*a^2*b^3*x^3 + 10*a^3*b^2*x - 1/3*(15*a^4*b*x^2 + a^5)/x^3$

**Fricas [A]** time = 0.193314, size = 80, normalized size = 1.33

$$\frac{3b^5x^{10} + 21ab^4x^8 + 70a^2b^3x^6 + 210a^3b^2x^4 - 105a^4bx^2 - 7a^5}{21x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^4,x, algorithm="fricas")`

[Out]  $1/21*(3*b^5*x^{10} + 21*a*b^4*x^8 + 70*a^2*b^3*x^6 + 210*a^3*b^2*x^4 - 105*a^4*b*x^2 - 7*a^5)/x^3$

**Sympy [A]** time = 1.31438, size = 58, normalized size = 0.97

$$10a^3b^2x + \frac{10a^2b^3x^3}{3} + ab^4x^5 + \frac{b^5x^7}{7} - \frac{a^5 + 15a^4bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x**4,x)`

[Out]  $10*a**3*b**2*x + 10*a**2*b**3*x**3/3 + a*b**4*x**5 + b**5*x**7/7 - (a**5 + 15*a**4*b*x**2)/(3*x**3)$

**GIAC/XCAS [A]** time = 0.206814, size = 74, normalized size = 1.23

$$\frac{1}{7}b^5x^7 + ab^4x^5 + \frac{10}{3}a^2b^3x^3 + 10a^3b^2x - \frac{15a^4bx^2 + a^5}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^4,x, algorithm="giac")`

[Out]  $1/7*b^5*x^7 + a*b^4*x^5 + 10/3*a^2*b^3*x^3 + 10*a^3*b^2*x - 1/3*(15*a^4*b*x^2 + a^5)/x^3$

$$3.77 \quad \int \frac{(a+bx^2)^5}{x^6} dx$$

**Optimal.** Leaf size=63

$$-\frac{a^5}{5x^5} - \frac{5a^4b}{3x^3} - \frac{10a^3b^2}{x} + 10a^2b^3x + \frac{5}{3}ab^4x^3 + \frac{b^5x^5}{5}$$

[Out]  $-a^5/(5*x^5) - (5*a^4*b)/(3*x^3) - (10*a^3*b^2)/x + 10*a^2*b^3*x + (5*a*b^4*x^3)/3 + (b^5*x^5)/5$

**Rubi [A]** time = 0.0638267, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^5}{5x^5} - \frac{5a^4b}{3x^3} - \frac{10a^3b^2}{x} + 10a^2b^3x + \frac{5}{3}ab^4x^3 + \frac{b^5x^5}{5}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x^6, x]

[Out]  $-a^5/(5*x^5) - (5*a^4*b)/(3*x^3) - (10*a^3*b^2)/x + 10*a^2*b^3*x + (5*a*b^4*x^3)/3 + (b^5*x^5)/5$

**Rubi in Sympy [A]** time = 11.5522, size = 60, normalized size = 0.95

$$-\frac{a^5}{5x^5} - \frac{5a^4b}{3x^3} - \frac{10a^3b^2}{x} + 10a^2b^3x + \frac{5ab^4x^3}{3} + \frac{b^5x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x\*\*6, x)

[Out]  $-a**5/(5*x**5) - 5*a**4*b/(3*x**3) - 10*a**3*b**2/x + 10*a**2*b**3*x + 5*a*b**4*x**3/3 + b**5*x**5/5$

**Mathematica [A]** time = 0.00767607, size = 63, normalized size = 1.

$$-\frac{a^5}{5x^5} - \frac{5a^4b}{3x^3} - \frac{10a^3b^2}{x} + 10a^2b^3x + \frac{5}{3}ab^4x^3 + \frac{b^5x^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x^6, x]

[Out]  $-a^5/(5*x^5) - (5*a^4*b)/(3*x^3) - (10*a^3*b^2)/x + 10*a^2*b^3*x + (5*a*b^4*x^3)/3 + (b^5*x^5)/5$

**Maple [A]** time = 0.008, size = 56, normalized size = 0.9

$$-\frac{a^5}{5x^5} - \frac{5a^4b}{3x^3} - 10\frac{a^3b^2}{x} + 10a^2b^3x + \frac{5ab^4x^3}{3} + \frac{b^5x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^5/x^6,x)`

[Out]  $-1/5*a^5/x^5-5/3*a^4*b/x^3-10*a^3*b^2/x+10*a^2*b^3*x+5/3*a*b^4*x^3+1/5*b^5*x^5$

**Maxima [A]** time = 1.35502, size = 78, normalized size = 1.24

$$\frac{1}{5}b^5x^5 + \frac{5}{3}ab^4x^3 + 10a^2b^3x - \frac{150a^3b^2x^4 + 25a^4bx^2 + 3a^5}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^6,x, algorithm="maxima")`

[Out]  $1/5*b^5*x^5 + 5/3*a*b^4*x^3 + 10*a^2*b^3*x - 1/15*(150*a^3*b^2*x^4 + 25*a^4*b*x^2 + 3*a^5)/x^5$

**Fricas [A]** time = 0.194738, size = 80, normalized size = 1.27

$$\frac{3b^5x^{10} + 25ab^4x^8 + 150a^2b^3x^6 - 150a^3b^2x^4 - 25a^4bx^2 - 3a^5}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^6,x, algorithm="fricas")`

[Out]  $1/15*(3*b^5*x^{10} + 25*a*b^4*x^8 + 150*a^2*b^3*x^6 - 150*a^3*b^2*x^4 - 25*a^4*b*x^2 - 3*a^5)/x^5$

**Sympy [A]** time = 1.56434, size = 61, normalized size = 0.97

$$10a^2b^3x + \frac{5ab^4x^3}{3} + \frac{b^5x^5}{5} - \frac{3a^5 + 25a^4bx^2 + 150a^3b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x**6,x)`

[Out]  $10*a**2*b**3*x + 5*a*b**4*x**3/3 + b**5*x**5/5 - (3*a**5 + 25*a**4*b*x**2 + 150*a**3*b**2*x**4)/(15*x**5)$

**GIAC/XCAS [A]** time = 0.207293, size = 78, normalized size = 1.24

$$\frac{1}{5}b^5x^5 + \frac{5}{3}ab^4x^3 + 10a^2b^3x - \frac{150a^3b^2x^4 + 25a^4bx^2 + 3a^5}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^6,x, algorithm="giac")`

[Out]  $1/5*b^5*x^5 + 5/3*a*b^4*x^3 + 10*a^2*b^3*x - 1/15*(150*a^3*b^2*x^4 + 25*a^4*b*x^2 + 3*a^5)/x^5$

$$3.78 \quad \int \frac{(a+bx^2)^5}{x^8} dx$$

**Optimal.** Leaf size=61

$$-\frac{a^5}{7x^7} - \frac{a^4b}{x^5} - \frac{10a^3b^2}{3x^3} - \frac{10a^2b^3}{x} + 5ab^4x + \frac{b^5x^3}{3}$$

[Out]  $-a^5/(7*x^7) - (a^4*b)/x^5 - (10*a^3*b^2)/(3*x^3) - (10*a^2*b^3)/x + 5*a*b^4*x + (b^5*x^3)/3$

**Rubi [A]** time = 0.0636641, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^5}{7x^7} - \frac{a^4b}{x^5} - \frac{10a^3b^2}{3x^3} - \frac{10a^2b^3}{x} + 5ab^4x + \frac{b^5x^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x^8, x]

[Out]  $-a^5/(7*x^7) - (a^4*b)/x^5 - (10*a^3*b^2)/(3*x^3) - (10*a^2*b^3)/x + 5*a*b^4*x + (b^5*x^3)/3$

**Rubi in Sympy [A]** time = 11.6714, size = 56, normalized size = 0.92

$$-\frac{a^5}{7x^7} - \frac{a^4b}{x^5} - \frac{10a^3b^2}{3x^3} - \frac{10a^2b^3}{x} + 5ab^4x + \frac{b^5x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x\*\*8, x)

[Out]  $-a**5/(7*x**7) - a**4*b/x**5 - 10*a**3*b**2/(3*x**3) - 10*a**2*b**3/x + 5*a*b**4*x + b**5*x**3/3$

**Mathematica [A]** time = 0.00717114, size = 61, normalized size = 1.

$$-\frac{a^5}{7x^7} - \frac{a^4b}{x^5} - \frac{10a^3b^2}{3x^3} - \frac{10a^2b^3}{x} + 5ab^4x + \frac{b^5x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x^8, x]

[Out]  $-a^5/(7*x^7) - (a^4*b)/x^5 - (10*a^3*b^2)/(3*x^3) - (10*a^2*b^3)/x + 5*a*b^4*x + (b^5*x^3)/3$

**Maple [A]** time = 0.008, size = 56, normalized size = 0.9

$$-\frac{a^5}{7x^7} - \frac{a^4b}{x^5} - \frac{10a^3b^2}{3x^3} - 10\frac{a^2b^3}{x} + 5ab^4x + \frac{b^5x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((b*x^2+a)^5/x^8,x)`

[Out]  $-1/7*a^5/x^7 - a^4*b/x^5 - 10/3*a^3*b^2/x^3 - 10*a^2*b^3/x + 5*a*b^4*x + 1/3*b^5*x^3$

**Maxima [A]** time = 1.35409, size = 78, normalized size = 1.28

$$\frac{1}{3}b^5x^3 + 5ab^4x - \frac{210a^2b^3x^6 + 70a^3b^2x^4 + 21a^4bx^2 + 3a^5}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^8,x, algorithm="maxima")`

[Out]  $1/3*b^5*x^3 + 5*a*b^4*x - 1/21*(210*a^2*b^3*x^6 + 70*a^3*b^2*x^4 + 21*a^4*b*x^2 + 3*a^5)/x^7$

**Fricas [A]** time = 0.194119, size = 80, normalized size = 1.31

$$\frac{7b^5x^{10} + 105ab^4x^8 - 210a^2b^3x^6 - 70a^3b^2x^4 - 21a^4bx^2 - 3a^5}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^8,x, algorithm="fricas")`

[Out]  $1/21*(7*b^5*x^{10} + 105*a*b^4*x^8 - 210*a^2*b^3*x^6 - 70*a^3*b^2*x^4 - 21*a^4*b*x^2 - 3*a^5)/x^7$

**Sympy [A]** time = 1.757, size = 60, normalized size = 0.98

$$5ab^4x + \frac{b^5x^3}{3} - \frac{3a^5 + 21a^4bx^2 + 70a^3b^2x^4 + 210a^2b^3x^6}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x**8,x)`

[Out]  $5*a*b**4*x + b**5*x**3/3 - (3*a**5 + 21*a**4*b*x**2 + 70*a**3*b**2*x**4 + 210*a**2*b**3*x**6)/(21*x**7)$

**GIAC/XCAS [A]** time = 0.205928, size = 78, normalized size = 1.28

$$\frac{1}{3}b^5x^3 + 5ab^4x - \frac{210a^2b^3x^6 + 70a^3b^2x^4 + 21a^4bx^2 + 3a^5}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^8,x, algorithm="giac")`

[Out]  $1/3*b^5*x^3 + 5*a*b^4*x - 1/21*(210*a^2*b^3*x^6 + 70*a^3*b^2*x^4 + 21*a^4*b*x^2 + 3*a^5)/x^7$

$$3.79 \quad \int \frac{(a+bx^2)^5}{x^{10}} dx$$

**Optimal.** Leaf size=60

$$-\frac{a^5}{9x^9} - \frac{5a^4b}{7x^7} - \frac{2a^3b^2}{x^5} - \frac{10a^2b^3}{3x^3} - \frac{5ab^4}{x} + b^5x$$

[Out]  $-a^5/(9*x^9) - (5*a^4*b)/(7*x^7) - (2*a^3*b^2)/x^5 - (10*a^2*b^3)/(3*x^3) - (5*a*b^4)/x + b^5*x$

**Rubi [A]** time = 0.0631125, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^5}{9x^9} - \frac{5a^4b}{7x^7} - \frac{2a^3b^2}{x^5} - \frac{10a^2b^3}{3x^3} - \frac{5ab^4}{x} + b^5x$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x^10, x]

[Out]  $-a^5/(9*x^9) - (5*a^4*b)/(7*x^7) - (2*a^3*b^2)/x^5 - (10*a^2*b^3)/(3*x^3) - (5*a*b^4)/x + b^5*x$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^5}{9x^9} - \frac{5a^4b}{7x^7} - \frac{2a^3b^2}{x^5} - \frac{10a^2b^3}{3x^3} - \frac{5ab^4}{x} + \int b^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x\*\*10, x)

[Out]  $-a**5/(9*x**9) - 5*a**4*b/(7*x**7) - 2*a**3*b**2/x**5 - 10*a**2*b**3/(3*x**3) - 5*a*b**4/x + \text{Integral}(b**5, x)$

**Mathematica [A]** time = 0.00981196, size = 60, normalized size = 1.

$$-\frac{a^5}{9x^9} - \frac{5a^4b}{7x^7} - \frac{2a^3b^2}{x^5} - \frac{10a^2b^3}{3x^3} - \frac{5ab^4}{x} + b^5x$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x^10, x]

[Out]  $-a^5/(9*x^9) - (5*a^4*b)/(7*x^7) - (2*a^3*b^2)/x^5 - (10*a^2*b^3)/(3*x^3) - (5*a*b^4)/x + b^5*x$

**Maple [A]** time = 0.009, size = 55, normalized size = 0.9

$$-\frac{a^5}{9x^9} - \frac{5a^4b}{7x^7} - 2\frac{a^3b^2}{x^5} - \frac{10a^2b^3}{3x^3} - 5\frac{ab^4}{x} + b^5x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^5/x^10, x)

[Out]  $-1/9*a^5/x^9-5/7*a^4*b/x^7-2*a^3*b^2/x^5-10/3*a^2*b^3/x^3-5*a*b^4/x+b^5*x$

**Maxima [A]** time = 1.34601, size = 77, normalized size = 1.28

$$b^5x - \frac{315ab^4x^8 + 210a^2b^3x^6 + 126a^3b^2x^4 + 45a^4bx^2 + 7a^5}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^10,x, algorithm="maxima")`

[Out]  $b^5*x - 1/63*(315*a*b^4*x^8 + 210*a^2*b^3*x^6 + 126*a^3*b^2*x^4 + 45*a^4*b*x^2 + 7*a^5)/x^9$

**Fricas [A]** time = 0.194458, size = 80, normalized size = 1.33

$$\frac{63b^5x^{10} - 315ab^4x^8 - 210a^2b^3x^6 - 126a^3b^2x^4 - 45a^4bx^2 - 7a^5}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^10,x, algorithm="fricas")`

[Out]  $1/63*(63*b^5*x^{10} - 315*a*b^4*x^8 - 210*a^2*b^3*x^6 - 126*a^3*b^2*x^4 - 45*a^4*b*x^2 - 7*a^5)/x^9$

**Sympy [A]** time = 2.04282, size = 58, normalized size = 0.97

$$b^5x - \frac{7a^5 + 45a^4bx^2 + 126a^3b^2x^4 + 210a^2b^3x^6 + 315ab^4x^8}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x**10,x)`

[Out]  $b**5*x - (7*a**5 + 45*a**4*b*x**2 + 126*a**3*b**2*x**4 + 210*a**2*b**3*x**6 + 315*a*b**4*x**8)/(63*x**9)$

**GIAC/XCAS [A]** time = 0.207153, size = 77, normalized size = 1.28

$$b^5x - \frac{315ab^4x^8 + 210a^2b^3x^6 + 126a^3b^2x^4 + 45a^4bx^2 + 7a^5}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^10,x, algorithm="giac")`

[Out]  $b^5*x - 1/63*(315*a*b^4*x^8 + 210*a^2*b^3*x^6 + 126*a^3*b^2*x^4 + 45*a^4*b*x^2 + 7*a^5)/x^9$

$$3.80 \quad \int \frac{(a+bx^2)^5}{x^{12}} dx$$

**Optimal.** Leaf size=65

$$-\frac{a^5}{11x^{11}} - \frac{5a^4b}{9x^9} - \frac{10a^3b^2}{7x^7} - \frac{2a^2b^3}{x^5} - \frac{5ab^4}{3x^3} - \frac{b^5}{x}$$

[Out]  $-a^5/(11*x^{11}) - (5*a^4*b)/(9*x^9) - (10*a^3*b^2)/(7*x^7) - (2*a^2*b^3)/x^5 - (5*a*b^4)/(3*x^3) - b^5/x$

**Rubi [A]** time = 0.0640907, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^5}{11x^{11}} - \frac{5a^4b}{9x^9} - \frac{10a^3b^2}{7x^7} - \frac{2a^2b^3}{x^5} - \frac{5ab^4}{3x^3} - \frac{b^5}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x^12, x]

[Out]  $-a^5/(11*x^{11}) - (5*a^4*b)/(9*x^9) - (10*a^3*b^2)/(7*x^7) - (2*a^2*b^3)/x^5 - (5*a*b^4)/(3*x^3) - b^5/x$

**Rubi in Sympy [A]** time = 12.106, size = 63, normalized size = 0.97

$$-\frac{a^5}{11x^{11}} - \frac{5a^4b}{9x^9} - \frac{10a^3b^2}{7x^7} - \frac{2a^2b^3}{x^5} - \frac{5ab^4}{3x^3} - \frac{b^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x\*\*12, x)

[Out]  $-a**5/(11*x**11) - 5*a**4*b/(9*x**9) - 10*a**3*b**2/(7*x**7) - 2*a**2*b**3/x**5 - 5*a*b**4/(3*x**3) - b**5/x$

**Mathematica [A]** time = 0.00754584, size = 65, normalized size = 1.

$$-\frac{a^5}{11x^{11}} - \frac{5a^4b}{9x^9} - \frac{10a^3b^2}{7x^7} - \frac{2a^2b^3}{x^5} - \frac{5ab^4}{3x^3} - \frac{b^5}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x^12, x]

[Out]  $-a^5/(11*x^{11}) - (5*a^4*b)/(9*x^9) - (10*a^3*b^2)/(7*x^7) - (2*a^2*b^3)/x^5 - (5*a*b^4)/(3*x^3) - b^5/x$

**Maple [A]** time = 0.008, size = 58, normalized size = 0.9

$$-\frac{a^5}{11x^{11}} - \frac{5a^4b}{9x^9} - \frac{10a^3b^2}{7x^7} - 2\frac{a^2b^3}{x^5} - \frac{5ab^4}{3x^3} - \frac{b^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^5/x^12, x)

[Out]  $-1/11*a^5/x^{11}-5/9*a^4*b/x^9-10/7*a^3*b^2/x^7-2*a^2*b^3/x^5-5/3*a*b^4/x^3-b^5/x$

**Maxima [A]** time = 1.34133, size = 80, normalized size = 1.23

$$\frac{693 b^5 x^{10} + 1155 a b^4 x^8 + 1386 a^2 b^3 x^6 + 990 a^3 b^2 x^4 + 385 a^4 b x^2 + 63 a^5}{693 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^12,x, algorithm="maxima")`

[Out]  $-1/693*(693*b^5*x^{10} + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4 + 385*a^4*b*x^2 + 63*a^5)/x^{11}$

**Fricas [A]** time = 0.195432, size = 80, normalized size = 1.23

$$\frac{693 b^5 x^{10} + 1155 a b^4 x^8 + 1386 a^2 b^3 x^6 + 990 a^3 b^2 x^4 + 385 a^4 b x^2 + 63 a^5}{693 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^12,x, algorithm="fricas")`

[Out]  $-1/693*(693*b^5*x^{10} + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4 + 385*a^4*b*x^2 + 63*a^5)/x^{11}$

**Sympy [A]** time = 2.23197, size = 63, normalized size = 0.97

$$\frac{63a^5 + 385a^4bx^2 + 990a^3b^2x^4 + 1386a^2b^3x^6 + 1155ab^4x^8 + 693b^5x^{10}}{693x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x**12,x)`

[Out]  $-(63*a**5 + 385*a**4*b*x**2 + 990*a**3*b**2*x**4 + 1386*a**2*b**3*x**6 + 1155*a*b**4*x**8 + 693*b**5*x**10)/(693*x**11)$

**GIAC/XCAS [A]** time = 0.206826, size = 80, normalized size = 1.23

$$\frac{693 b^5 x^{10} + 1155 a b^4 x^8 + 1386 a^2 b^3 x^6 + 990 a^3 b^2 x^4 + 385 a^4 b x^2 + 63 a^5}{693 x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^12,x, algorithm="giac")`

[Out]  $-1/693*(693*b^5*x^{10} + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4 + 385*a^4*b*x^2 + 63*a^5)/x^{11}$

$$3.81 \quad \int \frac{(a+bx^2)^5}{x^{14}} dx$$

Optimal. Leaf size=67

$$-\frac{a^5}{13x^{13}} - \frac{5a^4b}{11x^{11}} - \frac{10a^3b^2}{9x^9} - \frac{10a^2b^3}{7x^7} - \frac{ab^4}{x^5} - \frac{b^5}{3x^3}$$

[Out]  $-a^5/(13*x^{13}) - (5*a^4*b)/(11*x^{11}) - (10*a^3*b^2)/(9*x^9) - (10*a^2*b^3)/(7*x^7) - (a*b^4)/x^5 - b^5/(3*x^3)$

Rubi [A] time = 0.0647617, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^5}{13x^{13}} - \frac{5a^4b}{11x^{11}} - \frac{10a^3b^2}{9x^9} - \frac{10a^2b^3}{7x^7} - \frac{ab^4}{x^5} - \frac{b^5}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x^14, x]

[Out]  $-a^5/(13*x^{13}) - (5*a^4*b)/(11*x^{11}) - (10*a^3*b^2)/(9*x^9) - (10*a^2*b^3)/(7*x^7) - (a*b^4)/x^5 - b^5/(3*x^3)$

Rubi in Sympy [A] time = 12.133, size = 65, normalized size = 0.97

$$-\frac{a^5}{13x^{13}} - \frac{5a^4b}{11x^{11}} - \frac{10a^3b^2}{9x^9} - \frac{10a^2b^3}{7x^7} - \frac{ab^4}{x^5} - \frac{b^5}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x\*\*14, x)

[Out]  $-a**5/(13*x**13) - 5*a**4*b/(11*x**11) - 10*a**3*b**2/(9*x**9) - 10*a**2*b**3/(7*x**7) - a*b**4/x**5 - b**5/(3*x**3)$

Mathematica [A] time = 0.00749208, size = 67, normalized size = 1.

$$-\frac{a^5}{13x^{13}} - \frac{5a^4b}{11x^{11}} - \frac{10a^3b^2}{9x^9} - \frac{10a^2b^3}{7x^7} - \frac{ab^4}{x^5} - \frac{b^5}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x^14, x]

[Out]  $-a^5/(13*x^{13}) - (5*a^4*b)/(11*x^{11}) - (10*a^3*b^2)/(9*x^9) - (10*a^2*b^3)/(7*x^7) - (a*b^4)/x^5 - b^5/(3*x^3)$

Maple [A] time = 0.008, size = 58, normalized size = 0.9

$$-\frac{a^5}{13x^{13}} - \frac{5a^4b}{11x^{11}} - \frac{10a^3b^2}{9x^9} - \frac{10a^2b^3}{7x^7} - \frac{ab^4}{x^5} - \frac{b^5}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^5/x^14,x)`

[Out]  $-1/13*a^5/x^{13}-5/11*a^4*b/x^{11}-10/9*a^3*b^2/x^9-10/7*a^2*b^3/x^7-a*b^4/x^5-1/3*b^5/x^3$

**Maxima [A]** time = 1.34262, size = 80, normalized size = 1.19

$$\frac{3003 b^5 x^{10} + 9009 a b^4 x^8 + 12870 a^2 b^3 x^6 + 10010 a^3 b^2 x^4 + 4095 a^4 b x^2 + 693 a^5}{9009 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^14,x, algorithm="maxima")`

[Out]  $-1/9009*(3003*b^5*x^{10} + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^2*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^{13}$

**Fricas [A]** time = 0.194568, size = 80, normalized size = 1.19

$$\frac{3003 b^5 x^{10} + 9009 a b^4 x^8 + 12870 a^2 b^3 x^6 + 10010 a^3 b^2 x^4 + 4095 a^4 b x^2 + 693 a^5}{9009 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^14,x, algorithm="fricas")`

[Out]  $-1/9009*(3003*b^5*x^{10} + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^2*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^{13}$

**Sympy [A]** time = 2.36034, size = 63, normalized size = 0.94

$$\frac{693 a^5 + 4095 a^4 b x^2 + 10010 a^3 b^2 x^4 + 12870 a^2 b^3 x^6 + 9009 a b^4 x^8 + 3003 b^5 x^{10}}{9009 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x**14,x)`

[Out]  $-(693*a**5 + 4095*a**4*b*x**2 + 10010*a**3*b**2*x**4 + 12870*a**2*b**3*x**6 + 9009*a*b**4*x**8 + 3003*b**5*x**10)/(9009*x**13)$

**GIAC/XCAS [A]** time = 0.207883, size = 80, normalized size = 1.19

$$\frac{3003 b^5 x^{10} + 9009 a b^4 x^8 + 12870 a^2 b^3 x^6 + 10010 a^3 b^2 x^4 + 4095 a^4 b x^2 + 693 a^5}{9009 x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^14,x, algorithm="giac")`

[Out]  $-1/9009*(3003*b^5*x^{10} + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^2*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^{13}$

$$3.82 \quad \int \frac{(a+bx^2)^5}{x^{16}} dx$$

**Optimal.** Leaf size=69

$$-\frac{a^5}{15x^{15}} - \frac{5a^4b}{13x^{13}} - \frac{10a^3b^2}{11x^{11}} - \frac{10a^2b^3}{9x^9} - \frac{5ab^4}{7x^7} - \frac{b^5}{5x^5}$$

[Out]  $-a^5/(15*x^{15}) - (5*a^4*b)/(13*x^{13}) - (10*a^3*b^2)/(11*x^{11}) - (10*a^2*b^3)/(9*x^9) - (5*a*b^4)/(7*x^7) - b^5/(5*x^5)$

**Rubi [A]** time = 0.0656212, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^5}{15x^{15}} - \frac{5a^4b}{13x^{13}} - \frac{10a^3b^2}{11x^{11}} - \frac{10a^2b^3}{9x^9} - \frac{5ab^4}{7x^7} - \frac{b^5}{5x^5}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x^16, x]

[Out]  $-a^5/(15*x^{15}) - (5*a^4*b)/(13*x^{13}) - (10*a^3*b^2)/(11*x^{11}) - (10*a^2*b^3)/(9*x^9) - (5*a*b^4)/(7*x^7) - b^5/(5*x^5)$

**Rubi in Sympy [A]** time = 12.4414, size = 68, normalized size = 0.99

$$-\frac{a^5}{15x^{15}} - \frac{5a^4b}{13x^{13}} - \frac{10a^3b^2}{11x^{11}} - \frac{10a^2b^3}{9x^9} - \frac{5ab^4}{7x^7} - \frac{b^5}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x\*\*16, x)

[Out]  $-a**5/(15*x**15) - 5*a**4*b/(13*x**13) - 10*a**3*b**2/(11*x**11) - 10*a**2*b**3/(9*x**9) - 5*a*b**4/(7*x**7) - b**5/(5*x**5)$

**Mathematica [A]** time = 0.00807989, size = 69, normalized size = 1.

$$-\frac{a^5}{15x^{15}} - \frac{5a^4b}{13x^{13}} - \frac{10a^3b^2}{11x^{11}} - \frac{10a^2b^3}{9x^9} - \frac{5ab^4}{7x^7} - \frac{b^5}{5x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x^16, x]

[Out]  $-a^5/(15*x^{15}) - (5*a^4*b)/(13*x^{13}) - (10*a^3*b^2)/(11*x^{11}) - (10*a^2*b^3)/(9*x^9) - (5*a*b^4)/(7*x^7) - b^5/(5*x^5)$

**Maple [A]** time = 0.008, size = 58, normalized size = 0.8

$$-\frac{a^5}{15x^{15}} - \frac{5a^4b}{13x^{13}} - \frac{10a^3b^2}{11x^{11}} - \frac{10a^2b^3}{9x^9} - \frac{5ab^4}{7x^7} - \frac{b^5}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((b*x^2+a)^5/x^16,x)`

[Out]  $-1/15*a^5/x^{15}-5/13*a^4*b/x^{13}-10/11*a^3*b^2/x^{11}-10/9*a^2*b^3/x^9-5/7*a*b^4/x^7-1/5*b^5/x^5$

**Maxima [A]** time = 1.34527, size = 80, normalized size = 1.16

$$\frac{9009b^5x^{10} + 32175ab^4x^8 + 50050a^2b^3x^6 + 40950a^3b^2x^4 + 17325a^4bx^2 + 3003a^5}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^16,x, algorithm="maxima")`

[Out]  $-1/45045*(9009*b^5*x^{10} + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*b^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^{15}$

**Fricas [A]** time = 0.192023, size = 80, normalized size = 1.16

$$\frac{9009b^5x^{10} + 32175ab^4x^8 + 50050a^2b^3x^6 + 40950a^3b^2x^4 + 17325a^4bx^2 + 3003a^5}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^16,x, algorithm="fricas")`

[Out]  $-1/45045*(9009*b^5*x^{10} + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*b^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^{15}$

**Sympy [A]** time = 2.51209, size = 63, normalized size = 0.91

$$\frac{3003a^5 + 17325a^4bx^2 + 40950a^3b^2x^4 + 50050a^2b^3x^6 + 32175ab^4x^8 + 9009b^5x^{10}}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x**16,x)`

[Out]  $-(3003*a^{**5} + 17325*a^{**4}*b*x^{**2} + 40950*a^{**3}*b^{**2}*x^{**4} + 50050*a^{**2}*b^{**3}*x^{**6} + 32175*a*b^{**4}*x^{**8} + 9009*b^{**5}*x^{**10})/(45045*x^{**15})$

**GIAC/XCAS [A]** time = 0.20607, size = 80, normalized size = 1.16

$$\frac{9009b^5x^{10} + 32175ab^4x^8 + 50050a^2b^3x^6 + 40950a^3b^2x^4 + 17325a^4bx^2 + 3003a^5}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^16,x, algorithm="giac")`

[Out]  $-1/45045*(9009*b^5*x^{10} + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*b^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^{15}$

$$3.83 \quad \int \frac{(a+bx^2)^5}{x^{18}} dx$$

**Optimal.** Leaf size=69

$$-\frac{a^5}{17x^{17}} - \frac{a^4b}{3x^{15}} - \frac{10a^3b^2}{13x^{13}} - \frac{10a^2b^3}{11x^{11}} - \frac{5ab^4}{9x^9} - \frac{b^5}{7x^7}$$

[Out]  $-a^5/(17*x^{17}) - (a^4*b)/(3*x^{15}) - (10*a^3*b^2)/(13*x^{13}) - (10*a^2*b^3)/(11*x^{11}) - (5*a*b^4)/(9*x^9) - b^5/(7*x^7)$

**Rubi [A]** time = 0.065953, antiderivative size = 69, normalized size of antiderivative = 1., number of rules used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^5}{17x^{17}} - \frac{a^4b}{3x^{15}} - \frac{10a^3b^2}{13x^{13}} - \frac{10a^2b^3}{11x^{11}} - \frac{5ab^4}{9x^9} - \frac{b^5}{7x^7}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x^18, x]

[Out]  $-a^5/(17*x^{17}) - (a^4*b)/(3*x^{15}) - (10*a^3*b^2)/(13*x^{13}) - (10*a^2*b^3)/(11*x^{11}) - (5*a*b^4)/(9*x^9) - b^5/(7*x^7)$

**Rubi in Sympy [A]** time = 12.6108, size = 66, normalized size = 0.96

$$-\frac{a^5}{17x^{17}} - \frac{a^4b}{3x^{15}} - \frac{10a^3b^2}{13x^{13}} - \frac{10a^2b^3}{11x^{11}} - \frac{5ab^4}{9x^9} - \frac{b^5}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x\*\*18, x)

[Out]  $-a^{**5}/(17*x^{**17}) - a^{**4}*b/(3*x^{**15}) - 10*a^{**3}*b^{**2}/(13*x^{**13}) - 10*a^{**2}*b^{**3}/(11*x^{**11}) - 5*a*b^{**4}/(9*x^{**9}) - b^{**5}/(7*x^{**7})$

**Mathematica [A]** time = 0.00769079, size = 69, normalized size = 1.

$$-\frac{a^5}{17x^{17}} - \frac{a^4b}{3x^{15}} - \frac{10a^3b^2}{13x^{13}} - \frac{10a^2b^3}{11x^{11}} - \frac{5ab^4}{9x^9} - \frac{b^5}{7x^7}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x^18, x]

[Out]  $-a^5/(17*x^{17}) - (a^4*b)/(3*x^{15}) - (10*a^3*b^2)/(13*x^{13}) - (10*a^2*b^3)/(11*x^{11}) - (5*a*b^4)/(9*x^9) - b^5/(7*x^7)$

**Maple [A]** time = 0.008, size = 58, normalized size = 0.8

$$-\frac{a^5}{17x^{17}} - \frac{a^4b}{3x^{15}} - \frac{10a^3b^2}{13x^{13}} - \frac{10a^2b^3}{11x^{11}} - \frac{5ab^4}{9x^9} - \frac{b^5}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^5/x^18,x)`

[Out]  $-1/17*a^5/x^{17}-1/3*a^4*b/x^{15}-10/13*a^3*b^2/x^{13}-10/11*a^2*b^3/x^{11}-5/9*a*b^4/x^9-1/7*b^5/x^7$

**Maxima [A]** time = 1.34645, size = 80, normalized size = 1.16

$$\frac{21879b^5x^{10} + 85085ab^4x^8 + 139230a^2b^3x^6 + 117810a^3b^2x^4 + 51051a^4bx^2 + 9009a^5}{153153x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^18,x, algorithm="maxima")`

[Out]  $-1/153153*(21879*b^5*x^{10} + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*a^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^{17}$

**Fricas [A]** time = 0.193724, size = 80, normalized size = 1.16

$$\frac{21879b^5x^{10} + 85085ab^4x^8 + 139230a^2b^3x^6 + 117810a^3b^2x^4 + 51051a^4bx^2 + 9009a^5}{153153x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^18,x, algorithm="fricas")`

[Out]  $-1/153153*(21879*b^5*x^{10} + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*a^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^{17}$

**Sympy [A]** time = 2.63696, size = 63, normalized size = 0.91

$$\frac{9009a^5 + 51051a^4bx^2 + 117810a^3b^2x^4 + 139230a^2b^3x^6 + 85085ab^4x^8 + 21879b^5x^{10}}{153153x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x**18,x)`

[Out]  $-(9009*a^{**5} + 51051*a^{**4}*b*x^{**2} + 117810*a^{**3}*b^{**2}*x^{**4} + 139230*a^{**2}*b^{**3}*x^{**6} + 85085*a*b^{**4}*x^{**8} + 21879*b^{**5}*x^{**10})/(153153*x^{**17})$

**GIAC/XCAS [A]** time = 0.211336, size = 80, normalized size = 1.16

$$\frac{21879b^5x^{10} + 85085ab^4x^8 + 139230a^2b^3x^6 + 117810a^3b^2x^4 + 51051a^4bx^2 + 9009a^5}{153153x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^18,x, algorithm="giac")`

[Out]  $-1/153153*(21879*b^5*x^{10} + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*a^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^{17}$

$$3.84 \quad \int \frac{(a+bx^2)^5}{x^{20}} dx$$

**Optimal.** Leaf size=69

$$-\frac{a^5}{19x^{19}} - \frac{5a^4b}{17x^{17}} - \frac{2a^3b^2}{3x^{15}} - \frac{10a^2b^3}{13x^{13}} - \frac{5ab^4}{11x^{11}} - \frac{b^5}{9x^9}$$

[Out]  $-a^5/(19*x^{19}) - (5*a^4*b)/(17*x^{17}) - (2*a^3*b^2)/(3*x^{15}) - (10*a^2*b^3)/(13*x^{13}) - (5*a*b^4)/(11*x^{11}) - b^5/(9*x^9)$

**Rubi [A]** time = 0.0669264, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^5}{19x^{19}} - \frac{5a^4b}{17x^{17}} - \frac{2a^3b^2}{3x^{15}} - \frac{10a^2b^3}{13x^{13}} - \frac{5ab^4}{11x^{11}} - \frac{b^5}{9x^9}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^5/x^20, x]

[Out]  $-a^5/(19*x^{19}) - (5*a^4*b)/(17*x^{17}) - (2*a^3*b^2)/(3*x^{15}) - (10*a^2*b^3)/(13*x^{13}) - (5*a*b^4)/(11*x^{11}) - b^5/(9*x^9)$

**Rubi in Sympy [A]** time = 12.6117, size = 68, normalized size = 0.99

$$-\frac{a^5}{19x^{19}} - \frac{5a^4b}{17x^{17}} - \frac{2a^3b^2}{3x^{15}} - \frac{10a^2b^3}{13x^{13}} - \frac{5ab^4}{11x^{11}} - \frac{b^5}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*5/x\*\*20, x)

[Out]  $-a**5/(19*x**19) - 5*a**4*b/(17*x**17) - 2*a**3*b**2/(3*x**15) - 10*a**2*b**3/(13*x**13) - 5*a*b**4/(11*x**11) - b**5/(9*x**9)$

**Mathematica [A]** time = 0.0112039, size = 69, normalized size = 1.

$$-\frac{a^5}{19x^{19}} - \frac{5a^4b}{17x^{17}} - \frac{2a^3b^2}{3x^{15}} - \frac{10a^2b^3}{13x^{13}} - \frac{5ab^4}{11x^{11}} - \frac{b^5}{9x^9}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^5/x^20, x]

[Out]  $-a^5/(19*x^{19}) - (5*a^4*b)/(17*x^{17}) - (2*a^3*b^2)/(3*x^{15}) - (10*a^2*b^3)/(13*x^{13}) - (5*a*b^4)/(11*x^{11}) - b^5/(9*x^9)$

**Maple [A]** time = 0.008, size = 58, normalized size = 0.8

$$-\frac{a^5}{19x^{19}} - \frac{5a^4b}{17x^{17}} - \frac{2a^3b^2}{3x^{15}} - \frac{10a^2b^3}{13x^{13}} - \frac{5ab^4}{11x^{11}} - \frac{b^5}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^5/x^20,x)`

[Out]  $-1/19*a^5/x^{19}-5/17*a^4*b/x^{17}-2/3*a^3*b^2/x^{15}-10/13*a^2*b^3/x^{13}-5/11*a*b^4/x^{11}-1/9*b^5/x^9$

**Maxima [A]** time = 1.34837, size = 80, normalized size = 1.16

$$\frac{46189b^5x^{10} + 188955ab^4x^8 + 319770a^2b^3x^6 + 277134a^3b^2x^4 + 122265a^4bx^2 + 21879a^5}{415701x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^20,x, algorithm="maxima")`

[Out]  $-1/415701*(46189*b^5*x^{10} + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134*a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^{19}$

**Fricas [A]** time = 0.191566, size = 80, normalized size = 1.16

$$\frac{46189b^5x^{10} + 188955ab^4x^8 + 319770a^2b^3x^6 + 277134a^3b^2x^4 + 122265a^4bx^2 + 21879a^5}{415701x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^20,x, algorithm="fricas")`

[Out]  $-1/415701*(46189*b^5*x^{10} + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134*a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^{19}$

**Sympy [A]** time = 2.80634, size = 63, normalized size = 0.91

$$\frac{21879a^5 + 122265a^4bx^2 + 277134a^3b^2x^4 + 319770a^2b^3x^6 + 188955ab^4x^8 + 46189b^5x^{10}}{415701x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**5/x**20,x)`

[Out]  $-(21879*a**5 + 122265*a**4*b*x**2 + 277134*a**3*b**2*x**4 + 319770*a**2*b**3*x**6 + 188955*a*b**4*x**8 + 46189*b**5*x**10)/(415701*x**19)$

**GIAC/XCAS [A]** time = 0.207413, size = 80, normalized size = 1.16

$$\frac{46189b^5x^{10} + 188955ab^4x^8 + 319770a^2b^3x^6 + 277134a^3b^2x^4 + 122265a^4bx^2 + 21879a^5}{415701x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5/x^20,x, algorithm="giac")`

[Out]  $-1/415701*(46189*b^5*x^{10} + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134*a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^{19}$

### 3.85 $\int x^{13} (a + bx^2)^8 dx$

**Optimal.** Leaf size=129

$$\frac{a^6 (a + bx^2)^9}{18b^7} - \frac{3a^5 (a + bx^2)^{10}}{10b^7} + \frac{15a^4 (a + bx^2)^{11}}{22b^7} - \frac{5a^3 (a + bx^2)^{12}}{6b^7} \\ + \frac{15a^2 (a + bx^2)^{13}}{26b^7} + \frac{(a + bx^2)^{15}}{30b^7} - \frac{3a (a + bx^2)^{14}}{14b^7}$$

[Out]  $(a^6 (a + b^2 x^2)^9)/(18 b^7) - (3 a^5 (a + b^2 x^2)^{10})/(10 b^7) + (15 a^4 (a + b^2 x^2)^{11})/(22 b^7) - (5 a^3 (a + b^2 x^2)^{12})/(6 b^7) \\ + (15 a^2 (a + b^2 x^2)^{13})/(26 b^7) - (3 a (a + b^2 x^2)^{14})/(14 b^7) + (a + b^2 x^2)^{15}/(30 b^7)$

**Rubi [A]** time = 0.469701, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^6 (a + bx^2)^9}{18b^7} - \frac{3a^5 (a + bx^2)^{10}}{10b^7} + \frac{15a^4 (a + bx^2)^{11}}{22b^7} - \frac{5a^3 (a + bx^2)^{12}}{6b^7} \\ + \frac{15a^2 (a + bx^2)^{13}}{26b^7} + \frac{(a + bx^2)^{15}}{30b^7} - \frac{3a (a + bx^2)^{14}}{14b^7}$$

Antiderivative was successfully verified.

[In] Int[x^13\*(a + b\*x^2)^8,x]

[Out]  $(a^6 (a + b^2 x^2)^9)/(18 b^7) - (3 a^5 (a + b^2 x^2)^{10})/(10 b^7) + (15 a^4 (a + b^2 x^2)^{11})/(22 b^7) - (5 a^3 (a + b^2 x^2)^{12})/(6 b^7) \\ + (15 a^2 (a + b^2 x^2)^{13})/(26 b^7) - (3 a (a + b^2 x^2)^{14})/(14 b^7) + (a + b^2 x^2)^{15}/(30 b^7)$

**Rubi in Sympy [A]** time = 26.6533, size = 105, normalized size = 0.81

$$\frac{a^8 x^{14}}{14} + \frac{a^7 b x^{16}}{2} + \frac{14 a^6 b^2 x^{18}}{9} + \frac{14 a^5 b^3 x^{20}}{5} + \frac{35 a^4 b^4 x^{22}}{11} + \frac{7 a^3 b^5 x^{24}}{3} + \frac{14 a^2 b^6 x^{26}}{13} + \frac{2 a b^7 x^{28}}{7} + \frac{b^8 x^{30}}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*13\*(b\*x\*\*2+a)\*\*8,x)

[Out]  $a^8 x^{14}/14 + a^7 b x^{16}/2 + 14 a^6 b^2 x^{18}/9 + 14 a^5 b^3 x^{20}/5 + 35 a^4 b^4 x^{22}/11 + 7 a^3 b^5 x^{24}/3 + 14 a^2 b^6 x^{26}/13 \\ + 2 a b^7 x^{28}/7 + b^8 x^{30}/30$

**Mathematica [A]** time = 0.00495942, size = 108, normalized size = 0.84

$$\frac{a^8 x^{14}}{14} + \frac{1}{2} a^7 b x^{16} + \frac{14}{9} a^6 b^2 x^{18} + \frac{14}{5} a^5 b^3 x^{20} + \frac{35}{11} a^4 b^4 x^{22} + \frac{7}{3} a^3 b^5 x^{24} + \frac{14}{13} a^2 b^6 x^{26} + \frac{2}{7} a b^7 x^{28} + \frac{b^8 x^{30}}{30}$$

Antiderivative was successfully verified.

[In] Integrate[x^13\*(a + b\*x^2)^8,x]

[Out]  $(a^8 x^{14})/14 + (a^7 b x^{16})/2 + (14 a^6 b^2 x^{18})/9 + (14 a^5 b^3 x^{20})/5 + (35 a^4 b^4 x^{22})/11 + (7 a^3 b^5 x^{24})/3 + (14 a^2 b^6 x^{26})/13 \\ + (2 a b^7 x^{28})/7 + (b^8 x^{30})/30$

**Maple [A]** time = 0.002, size = 91, normalized size = 0.7

$$\frac{b^8 x^{30}}{30} + \frac{2ab^7 x^{28}}{7} + \frac{14a^2 b^6 x^{26}}{13} + \frac{7a^3 b^5 x^{24}}{3} + \frac{35a^4 b^4 x^{22}}{11} + \frac{14a^5 b^3 x^{20}}{5} + \frac{14a^6 b^2 x^{18}}{9} + \frac{a^7 b x^{16}}{2} + \frac{a^8 x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^13\*(b\*x^2+a)^8,x)

[Out] 1/30\*b^8\*x^30+2/7\*a\*b^7\*x^28+14/13\*a^2\*b^6\*x^26+7/3\*a^3\*b^5\*x^24+35/11\*a^4\*b^4\*x^22+14/5\*a^5\*b^3\*x^20+14/9\*a^6\*b^2\*x^18+1/2\*a^7\*b\*x^16+1/14\*a^8\*x^14

**Maxima [A]** time = 1.3442, size = 122, normalized size = 0.95

$$\frac{1}{30} b^8 x^{30} + \frac{2}{7} ab^7 x^{28} + \frac{14}{13} a^2 b^6 x^{26} + \frac{7}{3} a^3 b^5 x^{24} + \frac{35}{11} a^4 b^4 x^{22} + \frac{14}{5} a^5 b^3 x^{20} + \frac{14}{9} a^6 b^2 x^{18} + \frac{1}{2} a^7 b x^{16} + \frac{1}{14} a^8 x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^8\*x^13,x, algorithm="maxima")

[Out] 1/30\*b^8\*x^30 + 2/7\*a\*b^7\*x^28 + 14/13\*a^2\*b^6\*x^26 + 7/3\*a^3\*b^5\*x^24 + 35/11\*a^4\*b^4\*x^22 + 14/5\*a^5\*b^3\*x^20 + 14/9\*a^6\*b^2\*x^18 + 1/2\*a^7\*b\*x^16 + 1/14\*a^8\*x^14

**Fricas [A]** time = 0.177387, size = 1, normalized size = 0.01

$$\frac{1}{30} x^{30} b^8 + \frac{2}{7} x^{28} b^7 a + \frac{14}{13} x^{26} b^6 a^2 + \frac{7}{3} x^{24} b^5 a^3 + \frac{35}{11} x^{22} b^4 a^4 + \frac{14}{5} x^{20} b^3 a^5 + \frac{14}{9} x^{18} b^2 a^6 + \frac{1}{2} x^{16} b a^7 + \frac{1}{14} x^{14} a^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^8\*x^13,x, algorithm="fricas")

[Out] 1/30\*x^30\*b^8 + 2/7\*x^28\*b^7\*a + 14/13\*x^26\*b^6\*a^2 + 7/3\*x^24\*b^5\*a^3 + 35/11\*x^22\*b^4\*a^4 + 14/5\*x^20\*b^3\*a^5 + 14/9\*x^18\*b^2\*a^6 + 1/2\*x^16\*b\*a^7 + 1/14\*x^14\*a^8

**Sympy [A]** time = 0.164373, size = 105, normalized size = 0.81

$$\frac{a^8 x^{14}}{14} + \frac{a^7 b x^{16}}{2} + \frac{14 a^6 b^2 x^{18}}{9} + \frac{14 a^5 b^3 x^{20}}{5} + \frac{35 a^4 b^4 x^{22}}{11} + \frac{7 a^3 b^5 x^{24}}{3} + \frac{14 a^2 b^6 x^{26}}{13} + \frac{2 a b^7 x^{28}}{7} + \frac{b^8 x^{30}}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*13\*(b\*x\*\*2+a)\*\*8,x)

[Out] a\*\*8\*x\*\*14/14 + a\*\*7\*b\*x\*\*16/2 + 14\*a\*\*6\*b\*\*2\*x\*\*18/9 + 14\*a\*\*5\*b\*\*3\*x\*\*20/5 + 35\*a\*\*4\*b\*\*4\*x\*\*22/11 + 7\*a\*\*3\*b\*\*5\*x\*\*24/3 + 14\*a\*\*2\*b\*\*6\*x\*\*26/13 + 2\*a\*b\*\*7\*x\*\*28/7 + b\*\*8\*x\*\*30/30

**GIAC/XCAS [A]** time = 0.210322, size = 122, normalized size = 0.95

$$\frac{1}{30} b^8 x^{30} + \frac{2}{7} ab^7 x^{28} + \frac{14}{13} a^2 b^6 x^{26} + \frac{7}{3} a^3 b^5 x^{24} + \frac{35}{11} a^4 b^4 x^{22} + \frac{14}{5} a^5 b^3 x^{20} + \frac{14}{9} a^6 b^2 x^{18} + \frac{1}{2} a^7 b x^{16} + \frac{1}{14} a^8 x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^8*x^13,x, algorithm="giac")
```

```
[Out] 1/30*b^8*x^30 + 2/7*a*b^7*x^28 + 14/13*a^2*b^6*x^26 + 7/3*a^3*b^5*x^24 + 35/11*a^4*b^4*x^22 + 14/5*a^5*b^3*x^20 + 14/9*a^6*b^2*x^18 + 1/2*a^7*b*x^16 + 1/14*a^8*x^14
```



### 3.86 $\int x^{11} (a + bx^2)^8 dx$

**Optimal.** Leaf size=110

$$-\frac{a^5 (a + bx^2)^9}{18b^6} + \frac{a^4 (a + bx^2)^{10}}{4b^6} - \frac{5a^3 (a + bx^2)^{11}}{11b^6} + \frac{5a^2 (a + bx^2)^{12}}{12b^6} + \frac{(a + bx^2)^{14}}{28b^6} - \frac{5a (a + bx^2)^{13}}{26b^6}$$

[Out]  $-(a^5*(a + b*x^2)^9)/(18*b^6) + (a^4*(a + b*x^2)^{10})/(4*b^6) - (5*a^3*(a + b*x^2)^{11})/(11*b^6) + (5*a^2*(a + b*x^2)^{12})/(12*b^6) - (5*a*(a + b*x^2)^{13})/(26*b^6) + (a + b*x^2)^{14}/(28*b^6)$

**Rubi [A]** time = 0.391636, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^5 (a + bx^2)^9}{18b^6} + \frac{a^4 (a + bx^2)^{10}}{4b^6} - \frac{5a^3 (a + bx^2)^{11}}{11b^6} + \frac{5a^2 (a + bx^2)^{12}}{12b^6} + \frac{(a + bx^2)^{14}}{28b^6} - \frac{5a (a + bx^2)^{13}}{26b^6}$$

Antiderivative was successfully verified.

[In] Int[x^11\*(a + b\*x^2)^8,x]

[Out]  $-(a^5*(a + b*x^2)^9)/(18*b^6) + (a^4*(a + b*x^2)^{10})/(4*b^6) - (5*a^3*(a + b*x^2)^{11})/(11*b^6) + (5*a^2*(a + b*x^2)^{12})/(12*b^6) - (5*a*(a + b*x^2)^{13})/(26*b^6) + (a + b*x^2)^{14}/(28*b^6)$

**Rubi in Sympy [A]** time = 29.7974, size = 100, normalized size = 0.91

$$-\frac{a^5 (a + bx^2)^9}{18b^6} + \frac{a^4 (a + bx^2)^{10}}{4b^6} - \frac{5a^3 (a + bx^2)^{11}}{11b^6} + \frac{5a^2 (a + bx^2)^{12}}{12b^6} - \frac{5a (a + bx^2)^{13}}{26b^6} + \frac{(a + bx^2)^{14}}{28b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*11\*(b\*x\*\*2+a)\*\*8,x)

[Out]  $-a**5*(a + b*x**2)**9/(18*b**6) + a**4*(a + b*x**2)**10/(4*b**6) - 5*a**3*(a + b*x**2)**11/(11*b**6) + 5*a**2*(a + b*x**2)**12/(12*b**6) - 5*a*(a + b*x**2)**13/(26*b**6) + (a + b*x**2)**14/(28*b**6)$

**Mathematica [A]** time = 0.00470023, size = 108, normalized size = 0.98

$$\frac{a^8 x^{12}}{12} + \frac{4}{7} a^7 b x^{14} + \frac{7}{4} a^6 b^2 x^{16} + \frac{28}{9} a^5 b^3 x^{18} + \frac{7}{2} a^4 b^4 x^{20} + \frac{28}{11} a^3 b^5 x^{22} + \frac{7}{6} a^2 b^6 x^{24} + \frac{4}{13} a b^7 x^{26} + \frac{b^8 x^{28}}{28}$$

Antiderivative was successfully verified.

[In] Integrate[x^11\*(a + b\*x^2)^8,x]

[Out]  $(a^8*x^{12})/12 + (4*a^7*b*x^{14})/7 + (7*a^6*b^2*x^{16})/4 + (28*a^5*b^3*x^{18})/9 + (7*a^4*b^4*x^{20})/2 + (28*a^3*b^5*x^{22})/11 + (7*a^2*b^6*x^{24})/6 + (4*a*b^7*x^{26})/13 + (b^8*x^{28})/28$

**Maple [A]** time = 0.002, size = 91, normalized size = 0.8

$$\frac{b^8 x^{28}}{28} + \frac{4 a b^7 x^{26}}{13} + \frac{7 a^2 b^6 x^{24}}{6} + \frac{28 a^3 b^5 x^{22}}{11} + \frac{7 a^4 b^4 x^{20}}{2} + \frac{28 a^5 b^3 x^{18}}{9} + \frac{7 a^6 b^2 x^{16}}{4} + \frac{4 a^7 b x^{14}}{7} + \frac{a^8 x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11*(b*x^2+a)^8,x)`

[Out]  $\frac{1}{28}b^8x^{28} + \frac{4}{13}ab^7x^{26} + \frac{7}{6}a^2b^6x^{24} + \frac{28}{11}a^3b^5x^{22} + \frac{7}{2}a^4b^4x^{20} + \frac{28}{9}a^5b^3x^{18} + \frac{7}{4}a^6b^2x^{16} + \frac{4}{7}a^7bx^{14} + \frac{1}{12}a^8x^{12}$

**Maxima [A]** time = 1.35057, size = 122, normalized size = 1.11

$$\frac{1}{28}b^8x^{28} + \frac{4}{13}ab^7x^{26} + \frac{7}{6}a^2b^6x^{24} + \frac{28}{11}a^3b^5x^{22} + \frac{7}{2}a^4b^4x^{20} + \frac{28}{9}a^5b^3x^{18} + \frac{7}{4}a^6b^2x^{16} + \frac{4}{7}a^7bx^{14} + \frac{1}{12}a^8x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^11,x, algorithm="maxima")`

[Out]  $\frac{1}{28}b^8x^{28} + \frac{4}{13}ab^7x^{26} + \frac{7}{6}a^2b^6x^{24} + \frac{28}{11}a^3b^5x^{22} + \frac{7}{2}a^4b^4x^{20} + \frac{28}{9}a^5b^3x^{18} + \frac{7}{4}a^6b^2x^{16} + \frac{4}{7}a^7bx^{14} + \frac{1}{12}a^8x^{12}$

**Fricas [A]** time = 0.175537, size = 1, normalized size = 0.01

$$\frac{1}{28}x^{28}b^8 + \frac{4}{13}x^{26}b^7a + \frac{7}{6}x^{24}b^6a^2 + \frac{28}{11}x^{22}b^5a^3 + \frac{7}{2}x^{20}b^4a^4 + \frac{28}{9}x^{18}b^3a^5 + \frac{7}{4}x^{16}b^2a^6 + \frac{4}{7}x^{14}ba^7 + \frac{1}{12}x^{12}a^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^11,x, algorithm="fricas")`

[Out]  $\frac{1}{28}x^{28}b^8 + \frac{4}{13}x^{26}b^7a + \frac{7}{6}x^{24}b^6a^2 + \frac{28}{11}x^{22}b^5a^3 + \frac{7}{2}x^{20}b^4a^4 + \frac{28}{9}x^{18}b^3a^5 + \frac{7}{4}x^{16}b^2a^6 + \frac{4}{7}x^{14}ba^7 + \frac{1}{12}x^{12}a^8$

**Sympy [A]** time = 0.16385, size = 107, normalized size = 0.97

$$\frac{a^8x^{12}}{12} + \frac{4a^7bx^{14}}{7} + \frac{7a^6b^2x^{16}}{4} + \frac{28a^5b^3x^{18}}{9} + \frac{7a^4b^4x^{20}}{2} + \frac{28a^3b^5x^{22}}{11} + \frac{7a^2b^6x^{24}}{6} + \frac{4ab^7x^{26}}{13} + \frac{b^8x^{28}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**11*(b*x**2+a)**8,x)`

[Out]  $a^{**8}x^{**12}/12 + 4*a^{**7}b*x^{**14}/7 + 7*a^{**6}b^{**2}x^{**16}/4 + 28*a^{**5}b^{**3}x^{**18}/9 + 7*a^{**4}b^{**4}x^{**20}/2 + 28*a^{**3}b^{**5}x^{**22}/11 + 7*a^{**2}b^{**6}x^{**24}/6 + 4*a*b^{**7}x^{**26}/13 + b^{**8}x^{**28}/28$

**GIAC/XCAS [A]** time = 0.209367, size = 122, normalized size = 1.11

$$\frac{1}{28}b^8x^{28} + \frac{4}{13}ab^7x^{26} + \frac{7}{6}a^2b^6x^{24} + \frac{28}{11}a^3b^5x^{22} + \frac{7}{2}a^4b^4x^{20} + \frac{28}{9}a^5b^3x^{18} + \frac{7}{4}a^6b^2x^{16} + \frac{4}{7}a^7bx^{14} + \frac{1}{12}a^8x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^11,x, algorithm="giac")`

```
[Out] 1/28*b^8*x^28 + 4/13*a*b^7*x^26 + 7/6*a^2*b^6*x^24 + 28/11*a^3*b^5*x^22 + 7/2*a^4*b^4*x^20 + 28/9*a^5*b^3*x^18 + 7/4*a^6*b^2*x^16 + 4/7*a^7*b*x^14 + 1/12*a^8*x^12
```

### 3.87 $\int x^9 (a + bx^2)^8 dx$

**Optimal.** Leaf size=91

$$\frac{a^4 (a + bx^2)^9}{18b^5} - \frac{a^3 (a + bx^2)^{10}}{5b^5} + \frac{3a^2 (a + bx^2)^{11}}{11b^5} + \frac{(a + bx^2)^{13}}{26b^5} - \frac{a (a + bx^2)^{12}}{6b^5}$$

[Out] (a^4\*(a + b\*x^2)^9)/(18\*b^5) - (a^3\*(a + b\*x^2)^10)/(5\*b^5) + (3\*a^2\*(a + b\*x^2)^11)/(11\*b^5) - (a\*(a + b\*x^2)^12)/(6\*b^5) + (a + b\*x^2)^13/(26\*b^5)

**Rubi [A]** time = 0.323912, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^4 (a + bx^2)^9}{18b^5} - \frac{a^3 (a + bx^2)^{10}}{5b^5} + \frac{3a^2 (a + bx^2)^{11}}{11b^5} + \frac{(a + bx^2)^{13}}{26b^5} - \frac{a (a + bx^2)^{12}}{6b^5}$$

Antiderivative was successfully verified.

[In] Int[x^9\*(a + b\*x^2)^8,x]

[Out] (a^4\*(a + b\*x^2)^9)/(18\*b^5) - (a^3\*(a + b\*x^2)^10)/(5\*b^5) + (3\*a^2\*(a + b\*x^2)^11)/(11\*b^5) - (a\*(a + b\*x^2)^12)/(6\*b^5) + (a + b\*x^2)^13/(26\*b^5)

**Rubi in Sympy [A]** time = 25.4978, size = 80, normalized size = 0.88

$$\frac{a^4 (a + bx^2)^9}{18b^5} - \frac{a^3 (a + bx^2)^{10}}{5b^5} + \frac{3a^2 (a + bx^2)^{11}}{11b^5} - \frac{a (a + bx^2)^{12}}{6b^5} + \frac{(a + bx^2)^{13}}{26b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*9\*(b\*x\*\*2+a)\*\*8,x)

[Out] a\*\*4\*(a + b\*x\*\*2)\*\*9/(18\*b\*\*5) - a\*\*3\*(a + b\*x\*\*2)\*\*10/(5\*b\*\*5) + 3\*a\*\*2\*(a + b\*x\*\*2)\*\*11/(11\*b\*\*5) - a\*(a + b\*x\*\*2)\*\*12/(6\*b\*\*5) + (a + b\*x\*\*2)\*\*13/(26\*b\*\*5)

**Mathematica [A]** time = 0.00446184, size = 106, normalized size = 1.16

$$\frac{b^8 x^{10}}{10} + \frac{2}{3} a^7 b x^{12} + 2 a^6 b^2 x^{14} + \frac{7}{2} a^5 b^3 x^{16} + \frac{35}{9} a^4 b^4 x^{18} + \frac{14}{5} a^3 b^5 x^{20} + \frac{14}{11} a^2 b^6 x^{22} + \frac{1}{3} a b^7 x^{24} + \frac{b^8 x^{26}}{26}$$

Antiderivative was successfully verified.

[In] Integrate[x^9\*(a + b\*x^2)^8,x]

[Out] (a^8\*x^10)/10 + (2\*a^7\*b\*x^12)/3 + 2\*a^6\*b^2\*x^14 + (7\*a^5\*b^3\*x^16)/2 + (35\*a^4\*b^4\*x^18)/9 + (14\*a^3\*b^5\*x^20)/5 + (14\*a^2\*b^6\*x^22)/11 + (a\*b^7\*x^24)/3 + (b^8\*x^26)/26

**Maple [A]** time = 0.002, size = 91, normalized size = 1.

$$\frac{b^8 x^{26}}{26} + \frac{a b^7 x^{24}}{3} + \frac{14 a^2 b^6 x^{22}}{11} + \frac{14 a^3 b^5 x^{20}}{5} + \frac{35 a^4 b^4 x^{18}}{9} + \frac{7 a^5 b^3 x^{16}}{2} + 2 a^6 b^2 x^{14} + \frac{2 a^7 b x^{12}}{3} + \frac{a^8 x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9*(b*x^2+a)^8,x)`

[Out]  $\frac{1}{26}b^8x^{26} + \frac{1}{3}ab^7x^{24} + \frac{14}{11}a^2b^6x^{22} + \frac{14}{5}a^3b^5x^{20} + \frac{35}{9}a^4b^4x^{18} + \frac{7}{2}a^5b^3x^{16} + 2a^6b^2x^{14} + \frac{2}{3}a^7b^1x^{12} + \frac{1}{10}a^8x^{10}$

**Maxima [A]** time = 1.34849, size = 122, normalized size = 1.34

$$\frac{1}{26}b^8x^{26} + \frac{1}{3}ab^7x^{24} + \frac{14}{11}a^2b^6x^{22} + \frac{14}{5}a^3b^5x^{20} + \frac{35}{9}a^4b^4x^{18} + \frac{7}{2}a^5b^3x^{16} + 2a^6b^2x^{14} + \frac{2}{3}a^7bx^{12} + \frac{1}{10}a^8x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^9,x, algorithm="maxima")`

[Out]  $\frac{1}{26}b^8x^{26} + \frac{1}{3}ab^7x^{24} + \frac{14}{11}a^2b^6x^{22} + \frac{14}{5}a^3b^5x^{20} + \frac{35}{9}a^4b^4x^{18} + \frac{7}{2}a^5b^3x^{16} + 2a^6b^2x^{14} + \frac{2}{3}a^7bx^{12} + \frac{1}{10}a^8x^{10}$

**Fricas [A]** time = 0.17703, size = 1, normalized size = 0.01

$$\frac{1}{26}x^{26}b^8 + \frac{1}{3}x^{24}b^7a + \frac{14}{11}x^{22}b^6a^2 + \frac{14}{5}x^{20}b^5a^3 + \frac{35}{9}x^{18}b^4a^4 + \frac{7}{2}x^{16}b^3a^5 + 2x^{14}b^2a^6 + \frac{2}{3}x^{12}ba^7 + \frac{1}{10}x^{10}a^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^9,x, algorithm="fricas")`

[Out]  $\frac{1}{26}x^{26}b^8 + \frac{1}{3}x^{24}b^7a + \frac{14}{11}x^{22}b^6a^2 + \frac{14}{5}x^{20}b^5a^3 + \frac{35}{9}x^{18}b^4a^4 + \frac{7}{2}x^{16}b^3a^5 + 2x^{14}b^2a^6 + \frac{2}{3}x^{12}ba^7 + \frac{1}{10}x^{10}a^8$

**Sympy [A]** time = 0.160263, size = 104, normalized size = 1.14

$$\frac{a^8x^{10}}{10} + \frac{2a^7bx^{12}}{3} + 2a^6b^2x^{14} + \frac{7a^5b^3x^{16}}{2} + \frac{35a^4b^4x^{18}}{9} + \frac{14a^3b^5x^{20}}{5} + \frac{14a^2b^6x^{22}}{11} + \frac{ab^7x^{24}}{3} + \frac{b^8x^{26}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9*(b*x**2+a)**8,x)`

[Out]  $a^8x^{10}/10 + 2a^7bx^{12}/3 + 2a^6b^2x^{14} + 7a^5b^3x^{16}/2 + 35a^4b^4x^{18}/9 + 14a^3b^5x^{20}/5 + 14a^2b^6x^{22}/11 + ab^7x^{24}/3 + b^8x^{26}/26$

**GIAC/XCAS [A]** time = 0.207539, size = 122, normalized size = 1.34

$$\frac{1}{26}b^8x^{26} + \frac{1}{3}ab^7x^{24} + \frac{14}{11}a^2b^6x^{22} + \frac{14}{5}a^3b^5x^{20} + \frac{35}{9}a^4b^4x^{18} + \frac{7}{2}a^5b^3x^{16} + 2a^6b^2x^{14} + \frac{2}{3}a^7bx^{12} + \frac{1}{10}a^8x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^9,x, algorithm="giac")`

[Out]  $\frac{1}{26}b^8x^{26} + \frac{1}{3}ab^7x^{24} + \frac{14}{11}a^2b^6x^{22} + \frac{14}{5}a^3b^5x^{20} + \frac{35}{9}a^4b^4x^{18} + \frac{7}{2}a^5b^3x^{16} + 2a^6b^2x^{14} + \frac{2}{3}a^7bx^{12} + \frac{1}{10}a^8x^{10}$

### 3.88 $\int x^7 (a + bx^2)^8 dx$

**Optimal.** Leaf size=72

$$-\frac{a^3 (a + bx^2)^9}{18b^4} + \frac{3a^2 (a + bx^2)^{10}}{20b^4} + \frac{(a + bx^2)^{12}}{24b^4} - \frac{3a (a + bx^2)^{11}}{22b^4}$$

[Out]  $-(a^3*(a + b*x^2)^9)/(18*b^4) + (3*a^2*(a + b*x^2)^{10})/(20*b^4) - (3*a*(a + b*x^2)^{11})/(22*b^4) + (a + b*x^2)^{12}/(24*b^4)$

**Rubi [A]** time = 0.262634, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^3 (a + bx^2)^9}{18b^4} + \frac{3a^2 (a + bx^2)^{10}}{20b^4} + \frac{(a + bx^2)^{12}}{24b^4} - \frac{3a (a + bx^2)^{11}}{22b^4}$$

Antiderivative was successfully verified.

[In] Int[x^7\*(a + b\*x^2)^8, x]

[Out]  $-(a^3*(a + b*x^2)^9)/(18*b^4) + (3*a^2*(a + b*x^2)^{10})/(20*b^4) - (3*a*(a + b*x^2)^{11})/(22*b^4) + (a + b*x^2)^{12}/(24*b^4)$

**Rubi in Sympy [A]** time = 21.8261, size = 65, normalized size = 0.9

$$-\frac{a^3 (a + bx^2)^9}{18b^4} + \frac{3a^2 (a + bx^2)^{10}}{20b^4} - \frac{3a (a + bx^2)^{11}}{22b^4} + \frac{(a + bx^2)^{12}}{24b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*7\*(b\*x\*\*2+a)\*\*8, x)

[Out]  $-a**3*(a + b*x**2)**9/(18*b**4) + 3*a**2*(a + b*x**2)**10/(20*b**4) - 3*a*(a + b*x**2)**11/(22*b**4) + (a + b*x**2)**12/(24*b**4)$

**Mathematica [A]** time = 0.00506821, size = 106, normalized size = 1.47

$$\frac{a^8 x^8}{8} + \frac{4}{5} a^7 b x^{10} + \frac{7}{3} a^6 b^2 x^{12} + 4 a^5 b^3 x^{14} + \frac{35}{8} a^4 b^4 x^{16} + \frac{28}{9} a^3 b^5 x^{18} + \frac{7}{5} a^2 b^6 x^{20} + \frac{4}{11} a b^7 x^{22} + \frac{b^8 x^{24}}{24}$$

Antiderivative was successfully verified.

[In] Integrate[x^7\*(a + b\*x^2)^8, x]

[Out]  $(a^8*x^8)/8 + (4*a^7*b*x^10)/5 + (7*a^6*b^2*x^12)/3 + 4*a^5*b^3*x^14 + (35*a^4*b^4*x^16)/8 + (28*a^3*b^5*x^18)/9 + (7*a^2*b^6*x^20)/5 + (4*a*b^7*x^22)/11 + (b^8*x^24)/24$

**Maple [A]** time = 0.002, size = 91, normalized size = 1.3

$$\frac{b^8 x^{24}}{24} + \frac{4 a b^7 x^{22}}{11} + \frac{7 a^2 b^6 x^{20}}{5} + \frac{28 a^3 b^5 x^{18}}{9} + \frac{35 a^4 b^4 x^{16}}{8} + 4 a^5 b^3 x^{14} + \frac{7 a^6 b^2 x^{12}}{3} + \frac{4 a^7 b x^{10}}{5} + \frac{a^8 x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(b*x^2+a)^8,x)`

[Out]  $\frac{1}{24}b^8x^{24} + \frac{4}{11}ab^7x^{22} + \frac{7}{5}a^2b^6x^{20} + \frac{28}{9}a^3b^5x^{18} + \frac{35}{8}a^4b^4x^{16} + 4a^5b^3x^{14} + \frac{7}{3}a^6b^2x^{12} + \frac{4}{5}a^7bx^{10} + \frac{1}{8}a^8x^8$

**Maxima [A]** time = 1.34501, size = 122, normalized size = 1.69

$$\frac{1}{24}b^8x^{24} + \frac{4}{11}ab^7x^{22} + \frac{7}{5}a^2b^6x^{20} + \frac{28}{9}a^3b^5x^{18} + \frac{35}{8}a^4b^4x^{16} + 4a^5b^3x^{14} + \frac{7}{3}a^6b^2x^{12} + \frac{4}{5}a^7bx^{10} + \frac{1}{8}a^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^7,x, algorithm="maxima")`

[Out]  $\frac{1}{24}b^8x^{24} + \frac{4}{11}ab^7x^{22} + \frac{7}{5}a^2b^6x^{20} + \frac{28}{9}a^3b^5x^{18} + \frac{35}{8}a^4b^4x^{16} + 4a^5b^3x^{14} + \frac{7}{3}a^6b^2x^{12} + \frac{4}{5}a^7bx^{10} + \frac{1}{8}a^8x^8$

**Fricas [A]** time = 0.178341, size = 1, normalized size = 0.01

$$\frac{1}{24}x^{24}b^8 + \frac{4}{11}x^{22}b^7a + \frac{7}{5}x^{20}b^6a^2 + \frac{28}{9}x^{18}b^5a^3 + \frac{35}{8}x^{16}b^4a^4 + 4x^{14}b^3a^5 + \frac{7}{3}x^{12}b^2a^6 + \frac{4}{5}x^{10}ba^7 + \frac{1}{8}x^8a^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^7,x, algorithm="fricas")`

[Out]  $\frac{1}{24}x^{24}b^8 + \frac{4}{11}x^{22}b^7a + \frac{7}{5}x^{20}b^6a^2 + \frac{28}{9}x^{18}b^5a^3 + \frac{35}{8}x^{16}b^4a^4 + 4x^{14}b^3a^5 + \frac{7}{3}x^{12}b^2a^6 + \frac{4}{5}x^{10}ba^7 + \frac{1}{8}x^8a^8$

**Sympy [A]** time = 0.165065, size = 105, normalized size = 1.46

$$\frac{a^8x^8}{8} + \frac{4a^7bx^{10}}{5} + \frac{7a^6b^2x^{12}}{3} + 4a^5b^3x^{14} + \frac{35a^4b^4x^{16}}{8} + \frac{28a^3b^5x^{18}}{9} + \frac{7a^2b^6x^{20}}{5} + \frac{4ab^7x^{22}}{11} + \frac{b^8x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(b*x**2+a)**8,x)`

[Out]  $a^8x^8/8 + 4a^7bx^{10}/5 + 7a^6b^2x^{12}/3 + 4a^5b^3x^{14} + 35a^4b^4x^{16}/8 + 28a^3b^5x^{18}/9 + 7a^2b^6x^{20}/5 + 4ab^7x^{22}/11 + b^8x^{24}/24$

**GIAC/XCAS [A]** time = 0.207903, size = 122, normalized size = 1.69

$$\frac{1}{24}b^8x^{24} + \frac{4}{11}ab^7x^{22} + \frac{7}{5}a^2b^6x^{20} + \frac{28}{9}a^3b^5x^{18} + \frac{35}{8}a^4b^4x^{16} + 4a^5b^3x^{14} + \frac{7}{3}a^6b^2x^{12} + \frac{4}{5}a^7bx^{10} + \frac{1}{8}a^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^7,x, algorithm="giac")`

[Out]  $\frac{1}{24}b^8x^{24} + \frac{4}{11}ab^7x^{22} + \frac{7}{5}a^2b^6x^{20} + \frac{28}{9}a^3b^5x^{18} + \frac{35}{8}a^4b^4x^{16} + 4a^5b^3x^{14} + \frac{7}{3}a^6b^2x^{12} + \frac{4}{5}a^7bx^{10} + \frac{1}{8}a^8x^8$



### 3.89 $\int x^5 (a + bx^2)^8 dx$

**Optimal.** Leaf size=53

$$\frac{a^2 (a + bx^2)^9}{18b^3} + \frac{(a + bx^2)^{11}}{22b^3} - \frac{a (a + bx^2)^{10}}{10b^3}$$

[Out]  $(a^2*(a + b*x^2)^9)/(18*b^3) - (a*(a + b*x^2)^{10})/(10*b^3) + (a + b*x^2)^{11}/(22*b^3)$

**Rubi [A]** time = 0.205004, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^2 (a + bx^2)^9}{18b^3} + \frac{(a + bx^2)^{11}}{22b^3} - \frac{a (a + bx^2)^{10}}{10b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(a + b\*x^2)^8,x]

[Out]  $(a^2*(a + b*x^2)^9)/(18*b^3) - (a*(a + b*x^2)^{10})/(10*b^3) + (a + b*x^2)^{11}/(22*b^3)$

**Rubi in Sympy [A]** time = 17.4834, size = 44, normalized size = 0.83

$$\frac{a^2 (a + bx^2)^9}{18b^3} - \frac{a (a + bx^2)^{10}}{10b^3} + \frac{(a + bx^2)^{11}}{22b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(b\*x\*\*2+a)\*\*8,x)

[Out]  $a**2*(a + b*x**2)**9/(18*b**3) - a*(a + b*x**2)**10/(10*b**3) + (a + b*x**2)**11/(22*b**3)$

**Mathematica [A]** time = 0.00456712, size = 103, normalized size = 1.94

$$\frac{a^8 x^6}{6} + a^7 b x^8 + \frac{14}{5} a^6 b^2 x^{10} + \frac{14}{3} a^5 b^3 x^{12} + 5 a^4 b^4 x^{14} + \frac{7}{2} a^3 b^5 x^{16} + \frac{14}{9} a^2 b^6 x^{18} + \frac{2}{5} a b^7 x^{20} + \frac{b^8 x^{22}}{22}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*x^2)^8,x]

[Out]  $(a^8*x^6)/6 + a^7*b*x^8 + (14*a^6*b^2*x^{10})/5 + (14*a^5*b^3*x^{12})/3 + 5*a^4*b^4*x^{14} + (7*a^3*b^5*x^{16})/2 + (14*a^2*b^6*x^{18})/9 + (2*a*b^7*x^{20})/5 + (b^8*x^{22})/22$

**Maple [A]** time = 0.001, size = 90, normalized size = 1.7

$$\frac{b^8 x^{22}}{22} + \frac{2 a b^7 x^{20}}{5} + \frac{14 a^2 b^6 x^{18}}{9} + \frac{7 a^3 b^5 x^{16}}{2} + 5 a^4 b^4 x^{14} + \frac{14 a^5 b^3 x^{12}}{3} + \frac{14 a^6 b^2 x^{10}}{5} + a^7 b x^8 + \frac{a^8 x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(b*x^2+a)^8,x)`

[Out]  $\frac{1}{22}b^8x^{22} + \frac{2}{5}ab^7x^{20} + \frac{14}{9}a^2b^6x^{18} + \frac{7}{2}a^3b^5x^{16} + 5a^4b^4x^{14} + \frac{14}{3}a^5b^3x^{12} + \frac{14}{5}a^6b^2x^{10} + a^7bx^8 + \frac{1}{6}a^8x^6$

**Maxima [A]** time = 1.34853, size = 120, normalized size = 2.26

$$\frac{1}{22}b^8x^{22} + \frac{2}{5}ab^7x^{20} + \frac{14}{9}a^2b^6x^{18} + \frac{7}{2}a^3b^5x^{16} + 5a^4b^4x^{14} + \frac{14}{3}a^5b^3x^{12} + \frac{14}{5}a^6b^2x^{10} + a^7bx^8 + \frac{1}{6}a^8x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^5,x, algorithm="maxima")`

[Out]  $\frac{1}{22}b^8x^{22} + \frac{2}{5}a^5b^7x^{20} + \frac{14}{9}a^2b^6x^{18} + \frac{7}{2}a^3b^5x^{16} + 5a^4b^4x^{14} + \frac{14}{3}a^5b^3x^{12} + \frac{14}{5}a^6b^2x^{10} + a^7bx^8 + \frac{1}{6}a^8x^6$

**Fricas [A]** time = 0.174142, size = 1, normalized size = 0.02

$$\frac{1}{22}x^{22}b^8 + \frac{2}{5}x^{20}b^7a + \frac{14}{9}x^{18}b^6a^2 + \frac{7}{2}x^{16}b^5a^3 + 5x^{14}b^4a^4 + \frac{14}{3}x^{12}b^3a^5 + \frac{14}{5}x^{10}b^2a^6 + x^8ba^7 + \frac{1}{6}x^6a^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^5,x, algorithm="fricas")`

[Out]  $\frac{1}{22}x^{22}b^8 + \frac{2}{5}x^{20}b^7a + \frac{14}{9}x^{18}b^6a^2 + \frac{7}{2}x^{16}b^5a^3 + 5x^{14}b^4a^4 + \frac{14}{3}x^{12}b^3a^5 + \frac{14}{5}x^{10}b^2a^6 + x^8ba^7 + \frac{1}{6}x^6a^8$

**Sympy [A]** time = 0.157364, size = 102, normalized size = 1.92

$$\frac{a^8x^6}{6} + a^7bx^8 + \frac{14a^6b^2x^{10}}{5} + \frac{14a^5b^3x^{12}}{3} + 5a^4b^4x^{14} + \frac{7a^3b^5x^{16}}{2} + \frac{14a^2b^6x^{18}}{9} + \frac{2ab^7x^{20}}{5} + \frac{b^8x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(b*x**2+a)**8,x)`

[Out]  $a^8x^6/6 + a^7bx^8 + 14a^6b^2x^{10}/5 + 14a^5b^3x^{12}/3 + 5a^4b^4x^{14} + 7a^3b^5x^{16}/2 + 14a^2b^6x^{18}/9 + 2ab^7x^{20}/5 + b^8x^{22}/22$

**GIAC/XCAS [A]** time = 0.205505, size = 120, normalized size = 2.26

$$\frac{1}{22}b^8x^{22} + \frac{2}{5}ab^7x^{20} + \frac{14}{9}a^2b^6x^{18} + \frac{7}{2}a^3b^5x^{16} + 5a^4b^4x^{14} + \frac{14}{3}a^5b^3x^{12} + \frac{14}{5}a^6b^2x^{10} + a^7bx^8 + \frac{1}{6}a^8x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^5,x, algorithm="giac")`

[Out]  $\frac{1}{22}b^8x^{22} + \frac{2}{5}a^5b^7x^{20} + \frac{14}{9}a^2b^6x^{18} + \frac{7}{2}a^3b^5x^{16} + 5a^4b^4x^{14} + \frac{14}{3}a^5b^3x^{12} + \frac{14}{5}a^6b^2x^{10} + a^7bx^8 + \frac{1}{6}a^8x^6$

### 3.90 $\int x^3 (a + bx^2)^8 dx$

**Optimal.** Leaf size=34

$$\frac{(a + bx^2)^{10}}{20b^2} - \frac{a(a + bx^2)^9}{18b^2}$$

[Out]  $-(a*(a + b*x^2)^9)/(18*b^2) + (a + b*x^2)^{10}/(20*b^2)$

**Rubi [A]** time = 0.118407, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{(a + bx^2)^{10}}{20b^2} - \frac{a(a + bx^2)^9}{18b^2}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*x^2)^8, x]

[Out]  $-(a*(a + b*x^2)^9)/(18*b^2) + (a + b*x^2)^{10}/(20*b^2)$

**Rubi in Sympy [A]** time = 13.6076, size = 27, normalized size = 0.79

$$-\frac{a(a + bx^2)^9}{18b^2} + \frac{(a + bx^2)^{10}}{20b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(b\*x\*\*2+a)\*\*8, x)

[Out]  $-a*(a + b*x**2)**9/(18*b**2) + (a + b*x**2)**10/(20*b**2)$

**Mathematica [B]** time = 0.00426665, size = 106, normalized size = 3.12

$$\frac{a^8 x^4}{4} + \frac{4}{3} a^7 b x^6 + \frac{7}{2} a^6 b^2 x^8 + \frac{28}{5} a^5 b^3 x^{10} + \frac{35}{6} a^4 b^4 x^{12} + 4 a^3 b^5 x^{14} + \frac{7}{4} a^2 b^6 x^{16} + \frac{4}{9} a b^7 x^{18} + \frac{b^8 x^{20}}{20}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*x^2)^8, x]

[Out]  $(a^8*x^4)/4 + (4*a^7*b*x^6)/3 + (7*a^6*b^2*x^8)/2 + (28*a^5*b^3*x^{10})/5 + (35*a^4*b^4*x^{12})/6 + 4*a^3*b^5*x^{14} + (7*a^2*b^6*x^{16})/4 + (4*a*b^7*x^{18})/9 + (b^8*x^{20})/20$

**Maple [B]** time = 0.003, size = 91, normalized size = 2.7

$$\frac{b^8 x^{20}}{20} + \frac{4 a b^7 x^{18}}{9} + \frac{7 a^2 b^6 x^{16}}{4} + 4 a^3 b^5 x^{14} + \frac{35 a^4 b^4 x^{12}}{6} + \frac{28 a^5 b^3 x^{10}}{5} + \frac{7 a^6 b^2 x^8}{2} + \frac{4 a^7 b x^6}{3} + \frac{a^8 x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(b\*x^2+a)^8, x)

[Out]  $\frac{1}{20}b^8x^{20} + \frac{4}{9}ab^7x^{18} + \frac{7}{4}a^2b^6x^{16} + 4a^3b^5x^{14} + \frac{35}{6}a^4b^4x^{12} + \frac{28}{5}a^5b^3x^{10} + \frac{7}{2}a^6b^2x^8 + \frac{4}{3}a^7bx^6 + \frac{1}{4}a^8x^4$

**Maxima [A]** time = 1.34712, size = 122, normalized size = 3.59

$$\frac{1}{20}b^8x^{20} + \frac{4}{9}ab^7x^{18} + \frac{7}{4}a^2b^6x^{16} + 4a^3b^5x^{14} + \frac{35}{6}a^4b^4x^{12} + \frac{28}{5}a^5b^3x^{10} + \frac{7}{2}a^6b^2x^8 + \frac{4}{3}a^7bx^6 + \frac{1}{4}a^8x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^3,x, algorithm="maxima")`

[Out]  $\frac{1}{20}b^8x^{20} + \frac{4}{9}a^4b^4x^{12} + \frac{28}{5}a^5b^3x^{10} + \frac{7}{2}a^6b^2x^8 + \frac{4}{3}a^7bx^6 + \frac{1}{4}a^8x^4$

**Fricas [A]** time = 0.175892, size = 1, normalized size = 0.03

$$\frac{1}{20}x^{20}b^8 + \frac{4}{9}x^{18}b^7a + \frac{7}{4}x^{16}b^6a^2 + 4x^{14}b^5a^3 + \frac{35}{6}x^{12}b^4a^4 + \frac{28}{5}x^{10}b^3a^5 + \frac{7}{2}x^8b^2a^6 + \frac{4}{3}x^6ba^7 + \frac{1}{4}x^4a^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^3,x, algorithm="fricas")`

[Out]  $\frac{1}{20}x^{20}b^8 + \frac{4}{9}x^{18}b^7a + \frac{7}{4}x^{16}b^6a^2 + 4x^{14}b^5a^3 + \frac{35}{6}x^{12}b^4a^4 + \frac{28}{5}x^{10}b^3a^5 + \frac{7}{2}x^8b^2a^6 + \frac{4}{3}x^6ba^7 + \frac{1}{4}x^4a^8$

**Sympy [A]** time = 0.154378, size = 105, normalized size = 3.09

$$\frac{a^8x^4}{4} + \frac{4a^7bx^6}{3} + \frac{7a^6b^2x^8}{2} + \frac{28a^5b^3x^{10}}{5} + \frac{35a^4b^4x^{12}}{6} + 4a^3b^5x^{14} + \frac{7a^2b^6x^{16}}{4} + \frac{4ab^7x^{18}}{9} + \frac{b^8x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**2+a)**8,x)`

[Out]  $a^{**8}x^{**4}/4 + 4*a^{**7}b*x^{**6}/3 + 7*a^{**6}b^{**2}x^{**8}/2 + 28*a^{**5}b^{**3}x^{**10}/5 + 35*a^{**4}b^{**4}x^{**12}/6 + 4*a^{**3}b^{**5}x^{**14} + 7*a^{**2}b^{**6}x^{**16}/4 + 4*a*b^{**7}x^{**18}/9 + b^{**8}x^{**20}/20$

**GIAC/XCAS [A]** time = 0.207723, size = 122, normalized size = 3.59

$$\frac{1}{20}b^8x^{20} + \frac{4}{9}ab^7x^{18} + \frac{7}{4}a^2b^6x^{16} + 4a^3b^5x^{14} + \frac{35}{6}a^4b^4x^{12} + \frac{28}{5}a^5b^3x^{10} + \frac{7}{2}a^6b^2x^8 + \frac{4}{3}a^7bx^6 + \frac{1}{4}a^8x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^3,x, algorithm="giac")`

[Out]  $\frac{1}{20}b^8x^{20} + \frac{4}{9}a^4b^4x^{12} + \frac{28}{5}a^5b^3x^{10} + \frac{7}{2}a^6b^2x^8 + \frac{4}{3}a^7bx^6 + \frac{1}{4}a^8x^4$

### 3.91 $\int x (a + bx^2)^8 dx$

**Optimal.** Leaf size=16

$$\frac{(a + bx^2)^9}{18b}$$

[Out]  $(a + b*x^2)^9/(18*b)$

**Rubi [A]** time = 0.0135183, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{(a + bx^2)^9}{18b}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*x^2)^8, x]

[Out]  $(a + b*x^2)^9/(18*b)$

**Rubi in Sympy [A]** time = 2.17894, size = 10, normalized size = 0.62

$$\frac{(a + bx^2)^9}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(b\*x\*\*2+a)\*\*8, x)

[Out]  $(a + b*x**2)**9/(18*b)$

**Mathematica [A]** time = 0.00397643, size = 16, normalized size = 1.

$$\frac{(a + bx^2)^9}{18b}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*x^2)^8, x]

[Out]  $(a + b*x^2)^9/(18*b)$

**Maple [B]** time = 0.001, size = 91, normalized size = 5.7

$$\frac{b^8 x^{18}}{18} + \frac{ab^7 x^{16}}{2} + 2a^2 b^6 x^{14} + \frac{14a^3 b^5 x^{12}}{3} + 7a^4 b^4 x^{10} + 7a^5 b^3 x^8 + \frac{14a^6 b^2 x^6}{3} + 2a^7 b x^4 + \frac{a^8 x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(b\*x^2+a)^8, x)

[Out]  $1/18*b^8*x^18+1/2*a*b^7*x^16+2*a^2*b^6*x^14+14/3*a^3*b^5*x^12+7*a^4*b^4*x^10+7*a^5*b^3*x^8+14/3*a^6*b^2*x^6+2*a^7*b*x^4+1/2*a^8*x^2$

2

**Maxima [A]** time = 1.3445, size = 19, normalized size = 1.19

$$\frac{(bx^2 + a)^9}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^8\*x,x, algorithm="maxima")

[Out] 1/18\*(b\*x^2 + a)^9/b

**Fricas [A]** time = 0.176763, size = 1, normalized size = 0.06

$$\frac{1}{18}x^{18}b^8 + \frac{1}{2}x^{16}b^7a + 2x^{14}b^6a^2 + \frac{14}{3}x^{12}b^5a^3 + 7x^{10}b^4a^4 + 7x^8b^3a^5 + \frac{14}{3}x^6b^2a^6 + 2x^4ba^7 + \frac{1}{2}x^2a^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^8\*x,x, algorithm="fricas")

[Out] 1/18\*x^18\*b^8 + 1/2\*x^16\*b^7\*a + 2\*x^14\*b^6\*a^2 + 14/3\*x^12\*b^5\*a^3 + 7\*x^10\*b^4\*a^4 + 7\*x^8\*b^3\*a^5 + 14/3\*x^6\*b^2\*a^6 + 2\*x^4\*b\*a^7 + 1/2\*x^2\*a^8

**Sympy [A]** time = 0.153104, size = 99, normalized size = 6.19

$$\frac{a^8x^2}{2} + 2a^7bx^4 + \frac{14a^6b^2x^6}{3} + 7a^5b^3x^8 + 7a^4b^4x^{10} + \frac{14a^3b^5x^{12}}{3} + 2a^2b^6x^{14} + \frac{ab^7x^{16}}{2} + \frac{b^8x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x\*\*2+a)\*\*8,x)

[Out] a\*\*8\*x\*\*2/2 + 2\*a\*\*7\*b\*x\*\*4 + 14\*a\*\*6\*b\*\*2\*x\*\*6/3 + 7\*a\*\*5\*b\*\*3\*x\*\*8 + 7\*a\*\*4\*b\*\*4\*x\*\*10 + 14\*a\*\*3\*b\*\*5\*x\*\*12/3 + 2\*a\*\*2\*b\*\*6\*x\*\*14 + a\*b\*\*7\*x\*\*16/2 + b\*\*8\*x\*\*18/18

**GIAC/XCAS [A]** time = 0.206834, size = 19, normalized size = 1.19

$$\frac{(bx^2 + a)^9}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^8\*x,x, algorithm="giac")

[Out] 1/18\*(b\*x^2 + a)^9/b

$$3.92 \quad \int \frac{(a+bx^2)^8}{x} dx$$

**Optimal.** Leaf size=100

$$a^8 \log(x) + 4a^7bx^2 + 7a^6b^2x^4 + \frac{28}{3}a^5b^3x^6 + \frac{35}{4}a^4b^4x^8 + \frac{28}{5}a^3b^5x^{10} + \frac{7}{3}a^2b^6x^{12} + \frac{4}{7}ab^7x^{14} + \frac{b^8x^{16}}{16}$$

[Out]  $4*a^7*b*x^2 + 7*a^6*b^2*x^4 + (28*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/4 + (28*a^3*b^5*x^{10})/5 + (7*a^2*b^6*x^{12})/3 + (4*a*b^7*x^{14})/7 + (b^8*x^{16})/16 + a^8*\text{Log}[x]$

**Rubi [A]** time = 0.132441, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$a^8 \log(x) + 4a^7bx^2 + 7a^6b^2x^4 + \frac{28}{3}a^5b^3x^6 + \frac{35}{4}a^4b^4x^8 + \frac{28}{5}a^3b^5x^{10} + \frac{7}{3}a^2b^6x^{12} + \frac{4}{7}ab^7x^{14} + \frac{b^8x^{16}}{16}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x, x]

[Out]  $4*a^7*b*x^2 + 7*a^6*b^2*x^4 + (28*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/4 + (28*a^3*b^5*x^{10})/5 + (7*a^2*b^6*x^{12})/3 + (4*a*b^7*x^{14})/7 + (b^8*x^{16})/16 + a^8*\text{Log}[x]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^8 \log(x^2)}{2} + 4a^7bx^2 + 14a^6b^2 \int^{x^2} x dx + \frac{28a^5b^3x^6}{3} + \frac{35a^4b^4x^8}{4} + \frac{28a^3b^5x^{10}}{5} + \frac{7a^2b^6x^{12}}{3} + \frac{4ab^7x^{14}}{7} + \frac{b^8x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x, x)

[Out]  $a**8*\log(x**2)/2 + 4*a**7*b*x**2 + 14*a**6*b**2*\text{Integral}(x, (x, x**2)) + 28*a**5*b**3*x**6/3 + 35*a**4*b**4*x**8/4 + 28*a**3*b**5*x**10/5 + 7*a**2*b**6*x**12/3 + 4*a*b**7*x**14/7 + b**8*x**16/16$

**Mathematica [A]** time = 0.00805781, size = 100, normalized size = 1.

$$a^8 \log(x) + 4a^7bx^2 + 7a^6b^2x^4 + \frac{28}{3}a^5b^3x^6 + \frac{35}{4}a^4b^4x^8 + \frac{28}{5}a^3b^5x^{10} + \frac{7}{3}a^2b^6x^{12} + \frac{4}{7}ab^7x^{14} + \frac{b^8x^{16}}{16}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x, x]

[Out]  $4*a^7*b*x^2 + 7*a^6*b^2*x^4 + (28*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/4 + (28*a^3*b^5*x^{10})/5 + (7*a^2*b^6*x^{12})/3 + (4*a*b^7*x^{14})/7 + (b^8*x^{16})/16 + a^8*\text{Log}[x]$

**Maple [A]** time = 0.004, size = 89, normalized size = 0.9

$$4a^7bx^2 + 7a^6b^2x^4 + \frac{28a^5b^3x^6}{3} + \frac{35a^4b^4x^8}{4} + \frac{28a^3b^5x^{10}}{5} + \frac{7a^2b^6x^{12}}{3} + \frac{4ab^7x^{14}}{7} + \frac{b^8x^{16}}{16} + a^8 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x, x)`

[Out]  $4*a^7*b*x^2+7*a^6*b^2*x^4+28/3*a^5*b^3*x^6+35/4*a^4*b^4*x^8+28/5*a^3*b^5*x^{10}+7/3*a^2*b^6*x^{12}+4/7*a*b^7*x^{14}+1/16*b^8*x^{16}+a^8*\ln(x)$

**Maxima [A]** time = 1.34857, size = 123, normalized size = 1.23

$$\frac{1}{16}b^8x^{16} + \frac{4}{7}ab^7x^{14} + \frac{7}{3}a^2b^6x^{12} + \frac{28}{5}a^3b^5x^{10} + \frac{35}{4}a^4b^4x^8 + \frac{28}{3}a^5b^3x^6 + 7a^6b^2x^4 + 4a^7bx^2 + \frac{1}{2}a^8\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x, x, algorithm="maxima")`

[Out]  $1/16*b^8*x^{16} + 4/7*a*b^7*x^{14} + 7/3*a^2*b^6*x^{12} + 28/5*a^3*b^5*x^{10} + 35/4*a^4*b^4*x^8 + 28/3*a^5*b^3*x^6 + 7*a^6*b^2*x^4 + 4*a^7*b*x^2 + 1/2*a^8*\log(x^2)$

**Fricas [A]** time = 0.202509, size = 119, normalized size = 1.19

$$\frac{1}{16}b^8x^{16} + \frac{4}{7}ab^7x^{14} + \frac{7}{3}a^2b^6x^{12} + \frac{28}{5}a^3b^5x^{10} + \frac{35}{4}a^4b^4x^8 + \frac{28}{3}a^5b^3x^6 + 7a^6b^2x^4 + 4a^7bx^2 + a^8\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x, x, algorithm="fricas")`

[Out]  $1/16*b^8*x^{16} + 4/7*a*b^7*x^{14} + 7/3*a^2*b^6*x^{12} + 28/5*a^3*b^5*x^{10} + 35/4*a^4*b^4*x^8 + 28/3*a^5*b^3*x^6 + 7*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8*\log(x)$

**Sympy [A]** time = 1.33351, size = 102, normalized size = 1.02

$$a^8\log(x) + 4a^7bx^2 + 7a^6b^2x^4 + \frac{28a^5b^3x^6}{3} + \frac{35a^4b^4x^8}{4} + \frac{28a^3b^5x^{10}}{5} + \frac{7a^2b^6x^{12}}{3} + \frac{4ab^7x^{14}}{7} + \frac{b^8x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x, x)`

[Out]  $a**8*\log(x) + 4*a**7*b*x**2 + 7*a**6*b**2*x**4 + 28*a**5*b**3*x**6/3 + 35*a**4*b**4*x**8/4 + 28*a**3*b**5*x**10/5 + 7*a**2*b**6*x**12/3 + 4*a*b**7*x**14/7 + b**8*x**16/16$

**GIAC/XCAS [A]** time = 0.209267, size = 123, normalized size = 1.23

$$\frac{1}{16}b^8x^{16} + \frac{4}{7}ab^7x^{14} + \frac{7}{3}a^2b^6x^{12} + \frac{28}{5}a^3b^5x^{10} + \frac{35}{4}a^4b^4x^8 + \frac{28}{3}a^5b^3x^6 + 7a^6b^2x^4 + 4a^7bx^2 + \frac{1}{2}a^8\ln(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x, x, algorithm="giac")`



[Out]  $\frac{1}{16}b^8x^{16} + \frac{4}{7}ab^7x^{14} + \frac{7}{3}a^2b^6x^{12} + \frac{28}{5}a^3b^5x^{10} + \frac{35}{4}a^4b^4x^8 + \frac{28}{3}a^5b^3x^6 + 7a^6b^2x^4 + 4a^7bx^2 + \frac{1}{2}a^8\ln(x^2)$

$$3.93 \quad \int \frac{(a+bx^2)^8}{x^3} dx$$

**Optimal.** Leaf size=99

$$-\frac{a^8}{2x^2} + 8a^7b \log(x) + 14a^6b^2x^2 + 14a^5b^3x^4 + \frac{35}{3}a^4b^4x^6 + 7a^3b^5x^8 + \frac{14}{5}a^2b^6x^{10} + \frac{2}{3}ab^7x^{12} + \frac{b^8x^{14}}{14}$$

[Out]  $-a^8/(2*x^2) + 14*a^6*b^2*x^2 + 14*a^5*b^3*x^4 + (35*a^4*b^4*x^6)/3 + 7*a^3*b^5*x^8 + (14*a^2*b^6*x^{10})/5 + (2*a*b^7*x^{12})/3 + (b^8*x^{14})/14 + 8*a^7*b*\text{Log}[x]$

**Rubi [A]** time = 0.154265, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^8}{2x^2} + 8a^7b \log(x) + 14a^6b^2x^2 + 14a^5b^3x^4 + \frac{35}{3}a^4b^4x^6 + 7a^3b^5x^8 + \frac{14}{5}a^2b^6x^{10} + \frac{2}{3}ab^7x^{12} + \frac{b^8x^{14}}{14}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^3, x]

[Out]  $-a^8/(2*x^2) + 14*a^6*b^2*x^2 + 14*a^5*b^3*x^4 + (35*a^4*b^4*x^6)/3 + 7*a^3*b^5*x^8 + (14*a^2*b^6*x^{10})/5 + (2*a*b^7*x^{12})/3 + (b^8*x^{14})/14 + 8*a^7*b*\text{Log}[x]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^8}{2x^2} + 4a^7b \log(x^2) + 14a^6b^2x^2 + 28a^5b^3 \int^{x^2} x dx + \frac{35a^4b^4x^6}{3} + 7a^3b^5x^8 + \frac{14a^2b^6x^{10}}{5} + \frac{2ab^7x^{12}}{3} + \frac{b^8x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*3, x)

[Out]  $-a**8/(2*x**2) + 4*a**7*b*\text{log}(x**2) + 14*a**6*b**2*x**2 + 28*a**5*b**3*\text{Integral}(x, (x, x**2)) + 35*a**4*b**4*x**6/3 + 7*a**3*b**5*x**8 + 14*a**2*b**6*x**10/5 + 2*a*b**7*x**12/3 + b**8*x**14/14$

**Mathematica [A]** time = 0.00865362, size = 99, normalized size = 1.

$$-\frac{a^8}{2x^2} + 8a^7b \log(x) + 14a^6b^2x^2 + 14a^5b^3x^4 + \frac{35}{3}a^4b^4x^6 + 7a^3b^5x^8 + \frac{14}{5}a^2b^6x^{10} + \frac{2}{3}ab^7x^{12} + \frac{b^8x^{14}}{14}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^3, x]

[Out]  $-a^8/(2*x^2) + 14*a^6*b^2*x^2 + 14*a^5*b^3*x^4 + (35*a^4*b^4*x^6)/3 + 7*a^3*b^5*x^8 + (14*a^2*b^6*x^{10})/5 + (2*a*b^7*x^{12})/3 + (b^8*x^{14})/14 + 8*a^7*b*\text{Log}[x]$

**Maple [A]** time = 0.01, size = 90, normalized size = 0.9

$$-\frac{a^8}{2x^2} + 14a^6b^2x^2 + 14a^5b^3x^4 + \frac{35a^4b^4x^6}{3} + 7a^3b^5x^8 + \frac{14a^2b^6x^{10}}{5} + \frac{2ab^7x^{12}}{3} + \frac{b^8x^{14}}{14} + 8a^7b \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^3,x)`

[Out] 
$$-1/2*a^8/x^2+14*a^6*b^2*x^2+14*a^5*b^3*x^4+35/3*a^4*b^4*x^6+7*a^3*b^5*x^8+14/5*a^2*b^6*x^{10}+2/3*a*b^7*x^{12}+1/14*b^8*x^{14}+8*a^7*b*\ln(x)$$

**Maxima [A]** time = 1.34283, size = 123, normalized size = 1.24

$$\frac{1}{14}b^8x^{14} + \frac{2}{3}ab^7x^{12} + \frac{14}{5}a^2b^6x^{10} + 7a^3b^5x^8 + \frac{35}{3}a^4b^4x^6 + 14a^5b^3x^4 + 14a^6b^2x^2 + 4a^7b\log(x^2) - \frac{a^8}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^3,x, algorithm="maxima")`

[Out] 
$$1/14*b^8*x^{14} + 2/3*a*b^7*x^{12} + 14/5*a^2*b^6*x^{10} + 7*a^3*b^5*x^8 + 35/3*a^4*b^4*x^6 + 14*a^5*b^3*x^4 + 14*a^6*b^2*x^2 + 4*a^7*b*\log(x^2) - 1/2*a^8/x^2$$

**Fricas [A]** time = 0.202012, size = 127, normalized size = 1.28

$$\frac{15b^8x^{16} + 140ab^7x^{14} + 588a^2b^6x^{12} + 1470a^3b^5x^{10} + 2450a^4b^4x^8 + 2940a^5b^3x^6 + 2940a^6b^2x^4 + 1680a^7bx^2\log(x) - 105a^8}{210x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^3,x, algorithm="fricas")`

[Out] 
$$1/210*(15*b^8*x^{16} + 140*a*b^7*x^{14} + 588*a^2*b^6*x^{12} + 1470*a^3*b^5*x^{10} + 2450*a^4*b^4*x^8 + 2940*a^5*b^3*x^6 + 2940*a^6*b^2*x^4 + 1680*a^7*b*x^2*\log(x) - 105*a^8)/x^2$$

**Sympy [A]** time = 1.43758, size = 100, normalized size = 1.01

$$-\frac{a^8}{2x^2} + 8a^7b\log(x) + 14a^6b^2x^2 + 14a^5b^3x^4 + \frac{35a^4b^4x^6}{3} + 7a^3b^5x^8 + \frac{14a^2b^6x^{10}}{5} + \frac{2ab^7x^{12}}{3} + \frac{b^8x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**3,x)`

[Out] 
$$-a^{**8}/(2*x^{**2}) + 8*a^{**7}*b*\log(x) + 14*a^{**6}*b^{**2}*x^{**2} + 14*a^{**5}*b^{**3}*x^{**4} + 35*a^{**4}*b^{**4}*x^{**6}/3 + 7*a^{**3}*b^{**5}*x^{**8} + 14*a^{**2}*b^{**6}*x^{**10}/5 + 2*a*b^{**7}*x^{**12}/3 + b^{**8}*x^{**14}/14$$

**GIAC/XCAS [A]** time = 0.208561, size = 136, normalized size = 1.37

$$\frac{1}{14}b^8x^{14} + \frac{2}{3}ab^7x^{12} + \frac{14}{5}a^2b^6x^{10} + 7a^3b^5x^8 + \frac{35}{3}a^4b^4x^6 + 14a^5b^3x^4 + 14a^6b^2x^2 + 4a^7b\ln(x^2) - \frac{8a^7bx^2 + a^8}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^8/x^3,x, algorithm="giac")
```

```
[Out] 1/14*b^8*x^14 + 2/3*a*b^7*x^12 + 14/5*a^2*b^6*x^10 + 7*a^3*b^5*x^8 + 35/3*a^4*b^4*x^6 + 14*a^5*b^3*x^4 + 14*a^6*b^2*x^2 + 4*a^7*b*ln(x^2) - 1/2*(8*a^7*b*x^2 + a^8)/x^2
```

$$3.94 \quad \int \frac{(a+bx^2)^8}{x^5} dx$$

**Optimal.** Leaf size=101

$$-\frac{a^8}{4x^4} - \frac{4a^7b}{x^2} + 28a^6b^2 \log(x) + 28a^5b^3x^2 + \frac{35}{2}a^4b^4x^4 + \frac{28}{3}a^3b^5x^6 + \frac{7}{2}a^2b^6x^8 + \frac{4}{5}ab^7x^{10} + \frac{b^8x^{12}}{12}$$

[Out]  $-a^8/(4*x^4) - (4*a^7*b)/x^2 + 28*a^5*b^3*x^2 + (35*a^4*b^4*x^4)/2 + (28*a^3*b^5*x^6)/3 + (7*a^2*b^6*x^8)/2 + (4*a*b^7*x^{10})/5 + (b^8*x^{12})/12 + 28*a^6*b^2*Log[x]$

**Rubi [A]** time = 0.150214, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^8}{4x^4} - \frac{4a^7b}{x^2} + 28a^6b^2 \log(x) + 28a^5b^3x^2 + \frac{35}{2}a^4b^4x^4 + \frac{28}{3}a^3b^5x^6 + \frac{7}{2}a^2b^6x^8 + \frac{4}{5}ab^7x^{10} + \frac{b^8x^{12}}{12}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^5, x]

[Out]  $-a^8/(4*x^4) - (4*a^7*b)/x^2 + 28*a^5*b^3*x^2 + (35*a^4*b^4*x^4)/2 + (28*a^3*b^5*x^6)/3 + (7*a^2*b^6*x^8)/2 + (4*a*b^7*x^{10})/5 + (b^8*x^{12})/12 + 28*a^6*b^2*Log[x]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^8}{4x^4} - \frac{4a^7b}{x^2} + 14a^6b^2 \log(x^2) + 28a^5b^3x^2 + 35a^4b^4 \int^{x^2} x dx + \frac{28a^3b^5x^6}{3} + \frac{7a^2b^6x^8}{2} + \frac{4ab^7x^{10}}{5} + \frac{b^8x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*5, x)

[Out]  $-a**8/(4*x**4) - 4*a**7*b/x**2 + 14*a**6*b**2*log(x**2) + 28*a**5*b**3*x**2 + 35*a**4*b**4*Integral(x, (x, x**2)) + 28*a**3*b**5*x**6/3 + 7*a**2*b**6*x**8/2 + 4*a*b**7*x**10/5 + b**8*x**12/12$

**Mathematica [A]** time = 0.00882161, size = 101, normalized size = 1.

$$-\frac{a^8}{4x^4} - \frac{4a^7b}{x^2} + 28a^6b^2 \log(x) + 28a^5b^3x^2 + \frac{35}{2}a^4b^4x^4 + \frac{28}{3}a^3b^5x^6 + \frac{7}{2}a^2b^6x^8 + \frac{4}{5}ab^7x^{10} + \frac{b^8x^{12}}{12}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^5, x]

[Out]  $-a^8/(4*x^4) - (4*a^7*b)/x^2 + 28*a^5*b^3*x^2 + (35*a^4*b^4*x^4)/2 + (28*a^3*b^5*x^6)/3 + (7*a^2*b^6*x^8)/2 + (4*a*b^7*x^{10})/5 + (b^8*x^{12})/12 + 28*a^6*b^2*Log[x]$

**Maple [A]** time = 0.01, size = 90, normalized size = 0.9

$$-\frac{a^8}{4x^4} - 4\frac{a^7b}{x^2} + 28a^5b^3x^2 + \frac{35a^4b^4x^4}{2} + \frac{28a^3b^5x^6}{3} + \frac{7a^2b^6x^8}{2} + \frac{4ab^7x^{10}}{5} + \frac{b^8x^{12}}{12} + 28a^6b^2 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^5,x)`

[Out]  $-1/4*a^8/x^4-4*a^7*b/x^2+28*a^5*b^3*x^2+35/2*a^4*b^4*x^4+28/3*a^3*b^5*x^6+7/2*a^2*b^6*x^8+4/5*a*b^7*x^{10}+1/12*b^8*x^{12}+28*a^6*b^2*\ln(x)$

**Maxima [A]** time = 1.34527, size = 124, normalized size = 1.23

$$\frac{1}{12}b^8x^{12} + \frac{4}{5}ab^7x^{10} + \frac{7}{2}a^2b^6x^8 + \frac{28}{3}a^3b^5x^6 + \frac{35}{2}a^4b^4x^4 + 28a^5b^3x^2 + 14a^6b^2\log(x^2) - \frac{16a^7bx^2 + a^8}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^5,x, algorithm="maxima")`

[Out]  $1/12*b^8*x^{12} + 4/5*a*b^7*x^{10} + 7/2*a^2*b^6*x^8 + 28/3*a^3*b^5*x^6 + 35/2*a^4*b^4*x^4 + 28*a^5*b^3*x^2 + 14*a^6*b^2*\log(x^2) - 1/4*(16*a^7*b*x^2 + a^8)/x^4$

**Fricas [A]** time = 0.202054, size = 127, normalized size = 1.26

$$\frac{5b^8x^{16} + 48ab^7x^{14} + 210a^2b^6x^{12} + 560a^3b^5x^{10} + 1050a^4b^4x^8 + 1680a^5b^3x^6 + 1680a^6b^2x^4\log(x) - 240a^7bx^2 - 15a^8}{60x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^5,x, algorithm="fricas")`

[Out]  $1/60*(5*b^8*x^{16} + 48*a*b^7*x^{14} + 210*a^2*b^6*x^{12} + 560*a^3*b^5*x^{10} + 1050*a^4*b^4*x^8 + 1680*a^5*b^3*x^6 + 1680*a^6*b^2*x^4*\log(x) - 240*a^7*b*x^2 - 15*a^8)/x^4$

**Sympy [A]** time = 1.62975, size = 102, normalized size = 1.01

$$28a^6b^2\log(x) + 28a^5b^3x^2 + \frac{35a^4b^4x^4}{2} + \frac{28a^3b^5x^6}{3} + \frac{7a^2b^6x^8}{2} + \frac{4ab^7x^{10}}{5} + \frac{b^8x^{12}}{12} - \frac{a^8 + 16a^7bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**5,x)`

[Out]  $28*a**6*b**2*\log(x) + 28*a**5*b**3*x**2 + 35*a**4*b**4*x**4/2 + 28*a**3*b**5*x**6/3 + 7*a**2*b**6*x**8/2 + 4*a*b**7*x**10/5 + b**8*x**12/12 - (a**8 + 16*a**7*b*x**2)/(4*x**4)$

**GIAC/XCAS [A]** time = 0.207938, size = 139, normalized size = 1.38

$$\frac{1}{12}b^8x^{12} + \frac{4}{5}ab^7x^{10} + \frac{7}{2}a^2b^6x^8 + \frac{28}{3}a^3b^5x^6 + \frac{35}{2}a^4b^4x^4 + 28a^5b^3x^2 + 14a^6b^2\ln(x^2) - \frac{84a^6b^2x^4 + 16a^7bx^2 + a^8}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^8/x^5,x, algorithm="giac")
```

```
[Out] 1/12*b^8*x^12 + 4/5*a*b^7*x^10 + 7/2*a^2*b^6*x^8 + 28/3*a^3*b^5*x^6 + 35/2*a^4*b^4*x^4 + 28*a^5*b^3*x^2 + 14*a^6*b^2*ln(x^2) - 1/4*(84*a^6*b^2*x^4 + 16*a^7*b*x^2 + a^8)/x^4
```

$$3.95 \quad \int \frac{(a+bx^2)^8}{x^7} dx$$

**Optimal.** Leaf size=94

$$-\frac{a^8}{6x^6} - \frac{2a^7b}{x^4} - \frac{14a^6b^2}{x^2} + 56a^5b^3 \log(x) + 35a^4b^4x^2 + 14a^3b^5x^4 + \frac{14}{3}a^2b^6x^6 + ab^7x^8 + \frac{b^8x^{10}}{10}$$

[Out]  $-a^8/(6*x^6) - (2*a^7*b)/x^4 - (14*a^6*b^2)/x^2 + 35*a^4*b^4*x^2 + 14*a^3*b^5*x^4 + (14*a^2*b^6*x^6)/3 + a*b^7*x^8 + (b^8*x^{10})/10 + 56*a^5*b^3*\text{Log}[x]$

**Rubi [A]** time = 0.147566, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^8}{6x^6} - \frac{2a^7b}{x^4} - \frac{14a^6b^2}{x^2} + 56a^5b^3 \log(x) + 35a^4b^4x^2 + 14a^3b^5x^4 + \frac{14}{3}a^2b^6x^6 + ab^7x^8 + \frac{b^8x^{10}}{10}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^7, x]

[Out]  $-a^8/(6*x^6) - (2*a^7*b)/x^4 - (14*a^6*b^2)/x^2 + 35*a^4*b^4*x^2 + 14*a^3*b^5*x^4 + (14*a^2*b^6*x^6)/3 + a*b^7*x^8 + (b^8*x^{10})/10 + 56*a^5*b^3*\text{Log}[x]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^8}{6x^6} - \frac{2a^7b}{x^4} - \frac{14a^6b^2}{x^2} + 28a^5b^3 \log(x^2) + 35a^4b^4x^2 + 28a^3b^5 \int^{x^2} x dx + \frac{14a^2b^6x^6}{3} + ab^7x^8 + \frac{b^8x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*7, x)

[Out]  $-a**8/(6*x**6) - 2*a**7*b/x**4 - 14*a**6*b**2/x**2 + 28*a**5*b**3*\log(x**2) + 35*a**4*b**4*x**2 + 28*a**3*b**5*\text{Integral}(x, (x, x**2)) + 14*a**2*b**6*x**6/3 + a*b**7*x**8 + b**8*x**10/10$

**Mathematica [A]** time = 0.00846451, size = 94, normalized size = 1.

$$-\frac{a^8}{6x^6} - \frac{2a^7b}{x^4} - \frac{14a^6b^2}{x^2} + 56a^5b^3 \log(x) + 35a^4b^4x^2 + 14a^3b^5x^4 + \frac{14}{3}a^2b^6x^6 + ab^7x^8 + \frac{b^8x^{10}}{10}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^7, x]

[Out]  $-a^8/(6*x^6) - (2*a^7*b)/x^4 - (14*a^6*b^2)/x^2 + 35*a^4*b^4*x^2 + 14*a^3*b^5*x^4 + (14*a^2*b^6*x^6)/3 + a*b^7*x^8 + (b^8*x^{10})/10 + 56*a^5*b^3*\text{Log}[x]$

**Maple [A]** time = 0.01, size = 89, normalized size = 1.

$$-\frac{a^8}{6x^6} - 2\frac{a^7b}{x^4} - 14\frac{a^6b^2}{x^2} + 35a^4b^4x^2 + 14a^3b^5x^4 + \frac{14a^2b^6x^6}{3} + ab^7x^8 + \frac{b^8x^{10}}{10} + 56a^5b^3 \ln(x)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^7,x)`

[Out]  $-1/6*a^8/x^6-2*a^7*b/x^4-14*a^6*b^2/x^2+35*a^4*b^4*x^2+14*a^3*b^5*x^4+14/3*a^2*b^6*x^6+a*b^7*x^8+1/10*b^8*x^{10}+56*a^5*b^3*\ln(x)$

**Maxima [A]** time = 1.3527, size = 123, normalized size = 1.31

$$\frac{1}{10}b^8x^{10} + ab^7x^8 + \frac{14}{3}a^2b^6x^6 + 14a^3b^5x^4 + 35a^4b^4x^2 + 28a^5b^3\log(x^2) - \frac{84a^6b^2x^4 + 12a^7bx^2 + a^8}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^7,x, algorithm="maxima")`

[Out]  $1/10*b^8*x^{10} + a*b^7*x^8 + 14/3*a^2*b^6*x^6 + 14*a^3*b^5*x^4 + 35*a^4*b^4*x^2 + 28*a^5*b^3*\log(x^2) - 1/6*(84*a^6*b^2*x^4 + 12*a^7*b*x^2 + a^8)/x^6$

**Fricas [A]** time = 0.201199, size = 127, normalized size = 1.35

$$\frac{3b^8x^{16} + 30ab^7x^{14} + 140a^2b^6x^{12} + 420a^3b^5x^{10} + 1050a^4b^4x^8 + 1680a^5b^3x^6\log(x) - 420a^6b^2x^4 - 60a^7bx^2 - 5a^8}{30x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^7,x, algorithm="fricas")`

[Out]  $1/30*(3*b^8*x^{16} + 30*a*b^7*x^{14} + 140*a^2*b^6*x^{12} + 420*a^3*b^5*x^{10} + 1050*a^4*b^4*x^8 + 1680*a^5*b^3*x^6*\log(x) - 420*a^6*b^2*x^4 - 60*a^7*b*x^2 - 5*a^8)/x^6$

**Sympy [A]** time = 1.88127, size = 95, normalized size = 1.01

$$56a^5b^3\log(x) + 35a^4b^4x^2 + 14a^3b^5x^4 + \frac{14a^2b^6x^6}{3} + ab^7x^8 + \frac{b^8x^{10}}{10} - \frac{a^8 + 12a^7bx^2 + 84a^6b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**7,x)`

[Out]  $56*a**5*b**3*\log(x) + 35*a**4*b**4*x**2 + 14*a**3*b**5*x**4 + 14*a**2*b**6*x**6/3 + a*b**7*x**8 + b**8*x**10/10 - (a**8 + 12*a**7*b*x**2 + 84*a**6*b**2*x**4)/(6*x**6)$

**GIAC/XCAS [A]** time = 0.209781, size = 138, normalized size = 1.47

$$\frac{1}{10}b^8x^{10} + ab^7x^8 + \frac{14}{3}a^2b^6x^6 + 14a^3b^5x^4 + 35a^4b^4x^2 + 28a^5b^3\ln(x^2) - \frac{308a^5b^3x^6 + 84a^6b^2x^4 + 12a^7bx^2 + a^8}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^7,x, algorithm="giac")`

[Out]  $\frac{1}{10}b^8x^{10} + ab^7x^8 + \frac{14}{3}a^2b^6x^6 + 14a^3b^5x^4 + 35a^4b^4x^2 + 28a^5b^3\ln(x^2) - \frac{1}{6}(308a^5b^3x^6 + 84a^6b^2x^4 + 12a^7bx^2 + a^8)/x^6$

$$3.96 \quad \int \frac{(a+bx^2)^8}{x^9} dx$$

**Optimal.** Leaf size=97

$$-\frac{a^8}{8x^8} - \frac{4a^7b}{3x^6} - \frac{7a^6b^2}{x^4} - \frac{28a^5b^3}{x^2} + 70a^4b^4 \log(x) + 28a^3b^5x^2 + 7a^2b^6x^4 + \frac{4}{3}ab^7x^6 + \frac{b^8x^8}{8}$$

[Out]  $-a^8/(8*x^8) - (4*a^7*b)/(3*x^6) - (7*a^6*b^2)/x^4 - (28*a^5*b^3)/x^2 + 28*a^3*b^5*x^2 + 7*a^2*b^6*x^4 + (4*a*b^7*x^6)/3 + (b^8*x^8)/8 + 70*a^4*b^4*Log[x]$

**Rubi [A]** time = 0.144796, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^8}{8x^8} - \frac{4a^7b}{3x^6} - \frac{7a^6b^2}{x^4} - \frac{28a^5b^3}{x^2} + 70a^4b^4 \log(x) + 28a^3b^5x^2 + 7a^2b^6x^4 + \frac{4}{3}ab^7x^6 + \frac{b^8x^8}{8}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^9, x]

[Out]  $-a^8/(8*x^8) - (4*a^7*b)/(3*x^6) - (7*a^6*b^2)/x^4 - (28*a^5*b^3)/x^2 + 28*a^3*b^5*x^2 + 7*a^2*b^6*x^4 + (4*a*b^7*x^6)/3 + (b^8*x^8)/8 + 70*a^4*b^4*Log[x]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^8}{8x^8} - \frac{4a^7b}{3x^6} - \frac{7a^6b^2}{x^4} - \frac{28a^5b^3}{x^2} + 35a^4b^4 \log(x^2) + 28a^3b^5x^2 + 14a^2b^6 \int^{x^2} x dx + \frac{4ab^7x^6}{3} + \frac{b^8x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*9, x)

[Out]  $-a**8/(8*x**8) - 4*a**7*b/(3*x**6) - 7*a**6*b**2/x**4 - 28*a**5*b**3/x**2 + 35*a**4*b**4*log(x**2) + 28*a**3*b**5*x**2 + 14*a**2*b**6*Integral(x, (x, x**2)) + 4*a*b**7*x**6/3 + b**8*x**8/8$

**Mathematica [A]** time = 0.00848819, size = 97, normalized size = 1.

$$-\frac{a^8}{8x^8} - \frac{4a^7b}{3x^6} - \frac{7a^6b^2}{x^4} - \frac{28a^5b^3}{x^2} + 70a^4b^4 \log(x) + 28a^3b^5x^2 + 7a^2b^6x^4 + \frac{4}{3}ab^7x^6 + \frac{b^8x^8}{8}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^9, x]

[Out]  $-a^8/(8*x^8) - (4*a^7*b)/(3*x^6) - (7*a^6*b^2)/x^4 - (28*a^5*b^3)/x^2 + 28*a^3*b^5*x^2 + 7*a^2*b^6*x^4 + (4*a*b^7*x^6)/3 + (b^8*x^8)/8 + 70*a^4*b^4*Log[x]$

**Maple [A]** time = 0.01, size = 90, normalized size = 0.9

$$-\frac{a^8}{8x^8} - \frac{4a^7b}{3x^6} - 7\frac{a^6b^2}{x^4} - 28\frac{a^5b^3}{x^2} + 28a^3b^5x^2 + 7a^2b^6x^4 + \frac{4ab^7x^6}{3} + \frac{b^8x^8}{8} + 70a^4b^4 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^9,x)`

[Out]  $-1/8*a^8/x^8-4/3*a^7*b/x^6-7*a^6*b^2/x^4-28*a^5*b^3/x^2+28*a^3*b^5*x^2+7*a^2*b^6*x^4+4/3*a*b^7*x^6+1/8*b^8*x^8+70*a^4*b^4*\ln(x)$

**Maxima [A]** time = 1.34207, size = 127, normalized size = 1.31

$$\frac{1}{8}b^8x^8 + \frac{4}{3}ab^7x^6 + 7a^2b^6x^4 + 28a^3b^5x^2 + 35a^4b^4\log(x^2) - \frac{672a^5b^3x^6 + 168a^6b^2x^4 + 32a^7bx^2 + 3a^8}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^9,x, algorithm="maxima")`

[Out]  $1/8*b^8*x^8 + 4/3*a*b^7*x^6 + 7*a^2*b^6*x^4 + 28*a^3*b^5*x^2 + 35*a^4*b^4*\log(x^2) - 1/24*(672*a^5*b^3*x^6 + 168*a^6*b^2*x^4 + 32*a^7*b*x^2 + 3*a^8)/x^8$

**Fricas [A]** time = 0.201373, size = 127, normalized size = 1.31

$$\frac{3b^8x^{16} + 32ab^7x^{14} + 168a^2b^6x^{12} + 672a^3b^5x^{10} + 1680a^4b^4x^8\log(x) - 672a^5b^3x^6 - 168a^6b^2x^4 - 32a^7bx^2 - 3a^8}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^9,x, algorithm="fricas")`

[Out]  $1/24*(3*b^8*x^{16} + 32*a*b^7*x^{14} + 168*a^2*b^6*x^{12} + 672*a^3*b^5*x^{10} + 1680*a^4*b^4*x^8*\log(x) - 672*a^5*b^3*x^6 - 168*a^6*b^2*x^4 - 32*a^7*b*x^2 - 3*a^8)/x^8$

**Sympy [A]** time = 2.17249, size = 99, normalized size = 1.02

$$70a^4b^4\log(x) + 28a^3b^5x^2 + 7a^2b^6x^4 + \frac{4ab^7x^6}{3} + \frac{b^8x^8}{8} - \frac{3a^8 + 32a^7bx^2 + 168a^6b^2x^4 + 672a^5b^3x^6}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**9,x)`

[Out]  $70*a**4*b**4*\log(x) + 28*a**3*b**5*x**2 + 7*a**2*b**6*x**4 + 4*a*b**7*x**6/3 + b**8*x**8/8 - (3*a**8 + 32*a**7*b*x**2 + 168*a**6*b**2*x**4 + 672*a**5*b**3*x**6)/(24*x**8)$

**GIAC/XCAS [A]** time = 0.210028, size = 142, normalized size = 1.46

$$\frac{1}{8}b^8x^8 + \frac{4}{3}ab^7x^6 + 7a^2b^6x^4 + 28a^3b^5x^2 + 35a^4b^4\ln(x^2) - \frac{1750a^4b^4x^8 + 672a^5b^3x^6 + 168a^6b^2x^4 + 32a^7bx^2 + 3a^8}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^9,x, algorithm="giac")`

```
[Out] 1/8*b^8*x^8 + 4/3*a*b^7*x^6 + 7*a^2*b^6*x^4 + 28*a^3*b^5*x^2 + 35
*a^4*b^4*ln(x^2) - 1/24*(1750*a^4*b^4*x^8 + 672*a^5*b^3*x^6 + 168
*a^6*b^2*x^4 + 32*a^7*b*x^2 + 3*a^8)/x^8
```

$$3.97 \quad \int \frac{(a+bx^2)^8}{x^{11}} dx$$

**Optimal.** Leaf size=95

$$-\frac{a^8}{10x^{10}} - \frac{a^7b}{x^8} - \frac{14a^6b^2}{3x^6} - \frac{14a^5b^3}{x^4} - \frac{35a^4b^4}{x^2} + 56a^3b^5 \log(x) + 14a^2b^6x^2 + 2ab^7x^4 + \frac{b^8x^6}{6}$$

[Out]  $-a^8/(10*x^{10}) - (a^7*b)/x^8 - (14*a^6*b^2)/(3*x^6) - (14*a^5*b^3)/x^4 - (35*a^4*b^4)/x^2 + 14*a^2*b^6*x^2 + 2*a*b^7*x^4 + (b^8*x^6)/6 + 56*a^3*b^5*Log[x]$

**Rubi [A]** time = 0.14399, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^8}{10x^{10}} - \frac{a^7b}{x^8} - \frac{14a^6b^2}{3x^6} - \frac{14a^5b^3}{x^4} - \frac{35a^4b^4}{x^2} + 56a^3b^5 \log(x) + 14a^2b^6x^2 + 2ab^7x^4 + \frac{b^8x^6}{6}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^11, x]

[Out]  $-a^8/(10*x^{10}) - (a^7*b)/x^8 - (14*a^6*b^2)/(3*x^6) - (14*a^5*b^3)/x^4 - (35*a^4*b^4)/x^2 + 14*a^2*b^6*x^2 + 2*a*b^7*x^4 + (b^8*x^6)/6 + 56*a^3*b^5*Log[x]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^8}{10x^{10}} - \frac{a^7b}{x^8} - \frac{14a^6b^2}{3x^6} - \frac{14a^5b^3}{x^4} - \frac{35a^4b^4}{x^2} + 28a^3b^5 \log(x^2) + 14a^2b^6x^2 + 4ab^7 \int^{x^2} x dx + \frac{b^8x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*11, x)

[Out]  $-a**8/(10*x**10) - a**7*b/x**8 - 14*a**6*b**2/(3*x**6) - 14*a**5*b**3/x**4 - 35*a**4*b**4/x**2 + 28*a**3*b**5*log(x**2) + 14*a**2*b**6*x**2 + 4*a*b**7*Integral(x, (x, x**2)) + b**8*x**6/6$

**Mathematica [A]** time = 0.00847667, size = 95, normalized size = 1.

$$-\frac{a^8}{10x^{10}} - \frac{a^7b}{x^8} - \frac{14a^6b^2}{3x^6} - \frac{14a^5b^3}{x^4} - \frac{35a^4b^4}{x^2} + 56a^3b^5 \log(x) + 14a^2b^6x^2 + 2ab^7x^4 + \frac{b^8x^6}{6}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^11, x]

[Out]  $-a^8/(10*x^{10}) - (a^7*b)/x^8 - (14*a^6*b^2)/(3*x^6) - (14*a^5*b^3)/x^4 - (35*a^4*b^4)/x^2 + 14*a^2*b^6*x^2 + 2*a*b^7*x^4 + (b^8*x^6)/6 + 56*a^3*b^5*Log[x]$

**Maple [A]** time = 0.011, size = 90, normalized size = 1.

$$-\frac{a^8}{10x^{10}} - \frac{a^7b}{x^8} - \frac{14a^6b^2}{3x^6} - 14\frac{a^5b^3}{x^4} - 35\frac{a^4b^4}{x^2} + 14a^2b^6x^2 + 2ab^7x^4 + \frac{b^8x^6}{6} + 56a^3b^5 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^11,x)`

[Out]  $-1/10*a^8/x^{10}-a^7*b/x^8-14/3*a^6*b^2/x^6-14*a^5*b^3/x^4-35*a^4*b^4/x^2+14*a^2*b^6*x^2+2*a*b^7*x^4+1/6*b^8*x^6+56*a^3*b^5*\ln(x)$

**Maxima [A]** time = 1.3438, size = 127, normalized size = 1.34

$$\frac{1}{6}b^8x^6 + 2ab^7x^4 + 14a^2b^6x^2 + 28a^3b^5\log(x^2) - \frac{1050a^4b^4x^8 + 420a^5b^3x^6 + 140a^6b^2x^4 + 30a^7bx^2 + 3a^8}{30x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^11,x, algorithm="maxima")`

[Out]  $1/6*b^8*x^6 + 2*a*b^7*x^4 + 14*a^2*b^6*x^2 + 28*a^3*b^5*\log(x^2) - 1/30*(1050*a^4*b^4*x^8 + 420*a^5*b^3*x^6 + 140*a^6*b^2*x^4 + 30*a^7*b*x^2 + 3*a^8)/x^{10}$

**Fricas [A]** time = 0.202421, size = 127, normalized size = 1.34

$$\frac{5b^8x^{16} + 60ab^7x^{14} + 420a^2b^6x^{12} + 1680a^3b^5x^{10}\log(x) - 1050a^4b^4x^8 - 420a^5b^3x^6 - 140a^6b^2x^4 - 30a^7bx^2 - 3a^8}{30x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^11,x, algorithm="fricas")`

[Out]  $1/30*(5*b^8*x^{16} + 60*a*b^7*x^{14} + 420*a^2*b^6*x^{12} + 1680*a^3*b^5*x^{10}*\log(x) - 1050*a^4*b^4*x^8 - 420*a^5*b^3*x^6 - 140*a^6*b^2*x^4 - 30*a^7*b*x^2 - 3*a^8)/x^{10}$

**Sympy [A]** time = 2.55012, size = 97, normalized size = 1.02

$$56a^3b^5\log(x) + 14a^2b^6x^2 + 2ab^7x^4 + \frac{b^8x^6}{6} - \frac{3a^8 + 30a^7bx^2 + 140a^6b^2x^4 + 420a^5b^3x^6 + 1050a^4b^4x^8}{30x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**11,x)`

[Out]  $56*a**3*b**5*\log(x) + 14*a**2*b**6*x**2 + 2*a*b**7*x**4 + b**8*x**6/6 - (3*a**8 + 30*a**7*b*x**2 + 140*a**6*b**2*x**4 + 420*a**5*b**3*x**6 + 1050*a**4*b**4*x**8)/(30*x**10)$

**GIAC/XCAS [A]** time = 0.207923, size = 142, normalized size = 1.49

$$\frac{1}{6}b^8x^6 + 2ab^7x^4 + 14a^2b^6x^2 + 28a^3b^5\ln(x^2) - \frac{1918a^3b^5x^{10} + 1050a^4b^4x^8 + 420a^5b^3x^6 + 140a^6b^2x^4 + 30a^7bx^2 + 3a^8}{30x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^11,x, algorithm="giac")`

[Out]  $\frac{1}{6}b^8x^6 + 2ab^7x^4 + 14a^2b^6x^2 + 28a^3b^5\ln(x^2) - \frac{1}{30}(1918a^3b^5x^{10} + 1050a^4b^4x^8 + 420a^5b^3x^6 + 140a^6b^2x^4 + 30a^7bx^2 + 3a^8)/x^{10}$



$$3.98 \quad \int \frac{(a+bx^2)^8}{x^{13}} dx$$

**Optimal.** Leaf size=101

$$-\frac{a^8}{12x^{12}} - \frac{4a^7b}{5x^{10}} - \frac{7a^6b^2}{2x^8} - \frac{28a^5b^3}{3x^6} - \frac{35a^4b^4}{2x^4} - \frac{28a^3b^5}{x^2} + 28a^2b^6 \log(x) + 4ab^7x^2 + \frac{b^8x^4}{4}$$

[Out]  $-a^8/(12*x^{12}) - (4*a^7*b)/(5*x^{10}) - (7*a^6*b^2)/(2*x^8) - (28*a^5*b^3)/(3*x^6) - (35*a^4*b^4)/(2*x^4) - (28*a^3*b^5)/x^2 + 4*a^2*b^6 \log(x) + 4*a*b^7*x^2 + (b^8*x^4)/4$

**Rubi [A]** time = 0.142981, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^8}{12x^{12}} - \frac{4a^7b}{5x^{10}} - \frac{7a^6b^2}{2x^8} - \frac{28a^5b^3}{3x^6} - \frac{35a^4b^4}{2x^4} - \frac{28a^3b^5}{x^2} + 28a^2b^6 \log(x) + 4ab^7x^2 + \frac{b^8x^4}{4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^13, x]

[Out]  $-a^8/(12*x^{12}) - (4*a^7*b)/(5*x^{10}) - (7*a^6*b^2)/(2*x^8) - (28*a^5*b^3)/(3*x^6) - (35*a^4*b^4)/(2*x^4) - (28*a^3*b^5)/x^2 + 4*a^2*b^6 \log(x) + 4*a*b^7*x^2 + (b^8*x^4)/4$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^8}{12x^{12}} - \frac{4a^7b}{5x^{10}} - \frac{7a^6b^2}{2x^8} - \frac{28a^5b^3}{3x^6} - \frac{35a^4b^4}{2x^4} - \frac{28a^3b^5}{x^2} + 14a^2b^6 \log(x^2) + 4ab^7x^2 + \frac{b^8 \int x dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*13, x)

[Out]  $-a^{**8}/(12*x^{**12}) - 4*a^{**7}*b/(5*x^{**10}) - 7*a^{**6}*b^{**2}/(2*x^{**8}) - 28*a^{**5}*b^{**3}/(3*x^{**6}) - 35*a^{**4}*b^{**4}/(2*x^{**4}) - 28*a^{**3}*b^{**5}/x^{**2} + 14*a^{**2}*b^{**6} \log(x^{**2}) + 4*a*b^{**7}*x^{**2} + b^{**8} \text{Integral}(x, (x, x^{**2}))/2$

**Mathematica [A]** time = 0.00851347, size = 101, normalized size = 1.

$$-\frac{a^8}{12x^{12}} - \frac{4a^7b}{5x^{10}} - \frac{7a^6b^2}{2x^8} - \frac{28a^5b^3}{3x^6} - \frac{35a^4b^4}{2x^4} - \frac{28a^3b^5}{x^2} + 28a^2b^6 \log(x) + 4ab^7x^2 + \frac{b^8x^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^13, x]

[Out]  $-a^8/(12*x^{12}) - (4*a^7*b)/(5*x^{10}) - (7*a^6*b^2)/(2*x^8) - (28*a^5*b^3)/(3*x^6) - (35*a^4*b^4)/(2*x^4) - (28*a^3*b^5)/x^2 + 4*a^2*b^6 \log(x) + 4*a*b^7*x^2 + (b^8*x^4)/4$

**Maple [A]** time = 0.013, size = 90, normalized size = 0.9

$$-\frac{a^8}{12x^{12}} - \frac{4a^7b}{5x^{10}} - \frac{7a^6b^2}{2x^8} - \frac{28a^5b^3}{3x^6} - \frac{35a^4b^4}{2x^4} - 28 \frac{a^3b^5}{x^2} + 4ab^7x^2 + \frac{b^8x^4}{4} + 28a^2b^6 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^13,x)`

[Out] 
$$-1/12*a^8/x^12-4/5*a^7*b/x^10-7/2*a^6*b^2/x^8-28/3*a^5*b^3/x^6-35/2*a^4*b^4/x^4-28*a^3*b^5/x^2+4*a*b^7*x^2+1/4*b^8*x^4+28*a^2*b^6*\ln(x)$$

**Maxima [A]** time = 1.33487, size = 127, normalized size = 1.26

$$\frac{1}{4}b^8x^4+4ab^7x^2+14a^2b^6\log(x^2)-\frac{1680a^3b^5x^{10}+1050a^4b^4x^8+560a^5b^3x^6+210a^6b^2x^4+48a^7bx^2+5a^8}{60x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^13,x, algorithm="maxima")`

[Out] 
$$1/4*b^8*x^4 + 4*a*b^7*x^2 + 14*a^2*b^6*\log(x^2) - 1/60*(1680*a^3*b^5*x^{10} + 1050*a^4*b^4*x^8 + 560*a^5*b^3*x^6 + 210*a^6*b^2*x^4 + 48*a^7*b*x^2 + 5*a^8)/x^{12}$$

**Fricas [A]** time = 0.201881, size = 127, normalized size = 1.26

$$\frac{15b^8x^{16}+240ab^7x^{14}+1680a^2b^6x^{12}\log(x)-1680a^3b^5x^{10}-1050a^4b^4x^8-560a^5b^3x^6-210a^6b^2x^4-48a^7bx^2-5a^8}{60x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^13,x, algorithm="fricas")`

[Out] 
$$1/60*(15*b^8*x^{16} + 240*a*b^7*x^{14} + 1680*a^2*b^6*x^{12}*\log(x) - 1680*a^3*b^5*x^{10} - 1050*a^4*b^4*x^8 - 560*a^5*b^3*x^6 - 210*a^6*b^2*x^4 - 48*a^7*b*x^2 - 5*a^8)/x^{12}$$

**Sympy [A]** time = 2.85862, size = 97, normalized size = 0.96

$$28a^2b^6\log(x)+4ab^7x^2+\frac{b^8x^4}{4}-\frac{5a^8+48a^7bx^2+210a^6b^2x^4+560a^5b^3x^6+1050a^4b^4x^8+1680a^3b^5x^{10}}{60x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**13,x)`

[Out] 
$$28*a**2*b**6*\log(x) + 4*a*b**7*x**2 + b**8*x**4/4 - (5*a**8 + 48*a**7*b*x**2 + 210*a**6*b**2*x**4 + 560*a**5*b**3*x**6 + 1050*a**4*b**4*x**8 + 1680*a**3*b**5*x**10)/(60*x**12)$$

**GIAC/XCAS [A]** time = 0.212697, size = 142, normalized size = 1.41

$$\frac{1}{4}b^8x^4+4ab^7x^2+14a^2b^6\ln(x^2)-\frac{2058a^2b^6x^{12}+1680a^3b^5x^{10}+1050a^4b^4x^8+560a^5b^3x^6+210a^6b^2x^4+48a^7bx^2+5a^8}{60x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^8/x^13,x, algorithm="giac")
```

```
[Out] 1/4*b^8*x^4 + 4*a*b^7*x^2 + 14*a^2*b^6*ln(x^2) - 1/60*(2058*a^2*b^6*x^12 + 1680*a^3*b^5*x^10 + 1050*a^4*b^4*x^8 + 560*a^5*b^3*x^6 + 210*a^6*b^2*x^4 + 48*a^7*b*x^2 + 5*a^8)/x^12
```

$$3.99 \quad \int \frac{(a+bx^2)^8}{x^{15}} dx$$

**Optimal.** Leaf size=99

$$-\frac{a^8}{14x^{14}} - \frac{2a^7b}{3x^{12}} - \frac{14a^6b^2}{5x^{10}} - \frac{7a^5b^3}{x^8} - \frac{35a^4b^4}{3x^6} - \frac{14a^3b^5}{x^4} - \frac{14a^2b^6}{x^2} + 8ab^7 \log(x) + \frac{b^8x^2}{2}$$

[Out]  $-a^8/(14*x^{14}) - (2*a^7*b)/(3*x^{12}) - (14*a^6*b^2)/(5*x^{10}) - (7*a^5*b^3)/x^8 - (35*a^4*b^4)/(3*x^6) - (14*a^3*b^5)/x^4 - (14*a^2*b^6)/x^2 + (b^8*x^2)/2 + 8*a*b^7*Log[x]$

**Rubi [A]** time = 0.138245, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^8}{14x^{14}} - \frac{2a^7b}{3x^{12}} - \frac{14a^6b^2}{5x^{10}} - \frac{7a^5b^3}{x^8} - \frac{35a^4b^4}{3x^6} - \frac{14a^3b^5}{x^4} - \frac{14a^2b^6}{x^2} + 8ab^7 \log(x) + \frac{b^8x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^15, x]

[Out]  $-a^8/(14*x^{14}) - (2*a^7*b)/(3*x^{12}) - (14*a^6*b^2)/(5*x^{10}) - (7*a^5*b^3)/x^8 - (35*a^4*b^4)/(3*x^6) - (14*a^3*b^5)/x^4 - (14*a^2*b^6)/x^2 + (b^8*x^2)/2 + 8*a*b^7*Log[x]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^8}{14x^{14}} - \frac{2a^7b}{3x^{12}} - \frac{14a^6b^2}{5x^{10}} - \frac{7a^5b^3}{x^8} - \frac{35a^4b^4}{3x^6} - \frac{14a^3b^5}{x^4} - \frac{14a^2b^6}{x^2} + 4ab^7 \log(x^2) + \frac{\int^{x^2} b^8 dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*15, x)

[Out]  $-a**8/(14*x**14) - 2*a**7*b/(3*x**12) - 14*a**6*b**2/(5*x**10) - 7*a**5*b**3/x**8 - 35*a**4*b**4/(3*x**6) - 14*a**3*b**5/x**4 - 14*a**2*b**6/x**2 + 4*a*b**7*log(x**2) + Integral(b**8, (x, x**2))/2$

**Mathematica [A]** time = 0.00874418, size = 99, normalized size = 1.

$$-\frac{a^8}{14x^{14}} - \frac{2a^7b}{3x^{12}} - \frac{14a^6b^2}{5x^{10}} - \frac{7a^5b^3}{x^8} - \frac{35a^4b^4}{3x^6} - \frac{14a^3b^5}{x^4} - \frac{14a^2b^6}{x^2} + 8ab^7 \log(x) + \frac{b^8x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^15, x]

[Out]  $-a^8/(14*x^{14}) - (2*a^7*b)/(3*x^{12}) - (14*a^6*b^2)/(5*x^{10}) - (7*a^5*b^3)/x^8 - (35*a^4*b^4)/(3*x^6) - (14*a^3*b^5)/x^4 - (14*a^2*b^6)/x^2 + (b^8*x^2)/2 + 8*a*b^7*Log[x]$

**Maple [A]** time = 0.012, size = 90, normalized size = 0.9

$$-\frac{a^8}{14x^{14}} - \frac{2a^7b}{3x^{12}} - \frac{14a^6b^2}{5x^{10}} - 7\frac{a^5b^3}{x^8} - \frac{35a^4b^4}{3x^6} - 14\frac{a^3b^5}{x^4} - 14\frac{a^2b^6}{x^2} + \frac{b^8x^2}{2} + 8ab^7 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^15,x)`

[Out] 
$$-1/14*a^8/x^14 - 2/3*a^7*b/x^12 - 14/5*a^6*b^2/x^10 - 7*a^5*b^3/x^8 - 35/3*a^4*b^4/x^6 - 14*a^3*b^5/x^4 - 14*a^2*b^6/x^2 + 1/2*b^8*x^2 + 8*a*b^7*\ln(x)$$

**Maxima [A]** time = 1.34748, size = 127, normalized size = 1.28

$$\frac{\frac{1}{2} b^8 x^2 + 4 a b^7 \log(x^2)}{210 x^{14} + 2940 a^2 b^6 x^{12} + 2940 a^3 b^5 x^{10} + 2450 a^4 b^4 x^8 + 1470 a^5 b^3 x^6 + 588 a^6 b^2 x^4 + 140 a^7 b x^2 + 15 a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^15,x, algorithm="maxima")`

[Out] 
$$1/2*b^8*x^2 + 4*a*b^7*\log(x^2) - 1/210*(2940*a^2*b^6*x^12 + 2940*a^3*b^5*x^10 + 2450*a^4*b^4*x^8 + 1470*a^5*b^3*x^6 + 588*a^6*b^2*x^4 + 140*a^7*b*x^2 + 15*a^8)/x^14$$

**Fricas [A]** time = 0.200214, size = 127, normalized size = 1.28

$$\frac{105 b^8 x^{16} + 1680 a b^7 x^{14} \log(x) - 2940 a^2 b^6 x^{12} - 2940 a^3 b^5 x^{10} - 2450 a^4 b^4 x^8 - 1470 a^5 b^3 x^6 - 588 a^6 b^2 x^4 - 140 a^7 b x^2 - 15 a^8}{210 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^15,x, algorithm="fricas")`

[Out] 
$$1/210*(105*b^8*x^16 + 1680*a*b^7*x^14*\log(x) - 2940*a^2*b^6*x^12 - 2940*a^3*b^5*x^10 - 2450*a^4*b^4*x^8 - 1470*a^5*b^3*x^6 - 588*a^6*b^2*x^4 - 140*a^7*b*x^2 - 15*a^8)/x^14$$

**Sympy [A]** time = 3.31881, size = 97, normalized size = 0.98

$$8 a b^7 \log(x) + \frac{b^8 x^2}{2} - \frac{15 a^8 + 140 a^7 b x^2 + 588 a^6 b^2 x^4 + 1470 a^5 b^3 x^6 + 2450 a^4 b^4 x^8 + 2940 a^3 b^5 x^{10} + 2940 a^2 b^6 x^{12}}{210 x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**15,x)`

[Out] 
$$8*a*b^7*\log(x) + b^8*x^2/2 - (15*a^8 + 140*a^7*b*x^2 + 588*a^6*b^2*x^4 + 1470*a^5*b^3*x^6 + 2450*a^4*b^4*x^8 + 2940*a^3*b^5*x^{10} + 2940*a^2*b^6*x^{12})/(210*x^{14})$$

**GIAC/XCAS [A]** time = 0.207953, size = 139, normalized size = 1.4

$$\frac{\frac{1}{2} b^8 x^2 + 4 a b^7 \ln(x^2)}{210 x^{14} + 2178 a b^7 x^{14} + 2940 a^2 b^6 x^{12} + 2940 a^3 b^5 x^{10} + 2450 a^4 b^4 x^8 + 1470 a^5 b^3 x^6 + 588 a^6 b^2 x^4 + 140 a^7 b x^2 + 15 a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^8/x^15,x, algorithm="giac")
```

```
[Out] 1/2*b^8*x^2 + 4*a*b^7*ln(x^2) - 1/210*(2178*a*b^7*x^14 + 2940*a^2
*b^6*x^12 + 2940*a^3*b^5*x^10 + 2450*a^4*b^4*x^8 + 1470*a^5*b^3*x
^6 + 588*a^6*b^2*x^4 + 140*a^7*b*x^2 + 15*a^8)/x^14
```

$$3.100 \quad \int \frac{(a+bx^2)^8}{x^{17}} dx$$

**Optimal.** Leaf size=100

$$-\frac{a^8}{16x^{16}} - \frac{4a^7b}{7x^{14}} - \frac{7a^6b^2}{3x^{12}} - \frac{28a^5b^3}{5x^{10}} - \frac{35a^4b^4}{4x^8} - \frac{28a^3b^5}{3x^6} - \frac{7a^2b^6}{x^4} - \frac{4ab^7}{x^2} + b^8 \log(x)$$

[Out]  $-a^8/(16*x^{16}) - (4*a^7*b)/(7*x^{14}) - (7*a^6*b^2)/(3*x^{12}) - (28*a^5*b^3)/(5*x^{10}) - (35*a^4*b^4)/(4*x^8) - (28*a^3*b^5)/(3*x^6) - (7*a^2*b^6)/x^4 - (4*a*b^7)/x^2 + b^8*\text{Log}[x]$

**Rubi [A]** time = 0.135663, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^8}{16x^{16}} - \frac{4a^7b}{7x^{14}} - \frac{7a^6b^2}{3x^{12}} - \frac{28a^5b^3}{5x^{10}} - \frac{35a^4b^4}{4x^8} - \frac{28a^3b^5}{3x^6} - \frac{7a^2b^6}{x^4} - \frac{4ab^7}{x^2} + b^8 \log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^17, x]

[Out]  $-a^8/(16*x^{16}) - (4*a^7*b)/(7*x^{14}) - (7*a^6*b^2)/(3*x^{12}) - (28*a^5*b^3)/(5*x^{10}) - (35*a^4*b^4)/(4*x^8) - (28*a^3*b^5)/(3*x^6) - (7*a^2*b^6)/x^4 - (4*a*b^7)/x^2 + b^8*\text{Log}[x]$

**Rubi in Sympy [A]** time = 23.8249, size = 105, normalized size = 1.05

$$-\frac{a^8}{16x^{16}} - \frac{4a^7b}{7x^{14}} - \frac{7a^6b^2}{3x^{12}} - \frac{28a^5b^3}{5x^{10}} - \frac{35a^4b^4}{4x^8} - \frac{28a^3b^5}{3x^6} - \frac{7a^2b^6}{x^4} - \frac{4ab^7}{x^2} + \frac{b^8 \log(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*17, x)

[Out]  $-a**8/(16*x**16) - 4*a**7*b/(7*x**14) - 7*a**6*b**2/(3*x**12) - 28*a**5*b**3/(5*x**10) - 35*a**4*b**4/(4*x**8) - 28*a**3*b**5/(3*x**6) - 7*a**2*b**6/x**4 - 4*a*b**7/x**2 + b**8*\log(x**2)/2$

**Mathematica [A]** time = 0.00877329, size = 100, normalized size = 1.

$$-\frac{a^8}{16x^{16}} - \frac{4a^7b}{7x^{14}} - \frac{7a^6b^2}{3x^{12}} - \frac{28a^5b^3}{5x^{10}} - \frac{35a^4b^4}{4x^8} - \frac{28a^3b^5}{3x^6} - \frac{7a^2b^6}{x^4} - \frac{4ab^7}{x^2} + b^8 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^17, x]

[Out]  $-a^8/(16*x^{16}) - (4*a^7*b)/(7*x^{14}) - (7*a^6*b^2)/(3*x^{12}) - (28*a^5*b^3)/(5*x^{10}) - (35*a^4*b^4)/(4*x^8) - (28*a^3*b^5)/(3*x^6) - (7*a^2*b^6)/x^4 - (4*a*b^7)/x^2 + b^8*\text{Log}[x]$

**Maple [A]** time = 0.011, size = 89, normalized size = 0.9

$$-\frac{a^8}{16x^{16}} - \frac{4a^7b}{7x^{14}} - \frac{7a^6b^2}{3x^{12}} - \frac{28a^5b^3}{5x^{10}} - \frac{35a^4b^4}{4x^8} - \frac{28a^3b^5}{3x^6} - 7\frac{a^2b^6}{x^4} - 4\frac{ab^7}{x^2} + b^8 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^17,x)`

[Out] 
$$-1/16*a^8/x^{16}-4/7*a^7*b/x^{14}-7/3*a^6*b^2/x^{12}-28/5*a^5*b^3/x^{10}-35/4*a^4*b^4/x^8-28/3*a^3*b^5/x^6-7*a^2*b^6/x^4-4*a*b^7/x^2+b^8*1$$
  
n(x)

**Maxima [A]** time = 1.35059, size = 127, normalized size = 1.27

$$\frac{1}{2}b^8 \log(x^2) + \frac{6720ab^7x^{14} + 11760a^2b^6x^{12} + 15680a^3b^5x^{10} + 14700a^4b^4x^8 + 9408a^5b^3x^6 + 3920a^6b^2x^4 + 960a^7bx^2 + 105a^8}{1680x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^17,x, algorithm="maxima")`

[Out] 
$$1/2*b^8*\log(x^2) - 1/1680*(6720*a*b^7*x^{14} + 11760*a^2*b^6*x^{12} + 15680*a^3*b^5*x^{10} + 14700*a^4*b^4*x^8 + 9408*a^5*b^3*x^6 + 3920*a^6*b^2*x^4 + 960*a^7*b*x^2 + 105*a^8)/x^{16}$$

**Fricas [A]** time = 0.199327, size = 127, normalized size = 1.27

$$\frac{1680b^8x^{16} \log(x) - 6720ab^7x^{14} - 11760a^2b^6x^{12} - 15680a^3b^5x^{10} - 14700a^4b^4x^8 - 9408a^5b^3x^6 - 3920a^6b^2x^4 - 960a^7bx^2 - 105a^8}{1680x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^17,x, algorithm="fricas")`

[Out] 
$$1/1680*(1680*b^8*x^{16}*\log(x) - 6720*a*b^7*x^{14} - 11760*a^2*b^6*x^{12} - 15680*a^3*b^5*x^{10} - 14700*a^4*b^4*x^8 - 9408*a^5*b^3*x^6 - 3920*a^6*b^2*x^4 - 960*a^7*b*x^2 - 105*a^8)/x^{16}$$

**Sympy [A]** time = 3.7118, size = 95, normalized size = 0.95

$$b^8 \log(x) + \frac{105a^8 + 960a^7bx^2 + 3920a^6b^2x^4 + 9408a^5b^3x^6 + 14700a^4b^4x^8 + 15680a^3b^5x^{10} + 11760a^2b^6x^{12} + 6720ab^7x^{14}}{1680x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**17,x)`

[Out] 
$$b^{**8}*\log(x) - (105*a^{**8} + 960*a^{**7}*b*x^{**2} + 3920*a^{**6}*b^{**2}*x^{**4} + 9408*a^{**5}*b^{**3}*x^{**6} + 14700*a^{**4}*b^{**4}*x^{**8} + 15680*a^{**3}*b^{**5}*x^{**10} + 11760*a^{**2}*b^{**6}*x^{**12} + 6720*a*b^{**7}*x^{**14})/(1680*x^{**16})$$

**GIAC/XCAS [A]** time = 0.208714, size = 138, normalized size = 1.38

$$\frac{1}{2}b^8 \ln(x^2) + \frac{2283b^8x^{16} + 6720ab^7x^{14} + 11760a^2b^6x^{12} + 15680a^3b^5x^{10} + 14700a^4b^4x^8 + 9408a^5b^3x^6 + 3920a^6b^2x^4 + 960a^7bx^2 + 105a^8}{1680x^{16}}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^8/x^17,x, algorithm="giac")
```

```
[Out] 1/2*b^8*ln(x^2) - 1/1680*(2283*b^8*x^16 + 6720*a*b^7*x^14 + 11760
*a^2*b^6*x^12 + 15680*a^3*b^5*x^10 + 14700*a^4*b^4*x^8 + 9408*a^5
*b^3*x^6 + 3920*a^6*b^2*x^4 + 960*a^7*b*x^2 + 105*a^8)/x^16
```

$$3.101 \quad \int \frac{(a+bx^2)^8}{x^{19}} dx$$

**Optimal.** Leaf size=19

$$-\frac{(a+bx^2)^9}{18ax^{18}}$$

[Out]  $-(a + b*x^2)^9/(18*a*x^{18})$

**Rubi [A]** time = 0.0203048, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{(a+bx^2)^9}{18ax^{18}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^19, x]

[Out]  $-(a + b*x^2)^9/(18*a*x^{18})$

**Rubi in Sympy [A]** time = 3.26904, size = 15, normalized size = 0.79

$$-\frac{(a+bx^2)^9}{18ax^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*19, x)

[Out]  $-(a + b*x**2)**9/(18*a*x**18)$

**Mathematica [B]** time = 0.00799446, size = 100, normalized size = 5.26

$$-\frac{a^8}{18x^{18}} - \frac{a^7b}{2x^{16}} - \frac{2a^6b^2}{x^{14}} - \frac{14a^5b^3}{3x^{12}} - \frac{7a^4b^4}{x^{10}} - \frac{7a^3b^5}{x^8} - \frac{14a^2b^6}{3x^6} - \frac{2ab^7}{x^4} - \frac{b^8}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^19, x]

[Out]  $-a^8/(18*x^{18}) - (a^7*b)/(2*x^{16}) - (2*a^6*b^2)/x^{14} - (14*a^5*b^3)/(3*x^{12}) - (7*a^4*b^4)/x^{10} - (7*a^3*b^5)/x^8 - (14*a^2*b^6)/(3*x^6) - (2*a*b^7)/x^4 - b^8/(2*x^2)$

**Maple [B]** time = 0.01, size = 91, normalized size = 4.8

$$-\frac{14a^5b^3}{3x^{12}} - \frac{14a^2b^6}{3x^6} - 2\frac{ab^7}{x^4} - 7\frac{a^4b^4}{x^{10}} - 7\frac{a^3b^5}{x^8} - 2\frac{a^6b^2}{x^{14}} - \frac{a^8}{18x^{18}} - \frac{a^7b}{2x^{16}} - \frac{b^8}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^8/x^19, x)

[Out]  $-14/3*a^5*b^3/x^{12}-14/3*a^2*b^6/x^6-2*a*b^7/x^4-7*a^4*b^4/x^{10}-7*a^3*b^5/x^8-2*a^6*b^2/x^{14}-1/18*a^8/x^{18}-1/2*a^7*b/x^{16}-1/2*b^8/x^2$

**Maxima [A]** time = 1.41416, size = 122, normalized size = 6.42

$$\frac{9b^8x^{16} + 36ab^7x^{14} + 84a^2b^6x^{12} + 126a^3b^5x^{10} + 126a^4b^4x^8 + 84a^5b^3x^6 + 36a^6b^2x^4 + 9a^7bx^2 + a^8}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^19,x, algorithm="maxima")`

[Out]  $-1/18*(9*b^8*x^{16} + 36*a*b^7*x^{14} + 84*a^2*b^6*x^{12} + 126*a^3*b^5*x^{10} + 126*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/x^{18}$

**Fricas [A]** time = 0.195471, size = 122, normalized size = 6.42

$$\frac{9b^8x^{16} + 36ab^7x^{14} + 84a^2b^6x^{12} + 126a^3b^5x^{10} + 126a^4b^4x^8 + 84a^5b^3x^6 + 36a^6b^2x^4 + 9a^7bx^2 + a^8}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^19,x, algorithm="fricas")`

[Out]  $-1/18*(9*b^8*x^{16} + 36*a*b^7*x^{14} + 84*a^2*b^6*x^{12} + 126*a^3*b^5*x^{10} + 126*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/x^{18}$

**Sympy [A]** time = 3.9602, size = 97, normalized size = 5.11

$$\frac{a^8 + 9a^7bx^2 + 36a^6b^2x^4 + 84a^5b^3x^6 + 126a^4b^4x^8 + 126a^3b^5x^{10} + 84a^2b^6x^{12} + 36ab^7x^{14} + 9b^8x^{16}}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**19,x)`

[Out]  $-(a**8 + 9*a**7*b*x**2 + 36*a**6*b**2*x**4 + 84*a**5*b**3*x**6 + 126*a**4*b**4*x**8 + 126*a**3*b**5*x**10 + 84*a**2*b**6*x**12 + 36*a*b**7*x**14 + 9*b**8*x**16)/(18*x**18)$

**GIAC/XCAS [A]** time = 0.209354, size = 122, normalized size = 6.42

$$\frac{9b^8x^{16} + 36ab^7x^{14} + 84a^2b^6x^{12} + 126a^3b^5x^{10} + 126a^4b^4x^8 + 84a^5b^3x^6 + 36a^6b^2x^4 + 9a^7bx^2 + a^8}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^19,x, algorithm="giac")`

[Out]  $-1/18*(9*b^8*x^{16} + 36*a*b^7*x^{14} + 84*a^2*b^6*x^{12} + 126*a^3*b^5*x^{10} + 126*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/x^{18}$

$$3.102 \quad \int \frac{(a+bx^2)^8}{x^{21}} dx$$

**Optimal.** Leaf size=40

$$\frac{b(a+bx^2)^9}{180a^2x^{18}} - \frac{(a+bx^2)^9}{20ax^{20}}$$

[Out]  $-(a + b*x^2)^9/(20*a*x^{20}) + (b*(a + b*x^2)^9)/(180*a^2*x^{18})$

**Rubi [A]** time = 0.0600336, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{b(a+bx^2)^9}{180a^2x^{18}} - \frac{(a+bx^2)^9}{20ax^{20}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^21, x]

[Out]  $-(a + b*x^2)^9/(20*a*x^{20}) + (b*(a + b*x^2)^9)/(180*a^2*x^{18})$

**Rubi in Sympy [A]** time = 6.56769, size = 32, normalized size = 0.8

$$-\frac{(a+bx^2)^9}{20ax^{20}} + \frac{b(a+bx^2)^9}{180a^2x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*21, x)

[Out]  $-(a + b*x**2)**9/(20*a*x**20) + b*(a + b*x**2)**9/(180*a**2*x**18)$

**Mathematica [B]** time = 0.00772375, size = 106, normalized size = 2.65

$$-\frac{a^8}{20x^{20}} - \frac{4a^7b}{9x^{18}} - \frac{7a^6b^2}{4x^{16}} - \frac{4a^5b^3}{x^{14}} - \frac{35a^4b^4}{6x^{12}} - \frac{28a^3b^5}{5x^{10}} - \frac{7a^2b^6}{2x^8} - \frac{4ab^7}{3x^6} - \frac{b^8}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^21, x]

[Out]  $-a^8/(20*x^{20}) - (4*a^7*b)/(9*x^{18}) - (7*a^6*b^2)/(4*x^{16}) - (4*a^5*b^3)/x^{14} - (35*a^4*b^4)/(6*x^{12}) - (28*a^3*b^5)/(5*x^{10}) - (7*a^2*b^6)/(2*x^8) - (4*a*b^7)/(3*x^6) - b^8/(4*x^4)$

**Maple [B]** time = 0.009, size = 91, normalized size = 2.3

$$-\frac{35a^4b^4}{6x^{12}} - \frac{7a^6b^2}{4x^{16}} - \frac{a^8}{20x^{20}} - \frac{4ab^7}{3x^6} - \frac{b^8}{4x^4} - \frac{28a^3b^5}{5x^{10}} - \frac{7a^2b^6}{2x^8} - 4\frac{a^5b^3}{x^{14}} - \frac{4a^7b}{9x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^8/x^21, x)

[Out] 
$$-35/6 * a^4 * b^4 / x^{12} - 7/4 * a^6 * b^2 / x^{16} - 1/20 * a^8 / x^{20} - 4/3 * a * b^7 / x^6 - 1/4 * b^8 / x^4 - 28/5 * a^3 * b^5 / x^{10} - 7/2 * a^2 * b^6 / x^8 - 4 * a^5 * b^3 / x^{14} - 4/9 * a^7 * b / x^{18}$$

**Maxima [A]** time = 1.35034, size = 124, normalized size = 3.1

$$\frac{45 b^8 x^{16} + 240 a b^7 x^{14} + 630 a^2 b^6 x^{12} + 1008 a^3 b^5 x^{10} + 1050 a^4 b^4 x^8 + 720 a^5 b^3 x^6 + 315 a^6 b^2 x^4 + 80 a^7 b x^2 + 9 a^8}{180 x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^21,x, algorithm="maxima")`

[Out] 
$$-1/180 * (45 * b^8 * x^{16} + 240 * a * b^7 * x^{14} + 630 * a^2 * b^6 * x^{12} + 1008 * a^3 * b^5 * x^{10} + 1050 * a^4 * b^4 * x^8 + 720 * a^5 * b^3 * x^6 + 315 * a^6 * b^2 * x^4 + 80 * a^7 * b * x^2 + 9 * a^8) / x^{20}$$

**Fricas [A]** time = 0.195605, size = 124, normalized size = 3.1

$$\frac{45 b^8 x^{16} + 240 a b^7 x^{14} + 630 a^2 b^6 x^{12} + 1008 a^3 b^5 x^{10} + 1050 a^4 b^4 x^8 + 720 a^5 b^3 x^6 + 315 a^6 b^2 x^4 + 80 a^7 b x^2 + 9 a^8}{180 x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^21,x, algorithm="fricas")`

[Out] 
$$-1/180 * (45 * b^8 * x^{16} + 240 * a * b^7 * x^{14} + 630 * a^2 * b^6 * x^{12} + 1008 * a^3 * b^5 * x^{10} + 1050 * a^4 * b^4 * x^8 + 720 * a^5 * b^3 * x^6 + 315 * a^6 * b^2 * x^4 + 80 * a^7 * b * x^2 + 9 * a^8) / x^{20}$$

**Sympy [A]** time = 4.20112, size = 99, normalized size = 2.48

$$\frac{9 a^8 + 80 a^7 b x^2 + 315 a^6 b^2 x^4 + 720 a^5 b^3 x^6 + 1050 a^4 b^4 x^8 + 1008 a^3 b^5 x^{10} + 630 a^2 b^6 x^{12} + 240 a b^7 x^{14} + 45 b^8 x^{16}}{180 x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**21,x)`

[Out] 
$$-(9 * a^{**8} + 80 * a^{**7} * b * x^{**2} + 315 * a^{**6} * b^{**2} * x^{**4} + 720 * a^{**5} * b^{**3} * x^{**6} + 1050 * a^{**4} * b^{**4} * x^{**8} + 1008 * a^{**3} * b^{**5} * x^{**10} + 630 * a^{**2} * b^{**6} * x^{**12} + 240 * a * b^{**7} * x^{**14} + 45 * b^{**8} * x^{**16}) / (180 * x^{**20})$$

**GIAC/XCAS [A]** time = 0.210059, size = 124, normalized size = 3.1

$$\frac{45 b^8 x^{16} + 240 a b^7 x^{14} + 630 a^2 b^6 x^{12} + 1008 a^3 b^5 x^{10} + 1050 a^4 b^4 x^8 + 720 a^5 b^3 x^6 + 315 a^6 b^2 x^4 + 80 a^7 b x^2 + 9 a^8}{180 x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^21,x, algorithm="giac")`

[Out] 
$$-1/180 * (45 * b^8 * x^{16} + 240 * a * b^7 * x^{14} + 630 * a^2 * b^6 * x^{12} + 1008 * a^3 * b^5 * x^{10} + 1050 * a^4 * b^4 * x^8 + 720 * a^5 * b^3 * x^6 + 315 * a^6 * b^2 * x^4 + 80 * a^7 * b * x^2 + 9 * a^8) / x^{20}$$

$$3.103 \quad \int \frac{(a+bx^2)^8}{x^{23}} dx$$

**Optimal.** Leaf size=62

$$-\frac{b^2(a+bx^2)^9}{990a^3x^{18}} + \frac{b(a+bx^2)^9}{110a^2x^{20}} - \frac{(a+bx^2)^9}{22ax^{22}}$$

[Out]  $-(a + b*x^2)^9/(22*a*x^{22}) + (b*(a + b*x^2)^9)/(110*a^2*x^{20}) - (b^2*(a + b*x^2)^9)/(990*a^3*x^{18})$

**Rubi [A]** time = 0.0872331, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{b^2(a+bx^2)^9}{990a^3x^{18}} + \frac{b(a+bx^2)^9}{110a^2x^{20}} - \frac{(a+bx^2)^9}{22ax^{22}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^23, x]

[Out]  $-(a + b*x^2)^9/(22*a*x^{22}) + (b*(a + b*x^2)^9)/(110*a^2*x^{20}) - (b^2*(a + b*x^2)^9)/(990*a^3*x^{18})$

**Rubi in Sympy [A]** time = 9.47911, size = 53, normalized size = 0.85

$$-\frac{(a+bx^2)^9}{22ax^{22}} + \frac{b(a+bx^2)^9}{110a^2x^{20}} - \frac{b^2(a+bx^2)^9}{990a^3x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*23, x)

[Out]  $-(a + b*x^2)**9/(22*a*x**22) + b*(a + b*x^2)**9/(110*a**2*x**20) - b**2*(a + b*x^2)**9/(990*a**3*x**18)$

**Mathematica [A]** time = 0.00854995, size = 104, normalized size = 1.68

$$-\frac{a^8}{22x^{22}} - \frac{2a^7b}{5x^{20}} - \frac{14a^6b^2}{9x^{18}} - \frac{7a^5b^3}{2x^{16}} - \frac{5a^4b^4}{x^{14}} - \frac{14a^3b^5}{3x^{12}} - \frac{14a^2b^6}{5x^{10}} - \frac{ab^7}{x^8} - \frac{b^8}{6x^6}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^23, x]

[Out]  $-a^8/(22*x^{22}) - (2*a^7*b)/(5*x^{20}) - (14*a^6*b^2)/(9*x^{18}) - (7*a^5*b^3)/(2*x^{16}) - (5*a^4*b^4)/x^{14} - (14*a^3*b^5)/(3*x^{12}) - (14*a^2*b^6)/(5*x^{10}) - (a*b^7)/x^8 - b^8/(6*x^6)$

**Maple [A]** time = 0.008, size = 91, normalized size = 1.5

$$-\frac{14a^3b^5}{3x^{12}} - \frac{b^8}{6x^6} - \frac{14a^2b^6}{5x^{10}} - \frac{2a^7b}{5x^{20}} - \frac{ab^7}{x^8} - \frac{14a^6b^2}{9x^{18}} - \frac{7a^5b^3}{2x^{16}} - 5\frac{a^4b^4}{x^{14}} - \frac{a^8}{22x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^23,x)`

[Out]  $-14/3*a^3*b^5/x^{12}-1/6*b^8/x^6-14/5*a^2*b^6/x^{10}-2/5*a^7*b/x^{20}-a*b^7/x^8-14/9*a^6*b^2/x^{18}-7/2*a^5*b^3/x^{16}-5*a^4*b^4/x^{14}-1/22*a^8/x^{22}$

**Maxima [A]** time = 1.3436, size = 124, normalized size = 2.

$$\frac{165 b^8 x^{16} + 990 a b^7 x^{14} + 2772 a^2 b^6 x^{12} + 4620 a^3 b^5 x^{10} + 4950 a^4 b^4 x^8 + 3465 a^5 b^3 x^6 + 1540 a^6 b^2 x^4 + 396 a^7 b x^2 + 45 a^8}{990 x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^23,x, algorithm="maxima")`

[Out]  $-1/990*(165*b^8*x^{16} + 990*a*b^7*x^{14} + 2772*a^2*b^6*x^{12} + 4620*a^3*b^5*x^{10} + 4950*a^4*b^4*x^8 + 3465*a^5*b^3*x^6 + 1540*a^6*b^2*x^4 + 396*a^7*b*x^2 + 45*a^8)/x^{22}$

**Fricas [A]** time = 0.196308, size = 124, normalized size = 2.

$$\frac{165 b^8 x^{16} + 990 a b^7 x^{14} + 2772 a^2 b^6 x^{12} + 4620 a^3 b^5 x^{10} + 4950 a^4 b^4 x^8 + 3465 a^5 b^3 x^6 + 1540 a^6 b^2 x^4 + 396 a^7 b x^2 + 45 a^8}{990 x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^23,x, algorithm="fricas")`

[Out]  $-1/990*(165*b^8*x^{16} + 990*a*b^7*x^{14} + 2772*a^2*b^6*x^{12} + 4620*a^3*b^5*x^{10} + 4950*a^4*b^4*x^8 + 3465*a^5*b^3*x^6 + 1540*a^6*b^2*x^4 + 396*a^7*b*x^2 + 45*a^8)/x^{22}$

**Sympy [A]** time = 4.40976, size = 99, normalized size = 1.6

$$\frac{45 a^8 + 396 a^7 b x^2 + 1540 a^6 b^2 x^4 + 3465 a^5 b^3 x^6 + 4950 a^4 b^4 x^8 + 4620 a^3 b^5 x^{10} + 2772 a^2 b^6 x^{12} + 990 a b^7 x^{14} + 165 b^8 x^{16}}{990 x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**23,x)`

[Out]  $-(45*a**8 + 396*a**7*b*x**2 + 1540*a**6*b**2*x**4 + 3465*a**5*b**3*x**6 + 4950*a**4*b**4*x**8 + 4620*a**3*b**5*x**10 + 2772*a**2*b**6*x**12 + 990*a*b**7*x**14 + 165*b**8*x**16)/(990*x**22)$

**GIAC/XCAS [A]** time = 0.210089, size = 124, normalized size = 2.

$$\frac{165 b^8 x^{16} + 990 a b^7 x^{14} + 2772 a^2 b^6 x^{12} + 4620 a^3 b^5 x^{10} + 4950 a^4 b^4 x^8 + 3465 a^5 b^3 x^6 + 1540 a^6 b^2 x^4 + 396 a^7 b x^2 + 45 a^8}{990 x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^23,x, algorithm="giac")`

[Out]  $-1/990*(165*b^8*x^{16} + 990*a*b^7*x^{14} + 2772*a^2*b^6*x^{12} + 4620*a^3*b^5*x^{10} + 4950*a^4*b^4*x^8 + 3465*a^5*b^3*x^6 + 1540*a^6*b^2*x^4 + 396*a^7*b*x^2 + 45*a^8)/x^{22}$

$$3.104 \quad \int \frac{(a+bx^2)^8}{x^{25}} dx$$

**Optimal.** Leaf size=84

$$\frac{b^3 (a+bx^2)^9}{3960a^4x^{18}} - \frac{b^2 (a+bx^2)^9}{440a^3x^{20}} + \frac{b (a+bx^2)^9}{88a^2x^{22}} - \frac{(a+bx^2)^9}{24ax^{24}}$$

[Out]  $-(a + b*x^2)^9/(24*a*x^{24}) + (b*(a + b*x^2)^9)/(88*a^2*x^{22}) - (b^2*(a + b*x^2)^9)/(440*a^3*x^{20}) + (b^3*(a + b*x^2)^9)/(3960*a^4*x^{18})$

**Rubi [A]** time = 0.120124, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{b^3 (a+bx^2)^9}{3960a^4x^{18}} - \frac{b^2 (a+bx^2)^9}{440a^3x^{20}} + \frac{b (a+bx^2)^9}{88a^2x^{22}} - \frac{(a+bx^2)^9}{24ax^{24}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^25, x]

[Out]  $-(a + b*x^2)^9/(24*a*x^{24}) + (b*(a + b*x^2)^9)/(88*a^2*x^{22}) - (b^2*(a + b*x^2)^9)/(440*a^3*x^{20}) + (b^3*(a + b*x^2)^9)/(3960*a^4*x^{18})$

**Rubi in Sympy [A]** time = 13.1662, size = 73, normalized size = 0.87

$$-\frac{(a+bx^2)^9}{24ax^{24}} + \frac{b(a+bx^2)^9}{88a^2x^{22}} - \frac{b^2(a+bx^2)^9}{440a^3x^{20}} + \frac{b^3(a+bx^2)^9}{3960a^4x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*25, x)

[Out]  $-(a + b*x^2)^9/(24*a*x^{24}) + b*(a + b*x^2)^9/(88*a^2*x^{22}) - b^2*(a + b*x^2)^9/(440*a^3*x^{20}) + b^3*(a + b*x^2)^9/(3960*a^4*x^{18})$

**Mathematica [A]** time = 0.00809301, size = 106, normalized size = 1.26

$$-\frac{a^8}{24x^{24}} - \frac{4a^7b}{11x^{22}} - \frac{7a^6b^2}{5x^{20}} - \frac{28a^5b^3}{9x^{18}} - \frac{35a^4b^4}{8x^{16}} - \frac{4a^3b^5}{x^{14}} - \frac{7a^2b^6}{3x^{12}} - \frac{4ab^7}{5x^{10}} - \frac{b^8}{8x^8}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^25, x]

[Out]  $-a^8/(24*x^{24}) - (4*a^7*b)/(11*x^{22}) - (7*a^6*b^2)/(5*x^{20}) - (28*a^5*b^3)/(9*x^{18}) - (35*a^4*b^4)/(8*x^{16}) - (4*a^3*b^5)/x^{14} - (7*a^2*b^6)/(3*x^{12}) - (4*a*b^7)/(5*x^{10}) - b^8/(8*x^8)$

**Maple [A]** time = 0.01, size = 91, normalized size = 1.1

$$-\frac{7a^2b^6}{3x^{12}} - \frac{35a^4b^4}{8x^{16}} - \frac{4ab^7}{5x^{10}} - \frac{7a^6b^2}{5x^{20}} - \frac{a^8}{24x^{24}} - \frac{b^8}{8x^8} - \frac{28a^5b^3}{9x^{18}} - 4\frac{a^3b^5}{x^{14}} - \frac{4a^7b}{11x^{22}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^25,x)`

[Out] 
$$\frac{-7/3*a^2*b^6/x^{12}-35/8*a^4*b^4/x^{16}-4/5*a*b^7/x^{10}-7/5*a^6*b^2/x^{20}-1/24*a^8/x^{24}-1/8*b^8/x^8-28/9*a^5*b^3/x^{18}-4*a^3*b^5/x^{14}-4/11*a^7*b/x^{22}}$$

---

**Maxima [A]** time = 1.34394, size = 124, normalized size = 1.48

$$\frac{495 b^8 x^{16} + 3168 a b^7 x^{14} + 9240 a^2 b^6 x^{12} + 15840 a^3 b^5 x^{10} + 17325 a^4 b^4 x^8 + 12320 a^5 b^3 x^6 + 5544 a^6 b^2 x^4 + 1440 a^7 b x^2 + 165 a^8}{3960 x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^25,x, algorithm="maxima")`

[Out] 
$$-1/3960*(495*b^8*x^{16} + 3168*a*b^7*x^{14} + 9240*a^2*b^6*x^{12} + 15840*a^3*b^5*x^{10} + 17325*a^4*b^4*x^8 + 12320*a^5*b^3*x^6 + 5544*a^6*b^2*x^4 + 1440*a^7*b*x^2 + 165*a^8)/x^{24}$$

---

**Fricas [A]** time = 0.194779, size = 124, normalized size = 1.48

$$\frac{495 b^8 x^{16} + 3168 a b^7 x^{14} + 9240 a^2 b^6 x^{12} + 15840 a^3 b^5 x^{10} + 17325 a^4 b^4 x^8 + 12320 a^5 b^3 x^6 + 5544 a^6 b^2 x^4 + 1440 a^7 b x^2 + 165 a^8}{3960 x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^25,x, algorithm="fricas")`

[Out] 
$$-1/3960*(495*b^8*x^{16} + 3168*a*b^7*x^{14} + 9240*a^2*b^6*x^{12} + 15840*a^3*b^5*x^{10} + 17325*a^4*b^4*x^8 + 12320*a^5*b^3*x^6 + 5544*a^6*b^2*x^4 + 1440*a^7*b*x^2 + 165*a^8)/x^{24}$$

---

**Sympy [A]** time = 4.71054, size = 99, normalized size = 1.18

$$\frac{165 a^8 + 1440 a^7 b x^2 + 5544 a^6 b^2 x^4 + 12320 a^5 b^3 x^6 + 17325 a^4 b^4 x^8 + 15840 a^3 b^5 x^{10} + 9240 a^2 b^6 x^{12} + 3168 a b^7 x^{14} + 495 b^8 x^{16}}{3960 x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**25,x)`

[Out] 
$$-(165*a**8 + 1440*a**7*b*x**2 + 5544*a**6*b**2*x**4 + 12320*a**5*b**3*x**6 + 17325*a**4*b**4*x**8 + 15840*a**3*b**5*x**10 + 9240*a**2*b**6*x**12 + 3168*a*b**7*x**14 + 495*b**8*x**16)/(3960*x**24)$$

---

**GIAC/XCAS [A]** time = 0.212724, size = 124, normalized size = 1.48

$$\frac{495 b^8 x^{16} + 3168 a b^7 x^{14} + 9240 a^2 b^6 x^{12} + 15840 a^3 b^5 x^{10} + 17325 a^4 b^4 x^8 + 12320 a^5 b^3 x^6 + 5544 a^6 b^2 x^4 + 1440 a^7 b x^2 + 165 a^8}{3960 x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^25,x, algorithm="giac")`

```
[Out] -1/3960*(495*b^8*x^16 + 3168*a*b^7*x^14 + 9240*a^2*b^6*x^12 + 15840*a^3*b^5*x^10 + 17325*a^4*b^4*x^8 + 12320*a^5*b^3*x^6 + 5544*a^6*b^2*x^4 + 1440*a^7*b*x^2 + 165*a^8)/x^24
```

$$3.105 \quad \int \frac{(a+bx^2)^8}{x^{27}} dx$$

Optimal. Leaf size=106

$$-\frac{b^4 (a+bx^2)^9}{12870a^5x^{18}} + \frac{b^3 (a+bx^2)^9}{1430a^4x^{20}} - \frac{b^2 (a+bx^2)^9}{286a^3x^{22}} + \frac{b (a+bx^2)^9}{78a^2x^{24}} - \frac{(a+bx^2)^9}{26ax^{26}}$$

[Out]  $-(a + b*x^2)^9/(26*a*x^{26}) + (b*(a + b*x^2)^9)/(78*a^2*x^{24}) - (b^2*(a + b*x^2)^9)/(286*a^3*x^{22}) + (b^3*(a + b*x^2)^9)/(1430*a^4*x^{20}) - (b^4*(a + b*x^2)^9)/(12870*a^5*x^{18})$

Rubi [A] time = 0.154255, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{b^4 (a+bx^2)^9}{12870a^5x^{18}} + \frac{b^3 (a+bx^2)^9}{1430a^4x^{20}} - \frac{b^2 (a+bx^2)^9}{286a^3x^{22}} + \frac{b (a+bx^2)^9}{78a^2x^{24}} - \frac{(a+bx^2)^9}{26ax^{26}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^27, x]

[Out]  $-(a + b*x^2)^9/(26*a*x^{26}) + (b*(a + b*x^2)^9)/(78*a^2*x^{24}) - (b^2*(a + b*x^2)^9)/(286*a^3*x^{22}) + (b^3*(a + b*x^2)^9)/(1430*a^4*x^{20}) - (b^4*(a + b*x^2)^9)/(12870*a^5*x^{18})$

Rubi in Sympy [A] time = 24.3363, size = 105, normalized size = 0.99

$$-\frac{a^8}{26x^{26}} - \frac{a^7b}{3x^{24}} - \frac{14a^6b^2}{11x^{22}} - \frac{14a^5b^3}{5x^{20}} - \frac{35a^4b^4}{9x^{18}} - \frac{7a^3b^5}{2x^{16}} - \frac{2a^2b^6}{x^{14}} - \frac{2ab^7}{3x^{12}} - \frac{b^8}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*27, x)

[Out]  $-a**8/(26*x**26) - a**7*b/(3*x**24) - 14*a**6*b**2/(11*x**22) - 14*a**5*b**3/(5*x**20) - 35*a**4*b**4/(9*x**18) - 7*a**3*b**5/(2*x**16) - 2*a**2*b**6/x**14 - 2*a*b**7/(3*x**12) - b**8/(10*x**10)$

Mathematica [A] time = 0.00835444, size = 106, normalized size = 1.

$$-\frac{a^8}{26x^{26}} - \frac{a^7b}{3x^{24}} - \frac{14a^6b^2}{11x^{22}} - \frac{14a^5b^3}{5x^{20}} - \frac{35a^4b^4}{9x^{18}} - \frac{7a^3b^5}{2x^{16}} - \frac{2a^2b^6}{x^{14}} - \frac{2ab^7}{3x^{12}} - \frac{b^8}{10x^{10}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^27, x]

[Out]  $-a^8/(26*x^{26}) - (a^7*b)/(3*x^{24}) - (14*a^6*b^2)/(11*x^{22}) - (14*a^5*b^3)/(5*x^{20}) - (35*a^4*b^4)/(9*x^{18}) - (7*a^3*b^5)/(2*x^{16}) - (2*a^2*b^6)/x^{14} - (2*a*b^7)/(3*x^{12}) - b^8/(10*x^{10})$

Maple [A] time = 0.01, size = 91, normalized size = 0.9

$$-\frac{a^8}{26x^{26}} - \frac{14a^6b^2}{11x^{22}} - \frac{2ab^7}{3x^{12}} - \frac{b^8}{10x^{10}} - \frac{35a^4b^4}{9x^{18}} - \frac{a^7b}{3x^{24}} - \frac{14a^5b^3}{5x^{20}} - 2\frac{a^2b^6}{x^{14}} - \frac{7a^3b^5}{2x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^27,x)`

[Out] 
$$\frac{-1/26*a^8/x^{26}-14/11*a^6*b^2/x^{22}-2/3*a*b^7/x^{12}-1/10*b^8/x^{10}-35/9*a^4*b^4/x^{18}-1/3*a^7*b/x^{24}-14/5*a^5*b^3/x^{20}-2*a^2*b^6/x^{14}-7/2*a^3*b^5/x^{16}}$$

**Maxima [A]** time = 1.36105, size = 124, normalized size = 1.17

$$\frac{1287 b^8 x^{16} + 8580 a b^7 x^{14} + 25740 a^2 b^6 x^{12} + 45045 a^3 b^5 x^{10} + 50050 a^4 b^4 x^8 + 36036 a^5 b^3 x^6 + 16380 a^6 b^2 x^4 + 4290 a^7 b x^2 + 12870 a^8}{12870 x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^27,x, algorithm="maxima")`

[Out] 
$$\frac{-1/12870*(1287*b^8*x^{16} + 8580*a*b^7*x^{14} + 25740*a^2*b^6*x^{12} + 45045*a^3*b^5*x^{10} + 50050*a^4*b^4*x^8 + 36036*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 4290*a^7*b*x^2 + 495*a^8)/x^{26}}$$

**Fricas [A]** time = 0.197092, size = 124, normalized size = 1.17

$$\frac{1287 b^8 x^{16} + 8580 a b^7 x^{14} + 25740 a^2 b^6 x^{12} + 45045 a^3 b^5 x^{10} + 50050 a^4 b^4 x^8 + 36036 a^5 b^3 x^6 + 16380 a^6 b^2 x^4 + 4290 a^7 b x^2 + 12870 a^8}{12870 x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^27,x, algorithm="fricas")`

[Out] 
$$\frac{-1/12870*(1287*b^8*x^{16} + 8580*a*b^7*x^{14} + 25740*a^2*b^6*x^{12} + 45045*a^3*b^5*x^{10} + 50050*a^4*b^4*x^8 + 36036*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 4290*a^7*b*x^2 + 495*a^8)/x^{26}}$$

**Sympy [A]** time = 4.95066, size = 99, normalized size = 0.93

$$\frac{495 a^8 + 4290 a^7 b x^2 + 16380 a^6 b^2 x^4 + 36036 a^5 b^3 x^6 + 50050 a^4 b^4 x^8 + 45045 a^3 b^5 x^{10} + 25740 a^2 b^6 x^{12} + 8580 a b^7 x^{14} + 1287 b^8}{12870 x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**27,x)`

[Out] 
$$-(495*a**8 + 4290*a**7*b*x**2 + 16380*a**6*b**2*x**4 + 36036*a**5*b**3*x**6 + 50050*a**4*b**4*x**8 + 45045*a**3*b**5*x**10 + 25740*a**2*b**6*x**12 + 8580*a*b**7*x**14 + 1287*b**8*x**16)/(12870*x**26)$$

**GIAC/XCAS [A]** time = 0.210294, size = 124, normalized size = 1.17

$$\frac{1287 b^8 x^{16} + 8580 a b^7 x^{14} + 25740 a^2 b^6 x^{12} + 45045 a^3 b^5 x^{10} + 50050 a^4 b^4 x^8 + 36036 a^5 b^3 x^6 + 16380 a^6 b^2 x^4 + 4290 a^7 b x^2 + 12870 a^8}{12870 x^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^27,x, algorithm="giac")`

```
[Out] -1/12870*(1287*b^8*x^16 + 8580*a*b^7*x^14 + 25740*a^2*b^6*x^12 +  
45045*a^3*b^5*x^10 + 50050*a^4*b^4*x^8 + 36036*a^5*b^3*x^6 + 1638  
0*a^6*b^2*x^4 + 4290*a^7*b*x^2 + 495*a^8)/x^26
```

$$3.106 \quad \int \frac{(a+bx^2)^8}{x^{29}} dx$$

**Optimal.** Leaf size=108

$$-\frac{a^8}{28x^{28}} - \frac{4a^7b}{13x^{26}} - \frac{7a^6b^2}{6x^{24}} - \frac{28a^5b^3}{11x^{22}} - \frac{7a^4b^4}{2x^{20}} - \frac{28a^3b^5}{9x^{18}} - \frac{7a^2b^6}{4x^{16}} - \frac{4ab^7}{7x^{14}} - \frac{b^8}{12x^{12}}$$

[Out]  $-a^8/(28*x^{28}) - (4*a^7*b)/(13*x^{26}) - (7*a^6*b^2)/(6*x^{24}) - (28*a^5*b^3)/(11*x^{22}) - (7*a^4*b^4)/(2*x^{20}) - (28*a^3*b^5)/(9*x^{18}) - (7*a^2*b^6)/(4*x^{16}) - (4*a*b^7)/(7*x^{14}) - b^8/(12*x^{12})$

**Rubi [A]** time = 0.139638, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^8}{28x^{28}} - \frac{4a^7b}{13x^{26}} - \frac{7a^6b^2}{6x^{24}} - \frac{28a^5b^3}{11x^{22}} - \frac{7a^4b^4}{2x^{20}} - \frac{28a^3b^5}{9x^{18}} - \frac{7a^2b^6}{4x^{16}} - \frac{4ab^7}{7x^{14}} - \frac{b^8}{12x^{12}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^29, x]

[Out]  $-a^8/(28*x^{28}) - (4*a^7*b)/(13*x^{26}) - (7*a^6*b^2)/(6*x^{24}) - (28*a^5*b^3)/(11*x^{22}) - (7*a^4*b^4)/(2*x^{20}) - (28*a^3*b^5)/(9*x^{18}) - (7*a^2*b^6)/(4*x^{16}) - (4*a*b^7)/(7*x^{14}) - b^8/(12*x^{12})$

**Rubi in Sympy [A]** time = 24.5745, size = 109, normalized size = 1.01

$$-\frac{a^8}{28x^{28}} - \frac{4a^7b}{13x^{26}} - \frac{7a^6b^2}{6x^{24}} - \frac{28a^5b^3}{11x^{22}} - \frac{7a^4b^4}{2x^{20}} - \frac{28a^3b^5}{9x^{18}} - \frac{7a^2b^6}{4x^{16}} - \frac{4ab^7}{7x^{14}} - \frac{b^8}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*29, x)

[Out]  $-a**8/(28*x**28) - 4*a**7*b/(13*x**26) - 7*a**6*b**2/(6*x**24) - 28*a**5*b**3/(11*x**22) - 7*a**4*b**4/(2*x**20) - 28*a**3*b**5/(9*x**18) - 7*a**2*b**6/(4*x**16) - 4*a*b**7/(7*x**14) - b**8/(12*x**12)$

**Mathematica [A]** time = 0.00801749, size = 108, normalized size = 1.

$$-\frac{a^8}{28x^{28}} - \frac{4a^7b}{13x^{26}} - \frac{7a^6b^2}{6x^{24}} - \frac{28a^5b^3}{11x^{22}} - \frac{7a^4b^4}{2x^{20}} - \frac{28a^3b^5}{9x^{18}} - \frac{7a^2b^6}{4x^{16}} - \frac{4ab^7}{7x^{14}} - \frac{b^8}{12x^{12}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^29, x]

[Out]  $-a^8/(28*x^{28}) - (4*a^7*b)/(13*x^{26}) - (7*a^6*b^2)/(6*x^{24}) - (28*a^5*b^3)/(11*x^{22}) - (7*a^4*b^4)/(2*x^{20}) - (28*a^3*b^5)/(9*x^{18}) - (7*a^2*b^6)/(4*x^{16}) - (4*a*b^7)/(7*x^{14}) - b^8/(12*x^{12})$

**Maple [A]** time = 0.009, size = 91, normalized size = 0.8

$$-\frac{a^8}{28x^{28}} - \frac{4a^7b}{13x^{26}} - \frac{7a^6b^2}{6x^{24}} - \frac{28a^5b^3}{11x^{22}} - \frac{7a^4b^4}{2x^{20}} - \frac{28a^3b^5}{9x^{18}} - \frac{7a^2b^6}{4x^{16}} - \frac{4ab^7}{7x^{14}} - \frac{b^8}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^29,x)`

[Out] 
$$-1/28*a^8/x^28 - 4/13*a^7*b/x^26 - 7/6*a^6*b^2/x^24 - 28/11*a^5*b^3/x^22 - 7/2*a^4*b^4/x^20 - 28/9*a^3*b^5/x^18 - 7/4*a^2*b^6/x^16 - 4/7*a*b^7/x^14 - 1/12*b^8/x^12$$

---

**Maxima [A]** time = 1.34309, size = 124, normalized size = 1.15

$$\frac{3003 b^8 x^{16} + 20592 a b^7 x^{14} + 63063 a^2 b^6 x^{12} + 112112 a^3 b^5 x^{10} + 126126 a^4 b^4 x^8 + 91728 a^5 b^3 x^6 + 42042 a^6 b^2 x^4 + 11088 a^7 b x^2 + 3003 b^8}{36036 x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^29,x, algorithm="maxima")`

[Out] 
$$-1/36036*(3003*b^8*x^{16} + 20592*a*b^7*x^{14} + 63063*a^2*b^6*x^{12} + 112112*a^3*b^5*x^{10} + 126126*a^4*b^4*x^8 + 91728*a^5*b^3*x^6 + 42042*a^6*b^2*x^4 + 11088*a^7*b*x^2 + 1287*a^8)/x^{28}$$

---

**Fricas [A]** time = 0.197155, size = 124, normalized size = 1.15

$$\frac{3003 b^8 x^{16} + 20592 a b^7 x^{14} + 63063 a^2 b^6 x^{12} + 112112 a^3 b^5 x^{10} + 126126 a^4 b^4 x^8 + 91728 a^5 b^3 x^6 + 42042 a^6 b^2 x^4 + 11088 a^7 b x^2 + 3003 b^8}{36036 x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^29,x, algorithm="fricas")`

[Out] 
$$-1/36036*(3003*b^8*x^{16} + 20592*a*b^7*x^{14} + 63063*a^2*b^6*x^{12} + 112112*a^3*b^5*x^{10} + 126126*a^4*b^4*x^8 + 91728*a^5*b^3*x^6 + 42042*a^6*b^2*x^4 + 11088*a^7*b*x^2 + 1287*a^8)/x^{28}$$

---

**Sympy [A]** time = 5.32416, size = 99, normalized size = 0.92

$$\frac{1287a^8 + 11088a^7bx^2 + 42042a^6b^2x^4 + 91728a^5b^3x^6 + 126126a^4b^4x^8 + 112112a^3b^5x^{10} + 63063a^2b^6x^{12} + 20592ab^7x^{14} + 3003b^8}{36036x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**29,x)`

[Out] 
$$-(1287*a**8 + 11088*a**7*b*x**2 + 42042*a**6*b**2*x**4 + 91728*a**5*b**3*x**6 + 126126*a**4*b**4*x**8 + 112112*a**3*b**5*x**10 + 63063*a**2*b**6*x**12 + 20592*a*b**7*x**14 + 3003*b**8*x**16)/(36036*x**28)$$

---

**GIAC/XCAS [A]** time = 0.213164, size = 124, normalized size = 1.15

$$\frac{3003 b^8 x^{16} + 20592 a b^7 x^{14} + 63063 a^2 b^6 x^{12} + 112112 a^3 b^5 x^{10} + 126126 a^4 b^4 x^8 + 91728 a^5 b^3 x^6 + 42042 a^6 b^2 x^4 + 11088 a^7 b x^2 + 3003 b^8}{36036 x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^29,x, algorithm="giac")`

```
[Out] -1/36036*(3003*b^8*x^16 + 20592*a*b^7*x^14 + 63063*a^2*b^6*x^12 +  
112112*a^3*b^5*x^10 + 126126*a^4*b^4*x^8 + 91728*a^5*b^3*x^6 + 4  
2042*a^6*b^2*x^4 + 11088*a^7*b*x^2 + 1287*a^8)/x^28
```



$$3.107 \quad \int \frac{(a+bx^2)^8}{x^{31}} dx$$

**Optimal.** Leaf size=108

$$-\frac{a^8}{30x^{30}} - \frac{2a^7b}{7x^{28}} - \frac{14a^6b^2}{13x^{26}} - \frac{7a^5b^3}{3x^{24}} - \frac{35a^4b^4}{11x^{22}} - \frac{14a^3b^5}{5x^{20}} - \frac{14a^2b^6}{9x^{18}} - \frac{ab^7}{2x^{16}} - \frac{b^8}{14x^{14}}$$

[Out]  $-a^8/(30*x^{30}) - (2*a^7*b)/(7*x^{28}) - (14*a^6*b^2)/(13*x^{26}) - (7*a^5*b^3)/(3*x^{24}) - (35*a^4*b^4)/(11*x^{22}) - (14*a^3*b^5)/(5*x^{20}) - (14*a^2*b^6)/(9*x^{18}) - (a*b^7)/(2*x^{16}) - b^8/(14*x^{14})$

**Rubi [A]** time = 0.13646, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^8}{30x^{30}} - \frac{2a^7b}{7x^{28}} - \frac{14a^6b^2}{13x^{26}} - \frac{7a^5b^3}{3x^{24}} - \frac{35a^4b^4}{11x^{22}} - \frac{14a^3b^5}{5x^{20}} - \frac{14a^2b^6}{9x^{18}} - \frac{ab^7}{2x^{16}} - \frac{b^8}{14x^{14}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^31, x]

[Out]  $-a^8/(30*x^{30}) - (2*a^7*b)/(7*x^{28}) - (14*a^6*b^2)/(13*x^{26}) - (7*a^5*b^3)/(3*x^{24}) - (35*a^4*b^4)/(11*x^{22}) - (14*a^3*b^5)/(5*x^{20}) - (14*a^2*b^6)/(9*x^{18}) - (a*b^7)/(2*x^{16}) - b^8/(14*x^{14})$

**Rubi in Sympy [A]** time = 24.4194, size = 107, normalized size = 0.99

$$-\frac{a^8}{30x^{30}} - \frac{2a^7b}{7x^{28}} - \frac{14a^6b^2}{13x^{26}} - \frac{7a^5b^3}{3x^{24}} - \frac{35a^4b^4}{11x^{22}} - \frac{14a^3b^5}{5x^{20}} - \frac{14a^2b^6}{9x^{18}} - \frac{ab^7}{2x^{16}} - \frac{b^8}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*31, x)

[Out]  $-a**8/(30*x**30) - 2*a**7*b/(7*x**28) - 14*a**6*b**2/(13*x**26) - 7*a**5*b**3/(3*x**24) - 35*a**4*b**4/(11*x**22) - 14*a**3*b**5/(5*x**20) - 14*a**2*b**6/(9*x**18) - a*b**7/(2*x**16) - b**8/(14*x**14)$

**Mathematica [A]** time = 0.00810901, size = 108, normalized size = 1.

$$-\frac{a^8}{30x^{30}} - \frac{2a^7b}{7x^{28}} - \frac{14a^6b^2}{13x^{26}} - \frac{7a^5b^3}{3x^{24}} - \frac{35a^4b^4}{11x^{22}} - \frac{14a^3b^5}{5x^{20}} - \frac{14a^2b^6}{9x^{18}} - \frac{ab^7}{2x^{16}} - \frac{b^8}{14x^{14}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^31, x]

[Out]  $-a^8/(30*x^{30}) - (2*a^7*b)/(7*x^{28}) - (14*a^6*b^2)/(13*x^{26}) - (7*a^5*b^3)/(3*x^{24}) - (35*a^4*b^4)/(11*x^{22}) - (14*a^3*b^5)/(5*x^{20}) - (14*a^2*b^6)/(9*x^{18}) - (a*b^7)/(2*x^{16}) - b^8/(14*x^{14})$

**Maple [A]** time = 0.009, size = 91, normalized size = 0.8

$$-\frac{a^8}{30x^{30}} - \frac{2a^7b}{7x^{28}} - \frac{14a^6b^2}{13x^{26}} - \frac{7a^5b^3}{3x^{24}} - \frac{35a^4b^4}{11x^{22}} - \frac{14a^3b^5}{5x^{20}} - \frac{14a^2b^6}{9x^{18}} - \frac{ab^7}{2x^{16}} - \frac{b^8}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^31,x)`

[Out] 
$$-1/30*a^8/x^30 - 2/7*a^7*b/x^28 - 14/13*a^6*b^2/x^26 - 7/3*a^5*b^3/x^24 - 35/11*a^4*b^4/x^22 - 14/5*a^3*b^5/x^20 - 14/9*a^2*b^6/x^18 - 1/2*a*b^7/x^16 - 1/14*b^8/x^14$$

---

**Maxima [A]** time = 1.32672, size = 124, normalized size = 1.15

$$\frac{6435 b^8 x^{16} + 45045 a b^7 x^{14} + 140140 a^2 b^6 x^{12} + 252252 a^3 b^5 x^{10} + 286650 a^4 b^4 x^8 + 210210 a^5 b^3 x^6 + 97020 a^6 b^2 x^4 + 25740 a^7 b x^2 + 3003 a^8}{90090 x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^31,x, algorithm="maxima")`

[Out] 
$$-1/90090*(6435*b^8*x^{16} + 45045*a*b^7*x^{14} + 140140*a^2*b^6*x^{12} + 252252*a^3*b^5*x^{10} + 286650*a^4*b^4*x^8 + 210210*a^5*b^3*x^6 + 97020*a^6*b^2*x^4 + 25740*a^7*b*x^2 + 3003*a^8)/x^{30}$$

---

**Fricas [A]** time = 0.193265, size = 124, normalized size = 1.15

$$\frac{6435 b^8 x^{16} + 45045 a b^7 x^{14} + 140140 a^2 b^6 x^{12} + 252252 a^3 b^5 x^{10} + 286650 a^4 b^4 x^8 + 210210 a^5 b^3 x^6 + 97020 a^6 b^2 x^4 + 25740 a^7 b x^2 + 3003 a^8}{90090 x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^31,x, algorithm="fricas")`

[Out] 
$$-1/90090*(6435*b^8*x^{16} + 45045*a*b^7*x^{14} + 140140*a^2*b^6*x^{12} + 252252*a^3*b^5*x^{10} + 286650*a^4*b^4*x^8 + 210210*a^5*b^3*x^6 + 97020*a^6*b^2*x^4 + 25740*a^7*b*x^2 + 3003*a^8)/x^{30}$$

---

**Sympy [A]** time = 5.58287, size = 99, normalized size = 0.92

$$\frac{3003 a^8 + 25740 a^7 b x^2 + 97020 a^6 b^2 x^4 + 210210 a^5 b^3 x^6 + 286650 a^4 b^4 x^8 + 252252 a^3 b^5 x^{10} + 140140 a^2 b^6 x^{12} + 45045 a b^7 x^{14} + 3003 a^8}{90090 x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**31,x)`

[Out] 
$$-(3003*a**8 + 25740*a**7*b*x**2 + 97020*a**6*b**2*x**4 + 210210*a**5*b**3*x**6 + 286650*a**4*b**4*x**8 + 252252*a**3*b**5*x**10 + 140140*a**2*b**6*x**12 + 45045*a*b**7*x**14 + 6435*b**8*x**16)/(90090*x**30)$$

---

**GIAC/XCAS [A]** time = 0.207922, size = 124, normalized size = 1.15

$$\frac{6435 b^8 x^{16} + 45045 a b^7 x^{14} + 140140 a^2 b^6 x^{12} + 252252 a^3 b^5 x^{10} + 286650 a^4 b^4 x^8 + 210210 a^5 b^3 x^6 + 97020 a^6 b^2 x^4 + 25740 a^7 b x^2 + 3003 a^8}{90090 x^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^31,x, algorithm="giac")`

```
[Out] -1/90090*(6435*b^8*x^16 + 45045*a*b^7*x^14 + 140140*a^2*b^6*x^12  
+ 252252*a^3*b^5*x^10 + 286650*a^4*b^4*x^8 + 210210*a^5*b^3*x^6 +  
97020*a^6*b^2*x^4 + 25740*a^7*b*x^2 + 3003*a^8)/x^30
```

$$3.108 \quad \int \frac{(a+bx^2)^8}{x^{33}} dx$$

**Optimal.** Leaf size=106

$$-\frac{a^8}{32x^{32}} - \frac{4a^7b}{15x^{30}} - \frac{a^6b^2}{x^{28}} - \frac{28a^5b^3}{13x^{26}} - \frac{35a^4b^4}{12x^{24}} - \frac{28a^3b^5}{11x^{22}} - \frac{7a^2b^6}{5x^{20}} - \frac{4ab^7}{9x^{18}} - \frac{b^8}{16x^{16}}$$

[Out]  $-a^8/(32*x^{32}) - (4*a^7*b)/(15*x^{30}) - (a^6*b^2)/x^{28} - (28*a^5*b^3)/(13*x^{26}) - (35*a^4*b^4)/(12*x^{24}) - (28*a^3*b^5)/(11*x^{22}) - (7*a^2*b^6)/(5*x^{20}) - (4*a*b^7)/(9*x^{18}) - b^8/(16*x^{16})$

**Rubi [A]** time = 0.136572, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^8}{32x^{32}} - \frac{4a^7b}{15x^{30}} - \frac{a^6b^2}{x^{28}} - \frac{28a^5b^3}{13x^{26}} - \frac{35a^4b^4}{12x^{24}} - \frac{28a^3b^5}{11x^{22}} - \frac{7a^2b^6}{5x^{20}} - \frac{4ab^7}{9x^{18}} - \frac{b^8}{16x^{16}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^33, x]

[Out]  $-a^8/(32*x^{32}) - (4*a^7*b)/(15*x^{30}) - (a^6*b^2)/x^{28} - (28*a^5*b^3)/(13*x^{26}) - (35*a^4*b^4)/(12*x^{24}) - (28*a^3*b^5)/(11*x^{22}) - (7*a^2*b^6)/(5*x^{20}) - (4*a*b^7)/(9*x^{18}) - b^8/(16*x^{16})$

**Rubi in Sympy [A]** time = 24.4739, size = 105, normalized size = 0.99

$$-\frac{a^8}{32x^{32}} - \frac{4a^7b}{15x^{30}} - \frac{a^6b^2}{x^{28}} - \frac{28a^5b^3}{13x^{26}} - \frac{35a^4b^4}{12x^{24}} - \frac{28a^3b^5}{11x^{22}} - \frac{7a^2b^6}{5x^{20}} - \frac{4ab^7}{9x^{18}} - \frac{b^8}{16x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*33, x)

[Out]  $-a^{**8}/(32*x^{**32}) - 4*a^{**7}*b/(15*x^{**30}) - a^{**6}*b^{**2}/x^{**28} - 28*a^{**5}*b^{**3}/(13*x^{**26}) - 35*a^{**4}*b^{**4}/(12*x^{**24}) - 28*a^{**3}*b^{**5}/(11*x^{**22}) - 7*a^{**2}*b^{**6}/(5*x^{**20}) - 4*a*b^{**7}/(9*x^{**18}) - b^{**8}/(16*x^{**16})$

**Mathematica [A]** time = 0.00780919, size = 106, normalized size = 1.

$$-\frac{a^8}{32x^{32}} - \frac{4a^7b}{15x^{30}} - \frac{a^6b^2}{x^{28}} - \frac{28a^5b^3}{13x^{26}} - \frac{35a^4b^4}{12x^{24}} - \frac{28a^3b^5}{11x^{22}} - \frac{7a^2b^6}{5x^{20}} - \frac{4ab^7}{9x^{18}} - \frac{b^8}{16x^{16}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^33, x]

[Out]  $-a^8/(32*x^{32}) - (4*a^7*b)/(15*x^{30}) - (a^6*b^2)/x^{28} - (28*a^5*b^3)/(13*x^{26}) - (35*a^4*b^4)/(12*x^{24}) - (28*a^3*b^5)/(11*x^{22}) - (7*a^2*b^6)/(5*x^{20}) - (4*a*b^7)/(9*x^{18}) - b^8/(16*x^{16})$

**Maple [A]** time = 0.01, size = 91, normalized size = 0.9

$$-\frac{a^8}{32x^{32}} - \frac{4a^7b}{15x^{30}} - \frac{a^6b^2}{x^{28}} - \frac{28a^5b^3}{13x^{26}} - \frac{35a^4b^4}{12x^{24}} - \frac{28a^3b^5}{11x^{22}} - \frac{7a^2b^6}{5x^{20}} - \frac{4ab^7}{9x^{18}} - \frac{b^8}{16x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^33,x)`

[Out] 
$$-1/32*a^8/x^32-4/15*a^7*b/x^30-a^6*b^2/x^28-28/13*a^5*b^3/x^26-35/12*a^4*b^4/x^24-28/11*a^3*b^5/x^22-7/5*a^2*b^6/x^20-4/9*a*b^7/x^18-1/16*b^8/x^16$$

**Maxima [A]** time = 1.34485, size = 124, normalized size = 1.17

$$\frac{12870 b^8 x^{16} + 91520 a b^7 x^{14} + 288288 a^2 b^6 x^{12} + 524160 a^3 b^5 x^{10} + 600600 a^4 b^4 x^8 + 443520 a^5 b^3 x^6 + 205920 a^6 b^2 x^4 + 54912 a^7 b x^2 + 6435 a^8}{205920 x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^33,x, algorithm="maxima")`

[Out] 
$$-1/205920*(12870*b^8*x^16 + 91520*a*b^7*x^14 + 288288*a^2*b^6*x^12 + 524160*a^3*b^5*x^10 + 600600*a^4*b^4*x^8 + 443520*a^5*b^3*x^6 + 205920*a^6*b^2*x^4 + 54912*a^7*b*x^2 + 6435*a^8)/x^32$$

**Fricas [A]** time = 0.19363, size = 124, normalized size = 1.17

$$\frac{12870 b^8 x^{16} + 91520 a b^7 x^{14} + 288288 a^2 b^6 x^{12} + 524160 a^3 b^5 x^{10} + 600600 a^4 b^4 x^8 + 443520 a^5 b^3 x^6 + 205920 a^6 b^2 x^4 + 54912 a^7 b x^2 + 6435 a^8}{205920 x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^33,x, algorithm="fricas")`

[Out] 
$$-1/205920*(12870*b^8*x^16 + 91520*a*b^7*x^14 + 288288*a^2*b^6*x^12 + 524160*a^3*b^5*x^10 + 600600*a^4*b^4*x^8 + 443520*a^5*b^3*x^6 + 205920*a^6*b^2*x^4 + 54912*a^7*b*x^2 + 6435*a^8)/x^32$$

**Sympy [A]** time = 5.85024, size = 99, normalized size = 0.93

$$\frac{6435 a^8 + 54912 a^7 b x^2 + 205920 a^6 b^2 x^4 + 443520 a^5 b^3 x^6 + 600600 a^4 b^4 x^8 + 524160 a^3 b^5 x^{10} + 288288 a^2 b^6 x^{12} + 91520 a b^7 x^{14} + 6435 a^8}{205920 x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**33,x)`

[Out] 
$$-(6435*a**8 + 54912*a**7*b*x**2 + 205920*a**6*b**2*x**4 + 443520*a**5*b**3*x**6 + 600600*a**4*b**4*x**8 + 524160*a**3*b**5*x**10 + 288288*a**2*b**6*x**12 + 91520*a*b**7*x**14 + 12870*b**8*x**16)/(205920*x**32)$$

**GIAC/XCAS [A]** time = 0.208936, size = 124, normalized size = 1.17

$$\frac{12870 b^8 x^{16} + 91520 a b^7 x^{14} + 288288 a^2 b^6 x^{12} + 524160 a^3 b^5 x^{10} + 600600 a^4 b^4 x^8 + 443520 a^5 b^3 x^6 + 205920 a^6 b^2 x^4 + 54912 a^7 b x^2 + 6435 a^8}{205920 x^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^33,x, algorithm="giac")`

```
[Out] -1/205920*(12870*b^8*x^16 + 91520*a*b^7*x^14 + 288288*a^2*b^6*x^12 + 524160*a^3*b^5*x^10 + 600600*a^4*b^4*x^8 + 443520*a^5*b^3*x^6 + 205920*a^6*b^2*x^4 + 54912*a^7*b*x^2 + 6435*a^8)/x^32
```

### 3.109 $\int x^8 (a + bx^2)^8 dx$

**Optimal.** Leaf size=108

$$\frac{a^8 x^9}{9} + \frac{8}{11} a^7 b x^{11} + \frac{28}{13} a^6 b^2 x^{13} + \frac{56}{15} a^5 b^3 x^{15} + \frac{70}{17} a^4 b^4 x^{17} + \frac{56}{19} a^3 b^5 x^{19} + \frac{4}{3} a^2 b^6 x^{21} + \frac{8}{23} a b^7 x^{23} + \frac{b^8 x^{25}}{25}$$

[Out] (a^8\*x^9)/9 + (8\*a^7\*b\*x^11)/11 + (28\*a^6\*b^2\*x^13)/13 + (56\*a^5\*b^3\*x^15)/15 + (70\*a^4\*b^4\*x^17)/17 + (56\*a^3\*b^5\*x^19)/19 + (4\*a^2\*b^6\*x^21)/3 + (8\*a\*b^7\*x^23)/23 + (b^8\*x^25)/25

**Rubi [A]** time = 0.122213, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{a^8 x^9}{9} + \frac{8}{11} a^7 b x^{11} + \frac{28}{13} a^6 b^2 x^{13} + \frac{56}{15} a^5 b^3 x^{15} + \frac{70}{17} a^4 b^4 x^{17} + \frac{56}{19} a^3 b^5 x^{19} + \frac{4}{3} a^2 b^6 x^{21} + \frac{8}{23} a b^7 x^{23} + \frac{b^8 x^{25}}{25}$$

Antiderivative was successfully verified.

[In] Int[x^8\*(a + b\*x^2)^8, x]

[Out] (a^8\*x^9)/9 + (8\*a^7\*b\*x^11)/11 + (28\*a^6\*b^2\*x^13)/13 + (56\*a^5\*b^3\*x^15)/15 + (70\*a^4\*b^4\*x^17)/17 + (56\*a^3\*b^5\*x^19)/19 + (4\*a^2\*b^6\*x^21)/3 + (8\*a\*b^7\*x^23)/23 + (b^8\*x^25)/25

**Rubi in Sympy [A]** time = 19.3627, size = 107, normalized size = 0.99

$$\frac{a^8 x^9}{9} + \frac{8 a^7 b x^{11}}{11} + \frac{28 a^6 b^2 x^{13}}{13} + \frac{56 a^5 b^3 x^{15}}{15} + \frac{70 a^4 b^4 x^{17}}{17} + \frac{56 a^3 b^5 x^{19}}{19} + \frac{4 a^2 b^6 x^{21}}{3} + \frac{8 a b^7 x^{23}}{23} + \frac{b^8 x^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*8\*(b\*x\*\*2+a)\*\*8, x)

[Out] a\*\*8\*x\*\*9/9 + 8\*a\*\*7\*b\*x\*\*11/11 + 28\*a\*\*6\*b\*\*2\*x\*\*13/13 + 56\*a\*\*5\*b\*\*3\*x\*\*15/15 + 70\*a\*\*4\*b\*\*4\*x\*\*17/17 + 56\*a\*\*3\*b\*\*5\*x\*\*19/19 + 4\*a\*\*2\*b\*\*6\*x\*\*21/3 + 8\*a\*b\*\*7\*x\*\*23/23 + b\*\*8\*x\*\*25/25

**Mathematica [A]** time = 0.00494918, size = 108, normalized size = 1.

$$\frac{a^8 x^9}{9} + \frac{8}{11} a^7 b x^{11} + \frac{28}{13} a^6 b^2 x^{13} + \frac{56}{15} a^5 b^3 x^{15} + \frac{70}{17} a^4 b^4 x^{17} + \frac{56}{19} a^3 b^5 x^{19} + \frac{4}{3} a^2 b^6 x^{21} + \frac{8}{23} a b^7 x^{23} + \frac{b^8 x^{25}}{25}$$

Antiderivative was successfully verified.

[In] Integrate[x^8\*(a + b\*x^2)^8, x]

[Out] (a^8\*x^9)/9 + (8\*a^7\*b\*x^11)/11 + (28\*a^6\*b^2\*x^13)/13 + (56\*a^5\*b^3\*x^15)/15 + (70\*a^4\*b^4\*x^17)/17 + (56\*a^3\*b^5\*x^19)/19 + (4\*a^2\*b^6\*x^21)/3 + (8\*a\*b^7\*x^23)/23 + (b^8\*x^25)/25

**Maple [A]** time = 0.002, size = 91, normalized size = 0.8

$$\frac{a^8 x^9}{9} + \frac{8 a^7 b x^{11}}{11} + \frac{28 a^6 b^2 x^{13}}{13} + \frac{56 a^5 b^3 x^{15}}{15} + \frac{70 a^4 b^4 x^{17}}{17} + \frac{56 a^3 b^5 x^{19}}{19} + \frac{4 a^2 b^6 x^{21}}{3} + \frac{8 a b^7 x^{23}}{23} + \frac{b^8 x^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8*(b*x^2+a)^8,x)`

[Out]  $\frac{1}{9}a^8x^9 + \frac{8}{11}a^7bx^{11} + \frac{28}{13}a^6b^2x^{13} + \frac{56}{15}a^5b^3x^{15} + \frac{70}{17}a^4b^4x^{17} + \frac{56}{19}a^3b^5x^{19} + \frac{4}{3}a^2b^6x^{21} + \frac{8}{23}ab^7x^{23} + \frac{1}{25}b^8x^{25}$

**Maxima [A]** time = 1.34413, size = 122, normalized size = 1.13

$$\frac{1}{25}b^8x^{25} + \frac{8}{23}ab^7x^{23} + \frac{4}{3}a^2b^6x^{21} + \frac{56}{19}a^3b^5x^{19} + \frac{70}{17}a^4b^4x^{17} + \frac{56}{15}a^5b^3x^{15} + \frac{28}{13}a^6b^2x^{13} + \frac{8}{11}a^7bx^{11} + \frac{1}{9}a^8x^9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^8,x, algorithm="maxima")`

[Out]  $\frac{1}{25}b^8x^{25} + \frac{8}{23}a^7bx^{23} + \frac{4}{3}a^2b^6x^{21} + \frac{56}{19}a^3b^5x^{19} + \frac{70}{17}a^4b^4x^{17} + \frac{56}{15}a^5b^3x^{15} + \frac{28}{13}a^6b^2x^{13} + \frac{8}{11}a^7bx^{11} + \frac{1}{9}a^8x^9$

**Fricas [A]** time = 0.179939, size = 1, normalized size = 0.01

$$\frac{1}{25}x^{25}b^8 + \frac{8}{23}x^{23}b^7a + \frac{4}{3}x^{21}b^6a^2 + \frac{56}{19}x^{19}b^5a^3 + \frac{70}{17}x^{17}b^4a^4 + \frac{56}{15}x^{15}b^3a^5 + \frac{28}{13}x^{13}b^2a^6 + \frac{8}{11}x^{11}ba^7 + \frac{1}{9}x^9a^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^8,x, algorithm="fricas")`

[Out]  $\frac{1}{25}x^{25}b^8 + \frac{8}{23}x^{23}b^7a + \frac{4}{3}x^{21}b^6a^2 + \frac{56}{19}x^{19}b^5a^3 + \frac{70}{17}x^{17}b^4a^4 + \frac{56}{15}x^{15}b^3a^5 + \frac{28}{13}x^{13}b^2a^6 + \frac{8}{11}x^{11}ba^7 + \frac{1}{9}x^9a^8$

**Sympy [A]** time = 0.163643, size = 107, normalized size = 0.99

$$\frac{a^8x^9}{9} + \frac{8a^7bx^{11}}{11} + \frac{28a^6b^2x^{13}}{13} + \frac{56a^5b^3x^{15}}{15} + \frac{70a^4b^4x^{17}}{17} + \frac{56a^3b^5x^{19}}{19} + \frac{4a^2b^6x^{21}}{3} + \frac{8ab^7x^{23}}{23} + \frac{b^8x^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*(b*x**2+a)**8,x)`

[Out]  $a^8x^9/9 + 8a^7bx^{11}/11 + 28a^6b^2x^{13}/13 + 56a^5b^3x^{15}/15 + 70a^4b^4x^{17}/17 + 56a^3b^5x^{19}/19 + 4a^2b^6x^{21}/3 + 8a^7bx^{23}/23 + b^8x^{25}/25$

**GIAC/XCAS [A]** time = 0.206783, size = 122, normalized size = 1.13

$$\frac{1}{25}b^8x^{25} + \frac{8}{23}ab^7x^{23} + \frac{4}{3}a^2b^6x^{21} + \frac{56}{19}a^3b^5x^{19} + \frac{70}{17}a^4b^4x^{17} + \frac{56}{15}a^5b^3x^{15} + \frac{28}{13}a^6b^2x^{13} + \frac{8}{11}a^7bx^{11} + \frac{1}{9}a^8x^9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^8,x, algorithm="giac")`



[Out]  $\frac{1}{25}b^8x^{25} + \frac{8}{23}ab^7x^{23} + \frac{4}{3}a^2b^6x^{21} + \frac{56}{19}a^3b^5x^{19} + \frac{70}{17}a^4b^4x^{17} + \frac{56}{15}a^5b^3x^{15} + \frac{28}{13}a^6b^2x^{13} + \frac{8}{11}a^7bx^{11} + \frac{1}{9}a^8x^9$

### 3.110 $\int x^6 (a + bx^2)^8 dx$

**Optimal.** Leaf size=108

$$\frac{a^8 x^7}{7} + \frac{8}{9} a^7 b x^9 + \frac{28}{11} a^6 b^2 x^{11} + \frac{56}{13} a^5 b^3 x^{13} + \frac{14}{3} a^4 b^4 x^{15} + \frac{56}{17} a^3 b^5 x^{17} + \frac{28}{19} a^2 b^6 x^{19} + \frac{8}{21} a b^7 x^{21} + \frac{b^8 x^{23}}{23}$$

[Out] (a^8\*x^7)/7 + (8\*a^7\*b\*x^9)/9 + (28\*a^6\*b^2\*x^11)/11 + (56\*a^5\*b^3\*x^13)/13 + (14\*a^4\*b^4\*x^15)/3 + (56\*a^3\*b^5\*x^17)/17 + (28\*a^2\*b^6\*x^19)/19 + (8\*a\*b^7\*x^21)/21 + (b^8\*x^23)/23

**Rubi [A]** time = 0.113772, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{a^8 x^7}{7} + \frac{8}{9} a^7 b x^9 + \frac{28}{11} a^6 b^2 x^{11} + \frac{56}{13} a^5 b^3 x^{13} + \frac{14}{3} a^4 b^4 x^{15} + \frac{56}{17} a^3 b^5 x^{17} + \frac{28}{19} a^2 b^6 x^{19} + \frac{8}{21} a b^7 x^{21} + \frac{b^8 x^{23}}{23}$$

Antiderivative was successfully verified.

[In] Int[x^6\*(a + b\*x^2)^8, x]

[Out] (a^8\*x^7)/7 + (8\*a^7\*b\*x^9)/9 + (28\*a^6\*b^2\*x^11)/11 + (56\*a^5\*b^3\*x^13)/13 + (14\*a^4\*b^4\*x^15)/3 + (56\*a^3\*b^5\*x^17)/17 + (28\*a^2\*b^6\*x^19)/19 + (8\*a\*b^7\*x^21)/21 + (b^8\*x^23)/23

**Rubi in Sympy [A]** time = 19.562, size = 107, normalized size = 0.99

$$\frac{a^8 x^7}{7} + \frac{8 a^7 b x^9}{9} + \frac{28 a^6 b^2 x^{11}}{11} + \frac{56 a^5 b^3 x^{13}}{13} + \frac{14 a^4 b^4 x^{15}}{3} + \frac{56 a^3 b^5 x^{17}}{17} + \frac{28 a^2 b^6 x^{19}}{19} + \frac{8 a b^7 x^{21}}{21} + \frac{b^8 x^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6\*(b\*x\*\*2+a)\*\*8, x)

[Out] a\*\*8\*x\*\*7/7 + 8\*a\*\*7\*b\*x\*\*9/9 + 28\*a\*\*6\*b\*\*2\*x\*\*11/11 + 56\*a\*\*5\*b\*\*3\*x\*\*13/13 + 14\*a\*\*4\*b\*\*4\*x\*\*15/3 + 56\*a\*\*3\*b\*\*5\*x\*\*17/17 + 28\*a\*\*2\*b\*\*6\*x\*\*19/19 + 8\*a\*b\*\*7\*x\*\*21/21 + b\*\*8\*x\*\*23/23

**Mathematica [A]** time = 0.00446472, size = 108, normalized size = 1.

$$\frac{a^8 x^7}{7} + \frac{8}{9} a^7 b x^9 + \frac{28}{11} a^6 b^2 x^{11} + \frac{56}{13} a^5 b^3 x^{13} + \frac{14}{3} a^4 b^4 x^{15} + \frac{56}{17} a^3 b^5 x^{17} + \frac{28}{19} a^2 b^6 x^{19} + \frac{8}{21} a b^7 x^{21} + \frac{b^8 x^{23}}{23}$$

Antiderivative was successfully verified.

[In] Integrate[x^6\*(a + b\*x^2)^8, x]

[Out] (a^8\*x^7)/7 + (8\*a^7\*b\*x^9)/9 + (28\*a^6\*b^2\*x^11)/11 + (56\*a^5\*b^3\*x^13)/13 + (14\*a^4\*b^4\*x^15)/3 + (56\*a^3\*b^5\*x^17)/17 + (28\*a^2\*b^6\*x^19)/19 + (8\*a\*b^7\*x^21)/21 + (b^8\*x^23)/23

**Maple [A]** time = 0.001, size = 91, normalized size = 0.8

$$\frac{a^8 x^7}{7} + \frac{8 a^7 b x^9}{9} + \frac{28 a^6 b^2 x^{11}}{11} + \frac{56 a^5 b^3 x^{13}}{13} + \frac{14 a^4 b^4 x^{15}}{3} + \frac{56 a^3 b^5 x^{17}}{17} + \frac{28 a^2 b^6 x^{19}}{19} + \frac{8 a b^7 x^{21}}{21} + \frac{b^8 x^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*(b*x^2+a)^8,x)`

[Out]  $\frac{1}{7}a^8x^7 + \frac{8}{9}a^7bx^9 + \frac{28}{11}a^6b^2x^{11} + \frac{56}{13}a^5b^3x^{13} + \frac{14}{3}a^4b^4x^{15} + \frac{56}{17}a^3b^5x^{17} + \frac{28}{19}a^2b^6x^{19} + \frac{8}{21}ab^7x^{21} + \frac{1}{23}b^8x^{23}$

---

**Maxima [A]** time = 1.34351, size = 122, normalized size = 1.13

$$\frac{1}{23}b^8x^{23} + \frac{8}{21}ab^7x^{21} + \frac{28}{19}a^2b^6x^{19} + \frac{56}{17}a^3b^5x^{17} + \frac{14}{3}a^4b^4x^{15} + \frac{56}{13}a^5b^3x^{13} + \frac{28}{11}a^6b^2x^{11} + \frac{8}{9}a^7bx^9 + \frac{1}{7}a^8x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^6,x, algorithm="maxima")`

[Out]  $\frac{1}{23}b^8x^{23} + \frac{8}{21}ab^7x^{21} + \frac{28}{19}a^2b^6x^{19} + \frac{56}{17}a^3b^5x^{17} + \frac{14}{3}a^4b^4x^{15} + \frac{56}{13}a^5b^3x^{13} + \frac{28}{11}a^6b^2x^{11} + \frac{8}{9}a^7bx^9 + \frac{1}{7}a^8x^7$

---

**Fricas [A]** time = 0.180036, size = 1, normalized size = 0.01

$$\frac{1}{23}x^{23}b^8 + \frac{8}{21}x^{21}b^7a + \frac{28}{19}x^{19}b^6a^2 + \frac{56}{17}x^{17}b^5a^3 + \frac{14}{3}x^{15}b^4a^4 + \frac{56}{13}x^{13}b^3a^5 + \frac{28}{11}x^{11}b^2a^6 + \frac{8}{9}x^9ba^7 + \frac{1}{7}x^7a^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^6,x, algorithm="fricas")`

[Out]  $\frac{1}{23}x^{23}b^8 + \frac{8}{21}x^{21}b^7a + \frac{28}{19}x^{19}b^6a^2 + \frac{56}{17}x^{17}b^5a^3 + \frac{14}{3}x^{15}b^4a^4 + \frac{56}{13}x^{13}b^3a^5 + \frac{28}{11}x^{11}b^2a^6 + \frac{8}{9}x^9ba^7 + \frac{1}{7}x^7a^8$

---

**Sympy [A]** time = 0.157169, size = 107, normalized size = 0.99

$$\frac{a^8x^7}{7} + \frac{8a^7bx^9}{9} + \frac{28a^6b^2x^{11}}{11} + \frac{56a^5b^3x^{13}}{13} + \frac{14a^4b^4x^{15}}{3} + \frac{56a^3b^5x^{17}}{17} + \frac{28a^2b^6x^{19}}{19} + \frac{8ab^7x^{21}}{21} + \frac{b^8x^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6*(b*x**2+a)**8,x)`

[Out]  $a^{**8}x^{**7}/7 + 8*a^{**7}b*x^{**9}/9 + 28*a^{**6}b^{**2}x^{**11}/11 + 56*a^{**5}b^{**3}x^{**13}/13 + 14*a^{**4}b^{**4}x^{**15}/3 + 56*a^{**3}b^{**5}x^{**17}/17 + 28*a^{**2}b^{**6}x^{**19}/19 + 8*a*b^{**7}x^{**21}/21 + b^{**8}x^{**23}/23$

---

**GIAC/XCAS [A]** time = 0.209062, size = 122, normalized size = 1.13

$$\frac{1}{23}b^8x^{23} + \frac{8}{21}ab^7x^{21} + \frac{28}{19}a^2b^6x^{19} + \frac{56}{17}a^3b^5x^{17} + \frac{14}{3}a^4b^4x^{15} + \frac{56}{13}a^5b^3x^{13} + \frac{28}{11}a^6b^2x^{11} + \frac{8}{9}a^7bx^9 + \frac{1}{7}a^8x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^6,x, algorithm="giac")`

[Out]  $\frac{1}{23}b^8x^{23} + \frac{8}{21}ab^7x^{21} + \frac{28}{19}a^2b^6x^{19} + \frac{56}{17}a^3b^5x^{17} + \frac{14}{3}a^4b^4x^{15} + \frac{56}{13}a^5b^3x^{13} + \frac{28}{11}a^6b^2x^{11} + \frac{8}{9}a^7bx^9 + \frac{1}{7}a^8x^7$

### 3.111 $\int x^4 (a + bx^2)^8 dx$

**Optimal.** Leaf size=108

$$\frac{a^8 x^5}{5} + \frac{8}{7} a^7 b x^7 + \frac{28}{9} a^6 b^2 x^9 + \frac{56}{11} a^5 b^3 x^{11} + \frac{70}{13} a^4 b^4 x^{13} + \frac{56}{15} a^3 b^5 x^{15} + \frac{28}{17} a^2 b^6 x^{17} + \frac{8}{19} a b^7 x^{19} + \frac{b^8 x^{21}}{21}$$

[Out] (a^8\*x^5)/5 + (8\*a^7\*b\*x^7)/7 + (28\*a^6\*b^2\*x^9)/9 + (56\*a^5\*b^3\*x^11)/11 + (70\*a^4\*b^4\*x^13)/13 + (56\*a^3\*b^5\*x^15)/15 + (28\*a^2\*b^6\*x^17)/17 + (8\*a\*b^7\*x^19)/19 + (b^8\*x^21)/21

**Rubi [A]** time = 0.113855, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{a^8 x^5}{5} + \frac{8}{7} a^7 b x^7 + \frac{28}{9} a^6 b^2 x^9 + \frac{56}{11} a^5 b^3 x^{11} + \frac{70}{13} a^4 b^4 x^{13} + \frac{56}{15} a^3 b^5 x^{15} + \frac{28}{17} a^2 b^6 x^{17} + \frac{8}{19} a b^7 x^{19} + \frac{b^8 x^{21}}{21}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a + b\*x^2)^8, x]

[Out] (a^8\*x^5)/5 + (8\*a^7\*b\*x^7)/7 + (28\*a^6\*b^2\*x^9)/9 + (56\*a^5\*b^3\*x^11)/11 + (70\*a^4\*b^4\*x^13)/13 + (56\*a^3\*b^5\*x^15)/15 + (28\*a^2\*b^6\*x^17)/17 + (8\*a\*b^7\*x^19)/19 + (b^8\*x^21)/21

**Rubi in Sympy [A]** time = 19.9833, size = 107, normalized size = 0.99

$$\frac{a^8 x^5}{5} + \frac{8 a^7 b x^7}{7} + \frac{28 a^6 b^2 x^9}{9} + \frac{56 a^5 b^3 x^{11}}{11} + \frac{70 a^4 b^4 x^{13}}{13} + \frac{56 a^3 b^5 x^{15}}{15} + \frac{28 a^2 b^6 x^{17}}{17} + \frac{8 a b^7 x^{19}}{19} + \frac{b^8 x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(b\*x\*\*2+a)\*\*8, x)

[Out] a\*\*8\*x\*\*5/5 + 8\*a\*\*7\*b\*x\*\*7/7 + 28\*a\*\*6\*b\*\*2\*x\*\*9/9 + 56\*a\*\*5\*b\*\*3\*x\*\*11/11 + 70\*a\*\*4\*b\*\*4\*x\*\*13/13 + 56\*a\*\*3\*b\*\*5\*x\*\*15/15 + 28\*a\*\*2\*b\*\*6\*x\*\*17/17 + 8\*a\*b\*\*7\*x\*\*19/19 + b\*\*8\*x\*\*21/21

**Mathematica [A]** time = 0.00457256, size = 108, normalized size = 1.

$$\frac{a^8 x^5}{5} + \frac{8}{7} a^7 b x^7 + \frac{28}{9} a^6 b^2 x^9 + \frac{56}{11} a^5 b^3 x^{11} + \frac{70}{13} a^4 b^4 x^{13} + \frac{56}{15} a^3 b^5 x^{15} + \frac{28}{17} a^2 b^6 x^{17} + \frac{8}{19} a b^7 x^{19} + \frac{b^8 x^{21}}{21}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a + b\*x^2)^8, x]

[Out] (a^8\*x^5)/5 + (8\*a^7\*b\*x^7)/7 + (28\*a^6\*b^2\*x^9)/9 + (56\*a^5\*b^3\*x^11)/11 + (70\*a^4\*b^4\*x^13)/13 + (56\*a^3\*b^5\*x^15)/15 + (28\*a^2\*b^6\*x^17)/17 + (8\*a\*b^7\*x^19)/19 + (b^8\*x^21)/21

**Maple [A]** time = 0.002, size = 91, normalized size = 0.8

$$\frac{a^8 x^5}{5} + \frac{8 a^7 b x^7}{7} + \frac{28 a^6 b^2 x^9}{9} + \frac{56 a^5 b^3 x^{11}}{11} + \frac{70 a^4 b^4 x^{13}}{13} + \frac{56 a^3 b^5 x^{15}}{15} + \frac{28 a^2 b^6 x^{17}}{17} + \frac{8 a b^7 x^{19}}{19} + \frac{b^8 x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(b*x^2+a)^8,x)`

[Out]  $\frac{1}{5}a^8x^5 + \frac{8}{7}a^7bx^7 + \frac{28}{9}a^6b^2x^9 + \frac{56}{11}a^5b^3x^{11} + \frac{70}{13}a^4b^4x^{13} + \frac{56}{15}a^3b^5x^{15} + \frac{28}{17}a^2b^6x^{17} + \frac{8}{19}ab^7x^{19} + \frac{1}{21}b^8x^{21}$

**Maxima [A]** time = 1.3507, size = 122, normalized size = 1.13

$$\frac{1}{21}b^8x^{21} + \frac{8}{19}ab^7x^{19} + \frac{28}{17}a^2b^6x^{17} + \frac{56}{15}a^3b^5x^{15} + \frac{70}{13}a^4b^4x^{13} + \frac{56}{11}a^5b^3x^{11} + \frac{28}{9}a^6b^2x^9 + \frac{8}{7}a^7bx^7 + \frac{1}{5}a^8x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^4,x, algorithm="maxima")`

[Out]  $\frac{1}{21}b^8x^{21} + \frac{8}{19}ab^7x^{19} + \frac{28}{17}a^2b^6x^{17} + \frac{56}{15}a^3b^5x^{15} + \frac{70}{13}a^4b^4x^{13} + \frac{56}{11}a^5b^3x^{11} + \frac{28}{9}a^6b^2x^9 + \frac{8}{7}a^7bx^7 + \frac{1}{5}a^8x^5$

**Fricas [A]** time = 0.187574, size = 1, normalized size = 0.01

$$\frac{1}{21}x^{21}b^8 + \frac{8}{19}x^{19}b^7a + \frac{28}{17}x^{17}b^6a^2 + \frac{56}{15}x^{15}b^5a^3 + \frac{70}{13}x^{13}b^4a^4 + \frac{56}{11}x^{11}b^3a^5 + \frac{28}{9}x^9b^2a^6 + \frac{8}{7}x^7ba^7 + \frac{1}{5}x^5a^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^4,x, algorithm="fricas")`

[Out]  $\frac{1}{21}x^{21}b^8 + \frac{8}{19}x^{19}b^7a + \frac{28}{17}x^{17}b^6a^2 + \frac{56}{15}x^{15}b^5a^3 + \frac{70}{13}x^{13}b^4a^4 + \frac{56}{11}x^{11}b^3a^5 + \frac{28}{9}x^9b^2a^6 + \frac{8}{7}x^7ba^7 + \frac{1}{5}x^5a^8$

**Sympy [A]** time = 0.159621, size = 107, normalized size = 0.99

$$\frac{a^8x^5}{5} + \frac{8a^7bx^7}{7} + \frac{28a^6b^2x^9}{9} + \frac{56a^5b^3x^{11}}{11} + \frac{70a^4b^4x^{13}}{13} + \frac{56a^3b^5x^{15}}{15} + \frac{28a^2b^6x^{17}}{17} + \frac{8ab^7x^{19}}{19} + \frac{b^8x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(b*x**2+a)**8,x)`

[Out]  $a^8x^5/5 + 8a^7bx^7/7 + 28a^6b^2x^9/9 + 56a^5b^3x^{11}/11 + 70a^4b^4x^{13}/13 + 56a^3b^5x^{15}/15 + 28a^2b^6x^{17}/17 + 8ab^7x^{19}/19 + b^8x^{21}/21$

**GIAC/XCAS [A]** time = 0.207901, size = 122, normalized size = 1.13

$$\frac{1}{21}b^8x^{21} + \frac{8}{19}ab^7x^{19} + \frac{28}{17}a^2b^6x^{17} + \frac{56}{15}a^3b^5x^{15} + \frac{70}{13}a^4b^4x^{13} + \frac{56}{11}a^5b^3x^{11} + \frac{28}{9}a^6b^2x^9 + \frac{8}{7}a^7bx^7 + \frac{1}{5}a^8x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^4,x, algorithm="giac")`

```
[Out] 1/21*b^8*x^21 + 8/19*a*b^7*x^19 + 28/17*a^2*b^6*x^17 + 56/15*a^3*  
b^5*x^15 + 70/13*a^4*b^4*x^13 + 56/11*a^5*b^3*x^11 + 28/9*a^6*b^2  
*x^9 + 8/7*a^7*b*x^7 + 1/5*a^8*x^5
```

### 3.112 $\int x^2 (a + bx^2)^8 dx$

**Optimal.** Leaf size=106

$$\frac{a^8 x^3}{3} + \frac{8}{5} a^7 b x^5 + 4a^6 b^2 x^7 + \frac{56}{9} a^5 b^3 x^9 + \frac{70}{11} a^4 b^4 x^{11} + \frac{56}{13} a^3 b^5 x^{13} + \frac{28}{15} a^2 b^6 x^{15} + \frac{8}{17} a b^7 x^{17} + \frac{b^8 x^{19}}{19}$$

[Out]  $(a^8 x^3)/3 + (8 a^7 b x^5)/5 + 4 a^6 b^2 x^7 + (56 a^5 b^3 x^9)/9 + (70 a^4 b^4 x^{11})/11 + (56 a^3 b^5 x^{13})/13 + (28 a^2 b^6 x^{15})/15 + (8 a b^7 x^{17})/17 + (b^8 x^{19})/19$

**Rubi [A]** time = 0.110924, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{a^8 x^3}{3} + \frac{8}{5} a^7 b x^5 + 4a^6 b^2 x^7 + \frac{56}{9} a^5 b^3 x^9 + \frac{70}{11} a^4 b^4 x^{11} + \frac{56}{13} a^3 b^5 x^{13} + \frac{28}{15} a^2 b^6 x^{15} + \frac{8}{17} a b^7 x^{17} + \frac{b^8 x^{19}}{19}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*x^2)^8, x]

[Out]  $(a^8 x^3)/3 + (8 a^7 b x^5)/5 + 4 a^6 b^2 x^7 + (56 a^5 b^3 x^9)/9 + (70 a^4 b^4 x^{11})/11 + (56 a^3 b^5 x^{13})/13 + (28 a^2 b^6 x^{15})/15 + (8 a b^7 x^{17})/17 + (b^8 x^{19})/19$

**Rubi in Sympy [A]** time = 19.6931, size = 105, normalized size = 0.99

$$\frac{a^8 x^3}{3} + \frac{8 a^7 b x^5}{5} + 4 a^6 b^2 x^7 + \frac{56 a^5 b^3 x^9}{9} + \frac{70 a^4 b^4 x^{11}}{11} + \frac{56 a^3 b^5 x^{13}}{13} + \frac{28 a^2 b^6 x^{15}}{15} + \frac{8 a b^7 x^{17}}{17} + \frac{b^8 x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(b\*x\*\*2+a)\*\*8, x)

[Out]  $a**8*x**3/3 + 8*a**7*b*x**5/5 + 4*a**6*b**2*x**7 + 56*a**5*b**3*x**9/9 + 70*a**4*b**4*x**11/11 + 56*a**3*b**5*x**13/13 + 28*a**2*b**6*x**15/15 + 8*a*b**7*x**17/17 + b**8*x**19/19$

**Mathematica [A]** time = 0.0045652, size = 106, normalized size = 1.

$$\frac{a^8 x^3}{3} + \frac{8}{5} a^7 b x^5 + 4a^6 b^2 x^7 + \frac{56}{9} a^5 b^3 x^9 + \frac{70}{11} a^4 b^4 x^{11} + \frac{56}{13} a^3 b^5 x^{13} + \frac{28}{15} a^2 b^6 x^{15} + \frac{8}{17} a b^7 x^{17} + \frac{b^8 x^{19}}{19}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*x^2)^8, x]

[Out]  $(a^8 x^3)/3 + (8 a^7 b x^5)/5 + 4 a^6 b^2 x^7 + (56 a^5 b^3 x^9)/9 + (70 a^4 b^4 x^{11})/11 + (56 a^3 b^5 x^{13})/13 + (28 a^2 b^6 x^{15})/15 + (8 a b^7 x^{17})/17 + (b^8 x^{19})/19$

**Maple [A]** time = 0.003, size = 91, normalized size = 0.9

$$\frac{a^8 x^3}{3} + \frac{8 a^7 b x^5}{5} + 4 a^6 b^2 x^7 + \frac{56 a^5 b^3 x^9}{9} + \frac{70 a^4 b^4 x^{11}}{11} + \frac{56 a^3 b^5 x^{13}}{13} + \frac{28 a^2 b^6 x^{15}}{15} + \frac{8 a b^7 x^{17}}{17} + \frac{b^8 x^{19}}{19}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^2+a)^8,x)`

[Out]  $\frac{1}{3}a^8x^3 + \frac{8}{5}a^7bx^5 + 4a^6b^2x^7 + \frac{56}{9}a^5b^3x^9 + \frac{70}{11}a^4b^4x^{11} + \frac{56}{13}a^3b^5x^{13} + \frac{28}{15}a^2b^6x^{15} + \frac{8}{17}ab^7x^{17} + \frac{1}{19}b^8x^{19}$

**Maxima [A]** time = 1.34676, size = 122, normalized size = 1.15

$$\frac{1}{19}b^8x^{19} + \frac{8}{17}ab^7x^{17} + \frac{28}{15}a^2b^6x^{15} + \frac{56}{13}a^3b^5x^{13} + \frac{70}{11}a^4b^4x^{11} + \frac{56}{9}a^5b^3x^9 + 4a^6b^2x^7 + \frac{8}{5}a^7bx^5 + \frac{1}{3}a^8x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^2,x, algorithm="maxima")`

[Out]  $\frac{1}{19}b^8x^{19} + \frac{8}{17}a^7bx^{17} + \frac{28}{15}a^2b^6x^{15} + \frac{56}{13}a^3b^5x^{13} + \frac{70}{11}a^4b^4x^{11} + \frac{56}{9}a^5b^3x^9 + 4a^6b^2x^7 + \frac{8}{5}a^7bx^5 + \frac{1}{3}a^8x^3$

**Fricas [A]** time = 0.183043, size = 1, normalized size = 0.01

$$\frac{1}{19}x^{19}b^8 + \frac{8}{17}x^{17}b^7a + \frac{28}{15}x^{15}b^6a^2 + \frac{56}{13}x^{13}b^5a^3 + \frac{70}{11}x^{11}b^4a^4 + \frac{56}{9}x^9b^3a^5 + 4x^7b^2a^6 + \frac{8}{5}x^5ba^7 + \frac{1}{3}x^3a^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^2,x, algorithm="fricas")`

[Out]  $\frac{1}{19}x^{19}b^8 + \frac{8}{17}x^{17}b^7a + \frac{28}{15}x^{15}b^6a^2 + \frac{56}{13}x^{13}b^5a^3 + \frac{70}{11}x^{11}b^4a^4 + \frac{56}{9}x^9b^3a^5 + 4x^7b^2a^6 + \frac{8}{5}x^5ba^7 + \frac{1}{3}x^3a^8$

**Sympy [A]** time = 0.161441, size = 105, normalized size = 0.99

$$\frac{a^8x^3}{3} + \frac{8a^7bx^5}{5} + 4a^6b^2x^7 + \frac{56a^5b^3x^9}{9} + \frac{70a^4b^4x^{11}}{11} + \frac{56a^3b^5x^{13}}{13} + \frac{28a^2b^6x^{15}}{15} + \frac{8ab^7x^{17}}{17} + \frac{b^8x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**2+a)**8,x)`

[Out]  $a^{**8}x^{**3}/3 + 8*a^{**7}b*x^{**5}/5 + 4*a^{**6}b^{**2}x^{**7} + 56*a^{**5}b^{**3}x^{**9}/9 + 70*a^{**4}b^{**4}x^{**11}/11 + 56*a^{**3}b^{**5}x^{**13}/13 + 28*a^{**2}b^{**6}x^{**15}/15 + 8*a*b^{**7}x^{**17}/17 + b^{**8}x^{**19}/19$

**GIAC/XCAS [A]** time = 0.208456, size = 122, normalized size = 1.15

$$\frac{1}{19}b^8x^{19} + \frac{8}{17}ab^7x^{17} + \frac{28}{15}a^2b^6x^{15} + \frac{56}{13}a^3b^5x^{13} + \frac{70}{11}a^4b^4x^{11} + \frac{56}{9}a^5b^3x^9 + 4a^6b^2x^7 + \frac{8}{5}a^7bx^5 + \frac{1}{3}a^8x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8*x^2,x, algorithm="giac")`

```
[Out] 1/19*b^8*x^19 + 8/17*a*b^7*x^17 + 28/15*a^2*b^6*x^15 + 56/13*a^3*  
b^5*x^13 + 70/11*a^4*b^4*x^11 + 56/9*a^5*b^3*x^9 + 4*a^6*b^2*x^7  
+ 8/5*a^7*b*x^5 + 1/3*a^8*x^3
```

### 3.113 $\int (a + bx^2)^8 dx$

**Optimal.** Leaf size=101

$$a^8x + \frac{8}{3}a^7bx^3 + \frac{28}{5}a^6b^2x^5 + 8a^5b^3x^7 + \frac{70}{9}a^4b^4x^9 + \frac{56}{11}a^3b^5x^{11} + \frac{28}{13}a^2b^6x^{13} + \frac{8}{15}ab^7x^{15} + \frac{b^8x^{17}}{17}$$

[Out]  $a^8x + (8*a^7*b*x^3)/3 + (28*a^6*b^2*x^5)/5 + 8*a^5*b^3*x^7 + (70*a^4*b^4*x^9)/9 + (56*a^3*b^5*x^{11})/11 + (28*a^2*b^6*x^{13})/13 + (8*a*b^7*x^{15})/15 + (b^8*x^{17})/17$

**Rubi [A]** time = 0.0923036, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$a^8x + \frac{8}{3}a^7bx^3 + \frac{28}{5}a^6b^2x^5 + 8a^5b^3x^7 + \frac{70}{9}a^4b^4x^9 + \frac{56}{11}a^3b^5x^{11} + \frac{28}{13}a^2b^6x^{13} + \frac{8}{15}ab^7x^{15} + \frac{b^8x^{17}}{17}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8, x]

[Out]  $a^8x + (8*a^7*b*x^3)/3 + (28*a^6*b^2*x^5)/5 + 8*a^5*b^3*x^7 + (70*a^4*b^4*x^9)/9 + (56*a^3*b^5*x^{11})/11 + (28*a^2*b^6*x^{13})/13 + (8*a*b^7*x^{15})/15 + (b^8*x^{17})/17$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{8a^7bx^3}{3} + \frac{28a^6b^2x^5}{5} + 8a^5b^3x^7 + \frac{70a^4b^4x^9}{9} + \frac{56a^3b^5x^{11}}{11} + \frac{28a^2b^6x^{13}}{13} + \frac{8ab^7x^{15}}{15} + \frac{b^8x^{17}}{17} + \int a^8 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8, x)

[Out]  $8*a**7*b*x**3/3 + 28*a**6*b**2*x**5/5 + 8*a**5*b**3*x**7 + 70*a**4*b**4*x**9/9 + 56*a**3*b**5*x**11/11 + 28*a**2*b**6*x**13/13 + 8*a*b**7*x**15/15 + b**8*x**17/17 + \text{Integral}(a**8, x)$

**Mathematica [A]** time = 0.00242195, size = 101, normalized size = 1.

$$a^8x + \frac{8}{3}a^7bx^3 + \frac{28}{5}a^6b^2x^5 + 8a^5b^3x^7 + \frac{70}{9}a^4b^4x^9 + \frac{56}{11}a^3b^5x^{11} + \frac{28}{13}a^2b^6x^{13} + \frac{8}{15}ab^7x^{15} + \frac{b^8x^{17}}{17}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8, x]

[Out]  $a^8x + (8*a^7*b*x^3)/3 + (28*a^6*b^2*x^5)/5 + 8*a^5*b^3*x^7 + (70*a^4*b^4*x^9)/9 + (56*a^3*b^5*x^{11})/11 + (28*a^2*b^6*x^{13})/13 + (8*a*b^7*x^{15})/15 + (b^8*x^{17})/17$

**Maple [A]** time = 0.002, size = 88, normalized size = 0.9

$$a^8x + \frac{8a^7bx^3}{3} + \frac{28a^6b^2x^5}{5} + 8a^5b^3x^7 + \frac{70a^4b^4x^9}{9} + \frac{56a^3b^5x^{11}}{11} + \frac{28a^2b^6x^{13}}{13} + \frac{8ab^7x^{15}}{15} + \frac{b^8x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8,x)`

[Out]  $a^8x + \frac{8}{3}a^7bx^3 + \frac{28}{5}a^6b^2x^5 + 8a^5b^3x^7 + \frac{70}{9}a^4b^4x^9 + \frac{56}{11}a^3b^5x^{11} + \frac{28}{13}a^2b^6x^{13} + \frac{8}{15}ab^7x^{15} + \frac{1}{17}b^8x^{17}$

**Maxima [A]** time = 1.34273, size = 117, normalized size = 1.16

$$\frac{1}{17}b^8x^{17} + \frac{8}{15}ab^7x^{15} + \frac{28}{13}a^2b^6x^{13} + \frac{56}{11}a^3b^5x^{11} + \frac{70}{9}a^4b^4x^9 + 8a^5b^3x^7 + \frac{28}{5}a^6b^2x^5 + \frac{8}{3}a^7bx^3 + a^8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8,x, algorithm="maxima")`

[Out]  $\frac{1}{17}b^8x^{17} + \frac{8}{15}ab^7x^{15} + \frac{28}{13}a^2b^6x^{13} + \frac{56}{11}a^3b^5x^{11} + \frac{70}{9}a^4b^4x^9 + 8a^5b^3x^7 + \frac{28}{5}a^6b^2x^5 + \frac{8}{3}a^7bx^3 + a^8x$

**Fricas [A]** time = 0.179981, size = 1, normalized size = 0.01

$$\frac{1}{17}x^{17}b^8 + \frac{8}{15}x^{15}b^7a + \frac{28}{13}x^{13}b^6a^2 + \frac{56}{11}x^{11}b^5a^3 + \frac{70}{9}x^9b^4a^4 + 8x^7b^3a^5 + \frac{28}{5}x^5b^2a^6 + \frac{8}{3}x^3ba^7 + xa^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8,x, algorithm="fricas")`

[Out]  $\frac{1}{17}x^{17}b^8 + \frac{8}{15}x^{15}b^7a + \frac{28}{13}x^{13}b^6a^2 + \frac{56}{11}x^{11}b^5a^3 + \frac{70}{9}x^9b^4a^4 + 8x^7b^3a^5 + \frac{28}{5}x^5b^2a^6 + \frac{8}{3}x^3ba^7 + xa^8$

**Sympy [A]** time = 0.145078, size = 102, normalized size = 1.01

$$a^8x + \frac{8a^7bx^3}{3} + \frac{28a^6b^2x^5}{5} + 8a^5b^3x^7 + \frac{70a^4b^4x^9}{9} + \frac{56a^3b^5x^{11}}{11} + \frac{28a^2b^6x^{13}}{13} + \frac{8ab^7x^{15}}{15} + \frac{b^8x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8,x)`

[Out]  $a^8x + \frac{8a^7bx^3}{3} + \frac{28a^6b^2x^5}{5} + 8a^5b^3x^7 + \frac{70a^4b^4x^9}{9} + \frac{56a^3b^5x^{11}}{11} + \frac{28a^2b^6x^{13}}{13} + \frac{8ab^7x^{15}}{15} + \frac{b^8x^{17}}{17}$

**GIAC/XCAS [A]** time = 0.205345, size = 117, normalized size = 1.16

$$\frac{1}{17}b^8x^{17} + \frac{8}{15}ab^7x^{15} + \frac{28}{13}a^2b^6x^{13} + \frac{56}{11}a^3b^5x^{11} + \frac{70}{9}a^4b^4x^9 + 8a^5b^3x^7 + \frac{28}{5}a^6b^2x^5 + \frac{8}{3}a^7bx^3 + a^8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8,x, algorithm="giac")`

[Out]  $1/17*b^8*x^{17} + 8/15*a*b^7*x^{15} + 28/13*a^2*b^6*x^{13} + 56/11*a^3*b^5*x^{11} + 70/9*a^4*b^4*x^9 + 8*a^5*b^3*x^7 + 28/5*a^6*b^2*x^5 + 8/3*a^7*b*x^3 + a^8*x$

$$3.114 \quad \int \frac{(a+bx^2)^8}{x^2} dx$$

**Optimal.** Leaf size=100

$$-\frac{a^8}{x} + 8a^7bx + \frac{28}{3}a^6b^2x^3 + \frac{56}{5}a^5b^3x^5 + 10a^4b^4x^7 + \frac{56}{9}a^3b^5x^9 + \frac{28}{11}a^2b^6x^{11} + \frac{8}{13}ab^7x^{13} + \frac{b^8x^{15}}{15}$$

[Out]  $-(a^8/x) + 8*a^7*b*x + (28*a^6*b^2*x^3)/3 + (56*a^5*b^3*x^5)/5 + 10*a^4*b^4*x^7 + (56*a^3*b^5*x^9)/9 + (28*a^2*b^6*x^{11})/11 + (8*a*b^7*x^{13})/13 + (b^8*x^{15})/15$

**Rubi [A]** time = 0.102431, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^8}{x} + 8a^7bx + \frac{28}{3}a^6b^2x^3 + \frac{56}{5}a^5b^3x^5 + 10a^4b^4x^7 + \frac{56}{9}a^3b^5x^9 + \frac{28}{11}a^2b^6x^{11} + \frac{8}{13}ab^7x^{13} + \frac{b^8x^{15}}{15}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^2, x]

[Out]  $-(a^8/x) + 8*a^7*b*x + (28*a^6*b^2*x^3)/3 + (56*a^5*b^3*x^5)/5 + 10*a^4*b^4*x^7 + (56*a^3*b^5*x^9)/9 + (28*a^2*b^6*x^{11})/11 + (8*a*b^7*x^{13})/13 + (b^8*x^{15})/15$

**Rubi in Sympy [A]** time = 19.0815, size = 99, normalized size = 0.99

$$-\frac{a^8}{x} + 8a^7bx + \frac{28a^6b^2x^3}{3} + \frac{56a^5b^3x^5}{5} + 10a^4b^4x^7 + \frac{56a^3b^5x^9}{9} + \frac{28a^2b^6x^{11}}{11} + \frac{8ab^7x^{13}}{13} + \frac{b^8x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*2, x)

[Out]  $-a**8/x + 8*a**7*b*x + 28*a**6*b**2*x**3/3 + 56*a**5*b**3*x**5/5 + 10*a**4*b**4*x**7 + 56*a**3*b**5*x**9/9 + 28*a**2*b**6*x**11/11 + 8*a*b**7*x**13/13 + b**8*x**15/15$

**Mathematica [A]** time = 0.0175888, size = 100, normalized size = 1.

$$-\frac{a^8}{x} + 8a^7bx + \frac{28}{3}a^6b^2x^3 + \frac{56}{5}a^5b^3x^5 + 10a^4b^4x^7 + \frac{56}{9}a^3b^5x^9 + \frac{28}{11}a^2b^6x^{11} + \frac{8}{13}ab^7x^{13} + \frac{b^8x^{15}}{15}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^2, x]

[Out]  $-(a^8/x) + 8*a^7*b*x + (28*a^6*b^2*x^3)/3 + (56*a^5*b^3*x^5)/5 + 10*a^4*b^4*x^7 + (56*a^3*b^5*x^9)/9 + (28*a^2*b^6*x^{11})/11 + (8*a*b^7*x^{13})/13 + (b^8*x^{15})/15$

**Maple [A]** time = 0.006, size = 89, normalized size = 0.9

$$-\frac{a^8}{x} + 8a^7bx + \frac{28a^6b^2x^3}{3} + \frac{56a^5b^3x^5}{5} + 10a^4b^4x^7 + \frac{56a^3b^5x^9}{9} + \frac{28a^2b^6x^{11}}{11} + \frac{8ab^7x^{13}}{13} + \frac{b^8x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^2,x)`

[Out]  $-a^8/x + 8a^7bx + 28/3a^6b^2x^3 + 56/5a^5b^3x^5 + 10a^4b^4x^7 + 56/9a^3b^5x^9 + 28/11a^2b^6x^{11} + 8/13ab^7x^{13} + 1/15b^8x^{15}$

**Maxima [A]** time = 1.35774, size = 119, normalized size = 1.19

$$\frac{1}{15}b^8x^{15} + \frac{8}{13}ab^7x^{13} + \frac{28}{11}a^2b^6x^{11} + \frac{56}{9}a^3b^5x^9 + 10a^4b^4x^7 + \frac{56}{5}a^5b^3x^5 + \frac{28}{3}a^6b^2x^3 + 8a^7bx - \frac{a^8}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^2,x, algorithm="maxima")`

[Out]  $1/15b^8x^{15} + 8/13ab^7x^{13} + 28/11a^2b^6x^{11} + 56/9a^3b^5x^9 + 10a^4b^4x^7 + 56/5a^5b^3x^5 + 28/3a^6b^2x^3 + 8a^7bx - a^8/x$

**Fricas [A]** time = 0.1956, size = 124, normalized size = 1.24

$$\frac{429b^8x^{16} + 3960ab^7x^{14} + 16380a^2b^6x^{12} + 40040a^3b^5x^{10} + 64350a^4b^4x^8 + 72072a^5b^3x^6 + 60060a^6b^2x^4 + 51480a^7bx^2 - 6435a^8}{6435x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^2,x, algorithm="fricas")`

[Out]  $1/6435(429b^8x^{16} + 3960a^7bx^{14} + 16380a^2b^6x^{12} + 40040a^3b^5x^{10} + 64350a^4b^4x^8 + 72072a^5b^3x^6 + 60060a^6b^2x^4 + 51480a^7bx^2 - 6435a^8)/x$

**Sympy [A]** time = 1.31963, size = 99, normalized size = 0.99

$$-\frac{a^8}{x} + 8a^7bx + \frac{28a^6b^2x^3}{3} + \frac{56a^5b^3x^5}{5} + 10a^4b^4x^7 + \frac{56a^3b^5x^9}{9} + \frac{28a^2b^6x^{11}}{11} + \frac{8ab^7x^{13}}{13} + \frac{b^8x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**2,x)`

[Out]  $-a**8/x + 8*a**7*b*x + 28*a**6*b**2*x**3/3 + 56*a**5*b**3*x**5/5 + 10*a**4*b**4*x**7 + 56*a**3*b**5*x**9/9 + 28*a**2*b**6*x**11/11 + 8*a*b**7*x**13/13 + b**8*x**15/15$

**GIAC/XCAS [A]** time = 0.207365, size = 119, normalized size = 1.19

$$\frac{1}{15}b^8x^{15} + \frac{8}{13}ab^7x^{13} + \frac{28}{11}a^2b^6x^{11} + \frac{56}{9}a^3b^5x^9 + 10a^4b^4x^7 + \frac{56}{5}a^5b^3x^5 + \frac{28}{3}a^6b^2x^3 + 8a^7bx - \frac{a^8}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^2,x, algorithm="giac")`

```
[Out] 1/15*b^8*x^15 + 8/13*a*b^7*x^13 + 28/11*a^2*b^6*x^11 + 56/9*a^3*b^5*x^9 + 10*a^4*b^4*x^7 + 56/5*a^5*b^3*x^5 + 28/3*a^6*b^2*x^3 + 8*a^7*b*x - a^8/x
```



$$3.115 \quad \int \frac{(a+bx^2)^8}{x^4} dx$$

**Optimal.** Leaf size=98

$$-\frac{a^8}{3x^3} - \frac{8a^7b}{x} + 28a^6b^2x + \frac{56}{3}a^5b^3x^3 + 14a^4b^4x^5 + 8a^3b^5x^7 + \frac{28}{9}a^2b^6x^9 + \frac{8}{11}ab^7x^{11} + \frac{b^8x^{13}}{13}$$

[Out]  $-a^8/(3*x^3) - (8*a^7*b)/x + 28*a^6*b^2*x + (56*a^5*b^3*x^3)/3 + 14*a^4*b^4*x^5 + 8*a^3*b^5*x^7 + (28*a^2*b^6*x^9)/9 + (8*a*b^7*x^{11})/11 + (b^8*x^{13})/13$

**Rubi [A]** time = 0.102708, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^8}{3x^3} - \frac{8a^7b}{x} + 28a^6b^2x + \frac{56}{3}a^5b^3x^3 + 14a^4b^4x^5 + 8a^3b^5x^7 + \frac{28}{9}a^2b^6x^9 + \frac{8}{11}ab^7x^{11} + \frac{b^8x^{13}}{13}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^4, x]

[Out]  $-a^8/(3*x^3) - (8*a^7*b)/x + 28*a^6*b^2*x + (56*a^5*b^3*x^3)/3 + 14*a^4*b^4*x^5 + 8*a^3*b^5*x^7 + (28*a^2*b^6*x^9)/9 + (8*a*b^7*x^{11})/11 + (b^8*x^{13})/13$

**Rubi in Sympy [A]** time = 19.5012, size = 97, normalized size = 0.99

$$-\frac{a^8}{3x^3} - \frac{8a^7b}{x} + 28a^6b^2x + \frac{56a^5b^3x^3}{3} + 14a^4b^4x^5 + 8a^3b^5x^7 + \frac{28a^2b^6x^9}{9} + \frac{8ab^7x^{11}}{11} + \frac{b^8x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*4, x)

[Out]  $-a**8/(3*x**3) - 8*a**7*b/x + 28*a**6*b**2*x + 56*a**5*b**3*x**3/3 + 14*a**4*b**4*x**5 + 8*a**3*b**5*x**7 + 28*a**2*b**6*x**9/9 + 8*a*b**7*x**11/11 + b**8*x**13/13$

**Mathematica [A]** time = 0.0151192, size = 98, normalized size = 1.

$$-\frac{a^8}{3x^3} - \frac{8a^7b}{x} + 28a^6b^2x + \frac{56}{3}a^5b^3x^3 + 14a^4b^4x^5 + 8a^3b^5x^7 + \frac{28}{9}a^2b^6x^9 + \frac{8}{11}ab^7x^{11} + \frac{b^8x^{13}}{13}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^4, x]

[Out]  $-a^8/(3*x^3) - (8*a^7*b)/x + 28*a^6*b^2*x + (56*a^5*b^3*x^3)/3 + 14*a^4*b^4*x^5 + 8*a^3*b^5*x^7 + (28*a^2*b^6*x^9)/9 + (8*a*b^7*x^{11})/11 + (b^8*x^{13})/13$

**Maple [A]** time = 0.008, size = 89, normalized size = 0.9

$$-\frac{a^8}{3x^3} - 8\frac{a^7b}{x} + 28a^6b^2x + \frac{56a^5b^3x^3}{3} + 14a^4b^4x^5 + 8a^3b^5x^7 + \frac{28a^2b^6x^9}{9} + \frac{8ab^7x^{11}}{11} + \frac{b^8x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^4,x)`

[Out]  $-1/3*a^8/x^3-8*a^7*b/x+28*a^6*b^2*x+56/3*a^5*b^3*x^3+14*a^4*b^4*x^5+8*a^3*b^5*x^7+28/9*a^2*b^6*x^9+8/11*a*b^7*x^11+1/13*b^8*x^13$

**Maxima [A]** time = 1.34737, size = 120, normalized size = 1.22

$$\frac{1}{13} b^8 x^{13} + \frac{8}{11} a b^7 x^{11} + \frac{28}{9} a^2 b^6 x^9 + 8 a^3 b^5 x^7 + 14 a^4 b^4 x^5 + \frac{56}{3} a^5 b^3 x^3 + 28 a^6 b^2 x - \frac{24 a^7 b x^2 + a^8}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^4,x, algorithm="maxima")`

[Out]  $1/13*b^8*x^13 + 8/11*a*b^7*x^11 + 28/9*a^2*b^6*x^9 + 8*a^3*b^5*x^7 + 14*a^4*b^4*x^5 + 56/3*a^5*b^3*x^3 + 28*a^6*b^2*x - 1/3*(24*a^7*b*x^2 + a^8)/x^3$

**Fricas [A]** time = 0.206064, size = 124, normalized size = 1.27

$$\frac{99 b^8 x^{16} + 936 a b^7 x^{14} + 4004 a^2 b^6 x^{12} + 10296 a^3 b^5 x^{10} + 18018 a^4 b^4 x^8 + 24024 a^5 b^3 x^6 + 36036 a^6 b^2 x^4 - 10296 a^7 b x^2 - 429 a^8}{1287 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^4,x, algorithm="fricas")`

[Out]  $1/1287*(99*b^8*x^16 + 936*a*b^7*x^14 + 4004*a^2*b^6*x^12 + 10296*a^3*b^5*x^10 + 18018*a^4*b^4*x^8 + 24024*a^5*b^3*x^6 + 36036*a^6*b^2*x^4 - 10296*a^7*b*x^2 - 429*a^8)/x^3$

**Sympy [A]** time = 1.46653, size = 99, normalized size = 1.01

$$28 a^6 b^2 x + \frac{56 a^5 b^3 x^3}{3} + 14 a^4 b^4 x^5 + 8 a^3 b^5 x^7 + \frac{28 a^2 b^6 x^9}{9} + \frac{8 a b^7 x^{11}}{11} + \frac{b^8 x^{13}}{13} - \frac{a^8 + 24 a^7 b x^2}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**4,x)`

[Out]  $28*a**6*b**2*x + 56*a**5*b**3*x**3/3 + 14*a**4*b**4*x**5 + 8*a**3*b**5*x**7 + 28*a**2*b**6*x**9/9 + 8*a*b**7*x**11/11 + b**8*x**13/13 - (a**8 + 24*a**7*b*x**2)/(3*x**3)$

**GIAC/XCAS [A]** time = 0.206984, size = 120, normalized size = 1.22

$$\frac{1}{13} b^8 x^{13} + \frac{8}{11} a b^7 x^{11} + \frac{28}{9} a^2 b^6 x^9 + 8 a^3 b^5 x^7 + 14 a^4 b^4 x^5 + \frac{56}{3} a^5 b^3 x^3 + 28 a^6 b^2 x - \frac{24 a^7 b x^2 + a^8}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^4,x, algorithm="giac")`

[Out]  $\frac{1}{13}b^8x^{13} + \frac{8}{11}ab^7x^{11} + \frac{28}{9}a^2b^6x^9 + 8a^3b^5x^7 + 14a^4b^4x^5 + \frac{56}{3}a^5b^3x^3 + 28a^6b^2x - \frac{1}{3}(24a^7b^2x^2 + a^8)/x^3$

$$3.116 \quad \int \frac{(a+bx^2)^8}{x^6} dx$$

**Optimal.** Leaf size=100

$$-\frac{a^8}{5x^5} - \frac{8a^7b}{3x^3} - \frac{28a^6b^2}{x} + 56a^5b^3x + \frac{70}{3}a^4b^4x^3 + \frac{56}{5}a^3b^5x^5 + 4a^2b^6x^7 + \frac{8}{9}ab^7x^9 + \frac{b^8x^{11}}{11}$$

[Out]  $-a^8/(5*x^5) - (8*a^7*b)/(3*x^3) - (28*a^6*b^2)/x + 56*a^5*b^3*x + (70*a^4*b^4*x^3)/3 + (56*a^3*b^5*x^5)/5 + 4*a^2*b^6*x^7 + (8*a*b^7*x^9)/9 + (b^8*x^11)/11$

**Rubi [A]** time = 0.102779, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^8}{5x^5} - \frac{8a^7b}{3x^3} - \frac{28a^6b^2}{x} + 56a^5b^3x + \frac{70}{3}a^4b^4x^3 + \frac{56}{5}a^3b^5x^5 + 4a^2b^6x^7 + \frac{8}{9}ab^7x^9 + \frac{b^8x^{11}}{11}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^6, x]

[Out]  $-a^8/(5*x^5) - (8*a^7*b)/(3*x^3) - (28*a^6*b^2)/x + 56*a^5*b^3*x + (70*a^4*b^4*x^3)/3 + (56*a^3*b^5*x^5)/5 + 4*a^2*b^6*x^7 + (8*a*b^7*x^9)/9 + (b^8*x^11)/11$

**Rubi in Sympy [A]** time = 19.2669, size = 99, normalized size = 0.99

$$-\frac{a^8}{5x^5} - \frac{8a^7b}{3x^3} - \frac{28a^6b^2}{x} + 56a^5b^3x + \frac{70a^4b^4x^3}{3} + \frac{56a^3b^5x^5}{5} + 4a^2b^6x^7 + \frac{8ab^7x^9}{9} + \frac{b^8x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*6, x)

[Out]  $-a**8/(5*x**5) - 8*a**7*b/(3*x**3) - 28*a**6*b**2/x + 56*a**5*b**3*x + 70*a**4*b**4*x**3/3 + 56*a**3*b**5*x**5/5 + 4*a**2*b**6*x**7 + 8*a*b**7*x**9/9 + b**8*x**11/11$

**Mathematica [A]** time = 0.0171748, size = 100, normalized size = 1.

$$-\frac{a^8}{5x^5} - \frac{8a^7b}{3x^3} - \frac{28a^6b^2}{x} + 56a^5b^3x + \frac{70}{3}a^4b^4x^3 + \frac{56}{5}a^3b^5x^5 + 4a^2b^6x^7 + \frac{8}{9}ab^7x^9 + \frac{b^8x^{11}}{11}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^6, x]

[Out]  $-a^8/(5*x^5) - (8*a^7*b)/(3*x^3) - (28*a^6*b^2)/x + 56*a^5*b^3*x + (70*a^4*b^4*x^3)/3 + (56*a^3*b^5*x^5)/5 + 4*a^2*b^6*x^7 + (8*a*b^7*x^9)/9 + (b^8*x^11)/11$

**Maple [A]** time = 0.009, size = 89, normalized size = 0.9

$$-\frac{a^8}{5x^5} - \frac{8a^7b}{3x^3} - 28\frac{a^6b^2}{x} + 56a^5b^3x + \frac{70a^4b^4x^3}{3} + \frac{56a^3b^5x^5}{5} + 4a^2b^6x^7 + \frac{8ab^7x^9}{9} + \frac{b^8x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^6,x)`

[Out]  $-1/5*a^8/x^5-8/3*a^7*b/x^3-28*a^6*b^2/x+56*a^5*b^3*x+70/3*a^4*b^4*x^3+56/5*a^3*b^5*x^5+4*a^2*b^6*x^7+8/9*a*b^7*x^9+1/11*b^8*x^11$

**Maxima [A]** time = 1.34194, size = 123, normalized size = 1.23

$$\frac{1}{11}b^8x^{11} + \frac{8}{9}ab^7x^9 + 4a^2b^6x^7 + \frac{56}{5}a^3b^5x^5 + \frac{70}{3}a^4b^4x^3 + 56a^5b^3x - \frac{420a^6b^2x^4 + 40a^7bx^2 + 3a^8}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^6,x, algorithm="maxima")`

[Out]  $1/11*b^8*x^11 + 8/9*a*b^7*x^9 + 4*a^2*b^6*x^7 + 56/5*a^3*b^5*x^5 + 70/3*a^4*b^4*x^3 + 56*a^5*b^3*x - 1/15*(420*a^6*b^2*x^4 + 40*a^7*b*x^2 + 3*a^8)/x^5$

**Fricas [A]** time = 0.198182, size = 124, normalized size = 1.24

$$\frac{45b^8x^{16} + 440ab^7x^{14} + 1980a^2b^6x^{12} + 5544a^3b^5x^{10} + 11550a^4b^4x^8 + 27720a^5b^3x^6 - 13860a^6b^2x^4 - 1320a^7bx^2 - 99a^8}{495x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^6,x, algorithm="fricas")`

[Out]  $1/495*(45*b^8*x^16 + 440*a*b^7*x^14 + 1980*a^2*b^6*x^12 + 5544*a^3*b^5*x^10 + 11550*a^4*b^4*x^8 + 27720*a^5*b^3*x^6 - 13860*a^6*b^2*x^4 - 1320*a^7*b*x^2 - 99*a^8)/x^5$

**Sympy [A]** time = 1.68407, size = 100, normalized size = 1.

$$56a^5b^3x + \frac{70a^4b^4x^3}{3} + \frac{56a^3b^5x^5}{5} + 4a^2b^6x^7 + \frac{8ab^7x^9}{9} + \frac{b^8x^{11}}{11} - \frac{3a^8 + 40a^7bx^2 + 420a^6b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**6,x)`

[Out]  $56*a**5*b**3*x + 70*a**4*b**4*x**3/3 + 56*a**3*b**5*x**5/5 + 4*a**2*b**6*x**7 + 8*a*b**7*x**9/9 + b**8*x**11/11 - (3*a**8 + 40*a**7*b*x**2 + 420*a**6*b**2*x**4)/(15*x**5)$

**GIAC/XCAS [A]** time = 0.208788, size = 123, normalized size = 1.23

$$\frac{1}{11}b^8x^{11} + \frac{8}{9}ab^7x^9 + 4a^2b^6x^7 + \frac{56}{5}a^3b^5x^5 + \frac{70}{3}a^4b^4x^3 + 56a^5b^3x - \frac{420a^6b^2x^4 + 40a^7bx^2 + 3a^8}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^6,x, algorithm="giac")`

```
[Out] 1/11*b^8*x^11 + 8/9*a*b^7*x^9 + 4*a^2*b^6*x^7 + 56/5*a^3*b^5*x^5  
+ 70/3*a^4*b^4*x^3 + 56*a^5*b^3*x - 1/15*(420*a^6*b^2*x^4 + 40*a^  
7*b*x^2 + 3*a^8)/x^5
```

$$3.117 \quad \int \frac{(a+bx^2)^8}{x^8} dx$$

**Optimal.** Leaf size=102

$$-\frac{a^8}{7x^7} - \frac{8a^7b}{5x^5} - \frac{28a^6b^2}{3x^3} - \frac{56a^5b^3}{x} + 70a^4b^4x + \frac{56}{3}a^3b^5x^3 + \frac{28}{5}a^2b^6x^5 + \frac{8}{7}ab^7x^7 + \frac{b^8x^9}{9}$$

[Out]  $-a^8/(7*x^7) - (8*a^7*b)/(5*x^5) - (28*a^6*b^2)/(3*x^3) - (56*a^5*b^3)/x + 70*a^4*b^4*x + (56*a^3*b^5*x^3)/3 + (28*a^2*b^6*x^5)/5 + (8*a*b^7*x^7)/7 + (b^8*x^9)/9$

**Rubi [A]** time = 0.104669, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^8}{7x^7} - \frac{8a^7b}{5x^5} - \frac{28a^6b^2}{3x^3} - \frac{56a^5b^3}{x} + 70a^4b^4x + \frac{56}{3}a^3b^5x^3 + \frac{28}{5}a^2b^6x^5 + \frac{8}{7}ab^7x^7 + \frac{b^8x^9}{9}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^8, x]

[Out]  $-a^8/(7*x^7) - (8*a^7*b)/(5*x^5) - (28*a^6*b^2)/(3*x^3) - (56*a^5*b^3)/x + 70*a^4*b^4*x + (56*a^3*b^5*x^3)/3 + (28*a^2*b^6*x^5)/5 + (8*a*b^7*x^7)/7 + (b^8*x^9)/9$

**Rubi in Sympy [A]** time = 19.4764, size = 100, normalized size = 0.98

$$-\frac{a^8}{7x^7} - \frac{8a^7b}{5x^5} - \frac{28a^6b^2}{3x^3} - \frac{56a^5b^3}{x} + 70a^4b^4x + \frac{56a^3b^5x^3}{3} + \frac{28a^2b^6x^5}{5} + \frac{8ab^7x^7}{7} + \frac{b^8x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*8, x)

[Out]  $-a**8/(7*x**7) - 8*a**7*b/(5*x**5) - 28*a**6*b**2/(3*x**3) - 56*a**5*b**3/x + 70*a**4*b**4*x + 56*a**3*b**5*x**3/3 + 28*a**2*b**6*x**5/5 + 8*a*b**7*x**7/7 + b**8*x**9/9$

**Mathematica [A]** time = 0.0110263, size = 102, normalized size = 1.

$$-\frac{a^8}{7x^7} - \frac{8a^7b}{5x^5} - \frac{28a^6b^2}{3x^3} - \frac{56a^5b^3}{x} + 70a^4b^4x + \frac{56}{3}a^3b^5x^3 + \frac{28}{5}a^2b^6x^5 + \frac{8}{7}ab^7x^7 + \frac{b^8x^9}{9}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^8, x]

[Out]  $-a^8/(7*x^7) - (8*a^7*b)/(5*x^5) - (28*a^6*b^2)/(3*x^3) - (56*a^5*b^3)/x + 70*a^4*b^4*x + (56*a^3*b^5*x^3)/3 + (28*a^2*b^6*x^5)/5 + (8*a*b^7*x^7)/7 + (b^8*x^9)/9$

**Maple [A]** time = 0.009, size = 89, normalized size = 0.9

$$-\frac{a^8}{7x^7} - \frac{8a^7b}{5x^5} - \frac{28a^6b^2}{3x^3} - 56\frac{a^5b^3}{x} + 70a^4b^4x + \frac{56a^3b^5x^3}{3} + \frac{28a^2b^6x^5}{5} + \frac{8ab^7x^7}{7} + \frac{b^8x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^8,x)`

[Out]  $-1/7*a^8/x^7-8/5*a^7*b/x^5-28/3*a^6*b^2/x^3-56*a^5*b^3/x+70*a^4*b^4*x+56/3*a^3*b^5*x^3+28/5*a^2*b^6*x^5+8/7*a*b^7*x^7+1/9*b^8*x^9$

**Maxima [A]** time = 1.35642, size = 123, normalized size = 1.21

$$\frac{1}{9}b^8x^9 + \frac{8}{7}ab^7x^7 + \frac{28}{5}a^2b^6x^5 + \frac{56}{3}a^3b^5x^3 + 70a^4b^4x - \frac{5880a^5b^3x^6 + 980a^6b^2x^4 + 168a^7bx^2 + 15a^8}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^8,x, algorithm="maxima")`

[Out]  $1/9*b^8*x^9 + 8/7*a*b^7*x^7 + 28/5*a^2*b^6*x^5 + 56/3*a^3*b^5*x^3 + 70*a^4*b^4*x - 1/105*(5880*a^5*b^3*x^6 + 980*a^6*b^2*x^4 + 168*a^7*b*x^2 + 15*a^8)/x^7$

**Fricas [A]** time = 0.196614, size = 124, normalized size = 1.22

$$\frac{35b^8x^{16} + 360ab^7x^{14} + 1764a^2b^6x^{12} + 5880a^3b^5x^{10} + 22050a^4b^4x^8 - 17640a^5b^3x^6 - 2940a^6b^2x^4 - 504a^7bx^2 - 45a^8}{315x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^8,x, algorithm="fricas")`

[Out]  $1/315*(35*b^8*x^{16} + 360*a*b^7*x^{14} + 1764*a^2*b^6*x^{12} + 5880*a^3*b^5*x^{10} + 22050*a^4*b^4*x^8 - 17640*a^5*b^3*x^6 - 2940*a^6*b^2*x^4 - 504*a^7*b*x^2 - 45*a^8)/x^7$

**Sympy [A]** time = 1.90107, size = 100, normalized size = 0.98

$$70a^4b^4x + \frac{56a^3b^5x^3}{3} + \frac{28a^2b^6x^5}{5} + \frac{8ab^7x^7}{7} + \frac{b^8x^9}{9} - \frac{15a^8 + 168a^7bx^2 + 980a^6b^2x^4 + 5880a^5b^3x^6}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**8,x)`

[Out]  $70*a**4*b**4*x + 56*a**3*b**5*x**3/3 + 28*a**2*b**6*x**5/5 + 8*a*b**7*x**7/7 + b**8*x**9/9 - (15*a**8 + 168*a**7*b*x**2 + 980*a**6*b**2*x**4 + 5880*a**5*b**3*x**6)/(105*x**7)$

**GIAC/XCAS [A]** time = 0.209392, size = 123, normalized size = 1.21

$$\frac{1}{9}b^8x^9 + \frac{8}{7}ab^7x^7 + \frac{28}{5}a^2b^6x^5 + \frac{56}{3}a^3b^5x^3 + 70a^4b^4x - \frac{5880a^5b^3x^6 + 980a^6b^2x^4 + 168a^7bx^2 + 15a^8}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^8,x, algorithm="giac")`



```
[Out] 1/9*b^8*x^9 + 8/7*a*b^7*x^7 + 28/5*a^2*b^6*x^5 + 56/3*a^3*b^5*x^3  
+ 70*a^4*b^4*x - 1/105*(5880*a^5*b^3*x^6 + 980*a^6*b^2*x^4 + 168  
*a^7*b*x^2 + 15*a^8)/x^7
```

$$3.118 \quad \int \frac{(a+bx^2)^8}{x^{10}} dx$$

**Optimal.** Leaf size=102

$$-\frac{a^8}{9x^9} - \frac{8a^7b}{7x^7} - \frac{28a^6b^2}{5x^5} - \frac{56a^5b^3}{3x^3} - \frac{70a^4b^4}{x} + 56a^3b^5x + \frac{28}{3}a^2b^6x^3 + \frac{8}{5}ab^7x^5 + \frac{b^8x^7}{7}$$

[Out]  $-a^8/(9*x^9) - (8*a^7*b)/(7*x^7) - (28*a^6*b^2)/(5*x^5) - (56*a^5*b^3)/(3*x^3) - (70*a^4*b^4)/x + 56*a^3*b^5*x + (28*a^2*b^6*x^3)/3 + (8*a*b^7*x^5)/5 + (b^8*x^7)/7$

**Rubi [A]** time = 0.105443, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^8}{9x^9} - \frac{8a^7b}{7x^7} - \frac{28a^6b^2}{5x^5} - \frac{56a^5b^3}{3x^3} - \frac{70a^4b^4}{x} + 56a^3b^5x + \frac{28}{3}a^2b^6x^3 + \frac{8}{5}ab^7x^5 + \frac{b^8x^7}{7}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^10, x]

[Out]  $-a^8/(9*x^9) - (8*a^7*b)/(7*x^7) - (28*a^6*b^2)/(5*x^5) - (56*a^5*b^3)/(3*x^3) - (70*a^4*b^4)/x + 56*a^3*b^5*x + (28*a^2*b^6*x^3)/3 + (8*a*b^7*x^5)/5 + (b^8*x^7)/7$

**Rubi in Sympy [A]** time = 19.801, size = 100, normalized size = 0.98

$$-\frac{a^8}{9x^9} - \frac{8a^7b}{7x^7} - \frac{28a^6b^2}{5x^5} - \frac{56a^5b^3}{3x^3} - \frac{70a^4b^4}{x} + 56a^3b^5x + \frac{28a^2b^6x^3}{3} + \frac{8ab^7x^5}{5} + \frac{b^8x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*10, x)

[Out]  $-a**8/(9*x**9) - 8*a**7*b/(7*x**7) - 28*a**6*b**2/(5*x**5) - 56*a**5*b**3/(3*x**3) - 70*a**4*b**4/x + 56*a**3*b**5*x + 28*a**2*b**6*x^3/3 + 8*a*b**7*x^5/5 + b**8*x**7/7$

**Mathematica [A]** time = 0.0217153, size = 102, normalized size = 1.

$$-\frac{a^8}{9x^9} - \frac{8a^7b}{7x^7} - \frac{28a^6b^2}{5x^5} - \frac{56a^5b^3}{3x^3} - \frac{70a^4b^4}{x} + 56a^3b^5x + \frac{28}{3}a^2b^6x^3 + \frac{8}{5}ab^7x^5 + \frac{b^8x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^10, x]

[Out]  $-a^8/(9*x^9) - (8*a^7*b)/(7*x^7) - (28*a^6*b^2)/(5*x^5) - (56*a^5*b^3)/(3*x^3) - (70*a^4*b^4)/x + 56*a^3*b^5*x + (28*a^2*b^6*x^3)/3 + (8*a*b^7*x^5)/5 + (b^8*x^7)/7$

**Maple [A]** time = 0.009, size = 89, normalized size = 0.9

$$-\frac{a^8}{9x^9} - \frac{8a^7b}{7x^7} - \frac{28a^6b^2}{5x^5} - \frac{56a^5b^3}{3x^3} - 70\frac{a^4b^4}{x} + 56a^3b^5x + \frac{28a^2b^6x^3}{3} + \frac{8ab^7x^5}{5} + \frac{b^8x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^10,x)`

[Out]  $-1/9*a^8/x^9 - 8/7*a^7*b/x^7 - 28/5*a^6*b^2/x^5 - 56/3*a^5*b^3/x^3 - 70*a^4*b^4/x + 56*a^3*b^5*x + 28/3*a^2*b^6*x^3 + 8/5*a*b^7*x^5 + 1/7*b^8*x^7$

**Maxima [A]** time = 1.32951, size = 123, normalized size = 1.21

$$\frac{1}{7}b^8x^7 + \frac{8}{5}ab^7x^5 + \frac{28}{3}a^2b^6x^3 + 56a^3b^5x - \frac{22050a^4b^4x^8 + 5880a^5b^3x^6 + 1764a^6b^2x^4 + 360a^7bx^2 + 35a^8}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^10,x, algorithm="maxima")`

[Out]  $1/7*b^8*x^7 + 8/5*a*b^7*x^5 + 28/3*a^2*b^6*x^3 + 56*a^3*b^5*x - 1/315*(22050*a^4*b^4*x^8 + 5880*a^5*b^3*x^6 + 1764*a^6*b^2*x^4 + 360*a^7*b*x^2 + 35*a^8)/x^9$

**Fricas [A]** time = 0.195142, size = 124, normalized size = 1.22

$$\frac{45b^8x^{16} + 504ab^7x^{14} + 2940a^2b^6x^{12} + 17640a^3b^5x^{10} - 22050a^4b^4x^8 - 5880a^5b^3x^6 - 1764a^6b^2x^4 - 360a^7bx^2 - 35a^8}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^10,x, algorithm="fricas")`

[Out]  $1/315*(45*b^8*x^{16} + 504*a*b^7*x^{14} + 2940*a^2*b^6*x^{12} + 17640*a^3*b^5*x^{10} - 22050*a^4*b^4*x^8 - 5880*a^5*b^3*x^6 - 1764*a^6*b^2*x^4 - 360*a^7*b*x^2 - 35*a^8)/x^9$

**Sympy [A]** time = 2.22866, size = 99, normalized size = 0.97

$$56a^3b^5x + \frac{28a^2b^6x^3}{3} + \frac{8ab^7x^5}{5} + \frac{b^8x^7}{7} - \frac{35a^8 + 360a^7bx^2 + 1764a^6b^2x^4 + 5880a^5b^3x^6 + 22050a^4b^4x^8}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**10,x)`

[Out]  $56*a**3*b**5*x + 28*a**2*b**6*x**3/3 + 8*a*b**7*x**5/5 + b**8*x**7/7 - (35*a**8 + 360*a**7*b*x**2 + 1764*a**6*b**2*x**4 + 5880*a**5*b**3*x**6 + 22050*a**4*b**4*x**8)/(315*x**9)$

**GIAC/XCAS [A]** time = 0.211474, size = 123, normalized size = 1.21

$$\frac{1}{7}b^8x^7 + \frac{8}{5}ab^7x^5 + \frac{28}{3}a^2b^6x^3 + 56a^3b^5x - \frac{22050a^4b^4x^8 + 5880a^5b^3x^6 + 1764a^6b^2x^4 + 360a^7bx^2 + 35a^8}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^10,x, algorithm="giac")`

[Out]  $\frac{1}{7}b^8x^7 + \frac{8}{5}ab^7x^5 + \frac{28}{3}a^2b^6x^3 + 56a^3b^5x - \frac{1}{315}(22050a^4b^4x^8 + 5880a^5b^3x^6 + 1764a^6b^2x^4 + 360a^7bx^2 + 35a^8)/x^9$

$$3.119 \quad \int \frac{(a+bx^2)^8}{x^{12}} dx$$

**Optimal.** Leaf size=100

$$-\frac{a^8}{11x^{11}} - \frac{8a^7b}{9x^9} - \frac{4a^6b^2}{x^7} - \frac{56a^5b^3}{5x^5} - \frac{70a^4b^4}{3x^3} - \frac{56a^3b^5}{x} + 28a^2b^6x + \frac{8ab^7x^3}{3} + \frac{b^8x^5}{5}$$

[Out]  $-a^8/(11*x^{11}) - (8*a^7*b)/(9*x^9) - (4*a^6*b^2)/x^7 - (56*a^5*b^3)/(5*x^5) - (70*a^4*b^4)/(3*x^3) - (56*a^3*b^5)/x + 28*a^2*b^6*x + (8*a*b^7*x^3)/3 + (b^8*x^5)/5$

**Rubi [A]** time = 0.105073, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^8}{11x^{11}} - \frac{8a^7b}{9x^9} - \frac{4a^6b^2}{x^7} - \frac{56a^5b^3}{5x^5} - \frac{70a^4b^4}{3x^3} - \frac{56a^3b^5}{x} + 28a^2b^6x + \frac{8ab^7x^3}{3} + \frac{b^8x^5}{5}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^12, x]

[Out]  $-a^8/(11*x^{11}) - (8*a^7*b)/(9*x^9) - (4*a^6*b^2)/x^7 - (56*a^5*b^3)/(5*x^5) - (70*a^4*b^4)/(3*x^3) - (56*a^3*b^5)/x + 28*a^2*b^6*x + (8*a*b^7*x^3)/3 + (b^8*x^5)/5$

**Rubi in Sympy [A]** time = 19.6947, size = 99, normalized size = 0.99

$$-\frac{a^8}{11x^{11}} - \frac{8a^7b}{9x^9} - \frac{4a^6b^2}{x^7} - \frac{56a^5b^3}{5x^5} - \frac{70a^4b^4}{3x^3} - \frac{56a^3b^5}{x} + 28a^2b^6x + \frac{8ab^7x^3}{3} + \frac{b^8x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*12, x)

[Out]  $-a**8/(11*x**11) - 8*a**7*b/(9*x**9) - 4*a**6*b**2/x**7 - 56*a**5*b**3/(5*x**5) - 70*a**4*b**4/(3*x**3) - 56*a**3*b**5/x + 28*a**2*b**6*x + 8*a*b**7*x**3/3 + b**8*x**5/5$

**Mathematica [A]** time = 0.0151624, size = 100, normalized size = 1.

$$-\frac{a^8}{11x^{11}} - \frac{8a^7b}{9x^9} - \frac{4a^6b^2}{x^7} - \frac{56a^5b^3}{5x^5} - \frac{70a^4b^4}{3x^3} - \frac{56a^3b^5}{x} + 28a^2b^6x + \frac{8ab^7x^3}{3} + \frac{b^8x^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^12, x]

[Out]  $-a^8/(11*x^{11}) - (8*a^7*b)/(9*x^9) - (4*a^6*b^2)/x^7 - (56*a^5*b^3)/(5*x^5) - (70*a^4*b^4)/(3*x^3) - (56*a^3*b^5)/x + 28*a^2*b^6*x + (8*a*b^7*x^3)/3 + (b^8*x^5)/5$

**Maple [A]** time = 0.009, size = 89, normalized size = 0.9

$$-\frac{a^8}{11x^{11}} - \frac{8a^7b}{9x^9} - 4\frac{a^6b^2}{x^7} - \frac{56a^5b^3}{5x^5} - \frac{70a^4b^4}{3x^3} - 56\frac{a^3b^5}{x} + 28a^2b^6x + \frac{8ab^7x^3}{3} + \frac{b^8x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^12,x)`

[Out]  $-1/11*a^8/x^{11}-8/9*a^7*b/x^9-4*a^6*b^2/x^7-56/5*a^5*b^3/x^5-70/3*a^4*b^4/x^3-56*a^3*b^5/x+28*a^2*b^6*x+8/3*a*b^7*x^3+1/5*b^8*x^5$

**Maxima [A]** time = 1.3232, size = 123, normalized size = 1.23

$$\frac{1}{5}b^8x^5 + \frac{8}{3}ab^7x^3 + 28a^2b^6x - \frac{27720a^3b^5x^{10} + 11550a^4b^4x^8 + 5544a^5b^3x^6 + 1980a^6b^2x^4 + 440a^7bx^2 + 45a^8}{495x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^12,x, algorithm="maxima")`

[Out]  $1/5*b^8*x^5 + 8/3*a*b^7*x^3 + 28*a^2*b^6*x - 1/495*(27720*a^3*b^5*x^{10} + 11550*a^4*b^4*x^8 + 5544*a^5*b^3*x^6 + 1980*a^6*b^2*x^4 + 440*a^7*b*x^2 + 45*a^8)/x^{11}$

**Fricas [A]** time = 0.198086, size = 124, normalized size = 1.24

$$\frac{99b^8x^{16} + 1320ab^7x^{14} + 13860a^2b^6x^{12} - 27720a^3b^5x^{10} - 11550a^4b^4x^8 - 5544a^5b^3x^6 - 1980a^6b^2x^4 - 440a^7bx^2 - 45a^8}{495x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^12,x, algorithm="fricas")`

[Out]  $1/495*(99*b^8*x^{16} + 1320*a*b^7*x^{14} + 13860*a^2*b^6*x^{12} - 27720*a^3*b^5*x^{10} - 11550*a^4*b^4*x^8 - 5544*a^5*b^3*x^6 - 1980*a^6*b^2*x^4 - 440*a^7*b*x^2 - 45*a^8)/x^{11}$

**Sympy [A]** time = 2.53334, size = 97, normalized size = 0.97

$$28a^2b^6x + \frac{8ab^7x^3}{3} + \frac{b^8x^5}{5} - \frac{45a^8 + 440a^7bx^2 + 1980a^6b^2x^4 + 5544a^5b^3x^6 + 11550a^4b^4x^8 + 27720a^3b^5x^{10}}{495x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**12,x)`

[Out]  $28*a**2*b**6*x + 8*a*b**7*x**3/3 + b**8*x**5/5 - (45*a**8 + 440*a**7*b*x**2 + 1980*a**6*b**2*x**4 + 5544*a**5*b**3*x**6 + 11550*a**4*b**4*x**8 + 27720*a**3*b**5*x**10)/(495*x**11)$

**GIAC/XCAS [A]** time = 0.219101, size = 123, normalized size = 1.23

$$\frac{1}{5}b^8x^5 + \frac{8}{3}ab^7x^3 + 28a^2b^6x - \frac{27720a^3b^5x^{10} + 11550a^4b^4x^8 + 5544a^5b^3x^6 + 1980a^6b^2x^4 + 440a^7bx^2 + 45a^8}{495x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^12,x, algorithm="giac")`

```
[Out] 1/5*b^8*x^5 + 8/3*a*b^7*x^3 + 28*a^2*b^6*x - 1/495*(27720*a^3*b^5
*x^10 + 11550*a^4*b^4*x^8 + 5544*a^5*b^3*x^6 + 1980*a^6*b^2*x^4 +
440*a^7*b*x^2 + 45*a^8)/x^11
```

$$3.120 \quad \int \frac{(a+bx^2)^8}{x^{14}} dx$$

**Optimal.** Leaf size=98

$$-\frac{a^8}{13x^{13}} - \frac{8a^7b}{11x^{11}} - \frac{28a^6b^2}{9x^9} - \frac{8a^5b^3}{x^7} - \frac{14a^4b^4}{x^5} - \frac{56a^3b^5}{3x^3} - \frac{28a^2b^6}{x} + 8ab^7x + \frac{b^8x^3}{3}$$

[Out]  $-a^8/(13*x^{13}) - (8*a^7*b)/(11*x^{11}) - (28*a^6*b^2)/(9*x^9) - (8*a^5*b^3)/x^7 - (14*a^4*b^4)/x^5 - (56*a^3*b^5)/(3*x^3) - (28*a^2*b^6)/x + 8*a*b^7*x + (b^8*x^3)/3$

**Rubi [A]** time = 0.105774, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^8}{13x^{13}} - \frac{8a^7b}{11x^{11}} - \frac{28a^6b^2}{9x^9} - \frac{8a^5b^3}{x^7} - \frac{14a^4b^4}{x^5} - \frac{56a^3b^5}{3x^3} - \frac{28a^2b^6}{x} + 8ab^7x + \frac{b^8x^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^14, x]

[Out]  $-a^8/(13*x^{13}) - (8*a^7*b)/(11*x^{11}) - (28*a^6*b^2)/(9*x^9) - (8*a^5*b^3)/x^7 - (14*a^4*b^4)/x^5 - (56*a^3*b^5)/(3*x^3) - (28*a^2*b^6)/x + 8*a*b^7*x + (b^8*x^3)/3$

**Rubi in Sympy [A]** time = 19.7353, size = 97, normalized size = 0.99

$$-\frac{a^8}{13x^{13}} - \frac{8a^7b}{11x^{11}} - \frac{28a^6b^2}{9x^9} - \frac{8a^5b^3}{x^7} - \frac{14a^4b^4}{x^5} - \frac{56a^3b^5}{3x^3} - \frac{28a^2b^6}{x} + 8ab^7x + \frac{b^8x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*14, x)

[Out]  $-a**8/(13*x**13) - 8*a**7*b/(11*x**11) - 28*a**6*b**2/(9*x**9) - 8*a**5*b**3/x**7 - 14*a**4*b**4/x**5 - 56*a**3*b**5/(3*x**3) - 28*a**2*b**6/x + 8*a*b**7*x + b**8*x**3/3$

**Mathematica [A]** time = 0.017312, size = 98, normalized size = 1.

$$-\frac{a^8}{13x^{13}} - \frac{8a^7b}{11x^{11}} - \frac{28a^6b^2}{9x^9} - \frac{8a^5b^3}{x^7} - \frac{14a^4b^4}{x^5} - \frac{56a^3b^5}{3x^3} - \frac{28a^2b^6}{x} + 8ab^7x + \frac{b^8x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^14, x]

[Out]  $-a^8/(13*x^{13}) - (8*a^7*b)/(11*x^{11}) - (28*a^6*b^2)/(9*x^9) - (8*a^5*b^3)/x^7 - (14*a^4*b^4)/x^5 - (56*a^3*b^5)/(3*x^3) - (28*a^2*b^6)/x + 8*a*b^7*x + (b^8*x^3)/3$

**Maple [A]** time = 0.009, size = 89, normalized size = 0.9

$$-\frac{a^8}{13x^{13}} - \frac{8a^7b}{11x^{11}} - \frac{28a^6b^2}{9x^9} - 8\frac{a^5b^3}{x^7} - 14\frac{a^4b^4}{x^5} - \frac{56a^3b^5}{3x^3} - 28\frac{a^2b^6}{x} + 8ab^7x + \frac{b^8x^3}{3}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^14,x)`

[Out]  $-1/13*a^8/x^{13}-8/11*a^7*b/x^{11}-28/9*a^6*b^2/x^9-8*a^5*b^3/x^7-14*a^4*b^4/x^5-56/3*a^3*b^5/x^3-28*a^2*b^6/x+8*a*b^7*x+1/3*b^8*x^3$

**Maxima [A]** time = 1.51084, size = 123, normalized size = 1.26

$$\frac{\frac{1}{3}b^8x^3 + 8ab^7x}{36036a^2b^6x^{12} + 24024a^3b^5x^{10} + 18018a^4b^4x^8 + 10296a^5b^3x^6 + 4004a^6b^2x^4 + 936a^7bx^2 + 99a^8} \cdot 1287x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^14,x, algorithm="maxima")`

[Out]  $1/3*b^8*x^3 + 8*a*b^7*x - 1/1287*(36036*a^2*b^6*x^{12} + 24024*a^3*b^5*x^{10} + 18018*a^4*b^4*x^8 + 10296*a^5*b^3*x^6 + 4004*a^6*b^2*x^4 + 936*a^7*b*x^2 + 99*a^8)/x^{13}$

**Fricas [A]** time = 0.200349, size = 124, normalized size = 1.27

$$\frac{429b^8x^{16} + 10296ab^7x^{14} - 36036a^2b^6x^{12} - 24024a^3b^5x^{10} - 18018a^4b^4x^8 - 10296a^5b^3x^6 - 4004a^6b^2x^4 - 936a^7bx^2 - 99a^8}{1287x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^14,x, algorithm="fricas")`

[Out]  $1/1287*(429*b^8*x^{16} + 10296*a*b^7*x^{14} - 36036*a^2*b^6*x^{12} - 24024*a^3*b^5*x^{10} - 18018*a^4*b^4*x^8 - 10296*a^5*b^3*x^6 - 4004*a^6*b^2*x^4 - 936*a^7*b*x^2 - 99*a^8)/x^{13}$

**Sympy [A]** time = 2.88981, size = 95, normalized size = 0.97

$$8ab^7x + \frac{b^8x^3}{3} - \frac{99a^8 + 936a^7bx^2 + 4004a^6b^2x^4 + 10296a^5b^3x^6 + 18018a^4b^4x^8 + 24024a^3b^5x^{10} + 36036a^2b^6x^{12}}{1287x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**14,x)`

[Out]  $8*a*b**7*x + b**8*x**3/3 - (99*a**8 + 936*a**7*b*x**2 + 4004*a**6*b**2*x**4 + 10296*a**5*b**3*x**6 + 18018*a**4*b**4*x**8 + 24024*a**3*b**5*x**10 + 36036*a**2*b**6*x**12)/(1287*x**13)$

**GIAC/XCAS [A]** time = 0.206847, size = 123, normalized size = 1.26

$$\frac{\frac{1}{3}b^8x^3 + 8ab^7x}{36036a^2b^6x^{12} + 24024a^3b^5x^{10} + 18018a^4b^4x^8 + 10296a^5b^3x^6 + 4004a^6b^2x^4 + 936a^7bx^2 + 99a^8} \cdot 1287x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^8/x^14,x, algorithm="giac")
```

```
[Out] 1/3*b^8*x^3 + 8*a*b^7*x - 1/1287*(36036*a^2*b^6*x^12 + 24024*a^3*  
b^5*x^10 + 18018*a^4*b^4*x^8 + 10296*a^5*b^3*x^6 + 4004*a^6*b^2*x  
^4 + 936*a^7*b*x^2 + 99*a^8)/x^13
```

$$3.121 \quad \int \frac{(a+bx^2)^8}{x^{16}} dx$$

**Optimal.** Leaf size=99

$$-\frac{a^8}{15x^{15}} - \frac{8a^7b}{13x^{13}} - \frac{28a^6b^2}{11x^{11}} - \frac{56a^5b^3}{9x^9} - \frac{10a^4b^4}{x^7} - \frac{56a^3b^5}{5x^5} - \frac{28a^2b^6}{3x^3} - \frac{8ab^7}{x} + b^8x$$

[Out]  $-a^8/(15*x^{15}) - (8*a^7*b)/(13*x^{13}) - (28*a^6*b^2)/(11*x^{11}) - (56*a^5*b^3)/(9*x^9) - (10*a^4*b^4)/x^7 - (56*a^3*b^5)/(5*x^5) - (28*a^2*b^6)/(3*x^3) - (8*a*b^7)/x + b^8*x$

**Rubi [A]** time = 0.107473, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^8}{15x^{15}} - \frac{8a^7b}{13x^{13}} - \frac{28a^6b^2}{11x^{11}} - \frac{56a^5b^3}{9x^9} - \frac{10a^4b^4}{x^7} - \frac{56a^3b^5}{5x^5} - \frac{28a^2b^6}{3x^3} - \frac{8ab^7}{x} + b^8x$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^16, x]

[Out]  $-a^8/(15*x^{15}) - (8*a^7*b)/(13*x^{13}) - (28*a^6*b^2)/(11*x^{11}) - (56*a^5*b^3)/(9*x^9) - (10*a^4*b^4)/x^7 - (56*a^3*b^5)/(5*x^5) - (28*a^2*b^6)/(3*x^3) - (8*a*b^7)/x + b^8*x$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^8}{15x^{15}} - \frac{8a^7b}{13x^{13}} - \frac{28a^6b^2}{11x^{11}} - \frac{56a^5b^3}{9x^9} - \frac{10a^4b^4}{x^7} - \frac{56a^3b^5}{5x^5} - \frac{28a^2b^6}{3x^3} - \frac{8ab^7}{x} + \int b^8 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*16, x)

[Out]  $-a**8/(15*x**15) - 8*a**7*b/(13*x**13) - 28*a**6*b**2/(11*x**11) - 56*a**5*b**3/(9*x**9) - 10*a**4*b**4/x**7 - 56*a**3*b**5/(5*x**5) - 28*a**2*b**6/(3*x**3) - 8*a*b**7/x + \text{Integral}(b**8, x)$

**Mathematica [A]** time = 0.00982412, size = 99, normalized size = 1.

$$-\frac{a^8}{15x^{15}} - \frac{8a^7b}{13x^{13}} - \frac{28a^6b^2}{11x^{11}} - \frac{56a^5b^3}{9x^9} - \frac{10a^4b^4}{x^7} - \frac{56a^3b^5}{5x^5} - \frac{28a^2b^6}{3x^3} - \frac{8ab^7}{x} + b^8x$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^16, x]

[Out]  $-a^8/(15*x^{15}) - (8*a^7*b)/(13*x^{13}) - (28*a^6*b^2)/(11*x^{11}) - (56*a^5*b^3)/(9*x^9) - (10*a^4*b^4)/x^7 - (56*a^3*b^5)/(5*x^5) - (28*a^2*b^6)/(3*x^3) - (8*a*b^7)/x + b^8*x$

**Maple [A]** time = 0.01, size = 88, normalized size = 0.9

$$-\frac{a^8}{15x^{15}} - \frac{8a^7b}{13x^{13}} - \frac{28a^6b^2}{11x^{11}} - \frac{56a^5b^3}{9x^9} - 10\frac{a^4b^4}{x^7} - \frac{56a^3b^5}{5x^5} - \frac{28a^2b^6}{3x^3} - 8\frac{ab^7}{x} + b^8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^16,x)`

[Out] 
$$-1/15*a^8/x^15 - 8/13*a^7*b/x^13 - 28/11*a^6*b^2/x^11 - 56/9*a^5*b^3/x^9 - 10*a^4*b^4/x^7 - 56/5*a^3*b^5/x^5 - 28/3*a^2*b^6/x^3 - 8*a*b^7/x + b^8/x$$

**Maxima [A]** time = 1.34659, size = 122, normalized size = 1.23

$$\frac{b^8 x}{51480 a b^7 x^{14} + 60060 a^2 b^6 x^{12} + 72072 a^3 b^5 x^{10} + 64350 a^4 b^4 x^8 + 40040 a^5 b^3 x^6 + 16380 a^6 b^2 x^4 + 3960 a^7 b x^2 + 429 a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^16,x, algorithm="maxima")`

[Out] 
$$b^8 x - \frac{1}{6435} (51480 a b^7 x^{14} + 60060 a^2 b^6 x^{12} + 72072 a^3 b^5 x^{10} + 64350 a^4 b^4 x^8 + 40040 a^5 b^3 x^6 + 16380 a^6 b^2 x^4 + 3960 a^7 b x^2 + 429 a^8) / x^{15}$$

**Fricas [A]** time = 0.199813, size = 124, normalized size = 1.25

$$\frac{6435 b^8 x^{16} - 51480 a b^7 x^{14} - 60060 a^2 b^6 x^{12} - 72072 a^3 b^5 x^{10} - 64350 a^4 b^4 x^8 - 40040 a^5 b^3 x^6 - 16380 a^6 b^2 x^4 - 3960 a^7 b x^2 - 429 a^8}{6435 x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^16,x, algorithm="fricas")`

[Out] 
$$\frac{1}{6435} (6435 b^8 x^{16} - 51480 a b^7 x^{14} - 60060 a^2 b^6 x^{12} - 72072 a^3 b^5 x^{10} - 64350 a^4 b^4 x^8 - 40040 a^5 b^3 x^6 - 16380 a^6 b^2 x^4 - 3960 a^7 b x^2 - 429 a^8) / x^{15}$$

**Sympy [A]** time = 3.31311, size = 94, normalized size = 0.95

$$\frac{b^8 x}{429 a^8 + 3960 a^7 b x^2 + 16380 a^6 b^2 x^4 + 40040 a^5 b^3 x^6 + 64350 a^4 b^4 x^8 + 72072 a^3 b^5 x^{10} + 60060 a^2 b^6 x^{12} + 51480 a b^7 x^{14} + 429 a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**16,x)`

[Out] 
$$b^{**8} x - (429 a^{**8} + 3960 a^{**7} b x^{**2} + 16380 a^{**6} b^{**2} x^{**4} + 40040 a^{**5} b^{**3} x^{**6} + 64350 a^{**4} b^{**4} x^{**8} + 72072 a^{**3} b^{**5} x^{**10} + 60060 a^{**2} b^{**6} x^{**12} + 51480 a b^{**7} x^{**14}) / (6435 x^{**15})$$

**GIAC/XCAS [A]** time = 0.213568, size = 122, normalized size = 1.23

$$\frac{b^8 x}{51480 a b^7 x^{14} + 60060 a^2 b^6 x^{12} + 72072 a^3 b^5 x^{10} + 64350 a^4 b^4 x^8 + 40040 a^5 b^3 x^6 + 16380 a^6 b^2 x^4 + 3960 a^7 b x^2 + 429 a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^8/x^16,x, algorithm="giac")
```

```
[Out] b^8*x - 1/6435*(51480*a*b^7*x^14 + 60060*a^2*b^6*x^12 + 72072*a^3*  
b^5*x^10 + 64350*a^4*b^4*x^8 + 40040*a^5*b^3*x^6 + 16380*a^6*b^2*  
x^4 + 3960*a^7*b*x^2 + 429*a^8)/x^15
```

$$3.122 \quad \int \frac{(a+bx^2)^8}{x^{18}} dx$$

**Optimal.** Leaf size=104

$$-\frac{a^8}{17x^{17}} - \frac{8a^7b}{15x^{15}} - \frac{28a^6b^2}{13x^{13}} - \frac{56a^5b^3}{11x^{11}} - \frac{70a^4b^4}{9x^9} - \frac{8a^3b^5}{x^7} - \frac{28a^2b^6}{5x^5} - \frac{8ab^7}{3x^3} - \frac{b^8}{x}$$

[Out]  $-a^8/(17*x^{17}) - (8*a^7*b)/(15*x^{15}) - (28*a^6*b^2)/(13*x^{13}) - (56*a^5*b^3)/(11*x^{11}) - (70*a^4*b^4)/(9*x^9) - (8*a^3*b^5)/x^7 - (28*a^2*b^6)/(5*x^5) - (8*a*b^7)/(3*x^3) - b^8/x$

**Rubi [A]** time = 0.108001, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^8}{17x^{17}} - \frac{8a^7b}{15x^{15}} - \frac{28a^6b^2}{13x^{13}} - \frac{56a^5b^3}{11x^{11}} - \frac{70a^4b^4}{9x^9} - \frac{8a^3b^5}{x^7} - \frac{28a^2b^6}{5x^5} - \frac{8ab^7}{3x^3} - \frac{b^8}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^18, x]

[Out]  $-a^8/(17*x^{17}) - (8*a^7*b)/(15*x^{15}) - (28*a^6*b^2)/(13*x^{13}) - (56*a^5*b^3)/(11*x^{11}) - (70*a^4*b^4)/(9*x^9) - (8*a^3*b^5)/x^7 - (28*a^2*b^6)/(5*x^5) - (8*a*b^7)/(3*x^3) - b^8/x$

**Rubi in Sympy [A]** time = 20.2036, size = 104, normalized size = 1.

$$-\frac{a^8}{17x^{17}} - \frac{8a^7b}{15x^{15}} - \frac{28a^6b^2}{13x^{13}} - \frac{56a^5b^3}{11x^{11}} - \frac{70a^4b^4}{9x^9} - \frac{8a^3b^5}{x^7} - \frac{28a^2b^6}{5x^5} - \frac{8ab^7}{3x^3} - \frac{b^8}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*18, x)

[Out]  $-a**8/(17*x**17) - 8*a**7*b/(15*x**15) - 28*a**6*b**2/(13*x**13) - 56*a**5*b**3/(11*x**11) - 70*a**4*b**4/(9*x**9) - 8*a**3*b**5/x**7 - 28*a**2*b**6/(5*x**5) - 8*a*b**7/(3*x**3) - b**8/x$

**Mathematica [A]** time = 0.0171137, size = 104, normalized size = 1.

$$-\frac{a^8}{17x^{17}} - \frac{8a^7b}{15x^{15}} - \frac{28a^6b^2}{13x^{13}} - \frac{56a^5b^3}{11x^{11}} - \frac{70a^4b^4}{9x^9} - \frac{8a^3b^5}{x^7} - \frac{28a^2b^6}{5x^5} - \frac{8ab^7}{3x^3} - \frac{b^8}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^18, x]

[Out]  $-a^8/(17*x^{17}) - (8*a^7*b)/(15*x^{15}) - (28*a^6*b^2)/(13*x^{13}) - (56*a^5*b^3)/(11*x^{11}) - (70*a^4*b^4)/(9*x^9) - (8*a^3*b^5)/x^7 - (28*a^2*b^6)/(5*x^5) - (8*a*b^7)/(3*x^3) - b^8/x$

**Maple [A]** time = 0.008, size = 91, normalized size = 0.9

$$-\frac{a^8}{17x^{17}} - \frac{8a^7b}{15x^{15}} - \frac{28a^6b^2}{13x^{13}} - \frac{56a^5b^3}{11x^{11}} - \frac{70a^4b^4}{9x^9} - 8\frac{a^3b^5}{x^7} - \frac{28a^2b^6}{5x^5} - \frac{8ab^7}{3x^3} - \frac{b^8}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^18,x)`

[Out] 
$$-1/17*a^8/x^{17}-8/15*a^7*b/x^{15}-28/13*a^6*b^2/x^{13}-56/11*a^5*b^3/x^{11}-70/9*a^4*b^4/x^9-8*a^3*b^5/x^7-28/5*a^2*b^6/x^5-8/3*a*b^7/x^3-b^8/x$$

**Maxima [A]** time = 1.35327, size = 124, normalized size = 1.19

$$\frac{109395 b^8 x^{16} + 291720 a b^7 x^{14} + 612612 a^2 b^6 x^{12} + 875160 a^3 b^5 x^{10} + 850850 a^4 b^4 x^8 + 556920 a^5 b^3 x^6 + 235620 a^6 b^2 x^4 + 58344 a^7 b x^2 + 6435 a^8}{109395 x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^18,x, algorithm="maxima")`

[Out] 
$$-1/109395*(109395*b^8*x^{16} + 291720*a*b^7*x^{14} + 612612*a^2*b^6*x^{12} + 875160*a^3*b^5*x^{10} + 850850*a^4*b^4*x^8 + 556920*a^5*b^3*x^6 + 235620*a^6*b^2*x^4 + 58344*a^7*b*x^2 + 6435*a^8)/x^{17}$$

**Fricas [A]** time = 0.196853, size = 124, normalized size = 1.19

$$\frac{109395 b^8 x^{16} + 291720 a b^7 x^{14} + 612612 a^2 b^6 x^{12} + 875160 a^3 b^5 x^{10} + 850850 a^4 b^4 x^8 + 556920 a^5 b^3 x^6 + 235620 a^6 b^2 x^4 + 58344 a^7 b x^2 + 6435 a^8}{109395 x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^18,x, algorithm="fricas")`

[Out] 
$$-1/109395*(109395*b^8*x^{16} + 291720*a*b^7*x^{14} + 612612*a^2*b^6*x^{12} + 875160*a^3*b^5*x^{10} + 850850*a^4*b^4*x^8 + 556920*a^5*b^3*x^6 + 235620*a^6*b^2*x^4 + 58344*a^7*b*x^2 + 6435*a^8)/x^{17}$$

**Sympy [A]** time = 3.62763, size = 99, normalized size = 0.95

$$\frac{6435 a^8 + 58344 a^7 b x^2 + 235620 a^6 b^2 x^4 + 556920 a^5 b^3 x^6 + 850850 a^4 b^4 x^8 + 875160 a^3 b^5 x^{10} + 612612 a^2 b^6 x^{12} + 291720 a b^7 x^{14} + 6435 a^8}{109395 x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**18,x)`

[Out] 
$$-(6435*a^{**8} + 58344*a^{**7}*b*x^{**2} + 235620*a^{**6}*b^{**2}*x^{**4} + 556920*a^{**5}*b^{**3}*x^{**6} + 850850*a^{**4}*b^{**4}*x^{**8} + 875160*a^{**3}*b^{**5}*x^{**10} + 612612*a^{**2}*b^{**6}*x^{**12} + 291720*a*b^{**7}*x^{**14} + 109395*b^{**8}*x^{**16})/(109395*x^{**17})$$

**GIAC/XCAS [A]** time = 0.227165, size = 124, normalized size = 1.19

$$\frac{109395 b^8 x^{16} + 291720 a b^7 x^{14} + 612612 a^2 b^6 x^{12} + 875160 a^3 b^5 x^{10} + 850850 a^4 b^4 x^8 + 556920 a^5 b^3 x^6 + 235620 a^6 b^2 x^4 + 58344 a^7 b x^2 + 6435 a^8}{109395 x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^18,x, algorithm="giac")`

```
[Out] -1/109395*(109395*b^8*x^16 + 291720*a*b^7*x^14 + 612612*a^2*b^6*x  
^12 + 875160*a^3*b^5*x^10 + 850850*a^4*b^4*x^8 + 556920*a^5*b^3*x  
^6 + 235620*a^6*b^2*x^4 + 58344*a^7*b*x^2 + 6435*a^8)/x^17
```



$$3.123 \quad \int \frac{(a+bx^2)^8}{x^{20}} dx$$

**Optimal.** Leaf size=106

$$-\frac{a^8}{19x^{19}} - \frac{8a^7b}{17x^{17}} - \frac{28a^6b^2}{15x^{15}} - \frac{56a^5b^3}{13x^{13}} - \frac{70a^4b^4}{11x^{11}} - \frac{56a^3b^5}{9x^9} - \frac{4a^2b^6}{x^7} - \frac{8ab^7}{5x^5} - \frac{b^8}{3x^3}$$

[Out]  $-a^8/(19*x^{19}) - (8*a^7*b)/(17*x^{17}) - (28*a^6*b^2)/(15*x^{15}) - (56*a^5*b^3)/(13*x^{13}) - (70*a^4*b^4)/(11*x^{11}) - (56*a^3*b^5)/(9*x^9) - (4*a^2*b^6)/x^7 - (8*a*b^7)/(5*x^5) - b^8/(3*x^3)$

**Rubi [A]** time = 0.110195, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a^8}{19x^{19}} - \frac{8a^7b}{17x^{17}} - \frac{28a^6b^2}{15x^{15}} - \frac{56a^5b^3}{13x^{13}} - \frac{70a^4b^4}{11x^{11}} - \frac{56a^3b^5}{9x^9} - \frac{4a^2b^6}{x^7} - \frac{8ab^7}{5x^5} - \frac{b^8}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^8/x^20, x]

[Out]  $-a^8/(19*x^{19}) - (8*a^7*b)/(17*x^{17}) - (28*a^6*b^2)/(15*x^{15}) - (56*a^5*b^3)/(13*x^{13}) - (70*a^4*b^4)/(11*x^{11}) - (56*a^3*b^5)/(9*x^9) - (4*a^2*b^6)/x^7 - (8*a*b^7)/(5*x^5) - b^8/(3*x^3)$

**Rubi in Sympy [A]** time = 20.2337, size = 107, normalized size = 1.01

$$-\frac{a^8}{19x^{19}} - \frac{8a^7b}{17x^{17}} - \frac{28a^6b^2}{15x^{15}} - \frac{56a^5b^3}{13x^{13}} - \frac{70a^4b^4}{11x^{11}} - \frac{56a^3b^5}{9x^9} - \frac{4a^2b^6}{x^7} - \frac{8ab^7}{5x^5} - \frac{b^8}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*8/x\*\*20, x)

[Out]  $-a**8/(19*x**19) - 8*a**7*b/(17*x**17) - 28*a**6*b**2/(15*x**15) - 56*a**5*b**3/(13*x**13) - 70*a**4*b**4/(11*x**11) - 56*a**3*b**5/(9*x**9) - 4*a**2*b**6/x**7 - 8*a*b**7/(5*x**5) - b**8/(3*x**3)$

**Mathematica [A]** time = 0.0158849, size = 106, normalized size = 1.

$$-\frac{a^8}{19x^{19}} - \frac{8a^7b}{17x^{17}} - \frac{28a^6b^2}{15x^{15}} - \frac{56a^5b^3}{13x^{13}} - \frac{70a^4b^4}{11x^{11}} - \frac{56a^3b^5}{9x^9} - \frac{4a^2b^6}{x^7} - \frac{8ab^7}{5x^5} - \frac{b^8}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^8/x^20, x]

[Out]  $-a^8/(19*x^{19}) - (8*a^7*b)/(17*x^{17}) - (28*a^6*b^2)/(15*x^{15}) - (56*a^5*b^3)/(13*x^{13}) - (70*a^4*b^4)/(11*x^{11}) - (56*a^3*b^5)/(9*x^9) - (4*a^2*b^6)/x^7 - (8*a*b^7)/(5*x^5) - b^8/(3*x^3)$

**Maple [A]** time = 0.009, size = 91, normalized size = 0.9

$$-\frac{a^8}{19x^{19}} - \frac{8a^7b}{17x^{17}} - \frac{28a^6b^2}{15x^{15}} - \frac{56a^5b^3}{13x^{13}} - \frac{70a^4b^4}{11x^{11}} - \frac{56a^3b^5}{9x^9} - 4\frac{a^2b^6}{x^7} - \frac{8ab^7}{5x^5} - \frac{b^8}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^8/x^20,x)`

[Out] 
$$\frac{-1/19*a^8/x^{19}-8/17*a^7*b/x^{17}-28/15*a^6*b^2/x^{15}-56/13*a^5*b^3/x^{13}-70/11*a^4*b^4/x^{11}-56/9*a^3*b^5/x^9-4*a^2*b^6/x^7-8/5*a*b^7/x^5-1/3*b^8/x^3}$$

**Maxima [A]** time = 1.352, size = 124, normalized size = 1.17

$$\frac{692835 b^8 x^{16} + 3325608 a b^7 x^{14} + 8314020 a^2 b^6 x^{12} + 12932920 a^3 b^5 x^{10} + 13226850 a^4 b^4 x^8 + 8953560 a^5 b^3 x^6 + 3879876 a^6 b^2 x^4 + 978120 a^7 b x^2 + 109395 a^8}{2078505 x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^20,x, algorithm="maxima")`

[Out] 
$$\frac{-1/2078505*(692835*b^8*x^{16} + 3325608*a*b^7*x^{14} + 8314020*a^2*b^6*x^{12} + 12932920*a^3*b^5*x^{10} + 13226850*a^4*b^4*x^8 + 8953560*a^5*b^3*x^6 + 3879876*a^6*b^2*x^4 + 978120*a^7*b*x^2 + 109395*a^8)}{x^{19}}$$

**Fricas [A]** time = 0.19832, size = 124, normalized size = 1.17

$$\frac{692835 b^8 x^{16} + 3325608 a b^7 x^{14} + 8314020 a^2 b^6 x^{12} + 12932920 a^3 b^5 x^{10} + 13226850 a^4 b^4 x^8 + 8953560 a^5 b^3 x^6 + 3879876 a^6 b^2 x^4 + 978120 a^7 b x^2 + 109395 a^8}{2078505 x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^8/x^20,x, algorithm="fricas")`

[Out] 
$$\frac{-1/2078505*(692835*b^8*x^{16} + 3325608*a*b^7*x^{14} + 8314020*a^2*b^6*x^{12} + 12932920*a^3*b^5*x^{10} + 13226850*a^4*b^4*x^8 + 8953560*a^5*b^3*x^6 + 3879876*a^6*b^2*x^4 + 978120*a^7*b*x^2 + 109395*a^8)}{x^{19}}$$

**Sympy [A]** time = 3.96229, size = 99, normalized size = 0.93

$$\frac{109395 a^8 + 978120 a^7 b x^2 + 3879876 a^6 b^2 x^4 + 8953560 a^5 b^3 x^6 + 13226850 a^4 b^4 x^8 + 12932920 a^3 b^5 x^{10} + 8314020 a^2 b^6 x^{12} + 3325608 a b^7 x^{14} + 692835 b^8 x^{16}}{2078505 x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**8/x**20,x)`

[Out] 
$$\frac{-(109395*a**8 + 978120*a**7*b*x**2 + 3879876*a**6*b**2*x**4 + 8953560*a**5*b**3*x**6 + 13226850*a**4*b**4*x**8 + 12932920*a**3*b**5*x**10 + 8314020*a**2*b**6*x**12 + 3325608*a*b**7*x**14 + 692835*b**8*x**16)/(2078505*x**19)}$$

**GIAC/XCAS [A]** time = 0.218066, size = 124, normalized size = 1.17

$$\frac{692835 b^8 x^{16} + 3325608 a b^7 x^{14} + 8314020 a^2 b^6 x^{12} + 12932920 a^3 b^5 x^{10} + 13226850 a^4 b^4 x^8 + 8953560 a^5 b^3 x^6 + 3879876 a^6 b^2 x^4 + 978120 a^7 b x^2 + 109395 a^8}{2078505 x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^8/x^20,x, algorithm="giac")
```

```
[Out] -1/2078505*(692835*b^8*x^16 + 3325608*a*b^7*x^14 + 8314020*a^2*b^6*x^12 + 12932920*a^3*b^5*x^10 + 13226850*a^4*b^4*x^8 + 8953560*a^5*b^3*x^6 + 3879876*a^6*b^2*x^4 + 978120*a^7*b*x^2 + 109395*a^8)/x^19
```

### 3.124 $\int \frac{x^{11}}{a+bx^2} dx$

**Optimal.** Leaf size=79

$$-\frac{a^5 \log(a+bx^2)}{2b^6} + \frac{a^4 x^2}{2b^5} - \frac{a^3 x^4}{4b^4} + \frac{a^2 x^6}{6b^3} - \frac{ax^8}{8b^2} + \frac{x^{10}}{10b}$$

[Out]  $(a^4 x^2)/(2 b^5) - (a^3 x^4)/(4 b^4) + (a^2 x^6)/(6 b^3) - (a x^8)/(8 b^2) + x^{10}/(10 b) - (a^5 \text{Log}[a + b x^2])/(2 b^6)$

**Rubi [A]** time = 0.13527, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^5 \log(a+bx^2)}{2b^6} + \frac{a^4 x^2}{2b^5} - \frac{a^3 x^4}{4b^4} + \frac{a^2 x^6}{6b^3} - \frac{ax^8}{8b^2} + \frac{x^{10}}{10b}$$

Antiderivative was successfully verified.

[In] Int[x^11/(a + b\*x^2), x]

[Out]  $(a^4 x^2)/(2 b^5) - (a^3 x^4)/(4 b^4) + (a^2 x^6)/(6 b^3) - (a x^8)/(8 b^2) + x^{10}/(10 b) - (a^5 \text{Log}[a + b x^2])/(2 b^6)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^5 \log(a+bx^2)}{2b^6} - \frac{a^3 \int^{x^2} x dx}{2b^4} + \frac{a^2 x^6}{6b^3} - \frac{ax^8}{8b^2} + \frac{x^{10}}{10b} + \frac{\int^{x^2} a^4 dx}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*11/(b\*x\*\*2+a), x)

[Out]  $-a^{**5} \log(a + b*x^{**2})/(2*b^{**6}) - a^{**3} \text{Integral}(x, (x, x^{**2}))/ (2*b^{**4}) + a^{**2} x^{**6}/(6*b^{**3}) - a*x^{**8}/(8*b^{**2}) + x^{**10}/(10*b) + \text{Integral}(a^{**4}, (x, x^{**2}))/ (2*b^{**5})$

**Mathematica [A]** time = 0.00985356, size = 79, normalized size = 1.

$$-\frac{a^5 \log(a+bx^2)}{2b^6} + \frac{a^4 x^2}{2b^5} - \frac{a^3 x^4}{4b^4} + \frac{a^2 x^6}{6b^3} - \frac{ax^8}{8b^2} + \frac{x^{10}}{10b}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/(a + b\*x^2), x]

[Out]  $(a^4 x^2)/(2 b^5) - (a^3 x^4)/(4 b^4) + (a^2 x^6)/(6 b^3) - (a x^8)/(8 b^2) + x^{10}/(10 b) - (a^5 \text{Log}[a + b x^2])/(2 b^6)$

**Maple [A]** time = 0.005, size = 68, normalized size = 0.9

$$\frac{a^4 x^2}{2 b^5} - \frac{a^3 x^4}{4 b^4} + \frac{a^2 x^6}{6 b^3} - \frac{ax^8}{8 b^2} + \frac{x^{10}}{10 b} - \frac{a^5 \ln(bx^2 + a)}{2 b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11/(b*x^2+a), x)`

[Out]  $\frac{1}{2}a^4x^2/b^5 - \frac{1}{4}a^3x^4/b^4 + \frac{1}{6}a^2x^6/b^3 - \frac{1}{8}ax^8/b^2 + \frac{1}{10}x^{10}/b - \frac{1}{2}a^5 \ln(bx^2+a)/b^6$

**Maxima [A]** time = 1.34686, size = 92, normalized size = 1.16

$$-\frac{a^5 \log(bx^2 + a)}{2b^6} + \frac{12b^4x^{10} - 15ab^3x^8 + 20a^2b^2x^6 - 30a^3bx^4 + 60a^4x^2}{120b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(b*x^2 + a), x, algorithm="maxima")`

[Out]  $-\frac{1}{2}a^5 \log(bx^2 + a)/b^6 + \frac{1}{120}(12b^4x^{10} - 15a^3b^3x^8 + 20a^2b^2x^6 - 30a^3bx^4 + 60a^4x^2)/b^5$

**Fricas [A]** time = 0.200843, size = 90, normalized size = 1.14

$$\frac{12b^5x^{10} - 15ab^4x^8 + 20a^2b^3x^6 - 30a^3b^2x^4 + 60a^4bx^2 - 60a^5 \log(bx^2 + a)}{120b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(b*x^2 + a), x, algorithm="fricas")`

[Out]  $\frac{1}{120}(12b^5x^{10} - 15a^3b^4x^8 + 20a^2b^3x^6 - 30a^3b^2x^4 + 60a^4bx^2 - 60a^5 \log(bx^2 + a))/b^6$

**Sympy [A]** time = 1.3307, size = 68, normalized size = 0.86

$$-\frac{a^5 \log(a + bx^2)}{2b^6} + \frac{a^4x^2}{2b^5} - \frac{a^3x^4}{4b^4} + \frac{a^2x^6}{6b^3} - \frac{ax^8}{8b^2} + \frac{x^{10}}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**11/(b*x**2+a), x)`

[Out]  $-a^{**5} \log(a + b*x^{**2})/(2*b^{**6}) + a^{**4}x^{**2}/(2*b^{**5}) - a^{**3}x^{**4}/(4*b^{**4}) + a^{**2}x^{**6}/(6*b^{**3}) - a*x^{**8}/(8*b^{**2}) + x^{**10}/(10*b)$

**GIAC/XCAS [A]** time = 0.214059, size = 93, normalized size = 1.18

$$-\frac{a^5 \ln(|bx^2 + a|)}{2b^6} + \frac{12b^4x^{10} - 15ab^3x^8 + 20a^2b^2x^6 - 30a^3bx^4 + 60a^4x^2}{120b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(b*x^2 + a), x, algorithm="giac")`

[Out]  $-\frac{1}{2}a^5 \ln(\text{abs}(bx^2 + a))/b^6 + \frac{1}{120}(12b^4x^{10} - 15a^3b^3x^8 + 20a^2b^2x^6 - 30a^3bx^4 + 60a^4x^2)/b^5$

### 3.125 $\int \frac{x^{10}}{a+bx^2} dx$

**Optimal.** Leaf size=81

$$-\frac{a^{9/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{11/2}} + \frac{a^4 x}{b^5} - \frac{a^3 x^3}{3b^4} + \frac{a^2 x^5}{5b^3} - \frac{ax^7}{7b^2} + \frac{x^9}{9b}$$

[Out]  $(a^4 x)/b^5 - (a^3 x^3)/(3 b^4) + (a^2 x^5)/(5 b^3) - (a x^7)/(7 b^2) + x^9/(9 b) - (a^{(9/2)} \text{ArcTan}[(\text{Sqrt}[b] x)/\text{Sqrt}[a]])/b^{(11/2)}$

**Rubi [A]** time = 0.0924629, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^{9/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{11/2}} + \frac{a^4 x}{b^5} - \frac{a^3 x^3}{3b^4} + \frac{a^2 x^5}{5b^3} - \frac{ax^7}{7b^2} + \frac{x^9}{9b}$$

Antiderivative was successfully verified.

[In] Int[x^10/(a + b\*x^2), x]

[Out]  $(a^4 x)/b^5 - (a^3 x^3)/(3 b^4) + (a^2 x^5)/(5 b^3) - (a x^7)/(7 b^2) + x^9/(9 b) - (a^{(9/2)} \text{ArcTan}[(\text{Sqrt}[b] x)/\text{Sqrt}[a]])/b^{(11/2)}$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^{9/2} \text{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{11/2}} - \frac{a^3 x^3}{3b^4} + \frac{a^2 x^5}{5b^3} - \frac{ax^7}{7b^2} + \frac{x^9}{9b} + \frac{\int a^4 dx}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*10/(b\*x\*\*2+a), x)

[Out]  $-a^{(9/2)} \text{atan}(\text{sqrt}(b) x/\text{sqrt}(a))/b^{(11/2)} - a^{(3)} x^{(3)}/(3 b^{(4)}) + a^{(2)} x^{(5)}/(5 b^{(3)}) - a x^{(7)}/(7 b^{(2)}) + x^{(9)}/(9 b) + \text{Integral}(a^{(4)}, x)/b^{(5)}$

**Mathematica [A]** time = 0.0553004, size = 81, normalized size = 1.

$$-\frac{a^{9/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{11/2}} + \frac{a^4 x}{b^5} - \frac{a^3 x^3}{3b^4} + \frac{a^2 x^5}{5b^3} - \frac{ax^7}{7b^2} + \frac{x^9}{9b}$$

Antiderivative was successfully verified.

[In] Integrate[x^10/(a + b\*x^2), x]

[Out]  $(a^4 x)/b^5 - (a^3 x^3)/(3 b^4) + (a^2 x^5)/(5 b^3) - (a x^7)/(7 b^2) + x^9/(9 b) - (a^{(9/2)} \text{ArcTan}[(\text{Sqrt}[b] x)/\text{Sqrt}[a]])/b^{(11/2)}$

**Maple [A]** time = 0.009, size = 71, normalized size = 0.9

$$\frac{x^9}{9b} - \frac{ax^7}{7b^2} + \frac{a^2 x^5}{5b^3} - \frac{a^3 x^3}{3b^4} + \frac{a^4 x}{b^5} - \frac{a^5}{b^5} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^10/(b*x^2+a), x)`

[Out]  $1/9*x^9/b - 1/7*a*x^7/b^2 + 1/5*a^2*x^5/b^3 - 1/3*a^3*x^3/b^4 + a^4*x/b^5 - a^5/b^5/(a*b)^{(1/2)}*\arctan(x*b/(a*b)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^10/(b*x^2 + a), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.212479, size = 1, normalized size = 0.01

$$\left[ \frac{70 b^4 x^9 - 90 a b^3 x^7 + 126 a^2 b^2 x^5 - 210 a^3 b x^3 + 315 a^4 \sqrt{-\frac{a}{b}} \log\left(\frac{b x^2 - 2 b x \sqrt{-\frac{a}{b}} - a}{b x^2 + a}\right) + 630 a^4 x}{630 b^5}, \frac{35 b^4 x^9 - 45 a b^3 x^7 + 63 a^2 b^2 x^5 - 105 a^3 b x^3 - 315 a^4 \sqrt{\frac{a}{b}} \arctan\left(\frac{x}{\sqrt{\frac{a}{b}}}\right) + 315 a^4 x}{630 b^5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^10/(b*x^2 + a), x, algorithm="fricas")`

[Out]  $[1/630*(70*b^4*x^9 - 90*a*b^3*x^7 + 126*a^2*b^2*x^5 - 210*a^3*b*x^3 + 315*a^4*\sqrt{-a/b}*\log((b*x^2 - 2*b*x*\sqrt{-a/b} - a)/(b*x^2 + a)) + 630*a^4*x)/b^5, 1/315*(35*b^4*x^9 - 45*a*b^3*x^7 + 63*a^2*b^2*x^5 - 105*a^3*b*x^3 - 315*a^4*\sqrt{a/b}*\arctan(x/\sqrt{a/b}) + 315*a^4*x)/b^5]$

**Sympy [A]** time = 1.38741, size = 119, normalized size = 1.47

$$\frac{a^4 x}{b^5} - \frac{a^3 x^3}{3b^4} + \frac{a^2 x^5}{5b^3} - \frac{ax^7}{7b^2} + \frac{\sqrt{-\frac{a^9}{b^{11}}} \log\left(x - \frac{b^5 \sqrt{-\frac{a^9}{b^{11}}}}{a^4}\right)}{2} - \frac{\sqrt{-\frac{a^9}{b^{11}}} \log\left(x + \frac{b^5 \sqrt{-\frac{a^9}{b^{11}}}}{a^4}\right)}{2} + \frac{x^9}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**10/(b*x**2+a), x)`

[Out]  $a**4*x/b**5 - a**3*x**3/(3*b**4) + a**2*x**5/(5*b**3) - a*x**7/(7*b**2) + \sqrt{-a**9/b**11}*\log(x - b**5*\sqrt{-a**9/b**11}/a**4)/2 - \sqrt{-a**9/b**11}*\log(x + b**5*\sqrt{-a**9/b**11}/a**4)/2 + x**9/(9*b)$

**GIAC/XCAS [A]** time = 0.212045, size = 104, normalized size = 1.28

$$-\frac{a^5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{abb^5}} + \frac{35 b^8 x^9 - 45 a b^7 x^7 + 63 a^2 b^6 x^5 - 105 a^3 b^5 x^3 + 315 a^4 b^4 x}{315 b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] -a^5*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/315*(35*b^8*x^9 -  
45*a*b^7*x^7 + 63*a^2*b^6*x^5 - 105*a^3*b^5*x^3 + 315*a^4*b^4*x)/  
b^9
```



### 3.126 $\int \frac{x^9}{a+bx^2} dx$

**Optimal.** Leaf size=66

$$\frac{a^4 \log(a+bx^2)}{2b^5} - \frac{a^3x^2}{2b^4} + \frac{a^2x^4}{4b^3} - \frac{ax^6}{6b^2} + \frac{x^8}{8b}$$

[Out]  $-(a^3*x^2)/(2*b^4) + (a^2*x^4)/(4*b^3) - (a*x^6)/(6*b^2) + x^8/(8*b) + (a^4*Log[a + b*x^2])/(2*b^5)$

**Rubi [A]** time = 0.103514, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^4 \log(a+bx^2)}{2b^5} - \frac{a^3x^2}{2b^4} + \frac{a^2x^4}{4b^3} - \frac{ax^6}{6b^2} + \frac{x^8}{8b}$$

Antiderivative was successfully verified.

[In] Int[x^9/(a + b\*x^2), x]

[Out]  $-(a^3*x^2)/(2*b^4) + (a^2*x^4)/(4*b^3) - (a*x^6)/(6*b^2) + x^8/(8*b) + (a^4*Log[a + b*x^2])/(2*b^5)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^4 \log(a+bx^2)}{2b^5} + \frac{a^2 \int^{x^2} x dx}{2b^3} - \frac{ax^6}{6b^2} + \frac{x^8}{8b} - \frac{\int^{x^2} a^3 dx}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*9/(b\*x\*\*2+a), x)

[Out]  $a**4*log(a + b*x**2)/(2*b**5) + a**2*Integral(x, (x, x**2))/(2*b**3) - a*x**6/(6*b**2) + x**8/(8*b) - Integral(a**3, (x, x**2))/(2*b**4)$

**Mathematica [A]** time = 0.0111668, size = 66, normalized size = 1.

$$\frac{a^4 \log(a+bx^2)}{2b^5} - \frac{a^3x^2}{2b^4} + \frac{a^2x^4}{4b^3} - \frac{ax^6}{6b^2} + \frac{x^8}{8b}$$

Antiderivative was successfully verified.

[In] Integrate[x^9/(a + b\*x^2), x]

[Out]  $-(a^3*x^2)/(2*b^4) + (a^2*x^4)/(4*b^3) - (a*x^6)/(6*b^2) + x^8/(8*b) + (a^4*Log[a + b*x^2])/(2*b^5)$

**Maple [A]** time = 0.004, size = 57, normalized size = 0.9

$$-\frac{a^3x^2}{2b^4} + \frac{a^2x^4}{4b^3} - \frac{ax^6}{6b^2} + \frac{x^8}{8b} + \frac{a^4 \ln(bx^2 + a)}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9/(b*x^2+a),x)`

[Out]  $-1/2*a^3*x^2/b^4+1/4*a^2*x^4/b^3-1/6*a*x^6/b^2+1/8*x^8/b+1/2*a^4*\ln(b*x^2+a)/b^5$

**Maxima [A]** time = 1.34606, size = 77, normalized size = 1.17

$$\frac{a^4 \log(bx^2 + a)}{2b^5} + \frac{3b^3x^8 - 4ab^2x^6 + 6a^2bx^4 - 12a^3x^2}{24b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9/(b*x^2 + a),x, algorithm="maxima")`

[Out]  $1/2*a^4*\log(b*x^2 + a)/b^5 + 1/24*(3*b^3*x^8 - 4*a*b^2*x^6 + 6*a^2*b*x^4 - 12*a^3*x^2)/b^4$

**Fricas [A]** time = 0.200549, size = 76, normalized size = 1.15

$$\frac{3b^4x^8 - 4ab^3x^6 + 6a^2b^2x^4 - 12a^3bx^2 + 12a^4 \log(bx^2 + a)}{24b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9/(b*x^2 + a),x, algorithm="fricas")`

[Out]  $1/24*(3*b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 12*a^3*b*x^2 + 12*a^4*\log(b*x^2 + a))/b^5$

**Sympy [A]** time = 1.2525, size = 56, normalized size = 0.85

$$\frac{a^4 \log(a + bx^2)}{2b^5} - \frac{a^3x^2}{2b^4} + \frac{a^2x^4}{4b^3} - \frac{ax^6}{6b^2} + \frac{x^8}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9/(b*x**2+a),x)`

[Out]  $a**4*\log(a + b*x**2)/(2*b**5) - a**3*x**2/(2*b**4) + a**2*x**4/(4*b**3) - a*x**6/(6*b**2) + x**8/(8*b)$

**GIAC/XCAS [A]** time = 0.210473, size = 78, normalized size = 1.18

$$\frac{a^4 \ln(|bx^2 + a|)}{2b^5} + \frac{3b^3x^8 - 4ab^2x^6 + 6a^2bx^4 - 12a^3x^2}{24b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9/(b*x^2 + a),x, algorithm="giac")`

[Out]  $1/2*a^4*\ln(\text{abs}(b*x^2 + a))/b^5 + 1/24*(3*b^3*x^8 - 4*a*b^2*x^6 + 6*a^2*b*x^4 - 12*a^3*x^2)/b^4$

$$3.127 \quad \int \frac{x^8}{a+bx^2} dx$$

**Optimal.** Leaf size=68

$$\frac{a^{7/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{9/2}} - \frac{a^3 x}{b^4} + \frac{a^2 x^3}{3b^3} - \frac{ax^5}{5b^2} + \frac{x^7}{7b}$$

[Out]  $-\left(\frac{a^3 x}{b^4}\right) + \left(\frac{a^2 x^3}{3b^3}\right) - \left(\frac{a x^5}{5b^2}\right) + \frac{x^7}{7b} + \left(\frac{a^{7/2} \operatorname{ArcTan}\left[\frac{\operatorname{Sqrt}[b] x}{\operatorname{Sqrt}[a]}\right]}{b^{9/2}}\right)$

**Rubi [A]** time = 0.0769098, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^{7/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{9/2}} - \frac{a^3 x}{b^4} + \frac{a^2 x^3}{3b^3} - \frac{ax^5}{5b^2} + \frac{x^7}{7b}$$

Antiderivative was successfully verified.

[In] Int[x^8/(a + b\*x^2), x]

[Out]  $-\left(\frac{a^3 x}{b^4}\right) + \left(\frac{a^2 x^3}{3b^3}\right) - \left(\frac{a x^5}{5b^2}\right) + \frac{x^7}{7b} + \left(\frac{a^{7/2} \operatorname{ArcTan}\left[\frac{\operatorname{Sqrt}[b] x}{\operatorname{Sqrt}[a]}\right]}{b^{9/2}}\right)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^{7/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{9/2}} + \frac{a^2 x^3}{3b^3} - \frac{ax^5}{5b^2} + \frac{x^7}{7b} - \frac{\int a^3 dx}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*8/(b\*x\*\*2+a), x)

[Out]  $a^{7/2} \operatorname{atan}\left(\frac{\operatorname{sqrt}(b) x}{\operatorname{sqrt}(a)}\right) / b^{9/2} + a^2 x^3 / (3b^3) - a x^5 / (5b^2) + x^7 / (7b) - \operatorname{Integral}(a^3, x) / b^4$

**Mathematica [A]** time = 0.0448975, size = 68, normalized size = 1.

$$\frac{a^{7/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{9/2}} - \frac{a^3 x}{b^4} + \frac{a^2 x^3}{3b^3} - \frac{ax^5}{5b^2} + \frac{x^7}{7b}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(a + b\*x^2), x]

[Out]  $-\left(\frac{a^3 x}{b^4}\right) + \left(\frac{a^2 x^3}{3b^3}\right) - \left(\frac{a x^5}{5b^2}\right) + \frac{x^7}{7b} + \left(\frac{a^{7/2} \operatorname{ArcTan}\left[\frac{\operatorname{Sqrt}[b] x}{\operatorname{Sqrt}[a]}\right]}{b^{9/2}}\right)$

**Maple [A]** time = 0.004, size = 60, normalized size = 0.9

$$\frac{x^7}{7b} - \frac{ax^5}{5b^2} + \frac{a^2 x^3}{3b^3} - \frac{a^3 x}{b^4} + \frac{a^4}{b^4} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8/(b*x^2+a),x)`

[Out]  $\frac{1}{7}x^7/b - \frac{1}{5}a^2x^5/b^2 + \frac{1}{3}a^2x^3/b^3 - a^3x/b^4 + a^4/b^4 / (a^3b)^{1/2} \arctan(x\sqrt{b}/(a^3b)^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8/(b*x^2 + a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.206369, size = 1, normalized size = 0.01

$$\left[ \frac{30b^3x^7 - 42ab^2x^5 + 70a^2bx^3 + 105a^3\sqrt{-\frac{a}{b}} \log\left(\frac{bx^2+2bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) - 210a^3x}{210b^4}, \frac{15b^3x^7 - 21ab^2x^5 + 35a^2bx^3 + 105a^3\sqrt{\frac{a}{b}} \arctan\left(\frac{x\sqrt{a/b}}{\sqrt{bx^2+a}}\right) - 210a^3x}{105b^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8/(b*x^2 + a),x, algorithm="fricas")`

[Out]  $\left[ \frac{1}{210} \left( 30b^3x^7 - 42a^2b^2x^5 + 70a^2b^2x^3 + 105a^3\sqrt{-\frac{a}{b}} \log\left(\frac{bx^2+2bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) - 210a^3x \right) / b^4, \frac{1}{105} \left( 15b^3x^7 - 21a^2b^2x^5 + 35a^2b^2x^3 + 105a^3\sqrt{\frac{a}{b}} \arctan\left(\frac{x\sqrt{a/b}}{\sqrt{bx^2+a}}\right) - 210a^3x \right) / b^4 \right]$

**Sympy [A]** time = 1.31395, size = 107, normalized size = 1.57

$$-\frac{a^3x}{b^4} + \frac{a^2x^3}{3b^3} - \frac{ax^5}{5b^2} - \frac{\sqrt{-\frac{a^7}{b^9}} \log\left(x - \frac{b^4\sqrt{-\frac{a^7}{b^9}}}{a^3}\right)}{2} + \frac{\sqrt{-\frac{a^7}{b^9}} \log\left(x + \frac{b^4\sqrt{-\frac{a^7}{b^9}}}{a^3}\right)}{2} + \frac{x^7}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8/(b*x**2+a),x)`

[Out]  $-\frac{a^3x}{b^4} + \frac{a^2x^3}{3b^3} - \frac{ax^5}{5b^2} - \frac{\sqrt{-a^7/b^9} \log(x - b^4\sqrt{-a^7/b^9}/a^3)/2 + \sqrt{-a^7/b^9} \log(x + b^4\sqrt{-a^7/b^9}/a^3)/2 + x^7/(7b)}$

**GIAC/XCAS [A]** time = 0.210717, size = 88, normalized size = 1.29

$$\frac{a^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{abb^4}} + \frac{15b^6x^7 - 21ab^5x^5 + 35a^2b^4x^3 - 105a^3b^3x}{105b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8/(b*x^2 + a),x, algorithm="giac")`

```
[Out] a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^6*x^7 - 21*a*b^5*x^5 + 35*a^2*b^4*x^3 - 105*a^3*b^3*x)/b^7
```

$$3.128 \quad \int \frac{x^7}{a+bx^2} dx$$

**Optimal.** Leaf size=53

$$-\frac{a^3 \log(a+bx^2)}{2b^4} + \frac{a^2x^2}{2b^3} - \frac{ax^4}{4b^2} + \frac{x^6}{6b}$$

[Out]  $(a^2x^2)/(2b^3) - (ax^4)/(4b^2) + x^6/(6b) - (a^3 \text{Log}[a + bx^2])/(2b^4)$

**Rubi [A]** time = 0.0849097, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^3 \log(a+bx^2)}{2b^4} + \frac{a^2x^2}{2b^3} - \frac{ax^4}{4b^2} + \frac{x^6}{6b}$$

Antiderivative was successfully verified.

[In] Int[x^7/(a + b\*x^2), x]

[Out]  $(a^2x^2)/(2b^3) - (ax^4)/(4b^2) + x^6/(6b) - (a^3 \text{Log}[a + bx^2])/(2b^4)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^3 \log(a+bx^2)}{2b^4} - \frac{a \int^{x^2} x dx}{2b^2} + \frac{x^6}{6b} + \frac{\int^{x^2} a^2 dx}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*7/(b\*x\*\*2+a), x)

[Out]  $-a^{**3} \log(a + b*x^{**2})/(2*b^{**4}) - a*Integral(x, (x, x^{**2}))/(2*b^{**2}) + x^{**6}/(6*b) + Integral(a^{**2}, (x, x^{**2}))/(2*b^{**3})$

**Mathematica [A]** time = 0.0090984, size = 53, normalized size = 1.

$$-\frac{a^3 \log(a+bx^2)}{2b^4} + \frac{a^2x^2}{2b^3} - \frac{ax^4}{4b^2} + \frac{x^6}{6b}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a + b\*x^2), x]

[Out]  $(a^2x^2)/(2b^3) - (ax^4)/(4b^2) + x^6/(6b) - (a^3 \text{Log}[a + bx^2])/(2b^4)$

**Maple [A]** time = 0.003, size = 46, normalized size = 0.9

$$\frac{a^2x^2}{2b^3} - \frac{ax^4}{4b^2} + \frac{x^6}{6b} - \frac{a^3 \ln(bx^2 + a)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7/(b*x^2+a),x)`

[Out]  $1/2*a^2*x^2/b^3-1/4*a*x^4/b^2+1/6*x^6/b-1/2*a^3*\ln(b*x^2+a)/b^4$

**Maxima [A]** time = 1.34441, size = 62, normalized size = 1.17

$$-\frac{a^3 \log(bx^2 + a)}{2b^4} + \frac{2b^2x^6 - 3abx^4 + 6a^2x^2}{12b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^2 + a),x, algorithm="maxima")`

[Out]  $-1/2*a^3*\log(b*x^2 + a)/b^4 + 1/12*(2*b^2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)/b^3$

**Fricas [A]** time = 0.19787, size = 61, normalized size = 1.15

$$\frac{2b^3x^6 - 3ab^2x^4 + 6a^2bx^2 - 6a^3 \log(bx^2 + a)}{12b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^2 + a),x, algorithm="fricas")`

[Out]  $1/12*(2*b^3*x^6 - 3*a*b^2*x^4 + 6*a^2*b*x^2 - 6*a^3*\log(b*x^2 + a))/b^4$

**Sympy [A]** time = 1.28111, size = 44, normalized size = 0.83

$$-\frac{a^3 \log(a + bx^2)}{2b^4} + \frac{a^2x^2}{2b^3} - \frac{ax^4}{4b^2} + \frac{x^6}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7/(b*x**2+a),x)`

[Out]  $-a**3*\log(a + b*x**2)/(2*b**4) + a**2*x**2/(2*b**3) - a*x**4/(4*b**2) + x**6/(6*b)$

**GIAC/XCAS [A]** time = 0.209284, size = 63, normalized size = 1.19

$$-\frac{a^3 \ln(|bx^2 + a|)}{2b^4} + \frac{2b^2x^6 - 3abx^4 + 6a^2x^2}{12b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^2 + a),x, algorithm="giac")`

[Out]  $-1/2*a^3*\ln(\text{abs}(b*x^2 + a))/b^4 + 1/12*(2*b^2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)/b^3$

$$3.129 \quad \int \frac{x^6}{a+bx^2} dx$$

**Optimal.** Leaf size=55

$$-\frac{a^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{7/2}} + \frac{a^2x}{b^3} - \frac{ax^3}{3b^2} + \frac{x^5}{5b}$$

[Out]  $(a^2x)/b^3 - (ax^3)/(3b^2) + x^5/(5b) - (a^{5/2}) \operatorname{ArcTan}[(\operatorname{Sqrt}[b]x)/\operatorname{Sqrt}[a]]/b^{7/2}$

**Rubi [A]** time = 0.067159, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{7/2}} + \frac{a^2x}{b^3} - \frac{ax^3}{3b^2} + \frac{x^5}{5b}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a + b\*x^2), x]

[Out]  $(a^2x)/b^3 - (ax^3)/(3b^2) + x^5/(5b) - (a^{5/2}) \operatorname{ArcTan}[(\operatorname{Sqrt}[b]x)/\operatorname{Sqrt}[a]]/b^{7/2}$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^{5/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{7/2}} - \frac{ax^3}{3b^2} + \frac{x^5}{5b} + \frac{\int a^2 dx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6/(b\*x\*\*2+a), x)

[Out]  $-a^{5/2} \operatorname{atan}(\operatorname{sqrt}(b)x/\operatorname{sqrt}(a))/b^{7/2} - ax^3/(3b^2) + x^5/(5b) + \operatorname{Integral}(a^2, x)/b^3$

**Mathematica [A]** time = 0.0427939, size = 55, normalized size = 1.

$$-\frac{a^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{7/2}} + \frac{a^2x}{b^3} - \frac{ax^3}{3b^2} + \frac{x^5}{5b}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a + b\*x^2), x]

[Out]  $(a^2x)/b^3 - (ax^3)/(3b^2) + x^5/(5b) - (a^{5/2}) \operatorname{ArcTan}[(\operatorname{Sqrt}[b]x)/\operatorname{Sqrt}[a]]/b^{7/2}$

**Maple [A]** time = 0.004, size = 49, normalized size = 0.9

$$\frac{x^5}{5b} - \frac{ax^3}{3b^2} + \frac{a^2x}{b^3} - \frac{a^3}{b^3} \operatorname{arctan}\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(x^6/(b*x^2+a),x)`

[Out]  $1/5*x^5/b-1/3*a*x^3/b^2+a^2*x/b^3-a^3/b^3/(a*b)^{(1/2)}*\arctan(x*b/(a*b)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^2 + a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.205547, size = 1, normalized size = 0.02

$$\left[ \frac{6b^2x^5 - 10abx^3 + 15a^2\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) + 30a^2x}{30b^3}, \frac{3b^2x^5 - 5abx^3 - 15a^2\sqrt{\frac{a}{b}}\arctan\left(\frac{x}{\sqrt{\frac{a}{b}}}\right) + 15a^2x}{15b^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^2 + a),x, algorithm="fricas")`

[Out]  $[1/30*(6*b^2*x^5 - 10*a*b*x^3 + 15*a^2*\sqrt{-a/b}*\log((b*x^2 - 2*b*x*\sqrt{-a/b}) - a)/(b*x^2 + a)) + 30*a^2*x)/b^3, 1/15*(3*b^2*x^5 - 5*a*b*x^3 - 15*a^2*\sqrt{a/b}*\arctan(x/\sqrt{a/b}) + 15*a^2*x)/b^3]$

**Sympy [A]** time = 1.30463, size = 95, normalized size = 1.73

$$\frac{a^2x}{b^3} - \frac{ax^3}{3b^2} + \frac{\sqrt{-\frac{a^5}{b^7}}\log\left(x - \frac{b^3\sqrt{-\frac{a^5}{b^7}}}{a^2}\right)}{2} - \frac{\sqrt{-\frac{a^5}{b^7}}\log\left(x + \frac{b^3\sqrt{-\frac{a^5}{b^7}}}{a^2}\right)}{2} + \frac{x^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(b*x**2+a),x)`

[Out]  $a**2*x/b**3 - a*x**3/(3*b**2) + \sqrt{-a**5/b**7}*\log(x - b**3*\sqrt{-a**5/b**7}/a**2)/2 - \sqrt{-a**5/b**7}*\log(x + b**3*\sqrt{-a**5/b**7}/a**2)/2 + x**5/(5*b)$

**GIAC/XCAS [A]** time = 0.222634, size = 74, normalized size = 1.35

$$-\frac{a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{abb^3}} + \frac{3b^4x^5 - 5ab^3x^3 + 15a^2b^2x}{15b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^2 + a),x, algorithm="giac")`

```
[Out] -a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*b^4*x^5 - 5*  
a*b^3*x^3 + 15*a^2*b^2*x)/b^5
```

$$3.130 \quad \int \frac{x^5}{a+bx^2} dx$$

**Optimal.** Leaf size=40

$$\frac{a^2 \log(a+bx^2)}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b}$$

[Out]  $-(a*x^2)/(2*b^2) + x^4/(4*b) + (a^2*Log[a + b*x^2])/(2*b^3)$

**Rubi [A]** time = 0.0683234, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^2 \log(a+bx^2)}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a + b\*x^2), x]

[Out]  $-(a*x^2)/(2*b^2) + x^4/(4*b) + (a^2*Log[a + b*x^2])/(2*b^3)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^2 \log(a+bx^2)}{2b^3} + \frac{\int^{x^2} x dx}{2b} - \frac{\int^{x^2} a dx}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5/(b\*x\*\*2+a), x)

[Out]  $a**2*log(a + b*x**2)/(2*b**3) + Integral(x, (x, x**2))/(2*b) - Integral(a, (x, x**2))/(2*b**2)$

**Mathematica [A]** time = 0.0086645, size = 40, normalized size = 1.

$$\frac{a^2 \log(a+bx^2)}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a + b\*x^2), x]

[Out]  $-(a*x^2)/(2*b^2) + x^4/(4*b) + (a^2*Log[a + b*x^2])/(2*b^3)$

**Maple [A]** time = 0.003, size = 35, normalized size = 0.9

$$-\frac{ax^2}{2b^2} + \frac{x^4}{4b} + \frac{a^2 \ln(bx^2 + a)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b\*x^2+a), x)

[Out]  $-1/2*a*x^2/b^2+1/4*x^4/b+1/2*a^2*\ln(b*x^2+a)/b^3$

**Maxima [A]** time = 1.34284, size = 46, normalized size = 1.15

$$\frac{a^2 \log(bx^2 + a)}{2b^3} + \frac{bx^4 - 2ax^2}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2 + a), x, algorithm="maxima")`

[Out]  $1/2*a^2*\log(b*x^2 + a)/b^3 + 1/4*(b*x^4 - 2*a*x^2)/b^2$

**Fricas [A]** time = 0.198776, size = 45, normalized size = 1.12

$$\frac{b^2x^4 - 2abx^2 + 2a^2 \log(bx^2 + a)}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2 + a), x, algorithm="fricas")`

[Out]  $1/4*(b^2*x^4 - 2*a*b*x^2 + 2*a^2*\log(b*x^2 + a))/b^3$

**Sympy [A]** time = 1.19195, size = 32, normalized size = 0.8

$$\frac{a^2 \log(a + bx^2)}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(b*x**2+a), x)`

[Out]  $a**2*\log(a + b*x**2)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b)$

**GIAC/XCAS [A]** time = 0.243686, size = 47, normalized size = 1.18

$$\frac{a^2 \ln(|bx^2 + a|)}{2b^3} + \frac{bx^4 - 2ax^2}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2 + a), x, algorithm="giac")`

[Out]  $1/2*a^2*\ln(\text{abs}(b*x^2 + a))/b^3 + 1/4*(b*x^4 - 2*a*x^2)/b^2$

$$3.131 \quad \int \frac{x^4}{a+bx^2} dx$$

**Optimal.** Leaf size=42

$$\frac{a^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{5/2}} - \frac{ax}{b^2} + \frac{x^3}{3b}$$

[Out]  $-\left(\frac{a \cdot x}{b^2}\right) + \frac{x^3}{3 \cdot b} + \frac{a^{3/2} \cdot \text{ArcTan}\left[\frac{\text{Sqrt}[b] \cdot x}{\text{Sqrt}[a]}\right]}{b^{5/2}}$

**Rubi [A]** time = 0.056853, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{5/2}} - \frac{ax}{b^2} + \frac{x^3}{3b}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a + b\*x^2), x]

[Out]  $-\left(\frac{a \cdot x}{b^2}\right) + \frac{x^3}{3 \cdot b} + \frac{a^{3/2} \cdot \text{ArcTan}\left[\frac{\text{Sqrt}[b] \cdot x}{\text{Sqrt}[a]}\right]}{b^{5/2}}$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^{3/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{5/2}} + \frac{x^3}{3b} - \frac{\int a dx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(b\*x\*\*2+a), x)

[Out]  $a^{3/2} \cdot \operatorname{atan}(\text{sqrt}(b) \cdot x / \text{sqrt}(a)) / b^{5/2} + x^3 / (3 \cdot b) - \text{Integral}(a, x) / b^2$

**Mathematica [A]** time = 0.0337588, size = 42, normalized size = 1.

$$\frac{a^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{5/2}} - \frac{ax}{b^2} + \frac{x^3}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a + b\*x^2), x]

[Out]  $-\left(\frac{a \cdot x}{b^2}\right) + \frac{x^3}{3 \cdot b} + \frac{a^{3/2} \cdot \text{ArcTan}\left[\frac{\text{Sqrt}[b] \cdot x}{\text{Sqrt}[a]}\right]}{b^{5/2}}$

**Maple [A]** time = 0.004, size = 38, normalized size = 0.9

$$\frac{x^3}{3b} - \frac{ax}{b^2} + \frac{a^2}{b^2} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(b*x^2+a),x)`

[Out]  $1/3*x^3/b - a*x/b^2 + a^2/b^2/(a*b)^{(1/2)}*\arctan(x*b/(a*b)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.206135, size = 1, normalized size = 0.02

$$\left[ \frac{2bx^3 + 3a\sqrt{-\frac{a}{b}} \log\left(\frac{bx^2 + 2bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) - 6ax}{6b^2}, \frac{bx^3 + 3a\sqrt{\frac{a}{b}} \arctan\left(\frac{x}{\sqrt{\frac{a}{b}}}\right) - 3ax}{3b^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a),x, algorithm="fricas")`

[Out]  $[1/6*(2*b*x^3 + 3*a*\sqrt{-a/b})*\log((b*x^2 + 2*b*x*\sqrt{-a/b} - a)/(b*x^2 + a)) - 6*a*x)/b^2, 1/3*(b*x^3 + 3*a*\sqrt{a/b})*\arctan(x/\sqrt{a/b}) - 3*a*x)/b^2]$

**Sympy [A]** time = 1.25336, size = 80, normalized size = 1.9

$$-\frac{ax}{b^2} - \frac{\sqrt{-\frac{a^3}{b^5}} \log\left(x - \frac{b^2\sqrt{-\frac{a^3}{b^5}}}{a}\right)}{2} + \frac{\sqrt{-\frac{a^3}{b^5}} \log\left(x + \frac{b^2\sqrt{-\frac{a^3}{b^5}}}{a}\right)}{2} + \frac{x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(b*x**2+a),x)`

[Out]  $-a*x/b**2 - \sqrt{-a**3/b**5}*\log(x - b**2*\sqrt{-a**3/b**5}/a)/2 + \sqrt{-a**3/b**5}*\log(x + b**2*\sqrt{-a**3/b**5}/a)/2 + x**3/(3*b)$

**GIAC/XCAS [A]** time = 0.2233, size = 54, normalized size = 1.29

$$\frac{a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{abb^2}} + \frac{b^2x^3 - 3abx}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a),x, algorithm="giac")`

[Out]  $a^2*\arctan(b*x/\sqrt{a*b})/(\sqrt{a*b})*b^2 + 1/3*(b^2*x^3 - 3*a*b*x)/b^3$

$$3.132 \quad \int \frac{x^3}{a+bx^2} dx$$

**Optimal.** Leaf size=27

$$\frac{x^2}{2b} - \frac{a \log(a + bx^2)}{2b^2}$$

[Out]  $x^2/(2*b) - (a*\text{Log}[a + b*x^2])/(2*b^2)$

**Rubi [A]** time = 0.04987, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{x^2}{2b} - \frac{a \log(a + bx^2)}{2b^2}$$

Antiderivative was successfully verified.

[In] `Int[x^3/(a + b*x^2), x]`

[Out]  $x^2/(2*b) - (a*\text{Log}[a + b*x^2])/(2*b^2)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a \log(a + bx^2)}{2b^2} + \frac{\int^{x^2} \frac{1}{b} dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3/(b*x**2+a), x)`

[Out]  $-a*\log(a + b*x**2)/(2*b**2) + \text{Integral}(1/b, (x, x**2))/2$

**Mathematica [A]** time = 0.00759544, size = 27, normalized size = 1.

$$\frac{x^2}{2b} - \frac{a \log(a + bx^2)}{2b^2}$$

Antiderivative was successfully verified.

[In] `Integrate[x^3/(a + b*x^2), x]`

[Out]  $x^2/(2*b) - (a*\text{Log}[a + b*x^2])/(2*b^2)$

**Maple [A]** time = 0.003, size = 24, normalized size = 0.9

$$\frac{x^2}{2b} - \frac{a \ln(bx^2 + a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(b*x^2+a), x)`

[Out]  $1/2*x^2/b - 1/2*a*\ln(b*x^2+a)/b^2$

---

**Maxima [A]** time = 1.34588, size = 31, normalized size = 1.15

$$\frac{x^2}{2b} - \frac{a \log(bx^2 + a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b\*x^2 + a), x, algorithm="maxima")

[Out] 1/2\*x^2/b - 1/2\*a\*log(b\*x^2 + a)/b^2

---

**Fricas [A]** time = 0.198994, size = 30, normalized size = 1.11

$$\frac{bx^2 - a \log(bx^2 + a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b\*x^2 + a), x, algorithm="fricas")

[Out] 1/2\*(b\*x^2 - a\*log(b\*x^2 + a))/b^2

---

**Sympy [A]** time = 1.16468, size = 20, normalized size = 0.74

$$-\frac{a \log(a + bx^2)}{2b^2} + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(b\*x\*\*2+a), x)

[Out] -a\*log(a + b\*x\*\*2)/(2\*b\*\*2) + x\*\*2/(2\*b)

---

**GIAC/XCAS [A]** time = 0.224063, size = 32, normalized size = 1.19

$$\frac{x^2}{2b} - \frac{a \ln(|bx^2 + a|)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b\*x^2 + a), x, algorithm="giac")

[Out] 1/2\*x^2/b - 1/2\*a\*ln(abs(b\*x^2 + a))/b^2



$$3.133 \quad \int \frac{x^2}{a+bx^2} dx$$

**Optimal.** Leaf size=31

$$\frac{x}{b} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{3/2}}$$

[Out]  $x/b - (\text{Sqrt}[a] * \text{ArcTan}[(\text{Sqrt}[b] * x)/\text{Sqrt}[a]])/b^{(3/2)}$

**Rubi [A]** time = 0.0398427, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{x}{b} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*x^2), x]

[Out]  $x/b - (\text{Sqrt}[a] * \text{ArcTan}[(\text{Sqrt}[b] * x)/\text{Sqrt}[a]])/b^{(3/2)}$

**Rubi in Sympy [A]** time = 6.99731, size = 26, normalized size = 0.84

$$-\frac{\sqrt{a} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{3/2}} + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(b\*x\*\*2+a), x)

[Out]  $-\text{sqrt}(a) * \operatorname{atan}(\text{sqrt}(b) * x/\text{sqrt}(a))/b^{(3/2)} + x/b$

**Mathematica [A]** time = 0.0157188, size = 31, normalized size = 1.

$$\frac{x}{b} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*x^2), x]

[Out]  $x/b - (\text{Sqrt}[a] * \text{ArcTan}[(\text{Sqrt}[b] * x)/\text{Sqrt}[a]])/b^{(3/2)}$

**Maple [A]** time = 0.004, size = 27, normalized size = 0.9

$$\frac{x}{b} - \frac{a}{b} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b\*x^2+a), x)

[Out]  $x/b - a/b / (a*b)^{(1/2)} * \arctan(x*b / (a*b)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.204689, size = 1, normalized size = 0.03

$$\left[ \frac{\sqrt{-\frac{a}{b}} \log\left(\frac{bx^2 - 2bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) + 2x}{2b}, -\frac{\sqrt{\frac{a}{b}} \arctan\left(\frac{x}{\sqrt{\frac{a}{b}}}\right) - x}{b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a), x, algorithm="fricas")`

[Out]  $[1/2 * (\sqrt{-a/b} * \log((b*x^2 - 2*b*x*\sqrt{-a/b} - a)/(b*x^2 + a)) + 2*x)/b, -(\sqrt{a/b} * \arctan(x/\sqrt{a/b}) - x)/b]$

**Sympy [A]** time = 1.16809, size = 56, normalized size = 1.81

$$\frac{\sqrt{-\frac{a}{b^3}} \log\left(-b\sqrt{-\frac{a}{b^3}} + x\right)}{2} - \frac{\sqrt{-\frac{a}{b^3}} \log\left(b\sqrt{-\frac{a}{b^3}} + x\right)}{2} + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**2+a), x)`

[Out]  $\sqrt{-a/b^3} * \log(-b*\sqrt{-a/b^3} + x)/2 - \sqrt{-a/b^3} * \log(b*\sqrt{-a/b^3} + x)/2 + x/b$

**GIAC/XCAS [A]** time = 0.221352, size = 35, normalized size = 1.13

$$-\frac{a \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{abb}} + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a), x, algorithm="giac")`

[Out]  $-a*\arctan(b*x/\sqrt{a*b})/(\sqrt{a*b}*b) + x/b$

$$3.134 \quad \int \frac{x}{a+bx^2} dx$$

**Optimal.** Leaf size=15

$$\frac{\log(a+bx^2)}{2b}$$

[Out] Log[a + b\*x^2]/(2\*b)

---

**Rubi [A]** time = 0.0101463, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{\log(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*x^2), x]

[Out] Log[a + b\*x^2]/(2\*b)

---

**Rubi in Sympy [A]** time = 2.25209, size = 10, normalized size = 0.67

$$\frac{\log(a+bx^2)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(b\*x\*\*2+a), x)

[Out] log(a + b\*x\*\*2)/(2\*b)

---

**Mathematica [A]** time = 0.00313871, size = 15, normalized size = 1.

$$\frac{\log(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*x^2), x]

[Out] Log[a + b\*x^2]/(2\*b)

---

**Maple [A]** time = 0.002, size = 14, normalized size = 0.9

$$\frac{\ln(bx^2+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b\*x^2+a), x)

[Out] 1/2\*ln(b\*x^2+a)/b

---

**Maxima [A]** time = 1.35078, size = 18, normalized size = 1.2

$$\frac{\log(bx^2 + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b\*x^2 + a),x, algorithm="maxima")

[Out] 1/2\*log(b\*x^2 + a)/b

---

**Fricas [A]** time = 0.194934, size = 18, normalized size = 1.2

$$\frac{\log(bx^2 + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b\*x^2 + a),x, algorithm="fricas")

[Out] 1/2\*log(b\*x^2 + a)/b

---

**Sympy [A]** time = 0.227436, size = 10, normalized size = 0.67

$$\frac{\log(a + bx^2)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b\*x\*\*2+a),x)

[Out] log(a + b\*x\*\*2)/(2\*b)

---

**GIAC/XCAS [A]** time = 0.216785, size = 19, normalized size = 1.27

$$\frac{\ln(|bx^2 + a|)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b\*x^2 + a),x, algorithm="giac")

[Out] 1/2\*ln(abs(b\*x^2 + a))/b

$$3.135 \quad \int \frac{1}{a+bx^2} dx$$

**Optimal.** Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

[Out] ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/(Sqrt[a]\*Sqrt[b])

**Rubi [A]** time = 0.018048, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(-1), x]

[Out] ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/(Sqrt[a]\*Sqrt[b])

**Rubi in Sympy [A]** time = 2.4669, size = 22, normalized size = 0.92

$$\frac{\text{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+a), x)

[Out] atan(sqrt(b)\*x/sqrt(a))/(sqrt(a)\*sqrt(b))

**Mathematica [A]** time = 0.0076092, size = 24, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(-1), x]

[Out] ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/(Sqrt[a]\*Sqrt[b])

**Maple [A]** time = 0.002, size = 16, normalized size = 0.7

$$1 \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+a), x)

[Out]  $1/(a*b)^{(1/2)}*\arctan(x*b/(a*b)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^2 + a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.203606, size = 1, normalized size = 0.04

$$\left[ \frac{\log\left(\frac{2abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)}{2\sqrt{-ab}}, \frac{\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{\sqrt{ab}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^2 + a),x, algorithm="fricas")`

[Out]  $[1/2*\log((2*a*b*x + (b*x^2 - a)*\sqrt{-a*b})/(b*x^2 + a))/\sqrt{-a*b}, \arctan(\sqrt{a*b}*x/a)/\sqrt{a*b}]$

**Sympy [A]** time = 0.286827, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}} \log\left(-a\sqrt{-\frac{1}{ab}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{ab}} \log\left(a\sqrt{-\frac{1}{ab}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**2+a),x)`

[Out]  $-\sqrt{-1/(a*b)}*\log(-a*\sqrt{-1/(a*b)} + x)/2 + \sqrt{-1/(a*b)}*\log(a*\sqrt{-1/(a*b)} + x)/2$

**GIAC/XCAS [A]** time = 0.207529, size = 20, normalized size = 0.83

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^2 + a),x, algorithm="giac")`

[Out]  $\arctan(b*x/\sqrt{a*b})/\sqrt{a*b}$

$$3.136 \quad \int \frac{1}{x(a+bx^2)} dx$$

**Optimal.** Leaf size=22

$$\frac{\log(x)}{a} - \frac{\log(a+bx^2)}{2a}$$

[Out] Log[x]/a - Log[a + b\*x^2]/(2\*a)

**Rubi [A]** time = 0.0357655, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$

$$\frac{\log(x)}{a} - \frac{\log(a+bx^2)}{2a}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(a + b\*x^2)), x]

[Out] Log[x]/a - Log[a + b\*x^2]/(2\*a)

**Rubi in Sympy [A]** time = 6.28996, size = 19, normalized size = 0.86

$$\frac{\log(x^2)}{2a} - \frac{\log(a+bx^2)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(b\*x\*\*2+a), x)

[Out] log(x\*\*2)/(2\*a) - log(a + b\*x\*\*2)/(2\*a)

**Mathematica [A]** time = 0.00882065, size = 22, normalized size = 1.

$$\frac{\log(x)}{a} - \frac{\log(a+bx^2)}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(a + b\*x^2)), x]

[Out] Log[x]/a - Log[a + b\*x^2]/(2\*a)

**Maple [A]** time = 0.006, size = 21, normalized size = 1.

$$\frac{\ln(x)}{a} - \frac{\ln(bx^2+a)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b\*x^2+a), x)

[Out] ln(x)/a-1/2\*ln(b\*x^2+a)/a

---

**Maxima [A]** time = 1.35572, size = 31, normalized size = 1.41

$$-\frac{\log(bx^2 + a)}{2a} + \frac{\log(x^2)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)\*x),x, algorithm="maxima")

[Out] -1/2\*log(b\*x^2 + a)/a + 1/2\*log(x^2)/a

---

**Fricas [A]** time = 0.201445, size = 24, normalized size = 1.09

$$\frac{\log(bx^2 + a) - 2 \log(x)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)\*x),x, algorithm="fricas")

[Out] -1/2\*(log(b\*x^2 + a) - 2\*log(x))/a

---

**Sympy [A]** time = 0.496404, size = 15, normalized size = 0.68

$$\frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b\*x\*\*2+a),x)

[Out] log(x)/a - log(a/b + x\*\*2)/(2\*a)

---

**GIAC/XCAS [A]** time = 0.21102, size = 32, normalized size = 1.45

$$\frac{\ln(x^2)}{2a} - \frac{\ln(|bx^2 + a|)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)\*x),x, algorithm="giac")

[Out] 1/2\*ln(x^2)/a - 1/2\*ln(abs(b\*x^2 + a))/a



$$3.137 \quad \int \frac{1}{x^2(a+bx^2)} dx$$

**Optimal.** Leaf size=34

$$-\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{ax}$$

[Out]  $-(1/(a*x)) - (\text{Sqrt}[b]*\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/a^{(3/2)}$

**Rubi [A]** time = 0.0370384, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{ax}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^2*(a + b*x^2)), x]`

[Out]  $-(1/(a*x)) - (\text{Sqrt}[b]*\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/a^{(3/2)}$

**Rubi in Sympy [A]** time = 6.61247, size = 29, normalized size = 0.85

$$-\frac{1}{ax} - \frac{\sqrt{b} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**2/(b*x**2+a), x)`

[Out]  $-1/(a*x) - \text{sqrt}(b)*\text{atan}(\text{sqrt}(b)*x/\text{sqrt}(a))/a^{(3/2)}$

**Mathematica [A]** time = 0.0239622, size = 34, normalized size = 1.

$$-\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{ax}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^2*(a + b*x^2)), x]`

[Out]  $-(1/(a*x)) - (\text{Sqrt}[b]*\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/a^{(3/2)}$

**Maple [A]** time = 0.006, size = 30, normalized size = 0.9

$$-\frac{b}{a} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(b*x^2+a), x)`

[Out]  $-b/a/(a*b)^{(1/2)}*\arctan(x*b/(a*b)^{(1/2)})-1/a/x$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.207094, size = 1, normalized size = 0.03

$$\left[ \frac{x\sqrt{-\frac{b}{a}} \log\left(\frac{bx^2 - 2ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right) - 2}{2ax}, -\frac{x\sqrt{\frac{b}{a}} \arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right) + 1}{ax} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^2),x, algorithm="fricas")`

[Out]  $[1/2*(x*\sqrt{-b/a})*\log((b*x^2 - 2*a*x*\sqrt{-b/a} - a)/(b*x^2 + a) - 2)/(a*x), -(x*\sqrt{b/a})*\arctan(b*x/(a*\sqrt{b/a})) + 1)/(a*x)]$

**Sympy [A]** time = 1.29847, size = 65, normalized size = 1.91

$$\frac{\sqrt{-\frac{b}{a^3}} \log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b} + x\right)}{2} - \frac{\sqrt{-\frac{b}{a^3}} \log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b} + x\right)}{2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(b*x**2+a),x)`

[Out]  $\sqrt{-b/a**3}*\log(-a**2*\sqrt{-b/a**3}/b + x)/2 - \sqrt{-b/a**3}*\log(a**2*\sqrt{-b/a**3}/b + x)/2 - 1/(a*x)$

**GIAC/XCAS [A]** time = 0.215815, size = 39, normalized size = 1.15

$$-\frac{b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{aba}} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^2),x, algorithm="giac")`

[Out]  $-b*\arctan(b*x/\sqrt{a*b})/(\sqrt{a*b}*a) - 1/(a*x)$

$$3.138 \quad \int \frac{1}{x^3(a+bx^2)} dx$$

**Optimal.** Leaf size=35

$$\frac{b \log(a+bx^2)}{2a^2} - \frac{b \log(x)}{a^2} - \frac{1}{2ax^2}$$

[Out]  $-1/(2*a*x^2) - (b*\text{Log}[x])/a^2 + (b*\text{Log}[a + b*x^2])/(2*a^2)$

**Rubi [A]** time = 0.0594205, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{b \log(a+bx^2)}{2a^2} - \frac{b \log(x)}{a^2} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^3*(a + b*x^2)), x]`

[Out]  $-1/(2*a*x^2) - (b*\text{Log}[x])/a^2 + (b*\text{Log}[a + b*x^2])/(2*a^2)$

**Rubi in Sympy [A]** time = 9.18644, size = 34, normalized size = 0.97

$$-\frac{1}{2ax^2} - \frac{b \log(x^2)}{2a^2} + \frac{b \log(a+bx^2)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**3/(b*x**2+a), x)`

[Out]  $-1/(2*a*x**2) - b*\log(x**2)/(2*a**2) + b*\log(a + b*x**2)/(2*a**2)$

**Mathematica [A]** time = 0.0121421, size = 35, normalized size = 1.

$$\frac{b \log(a+bx^2)}{2a^2} - \frac{b \log(x)}{a^2} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^3*(a + b*x^2)), x]`

[Out]  $-1/(2*a*x^2) - (b*\text{Log}[x])/a^2 + (b*\text{Log}[a + b*x^2])/(2*a^2)$

**Maple [A]** time = 0.009, size = 32, normalized size = 0.9

$$-\frac{1}{2ax^2} - \frac{b \ln(x)}{a^2} + \frac{b \ln(bx^2+a)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(b*x^2+a), x)`

[Out]  $-1/2/a/x^2 - b*\ln(x)/a^2 + 1/2*b*\ln(b*x^2+a)/a^2$

---

**Maxima [A]** time = 1.3471, size = 45, normalized size = 1.29

$$\frac{b \log(bx^2 + a)}{2a^2} - \frac{b \log(x^2)}{2a^2} - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^3),x, algorithm="maxima")`

[Out] `1/2*b*log(b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 - 1/2/(a*x^2)`

---

**Fricas [A]** time = 0.206945, size = 45, normalized size = 1.29

$$\frac{bx^2 \log(bx^2 + a) - 2bx^2 \log(x) - a}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^3),x, algorithm="fricas")`

[Out] `1/2*(b*x^2*log(b*x^2 + a) - 2*b*x^2*log(x) - a)/(a^2*x^2)`

---

**Sympy [A]** time = 1.58851, size = 31, normalized size = 0.89

$$-\frac{1}{2ax^2} - \frac{b \log(x)}{a^2} + \frac{b \log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(b*x**2+a),x)`

[Out] `-1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2)`

---

**GIAC/XCAS [A]** time = 0.211086, size = 58, normalized size = 1.66

$$-\frac{b \ln(x^2)}{2a^2} + \frac{b \ln(|bx^2 + a|)}{2a^2} + \frac{bx^2 - a}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^3),x, algorithm="giac")`

[Out] `-1/2*b*ln(x^2)/a^2 + 1/2*b*ln(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 - a)/(a^2*x^2)`

$$3.139 \quad \int \frac{1}{x^4(a+bx^2)} dx$$

**Optimal.** Leaf size=43

$$\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{b}{a^2x} - \frac{1}{3ax^3}$$

[Out]  $-1/(3*a*x^3) + b/(a^2*x) + (b^{(3/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^{(5/2)}$

**Rubi [A]** time = 0.0535908, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{b}{a^2x} - \frac{1}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a + b\*x^2)), x]

[Out]  $-1/(3*a*x^3) + b/(a^2*x) + (b^{(3/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^{(5/2)}$

**Rubi in Sympy [A]** time = 10.5003, size = 37, normalized size = 0.86

$$-\frac{1}{3ax^3} + \frac{b}{a^2x} + \frac{b^{3/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(b\*x\*\*2+a), x)

[Out]  $-1/(3*a*x^3) + b/(a^2*x) + b^{(3/2)}*atan(sqrt(b)*x/sqrt(a))/a^{(5/2)}$

**Mathematica [A]** time = 0.0387371, size = 43, normalized size = 1.

$$\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{b}{a^2x} - \frac{1}{3ax^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a + b\*x^2)), x]

[Out]  $-1/(3*a*x^3) + b/(a^2*x) + (b^{(3/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^{(5/2)}$

**Maple [A]** time = 0.008, size = 39, normalized size = 0.9

$$-\frac{1}{3ax^3} + \frac{b}{a^2x} + \frac{b^2}{a^2} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(b*x^2+a), x)`

[Out]  $-1/3/a/x^3+b/a^2/x+b^2/a^2/(a*b)^{(1/2)}*\arctan(x*b/(a*b)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^4), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.208656, size = 1, normalized size = 0.02

$$\left[ \frac{3bx^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right) + 6bx^2 - 2a}{6a^2x^3}, \frac{3bx^3\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right) + 3bx^2 - a}{3a^2x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^4), x, algorithm="fricas")`

[Out]  $[1/6*(3*b*x^3*\sqrt{-b/a}*\log((b*x^2 + 2*a*x*\sqrt{-b/a} - a)/(b*x^2 + a)) + 6*b*x^2 - 2*a)/(a^2*x^3), 1/3*(3*b*x^3*\sqrt{b/a}*\arctan(b*x/(a*\sqrt{b/a})) + 3*b*x^2 - a)/(a^2*x^3)]$

**Sympy [A]** time = 1.47088, size = 87, normalized size = 2.02

$$-\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2} + x\right)}{2} + \frac{\sqrt{-\frac{b^3}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2} + x\right)}{2} + \frac{-a + 3bx^2}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(b*x**2+a), x)`

[Out]  $-\sqrt{-b**3/a**5}*\log(-a**3*\sqrt{-b**3/a**5}/b**2 + x)/2 + \sqrt{-b**3/a**5}*\log(a**3*\sqrt{-b**3/a**5}/b**2 + x)/2 + (-a + 3*b*x**2)/(3*a**2*x**3)$

**GIAC/XCAS [A]** time = 0.210645, size = 54, normalized size = 1.26

$$\frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{3bx^2 - a}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^4), x, algorithm="giac")`

[Out]  $b^2*\arctan(b*x/\sqrt{a*b})/(\sqrt{a*b}*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)$

$$3.140 \quad \int \frac{1}{x^5(a+bx^2)} dx$$

**Optimal.** Leaf size=49

$$-\frac{b^2 \log(a+bx^2)}{2a^3} + \frac{b^2 \log(x)}{a^3} + \frac{b}{2a^2x^2} - \frac{1}{4ax^4}$$

[Out]  $-1/(4*a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2*a^3)$

**Rubi [A]** time = 0.0739861, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{b^2 \log(a+bx^2)}{2a^3} + \frac{b^2 \log(x)}{a^3} + \frac{b}{2a^2x^2} - \frac{1}{4ax^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5\*(a + b\*x^2)), x]

[Out]  $-1/(4*a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2*a^3)$

**Rubi in Sympy [A]** time = 11.9459, size = 48, normalized size = 0.98

$$-\frac{1}{4ax^4} + \frac{b}{2a^2x^2} + \frac{b^2 \log(x^2)}{2a^3} - \frac{b^2 \log(a+bx^2)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*5/(b\*x\*\*2+a), x)

[Out]  $-1/(4*a*x**4) + b/(2*a**2*x**2) + b**2*log(x**2)/(2*a**3) - b**2*log(a + b*x**2)/(2*a**3)$

**Mathematica [A]** time = 0.0127184, size = 49, normalized size = 1.

$$-\frac{b^2 \log(a+bx^2)}{2a^3} + \frac{b^2 \log(x)}{a^3} + \frac{b}{2a^2x^2} - \frac{1}{4ax^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5\*(a + b\*x^2)), x]

[Out]  $-1/(4*a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2*a^3)$

**Maple [A]** time = 0.01, size = 44, normalized size = 0.9

$$-\frac{1}{4ax^4} + \frac{b}{2a^2x^2} + \frac{b^2 \ln(x)}{a^3} - \frac{b^2 \ln(bx^2+a)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(b\*x^2+a), x)

[Out]  $-1/4/a/x^4+1/2*b/a^2/x^2+b^2*\ln(x)/a^3-1/2*b^2*\ln(b*x^2+a)/a^3$

**Maxima [A]** time = 1.35397, size = 63, normalized size = 1.29

$$-\frac{b^2 \log(bx^2 + a)}{2a^3} + \frac{b^2 \log(x^2)}{2a^3} + \frac{2bx^2 - a}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^5),x, algorithm="maxima")`

[Out]  $-1/2*b^2*\log(b*x^2 + a)/a^3 + 1/2*b^2*\log(x^2)/a^3 + 1/4*(2*b*x^2 - a)/(a^2*x^4)$

**Fricas [A]** time = 0.204541, size = 61, normalized size = 1.24

$$-\frac{2b^2x^4 \log(bx^2 + a) - 4b^2x^4 \log(x) - 2abx^2 + a^2}{4a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^5),x, algorithm="fricas")`

[Out]  $-1/4*(2*b^2*x^4*\log(b*x^2 + a) - 4*b^2*x^4*\log(x) - 2*a*b*x^2 + a^2)/(a^3*x^4)$

**Sympy [A]** time = 1.78807, size = 42, normalized size = 0.86

$$\frac{-a + 2bx^2}{4a^2x^4} + \frac{b^2 \log(x)}{a^3} - \frac{b^2 \log\left(\frac{a}{b} + x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(b*x**2+a),x)`

[Out]  $(-a + 2*b*x**2)/(4*a**2*x**4) + b**2*\log(x)/a**3 - b**2*\log(a/b + x**2)/(2*a**3)$

**GIAC/XCAS [A]** time = 0.210879, size = 77, normalized size = 1.57

$$\frac{b^2 \ln(x^2)}{2a^3} - \frac{b^2 \ln(|bx^2 + a|)}{2a^3} - \frac{3b^2x^4 - 2abx^2 + a^2}{4a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^5),x, algorithm="giac")`

[Out]  $1/2*b^2*\ln(x^2)/a^3 - 1/2*b^2*\ln(\text{abs}(b*x^2 + a))/a^3 - 1/4*(3*b^2*x^4 - 2*a*b*x^2 + a^2)/(a^3*x^4)$



$$3.141 \quad \int \frac{1}{x^6(a+bx^2)} dx$$

**Optimal.** Leaf size=58

$$-\frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{7/2}} - \frac{b^2}{a^3x} + \frac{b}{3a^2x^3} - \frac{1}{5ax^5}$$

[Out]  $-1/(5*a*x^5) + b/(3*a^2*x^3) - b^2/(a^3*x) - (b^{5/2})*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/a^{7/2}$

**Rubi [A]** time = 0.0754309, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{7/2}} - \frac{b^2}{a^3x} + \frac{b}{3a^2x^3} - \frac{1}{5ax^5}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(a + b\*x^2)), x]

[Out]  $-1/(5*a*x^5) + b/(3*a^2*x^3) - b^2/(a^3*x) - (b^{5/2})*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/a^{7/2}$

**Rubi in Sympy [A]** time = 14.989, size = 49, normalized size = 0.84

$$-\frac{1}{5ax^5} + \frac{b}{3a^2x^3} - \frac{b^2}{a^3x} - \frac{b^{5/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*6/(b\*x\*\*2+a), x)

[Out]  $-1/(5*a*x^5) + b/(3*a^2*x^3) - b^2/(a^3*x) - b^{5/2}*atan(sqrt(b)*x/sqrt(a))/a^{7/2}$

**Mathematica [A]** time = 0.0455841, size = 58, normalized size = 1.

$$-\frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{7/2}} - \frac{b^2}{a^3x} + \frac{b}{3a^2x^3} - \frac{1}{5ax^5}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6\*(a + b\*x^2)), x]

[Out]  $-1/(5*a*x^5) + b/(3*a^2*x^3) - b^2/(a^3*x) - (b^{5/2})*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/a^{7/2}$

**Maple [A]** time = 0.009, size = 52, normalized size = 0.9

$$-\frac{1}{5ax^5} - \frac{b^2}{a^3x} + \frac{b}{3a^2x^3} - \frac{b^3}{a^3} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^6/(b*x^2+a), x)`

[Out]  $-1/5/a/x^5 - b^2/a^3/x + 1/3*b/a^2/x^3 - b^3/a^3/(a*b)^{(1/2)}*\arctan(x*b/(a*b)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^6), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.207747, size = 1, normalized size = 0.02

$$\left[ \frac{15 b^2 x^5 \sqrt{-\frac{b}{a}} \log\left(\frac{b x^2 - 2 a x \sqrt{-\frac{b}{a}} - a}{b x^2 + a}\right) - 30 b^2 x^4 + 10 a b x^2 - 6 a^2}{30 a^3 x^5}, \right. \\ \left. - \frac{15 b^2 x^5 \sqrt{\frac{b}{a}} \arctan\left(\frac{b x}{a \sqrt{\frac{b}{a}}}\right) + 15 b^2 x^4 - 5 a b x^2 + 3 a^2}{15 a^3 x^5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^6), x, algorithm="fricas")`

[Out]  $[1/30*(15*b^2*x^5*\sqrt{-b/a}*\log((b*x^2 - 2*a*x*\sqrt{-b/a} - a)/(b*x^2 + a)) - 30*b^2*x^4 + 10*a*b*x^2 - 6*a^2)/(a^3*x^5), -1/15*(15*b^2*x^5*\sqrt{b/a}*\arctan(b*x/(a*\sqrt{b/a})) + 15*b^2*x^4 - 5*a*b*x^2 + 3*a^2)/(a^3*x^5)]$

**Sympy [A]** time = 1.74768, size = 100, normalized size = 1.72

$$\frac{\sqrt{-\frac{b^5}{a^7}} \log\left(-\frac{a^4 \sqrt{-\frac{b^5}{a^7}}}{b^3} + x\right)}{2} - \frac{\sqrt{-\frac{b^5}{a^7}} \log\left(\frac{a^4 \sqrt{-\frac{b^5}{a^7}}}{b^3} + x\right)}{2} - \frac{3a^2 - 5abx^2 + 15b^2x^4}{15a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**6/(b*x**2+a), x)`

[Out]  $\sqrt{-b^{**5}/a^{**7}}*\log(-a^{**4}*\sqrt{-b^{**5}/a^{**7}}/b^{**3} + x)/2 - \sqrt{-b^{**5}/a^{**7}}*\log(a^{**4}*\sqrt{-b^{**5}/a^{**7}}/b^{**3} + x)/2 - (3*a^{**2} - 5*a*b*x^{**2} + 15*b^{**2}*x^{**4})/(15*a^{**3}*x^{**5})$

**GIAC/XCAS [A]** time = 0.207905, size = 70, normalized size = 1.21

$$\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{15 b^2 x^4 - 5 a b x^2 + 3 a^2}{15 a^3 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*x^6),x, algorithm="giac")
```

```
[Out] -b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/15*(15*b^2*x^4 - 5  
*a*b*x^2 + 3*a^2)/(a^3*x^5)
```

$$3.142 \quad \int \frac{1}{x^7(a+bx^2)} dx$$

**Optimal.** Leaf size=63

$$\frac{b^3 \log(a+bx^2)}{2a^4} - \frac{b^3 \log(x)}{a^4} - \frac{b^2}{2a^3x^2} + \frac{b}{4a^2x^4} - \frac{1}{6ax^6}$$

[Out]  $-1/(6*a*x^6) + b/(4*a^2*x^4) - b^2/(2*a^3*x^2) - (b^3*\text{Log}[x])/a^4 + (b^3*\text{Log}[a + b*x^2])/(2*a^4)$

**Rubi [A]** time = 0.0885751, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{b^3 \log(a+bx^2)}{2a^4} - \frac{b^3 \log(x)}{a^4} - \frac{b^2}{2a^3x^2} + \frac{b}{4a^2x^4} - \frac{1}{6ax^6}$$

Antiderivative was successfully verified.

[In] Int[1/(x^7\*(a + b\*x^2)), x]

[Out]  $-1/(6*a*x^6) + b/(4*a^2*x^4) - b^2/(2*a^3*x^2) - (b^3*\text{Log}[x])/a^4 + (b^3*\text{Log}[a + b*x^2])/(2*a^4)$

**Rubi in Sympy [A]** time = 13.9522, size = 60, normalized size = 0.95

$$-\frac{1}{6ax^6} + \frac{b}{4a^2x^4} - \frac{b^2}{2a^3x^2} - \frac{b^3 \log(x^2)}{2a^4} + \frac{b^3 \log(a+bx^2)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*7/(b\*x\*\*2+a), x)

[Out]  $-1/(6*a*x**6) + b/(4*a**2*x**4) - b**2/(2*a**3*x**2) - b**3*\log(x**2)/(2*a**4) + b**3*\log(a + b*x**2)/(2*a**4)$

**Mathematica [A]** time = 0.0127059, size = 63, normalized size = 1.

$$\frac{b^3 \log(a+bx^2)}{2a^4} - \frac{b^3 \log(x)}{a^4} - \frac{b^2}{2a^3x^2} + \frac{b}{4a^2x^4} - \frac{1}{6ax^6}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^7\*(a + b\*x^2)), x]

[Out]  $-1/(6*a*x^6) + b/(4*a^2*x^4) - b^2/(2*a^3*x^2) - (b^3*\text{Log}[x])/a^4 + (b^3*\text{Log}[a + b*x^2])/(2*a^4)$

**Maple [A]** time = 0.01, size = 56, normalized size = 0.9

$$-\frac{1}{6ax^6} + \frac{b}{4a^2x^4} - \frac{b^2}{2a^3x^2} - \frac{b^3 \ln(x)}{a^4} + \frac{b^3 \ln(bx^2+a)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^7/(b*x^2+a), x)`

[Out]  $-1/6/a/x^6 + 1/4*b/a^2/x^4 - 1/2*b^2/a^3/x^2 - b^3 \ln(x)/a^4 + 1/2*b^3 \ln(b*x^2+a)/a^4$

**Maxima [A]** time = 1.35327, size = 78, normalized size = 1.24

$$\frac{b^3 \log(bx^2 + a)}{2a^4} - \frac{b^3 \log(x^2)}{2a^4} - \frac{6b^2x^4 - 3abx^2 + 2a^2}{12a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^7), x, algorithm="maxima")`

[Out]  $1/2*b^3 \log(b*x^2 + a)/a^4 - 1/2*b^3 \log(x^2)/a^4 - 1/12*(6*b^2*x^4 - 3*a*b*x^2 + 2*a^2)/(a^3*x^6)$

**Fricas [A]** time = 0.20758, size = 78, normalized size = 1.24

$$\frac{6b^3x^6 \log(bx^2 + a) - 12b^3x^6 \log(x) - 6ab^2x^4 + 3a^2bx^2 - 2a^3}{12a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^7), x, algorithm="fricas")`

[Out]  $1/12*(6*b^3*x^6*\log(b*x^2 + a) - 12*b^3*x^6*\log(x) - 6*a*b^2*x^4 + 3*a^2*b*x^2 - 2*a^3)/(a^4*x^6)$

**Sympy [A]** time = 2.18698, size = 56, normalized size = 0.89

$$-\frac{2a^2 - 3abx^2 + 6b^2x^4}{12a^3x^6} - \frac{b^3 \log(x)}{a^4} + \frac{b^3 \log\left(\frac{a}{b} + x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**7/(b*x**2+a), x)`

[Out]  $-(2*a**2 - 3*a*b*x**2 + 6*b**2*x**4)/(12*a**3*x**6) - b**3*\log(x)/a**4 + b**3*\log(a/b + x**2)/(2*a**4)$

**GIAC/XCAS [A]** time = 0.209489, size = 95, normalized size = 1.51

$$-\frac{b^3 \ln(x^2)}{2a^4} + \frac{b^3 \ln(|bx^2 + a|)}{2a^4} + \frac{11b^3x^6 - 6ab^2x^4 + 3a^2bx^2 - 2a^3}{12a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^7), x, algorithm="giac")`

[Out]  $-1/2*b^3 \ln(x^2)/a^4 + 1/2*b^3 \ln(\text{abs}(b*x^2 + a))/a^4 + 1/12*(11*b^3*x^6 - 6*a*b^2*x^4 + 3*a^2*b*x^2 - 2*a^3)/(a^4*x^6)$

$$3.143 \quad \int \frac{1}{x^8(a+bx^2)} dx$$

**Optimal.** Leaf size=69

$$\frac{b^{7/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{9/2}} + \frac{b^3}{a^4x} - \frac{b^2}{3a^3x^3} + \frac{b}{5a^2x^5} - \frac{1}{7ax^7}$$

[Out]  $-1/(7*a*x^7) + b/(5*a^2*x^5) - b^2/(3*a^3*x^3) + b^3/(a^4*x) + (b^{7/2})*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/a^{9/2}$

**Rubi [A]** time = 0.0962509, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{b^{7/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{9/2}} + \frac{b^3}{a^4x} - \frac{b^2}{3a^3x^3} + \frac{b}{5a^2x^5} - \frac{1}{7ax^7}$$

Antiderivative was successfully verified.

[In] Int[1/(x^8\*(a + b\*x^2)), x]

[Out]  $-1/(7*a*x^7) + b/(5*a^2*x^5) - b^2/(3*a^3*x^3) + b^3/(a^4*x) + (b^{7/2})*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/a^{9/2}$

**Rubi in Sympy [A]** time = 20.2115, size = 61, normalized size = 0.88

$$-\frac{1}{7ax^7} + \frac{b}{5a^2x^5} - \frac{b^2}{3a^3x^3} + \frac{b^3}{a^4x} + \frac{b^{7/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*8/(b\*x\*\*2+a), x)

[Out]  $-1/(7*a*x**7) + b/(5*a**2*x**5) - b**2/(3*a**3*x**3) + b**3/(a**4*x) + b**(7/2)*atan(sqrt(b)*x/sqrt(a))/a**(9/2)$

**Mathematica [A]** time = 0.0563356, size = 69, normalized size = 1.

$$\frac{b^{7/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{9/2}} + \frac{b^3}{a^4x} - \frac{b^2}{3a^3x^3} + \frac{b}{5a^2x^5} - \frac{1}{7ax^7}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^8\*(a + b\*x^2)), x]

[Out]  $-1/(7*a*x^7) + b/(5*a^2*x^5) - b^2/(3*a^3*x^3) + b^3/(a^4*x) + (b^{7/2})*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/a^{9/2}$

**Maple [A]** time = 0.01, size = 61, normalized size = 0.9

$$-\frac{1}{7ax^7} - \frac{b^2}{3a^3x^3} + \frac{b}{5a^2x^5} + \frac{b^3}{a^4x} + \frac{b^4}{a^4} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^8/(b*x^2+a), x)`

[Out]  $-1/7/a/x^7 - 1/3*b^2/a^3/x^3 + 1/5*b/a^2/x^5 + b^3/a^4/x + b^4/a^4/(a*b)^{1/2} \arctan(x*b/(a*b)^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^8), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.215311, size = 1, normalized size = 0.01

$$\left[ \frac{105 b^3 x^7 \sqrt{-\frac{b}{a}} \log\left(\frac{bx^2 + 2ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right) + 210 b^3 x^6 - 70 ab^2 x^4 + 42 a^2 b x^2 - 30 a^3}{210 a^4 x^7}, \frac{105 b^3 x^7 \sqrt{\frac{b}{a}} \arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right) + 105 b^3 x^6 - 35 a^2 b x^4 + 21 a^2 b x^2 - 15 a^3}{105 a^4 x^7} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^8), x, algorithm="fricas")`

[Out]  $[1/210*(105*b^3*x^7*\sqrt{-b/a}*\log((b*x^2 + 2*a*x*\sqrt{-b/a} - a)/(b*x^2 + a)) + 210*b^3*x^6 - 70*a*b^2*x^4 + 42*a^2*b*x^2 - 30*a^3)/(a^4*x^7), 1/105*(105*b^3*x^7*\sqrt{b/a}*\arctan(b*x/(a*\sqrt{b/a}))) + 105*b^3*x^6 - 35*a*b^2*x^4 + 21*a^2*b*x^2 - 15*a^3)/(a^4*x^7)]$

**Sympy [A]** time = 2.25839, size = 112, normalized size = 1.62

$$-\frac{\sqrt{-\frac{b^7}{a^9}} \log\left(-\frac{a^5 \sqrt{-\frac{b^7}{a^9}}}{b^4} + x\right)}{2} + \frac{\sqrt{-\frac{b^7}{a^9}} \log\left(\frac{a^5 \sqrt{-\frac{b^7}{a^9}}}{b^4} + x\right)}{2} + \frac{-15a^3 + 21a^2bx^2 - 35ab^2x^4 + 105b^3x^6}{105a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**8/(b*x**2+a), x)`

[Out]  $-\sqrt{-b^{**7}/a^{**9}}*\log(-a^{**5}*\sqrt{-b^{**7}/a^{**9}}/b^{**4} + x)/2 + \sqrt{-b^{**7}/a^{**9}}*\log(a^{**5}*\sqrt{-b^{**7}/a^{**9}}/b^{**4} + x)/2 + (-15*a^{**3} + 21*a^{**2}*b*x^{**2} - 35*a*b^{**2}*x^{**4} + 105*b^{**3}*x^{**6})/(105*a^{**4}*x^{**7})$

**GIAC/XCAS [A]** time = 0.208479, size = 84, normalized size = 1.22

$$\frac{b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{aba^4}} + \frac{105 b^3 x^6 - 35 ab^2 x^4 + 21 a^2 b x^2 - 15 a^3}{105 a^4 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*x^8),x, algorithm="giac")
```

```
[Out] b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/105*(105*b^3*x^6 -  
35*a*b^2*x^4 + 21*a^2*b*x^2 - 15*a^3)/(a^4*x^7)
```



$$3.144 \quad \int \frac{1}{x^9(a+bx^2)} dx$$

**Optimal.** Leaf size=75

$$-\frac{b^4 \log(a+bx^2)}{2a^5} + \frac{b^4 \log(x)}{a^5} + \frac{b^3}{2a^4x^2} - \frac{b^2}{4a^3x^4} + \frac{b}{6a^2x^6} - \frac{1}{8ax^8}$$

[Out]  $-1/(8*a*x^8) + b/(6*a^2*x^6) - b^2/(4*a^3*x^4) + b^3/(2*a^4*x^2) + (b^4*Log[x])/a^5 - (b^4*Log[a + b*x^2])/(2*a^5)$

**Rubi [A]** time = 0.103129, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{b^4 \log(a+bx^2)}{2a^5} + \frac{b^4 \log(x)}{a^5} + \frac{b^3}{2a^4x^2} - \frac{b^2}{4a^3x^4} + \frac{b}{6a^2x^6} - \frac{1}{8ax^8}$$

Antiderivative was successfully verified.

[In] Int[1/(x^9\*(a + b\*x^2)), x]

[Out]  $-1/(8*a*x^8) + b/(6*a^2*x^6) - b^2/(4*a^3*x^4) + b^3/(2*a^4*x^2) + (b^4*Log[x])/a^5 - (b^4*Log[a + b*x^2])/(2*a^5)$

**Rubi in Sympy [A]** time = 16.4612, size = 71, normalized size = 0.95

$$-\frac{1}{8ax^8} + \frac{b}{6a^2x^6} - \frac{b^2}{4a^3x^4} + \frac{b^3}{2a^4x^2} + \frac{b^4 \log(x^2)}{2a^5} - \frac{b^4 \log(a+bx^2)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*9/(b\*x\*\*2+a), x)

[Out]  $-1/(8*a*x**8) + b/(6*a**2*x**6) - b**2/(4*a**3*x**4) + b**3/(2*a**4*x**2) + b**4*log(x**2)/(2*a**5) - b**4*log(a + b*x**2)/(2*a**5)$

**Mathematica [A]** time = 0.0114208, size = 75, normalized size = 1.

$$-\frac{b^4 \log(a+bx^2)}{2a^5} + \frac{b^4 \log(x)}{a^5} + \frac{b^3}{2a^4x^2} - \frac{b^2}{4a^3x^4} + \frac{b}{6a^2x^6} - \frac{1}{8ax^8}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^9\*(a + b\*x^2)), x]

[Out]  $-1/(8*a*x^8) + b/(6*a^2*x^6) - b^2/(4*a^3*x^4) + b^3/(2*a^4*x^2) + (b^4*Log[x])/a^5 - (b^4*Log[a + b*x^2])/(2*a^5)$

**Maple [A]** time = 0.01, size = 66, normalized size = 0.9

$$-\frac{1}{8ax^8} + \frac{b}{6a^2x^6} - \frac{b^2}{4a^3x^4} + \frac{b^3}{2a^4x^2} + \frac{b^4 \ln(x)}{a^5} - \frac{b^4 \ln(bx^2+a)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^9/(b*x^2+a), x)`

[Out]  $-1/8/a/x^8 + 1/6*b/a^2/x^6 - 1/4*b^2/a^3/x^4 + 1/2*b^3/a^4/x^2 + b^4 \ln(x)/a^5 - 1/2*b^4 \ln(b*x^2+a)/a^5$

**Maxima [A]** time = 1.34439, size = 93, normalized size = 1.24

$$-\frac{b^4 \log(bx^2 + a)}{2a^5} + \frac{b^4 \log(x^2)}{2a^5} + \frac{12b^3x^6 - 6ab^2x^4 + 4a^2bx^2 - 3a^3}{24a^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^9), x, algorithm="maxima")`

[Out]  $-1/2*b^4 \log(b*x^2 + a)/a^5 + 1/2*b^4 \log(x^2)/a^5 + 1/24*(12*b^3*x^6 - 6*a*b^2*x^4 + 4*a^2*b*x^2 - 3*a^3)/(a^4*x^8)$

**Fricas [A]** time = 0.202656, size = 93, normalized size = 1.24

$$-\frac{12b^4x^8 \log(bx^2 + a) - 24b^4x^8 \log(x) - 12ab^3x^6 + 6a^2b^2x^4 - 4a^3bx^2 + 3a^4}{24a^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^9), x, algorithm="fricas")`

[Out]  $-1/24*(12*b^4*x^8 \log(b*x^2 + a) - 24*b^4*x^8 \log(x) - 12*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + 3*a^4)/(a^5*x^8)$

**Sympy [A]** time = 2.91635, size = 68, normalized size = 0.91

$$\frac{-3a^3 + 4a^2bx^2 - 6ab^2x^4 + 12b^3x^6}{24a^4x^8} + \frac{b^4 \log(x)}{a^5} - \frac{b^4 \log\left(\frac{a}{b} + x^2\right)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**9/(b*x**2+a), x)`

[Out]  $(-3*a**3 + 4*a**2*b*x**2 - 6*a*b**2*x**4 + 12*b**3*x**6)/(24*a**4*x**8) + b**4 \log(x)/a**5 - b**4 \log(a/b + x**2)/(2*a**5)$

**GIAC/XCAS [A]** time = 0.212348, size = 109, normalized size = 1.45

$$\frac{b^4 \ln(x^2)}{2a^5} - \frac{b^4 \ln(|bx^2 + a|)}{2a^5} - \frac{25b^4x^8 - 12ab^3x^6 + 6a^2b^2x^4 - 4a^3bx^2 + 3a^4}{24a^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^9), x, algorithm="giac")`

[Out]  $1/2*b^4 \ln(x^2)/a^5 - 1/2*b^4 \ln(\text{abs}(b*x^2 + a))/a^5 - 1/24*(25*b^4*x^8 - 12*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + 3*a^4)/(a^5*x^8)$

$$3.145 \quad \int \frac{x^{13}}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=94

$$-\frac{a^6}{2b^7(a+bx^2)} - \frac{3a^5 \log(a+bx^2)}{b^7} + \frac{5a^4x^2}{2b^6} - \frac{a^3x^4}{b^5} + \frac{a^2x^6}{2b^4} - \frac{ax^8}{4b^3} + \frac{x^{10}}{10b^2}$$

[Out] (5\*a^4\*x^2)/(2\*b^6) - (a^3\*x^4)/b^5 + (a^2\*x^6)/(2\*b^4) - (a\*x^8)/(4\*b^3) + x^10/(10\*b^2) - a^6/(2\*b^7\*(a + b\*x^2)) - (3\*a^5\*Log[a + b\*x^2])/b^7

**Rubi [A]** time = 0.181404, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^6}{2b^7(a+bx^2)} - \frac{3a^5 \log(a+bx^2)}{b^7} + \frac{5a^4x^2}{2b^6} - \frac{a^3x^4}{b^5} + \frac{a^2x^6}{2b^4} - \frac{ax^8}{4b^3} + \frac{x^{10}}{10b^2}$$

Antiderivative was successfully verified.

[In] Int[x^13/(a + b\*x^2)^2, x]

[Out] (5\*a^4\*x^2)/(2\*b^6) - (a^3\*x^4)/b^5 + (a^2\*x^6)/(2\*b^4) - (a\*x^8)/(4\*b^3) + x^10/(10\*b^2) - a^6/(2\*b^7\*(a + b\*x^2)) - (3\*a^5\*Log[a + b\*x^2])/b^7

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^6}{2b^7(a+bx^2)} - \frac{3a^5 \log(a+bx^2)}{b^7} + \frac{5a^4x^2}{2b^6} - \frac{2a^3 \int^{x^2} x dx}{b^5} + \frac{a^2x^6}{2b^4} - \frac{ax^8}{4b^3} + \frac{x^{10}}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*13/(b\*x\*\*2+a)\*\*2, x)

[Out] -a\*\*6/(2\*b\*\*7\*(a + b\*x\*\*2)) - 3\*a\*\*5\*log(a + b\*x\*\*2)/b\*\*7 + 5\*a\*\*4\*x\*\*2/(2\*b\*\*6) - 2\*a\*\*3\*Integral(x, (x, x\*\*2))/b\*\*5 + a\*\*2\*x\*\*6/(2\*b\*\*4) - a\*x\*\*8/(4\*b\*\*3) + x\*\*10/(10\*b\*\*2)

**Mathematica [A]** time = 0.0565087, size = 83, normalized size = 0.88

$$\frac{-\frac{10a^6}{a+bx^2} - 60a^5 \log(a+bx^2) + 50a^4bx^2 - 20a^3b^2x^4 + 10a^2b^3x^6 - 5ab^4x^8 + 2b^5x^{10}}{20b^7}$$

Antiderivative was successfully verified.

[In] Integrate[x^13/(a + b\*x^2)^2, x]

[Out] (50\*a^4\*b\*x^2 - 20\*a^3\*b^2\*x^4 + 10\*a^2\*b^3\*x^6 - 5\*a\*b^4\*x^8 + 2\*b^5\*x^10 - (10\*a^6)/(a + b\*x^2) - 60\*a^5\*Log[a + b\*x^2])/(20\*b^7)

**Maple [A]** time = 0.014, size = 85, normalized size = 0.9

$$\frac{5a^4x^2}{2b^6} - \frac{a^3x^4}{b^5} + \frac{a^2x^6}{2b^4} - \frac{ax^8}{4b^3} + \frac{x^{10}}{10b^2} - \frac{a^6}{2b^7(bx^2+a)} - 3 \frac{a^5 \ln(bx^2+a)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^13/(b*x^2+a)^2,x)`

[Out]  $5/2*a^4*x^2/b^6 - a^3*x^4/b^5 + 1/2*a^2*x^6/b^4 - 1/4*a*x^8/b^3 + 1/10*x^10/b^2 - 1/2*a^6/b^7/(b*x^2+a) - 3*a^5*\ln(b*x^2+a)/b^7$

**Maxima [A]** time = 1.34635, size = 119, normalized size = 1.27

$$-\frac{a^6}{2(b^8x^2 + ab^7)} - \frac{3a^5 \log(bx^2 + a)}{b^7} + \frac{2b^4x^{10} - 5ab^3x^8 + 10a^2b^2x^6 - 20a^3bx^4 + 50a^4x^2}{20b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^13/(b*x^2 + a)^2,x, algorithm="maxima")`

[Out]  $-1/2*a^6/(b^8*x^2 + a*b^7) - 3*a^5*\log(b*x^2 + a)/b^7 + 1/20*(2*b^4*x^{10} - 5*a*b^3*x^8 + 10*a^2*b^2*x^6 - 20*a^3*b*x^4 + 50*a^4*x^2)/b^6$

**Fricas [A]** time = 0.197825, size = 140, normalized size = 1.49

$$\frac{2b^6x^{12} - 3ab^5x^{10} + 5a^2b^4x^8 - 10a^3b^3x^6 + 30a^4b^2x^4 + 50a^5bx^2 - 10a^6 - 60(a^5bx^2 + a^6)\log(bx^2 + a)}{20(b^8x^2 + ab^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^13/(b*x^2 + a)^2,x, algorithm="fricas")`

[Out]  $1/20*(2*b^6*x^{12} - 3*a*b^5*x^{10} + 5*a^2*b^4*x^8 - 10*a^3*b^3*x^6 + 30*a^4*b^2*x^4 + 50*a^5*b*x^2 - 10*a^6 - 60*(a^5*b*x^2 + a^6)*\log(b*x^2 + a))/(b^8*x^2 + a*b^7)$

**Sympy [A]** time = 1.74405, size = 88, normalized size = 0.94

$$-\frac{a^6}{2ab^7 + 2b^8x^2} - \frac{3a^5 \log(a + bx^2)}{b^7} + \frac{5a^4x^2}{2b^6} - \frac{a^3x^4}{b^5} + \frac{a^2x^6}{2b^4} - \frac{ax^8}{4b^3} + \frac{x^{10}}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**13/(b*x**2+a)**2,x)`

[Out]  $-a**6/(2*a*b**7 + 2*b**8*x**2) - 3*a**5*\log(a + b*x**2)/b**7 + 5*a**4*x**2/(2*b**6) - a**3*x**4/b**5 + a**2*x**6/(2*b**4) - a*x**8/(4*b**3) + x**10/(10*b**2)$

**GIAC/XCAS [A]** time = 0.211721, size = 139, normalized size = 1.48

$$-\frac{3a^5 \ln(|bx^2 + a|)}{b^7} + \frac{6a^5bx^2 + 5a^6}{2(bx^2 + a)b^7} + \frac{2b^8x^{10} - 5ab^7x^8 + 10a^2b^6x^6 - 20a^3b^5x^4 + 50a^4b^4x^2}{20b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^13/(b*x^2 + a)^2,x, algorithm="giac")`

```
[Out] -3*a^5*ln(abs(b*x^2 + a))/b^7 + 1/2*(6*a^5*b*x^2 + 5*a^6)/((b*x^2 + a)*b^7) + 1/20*(2*b^8*x^10 - 5*a*b^7*x^8 + 10*a^2*b^6*x^6 - 20*a^3*b^5*x^4 + 50*a^4*b^4*x^2)/b^10
```

$$3.146 \quad \int \frac{x^{12}}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=105

$$-\frac{11a^{9/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{13/2}} + \frac{11a^4x}{2b^6} - \frac{11a^3x^3}{6b^5} + \frac{11a^2x^5}{10b^4} - \frac{11ax^7}{14b^3} - \frac{x^{11}}{2b(a+bx^2)} + \frac{11x^9}{18b^2}$$

[Out]  $(11*a^4*x)/(2*b^6) - (11*a^3*x^3)/(6*b^5) + (11*a^2*x^5)/(10*b^4) - (11*a*x^7)/(14*b^3) + (11*x^9)/(18*b^2) - x^{11}/(2*b*(a + b*x^2)) - (11*a^{(9/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^{(13/2)})$

**Rubi [A]** time = 0.120013, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{11a^{9/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{13/2}} + \frac{11a^4x}{2b^6} - \frac{11a^3x^3}{6b^5} + \frac{11a^2x^5}{10b^4} - \frac{11ax^7}{14b^3} - \frac{x^{11}}{2b(a+bx^2)} + \frac{11x^9}{18b^2}$$

Antiderivative was successfully verified.

[In] Int[x^12/(a + b\*x^2)^2, x]

[Out]  $(11*a^4*x)/(2*b^6) - (11*a^3*x^3)/(6*b^5) + (11*a^2*x^5)/(10*b^4) - (11*a*x^7)/(14*b^3) + (11*x^9)/(18*b^2) - x^{11}/(2*b*(a + b*x^2)) - (11*a^{(9/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^{(13/2)})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{11a^{9/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{13/2}} - \frac{11a^3x^3}{6b^5} + \frac{11a^2x^5}{10b^4} - \frac{11ax^7}{14b^3} - \frac{x^{11}}{2b(a+bx^2)} + \frac{11x^9}{18b^2} + \frac{11 \int a^4 dx}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*12/(b\*x\*\*2+a)\*\*2, x)

[Out]  $-11*a^{(9/2)}*atan(sqrt(b)*x/sqrt(a))/(2*b^{(13/2)}) - 11*a^3*x^3/(6*b^5) + 11*a^2*x^5/(10*b^4) - 11*a*x^7/(14*b^3) - x^{11}/(2*b*(a + b*x^2)) + 11*x^9/(18*b^2) + 11*Integral(a^4, x)/(2*b^6)$

**Mathematica [A]** time = 0.131652, size = 93, normalized size = 0.89

$$\frac{x \left( \frac{315a^5}{a+bx^2} + 3150a^4 - 840a^3bx^2 + 378a^2b^2x^4 - 180ab^3x^6 + 70b^4x^8 \right)}{630b^6} - \frac{11a^{9/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{13/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^12/(a + b\*x^2)^2, x]

[Out]  $(x*(3150*a^4 - 840*a^3*b*x^2 + 378*a^2*b^2*x^4 - 180*a*b^3*x^6 + 70*b^4*x^8 + (315*a^5)/(a + b*x^2)))/(630*b^6) - (11*a^{(9/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^{(13/2)})$

**Maple [A]** time = 0.013, size = 90, normalized size = 0.9

$$\frac{x^9}{9b^2} - \frac{2ax^7}{7b^3} + \frac{3a^2x^5}{5b^4} - \frac{4a^3x^3}{3b^5} + 5\frac{a^4x}{b^6} + \frac{a^5x}{2b^6(bx^2+a)} - \frac{11a^5}{2b^6} \arctan\left(\frac{bx}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^12/(b\*x^2+a)^2,x)

[Out] 1/9\*x^9/b^2-2/7\*a\*x^7/b^3+3/5\*a^2\*x^5/b^4-4/3\*a^3\*x^3/b^5+5\*a^4\*x/b^6+1/2/b^6\*a^5\*x/(b\*x^2+a)-11/2/b^6\*a^5/(a\*b)^(1/2)\*arctan(x\*b/(a\*b)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12/(b\*x^2 + a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.208047, size = 1, normalized size = 0.01

$$\frac{140b^5x^{11} - 220ab^4x^9 + 396a^2b^3x^7 - 924a^3b^2x^5 + 4620a^4bx^3 + 6930a^5x + 3465(a^4bx^2 + a^5)\sqrt{-\frac{a}{b}} \log\left(\frac{bx^2 - 2bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right)}{1260(b^7x^2 + ab^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12/(b\*x^2 + a)^2,x, algorithm="fricas")

[Out] [1/1260\*(140\*b^5\*x^11 - 220\*a\*b^4\*x^9 + 396\*a^2\*b^3\*x^7 - 924\*a^3\*b^2\*x^5 + 4620\*a^4\*b\*x^3 + 6930\*a^5\*x + 3465\*(a^4\*b\*x^2 + a^5)\*sqrt(-a/b)\*log((b\*x^2 - 2\*b\*x\*sqrt(-a/b) - a)/(b\*x^2 + a)))/(b^7\*x^2 + a\*b^6), 1/630\*(70\*b^5\*x^11 - 110\*a\*b^4\*x^9 + 198\*a^2\*b^3\*x^7 - 462\*a^3\*b^2\*x^5 + 2310\*a^4\*b\*x^3 + 3465\*a^5\*x - 3465\*(a^4\*b\*x^2 + a^5)\*sqrt(a/b)\*arctan(x/sqrt(a/b)))/(b^7\*x^2 + a\*b^6)]

**Sympy [A]** time = 1.83144, size = 151, normalized size = 1.44

$$\frac{a^5x}{2ab^6 + 2b^7x^2} + \frac{5a^4x}{b^6} - \frac{4a^3x^3}{3b^5} + \frac{3a^2x^5}{5b^4} - \frac{2ax^7}{7b^3} + \frac{11\sqrt{-\frac{a^9}{b^{13}}}\log\left(x - \frac{b^6\sqrt{-\frac{a^9}{b^{13}}}}{a^4}\right)}{4} - \frac{11\sqrt{-\frac{a^9}{b^{13}}}\log\left(x + \frac{b^6\sqrt{-\frac{a^9}{b^{13}}}}{a^4}\right)}{4} + \frac{x^9}{9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*12/(b\*x\*\*2+a)\*\*2,x)

[Out] a\*\*5\*x/(2\*a\*b\*\*6 + 2\*b\*\*7\*x\*\*2) + 5\*a\*\*4\*x/b\*\*6 - 4\*a\*\*3\*x\*\*3/(3\*b\*\*5) + 3\*a\*\*2\*x\*\*5/(5\*b\*\*4) - 2\*a\*x\*\*7/(7\*b\*\*3) + 11\*sqrt(-a\*\*9/

$$b^{13} \log(x - b^6 \sqrt{-a^9/b^{13}}/a^4)/4 - 11 \sqrt{-a^9/b^{13}} \log(x + b^6 \sqrt{-a^9/b^{13}}/a^4)/4 + x^9/(9b^2)$$

**GIAC/XCAS [A]** time = 0.207905, size = 128, normalized size = 1.22

$$-\frac{11 a^5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2 \sqrt{ab} b^6} + \frac{a^5 x}{2 (bx^2 + a) b^6} + \frac{35 b^{16} x^9 - 90 a b^{15} x^7 + 189 a^2 b^{14} x^5 - 420 a^3 b^{13} x^3 + 1575 a^4 b^{12} x}{315 b^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12/(b\*x^2 + a)^2,x, algorithm="giac")

[Out] -11/2\*a^5\*arctan(b\*x/sqrt(a\*b))/(sqrt(a\*b)\*b^6) + 1/2\*a^5\*x/((b\*x^2 + a)\*b^6) + 1/315\*(35\*b^16\*x^9 - 90\*a\*b^15\*x^7 + 189\*a^2\*b^14\*x^5 - 420\*a^3\*b^13\*x^3 + 1575\*a^4\*b^12\*x)/b^18



$$3.147 \quad \int \frac{x^{11}}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=83

$$\frac{a^5}{2b^6(a+bx^2)} + \frac{5a^4 \log(a+bx^2)}{2b^6} - \frac{2a^3x^2}{b^5} + \frac{3a^2x^4}{4b^4} - \frac{ax^6}{3b^3} + \frac{x^8}{8b^2}$$

[Out]  $(-2*a^3*x^2)/b^5 + (3*a^2*x^4)/(4*b^4) - (a*x^6)/(3*b^3) + x^8/(8*b^2) + a^5/(2*b^6*(a + b*x^2)) + (5*a^4*Log[a + b*x^2])/(2*b^6)$

**Rubi [A]** time = 0.152003, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^5}{2b^6(a+bx^2)} + \frac{5a^4 \log(a+bx^2)}{2b^6} - \frac{2a^3x^2}{b^5} + \frac{3a^2x^4}{4b^4} - \frac{ax^6}{3b^3} + \frac{x^8}{8b^2}$$

Antiderivative was successfully verified.

[In] Int[x^11/(a + b\*x^2)^2, x]

[Out]  $(-2*a^3*x^2)/b^5 + (3*a^2*x^4)/(4*b^4) - (a*x^6)/(3*b^3) + x^8/(8*b^2) + a^5/(2*b^6*(a + b*x^2)) + (5*a^4*Log[a + b*x^2])/(2*b^6)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^5}{2b^6(a+bx^2)} + \frac{5a^4 \log(a+bx^2)}{2b^6} - \frac{2a^3x^2}{b^5} + \frac{3a^2 \int^{x^2} x dx}{2b^4} - \frac{ax^6}{3b^3} + \frac{x^8}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*11/(b\*x\*\*2+a)\*\*2, x)

[Out]  $a**5/(2*b**6*(a + b*x**2)) + 5*a**4*log(a + b*x**2)/(2*b**6) - 2*a**3*x**2/b**5 + 3*a**2*Integral(x, (x, x**2))/(2*b**4) - a*x**6/(3*b**3) + x**8/(8*b**2)$

**Mathematica [A]** time = 0.0376604, size = 72, normalized size = 0.87

$$\frac{\frac{12a^5}{a+bx^2} + 60a^4 \log(a+bx^2) - 48a^3bx^2 + 18a^2b^2x^4 - 8ab^3x^6 + 3b^4x^8}{24b^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/(a + b\*x^2)^2, x]

[Out]  $(-48*a^3*b*x^2 + 18*a^2*b^2*x^4 - 8*a*b^3*x^6 + 3*b^4*x^8 + (12*a^5)/(a + b*x^2) + 60*a^4*Log[a + b*x^2])/(24*b^6)$

**Maple [A]** time = 0.014, size = 74, normalized size = 0.9

$$-2 \frac{a^3x^2}{b^5} + \frac{3a^2x^4}{4b^4} - \frac{ax^6}{3b^3} + \frac{x^8}{8b^2} + \frac{a^5}{2b^6(bx^2+a)} + \frac{5a^4 \ln(bx^2+a)}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11/(b*x^2+a)^2,x)`

[Out]  $-2*a^3*x^2/b^5+3/4*a^2*x^4/b^4-1/3*a*x^6/b^3+1/8*x^8/b^2+1/2*a^5/b^6/(b*x^2+a)+5/2*a^4*\ln(b*x^2+a)/b^6$

**Maxima [A]** time = 1.35074, size = 104, normalized size = 1.25

$$\frac{a^5}{2(b^7x^2 + ab^6)} + \frac{5a^4 \log(bx^2 + a)}{2b^6} + \frac{3b^3x^8 - 8ab^2x^6 + 18a^2bx^4 - 48a^3x^2}{24b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(b*x^2 + a)^2,x, algorithm="maxima")`

[Out]  $1/2*a^5/(b^7*x^2 + a*b^6) + 5/2*a^4*\log(b*x^2 + a)/b^6 + 1/24*(3*b^3*x^8 - 8*a*b^2*x^6 + 18*a^2*b*x^4 - 48*a^3*x^2)/b^5$

**Fricas [A]** time = 0.200281, size = 126, normalized size = 1.52

$$\frac{3b^5x^{10} - 5ab^4x^8 + 10a^2b^3x^6 - 30a^3b^2x^4 - 48a^4bx^2 + 12a^5 + 60(a^4bx^2 + a^5)\log(bx^2 + a)}{24(b^7x^2 + ab^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(b*x^2 + a)^2,x, algorithm="fricas")`

[Out]  $1/24*(3*b^5*x^{10} - 5*a*b^4*x^8 + 10*a^2*b^3*x^6 - 30*a^3*b^2*x^4 - 48*a^4*b*x^2 + 12*a^5 + 60*(a^4*b*x^2 + a^5)*\log(b*x^2 + a))/(b^7*x^2 + a*b^6)$

**Sympy [A]** time = 1.70396, size = 80, normalized size = 0.96

$$\frac{a^5}{2ab^6 + 2b^7x^2} + \frac{5a^4 \log(a + bx^2)}{2b^6} - \frac{2a^3x^2}{b^5} + \frac{3a^2x^4}{4b^4} - \frac{ax^6}{3b^3} + \frac{x^8}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**11/(b*x**2+a)**2,x)`

[Out]  $a**5/(2*a*b**6 + 2*b**7*x**2) + 5*a**4*\log(a + b*x**2)/(2*b**6) - 2*a**3*x**2/b**5 + 3*a**2*x**4/(4*b**4) - a*x**6/(3*b**3) + x**8/(8*b**2)$

**GIAC/XCAS [A]** time = 0.211143, size = 124, normalized size = 1.49

$$\frac{5a^4 \ln(|bx^2 + a|)}{2b^6} - \frac{5a^4bx^2 + 4a^5}{2(bx^2 + a)b^6} + \frac{3b^6x^8 - 8ab^5x^6 + 18a^2b^4x^4 - 48a^3b^3x^2}{24b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(b*x^2 + a)^2,x, algorithm="giac")`

```
[Out] 5/2*a^4*ln(abs(b*x^2 + a))/b^6 - 1/2*(5*a^4*b*x^2 + 4*a^5)/((b*x^
2 + a)*b^6) + 1/24*(3*b^6*x^8 - 8*a*b^5*x^6 + 18*a^2*b^4*x^4 - 48
*a^3*b^3*x^2)/b^8
```

$$3.148 \quad \int \frac{x^{10}}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=92

$$\frac{9a^{7/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{11/2}} - \frac{9a^3x}{2b^5} + \frac{3a^2x^3}{2b^4} - \frac{9ax^5}{10b^3} - \frac{x^9}{2b(a+bx^2)} + \frac{9x^7}{14b^2}$$

[Out]  $(-9*a^3*x)/(2*b^5) + (3*a^2*x^3)/(2*b^4) - (9*a*x^5)/(10*b^3) + (9*x^7)/(14*b^2) - x^9/(2*b*(a + b*x^2)) + (9*a^{(7/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^{(11/2)})$

**Rubi [A]** time = 0.101984, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{9a^{7/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{11/2}} - \frac{9a^3x}{2b^5} + \frac{3a^2x^3}{2b^4} - \frac{9ax^5}{10b^3} - \frac{x^9}{2b(a+bx^2)} + \frac{9x^7}{14b^2}$$

Antiderivative was successfully verified.

[In] Int[x^10/(a + b\*x^2)^2, x]

[Out]  $(-9*a^3*x)/(2*b^5) + (3*a^2*x^3)/(2*b^4) - (9*a*x^5)/(10*b^3) + (9*x^7)/(14*b^2) - x^9/(2*b*(a + b*x^2)) + (9*a^{(7/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^{(11/2)})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{9a^{7/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{11/2}} + \frac{3a^2x^3}{2b^4} - \frac{9ax^5}{10b^3} - \frac{x^9}{2b(a+bx^2)} + \frac{9x^7}{14b^2} - \frac{9 \int a^3 dx}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*10/(b\*x\*\*2+a)\*\*2, x)

[Out]  $9*a^{(7/2)}*atan(sqrt(b)*x/sqrt(a))/(2*b^{(11/2)}) + 3*a^2*x^3/(2*b^4) - 9*a*x^5/(10*b^3) - x^9/(2*b*(a + b*x^2)) + 9*x^7/(14*b^2) - 9*Integral(a^3, x)/(2*b^5)$

**Mathematica [A]** time = 0.0970512, size = 82, normalized size = 0.89

$$\frac{9a^{7/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{11/2}} + \frac{x\left(-\frac{35a^4}{a+bx^2} - 280a^3 + 70a^2bx^2 - 28ab^2x^4 + 10b^3x^6\right)}{70b^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^10/(a + b\*x^2)^2, x]

[Out]  $(x*(-280*a^3 + 70*a^2*b*x^2 - 28*a*b^2*x^4 + 10*b^3*x^6 - (35*a^4)/(a + b*x^2)))/(70*b^5) + (9*a^{(7/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^{(11/2)})$

**Maple [A]** time = 0.023, size = 78, normalized size = 0.9

$$\frac{x^7}{7b^2} - \frac{2ax^5}{5b^3} + \frac{a^2x^3}{b^4} - 4\frac{a^3x}{b^5} - \frac{a^4x}{2b^5(bx^2+a)} + \frac{9a^4}{2b^5} \arctan\left(bx\frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^10/(b*x^2+a)^2,x)`

[Out]  $\frac{1}{7}x^7/b^2 - \frac{2}{5}a*x^5/b^3 + \frac{a^2*x^3}{b^4} - 4*a^3*x/b^5 - \frac{1}{2}/b^5*a^4*x/(b*x^2+a) + \frac{9}{2}/b^5*a^4/(a*b)^{(1/2)}*\arctan(x*b/(a*b)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^10/(b*x^2 + a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.208585, size = 1, normalized size = 0.01

$$\left[ \frac{20b^4x^9 - 36ab^3x^7 + 84a^2b^2x^5 - 420a^3bx^3 - 630a^4x + 315(a^3bx^2 + a^4)\sqrt{-\frac{a}{b}} \log\left(\frac{bx^2+2bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{140(b^6x^2 + ab^5)}, \frac{10b^4x^9 - 18ab^3x^7 + \dots}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^10/(b*x^2 + a)^2,x, algorithm="fricas")`

[Out]  $[1/140*(20*b^4*x^9 - 36*a*b^3*x^7 + 84*a^2*b^2*x^5 - 420*a^3*b*x^3 - 630*a^4*x + 315*(a^3*b*x^2 + a^4)*\sqrt{-a/b}*\log((b*x^2 + 2*b*x*\sqrt{-a/b} - a)/(b*x^2 + a)))/(b^6*x^2 + a*b^5), 1/70*(10*b^4*x^9 - 18*a*b^3*x^7 + 42*a^2*b^2*x^5 - 210*a^3*b*x^3 - 315*a^4*x + 315*(a^3*b*x^2 + a^4)*\sqrt{a/b}*\arctan(x/\sqrt{a/b}))/ (b^6*x^2 + a*b^5)]$

**Sympy [A]** time = 1.76512, size = 134, normalized size = 1.46

$$-\frac{a^4x}{2ab^5 + 2b^6x^2} - \frac{4a^3x}{b^5} + \frac{a^2x^3}{b^4} - \frac{2ax^5}{5b^3} - \frac{9\sqrt{-\frac{a^7}{b^{11}}}\log\left(x - \frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4} + \frac{9\sqrt{-\frac{a^7}{b^{11}}}\log\left(x + \frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4} + \frac{x^7}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**10/(b*x**2+a)**2,x)`

[Out]  $-a**4*x/(2*a*b**5 + 2*b**6*x**2) - 4*a**3*x/b**5 + a**2*x**3/b**4 - 2*a*x**5/(5*b**3) - 9*\sqrt{-a**7/b**11}*\log(x - b**5*\sqrt{-a**7/b**11})/a**3/4 + 9*\sqrt{-a**7/b**11}*\log(x + b**5*\sqrt{-a**7/b**11})/a**3/4 + x**7/(7*b**2)$

---

**GIAC/XCAS [A]** time = 0.208516, size = 113, normalized size = 1.23

$$\frac{9 a^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2 \sqrt{ab} b^5} - \frac{a^4 x}{2 (bx^2 + a) b^5} + \frac{5 b^{12} x^7 - 14 a b^{11} x^5 + 35 a^2 b^{10} x^3 - 140 a^3 b^9 x}{35 b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10/(b\*x^2 + a)^2,x, algorithm="giac")

[Out] 9/2\*a^4\*arctan(b\*x/sqrt(a\*b))/(sqrt(a\*b)\*b^5) - 1/2\*a^4\*x/((b\*x^2 + a)\*b^5) + 1/35\*(5\*b^12\*x^7 - 14\*a\*b^11\*x^5 + 35\*a^2\*b^10\*x^3 - 140\*a^3\*b^9\*x)/b^14

$$3.149 \quad \int \frac{x^9}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=70

$$-\frac{a^4}{2b^5(a+bx^2)} - \frac{2a^3 \log(a+bx^2)}{b^5} + \frac{3a^2x^2}{2b^4} - \frac{ax^4}{2b^3} + \frac{x^6}{6b^2}$$

[Out]  $(3*a^2*x^2)/(2*b^4) - (a*x^4)/(2*b^3) + x^6/(6*b^2) - a^4/(2*b^5*(a + b*x^2)) - (2*a^3*Log[a + b*x^2])/b^5$

**Rubi [A]** time = 0.128919, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^4}{2b^5(a+bx^2)} - \frac{2a^3 \log(a+bx^2)}{b^5} + \frac{3a^2x^2}{2b^4} - \frac{ax^4}{2b^3} + \frac{x^6}{6b^2}$$

Antiderivative was successfully verified.

[In] Int[x^9/(a + b\*x^2)^2, x]

[Out]  $(3*a^2*x^2)/(2*b^4) - (a*x^4)/(2*b^3) + x^6/(6*b^2) - a^4/(2*b^5*(a + b*x^2)) - (2*a^3*Log[a + b*x^2])/b^5$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^4}{2b^5(a+bx^2)} - \frac{2a^3 \log(a+bx^2)}{b^5} + \frac{3a^2x^2}{2b^4} - \frac{a \int^{x^2} x dx}{b^3} + \frac{x^6}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*9/(b\*x\*\*2+a)\*\*2, x)

[Out]  $-a**4/(2*b**5*(a + b*x**2)) - 2*a**3*log(a + b*x**2)/b**5 + 3*a**2*x**2/(2*b**4) - a*Integral(x, (x, x**2))/b**3 + x**6/(6*b**2)$

**Mathematica [A]** time = 0.0388479, size = 60, normalized size = 0.86

$$\frac{-\frac{3a^4}{a+bx^2} - 12a^3 \log(a+bx^2) + 9a^2bx^2 - 3ab^2x^4 + b^3x^6}{6b^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^9/(a + b\*x^2)^2, x]

[Out]  $(9*a^2*b*x^2 - 3*a*b^2*x^4 + b^3*x^6 - (3*a^4)/(a + b*x^2) - 12*a^3*Log[a + b*x^2])/(6*b^5)$

**Maple [A]** time = 0.028, size = 63, normalized size = 0.9

$$\frac{3a^2x^2}{2b^4} - \frac{ax^4}{2b^3} + \frac{x^6}{6b^2} - \frac{a^4}{2b^5(bx^2+a)} - 2\frac{a^3 \ln(bx^2+a)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9/(b*x^2+a)^2,x)`

[Out]  $\frac{3}{2}a^2x^2/b^4 - \frac{1}{2}ax^4/b^3 + \frac{1}{6}x^6/b^2 - \frac{1}{2}a^4/b^5/(b*x^2+a) - 2a^3 \ln(b*x^2+a)/b^5$

**Maxima [A]** time = 1.33699, size = 88, normalized size = 1.26

$$-\frac{a^4}{2(b^6x^2 + ab^5)} - \frac{2a^3 \log(bx^2 + a)}{b^5} + \frac{b^2x^6 - 3abx^4 + 9a^2x^2}{6b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9/(b*x^2 + a)^2,x, algorithm="maxima")`

[Out]  $-\frac{1}{2}a^4/(b^6x^2 + a*b^5) - 2a^3 \log(b*x^2 + a)/b^5 + \frac{1}{6}(b^2x^6 - 3a*b*x^4 + 9a^2x^2)/b^4$

**Fricas [A]** time = 0.200549, size = 109, normalized size = 1.56

$$\frac{b^4x^8 - 2ab^3x^6 + 6a^2b^2x^4 + 9a^3bx^2 - 3a^4 - 12(a^3bx^2 + a^4) \log(bx^2 + a)}{6(b^6x^2 + ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9/(b*x^2 + a)^2,x, algorithm="fricas")`

[Out]  $\frac{1}{6}(b^4x^8 - 2a*b^3x^6 + 6a^2b^2x^4 + 9a^3b*x^2 - 3a^4 - 12(a^3b*x^2 + a^4) \log(b*x^2 + a))/(b^6x^2 + a*b^5)$

**Sympy [A]** time = 1.62338, size = 66, normalized size = 0.94

$$-\frac{a^4}{2ab^5 + 2b^6x^2} - \frac{2a^3 \log(a + bx^2)}{b^5} + \frac{3a^2x^2}{2b^4} - \frac{ax^4}{2b^3} + \frac{x^6}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9/(b*x**2+a)**2,x)`

[Out]  $-a**4/(2*a*b**5 + 2*b**6*x**2) - 2*a**3 \log(a + b*x**2)/b**5 + 3*a**2*x**2/(2*b**4) - a*x**4/(2*b**3) + x**6/(6*b**2)$

**GIAC/XCAS [A]** time = 0.212047, size = 108, normalized size = 1.54

$$-\frac{2a^3 \ln(|bx^2 + a|)}{b^5} + \frac{b^4x^6 - 3ab^3x^4 + 9a^2b^2x^2}{6b^6} + \frac{4a^3bx^2 + 3a^4}{2(bx^2 + a)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9/(b*x^2 + a)^2,x, algorithm="giac")`

[Out]  $-2a^3 \ln(\text{abs}(b*x^2 + a))/b^5 + \frac{1}{6}(b^4x^6 - 3a*b^3x^4 + 9a^2b^2x^2)/b^6 + \frac{1}{2}(4a^3b*x^2 + 3a^4)/((b*x^2 + a)*b^5)$



$$3.150 \quad \int \frac{x^8}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=79

$$-\frac{7a^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{9/2}} + \frac{7a^2x}{2b^4} - \frac{7ax^3}{6b^3} - \frac{x^7}{2b(a+bx^2)} + \frac{7x^5}{10b^2}$$

[Out] (7\*a^2\*x)/(2\*b^4) - (7\*a\*x^3)/(6\*b^3) + (7\*x^5)/(10\*b^2) - x^7/(2\*b\*(a + b\*x^2)) - (7\*a^(5/2)\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(2\*b^(9/2))

**Rubi [A]** time = 0.0911804, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{7a^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{9/2}} + \frac{7a^2x}{2b^4} - \frac{7ax^3}{6b^3} - \frac{x^7}{2b(a+bx^2)} + \frac{7x^5}{10b^2}$$

Antiderivative was successfully verified.

[In] Int[x^8/(a + b\*x^2)^2, x]

[Out] (7\*a^2\*x)/(2\*b^4) - (7\*a\*x^3)/(6\*b^3) + (7\*x^5)/(10\*b^2) - x^7/(2\*b\*(a + b\*x^2)) - (7\*a^(5/2)\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(2\*b^(9/2))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{7a^{5/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{9/2}} - \frac{7ax^3}{6b^3} - \frac{x^7}{2b(a+bx^2)} + \frac{7x^5}{10b^2} + \frac{7 \int a^2 dx}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*8/(b\*x\*\*2+a)\*\*2, x)

[Out] -7\*a\*\*(5/2)\*atan(sqrt(b)\*x/sqrt(a))/(2\*b\*\*(9/2)) - 7\*a\*x\*\*3/(6\*b\*\*3) - x\*\*7/(2\*b\*(a + b\*x\*\*2)) + 7\*x\*\*5/(10\*b\*\*2) + 7\*Integral(a\*\*2, x)/(2\*b\*\*4)

**Mathematica [A]** time = 0.0877336, size = 71, normalized size = 0.9

$$\frac{x\left(\frac{15a^3}{a+bx^2} + 90a^2 - 20abx^2 + 6b^2x^4\right)}{30b^4} - \frac{7a^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(a + b\*x^2)^2, x]

[Out] (x\*(90\*a^2 - 20\*a\*b\*x^2 + 6\*b^2\*x^4 + (15\*a^3)/(a + b\*x^2)))/(30\*b^4) - (7\*a^(5/2)\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(2\*b^(9/2))

**Maple [A]** time = 0.012, size = 68, normalized size = 0.9

$$\frac{x^5}{5b^2} - \frac{2ax^3}{3b^3} + 3\frac{a^2x}{b^4} + \frac{a^3x}{2b^4(bx^2+a)} - \frac{7a^3}{2b^4} \arctan\left(bx\frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8/(b*x^2+a)^2,x)`

[Out]  $\frac{1}{5}x^5/b^2 - \frac{2}{3}a^2x^3/b^3 + 3a^2x/b^4 + \frac{1}{2}a^3x/(b^4(x^2+a)) - \frac{7}{2}a^3/(b^4(a^2b)^{1/2}) \arctan(xb/(a^2b)^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8/(b*x^2 + a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.207286, size = 1, normalized size = 0.01

$$\left[ \frac{12b^3x^7 - 28ab^2x^5 + 140a^2bx^3 + 210a^3x + 105(a^2bx^2 + a^3)\sqrt{-\frac{a}{b}} \log\left(\frac{bx^2 - 2bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right)}{60(b^5x^2 + ab^4)}, \frac{6b^3x^7 - 14ab^2x^5 + 70a^2bx^3 + 105a^3x}{30} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8/(b*x^2 + a)^2,x, algorithm="fricas")`

[Out]  $\left[ \frac{1}{60} (12b^3x^7 - 28a^2b^2x^5 + 140a^2bx^3 + 210a^3x + 105(a^2bx^2 + a^3)\sqrt{-a/b} \log((bx^2 - 2bx\sqrt{-a/b} - a)/(bx^2 + a)))/(b^5x^2 + ab^4), \frac{1}{30} (6b^3x^7 - 14a^2b^2x^5 + 70a^2bx^3 + 105a^3x - 105(a^2bx^2 + a^3)\sqrt{a/b} \arctan(x/\sqrt{a/b}))/ (b^5x^2 + ab^4) \right]$

**Sympy [A]** time = 1.71951, size = 124, normalized size = 1.57

$$\frac{a^3x}{2ab^4 + 2b^5x^2} + \frac{3a^2x}{b^4} - \frac{2ax^3}{3b^3} + \frac{7\sqrt{-\frac{a^5}{b^9}} \log\left(x - \frac{b^4\sqrt{-\frac{a^5}{b^9}}}{a^2}\right)}{4} - \frac{7\sqrt{-\frac{a^5}{b^9}} \log\left(x + \frac{b^4\sqrt{-\frac{a^5}{b^9}}}{a^2}\right)}{4} + \frac{x^5}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8/(b*x**2+a)**2,x)`

[Out]  $a^3x/(2a^2b^4 + 2b^5x^2) + 3a^2x/b^4 - 2a^3x^3/(3b^3) + 7\sqrt{-a^5/b^9} \log(x - b^4\sqrt{-a^5/b^9}/a^2)/4 - 7\sqrt{-a^5/b^9} \log(x + b^4\sqrt{-a^5/b^9}/a^2)/4 + x^5/(5b^2)$

**GIAC/XCAS [A]** time = 0.214181, size = 99, normalized size = 1.25

$$-\frac{7a^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{abb^4}} + \frac{a^3x}{2(bx^2 + a)b^4} + \frac{3b^8x^5 - 10ab^7x^3 + 45a^2b^6x}{15b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] -7/2*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/2*a^3*x/((b*x^2 + a)*b^4) + 1/15*(3*b^8*x^5 - 10*a*b^7*x^3 + 45*a^2*b^6*x)/b^10
```

$$3.151 \quad \int \frac{x^7}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=57

$$\frac{a^3}{2b^4(a+bx^2)} + \frac{3a^2 \log(a+bx^2)}{2b^4} - \frac{ax^2}{b^3} + \frac{x^4}{4b^2}$$

[Out]  $-\left(\frac{a^3 x^2}{b^3} + \frac{x^4}{4b^2}\right) + \frac{a^3}{2b^4(a+bx^2)} + \frac{3a^2 \log[a+bx^2]}{2b^4}$

**Rubi [A]** time = 0.106064, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^3}{2b^4(a+bx^2)} + \frac{3a^2 \log(a+bx^2)}{2b^4} - \frac{ax^2}{b^3} + \frac{x^4}{4b^2}$$

Antiderivative was successfully verified.

[In] Int[x^7/(a + b\*x^2)^2, x]

[Out]  $-\left(\frac{a^3 x^2}{b^3} + \frac{x^4}{4b^2}\right) + \frac{a^3}{2b^4(a+bx^2)} + \frac{3a^2 \log[a+bx^2]}{2b^4}$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^3}{2b^4(a+bx^2)} + \frac{3a^2 \log(a+bx^2)}{2b^4} - \frac{ax^2}{b^3} + \frac{\int^{x^2} x dx}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*7/(b\*x\*\*2+a)\*\*2, x)

[Out]  $\frac{a^3}{2b^4(a+bx^2)} + \frac{3a^2 \log(a+bx^2)}{2b^4} - \frac{ax^2}{b^3} + \text{Integral}(x, (x, x^2))/(2b^2)$

**Mathematica [A]** time = 0.0289825, size = 49, normalized size = 0.86

$$\frac{\frac{2a^3}{a+bx^2} + 6a^2 \log(a+bx^2) - 4abx^2 + b^2x^4}{4b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a + b\*x^2)^2, x]

[Out]  $\frac{-4abx^2 + b^2x^4 + (2a^3)}{(a+bx^2)} + \frac{6a^2 \text{Log}[a+bx^2]}{4b^4}$

**Maple [A]** time = 0.014, size = 52, normalized size = 0.9

$$-\frac{ax^2}{b^3} + \frac{x^4}{4b^2} + \frac{a^3}{2b^4(bx^2+a)} + \frac{3a^2 \ln(bx^2+a)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7/(b*x^2+a)^2,x)`

[Out]  $-a*x^2/b^3+1/4*x^4/b^2+1/2*a^3/b^4/(b*x^2+a)+3/2*a^2*\ln(b*x^2+a)/b^4$

**Maxima [A]** time = 1.34014, size = 73, normalized size = 1.28

$$\frac{a^3}{2(b^5x^2+ab^4)} + \frac{3a^2 \log(bx^2+a)}{2b^4} + \frac{bx^4-4ax^2}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^2+a)^2,x, algorithm="maxima")`

[Out]  $1/2*a^3/(b^5*x^2+a*b^4)+3/2*a^2*\log(b*x^2+a)/b^4+1/4*(b*x^4-4*a*x^2)/b^3$

**Fricas [A]** time = 0.201234, size = 95, normalized size = 1.67

$$\frac{b^3x^6-3ab^2x^4-4a^2bx^2+2a^3+6(a^2bx^2+a^3)\log(bx^2+a)}{4(b^5x^2+ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^2+a)^2,x, algorithm="fricas")`

[Out]  $1/4*(b^3*x^6-3*a*b^2*x^4-4*a^2*b*x^2+2*a^3+6*(a^2*b*x^2+a^3)*\log(b*x^2+a))/(b^5*x^2+a*b^4)$

**Sympy [A]** time = 1.58859, size = 53, normalized size = 0.93

$$\frac{a^3}{2ab^4+2b^5x^2} + \frac{3a^2 \log(a+bx^2)}{2b^4} - \frac{ax^2}{b^3} + \frac{x^4}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7/(b*x**2+a)**2,x)`

[Out]  $a**3/(2*a*b**4+2*b**5*x**2)+3*a**2*\log(a+b*x**2)/(2*b**4)-a*x**2/b**3+x**4/(4*b**2)$

**GIAC/XCAS [A]** time = 0.213685, size = 90, normalized size = 1.58

$$\frac{3a^2 \ln(|bx^2+a|)}{2b^4} + \frac{b^2x^4-4abx^2}{4b^4} - \frac{3a^2bx^2+2a^3}{2(bx^2+a)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^2+a)^2,x, algorithm="giac")`

[Out]  $3/2*a^2*\ln(\text{abs}(b*x^2+a))/b^4+1/4*(b^2*x^4-4*a*b*x^2)/b^4-1/2*(3*a^2*b*x^2+2*a^3)/((b*x^2+a)*b^4)$

$$3.152 \quad \int \frac{x^6}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=66

$$\frac{5a^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{7/2}} - \frac{5ax}{2b^3} - \frac{x^5}{2b(a+bx^2)} + \frac{5x^3}{6b^2}$$

[Out]  $(-5*a*x)/(2*b^3) + (5*x^3)/(6*b^2) - x^5/(2*b*(a + b*x^2)) + (5*a^{3/2})*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*b^{(7/2)})$

**Rubi [A]** time = 0.07645, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{5a^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{7/2}} - \frac{5ax}{2b^3} - \frac{x^5}{2b(a+bx^2)} + \frac{5x^3}{6b^2}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a + b\*x^2)^2, x]

[Out]  $(-5*a*x)/(2*b^3) + (5*x^3)/(6*b^2) - x^5/(2*b*(a + b*x^2)) + (5*a^{3/2})*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*b^{(7/2)})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{5a^{3/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{7/2}} - \frac{x^5}{2b(a+bx^2)} + \frac{5x^3}{6b^2} - \frac{5 \int a dx}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6/(b\*x\*\*2+a)\*\*2, x)

[Out]  $5*a^{3/2}*atan(sqrt(b)*x/sqrt(a))/(2*b^{(7/2)}) - x^5/(2*b*(a + b*x^2)) + 5*x^3/(6*b^2) - 5*Integral(a, x)/(2*b^3)$

**Mathematica [A]** time = 0.0773642, size = 60, normalized size = 0.91

$$\frac{5a^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{7/2}} + \frac{x\left(-\frac{3a^2}{a+bx^2} - 12a + 2bx^2\right)}{6b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a + b\*x^2)^2, x]

[Out]  $(x*(-12*a + 2*b*x^2 - (3*a^2)/(a + b*x^2)))/(6*b^3) + (5*a^{(3/2)})*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*b^{(7/2)})$

**Maple [A]** time = 0.011, size = 57, normalized size = 0.9

$$\frac{x^3}{3b^2} - 2\frac{ax}{b^3} - \frac{a^2x}{2b^3(bx^2+a)} + \frac{5a^2}{2b^3} \arctan\left(bx\frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(b*x^2+a)^2,x)`

[Out]  $\frac{1}{3}x^3/b^2 - 2ax/b^3 - \frac{1}{2}b^3 a^2 x / (bx^2+a) + \frac{5}{2}b^3 a^2 / (ab)^{1/2} \arctan(xb/(ab)^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^2 + a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.206657, size = 1, normalized size = 0.02

$$\left[ \frac{4b^2x^5 - 20abx^3 - 30a^2x + 15(abx^2 + a^2)\sqrt{-\frac{a}{b}} \log\left(\frac{bx^2 + 2bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right)}{12(b^4x^2 + ab^3)}, \frac{2b^2x^5 - 10abx^3 - 15a^2x + 15(abx^2 + a^2)\sqrt{\frac{a}{b}} \arctan\left(\frac{x}{\sqrt{a/b}}\right)}{6(b^4x^2 + ab^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^2 + a)^2,x, algorithm="fricas")`

[Out]  $\left[ \frac{1}{12} \left( 4b^2x^5 - 20a^2bx^3 - 30a^2x + 15(abx^2 + a^2) \sqrt{-\frac{a}{b}} \log\left(\frac{bx^2 + 2bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) \right) / (b^4x^2 + ab^3), \frac{1}{6} \left( 2b^2x^5 - 10a^2bx^3 - 15a^2x + 15(abx^2 + a^2) \sqrt{\frac{a}{b}} \arctan\left(\frac{x}{\sqrt{a/b}}\right) \right) / (b^4x^2 + ab^3) \right]$

**Sympy [A]** time = 1.66467, size = 107, normalized size = 1.62

$$-\frac{a^2x}{2ab^3 + 2b^4x^2} - \frac{2ax}{b^3} - \frac{5\sqrt{-\frac{a^3}{b^7}} \log\left(x - \frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4} + \frac{5\sqrt{-\frac{a^3}{b^7}} \log\left(x + \frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4} + \frac{x^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(b*x**2+a)**2,x)`

[Out]  $-a^2x/(2a^2b^3 + 2b^4x^2) - 2ax/b^3 - 5\sqrt{-a^3/b^7} \log(x - b^3\sqrt{-a^3/b^7}/a)/4 + 5\sqrt{-a^3/b^7} \log(x + b^3\sqrt{-a^3/b^7}/a)/4 + x^3/(3b^2)$

**GIAC/XCAS [A]** time = 0.20972, size = 82, normalized size = 1.24

$$\frac{5a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{abb^3}} - \frac{a^2x}{2(bx^2 + a)b^3} + \frac{b^4x^3 - 6ab^3x}{3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 5/2*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*a^2*x/((b*x^2 + a)*b^3) + 1/3*(b^4*x^3 - 6*a*b^3*x)/b^6
```



$$3.153 \quad \int \frac{x^5}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=44

$$-\frac{a^2}{2b^3(a+bx^2)} - \frac{a \log(a+bx^2)}{b^3} + \frac{x^2}{2b^2}$$

[Out]  $x^2/(2*b^2) - a^2/(2*b^3*(a + b*x^2)) - (a*Log[a + b*x^2])/b^3$

**Rubi [A]** time = 0.0808587, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^2}{2b^3(a+bx^2)} - \frac{a \log(a+bx^2)}{b^3} + \frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a + b\*x^2)^2, x]

[Out]  $x^2/(2*b^2) - a^2/(2*b^3*(a + b*x^2)) - (a*Log[a + b*x^2])/b^3$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^2}{2b^3(a+bx^2)} - \frac{a \log(a+bx^2)}{b^3} + \int \frac{1}{b^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5/(b\*x\*\*2+a)\*\*2, x)

[Out]  $-a**2/(2*b**3*(a + b*x**2)) - a*log(a + b*x**2)/b**3 + Integral(b**(-2), (x, x**2))/2$

**Mathematica [A]** time = 0.0331013, size = 38, normalized size = 0.86

$$\frac{-\frac{a^2}{a+bx^2} - 2a \log(a+bx^2) + bx^2}{2b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a + b\*x^2)^2, x]

[Out]  $(b*x^2 - a^2/(a + b*x^2) - 2*a*Log[a + b*x^2])/(2*b^3)$

**Maple [A]** time = 0.015, size = 41, normalized size = 0.9

$$\frac{x^2}{2b^2} - \frac{a^2}{2b^3(bx^2+a)} - \frac{a \ln(bx^2+a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b\*x^2+a)^2, x)

[Out]  $1/2*x^2/b^2-1/2*a^2/b^3/(b*x^2+a)-a*\ln(b*x^2+a)/b^3$

**Maxima [A]** time = 1.35656, size = 58, normalized size = 1.32

$$-\frac{a^2}{2(b^4x^2+ab^3)} + \frac{x^2}{2b^2} - \frac{a \log(bx^2+a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2 + a)^2,x, algorithm="maxima")`

[Out]  $-1/2*a^2/(b^4*x^2 + a*b^3) + 1/2*x^2/b^2 - a*\log(b*x^2 + a)/b^3$

**Fricas [A]** time = 0.197063, size = 76, normalized size = 1.73

$$\frac{b^2x^4 + abx^2 - a^2 - 2(abx^2 + a^2) \log(bx^2 + a)}{2(b^4x^2 + ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2 + a)^2,x, algorithm="fricas")`

[Out]  $1/2*(b^2*x^4 + a*b*x^2 - a^2 - 2*(a*b*x^2 + a^2)*\log(b*x^2 + a))/(b^4*x^2 + a*b^3)$

**Sympy [A]** time = 1.51599, size = 39, normalized size = 0.89

$$-\frac{a^2}{2ab^3 + 2b^4x^2} - \frac{a \log(a + bx^2)}{b^3} + \frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(b*x**2+a)**2,x)`

[Out]  $-a**2/(2*a*b**3 + 2*b**4*x**2) - a*\log(a + b*x**2)/b**3 + x**2/(2*b**2)$

**GIAC/XCAS [A]** time = 0.213194, size = 66, normalized size = 1.5

$$\frac{x^2}{2b^2} - \frac{a \ln(|bx^2 + a|)}{b^3} + \frac{2abx^2 + a^2}{2(bx^2 + a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2 + a)^2,x, algorithm="giac")`

[Out]  $1/2*x^2/b^2 - a*\ln(\text{abs}(b*x^2 + a))/b^3 + 1/2*(2*a*b*x^2 + a^2)/((b*x^2 + a)*b^3)$

$$3.154 \quad \int \frac{x^4}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=55

$$-\frac{3\sqrt{a} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{5/2}} - \frac{x^3}{2b(a+bx^2)} + \frac{3x}{2b^2}$$

[Out] (3\*x)/(2\*b^2) - x^3/(2\*b\*(a + b\*x^2)) - (3\*Sqrt[a]\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(2\*b^(5/2))

**Rubi [A]** time = 0.0586503, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{3\sqrt{a} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{5/2}} - \frac{x^3}{2b(a+bx^2)} + \frac{3x}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a + b\*x^2)^2, x]

[Out] (3\*x)/(2\*b^2) - x^3/(2\*b\*(a + b\*x^2)) - (3\*Sqrt[a]\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(2\*b^(5/2))

**Rubi in Sympy [A]** time = 11.1678, size = 48, normalized size = 0.87

$$-\frac{3\sqrt{a} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{5/2}} - \frac{x^3}{2b(a+bx^2)} + \frac{3x}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(b\*x\*\*2+a)\*\*2, x)

[Out] -3\*sqrt(a)\*atan(sqrt(b)\*x/sqrt(a))/(2\*b\*\*(5/2)) - x\*\*3/(2\*b\*(a + b\*x\*\*2)) + 3\*x/(2\*b\*\*2)

**Mathematica [A]** time = 0.0590145, size = 51, normalized size = 0.93

$$-\frac{3\sqrt{a} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{5/2}} + \frac{ax}{2b^2(a+bx^2)} + \frac{x}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a + b\*x^2)^2, x]

[Out] x/b^2 + (a\*x)/(2\*b^2\*(a + b\*x^2)) - (3\*Sqrt[a]\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(2\*b^(5/2))

**Maple [A]** time = 0.01, size = 43, normalized size = 0.8

$$\frac{x}{b^2} + \frac{ax}{2b^2(bx^2+a)} - \frac{3a}{2b^2} \operatorname{arctan}\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(b*x^2+a)^2,x)`

[Out]  $x/b^2 + 1/2/b^2 * a * x / (b * x^2 + a) - 3/2/b^2 * a / (a * b)^{1/2} * \arctan(x * b / (a * b)^{1/2})$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.204533, size = 1, normalized size = 0.02

$$\left[ \frac{4bx^3 + 3(bx^2 + a)\sqrt{-\frac{a}{b}} \log\left(\frac{bx^2 - 2bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) + 6ax}{4(b^3x^2 + ab^2)}, \frac{2bx^3 - 3(bx^2 + a)\sqrt{\frac{a}{b}} \arctan\left(\frac{x}{\sqrt{\frac{a}{b}}}\right) + 3ax}{2(b^3x^2 + ab^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^2,x, algorithm="fricas")`

[Out]  $[1/4 * (4 * b * x^3 + 3 * (b * x^2 + a) * \sqrt{-a/b}) * \log((b * x^2 - 2 * b * x * \sqrt{-a/b}) - a / (b * x^2 + a)) + 6 * a * x / (b^3 * x^2 + a * b^2), 1/2 * (2 * b * x^3 - 3 * (b * x^2 + a) * \sqrt{a/b}) * \arctan(x / \sqrt{a/b}) + 3 * a * x / (b^3 * x^2 + a * b^2)]$

**Sympy** [A] time = 1.54405, size = 83, normalized size = 1.51

$$\frac{ax}{2ab^2 + 2b^3x^2} + \frac{3\sqrt{-\frac{a}{b^5}} \log\left(-b^2\sqrt{-\frac{a}{b^5}} + x\right)}{4} - \frac{3\sqrt{-\frac{a}{b^5}} \log\left(b^2\sqrt{-\frac{a}{b^5}} + x\right)}{4} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(b*x**2+a)**2,x)`

[Out]  $a * x / (2 * a * b^2 + 2 * b^3 * x^2) + 3 * \sqrt{-a/b^5} * \log(-b^2 * \sqrt{-a/b^5} + x) / 4 - 3 * \sqrt{-a/b^5} * \log(b^2 * \sqrt{-a/b^5} + x) / 4 + x / b^2$

**GIAC/XCAS** [A] time = 0.206647, size = 57, normalized size = 1.04

$$-\frac{3a \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}b^2} + \frac{ax}{2(bx^2 + a)b^2} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] -3/2*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*a*x/((b*x^2 +  
a)*b^2) + x/b^2
```

$$3.155 \quad \int \frac{x^3}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=33

$$\frac{a}{2b^2(a+bx^2)} + \frac{\log(a+bx^2)}{2b^2}$$

[Out]  $a/(2*b^2*(a + b*x^2)) + \text{Log}[a + b*x^2]/(2*b^2)$

**Rubi [A]** time = 0.0615391, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a}{2b^2(a+bx^2)} + \frac{\log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

[In] `Int[x^3/(a + b*x^2)^2, x]`

[Out]  $a/(2*b^2*(a + b*x^2)) + \text{Log}[a + b*x^2]/(2*b^2)$

**Rubi in Sympy [A]** time = 8.4684, size = 26, normalized size = 0.79

$$\frac{a}{2b^2(a+bx^2)} + \frac{\log(a+bx^2)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3/(b*x**2+a)**2, x)`

[Out]  $a/(2*b**2*(a + b*x**2)) + \log(a + b*x**2)/(2*b**2)$

**Mathematica [A]** time = 0.0218097, size = 27, normalized size = 0.82

$$\frac{\frac{a}{a+bx^2} + \log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

[In] `Integrate[x^3/(a + b*x^2)^2, x]`

[Out]  $(a/(a + b*x^2) + \text{Log}[a + b*x^2])/(2*b^2)$

**Maple [A]** time = 0.012, size = 30, normalized size = 0.9

$$\frac{a}{2b^2(bx^2+a)} + \frac{\ln(bx^2+a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(b*x^2+a)^2, x)`

[Out]  $1/2 * a/b^2/(b * x^2+a)+1/2 * \ln(b * x^2+a)/b^2$

**Maxima [A]** time = 1.33363, size = 43, normalized size = 1.3

$$\frac{a}{2(b^3x^2 + ab^2)} + \frac{\log(bx^2 + a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 + a)^2,x, algorithm="maxima")`

[Out]  $1/2 * a/(b^3 * x^2 + a * b^2) + 1/2 * \log(b * x^2 + a)/b^2$

**Fricas [A]** time = 0.200882, size = 47, normalized size = 1.42

$$\frac{(bx^2 + a) \log(bx^2 + a) + a}{2(b^3x^2 + ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 + a)^2,x, algorithm="fricas")`

[Out]  $1/2 * ((b * x^2 + a) * \log(b * x^2 + a) + a)/(b^3 * x^2 + a * b^2)$

**Sympy [A]** time = 1.35504, size = 29, normalized size = 0.88

$$\frac{a}{2ab^2 + 2b^3x^2} + \frac{\log(a + bx^2)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(b*x**2+a)**2,x)`

[Out]  $a/(2 * a * b^2 + 2 * b^3 * x^2) + \log(a + b * x^2)/(2 * b^2)$

**GIAC/XCAS [A]** time = 0.215871, size = 65, normalized size = 1.97

$$\frac{\ln\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b} - \frac{a}{(bx^2+a)b}$$

$$- \frac{1}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 + a)^2,x, algorithm="giac")`

[Out]  $-1/2 * (\ln(\text{abs}(b * x^2 + a)/((b * x^2 + a)^2 * \text{abs}(b))))/b - a/((b * x^2 + a) * b)/b$

$$3.156 \quad \int \frac{x^2}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2\sqrt{ab}^{3/2}} - \frac{x}{2b(a+bx^2)}$$

[Out]  $-x/(2*b*(a + b*x^2)) + \text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/(2*\text{Sqrt}[a]*b^{(3/2)})$

**Rubi [A]** time = 0.0404158, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2\sqrt{ab}^{3/2}} - \frac{x}{2b(a+bx^2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2/(a + b*x^2)^2, x]$

[Out]  $-x/(2*b*(a + b*x^2)) + \text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/(2*\text{Sqrt}[a]*b^{(3/2)})$

**Rubi in Sympy [A]** time = 6.34838, size = 36, normalized size = 0.8

$$-\frac{x}{2b(a+bx^2)} + \frac{\text{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2\sqrt{ab}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**2/(b*x**2+a)**2, x)$

[Out]  $-x/(2*b*(a + b*x**2)) + \text{atan}(\text{sqrt}(b)*x/\text{sqrt}(a))/(2*\text{sqrt}(a)*b**(3/2))$

**Mathematica [A]** time = 0.0369481, size = 45, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2\sqrt{ab}^{3/2}} - \frac{x}{2b(a+bx^2)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^2/(a + b*x^2)^2, x]$

[Out]  $-x/(2*b*(a + b*x^2)) + \text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/(2*\text{Sqrt}[a]*b^{(3/2)})$

**Maple [A]** time = 0.009, size = 36, normalized size = 0.8

$$-\frac{x}{2b(bx^2+a)} + \frac{1}{2b} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*x^2+a)^2,x)`

[Out]  $-1/2*x/b/(b*x^2+a)+1/2/b/(a*b)^{(1/2)}*\arctan(x*b/(a*b)^{(1/2)})$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.210355, size = 1, normalized size = 0.02

$$\left[ \frac{(bx^2 + a) \log\left(\frac{2abx + (bx^2 - a)\sqrt{-ab}}{bx^2 + a}\right) - 2\sqrt{-ab}x}{4(b^2x^2 + ab)\sqrt{-ab}}, \frac{(bx^2 + a) \arctan\left(\frac{\sqrt{ab}x}{a}\right) - \sqrt{ab}x}{2(b^2x^2 + ab)\sqrt{ab}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^2,x, algorithm="fricas")`

[Out]  $[1/4*((b*x^2 + a)*\log((2*a*b*x + (b*x^2 - a)*\sqrt{-a*b}))/((b*x^2 + a) - 2*\sqrt{-a*b}*x)/((b^2*x^2 + a*b)*\sqrt{-a*b}), 1/2*((b*x^2 + a)*\arctan(\sqrt{a*b}*x/a) - \sqrt{a*b}*x)/((b^2*x^2 + a*b)*\sqrt{a*b})]$

**Sympy** [A] time = 1.36332, size = 78, normalized size = 1.73

$$-\frac{x}{2ab + 2b^2x^2} - \frac{\sqrt{-\frac{1}{ab^3}} \log\left(-ab\sqrt{-\frac{1}{ab^3}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{ab^3}} \log\left(ab\sqrt{-\frac{1}{ab^3}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**2+a)**2,x)`

[Out]  $-x/(2*a*b + 2*b**2*x**2) - \sqrt{-1/(a*b**3)}*\log(-a*b*\sqrt{-1/(a*b**3)} + x)/4 + \sqrt{-1/(a*b**3)}*\log(a*b*\sqrt{-1/(a*b**3)} + x)/4$

**GIAC/XCAS** [A] time = 0.207177, size = 47, normalized size = 1.04

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{abb}} - \frac{x}{2(bx^2 + a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^2,x, algorithm="giac")`

[Out]  $\frac{1}{2} \arctan\left(\frac{b x}{\sqrt{a b}}\right) / (\sqrt{a b})^b - \frac{1}{2} x / ((b x^2 + a) b)$

$$3.157 \quad \int \frac{x}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=16

$$-\frac{1}{2b(a+bx^2)}$$

[Out] -1/(2\*b\*(a + b\*x^2))

**Rubi [A]** time = 0.0109041, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{1}{2b(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*x^2)^2, x]

[Out] -1/(2\*b\*(a + b\*x^2))

**Rubi in Sympy [A]** time = 2.28912, size = 12, normalized size = 0.75

$$-\frac{1}{2b(a+bx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(b\*x\*\*2+a)\*\*2, x)

[Out] -1/(2\*b\*(a + b\*x\*\*2))

**Mathematica [A]** time = 0.00378476, size = 16, normalized size = 1.

$$-\frac{1}{2b(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*x^2)^2, x]

[Out] -1/(2\*b\*(a + b\*x^2))

**Maple [A]** time = 0.001, size = 15, normalized size = 0.9

$$-\frac{1}{2b(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b\*x^2+a)^2, x)

[Out] -1/2/b/(b\*x^2+a)

---

**Maxima [A]** time = 1.3769, size = 19, normalized size = 1.19

$$-\frac{1}{2(bx^2 + a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^2,x, algorithm="maxima")`

[Out] `-1/2/((b*x^2 + a)*b)`

---

**Fricas [A]** time = 0.198296, size = 20, normalized size = 1.25

$$-\frac{1}{2(b^2x^2 + ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^2,x, algorithm="fricas")`

[Out] `-1/2/(b^2*x^2 + a*b)`

---

**Sympy [A]** time = 1.22242, size = 15, normalized size = 0.94

$$-\frac{1}{2ab + 2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x**2+a)**2,x)`

[Out] `-1/(2*a*b + 2*b**2*x**2)`

---

**GIAC/XCAS [A]** time = 0.215638, size = 19, normalized size = 1.19

$$-\frac{1}{2(bx^2 + a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^2,x, algorithm="giac")`

[Out] `-1/2/((b*x^2 + a)*b)`

$$3.158 \quad \int \frac{1}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{x}{2a(a+bx^2)}$$

[Out]  $x/(2*a*(a + b*x^2)) + \text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/(2*a^{(3/2)}*\text{Sqrt}[b])$

**Rubi [A]** time = 0.0317654, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{x}{2a(a+bx^2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^{-2}, x]$

[Out]  $x/(2*a*(a + b*x^2)) + \text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/(2*a^{(3/2)}*\text{Sqrt}[b])$

**Rubi in Sympy [A]** time = 4.0129, size = 36, normalized size = 0.8

$$\frac{x}{2a(a+bx^2)} + \frac{\text{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/(b*x**2+a)**2, x)$

[Out]  $x/(2*a*(a + b*x**2)) + \text{atan}(\text{sqrt}(b)*x/\text{sqrt}(a))/(2*a^{(3/2)}*\text{sqrt}(b))$

**Mathematica [A]** time = 0.0454433, size = 45, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{x}{2a(a+bx^2)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b*x^2)^{-2}, x]$

[Out]  $x/(2*a*(a + b*x^2)) + \text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/(2*a^{(3/2)}*\text{Sqrt}[b])$

**Maple [A]** time = 0.004, size = 36, normalized size = 0.8

$$\frac{x}{2a(bx^2+a)} + \frac{1}{2a} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^2+a)^2, x)`

[Out]  $1/2 * x/a / (b * x^2 + a) + 1/2/a / (a * b)^{(1/2)} * \arctan(x * b / (a * b)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(-2), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.210381, size = 1, normalized size = 0.02

$$\left[ \frac{(bx^2 + a) \log\left(\frac{2abx + (bx^2 - a)\sqrt{-ab}}{bx^2 + a}\right) + 2\sqrt{-ab}x}{4(abx^2 + a^2)\sqrt{-ab}}, \frac{(bx^2 + a) \arctan\left(\frac{\sqrt{ab}x}{a}\right) + \sqrt{ab}x}{2(abx^2 + a^2)\sqrt{ab}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(-2), x, algorithm="fricas")`

[Out]  $[1/4 * ((b * x^2 + a) * \log((2 * a * b * x + (b * x^2 - a) * \sqrt{-a * b}) / (b * x^2 + a)) + 2 * \sqrt{-a * b} * x) / ((a * b * x^2 + a^2) * \sqrt{-a * b}), 1/2 * ((b * x^2 + a) * \arctan(\sqrt{a * b} * x / a) + \sqrt{a * b} * x) / ((a * b * x^2 + a^2) * \sqrt{a * b})]$

**Sympy [A]** time = 1.4196, size = 78, normalized size = 1.73

$$\frac{x}{2a^2 + 2abx^2} - \frac{\sqrt{-\frac{1}{a^3b}} \log\left(-a^2 \sqrt{-\frac{1}{a^3b}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}} \log\left(a^2 \sqrt{-\frac{1}{a^3b}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**2+a)**2, x)`

[Out]  $x / (2 * a ** 2 + 2 * a * b * x ** 2) - \sqrt{-1 / (a ** 3 * b)} * \log(-a ** 2 * \sqrt{-1 / (a ** 3 * b)} + x) / 4 + \sqrt{-1 / (a ** 3 * b)} * \log(a ** 2 * \sqrt{-1 / (a ** 3 * b)} + x) / 4$

**GIAC/XCAS [A]** time = 0.206082, size = 47, normalized size = 1.04

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{aba}} + \frac{x}{2(bx^2 + a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(-2), x, algorithm="giac")`

[Out]  $\frac{1}{2} \arctan\left(\frac{b x}{\sqrt{a b}}\right) / (\sqrt{a b})^a + \frac{1}{2} x / ((b x^2 + a)^a)$

$$3.159 \quad \int \frac{1}{x(a+bx^2)^2} dx$$

**Optimal.** Leaf size=38

$$-\frac{\log(a+bx^2)}{2a^2} + \frac{\log(x)}{a^2} + \frac{1}{2a(a+bx^2)}$$

[Out]  $1/(2*a*(a + b*x^2)) + \text{Log}[x]/a^2 - \text{Log}[a + b*x^2]/(2*a^2)$

**Rubi [A]** time = 0.0650545, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{\log(a+bx^2)}{2a^2} + \frac{\log(x)}{a^2} + \frac{1}{2a(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(a + b\*x^2)^2), x]

[Out]  $1/(2*a*(a + b*x^2)) + \text{Log}[x]/a^2 - \text{Log}[a + b*x^2]/(2*a^2)$

**Rubi in Sympy [A]** time = 9.59399, size = 34, normalized size = 0.89

$$\frac{1}{2a(a+bx^2)} + \frac{\log(x^2)}{2a^2} - \frac{\log(a+bx^2)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(b\*x\*\*2+a)\*\*2, x)

[Out]  $1/(2*a*(a + b*x**2)) + \log(x**2)/(2*a**2) - \log(a + b*x**2)/(2*a**2)$

**Mathematica [A]** time = 0.0270379, size = 33, normalized size = 0.87

$$\frac{\frac{a}{a+bx^2} - \log(a+bx^2) + 2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(a + b\*x^2)^2), x]

[Out]  $(a/(a + b*x^2) + 2*\text{Log}[x] - \text{Log}[a + b*x^2])/ (2*a^2)$

**Maple [A]** time = 0.016, size = 35, normalized size = 0.9

$$\frac{1}{2a(bx^2+a)} + \frac{\ln(x)}{a^2} - \frac{\ln(bx^2+a)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b\*x^2+a)^2, x)



[Out]  $1/2/a/(b*x^2+a)+\ln(x)/a^2-1/2*\ln(b*x^2+a)/a^2$

**Maxima [A]** time = 1.55198, size = 50, normalized size = 1.32

$$\frac{1}{2(abx^2 + a^2)} - \frac{\log(bx^2 + a)}{2a^2} + \frac{\log(x^2)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^2*x),x, algorithm="maxima")`

[Out]  $1/2/(a*b*x^2 + a^2) - 1/2*\log(b*x^2 + a)/a^2 + 1/2*\log(x^2)/a^2$

**Fricas [A]** time = 0.213054, size = 63, normalized size = 1.66

$$-\frac{(bx^2 + a) \log(bx^2 + a) - 2(bx^2 + a) \log(x) - a}{2(a^2bx^2 + a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^2*x),x, algorithm="fricas")`

[Out]  $-1/2*((b*x^2 + a)*\log(b*x^2 + a) - 2*(b*x^2 + a)*\log(x) - a)/(a^2*b*x^2 + a^3)$

**Sympy [A]** time = 1.6692, size = 34, normalized size = 0.89

$$\frac{1}{2a^2 + 2abx^2} + \frac{\log(x)}{a^2} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x**2+a)**2,x)`

[Out]  $1/(2*a**2 + 2*a*b*x**2) + \log(x)/a**2 - \log(a/b + x**2)/(2*a**2)$

**GIAC/XCAS [A]** time = 0.211098, size = 63, normalized size = 1.66

$$\frac{\ln(x^2)}{2a^2} - \frac{\ln(|bx^2 + a|)}{2a^2} + \frac{bx^2 + 2a}{2(bx^2 + a)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^2*x),x, algorithm="giac")`

[Out]  $1/2*\ln(x^2)/a^2 - 1/2*\ln(\text{abs}(b*x^2 + a))/a^2 + 1/2*(b*x^2 + 2*a)/((b*x^2 + a)*a^2)$

$$3.160 \quad \int \frac{1}{x^2(a+bx^2)^2} dx$$

**Optimal.** Leaf size=57

$$-\frac{3\sqrt{b} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{3}{2a^2x} + \frac{1}{2ax(a+bx^2)}$$

[Out]  $-3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*\text{Sqrt}[b]*\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/(2*a^{(5/2)})$

**Rubi [A]** time = 0.0545213, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{3\sqrt{b} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{3}{2a^2x} + \frac{1}{2ax(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a + b\*x^2)^2), x]

[Out]  $-3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*\text{Sqrt}[b]*\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/(2*a^{(5/2)})$

**Rubi in Sympy [A]** time = 10.2146, size = 48, normalized size = 0.84

$$\frac{1}{2ax(a+bx^2)} - \frac{3}{2a^2x} - \frac{3\sqrt{b} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(b\*x\*\*2+a)\*\*2, x)

[Out]  $1/(2*a*x*(a + b*x^2)) - 3/(2*a^2*x) - 3*\text{sqrt}(b)*\text{atan}(\text{sqrt}(b)*x/\text{sqrt}(a))/(2*a^{(5/2)})$

**Mathematica [A]** time = 0.0674585, size = 54, normalized size = 0.95

$$-\frac{3\sqrt{b} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{bx}{2a^2(a+bx^2)} - \frac{1}{a^2x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a + b\*x^2)^2), x]

[Out]  $-(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*\text{Sqrt}[b]*\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/(2*a^{(5/2)})$

**Maple [A]** time = 0.013, size = 46, normalized size = 0.8

$$-\frac{1}{a^2x} - \frac{bx}{2a^2(bx^2+a)} - \frac{3b}{2a^2} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(b*x^2+a)^2,x)`

[Out]  $-1/a^2/x - 1/2*b/a^2*x/(b*x^2+a) - 3/2*b/a^2/(a*b)^{(1/2)}*\arctan(x*b/(a*b)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^2*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.214271, size = 1, normalized size = 0.02

$$\left[ \frac{6bx^2 - 3(bx^3 + ax)\sqrt{-\frac{b}{a}} \log\left(\frac{bx^2 - 2ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right) + 4a}{4(a^2bx^3 + a^3x)}, -\frac{3bx^2 + 3(bx^3 + ax)\sqrt{\frac{b}{a}} \arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right) + 2a}{2(a^2bx^3 + a^3x)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^2*x^2),x, algorithm="fricas")`

[Out]  $[-1/4*(6*b*x^2 - 3*(b*x^3 + a*x)*\sqrt{-b/a}*\log((b*x^2 - 2*a*x*\sqrt{-b/a} - a)/(b*x^2 + a)) + 4*a)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 + a*x)*\sqrt{b/a}*\arctan(b*x/(a*\sqrt{b/a}))) + 2*a)/(a^2*b*x^3 + a^3*x]$

**Sympy [A]** time = 1.69838, size = 90, normalized size = 1.58

$$\frac{3\sqrt{-\frac{b}{a^5}} \log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b} + x\right)}{4} - \frac{3\sqrt{-\frac{b}{a^5}} \log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b} + x\right)}{4} - \frac{2a + 3bx^2}{2a^3x + 2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(b*x**2+a)**2,x)`

[Out]  $3*\sqrt{-b/a**5}*\log(-a**3*\sqrt{-b/a**5}/b + x)/4 - 3*\sqrt{-b/a**5}*\log(a**3*\sqrt{-b/a**5}/b + x)/4 - (2*a + 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3)$

**GIAC/XCAS [A]** time = 0.208128, size = 63, normalized size = 1.11

$$-\frac{3b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{aba^2}} - \frac{3bx^2 + 2a}{2(bx^3 + ax)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*x^2),x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*x^2 + 2*a)
)/((b*x^3 + a*x)*a^2)
```

$$3.161 \quad \int \frac{1}{x^3(a+bx^2)^2} dx$$

**Optimal.** Leaf size=49

$$\frac{b \log(a+bx^2)}{a^3} - \frac{2b \log(x)}{a^3} - \frac{b}{2a^2(a+bx^2)} - \frac{1}{2a^2x^2}$$

[Out]  $-1/(2*a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^2])/a^3$

**Rubi [A]** time = 0.087268, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{b \log(a+bx^2)}{a^3} - \frac{2b \log(x)}{a^3} - \frac{b}{2a^2(a+bx^2)} - \frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(a + b\*x^2)^2), x]

[Out]  $-1/(2*a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^2])/a^3$

**Rubi in Sympy [A]** time = 12.2934, size = 46, normalized size = 0.94

$$-\frac{b}{2a^2(a+bx^2)} - \frac{1}{2a^2x^2} - \frac{b \log(x^2)}{a^3} + \frac{b \log(a+bx^2)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(b\*x\*\*2+a)\*\*2, x)

[Out]  $-b/(2*a**2*(a + b*x**2)) - 1/(2*a**2*x**2) - b*log(x**2)/a**3 + b*log(a + b*x**2)/a**3$

**Mathematica [A]** time = 0.0723158, size = 41, normalized size = 0.84

$$\frac{a \left( \frac{b}{a+bx^2} + \frac{1}{x^2} \right) - 2b \log(a+bx^2) + 4b \log(x)}{2a^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a + b\*x^2)^2), x]

[Out]  $-(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2])/(2*a^3)$

**Maple [A]** time = 0.018, size = 46, normalized size = 0.9

$$-\frac{1}{2a^2x^2} - \frac{b}{2a^2(bx^2+a)} - 2\frac{b \ln(x)}{a^3} + \frac{b \ln(bx^2+a)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(b*x^2+a)^2,x)`

[Out]  $-1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*\ln(x)/a^3+b*\ln(b*x^2+a)/a^3$

**Maxima [A]** time = 1.33551, size = 70, normalized size = 1.43

$$-\frac{2bx^2+a}{2(a^2bx^4+a^3x^2)} + \frac{b\log(bx^2+a)}{a^3} - \frac{b\log(x^2)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2+a)^2*x^3),x, algorithm="maxima")`

[Out]  $-1/2*(2*b*x^2+a)/(a^2*b*x^4+a^3*x^2)+b*\log(b*x^2+a)/a^3-b*\log(x^2)/a^3$

**Fricas [A]** time = 0.213269, size = 99, normalized size = 2.02

$$-\frac{2abx^2+a^2-2(b^2x^4+abx^2)\log(bx^2+a)+4(b^2x^4+abx^2)\log(x)}{2(a^3bx^4+a^4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2+a)^2*x^3),x, algorithm="fricas")`

[Out]  $-1/2*(2*a*b*x^2+a^2-2*(b^2*x^4+a*b*x^2)*\log(b*x^2+a)+4*(b^2*x^4+a*b*x^2)*\log(x))/(a^3*b*x^4+a^4*x^2)$

**Sympy [A]** time = 2.04719, size = 49, normalized size = 1.

$$-\frac{a+2bx^2}{2a^3x^2+2a^2bx^4} - \frac{2b\log(x)}{a^3} + \frac{b\log(\frac{a}{b}+x^2)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(b*x**2+a)**2,x)`

[Out]  $-(a+2*b*x**2)/(2*a**3*x**2+2*a**2*b*x**4)-2*b*\log(x)/a**3+b*\log(a/b+x**2)/a**3$

**GIAC/XCAS [A]** time = 0.221261, size = 69, normalized size = 1.41

$$-\frac{b\ln(x^2)}{a^3} + \frac{b\ln(|bx^2+a|)}{a^3} - \frac{2bx^2+a}{2(bx^4+ax^2)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2+a)^2*x^3),x, algorithm="giac")`

[Out]  $-b*\ln(x^2)/a^3+b*\ln(\text{abs}(b*x^2+a))/a^3-1/2*(2*b*x^2+a)/((b*x^4+a*x^2)*a^2)$

$$3.162 \quad \int \frac{1}{x^4(a+bx^2)^2} dx$$

**Optimal.** Leaf size=68

$$\frac{5b^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{7/2}} + \frac{5b}{2a^3x} - \frac{5}{6a^2x^3} + \frac{1}{2ax^3(a+bx^2)}$$

[Out]  $-5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b^{3/2})*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^{7/2})$

**Rubi [A]** time = 0.0732291, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{5b^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{7/2}} + \frac{5b}{2a^3x} - \frac{5}{6a^2x^3} + \frac{1}{2ax^3(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a + b\*x^2)^2), x]

[Out]  $-5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b^{3/2})*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^{7/2})$

**Rubi in Sympy [A]** time = 14.2898, size = 61, normalized size = 0.9

$$\frac{1}{2ax^3(a+bx^2)} - \frac{5}{6a^2x^3} + \frac{5b}{2a^3x} + \frac{5b^{3/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(b\*x\*\*2+a)\*\*2, x)

[Out]  $1/(2*a*x^3*(a + b*x^2)) - 5/(6*a^2*x^3) + 5*b/(2*a^3*x) + 5*b^{3/2}*atan(sqrt(b)*x/sqrt(a))/(2*a^{7/2})$

**Mathematica [A]** time = 0.0769354, size = 67, normalized size = 0.99

$$\frac{5b^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{7/2}} + \frac{b^2x}{2a^3(a+bx^2)} + \frac{2b}{a^3x} - \frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a + b\*x^2)^2), x]

[Out]  $-1/(3*a^2*x^3) + (2*b)/(a^3*x) + (b^2*x)/(2*a^3*(a + b*x^2)) + (5*b^{3/2})*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^{7/2})$

**Maple [A]** time = 0.016, size = 59, normalized size = 0.9

$$-\frac{1}{3a^2x^3} + 2\frac{b}{a^3x} + \frac{b^2x}{2a^3(bx^2+a)} + \frac{5b^2}{2a^3} \arctan\left(bx\frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(b*x^2+a)^2,x)`

[Out]  $-1/3/a^2/x^3+2*b/a^3/x+1/2*b^2/a^3*x/(b*x^2+a)+5/2*b^2/a^3/(a*b)^{(1/2)*\arctan(x*b/(a*b)^{(1/2)})}$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^2*x^4),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.211653, size = 1, normalized size = 0.01

$$\left[ \frac{30b^2x^4 + 20abx^2 + 15(b^2x^5 + abx^3)\sqrt{-\frac{b}{a}} \log\left(\frac{bx^2+2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right) - 4a^2}{12(a^3bx^5 + a^4x^3)}, \frac{15b^2x^4 + 10abx^2 + 15(b^2x^5 + abx^3)\sqrt{\frac{b}{a}} \arctan\left(\sqrt{\frac{b}{a}}\right)}{6(a^3bx^5 + a^4x^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^2*x^4),x, algorithm="fricas")`

[Out]  $[1/12*(30*b^2*x^4 + 20*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*\sqrt{-b/a})*\log((b*x^2 + 2*a*x*\sqrt{-b/a} - a)/(b*x^2 + a)) - 4*a^2)/(a^3*b*x^5 + a^4*x^3), 1/6*(15*b^2*x^4 + 10*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*\sqrt{b/a}*\arctan(b*x/(a*\sqrt{b/a}))) - 2*a^2)/(a^3*b*x^5 + a^4*x^3)]$

**Sympy** [A] time = 2.07963, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{b^3}{a^7}} \log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2} + x\right)}{4} + \frac{5\sqrt{-\frac{b^3}{a^7}} \log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2} + x\right)}{4} + \frac{-2a^2 + 10abx^2 + 15b^2x^4}{6a^4x^3 + 6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(b*x**2+a)**2,x)`

[Out]  $-5*\sqrt{-b**3/a**7}*\log(-a**4*\sqrt{-b**3/a**7}/b**2 + x)/4 + 5*\sqrt{-b**3/a**7}*\log(a**4*\sqrt{-b**3/a**7}/b**2 + x)/4 + (-2*a**2 + 10*a*b*x**2 + 15*b**2*x**4)/(6*a**4*x**3 + 6*a**3*b*x**5)$

**GIAC/XCAS** [A] time = 0.206147, size = 80, normalized size = 1.18

$$\frac{5b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a^3} + \frac{b^2x}{2(bx^2 + a)a^3} + \frac{6bx^2 - a}{3a^3x^3}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*x^4),x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*b^2*x/((b*x^2 + a)*a^3) + 1/3*(6*b*x^2 - a)/(a^3*x^3)
```

$$3.163 \quad \int \frac{1}{x^5(a+bx^2)^2} dx$$

**Optimal.** Leaf size=66

$$-\frac{3b^2 \log(a+bx^2)}{2a^4} + \frac{3b^2 \log(x)}{a^4} + \frac{b^2}{2a^3(a+bx^2)} + \frac{b}{a^3x^2} - \frac{1}{4a^2x^4}$$

[Out]  $-1/(4*a^2*x^4) + b/(a^3*x^2) + b^2/(2*a^3*(a + b*x^2)) + (3*b^2*\text{Log}[x])/a^4 - (3*b^2*\text{Log}[a + b*x^2])/(2*a^4)$

**Rubi [A]** time = 0.113482, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{3b^2 \log(a+bx^2)}{2a^4} + \frac{3b^2 \log(x)}{a^4} + \frac{b^2}{2a^3(a+bx^2)} + \frac{b}{a^3x^2} - \frac{1}{4a^2x^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5\*(a + b\*x^2)^2), x]

[Out]  $-1/(4*a^2*x^4) + b/(a^3*x^2) + b^2/(2*a^3*(a + b*x^2)) + (3*b^2*\text{Log}[x])/a^4 - (3*b^2*\text{Log}[a + b*x^2])/(2*a^4)$

**Rubi in Sympy [A]** time = 15.8887, size = 66, normalized size = 1.

$$-\frac{1}{4a^2x^4} + \frac{b^2}{2a^3(a+bx^2)} + \frac{b}{a^3x^2} + \frac{3b^2 \log(x^2)}{2a^4} - \frac{3b^2 \log(a+bx^2)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*5/(b\*x\*\*2+a)\*\*2, x)

[Out]  $-1/(4*a**2*x**4) + b**2/(2*a**3*(a + b*x**2)) + b/(a**3*x**2) + 3*b**2*\log(x**2)/(2*a**4) - 3*b**2*\log(a + b*x**2)/(2*a**4)$

**Mathematica [A]** time = 0.13208, size = 57, normalized size = 0.86

$$\frac{-6b^2 \log(a+bx^2) + a \left( \frac{2b^2}{a+bx^2} - \frac{a}{x^4} + \frac{4b}{x^2} \right) + 12b^2 \log(x)}{4a^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5\*(a + b\*x^2)^2), x]

[Out]  $(a*(-(a/x^4) + (4*b)/x^2 + (2*b^2)/(a + b*x^2)) + 12*b^2*\text{Log}[x] - 6*b^2*\text{Log}[a + b*x^2])/(4*a^4)$

**Maple [A]** time = 0.02, size = 61, normalized size = 0.9

$$-\frac{1}{4a^2x^4} + \frac{b}{a^3x^2} + \frac{b^2}{2a^3(bx^2+a)} + 3\frac{b^2 \ln(x)}{a^4} - \frac{3b^2 \ln(bx^2+a)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^5/(b*x^2+a)^2,x)`

[Out]  $-1/4/a^2/x^4+b/a^3/x^2+1/2*b^2/a^3/(b*x^2+a)+3*b^2*\ln(x)/a^4-3/2*b^2*\ln(b*x^2+a)/a^4$

**Maxima [A]** time = 1.3425, size = 95, normalized size = 1.44

$$\frac{6b^2x^4 + 3abx^2 - a^2}{4(a^3bx^6 + a^4x^4)} - \frac{3b^2 \log(bx^2 + a)}{2a^4} + \frac{3b^2 \log(x^2)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^2*x^5),x, algorithm="maxima")`

[Out]  $1/4*(6*b^2*x^4 + 3*a*b*x^2 - a^2)/(a^3*b*x^6 + a^4*x^4) - 3/2*b^2*\log(b*x^2 + a)/a^4 + 3/2*b^2*\log(x^2)/a^4$

**Fricas [A]** time = 0.210505, size = 122, normalized size = 1.85

$$\frac{6ab^2x^4 + 3a^2bx^2 - a^3 - 6(b^3x^6 + ab^2x^4) \log(bx^2 + a) + 12(b^3x^6 + ab^2x^4) \log(x)}{4(a^4bx^6 + a^5x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^2*x^5),x, algorithm="fricas")`

[Out]  $1/4*(6*a*b^2*x^4 + 3*a^2*b*x^2 - a^3 - 6*(b^3*x^6 + a*b^2*x^4)*\log(b*x^2 + a) + 12*(b^3*x^6 + a*b^2*x^4)*\log(x))/(a^4*b*x^6 + a^5*x^4)$

**Sympy [A]** time = 2.56336, size = 68, normalized size = 1.03

$$\frac{-a^2 + 3abx^2 + 6b^2x^4}{4a^4x^4 + 4a^3bx^6} + \frac{3b^2 \log(x)}{a^4} - \frac{3b^2 \log\left(\frac{a}{b} + x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(b*x**2+a)**2,x)`

[Out]  $(-a**2 + 3*a*b*x**2 + 6*b**2*x**4)/(4*a**4*x**4 + 4*a**3*b*x**6) + 3*b**2*\log(x)/a**4 - 3*b**2*\log(a/b + x**2)/(2*a**4)$

**GIAC/XCAS [A]** time = 0.209196, size = 116, normalized size = 1.76

$$\frac{3b^2 \ln(x^2)}{2a^4} - \frac{3b^2 \ln(|bx^2 + a|)}{2a^4} + \frac{3b^3x^2 + 4ab^2}{2(bx^2 + a)a^4} - \frac{9b^2x^4 - 4abx^2 + a^2}{4a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^2*x^5),x, algorithm="giac")`

[Out]  $3/2*b^2*\ln(x^2)/a^4 - 3/2*b^2*\ln(\text{abs}(b*x^2 + a))/a^4 + 1/2*(3*b^3*x^2 + 4*a*b^2)/((b*x^2 + a)*a^4) - 1/4*(9*b^2*x^4 - 4*a*b*x^2 + a^2)/(a^4*x^4)$

$$3.164 \quad \int \frac{1}{x^6(a+bx^2)^2} dx$$

**Optimal.** Leaf size=81

$$-\frac{7b^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{9/2}} - \frac{7b^2}{2a^4x} + \frac{7b}{6a^3x^3} - \frac{7}{10a^2x^5} + \frac{1}{2ax^5(a+bx^2)}$$

[Out]  $-7/(10*a^2*x^5) + (7*b)/(6*a^3*x^3) - (7*b^2)/(2*a^4*x) + 1/(2*a*x^5*(a + b*x^2)) - (7*b^{(5/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^{(9/2)})$

**Rubi [A]** time = 0.101228, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{7b^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{9/2}} - \frac{7b^2}{2a^4x} + \frac{7b}{6a^3x^3} - \frac{7}{10a^2x^5} + \frac{1}{2ax^5(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(a + b\*x^2)^2), x]

[Out]  $-7/(10*a^2*x^5) + (7*b)/(6*a^3*x^3) - (7*b^2)/(2*a^4*x) + 1/(2*a*x^5*(a + b*x^2)) - (7*b^{(5/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^{(9/2)})$

**Rubi in Sympy [A]** time = 22.1236, size = 75, normalized size = 0.93

$$\frac{1}{2ax^5(a+bx^2)} - \frac{7}{10a^2x^5} + \frac{7b}{6a^3x^3} - \frac{7b^2}{2a^4x} - \frac{7b^{5/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*6/(b\*x\*\*2+a)\*\*2, x)

[Out]  $1/(2*a*x^5*(a + b*x^2)) - 7/(10*a^2*x^5) + 7*b/(6*a^3*x^3) - 7*b^2/(2*a^4*x) - 7*b^{(5/2)}*atan(sqrt(b)*x/sqrt(a))/(2*a^{(9/2)})$

**Mathematica [A]** time = 0.0819537, size = 80, normalized size = 0.99

$$-\frac{7b^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{9/2}} - \frac{b^3x}{2a^4(a+bx^2)} - \frac{3b^2}{a^4x} + \frac{2b}{3a^3x^3} - \frac{1}{5a^2x^5}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6\*(a + b\*x^2)^2), x]

[Out]  $-1/(5*a^2*x^5) + (2*b)/(3*a^3*x^3) - (3*b^2)/(a^4*x) - (b^3*x)/(2*a^4*(a + b*x^2)) - (7*b^{(5/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^{(9/2)})$

**Maple [A]** time = 0.017, size = 70, normalized size = 0.9

$$-\frac{1}{5a^2x^5} - 3\frac{b^2}{a^4x} + \frac{2b}{3a^3x^3} - \frac{b^3x}{2a^4(bx^2+a)} - \frac{7b^3}{2a^4} \arctan\left(bx\frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^6/(b*x^2+a)^2,x)`

[Out] `-1/5/a^2/x^5-3*b^2/a^4/x+2/3*b/a^3/x^3-1/2*b^3/a^4*x/(b*x^2+a)-7/2*b^3/a^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^2*x^6),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.209935, size = 1, normalized size = 0.01

$$\left[ \begin{array}{l} \frac{210b^3x^6 + 140ab^2x^4 - 28a^2bx^2 + 12a^3 - 105(b^3x^7 + ab^2x^5)\sqrt{-\frac{b}{a}} \log\left(\frac{bx^2 - 2ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right)}{60(a^4bx^7 + a^5x^5)}, \\ \frac{105b^3x^6 + 70ab^2x^4 - 14a^2bx^2 + 6a^3 + 105(b^3x^7 + ab^2x^5)\sqrt{\frac{b}{a}} \arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)}{30(a^4bx^7 + a^5x^5)} \end{array} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^2*x^6),x, algorithm="fricas")`

[Out] `[-1/60*(210*b^3*x^6 + 140*a*b^2*x^4 - 28*a^2*b*x^2 + 12*a^3 - 105*(b^3*x^7 + a*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^4*b*x^7 + a^5*x^5), -1/30*(105*b^3*x^6 + 70*a*b^2*x^4 - 14*a^2*b*x^2 + 6*a^3 + 105*(b^3*x^7 + a*b^2*x^5)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a)))/(a^4*b*x^7 + a^5*x^5)]`

**Sympy [A]** time = 2.80513, size = 126, normalized size = 1.56

$$\frac{7\sqrt{-\frac{b^5}{a^9}} \log\left(-\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3} + x\right)}{4} - \frac{7\sqrt{-\frac{b^5}{a^9}} \log\left(\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3} + x\right)}{4} - \frac{6a^3 - 14a^2bx^2 + 70ab^2x^4 + 105b^3x^6}{30a^5x^5 + 30a^4bx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**6/(b*x**2+a)**2,x)`

```
[Out] 7*sqrt(-b**5/a**9)*log(-a**5*sqrt(-b**5/a**9)/b**3 + x)/4 - 7*sqrt(-b**5/a**9)*log(a**5*sqrt(-b**5/a**9)/b**3 + x)/4 - (6*a**3 - 14*a**2*b*x**2 + 70*a*b**2*x**4 + 105*b**3*x**6)/(30*a**5*x**5 + 30*a**4*b*x**7)
```

**GIAC/XCAS [A]** time = 0.211402, size = 95, normalized size = 1.17

$$-\frac{7b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a^4} - \frac{b^3x}{2(bx^2 + a)a^4} - \frac{45b^2x^4 - 10abx^2 + 3a^2}{15a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*x^6),x, algorithm="giac")
```

```
[Out] -7/2*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/2*b^3*x/((b*x^2 + a)*a^4) - 1/15*(45*b^2*x^4 - 10*a*b*x^2 + 3*a^2)/(a^4*x^5)
```

$$3.165 \quad \int \frac{1}{x^7(a+bx^2)^2} dx$$

**Optimal.** Leaf size=80

$$\frac{2b^3 \log(a+bx^2)}{a^5} - \frac{4b^3 \log(x)}{a^5} - \frac{b^3}{2a^4(a+bx^2)} - \frac{3b^2}{2a^4x^2} + \frac{b}{2a^3x^4} - \frac{1}{6a^2x^6}$$

[Out]  $-1/(6*a^2*x^6) + b/(2*a^3*x^4) - (3*b^2)/(2*a^4*x^2) - b^3/(2*a^4*(a + b*x^2)) - (4*b^3*Log[x])/a^5 + (2*b^3*Log[a + b*x^2])/a^5$

**Rubi [A]** time = 0.130432, antiderivative size = 80, normalized size of antiderivative = 1., number of rules used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{2b^3 \log(a+bx^2)}{a^5} - \frac{4b^3 \log(x)}{a^5} - \frac{b^3}{2a^4(a+bx^2)} - \frac{3b^2}{2a^4x^2} + \frac{b}{2a^3x^4} - \frac{1}{6a^2x^6}$$

Antiderivative was successfully verified.

[In] Int[1/(x^7\*(a + b\*x^2)^2), x]

[Out]  $-1/(6*a^2*x^6) + b/(2*a^3*x^4) - (3*b^2)/(2*a^4*x^2) - b^3/(2*a^4*(a + b*x^2)) - (4*b^3*Log[x])/a^5 + (2*b^3*Log[a + b*x^2])/a^5$

**Rubi in Sympy [A]** time = 19.0687, size = 78, normalized size = 0.98

$$-\frac{1}{6a^2x^6} + \frac{b}{2a^3x^4} - \frac{b^3}{2a^4(a+bx^2)} - \frac{3b^2}{2a^4x^2} - \frac{2b^3 \log(x^2)}{a^5} + \frac{2b^3 \log(a+bx^2)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*7/(b\*x\*\*2+a)\*\*2, x)

[Out]  $-1/(6*a**2*x**6) + b/(2*a**3*x**4) - b**3/(2*a**4*(a + b*x**2)) - 3*b**2/(2*a**4*x**2) - 2*b**3*log(x**2)/a**5 + 2*b**3*log(a + b*x**2)/a**5$

**Mathematica [A]** time = 0.113597, size = 68, normalized size = 0.85

$$\frac{a \left( -\frac{a^2}{x^6} - \frac{3b^3}{a+bx^2} + \frac{3ab}{x^4} - \frac{9b^2}{x^2} \right) + 12b^3 \log(a+bx^2) - 24b^3 \log(x)}{6a^5}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^7\*(a + b\*x^2)^2), x]

[Out]  $(a*(-(a^2/x^6) + (3*a*b)/x^4 - (9*b^2)/x^2 - (3*b^3)/(a + b*x^2)) - 24*b^3*Log[x] + 12*b^3*Log[a + b*x^2])/(6*a^5)$

**Maple [A]** time = 0.02, size = 73, normalized size = 0.9

$$-\frac{1}{6a^2x^6} + \frac{b}{2a^3x^4} - \frac{3b^2}{2a^4x^2} - \frac{b^3}{2a^4(bx^2+a)} - 4\frac{b^3 \ln(x)}{a^5} + 2\frac{b^3 \ln(bx^2+a)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^7/(b*x^2+a)^2,x)`

[Out] 
$$-1/6/a^2/x^6 + 1/2*b/a^3/x^4 - 3/2*b^2/a^4/x^2 - 1/2*b^3/a^4/(b*x^2+a) - 4*b^3*ln(x)/a^5 + 2*b^3*ln(b*x^2+a)/a^5$$

**Maxima [A]** time = 1.34937, size = 107, normalized size = 1.34

$$-\frac{12b^3x^6 + 6ab^2x^4 - 2a^2bx^2 + a^3}{6(a^4bx^8 + a^5x^6)} + \frac{2b^3 \log(bx^2 + a)}{a^5} - \frac{2b^3 \log(x^2)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^2*x^7),x, algorithm="maxima")`

[Out] 
$$-1/6*(12*b^3*x^6 + 6*a*b^2*x^4 - 2*a^2*b*x^2 + a^3)/(a^4*b*x^8 + a^5*x^6) + 2*b^3*log(b*x^2 + a)/a^5 - 2*b^3*log(x^2)/a^5$$

**Fricas [A]** time = 0.204914, size = 134, normalized size = 1.68

$$-\frac{12ab^3x^6 + 6a^2b^2x^4 - 2a^3bx^2 + a^4 - 12(b^4x^8 + ab^3x^6) \log(bx^2 + a) + 24(b^4x^8 + ab^3x^6) \log(x)}{6(a^5bx^8 + a^6x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^2*x^7),x, algorithm="fricas")`

[Out] 
$$-1/6*(12*a*b^3*x^6 + 6*a^2*b^2*x^4 - 2*a^3*b*x^2 + a^4 - 12*(b^4*x^8 + a*b^3*x^6)*\log(b*x^2 + a) + 24*(b^4*x^8 + a*b^3*x^6)*\log(x))/(a^5*b*x^8 + a^6*x^6)$$

**Sympy [A]** time = 3.62777, size = 78, normalized size = 0.98

$$-\frac{a^3 - 2a^2bx^2 + 6ab^2x^4 + 12b^3x^6}{6a^5x^6 + 6a^4bx^8} - \frac{4b^3 \log(x)}{a^5} + \frac{2b^3 \log\left(\frac{a}{b} + x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**7/(b*x**2+a)**2,x)`

[Out] 
$$-(a**3 - 2*a**2*b*x**2 + 6*a*b**2*x**4 + 12*b**3*x**6)/(6*a**5*x**6 + 6*a**4*b*x**8) - 4*b**3*log(x)/a**5 + 2*b**3*log(a/b + x**2)/a**5$$

**GIAC/XCAS [A]** time = 0.207964, size = 134, normalized size = 1.68

$$-\frac{2b^3 \ln(x^2)}{a^5} + \frac{2b^3 \ln(|bx^2 + a|)}{a^5} - \frac{4b^4x^2 + 5ab^3}{2(bx^2 + a)a^5} + \frac{22b^3x^6 - 9ab^2x^4 + 3a^2bx^2 - a^3}{6a^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^2*x^7),x, algorithm="giac")`



```
[Out] -2*b^3*ln(x^2)/a^5 + 2*b^3*ln(abs(b*x^2 + a))/a^5 - 1/2*(4*b^4*x^2 + 5*a*b^3)/((b*x^2 + a)*a^5) + 1/6*(22*b^3*x^6 - 9*a*b^2*x^4 + 3*a^2*b*x^2 - a^3)/(a^5*x^6)
```

$$3.166 \quad \int \frac{1}{x^8(a+bx^2)^2} dx$$

**Optimal.** Leaf size=94

$$\frac{9b^{7/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{11/2}} + \frac{9b^3}{2a^5x} - \frac{3b^2}{2a^4x^3} + \frac{9b}{10a^3x^5} - \frac{9}{14a^2x^7} + \frac{1}{2ax^7(a+bx^2)}$$

[Out]  $-9/(14*a^2*x^7) + (9*b)/(10*a^3*x^5) - (3*b^2)/(2*a^4*x^3) + (9*b^3)/(2*a^5*x) + 1/(2*a*x^7*(a + b*x^2)) + (9*b^{(7/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^{(11/2)})$

**Rubi [A]** time = 0.126213, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{9b^{7/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{11/2}} + \frac{9b^3}{2a^5x} - \frac{3b^2}{2a^4x^3} + \frac{9b}{10a^3x^5} - \frac{9}{14a^2x^7} + \frac{1}{2ax^7(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^8\*(a + b\*x^2)^2), x]

[Out]  $-9/(14*a^2*x^7) + (9*b)/(10*a^3*x^5) - (3*b^2)/(2*a^4*x^3) + (9*b^3)/(2*a^5*x) + 1/(2*a*x^7*(a + b*x^2)) + (9*b^{(7/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^{(11/2)})$

**Rubi in Sympy [A]** time = 27.6216, size = 88, normalized size = 0.94

$$\frac{1}{2ax^7(a+bx^2)} - \frac{9}{14a^2x^7} + \frac{9b}{10a^3x^5} - \frac{3b^2}{2a^4x^3} + \frac{9b^3}{2a^5x} + \frac{9b^{7/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*8/(b\*x\*\*2+a)\*\*2, x)

[Out]  $1/(2*a*x**7*(a + b*x**2)) - 9/(14*a**2*x**7) + 9*b/(10*a**3*x**5) - 3*b**2/(2*a**4*x**3) + 9*b**3/(2*a**5*x) + 9*b**(7/2)*atan(sqrt(b)*x/sqrt(a))/(2*a**(11/2))$

**Mathematica [A]** time = 0.0891402, size = 91, normalized size = 0.97

$$\frac{9b^{7/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{11/2}} + \frac{b^4x}{2a^5(a+bx^2)} + \frac{4b^3}{a^5x} - \frac{b^2}{a^4x^3} + \frac{2b}{5a^3x^5} - \frac{1}{7a^2x^7}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^8\*(a + b\*x^2)^2), x]

[Out]  $-1/(7*a^2*x^7) + (2*b)/(5*a^3*x^5) - b^2/(a^4*x^3) + (4*b^3)/(a^5*x) + (b^4*x)/(2*a^5*(a + b*x^2)) + (9*b^{(7/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^{(11/2)})$

**Maple [A]** time = 0.017, size = 81, normalized size = 0.9

$$-\frac{1}{7a^2x^7} + 4\frac{b^3}{a^5x} - \frac{b^2}{a^4x^3} + \frac{2b}{5a^3x^5} + \frac{b^4x}{2a^5(bx^2+a)} + \frac{9b^4}{2a^5} \arctan\left(\frac{bx}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^8/(b\*x^2+a)^2,x)

[Out] -1/7/a^2/x^7+4\*b^3/a^5/x-b^2/a^4/x^3+2/5\*b/a^3/x^5+1/2\*b^4/a^5\*x/(b\*x^2+a)+9/2\*b^4/a^5/(a\*b)^(1/2)\*arctan(x\*b/(a\*b)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^2\*x^8),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.210764, size = 1, normalized size = 0.01

$$\left[ \frac{630b^4x^8 + 420ab^3x^6 - 84a^2b^2x^4 + 36a^3bx^2 - 20a^4 + 315(b^4x^9 + ab^3x^7)\sqrt{-\frac{b}{a}} \log\left(\frac{bx^2+2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{140(a^5bx^9 + a^6x^7)}, \frac{315b^4x^8 + 210ab^3x^6}{140(a^5bx^9 + a^6x^7)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^2\*x^8),x, algorithm="fricas")

[Out] [1/140\*(630\*b^4\*x^8 + 420\*a\*b^3\*x^6 - 84\*a^2\*b^2\*x^4 + 36\*a^3\*b\*x^2 - 20\*a^4 + 315\*(b^4\*x^9 + a\*b^3\*x^7)\*sqrt(-b/a)\*log((b\*x^2 + 2\*a\*x\*sqrt(-b/a) - a)/(b\*x^2 + a)))/(a^5\*b\*x^9 + a^6\*x^7), 1/70\*(315\*b^4\*x^8 + 210\*a\*b^3\*x^6 - 42\*a^2\*b^2\*x^4 + 18\*a^3\*b\*x^2 - 10\*a^4 + 315\*(b^4\*x^9 + a\*b^3\*x^7)\*sqrt(b/a)\*arctan(b\*x/(a\*sqrt(b/a)))/(a^5\*b\*x^9 + a^6\*x^7)]

**Sympy [A]** time = 4.21049, size = 138, normalized size = 1.47

$$-\frac{9\sqrt{-\frac{b^7}{a^{11}}}\log\left(-\frac{a^6\sqrt{-\frac{b^7}{a^{11}}}}{b^4}+x\right)}{4} + \frac{9\sqrt{-\frac{b^7}{a^{11}}}\log\left(\frac{a^6\sqrt{-\frac{b^7}{a^{11}}}}{b^4}+x\right)}{4} + \frac{-10a^4 + 18a^3bx^2 - 42a^2b^2x^4 + 210ab^3x^6 + 315b^4x^8}{70a^6x^7 + 70a^5bx^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*8/(b\*x\*\*2+a)\*\*2,x)

[Out] -9\*sqrt(-b\*\*7/a\*\*11)\*log(-a\*\*6\*sqrt(-b\*\*7/a\*\*11)/b\*\*4 + x)/4 + 9\*sqrt(-b\*\*7/a\*\*11)\*log(a\*\*6\*sqrt(-b\*\*7/a\*\*11)/b\*\*4 + x)/4 + (-10\*a\*\*4 + 18\*a\*\*3\*b\*x\*\*2 - 42\*a\*\*2\*b\*\*2\*x\*\*4 + 210\*a\*b\*\*3\*x\*\*6 + 315\*b\*\*4\*x\*\*8)/(70\*a\*\*6\*x\*\*7 + 70\*a\*\*5\*b\*x\*\*9)

$$b^{*4}x^{*8})/(70*a^{*6}x^{*7} + 70*a^{*5}b*x^{*9})$$

---

**GIAC/XCAS [A]** time = 0.209696, size = 109, normalized size = 1.16

$$\frac{9b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a^5} + \frac{b^4x}{2(bx^2 + a)a^5} + \frac{140b^3x^6 - 35ab^2x^4 + 14a^2bx^2 - 5a^3}{35a^5x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^2\*x^8),x, algorithm="giac")

[Out] 9/2\*b^4\*arctan(b\*x/sqrt(a\*b))/(sqrt(a\*b)\*a^5) + 1/2\*b^4\*x/((b\*x^2 + a)\*a^5) + 1/35\*(140\*b^3\*x^6 - 35\*a\*b^2\*x^4 + 14\*a^2\*b\*x^2 - 5\*a^3)/(a^5\*x^7)

$$3.167 \quad \int \frac{1}{x^9(a+bx^2)^2} dx$$

**Optimal.** Leaf size=93

$$-\frac{5b^4 \log(a+bx^2)}{2a^6} + \frac{5b^4 \log(x)}{a^6} + \frac{b^4}{2a^5(a+bx^2)} + \frac{2b^3}{a^5x^2} - \frac{3b^2}{4a^4x^4} + \frac{b}{3a^3x^6} - \frac{1}{8a^2x^8}$$

[Out]  $-1/(8*a^2*x^8) + b/(3*a^3*x^6) - (3*b^2)/(4*a^4*x^4) + (2*b^3)/(a^5*x^2) + b^4/(2*a^5*(a + b*x^2)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x^2])/(2*a^6)$

**Rubi [A]** time = 0.149793, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{5b^4 \log(a+bx^2)}{2a^6} + \frac{5b^4 \log(x)}{a^6} + \frac{b^4}{2a^5(a+bx^2)} + \frac{2b^3}{a^5x^2} - \frac{3b^2}{4a^4x^4} + \frac{b}{3a^3x^6} - \frac{1}{8a^2x^8}$$

Antiderivative was successfully verified.

[In] Int[1/(x^9\*(a + b\*x^2)^2), x]

[Out]  $-1/(8*a^2*x^8) + b/(3*a^3*x^6) - (3*b^2)/(4*a^4*x^4) + (2*b^3)/(a^5*x^2) + b^4/(2*a^5*(a + b*x^2)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x^2])/(2*a^6)$

**Rubi in Sympy [A]** time = 22.3758, size = 94, normalized size = 1.01

$$-\frac{1}{8a^2x^8} + \frac{b}{3a^3x^6} - \frac{3b^2}{4a^4x^4} + \frac{b^4}{2a^5(a+bx^2)} + \frac{2b^3}{a^5x^2} + \frac{5b^4 \log(x^2)}{2a^6} - \frac{5b^4 \log(a+bx^2)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*9/(b\*x\*\*2+a)\*\*2, x)

[Out]  $-1/(8*a**2*x**8) + b/(3*a**3*x**6) - 3*b**2/(4*a**4*x**4) + b**4/(2*a**5*(a + b*x**2)) + 2*b**3/(a**5*x**2) + 5*b**4*log(x**2)/(2*a**6) - 5*b**4*log(a + b*x**2)/(2*a**6)$

**Mathematica [A]** time = 0.132883, size = 79, normalized size = 0.85

$$\frac{a \left( -\frac{3a^3}{x^8} + \frac{8a^2b}{x^6} + 12b^3 \left( \frac{b}{a+bx^2} + \frac{4}{x^2} \right) - \frac{18ab^2}{x^4} \right) - 60b^4 \log(a+bx^2) + 120b^4 \log(x)}{24a^6}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^9\*(a + b\*x^2)^2), x]

[Out]  $(a*((-3*a^3)/x^8 + (8*a^2*b)/x^6 - (18*a*b^2)/x^4 + 12*b^3*(4/x^2 + b/(a + b*x^2)))) + 120*b^4*Log[x] - 60*b^4*Log[a + b*x^2])/(24*a^6)$

**Maple [A]** time = 0.021, size = 84, normalized size = 0.9

$$-\frac{1}{8a^2x^8} + \frac{b}{3a^3x^6} - \frac{3b^2}{4a^4x^4} + 2\frac{b^3}{a^5x^2} + \frac{b^4}{2a^5(bx^2+a)} + 5\frac{b^4 \ln(x)}{a^6} - \frac{5b^4 \ln(bx^2+a)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^9/(b*x^2+a)^2,x)`

[Out] 
$$-1/8/a^2/x^8 + 1/3*b/a^3/x^6 - 3/4*b^2/a^4/x^4 + 2*b^3/a^5/x^2 + 1/2*b^4/a^5/(b*x^2+a) + 5*b^4*\ln(x)/a^6 - 5/2*b^4*\ln(b*x^2+a)/a^6$$

**Maxima [A]** time = 1.34696, size = 124, normalized size = 1.33

$$\frac{60b^4x^8 + 30ab^3x^6 - 10a^2b^2x^4 + 5a^3bx^2 - 3a^4}{24(a^5bx^{10} + a^6x^8)} - \frac{5b^4\log(bx^2 + a)}{2a^6} + \frac{5b^4\log(x^2)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^2*x^9),x, algorithm="maxima")`

[Out] 
$$1/24*(60*b^4*x^8 + 30*a*b^3*x^6 - 10*a^2*b^2*x^4 + 5*a^3*b*x^2 - 3*a^4)/(a^5*b*x^{10} + a^6*x^8) - 5/2*b^4*\log(b*x^2 + a)/a^6 + 5/2*b^4*\log(x^2)/a^6$$

**Fricas [A]** time = 0.204681, size = 151, normalized size = 1.62

$$\frac{60ab^4x^8 + 30a^2b^3x^6 - 10a^3b^2x^4 + 5a^4bx^2 - 3a^5 - 60(b^5x^{10} + ab^4x^8)\log(bx^2 + a) + 120(b^5x^{10} + ab^4x^8)\log(x)}{24(a^6bx^{10} + a^7x^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^2*x^9),x, algorithm="fricas")`

[Out] 
$$1/24*(60*a*b^4*x^8 + 30*a^2*b^3*x^6 - 10*a^3*b^2*x^4 + 5*a^4*b*x^2 - 3*a^5 - 60*(b^5*x^{10} + a*b^4*x^8)*\log(b*x^2 + a) + 120*(b^5*x^{10} + a*b^4*x^8)*\log(x))/(a^6*b*x^{10} + a^7*x^8)$$

**Sympy [A]** time = 5.71611, size = 94, normalized size = 1.01

$$\frac{-3a^4 + 5a^3bx^2 - 10a^2b^2x^4 + 30ab^3x^6 + 60b^4x^8}{24a^6x^8 + 24a^5bx^{10}} + \frac{5b^4\log(x)}{a^6} - \frac{5b^4\log\left(\frac{a}{b} + x^2\right)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**9/(b*x**2+a)**2,x)`

[Out] 
$$(-3*a**4 + 5*a**3*b*x**2 - 10*a**2*b**2*x**4 + 30*a*b**3*x**6 + 60*b**4*x**8)/(24*a**6*x**8 + 24*a**5*b*x**10) + 5*b**4*\log(x)/a**6 - 5*b**4*\log(a/b + x**2)/(2*a**6)$$

**GIAC/XCAS [A]** time = 0.211428, size = 149, normalized size = 1.6

$$\frac{5b^4\ln(x^2)}{2a^6} - \frac{5b^4\ln(|bx^2 + a|)}{2a^6} + \frac{5b^5x^2 + 6ab^4}{2(bx^2 + a)a^6} - \frac{125b^4x^8 - 48ab^3x^6 + 18a^2b^2x^4 - 8a^3bx^2 + 3a^4}{24a^6x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^2*x^9),x, algorithm="giac")`

```
[Out] 5/2*b^4*ln(x^2)/a^6 - 5/2*b^4*ln(abs(b*x^2 + a))/a^6 + 1/2*(5*b^5
*x^2 + 6*a*b^4)/((b*x^2 + a)*a^6) - 1/24*(125*b^4*x^8 - 48*a*b^3*
x^6 + 18*a^2*b^2*x^4 - 8*a^3*b*x^2 + 3*a^4)/(a^6*x^8)
```

$$3.168 \quad \int \frac{x^{15}}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=114

$$\frac{a^7}{4b^8(a+bx^2)^2} - \frac{7a^6}{2b^8(a+bx^2)} - \frac{21a^5 \log(a+bx^2)}{2b^8} + \frac{15a^4x^2}{2b^7} - \frac{5a^3x^4}{2b^6} + \frac{a^2x^6}{b^5} - \frac{3ax^8}{8b^4} + \frac{x^{10}}{10b^3}$$

[Out]  $(15*a^4*x^2)/(2*b^7) - (5*a^3*x^4)/(2*b^6) + (a^2*x^6)/b^5 - (3*a*x^8)/(8*b^4) + x^{10}/(10*b^3) + a^7/(4*b^8*(a + b*x^2)^2) - (7*a^6)/(2*b^8*(a + b*x^2)) - (21*a^5*Log[a + b*x^2])/(2*b^8)$

**Rubi [A]** time = 0.231027, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^7}{4b^8(a+bx^2)^2} - \frac{7a^6}{2b^8(a+bx^2)} - \frac{21a^5 \log(a+bx^2)}{2b^8} + \frac{15a^4x^2}{2b^7} - \frac{5a^3x^4}{2b^6} + \frac{a^2x^6}{b^5} - \frac{3ax^8}{8b^4} + \frac{x^{10}}{10b^3}$$

Antiderivative was successfully verified.

[In] Int[x^15/(a + b\*x^2)^3, x]

[Out]  $(15*a^4*x^2)/(2*b^7) - (5*a^3*x^4)/(2*b^6) + (a^2*x^6)/b^5 - (3*a*x^8)/(8*b^4) + x^{10}/(10*b^3) + a^7/(4*b^8*(a + b*x^2)^2) - (7*a^6)/(2*b^8*(a + b*x^2)) - (21*a^5*Log[a + b*x^2])/(2*b^8)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^7}{4b^8(a+bx^2)^2} - \frac{7a^6}{2b^8(a+bx^2)} - \frac{21a^5 \log(a+bx^2)}{2b^8} + \frac{15a^4x^2}{2b^7} - \frac{5a^3 \int^{x^2} x dx}{b^6} + \frac{a^2x^6}{b^5} - \frac{3ax^8}{8b^4} + \frac{x^{10}}{10b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*15/(b\*x\*\*2+a)\*\*3, x)

[Out]  $a**7/(4*b**8*(a + b*x**2)**2) - 7*a**6/(2*b**8*(a + b*x**2)) - 21*a**5*log(a + b*x**2)/(2*b**8) + 15*a**4*x**2/(2*b**7) - 5*a**3*Integral(x, (x, x**2))/b**6 + a**2*x**6/b**5 - 3*a*x**8/(8*b**4) + x**10/(10*b**3)$

**Mathematica [A]** time = 0.0549302, size = 97, normalized size = 0.85

$$\frac{\frac{10a^7}{(a+bx^2)^2} - \frac{140a^6}{a+bx^2} - 420a^5 \log(a+bx^2) + 300a^4bx^2 - 100a^3b^2x^4 + 40a^2b^3x^6 - 15ab^4x^8 + 4b^5x^{10}}{40b^8}$$

Antiderivative was successfully verified.

[In] Integrate[x^15/(a + b\*x^2)^3, x]

[Out]  $(300*a^4*b*x^2 - 100*a^3*b^2*x^4 + 40*a^2*b^3*x^6 - 15*a*b^4*x^8 + 4*b^5*x^{10} + (10*a^7)/(a + b*x^2)^2 - (140*a^6)/(a + b*x^2) - 420*a^5*Log[a + b*x^2])/(40*b^8)$



**Maple [A]** time = 0.016, size = 101, normalized size = 0.9

$$\frac{15a^4x^2}{2b^7} - \frac{5a^3x^4}{2b^6} + \frac{a^2x^6}{b^5} - \frac{3ax^8}{8b^4} + \frac{x^{10}}{10b^3} + \frac{a^7}{4b^8(bx^2+a)^2} - \frac{7a^6}{2b^8(bx^2+a)} - \frac{21a^5 \ln(bx^2+a)}{2b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^15/(b\*x^2+a)^3, x)

[Out] 15/2\*a^4\*x^2/b^7-5/2\*a^3\*x^4/b^6+a^2\*x^6/b^5-3/8\*a\*x^8/b^4+1/10\*x^10/b^3+1/4\*a^7/b^8/(b\*x^2+a)^2-7/2\*a^6/b^8/(b\*x^2+a)-21/2\*a^5\*ln(b\*x^2+a)/b^8

**Maxima [A]** time = 1.34626, size = 150, normalized size = 1.32

$$\frac{14a^6bx^2 + 13a^7}{4(b^{10}x^4 + 2ab^9x^2 + a^2b^8)} - \frac{21a^5 \log(bx^2 + a)}{2b^8} + \frac{4b^4x^{10} - 15ab^3x^8 + 40a^2b^2x^6 - 100a^3bx^4 + 300a^4x^2}{40b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^15/(b\*x^2 + a)^3, x, algorithm="maxima")

[Out] -1/4\*(14\*a^6\*b\*x^2 + 13\*a^7)/(b^10\*x^4 + 2\*a\*b^9\*x^2 + a^2\*b^8) - 21/2\*a^5\*log(b\*x^2 + a)/b^8 + 1/40\*(4\*b^4\*x^10 - 15\*a\*b^3\*x^8 + 40\*a^2\*b^2\*x^6 - 100\*a^3\*b\*x^4 + 300\*a^4\*x^2)/b^7

**Fricas [A]** time = 0.199038, size = 185, normalized size = 1.62

$$\frac{4b^7x^{14} - 7ab^6x^{12} + 14a^2b^5x^{10} - 35a^3b^4x^8 + 140a^4b^3x^6 + 500a^5b^2x^4 + 160a^6bx^2 - 130a^7 - 420(a^5b^2x^4 + 2a^6bx^2 + a^7)}{40(b^{10}x^4 + 2ab^9x^2 + a^2b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^15/(b\*x^2 + a)^3, x, algorithm="fricas")

[Out] 1/40\*(4\*b^7\*x^14 - 7\*a\*b^6\*x^12 + 14\*a^2\*b^5\*x^10 - 35\*a^3\*b^4\*x^8 + 140\*a^4\*b^3\*x^6 + 500\*a^5\*b^2\*x^4 + 160\*a^6\*b\*x^2 - 130\*a^7 - 420\*(a^5\*b^2\*x^4 + 2\*a^6\*b\*x^2 + a^7)\*log(b\*x^2 + a))/(b^10\*x^4 + 2\*a\*b^9\*x^2 + a^2\*b^8)

**Sympy [A]** time = 2.40879, size = 117, normalized size = 1.03

$$-\frac{21a^5 \log(a + bx^2)}{2b^8} + \frac{15a^4x^2}{2b^7} - \frac{5a^3x^4}{2b^6} + \frac{a^2x^6}{b^5} - \frac{3ax^8}{8b^4} - \frac{13a^7 + 14a^6bx^2}{4a^2b^8 + 8ab^9x^2 + 4b^{10}x^4} + \frac{x^{10}}{10b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*15/(b\*x\*\*2+a)\*\*3, x)

[Out] -21\*a\*\*5\*log(a + b\*x\*\*2)/(2\*b\*\*8) + 15\*a\*\*4\*x\*\*2/(2\*b\*\*7) - 5\*a\*\*3\*x\*\*4/(2\*b\*\*6) + a\*\*2\*x\*\*6/b\*\*5 - 3\*a\*x\*\*8/(8\*b\*\*4) - (13\*a\*\*7 + 14\*a\*\*6\*b\*x\*\*2)/(4\*a\*\*2\*b\*\*8 + 8\*a\*b\*\*9\*x\*\*2 + 4\*b\*\*10\*x\*\*4) + x\*\*10/(10\*b\*\*3)

**GIAC/XCAS [A]** time = 0.211518, size = 154, normalized size = 1.35

$$-\frac{21 a^5 \ln(|bx^2 + a|)}{2 b^8} + \frac{63 a^5 b^2 x^4 + 112 a^6 b x^2 + 50 a^7}{4 (bx^2 + a)^2 b^8} + \frac{4 b^{12} x^{10} - 15 a b^{11} x^8 + 40 a^2 b^{10} x^6 - 100 a^3 b^9 x^4 + 300 a^4 b^8 x^2}{40 b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^15/(b\*x^2 + a)^3,x, algorithm="giac")

[Out] -21/2\*a^5\*ln(abs(b\*x^2 + a))/b^8 + 1/4\*(63\*a^5\*b^2\*x^4 + 112\*a^6\*b\*x^2 + 50\*a^7)/((b\*x^2 + a)^2\*b^8) + 1/40\*(4\*b^12\*x^10 - 15\*a\*b^11\*x^8 + 40\*a^2\*b^10\*x^6 - 100\*a^3\*b^9\*x^4 + 300\*a^4\*b^8\*x^2)/b^15

$$3.169 \quad \int \frac{x^{13}}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=100

$$-\frac{a^6}{4b^7(a+bx^2)^2} + \frac{3a^5}{b^7(a+bx^2)} + \frac{15a^4 \log(a+bx^2)}{2b^7} - \frac{5a^3x^2}{b^6} + \frac{3a^2x^4}{2b^5} - \frac{ax^6}{2b^4} + \frac{x^8}{8b^3}$$

[Out]  $(-5*a^3*x^2)/b^6 + (3*a^2*x^4)/(2*b^5) - (a*x^6)/(2*b^4) + x^8/(8*b^3) - a^6/(4*b^7*(a + b*x^2)^2) + (3*a^5)/(b^7*(a + b*x^2)) + (15*a^4*Log[a + b*x^2])/(2*b^7)$

**Rubi [A]** time = 0.197011, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^6}{4b^7(a+bx^2)^2} + \frac{3a^5}{b^7(a+bx^2)} + \frac{15a^4 \log(a+bx^2)}{2b^7} - \frac{5a^3x^2}{b^6} + \frac{3a^2x^4}{2b^5} - \frac{ax^6}{2b^4} + \frac{x^8}{8b^3}$$

Antiderivative was successfully verified.

[In] Int[x^13/(a + b\*x^2)^3, x]

[Out]  $(-5*a^3*x^2)/b^6 + (3*a^2*x^4)/(2*b^5) - (a*x^6)/(2*b^4) + x^8/(8*b^3) - a^6/(4*b^7*(a + b*x^2)^2) + (3*a^5)/(b^7*(a + b*x^2)) + (15*a^4*Log[a + b*x^2])/(2*b^7)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^6}{4b^7(a+bx^2)^2} + \frac{3a^5}{b^7(a+bx^2)} + \frac{15a^4 \log(a+bx^2)}{2b^7} - \frac{5a^3x^2}{b^6} + \frac{3a^2 \int^{x^2} x dx}{b^5} - \frac{ax^6}{2b^4} + \frac{x^8}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*13/(b\*x\*\*2+a)\*\*3, x)

[Out]  $-a**6/(4*b**7*(a + b*x**2)**2) + 3*a**5/(b**7*(a + b*x**2)) + 15*a**4*log(a + b*x**2)/(2*b**7) - 5*a**3*x**2/b**6 + 3*a**2*Integral(x, (x, x**2))/b**5 - a*x**6/(2*b**4) + x**8/(8*b**3)$

**Mathematica [A]** time = 0.0538615, size = 85, normalized size = 0.85

$$\frac{-\frac{2a^6}{(a+bx^2)^2} + \frac{24a^5}{a+bx^2} + 60a^4 \log(a+bx^2) - 40a^3bx^2 + 12a^2b^2x^4 - 4ab^3x^6 + b^4x^8}{8b^7}$$

Antiderivative was successfully verified.

[In] Integrate[x^13/(a + b\*x^2)^3, x]

[Out]  $(-40*a^3*b*x^2 + 12*a^2*b^2*x^4 - 4*a*b^3*x^6 + b^4*x^8 - (2*a^6)/(a + b*x^2)^2 + (24*a^5)/(a + b*x^2) + 60*a^4*Log[a + b*x^2])/(8*b^7)$

**Maple [A]** time = 0.016, size = 91, normalized size = 0.9

$$-5 \frac{a^3x^2}{b^6} + \frac{3a^2x^4}{2b^5} - \frac{ax^6}{2b^4} + \frac{x^8}{8b^3} - \frac{a^6}{4b^7(bx^2+a)^2} + 3 \frac{a^5}{b^7(bx^2+a)} + \frac{15a^4 \ln(bx^2+a)}{2b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^13/(b*x^2+a)^3,x)`

[Out]  $-5*a^3*x^2/b^6+3/2*a^2*x^4/b^5-1/2*a*x^6/b^4+1/8*x^8/b^3-1/4*a^6/b^7/(b*x^2+a)^2+3*a^5/b^7/(b*x^2+a)+15/2*a^4*\ln(b*x^2+a)/b^7$

**Maxima [A]** time = 1.35931, size = 134, normalized size = 1.34

$$\frac{12a^5bx^2 + 11a^6}{4(b^9x^4 + 2ab^8x^2 + a^2b^7)} + \frac{15a^4 \log(bx^2 + a)}{2b^7} + \frac{b^3x^8 - 4ab^2x^6 + 12a^2bx^4 - 40a^3x^2}{8b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^13/(b*x^2 + a)^3,x, algorithm="maxima")`

[Out]  $1/4*(12*a^5*b*x^2 + 11*a^6)/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7) + 15/2*a^4*\log(b*x^2 + a)/b^7 + 1/8*(b^3*x^8 - 4*a*b^2*x^6 + 12*a^2*b*x^4 - 40*a^3*x^2)/b^6$

**Fricas [A]** time = 0.199089, size = 169, normalized size = 1.69

$$\frac{b^6x^{12} - 2ab^5x^{10} + 5a^2b^4x^8 - 20a^3b^3x^6 - 68a^4b^2x^4 - 16a^5bx^2 + 22a^6 + 60(a^4b^2x^4 + 2a^5bx^2 + a^6) \log(bx^2 + a)}{8(b^9x^4 + 2ab^8x^2 + a^2b^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^13/(b*x^2 + a)^3,x, algorithm="fricas")`

[Out]  $1/8*(b^6*x^12 - 2*a*b^5*x^10 + 5*a^2*b^4*x^8 - 20*a^3*b^3*x^6 - 68*a^4*b^2*x^4 - 16*a^5*b*x^2 + 22*a^6 + 60*(a^4*b^2*x^4 + 2*a^5*b*x^2 + a^6)*\log(b*x^2 + a))/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7)$

**Sympy [A]** time = 2.33866, size = 104, normalized size = 1.04

$$\frac{15a^4 \log(a + bx^2)}{2b^7} - \frac{5a^3x^2}{b^6} + \frac{3a^2x^4}{2b^5} - \frac{ax^6}{2b^4} + \frac{11a^6 + 12a^5bx^2}{4a^2b^7 + 8ab^8x^2 + 4b^9x^4} + \frac{x^8}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**13/(b*x**2+a)**3,x)`

[Out]  $15*a**4*\log(a + b*x**2)/(2*b**7) - 5*a**3*x**2/b**6 + 3*a**2*x**4/(2*b**5) - a*x**6/(2*b**4) + (11*a**6 + 12*a**5*b*x**2)/(4*a**2*b**7 + 8*a*b**8*x**2 + 4*b**9*x**4) + x**8/(8*b**3)$

**GIAC/XCAS [A]** time = 0.207434, size = 138, normalized size = 1.38

$$\frac{15a^4 \ln(|bx^2 + a|)}{2b^7} - \frac{45a^4b^2x^4 + 78a^5bx^2 + 34a^6}{4(bx^2 + a)^2b^7} + \frac{b^9x^8 - 4ab^8x^6 + 12a^2b^7x^4 - 40a^3b^6x^2}{8b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^13/(b*x^2 + a)^3,x, algorithm="giac")`

```
[Out] 15/2*a^4*ln(abs(b*x^2 + a))/b^7 - 1/4*(45*a^4*b^2*x^4 + 78*a^5*b*  
x^2 + 34*a^6)/((b*x^2 + a)^2*b^7) + 1/8*(b^9*x^8 - 4*a*b^8*x^6 +  
12*a^2*b^7*x^4 - 40*a^3*b^6*x^2)/b^12
```

$$3.170 \quad \int \frac{x^{11}}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=87

$$\frac{a^5}{4b^6(a+bx^2)^2} - \frac{5a^4}{2b^6(a+bx^2)} - \frac{5a^3 \log(a+bx^2)}{b^6} + \frac{3a^2x^2}{b^5} - \frac{3ax^4}{4b^4} + \frac{x^6}{6b^3}$$

[Out] (3\*a^2\*x^2)/b^5 - (3\*a\*x^4)/(4\*b^4) + x^6/(6\*b^3) + a^5/(4\*b^6\*(a + b\*x^2)^2) - (5\*a^4)/(2\*b^6\*(a + b\*x^2)) - (5\*a^3\*Log[a + b\*x^2])/b^6

**Rubi [A]** time = 0.166276, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^5}{4b^6(a+bx^2)^2} - \frac{5a^4}{2b^6(a+bx^2)} - \frac{5a^3 \log(a+bx^2)}{b^6} + \frac{3a^2x^2}{b^5} - \frac{3ax^4}{4b^4} + \frac{x^6}{6b^3}$$

Antiderivative was successfully verified.

[In] Int[x^11/(a + b\*x^2)^3, x]

[Out] (3\*a^2\*x^2)/b^5 - (3\*a\*x^4)/(4\*b^4) + x^6/(6\*b^3) + a^5/(4\*b^6\*(a + b\*x^2)^2) - (5\*a^4)/(2\*b^6\*(a + b\*x^2)) - (5\*a^3\*Log[a + b\*x^2])/b^6

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^5}{4b^6(a+bx^2)^2} - \frac{5a^4}{2b^6(a+bx^2)} - \frac{5a^3 \log(a+bx^2)}{b^6} + \frac{3a^2x^2}{b^5} - \frac{3a \int^{x^2} x dx}{2b^4} + \frac{x^6}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*11/(b\*x\*\*2+a)\*\*3, x)

[Out] a\*\*5/(4\*b\*\*6\*(a + b\*x\*\*2)\*\*2) - 5\*a\*\*4/(2\*b\*\*6\*(a + b\*x\*\*2)) - 5\*a\*\*3\*log(a + b\*x\*\*2)/b\*\*6 + 3\*a\*\*2\*x\*\*2/b\*\*5 - 3\*a\*Integral(x, (x , x\*\*2))/(2\*b\*\*4) + x\*\*6/(6\*b\*\*3)

**Mathematica [A]** time = 0.0437977, size = 75, normalized size = 0.86

$$\frac{\frac{3a^5}{(a+bx^2)^2} - \frac{30a^4}{a+bx^2} - 60a^3 \log(a+bx^2) + 36a^2bx^2 - 9ab^2x^4 + 2b^3x^6}{12b^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/(a + b\*x^2)^3, x]

[Out] (36\*a^2\*b\*x^2 - 9\*a\*b^2\*x^4 + 2\*b^3\*x^6 + (3\*a^5)/(a + b\*x^2)^2 - (30\*a^4)/(a + b\*x^2) - 60\*a^3\*Log[a + b\*x^2])/(12\*b^6)

**Maple [A]** time = 0.015, size = 80, normalized size = 0.9

$$3 \frac{a^2x^2}{b^5} - \frac{3ax^4}{4b^4} + \frac{x^6}{6b^3} + \frac{a^5}{4b^6(bx^2+a)^2} - \frac{5a^4}{2b^6(bx^2+a)} - 5 \frac{a^3 \ln(bx^2+a)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11/(b*x^2+a)^3,x)`

[Out]  $3*a^2*x^2/b^5 - 3/4*a*x^4/b^4 + 1/6*x^6/b^3 + 1/4*a^5/b^6/(b*x^2+a)^2 - 5/2*a^4/b^6/(b*x^2+a) - 5*a^3*\ln(b*x^2+a)/b^6$

**Maxima [A]** time = 1.35859, size = 120, normalized size = 1.38

$$-\frac{10a^4bx^2 + 9a^5}{4(b^8x^4 + 2ab^7x^2 + a^2b^6)} - \frac{5a^3 \log(bx^2 + a)}{b^6} + \frac{2b^2x^6 - 9abx^4 + 36a^2x^2}{12b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(b*x^2 + a)^3,x, algorithm="maxima")`

[Out]  $-1/4*(10*a^4*b*x^2 + 9*a^5)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6) - 5*a^3*\log(b*x^2 + a)/b^6 + 1/12*(2*b^2*x^6 - 9*a*b*x^4 + 36*a^2*x^2)/b^5$

**Fricas [A]** time = 0.206862, size = 155, normalized size = 1.78

$$\frac{2b^5x^{10} - 5ab^4x^8 + 20a^2b^3x^6 + 63a^3b^2x^4 + 6a^4bx^2 - 27a^5 - 60(a^3b^2x^4 + 2a^4bx^2 + a^5) \log(bx^2 + a)}{12(b^8x^4 + 2ab^7x^2 + a^2b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(b*x^2 + a)^3,x, algorithm="fricas")`

[Out]  $1/12*(2*b^5*x^{10} - 5*a*b^4*x^8 + 20*a^2*b^3*x^6 + 63*a^3*b^2*x^4 + 6*a^4*b*x^2 - 27*a^5 - 60*(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)*\log(b*x^2 + a))/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6)$

**Sympy [A]** time = 2.24815, size = 90, normalized size = 1.03

$$-\frac{5a^3 \log(a + bx^2)}{b^6} + \frac{3a^2x^2}{b^5} - \frac{3ax^4}{4b^4} - \frac{9a^5 + 10a^4bx^2}{4a^2b^6 + 8ab^7x^2 + 4b^8x^4} + \frac{x^6}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**11/(b*x**2+a)**3,x)`

[Out]  $-5*a**3*\log(a + b*x**2)/b**6 + 3*a**2*x**2/b**5 - 3*a*x**4/(4*b**4) - (9*a**5 + 10*a**4*b*x**2)/(4*a**2*b**6 + 8*a*b**7*x**2 + 4*b**8*x**4) + x**6/(6*b**3)$

**GIAC/XCAS [A]** time = 0.213021, size = 124, normalized size = 1.43

$$-\frac{5a^3 \ln(|bx^2 + a|)}{b^6} + \frac{30a^3b^2x^4 + 50a^4bx^2 + 21a^5}{4(bx^2 + a)^2b^6} + \frac{2b^6x^6 - 9ab^5x^4 + 36a^2b^4x^2}{12b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(b*x^2 + a)^3,x, algorithm="giac")`

```
[Out] -5*a^3*ln(abs(b*x^2 + a))/b^6 + 1/4*(30*a^3*b^2*x^4 + 50*a^4*b*x^2 + 21*a^5)/((b*x^2 + a)^2*b^6) + 1/12*(2*b^6*x^6 - 9*a*b^5*x^4 + 36*a^2*b^4*x^2)/b^9
```



$$3.171 \quad \int \frac{x^9}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=74

$$-\frac{a^4}{4b^5(a+bx^2)^2} + \frac{2a^3}{b^5(a+bx^2)} + \frac{3a^2 \log(a+bx^2)}{b^5} - \frac{3ax^2}{2b^4} + \frac{x^4}{4b^3}$$

[Out]  $(-3*a*x^2)/(2*b^4) + x^4/(4*b^3) - a^4/(4*b^5*(a + b*x^2)^2) + (2*a^3)/(b^5*(a + b*x^2)) + (3*a^2*Log[a + b*x^2])/b^5$

**Rubi [A]** time = 0.140141, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^4}{4b^5(a+bx^2)^2} + \frac{2a^3}{b^5(a+bx^2)} + \frac{3a^2 \log(a+bx^2)}{b^5} - \frac{3ax^2}{2b^4} + \frac{x^4}{4b^3}$$

Antiderivative was successfully verified.

[In] Int[x^9/(a + b\*x^2)^3, x]

[Out]  $(-3*a*x^2)/(2*b^4) + x^4/(4*b^3) - a^4/(4*b^5*(a + b*x^2)^2) + (2*a^3)/(b^5*(a + b*x^2)) + (3*a^2*Log[a + b*x^2])/b^5$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a^4}{4b^5(a+bx^2)^2} + \frac{2a^3}{b^5(a+bx^2)} + \frac{3a^2 \log(a+bx^2)}{b^5} - \frac{3ax^2}{2b^4} + \frac{\int x^2 x dx}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*9/(b\*x\*\*2+a)\*\*3, x)

[Out]  $-a**4/(4*b**5*(a + b*x**2)**2) + 2*a**3/(b**5*(a + b*x**2)) + 3*a**2*log(a + b*x**2)/b**5 - 3*a*x**2/(2*b**4) + Integral(x, (x, x**2))/(2*b**3)$

**Mathematica [A]** time = 0.0444069, size = 63, normalized size = 0.85

$$\frac{-\frac{a^4}{(a+bx^2)^2} + \frac{8a^3}{a+bx^2} + 12a^2 \log(a+bx^2) - 6abx^2 + b^2x^4}{4b^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^9/(a + b\*x^2)^3, x]

[Out]  $(-6*a*b*x^2 + b^2*x^4 - a^4/(a + b*x^2)^2 + (8*a^3)/(a + b*x^2) + 12*a^2*Log[a + b*x^2])/(4*b^5)$

**Maple [A]** time = 0.014, size = 69, normalized size = 0.9

$$-\frac{3ax^2}{2b^4} + \frac{x^4}{4b^3} - \frac{a^4}{4b^5(bx^2+a)^2} + 2\frac{a^3}{b^5(bx^2+a)} + 3\frac{a^2 \ln(bx^2+a)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9/(b*x^2+a)^3,x)`

[Out]  $-3/2*a*x^2/b^4+1/4*x^4/b^3-1/4*a^4/b^5/(b*x^2+a)^2+2*a^3/b^5/(b*x^2+a)+3*a^2*ln(b*x^2+a)/b^5$

**Maxima [A]** time = 1.34629, size = 104, normalized size = 1.41

$$\frac{8a^3bx^2 + 7a^4}{4(b^7x^4 + 2ab^6x^2 + a^2b^5)} + \frac{3a^2 \log(bx^2 + a)}{b^5} + \frac{bx^4 - 6ax^2}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9/(b*x^2 + a)^3,x, algorithm="maxima")`

[Out]  $1/4*(8*a^3*b*x^2 + 7*a^4)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5) + 3*a^2*log(b*x^2 + a)/b^5 + 1/4*(b*x^4 - 6*a*x^2)/b^4$

**Fricas [A]** time = 0.215265, size = 139, normalized size = 1.88

$$\frac{b^4x^8 - 4ab^3x^6 - 11a^2b^2x^4 + 2a^3bx^2 + 7a^4 + 12(a^2b^2x^4 + 2a^3bx^2 + a^4) \log(bx^2 + a)}{4(b^7x^4 + 2ab^6x^2 + a^2b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9/(b*x^2 + a)^3,x, algorithm="fricas")`

[Out]  $1/4*(b^4*x^8 - 4*a*b^3*x^6 - 11*a^2*b^2*x^4 + 2*a^3*b*x^2 + 7*a^4 + 12*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*log(b*x^2 + a))/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5)$

**Sympy [A]** time = 2.14198, size = 78, normalized size = 1.05

$$\frac{3a^2 \log(a + bx^2)}{b^5} - \frac{3ax^2}{2b^4} + \frac{7a^4 + 8a^3bx^2}{4a^2b^5 + 8ab^6x^2 + 4b^7x^4} + \frac{x^4}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9/(b*x**2+a)**3,x)`

[Out]  $3*a**2*log(a + b*x**2)/b**5 - 3*a*x**2/(2*b**4) + (7*a**4 + 8*a**3*b*x**2)/(4*a**2*b**5 + 8*a*b**6*x**2 + 4*b**7*x**4) + x**4/(4*b**3)$

**GIAC/XCAS [A]** time = 0.214259, size = 108, normalized size = 1.46

$$\frac{3a^2 \ln(|bx^2 + a|)}{b^5} + \frac{b^3x^4 - 6ab^2x^2}{4b^6} - \frac{18a^2b^2x^4 + 28a^3bx^2 + 11a^4}{4(bx^2 + a)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9/(b*x^2 + a)^3,x, algorithm="giac")`

[Out]  $3*a^2*ln(abs(b*x^2 + a))/b^5 + 1/4*(b^3*x^4 - 6*a*b^2*x^2)/b^6 - 1/4*(18*a^2*b^2*x^4 + 28*a^3*b*x^2 + 11*a^4)/((b*x^2 + a)^2*b^5)$

$$3.172 \quad \int \frac{x^7}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=65

$$\frac{a^3}{4b^4(a+bx^2)^2} - \frac{3a^2}{2b^4(a+bx^2)} - \frac{3a \log(a+bx^2)}{2b^4} + \frac{x^2}{2b^3}$$

[Out]  $x^2/(2*b^3) + a^3/(4*b^4*(a + b*x^2)^2) - (3*a^2)/(2*b^4*(a + b*x^2)) - (3*a*Log[a + b*x^2])/(2*b^4)$

**Rubi [A]** time = 0.115182, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^3}{4b^4(a+bx^2)^2} - \frac{3a^2}{2b^4(a+bx^2)} - \frac{3a \log(a+bx^2)}{2b^4} + \frac{x^2}{2b^3}$$

Antiderivative was successfully verified.

[In] Int[x^7/(a + b\*x^2)^3, x]

[Out]  $x^2/(2*b^3) + a^3/(4*b^4*(a + b*x^2)^2) - (3*a^2)/(2*b^4*(a + b*x^2)) - (3*a*Log[a + b*x^2])/(2*b^4)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^3}{4b^4(a+bx^2)^2} - \frac{3a^2}{2b^4(a+bx^2)} - \frac{3a \log(a+bx^2)}{2b^4} + \frac{\int^{x^2} \frac{1}{b^3} dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*7/(b\*x\*\*2+a)\*\*3, x)

[Out]  $a**3/(4*b**4*(a + b*x**2)**2) - 3*a**2/(2*b**4*(a + b*x**2)) - 3*a*log(a + b*x**2)/(2*b**4) + Integral(b**(-3), (x, x**2))/2$

**Mathematica [A]** time = 0.110546, size = 48, normalized size = 0.74

$$\frac{\frac{a^2(5a+6bx^2)}{(a+bx^2)^2} + 6a \log(a+bx^2) - 2bx^2}{4b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a + b\*x^2)^3, x]

[Out]  $-(-2*b*x^2 + (a^2*(5*a + 6*b*x^2)))/(a + b*x^2)^2 + 6*a*Log[a + b*x^2])/(4*b^4)$

**Maple [A]** time = 0.013, size = 58, normalized size = 0.9

$$\frac{x^2}{2b^3} + \frac{a^3}{4b^4(bx^2+a)^2} - \frac{3a^2}{2b^4(bx^2+a)} - \frac{3a \ln(bx^2+a)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7/(b*x^2+a)^3,x)`

[Out]  $\frac{1}{2}x^2/b^3 + 1/4a^3/b^4/(b^2x^2+a)^2 - 3/2a^2/b^4/(b^2x^2+a) - 3/2a \ln(b^2x^2+a)/b^4$

**Maxima [A]** time = 1.34784, size = 89, normalized size = 1.37

$$-\frac{6a^2bx^2 + 5a^3}{4(b^6x^4 + 2ab^5x^2 + a^2b^4)} + \frac{x^2}{2b^3} - \frac{3a \log(bx^2 + a)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^2 + a)^3,x, algorithm="maxima")`

[Out]  $-1/4*(6*a^2*b*x^2 + 5*a^3)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4) + 1/2*x^2/b^3 - 3/2*a \log(b*x^2 + a)/b^4$

**Fricas [A]** time = 0.212246, size = 123, normalized size = 1.89

$$\frac{2b^3x^6 + 4ab^2x^4 - 4a^2bx^2 - 5a^3 - 6(ab^2x^4 + 2a^2bx^2 + a^3) \log(bx^2 + a)}{4(b^6x^4 + 2ab^5x^2 + a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^2 + a)^3,x, algorithm="fricas")`

[Out]  $1/4*(2*b^3*x^6 + 4*a*b^2*x^4 - 4*a^2*b*x^2 - 5*a^3 - 6*(a*b^2*x^4 + 2*a^2*b*x^2 + a^3)*\log(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)$

**Sympy [A]** time = 2.06438, size = 66, normalized size = 1.02

$$-\frac{3a \log(a + bx^2)}{2b^4} - \frac{5a^3 + 6a^2bx^2}{4a^2b^4 + 8ab^5x^2 + 4b^6x^4} + \frac{x^2}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7/(b*x**2+a)**3,x)`

[Out]  $-3*a \log(a + b*x**2)/(2*b**4) - (5*a**3 + 6*a**2*b*x**2)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) + x**2/(2*b**3)$

**GIAC/XCAS [A]** time = 0.213641, size = 84, normalized size = 1.29

$$\frac{x^2}{2b^3} - \frac{3a \ln(|bx^2 + a|)}{2b^4} + \frac{9ab^2x^4 + 12a^2bx^2 + 4a^3}{4(bx^2 + a)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^2 + a)^3,x, algorithm="giac")`

[Out]  $1/2*x^2/b^3 - 3/2*a \ln(\text{abs}(b*x^2 + a))/b^4 + 1/4*(9*a*b^2*x^4 + 12*a^2*b*x^2 + 4*a^3)/((b*x^2 + a)^2*b^4)$

$$3.173 \quad \int \frac{x^5}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=49

$$-\frac{a^2}{4b^3(a+bx^2)^2} + \frac{a}{b^3(a+bx^2)} + \frac{\log(a+bx^2)}{2b^3}$$

[Out]  $-a^2/(4*b^3*(a + b*x^2)^2) + a/(b^3*(a + b*x^2)) + \text{Log}[a + b*x^2]/(2*b^3)$

**Rubi [A]** time = 0.0943527, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^2}{4b^3(a+bx^2)^2} + \frac{a}{b^3(a+bx^2)} + \frac{\log(a+bx^2)}{2b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a + b\*x^2)^3, x]

[Out]  $-a^2/(4*b^3*(a + b*x^2)^2) + a/(b^3*(a + b*x^2)) + \text{Log}[a + b*x^2]/(2*b^3)$

**Rubi in Sympy [A]** time = 12.7735, size = 41, normalized size = 0.84

$$-\frac{a^2}{4b^3(a+bx^2)^2} + \frac{a}{b^3(a+bx^2)} + \frac{\log(a+bx^2)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5/(b\*x\*\*2+a)\*\*3, x)

[Out]  $-a**2/(4*b**3*(a + b*x**2)**2) + a/(b**3*(a + b*x**2)) + \log(a + b*x**2)/(2*b**3)$

**Mathematica [A]** time = 0.0283806, size = 39, normalized size = 0.8

$$\frac{\frac{a(3a+4bx^2)}{(a+bx^2)^2} + 2 \log(a+bx^2)}{4b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a + b\*x^2)^3, x]

[Out]  $((a*(3*a + 4*b*x^2))/(a + b*x^2)^2 + 2*\text{Log}[a + b*x^2])/(4*b^3)$

**Maple [A]** time = 0.013, size = 46, normalized size = 0.9

$$-\frac{a^2}{4b^3(bx^2+a)^2} + \frac{a}{b^3(bx^2+a)} + \frac{\ln(bx^2+a)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(b*x^2+a)^3,x)`

[Out]  $-1/4*a^2/b^3/(b*x^2+a)^2+a/b^3/(b*x^2+a)+1/2*\ln(b*x^2+a)/b^3$

**Maxima [A]** time = 1.35084, size = 74, normalized size = 1.51

$$\frac{4abx^2 + 3a^2}{4(b^5x^4 + 2ab^4x^2 + a^2b^3)} + \frac{\log(bx^2 + a)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2 + a)^3,x, algorithm="maxima")`

[Out]  $1/4*(4*a*b*x^2 + 3*a^2)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) + 1/2*\log(b*x^2 + a)/b^3$

**Fricas [A]** time = 0.22632, size = 93, normalized size = 1.9

$$\frac{4abx^2 + 3a^2 + 2(b^2x^4 + 2abx^2 + a^2)\log(bx^2 + a)}{4(b^5x^4 + 2ab^4x^2 + a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2 + a)^3,x, algorithm="fricas")`

[Out]  $1/4*(4*a*b*x^2 + 3*a^2 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*\log(b*x^2 + a))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)$

**Sympy [A]** time = 1.82636, size = 53, normalized size = 1.08

$$\frac{3a^2 + 4abx^2}{4a^2b^3 + 8ab^4x^2 + 4b^5x^4} + \frac{\log(a + bx^2)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(b*x**2+a)**3,x)`

[Out]  $(3*a**2 + 4*a*b*x**2)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + \log(a + b*x**2)/(2*b**3)$

**GIAC/XCAS [A]** time = 0.221898, size = 57, normalized size = 1.16

$$\frac{\ln(|bx^2 + a|)}{2b^3} - \frac{3bx^4 + 2ax^2}{4(bx^2 + a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2 + a)^3,x, algorithm="giac")`

[Out]  $1/2*\ln(\text{abs}(b*x^2 + a))/b^3 - 1/4*(3*b*x^4 + 2*a*x^2)/((b*x^2 + a)^2*b^2)$

$$3.174 \quad \int \frac{x^3}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=19

$$\frac{x^4}{4a(a+bx^2)^2}$$

[Out]  $x^4/(4*a*(a+b*x^2)^2)$

**Rubi [A]** time = 0.0195922, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{x^4}{4a(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3/(a+b*x^2)^3, x]$

[Out]  $x^4/(4*a*(a+b*x^2)^2)$

**Rubi in Sympy [A]** time = 3.43923, size = 14, normalized size = 0.74

$$\frac{x^4}{4a(a+bx^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**3/(b*x**2+a)**3, x)$

[Out]  $x**4/(4*a*(a+b*x**2)**2)$

**Mathematica [A]** time = 0.0124752, size = 24, normalized size = 1.26

$$-\frac{a+2bx^2}{4b^2(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^3/(a+b*x^2)^3, x]$

[Out]  $-(a+2*b*x^2)/(4*b^2*(a+b*x^2)^2)$

**Maple [A]** time = 0.01, size = 31, normalized size = 1.6

$$\frac{a}{4b^2(bx^2+a)^2} - \frac{1}{(2bx^2+2a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^3/(b*x^2+a)^3, x)$

[Out]  $1/4 * a/b^2/(b*x^2+a)^2 - 1/2/(b*x^2+a)/b^2$

**Maxima [A]** time = 1.35181, size = 49, normalized size = 2.58

$$-\frac{2bx^2 + a}{4(b^4x^4 + 2ab^3x^2 + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 + a)^3,x, algorithm="maxima")`

[Out]  $-1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)$

**Fricas [A]** time = 0.206779, size = 49, normalized size = 2.58

$$-\frac{2bx^2 + a}{4(b^4x^4 + 2ab^3x^2 + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 + a)^3,x, algorithm="fricas")`

[Out]  $-1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)$

**Sympy [A]** time = 1.6534, size = 36, normalized size = 1.89

$$-\frac{a + 2bx^2}{4a^2b^2 + 8ab^3x^2 + 4b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(b*x**2+a)**3,x)`

[Out]  $-(a + 2*b*x^2)/(4*a^2*b^2 + 8*a*b^3*x^2 + 4*b^4*x^4)$

**GIAC/XCAS [A]** time = 0.210187, size = 30, normalized size = 1.58

$$-\frac{2bx^2 + a}{4(bx^2 + a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 + a)^3,x, algorithm="giac")`

[Out]  $-1/4*(2*b*x^2 + a)/((b*x^2 + a)^2*b^2)$



$$3.175 \quad \int \frac{x}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=16

$$-\frac{1}{4b(a+bx^2)^2}$$

[Out]  $-1/(4*b*(a + b*x^2)^2)$

**Rubi [A]** time = 0.0109076, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{1}{4b(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x/(a + b*x^2)^3, x]$

[Out]  $-1/(4*b*(a + b*x^2)^2)$

**Rubi in Sympy [A]** time = 2.2242, size = 14, normalized size = 0.88

$$-\frac{1}{4b(a+bx^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x/(b*x**2+a)**3, x)$

[Out]  $-1/(4*b*(a + b*x**2)**2)$

**Mathematica [A]** time = 0.00447464, size = 16, normalized size = 1.

$$-\frac{1}{4b(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x/(a + b*x^2)^3, x]$

[Out]  $-1/(4*b*(a + b*x^2)^2)$

**Maple [A]** time = 0.002, size = 15, normalized size = 0.9

$$-\frac{1}{4b(bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x/(b*x^2+a)^3, x)$

[Out]  $-1/4/b/(b*x^2+a)^2$

---

**Maxima [A]** time = 1.34332, size = 19, normalized size = 1.19

$$-\frac{1}{4(bx^2 + a)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^3,x, algorithm="maxima")`

[Out] `-1/4/((b*x^2 + a)^2*b)`

---

**Fricas [A]** time = 0.222624, size = 35, normalized size = 2.19

$$-\frac{1}{4(b^3x^4 + 2ab^2x^2 + a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^3,x, algorithm="fricas")`

[Out] `-1/4/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)`

---

**Sympy [A]** time = 1.52871, size = 27, normalized size = 1.69

$$-\frac{1}{4a^2b + 8ab^2x^2 + 4b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x**2+a)**3,x)`

[Out] `-1/(4*a**2*b + 8*a*b**2*x**2 + 4*b**3*x**4)`

---

**GIAC/XCAS [A]** time = 0.210801, size = 19, normalized size = 1.19

$$-\frac{1}{4(bx^2 + a)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^3,x, algorithm="giac")`

[Out] `-1/4/((b*x^2 + a)^2*b)`

$$3.176 \quad \int \frac{1}{x(a+bx^2)^3} dx$$

**Optimal.** Leaf size=54

$$-\frac{\log(a+bx^2)}{2a^3} + \frac{\log(x)}{a^3} + \frac{1}{2a^2(a+bx^2)} + \frac{1}{4a(a+bx^2)^2}$$

[Out]  $1/(4*a*(a+b*x^2)^2) + 1/(2*a^2*(a+b*x^2)) + \text{Log}[x]/a^3 - \text{Log}[a+b*x^2]/(2*a^3)$

**Rubi [A]** time = 0.092766, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{\log(a+bx^2)}{2a^3} + \frac{\log(x)}{a^3} + \frac{1}{2a^2(a+bx^2)} + \frac{1}{4a(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(a+b\*x^2)^3),x]

[Out]  $1/(4*a*(a+b*x^2)^2) + 1/(2*a^2*(a+b*x^2)) + \text{Log}[x]/a^3 - \text{Log}[a+b*x^2]/(2*a^3)$

**Rubi in Sympy [A]** time = 12.3087, size = 49, normalized size = 0.91

$$\frac{1}{4a(a+bx^2)^2} + \frac{1}{2a^2(a+bx^2)} + \frac{\log(x^2)}{2a^3} - \frac{\log(a+bx^2)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(b\*x\*\*2+a)\*\*3,x)

[Out]  $1/(4*a*(a+b*x**2)**2) + 1/(2*a**2*(a+b*x**2)) + \log(x**2)/(2*a**3) - \log(a+b*x**2)/(2*a**3)$

**Mathematica [A]** time = 0.0590055, size = 43, normalized size = 0.8

$$\frac{\frac{a(3a+2bx^2)}{(a+bx^2)^2} - 2 \log(a+bx^2) + 4 \log(x)}{4a^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(a+b\*x^2)^3),x]

[Out]  $((a*(3*a+2*b*x^2))/(a+b*x^2)^2 + 4*\text{Log}[x] - 2*\text{Log}[a+b*x^2])/(4*a^3)$

**Maple [A]** time = 0.015, size = 49, normalized size = 0.9

$$\frac{1}{4a(bx^2+a)^2} + \frac{1}{2a^2(bx^2+a)} + \frac{\ln(x)}{a^3} - \frac{\ln(bx^2+a)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(b*x^2+a)^3,x)`

[Out]  $1/4/a/(b*x^2+a)^2+1/2/a^2/(b*x^2+a)+\ln(x)/a^3-1/2*\ln(b*x^2+a)/a^3$

**Maxima [A]** time = 1.35146, size = 81, normalized size = 1.5

$$\frac{2bx^2 + 3a}{4(a^2b^2x^4 + 2a^3bx^2 + a^4)} - \frac{\log(bx^2 + a)}{2a^3} + \frac{\log(x^2)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^3*x),x, algorithm="maxima")`

[Out]  $1/4*(2*b*x^2 + 3*a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) - 1/2*\log(b*x^2 + a)/a^3 + 1/2*\log(x^2)/a^3$

**Fricas [A]** time = 0.232427, size = 122, normalized size = 2.26

$$\frac{2abx^2 + 3a^2 - 2(b^2x^4 + 2abx^2 + a^2)\log(bx^2 + a) + 4(b^2x^4 + 2abx^2 + a^2)\log(x)}{4(a^3b^2x^4 + 2a^4bx^2 + a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^3*x),x, algorithm="fricas")`

[Out]  $1/4*(2*a*b*x^2 + 3*a^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*\log(b*x^2 + a) + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)*\log(x))/(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)$

**Sympy [A]** time = 2.18848, size = 56, normalized size = 1.04

$$\frac{3a + 2bx^2}{4a^4 + 8a^3bx^2 + 4a^2b^2x^4} + \frac{\log(x)}{a^3} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x**2+a)**3,x)`

[Out]  $(3*a + 2*b*x**2)/(4*a**4 + 8*a**3*b*x**2 + 4*a**2*b**2*x**4) + \log(x)/a**3 - \log(a/b + x**2)/(2*a**3)$

**GIAC/XCAS [A]** time = 0.218653, size = 80, normalized size = 1.48

$$\frac{\ln(x^2)}{2a^3} - \frac{\ln(|bx^2 + a|)}{2a^3} + \frac{3b^2x^4 + 8abx^2 + 6a^2}{4(bx^2 + a)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^3*x),x, algorithm="giac")`

[Out]  $1/2*\ln(x^2)/a^3 - 1/2*\ln(\text{abs}(b*x^2 + a))/a^3 + 1/4*(3*b^2*x^4 + 8*a*b*x^2 + 6*a^2)/((b*x^2 + a)^2*a^3)$

$$3.177 \quad \int \frac{1}{x^3(a+bx^2)^3} dx$$

**Optimal.** Leaf size=67

$$\frac{3b \log(a+bx^2)}{2a^4} - \frac{3b \log(x)}{a^4} - \frac{b}{a^3(a+bx^2)} - \frac{1}{2a^3x^2} - \frac{b}{4a^2(a+bx^2)^2}$$

[Out]  $-1/(2*a^3*x^2) - b/(4*a^2*(a+b*x^2)^2) - b/(a^3*(a+b*x^2)) - (3*b*Log[x])/a^4 + (3*b*Log[a+b*x^2])/(2*a^4)$

**Rubi [A]** time = 0.11722, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{3b \log(a+bx^2)}{2a^4} - \frac{3b \log(x)}{a^4} - \frac{b}{a^3(a+bx^2)} - \frac{1}{2a^3x^2} - \frac{b}{4a^2(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(a+b\*x^2)^3),x]

[Out]  $-1/(2*a^3*x^2) - b/(4*a^2*(a+b*x^2)^2) - b/(a^3*(a+b*x^2)) - (3*b*Log[x])/a^4 + (3*b*Log[a+b*x^2])/(2*a^4)$

**Rubi in Sympy [A]** time = 16.1864, size = 66, normalized size = 0.99

$$-\frac{b}{4a^2(a+bx^2)^2} - \frac{b}{a^3(a+bx^2)} - \frac{1}{2a^3x^2} - \frac{3b \log(x^2)}{2a^4} + \frac{3b \log(a+bx^2)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(b\*x\*\*2+a)\*\*3,x)

[Out]  $-b/(4*a**2*(a+b*x**2)**2) - b/(a**3*(a+b*x**2)) - 1/(2*a**3*x**2) - 3*b*log(x**2)/(2*a**4) + 3*b*log(a+b*x**2)/(2*a**4)$

**Mathematica [A]** time = 0.0970057, size = 59, normalized size = 0.88

$$-\frac{\frac{a(2a^2+9abx^2+6b^2x^4)}{x^2(a+bx^2)^2} - 6b \log(a+bx^2) + 12b \log(x)}{4a^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a+b\*x^2)^3),x]

[Out]  $-((a*(2*a^2+9*a*b*x^2+6*b^2*x^4))/(x^2*(a+b*x^2)^2)+12*b*Log[x]-6*b*Log[a+b*x^2])/(4*a^4)$

**Maple [A]** time = 0.02, size = 62, normalized size = 0.9

$$-\frac{1}{2a^3x^2} - \frac{b}{4a^2(bx^2+a)^2} - \frac{b}{a^3(bx^2+a)} - 3\frac{b \ln(x)}{a^4} + \frac{3b \ln(bx^2+a)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(b*x^2+a)^3,x)`

[Out]  $-1/2/a^3/x^2 - 1/4*b/a^2/(b*x^2+a)^2 - b/a^3/(b*x^2+a) - 3*b*\ln(x)/a^4 + 3/2*b*\ln(b*x^2+a)/a^4$

**Maxima [A]** time = 1.33268, size = 104, normalized size = 1.55

$$-\frac{6b^2x^4 + 9abx^2 + 2a^2}{4(a^3b^2x^6 + 2a^4bx^4 + a^5x^2)} + \frac{3b \log(bx^2 + a)}{2a^4} - \frac{3b \log(x^2)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^3*x^3),x, algorithm="maxima")`

[Out]  $-1/4*(6*b^2*x^4 + 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2) + 3/2*b*\log(b*x^2 + a)/a^4 - 3/2*b*\log(x^2)/a^4$

**Fricas [A]** time = 0.218662, size = 161, normalized size = 2.4

$$\frac{6ab^2x^4 + 9a^2bx^2 + 2a^3 - 6(b^3x^6 + 2ab^2x^4 + a^2bx^2) \log(bx^2 + a) + 12(b^3x^6 + 2ab^2x^4 + a^2bx^2) \log(x)}{4(a^4b^2x^6 + 2a^5bx^4 + a^6x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^3*x^3),x, algorithm="fricas")`

[Out]  $-1/4*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3 - 6*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*\log(b*x^2 + a) + 12*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*\log(x))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)$

**Sympy [A]** time = 3.01331, size = 78, normalized size = 1.16

$$-\frac{2a^2 + 9abx^2 + 6b^2x^4}{4a^5x^2 + 8a^4bx^4 + 4a^3b^2x^6} - \frac{3b \log(x)}{a^4} + \frac{3b \log\left(\frac{a}{b} + x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(b*x**2+a)**3,x)`

[Out]  $-(2*a**2 + 9*a*b*x**2 + 6*b**2*x**4)/(4*a**5*x**2 + 8*a**4*b*x**4 + 4*a**3*b**2*x**6) - 3*b*\log(x)/a**4 + 3*b*\log(a/b + x**2)/(2*a**4)$

**GIAC/XCAS [A]** time = 0.215161, size = 111, normalized size = 1.66

$$-\frac{3b \ln(x^2)}{2a^4} + \frac{3b \ln(|bx^2 + a|)}{2a^4} - \frac{9b^3x^4 + 22ab^2x^2 + 14a^2b}{4(bx^2 + a)^2a^4} + \frac{3bx^2 - a}{2a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^3*x^3),x, algorithm="giac")`

[Out]  $-3/2*b*\ln(x^2)/a^4 + 3/2*b*\ln(\text{abs}(b*x^2 + a))/a^4 - 1/4*(9*b^3*x^4 + 22*a*b^2*x^2 + 14*a^2*b)/((b*x^2 + a)^2*a^4) + 1/2*(3*b*x^2 - a)/(a^4*x^2)$

$$3.178 \quad \int \frac{1}{x^5(a+bx^2)^3} dx$$

**Optimal.** Leaf size=86

$$-\frac{3b^2 \log(a+bx^2)}{a^5} + \frac{6b^2 \log(x)}{a^5} + \frac{3b^2}{2a^4(a+bx^2)} + \frac{3b}{2a^4x^2} + \frac{b^2}{4a^3(a+bx^2)^2} - \frac{1}{4a^3x^4}$$

[Out]  $-1/(4*a^3*x^4) + (3*b)/(2*a^4*x^2) + b^2/(4*a^3*(a+b*x^2)^2) + (3*b^2)/(2*a^4*(a+b*x^2)) + (6*b^2*Log[x])/a^5 - (3*b^2*Log[a+b*x^2])/a^5$

**Rubi [A]** time = 0.146007, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{3b^2 \log(a+bx^2)}{a^5} + \frac{6b^2 \log(x)}{a^5} + \frac{3b^2}{2a^4(a+bx^2)} + \frac{3b}{2a^4x^2} + \frac{b^2}{4a^3(a+bx^2)^2} - \frac{1}{4a^3x^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5\*(a+b\*x^2)^3),x]

[Out]  $-1/(4*a^3*x^4) + (3*b)/(2*a^4*x^2) + b^2/(4*a^3*(a+b*x^2)^2) + (3*b^2)/(2*a^4*(a+b*x^2)) + (6*b^2*Log[x])/a^5 - (3*b^2*Log[a+b*x^2])/a^5$

**Rubi in Sympy [A]** time = 20.4554, size = 85, normalized size = 0.99

$$\frac{b^2}{4a^3(a+bx^2)^2} - \frac{1}{4a^3x^4} + \frac{3b^2}{2a^4(a+bx^2)} + \frac{3b}{2a^4x^2} + \frac{3b^2 \log(x^2)}{a^5} - \frac{3b^2 \log(a+bx^2)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*5/(b\*x\*\*2+a)\*\*3,x)

[Out]  $b**2/(4*a**3*(a+b*x**2)**2) - 1/(4*a**3*x**4) + 3*b**2/(2*a**4*(a+b*x**2)) + 3*b/(2*a**4*x**2) + 3*b**2*log(x**2)/a**5 - 3*b**2*log(a+b*x**2)/a**5$

**Mathematica [A]** time = 0.09068, size = 74, normalized size = 0.86

$$\frac{a(-a^3+4a^2bx^2+18ab^2x^4+12b^3x^6)}{x^4(a+bx^2)^2} - \frac{12b^2 \log(a+bx^2) + 24b^2 \log(x)}{4a^5}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5\*(a+b\*x^2)^3),x]

[Out]  $((a*(-a^3+4*a^2*b*x^2+18*a*b^2*x^4+12*b^3*x^6))/(x^4*(a+b*x^2)^2) + 24*b^2*Log[x] - 12*b^2*Log[a+b*x^2])/(4*a^5)$

**Maple [A]** time = 0.02, size = 79, normalized size = 0.9

$$-\frac{1}{4a^3x^4} + \frac{3b}{2a^4x^2} + \frac{b^2}{4a^3(bx^2+a)^2} + \frac{3b^2}{2a^4(bx^2+a)} + 6\frac{b^2 \ln(x)}{a^5} - 3\frac{b^2 \ln(bx^2+a)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^5/(b*x^2+a)^3,x)`

[Out] 
$$-1/4/a^3/x^4 + 3/2*b/a^4/x^2 + 1/4*b^2/a^3/(b*x^2+a)^2 + 3/2*b^2/a^4/(b*x^2+a) + 6*b^2*\ln(x)/a^5 - 3*b^2*\ln(b*x^2+a)/a^5$$

**Maxima [A]** time = 1.34809, size = 124, normalized size = 1.44

$$\frac{12b^3x^6 + 18ab^2x^4 + 4a^2bx^2 - a^3}{4(a^4b^2x^8 + 2a^5bx^6 + a^6x^4)} - \frac{3b^2\log(bx^2 + a)}{a^5} + \frac{3b^2\log(x^2)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^3*x^5),x, algorithm="maxima")`

[Out] 
$$1/4*(12*b^3*x^6 + 18*a*b^2*x^4 + 4*a^2*b*x^2 - a^3)/(a^4*b^2*x^8 + 2*a^5*b*x^6 + a^6*x^4) - 3*b^2*\log(b*x^2 + a)/a^5 + 3*b^2*\log(x^2)/a^5$$

**Fricas [A]** time = 0.208426, size = 181, normalized size = 2.1

$$\frac{12ab^3x^6 + 18a^2b^2x^4 + 4a^3bx^2 - a^4 - 12(b^4x^8 + 2ab^3x^6 + a^2b^2x^4)\log(bx^2 + a) + 24(b^4x^8 + 2ab^3x^6 + a^2b^2x^4)\log(x)}{4(a^5b^2x^8 + 2a^6bx^6 + a^7x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^3*x^5),x, algorithm="fricas")`

[Out] 
$$1/4*(12*a*b^3*x^6 + 18*a^2*b^2*x^4 + 4*a^3*b*x^2 - a^4 - 12*(b^4*x^8 + 2*a*b^3*x^6 + a^2*b^2*x^4)*\log(b*x^2 + a) + 24*(b^4*x^8 + 2*a*b^3*x^6 + a^2*b^2*x^4)*\log(x))/(a^5*b^2*x^8 + 2*a^6*b*x^6 + a^7*x^4)$$

**Sympy [A]** time = 4.37381, size = 90, normalized size = 1.05

$$\frac{-a^3 + 4a^2bx^2 + 18ab^2x^4 + 12b^3x^6}{4a^6x^4 + 8a^5bx^6 + 4a^4b^2x^8} + \frac{6b^2\log(x)}{a^5} - \frac{3b^2\log\left(\frac{a}{b} + x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(b*x**2+a)**3,x)`

[Out] 
$$(-a**3 + 4*a**2*b*x**2 + 18*a*b**2*x**4 + 12*b**3*x**6)/(4*a**6*x**4 + 8*a**5*b*x**6 + 4*a**4*b**2*x**8) + 6*b**2*\log(x)/a**5 - 3*b**2*\log(a/b + x**2)/a**5$$

**GIAC/XCAS [A]** time = 0.216834, size = 108, normalized size = 1.26

$$\frac{3b^2\ln(x^2)}{a^5} - \frac{3b^2\ln(|bx^2 + a|)}{a^5} + \frac{12b^3x^6 + 18ab^2x^4 + 4a^2bx^2 - a^3}{4(bx^4 + ax^2)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^3*x^5),x, algorithm="giac")`



```
[Out] 3*b^2*ln(x^2)/a^5 - 3*b^2*ln(abs(b*x^2 + a))/a^5 + 1/4*(12*b^3*x^6 + 18*a*b^2*x^4 + 4*a^2*b*x^2 - a^3)/((b*x^4 + a*x^2)^2*a^4)
```

$$3.179 \quad \int \frac{1}{x^7(a+bx^2)^3} dx$$

**Optimal.** Leaf size=95

$$\frac{5b^3 \log(a+bx^2)}{a^6} - \frac{10b^3 \log(x)}{a^6} - \frac{2b^3}{a^5(a+bx^2)} - \frac{3b^2}{a^5x^2} - \frac{b^3}{4a^4(a+bx^2)^2} + \frac{3b}{4a^4x^4} - \frac{1}{6a^3x^6}$$

[Out]  $-1/(6*a^3*x^6) + (3*b)/(4*a^4*x^4) - (3*b^2)/(a^5*x^2) - b^3/(4*a^4*(a+b*x^2)^2) - (2*b^3)/(a^5*(a+b*x^2)) - (10*b^3*Log[x])/a^6 + (5*b^3*Log[a+b*x^2])/a^6$

**Rubi [A]** time = 0.167392, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{5b^3 \log(a+bx^2)}{a^6} - \frac{10b^3 \log(x)}{a^6} - \frac{2b^3}{a^5(a+bx^2)} - \frac{3b^2}{a^5x^2} - \frac{b^3}{4a^4(a+bx^2)^2} + \frac{3b}{4a^4x^4} - \frac{1}{6a^3x^6}$$

Antiderivative was successfully verified.

[In] Int[1/(x^7\*(a+b\*x^2)^3),x]

[Out]  $-1/(6*a^3*x^6) + (3*b)/(4*a^4*x^4) - (3*b^2)/(a^5*x^2) - b^3/(4*a^4*(a+b*x^2)^2) - (2*b^3)/(a^5*(a+b*x^2)) - (10*b^3*Log[x])/a^6 + (5*b^3*Log[a+b*x^2])/a^6$

**Rubi in Sympy [A]** time = 22.7697, size = 95, normalized size = 1.

$$-\frac{1}{6a^3x^6} - \frac{b^3}{4a^4(a+bx^2)^2} + \frac{3b}{4a^4x^4} - \frac{2b^3}{a^5(a+bx^2)} - \frac{3b^2}{a^5x^2} - \frac{5b^3 \log(x^2)}{a^6} + \frac{5b^3 \log(a+bx^2)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*7/(b\*x\*\*2+a)\*\*3,x)

[Out]  $-1/(6*a**3*x**6) - b**3/(4*a**4*(a+b*x**2)**2) + 3*b/(4*a**4*x**4) - 2*b**3/(a**5*(a+b*x**2)) - 3*b**2/(a**5*x**2) - 5*b**3*log(x**2)/a**6 + 5*b**3*log(a+b*x**2)/a**6$

**Mathematica [A]** time = 0.140014, size = 85, normalized size = 0.89

$$\frac{\frac{2a^4-5a^3bx^2+20a^2b^2x^4+90ab^3x^6+60b^4x^8}{x^6(a+bx^2)^2} - 60b^3 \log(a+bx^2) + 120b^3 \log(x)}{12a^6}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^7\*(a+b\*x^2)^3),x]

[Out]  $-((a*(2*a^4 - 5*a^3*b*x^2 + 20*a^2*b^2*x^4 + 90*a*b^3*x^6 + 60*b^4*x^8))/(x^6*(a+b*x^2)^2) + 120*b^3*Log[x] - 60*b^3*Log[a+b*x^2])/(12*a^6)$

**Maple [A]** time = 0.019, size = 90, normalized size = 1.

$$-\frac{1}{6a^3x^6} + \frac{3b}{4a^4x^4} - 3\frac{b^2}{a^5x^2} - \frac{b^3}{4a^4(bx^2+a)^2} - 2\frac{b^3}{a^5(bx^2+a)} - 10\frac{b^3 \ln(x)}{a^6} + 5\frac{b^3 \ln(bx^2+a)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^7/(b*x^2+a)^3,x)`

[Out] 
$$-1/6/a^3/x^6 + 3/4*b/a^4/x^4 - 3*b^2/a^5/x^2 - 1/4*b^3/a^4/(b*x^2+a)^2 - 2*b^3/a^5/(b*x^2+a) - 10*b^3*\ln(x)/a^6 + 5*b^3*\ln(b*x^2+a)/a^6$$

**Maxima [A]** time = 1.35816, size = 139, normalized size = 1.46

$$-\frac{60b^4x^8 + 90ab^3x^6 + 20a^2b^2x^4 - 5a^3bx^2 + 2a^4}{12(a^5b^2x^{10} + 2a^6bx^8 + a^7x^6)} + \frac{5b^3\log(bx^2 + a)}{a^6} - \frac{5b^3\log(x^2)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^3*x^7),x, algorithm="maxima")`

[Out] 
$$-1/12*(60*b^4*x^8 + 90*a*b^3*x^6 + 20*a^2*b^2*x^4 - 5*a^3*b*x^2 + 2*a^4)/(a^5*b^2*x^{10} + 2*a^6*b*x^8 + a^7*x^6) + 5*b^3*\log(b*x^2 + a)/a^6 - 5*b^3*\log(x^2)/a^6$$

**Fricas [A]** time = 0.218861, size = 196, normalized size = 2.06

$$\frac{60ab^4x^8 + 90a^2b^3x^6 + 20a^3b^2x^4 - 5a^4bx^2 + 2a^5 - 60(b^5x^{10} + 2ab^4x^8 + a^2b^3x^6)\log(bx^2 + a) + 120(b^5x^{10} + 2ab^4x^8 + a^2b^3x^6)\log(x)}{12(a^6b^2x^{10} + 2a^7bx^8 + a^8x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^3*x^7),x, algorithm="fricas")`

[Out] 
$$-1/12*(60*a*b^4*x^8 + 90*a^2*b^3*x^6 + 20*a^3*b^2*x^4 - 5*a^4*b*x^2 + 2*a^5 - 60*(b^5*x^{10} + 2*a*b^4*x^8 + a^2*b^3*x^6)*\log(b*x^2 + a) + 120*(b^5*x^{10} + 2*a*b^4*x^8 + a^2*b^3*x^6)*\log(x))/(a^6*b^2*x^{10} + 2*a^7*b*x^8 + a^8*x^6)$$

**Sympy [A]** time = 7.60983, size = 104, normalized size = 1.09

$$-\frac{2a^4 - 5a^3bx^2 + 20a^2b^2x^4 + 90ab^3x^6 + 60b^4x^8}{12a^7x^6 + 24a^6bx^8 + 12a^5b^2x^{10}} - \frac{10b^3\log(x)}{a^6} + \frac{5b^3\log\left(\frac{a}{b} + x^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**7/(b*x**2+a)**3,x)`

[Out] 
$$-(2*a**4 - 5*a**3*b*x**2 + 20*a**2*b**2*x**4 + 90*a*b**3*x**6 + 60*b**4*x**8)/(12*a**7*x**6 + 24*a**6*b*x**8 + 12*a**5*b**2*x**10) - 10*b**3*\log(x)/a**6 + 5*b**3*\log(a/b + x**2)/a**6$$

**GIAC/XCAS [A]** time = 0.211365, size = 149, normalized size = 1.57

$$-\frac{5b^3\ln(x^2)}{a^6} + \frac{5b^3\ln(|bx^2 + a|)}{a^6} - \frac{30b^5x^4 + 68ab^4x^2 + 39a^2b^3}{4(bx^2 + a)^2a^6} + \frac{110b^3x^6 - 36ab^2x^4 + 9a^2bx^2 - 2a^3}{12a^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^3*x^7),x, algorithm="giac")`

```
[Out] -5*b^3*ln(x^2)/a^6 + 5*b^3*ln(abs(b*x^2 + a))/a^6 - 1/4*(30*b^5*x  
^4 + 68*a*b^4*x^2 + 39*a^2*b^3)/((b*x^2 + a)^2*a^6) + 1/12*(110*b  
^3*x^6 - 36*a*b^2*x^4 + 9*a^2*b*x^2 - 2*a^3)/(a^6*x^6)
```

$$3.180 \quad \int \frac{1}{x^9(a+bx^2)^3} dx$$

**Optimal.** Leaf size=112

$$-\frac{15b^4 \log(a+bx^2)}{2a^7} + \frac{15b^4 \log(x)}{a^7} + \frac{5b^4}{2a^6(a+bx^2)} + \frac{5b^3}{a^6x^2} + \frac{b^4}{4a^5(a+bx^2)^2} - \frac{3b^2}{2a^5x^4} + \frac{b}{2a^4x^6} - \frac{1}{8a^3x^8}$$

[Out]  $-1/(8*a^3*x^8) + b/(2*a^4*x^6) - (3*b^2)/(2*a^5*x^4) + (5*b^3)/(a^6*x^2) + b^4/(4*a^5*(a+b*x^2)^2) + (5*b^4)/(2*a^6*(a+b*x^2)) + (15*b^4*Log[x])/a^7 - (15*b^4*Log[a+b*x^2])/(2*a^7)$

**Rubi [A]** time = 0.193891, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{15b^4 \log(a+bx^2)}{2a^7} + \frac{15b^4 \log(x)}{a^7} + \frac{5b^4}{2a^6(a+bx^2)} + \frac{5b^3}{a^6x^2} + \frac{b^4}{4a^5(a+bx^2)^2} - \frac{3b^2}{2a^5x^4} + \frac{b}{2a^4x^6} - \frac{1}{8a^3x^8}$$

Antiderivative was successfully verified.

[In] Int[1/(x^9\*(a+b\*x^2)^3),x]

[Out]  $-1/(8*a^3*x^8) + b/(2*a^4*x^6) - (3*b^2)/(2*a^5*x^4) + (5*b^3)/(a^6*x^2) + b^4/(4*a^5*(a+b*x^2)^2) + (5*b^4)/(2*a^6*(a+b*x^2)) + (15*b^4*Log[x])/a^7 - (15*b^4*Log[a+b*x^2])/(2*a^7)$

**Rubi in Sympy [A]** time = 27.0667, size = 112, normalized size = 1.

$$-\frac{1}{8a^3x^8} + \frac{b}{2a^4x^6} + \frac{b^4}{4a^5(a+bx^2)^2} - \frac{3b^2}{2a^5x^4} + \frac{5b^4}{2a^6(a+bx^2)} + \frac{5b^3}{a^6x^2} + \frac{15b^4 \log(x^2)}{2a^7} - \frac{15b^4 \log(a+bx^2)}{2a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*9/(b\*x\*\*2+a)\*\*3,x)

[Out]  $-1/(8*a**3*x**8) + b/(2*a**4*x**6) + b**4/(4*a**5*(a+b*x**2)**2) - 3*b**2/(2*a**5*x**4) + 5*b**4/(2*a**6*(a+b*x**2)) + 5*b**3/(a**6*x**2) + 15*b**4*log(x**2)/(2*a**7) - 15*b**4*log(a+b*x**2)/(2*a**7)$

**Mathematica [A]** time = 0.134365, size = 96, normalized size = 0.86

$$\frac{\frac{a(-a^5+2a^4bx^2-5a^3b^2x^4+20a^2b^3x^6+90ab^4x^8+60b^5x^{10})}{x^8(a+bx^2)^2} - 60b^4 \log(a+bx^2) + 120b^4 \log(x)}{8a^7}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^9\*(a+b\*x^2)^3),x]

[Out]  $((a*(-a^5+2*a^4*b*x^2-5*a^3*b^2*x^4+20*a^2*b^3*x^6+90*a*b^4*x^8+60*b^5*x^{10}))/x^8*(a+b*x^2)^2+120*b^4*Log[x]-60*b^4*Log[a+b*x^2])/(8*a^7)$

**Maple [A]** time = 0.022, size = 101, normalized size = 0.9

$$-\frac{1}{8a^3x^8} + \frac{b}{2a^4x^6} - \frac{3b^2}{2a^5x^4} + 5\frac{b^3}{a^6x^2} + \frac{b^4}{4a^5(bx^2+a)^2} + \frac{5b^4}{2a^6(bx^2+a)} + 15\frac{b^4 \ln(x)}{a^7} - \frac{15b^4 \ln(bx^2+a)}{2a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^9/(b*x^2+a)^3,x)`

[Out] 
$$-1/8/a^3/x^8 + 1/2*b/a^4/x^6 - 3/2*b^2/a^5/x^4 + 5*b^3/a^6/x^2 + 1/4*b^4/a^5/(b*x^2+a)^2 + 5/2*b^4/a^6/(b*x^2+a) + 15*b^4*ln(x)/a^7 - 15/2*b^4*ln(b*x^2+a)/a^7$$

**Maxima [A]** time = 1.35409, size = 154, normalized size = 1.38

$$\frac{60 b^5 x^{10} + 90 a b^4 x^8 + 20 a^2 b^3 x^6 - 5 a^3 b^2 x^4 + 2 a^4 b x^2 - a^5}{8 (a^6 b^2 x^{12} + 2 a^7 b x^{10} + a^8 x^8)} - \frac{15 b^4 \log (b x^2 + a)}{2 a^7} + \frac{15 b^4 \log (x^2)}{2 a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^3*x^9),x, algorithm="maxima")`

[Out] 
$$1/8*(60*b^5*x^{10} + 90*a*b^4*x^8 + 20*a^2*b^3*x^6 - 5*a^3*b^2*x^4 + 2*a^4*b*x^2 - a^5)/(a^6*b^2*x^{12} + 2*a^7*b*x^{10} + a^8*x^8) - 15/2*b^4*log(b*x^2 + a)/a^7 + 15/2*b^4*log(x^2)/a^7$$

**Fricas [A]** time = 0.212335, size = 211, normalized size = 1.88

$$\frac{60 a b^5 x^{10} + 90 a^2 b^4 x^8 + 20 a^3 b^3 x^6 - 5 a^4 b^2 x^4 + 2 a^5 b x^2 - a^6 - 60 (b^6 x^{12} + 2 a b^5 x^{10} + a^2 b^4 x^8) \log (b x^2 + a) + 120 (b^6 x^{12} + a^7 b^2 x^{12} + 2 a^8 b x^{10} + a^9 x^8)}{8 (a^7 b^2 x^{12} + 2 a^8 b x^{10} + a^9 x^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^3*x^9),x, algorithm="fricas")`

[Out] 
$$1/8*(60*a*b^5*x^{10} + 90*a^2*b^4*x^8 + 20*a^3*b^3*x^6 - 5*a^4*b^2*x^4 + 2*a^5*b*x^2 - a^6 - 60*(b^6*x^{12} + 2*a*b^5*x^{10} + a^2*b^4*x^8)*log(b*x^2 + a) + 120*(b^6*x^{12} + 2*a*b^5*x^{10} + a^2*b^4*x^8)*log(x))/(a^7*b^2*x^{12} + 2*a^8*b*x^{10} + a^9*x^8)$$

**Sympy [A]** time = 14.5117, size = 116, normalized size = 1.04

$$\frac{-a^5 + 2a^4bx^2 - 5a^3b^2x^4 + 20a^2b^3x^6 + 90ab^4x^8 + 60b^5x^{10}}{8a^8x^8 + 16a^7bx^{10} + 8a^6b^2x^{12}} + \frac{15b^4\log(x)}{a^7} - \frac{15b^4\log\left(\frac{a}{b} + x^2\right)}{2a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**9/(b*x**2+a)**3,x)`

[Out] 
$$(-a^{**5} + 2*a^{**4}*b*x^{**2} - 5*a^{**3}*b^{**2}*x^{**4} + 20*a^{**2}*b^{**3}*x^{**6} + 90*a*b^{**4}*x^{**8} + 60*b^{**5}*x^{**10})/(8*a^{**8}*x^{**8} + 16*a^{**7}*b*x^{**10} + 8*a^{**6}*b^{**2}*x^{**12}) + 15*b^{**4}*log(x)/a^{**7} - 15*b^{**4}*log(a/b + x^{**2})/(2*a^{**7})$$

**GIAC/XCAS [A]** time = 0.21146, size = 161, normalized size = 1.44

$$\frac{15 b^4 \ln (x^2)}{2 a^7} - \frac{15 b^4 \ln (|b x^2 + a|)}{2 a^7} + \frac{45 b^6 x^4 + 100 a b^5 x^2 + 56 a^2 b^4}{4 (b x^2 + a)^2 a^7} - \frac{125 b^4 x^8 - 40 a b^3 x^6 + 12 a^2 b^2 x^4 - 4 a^3 b x^2 + a^4}{8 a^7 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^3*x^9),x, algorithm="giac")
```

```
[Out] 15/2*b^4*ln(x^2)/a^7 - 15/2*b^4*ln(abs(b*x^2 + a))/a^7 + 1/4*(45*  
b^6*x^4 + 100*a*b^5*x^2 + 56*a^2*b^4)/((b*x^2 + a)^2*a^7) - 1/8*(  
125*b^4*x^8 - 40*a*b^3*x^6 + 12*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)/  
(a^7*x^8)
```

$$3.181 \quad \int \frac{x^{12}}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=111

$$\frac{99a^{7/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8b^{13/2}} - \frac{99a^3x}{8b^6} + \frac{33a^2x^3}{8b^5} - \frac{99ax^5}{40b^4} - \frac{11x^9}{8b^2(a+bx^2)} - \frac{x^{11}}{4b(a+bx^2)^2} + \frac{99x^7}{56b^3}$$

[Out]  $(-99*a^3*x)/(8*b^6) + (33*a^2*x^3)/(8*b^5) - (99*a*x^5)/(40*b^4) + (99*x^7)/(56*b^3) - x^{11}/(4*b*(a + b*x^2)^2) - (11*x^9)/(8*b^2*(a + b*x^2)) + (99*a^{7/2})*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(8*b^{13/2})$

**Rubi [A]** time = 0.143127, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{99a^{7/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8b^{13/2}} - \frac{99a^3x}{8b^6} + \frac{33a^2x^3}{8b^5} - \frac{99ax^5}{40b^4} - \frac{11x^9}{8b^2(a+bx^2)} - \frac{x^{11}}{4b(a+bx^2)^2} + \frac{99x^7}{56b^3}$$

Antiderivative was successfully verified.

[In] Int[x^12/(a + b\*x^2)^3, x]

[Out]  $(-99*a^3*x)/(8*b^6) + (33*a^2*x^3)/(8*b^5) - (99*a*x^5)/(40*b^4) + (99*x^7)/(56*b^3) - x^{11}/(4*b*(a + b*x^2)^2) - (11*x^9)/(8*b^2*(a + b*x^2)) + (99*a^{7/2})*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(8*b^{13/2})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{99a^{7/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8b^{13/2}} + \frac{33a^2x^3}{8b^5} - \frac{99ax^5}{40b^4} - \frac{x^{11}}{4b(a+bx^2)^2} - \frac{11x^9}{8b^2(a+bx^2)} + \frac{99x^7}{56b^3} - \frac{99 \int a^3 dx}{8b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*12/(b\*x\*\*2+a)\*\*3, x)

[Out]  $99*a^{7/2}*atan(sqrt(b)*x/sqrt(a))/(8*b^{13/2}) + 33*a^2*x^3/(8*b^5) - 99*a*x^5/(40*b^4) - x^{11}/(4*b*(a + b*x^2)^2) - 11*x^9/(8*b^2*(a + b*x^2)) + 99*x^7/(56*b^3) - 99*Integral(a^3, x)/(8*b^6)$

**Mathematica [A]** time = 0.146128, size = 99, normalized size = 0.89

$$\frac{99a^{7/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8b^{13/2}} - \frac{3465a^5x + 5775a^4bx^3 + 1848a^3b^2x^5 - 264a^2b^3x^7 + 88ab^4x^9 - 40b^5x^{11}}{280b^6(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^12/(a + b\*x^2)^3, x]

[Out]  $-(3465*a^5*x + 5775*a^4*b*x^3 + 1848*a^3*b^2*x^5 - 264*a^2*b^3*x^7 + 88*a*b^4*x^9 - 40*b^5*x^{11})/(280*b^6*(a + b*x^2)^2) + (99*a^{7/2})*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(8*b^{13/2})$



**Maple [A]** time = 0.013, size = 99, normalized size = 0.9

$$\frac{x^7}{7b^3} - \frac{3ax^5}{5b^4} + 2\frac{a^2x^3}{b^5} - 10\frac{a^3x}{b^6} - \frac{21a^4x^3}{8b^5(bx^2+a)^2} - \frac{19a^5x}{8b^6(bx^2+a)^2} + \frac{99a^4}{8b^6} \arctan\left(bx\frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^12/(b\*x^2+a)^3,x)

[Out] 1/7\*x^7/b^3-3/5\*a\*x^5/b^4+2\*a^2\*x^3/b^5-10\*a^3\*x/b^6-21/8/b^5\*a^4/(b\*x^2+a)^2\*x^3-19/8/b^6\*a^5/(b\*x^2+a)^2\*x+99/8/b^6\*a^4/(a\*b)^(1/2)\*arctan(x\*b/(a\*b)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12/(b\*x^2 + a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.212637, size = 1, normalized size = 0.01

$$\frac{80b^5x^{11} - 176ab^4x^9 + 528a^2b^3x^7 - 3696a^3b^2x^5 - 11550a^4bx^3 - 6930a^5x + 3465(a^3b^2x^4 + 2a^4bx^2 + a^5)\sqrt{-\frac{a}{b}} \log\left(\frac{bx^2 + a}{bx^2 + a}\right)}{560(b^8x^4 + 2ab^7x^2 + a^2b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12/(b\*x^2 + a)^3,x, algorithm="fricas")

[Out] [1/560\*(80\*b^5\*x^11 - 176\*a\*b^4\*x^9 + 528\*a^2\*b^3\*x^7 - 3696\*a^3\*b^2\*x^5 - 11550\*a^4\*b\*x^3 - 6930\*a^5\*x + 3465\*(a^3\*b^2\*x^4 + 2\*a^4\*b\*x^2 + a^5)\*sqrt(-a/b)\*log((b\*x^2 + 2\*b\*x\*sqrt(-a/b) - a)/(b\*x^2 + a)))/(b^8\*x^4 + 2\*a\*b^7\*x^2 + a^2\*b^6), 1/280\*(40\*b^5\*x^11 - 88\*a\*b^4\*x^9 + 264\*a^2\*b^3\*x^7 - 1848\*a^3\*b^2\*x^5 - 5775\*a^4\*b\*x^3 - 3465\*a^5\*x + 3465\*(a^3\*b^2\*x^4 + 2\*a^4\*b\*x^2 + a^5)\*sqrt(a/b)\*arctan(x/sqrt(a/b)))/(b^8\*x^4 + 2\*a\*b^7\*x^2 + a^2\*b^6)]

**Sympy [A]** time = 2.40492, size = 160, normalized size = 1.44

$$-\frac{10a^3x}{b^6} + \frac{2a^2x^3}{b^5} - \frac{3ax^5}{5b^4} - \frac{99\sqrt{-\frac{a^7}{b^{13}}}\log\left(x - \frac{b^6\sqrt{-\frac{a^7}{b^{13}}}}{a^3}\right)}{16} + \frac{99\sqrt{-\frac{a^7}{b^{13}}}\log\left(x + \frac{b^6\sqrt{-\frac{a^7}{b^{13}}}}{a^3}\right)}{16} - \frac{19a^5x + 21a^4bx^3}{8a^2b^6 + 16ab^7x^2 + 8b^8x^4} + \frac{x^7}{7b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*12/(b\*x\*\*2+a)\*\*3,x)

```
[Out] -10*a**3*x/b**6 + 2*a**2*x**3/b**5 - 3*a*x**5/(5*b**4) - 99*sqrt(-a**7/b**13)*log(x - b**6*sqrt(-a**7/b**13)/a**3)/16 + 99*sqrt(-a**7/b**13)*log(x + b**6*sqrt(-a**7/b**13)/a**3)/16 - (19*a**5*x + 21*a**4*b*x**3)/(8*a**2*b**6 + 16*a*b**7*x**2 + 8*b**8*x**4) + x**7/(7*b**3)
```

**GIAC/XCAS [A]** time = 0.239335, size = 130, normalized size = 1.17

$$\frac{99 a^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8 \sqrt{ab} b^6} - \frac{21 a^4 b x^3 + 19 a^5 x}{8 (bx^2 + a)^2 b^6} + \frac{5 b^{18} x^7 - 21 a b^{17} x^5 + 70 a^2 b^{16} x^3 - 350 a^3 b^{15} x}{35 b^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 99/8*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) - 1/8*(21*a^4*b*x^3 + 19*a^5*x)/((b*x^2 + a)^2*b^6) + 1/35*(5*b^18*x^7 - 21*a*b^17*x^5 + 70*a^2*b^16*x^3 - 350*a^3*b^15*x)/b^21
```

$$3.182 \quad \int \frac{x^{10}}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=98

$$-\frac{63a^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8b^{11/2}} + \frac{63a^2x}{8b^5} - \frac{21ax^3}{8b^4} - \frac{9x^7}{8b^2(a+bx^2)} - \frac{x^9}{4b(a+bx^2)^2} + \frac{63x^5}{40b^3}$$

[Out]  $(63*a^2*x)/(8*b^5) - (21*a*x^3)/(8*b^4) + (63*x^5)/(40*b^3) - x^9/(4*b*(a+b*x^2)^2) - (9*x^7)/(8*b^2*(a+b*x^2)) - (63*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(11/2))$

**Rubi [A]** time = 0.119709, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{63a^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8b^{11/2}} + \frac{63a^2x}{8b^5} - \frac{21ax^3}{8b^4} - \frac{9x^7}{8b^2(a+bx^2)} - \frac{x^9}{4b(a+bx^2)^2} + \frac{63x^5}{40b^3}$$

Antiderivative was successfully verified.

[In] Int[x^10/(a + b\*x^2)^3, x]

[Out]  $(63*a^2*x)/(8*b^5) - (21*a*x^3)/(8*b^4) + (63*x^5)/(40*b^3) - x^9/(4*b*(a+b*x^2)^2) - (9*x^7)/(8*b^2*(a+b*x^2)) - (63*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(11/2))$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{63a^{5/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8b^{11/2}} - \frac{21ax^3}{8b^4} - \frac{x^9}{4b(a+bx^2)^2} - \frac{9x^7}{8b^2(a+bx^2)} + \frac{63x^5}{40b^3} + \frac{63 \int a^2 dx}{8b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*10/(b\*x\*\*2+a)\*\*3, x)

[Out]  $-63*a**(5/2)*atan(sqrt(b)*x/sqrt(a))/(8*b**(11/2)) - 21*a*x**3/(8*b**4) - x**9/(4*b*(a+b*x**2)**2) - 9*x**7/(8*b**2*(a+b*x**2)) + 63*x**5/(40*b**3) + 63*Integral(a**2, x)/(8*b**5)$

**Mathematica [A]** time = 0.0970531, size = 88, normalized size = 0.9

$$\frac{315a^4x + 525a^3bx^3 + 168a^2b^2x^5 - 24ab^3x^7 + 8b^4x^9}{40b^5(a+bx^2)^2} - \frac{63a^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8b^{11/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^10/(a + b\*x^2)^3, x]

[Out]  $(315*a^4*x + 525*a^3*b*x^3 + 168*a^2*b^2*x^5 - 24*a*b^3*x^7 + 8*b^4*x^9)/(40*b^5*(a+b*x^2)^2) - (63*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(11/2))$

**Maple [A]** time = 0.014, size = 88, normalized size = 0.9

$$\frac{x^5}{5b^3} - \frac{ax^3}{b^4} + 6\frac{a^2x}{b^5} + \frac{17a^3x^3}{8b^4(bx^2+a)^2} + \frac{15a^4x}{8b^5(bx^2+a)^2} - \frac{63a^3}{8b^5} \arctan\left(bx\frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^10/(b\*x^2+a)^3,x)

[Out] 1/5\*x^5/b^3-a\*x^3/b^4+6\*a^2\*x/b^5+17/8/b^4\*a^3/(b\*x^2+a)^2\*x^3+15/8/b^5\*a^4/(b\*x^2+a)^2\*x-63/8/b^5\*a^3/(a\*b)^(1/2)\*arctan(x\*b/(a\*b)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10/(b\*x^2 + a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.20992, size = 1, normalized size = 0.01

$$\left[ \frac{16b^4x^9 - 48ab^3x^7 + 336a^2b^2x^5 + 1050a^3bx^3 + 630a^4x + 315(a^2b^2x^4 + 2a^3bx^2 + a^4)\sqrt{-\frac{a}{b}} \log\left(\frac{bx^2 - 2bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right)}{80(b^7x^4 + 2ab^6x^2 + a^2b^5)}, 8b^4x^9 \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10/(b\*x^2 + a)^3,x, algorithm="fricas")

[Out] [1/80\*(16\*b^4\*x^9 - 48\*a\*b^3\*x^7 + 336\*a^2\*b^2\*x^5 + 1050\*a^3\*b\*x^3 + 630\*a^4\*x + 315\*(a^2\*b^2\*x^4 + 2\*a^3\*b\*x^2 + a^4)\*sqrt(-a/b)\*log((b\*x^2 - 2\*b\*x\*sqrt(-a/b) - a)/(b\*x^2 + a)))/(b^7\*x^4 + 2\*a\*b^6\*x^2 + a^2\*b^5), 1/40\*(8\*b^4\*x^9 - 24\*a\*b^3\*x^7 + 168\*a^2\*b^2\*x^5 + 525\*a^3\*b\*x^3 + 315\*a^4\*x - 315\*(a^2\*b^2\*x^4 + 2\*a^3\*b\*x^2 + a^4)\*sqrt(a/b)\*arctan(x/sqrt(a/b)))/(b^7\*x^4 + 2\*a\*b^6\*x^2 + a^2\*b^5)]

**Sympy [A]** time = 2.2917, size = 144, normalized size = 1.47

$$\frac{6a^2x}{b^5} - \frac{ax^3}{b^4} + \frac{63\sqrt{-\frac{a^5}{b^{11}}}\log\left(x - \frac{b^5\sqrt{-\frac{a^5}{b^{11}}}}{a^2}\right)}{16} - \frac{63\sqrt{-\frac{a^5}{b^{11}}}\log\left(x + \frac{b^5\sqrt{-\frac{a^5}{b^{11}}}}{a^2}\right)}{16} + \frac{15a^4x + 17a^3bx^3}{8a^2b^5 + 16ab^6x^2 + 8b^7x^4} + \frac{x^5}{5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*10/(b\*x\*\*2+a)\*\*3,x)

[Out] 6\*a\*\*2\*x/b\*\*5 - a\*x\*\*3/b\*\*4 + 63\*sqrt(-a\*\*5/b\*\*11)\*log(x - b\*\*5\*sqrt(-a\*\*5/b\*\*11)/a\*\*2)/16 - 63\*sqrt(-a\*\*5/b\*\*11)\*log(x + b\*\*5\*sqrt(-a\*\*5/b\*\*11)/a\*\*2)/16 + (15\*a\*\*4\*x + 17\*a\*\*3\*b\*x\*\*3)/(8\*a\*\*2\*b\*\*5 + 16\*a\*b\*\*6\*x\*\*2 + 8\*b\*\*7\*x\*\*4) + x\*\*5/5\*b\*\*3

$$*5 + 16*a*b**6*x**2 + 8*b**7*x**4) + x**5/(5*b**3)$$

---

**GIAC/XCAS [A]** time = 0.231723, size = 113, normalized size = 1.15

$$-\frac{63 a^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8 \sqrt{ab} b^5} + \frac{17 a^3 b x^3 + 15 a^4 x}{8 (bx^2 + a)^2 b^5} + \frac{b^{12} x^5 - 5 ab^{11} x^3 + 30 a^2 b^{10} x}{5 b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10/(b\*x^2 + a)^3,x, algorithm="giac")

[Out] -63/8\*a^3\*arctan(b\*x/sqrt(a\*b))/(sqrt(a\*b)\*b^5) + 1/8\*(17\*a^3\*b\*x^3 + 15\*a^4\*x)/((b\*x^2 + a)^2\*b^5) + 1/5\*(b^12\*x^5 - 5\*a\*b^11\*x^3 + 30\*a^2\*b^10\*x)/b^15

$$3.183 \quad \int \frac{x^8}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=85

$$\frac{35a^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8b^{9/2}} - \frac{35ax}{8b^4} - \frac{7x^5}{8b^2(a+bx^2)} - \frac{x^7}{4b(a+bx^2)^2} + \frac{35x^3}{24b^3}$$

[Out]  $(-35*a*x)/(8*b^4) + (35*x^3)/(24*b^3) - x^7/(4*b*(a + b*x^2)^2) - (7*x^5)/(8*b^2*(a + b*x^2)) + (35*a^{(3/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^{(9/2)})$

**Rubi [A]** time = 0.102577, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{35a^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8b^{9/2}} - \frac{35ax}{8b^4} - \frac{7x^5}{8b^2(a+bx^2)} - \frac{x^7}{4b(a+bx^2)^2} + \frac{35x^3}{24b^3}$$

Antiderivative was successfully verified.

[In] Int[x^8/(a + b\*x^2)^3, x]

[Out]  $(-35*a*x)/(8*b^4) + (35*x^3)/(24*b^3) - x^7/(4*b*(a + b*x^2)^2) - (7*x^5)/(8*b^2*(a + b*x^2)) + (35*a^{(3/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^{(9/2)})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{35a^{3/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8b^{9/2}} - \frac{x^7}{4b(a+bx^2)^2} - \frac{7x^5}{8b^2(a+bx^2)} + \frac{35x^3}{24b^3} - \frac{35 \int a dx}{8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*8/(b\*x\*\*2+a)\*\*3, x)

[Out]  $35*a^{(3/2)}*\operatorname{atan}(\operatorname{sqrt}(b)*x/\operatorname{sqrt}(a))/(8*b^{(9/2)}) - x^{**7}/(4*b*(a + b*x^{**2})^{**2}) - 7*x^{**5}/(8*b^{**2}*(a + b*x^{**2})) + 35*x^{**3}/(24*b^{**3}) - 35*\operatorname{Integral}(a, x)/(8*b^{**4})$

**Mathematica [A]** time = 0.0857986, size = 77, normalized size = 0.91

$$\frac{35a^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8b^{9/2}} - \frac{105a^3x + 175a^2bx^3 + 56ab^2x^5 - 8b^3x^7}{24b^4(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(a + b\*x^2)^3, x]

[Out]  $-(105*a^3*x + 175*a^2*b*x^3 + 56*a*b^2*x^5 - 8*b^3*x^7)/(24*b^4*(a + b*x^2)^2) + (35*a^{(3/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^{(9/2)})$

**Maple [A]** time = 0.014, size = 77, normalized size = 0.9

$$\frac{x^3}{3b^3} - 3\frac{ax}{b^4} - \frac{13a^2x^3}{8b^3(bx^2+a)^2} - \frac{11a^3x}{8b^4(bx^2+a)^2} + \frac{35a^2}{8b^4} \arctan\left(bx\frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(b\*x^2+a)^3,x)

[Out] 1/3\*x^3/b^3-3\*a\*x/b^4-13/8/b^3\*a^2/(b\*x^2+a)^2\*x^3-11/8/b^4\*a^3/(b\*x^2+a)^2\*x+35/8/b^4\*a^2/(a\*b)^(1/2)\*arctan(x\*b/(a\*b)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b\*x^2 + a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.213456, size = 1, normalized size = 0.01

$$\left[ \frac{16b^3x^7 - 112ab^2x^5 - 350a^2bx^3 - 210a^3x + 105(ab^2x^4 + 2a^2bx^2 + a^3)\sqrt{-\frac{a}{b}} \log\left(\frac{bx^2+2bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{48(b^6x^4 + 2ab^5x^2 + a^2b^4)}, \frac{8b^3x^7 - 56ab^2x^5 - 105a^2bx^3 - 210a^3x + 105(ab^2x^4 + 2a^2bx^2 + a^3)\sqrt{\frac{a}{b}} \arctan\left(\frac{x\sqrt{a/b}}{\sqrt{bx^2+a}}\right)}{48(b^6x^4 + 2ab^5x^2 + a^2b^4)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b\*x^2 + a)^3,x, algorithm="fricas")

[Out] [1/48\*(16\*b^3\*x^7 - 112\*a\*b^2\*x^5 - 350\*a^2\*b\*x^3 - 210\*a^3\*x + 105\*(a\*b^2\*x^4 + 2\*a^2\*b\*x^2 + a^3)\*sqrt(-a/b)\*log((b\*x^2 + 2\*b\*x\*sqrt(-a/b) - a)/(b\*x^2 + a)))/(b^6\*x^4 + 2\*a\*b^5\*x^2 + a^2\*b^4), 1/24\*(8\*b^3\*x^7 - 56\*a\*b^2\*x^5 - 175\*a^2\*b\*x^3 - 105\*a^3\*x + 105\*(a\*b^2\*x^4 + 2\*a^2\*b\*x^2 + a^3)\*sqrt(a/b)\*arctan(x/sqrt(a/b)))/(b^6\*x^4 + 2\*a\*b^5\*x^2 + a^2\*b^4)]

**Sympy [A]** time = 2.27964, size = 131, normalized size = 1.54

$$-\frac{3ax}{b^4} - \frac{35\sqrt{-\frac{a^3}{b^9}} \log\left(x - \frac{b^4\sqrt{-\frac{a^3}{b^9}}}{a}\right)}{16} + \frac{35\sqrt{-\frac{a^3}{b^9}} \log\left(x + \frac{b^4\sqrt{-\frac{a^3}{b^9}}}{a}\right)}{16} - \frac{11a^3x + 13a^2bx^3}{8a^2b^4 + 16ab^5x^2 + 8b^6x^4} + \frac{x^3}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*8/(b\*x\*\*2+a)\*\*3,x)

[Out] -3\*a\*x/b\*\*4 - 35\*sqrt(-a\*\*3/b\*\*9)\*log(x - b\*\*4\*sqrt(-a\*\*3/b\*\*9)/a)/16 + 35\*sqrt(-a\*\*3/b\*\*9)\*log(x + b\*\*4\*sqrt(-a\*\*3/b\*\*9)/a)/16 - (11\*a\*\*3\*x + 13\*a\*\*2\*b\*x\*\*3)/(8\*a\*\*2\*b\*\*4 + 16\*a\*b\*\*5\*x\*\*2 + 8\*b\*\*6\*x\*\*4) + x\*\*3/(3\*b\*\*3)

---

**GIAC/XCAS [A]** time = 0.226472, size = 99, normalized size = 1.16

$$\frac{35 a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8 \sqrt{ab} b^4} - \frac{13 a^2 b x^3 + 11 a^3 x}{8 (bx^2 + a)^2 b^4} + \frac{b^6 x^3 - 9 ab^5 x}{3 b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b\*x^2 + a)^3,x, algorithm="giac")

[Out] 35/8\*a^2\*arctan(b\*x/sqrt(a\*b))/(sqrt(a\*b)\*b^4) - 1/8\*(13\*a^2\*b\*x^3 + 11\*a^3\*x)/((b\*x^2 + a)^2\*b^4) + 1/3\*(b^6\*x^3 - 9\*a\*b^5\*x)/b^9



$$3.184 \quad \int \frac{x^6}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=74

$$-\frac{15\sqrt{a} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8b^{7/2}} - \frac{5x^3}{8b^2(a+bx^2)} - \frac{x^5}{4b(a+bx^2)^2} + \frac{15x}{8b^3}$$

[Out] (15\*x)/(8\*b^3) - x^5/(4\*b\*(a + b\*x^2)^2) - (5\*x^3)/(8\*b^2\*(a + b\*x^2)) - (15\*Sqrt[a]\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(8\*b^(7/2))

**Rubi [A]** time = 0.0813861, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{15\sqrt{a} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8b^{7/2}} - \frac{5x^3}{8b^2(a+bx^2)} - \frac{x^5}{4b(a+bx^2)^2} + \frac{15x}{8b^3}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a + b\*x^2)^3, x]

[Out] (15\*x)/(8\*b^3) - x^5/(4\*b\*(a + b\*x^2)^2) - (5\*x^3)/(8\*b^2\*(a + b\*x^2)) - (15\*Sqrt[a]\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(8\*b^(7/2))

**Rubi in Sympy [A]** time = 15.0239, size = 66, normalized size = 0.89

$$-\frac{15\sqrt{a} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8b^{7/2}} - \frac{x^5}{4b(a+bx^2)^2} - \frac{5x^3}{8b^2(a+bx^2)} + \frac{15x}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6/(b\*x\*\*2+a)\*\*3, x)

[Out] -15\*sqrt(a)\*atan(sqrt(b)\*x/sqrt(a))/(8\*b\*\*(7/2)) - x\*\*5/(4\*b\*(a + b\*x\*\*2)\*\*2) - 5\*x\*\*3/(8\*b\*\*2\*(a + b\*x\*\*2)) + 15\*x/(8\*b\*\*3)

**Mathematica [A]** time = 0.0906355, size = 66, normalized size = 0.89

$$\frac{15a^2x + 25abx^3 + 8b^2x^5}{8b^3(a+bx^2)^2} - \frac{15\sqrt{a} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a + b\*x^2)^3, x]

[Out] (15\*a^2\*x + 25\*a\*b\*x^3 + 8\*b^2\*x^5)/(8\*b^3\*(a + b\*x^2)^2) - (15\*Sqrt[a]\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(8\*b^(7/2))

**Maple [A]** time = 0.014, size = 63, normalized size = 0.9

$$\frac{x}{b^3} + \frac{9ax^3}{8b^2(bx^2+a)^2} + \frac{7a^2x}{8b^3(bx^2+a)^2} - \frac{15a}{8b^3} \arctan\left(bx\frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(b*x^2+a)^3,x)`

[Out]  $x/b^3+9/8/b^2*a/(b*x^2+a)^2*x^3+7/8/b^3*a^2/(b*x^2+a)^2*x-15/8/b^3*a/(a*b)^{(1/2)}*\arctan(x*b/(a*b)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^2 + a)^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.209967, size = 1, normalized size = 0.01

$$\left[ \frac{16 b^2 x^5 + 50 a b x^3 + 30 a^2 x + 15 (b^2 x^4 + 2 a b x^2 + a^2) \sqrt{-\frac{a}{b}} \log\left(\frac{b x^2 - 2 b x \sqrt{-\frac{a}{b}} - a}{b x^2 + a}\right)}{16 (b^5 x^4 + 2 a b^4 x^2 + a^2 b^3)}, \frac{8 b^2 x^5 + 25 a b x^3 + 15 a^2 x - 15 (b^2 x^4 + 2 a b x^2 + a^2) \sqrt{a/b} \arctan\left(\frac{x \sqrt{a/b}}{\sqrt{b x^2 + a}}\right)}{8 (b^5 x^4 + 2 a b^4 x^2 + a^2 b^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^2 + a)^3,x, algorithm="fricas")`

[Out]  $[1/16*(16*b^2*x^5 + 50*a*b*x^3 + 30*a^2*x + 15*(b^2*x^4 + 2*a*b*x^2 + a^2)*\sqrt{-a/b}*\log((b*x^2 - 2*b*x*\sqrt{-a/b} - a)/(b*x^2 + a)))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3), 1/8*(8*b^2*x^5 + 25*a*b*x^3 + 15*a^2*x - 15*(b^2*x^4 + 2*a*b*x^2 + a^2)*\sqrt{a/b}*\arctan(x/\sqrt{a/b}))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)]$

**Sympy [A]** time = 2.10661, size = 107, normalized size = 1.45

$$\frac{15\sqrt{-\frac{a}{b^7}} \log\left(-b^3\sqrt{-\frac{a}{b^7}} + x\right)}{16} - \frac{15\sqrt{-\frac{a}{b^7}} \log\left(b^3\sqrt{-\frac{a}{b^7}} + x\right)}{16} + \frac{7a^2x + 9abx^3}{8a^2b^3 + 16ab^4x^2 + 8b^5x^4} + \frac{x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(b*x**2+a)**3,x)`

[Out]  $15*\sqrt{-a/b**7}*\log(-b**3*\sqrt{-a/b**7} + x)/16 - 15*\sqrt{-a/b**7}*\log(b**3*\sqrt{-a/b**7} + x)/16 + (7*a**2*x + 9*a*b*x**3)/(8*a**2*b**3 + 16*a*b**4*x**2 + 8*b**5*x**4) + x/b**3$

**GIAC/XCAS [A]** time = 0.210637, size = 73, normalized size = 0.99

$$-\frac{15 a \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{8 \sqrt{a b} b^3} + \frac{x}{b^3} + \frac{9 a b x^3 + 7 a^2 x}{8 (b x^2 + a)^2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] -15/8*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + x/b^3 + 1/8*(9*a*  
b*x^3 + 7*a^2*x)/((b*x^2 + a)^2*b^3)
```

$$3.185 \quad \int \frac{x^4}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=64

$$\frac{3 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8\sqrt{ab^{5/2}}} - \frac{3x}{8b^2(a+bx^2)} - \frac{x^3}{4b(a+bx^2)^2}$$

[Out]  $-x^3/(4*b*(a + b*x^2)^2) - (3*x)/(8*b^2*(a + b*x^2)) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(5/2))$

**Rubi [A]** time = 0.0625032, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{3 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8\sqrt{ab^{5/2}}} - \frac{3x}{8b^2(a+bx^2)} - \frac{x^3}{4b(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a + b\*x^2)^3, x]

[Out]  $-x^3/(4*b*(a + b*x^2)^2) - (3*x)/(8*b^2*(a + b*x^2)) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(5/2))$

**Rubi in Sympy [A]** time = 10.3164, size = 56, normalized size = 0.88

$$-\frac{x^3}{4b(a+bx^2)^2} - \frac{3x}{8b^2(a+bx^2)} + \frac{3 \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8\sqrt{ab^{\frac{5}{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(b\*x\*\*2+a)\*\*3, x)

[Out]  $-x**3/(4*b*(a + b*x**2)**2) - 3*x/(8*b**2*(a + b*x**2)) + 3*atan(sqrt(b)*x/sqrt(a))/(8*sqrt(a)*b**(5/2))$

**Mathematica [A]** time = 0.076753, size = 55, normalized size = 0.86

$$\frac{3 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8\sqrt{ab^{5/2}}} - \frac{3ax + 5bx^3}{8b^2(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a + b\*x^2)^3, x]

[Out]  $-(3*a*x + 5*b*x^3)/(8*b^2*(a + b*x^2)^2) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(5/2))$

**Maple [A]** time = 0.012, size = 47, normalized size = 0.7

$$\frac{1}{(bx^2+a)^2} \left( -\frac{5x^3}{8b} - \frac{3ax}{8b^2} \right) + \frac{3}{8b^2} \arctan\left( bx \frac{1}{\sqrt{ab}} \right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(b*x^2+a)^3,x)`

[Out]  $(-5/8*x^3/b-3/8*a*x/b^2)/(b*x^2+a)^2+3/8/b^2/(a*b)^{(1/2)}*\arctan(x*b/(a*b)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.211152, size = 1, normalized size = 0.02

$$\left[ \frac{3(b^2x^4 + 2abx^2 + a^2) \log\left(\frac{2abx + (bx^2 - a)\sqrt{-ab}}{bx^2 + a}\right) - 2(5bx^3 + 3ax)\sqrt{-ab}}{16(b^4x^4 + 2ab^3x^2 + a^2b^2)\sqrt{-ab}}, \frac{3(b^2x^4 + 2abx^2 + a^2) \arctan\left(\frac{\sqrt{ab}x}{a}\right) - (5bx^3 + 3ax)\sqrt{ab}}{8(b^4x^4 + 2ab^3x^2 + a^2b^2)\sqrt{ab}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^3,x, algorithm="fricas")`

[Out]  $[1/16*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*\log((2*a*b*x + (b*x^2 - a)*\sqrt{-a*b})/(b*x^2 + a)) - 2*(5*b*x^3 + 3*a*x)*\sqrt{-a*b})/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*\sqrt{-a*b}), 1/8*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*\arctan(\sqrt{a*b}*x/a) - (5*b*x^3 + 3*a*x)*\sqrt{a*b})/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*\sqrt{a*b}]$

**Sympy [A]** time = 1.84902, size = 109, normalized size = 1.7

$$-\frac{3\sqrt{-\frac{1}{ab^5}} \log\left(-ab^2\sqrt{-\frac{1}{ab^5}} + x\right)}{16} + \frac{3\sqrt{-\frac{1}{ab^5}} \log\left(ab^2\sqrt{-\frac{1}{ab^5}} + x\right)}{16} - \frac{3ax + 5bx^3}{8a^2b^2 + 16ab^3x^2 + 8b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(b*x**2+a)**3,x)`

[Out]  $-3*\sqrt{-1/(a*b**5)}*\log(-a*b**2*\sqrt{-1/(a*b**5)} + x)/16 + 3*\sqrt{-1/(a*b**5)}*\log(a*b**2*\sqrt{-1/(a*b**5)} + x)/16 - (3*a*x + 5*b*x**3)/(8*a**2*b**2 + 16*a*b**3*x**2 + 8*b**4*x**4)$

**GIAC/XCAS [A]** time = 0.211024, size = 61, normalized size = 0.95

$$\frac{3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{abb^2}} - \frac{5bx^3 + 3ax}{8(bx^2 + a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/8*(5*b*x^3 + 3*a*x)
/((b*x^2 + a)^2*b^2)
```

$$3.186 \quad \int \frac{x^2}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=65

$$\frac{\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}} + \frac{x}{8ab(a+bx^2)} - \frac{x}{4b(a+bx^2)^2}$$

[Out]  $-x/(4*b*(a + b*x^2)^2) + x/(8*a*b*(a + b*x^2)) + \text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/(8*a^{(3/2)}*b^{(3/2)})$

**Rubi [A]** time = 0.0577224, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}} + \frac{x}{8ab(a+bx^2)} - \frac{x}{4b(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*x^2)^3, x]

[Out]  $-x/(4*b*(a + b*x^2)^2) + x/(8*a*b*(a + b*x^2)) + \text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/(8*a^{(3/2)}*b^{(3/2)})$

**Rubi in Sympy [A]** time = 8.40751, size = 51, normalized size = 0.78

$$-\frac{x}{4b(a+bx^2)^2} + \frac{x}{8ab(a+bx^2)} + \frac{\text{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(b\*x\*\*2+a)\*\*3, x)

[Out]  $-x/(4*b*(a + b*x**2)**2) + x/(8*a*b*(a + b*x**2)) + \text{atan}(\text{sqrt}(b)*x/\text{sqrt}(a))/(8*a^{(3/2)}*b^{(3/2)})$

**Mathematica [A]** time = 0.0522065, size = 58, normalized size = 0.89

$$\frac{\frac{\sqrt{a}\sqrt{bx}(bx^2-a)}{(a+bx^2)^2} + \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*x^2)^3, x]

[Out]  $((\text{Sqrt}[a]*\text{Sqrt}[b]*x*(-a + b*x^2))/(a + b*x^2)^2 + \text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/(8*a^{(3/2)}*b^{(3/2)})$

**Maple [A]** time = 0.011, size = 49, normalized size = 0.8

$$\frac{1}{(bx^2+a)^2} \left( \frac{x^3}{8a} - \frac{x}{8b} \right) + \frac{1}{8ab} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*x^2+a)^3,x)`

[Out]  $(1/8/a*x^3-1/8*x/b)/(b*x^2+a)^2+1/8/b/a/(a*b)^{(1/2)}*\arctan(x*b/(a*b)^{(1/2)})$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.22503, size = 1, normalized size = 0.02

$$\left[ \frac{(b^2x^4 + 2abx^2 + a^2) \log\left(\frac{2abx + (bx^2 - a)\sqrt{-ab}}{bx^2 + a}\right) + 2(bx^3 - ax)\sqrt{-ab}}{16(ab^3x^4 + 2a^2b^2x^2 + a^3b)\sqrt{-ab}}, \frac{(b^2x^4 + 2abx^2 + a^2) \arctan\left(\frac{\sqrt{ab}x}{a}\right) + (bx^3 - ax)\sqrt{ab}}{8(ab^3x^4 + 2a^2b^2x^2 + a^3b)\sqrt{ab}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^3,x, algorithm="fricas")`

[Out]  $[1/16*((b^2*x^4 + 2*a*b*x^2 + a^2)*\log((2*a*b*x + (b*x^2 - a)*\sqrt{-a*b})/((a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*\sqrt{-a*b})), 1/8*((b^2*x^4 + 2*a*b*x^2 + a^2)*\arctan(\sqrt{a*b}*x/a) + (b*x^3 - a*x)*\sqrt{a*b})/((a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*\sqrt{a*b})]$

**Sympy** [A] time = 1.78902, size = 110, normalized size = 1.69

$$-\frac{\sqrt{-\frac{1}{a^3b^3}} \log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{a^3b^3}} \log\left(a^2b\sqrt{-\frac{1}{a^3b^3}} + x\right)}{16} + \frac{-ax + bx^3}{8a^3b + 16a^2b^2x^2 + 8ab^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**2+a)**3,x)`

[Out]  $-\sqrt{-1/(a**3*b**3)}*\log(-a**2*b*\sqrt{-1/(a**3*b**3)} + x)/16 + \sqrt{-1/(a**3*b**3)}*\log(a**2*b*\sqrt{-1/(a**3*b**3)} + x)/16 + (-a*x + b*x**3)/(8*a**3*b + 16*a**2*b**2*x**2 + 8*a*b**3*x**4)$

**GIAC/XCAS** [A] time = 0.214209, size = 68, normalized size = 1.05

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{abab}} + \frac{bx^3 - ax}{8(bx^2 + a)^2ab}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(x^2/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/8*(b*x^3 - a*x)/((b*x^2 + a)^2*a*b)
```

$$3.187 \quad \int \frac{1}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=62

$$\frac{3 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}} + \frac{3x}{8a^2(a+bx^2)} + \frac{x}{4a(a+bx^2)^2}$$

[Out]  $x/(4*a*(a+b*x^2)^2) + (3*x)/(8*a^2*(a+b*x^2)) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[b])$

**Rubi [A]** time = 0.0458219, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{3 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}} + \frac{3x}{8a^2(a+bx^2)} + \frac{x}{4a(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(-3), x]

[Out]  $x/(4*a*(a+b*x^2)^2) + (3*x)/(8*a^2*(a+b*x^2)) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[b])$

**Rubi in Sympy [A]** time = 5.70246, size = 54, normalized size = 0.87

$$\frac{x}{4a(a+bx^2)^2} + \frac{3x}{8a^2(a+bx^2)} + \frac{3 \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+a)\*\*3, x)

[Out]  $x/(4*a*(a+b*x**2)**2) + 3*x/(8*a**2*(a+b*x**2)) + 3*atan(sqrt(b)*x/sqrt(a))/(8*a**(5/2)*sqrt(b))$

**Mathematica [A]** time = 0.077456, size = 55, normalized size = 0.89

$$\frac{3 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}} + \frac{5ax + 3bx^3}{8a^2(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(-3), x]

[Out]  $(5*a*x + 3*b*x^3)/(8*a^2*(a+b*x^2)^2) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[b])$

**Maple [A]** time = 0.006, size = 51, normalized size = 0.8

$$\frac{x}{4a(bx^2+a)^2} + \frac{3x}{8a^2(bx^2+a)} + \frac{3}{8a^2} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^2+a)^3, x)`

[Out]  $\frac{1}{4} \frac{x}{a} (b x^2 + a)^2 + \frac{3}{8} \frac{x}{a^2} (b x^2 + a) + \frac{3}{8} \frac{1}{a^2} (a b)^{1/2} \arctan\left(\frac{x \sqrt{b}}{(a b)^{1/2}}\right)$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(-3), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.215088, size = 1, normalized size = 0.02

$$\left[ \frac{3(b^2x^4 + 2abx^2 + a^2) \log\left(\frac{2abx + (bx^2 - a)\sqrt{-ab}}{bx^2 + a}\right) + 2(3bx^3 + 5ax)\sqrt{-ab}}{16(a^2b^2x^4 + 2a^3bx^2 + a^4)\sqrt{-ab}}, \frac{3(b^2x^4 + 2abx^2 + a^2) \arctan\left(\frac{\sqrt{ab}x}{a}\right) + (3bx^3 + 5ax)\sqrt{ab}}{8(a^2b^2x^4 + 2a^3bx^2 + a^4)\sqrt{ab}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(-3), x, algorithm="fricas")`

[Out]  $\left[ \frac{1}{16} (3(b^2x^4 + 2abx^2 + a^2) \log((2abx + (bx^2 - a)\sqrt{-ab})/(bx^2 + a)) + 2(3bx^3 + 5ax)\sqrt{-ab}) / ((a^2b^2x^4 + 2a^3bx^2 + a^4)\sqrt{-ab}), \frac{1}{8} (3(b^2x^4 + 2abx^2 + a^2) \arctan(\sqrt{ab}x/a) + (3bx^3 + 5ax)\sqrt{ab}) / ((a^2b^2x^4 + 2a^3bx^2 + a^4)\sqrt{ab}) \right]$

**Sympy** [A] time = 1.84603, size = 105, normalized size = 1.69

$$-\frac{3\sqrt{-\frac{1}{a^5b}} \log\left(-a^3\sqrt{-\frac{1}{a^5b}} + x\right)}{16} + \frac{3\sqrt{-\frac{1}{a^5b}} \log\left(a^3\sqrt{-\frac{1}{a^5b}} + x\right)}{16} + \frac{5ax + 3bx^3}{8a^4 + 16a^3bx^2 + 8a^2b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**2+a)**3, x)`

[Out]  $-3\sqrt{-1/(a^5b)} \log(-a^3\sqrt{-1/(a^5b)} + x)/16 + 3\sqrt{-1/(a^5b)} \log(a^3\sqrt{-1/(a^5b)} + x)/16 + (5ax + 3bx^3)/(8a^4 + 16a^3bx^2 + 8a^2b^2x^4)$

**GIAC/XCAS** [A] time = 0.21021, size = 61, normalized size = 0.98

$$\frac{3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}a^2} + \frac{3bx^3 + 5ax}{8(bx^2 + a)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(-3),x, algorithm="giac")
```

```
[Out] 3/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/8*(3*b*x^3 + 5*a*x)
/((b*x^2 + a)^2*a^2)
```

$$3.188 \quad \int \frac{1}{x^2(a+bx^2)^3} dx$$

**Optimal.** Leaf size=76

$$-\frac{15\sqrt{b} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{7/2}} - \frac{15}{8a^3x} + \frac{5}{8a^2x(a+bx^2)} + \frac{1}{4ax(a+bx^2)^2}$$

[Out]  $-15/(8*a^3*x) + 1/(4*a*x*(a + b*x^2)^2) + 5/(8*a^2*x*(a + b*x^2)) - (15*\text{Sqrt}[b]*\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/(8*a^{(7/2)})$

**Rubi [A]** time = 0.0768378, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{15\sqrt{b} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{7/2}} - \frac{15}{8a^3x} + \frac{5}{8a^2x(a+bx^2)} + \frac{1}{4ax(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^2*(a + b*x^2)^3), x]$

[Out]  $-15/(8*a^3*x) + 1/(4*a*x*(a + b*x^2)^2) + 5/(8*a^2*x*(a + b*x^2)) - (15*\text{Sqrt}[b]*\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/(8*a^{(7/2)})$

**Rubi in Sympy [A]** time = 14.17, size = 65, normalized size = 0.86

$$\frac{1}{4ax(a+bx^2)^2} + \frac{5}{8a^2x(a+bx^2)} - \frac{15}{8a^3x} - \frac{15\sqrt{b} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{**2}/(b*x^{**2}+a)^{**3}, x)$

[Out]  $1/(4*a*x*(a + b*x^{**2})^{**2}) + 5/(8*a^{**2}*x*(a + b*x^{**2})) - 15/(8*a^{**3}*x) - 15*\text{sqrt}(b)*\text{atan}(\text{sqrt}(b)*x/\text{sqrt}(a))/(8*a^{**7/2})$

**Mathematica [A]** time = 0.0768561, size = 68, normalized size = 0.89

$$-\frac{15\sqrt{b} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{7/2}} - \frac{8a^2 + 25abx^2 + 15b^2x^4}{8a^3x(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/(x^2*(a + b*x^2)^3), x]$

[Out]  $-(8*a^2 + 25*a*b*x^2 + 15*b^2*x^4)/(8*a^3*x*(a + b*x^2)^2) - (15*\text{Sqrt}[b]*\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/(8*a^{(7/2)})$

**Maple [A]** time = 0.016, size = 66, normalized size = 0.9

$$-\frac{1}{a^3x} - \frac{7b^2x^3}{8a^3(bx^2+a)^2} - \frac{9bx}{8a^2(bx^2+a)^2} - \frac{15b}{8a^3} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(b*x^2+a)^3,x)`

[Out]  $-1/a^3/x - 7/8/a^3*b^2/(b*x^2+a)^2*x^3 - 9/8/a^2*b/(b*x^2+a)^2*x - 15/8/a^3*b/(a*b)^{(1/2)}*\arctan(x*b/(a*b)^{(1/2)})$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^3*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.226484, size = 1, normalized size = 0.01

$$\left[ \begin{array}{l} \frac{30 b^2 x^4 + 50 a b x^2 - 15 (b^2 x^5 + 2 a b x^3 + a^2 x) \sqrt{-\frac{b}{a}} \log\left(\frac{b x^2 - 2 a x \sqrt{-\frac{b}{a}} - a}{b x^2 + a}\right) + 16 a^2}{16 (a^3 b^2 x^5 + 2 a^4 b x^3 + a^5 x)}, \\ \frac{15 b^2 x^4 + 25 a b x^2 + 15 (b^2 x^5 + 2 a b x^3 + a^2 x) \sqrt{\frac{b}{a}} \arctan\left(\frac{b x}{a \sqrt{\frac{b}{a}}}\right) + 8 a^2}{8 (a^3 b^2 x^5 + 2 a^4 b x^3 + a^5 x)} \end{array} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^3*x^2),x, algorithm="fricas")`

[Out]  $[-1/16*(30*b^2*x^4 + 50*a*b*x^2 - 15*(b^2*x^5 + 2*a*b*x^3 + a^2*x)*\sqrt{-b/a}*\log((b*x^2 - 2*a*x*\sqrt{-b/a} - a)/(b*x^2 + a)) + 16*a^2)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x), -1/8*(15*b^2*x^4 + 25*a*b*x^2 + 15*(b^2*x^5 + 2*a*b*x^3 + a^2*x)*\sqrt{b/a}*\arctan(b*x/(a*\sqrt{b/a}))) + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)]$

**Sympy** [A] time = 2.41412, size = 114, normalized size = 1.5

$$\frac{15\sqrt{-\frac{b}{a^7}} \log\left(-\frac{a^4\sqrt{-\frac{b}{a^7}}}{b} + x\right)}{16} - \frac{15\sqrt{-\frac{b}{a^7}} \log\left(\frac{a^4\sqrt{-\frac{b}{a^7}}}{b} + x\right)}{16} - \frac{8a^2 + 25abx^2 + 15b^2x^4}{8a^5x + 16a^4bx^3 + 8a^3b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(b*x**2+a)**3,x)`

[Out]  $15*\sqrt{-b/a**7}*\log(-a**4*\sqrt{-b/a**7}/b + x)/16 - 15*\sqrt{-b/a**7}*\log(a**4*\sqrt{-b/a**7}/b + x)/16 - (8*a**2 + 25*a*b*x**2 + 15*b**2*x**4)/(8*a**5*x + 16*a**4*b*x**3 + 8*a**3*b**2*x**5)$

**GIAC/XCAS [A]** time = 0.206678, size = 77, normalized size = 1.01

$$-\frac{15 b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8 \sqrt{ab} a^3} - \frac{7 b^2 x^3 + 9 abx}{8 (bx^2 + a)^2 a^3} - \frac{1}{a^3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^3\*x^2),x, algorithm="giac")

[Out] -15/8\*b\*arctan(b\*x/sqrt(a\*b))/(sqrt(a\*b)\*a^3) - 1/8\*(7\*b^2\*x^3 + 9\*a\*b\*x)/((b\*x^2 + a)^2\*a^3) - 1/(a^3\*x)

$$3.189 \quad \int \frac{1}{x^4(a+bx^2)^3} dx$$

**Optimal.** Leaf size=87

$$\frac{35b^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{9/2}} + \frac{35b}{8a^4x} - \frac{35}{24a^3x^3} + \frac{7}{8a^2x^3(a+bx^2)} + \frac{1}{4ax^3(a+bx^2)^2}$$

[Out]  $-35/(24*a^3*x^3) + (35*b)/(8*a^4*x) + 1/(4*a*x^3*(a+b*x^2)^2) + 7/(8*a^2*x^3*(a+b*x^2)) + (35*b^{(3/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^{(9/2)})$

**Rubi [A]** time = 0.101161, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{35b^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{9/2}} + \frac{35b}{8a^4x} - \frac{35}{24a^3x^3} + \frac{7}{8a^2x^3(a+bx^2)} + \frac{1}{4ax^3(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a+b\*x^2)^3), x]

[Out]  $-35/(24*a^3*x^3) + (35*b)/(8*a^4*x) + 1/(4*a*x^3*(a+b*x^2)^2) + 7/(8*a^2*x^3*(a+b*x^2)) + (35*b^{(3/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^{(9/2)})$

**Rubi in Sympy [A]** time = 18.9718, size = 80, normalized size = 0.92

$$\frac{1}{4ax^3(a+bx^2)^2} + \frac{7}{8a^2x^3(a+bx^2)} - \frac{35}{24a^3x^3} + \frac{35b}{8a^4x} + \frac{35b^{3/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(b\*x\*\*2+a)\*\*3, x)

[Out]  $1/(4*a*x**3*(a+b*x**2)**2) + 7/(8*a**2*x**3*(a+b*x**2)) - 35/(24*a**3*x**3) + 35*b/(8*a**4*x) + 35*b**(3/2)*atan(sqrt(b)*x/sqrt(a))/(8*a**(9/2))$

**Mathematica [A]** time = 0.0877832, size = 79, normalized size = 0.91

$$\frac{35b^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{9/2}} + \frac{-8a^3 + 56a^2bx^2 + 175ab^2x^4 + 105b^3x^6}{24a^4x^3(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a+b\*x^2)^3), x]

[Out]  $(-8*a^3 + 56*a^2*b*x^2 + 175*a*b^2*x^4 + 105*b^3*x^6)/(24*a^4*x^3*(a+b*x^2)^2) + (35*b^{(3/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^{(9/2)})$



**Maple [A]** time = 0.019, size = 79, normalized size = 0.9

$$-\frac{1}{3a^3x^3} + 3\frac{b}{a^4x} + \frac{11b^3x^3}{8a^4(bx^2+a)^2} + \frac{13b^2x}{8a^3(bx^2+a)^2} + \frac{35b^2}{8a^4} \arctan\left(bx\frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(b*x^2+a)^3,x)`

[Out] 
$$-1/3/a^3/x^3+3*b/a^4/x+11/8/a^4*b^3/(b*x^2+a)^2*x^3+13/8/a^3*b^2/(b*x^2+a)^2*x+35/8/a^4*b^2/(a*b)^(1/2)*\arctan(x*b/(a*b)^(1/2))$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^3*x^4),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.22247, size = 1, normalized size = 0.01

$$\left[ \frac{210b^3x^6 + 350ab^2x^4 + 112a^2bx^2 - 16a^3 + 105(b^3x^7 + 2ab^2x^5 + a^2bx^3)\sqrt{-\frac{b}{a}} \log\left(\frac{bx^2+2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{48(a^4b^2x^7 + 2a^5bx^5 + a^6x^3)}, \frac{105b^3x^6 + 175ab^2x^4}{48(a^4b^2x^7 + 2a^5bx^5 + a^6x^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^3*x^4),x, algorithm="fricas")`

[Out] 
$$\left[ \frac{1}{48} \cdot (210 \cdot b^3 \cdot x^6 + 350 \cdot a \cdot b^2 \cdot x^4 + 112 \cdot a^2 \cdot b \cdot x^2 - 16 \cdot a^3 + 105 \cdot (b^3 \cdot x^7 + 2 \cdot a \cdot b^2 \cdot x^5 + a^2 \cdot b \cdot x^3) \cdot \sqrt{-b/a} \cdot \log((b \cdot x^2 + 2 \cdot a \cdot x \cdot \sqrt{-b/a} - a)/(b \cdot x^2 + a)))/(a^4 \cdot b^2 \cdot x^7 + 2 \cdot a^5 \cdot b \cdot x^5 + a^6 \cdot x^3), \frac{1}{24} \cdot (105 \cdot b^3 \cdot x^6 + 175 \cdot a \cdot b^2 \cdot x^4 + 56 \cdot a^2 \cdot b \cdot x^2 - 8 \cdot a^3 + 105 \cdot (b^3 \cdot x^7 + 2 \cdot a \cdot b^2 \cdot x^5 + a^2 \cdot b \cdot x^3) \cdot \sqrt{b/a} \cdot \arctan(b \cdot x/(a \cdot \sqrt{b/a}))) / (a^4 \cdot b^2 \cdot x^7 + 2 \cdot a^5 \cdot b \cdot x^5 + a^6 \cdot x^3) \right]$$

**Sympy [A]** time = 3.33679, size = 138, normalized size = 1.59

$$-\frac{35\sqrt{-\frac{b^3}{a^9}} \log\left(-\frac{a^5\sqrt{-\frac{b^3}{a^9}}}{b^2} + x\right)}{16} + \frac{35\sqrt{-\frac{b^3}{a^9}} \log\left(\frac{a^5\sqrt{-\frac{b^3}{a^9}}}{b^2} + x\right)}{16} + \frac{-8a^3 + 56a^2bx^2 + 175ab^2x^4 + 105b^3x^6}{24a^6x^3 + 48a^5bx^5 + 24a^4b^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(b*x**2+a)**3,x)`

[Out] 
$$-35*\sqrt{-b**3/a**9}*\log(-a**5*\sqrt{-b**3/a**9}/b**2 + x)/16 + 35*\sqrt{-b**3/a**9}*\log(a**5*\sqrt{-b**3/a**9}/b**2 + x)/16 + (-8*a**3 + 56*a**2*b*x**2 + 175*a*b**2*x**4 + 105*b**3*x**6)/(24*a**6*x**3 + 48*a**5*b*x**5 + 24*a**4*b**2*x**7)$$

---

**GIAC/XCAS [A]** time = 0.223054, size = 96, normalized size = 1.1

$$\frac{35 b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8 \sqrt{ab} a^4} + \frac{11 b^3 x^3 + 13 ab^2 x}{8 (bx^2 + a)^2 a^4} + \frac{9 bx^2 - a}{3 a^4 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^3*x^4),x, algorithm="giac")`

[Out] `35/8*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/8*(11*b^3*x^3 + 13*a*b^2*x)/((b*x^2 + a)^2*a^4) + 1/3*(9*b*x^2 - a)/(a^4*x^3)`

$$3.190 \quad \int \frac{1}{x^6(a+bx^2)^3} dx$$

**Optimal.** Leaf size=100

$$-\frac{63b^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{11/2}} - \frac{63b^2}{8a^5x} + \frac{21b}{8a^4x^3} - \frac{63}{40a^3x^5} + \frac{9}{8a^2x^5(a+bx^2)} + \frac{1}{4ax^5(a+bx^2)^2}$$

[Out]  $-63/(40*a^3*x^5) + (21*b)/(8*a^4*x^3) - (63*b^2)/(8*a^5*x) + 1/(4*a*x^5*(a+b*x^2)^2) + 9/(8*a^2*x^5*(a+b*x^2)) - (63*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(11/2))$

**Rubi [A]** time = 0.123551, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{63b^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{11/2}} - \frac{63b^2}{8a^5x} + \frac{21b}{8a^4x^3} - \frac{63}{40a^3x^5} + \frac{9}{8a^2x^5(a+bx^2)} + \frac{1}{4ax^5(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(a+b\*x^2)^3),x]

[Out]  $-63/(40*a^3*x^5) + (21*b)/(8*a^4*x^3) - (63*b^2)/(8*a^5*x) + 1/(4*a*x^5*(a+b*x^2)^2) + 9/(8*a^2*x^5*(a+b*x^2)) - (63*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(11/2))$

**Rubi in Sympy [A]** time = 24.4599, size = 94, normalized size = 0.94

$$\frac{1}{4ax^5(a+bx^2)^2} + \frac{9}{8a^2x^5(a+bx^2)} - \frac{63}{40a^3x^5} + \frac{21b}{8a^4x^3} - \frac{63b^2}{8a^5x} - \frac{63b^{5/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*6/(b\*x\*\*2+a)\*\*3,x)

[Out]  $1/(4*a*x**5*(a+b*x**2)**2) + 9/(8*a**2*x**5*(a+b*x**2)) - 63/(40*a**3*x**5) + 21*b/(8*a**4*x**3) - 63*b**2/(8*a**5*x) - 63*b**(5/2)*atan(sqrt(b)*x/sqrt(a))/(8*a**(11/2))$

**Mathematica [A]** time = 0.102001, size = 90, normalized size = 0.9

$$-\frac{63b^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{11/2}} - \frac{8a^4 - 24a^3bx^2 + 168a^2b^2x^4 + 525ab^3x^6 + 315b^4x^8}{40a^5x^5(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6\*(a+b\*x^2)^3),x]

[Out]  $-(8*a^4 - 24*a^3*b*x^2 + 168*a^2*b^2*x^4 + 525*a*b^3*x^6 + 315*b^4*x^8)/(40*a^5*x^5*(a+b*x^2)^2) - (63*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(11/2))$

**Maple [A]** time = 0.017, size = 89, normalized size = 0.9

$$-\frac{1}{5a^3x^5} - 6\frac{b^2}{a^5x} + \frac{b}{a^4x^3} - \frac{15b^4x^3}{8a^5(bx^2+a)^2} - \frac{17b^3x}{8a^4(bx^2+a)^2} - \frac{63b^3}{8a^5} \arctan\left(bx\frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(b\*x^2+a)^3,x)

[Out]  $-\frac{1}{5} \frac{1}{a^3 x^5} - 6 \frac{b^2}{a^5 x} + \frac{b}{a^4 x^3} - \frac{15}{8} \frac{b^4 x^3}{a^5 (bx^2 + a)^2} - \frac{17}{8} \frac{b^3 x}{a^4 (bx^2 + a)^2} - \frac{63}{8} \frac{b^3}{a^5} \arctan\left(\frac{bx}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^3\*x^6),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.222119, size = 1, normalized size = 0.01

$$\left[ \frac{630b^4x^8 + 1050ab^3x^6 + 336a^2b^2x^4 - 48a^3bx^2 + 16a^4 - 315(b^4x^9 + 2ab^3x^7 + a^2b^2x^5)\sqrt{-\frac{b}{a}} \log\left(\frac{bx^2 - 2ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right)}{80(a^5b^2x^9 + 2a^6bx^7 + a^7x^5)}, \right. \\ \left. \frac{315b^4x^8 + 525ab^3x^6 + 168a^2b^2x^4 - 24a^3bx^2 + 8a^4 + 315(b^4x^9 + 2ab^3x^7 + a^2b^2x^5)\sqrt{\frac{b}{a}} \arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)}{40(a^5b^2x^9 + 2a^6bx^7 + a^7x^5)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^3\*x^6),x, algorithm="fricas")

[Out]  $[-\frac{1}{80} * (630 * b^4 * x^8 + 1050 * a * b^3 * x^6 + 336 * a^2 * b^2 * x^4 - 48 * a^3 * b * x^2 + 16 * a^4 - 315 * (b^4 * x^9 + 2 * a * b^3 * x^7 + a^2 * b^2 * x^5) * \sqrt{-b/a} * \log((b * x^2 - 2 * a * x * \sqrt{-b/a} - a) / (b * x^2 + a))) / (a^5 * b^2 * x^9 + 2 * a^6 * b * x^7 + a^7 * x^5), -\frac{1}{40} * (315 * b^4 * x^8 + 525 * a * b^3 * x^6 + 168 * a^2 * b^2 * x^4 - 24 * a^3 * b * x^2 + 8 * a^4 + 315 * (b^4 * x^9 + 2 * a * b^3 * x^7 + a^2 * b^2 * x^5) * \sqrt{b/a} * \arctan(b * x / (a * \sqrt{b/a}))) / (a^5 * b^2 * x^9 + 2 * a^6 * b * x^7 + a^7 * x^5)]$

**Sympy [A]** time = 5.47564, size = 150, normalized size = 1.5

$$\frac{63\sqrt{-\frac{b^5}{a^{11}}}\log\left(-\frac{a^6\sqrt{-\frac{b^5}{a^{11}}}}{b^3} + x\right)}{16} - \frac{63\sqrt{-\frac{b^5}{a^{11}}}\log\left(\frac{a^6\sqrt{-\frac{b^5}{a^{11}}}}{b^3} + x\right)}{16} \\ - \frac{8a^4 - 24a^3bx^2 + 168a^2b^2x^4 + 525ab^3x^6 + 315b^4x^8}{40a^7x^5 + 80a^6bx^7 + 40a^5b^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*6/(b\*x\*\*2+a)\*\*3,x)

[Out]  $63\sqrt{-b^5/a^{11}}\log(-a^6\sqrt{-b^5/a^{11}}/b^3 + x)/16 - 63\sqrt{-b^5/a^{11}}\log(a^6\sqrt{-b^5/a^{11}}/b^3 + x)/16 - (8a^4 - 24a^3bx^2 + 168a^2b^2x^4 + 525ab^3x^6 + 315b^4x^8)/(40a^7x^5 + 80a^6bx^7 + 40a^5b^2x^9)$

**GIAC/XCAS [A]** time = 0.222618, size = 108, normalized size = 1.08

$$-\frac{63b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}a^5} - \frac{15b^4x^3 + 17ab^3x}{8(bx^2 + a)^2a^5} - \frac{30b^2x^4 - 5abx^2 + a^2}{5a^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^3\*x^6),x, algorithm="giac")

[Out]  $-63/8*b^3*\arctan(b*x/\sqrt{a*b})/(\sqrt{a*b})*a^5 - 1/8*(15*b^4*x^3 + 17*a*b^3*x)/((b*x^2 + a)^2*a^5) - 1/5*(30*b^2*x^4 - 5*a*b*x^2 + a^2)/(a^5*x^5)$

$$3.191 \quad \int \frac{1}{x^8(a+bx^2)^3} dx$$

**Optimal.** Leaf size=113

$$\frac{99b^{7/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{13/2}} + \frac{99b^3}{8a^6x} - \frac{33b^2}{8a^5x^3} + \frac{99b}{40a^4x^5} - \frac{99}{56a^3x^7} + \frac{11}{8a^2x^7(a+bx^2)} + \frac{1}{4ax^7(a+bx^2)^2}$$

[Out]  $-99/(56*a^3*x^7) + (99*b)/(40*a^4*x^5) - (33*b^2)/(8*a^5*x^3) + (99*b^3)/(8*a^6*x) + 1/(4*a*x^7*(a+b*x^2)^2) + 11/(8*a^2*x^7*(a+b*x^2)) + (99*b^{(7/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^{(13/2)})$

**Rubi [A]** time = 0.152835, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{99b^{7/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{13/2}} + \frac{99b^3}{8a^6x} - \frac{33b^2}{8a^5x^3} + \frac{99b}{40a^4x^5} - \frac{99}{56a^3x^7} + \frac{11}{8a^2x^7(a+bx^2)} + \frac{1}{4ax^7(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^8\*(a+b\*x^2)^3),x]

[Out]  $-99/(56*a^3*x^7) + (99*b)/(40*a^4*x^5) - (33*b^2)/(8*a^5*x^3) + (99*b^3)/(8*a^6*x) + 1/(4*a*x^7*(a+b*x^2)^2) + 11/(8*a^2*x^7*(a+b*x^2)) + (99*b^{(7/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^{(13/2)})$

**Rubi in Sympy [A]** time = 30.3363, size = 107, normalized size = 0.95

$$\frac{1}{4ax^7(a+bx^2)^2} + \frac{11}{8a^2x^7(a+bx^2)} - \frac{99}{56a^3x^7} + \frac{99b}{40a^4x^5} - \frac{33b^2}{8a^5x^3} + \frac{99b^3}{8a^6x} + \frac{99b^{7/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*8/(b\*x\*\*2+a)\*\*3,x)

[Out]  $1/(4*a*x**7*(a+b*x**2)**2) + 11/(8*a**2*x**7*(a+b*x**2)) - 99/(56*a**3*x**7) + 99*b/(40*a**4*x**5) - 33*b**2/(8*a**5*x**3) + 99*b**3/(8*a**6*x) + 99*b**7/2*atan(sqrt(b)*x/sqrt(a))/(8*a**13/2)$

**Mathematica [A]** time = 0.0996398, size = 101, normalized size = 0.89

$$\frac{99b^{7/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{13/2}} + \frac{-40a^5 + 88a^4bx^2 - 264a^3b^2x^4 + 1848a^2b^3x^6 + 5775ab^4x^8 + 3465b^5x^{10}}{280a^6x^7(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^8\*(a+b\*x^2)^3),x]

[Out]  $(-40*a^5 + 88*a^4*b*x^2 - 264*a^3*b^2*x^4 + 1848*a^2*b^3*x^6 + 5775*a*b^4*x^8 + 3465*b^5*x^{10})/(280*a^6*x^7*(a+b*x^2)^2) + (99*b^{(7/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^{(13/2)})$

**Maple [A]** time = 0.019, size = 101, normalized size = 0.9

$$-\frac{1}{7a^3x^7} + 10\frac{b^3}{a^6x} - 2\frac{b^2}{a^5x^3} + \frac{3b}{5a^4x^5} + \frac{19b^5x^3}{8a^6(bx^2+a)^2} + \frac{21b^4x}{8a^5(bx^2+a)^2} + \frac{99b^4}{8a^6} \arctan\left(bx\frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^8/(b\*x^2+a)^3,x)

[Out] -1/7/a^3/x^7+10\*b^3/a^6/x-2\*b^2/a^5/x^3+3/5\*b/a^4/x^5+19/8/a^6\*b^5/(b\*x^2+a)^2\*x^3+21/8/a^5\*b^4/(b\*x^2+a)^2\*x+99/8/a^6\*b^4/(a\*b)^(1/2)\*arctan(x\*b/(a\*b)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^3\*x^8),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.221491, size = 1, normalized size = 0.01

$$\frac{6930b^5x^{10} + 11550ab^4x^8 + 3696a^2b^3x^6 - 528a^3b^2x^4 + 176a^4bx^2 - 80a^5 + 3465(b^5x^{11} + 2ab^4x^9 + a^2b^3x^7)\sqrt{-\frac{b}{a}} \log\left(\frac{bx}{\sqrt{-\frac{b}{a}}}\right)}{560(a^6b^2x^{11} + 2a^7bx^9 + a^8x^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^3\*x^8),x, algorithm="fricas")

[Out] [1/560\*(6930\*b^5\*x^10 + 11550\*a\*b^4\*x^8 + 3696\*a^2\*b^3\*x^6 - 528\*a^3\*b^2\*x^4 + 176\*a^4\*b\*x^2 - 80\*a^5 + 3465\*(b^5\*x^11 + 2\*a\*b^4\*x^9 + a^2\*b^3\*x^7)\*sqrt(-b/a)\*log((b\*x^2 + 2\*a\*x\*sqrt(-b/a) - a)/(b\*x^2 + a)))/(a^6\*b^2\*x^11 + 2\*a^7\*b\*x^9 + a^8\*x^7), 1/280\*(3465\*b^5\*x^10 + 5775\*a\*b^4\*x^8 + 1848\*a^2\*b^3\*x^6 - 264\*a^3\*b^2\*x^4 + 88\*a^4\*b\*x^2 - 40\*a^5 + 3465\*(b^5\*x^11 + 2\*a\*b^4\*x^9 + a^2\*b^3\*x^7)\*sqrt(b/a)\*arctan(b\*x/(a\*sqrt(b/a)))/(a^6\*b^2\*x^11 + 2\*a^7\*b\*x^9 + a^8\*x^7)]

**Sympy [A]** time = 10.188, size = 162, normalized size = 1.43

$$-\frac{99\sqrt{-\frac{b^7}{a^{13}}}\log\left(-\frac{a^7\sqrt{-\frac{b^7}{a^{13}}}}{b^4} + x\right)}{16} + \frac{99\sqrt{-\frac{b^7}{a^{13}}}\log\left(\frac{a^7\sqrt{-\frac{b^7}{a^{13}}}}{b^4} + x\right)}{16} + \frac{-40a^5 + 88a^4bx^2 - 264a^3b^2x^4 + 1848a^2b^3x^6 + 5775ab^4x^8 + 3465b^5x^{10}}{280a^8x^7 + 560a^7bx^9 + 280a^6b^2x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*8/(b\*x\*\*2+a)\*\*3,x)

```
[Out] -99*sqrt(-b**7/a**13)*log(-a**7*sqrt(-b**7/a**13)/b**4 + x)/16 +
99*sqrt(-b**7/a**13)*log(a**7*sqrt(-b**7/a**13)/b**4 + x)/16 + (-
40*a**5 + 88*a**4*b*x**2 - 264*a**3*b**2*x**4 + 1848*a**2*b**3*x**
6 + 5775*a*b**4*x**8 + 3465*b**5*x**10)/(280*a**8*x**7 + 560*a**
7*b*x**9 + 280*a**6*b**2*x**11)
```

**GIAC/XCAS [A]** time = 0.222218, size = 126, normalized size = 1.12

$$\frac{99b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}a^6} + \frac{19b^5x^3 + 21ab^4x}{8(bx^2 + a)^2a^6} + \frac{350b^3x^6 - 70ab^2x^4 + 21a^2bx^2 - 5a^3}{35a^6x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^3*x^8),x, algorithm="giac")
```

```
[Out] 99/8*b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6) + 1/8*(19*b^5*x^3
+ 21*a*b^4*x)/((b*x^2 + a)^2*a^6) + 1/35*(350*b^3*x^6 - 70*a*b^2*
x^4 + 21*a^2*b*x^2 - 5*a^3)/(a^6*x^7)
```



$$3.192 \quad \int \frac{x^{25}}{(a+bx^2)^{10}} dx$$

Optimal. Leaf size=216

$$\begin{aligned} & -\frac{a^{12}}{18b^{13}(a+bx^2)^9} + \frac{3a^{11}}{4b^{13}(a+bx^2)^8} - \frac{33a^{10}}{7b^{13}(a+bx^2)^7} + \frac{55a^9}{3b^{13}(a+bx^2)^6} \\ & - \frac{99a^8}{2b^{13}(a+bx^2)^5} + \frac{99a^7}{b^{13}(a+bx^2)^4} - \frac{154a^6}{b^{13}(a+bx^2)^3} + \frac{198a^5}{b^{13}(a+bx^2)^2} \\ & - \frac{495a^4}{2b^{13}(a+bx^2)} - \frac{110a^3 \log(a+bx^2)}{b^{13}} + \frac{55a^2x^2}{2b^{12}} - \frac{5ax^4}{2b^{11}} + \frac{x^6}{6b^{10}} \end{aligned}$$

[Out] (55\*a^2\*x^2)/(2\*b^12) - (5\*a\*x^4)/(2\*b^11) + x^6/(6\*b^10) - a^12/(18\*b^13\*(a+b\*x^2)^9) + (3\*a^11)/(4\*b^13\*(a+b\*x^2)^8) - (33\*a^10)/(7\*b^13\*(a+b\*x^2)^7) + (55\*a^9)/(3\*b^13\*(a+b\*x^2)^6) - (99\*a^8)/(2\*b^13\*(a+b\*x^2)^5) + (99\*a^7)/(b^13\*(a+b\*x^2)^4) - (154\*a^6)/(b^13\*(a+b\*x^2)^3) + (198\*a^5)/(b^13\*(a+b\*x^2)^2) - (495\*a^4)/(2\*b^13\*(a+b\*x^2)) - (110\*a^3\*Log[a+b\*x^2])/b^13

Rubi [A] time = 0.534803, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\begin{aligned} & -\frac{a^{12}}{18b^{13}(a+bx^2)^9} + \frac{3a^{11}}{4b^{13}(a+bx^2)^8} - \frac{33a^{10}}{7b^{13}(a+bx^2)^7} + \frac{55a^9}{3b^{13}(a+bx^2)^6} \\ & - \frac{99a^8}{2b^{13}(a+bx^2)^5} + \frac{99a^7}{b^{13}(a+bx^2)^4} - \frac{154a^6}{b^{13}(a+bx^2)^3} + \frac{198a^5}{b^{13}(a+bx^2)^2} \\ & - \frac{495a^4}{2b^{13}(a+bx^2)} - \frac{110a^3 \log(a+bx^2)}{b^{13}} + \frac{55a^2x^2}{2b^{12}} - \frac{5ax^4}{2b^{11}} + \frac{x^6}{6b^{10}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^25/(a + b\*x^2)^10, x]

[Out] (55\*a^2\*x^2)/(2\*b^12) - (5\*a\*x^4)/(2\*b^11) + x^6/(6\*b^10) - a^12/(18\*b^13\*(a+b\*x^2)^9) + (3\*a^11)/(4\*b^13\*(a+b\*x^2)^8) - (33\*a^10)/(7\*b^13\*(a+b\*x^2)^7) + (55\*a^9)/(3\*b^13\*(a+b\*x^2)^6) - (99\*a^8)/(2\*b^13\*(a+b\*x^2)^5) + (99\*a^7)/(b^13\*(a+b\*x^2)^4) - (154\*a^6)/(b^13\*(a+b\*x^2)^3) + (198\*a^5)/(b^13\*(a+b\*x^2)^2) - (495\*a^4)/(2\*b^13\*(a+b\*x^2)) - (110\*a^3\*Log[a+b\*x^2])/b^13

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\begin{aligned} & -\frac{a^{12}}{18b^{13}(a+bx^2)^9} + \frac{3a^{11}}{4b^{13}(a+bx^2)^8} - \frac{33a^{10}}{7b^{13}(a+bx^2)^7} + \frac{55a^9}{3b^{13}(a+bx^2)^6} \\ & - \frac{99a^8}{2b^{13}(a+bx^2)^5} + \frac{99a^7}{b^{13}(a+bx^2)^4} - \frac{154a^6}{b^{13}(a+bx^2)^3} + \frac{198a^5}{b^{13}(a+bx^2)^2} \\ & - \frac{495a^4}{2b^{13}(a+bx^2)} - \frac{110a^3 \log(a+bx^2)}{b^{13}} + \frac{55a^2x^2}{2b^{12}} - \frac{5a \int^{x^2} x dx}{b^{11}} + \frac{x^6}{6b^{10}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*25/(b\*x\*\*2+a)\*\*10, x)

[Out] -a\*\*12/(18\*b\*\*13\*(a+b\*x\*\*2)\*\*9) + 3\*a\*\*11/(4\*b\*\*13\*(a+b\*x\*\*2)\*\*8) - 33\*a\*\*10/(7\*b\*\*13\*(a+b\*x\*\*2)\*\*7) + 55\*a\*\*9/(3\*b\*\*13\*(a+b\*x\*\*2)\*\*6) - 99\*a\*\*8/(2\*b\*\*13\*(a+b\*x\*\*2)\*\*5) + 99\*a\*\*7/(b\*\*13\*(a+b\*x\*\*2)\*\*4) - 154\*a\*\*6/(b\*\*13\*(a+b\*x\*\*2)\*\*3) + 198\*a\*\*5/(b\*\*13\*(a+b\*x\*\*2)\*\*2) - 495\*a\*\*4/(2\*b\*\*13\*(a+b\*x\*\*2)) - 110\*a\*\*3\*log(a+b\*x\*\*2)/b\*\*13 + 55\*a\*\*2\*x\*\*2/(2\*b\*\*12) - 5\*a\*Integral(

$x, (x, x^2))/b^{11} + x^6/(6b^{10})$

**Mathematica [A]** time = 0.0883611, size = 169, normalized size = 0.78

$$\frac{35201a^{12} + 289089a^{11}bx^2 + 1031616a^{10}b^2x^4 + 2074464a^9b^3x^6 + 2529576a^8b^4x^8 + 1831032a^7b^5x^{10} + 638568a^6b^6x^{12} - 58968a^5b^7x^{14} - 139482a^4b^8x^{16} - 43218a^3b^9x^{18} - 2772a^2b^{10}x^{20} + 252ab^{11}x^{22} - 42b^{12}x^{24} + 27720a^3(a + b^2x^2)^9 \text{Log}[a + b^2x^2]}{252b^{13}(a + b^2x^2)^9}$$

Antiderivative was successfully verified.

[In] Integrate[x^25/(a + b\*x^2)^10, x]

[Out]  $-(35201a^{12} + 289089a^{11}b^2x^2 + 1031616a^{10}b^4x^4 + 2074464a^9b^6x^6 + 2529576a^8b^8x^8 + 1831032a^7b^{10}x^{10} + 638568a^6b^{12}x^{12} - 58968a^5b^{14}x^{14} - 139482a^4b^{16}x^{16} - 43218a^3b^{18}x^{18} - 2772a^2b^{20}x^{20} + 252ab^{22}x^{22} - 42b^{24}x^{24} + 27720a^3(a + b^2x^2)^9 \text{Log}[a + b^2x^2])/(252b^{13}(a + b^2x^2)^9)$

**Maple [A]** time = 0.027, size = 199, normalized size = 0.9

$$\frac{55a^2x^2}{2b^{12}} - \frac{5ax^4}{2b^{11}} + \frac{x^6}{6b^{10}} - \frac{a^{12}}{18b^{13}(bx^2 + a)^9} + \frac{3a^{11}}{4b^{13}(bx^2 + a)^8} - \frac{33a^{10}}{7b^{13}(bx^2 + a)^7} + \frac{55a^9}{3b^{13}(bx^2 + a)^6} - \frac{99a^8}{2b^{13}(bx^2 + a)^5} + 99\frac{a^7}{b^{13}(bx^2 + a)^4} - 154\frac{a^6}{b^{13}(bx^2 + a)^3} + 198\frac{a^5}{b^{13}(bx^2 + a)^2} - \frac{495a^4}{2b^{13}(bx^2 + a)} - 110\frac{a^3 \ln(bx^2 + a)}{b^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^25/(b\*x^2+a)^10, x)

[Out]  $55/2*a^2*x^2/b^{12} - 5/2*a*x^4/b^{11} + 1/6*x^6/b^{10} - 1/18*a^{12}/b^{13}/(b*x^2+a)^9 + 3/4*a^{11}/b^{13}/(b*x^2+a)^8 - 33/7*a^{10}/b^{13}/(b*x^2+a)^7 + 55/3*a^9/b^{13}/(b*x^2+a)^6 - 99/2*a^8/b^{13}/(b*x^2+a)^5 + 99*a^7/b^{13}/(b*x^2+a)^4 - 154*a^6/b^{13}/(b*x^2+a)^3 + 198*a^5/b^{13}/(b*x^2+a)^2 - 495/2*a^4/b^{13}/(b*x^2+a) - 110*a^3*ln(b*x^2+a)/b^{13}$

**Maxima [A]** time = 1.4119, size = 327, normalized size = 1.51

$$\frac{62370a^4b^8x^{16} + 449064a^5b^7x^{14} + 1435896a^6b^6x^{12} + 2652804a^7b^5x^{10} + 3089394a^8b^4x^8 + 2318316a^9b^3x^6 + 1093356a^{10}b^2x^4 + 296019a^{11}b^2x^2 + 35201a^{12}}{252(b^2x^2 + a)^9} + \frac{110a^3 \log(bx^2 + a)}{b^{13}} + \frac{b^2x^6 - 15abx^4 + 165a^2x^2}{6b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^25/(b\*x^2 + a)^10, x, algorithm="maxima")

[Out]  $-1/252*(62370*a^4*b^8*x^{16} + 449064*a^5*b^7*x^{14} + 1435896*a^6*b^6*x^{12} + 2652804*a^7*b^5*x^{10} + 3089394*a^8*b^4*x^8 + 2318316*a^9*b^3*x^6 + 1093356*a^{10}*b^2*x^4 + 296019*a^{11}*b*x^2 + 35201*a^{12})/(b^{22}*x^{18} + 9*a*b^{21}*x^{16} + 36*a^2*b^{20}*x^{14} + 84*a^3*b^{19}*x^{12} + 126*a^4*b^{18}*x^{10} + 126*a^5*b^{17}*x^8 + 84*a^6*b^{16}*x^6 + 36*a^7*b^{15}*x^4 + 9*a^8*b^{14}*x^2 + a^9*b^{13}) - 110*a^3*log(b*x^2 + a)/b^{13} + 1/6*(b^2*x^6 - 15*a*b*x^4 + 165*a^2*x^2)/b^{12}$

**Fricas [A]** time = 0.208724, size = 467, normalized size = 2.16

$$\frac{42 b^{12} x^{24} - 252 a b^{11} x^{22} + 2772 a^2 b^{10} x^{20} + 43218 a^3 b^9 x^{18} + 139482 a^4 b^8 x^{16} + 58968 a^5 b^7 x^{14} - 638568 a^6 b^6 x^{12} - 1831032 a^7 b^5 x^{10} - 2529576 a^8 b^4 x^8 - 2074464 a^9 b^3 x^6 - 1031616 a^{10} b^2 x^4 - 289089 a^{11} b x^2 - 35201 a^{12}}{252 (b^2 x^2 + a)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^25/(b\*x^2 + a)^10,x, algorithm="fricas")

[Out] 1/252\*(42\*b^12\*x^24 - 252\*a\*b^11\*x^22 + 2772\*a^2\*b^10\*x^20 + 43218\*a^3\*b^9\*x^18 + 139482\*a^4\*b^8\*x^16 + 58968\*a^5\*b^7\*x^14 - 638568\*a^6\*b^6\*x^12 - 1831032\*a^7\*b^5\*x^10 - 2529576\*a^8\*b^4\*x^8 - 2074464\*a^9\*b^3\*x^6 - 1031616\*a^10\*b^2\*x^4 - 289089\*a^11\*b\*x^2 - 35201\*a^12 - 27720\*(a^3\*b^9\*x^18 + 9\*a^4\*b^8\*x^16 + 36\*a^5\*b^7\*x^14 + 84\*a^6\*b^6\*x^12 + 126\*a^7\*b^5\*x^10 + 126\*a^8\*b^4\*x^8 + 84\*a^9\*b^3\*x^6 + 36\*a^10\*b^2\*x^4 + 9\*a^11\*b\*x^2 + a^12)\*log(b\*x^2 + a))/(b^22\*x^18 + 9\*a\*b^21\*x^16 + 36\*a^2\*b^20\*x^14 + 84\*a^3\*b^19\*x^12 + 126\*a^4\*b^18\*x^10 + 126\*a^5\*b^17\*x^8 + 84\*a^6\*b^16\*x^6 + 36\*a^7\*b^15\*x^4 + 9\*a^8\*b^14\*x^2 + a^9\*b^13)

**Sympy [A]** time = 37.9672, size = 258, normalized size = 1.19

$$\frac{-\frac{110 a^3 \log(a + b x^2)}{b^{13}} + \frac{55 a^2 x^2}{2 b^{12}} - \frac{5 a x^4}{2 b^{11}}}{\frac{35201 a^{12} + 296019 a^{11} b x^2 + 1093356 a^{10} b^2 x^4 + 2318316 a^9 b^3 x^6 + 3089394 a^8 b^4 x^8 + 2652804 a^7 b^5 x^{10} + 1435896 a^6 b^6 x^{12} + 449064 a^5 b^7 x^{14} + 62370 a^4 b^8 x^{16} + 252 a^3 b^9 x^{18} + 2268 a^2 b^{10} x^{20} + 21168 a b^{11} x^{22} + 252 b^{12} x^{24}}{252 (b^2 x^2 + a)^{10}}} + \frac{x^6}{6 b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*25/(b\*x\*\*2+a)\*\*10,x)

[Out] -110\*a\*\*3\*log(a + b\*x\*\*2)/b\*\*13 + 55\*a\*\*2\*x\*\*2/(2\*b\*\*12) - 5\*a\*x\*\*4/(2\*b\*\*11) - (35201\*a\*\*12 + 296019\*a\*\*11\*b\*x\*\*2 + 1093356\*a\*\*10\*b\*\*2\*x\*\*4 + 2318316\*a\*\*9\*b\*\*3\*x\*\*6 + 3089394\*a\*\*8\*b\*\*4\*x\*\*8 + 2652804\*a\*\*7\*b\*\*5\*x\*\*10 + 1435896\*a\*\*6\*b\*\*6\*x\*\*12 + 449064\*a\*\*5\*b\*\*7\*x\*\*14 + 62370\*a\*\*4\*b\*\*8\*x\*\*16)/(252\*a\*\*9\*b\*\*13 + 2268\*a\*\*8\*b\*\*14\*x\*\*2 + 9072\*a\*\*7\*b\*\*15\*x\*\*4 + 21168\*a\*\*6\*b\*\*16\*x\*\*6 + 31752\*a\*\*5\*b\*\*17\*x\*\*8 + 31752\*a\*\*4\*b\*\*18\*x\*\*10 + 21168\*a\*\*3\*b\*\*19\*x\*\*12 + 9072\*a\*\*2\*b\*\*20\*x\*\*14 + 2268\*a\*b\*\*21\*x\*\*16 + 252\*b\*\*22\*x\*\*18) + x\*\*6/(6\*b\*\*10)

**GIAC/XCAS [A]** time = 0.217233, size = 227, normalized size = 1.05

$$\frac{\frac{110 a^3 \ln(|b x^2 + a|)}{b^{13}}}{\frac{78419 a^3 b^9 x^{18} + 643401 a^4 b^8 x^{16} + 2374020 a^5 b^7 x^{14} + 5151300 a^6 b^6 x^{12} + 7227990 a^7 b^5 x^{10} + 6791400 a^8 b^4 x^8 + 4268880 a^9 b^3 x^6 + 1729728 a^{10} b^2 x^4 + 409752 a^{11} b x^2 + 43218 a^{12}}{252 (b x^2 + a)^9 b^{13}}} + \frac{b^{20} x^6 - 15 a b^{19} x^4 + 165 a^2 b^{18} x^2}{6 b^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^25/(b\*x^2 + a)^10,x, algorithm="giac")

[Out] -110\*a^3\*ln(abs(b\*x^2 + a))/b^13 + 1/252\*(78419\*a^3\*b^9\*x^18 + 643401\*a^4\*b^8\*x^16 + 2374020\*a^5\*b^7\*x^14 + 5151300\*a^6\*b^6\*x^12 + 7227990\*a^7\*b^5\*x^10 + 6791400\*a^8\*b^4\*x^8 + 4268880\*a^9\*b^3\*x^6 + 1729728\*a^10\*b^2\*x^4 + 409752\*a^11\*b\*x^2 + 43218\*a^12)/(b\*x^2 + a)^9\*b^13) + 1/6\*(b^20\*x^6 - 15\*a\*b^19\*x^4 + 165\*a^2\*b^18\*x^2)/b^30

$$3.193 \quad \int \frac{x^{23}}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=205

$$\frac{a^{11}}{18b^{12}(a+bx^2)^9} - \frac{11a^{10}}{16b^{12}(a+bx^2)^8} + \frac{55a^9}{14b^{12}(a+bx^2)^7} - \frac{55a^8}{4b^{12}(a+bx^2)^6} + \frac{33a^7}{b^{12}(a+bx^2)^5} \\ - \frac{231a^6}{4b^{12}(a+bx^2)^4} + \frac{77a^5}{b^{12}(a+bx^2)^3} - \frac{165a^4}{2b^{12}(a+bx^2)^2} + \frac{165a^3}{2b^{12}(a+bx^2)} + \frac{55a^2 \log(a+bx^2)}{2b^{12}} - \frac{5ax^2}{b^{11}} + \frac{x^4}{4b^{10}}$$

[Out]  $(-5*a*x^2)/b^{11} + x^4/(4*b^{10}) + a^{11}/(18*b^{12}*(a + b*x^2)^9) - (11*a^{10})/(16*b^{12}*(a + b*x^2)^8) + (55*a^9)/(14*b^{12}*(a + b*x^2)^7) - (55*a^8)/(4*b^{12}*(a + b*x^2)^6) + (33*a^7)/(b^{12}*(a + b*x^2)^5) - (231*a^6)/(4*b^{12}*(a + b*x^2)^4) + (77*a^5)/(b^{12}*(a + b*x^2)^3) - (165*a^4)/(2*b^{12}*(a + b*x^2)^2) + (165*a^3)/(2*b^{12}*(a + b*x^2)) + (55*a^2*Log[a + b*x^2])/(2*b^{12})$

**Rubi [A]** time = 0.469183, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^{11}}{18b^{12}(a+bx^2)^9} - \frac{11a^{10}}{16b^{12}(a+bx^2)^8} + \frac{55a^9}{14b^{12}(a+bx^2)^7} - \frac{55a^8}{4b^{12}(a+bx^2)^6} + \frac{33a^7}{b^{12}(a+bx^2)^5} \\ - \frac{231a^6}{4b^{12}(a+bx^2)^4} + \frac{77a^5}{b^{12}(a+bx^2)^3} - \frac{165a^4}{2b^{12}(a+bx^2)^2} + \frac{165a^3}{2b^{12}(a+bx^2)} + \frac{55a^2 \log(a+bx^2)}{2b^{12}} - \frac{5ax^2}{b^{11}} + \frac{x^4}{4b^{10}}$$

Antiderivative was successfully verified.

[In] Int[x^23/(a + b\*x^2)^10, x]

[Out]  $(-5*a*x^2)/b^{11} + x^4/(4*b^{10}) + a^{11}/(18*b^{12}*(a + b*x^2)^9) - (11*a^{10})/(16*b^{12}*(a + b*x^2)^8) + (55*a^9)/(14*b^{12}*(a + b*x^2)^7) - (55*a^8)/(4*b^{12}*(a + b*x^2)^6) + (33*a^7)/(b^{12}*(a + b*x^2)^5) - (231*a^6)/(4*b^{12}*(a + b*x^2)^4) + (77*a^5)/(b^{12}*(a + b*x^2)^3) - (165*a^4)/(2*b^{12}*(a + b*x^2)^2) + (165*a^3)/(2*b^{12}*(a + b*x^2)) + (55*a^2*Log[a + b*x^2])/(2*b^{12})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^{11}}{18b^{12}(a+bx^2)^9} - \frac{11a^{10}}{16b^{12}(a+bx^2)^8} + \frac{55a^9}{14b^{12}(a+bx^2)^7} - \frac{55a^8}{4b^{12}(a+bx^2)^6} \\ + \frac{33a^7}{b^{12}(a+bx^2)^5} - \frac{231a^6}{4b^{12}(a+bx^2)^4} + \frac{77a^5}{b^{12}(a+bx^2)^3} - \frac{165a^4}{2b^{12}(a+bx^2)^2} \\ + \frac{165a^3}{2b^{12}(a+bx^2)} + \frac{55a^2 \log(a+bx^2)}{2b^{12}} - \frac{5ax^2}{b^{11}} + \frac{\int^{x^2} x dx}{2b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*23/(b\*x\*\*2+a)\*\*10, x)

[Out]  $a^{11}/(18*b^{12}*(a + b*x^2)^9) - 11*a^{10}/(16*b^{12}*(a + b*x^2)^8) + 55*a^9/(14*b^{12}*(a + b*x^2)^7) - 55*a^8/(4*b^{12}*(a + b*x^2)^6) + 33*a^7/(b^{12}*(a + b*x^2)^5) - 231*a^6/(4*b^{12}*(a + b*x^2)^4) + 77*a^5/(b^{12}*(a + b*x^2)^3) - 165*a^4/(2*b^{12}*(a + b*x^2)^2) + 165*a^3/(2*b^{12}*(a + b*x^2)) + 55*a^2*log(a + b*x^2)/(2*b^{12}) - 5*a*x^2/b^{11} + Integral(x, (x, x^2))/(2*b^{10})$

**Mathematica [A]** time = 0.0657834, size = 158, normalized size = 0.77

$$\frac{42131a^{11} + 351459a^{10}bx^2 + 1281096a^9b^2x^4 + 2656584a^8b^3x^6 + 3402756a^7b^4x^8 + 2704212a^6b^5x^{10} + 1220688a^5b^6x^{12} + 190512a^4b^7x^{14} - 77112a^3b^8x^{16} - 36288a^2b^9x^{18} - 2772ab^{10}x^{20} + 252b^{11}x^{22} + 27720a^2(a + b^2x^2)^9 \operatorname{Log}[a + b^2x^2]}{1008b^{12}(a + b^2x^2)^9}$$

Antiderivative was successfully verified.

[In] Integrate[x^23/(a + b\*x^2)^10,x]

[Out] (42131\*a^11 + 351459\*a^10\*b\*x^2 + 1281096\*a^9\*b^2\*x^4 + 2656584\*a^8\*b^3\*x^6 + 3402756\*a^7\*b^4\*x^8 + 2704212\*a^6\*b^5\*x^10 + 1220688\*a^5\*b^6\*x^12 + 190512\*a^4\*b^7\*x^14 - 77112\*a^3\*b^8\*x^16 - 36288\*a^2\*b^9\*x^18 - 2772\*a\*b^10\*x^20 + 252\*b^11\*x^22 + 27720\*a^2\*(a + b\*x^2)^9\*Log[a + b\*x^2])/(1008\*b^12\*(a + b\*x^2)^9)

**Maple [A]** time = 0.027, size = 188, normalized size = 0.9

$$\begin{aligned} & -5 \frac{ax^2}{b^{11}} + \frac{x^4}{4b^{10}} + \frac{a^{11}}{18b^{12}(bx^2+a)^9} - \frac{11a^{10}}{16b^{12}(bx^2+a)^8} + \frac{55a^9}{14b^{12}(bx^2+a)^7} \\ & - \frac{55a^8}{4b^{12}(bx^2+a)^6} + 33 \frac{a^7}{b^{12}(bx^2+a)^5} - \frac{231a^6}{4b^{12}(bx^2+a)^4} + 77 \frac{a^5}{b^{12}(bx^2+a)^3} \\ & - \frac{165a^4}{2b^{12}(bx^2+a)^2} + \frac{165a^3}{2b^{12}(bx^2+a)} + \frac{55a^2 \ln(bx^2+a)}{2b^{12}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^23/(b\*x^2+a)^10,x)

[Out] -5\*a\*x^2/b^11+1/4\*x^4/b^10+1/18\*a^11/b^12/(b\*x^2+a)^9-11/16\*a^10/b^12/(b\*x^2+a)^8+55/14\*a^9/b^12/(b\*x^2+a)^7-55/4\*a^8/b^12/(b\*x^2+a)^6+33\*a^7/b^12/(b\*x^2+a)^5-231/4\*a^6/b^12/(b\*x^2+a)^4+77\*a^5/b^12/(b\*x^2+a)^3-165/2\*a^4/b^12/(b\*x^2+a)^2+165/2\*a^3/b^12/(b\*x^2+a)+55/2\*a^2\*ln(b\*x^2+a)/b^12

**Maxima [A]** time = 1.38903, size = 312, normalized size = 1.52

$$\frac{83160a^3b^8x^{16} + 582120a^4b^7x^{14} + 1823976a^5b^6x^{12} + 3318084a^6b^5x^{10} + 3817044a^7b^4x^8 + 2835756a^8b^3x^6 + 1326204a^9b^2x^4 + 190512a^{10}b^1x^2 + 252b^{11}x^2 + 27720a^2(a + b^2x^2)^9 \operatorname{Log}[a + b^2x^2]}{1008(b^{21}x^{18} + 9ab^{20}x^{16} + 36a^2b^{19}x^{14} + 84a^3b^{18}x^{12} + 126a^4b^{17}x^{10} + 126a^5b^{16}x^8 + 84a^6b^{15}x^6 + 36a^7b^{14}x^4 + 9a^8b^{13}x^2 + a^9b^{12})} + \frac{55a^2 \log(bx^2+a)}{2b^{12}} + \frac{bx^4 - 20ax^2}{4b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^23/(b\*x^2 + a)^10,x, algorithm="maxima")

[Out] 1/1008\*(83160\*a^3\*b^8\*x^16 + 582120\*a^4\*b^7\*x^14 + 1823976\*a^5\*b^6\*x^12 + 3318084\*a^6\*b^5\*x^10 + 3817044\*a^7\*b^4\*x^8 + 2835756\*a^8\*b^3\*x^6 + 1326204\*a^9\*b^2\*x^4 + 356499\*a^10\*b\*x^2 + 42131\*a^11)/(b^21\*x^18 + 9\*a\*b^20\*x^16 + 36\*a^2\*b^19\*x^14 + 84\*a^3\*b^18\*x^12 + 126\*a^4\*b^17\*x^10 + 126\*a^5\*b^16\*x^8 + 84\*a^6\*b^15\*x^6 + 36\*a^7\*b^14\*x^4 + 9\*a^8\*b^13\*x^2 + a^9\*b^12) + 55/2\*a^2\*log(b\*x^2 + a)/b^12 + 1/4\*(b\*x^4 - 20\*a\*x^2)/b^11

**Ericsas [A]** time = 0.211142, size = 452, normalized size = 2.2

$$\frac{252b^{11}x^{22} - 2772ab^{10}x^{20} - 36288a^2b^9x^{18} - 77112a^3b^8x^{16} + 190512a^4b^7x^{14} + 1220688a^5b^6x^{12} + 2704212a^6b^5x^{10} + 3402756a^7b^4x^8 + 2704212a^8b^3x^6 + 190512a^9b^2x^4 + 42131a^{10}bx^2 + 252b^{11}x^2 + 27720a^2(a + b^2x^2)^9 \operatorname{Log}[a + b^2x^2]}{1008(b^{21}x^{18} + 9ab^{20}x^{16} + 36a^2b^{19}x^{14} + 84a^3b^{18}x^{12} + 126a^4b^{17}x^{10} + 126a^5b^{16}x^8 + 84a^6b^{15}x^6 + 36a^7b^{14}x^4 + 9a^8b^{13}x^2 + a^9b^{12})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^23/(b\*x^2 + a)^10,x, algorithm="fricas")

[Out]  $\frac{1}{1008} \cdot (252 \cdot b^{11} \cdot x^{22} - 2772 \cdot a \cdot b^{10} \cdot x^{20} - 36288 \cdot a^2 \cdot b^9 \cdot x^{18} - 77112 \cdot a^3 \cdot b^8 \cdot x^{16} + 190512 \cdot a^4 \cdot b^7 \cdot x^{14} + 1220688 \cdot a^5 \cdot b^6 \cdot x^{12} + 2704212 \cdot a^6 \cdot b^5 \cdot x^{10} + 3402756 \cdot a^7 \cdot b^4 \cdot x^8 + 2656584 \cdot a^8 \cdot b^3 \cdot x^6 + 1281096 \cdot a^9 \cdot b^2 \cdot x^4 + 351459 \cdot a^{10} \cdot b \cdot x^2 + 42131 \cdot a^{11} + 27720 \cdot (a^2 \cdot b^9 \cdot x^{18} + 9 \cdot a^3 \cdot b^8 \cdot x^{16} + 36 \cdot a^4 \cdot b^7 \cdot x^{14} + 84 \cdot a^5 \cdot b^6 \cdot x^{12} + 126 \cdot a^6 \cdot b^5 \cdot x^{10} + 126 \cdot a^7 \cdot b^4 \cdot x^8 + 84 \cdot a^8 \cdot b^3 \cdot x^6 + 36 \cdot a^9 \cdot b^2 \cdot x^4 + 9 \cdot a^{10} \cdot b \cdot x^2 + a^{11}) \cdot \log(b \cdot x^2 + a)) / (b^{21} \cdot x^{18} + 9 \cdot a \cdot b^{20} \cdot x^{16} + 36 \cdot a^2 \cdot b^{19} \cdot x^{14} + 84 \cdot a^3 \cdot b^{18} \cdot x^{12} + 126 \cdot a^4 \cdot b^{17} \cdot x^{10} + 126 \cdot a^5 \cdot b^{16} \cdot x^8 + 84 \cdot a^6 \cdot b^{15} \cdot x^6 + 36 \cdot a^7 \cdot b^{14} \cdot x^4 + 9 \cdot a^8 \cdot b^{13} \cdot x^2 + a^9 \cdot b^{12})$

**Sympy [A]** time = 37.7134, size = 245, normalized size = 1.2

$$\frac{55a^2 \log(a + bx^2)}{2b^{12}} - \frac{5ax^2}{b^{11}} + \frac{42131a^{11} + 356499a^{10}bx^2 + 1326204a^9b^2x^4 + 2835756a^8b^3x^6 + 3817044a^7b^4x^8 + 3318084a^6b^5x^{10} + 1823976a^5b^6x^{12} + 1008a^9b^{12} + 9072a^8b^{13}x^2 + 36288a^7b^{14}x^4 + 84672a^6b^{15}x^6 + 127008a^5b^{16}x^8 + 127008a^4b^{17}x^{10} + 84672a^3b^{18}x^{12} + 36288a^2b^{19}x^{14} + 36288a^{10}b^{19}x^{14} + 36288a^{11}b^{19}x^{14}}{4b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*23/(b\*x\*\*2+a)\*\*10,x)

[Out]  $55 \cdot a^{11} \cdot \log(a + b \cdot x^2) / (2 \cdot b^{12}) - 5 \cdot a \cdot x^2 / b^{11} + (42131 \cdot a^{11} + 356499 \cdot a^{10} \cdot b \cdot x^2 + 1326204 \cdot a^9 \cdot b^2 \cdot x^4 + 2835756 \cdot a^8 \cdot b^3 \cdot x^6 + 3817044 \cdot a^7 \cdot b^4 \cdot x^8 + 3318084 \cdot a^6 \cdot b^5 \cdot x^{10} + 1823976 \cdot a^5 \cdot b^6 \cdot x^{12} + 582120 \cdot a^4 \cdot b^7 \cdot x^{14} + 83160 \cdot a^3 \cdot b^8 \cdot x^{16}) / (1008 \cdot a^9 \cdot b^{12} + 9072 \cdot a^8 \cdot b^{13} \cdot x^2 + 36288 \cdot a^7 \cdot b^{14} \cdot x^4 + 84672 \cdot a^6 \cdot b^{15} \cdot x^6 + 127008 \cdot a^5 \cdot b^{16} \cdot x^8 + 127008 \cdot a^4 \cdot b^{17} \cdot x^{10} + 84672 \cdot a^3 \cdot b^{18} \cdot x^{12} + 36288 \cdot a^2 \cdot b^{19} \cdot x^{14} + 9072 \cdot a \cdot b^{20} \cdot x^{16} + 1008 \cdot b^{21} \cdot x^{18}) + x^4 / (4 \cdot b^{10})$

**GIAC/XCAS [A]** time = 0.220998, size = 212, normalized size = 1.03

$$\frac{55a^2 \ln(|bx^2 + a|)}{2b^{12}} + \frac{b^{10}x^4 - 20ab^9x^2}{4b^{20}} + \frac{78419a^2b^9x^{18} + 622611a^3b^8x^{16} + 2240964a^4b^7x^{14} + 4763220a^5b^6x^{12} + 6562710a^6b^5x^{10} + 6063750a^7b^4x^8 + 3751440a^8b^3x^6 + 1496880a^9b^2x^4 + 349272a^{10}bx^2 + 36288a^{11}}{1008(bx^2 + a)^9b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^23/(b\*x^2 + a)^10,x, algorithm="giac")

[Out]  $\frac{55}{2} \cdot a^2 \cdot \ln(\text{abs}(b \cdot x^2 + a)) / b^{12} + \frac{1}{4} \cdot (b^{10} \cdot x^4 - 20 \cdot a \cdot b^9 \cdot x^2) / b^{20} - \frac{1}{1008} \cdot (78419 \cdot a^2 \cdot b^9 \cdot x^{18} + 622611 \cdot a^3 \cdot b^8 \cdot x^{16} + 2240964 \cdot a^4 \cdot b^7 \cdot x^{14} + 4763220 \cdot a^5 \cdot b^6 \cdot x^{12} + 6562710 \cdot a^6 \cdot b^5 \cdot x^{10} + 6063750 \cdot a^7 \cdot b^4 \cdot x^8 + 3751440 \cdot a^8 \cdot b^3 \cdot x^6 + 1496880 \cdot a^9 \cdot b^2 \cdot x^4 + 349272 \cdot a^{10} \cdot b \cdot x^2 + 36288 \cdot a^{11}) / ((b \cdot x^2 + a)^9 \cdot b^{12})$

$$3.194 \quad \int \frac{x^{21}}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=188

$$\begin{aligned} & -\frac{a^{10}}{18b^{11}(a+bx^2)^9} + \frac{5a^9}{8b^{11}(a+bx^2)^8} - \frac{45a^8}{14b^{11}(a+bx^2)^7} + \frac{10a^7}{b^{11}(a+bx^2)^6} - \frac{21a^6}{b^{11}(a+bx^2)^5} \\ & + \frac{63a^5}{2b^{11}(a+bx^2)^4} - \frac{35a^4}{b^{11}(a+bx^2)^3} + \frac{30a^3}{b^{11}(a+bx^2)^2} - \frac{45a^2}{2b^{11}(a+bx^2)} - \frac{5a \log(a+bx^2)}{b^{11}} + \frac{x^2}{2b^{10}} \end{aligned}$$

[Out]  $x^2/(2*b^{10}) - a^{10}/(18*b^{11}*(a + b*x^2)^9) + (5*a^9)/(8*b^{11}*(a + b*x^2)^8) - (45*a^8)/(14*b^{11}*(a + b*x^2)^7) + (10*a^7)/(b^{11}*(a + b*x^2)^6) - (21*a^6)/(b^{11}*(a + b*x^2)^5) + (63*a^5)/(2*b^{11}*(a + b*x^2)^4) - (35*a^4)/(b^{11}*(a + b*x^2)^3) + (30*a^3)/(b^{11}*(a + b*x^2)^2) - (45*a^2)/(2*b^{11}*(a + b*x^2)) - (5*a*Log[a + b*x^2])/b^{11}$

**Rubi [A]** time = 0.409058, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\begin{aligned} & -\frac{a^{10}}{18b^{11}(a+bx^2)^9} + \frac{5a^9}{8b^{11}(a+bx^2)^8} - \frac{45a^8}{14b^{11}(a+bx^2)^7} + \frac{10a^7}{b^{11}(a+bx^2)^6} - \frac{21a^6}{b^{11}(a+bx^2)^5} \\ & + \frac{63a^5}{2b^{11}(a+bx^2)^4} - \frac{35a^4}{b^{11}(a+bx^2)^3} + \frac{30a^3}{b^{11}(a+bx^2)^2} - \frac{45a^2}{2b^{11}(a+bx^2)} - \frac{5a \log(a+bx^2)}{b^{11}} + \frac{x^2}{2b^{10}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^21/(a + b\*x^2)^10, x]

[Out]  $x^2/(2*b^{10}) - a^{10}/(18*b^{11}*(a + b*x^2)^9) + (5*a^9)/(8*b^{11}*(a + b*x^2)^8) - (45*a^8)/(14*b^{11}*(a + b*x^2)^7) + (10*a^7)/(b^{11}*(a + b*x^2)^6) - (21*a^6)/(b^{11}*(a + b*x^2)^5) + (63*a^5)/(2*b^{11}*(a + b*x^2)^4) - (35*a^4)/(b^{11}*(a + b*x^2)^3) + (30*a^3)/(b^{11}*(a + b*x^2)^2) - (45*a^2)/(2*b^{11}*(a + b*x^2)) - (5*a*Log[a + b*x^2])/b^{11}$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned} & -\frac{a^{10}}{18b^{11}(a+bx^2)^9} + \frac{5a^9}{8b^{11}(a+bx^2)^8} - \frac{45a^8}{14b^{11}(a+bx^2)^7} + \frac{10a^7}{b^{11}(a+bx^2)^6} - \frac{21a^6}{b^{11}(a+bx^2)^5} \\ & + \frac{63a^5}{2b^{11}(a+bx^2)^4} - \frac{35a^4}{b^{11}(a+bx^2)^3} + \frac{30a^3}{b^{11}(a+bx^2)^2} - \frac{45a^2}{2b^{11}(a+bx^2)} - \frac{5a \log(a+bx^2)}{b^{11}} + \frac{\int^{x^2} \frac{1}{b^{10}} dx}{2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*21/(b\*x\*\*2+a)\*\*10, x)

[Out]  $-a^{10}/(18*b^{11}*(a + b*x^2)^9) + 5*a^9/(8*b^{11}*(a + b*x^2)^8) - 45*a^8/(14*b^{11}*(a + b*x^2)^7) + 10*a^7/(b^{11}*(a + b*x^2)^6) - 21*a^6/(b^{11}*(a + b*x^2)^5) + 63*a^5/(2*b^{11}*(a + b*x^2)^4) - 35*a^4/(b^{11}*(a + b*x^2)^3) + 30*a^3/(b^{11}*(a + b*x^2)^2) - 45*a^2/(2*b^{11}*(a + b*x^2)) - 5*a*log(a + b*x^2)/b^{11} + Integral(b^{11}*(-10), (x, x^2))/2$

**Mathematica [A]** time = 0.0671353, size = 145, normalized size = 0.77

$$\frac{4861a^{10} + 41229a^9bx^2 + 153576a^8b^2x^4 + 328104a^7b^3x^6 + 439236a^6b^4x^8 + 375732a^5b^5x^{10} + 197568a^4b^6x^{12} + 54432a^3b^7x^{14}}{504b^{11}(a+bx^2)^9}$$

Antiderivative was successfully verified.

[In] Integrate[x^21/(a + b\*x^2)^10,x]

[Out]  $-(4861*a^{10} + 41229*a^9*b*x^2 + 153576*a^8*b^2*x^4 + 328104*a^7*b^3*x^6 + 439236*a^6*b^4*x^8 + 375732*a^5*b^5*x^{10} + 197568*a^4*b^6*x^{12} + 54432*a^3*b^7*x^{14} + 2268*a^2*b^8*x^{16} - 2268*a*b^9*x^{18} - 252*b^{10}*x^{20} + 2520*a*(a + b*x^2)^9*\text{Log}[a + b*x^2])/(504*b^{11}*(a + b*x^2)^9)$

**Maple [A]** time = 0.025, size = 177, normalized size = 0.9

$$\frac{x^2}{2b^{10}} - \frac{a^{10}}{18b^{11}(bx^2+a)^9} + \frac{5a^9}{8b^{11}(bx^2+a)^8} - \frac{45a^8}{14b^{11}(bx^2+a)^7} + 10\frac{a^7}{b^{11}(bx^2+a)^6} - 21\frac{a^6}{b^{11}(bx^2+a)^5} + \frac{63a^5}{2b^{11}(bx^2+a)^4} - 35\frac{a^4}{b^{11}(bx^2+a)^3} + 30\frac{a^3}{b^{11}(bx^2+a)^2} - \frac{45a^2}{2b^{11}(bx^2+a)} - 5\frac{a\ln(bx^2+a)}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^21/(b\*x^2+a)^10,x)

[Out]  $1/2*x^2/b^{10} - 1/18*a^{10}/b^{11}/(b*x^2+a)^9 + 5/8*a^9/b^{11}/(b*x^2+a)^8 - 45/14*a^8/b^{11}/(b*x^2+a)^7 + 10*a^7/b^{11}/(b*x^2+a)^6 - 21*a^6/b^{11}/(b*x^2+a)^5 + 63/2*a^5/b^{11}/(b*x^2+a)^4 - 35*a^4/b^{11}/(b*x^2+a)^3 + 30*a^3/b^{11}/(b*x^2+a)^2 - 45/2*a^2/b^{11}/(b*x^2+a) - 5*a*\ln(b*x^2+a)/b^{11}$

**Maxima [A]** time = 1.39763, size = 297, normalized size = 1.58

$$\frac{11340a^2b^8x^{16} + 75600a^3b^7x^{14} + 229320a^4b^6x^{12} + 407484a^5b^5x^{10} + 460404a^6b^4x^8 + 337176a^7b^3x^6 + 155844a^8b^2x^4 + 407484a^9b^1x^2 + 2520a^{10}}{504(b^{20}x^{18} + 9ab^{19}x^{16} + 36a^2b^{18}x^{14} + 84a^3b^{17}x^{12} + 126a^4b^{16}x^{10} + 126a^5b^{15}x^8 + 84a^6b^{14}x^6 + 36a^7b^{13}x^4 + 9a^8b^{12}x^2 + a^9b^{11})} + \frac{x^2}{2b^{10}} - \frac{5a\log(bx^2+a)}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^21/(b\*x^2 + a)^10,x, algorithm="maxima")

[Out]  $-1/504*(11340*a^2*b^8*x^{16} + 75600*a^3*b^7*x^{14} + 229320*a^4*b^6*x^{12} + 407484*a^5*b^5*x^{10} + 460404*a^6*b^4*x^8 + 337176*a^7*b^3*x^6 + 155844*a^8*b^2*x^4 + 41481*a^9*b^1*x^2 + 4861*a^{10})/(b^{20}*x^{18} + 9*a*b^{19}*x^{16} + 36*a^2*b^{18}*x^{14} + 84*a^3*b^{17}*x^{12} + 126*a^4*b^{16}*x^{10} + 126*a^5*b^{15}*x^8 + 84*a^6*b^{14}*x^6 + 36*a^7*b^{13}*x^4 + 9*a^8*b^{12}*x^2 + a^9*b^{11}) + 1/2*x^2/b^{10} - 5*a*\log(b*x^2 + a)/b^{11}$

**Fricas [A]** time = 0.216895, size = 435, normalized size = 2.31

$$\frac{252b^{10}x^{20} + 2268ab^9x^{18} - 2268a^2b^8x^{16} - 54432a^3b^7x^{14} - 197568a^4b^6x^{12} - 375732a^5b^5x^{10} - 439236a^6b^4x^8 - 328104a^7b^3x^6 - 155844a^8b^2x^4 - 407484a^9b^1x^2 + 2520a^{10}}{504(b^{20}x^{18} + 9ab^{19}x^{16} + 36a^2b^{18}x^{14} + 84a^3b^{17}x^{12} + 126a^4b^{16}x^{10} + 126a^5b^{15}x^8 + 84a^6b^{14}x^6 + 36a^7b^{13}x^4 + 9a^8b^{12}x^2 + a^9b^{11})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^21/(b\*x^2 + a)^10,x, algorithm="fricas")

[Out]  $1/504*(252*b^{10}*x^{20} + 2268*a*b^9*x^{18} - 2268*a^2*b^8*x^{16} - 54432*a^3*b^7*x^{14} - 197568*a^4*b^6*x^{12} - 375732*a^5*b^5*x^{10} - 439236*a^6*b^4*x^8 - 328104*a^7*b^3*x^6 - 153576*a^8*b^2*x^4 - 41229*a^9*b^1*x^2 - 4861*a^{10} - 2520*(a*b^9*x^{18} + 9*a^2*b^8*x^{16} + 36*a^3*b^7*x^{14} + 84*a^4*b^6*x^{12} + 126*a^5*b^5*x^{10} + 126*a^6*b^4*x^8 + 36*a^7*b^3*x^6 + 9*a^8*b^2*x^4 + a^9*b^1*x^2 + a^{10}))$



$$3*b^7*x^{14} + 84*a^4*b^6*x^{12} + 126*a^5*b^5*x^{10} + 126*a^6*b^4*x^8 + 84*a^7*b^3*x^6 + 36*a^8*b^2*x^4 + 9*a^9*b*x^2 + a^{10})*\log(b*x^2 + a)/(b^{20}*x^{18} + 9*a*b^{19}*x^{16} + 36*a^2*b^{18}*x^{14} + 84*a^3*b^{17}*x^{12} + 126*a^4*b^{16}*x^{10} + 126*a^5*b^{15}*x^8 + 84*a^6*b^{14}*x^6 + 36*a^7*b^{13}*x^4 + 9*a^8*b^{12}*x^2 + a^9*b^{11})$$

---

**Sympy [A]** time = 37.1459, size = 231, normalized size = 1.23

$$\frac{5a \log(a + bx^2)}{b^{11}} - \frac{4861a^{10} + 41481a^9bx^2 + 155844a^8b^2x^4 + 337176a^7b^3x^6 + 460404a^6b^4x^8 + 407484a^5b^5x^{10} + 229320a^4b^6x^{12} + 75600a^3b^7x^{14} + 11340a^2b^8x^{16}}{504a^9b^{11} + 4536a^8b^{12}x^2 + 18144a^7b^{13}x^4 + 42336a^6b^{14}x^6 + 63504a^5b^{15}x^8 + 63504a^4b^{16}x^{10} + 42336a^3b^{17}x^{12} + 18144a^2b^{18}x^{14} + 18144ab^{19}x^{16} + 504b^{20}x^{18}} + \frac{x^2}{2b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*21/(b\*x\*\*2+a)\*\*10,x)

[Out] -5\*a\*log(a + b\*x\*\*2)/b\*\*11 - (4861\*a\*\*10 + 41481\*a\*\*9\*b\*x\*\*2 + 155844\*a\*\*8\*b\*\*2\*x\*\*4 + 337176\*a\*\*7\*b\*\*3\*x\*\*6 + 460404\*a\*\*6\*b\*\*4\*x\*\*8 + 407484\*a\*\*5\*b\*\*5\*x\*\*10 + 229320\*a\*\*4\*b\*\*6\*x\*\*12 + 75600\*a\*\*3\*b\*\*7\*x\*\*14 + 11340\*a\*\*2\*b\*\*8\*x\*\*16)/(504\*a\*\*9\*b\*\*11 + 4536\*a\*\*8\*b\*\*12\*x\*\*2 + 18144\*a\*\*7\*b\*\*13\*x\*\*4 + 42336\*a\*\*6\*b\*\*14\*x\*\*6 + 63504\*a\*\*5\*b\*\*15\*x\*\*8 + 63504\*a\*\*4\*b\*\*16\*x\*\*10 + 42336\*a\*\*3\*b\*\*17\*x\*\*12 + 18144\*a\*\*2\*b\*\*18\*x\*\*14 + 18144\*a\*b\*\*19\*x\*\*16 + 504\*b\*\*20\*x\*\*18) + x\*\*2/(2\*b\*\*10)

---

**GIAC/XCAS [A]** time = 0.212303, size = 188, normalized size = 1.

$$\frac{x^2}{2b^{10}} - \frac{5 \operatorname{aln}(|bx^2 + a|)}{b^{11}} + \frac{7129ab^9x^{18} + 52821a^2b^8x^{16} + 181044a^3b^7x^{14} + 369516a^4b^6x^{12} + 490770a^5b^5x^{10} + 437850a^6b^4x^8 + 261660a^7b^3x^6 + 100800a^8b^2x^4 + 22680a^9b^1x^2 + 2268a^{10}}{504(bx^2 + a)^9b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^21/(b\*x^2 + a)^10,x, algorithm="giac")

[Out] 1/2\*x^2/b^10 - 5\*a\*ln(abs(b\*x^2 + a))/b^11 + 1/504\*(7129\*a\*b^9\*x^18 + 52821\*a^2\*b^8\*x^16 + 181044\*a^3\*b^7\*x^14 + 369516\*a^4\*b^6\*x^12 + 490770\*a^5\*b^5\*x^10 + 437850\*a^6\*b^4\*x^8 + 261660\*a^7\*b^3\*x^6 + 100800\*a^8\*b^2\*x^4 + 22680\*a^9\*b^1\*x^2 + 2268\*a^10)/((b\*x^2 + a)^9\*b^11)

$$3.195 \quad \int \frac{x^{19}}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=179

$$\frac{a^9}{18b^{10}(a+bx^2)^9} - \frac{9a^8}{16b^{10}(a+bx^2)^8} + \frac{18a^7}{7b^{10}(a+bx^2)^7} - \frac{7a^6}{b^{10}(a+bx^2)^6} + \frac{63a^5}{5b^{10}(a+bx^2)^5} \\ - \frac{63a^4}{4b^{10}(a+bx^2)^4} + \frac{14a^3}{b^{10}(a+bx^2)^3} - \frac{9a^2}{b^{10}(a+bx^2)^2} + \frac{9a}{2b^{10}(a+bx^2)} + \frac{\log(a+bx^2)}{2b^{10}}$$

[Out]  $a^9/(18*b^{10}*(a+b*x^2)^9) - (9*a^8)/(16*b^{10}*(a+b*x^2)^8) + (18*a^7)/(7*b^{10}*(a+b*x^2)^7) - (7*a^6)/(b^{10}*(a+b*x^2)^6) + (63*a^5)/(5*b^{10}*(a+b*x^2)^5) - (63*a^4)/(4*b^{10}*(a+b*x^2)^4) + (14*a^3)/(b^{10}*(a+b*x^2)^3) - (9*a^2)/(b^{10}*(a+b*x^2)^2) + (9*a)/(2*b^{10}*(a+b*x^2)) + \text{Log}[a+b*x^2]/(2*b^{10})$

**Rubi [A]** time = 0.378723, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^9}{18b^{10}(a+bx^2)^9} - \frac{9a^8}{16b^{10}(a+bx^2)^8} + \frac{18a^7}{7b^{10}(a+bx^2)^7} - \frac{7a^6}{b^{10}(a+bx^2)^6} + \frac{63a^5}{5b^{10}(a+bx^2)^5} \\ - \frac{63a^4}{4b^{10}(a+bx^2)^4} + \frac{14a^3}{b^{10}(a+bx^2)^3} - \frac{9a^2}{b^{10}(a+bx^2)^2} + \frac{9a}{2b^{10}(a+bx^2)} + \frac{\log(a+bx^2)}{2b^{10}}$$

Antiderivative was successfully verified.

[In] Int[x^19/(a + b\*x^2)^10, x]

[Out]  $a^9/(18*b^{10}*(a+b*x^2)^9) - (9*a^8)/(16*b^{10}*(a+b*x^2)^8) + (18*a^7)/(7*b^{10}*(a+b*x^2)^7) - (7*a^6)/(b^{10}*(a+b*x^2)^6) + (63*a^5)/(5*b^{10}*(a+b*x^2)^5) - (63*a^4)/(4*b^{10}*(a+b*x^2)^4) + (14*a^3)/(b^{10}*(a+b*x^2)^3) - (9*a^2)/(b^{10}*(a+b*x^2)^2) + (9*a)/(2*b^{10}*(a+b*x^2)) + \text{Log}[a+b*x^2]/(2*b^{10})$

**Rubi in Sympy [A]** time = 51.8683, size = 170, normalized size = 0.95

$$\frac{a^9}{18b^{10}(a+bx^2)^9} - \frac{9a^8}{16b^{10}(a+bx^2)^8} + \frac{18a^7}{7b^{10}(a+bx^2)^7} - \frac{7a^6}{b^{10}(a+bx^2)^6} + \frac{63a^5}{5b^{10}(a+bx^2)^5} \\ - \frac{63a^4}{4b^{10}(a+bx^2)^4} + \frac{14a^3}{b^{10}(a+bx^2)^3} - \frac{9a^2}{b^{10}(a+bx^2)^2} + \frac{9a}{2b^{10}(a+bx^2)} + \frac{\log(a+bx^2)}{2b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*19/(b\*x\*\*2+a)\*\*10, x)

[Out]  $a^{**9}/(18*b^{**10}*(a+b*x^{**2})^{**9}) - 9*a^{**8}/(16*b^{**10}*(a+b*x^{**2})^{**8}) + 18*a^{**7}/(7*b^{**10}*(a+b*x^{**2})^{**7}) - 7*a^{**6}/(b^{**10}*(a+b*x^{**2})^{**6}) + 63*a^{**5}/(5*b^{**10}*(a+b*x^{**2})^{**5}) - 63*a^{**4}/(4*b^{**10}*(a+b*x^{**2})^{**4}) + 14*a^{**3}/(b^{**10}*(a+b*x^{**2})^{**3}) - 9*a^{**2}/(b^{**10}*(a+b*x^{**2})^{**2}) + 9*a/(2*b^{**10}*(a+b*x^{**2})) + \log(a+b*x^{**2})/(2*b^{**10})$

**Mathematica [A]** time = 0.0638865, size = 116, normalized size = 0.65

$$\frac{a(7129a^8+61641a^7bx^2+235224a^6b^2x^4+518616a^5b^3x^6+725004a^4b^4x^8+661500a^3b^5x^{10}+388080a^2b^6x^{12}+136080ab^7x^{14}+22680b^8x^{16})}{(a+bx^2)^9} + 2520 \log(a+bx^2)$$

5040b<sup>10</sup>

Antiderivative was successfully verified.

[In] Integrate[x^19/(a + b\*x^2)^10,x]

[Out] ((a\*(7129\*a^8 + 61641\*a^7\*b\*x^2 + 235224\*a^6\*b^2\*x^4 + 518616\*a^5\*b^3\*x^6 + 725004\*a^4\*b^4\*x^8 + 661500\*a^3\*b^5\*x^10 + 388080\*a^2\*b^6\*x^12 + 136080\*a\*b^7\*x^14 + 22680\*b^8\*x^16))/(a + b\*x^2)^9 + 2520\*Log[a + b\*x^2])/(5040\*b^10)

**Maple [A]** time = 0.017, size = 166, normalized size = 0.9

$$\frac{a^9}{18 b^{10} (bx^2 + a)^9} - \frac{9 a^8}{16 b^{10} (bx^2 + a)^8} + \frac{18 a^7}{7 b^{10} (bx^2 + a)^7} - 7 \frac{a^6}{b^{10} (bx^2 + a)^6} + \frac{63 a^5}{5 b^{10} (bx^2 + a)^5} - \frac{63 a^4}{4 b^{10} (bx^2 + a)^4} + 14 \frac{a^3}{b^{10} (bx^2 + a)^3} - 9 \frac{a^2}{b^{10} (bx^2 + a)^2} + \frac{9 a}{2 b^{10} (bx^2 + a)} + \frac{\ln(bx^2 + a)}{2 b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^19/(b\*x^2+a)^10,x)

[Out] 1/18\*a^9/b^10/(b\*x^2+a)^9-9/16\*a^8/b^10/(b\*x^2+a)^8+18/7\*a^7/b^10/(b\*x^2+a)^7-7\*a^6/b^10/(b\*x^2+a)^6+63/5\*a^5/b^10/(b\*x^2+a)^5-63/4\*a^4/b^10/(b\*x^2+a)^4+14\*a^3/b^10/(b\*x^2+a)^3-9\*a^2/b^10/(b\*x^2+a)^2+9/2\*a/b^10/(b\*x^2+a)+1/2\*ln(b\*x^2+a)/b^10

**Maxima [A]** time = 1.36392, size = 282, normalized size = 1.58

$$\frac{22680 ab^8 x^{16} + 136080 a^2 b^7 x^{14} + 388080 a^3 b^6 x^{12} + 661500 a^4 b^5 x^{10} + 725004 a^5 b^4 x^8 + 518616 a^6 b^3 x^6 + 235224 a^7 b^2 x^4 + 61641 a^8 b x^2 + 7129 a^9}{5040 (b^{19} x^{18} + 9 ab^{18} x^{16} + 36 a^2 b^{17} x^{14} + 84 a^3 b^{16} x^{12} + 126 a^4 b^{15} x^{10} + 126 a^5 b^{14} x^8 + 84 a^6 b^{13} x^6 + 36 a^7 b^{12} x^4 + 9 a^8 b^{11} x^2 + a^9)} + \frac{\log(bx^2 + a)}{2 b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^19/(b\*x^2 + a)^10,x, algorithm="maxima")

[Out] 1/5040\*(22680\*a\*b^8\*x^16 + 136080\*a^2\*b^7\*x^14 + 388080\*a^3\*b^6\*x^12 + 661500\*a^4\*b^5\*x^10 + 725004\*a^5\*b^4\*x^8 + 518616\*a^6\*b^3\*x^6 + 235224\*a^7\*b^2\*x^4 + 61641\*a^8\*b\*x^2 + 7129\*a^9)/(b^19\*x^18 + 9\*a\*b^18\*x^16 + 36\*a^2\*b^17\*x^14 + 84\*a^3\*b^16\*x^12 + 126\*a^4\*b^15\*x^10 + 126\*a^5\*b^14\*x^8 + 84\*a^6\*b^13\*x^6 + 36\*a^7\*b^12\*x^4 + 9\*a^8\*b^11\*x^2 + a^9\*b^10) + 1/2\*log(b\*x^2 + a)/b^10

**Fricas [A]** time = 0.209447, size = 405, normalized size = 2.26

$$\frac{22680 ab^8 x^{16} + 136080 a^2 b^7 x^{14} + 388080 a^3 b^6 x^{12} + 661500 a^4 b^5 x^{10} + 725004 a^5 b^4 x^8 + 518616 a^6 b^3 x^6 + 235224 a^7 b^2 x^4 + 61641 a^8 b x^2 + 7129 a^9}{5040 (b^{19} x^{18} + 9 ab^{18} x^{16} + 36 a^2 b^{17} x^{14} + 84 a^3 b^{16} x^{12} + 126 a^4 b^{15} x^{10} + 126 a^5 b^{14} x^8 + 84 a^6 b^{13} x^6 + 36 a^7 b^{12} x^4 + 9 a^8 b^{11} x^2 + a^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^19/(b\*x^2 + a)^10,x, algorithm="fricas")

[Out] 1/5040\*(22680\*a\*b^8\*x^16 + 136080\*a^2\*b^7\*x^14 + 388080\*a^3\*b^6\*x^12 + 661500\*a^4\*b^5\*x^10 + 725004\*a^5\*b^4\*x^8 + 518616\*a^6\*b^3\*x^6 + 235224\*a^7\*b^2\*x^4 + 61641\*a^8\*b\*x^2 + 7129\*a^9 + 2520\*(b^9\*x^18 + 9\*a\*b^8\*x^16 + 36\*a^2\*b^7\*x^14 + 84\*a^3\*b^6\*x^12 + 126\*a^4\*b^5\*x^10 + 126\*a^5\*b^4\*x^8 + 84\*a^6\*b^3\*x^6 + 36\*a^7\*b^2\*x^4 + 9\*a^8\*b\*x^2 + a^9)\*log(b\*x^2 + a))/(b^19\*x^18 + 9\*a\*b^18\*x^16 + 36

$$*a^2*b^{17}*x^{14} + 84*a^3*b^{16}*x^{12} + 126*a^4*b^{15}*x^{10} + 126*a^5*b^{14}*x^8 + 84*a^6*b^{13}*x^6 + 36*a^7*b^{12}*x^4 + 9*a^8*b^{11}*x^2 + a^9*b^{10})$$

---

**Sympy [A]** time = 36.144, size = 219, normalized size = 1.22

$$\frac{7129a^9 + 61641a^8bx^2 + 235224a^7b^2x^4 + 518616a^6b^3x^6 + 725004a^5b^4x^8 + 661500a^4b^5x^{10} + 388080a^3b^6x^{12} + 136080a^2b^7x^{14} + 22680ab^8x^{16} + 5040a^9b^{10} + 45360a^8b^{11}x^2 + 181440a^7b^{12}x^4 + 423360a^6b^{13}x^6 + 635040a^5b^{14}x^8 + 635040a^4b^{15}x^{10} + 423360a^3b^{16}x^{12} + 181440a^2b^{17}x^{14} + 84a^3b^{16}x^{12} + 126a^4b^{15}x^{10} + 126a^5b^{14}x^8 + 84a^6b^{13}x^6 + 36a^7b^{12}x^4 + 9a^8b^{11}x^2 + a^9b^{10}}{2b^{10}} + \frac{\log(a + bx^2)}{2b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*19/(b\*x\*\*2+a)\*\*10,x)

[Out] (7129\*a\*\*9 + 61641\*a\*\*8\*b\*x\*\*2 + 235224\*a\*\*7\*b\*\*2\*x\*\*4 + 518616\*a\*\*6\*b\*\*3\*x\*\*6 + 725004\*a\*\*5\*b\*\*4\*x\*\*8 + 661500\*a\*\*4\*b\*\*5\*x\*\*10 + 388080\*a\*\*3\*b\*\*6\*x\*\*12 + 136080\*a\*\*2\*b\*\*7\*x\*\*14 + 22680\*a\*b\*\*8\*x\*\*16)/(5040\*a\*\*9\*b\*\*10 + 45360\*a\*\*8\*b\*\*11\*x\*\*2 + 181440\*a\*\*7\*b\*\*12\*x\*\*4 + 423360\*a\*\*6\*b\*\*13\*x\*\*6 + 635040\*a\*\*5\*b\*\*14\*x\*\*8 + 635040\*a\*\*4\*b\*\*15\*x\*\*10 + 423360\*a\*\*3\*b\*\*16\*x\*\*12 + 181440\*a\*\*2\*b\*\*17\*x\*\*14 + 45360\*a\*b\*\*18\*x\*\*16 + 5040\*b\*\*19\*x\*\*18) + log(a + b\*x\*\*2)/(2\*b\*\*10)

---

**GIAC/XCAS [A]** time = 0.211642, size = 161, normalized size = 0.9

$$\frac{\ln(|bx^2 + a|)}{2b^{10}} - \frac{7129b^8x^{18} + 41481ab^7x^{16} + 120564a^2b^6x^{14} + 210756a^3b^5x^{12} + 236754a^4b^4x^{10} + 173250a^5b^3x^8 + 80220a^6b^2x^6 + 21420a^7b^1x^4 + 2520a^8x^2}{5040(bx^2 + a)^9b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^19/(b\*x^2 + a)^10,x, algorithm="giac")

[Out] 1/2\*ln(abs(b\*x^2 + a))/b^10 - 1/5040\*(7129\*b^8\*x^18 + 41481\*a\*b^7\*x^16 + 120564\*a^2\*b^6\*x^14 + 210756\*a^3\*b^5\*x^12 + 236754\*a^4\*b^4\*x^10 + 173250\*a^5\*b^3\*x^8 + 80220\*a^6\*b^2\*x^6 + 21420\*a^7\*b^1\*x^4 + 2520\*a^8\*x^2)/((b\*x^2 + a)^9\*b^9)

$$3.196 \quad \int \frac{x^{17}}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=19

$$\frac{x^{18}}{18a(a+bx^2)^9}$$

[Out]  $x^{18}/(18*a*(a+b*x^2)^9)$

**Rubi [A]** time = 0.0196809, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{x^{18}}{18a(a+bx^2)^9}$$

Antiderivative was successfully verified.

[In] `Int[x^17/(a+b*x^2)^10,x]`

[Out]  $x^{18}/(18*a*(a+b*x^2)^9)$

**Rubi in Sympy [A]** time = 3.29056, size = 14, normalized size = 0.74

$$\frac{x^{18}}{18a(a+bx^2)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**17/(b*x**2+a)**10,x)`

[Out]  $x^{18}/(18*a*(a+b*x^2)^9)$

**Mathematica [B]** time = 0.0370214, size = 101, normalized size = 5.32

$$\frac{a^8 + 9a^7bx^2 + 36a^6b^2x^4 + 84a^5b^3x^6 + 126a^4b^4x^8 + 126a^3b^5x^{10} + 84a^2b^6x^{12} + 36ab^7x^{14} + 9b^8x^{16}}{18b^9(a+bx^2)^9}$$

Antiderivative was successfully verified.

[In] `Integrate[x^17/(a+b*x^2)^10,x]`

[Out]  $-(a^8 + 9a^7bx^2 + 36a^6b^2x^4 + 84a^5b^3x^6 + 126a^4b^4x^8 + 126a^3b^5x^{10} + 84a^2b^6x^{12} + 36ab^7x^{14} + 9b^8x^{16})/(18b^9(a+bx^2)^9)$

**Maple [B]** time = 0.014, size = 150, normalized size = 7.9

$$\frac{14a^5}{3b^9(bx^2+a)^6} - \frac{a^8}{18b^9(bx^2+a)^9} + \frac{a^7}{2b^9(bx^2+a)^8} - 2\frac{a^6}{b^9(bx^2+a)^7} - \frac{14a^2}{3b^9(bx^2+a)^3} + 2\frac{a}{b^9(bx^2+a)^2} + 7\frac{a^3}{b^9(bx^2+a)^4} - \frac{1}{(2bx^2+2a)b^9} - 7\frac{a^4}{b^9(bx^2+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^17/(b\*x^2+a)^10,x)

[Out]  $14/3*a^5/b^9/(b*x^2+a)^6 - 1/18*a^8/b^9/(b*x^2+a)^9 + 1/2*a^7/b^9/(b*x^2+a)^8 - 2*a^6/b^9/(b*x^2+a)^7 - 14/3*a^2/b^9/(b*x^2+a)^3 + 2*a/b^9/(b*x^2+a)^2 + 7*a^3/b^9/(b*x^2+a)^4 - 1/2/(b*x^2+a)/b^9 - 7*a^4/b^9/(b*x^2+a)^5$

**Maxima [A]** time = 1.36998, size = 257, normalized size = 13.53

$$\frac{9b^8x^{16} + 36ab^7x^{14} + 84a^2b^6x^{12} + 126a^3b^5x^{10} + 126a^4b^4x^8 + 84a^5b^3x^6 + 36a^6b^2x^4 + 9a^7bx^2 + a^8}{18(b^{18}x^{18} + 9ab^{17}x^{16} + 36a^2b^{16}x^{14} + 84a^3b^{15}x^{12} + 126a^4b^{14}x^{10} + 126a^5b^{13}x^8 + 84a^6b^{12}x^6 + 36a^7b^{11}x^4 + 9a^8b^{10}x^2 + a^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^17/(b\*x^2 + a)^10,x, algorithm="maxima")

[Out]  $-1/18*(9*b^8*x^{16} + 36*a*b^7*x^{14} + 84*a^2*b^6*x^{12} + 126*a^3*b^5*x^{10} + 126*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/(b^{18}*x^{18} + 9*a*b^{17}*x^{16} + 36*a^2*b^{16}*x^{14} + 84*a^3*b^{15}*x^{12} + 126*a^4*b^{14}*x^{10} + 126*a^5*b^{13}*x^8 + 84*a^6*b^{12}*x^6 + 36*a^7*b^{11}*x^4 + 9*a^8*b^{10}*x^2 + a^9*b^9)$

**Fricas [A]** time = 0.203905, size = 257, normalized size = 13.53

$$\frac{9b^8x^{16} + 36ab^7x^{14} + 84a^2b^6x^{12} + 126a^3b^5x^{10} + 126a^4b^4x^8 + 84a^5b^3x^6 + 36a^6b^2x^4 + 9a^7bx^2 + a^8}{18(b^{18}x^{18} + 9ab^{17}x^{16} + 36a^2b^{16}x^{14} + 84a^3b^{15}x^{12} + 126a^4b^{14}x^{10} + 126a^5b^{13}x^8 + 84a^6b^{12}x^6 + 36a^7b^{11}x^4 + 9a^8b^{10}x^2 + a^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^17/(b\*x^2 + a)^10,x, algorithm="fricas")

[Out]  $-1/18*(9*b^8*x^{16} + 36*a*b^7*x^{14} + 84*a^2*b^6*x^{12} + 126*a^3*b^5*x^{10} + 126*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/(b^{18}*x^{18} + 9*a*b^{17}*x^{16} + 36*a^2*b^{16}*x^{14} + 84*a^3*b^{15}*x^{12} + 126*a^4*b^{14}*x^{10} + 126*a^5*b^{13}*x^8 + 84*a^6*b^{12}*x^6 + 36*a^7*b^{11}*x^4 + 9*a^8*b^{10}*x^2 + a^9*b^9)$

**Sympy [A]** time = 35.8375, size = 202, normalized size = 10.63

$$\frac{a^8 + 9a^7bx^2 + 36a^6b^2x^4 + 84a^5b^3x^6 + 126a^4b^4x^8 + 126a^3b^5x^{10} + 84a^2b^6x^{12} + 36ab^7x^{14} + 9b^8x^{16}}{18a^9b^9 + 162a^8b^{10}x^2 + 648a^7b^{11}x^4 + 1512a^6b^{12}x^6 + 2268a^5b^{13}x^8 + 2268a^4b^{14}x^{10} + 1512a^3b^{15}x^{12} + 648a^2b^{16}x^{14} + 162ab^{17}x^{16} + b^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*17/(b\*x\*\*2+a)\*\*10,x)

[Out]  $-(a^{**8} + 9*a^{**7}*b*x^{**2} + 36*a^{**6}*b^{**2}*x^{**4} + 84*a^{**5}*b^{**3}*x^{**6} + 126*a^{**4}*b^{**4}*x^{**8} + 126*a^{**3}*b^{**5}*x^{**10} + 84*a^{**2}*b^{**6}*x^{**12} + 36*a^{**1}*b^{**7}*x^{**14} + 9*b^{**8}*x^{**16})/(18*a^{**9}*b^{**9} + 162*a^{**8}*b^{**10}*x^{**2} + 648*a^{**7}*b^{**11}*x^{**4} + 1512*a^{**6}*b^{**12}*x^{**6} + 2268*a^{**5}*b^{**13}*x^{**8} + 2268*a^{**4}*b^{**14}*x^{**10} + 1512*a^{**3}*b^{**15}*x^{**12} + 648*a^{**2}*b^{**16}*x^{**14} + 162*a^{**1}*b^{**17}*x^{**16} + 18*b^{**18}*x^{**18})$

**GIAC/XCAS [A]** time = 0.212008, size = 134, normalized size = 7.05

$$\frac{9b^8x^{16} + 36ab^7x^{14} + 84a^2b^6x^{12} + 126a^3b^5x^{10} + 126a^4b^4x^8 + 84a^5b^3x^6 + 36a^6b^2x^4 + 9a^7bx^2 + a^8}{18(bx^2 + a)^9b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^2 + a)^10,x, algorithm="giac")
```

```
[Out] -1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 126*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/((b*x^2 + a)^9*b^9)
```

$$3.197 \quad \int \frac{x^{15}}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=39

$$\frac{x^{16}}{144a^2(a+bx^2)^8} + \frac{x^{16}}{18a(a+bx^2)^9}$$

[Out]  $x^{16}/(18*a*(a+b*x^2)^9) + x^{16}/(144*a^2*(a+b*x^2)^8)$

**Rubi [A]** time = 0.0571246, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{x^{16}}{144a^2(a+bx^2)^8} + \frac{x^{16}}{18a(a+bx^2)^9}$$

Antiderivative was successfully verified.

[In] Int[x^15/(a + b\*x^2)^10, x]

[Out]  $x^{16}/(18*a*(a+b*x^2)^9) + x^{16}/(144*a^2*(a+b*x^2)^8)$

**Rubi in Sympy [A]** time = 7.04433, size = 31, normalized size = 0.79

$$\frac{x^{16}}{18a(a+bx^2)^9} + \frac{x^{16}}{144a^2(a+bx^2)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*15/(b\*x\*\*2+a)\*\*10, x)

[Out]  $x^{16}/(18*a*(a+b*x^2)^9) + x^{16}/(144*a^2*(a+b*x^2)^8)$

**Mathematica [B]** time = 0.0302042, size = 90, normalized size = 2.31

$$\frac{a^7 + 9a^6bx^2 + 36a^5b^2x^4 + 84a^4b^3x^6 + 126a^3b^4x^8 + 126a^2b^5x^{10} + 84ab^6x^{12} + 36b^7x^{14}}{144b^8(a+bx^2)^9}$$

Antiderivative was successfully verified.

[In] Integrate[x^15/(a + b\*x^2)^10, x]

[Out]  $-(a^7 + 9*a^6*b*x^2 + 36*a^5*b^2*x^4 + 84*a^4*b^3*x^6 + 126*a^3*b^4*x^8 + 126*a^2*b^5*x^{10} + 84*a*b^6*x^{12} + 36*b^7*x^{14})/(144*b^8*(a+b*x^2)^9)$

**Maple [B]** time = 0.015, size = 133, normalized size = 3.4

$$\frac{a^7}{18b^8(bx^2+a)^9} + \frac{7a}{6b^8(bx^2+a)^3} - \frac{21a^2}{8b^8(bx^2+a)^4} - \frac{35a^4}{12b^8(bx^2+a)^6} - \frac{1}{4(bx^2+a)^2b^8} - \frac{7a^6}{16b^8(bx^2+a)^8} + \frac{3a^5}{2b^8(bx^2+a)^7} + \frac{7a^3}{2b^8(bx^2+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(x^15/(b*x^2+a)^10,x)`

[Out]  $\frac{1}{18} \frac{a^7}{b^8} (b^2 x^2 + a)^9 + \frac{7}{6} \frac{a}{b^8} (b^2 x^2 + a)^3 - \frac{21}{8} \frac{a^2}{b^8} (b^2 x^2 + a)^4 - \frac{35}{12} \frac{a^4}{b^8} (b^2 x^2 + a)^6 - \frac{1}{4} (b^2 x^2 + a)^2 - \frac{7}{16} \frac{a^6}{b^8} (b^2 x^2 + a)^8 + \frac{3}{2} \frac{a^5}{b^8} (b^2 x^2 + a)^7 + \frac{7}{2} \frac{a^3}{b^8} (b^2 x^2 + a)^5$

**Maxima [A]** time = 1.36119, size = 242, normalized size = 6.21

$$\frac{36 b^7 x^{14} + 84 a b^6 x^{12} + 126 a^2 b^5 x^{10} + 126 a^3 b^4 x^8 + 84 a^4 b^3 x^6 + 36 a^5 b^2 x^4 + 9 a^6 b x^2 + a^7}{144 (b^{17} x^{18} + 9 a b^{16} x^{16} + 36 a^2 b^{15} x^{14} + 84 a^3 b^{14} x^{12} + 126 a^4 b^{13} x^{10} + 126 a^5 b^{12} x^8 + 84 a^6 b^{11} x^6 + 36 a^7 b^{10} x^4 + 9 a^8 b^9 x^2 + a^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^15/(b*x^2 + a)^10,x, algorithm="maxima")`

[Out]  $-\frac{1}{144} (36 b^7 x^{14} + 84 a b^6 x^{12} + 126 a^2 b^5 x^{10} + 126 a^3 b^4 x^8 + 84 a^4 b^3 x^6 + 36 a^5 b^2 x^4 + 9 a^6 b x^2 + a^7) / (b^{17} x^{18} + 9 a b^{16} x^{16} + 36 a^2 b^{15} x^{14} + 84 a^3 b^{14} x^{12} + 126 a^4 b^{13} x^{10} + 126 a^5 b^{12} x^8 + 84 a^6 b^{11} x^6 + 36 a^7 b^{10} x^4 + 9 a^8 b^9 x^2 + a^9)$

**Fricas [A]** time = 0.203765, size = 242, normalized size = 6.21

$$\frac{36 b^7 x^{14} + 84 a b^6 x^{12} + 126 a^2 b^5 x^{10} + 126 a^3 b^4 x^8 + 84 a^4 b^3 x^6 + 36 a^5 b^2 x^4 + 9 a^6 b x^2 + a^7}{144 (b^{17} x^{18} + 9 a b^{16} x^{16} + 36 a^2 b^{15} x^{14} + 84 a^3 b^{14} x^{12} + 126 a^4 b^{13} x^{10} + 126 a^5 b^{12} x^8 + 84 a^6 b^{11} x^6 + 36 a^7 b^{10} x^4 + 9 a^8 b^9 x^2 + a^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^15/(b*x^2 + a)^10,x, algorithm="fricas")`

[Out]  $-\frac{1}{144} (36 b^7 x^{14} + 84 a b^6 x^{12} + 126 a^2 b^5 x^{10} + 126 a^3 b^4 x^8 + 84 a^4 b^3 x^6 + 36 a^5 b^2 x^4 + 9 a^6 b x^2 + a^7) / (b^{17} x^{18} + 9 a b^{16} x^{16} + 36 a^2 b^{15} x^{14} + 84 a^3 b^{14} x^{12} + 126 a^4 b^{13} x^{10} + 126 a^5 b^{12} x^8 + 84 a^6 b^{11} x^6 + 36 a^7 b^{10} x^4 + 9 a^8 b^9 x^2 + a^9)$

**Sympy [A]** time = 35.1331, size = 190, normalized size = 4.87

$$\frac{a^7 + 9 a^6 b x^2 + 36 a^5 b^2 x^4 + 84 a^4 b^3 x^6 + 126 a^3 b^4 x^8 + 126 a^2 b^5 x^{10} + 84 a b^6 x^{12} + 36 b^7 x^{14}}{144 a^9 b^8 + 1296 a^8 b^9 x^2 + 5184 a^7 b^{10} x^4 + 12096 a^6 b^{11} x^6 + 18144 a^5 b^{12} x^8 + 18144 a^4 b^{13} x^{10} + 12096 a^3 b^{14} x^{12} + 5184 a^2 b^{15} x^{14} + 1296 a b^{16} x^{16} + 144 b^{17} x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**15/(b*x**2+a)**10,x)`

[Out]  $-\frac{(a^7 + 9 a^6 b x^2 + 36 a^5 b^2 x^4 + 84 a^4 b^3 x^6 + 126 a^3 b^4 x^8 + 126 a^2 b^5 x^{10} + 84 a b^6 x^{12} + 36 b^7 x^{14})}{(144 a^9 b^8 + 1296 a^8 b^9 x^2 + 5184 a^7 b^{10} x^4 + 12096 a^6 b^{11} x^6 + 18144 a^5 b^{12} x^8 + 18144 a^4 b^{13} x^{10} + 12096 a^3 b^{14} x^{12} + 5184 a^2 b^{15} x^{14} + 1296 a b^{16} x^{16} + 144 b^{17} x^{18})}$

**GIAC/XCAS [A]** time = 0.210622, size = 119, normalized size = 3.05

$$\frac{36 b^7 x^{14} + 84 a b^6 x^{12} + 126 a^2 b^5 x^{10} + 126 a^3 b^4 x^8 + 84 a^4 b^3 x^6 + 36 a^5 b^2 x^4 + 9 a^6 b x^2 + a^7}{144 (b x^2 + a)^9 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^2 + a)^10,x, algorithm="giac")
```

```
[Out] -1/144*(36*b^7*x^14 + 84*a*b^6*x^12 + 126*a^2*b^5*x^10 + 126*a^3*  
b^4*x^8 + 84*a^4*b^3*x^6 + 36*a^5*b^2*x^4 + 9*a^6*b*x^2 + a^7)/((  
b*x^2 + a)^9*b^8)
```

$$3.198 \quad \int \frac{x^{13}}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=58

$$\frac{x^{14}}{504a^3(a+bx^2)^7} + \frac{x^{14}}{72a^2(a+bx^2)^8} + \frac{x^{14}}{18a(a+bx^2)^9}$$

[Out]  $x^{14}/(18*a*(a+b*x^2)^9) + x^{14}/(72*a^2*(a+b*x^2)^8) + x^{14}/(504*a^3*(a+b*x^2)^7)$

**Rubi [A]** time = 0.0814398, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{x^{14}}{504a^3(a+bx^2)^7} + \frac{x^{14}}{72a^2(a+bx^2)^8} + \frac{x^{14}}{18a(a+bx^2)^9}$$

Antiderivative was successfully verified.

[In] Int[x^13/(a + b\*x^2)^10, x]

[Out]  $x^{14}/(18*a*(a+b*x^2)^9) + x^{14}/(72*a^2*(a+b*x^2)^8) + x^{14}/(504*a^3*(a+b*x^2)^7)$

**Rubi in Sympy [A]** time = 9.86219, size = 48, normalized size = 0.83

$$\frac{x^{14}}{18a(a+bx^2)^9} + \frac{x^{14}}{72a^2(a+bx^2)^8} + \frac{x^{14}}{504a^3(a+bx^2)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*13/(b\*x\*\*2+a)\*\*10, x)

[Out]  $x^{14}/(18*a*(a+b*x^2)^9) + x^{14}/(72*a^2*(a+b*x^2)^8) + x^{14}/(504*a^3*(a+b*x^2)^7)$

**Mathematica [A]** time = 0.031869, size = 79, normalized size = 1.36

$$\frac{a^6 + 9a^5bx^2 + 36a^4b^2x^4 + 84a^3b^3x^6 + 126a^2b^4x^8 + 126ab^5x^{10} + 84b^6x^{12}}{504b^7(a+bx^2)^9}$$

Antiderivative was successfully verified.

[In] Integrate[x^13/(a + b\*x^2)^10, x]

[Out]  $-(a^6 + 9*a^5*b*x^2 + 36*a^4*b^2*x^4 + 84*a^3*b^3*x^6 + 126*a^2*b^4*x^8 + 126*a*b^5*x^{10} + 84*b^6*x^{12})/(504*b^7*(a+b*x^2)^9)$

**Maple [B]** time = 0.014, size = 116, normalized size = 2.

$$\begin{aligned} & -\frac{3a^2}{2b^7(bx^2+a)^5} - \frac{a^6}{18b^7(bx^2+a)^9} - \frac{1}{6(bx^2+a)^3b^7} - \frac{15a^4}{14b^7(bx^2+a)^7} \\ & + \frac{5a^3}{3b^7(bx^2+a)^6} + \frac{3a^5}{8b^7(bx^2+a)^8} + \frac{3a}{4b^7(bx^2+a)^4} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^13/(b*x^2+a)^10,x)`

[Out] 
$$-3/2*a^2/b^7/(b*x^2+a)^5 - 1/18*a^6/b^7/(b*x^2+a)^9 - 1/6/(b*x^2+a)^3/b^7 - 15/14*a^4/b^7/(b*x^2+a)^7 + 5/3*a^3/b^7/(b*x^2+a)^6 + 3/8*a^5/b^7/(b*x^2+a)^8 + 3/4*a/b^7/(b*x^2+a)^4$$

**Maxima [A]** time = 1.34643, size = 227, normalized size = 3.91

$$\frac{84b^6x^{12} + 126ab^5x^{10} + 126a^2b^4x^8 + 84a^3b^3x^6 + 36a^4b^2x^4 + 9a^5bx^2 + a^6}{504(b^{16}x^{18} + 9ab^{15}x^{16} + 36a^2b^{14}x^{14} + 84a^3b^{13}x^{12} + 126a^4b^{12}x^{10} + 126a^5b^{11}x^8 + 84a^6b^{10}x^6 + 36a^7b^9x^4 + 9a^8b^8x^2 + a^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^13/(b*x^2 + a)^10,x, algorithm="maxima")`

[Out] 
$$-1/504*(84*b^6*x^12 + 126*a*b^5*x^10 + 126*a^2*b^4*x^8 + 84*a^3*b^3*x^6 + 36*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/(b^16*x^18 + 9*a*b^15*x^16 + 36*a^2*b^14*x^14 + 84*a^3*b^13*x^12 + 126*a^4*b^12*x^10 + 126*a^5*b^11*x^8 + 84*a^6*b^10*x^6 + 36*a^7*b^9*x^4 + 9*a^8*b^8*x^2 + a^9*b^7)$$

**Fricas [A]** time = 0.204256, size = 227, normalized size = 3.91

$$\frac{84b^6x^{12} + 126ab^5x^{10} + 126a^2b^4x^8 + 84a^3b^3x^6 + 36a^4b^2x^4 + 9a^5bx^2 + a^6}{504(b^{16}x^{18} + 9ab^{15}x^{16} + 36a^2b^{14}x^{14} + 84a^3b^{13}x^{12} + 126a^4b^{12}x^{10} + 126a^5b^{11}x^8 + 84a^6b^{10}x^6 + 36a^7b^9x^4 + 9a^8b^8x^2 + a^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^13/(b*x^2 + a)^10,x, algorithm="fricas")`

[Out] 
$$-1/504*(84*b^6*x^12 + 126*a*b^5*x^10 + 126*a^2*b^4*x^8 + 84*a^3*b^3*x^6 + 36*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/(b^16*x^18 + 9*a*b^15*x^16 + 36*a^2*b^14*x^14 + 84*a^3*b^13*x^12 + 126*a^4*b^12*x^10 + 126*a^5*b^11*x^8 + 84*a^6*b^10*x^6 + 36*a^7*b^9*x^4 + 9*a^8*b^8*x^2 + a^9*b^7)$$

**Sympy [A]** time = 34.8363, size = 178, normalized size = 3.07

$$\frac{a^6 + 9a^5bx^2 + 36a^4b^2x^4 + 84a^3b^3x^6 + 126a^2b^4x^8 + 126ab^5x^{10} + 84b^6x^{12}}{504a^9b^7 + 4536a^8b^8x^2 + 18144a^7b^9x^4 + 42336a^6b^{10}x^6 + 63504a^5b^{11}x^8 + 63504a^4b^{12}x^{10} + 42336a^3b^{13}x^{12} + 18144a^2b^{14}x^{14} + 4536ab^{15}x^{16} + b^{16}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**13/(b*x**2+a)**10,x)`

[Out] 
$$-(a^{**6} + 9*a^{**5}*b*x^{**2} + 36*a^{**4}*b^{**2}*x^{**4} + 84*a^{**3}*b^{**3}*x^{**6} + 126*a^{**2}*b^{**4}*x^{**8} + 126*a*b^{**5}*x^{**10} + 84*b^{**6}*x^{**12})/(504*a^{**9}*b^{**7} + 4536*a^{**8}*b^{**8}*x^{**2} + 18144*a^{**7}*b^{**9}*x^{**4} + 42336*a^{**6}*b^{**10}*x^{**6} + 63504*a^{**5}*b^{**11}*x^{**8} + 63504*a^{**4}*b^{**12}*x^{**10} + 42336*a^{**3}*b^{**13}*x^{**12} + 18144*a^{**2}*b^{**14}*x^{**14} + 4536*a*b^{**15}*x^{**16} + 504*b^{**16}*x^{**18})$$

**GIAC/XCAS [A]** time = 0.209252, size = 104, normalized size = 1.79

$$\frac{84b^6x^{12} + 126ab^5x^{10} + 126a^2b^4x^8 + 84a^3b^3x^6 + 36a^4b^2x^4 + 9a^5bx^2 + a^6}{504(bx^2 + a)^9b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^2 + a)^10,x, algorithm="giac")
```

```
[Out] -1/504*(84*b^6*x^12 + 126*a*b^5*x^10 + 126*a^2*b^4*x^8 + 84*a^3*b^3*x^6 + 36*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/((b*x^2 + a)^9*b^7)
```

$$3.199 \quad \int \frac{x^{11}}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=77

$$\frac{x^{12}}{1008a^4(a+bx^2)^6} + \frac{x^{12}}{168a^3(a+bx^2)^7} + \frac{x^{12}}{48a^2(a+bx^2)^8} + \frac{x^{12}}{18a(a+bx^2)^9}$$

[Out]  $x^{12}/(18*a*(a+b*x^2)^9) + x^{12}/(48*a^2*(a+b*x^2)^8) + x^{12}/(168*a^3*(a+b*x^2)^7) + x^{12}/(1008*a^4*(a+b*x^2)^6)$

**Rubi [A]** time = 0.108005, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{x^{12}}{1008a^4(a+bx^2)^6} + \frac{x^{12}}{168a^3(a+bx^2)^7} + \frac{x^{12}}{48a^2(a+bx^2)^8} + \frac{x^{12}}{18a(a+bx^2)^9}$$

Antiderivative was successfully verified.

[In] Int[x^11/(a + b\*x^2)^10, x]

[Out]  $x^{12}/(18*a*(a+b*x^2)^9) + x^{12}/(48*a^2*(a+b*x^2)^8) + x^{12}/(168*a^3*(a+b*x^2)^7) + x^{12}/(1008*a^4*(a+b*x^2)^6)$

**Rubi in Sympy [A]** time = 13.1752, size = 65, normalized size = 0.84

$$\frac{x^{12}}{18a(a+bx^2)^9} + \frac{x^{12}}{48a^2(a+bx^2)^8} + \frac{x^{12}}{168a^3(a+bx^2)^7} + \frac{x^{12}}{1008a^4(a+bx^2)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*11/(b\*x\*\*2+a)\*\*10, x)

[Out]  $x^{12}/(18*a*(a+b*x^2)^9) + x^{12}/(48*a^2*(a+b*x^2)^8) + x^{12}/(168*a^3*(a+b*x^2)^7) + x^{12}/(1008*a^4*(a+b*x^2)^6)$

**Mathematica [A]** time = 0.0346753, size = 68, normalized size = 0.88

$$\frac{a^5 + 9a^4bx^2 + 36a^3b^2x^4 + 84a^2b^3x^6 + 126ab^4x^8 + 126b^5x^{10}}{1008b^6(a+bx^2)^9}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/(a + b\*x^2)^10, x]

[Out]  $-(a^5 + 9*a^4*b*x^2 + 36*a^3*b^2*x^4 + 84*a^2*b^3*x^6 + 126*a*b^4*x^8 + 126*b^5*x^{10})/(1008*b^6*(a+b*x^2)^9)$

**Maple [A]** time = 0.013, size = 99, normalized size = 1.3

$$\frac{a^5}{18b^6(bx^2+a)^9} - \frac{5a^4}{16b^6(bx^2+a)^8} + \frac{5a^3}{7b^6(bx^2+a)^7} - \frac{1}{8(bx^2+a)^4b^6} - \frac{5a^2}{6b^6(bx^2+a)^6} + \frac{a}{2b^6(bx^2+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11/(b*x^2+a)^10,x)`

[Out]  $\frac{1}{18} \frac{a^5}{b^6} (b^2 x^2 + a)^9 - \frac{5}{16} \frac{a^4}{b^6} (b^2 x^2 + a)^8 + \frac{5}{7} \frac{a^3}{b^6} (b^2 x^2 + a)^7 - \frac{1}{8} \frac{a^2}{b^6} (b^2 x^2 + a)^6 + \frac{1}{2} \frac{a}{b^6} (b^2 x^2 + a)^5$

**Maxima [A]** time = 1.37778, size = 212, normalized size = 2.75

$$\frac{126 b^5 x^{10} + 126 a b^4 x^8 + 84 a^2 b^3 x^6 + 36 a^3 b^2 x^4 + 9 a^4 b x^2 + a^5}{1008 (b^{15} x^{18} + 9 a b^{14} x^{16} + 36 a^2 b^{13} x^{14} + 84 a^3 b^{12} x^{12} + 126 a^4 b^{11} x^{10} + 126 a^5 b^{10} x^8 + 84 a^6 b^9 x^6 + 36 a^7 b^8 x^4 + 9 a^8 b^7 x^2 + a^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(b*x^2 + a)^10,x, algorithm="maxima")`

[Out]  $-\frac{1}{1008} \frac{(126 b^5 x^{10} + 126 a b^4 x^8 + 84 a^2 b^3 x^6 + 36 a^3 b^2 x^4 + 9 a^4 b x^2 + a^5)}{(b^{15} x^{18} + 9 a b^{14} x^{16} + 36 a^2 b^{13} x^{14} + 84 a^3 b^{12} x^{12} + 126 a^4 b^{11} x^{10} + 126 a^5 b^{10} x^8 + 84 a^6 b^9 x^6 + 36 a^7 b^8 x^4 + 9 a^8 b^7 x^2 + a^9)}$

**Fricas [A]** time = 0.20364, size = 212, normalized size = 2.75

$$\frac{126 b^5 x^{10} + 126 a b^4 x^8 + 84 a^2 b^3 x^6 + 36 a^3 b^2 x^4 + 9 a^4 b x^2 + a^5}{1008 (b^{15} x^{18} + 9 a b^{14} x^{16} + 36 a^2 b^{13} x^{14} + 84 a^3 b^{12} x^{12} + 126 a^4 b^{11} x^{10} + 126 a^5 b^{10} x^8 + 84 a^6 b^9 x^6 + 36 a^7 b^8 x^4 + 9 a^8 b^7 x^2 + a^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11/(b*x^2 + a)^10,x, algorithm="fricas")`

[Out]  $-\frac{1}{1008} \frac{(126 b^5 x^{10} + 126 a b^4 x^8 + 84 a^2 b^3 x^6 + 36 a^3 b^2 x^4 + 9 a^4 b x^2 + a^5)}{(b^{15} x^{18} + 9 a b^{14} x^{16} + 36 a^2 b^{13} x^{14} + 84 a^3 b^{12} x^{12} + 126 a^4 b^{11} x^{10} + 126 a^5 b^{10} x^8 + 84 a^6 b^9 x^6 + 36 a^7 b^8 x^4 + 9 a^8 b^7 x^2 + a^9)}$

**Sympy [A]** time = 34.329, size = 167, normalized size = 2.17

$$\frac{a^5 + 9 a^4 b x^2 + 36 a^3 b^2 x^4 + 84 a^2 b^3 x^6 + 126 a b^4 x^8 + 126 b^5 x^{10}}{1008 a^9 b^6 + 9072 a^8 b^7 x^2 + 36288 a^7 b^8 x^4 + 84672 a^6 b^9 x^6 + 127008 a^5 b^{10} x^8 + 127008 a^4 b^{11} x^{10} + 84672 a^3 b^{12} x^{12} + 36288 a^2 b^{13} x^{14} + 9072 a b^{14} x^{16} + 1008 b^{15} x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**11/(b*x**2+a)**10,x)`

[Out]  $-\frac{(a^5 + 9 a^4 b x^2 + 36 a^3 b^2 x^4 + 84 a^2 b^3 x^6 + 126 a b^4 x^8 + 126 b^5 x^{10})}{(1008 a^9 b^6 + 9072 a^8 b^7 x^2 + 36288 a^7 b^8 x^4 + 84672 a^6 b^9 x^6 + 127008 a^5 b^{10} x^8 + 127008 a^4 b^{11} x^{10} + 84672 a^3 b^{12} x^{12} + 36288 a^2 b^{13} x^{14} + 9072 a b^{14} x^{16} + 1008 b^{15} x^{18})}$

**GIAC/XCAS [A]** time = 0.209055, size = 89, normalized size = 1.16

$$\frac{126 b^5 x^{10} + 126 a b^4 x^8 + 84 a^2 b^3 x^6 + 36 a^3 b^2 x^4 + 9 a^4 b x^2 + a^5}{1008 (b x^2 + a)^9 b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^2 + a)^10,x, algorithm="giac")
```

```
[Out] -1/1008*(126*b^5*x^10 + 126*a*b^4*x^8 + 84*a^2*b^3*x^6 + 36*a^3*b^2*x^4 + 9*a^4*b*x^2 + a^5)/((b*x^2 + a)^9*b^6)
```



$$3.200 \quad \int \frac{x^9}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=91

$$-\frac{a^4}{18b^5(a+bx^2)^9} + \frac{a^3}{4b^5(a+bx^2)^8} - \frac{3a^2}{7b^5(a+bx^2)^7} + \frac{a}{3b^5(a+bx^2)^6} - \frac{1}{10b^5(a+bx^2)^5}$$

[Out]  $-a^4/(18*b^5*(a+b*x^2)^9) + a^3/(4*b^5*(a+b*x^2)^8) - (3*a^2)/(7*b^5*(a+b*x^2)^7) + a/(3*b^5*(a+b*x^2)^6) - 1/(10*b^5*(a+b*x^2)^5)$

**Rubi [A]** time = 0.163549, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^4}{18b^5(a+bx^2)^9} + \frac{a^3}{4b^5(a+bx^2)^8} - \frac{3a^2}{7b^5(a+bx^2)^7} + \frac{a}{3b^5(a+bx^2)^6} - \frac{1}{10b^5(a+bx^2)^5}$$

Antiderivative was successfully verified.

[In] Int[x^9/(a+b\*x^2)^10,x]

[Out]  $-a^4/(18*b^5*(a+b*x^2)^9) + a^3/(4*b^5*(a+b*x^2)^8) - (3*a^2)/(7*b^5*(a+b*x^2)^7) + a/(3*b^5*(a+b*x^2)^6) - 1/(10*b^5*(a+b*x^2)^5)$

**Rubi in Sympy [A]** time = 23.7135, size = 82, normalized size = 0.9

$$-\frac{a^4}{18b^5(a+bx^2)^9} + \frac{a^3}{4b^5(a+bx^2)^8} - \frac{3a^2}{7b^5(a+bx^2)^7} + \frac{a}{3b^5(a+bx^2)^6} - \frac{1}{10b^5(a+bx^2)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*9/(b\*x\*\*2+a)\*\*10,x)

[Out]  $-a**4/(18*b**5*(a+b*x**2)**9) + a**3/(4*b**5*(a+b*x**2)**8) - 3*a**2/(7*b**5*(a+b*x**2)**7) + a/(3*b**5*(a+b*x**2)**6) - 1/(10*b**5*(a+b*x**2)**5)$

**Mathematica [A]** time = 0.0267643, size = 57, normalized size = 0.63

$$\frac{a^4 + 9a^3bx^2 + 36a^2b^2x^4 + 84ab^3x^6 + 126b^4x^8}{1260b^5(a+bx^2)^9}$$

Antiderivative was successfully verified.

[In] Integrate[x^9/(a+b\*x^2)^10,x]

[Out]  $-(a^4 + 9*a^3*b*x^2 + 36*a^2*b^2*x^4 + 84*a*b^3*x^6 + 126*b^4*x^8)/(1260*b^5*(a+b*x^2)^9)$

**Maple [A]** time = 0.011, size = 82, normalized size = 0.9

$$-\frac{a^4}{18b^5(bx^2+a)^9} + \frac{a^3}{4b^5(bx^2+a)^8} - \frac{3a^2}{7b^5(bx^2+a)^7} + \frac{a}{3b^5(bx^2+a)^6} - \frac{1}{10b^5(bx^2+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9/(b*x^2+a)^10,x)`

[Out] 
$$-1/18*a^4/b^5/(b*x^2+a)^9+1/4*a^3/b^5/(b*x^2+a)^8-3/7*a^2/b^5/(b*x^2+a)^7+1/3*a/b^5/(b*x^2+a)^6-1/10/b^5/(b*x^2+a)^5$$

**Maxima [A]** time = 1.36678, size = 197, normalized size = 2.16

$$\frac{126b^4x^8 + 84ab^3x^6 + 36a^2b^2x^4 + 9a^3bx^2 + a^4}{1260(b^{14}x^{18} + 9ab^{13}x^{16} + 36a^2b^{12}x^{14} + 84a^3b^{11}x^{12} + 126a^4b^{10}x^{10} + 126a^5b^9x^8 + 84a^6b^8x^6 + 36a^7b^7x^4 + 9a^8b^6x^2 + a^9b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9/(b*x^2 + a)^10,x, algorithm="maxima")`

[Out] 
$$-1/1260*(126*b^4*x^8 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4 + 9*a^3*b*x^2 + a^4)/(b^{14}*x^{18} + 9*a*b^{13}*x^{16} + 36*a^2*b^{12}*x^{14} + 84*a^3*b^{11}*x^{12} + 126*a^4*b^{10}*x^{10} + 126*a^5*b^9*x^8 + 84*a^6*b^8*x^6 + 36*a^7*b^7*x^4 + 9*a^8*b^6*x^2 + a^9*b^5)$$

**Fricas [A]** time = 0.206562, size = 197, normalized size = 2.16

$$\frac{126b^4x^8 + 84ab^3x^6 + 36a^2b^2x^4 + 9a^3bx^2 + a^4}{1260(b^{14}x^{18} + 9ab^{13}x^{16} + 36a^2b^{12}x^{14} + 84a^3b^{11}x^{12} + 126a^4b^{10}x^{10} + 126a^5b^9x^8 + 84a^6b^8x^6 + 36a^7b^7x^4 + 9a^8b^6x^2 + a^9b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9/(b*x^2 + a)^10,x, algorithm="fricas")`

[Out] 
$$-1/1260*(126*b^4*x^8 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4 + 9*a^3*b*x^2 + a^4)/(b^{14}*x^{18} + 9*a*b^{13}*x^{16} + 36*a^2*b^{12}*x^{14} + 84*a^3*b^{11}*x^{12} + 126*a^4*b^{10}*x^{10} + 126*a^5*b^9*x^8 + 84*a^6*b^8*x^6 + 36*a^7*b^7*x^4 + 9*a^8*b^6*x^2 + a^9*b^5)$$

**Sympy [A]** time = 33.929, size = 155, normalized size = 1.7

$$\frac{a^4 + 9a^3bx^2 + 36a^2b^2x^4 + 84ab^3x^6 + 126b^4x^8}{1260a^9b^5 + 11340a^8b^6x^2 + 45360a^7b^7x^4 + 105840a^6b^8x^6 + 158760a^5b^9x^8 + 158760a^4b^{10}x^{10} + 105840a^3b^{11}x^{12} + 45360a^2b^{12}x^{14} + 11340ab^{13}x^{16} + 1260b^{14}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9/(b*x**2+a)**10,x)`

[Out] 
$$-(a^{**4} + 9*a^{**3}*b*x^{**2} + 36*a^{**2}*b^{**2}*x^{**4} + 84*a*b^{**3}*x^{**6} + 126*b^{**4}*x^{**8})/(1260*a^{**9}*b^{**5} + 11340*a^{**8}*b^{**6}*x^{**2} + 45360*a^{**7}*b^{**7}*x^{**4} + 105840*a^{**6}*b^{**8}*x^{**6} + 158760*a^{**5}*b^{**9}*x^{**8} + 158760*a^{**4}*b^{**10}*x^{**10} + 105840*a^{**3}*b^{**11}*x^{**12} + 45360*a^{**2}*b^{**12}*x^{**14} + 11340*a*b^{**13}*x^{**16} + 1260*b^{**14}*x^{**18})$$

**GIAC/XCAS [A]** time = 0.211509, size = 74, normalized size = 0.81

$$\frac{126b^4x^8 + 84ab^3x^6 + 36a^2b^2x^4 + 9a^3bx^2 + a^4}{1260(bx^2 + a)^9b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^2 + a)^10,x, algorithm="giac")
```

```
[Out] -1/1260*(126*b^4*x^8 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4 + 9*a^3*b*x^2 + a^4)/((b*x^2 + a)^9*b^5)
```

$$3.201 \quad \int \frac{x^7}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=72

$$\frac{a^3}{18b^4(a+bx^2)^9} - \frac{3a^2}{16b^4(a+bx^2)^8} + \frac{3a}{14b^4(a+bx^2)^7} - \frac{1}{12b^4(a+bx^2)^6}$$

[Out]  $a^3/(18*b^4*(a + b*x^2)^9) - (3*a^2)/(16*b^4*(a + b*x^2)^8) + (3*a)/(14*b^4*(a + b*x^2)^7) - 1/(12*b^4*(a + b*x^2)^6)$

**Rubi [A]** time = 0.130764, antiderivative size = 72, normalized size of antiderivative = 1., number of rules used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^3}{18b^4(a+bx^2)^9} - \frac{3a^2}{16b^4(a+bx^2)^8} + \frac{3a}{14b^4(a+bx^2)^7} - \frac{1}{12b^4(a+bx^2)^6}$$

Antiderivative was successfully verified.

[In] Int[x^7/(a + b\*x^2)^10, x]

[Out]  $a^3/(18*b^4*(a + b*x^2)^9) - (3*a^2)/(16*b^4*(a + b*x^2)^8) + (3*a)/(14*b^4*(a + b*x^2)^7) - 1/(12*b^4*(a + b*x^2)^6)$

**Rubi in Sympy [A]** time = 19.3408, size = 66, normalized size = 0.92

$$\frac{a^3}{18b^4(a+bx^2)^9} - \frac{3a^2}{16b^4(a+bx^2)^8} + \frac{3a}{14b^4(a+bx^2)^7} - \frac{1}{12b^4(a+bx^2)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*7/(b\*x\*\*2+a)\*\*10, x)

[Out]  $a**3/(18*b**4*(a + b*x**2)**9) - 3*a**2/(16*b**4*(a + b*x**2)**8) + 3*a/(14*b**4*(a + b*x**2)**7) - 1/(12*b**4*(a + b*x**2)**6)$

**Mathematica [A]** time = 0.0198645, size = 46, normalized size = 0.64

$$-\frac{a^3 + 9a^2bx^2 + 36ab^2x^4 + 84b^3x^6}{1008b^4(a+bx^2)^9}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a + b\*x^2)^10, x]

[Out]  $-(a^3 + 9*a^2*b*x^2 + 36*a*b^2*x^4 + 84*b^3*x^6)/(1008*b^4*(a + b*x^2)^9)$

**Maple [A]** time = 0.01, size = 65, normalized size = 0.9

$$\frac{a^3}{18b^4(bx^2+a)^9} - \frac{3a^2}{16b^4(bx^2+a)^8} + \frac{3a}{14b^4(bx^2+a)^7} - \frac{1}{12b^4(bx^2+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7/(b*x^2+a)^10,x)`

[Out]  $1/18*a^3/b^4/(b*x^2+a)^9 - 3/16*a^2/b^4/(b*x^2+a)^8 + 3/14*a/b^4/(b*x^2+a)^7 - 1/12/b^4/(b*x^2+a)^6$

**Maxima [A]** time = 1.35434, size = 182, normalized size = 2.53

$$\frac{84b^3x^6 + 36ab^2x^4 + 9a^2bx^2 + a^3}{1008(b^{13}x^{18} + 9ab^{12}x^{16} + 36a^2b^{11}x^{14} + 84a^3b^{10}x^{12} + 126a^4b^9x^{10} + 126a^5b^8x^8 + 84a^6b^7x^6 + 36a^7b^6x^4 + 9a^8b^5x^2 + a^9b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^2 + a)^10,x, algorithm="maxima")`

[Out]  $-1/1008*(84*b^3*x^6 + 36*a*b^2*x^4 + 9*a^2*b*x^2 + a^3)/(b^{13}*x^{18} + 9*a*b^{12}*x^{16} + 36*a^2*b^{11}*x^{14} + 84*a^3*b^{10}*x^{12} + 126*a^4*b^9*x^{10} + 126*a^5*b^8*x^8 + 84*a^6*b^7*x^6 + 36*a^7*b^6*x^4 + 9*a^8*b^5*x^2 + a^9*b^4)$

**Fricas [A]** time = 0.199584, size = 182, normalized size = 2.53

$$\frac{84b^3x^6 + 36ab^2x^4 + 9a^2bx^2 + a^3}{1008(b^{13}x^{18} + 9ab^{12}x^{16} + 36a^2b^{11}x^{14} + 84a^3b^{10}x^{12} + 126a^4b^9x^{10} + 126a^5b^8x^8 + 84a^6b^7x^6 + 36a^7b^6x^4 + 9a^8b^5x^2 + a^9b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^2 + a)^10,x, algorithm="fricas")`

[Out]  $-1/1008*(84*b^3*x^6 + 36*a*b^2*x^4 + 9*a^2*b*x^2 + a^3)/(b^{13}*x^{18} + 9*a*b^{12}*x^{16} + 36*a^2*b^{11}*x^{14} + 84*a^3*b^{10}*x^{12} + 126*a^4*b^9*x^{10} + 126*a^5*b^8*x^8 + 84*a^6*b^7*x^6 + 36*a^7*b^6*x^4 + 9*a^8*b^5*x^2 + a^9*b^4)$

**Sympy [A]** time = 33.9324, size = 143, normalized size = 1.99

$$\frac{a^3 + 9a^2bx^2 + 36ab^2x^4 + 84b^3x^6}{1008a^9b^4 + 9072a^8b^5x^2 + 36288a^7b^6x^4 + 84672a^6b^7x^6 + 127008a^5b^8x^8 + 127008a^4b^9x^{10} + 84672a^3b^{10}x^{12} + 36288a^2b^{11}x^{14} + 1008b^{13}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7/(b*x**2+a)**10,x)`

[Out]  $-(a^{**3} + 9*a^{**2}*b*x^{**2} + 36*a*b^{**2}*x^{**4} + 84*b^{**3}*x^{**6})/(1008*a^{**9}*b^{**4} + 9072*a^{**8}*b^{**5}*x^{**2} + 36288*a^{**7}*b^{**6}*x^{**4} + 84672*a^{**6}*b^{**7}*x^{**6} + 127008*a^{**5}*b^{**8}*x^{**8} + 127008*a^{**4}*b^{**9}*x^{**10} + 84672*a^{**3}*b^{**10}*x^{**12} + 36288*a^{**2}*b^{**11}*x^{**14} + 9072*a*b^{**12}*x^{**16} + 1008*b^{**13}*x^{**18})$

**GIAC/XCAS [A]** time = 0.209534, size = 59, normalized size = 0.82

$$\frac{84b^3x^6 + 36ab^2x^4 + 9a^2bx^2 + a^3}{1008(bx^2 + a)^9b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^2 + a)^10,x, algorithm="giac")`

[Out]  $-1/1008 * (84 * b^3 * x^6 + 36 * a * b^2 * x^4 + 9 * a^2 * b * x^2 + a^3) / ((b * x^2 + a)^9 * b^4)$

$$3.202 \quad \int \frac{x^5}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=53

$$-\frac{a^2}{18b^3(a+bx^2)^9} + \frac{a}{8b^3(a+bx^2)^8} - \frac{1}{14b^3(a+bx^2)^7}$$

[Out]  $-a^2/(18*b^3*(a + b*x^2)^9) + a/(8*b^3*(a + b*x^2)^8) - 1/(14*b^3*(a + b*x^2)^7)$

**Rubi [A]** time = 0.100755, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^2}{18b^3(a+bx^2)^9} + \frac{a}{8b^3(a+bx^2)^8} - \frac{1}{14b^3(a+bx^2)^7}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a + b\*x^2)^10, x]

[Out]  $-a^2/(18*b^3*(a + b*x^2)^9) + a/(8*b^3*(a + b*x^2)^8) - 1/(14*b^3*(a + b*x^2)^7)$

**Rubi in Sympy [A]** time = 14.7641, size = 46, normalized size = 0.87

$$-\frac{a^2}{18b^3(a+bx^2)^9} + \frac{a}{8b^3(a+bx^2)^8} - \frac{1}{14b^3(a+bx^2)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5/(b\*x\*\*2+a)\*\*10, x)

[Out]  $-a**2/(18*b**3*(a + b*x**2)**9) + a/(8*b**3*(a + b*x**2)**8) - 1/(14*b**3*(a + b*x**2)**7)$

**Mathematica [A]** time = 0.0240419, size = 35, normalized size = 0.66

$$-\frac{a^2 + 9abx^2 + 36b^2x^4}{504b^3(a+bx^2)^9}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a + b\*x^2)^10, x]

[Out]  $-(a^2 + 9*a*b*x^2 + 36*b^2*x^4)/(504*b^3*(a + b*x^2)^9)$

**Maple [A]** time = 0.012, size = 48, normalized size = 0.9

$$-\frac{a^2}{18b^3(bx^2+a)^9} + \frac{a}{8b^3(bx^2+a)^8} - \frac{1}{14b^3(bx^2+a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(b*x^2+a)^10,x)`

[Out]  $-1/18*a^2/b^3/(b*x^2+a)^9+1/8*a/b^3/(b*x^2+a)^8-1/14/b^3/(b*x^2+a)^7$

**Maxima [A]** time = 1.38605, size = 167, normalized size = 3.15

$$\frac{36b^2x^4 + 9abx^2 + a^2}{504(b^{12}x^{18} + 9ab^{11}x^{16} + 36a^2b^{10}x^{14} + 84a^3b^9x^{12} + 126a^4b^8x^{10} + 126a^5b^7x^8 + 84a^6b^6x^6 + 36a^7b^5x^4 + 9a^8b^4x^2 + a^9b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2 + a)^10,x, algorithm="maxima")`

[Out]  $-1/504*(36*b^2*x^4 + 9*a*b*x^2 + a^2)/(b^{12}*x^{18} + 9*a*b^{11}*x^{16} + 36*a^2*b^{10}*x^{14} + 84*a^3*b^9*x^{12} + 126*a^4*b^8*x^{10} + 126*a^5*b^7*x^8 + 84*a^6*b^6*x^6 + 36*a^7*b^5*x^4 + 9*a^8*b^4*x^2 + a^9*b^3)$

**Fricas [A]** time = 0.204608, size = 167, normalized size = 3.15

$$\frac{36b^2x^4 + 9abx^2 + a^2}{504(b^{12}x^{18} + 9ab^{11}x^{16} + 36a^2b^{10}x^{14} + 84a^3b^9x^{12} + 126a^4b^8x^{10} + 126a^5b^7x^8 + 84a^6b^6x^6 + 36a^7b^5x^4 + 9a^8b^4x^2 + a^9b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2 + a)^10,x, algorithm="fricas")`

[Out]  $-1/504*(36*b^2*x^4 + 9*a*b*x^2 + a^2)/(b^{12}*x^{18} + 9*a*b^{11}*x^{16} + 36*a^2*b^{10}*x^{14} + 84*a^3*b^9*x^{12} + 126*a^4*b^8*x^{10} + 126*a^5*b^7*x^8 + 84*a^6*b^6*x^6 + 36*a^7*b^5*x^4 + 9*a^8*b^4*x^2 + a^9*b^3)$

**Sympy [A]** time = 33.5177, size = 131, normalized size = 2.47

$$\frac{a^2 + 9abx^2 + 36b^2x^4}{504a^9b^3 + 4536a^8b^4x^2 + 18144a^7b^5x^4 + 42336a^6b^6x^6 + 63504a^5b^7x^8 + 63504a^4b^8x^{10} + 42336a^3b^9x^{12} + 18144a^2b^{10}x^{14} + 4536ab^{11}x^{16} + 504b^{12}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(b*x**2+a)**10,x)`

[Out]  $-(a**2 + 9*a*b*x**2 + 36*b**2*x**4)/(504*a**9*b**3 + 4536*a**8*b**4*x**2 + 18144*a**7*b**5*x**4 + 42336*a**6*b**6*x**6 + 63504*a**5*b**7*x**8 + 63504*a**4*b**8*x**10 + 42336*a**3*b**9*x**12 + 18144*a**2*b**10*x**14 + 4536*a*b**11*x**16 + 504*b**12*x**18)$

**GIAC/XCAS [A]** time = 0.225358, size = 45, normalized size = 0.85

$$\frac{36b^2x^4 + 9abx^2 + a^2}{504(bx^2 + a)^9b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2 + a)^10,x, algorithm="giac")`

[Out]  $-1/504*(36*b^2*x^4 + 9*a*b*x^2 + a^2)/((b*x^2 + a)^9*b^3)$



$$3.203 \quad \int \frac{x^3}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=34

$$\frac{a}{18b^2(a+bx^2)^9} - \frac{1}{16b^2(a+bx^2)^8}$$

[Out]  $a/(18*b^2*(a + b*x^2)^9) - 1/(16*b^2*(a + b*x^2)^8)$

**Rubi [A]** time = 0.0660467, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a}{18b^2(a+bx^2)^9} - \frac{1}{16b^2(a+bx^2)^8}$$

Antiderivative was successfully verified.

[In] `Int[x^3/(a + b*x^2)^10, x]`

[Out]  $a/(18*b^2*(a + b*x^2)^9) - 1/(16*b^2*(a + b*x^2)^8)$

**Rubi in Sympy [A]** time = 11.0167, size = 29, normalized size = 0.85

$$\frac{a}{18b^2(a+bx^2)^9} - \frac{1}{16b^2(a+bx^2)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3/(b*x**2+a)**10, x)`

[Out]  $a/(18*b**2*(a + b*x**2)**9) - 1/(16*b**2*(a + b*x**2)**8)$

**Mathematica [A]** time = 0.0136156, size = 24, normalized size = 0.71

$$-\frac{a+9bx^2}{144b^2(a+bx^2)^9}$$

Antiderivative was successfully verified.

[In] `Integrate[x^3/(a + b*x^2)^10, x]`

[Out]  $-(a + 9*b*x^2)/(144*b^2*(a + b*x^2)^9)$

**Maple [A]** time = 0.01, size = 31, normalized size = 0.9

$$\frac{a}{18b^2(bx^2+a)^9} - \frac{1}{16b^2(bx^2+a)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(b*x^2+a)^10, x)`

[Out]  $1/18*a/b^2/(b*x^2+a)^9 - 1/16/b^2/(b*x^2+a)^8$

**Maxima [A]** time = 1.34846, size = 153, normalized size = 4.5

$$\frac{9bx^2 + a}{144(b^{11}x^{18} + 9ab^{10}x^{16} + 36a^2b^9x^{14} + 84a^3b^8x^{12} + 126a^4b^7x^{10} + 126a^5b^6x^8 + 84a^6b^5x^6 + 36a^7b^4x^4 + 9a^8b^3x^2 + a^9b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b\*x^2 + a)^10,x, algorithm="maxima")

[Out] -1/144\*(9\*b\*x^2 + a)/(b^11\*x^18 + 9\*a\*b^10\*x^16 + 36\*a^2\*b^9\*x^14 + 84\*a^3\*b^8\*x^12 + 126\*a^4\*b^7\*x^10 + 126\*a^5\*b^6\*x^8 + 84\*a^6\*b^5\*x^6 + 36\*a^7\*b^4\*x^4 + 9\*a^8\*b^3\*x^2 + a^9\*b^2)

**Fricas [A]** time = 0.201802, size = 153, normalized size = 4.5

$$\frac{9bx^2 + a}{144(b^{11}x^{18} + 9ab^{10}x^{16} + 36a^2b^9x^{14} + 84a^3b^8x^{12} + 126a^4b^7x^{10} + 126a^5b^6x^8 + 84a^6b^5x^6 + 36a^7b^4x^4 + 9a^8b^3x^2 + a^9b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b\*x^2 + a)^10,x, algorithm="fricas")

[Out] -1/144\*(9\*b\*x^2 + a)/(b^11\*x^18 + 9\*a\*b^10\*x^16 + 36\*a^2\*b^9\*x^14 + 84\*a^3\*b^8\*x^12 + 126\*a^4\*b^7\*x^10 + 126\*a^5\*b^6\*x^8 + 84\*a^6\*b^5\*x^6 + 36\*a^7\*b^4\*x^4 + 9\*a^8\*b^3\*x^2 + a^9\*b^2)

**Sympy [A]** time = 33.2872, size = 119, normalized size = 3.5

$$\frac{a + 9bx^2}{144a^9b^2 + 1296a^8b^3x^2 + 5184a^7b^4x^4 + 12096a^6b^5x^6 + 18144a^5b^6x^8 + 18144a^4b^7x^{10} + 12096a^3b^8x^{12} + 5184a^2b^9x^{14} + 1296ab^{10}x^{16} + 144b^{11}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(b\*x\*\*2+a)\*\*10,x)

[Out] -(a + 9\*b\*x\*\*2)/(144\*a\*\*9\*b\*\*2 + 1296\*a\*\*8\*b\*\*3\*x\*\*2 + 5184\*a\*\*7\*b\*\*4\*x\*\*4 + 12096\*a\*\*6\*b\*\*5\*x\*\*6 + 18144\*a\*\*5\*b\*\*6\*x\*\*8 + 18144\*a\*\*4\*b\*\*7\*x\*\*10 + 12096\*a\*\*3\*b\*\*8\*x\*\*12 + 5184\*a\*\*2\*b\*\*9\*x\*\*14 + 1296\*a\*b\*\*10\*x\*\*16 + 144\*b\*\*11\*x\*\*18)

**GIAC/XCAS [A]** time = 0.223859, size = 30, normalized size = 0.88

$$\frac{9bx^2 + a}{144(bx^2 + a)^9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b\*x^2 + a)^10,x, algorithm="giac")

[Out] -1/144\*(9\*b\*x^2 + a)/((b\*x^2 + a)^9\*b^2)

$$3.204 \quad \int \frac{x}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=16

$$-\frac{1}{18b(a+bx^2)^9}$$

[Out] -1/(18\*b\*(a + b\*x^2)^9)

**Rubi [A]** time = 0.010665, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{1}{18b(a+bx^2)^9}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*x^2)^10, x]

[Out] -1/(18\*b\*(a + b\*x^2)^9)

**Rubi in Sympy [A]** time = 2.23328, size = 14, normalized size = 0.88

$$-\frac{1}{18b(a+bx^2)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(b\*x\*\*2+a)\*\*10, x)

[Out] -1/(18\*b\*(a + b\*x\*\*2)\*\*9)

**Mathematica [A]** time = 0.00476551, size = 16, normalized size = 1.

$$-\frac{1}{18b(a+bx^2)^9}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*x^2)^10, x]

[Out] -1/(18\*b\*(a + b\*x^2)^9)

**Maple [A]** time = 0., size = 15, normalized size = 0.9

$$-\frac{1}{18b(bx^2+a)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b\*x^2+a)^10, x)

[Out] -1/18/b/(b\*x^2+a)^9

---

**Maxima [A]** time = 1.3297, size = 19, normalized size = 1.19

$$-\frac{1}{18(bx^2 + a)^9 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b\*x^2 + a)^10,x, algorithm="maxima")

[Out] -1/18/((b\*x^2 + a)^9\*b)

---

**Fricas [A]** time = 0.195207, size = 139, normalized size = 8.69

$$-\frac{1}{18(b^{10}x^{18} + 9ab^9x^{16} + 36a^2b^8x^{14} + 84a^3b^7x^{12} + 126a^4b^6x^{10} + 126a^5b^5x^8 + 84a^6b^4x^6 + 36a^7b^3x^4 + 9a^8b^2x^2 + a^9b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b\*x^2 + a)^10,x, algorithm="fricas")

[Out] -1/18/(b^10\*x^18 + 9\*a\*b^9\*x^16 + 36\*a^2\*b^8\*x^14 + 84\*a^3\*b^7\*x^12 + 126\*a^4\*b^6\*x^10 + 126\*a^5\*b^5\*x^8 + 84\*a^6\*b^4\*x^6 + 36\*a^7\*b^3\*x^4 + 9\*a^8\*b^2\*x^2 + a^9\*b)

---

**Sympy [A]** time = 33.1417, size = 110, normalized size = 6.88

$$-\frac{1}{18a^9b + 162a^8b^2x^2 + 648a^7b^3x^4 + 1512a^6b^4x^6 + 2268a^5b^5x^8 + 2268a^4b^6x^{10} + 1512a^3b^7x^{12} + 648a^2b^8x^{14} + 162ab^9x^{16} + 18b^{10}x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b\*x\*\*2+a)\*\*10,x)

[Out] -1/(18\*a\*\*9\*b + 162\*a\*\*8\*b\*\*2\*x\*\*2 + 648\*a\*\*7\*b\*\*3\*x\*\*4 + 1512\*a\*\*6\*b\*\*4\*x\*\*6 + 2268\*a\*\*5\*b\*\*5\*x\*\*8 + 2268\*a\*\*4\*b\*\*6\*x\*\*10 + 1512\*a\*\*3\*b\*\*7\*x\*\*12 + 648\*a\*\*2\*b\*\*8\*x\*\*14 + 162\*a\*b\*\*9\*x\*\*16 + 18\*b\*\*10\*x\*\*18)

---

**GIAC/XCAS [A]** time = 0.21295, size = 19, normalized size = 1.19

$$-\frac{1}{18(bx^2 + a)^9 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b\*x^2 + a)^10,x, algorithm="giac")

[Out] -1/18/((b\*x^2 + a)^9\*b)

$$3.205 \quad \int \frac{1}{x(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=166

$$\begin{aligned} & -\frac{\log(a+bx^2)}{2a^{10}} + \frac{\log(x)}{a^{10}} + \frac{1}{2a^9(a+bx^2)} + \frac{1}{4a^8(a+bx^2)^2} + \frac{1}{6a^7(a+bx^2)^3} + \frac{1}{8a^6(a+bx^2)^4} \\ & + \frac{1}{10a^5(a+bx^2)^5} + \frac{1}{12a^4(a+bx^2)^6} + \frac{1}{14a^3(a+bx^2)^7} + \frac{1}{16a^2(a+bx^2)^8} + \frac{1}{18a(a+bx^2)^9} \end{aligned}$$

[Out]  $1/(18*a*(a+b*x^2)^9) + 1/(16*a^2*(a+b*x^2)^8) + 1/(14*a^3*(a+b*x^2)^7) + 1/(12*a^4*(a+b*x^2)^6) + 1/(10*a^5*(a+b*x^2)^5) + 1/(8*a^6*(a+b*x^2)^4) + 1/(6*a^7*(a+b*x^2)^3) + 1/(4*a^8*(a+b*x^2)^2) + 1/(2*a^9*(a+b*x^2)) + \text{Log}[x]/a^{10} - \text{Log}[a+b*x^2]/(2*a^{10})$

**Rubi [A]** time = 0.279101, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\begin{aligned} & -\frac{\log(a+bx^2)}{2a^{10}} + \frac{\log(x)}{a^{10}} + \frac{1}{2a^9(a+bx^2)} + \frac{1}{4a^8(a+bx^2)^2} + \frac{1}{6a^7(a+bx^2)^3} + \frac{1}{8a^6(a+bx^2)^4} \\ & + \frac{1}{10a^5(a+bx^2)^5} + \frac{1}{12a^4(a+bx^2)^6} + \frac{1}{14a^3(a+bx^2)^7} + \frac{1}{16a^2(a+bx^2)^8} + \frac{1}{18a(a+bx^2)^9} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(a+b\*x^2)^10),x]

[Out]  $1/(18*a*(a+b*x^2)^9) + 1/(16*a^2*(a+b*x^2)^8) + 1/(14*a^3*(a+b*x^2)^7) + 1/(12*a^4*(a+b*x^2)^6) + 1/(10*a^5*(a+b*x^2)^5) + 1/(8*a^6*(a+b*x^2)^4) + 1/(6*a^7*(a+b*x^2)^3) + 1/(4*a^8*(a+b*x^2)^2) + 1/(2*a^9*(a+b*x^2)) + \text{Log}[x]/a^{10} - \text{Log}[a+b*x^2]/(2*a^{10})$

**Rubi in Sympy [A]** time = 56.3253, size = 156, normalized size = 0.94

$$\begin{aligned} & \frac{1}{18a(a+bx^2)^9} + \frac{1}{16a^2(a+bx^2)^8} + \frac{1}{14a^3(a+bx^2)^7} + \frac{1}{12a^4(a+bx^2)^6} + \frac{1}{10a^5(a+bx^2)^5} \\ & + \frac{1}{8a^6(a+bx^2)^4} + \frac{1}{6a^7(a+bx^2)^3} + \frac{1}{4a^8(a+bx^2)^2} + \frac{1}{2a^9(a+bx^2)} + \frac{\log(x^2)}{2a^{10}} - \frac{\log(a+bx^2)}{2a^{10}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(b\*x\*\*2+a)\*\*10,x)

[Out]  $1/(18*a*(a+b*x**2)**9) + 1/(16*a**2*(a+b*x**2)**8) + 1/(14*a**3*(a+b*x**2)**7) + 1/(12*a**4*(a+b*x**2)**6) + 1/(10*a**5*(a+b*x**2)**5) + 1/(8*a**6*(a+b*x**2)**4) + 1/(6*a**7*(a+b*x**2)**3) + 1/(4*a**8*(a+b*x**2)**2) + 1/(2*a**9*(a+b*x**2)) + \log(x**2)/(2*a**10) - \log(a+b*x**2)/(2*a**10)$

**Mathematica [A]** time = 0.154521, size = 120, normalized size = 0.72

$$\frac{a(7129a^8+41481a^7bx^2+120564a^6b^2x^4+210756a^5b^3x^6+236754a^4b^4x^8+173250a^3b^5x^{10}+80220a^2b^6x^{12}+21420ab^7x^{14}+2520b^8x^{16})}{(a+bx^2)^9} - 2520 \log(a+bx^2) + 5040a^{10}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(a + b\*x^2)^10),x]

[Out] ((a\*(7129\*a^8 + 41481\*a^7\*b\*x^2 + 120564\*a^6\*b^2\*x^4 + 210756\*a^5\*b^3\*x^6 + 236754\*a^4\*b^4\*x^8 + 173250\*a^3\*b^5\*x^10 + 80220\*a^2\*b^6\*x^12 + 21420\*a\*b^7\*x^14 + 2520\*b^8\*x^16))/(a + b\*x^2)^9 + 5040\*Log[x] - 2520\*Log[a + b\*x^2])/(5040\*a^10)

**Maple [A]** time = 0.026, size = 147, normalized size = 0.9

$$\frac{1}{18 a (b x^2 + a)^9} + \frac{1}{16 a^2 (b x^2 + a)^8} + \frac{1}{14 a^3 (b x^2 + a)^7} + \frac{1}{12 a^4 (b x^2 + a)^6} + \frac{1}{10 a^5 (b x^2 + a)^5} + \frac{1}{8 a^6 (b x^2 + a)^4} + \frac{1}{6 a^7 (b x^2 + a)^3} + \frac{1}{4 a^8 (b x^2 + a)^2} + \frac{1}{2 a^9 (b x^2 + a)} + \frac{\ln(x)}{a^{10}} - \frac{\ln(b x^2 + a)}{2 a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b\*x^2+a)^10,x)

[Out] 1/18/a/(b\*x^2+a)^9+1/16/a^2/(b\*x^2+a)^8+1/14/a^3/(b\*x^2+a)^7+1/12/a^4/(b\*x^2+a)^6+1/10/a^5/(b\*x^2+a)^5+1/8/a^6/(b\*x^2+a)^4+1/6/a^7/(b\*x^2+a)^3+1/4/a^8/(b\*x^2+a)^2+1/2/a^9/(b\*x^2+a)+ln(x)/a^10-1/2\*ln(b\*x^2+a)/a^10

**Maxima [A]** time = 1.37792, size = 289, normalized size = 1.74

$$\frac{2520 b^8 x^{16} + 21420 a b^7 x^{14} + 80220 a^2 b^6 x^{12} + 173250 a^3 b^5 x^{10} + 236754 a^4 b^4 x^8 + 210756 a^5 b^3 x^6 + 120564 a^6 b^2 x^4 + 41481 a^7 b x^2 + 2520 b^8 x^{16}}{5040 (a^9 b^9 x^{18} + 9 a^{10} b^8 x^{16} + 36 a^{11} b^7 x^{14} + 84 a^{12} b^6 x^{12} + 126 a^{13} b^5 x^{10} + 126 a^{14} b^4 x^8 + 84 a^{15} b^3 x^6 + 36 a^{16} b^2 x^4 + 9 a^{17} b x^2 + a^{18})} - \frac{\log(b x^2 + a)}{2 a^{10}} + \frac{\log(x^2)}{2 a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^10\*x),x, algorithm="maxima")

[Out] 1/5040\*(2520\*b^8\*x^16 + 21420\*a\*b^7\*x^14 + 80220\*a^2\*b^6\*x^12 + 173250\*a^3\*b^5\*x^10 + 236754\*a^4\*b^4\*x^8 + 210756\*a^5\*b^3\*x^6 + 120564\*a^6\*b^2\*x^4 + 41481\*a^7\*b\*x^2 + 2520\*b^8\*x^16)/(a^9\*b^9\*x^18 + 9\*a^10\*b^8\*x^16 + 36\*a^11\*b^7\*x^14 + 84\*a^12\*b^6\*x^12 + 126\*a^13\*b^5\*x^10 + 126\*a^14\*b^4\*x^8 + 84\*a^15\*b^3\*x^6 + 36\*a^16\*b^2\*x^4 + 9\*a^17\*b\*x^2 + a^18) - 1/2\*log(b\*x^2 + a)/a^10 + 1/2\*log(x^2)/a^10

**Fricas [A]** time = 0.220868, size = 537, normalized size = 3.23

$$\frac{2520 a b^8 x^{16} + 21420 a^2 b^7 x^{14} + 80220 a^3 b^6 x^{12} + 173250 a^4 b^5 x^{10} + 236754 a^5 b^4 x^8 + 210756 a^6 b^3 x^6 + 120564 a^7 b^2 x^4 + 41481 a^8 b x^2 + 2520 b^8 x^{16}}{5040 (a^9 b^9 x^{18} + 9 a^{10} b^8 x^{16} + 36 a^{11} b^7 x^{14} + 84 a^{12} b^6 x^{12} + 126 a^{13} b^5 x^{10} + 126 a^{14} b^4 x^8 + 84 a^{15} b^3 x^6 + 36 a^{16} b^2 x^4 + 9 a^{17} b x^2 + a^{18})} - \frac{\log(b x^2 + a)}{2 a^{10}} + \frac{\log(x^2)}{2 a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^10\*x),x, algorithm="fricas")

[Out] 1/5040\*(2520\*a\*b^8\*x^16 + 21420\*a^2\*b^7\*x^14 + 80220\*a^3\*b^6\*x^12 + 173250\*a^4\*b^5\*x^10 + 236754\*a^5\*b^4\*x^8 + 210756\*a^6\*b^3\*x^6 + 120564\*a^7\*b^2\*x^4 + 41481\*a^8\*b\*x^2 + 2520\*b^8\*x^16)/(a^9\*b^9\*x^18 + 9\*a^10\*b^8\*x^16 + 36\*a^11\*b^7\*x^14 + 84\*a^12\*b^6\*x^12 + 126\*a^13\*b^5\*x^10 + 126\*a^14\*b^4\*x^8 + 84\*a^15\*b^3\*x^6 + 36\*a^16\*b^2\*x^4 + 9\*a^17\*b\*x^2 + a^18) - 1/2\*log(b\*x^2 + a)/a^10 + 1/2\*log(x^2)/a^10

$$\left. \right) / (a^{10} b^9 x^{18} + 9 a^{11} b^8 x^{16} + 36 a^{12} b^7 x^{14} + 84 a^{13} b^6 x^{12} + 126 a^{14} b^5 x^{10} + 126 a^{15} b^4 x^8 + 84 a^{16} b^3 x^6 + 36 a^{17} b^2 x^4 + 9 a^{18} b x^2 + a^{19})$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b\*x\*\*2+a)\*\*10,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.214037, size = 184, normalized size = 1.11

$$\frac{\ln(x^2)}{2a^{10}} - \frac{\ln(|bx^2 + a|)}{2a^{10}} + \frac{7129b^9x^{18} + 66681ab^8x^{16} + 278064a^2b^7x^{14} + 679056a^3b^6x^{12} + 1071504a^4b^5x^{10} + 1135008a^5b^4x^8 + 809592a^6b^3x^6 + 377208a^7b^2x^4 + 105642a^8bx^2 + 14258a^9}{5040(bx^2 + a)^9a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^10\*x),x, algorithm="giac")

[Out] 1/2\*ln(x^2)/a^10 - 1/2\*ln(abs(b\*x^2 + a))/a^10 + 1/5040\*(7129\*b^9\*x^18 + 66681\*a\*b^8\*x^16 + 278064\*a^2\*b^7\*x^14 + 679056\*a^3\*b^6\*x^12 + 1071504\*a^4\*b^5\*x^10 + 1135008\*a^5\*b^4\*x^8 + 809592\*a^6\*b^3\*x^6 + 377208\*a^7\*b^2\*x^4 + 105642\*a^8\*b\*x^2 + 14258\*a^9)/((b\*x^2 + a)^9\*a^10)

$$3.206 \quad \int \frac{1}{x^3(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=184

$$\frac{5b \log(a+bx^2)}{a^{11}} - \frac{10b \log(x)}{a^{11}} - \frac{9b}{2a^{10}(a+bx^2)} - \frac{1}{2a^{10}x^2} - \frac{2b}{a^9(a+bx^2)^2} - \frac{7b}{6a^8(a+bx^2)^3} - \frac{3b}{4a^7(a+bx^2)^4} - \frac{b}{2a^6(a+bx^2)^5} - \frac{b}{3a^5(a+bx^2)^6} - \frac{3b}{14a^4(a+bx^2)^7} - \frac{b}{8a^3(a+bx^2)^8} - \frac{b}{18a^2(a+bx^2)^9}$$

[Out]  $-1/(2*a^{10}*x^2) - b/(18*a^2*(a + b*x^2)^9) - b/(8*a^3*(a + b*x^2)^8) - (3*b)/(14*a^4*(a + b*x^2)^7) - b/(3*a^5*(a + b*x^2)^6) - b/(2*a^6*(a + b*x^2)^5) - (3*b)/(4*a^7*(a + b*x^2)^4) - (7*b)/(6*a^8*(a + b*x^2)^3) - (2*b)/(a^9*(a + b*x^2)^2) - (9*b)/(2*a^{10}*(a + b*x^2)) - (10*b*Log[x])/a^{11} + (5*b*Log[a + b*x^2])/a^{11}$

**Rubi [A]** time = 0.411028, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{5b \log(a+bx^2)}{a^{11}} - \frac{10b \log(x)}{a^{11}} - \frac{9b}{2a^{10}(a+bx^2)} - \frac{1}{2a^{10}x^2} - \frac{2b}{a^9(a+bx^2)^2} - \frac{7b}{6a^8(a+bx^2)^3} - \frac{3b}{4a^7(a+bx^2)^4} - \frac{b}{2a^6(a+bx^2)^5} - \frac{b}{3a^5(a+bx^2)^6} - \frac{3b}{14a^4(a+bx^2)^7} - \frac{b}{8a^3(a+bx^2)^8} - \frac{b}{18a^2(a+bx^2)^9}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(a + b\*x^2)^10), x]

[Out]  $-1/(2*a^{10}*x^2) - b/(18*a^2*(a + b*x^2)^9) - b/(8*a^3*(a + b*x^2)^8) - (3*b)/(14*a^4*(a + b*x^2)^7) - b/(3*a^5*(a + b*x^2)^6) - b/(2*a^6*(a + b*x^2)^5) - (3*b)/(4*a^7*(a + b*x^2)^4) - (7*b)/(6*a^8*(a + b*x^2)^3) - (2*b)/(a^9*(a + b*x^2)^2) - (9*b)/(2*a^{10}*(a + b*x^2)) - (10*b*Log[x])/a^{11} + (5*b*Log[a + b*x^2])/a^{11}$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(b\*x\*\*2+a)\*\*10, x)

[Out] Timed out

**Mathematica [A]** time = 0.242227, size = 136, normalized size = 0.74

$$\frac{a(252a^9+7129a^8bx^2+41481a^7b^2x^4+120564a^6b^3x^6+210756a^5b^4x^8+236754a^4b^5x^{10}+173250a^3b^6x^{12}+80220a^2b^7x^{14}+21420ab^8x^{16}+2520b^9x^{18})}{x^2(a+bx^2)^9} - 2520b \log$$

$$504a^{11}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a + b\*x^2)^10), x]

[Out]  $-((a*(252*a^9 + 7129*a^8*b*x^2 + 41481*a^7*b^2*x^4 + 120564*a^6*b^3*x^6 + 210756*a^5*b^4*x^8 + 236754*a^4*b^5*x^{10} + 173250*a^3*b^6*x^{12} + 80220*a^2*b^7*x^{14} + 21420*a*b^8*x^{16} + 2520*b^9*x^{18}))/$



$$(x^2*(a + b*x^2)^9) + 5040*b*Log[x] - 2520*b*Log[a + b*x^2])/(504*a^11)$$

**Maple [A]** time = 0.029, size = 167, normalized size = 0.9

$$\begin{aligned} & -\frac{1}{2a^{10}x^2} - \frac{b}{18a^2(bx^2+a)^9} - \frac{b}{8a^3(bx^2+a)^8} - \frac{3b}{14a^4(bx^2+a)^7} - \frac{b}{3a^5(bx^2+a)^6} - \frac{b}{2a^6(bx^2+a)^5} \\ & - \frac{3b}{4a^7(bx^2+a)^4} - \frac{7b}{6a^8(bx^2+a)^3} - 2\frac{b}{a^9(bx^2+a)^2} - \frac{9b}{2a^{10}(bx^2+a)} - 10\frac{b\ln(x)}{a^{11}} + 5\frac{b\ln(bx^2+a)}{a^{11}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b\*x^2+a)^10,x)

[Out] -1/2/a^10/x^2-1/18\*b/a^2/(b\*x^2+a)^9-1/8\*b/a^3/(b\*x^2+a)^8-3/14\*b/a^4/(b\*x^2+a)^7-1/3\*b/a^5/(b\*x^2+a)^6-1/2\*b/a^6/(b\*x^2+a)^5-3/4\*b/a^7/(b\*x^2+a)^4-7/6\*b/a^8/(b\*x^2+a)^3-2\*b/a^9/(b\*x^2+a)^2-9/2\*b/a^10/(b\*x^2+a)-10\*b\*ln(x)/a^11+5\*b\*ln(b\*x^2+a)/a^11

**Maxima [A]** time = 1.3957, size = 312, normalized size = 1.7

$$\begin{aligned} & \frac{2520b^9x^{18} + 21420ab^8x^{16} + 80220a^2b^7x^{14} + 173250a^3b^6x^{12} + 236754a^4b^5x^{10} + 210756a^5b^4x^8 + 120564a^6b^3x^6 + 41481a^7b^2x^4 + 7129a^8b^2x^4 + 252a^9}{504(a^{10}b^9x^{20} + 9a^{11}b^8x^{18} + 36a^{12}b^7x^{16} + 84a^{13}b^6x^{14} + 126a^{14}b^5x^{12} + 126a^{15}b^4x^{10} + 84a^{16}b^3x^8 + 36a^{17}b^2x^6 + 12a^{18}bx^4 + 6a^{19})} \\ & + \frac{5b\log(bx^2+a)}{a^{11}} - \frac{5b\log(x^2)}{a^{11}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^10\*x^3),x, algorithm="maxima")

[Out] -1/504\*(2520\*b^9\*x^18 + 21420\*a\*b^8\*x^16 + 80220\*a^2\*b^7\*x^14 + 173250\*a^3\*b^6\*x^12 + 236754\*a^4\*b^5\*x^10 + 210756\*a^5\*b^4\*x^8 + 120564\*a^6\*b^3\*x^6 + 41481\*a^7\*b^2\*x^4 + 7129\*a^8\*b^2\*x^4 + 252\*a^9)/(a^10\*b^9\*x^20 + 9\*a^11\*b^8\*x^18 + 36\*a^12\*b^7\*x^16 + 84\*a^13\*b^6\*x^14 + 126\*a^14\*b^5\*x^12 + 126\*a^15\*b^4\*x^10 + 84\*a^16\*b^3\*x^8 + 36\*a^17\*b^2\*x^6 + 12\*a^18\*b\*x^4 + a^19\*x^2) + 5\*b\*log(b\*x^2 + a)/a^11 - 5\*b\*log(x^2)/a^11

**Fricas [A]** time = 0.240893, size = 576, normalized size = 3.13

$$\frac{2520ab^9x^{18} + 21420a^2b^8x^{16} + 80220a^3b^7x^{14} + 173250a^4b^6x^{12} + 236754a^5b^5x^{10} + 210756a^6b^4x^8 + 120564a^7b^3x^6 + 41481a^8b^2x^4 + 7129a^9b^2x^4 + 252a^{10}}{504(a^{10}b^9x^{20} + 9a^{11}b^8x^{18} + 36a^{12}b^7x^{16} + 84a^{13}b^6x^{14} + 126a^{14}b^5x^{12} + 126a^{15}b^4x^{10} + 84a^{16}b^3x^8 + 36a^{17}b^2x^6 + 12a^{18}bx^4 + 6a^{19})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^10\*x^3),x, algorithm="fricas")

[Out] -1/504\*(2520\*a\*b^9\*x^18 + 21420\*a^2\*b^8\*x^16 + 80220\*a^3\*b^7\*x^14 + 173250\*a^4\*b^6\*x^12 + 236754\*a^5\*b^5\*x^10 + 210756\*a^6\*b^4\*x^8 + 120564\*a^7\*b^3\*x^6 + 41481\*a^8\*b^2\*x^4 + 7129\*a^9\*b^2\*x^4 + 252\*a^10 - 2520\*(b^10\*x^20 + 9\*a\*b^9\*x^18 + 36\*a^2\*b^8\*x^16 + 84\*a^3\*b^7\*x^14 + 126\*a^4\*b^6\*x^12 + 126\*a^5\*b^5\*x^10 + 84\*a^6\*b^4\*x^8 + 36\*a^7\*b^3\*x^6 + 9\*a^8\*b^2\*x^4 + a^9\*b\*x^2)\*log(b\*x^2 + a) + 5040\*(b^10\*x^20 + 9\*a\*b^9\*x^18 + 36\*a^2\*b^8\*x^16 + 84\*a^3\*b^7\*x^14 + 126\*a^4\*b^6\*x^12 + 126\*a^5\*b^5\*x^10 + 84\*a^6\*b^4\*x^8 + 36\*a^7\*b^3\*x^6 + 9\*a^8\*b^2\*x^4 + a^9\*b\*x^2)\*log(x))/(a^11\*b^9\*x^20 + 9\*a^12\*b^8\*x^18 + 36\*a^13\*b^7\*x^16 + 84\*a^14\*b^6\*x^14 + 126\*a^15\*b^5\*x^12 + 126\*a^16\*b^4\*x^10 + 84\*a^17\*b^3\*x^8 + 36\*a^18\*b^2\*x^6 + 12\*a^19\*b\*x^4 + a^20)

$$^{19}b \cdot x^4 + a^{20} \cdot x^2)$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(b\*x\*\*2+a)\*\*10,x)

[Out] Timed out

---

**GIAC/XCAS [A]** time = 0.217368, size = 215, normalized size = 1.17

$$\frac{-\frac{5 b \ln(x^2)}{a^{11}} + \frac{5 b \ln(|bx^2 + a|)}{a^{11}} + \frac{10 bx^2 - a}{2 a^{11} x^2}}{7129 b^{10} x^{18} + 66429 ab^9 x^{16} + 275796 a^2 b^8 x^{14} + 669984 a^3 b^7 x^{12} + 1050336 a^4 b^6 x^{10} + 1103256 a^5 b^5 x^8 + 777840 a^6 b^4 x^6 + 356040 a^7 b^3 x^4 + 96570 a^8 b^2 x^2 + 11990 a^9 b} \cdot \frac{1}{504 (bx^2 + a)^9 a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^10\*x^3),x, algorithm="giac")

[Out] -5\*b\*ln(x^2)/a^11 + 5\*b\*ln(abs(b\*x^2 + a))/a^11 + 1/2\*(10\*b\*x^2 - a)/(a^11\*x^2) - 1/504\*(7129\*b^10\*x^18 + 66429\*a\*b^9\*x^16 + 275796\*a^2\*b^8\*x^14 + 669984\*a^3\*b^7\*x^12 + 1050336\*a^4\*b^6\*x^10 + 1103256\*a^5\*b^5\*x^8 + 777840\*a^6\*b^4\*x^6 + 356040\*a^7\*b^3\*x^4 + 96570\*a^8\*b^2\*x^2 + 11990\*a^9\*b)/(b\*x^2 + a)^9\*a^11)

$$3.207 \quad \int \frac{1}{x^5(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=217

$$\begin{aligned} & -\frac{55b^2 \log(a+bx^2)}{2a^{12}} + \frac{55b^2 \log(x)}{a^{12}} + \frac{45b^2}{2a^{11}(a+bx^2)} + \frac{5b}{a^{11}x^2} + \frac{9b^2}{a^{10}(a+bx^2)^2} - \frac{1}{4a^{10}x^4} + \frac{14b^2}{3a^9(a+bx^2)^3} \\ & + \frac{21b^2}{8a^8(a+bx^2)^4} + \frac{3b^2}{2a^7(a+bx^2)^5} + \frac{5b^2}{6a^6(a+bx^2)^6} + \frac{3b^2}{7a^5(a+bx^2)^7} + \frac{3b^2}{16a^4(a+bx^2)^8} + \frac{b^2}{18a^3(a+bx^2)^9} \end{aligned}$$

[Out]  $-1/(4*a^{10}*x^4) + (5*b)/(a^{11}*x^2) + b^2/(18*a^3*(a+b*x^2)^9) + (3*b^2)/(16*a^4*(a+b*x^2)^8) + (3*b^2)/(7*a^5*(a+b*x^2)^7) + (5*b^2)/(6*a^6*(a+b*x^2)^6) + (3*b^2)/(2*a^7*(a+b*x^2)^5) + (21*b^2)/(8*a^8*(a+b*x^2)^4) + (14*b^2)/(3*a^9*(a+b*x^2)^3) + (9*b^2)/(a^{10}*(a+b*x^2)^2) + (45*b^2)/(2*a^{11}*(a+b*x^2)) + (55*b^2*Log[x])/a^{12} - (55*b^2*Log[a+b*x^2])/(2*a^{12})$

**Rubi [A]** time = 0.485105, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\begin{aligned} & -\frac{55b^2 \log(a+bx^2)}{2a^{12}} + \frac{55b^2 \log(x)}{a^{12}} + \frac{45b^2}{2a^{11}(a+bx^2)} + \frac{5b}{a^{11}x^2} + \frac{9b^2}{a^{10}(a+bx^2)^2} - \frac{1}{4a^{10}x^4} + \frac{14b^2}{3a^9(a+bx^2)^3} \\ & + \frac{21b^2}{8a^8(a+bx^2)^4} + \frac{3b^2}{2a^7(a+bx^2)^5} + \frac{5b^2}{6a^6(a+bx^2)^6} + \frac{3b^2}{7a^5(a+bx^2)^7} + \frac{3b^2}{16a^4(a+bx^2)^8} + \frac{b^2}{18a^3(a+bx^2)^9} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5\*(a+b\*x^2)^10),x]

[Out]  $-1/(4*a^{10}*x^4) + (5*b)/(a^{11}*x^2) + b^2/(18*a^3*(a+b*x^2)^9) + (3*b^2)/(16*a^4*(a+b*x^2)^8) + (3*b^2)/(7*a^5*(a+b*x^2)^7) + (5*b^2)/(6*a^6*(a+b*x^2)^6) + (3*b^2)/(2*a^7*(a+b*x^2)^5) + (21*b^2)/(8*a^8*(a+b*x^2)^4) + (14*b^2)/(3*a^9*(a+b*x^2)^3) + (9*b^2)/(a^{10}*(a+b*x^2)^2) + (45*b^2)/(2*a^{11}*(a+b*x^2)) + (55*b^2*Log[x])/a^{12} - (55*b^2*Log[a+b*x^2])/(2*a^{12})$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*5/(b\*x\*\*2+a)\*\*10,x)

[Out] Timed out

**Mathematica [A]** time = 0.18622, size = 151, normalized size = 0.7

$$\frac{a(-252a^{10}+2772a^9bx^2+78419a^8b^2x^4+456291a^7b^3x^6+1326204a^6b^4x^8+2318316a^5b^5x^{10}+2604294a^4b^6x^{12}+1905750a^3b^7x^{14}+882420a^2b^8x^{16}+235620ab^9x^{18}+2772b^{10}x^{20})}{x^4(a+bx^2)^9} 1008a^{12}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5\*(a+b\*x^2)^10),x]

[Out]  $((a*(-252*a^{10} + 2772*a^9*b*x^2 + 78419*a^8*b^2*x^4 + 456291*a^7*b^3*x^6 + 1326204*a^6*b^4*x^8 + 2318316*a^5*b^5*x^{10} + 2604294*a^4*b^6*x^{12} + 1905750*a^3*b^7*x^{14} + 882420*a^2*b^8*x^{16} + 235620*a*b^9*x^{18} + 2772*b^{10}*x^{20}))/x^4(a+bx^2)^9$

$$4*b^6*x^{12} + 1905750*a^3*b^7*x^{14} + 882420*a^2*b^8*x^{16} + 235620*a*b^9*x^{18} + 27720*b^{10}*x^{20})/(x^4*(a + b*x^2)^9) + 55440*b^2*Log[x] - 27720*b^2*Log[a + b*x^2])/(1008*a^{12})$$

**Maple [A]** time = 0.029, size = 198, normalized size = 0.9

$$-\frac{1}{4a^{10}x^4} + \frac{5b}{a^{11}x^2} + \frac{b^2}{18a^3(bx^2+a)^9} + \frac{3b^2}{16a^4(bx^2+a)^8} + \frac{3b^2}{7a^5(bx^2+a)^7} + \frac{5b^2}{6a^6(bx^2+a)^6} + \frac{3b^2}{2a^7(bx^2+a)^5} + \frac{21b^2}{8a^8(bx^2+a)^4} + \frac{14b^2}{3a^9(bx^2+a)^3} + 9\frac{b^2}{a^{10}(bx^2+a)^2} + \frac{45b^2}{2a^{11}(bx^2+a)} + 55\frac{b^2 \ln(x)}{a^{12}} - \frac{55b^2 \ln(bx^2+a)}{2a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(b\*x^2+a)^10,x)

[Out] -1/4/a^10/x^4+5\*b/a^11/x^2+1/18\*b^2/a^3/(b\*x^2+a)^9+3/16\*b^2/a^4/(b\*x^2+a)^8+3/7\*b^2/a^5/(b\*x^2+a)^7+5/6\*b^2/a^6/(b\*x^2+a)^6+3/2\*b^2/a^7/(b\*x^2+a)^5+21/8\*b^2/a^8/(b\*x^2+a)^4+14/3\*b^2/a^9/(b\*x^2+a)^3+9\*b^2/a^10/(b\*x^2+a)^2+45/2\*b^2/a^11/(b\*x^2+a)+55\*b^2\*ln(x)/a^12-55/2\*b^2\*ln(b\*x^2+a)/a^12

**Maxima [A]** time = 1.37607, size = 332, normalized size = 1.53

$$\frac{27720b^{10}x^{20} + 235620ab^9x^{18} + 882420a^2b^8x^{16} + 1905750a^3b^7x^{14} + 2604294a^4b^6x^{12} + 2318316a^5b^5x^{10} + 1326204a^6b^4x^8 + 456291a^7b^3x^6 + 78419a^8b^2x^4 + 2772a^9b^1x^2 - 252a^{10}}{1008(a^{11}b^9x^{22} + 9a^{12}b^8x^{20} + 36a^{13}b^7x^{18} + 84a^{14}b^6x^{16} + 126a^{15}b^5x^{14} + 126a^{16}b^4x^{12} + 84a^{17}b^3x^{10} + 27a^{18}b^2x^8 + 9a^{19}b^1x^2 - 252a^{20})} - \frac{55b^2 \log(bx^2+a)}{2a^{12}} + \frac{55b^2 \log(x^2)}{2a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^10\*x^5),x, algorithm="maxima")

[Out] 1/1008\*(27720\*b^10\*x^20 + 235620\*a\*b^9\*x^18 + 882420\*a^2\*b^8\*x^16 + 1905750\*a^3\*b^7\*x^14 + 2604294\*a^4\*b^6\*x^12 + 2318316\*a^5\*b^5\*x^10 + 1326204\*a^6\*b^4\*x^8 + 456291\*a^7\*b^3\*x^6 + 78419\*a^8\*b^2\*x^4 + 2772\*a^9\*b\*x^2 - 252\*a^10)/(a^11\*b^9\*x^22 + 9\*a^12\*b^8\*x^20 + 36\*a^13\*b^7\*x^18 + 84\*a^14\*b^6\*x^16 + 126\*a^15\*b^5\*x^14 + 126\*a^16\*b^4\*x^12 + 84\*a^17\*b^3\*x^10 + 27\*a^18\*b^2\*x^8 + 9\*a^19\*b\*x^2 + a^20\*x^4) - 55/2\*b^2\*log(b\*x^2 + a)/a^12 + 55/2\*b^2\*log(x^2)/a^12

**Fricas [A]** time = 0.231136, size = 597, normalized size = 2.75

$$27720ab^{10}x^{20} + 235620a^2b^9x^{18} + 882420a^3b^8x^{16} + 1905750a^4b^7x^{14} + 2604294a^5b^6x^{12} + 2318316a^6b^5x^{10} + 1326204a^7b^4x^8 + 456291a^8b^3x^6 + 78419a^9b^2x^4 + 2772a^{10}b^1x^2 - 252a^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^10\*x^5),x, algorithm="fricas")

[Out] 1/1008\*(27720\*a\*b^10\*x^20 + 235620\*a^2\*b^9\*x^18 + 882420\*a^3\*b^8\*x^16 + 1905750\*a^4\*b^7\*x^14 + 2604294\*a^5\*b^6\*x^12 + 2318316\*a^6\*b^5\*x^10 + 1326204\*a^7\*b^4\*x^8 + 456291\*a^8\*b^3\*x^6 + 78419\*a^9\*b^2\*x^4 + 2772\*a^10\*b\*x^2 - 252\*a^11 - 27720\*(b^11\*x^22 + 9\*a\*b^10\*x^20 + 36\*a^2\*b^9\*x^18 + 84\*a^3\*b^8\*x^16 + 126\*a^4\*b^7\*x^14 + 126\*a^5\*b^6\*x^12 + 84\*a^6\*b^5\*x^10 + 36\*a^7\*b^4\*x^8 + 9\*a^8\*b^3\*x^6 + 27\*a^9\*b^2\*x^4 + 9\*a^10\*b\*x^2 - 252\*a^11))

$$\begin{aligned}
& + a^9 b^2 x^4) \log(b x^2 + a) + 55440 (b^{11} x^{22} + 9 a b^{10} x^{20} \\
& + 36 a^2 b^9 x^{18} + 84 a^3 b^8 x^{16} + 126 a^4 b^7 x^{14} + 126 a^5 \\
& * b^6 x^{12} + 84 a^6 b^5 x^{10} + 36 a^7 b^4 x^8 + 9 a^8 b^3 x^6 + a^9 \\
& * b^2 x^4) \log(x) / (a^{12} b^9 x^{22} + 9 a^{13} b^8 x^{20} + 36 a^{14} b^7 \\
& * x^{18} + 84 a^{15} b^6 x^{16} + 126 a^{16} b^5 x^{14} + 126 a^{17} b^4 x^{12} \\
& + 84 a^{18} b^3 x^{10} + 36 a^{19} b^2 x^8 + 9 a^{20} b x^6 + a^{21} x^4)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*5/(b\*x\*\*2+a)\*\*10,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.214434, size = 235, normalized size = 1.08

$$\begin{aligned}
& \frac{55 b^2 \ln(x^2)}{2 a^{12}} - \frac{55 b^2 \ln(|bx^2 + a|)}{2 a^{12}} - \frac{165 b^2 x^4 - 20 abx^2 + a^2}{4 a^{12} x^4} \\
& + \frac{78419 b^{11} x^{18} + 728451 ab^{10} x^{16} + 3013596 a^2 b^9 x^{14} + 7290444 a^3 b^8 x^{12} + 11372256 a^4 b^7 x^{10} + 11871216 a^5 b^6 x^8 + 8302224 a^6 b^5 x^6 + 3757680 a^7 b^4 x^4 + 1001790 a^8 b^3 x^2 + 120550 a^9 b^2}{1008 (bx^2 + a)^9 a^{12}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^10\*x^5),x, algorithm="giac")

[Out] 55/2\*b^2\*ln(x^2)/a^12 - 55/2\*b^2\*ln(abs(b\*x^2 + a))/a^12 - 1/4\*(165\*b^2\*x^4 - 20\*a\*b\*x^2 + a^2)/(a^12\*x^4) + 1/1008\*(78419\*b^11\*x^18 + 728451\*a\*b^10\*x^16 + 3013596\*a^2\*b^9\*x^14 + 7290444\*a^3\*b^8\*x^12 + 11372256\*a^4\*b^7\*x^10 + 11871216\*a^5\*b^6\*x^8 + 8302224\*a^6\*b^5\*x^6 + 3757680\*a^7\*b^4\*x^4 + 1001790\*a^8\*b^3\*x^2 + 120550\*a^9\*b^2)/((b\*x^2 + a)^9\*a^12)

$$3.208 \quad \int \frac{1}{x^7(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=226

$$\begin{aligned} & \frac{110b^3 \log(a+bx^2)}{a^{13}} - \frac{220b^3 \log(x)}{a^{13}} - \frac{165b^3}{2a^{12}(a+bx^2)} - \frac{55b^2}{2a^{12}x^2} - \frac{30b^3}{a^{11}(a+bx^2)^2} \\ & + \frac{5b}{2a^{11}x^4} - \frac{14b^3}{a^{10}(a+bx^2)^3} - \frac{1}{6a^{10}x^6} - \frac{7b^3}{a^9(a+bx^2)^4} - \frac{7b^3}{2a^8(a+bx^2)^5} \\ & - \frac{5b^3}{3a^7(a+bx^2)^6} - \frac{5b^3}{7a^6(a+bx^2)^7} - \frac{b^3}{4a^5(a+bx^2)^8} - \frac{b^3}{18a^4(a+bx^2)^9} \end{aligned}$$

[Out]  $-1/(6*a^{10}*x^6) + (5*b)/(2*a^{11}*x^4) - (55*b^2)/(2*a^{12}*x^2) - b^3/(18*a^4*(a+b*x^2)^9) - b^3/(4*a^5*(a+b*x^2)^8) - (5*b^3)/(7*a^6*(a+b*x^2)^7) - (5*b^3)/(3*a^7*(a+b*x^2)^6) - (7*b^3)/(2*a^8*(a+b*x^2)^5) - (7*b^3)/(a^9*(a+b*x^2)^4) - (14*b^3)/(a^{10}*(a+b*x^2)^3) - (30*b^3)/(a^{11}*(a+b*x^2)^2) - (165*b^3)/(2*a^{12}*(a+b*x^2)) - (220*b^3*Log[x])/a^{13} + (110*b^3*Log[a+b*x^2])/a^{13}$

**Rubi [A]** time = 0.525568, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\begin{aligned} & \frac{110b^3 \log(a+bx^2)}{a^{13}} - \frac{220b^3 \log(x)}{a^{13}} - \frac{165b^3}{2a^{12}(a+bx^2)} - \frac{55b^2}{2a^{12}x^2} - \frac{30b^3}{a^{11}(a+bx^2)^2} \\ & + \frac{5b}{2a^{11}x^4} - \frac{14b^3}{a^{10}(a+bx^2)^3} - \frac{1}{6a^{10}x^6} - \frac{7b^3}{a^9(a+bx^2)^4} - \frac{7b^3}{2a^8(a+bx^2)^5} \\ & - \frac{5b^3}{3a^7(a+bx^2)^6} - \frac{5b^3}{7a^6(a+bx^2)^7} - \frac{b^3}{4a^5(a+bx^2)^8} - \frac{b^3}{18a^4(a+bx^2)^9} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x^7\*(a+b\*x^2)^10),x]

[Out]  $-1/(6*a^{10}*x^6) + (5*b)/(2*a^{11}*x^4) - (55*b^2)/(2*a^{12}*x^2) - b^3/(18*a^4*(a+b*x^2)^9) - b^3/(4*a^5*(a+b*x^2)^8) - (5*b^3)/(7*a^6*(a+b*x^2)^7) - (5*b^3)/(3*a^7*(a+b*x^2)^6) - (7*b^3)/(2*a^8*(a+b*x^2)^5) - (7*b^3)/(a^9*(a+b*x^2)^4) - (14*b^3)/(a^{10}*(a+b*x^2)^3) - (30*b^3)/(a^{11}*(a+b*x^2)^2) - (165*b^3)/(2*a^{12}*(a+b*x^2)) - (220*b^3*Log[x])/a^{13} + (110*b^3*Log[a+b*x^2])/a^{13}$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*7/(b\*x\*\*2+a)\*\*10,x)

[Out] Timed out

**Mathematica [A]** time = 0.248067, size = 162, normalized size = 0.72

$$\frac{-27720b^3 \log(a+bx^2) + \frac{a(42a^{11} - 252a^{10}bx^2 + 2772a^9b^2x^4 + 78419a^8b^3x^6 + 456291a^7b^4x^8 + 1326204a^6b^5x^{10} + 2318316a^5b^6x^{12} + 2604294a^4b^7x^{14} + 190575a^3b^8x^{16} + 105285a^2b^9x^{18} + 35095ab^{10}x^{20} + 7019b^{11}x^{22})}{x^6(a+bx^2)^9}}{252a^{13}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^7\*(a + b\*x^2)^10),x]

[Out]  $-\left(\frac{a(42a^{11} - 252a^{10}b^2x^2 + 2772a^9b^4x^4 + 78419a^8b^6x^6 + 456291a^7b^8x^8 + 1326204a^6b^{10}x^{10} + 2318316a^5b^{12}x^{12} + 2604294a^4b^{14}x^{14} + 1905750a^3b^{16}x^{16} + 882420a^2b^{18}x^{18} + 235620ab^{20}x^{20} + 27720b^{22}x^{22})}{x^6(a + b^2x^2)^9} + 55440b^3\text{Log}[x] - 27720b^3\text{Log}[a + b^2x^2]\right)/(252a^{13})$

**Maple [A]** time = 0.03, size = 209, normalized size = 0.9

$$\begin{aligned} &-\frac{1}{6a^{10}x^6} + \frac{5b}{2a^{11}x^4} - \frac{55b^2}{2a^{12}x^2} - \frac{b^3}{18a^4(bx^2+a)^9} - \frac{b^3}{4a^5(bx^2+a)^8} - \frac{5b^3}{7a^6(bx^2+a)^7} \\ &-\frac{5b^3}{3a^7(bx^2+a)^6} - \frac{7b^3}{2a^8(bx^2+a)^5} - 7\frac{b^3}{a^9(bx^2+a)^4} - 14\frac{b^3}{a^{10}(bx^2+a)^3} \\ &-30\frac{b^3}{a^{11}(bx^2+a)^2} - \frac{165b^3}{2a^{12}(bx^2+a)} - 220\frac{b^3\ln(x)}{a^{13}} + 110\frac{b^3\ln(bx^2+a)}{a^{13}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^7/(b\*x^2+a)^10,x)

[Out]  $-1/6/a^{10}/x^6 + 5/2*b/a^{11}/x^4 - 55/2*b^2/a^{12}/x^2 - 1/18*b^3/a^4/(b*x^2+a)^9 - 1/4*b^3/a^5/(b*x^2+a)^8 - 5/7*b^3/a^6/(b*x^2+a)^7 - 5/3*b^3/a^7/(b*x^2+a)^6 - 7/2*b^3/a^8/(b*x^2+a)^5 - 7*b^3/a^9/(b*x^2+a)^4 - 14*b^3/a^{10}/(b*x^2+a)^3 - 30*b^3/a^{11}/(b*x^2+a)^2 - 165/2*b^3/a^{12}/(b*x^2+a) - 220*b^3*\ln(x)/a^{13} + 110*b^3*\ln(b*x^2+a)/a^{13}$

**Maxima [A]** time = 1.39422, size = 347, normalized size = 1.54

$$\begin{aligned} &\frac{27720b^{11}x^{22} + 235620ab^{10}x^{20} + 882420a^2b^9x^{18} + 1905750a^3b^8x^{16} + 2604294a^4b^7x^{14} + 2318316a^5b^6x^{12} + 1326204a^6b^5x^{10} + 784191a^7b^4x^8 + 456291a^8b^3x^6 + 27720a^9b^2x^4 + 252a^{10}b^2x^2 + 42a^{11}}{252(a^{12}b^9x^{24} + 9a^{13}b^8x^{22} + 36a^{14}b^7x^{20} + 84a^{15}b^6x^{18} + 126a^{16}b^5x^{16} + 126a^{17}b^4x^{14} + 84a^{18}b^3x^{12} + 36a^{19}b^2x^{10} + 9a^{20}b^2x^8 + a^{21}x^6)} \\ &+ \frac{110b^3\log(bx^2+a)}{a^{13}} - \frac{110b^3\log(x^2)}{a^{13}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^10\*x^7),x, algorithm="maxima")

[Out]  $-1/252*(27720b^{11}x^{22} + 235620a^2b^{10}x^{20} + 882420a^4b^9x^{18} + 1905750a^6b^8x^{16} + 2604294a^8b^7x^{14} + 2318316a^{10}b^6x^{12} + 1326204a^{12}b^5x^{10} + 784191a^{14}b^4x^8 + 456291a^{16}b^3x^6 + 27720a^{18}b^2x^4 - 252a^{20}b^2x^2 + 42a^{21})/(a^{12}b^9x^{24} + 9a^{13}b^8x^{22} + 36a^{14}b^7x^{20} + 84a^{15}b^6x^{18} + 126a^{16}b^5x^{16} + 126a^{17}b^4x^{14} + 84a^{18}b^3x^{12} + 36a^{19}b^2x^{10} + 9a^{20}b^2x^8 + a^{21}x^6) + 110b^3*\log(b*x^2 + a)/a^{13} - 110b^3*\log(x^2)/a^{13}$

**Fricas [A]** time = 0.234776, size = 612, normalized size = 2.71

$$\frac{27720ab^{11}x^{22} + 235620a^2b^{10}x^{20} + 882420a^3b^9x^{18} + 1905750a^4b^8x^{16} + 2604294a^5b^7x^{14} + 2318316a^6b^6x^{12} + 1326204a^7b^5x^{10} + 784191a^8b^4x^8 + 456291a^9b^3x^6 + 27720a^{10}b^3x^4 + 252a^{11}b^3x^2 + 42a^{12}}{252(a^{12}b^9x^{24} + 9a^{13}b^8x^{22} + 36a^{14}b^7x^{20} + 84a^{15}b^6x^{18} + 126a^{16}b^5x^{16} + 126a^{17}b^4x^{14} + 84a^{18}b^3x^{12} + 36a^{19}b^2x^{10} + 9a^{20}b^2x^8 + a^{21}x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^10\*x^7),x, algorithm="fricas")

[Out] 
$$-1/252 * (27720 * a * b^{11} * x^{22} + 235620 * a^2 * b^{10} * x^{20} + 882420 * a^3 * b^9 * x^{18} + 1905750 * a^4 * b^8 * x^{16} + 2604294 * a^5 * b^7 * x^{14} + 2318316 * a^6 * b^6 * x^{12} + 1326204 * a^7 * b^5 * x^{10} + 456291 * a^8 * b^4 * x^8 + 78419 * a^9 * b^3 * x^6 + 2772 * a^{10} * b^2 * x^4 - 252 * a^{11} * b * x^2 + 42 * a^{12} - 27720 * (b^{12} * x^{24} + 9 * a * b^{11} * x^{22} + 36 * a^2 * b^{10} * x^{20} + 84 * a^3 * b^9 * x^{18} + 126 * a^4 * b^8 * x^{16} + 126 * a^5 * b^7 * x^{14} + 84 * a^6 * b^6 * x^{12} + 36 * a^7 * b^5 * x^{10} + 9 * a^8 * b^4 * x^8 + a^9 * b^3 * x^6) * \log(b * x^2 + a) + 55440 * (b^{12} * x^{24} + 9 * a * b^{11} * x^{22} + 36 * a^2 * b^{10} * x^{20} + 84 * a^3 * b^9 * x^{18} + 126 * a^4 * b^8 * x^{16} + 126 * a^5 * b^7 * x^{14} + 84 * a^6 * b^6 * x^{12} + 36 * a^7 * b^5 * x^{10} + 9 * a^8 * b^4 * x^8 + a^9 * b^3 * x^6) * \log(x)) / (a^{13} * b^9 * x^{24} + 9 * a^{14} * b^8 * x^{22} + 36 * a^{15} * b^7 * x^{20} + 84 * a^{16} * b^6 * x^{18} + 126 * a^{17} * b^5 * x^{16} + 126 * a^{18} * b^4 * x^{14} + 84 * a^{19} * b^3 * x^{12} + 36 * a^{20} * b^2 * x^{10} + 9 * a^{21} * b * x^8 + a^{22} * x^6)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**7/(b*x**2+a)**10,x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.213004, size = 252, normalized size = 1.12

$$\frac{-\frac{110 b^3 \ln(x^2)}{a^{13}} + \frac{110 b^3 \ln(|bx^2 + a|)}{a^{13}} + \frac{1210 b^3 x^6 - 165 ab^2 x^4 + 15 a^2 bx^2 - a^3}{6 a^{13} x^6}}{78419 b^{12} x^{18} + 726561 ab^{11} x^{16} + 2996964 a^2 b^{10} x^{14} + 7225764 a^3 b^9 x^{12} + 11226726 a^4 b^8 x^{10} + 11663316 a^5 b^7 x^8 + 8108184 a^6 b^6 x^6 + 3641256 a^7 b^5 x^4 + 960210 a^8 b^4 x^2 + 113620 a^9 b^3} / (252 (bx^2 + a)^9 a^{13})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^10*x^7),x, algorithm="giac")`

[Out] 
$$-110 * b^3 * \ln(x^2) / a^{13} + 110 * b^3 * \ln(\text{abs}(b * x^2 + a)) / a^{13} + 1/6 * (1210 * b^3 * x^6 - 165 * a * b^2 * x^4 + 15 * a^2 * b * x^2 - a^3) / (a^{13} * x^6) - 1/252 * (78419 * b^{12} * x^{18} + 726561 * a * b^{11} * x^{16} + 2996964 * a^2 * b^{10} * x^{14} + 7225764 * a^3 * b^9 * x^{12} + 11226726 * a^4 * b^8 * x^{10} + 11663316 * a^5 * b^7 * x^8 + 8108184 * a^6 * b^6 * x^6 + 3641256 * a^7 * b^5 * x^4 + 960210 * a^8 * b^4 * x^2 + 113620 * a^9 * b^3) / ((b * x^2 + a)^9 * a^{13})$$



$$3.209 \quad \int \frac{x^{24}}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=231

$$\begin{aligned} & -\frac{7436429a^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536b^{25/2}} + \frac{7436429a^2x}{65536b^{12}} - \frac{7436429ax^3}{196608b^{11}} - \frac{1062347x^7}{65536b^9(a+bx^2)} \\ & - \frac{1062347x^9}{294912b^8(a+bx^2)^2} - \frac{96577x^{11}}{73728b^7(a+bx^2)^3} - \frac{7429x^{13}}{12288b^6(a+bx^2)^4} - \frac{7429x^{15}}{23040b^5(a+bx^2)^5} \\ & - \frac{437x^{17}}{2304b^4(a+bx^2)^6} - \frac{23x^{19}}{192b^3(a+bx^2)^7} - \frac{23x^{21}}{288b^2(a+bx^2)^8} - \frac{x^{23}}{18b(a+bx^2)^9} + \frac{7436429x^5}{327680b^{10}} \end{aligned}$$

[Out] (7436429\*a^2\*x)/(65536\*b^12) - (7436429\*a\*x^3)/(196608\*b^11) + (7436429\*x^5)/(327680\*b^10) - x^23/(18\*b\*(a+b\*x^2)^9) - (23\*x^21)/(288\*b^2\*(a+b\*x^2)^8) - (23\*x^19)/(192\*b^3\*(a+b\*x^2)^7) - (437\*x^17)/(2304\*b^4\*(a+b\*x^2)^6) - (7429\*x^15)/(23040\*b^5\*(a+b\*x^2)^5) - (7429\*x^13)/(12288\*b^6\*(a+b\*x^2)^4) - (96577\*x^11)/(73728\*b^7\*(a+b\*x^2)^3) - (1062347\*x^9)/(294912\*b^8\*(a+b\*x^2)^2) - (1062347\*x^7)/(65536\*b^9\*(a+b\*x^2)) - (7436429\*a^(5/2)\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(65536\*b^(25/2))

**Rubi [A]** time = 0.433758, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\begin{aligned} & -\frac{7436429a^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536b^{25/2}} + \frac{7436429a^2x}{65536b^{12}} - \frac{7436429ax^3}{196608b^{11}} - \frac{1062347x^7}{65536b^9(a+bx^2)} \\ & - \frac{1062347x^9}{294912b^8(a+bx^2)^2} - \frac{96577x^{11}}{73728b^7(a+bx^2)^3} - \frac{7429x^{13}}{12288b^6(a+bx^2)^4} - \frac{7429x^{15}}{23040b^5(a+bx^2)^5} \\ & - \frac{437x^{17}}{2304b^4(a+bx^2)^6} - \frac{23x^{19}}{192b^3(a+bx^2)^7} - \frac{23x^{21}}{288b^2(a+bx^2)^8} - \frac{x^{23}}{18b(a+bx^2)^9} + \frac{7436429x^5}{327680b^{10}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^24/(a + b\*x^2)^10, x]

[Out] (7436429\*a^2\*x)/(65536\*b^12) - (7436429\*a\*x^3)/(196608\*b^11) + (7436429\*x^5)/(327680\*b^10) - x^23/(18\*b\*(a+b\*x^2)^9) - (23\*x^21)/(288\*b^2\*(a+b\*x^2)^8) - (23\*x^19)/(192\*b^3\*(a+b\*x^2)^7) - (437\*x^17)/(2304\*b^4\*(a+b\*x^2)^6) - (7429\*x^15)/(23040\*b^5\*(a+b\*x^2)^5) - (7429\*x^13)/(12288\*b^6\*(a+b\*x^2)^4) - (96577\*x^11)/(73728\*b^7\*(a+b\*x^2)^3) - (1062347\*x^9)/(294912\*b^8\*(a+b\*x^2)^2) - (1062347\*x^7)/(65536\*b^9\*(a+b\*x^2)) - (7436429\*a^(5/2)\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(65536\*b^(25/2))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned} & -\frac{7436429a^{5/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536b^{25/2}} - \frac{7436429ax^3}{196608b^{11}} - \frac{x^{23}}{18b(a+bx^2)^9} - \frac{23x^{21}}{288b^2(a+bx^2)^8} \\ & - \frac{437x^{17}}{2304b^4(a+bx^2)^6} - \frac{7429x^{15}}{23040b^5(a+bx^2)^5} - \frac{7429x^{13}}{12288b^6(a+bx^2)^4} \\ & - \frac{96577x^{11}}{73728b^7(a+bx^2)^3} - \frac{1062347x^9}{294912b^8(a+bx^2)^2} - \frac{1062347x^7}{65536b^9(a+bx^2)} + \frac{7436429x^5}{327680b^{10}} + \frac{7436429 \int a^2 dx}{65536b^{12}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*24/(b\*x\*\*2+a)\*\*10, x)

[Out]  $-7436429 a^{5/2} \operatorname{atan}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right) / (65536 b^{25/2}) - 7436429 a^3 x^3 / (196608 b^{11}) - x^{23} / (18 b (a + b x^2)^9) - 23 x^{21} / (288 b^2 (a + b x^2)^8) - 23 x^{19} / (192 b^3 (a + b x^2)^7) - 437 x^{17} / (2304 b^4 (a + b x^2)^6) - 7429 x^{15} / (23040 b^5 (a + b x^2)^5) - 7429 x^{13} / (12288 b^6 (a + b x^2)^4) - 96577 x^{11} / (73728 b^7 (a + b x^2)^3) - 1062347 x^9 / (294912 b^8 (a + b x^2)^2) - 1062347 x^7 / (65536 b^9 (a + b x^2)) + 7436429 x^5 / (327680 b^{10}) + 7436429 \operatorname{Integral}(a^2, x) / (65536 b^{12})$

**Mathematica [A]** time = 0.166291, size = 166, normalized size = 0.72

$$\frac{\sqrt{b} x (334639305 a^{11} + 2900207310 a^{10} b x^2 + 11110024926 a^9 b^2 x^4 + 24648575094 a^8 b^3 x^6 + 34810986496 a^7 b^4 x^8 + 32314857354 a^6 b^5 x^{10} + 19562592546 a^5 b^6 x^{12} + 7323998514 a^4 b^7 x^{14} + 1469632311 a^3 b^8 x^{16} + 94961664 a^2 b^9 x^{18} - 4521984 a b^{10} x^{20} + 589824 b^{11} x^{22})}{(a + b x^2)^9}$$

2949120 b<sup>25/2</sup>

Antiderivative was successfully verified.

[In] Integrate[x^24/(a + b\*x^2)^10, x]

[Out]  $((\operatorname{Sqrt}[b] x (334639305 a^{11} + 2900207310 a^{10} b x^2 + 11110024926 a^9 b^2 x^4 + 24648575094 a^8 b^3 x^6 + 34810986496 a^7 b^4 x^8 + 32314857354 a^6 b^5 x^{10} + 19562592546 a^5 b^6 x^{12} + 7323998514 a^4 b^7 x^{14} + 1469632311 a^3 b^8 x^{16} + 94961664 a^2 b^9 x^{18} - 4521984 a b^{10} x^{20} + 589824 b^{11} x^{22})) / (a + b x^2)^9 - 334639305 a^{5/2} \operatorname{ArcTan}[\operatorname{Sqrt}[b] x / \operatorname{Sqrt}[a]]) / (2949120 b^{25/2})$

**Maple [A]** time = 0.027, size = 228, normalized size = 1.

$$\begin{aligned} & \frac{x^5}{5 b^{10}} - \frac{10 a x^3}{3 b^{11}} + 55 \frac{a^2 x}{b^{12}} + \frac{3831949 a^{11} x}{65536 b^{12} (b x^2 + a)^9} + \frac{48340777 a^{10} x^3}{98304 b^{11} (b x^2 + a)^9} + \frac{297702839 a^9 x^5}{163840 b^{10} (b x^2 + a)^9} \\ & + \frac{631790371 a^8 x^7}{163840 b^9 (b x^2 + a)^9} + \frac{463199 a^7 x^9}{90 b^8 (b x^2 + a)^9} + \frac{725918941 a^6 x^{11}}{163840 b^7 (b x^2 + a)^9} + \frac{394553929 a^5 x^{13}}{163840 b^6 (b x^2 + a)^9} \\ & + \frac{74539223 a^4 x^{15}}{98304 b^5 (b x^2 + a)^9} + \frac{6981491 a^3 x^{17}}{65536 b^4 (b x^2 + a)^9} - \frac{7436429 a^3}{65536 b^{12}} \operatorname{arctan}\left(b x \frac{1}{\sqrt{a b}}\right) \frac{1}{\sqrt{a b}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^24/(b\*x^2+a)^10, x)

[Out]  $1/5 x^5/b^{10} - 10/3 a x^3/b^{11} + 55 a^2 x/b^{12} + 3831949/65536 b^{12} a^{11} x / (b x^2 + a)^9 + 48340777/98304 b^{11} a^{10} x^3 / (b x^2 + a)^9 + 297702839/163840 b^{10} a^9 x^5 / (b x^2 + a)^9 + 631790371/163840 b^9 a^8 x^7 / (b x^2 + a)^9 + 463199/90 b^8 a^7 x^9 / (b x^2 + a)^9 + 725918941/163840 b^7 a^6 x^{11} / (b x^2 + a)^9 + 394553929/163840 b^6 a^5 x^{13} / (b x^2 + a)^9 + 74539223/98304 b^5 a^4 x^{15} / (b x^2 + a)^9 + 6981491/65536 b^4 a^3 x^{17} / (b x^2 + a)^9 - 7436429/65536 b^{12} a^3 / (a b)^{1/2} \operatorname{arctan}(x b / (a b)^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^24/(b\*x^2 + a)^10, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.214943, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^24/(b\*x^2 + a)^10,x, algorithm="fricas")

[Out] [1/5898240\*(1179648\*b^11\*x^23 - 9043968\*a\*b^10\*x^21 + 189923328\*a^2\*b^9\*x^19 + 2939264622\*a^3\*b^8\*x^17 + 14647997028\*a^4\*b^7\*x^15 + 39125185092\*a^5\*b^6\*x^13 + 64629714708\*a^6\*b^5\*x^11 + 69621972992\*a^7\*b^4\*x^9 + 49297150188\*a^8\*b^3\*x^7 + 22220049852\*a^9\*b^2\*x^5 + 5800414620\*a^10\*b\*x^3 + 669278610\*a^11\*x + 334639305\*(a^2\*b^9\*x^18 + 9\*a^3\*b^8\*x^16 + 36\*a^4\*b^7\*x^14 + 84\*a^5\*b^6\*x^12 + 126\*a^6\*b^5\*x^10 + 126\*a^7\*b^4\*x^8 + 84\*a^8\*b^3\*x^6 + 36\*a^9\*b^2\*x^4 + 9\*a^10\*b\*x^2 + a^11)\*sqrt(-a/b)\*log((b\*x^2 - 2\*b\*x\*sqrt(-a/b) - a)/(b\*x^2 + a)))/(b^21\*x^18 + 9\*a\*b^20\*x^16 + 36\*a^2\*b^19\*x^14 + 84\*a^3\*b^18\*x^12 + 126\*a^4\*b^17\*x^10 + 126\*a^5\*b^16\*x^8 + 84\*a^6\*b^15\*x^6 + 36\*a^7\*b^14\*x^4 + 9\*a^8\*b^13\*x^2 + a^9\*b^12), 1/2949120\*(589824\*b^11\*x^23 - 4521984\*a\*b^10\*x^21 + 94961664\*a^2\*b^9\*x^19 + 1469632311\*a^3\*b^8\*x^17 + 7323998514\*a^4\*b^7\*x^15 + 19562592546\*a^5\*b^6\*x^13 + 32314857354\*a^6\*b^5\*x^11 + 34810986496\*a^7\*b^4\*x^9 + 24648575094\*a^8\*b^3\*x^7 + 11110024926\*a^9\*b^2\*x^5 + 2900207310\*a^10\*b\*x^3 + 334639305\*a^11\*x - 334639305\*(a^2\*b^9\*x^18 + 9\*a^3\*b^8\*x^16 + 36\*a^4\*b^7\*x^14 + 84\*a^5\*b^6\*x^12 + 126\*a^6\*b^5\*x^10 + 126\*a^7\*b^4\*x^8 + 84\*a^8\*b^3\*x^6 + 36\*a^9\*b^2\*x^4 + 9\*a^10\*b\*x^2 + a^11)\*sqrt(a/b)\*arctan(x/sqrt(a/b)))/(b^21\*x^18 + 9\*a\*b^20\*x^16 + 36\*a^2\*b^19\*x^14 + 84\*a^3\*b^18\*x^12 + 126\*a^4\*b^17\*x^10 + 126\*a^5\*b^16\*x^8 + 84\*a^6\*b^15\*x^6 + 36\*a^7\*b^14\*x^4 + 9\*a^8\*b^13\*x^2 + a^9\*b^12)]

**Sympy [A]** time = 37.6168, size = 314, normalized size = 1.36

$$\frac{55a^2x}{b^{12}} - \frac{10ax^3}{3b^{11}} + \frac{7436429\sqrt{-\frac{a^5}{b^{25}}}\log\left(x - \frac{b^{12}\sqrt{-\frac{a^5}{b^{25}}}}{a^2}\right)}{131072} - \frac{7436429\sqrt{-\frac{a^5}{b^{25}}}\log\left(x + \frac{b^{12}\sqrt{-\frac{a^5}{b^{25}}}}{a^2}\right)}{131072} + \frac{172437705a^{11}x + 1450223310a^{10}bx^3 + 5358651102a^9b^2x^5 + 11372226678a^8b^3x^7 + 15178104832a^7b^4x^9 + 13066540938a^6b^5x^{11} + 7101970722a^5b^6x^{13} + 2236176690a^4b^7x^{15} + 314167095a^3b^8x^{17}}{(2949120a^9b^{12} + 26542080a^8b^{13}x^2 + 106168320a^7b^{14}x^4 + 247726080a^6b^{15}x^6 + 371589120a^5b^{16}x^8 + 371589120a^4b^{17}x^{10} + 2949120a^3b^{18}x^{12} + 106168320a^2b^{19}x^{14} + 26542080ab^{20}x^{16} + 2949120b^{21}x^{18})} + \frac{x^5}{5b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*24/(b\*x\*\*2+a)\*\*10,x)

[Out] 55\*a\*\*2\*x/b\*\*12 - 10\*a\*x\*\*3/(3\*b\*\*11) + 7436429\*sqrt(-a\*\*5/b\*\*25)\*log(x - b\*\*12\*sqrt(-a\*\*5/b\*\*25)/a\*\*2)/131072 - 7436429\*sqrt(-a\*\*5/b\*\*25)\*log(x + b\*\*12\*sqrt(-a\*\*5/b\*\*25)/a\*\*2)/131072 + (172437705\*a\*\*11\*x + 1450223310\*a\*\*10\*b\*x\*\*3 + 5358651102\*a\*\*9\*b\*\*2\*x\*\*5 + 11372226678\*a\*\*8\*b\*\*3\*x\*\*7 + 15178104832\*a\*\*7\*b\*\*4\*x\*\*9 + 13066540938\*a\*\*6\*b\*\*5\*x\*\*11 + 7101970722\*a\*\*5\*b\*\*6\*x\*\*13 + 2236176690\*a\*\*4\*b\*\*7\*x\*\*15 + 314167095\*a\*\*3\*b\*\*8\*x\*\*17)/(2949120\*a\*\*9\*b\*\*12 + 26542080\*a\*\*8\*b\*\*13\*x\*\*2 + 106168320\*a\*\*7\*b\*\*14\*x\*\*4 + 247726080\*a\*\*6\*b\*\*15\*x\*\*6 + 371589120\*a\*\*5\*b\*\*16\*x\*\*8 + 371589120\*a\*\*4\*b\*\*17\*x\*\*10 + 247726080\*a\*\*3\*b\*\*18\*x\*\*12 + 106168320\*a\*\*2\*b\*\*19\*x\*\*14 + 26542080\*a\*b\*\*20\*x\*\*16 + 2949120\*b\*\*21\*x\*\*18) + x\*\*5/(5\*b\*\*10)

**GIAC/XCAS [A]** time = 0.210995, size = 219, normalized size = 0.95

$$\frac{7436429 a^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab} b^{12}} + \frac{314167095 a^3 b^8 x^{17} + 2236176690 a^4 b^7 x^{15} + 7101970722 a^5 b^6 x^{13} + 13066540938 a^6 b^5 x^{11} + 15178104832 a^7 b^4 x^9 + 11372226678 a^8 b^3 x^7 + 5358651102 a^9 b^2 x^5 + 145022330 a^{10} b x^3 + 172437705 a^{11} x}{2949120 (bx^2 + a)^9 b^{12}} + \frac{3 b^{40} x^5 - 50 a b^{39} x^3 + 825 a^2 b^{38} x}{15 b^{50}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^24/(b\*x^2 + a)^10,x, algorithm="giac")

[Out] -7436429/65536\*a^3\*arctan(b\*x/sqrt(a\*b))/(sqrt(a\*b)\*b^12) + 1/2949120\*(314167095\*a^3\*b^8\*x^17 + 2236176690\*a^4\*b^7\*x^15 + 7101970722\*a^5\*b^6\*x^13 + 13066540938\*a^6\*b^5\*x^11 + 15178104832\*a^7\*b^4\*x^9 + 11372226678\*a^8\*b^3\*x^7 + 5358651102\*a^9\*b^2\*x^5 + 145022330\*a^10\*b\*x^3 + 172437705\*a^11\*x)/(b\*x^2 + a)^9\*b^12) + 1/15\*(3\*b^40\*x^5 - 50\*a\*b^39\*x^3 + 825\*a^2\*b^38\*x)/b^50

$$3.210 \quad \int \frac{x^{22}}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=218

$$\frac{1616615a^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536b^{23/2}} - \frac{1616615ax}{65536b^{11}} - \frac{323323x^5}{65536b^9(a+bx^2)} - \frac{46189x^7}{32768b^8(a+bx^2)^2}$$

$$- \frac{46189x^9}{73728b^7(a+bx^2)^3} - \frac{4199x^{11}}{12288b^6(a+bx^2)^4} - \frac{323x^{13}}{1536b^5(a+bx^2)^5} - \frac{323x^{15}}{2304b^4(a+bx^2)^6}$$

$$- \frac{19x^{17}}{192b^3(a+bx^2)^7} - \frac{7x^{19}}{96b^2(a+bx^2)^8} - \frac{x^{21}}{18b(a+bx^2)^9} + \frac{1616615x^3}{196608b^{10}}$$

[Out]  $(-1616615*a*x)/(65536*b^{11}) + (1616615*x^3)/(196608*b^{10}) - x^{21}/(18*b*(a+b*x^2)^9) - (7*x^{19})/(96*b^2*(a+b*x^2)^8) - (19*x^{17})/(192*b^3*(a+b*x^2)^7) - (323*x^{15})/(2304*b^4*(a+b*x^2)^6) - (323*x^{13})/(1536*b^5*(a+b*x^2)^5) - (4199*x^{11})/(12288*b^6*(a+b*x^2)^4) - (46189*x^9)/(73728*b^7*(a+b*x^2)^3) - (46189*x^7)/(32768*b^8*(a+b*x^2)^2) - (323323*x^5)/(65536*b^9*(a+b*x^2)) + (1616615*a^{(3/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*b^{(23/2)})$

**Rubi [A]** time = 0.372654, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{1616615a^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536b^{23/2}} - \frac{1616615ax}{65536b^{11}} - \frac{323323x^5}{65536b^9(a+bx^2)} - \frac{46189x^7}{32768b^8(a+bx^2)^2}$$

$$- \frac{46189x^9}{73728b^7(a+bx^2)^3} - \frac{4199x^{11}}{12288b^6(a+bx^2)^4} - \frac{323x^{13}}{1536b^5(a+bx^2)^5} - \frac{323x^{15}}{2304b^4(a+bx^2)^6}$$

$$- \frac{19x^{17}}{192b^3(a+bx^2)^7} - \frac{7x^{19}}{96b^2(a+bx^2)^8} - \frac{x^{21}}{18b(a+bx^2)^9} + \frac{1616615x^3}{196608b^{10}}$$

Antiderivative was successfully verified.

[In] Int[x^22/(a + b\*x^2)^10, x]

[Out]  $(-1616615*a*x)/(65536*b^{11}) + (1616615*x^3)/(196608*b^{10}) - x^{21}/(18*b*(a+b*x^2)^9) - (7*x^{19})/(96*b^2*(a+b*x^2)^8) - (19*x^{17})/(192*b^3*(a+b*x^2)^7) - (323*x^{15})/(2304*b^4*(a+b*x^2)^6) - (323*x^{13})/(1536*b^5*(a+b*x^2)^5) - (4199*x^{11})/(12288*b^6*(a+b*x^2)^4) - (46189*x^9)/(73728*b^7*(a+b*x^2)^3) - (46189*x^7)/(32768*b^8*(a+b*x^2)^2) - (323323*x^5)/(65536*b^9*(a+b*x^2)) + (1616615*a^{(3/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*b^{(23/2)})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1616615a^{3/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536b^{23/2}} - \frac{x^{21}}{18b(a+bx^2)^9} - \frac{7x^{19}}{96b^2(a+bx^2)^8} - \frac{19x^{17}}{192b^3(a+bx^2)^7}$$

$$- \frac{323x^{15}}{2304b^4(a+bx^2)^6} - \frac{323x^{13}}{1536b^5(a+bx^2)^5} - \frac{4199x^{11}}{12288b^6(a+bx^2)^4} - \frac{46189x^9}{73728b^7(a+bx^2)^3}$$

$$- \frac{46189x^7}{32768b^8(a+bx^2)^2} - \frac{323323x^5}{65536b^9(a+bx^2)} + \frac{1616615x^3}{196608b^{10}} - \frac{1616615 \int a dx}{65536b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*22/(b\*x\*\*2+a)\*\*10, x)

[Out]  $1616615*a^{(3/2)}*atan(sqrt(b)*x/sqrt(a))/(65536*b^{(23/2)}) - x^{**2}/(18*b*(a+b*x^{**2})^{**9}) - 7*x^{**19}/(96*b^{**2}*(a+b*x^{**2})^{**8}) - 19$

\*x\*\*17/(192\*b\*\*3\*(a + b\*x\*\*2)\*\*7) - 323\*x\*\*15/(2304\*b\*\*4\*(a + b\*x\*\*2)\*\*6) - 323\*x\*\*13/(1536\*b\*\*5\*(a + b\*x\*\*2)\*\*5) - 4199\*x\*\*11/(12288\*b\*\*6\*(a + b\*x\*\*2)\*\*4) - 46189\*x\*\*9/(73728\*b\*\*7\*(a + b\*x\*\*2)\*\*3) - 46189\*x\*\*7/(32768\*b\*\*8\*(a + b\*x\*\*2)\*\*2) - 323323\*x\*\*5/(65536\*b\*\*9\*(a + b\*x\*\*2)) + 1616615\*x\*\*3/(196608\*b\*\*10) - 1616615\*Integral(a, x)/(65536\*b\*\*11)

**Mathematica [A]** time = 0.156047, size = 155, normalized size = 0.71

$$\frac{14549535a^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) + \frac{\sqrt{bx}(-14549535a^{10} - 126095970a^9bx^2 - 483044562a^8b^2x^4 - 1071677178a^7b^3x^6 - 1513521152a^6b^4x^8 - 1404993798a^5b^5x^{10} - 850547502a^4b^6x^{12} - 318434718a^3b^7x^{14} - 63897057a^2b^8x^{16} - 4128768ab^9x^{18} + 196608b^{10}x^{20})}{(a+bx^2)^9}}{589824b^{23/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^22/(a + b\*x^2)^10, x]

[Out] ((Sqrt[b]\*x\*(-14549535\*a^10 - 126095970\*a^9\*b\*x^2 - 483044562\*a^8\*b^2\*x^4 - 1071677178\*a^7\*b^3\*x^6 - 1513521152\*a^6\*b^4\*x^8 - 1404993798\*a^5\*b^5\*x^10 - 850547502\*a^4\*b^6\*x^12 - 318434718\*a^3\*b^7\*x^14 - 63897057\*a^2\*b^8\*x^16 - 4128768\*a\*b^9\*x^18 + 196608\*b^10\*x^20))/(a + b\*x^2)^9 + 14549535\*a^(3/2)\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(589824\*b^(23/2))

**Maple [A]** time = 0.028, size = 217, normalized size = 1.

$$\begin{aligned} & \frac{x^3}{3b^{10}} - 10\frac{ax}{b^{11}} - \frac{961255a^{10}x}{65536b^{11}(bx^2+a)^9} - \frac{12201403a^9x^3}{98304b^{10}(bx^2+a)^9} - \frac{15137633a^8x^5}{32768b^9(bx^2+a)^9} \\ & - \frac{32405717a^7x^7}{32768b^8(bx^2+a)^9} - \frac{24013a^6x^9}{18b^7(bx^2+a)^9} - \frac{38143787a^5x^{11}}{32768b^6(bx^2+a)^9} - \frac{21103775a^4x^{13}}{32768b^5(bx^2+a)^9} \\ & - \frac{20435525a^3x^{15}}{98304b^4(bx^2+a)^9} - \frac{1987865a^2x^{17}}{65536b^3(bx^2+a)^9} + \frac{1616615a^2}{65536b^{11}} \arctan\left(bx\frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^22/(b\*x^2+a)^10, x)

[Out] 1/3\*x^3/b^10-10\*a\*x/b^11-961255/65536/b^11\*a^10/(b\*x^2+a)^9\*x-12201403/98304/b^10\*a^9/(b\*x^2+a)^9\*x^3-15137633/32768/b^9\*a^8/(b\*x^2+a)^9\*x^5-32405717/32768/b^8\*a^7/(b\*x^2+a)^9\*x^7-24013/18/b^7\*a^6/(b\*x^2+a)^9\*x^9-38143787/32768/b^6\*a^5/(b\*x^2+a)^9\*x^11-21103775/32768/b^5\*a^4/(b\*x^2+a)^9\*x^13-20435525/98304/b^4\*a^3/(b\*x^2+a)^9\*x^15-1987865/65536/b^3\*a^2/(b\*x^2+a)^9\*x^17+1616615/65536/b^11\*a^2/(a\*b)^(1/2)\*arctan(x\*b/(a\*b)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^22/(b\*x^2 + a)^10, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.217931, size = 1, normalized size = 0.

$$393216 b^{10} x^{21} - 8257536 a b^9 x^{19} - 127794114 a^2 b^8 x^{17} - 636869436 a^3 b^7 x^{15} - 1701095004 a^4 b^6 x^{13} - 2809987596 a^5 b^5 x^{11} - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^22/(b\*x^2 + a)^10,x, algorithm="fricas")

[Out] [1/1179648\*(393216\*b^10\*x^21 - 8257536\*a\*b^9\*x^19 - 127794114\*a^2\*b^8\*x^17 - 636869436\*a^3\*b^7\*x^15 - 1701095004\*a^4\*b^6\*x^13 - 2809987596\*a^5\*b^5\*x^11 - 3027042304\*a^6\*b^4\*x^9 - 2143354356\*a^7\*b^3\*x^7 - 966089124\*a^8\*b^2\*x^5 - 252191940\*a^9\*b\*x^3 - 29099070\*a^10\*x + 14549535\*(a\*b^9\*x^18 + 9\*a^2\*b^8\*x^16 + 36\*a^3\*b^7\*x^14 + 84\*a^4\*b^6\*x^12 + 126\*a^5\*b^5\*x^10 + 126\*a^6\*b^4\*x^8 + 84\*a^7\*b^3\*x^6 + 36\*a^8\*b^2\*x^4 + 9\*a^9\*b\*x^2 + a^10)\*sqrt(-a/b)\*log((b\*x^2 + 2\*b\*x\*sqrt(-a/b) - a)/(b\*x^2 + a)))/(b^20\*x^18 + 9\*a\*b^19\*x^16 + 36\*a^2\*b^18\*x^14 + 84\*a^3\*b^17\*x^12 + 126\*a^4\*b^16\*x^10 + 126\*a^5\*b^15\*x^8 + 84\*a^6\*b^14\*x^6 + 36\*a^7\*b^13\*x^4 + 9\*a^8\*b^12\*x^2 + a^9\*b^11), 1/589824\*(196608\*b^10\*x^21 - 4128768\*a\*b^9\*x^19 - 63897057\*a^2\*b^8\*x^17 - 318434718\*a^3\*b^7\*x^15 - 850547502\*a^4\*b^6\*x^13 - 1404993798\*a^5\*b^5\*x^11 - 1513521152\*a^6\*b^4\*x^9 - 1071677178\*a^7\*b^3\*x^7 - 483044562\*a^8\*b^2\*x^5 - 126095970\*a^9\*b\*x^3 - 14549535\*a^10\*x + 14549535\*(a\*b^9\*x^18 + 9\*a^2\*b^8\*x^16 + 36\*a^3\*b^7\*x^14 + 84\*a^4\*b^6\*x^12 + 126\*a^5\*b^5\*x^10 + 126\*a^6\*b^4\*x^8 + 84\*a^7\*b^3\*x^6 + 36\*a^8\*b^2\*x^4 + 9\*a^9\*b\*x^2 + a^10)\*sqrt(a/b)\*arctan(x/sqrt(a/b)))/(b^20\*x^18 + 9\*a\*b^19\*x^16 + 36\*a^2\*b^18\*x^14 + 84\*a^3\*b^17\*x^12 + 126\*a^4\*b^16\*x^10 + 126\*a^5\*b^15\*x^8 + 84\*a^6\*b^14\*x^6 + 36\*a^7\*b^13\*x^4 + 9\*a^8\*b^12\*x^2 + a^9\*b^11)]

**Sympy [A]** time = 37.1602, size = 298, normalized size = 1.37

$$\frac{10ax}{b^{11}} - \frac{1616615\sqrt{-\frac{a^3}{b^{23}}}\log\left(x - \frac{b^{11}\sqrt{-\frac{a^3}{b^{23}}}}{a}\right)}{131072} + \frac{1616615\sqrt{-\frac{a^3}{b^{23}}}\log\left(x + \frac{b^{11}\sqrt{-\frac{a^3}{b^{23}}}}{a}\right)}{131072} + \frac{8651295a^{10}x + 73208418a^9bx^3 + 272477394a^8b^2x^5 + 583302906a^7b^3x^7 + 786857984a^6b^4x^9 + 686588166a^5b^5x^{11} + \dots}{589824a^9b^{11} + 5308416a^8b^{12}x^2 + 21233664a^7b^{13}x^4 + 49545216a^6b^{14}x^6 + 74317824a^5b^{15}x^8 + 74317824a^4b^{16}x^{10} + 49545216a^3b^{17}x^{12} + 21233664a^2b^{18}x^{14} + 5308416ab^{19}x^{16} + 589824b^{20}x^{18}} + \frac{x^3}{3b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*22/(b\*x\*\*2+a)\*\*10,x)

[Out] -10\*a\*x/b\*\*11 - 1616615\*sqrt(-a\*\*3/b\*\*23)\*log(x - b\*\*11\*sqrt(-a\*\*3/b\*\*23)/a)/131072 + 1616615\*sqrt(-a\*\*3/b\*\*23)\*log(x + b\*\*11\*sqrt(-a\*\*3/b\*\*23)/a)/131072 - (8651295\*a\*\*10\*x + 73208418\*a\*\*9\*b\*x\*\*3 + 272477394\*a\*\*8\*b\*\*2\*x\*\*5 + 583302906\*a\*\*7\*b\*\*3\*x\*\*7 + 786857984\*a\*\*6\*b\*\*4\*x\*\*9 + 686588166\*a\*\*5\*b\*\*5\*x\*\*11 + 379867950\*a\*\*4\*b\*\*6\*x\*\*13 + 122613150\*a\*\*3\*b\*\*7\*x\*\*15 + 17890785\*a\*\*2\*b\*\*8\*x\*\*17)/(589824\*a\*\*9\*b\*\*11 + 5308416\*a\*\*8\*b\*\*12\*x\*\*2 + 21233664\*a\*\*7\*b\*\*13\*x\*\*4 + 49545216\*a\*\*6\*b\*\*14\*x\*\*6 + 74317824\*a\*\*5\*b\*\*15\*x\*\*8 + 74317824\*a\*\*4\*b\*\*16\*x\*\*10 + 49545216\*a\*\*3\*b\*\*17\*x\*\*12 + 21233664\*a\*\*2\*b\*\*18\*x\*\*14 + 5308416\*a\*b\*\*19\*x\*\*16 + 589824\*b\*\*20\*x\*\*18) + x\*\*3/(3\*b\*\*10)

**GIAC/XCAS [A]** time = 0.215538, size = 203, normalized size = 0.93

$$\frac{1616615 a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab} b^{11}} - \frac{17890785 a^2 b^8 x^{17} + 122613150 a^3 b^7 x^{15} + 379867950 a^4 b^6 x^{13} + 686588166 a^5 b^5 x^{11} + 786857984 a^6 b^4 x^9 + 583302906 a^7 b^3 x^7 + 272477394 a^8 b^2 x^5 + 73208418 a^9 b x^3 + 8651295 a^{10} x}{589824 (bx^2 + a)^9 b^{11}} + \frac{b^{20} x^3 - 30 ab^{19} x}{3 b^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^22/(b\*x^2 + a)^10,x, algorithm="giac")

[Out] 1616615/65536\*a^2\*arctan(b\*x/sqrt(a\*b))/(sqrt(a\*b)\*b^11) - 1/589824\*(17890785\*a^2\*b^8\*x^17 + 122613150\*a^3\*b^7\*x^15 + 379867950\*a^4\*b^6\*x^13 + 686588166\*a^5\*b^5\*x^11 + 786857984\*a^6\*b^4\*x^9 + 583302906\*a^7\*b^3\*x^7 + 272477394\*a^8\*b^2\*x^5 + 73208418\*a^9\*b\*x^3 + 8651295\*a^10\*x)/((b\*x^2 + a)^9\*b^11) + 1/3\*(b^20\*x^3 - 30\*a\*b^19\*x)/b^30



$$3.211 \quad \int \frac{x^{20}}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=207

$$\begin{aligned} & -\frac{230945\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536b^{21/2}} - \frac{230945x^3}{196608b^9(a+bx^2)} - \frac{46189x^5}{98304b^8(a+bx^2)^2} - \frac{46189x^7}{172032b^7(a+bx^2)^3} \\ & - \frac{46189x^9}{258048b^6(a+bx^2)^4} - \frac{4199x^{11}}{32256b^5(a+bx^2)^5} - \frac{1615x^{13}}{16128b^4(a+bx^2)^6} \\ & - \frac{323x^{15}}{4032b^3(a+bx^2)^7} - \frac{19x^{17}}{288b^2(a+bx^2)^8} - \frac{x^{19}}{18b(a+bx^2)^9} + \frac{230945x}{65536b^{10}} \end{aligned}$$

[Out] (230945\*x)/(65536\*b^10) - x^19/(18\*b\*(a+b\*x^2)^9) - (19\*x^17)/(288\*b^2\*(a+b\*x^2)^8) - (323\*x^15)/(4032\*b^3\*(a+b\*x^2)^7) - (1615\*x^13)/(16128\*b^4\*(a+b\*x^2)^6) - (4199\*x^11)/(32256\*b^5\*(a+b\*x^2)^5) - (46189\*x^9)/(258048\*b^6\*(a+b\*x^2)^4) - (46189\*x^7)/(172032\*b^7\*(a+b\*x^2)^3) - (46189\*x^5)/(98304\*b^8\*(a+b\*x^2)^2) - (230945\*x^3)/(196608\*b^9\*(a+b\*x^2)) - (230945\*sqrt[a]\*ArcTan[(sqrt[b]\*x)/sqrt[a]])/(65536\*b^(21/2))

**Rubi [A]** time = 0.332515, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\begin{aligned} & -\frac{230945\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536b^{21/2}} - \frac{230945x^3}{196608b^9(a+bx^2)} - \frac{46189x^5}{98304b^8(a+bx^2)^2} - \frac{46189x^7}{172032b^7(a+bx^2)^3} \\ & - \frac{46189x^9}{258048b^6(a+bx^2)^4} - \frac{4199x^{11}}{32256b^5(a+bx^2)^5} - \frac{1615x^{13}}{16128b^4(a+bx^2)^6} \\ & - \frac{323x^{15}}{4032b^3(a+bx^2)^7} - \frac{19x^{17}}{288b^2(a+bx^2)^8} - \frac{x^{19}}{18b(a+bx^2)^9} + \frac{230945x}{65536b^{10}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^20/(a+b\*x^2)^10,x]

[Out] (230945\*x)/(65536\*b^10) - x^19/(18\*b\*(a+b\*x^2)^9) - (19\*x^17)/(288\*b^2\*(a+b\*x^2)^8) - (323\*x^15)/(4032\*b^3\*(a+b\*x^2)^7) - (1615\*x^13)/(16128\*b^4\*(a+b\*x^2)^6) - (4199\*x^11)/(32256\*b^5\*(a+b\*x^2)^5) - (46189\*x^9)/(258048\*b^6\*(a+b\*x^2)^4) - (46189\*x^7)/(172032\*b^7\*(a+b\*x^2)^3) - (46189\*x^5)/(98304\*b^8\*(a+b\*x^2)^2) - (230945\*x^3)/(196608\*b^9\*(a+b\*x^2)) - (230945\*sqrt[a]\*ArcTan[(sqrt[b]\*x)/sqrt[a]])/(65536\*b^(21/2))

**Rubi in Sympy [A]** time = 57.4221, size = 197, normalized size = 0.95

$$\begin{aligned} & -\frac{230945\sqrt{a}\operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536b^{\frac{21}{2}}} - \frac{x^{19}}{18b(a+bx^2)^9} - \frac{19x^{17}}{288b^2(a+bx^2)^8} - \frac{323x^{15}}{4032b^3(a+bx^2)^7} \\ & - \frac{1615x^{13}}{16128b^4(a+bx^2)^6} - \frac{4199x^{11}}{32256b^5(a+bx^2)^5} - \frac{46189x^9}{258048b^6(a+bx^2)^4} \\ & - \frac{46189x^7}{172032b^7(a+bx^2)^3} - \frac{46189x^5}{98304b^8(a+bx^2)^2} - \frac{230945x^3}{196608b^9(a+bx^2)} + \frac{230945x}{65536b^{10}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*20/(b\*x\*\*2+a)\*\*10,x)

[Out] -230945\*sqrt(a)\*atan(sqrt(b)\*x/sqrt(a))/(65536\*b\*\*(21/2)) - x\*\*19/(18\*b\*(a+b\*x\*\*2)\*\*9) - 19\*x\*\*17/(288\*b\*\*2\*(a+b\*x\*\*2)\*\*8) - 3

$$23x^{15}/(4032b^3(a+bx^2)^7) - 1615x^{13}/(16128b^4(a+bx^2)^6) - 4199x^{11}/(32256b^5(a+bx^2)^5) - 46189x^9/(258048b^6(a+bx^2)^4) - 46189x^7/(172032b^7(a+bx^2)^3) - 46189x^5/(98304b^8(a+bx^2)^2) - 230945x^3/(196608b^9(a+bx^2)) + 230945x/(65536b^{10})$$

**Mathematica [A]** time = 0.142058, size = 144, normalized size = 0.7

$$\frac{\sqrt{bx}(14549535a^9+126095970a^8bx^2+483044562a^7b^2x^4+1071677178a^6b^3x^6+1513521152a^5b^4x^8+1404993798a^4b^5x^{10}+850547502a^3b^6x^{12}+318434718a^2b^7x^{14}+63894535ab^8x^{16}+4128768b^9x^{18})}{(a+bx^2)^9} 4128768b^{21/2}$$

Antiderivative was successfully verified.

[In] Integrate[x^20/(a + b\*x^2)^10, x]

[Out] ((Sqrt[b]\*x\*(14549535\*a^9 + 126095970\*a^8\*b\*x^2 + 483044562\*a^7\*b^2\*x^4 + 1071677178\*a^6\*b^3\*x^6 + 1513521152\*a^5\*b^4\*x^8 + 1404993798\*a^4\*b^5\*x^10 + 850547502\*a^3\*b^6\*x^12 + 318434718\*a^2\*b^7\*x^14 + 63897057\*a\*b^8\*x^16 + 4128768\*b^9\*x^18))/(a + b\*x^2)^9 - 14549535\*Sqrt[a]\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/(4128768\*b^(21/2)))

**Maple [A]** time = 0.027, size = 203, normalized size = 1.

$$\begin{aligned} & \frac{x}{b^{10}} + \frac{165409 a^9 x}{65536 b^{10} (bx^2 + a)^9} + \frac{2117549 a^8 x^3}{98304 b^9 (bx^2 + a)^9} + \frac{2654039 a^7 x^5}{32768 b^8 (bx^2 + a)^9} \\ & + \frac{40270037 a^6 x^7}{229376 b^7 (bx^2 + a)^9} + \frac{30313 a^5 x^9}{126 b^6 (bx^2 + a)^9} + \frac{49153835 a^4 x^{11}}{229376 b^5 (bx^2 + a)^9} + \frac{3997865 a^3 x^{13}}{32768 b^4 (bx^2 + a)^9} \\ & + \frac{4042835 a^2 x^{15}}{98304 b^3 (bx^2 + a)^9} + \frac{424415 a x^{17}}{65536 b^2 (bx^2 + a)^9} - \frac{230945 a}{65536 b^{10}} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^20/(b\*x^2+a)^10, x)

[Out] x/b^10+165409/65536/b^10\*a^9/(b\*x^2+a)^9\*x+2117549/98304/b^9\*a^8/(b\*x^2+a)^9\*x^3+2654039/32768/b^8\*a^7/(b\*x^2+a)^9\*x^5+40270037/229376/b^7\*a^6/(b\*x^2+a)^9\*x^7+30313/126/b^6\*a^5/(b\*x^2+a)^9\*x^9+49153835/229376/b^5\*a^4/(b\*x^2+a)^9\*x^11+3997865/32768/b^4\*a^3/(b\*x^2+a)^9\*x^13+4042835/98304/b^3\*a^2/(b\*x^2+a)^9\*x^15+424415/65536/b^2\*a/(b\*x^2+a)^9\*x^17-230945/65536/b^10\*a/(a\*b)^(1/2)\*arctan(x\*b/(a\*b)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^20/(b\*x^2 + a)^10,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.21359, size = 1, normalized size = 0.

$$\frac{8257536 b^9 x^{19} + 127794114 a b^8 x^{17} + 636869436 a^2 b^7 x^{15} + 1701095004 a^3 b^6 x^{13} + 2809987596 a^4 b^5 x^{11} + 3027042304 a^5 b^4 x^9 + 2143354356 a^6 b^3 x^7 + 966089124 a^7 b^2 x^5 + 252191940 a^8 b x^3 + 29099070 a^9 x + 14549535 (b^9 x^{18} + 9 a b^8 x^{16} + 36 a^2 b^7 x^{14} + 84 a^3 b^6 x^{12} + 126 a^4 b^5 x^{10} + 126 a^5 b^4 x^8 + 84 a^6 b^3 x^6 + 36 a^7 b^2 x^4 + 9 a^8 b x^2 + a^9) \sqrt{-a/b} \log((b x^2 - 2 b x \sqrt{-a/b} - a)/(b x^2 + a))}{(b^{19} x^{18} + 9 a b^{18} x^{16} + 36 a^2 b^{17} x^{14} + 84 a^3 b^{16} x^{12} + 126 a^4 b^{15} x^{10} + 126 a^5 b^{14} x^8 + 84 a^6 b^{13} x^6 + 36 a^7 b^{12} x^4 + 9 a^8 b^{11} x^2 + a^9 b^{10}), 1/4128768 (4128768 b^9 x^{19} + 63897057 a b^8 x^{17} + 318434718 a^2 b^7 x^{15} + 850547502 a^3 b^6 x^{13} + 1404993798 a^4 b^5 x^{11} + 1513521152 a^5 b^4 x^9 + 1071677178 a^6 b^3 x^7 + 483044562 a^7 b^2 x^5 + 126095970 a^8 b x^3 + 14549535 a^9 x - 14549535 (b^9 x^{18} + 9 a b^8 x^{16} + 36 a^2 b^7 x^{14} + 84 a^3 b^6 x^{12} + 126 a^4 b^5 x^{10} + 126 a^5 b^4 x^8 + 84 a^6 b^3 x^6 + 36 a^7 b^2 x^4 + 9 a^8 b x^2 + a^9) \sqrt{a/b} \arctan(x/\sqrt{a/b}))}{(b^{19} x^{18} + 9 a b^{18} x^{16} + 36 a^2 b^{17} x^{14} + 84 a^3 b^{16} x^{12} + 126 a^4 b^{15} x^{10} + 126 a^5 b^{14} x^8 + 84 a^6 b^{13} x^6 + 36 a^7 b^{12} x^4 + 9 a^8 b^{11} x^2 + a^9 b^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^20/(b\*x^2 + a)^10,x, algorithm="fricas")

[Out] [1/8257536\*(8257536\*b^9\*x^19 + 127794114\*a\*b^8\*x^17 + 636869436\*a^2\*b^7\*x^15 + 1701095004\*a^3\*b^6\*x^13 + 2809987596\*a^4\*b^5\*x^11 + 3027042304\*a^5\*b^4\*x^9 + 2143354356\*a^6\*b^3\*x^7 + 966089124\*a^7\*b^2\*x^5 + 252191940\*a^8\*b\*x^3 + 29099070\*a^9\*x + 14549535\*(b^9\*x^18 + 9\*a\*b^8\*x^16 + 36\*a^2\*b^7\*x^14 + 84\*a^3\*b^6\*x^12 + 126\*a^4\*b^5\*x^10 + 126\*a^5\*b^4\*x^8 + 84\*a^6\*b^3\*x^6 + 36\*a^7\*b^2\*x^4 + 9\*a^8\*b\*x^2 + a^9)\*sqrt(-a/b)\*log((b\*x^2 - 2\*b\*x\*sqrt(-a/b) - a)/(b\*x^2 + a)))/(b^19\*x^18 + 9\*a\*b^18\*x^16 + 36\*a^2\*b^17\*x^14 + 84\*a^3\*b^16\*x^12 + 126\*a^4\*b^15\*x^10 + 126\*a^5\*b^14\*x^8 + 84\*a^6\*b^13\*x^6 + 36\*a^7\*b^12\*x^4 + 9\*a^8\*b^11\*x^2 + a^9\*b^10), 1/4128768\*(4128768\*b^9\*x^19 + 63897057\*a\*b^8\*x^17 + 318434718\*a^2\*b^7\*x^15 + 850547502\*a^3\*b^6\*x^13 + 1404993798\*a^4\*b^5\*x^11 + 1513521152\*a^5\*b^4\*x^9 + 1071677178\*a^6\*b^3\*x^7 + 483044562\*a^7\*b^2\*x^5 + 126095970\*a^8\*b\*x^3 + 14549535\*a^9\*x - 14549535\*(b^9\*x^18 + 9\*a\*b^8\*x^16 + 36\*a^2\*b^7\*x^14 + 84\*a^3\*b^6\*x^12 + 126\*a^4\*b^5\*x^10 + 126\*a^5\*b^4\*x^8 + 84\*a^6\*b^3\*x^6 + 36\*a^7\*b^2\*x^4 + 9\*a^8\*b\*x^2 + a^9)\*sqrt(a/b)\*arctan(x/sqrt(a/b)))/(b^19\*x^18 + 9\*a\*b^18\*x^16 + 36\*a^2\*b^17\*x^14 + 84\*a^3\*b^16\*x^12 + 126\*a^4\*b^15\*x^10 + 126\*a^5\*b^14\*x^8 + 84\*a^6\*b^13\*x^6 + 36\*a^7\*b^12\*x^4 + 9\*a^8\*b^11\*x^2 + a^9\*b^10)]

**Sympy [A]** time = 36.8681, size = 274, normalized size = 1.32

$$\frac{230945 \sqrt{-\frac{a}{b^{21}}} \log\left(-b^{10} \sqrt{-\frac{a}{b^{21}}} + x\right)}{131072} - \frac{230945 \sqrt{-\frac{a}{b^{21}}} \log\left(b^{10} \sqrt{-\frac{a}{b^{21}}} + x\right)}{131072} + \frac{10420767 a^9 x + 88937058 a^8 b x^3 + 334408914 a^7 b^2 x^5 + 724860666 a^6 b^3 x^7 + 993296384 a^5 b^4 x^9 + 884769030 a^4 b^5 x^{11} + 4128768 a^9 b^{10} + 37158912 a^8 b^{11} x^2 + 148635648 a^7 b^{12} x^4 + 346816512 a^6 b^{13} x^6 + 520224768 a^5 b^{14} x^8 + 520224768 a^4 b^{15} x^{10} + 346816512 a^3 b^{16} x^{12} + 148635648 a^2 b^{17} x^{14} + 37158912 a b^{18} x^{16} + 4128768 b^{19} x^{18}}{4128768 (b x^2 + a)^9 b^{10}} + \frac{x}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*20/(b\*x\*\*2+a)\*\*10,x)

[Out] 230945\*sqrt(-a/b\*\*21)\*log(-b\*\*10\*sqrt(-a/b\*\*21) + x)/131072 - 230945\*sqrt(-a/b\*\*21)\*log(b\*\*10\*sqrt(-a/b\*\*21) + x)/131072 + (10420767\*a\*\*9\*x + 88937058\*a\*\*8\*b\*x\*\*3 + 334408914\*a\*\*7\*b\*\*2\*x\*\*5 + 724860666\*a\*\*6\*b\*\*3\*x\*\*7 + 993296384\*a\*\*5\*b\*\*4\*x\*\*9 + 884769030\*a\*\*4\*b\*\*5\*x\*\*11 + 503730990\*a\*\*3\*b\*\*6\*x\*\*13 + 169799070\*a\*\*2\*b\*\*7\*x\*\*15 + 26738145\*a\*b\*\*8\*x\*\*17)/(4128768\*a\*\*9\*b\*\*10 + 37158912\*a\*\*8\*b\*\*11\*x\*\*2 + 148635648\*a\*\*7\*b\*\*12\*x\*\*4 + 346816512\*a\*\*6\*b\*\*13\*x\*\*6 + 520224768\*a\*\*5\*b\*\*14\*x\*\*8 + 520224768\*a\*\*4\*b\*\*15\*x\*\*10 + 346816512\*a\*\*3\*b\*\*16\*x\*\*12 + 148635648\*a\*\*2\*b\*\*17\*x\*\*14 + 37158912\*a\*b\*\*18\*x\*\*16 + 4128768\*b\*\*19\*x\*\*18) + x/b\*\*10

**GIAC/XCAS [A]** time = 0.209494, size = 177, normalized size = 0.86

$$\frac{230945 a \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab} b^{10}} + \frac{x}{b^{10}} + \frac{26738145 a b^8 x^{17} + 169799070 a^2 b^7 x^{15} + 503730990 a^3 b^6 x^{13} + 884769030 a^4 b^5 x^{11} + 993296384 a^5 b^4 x^9 + 724860666 a^6 b^3 x^7 + 4128768 (b x^2 + a)^9 b^{10}}{4128768 (b x^2 + a)^9 b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^20/(b*x^2 + a)^10,x, algorithm="giac")`

[Out] 
$$\frac{-230945}{65536} a \arctan\left(\frac{b x}{\sqrt{a b}}\right) / (\sqrt{a b})^2 b^{10} + \frac{x}{b^{10}} + \frac{1}{4128768} (26738145 a^2 b^8 x^{17} + 169799070 a^2 b^7 x^{15} + 503730990 a^3 b^6 x^{13} + 884769030 a^4 b^5 x^{11} + 993296384 a^5 b^4 x^9 + 724860666 a^6 b^3 x^7 + 334408914 a^7 b^2 x^5 + 88937058 a^8 b x^3 + 10420767 a^9 x) / ((b x^2 + a)^9 b^{10})$$

$$3.212 \quad \int \frac{x^{18}}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=197

$$\begin{aligned} & \frac{12155 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536\sqrt{ab}^{19/2}} - \frac{12155x}{65536b^9(a+bx^2)} - \frac{12155x^3}{98304b^8(a+bx^2)^2} \\ & - \frac{2431x^5}{24576b^7(a+bx^2)^3} - \frac{2431x^7}{28672b^6(a+bx^2)^4} - \frac{2431x^9}{32256b^5(a+bx^2)^5} \\ & - \frac{1105x^{11}}{16128b^4(a+bx^2)^6} - \frac{85x^{13}}{1344b^3(a+bx^2)^7} - \frac{17x^{15}}{288b^2(a+bx^2)^8} - \frac{x^{17}}{18b(a+bx^2)^9} \end{aligned}$$

[Out]  $-x^{17}/(18*b*(a+b*x^2)^9) - (17*x^{15})/(288*b^2*(a+b*x^2)^8) - (85*x^{13})/(1344*b^3*(a+b*x^2)^7) - (1105*x^{11})/(16128*b^4*(a+b*x^2)^6) - (2431*x^9)/(32256*b^5*(a+b*x^2)^5) - (2431*x^7)/(28672*b^6*(a+b*x^2)^4) - (2431*x^5)/(24576*b^7*(a+b*x^2)^3) - (12155*x^3)/(98304*b^8*(a+b*x^2)^2) - (12155*x)/(65536*b^9*(a+b*x^2)) + (12155*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*Sqrt[a]*b^(19/2))$

**Rubi [A]** time = 0.310432, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\begin{aligned} & \frac{12155 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536\sqrt{ab}^{19/2}} - \frac{12155x}{65536b^9(a+bx^2)} - \frac{12155x^3}{98304b^8(a+bx^2)^2} \\ & - \frac{2431x^5}{24576b^7(a+bx^2)^3} - \frac{2431x^7}{28672b^6(a+bx^2)^4} - \frac{2431x^9}{32256b^5(a+bx^2)^5} \\ & - \frac{1105x^{11}}{16128b^4(a+bx^2)^6} - \frac{85x^{13}}{1344b^3(a+bx^2)^7} - \frac{17x^{15}}{288b^2(a+bx^2)^8} - \frac{x^{17}}{18b(a+bx^2)^9} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^18/(a + b\*x^2)^10, x]

[Out]  $-x^{17}/(18*b*(a+b*x^2)^9) - (17*x^{15})/(288*b^2*(a+b*x^2)^8) - (85*x^{13})/(1344*b^3*(a+b*x^2)^7) - (1105*x^{11})/(16128*b^4*(a+b*x^2)^6) - (2431*x^9)/(32256*b^5*(a+b*x^2)^5) - (2431*x^7)/(28672*b^6*(a+b*x^2)^4) - (2431*x^5)/(24576*b^7*(a+b*x^2)^3) - (12155*x^3)/(98304*b^8*(a+b*x^2)^2) - (12155*x)/(65536*b^9*(a+b*x^2)) + (12155*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*Sqrt[a]*b^(19/2))$

**Rubi in Sympy [A]** time = 50.6253, size = 187, normalized size = 0.95

$$\begin{aligned} & -\frac{x^{17}}{18b(a+bx^2)^9} - \frac{17x^{15}}{288b^2(a+bx^2)^8} - \frac{85x^{13}}{1344b^3(a+bx^2)^7} - \frac{1105x^{11}}{16128b^4(a+bx^2)^6} \\ & - \frac{2431x^9}{32256b^5(a+bx^2)^5} - \frac{2431x^7}{28672b^6(a+bx^2)^4} - \frac{2431x^5}{24576b^7(a+bx^2)^3} \\ & - \frac{12155x^3}{98304b^8(a+bx^2)^2} - \frac{12155x}{65536b^9(a+bx^2)} + \frac{12155 \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536\sqrt{ab}^{19/2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*18/(b\*x\*\*2+a)\*\*10, x)

[Out]  $-x^{17}/(18*b*(a+b*x^2)^9) - 17*x^{15}/(288*b^2*(a+b*x^2)^8) - 85*x^{13}/(1344*b^3*(a+b*x^2)^7) - 1105*x^{11}/(16128*b^4*(a+b*x^2)^6) - 2431*x^9/(32256*b^5*(a+b*x^2)^5) - 2431*x^7/(28672*b^6*(a+b*x^2)^4) - 2431*x^5/(24576*b^7*(a+b*x^2)^3) - 12155*x^3/(98304*b^8*(a+b*x^2)^2) - 12155*x/(65536*b^9*(a+b*x^2)) + 12155*atan(sqrt(b*x)/sqrt(a))/(65536*sqrt(a)*b^(19/2))$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^18/(b\*x^2 + a)^10,x, algorithm="fricas")

[Out] [1/8257536\*(765765\*(b^9\*x^18 + 9\*a\*b^8\*x^16 + 36\*a^2\*b^7\*x^14 + 84\*a^3\*b^6\*x^12 + 126\*a^4\*b^5\*x^10 + 126\*a^5\*b^4\*x^8 + 84\*a^6\*b^3\*x^6 + 36\*a^7\*b^2\*x^4 + 9\*a^8\*b\*x^2 + a^9)\*log((2\*a\*b\*x + (b\*x^2 - a)\*sqrt(-a\*b))/(b\*x^2 + a)) - 2\*(3363003\*b^8\*x^17 + 16759722\*a\*b^7\*x^15 + 44765658\*a^2\*b^6\*x^13 + 73947042\*a^3\*b^5\*x^11 + 79659008\*a^4\*b^4\*x^9 + 56404062\*a^5\*b^3\*x^7 + 25423398\*a^6\*b^2\*x^5 + 6636630\*a^7\*b\*x^3 + 765765\*a^8\*x)\*sqrt(-a\*b))/((b^18\*x^18 + 9\*a\*b^17\*x^16 + 36\*a^2\*b^16\*x^14 + 84\*a^3\*b^15\*x^12 + 126\*a^4\*b^14\*x^10 + 126\*a^5\*b^13\*x^8 + 84\*a^6\*b^12\*x^6 + 36\*a^7\*b^11\*x^4 + 9\*a^8\*b^10\*x^2 + a^9\*b^9)\*sqrt(-a\*b)), 1/4128768\*(765765\*(b^9\*x^18 + 9\*a\*b^8\*x^16 + 36\*a^2\*b^7\*x^14 + 84\*a^3\*b^6\*x^12 + 126\*a^4\*b^5\*x^10 + 126\*a^5\*b^4\*x^8 + 84\*a^6\*b^3\*x^6 + 36\*a^7\*b^2\*x^4 + 9\*a^8\*b\*x^2 + a^9)\*arctan(sqrt(a\*b)\*x/a) - (3363003\*b^8\*x^17 + 16759722\*a\*b^7\*x^15 + 44765658\*a^2\*b^6\*x^13 + 73947042\*a^3\*b^5\*x^11 + 79659008\*a^4\*b^4\*x^9 + 56404062\*a^5\*b^3\*x^7 + 25423398\*a^6\*b^2\*x^5 + 6636630\*a^7\*b\*x^3 + 765765\*a^8\*x)\*sqrt(a\*b))/((b^18\*x^18 + 9\*a\*b^17\*x^16 + 36\*a^2\*b^16\*x^14 + 84\*a^3\*b^15\*x^12 + 126\*a^4\*b^14\*x^10 + 126\*a^5\*b^13\*x^8 + 84\*a^6\*b^12\*x^6 + 36\*a^7\*b^11\*x^4 + 9\*a^8\*b^10\*x^2 + a^9\*b^9)\*sqrt(a\*b))]

**Sympy [A]** time = 35.9627, size = 275, normalized size = 1.4

$$\frac{12155\sqrt{-\frac{1}{ab^{19}}}\log\left(-ab^9\sqrt{-\frac{1}{ab^{19}}}+x\right)}{131072} + \frac{12155\sqrt{-\frac{1}{ab^{19}}}\log\left(ab^9\sqrt{-\frac{1}{ab^{19}}}+x\right)}{131072}$$

$$\frac{765765a^8x + 6636630a^7bx^3 + 25423398a^6b^2x^5 + 56404062a^5b^3x^7 + 79659008a^4b^4x^9 + 73947042a^3b^5x^{11} + 56404062a^2b^6x^{13} + 25423398a^1b^7x^{15} + 765765a^8x}{4128768a^9b^9 + 37158912a^8b^{10}x^2 + 148635648a^7b^{11}x^4 + 346816512a^6b^{12}x^6 + 520224768a^5b^{13}x^8 + 520224768a^4b^{14}x^{10} + 346816512a^3b^{15}x^{12} + 148635648a^2b^{16}x^{14} + 37158912a^1b^{17}x^{16} + 765765a^8x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*18/(b\*x\*\*2+a)\*\*10,x)

[Out] -12155\*sqrt(-1/(a\*b\*\*19))\*log(-a\*b\*\*9\*sqrt(-1/(a\*b\*\*19)) + x)/131072 + 12155\*sqrt(-1/(a\*b\*\*19))\*log(a\*b\*\*9\*sqrt(-1/(a\*b\*\*19)) + x)/131072 - (765765\*a\*\*8\*x + 6636630\*a\*\*7\*b\*x\*\*3 + 25423398\*a\*\*6\*b\*\*2\*x\*\*5 + 56404062\*a\*\*5\*b\*\*3\*x\*\*7 + 79659008\*a\*\*4\*b\*\*4\*x\*\*9 + 73947042\*a\*\*3\*b\*\*5\*x\*\*11 + 44765658\*a\*\*2\*b\*\*6\*x\*\*13 + 16759722\*a\*b\*\*7\*x\*\*15 + 3363003\*b\*\*8\*x\*\*17)/(4128768\*a\*\*9\*b\*\*9 + 37158912\*a\*\*8\*b\*\*10\*x\*\*2 + 148635648\*a\*\*7\*b\*\*11\*x\*\*4 + 346816512\*a\*\*6\*b\*\*12\*x\*\*6 + 520224768\*a\*\*5\*b\*\*13\*x\*\*8 + 520224768\*a\*\*4\*b\*\*14\*x\*\*10 + 346816512\*a\*\*3\*b\*\*15\*x\*\*12 + 148635648\*a\*\*2\*b\*\*16\*x\*\*14 + 37158912\*a\*\*1\*b\*\*17\*x\*\*16 + 4128768\*b\*\*18\*x\*\*18)

**GIAC/XCAS [A]** time = 0.209883, size = 165, normalized size = 0.84

$$\frac{12155 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{abb^9}}$$

$$\frac{3363003 b^8 x^{17} + 16759722 ab^7 x^{15} + 44765658 a^2 b^6 x^{13} + 73947042 a^3 b^5 x^{11} + 79659008 a^4 b^4 x^9 + 56404062 a^5 b^3 x^7 + 25423398 a^6 b^2 x^5 + 6636630 a^7 b x^3 + 765765 a^8 x}{4128768 (bx^2 + a)^9 b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^18/(b\*x^2 + a)^10,x, algorithm="giac")

[Out] 12155/65536\*arctan(b\*x/sqrt(a\*b))/(sqrt(a\*b)\*b^9) - 1/4128768\*(3363003\*b^8\*x^17 + 16759722\*a\*b^7\*x^15 + 44765658\*a^2\*b^6\*x^13 + 73947042\*a^3\*b^5\*x^11 + 79659008\*a^4\*b^4\*x^9 + 56404062\*a^5\*b^3\*x^7 + 25423398\*a^6\*b^2\*x^5 + 6636630\*a^7\*b\*x^3 + 765765\*a^8\*x)/((b\*x^2 + a)^9\*b^9)

$$3.213 \quad \int \frac{x^{16}}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=198

$$\frac{715 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{3/2}b^{17/2}} + \frac{715x}{65536ab^8(a+bx^2)} - \frac{715x}{32768b^8(a+bx^2)^2} - \frac{715x^3}{24576b^7(a+bx^2)^3} - \frac{143x^5}{4096b^6(a+bx^2)^4}$$

$$- \frac{143x^7}{3584b^5(a+bx^2)^5} - \frac{715x^9}{16128b^4(a+bx^2)^6} - \frac{65x^{11}}{1344b^3(a+bx^2)^7} - \frac{5x^{13}}{96b^2(a+bx^2)^8} - \frac{x^{15}}{18b(a+bx^2)^9}$$

[Out]  $-x^{15}/(18*b*(a+b*x^2)^9) - (5*x^{13})/(96*b^2*(a+b*x^2)^8) - (65*x^{11})/(1344*b^3*(a+b*x^2)^7) - (715*x^9)/(16128*b^4*(a+b*x^2)^6) - (143*x^7)/(3584*b^5*(a+b*x^2)^5) - (143*x^5)/(4096*b^6*(a+b*x^2)^4) - (715*x^3)/(24576*b^7*(a+b*x^2)^3) - (715*x)/(32768*b^8*(a+b*x^2)^2) + (715*x)/(65536*a*b^8*(a+b*x^2)) + (715*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(3/2)*b^(17/2))$

**Rubi [A]** time = 0.314153, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{715 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{3/2}b^{17/2}} + \frac{715x}{65536ab^8(a+bx^2)} - \frac{715x}{32768b^8(a+bx^2)^2} - \frac{715x^3}{24576b^7(a+bx^2)^3} - \frac{143x^5}{4096b^6(a+bx^2)^4}$$

$$- \frac{143x^7}{3584b^5(a+bx^2)^5} - \frac{715x^9}{16128b^4(a+bx^2)^6} - \frac{65x^{11}}{1344b^3(a+bx^2)^7} - \frac{5x^{13}}{96b^2(a+bx^2)^8} - \frac{x^{15}}{18b(a+bx^2)^9}$$

Antiderivative was successfully verified.

[In] Int[x^16/(a + b\*x^2)^10, x]

[Out]  $-x^{15}/(18*b*(a+b*x^2)^9) - (5*x^{13})/(96*b^2*(a+b*x^2)^8) - (65*x^{11})/(1344*b^3*(a+b*x^2)^7) - (715*x^9)/(16128*b^4*(a+b*x^2)^6) - (143*x^7)/(3584*b^5*(a+b*x^2)^5) - (143*x^5)/(4096*b^6*(a+b*x^2)^4) - (715*x^3)/(24576*b^7*(a+b*x^2)^3) - (715*x)/(32768*b^8*(a+b*x^2)^2) + (715*x)/(65536*a*b^8*(a+b*x^2)) + (715*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(3/2)*b^(17/2))$

**Rubi in Sympy [A]** time = 50.6954, size = 187, normalized size = 0.94

$$-\frac{x^{15}}{18b(a+bx^2)^9} - \frac{5x^{13}}{96b^2(a+bx^2)^8} - \frac{65x^{11}}{1344b^3(a+bx^2)^7} - \frac{715x^9}{16128b^4(a+bx^2)^6} - \frac{143x^7}{3584b^5(a+bx^2)^5}$$

$$- \frac{143x^5}{4096b^6(a+bx^2)^4} - \frac{715x^3}{24576b^7(a+bx^2)^3} - \frac{715x}{32768b^8(a+bx^2)^2} + \frac{715x}{65536ab^8(a+bx^2)} + \frac{715 \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{3/2}b^{17/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*16/(b\*x\*\*2+a)\*\*10, x)

[Out]  $-x^{15}/(18*b*(a+b*x^2)^9) - 5*x^{13}/(96*b^2*(a+b*x^2)^8) - 65*x^{11}/(1344*b^3*(a+b*x^2)^7) - 715*x^9/(16128*b^4*(a+b*x^2)^6) - 143*x^7/(3584*b^5*(a+b*x^2)^5) - 143*x^5/(4096*b^6*(a+b*x^2)^4) - 715*x^3/(24576*b^7*(a+b*x^2)^3) - 715*x/(32768*b^8*(a+b*x^2)^2) + 715*x/(65536*a*b^8*(a+b*x^2)) + 715*atan(sqrt(b)*x/sqrt(a))/(65536*a^(3/2)*b^(17/2))$



**Mathematica [A]** time = 0.139646, size = 138, normalized size = 0.7

$$\frac{\sqrt{a}\sqrt{bx}(-45045a^8-390390a^7bx^2-1495494a^6b^2x^4-3317886a^5b^3x^6-4685824a^4b^4x^8-4349826a^3b^5x^{10}-2633274a^2b^6x^{12}-985866ab^7x^{14}+45045b^8x^{16})}{(a+bx^2)^9} + 45045 \frac{4128768a^{3/2}b^{17/2}}{4128768a^{3/2}b^{17/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^16/(a + b\*x^2)^10,x]

[Out] ((Sqrt[a]\*Sqrt[b]\*x\*(-45045\*a^8 - 390390\*a^7\*b\*x^2 - 1495494\*a^6\*b^2\*x^4 - 3317886\*a^5\*b^3\*x^6 - 4685824\*a^4\*b^4\*x^8 - 4349826\*a^3\*b^5\*x^10 - 2633274\*a^2\*b^6\*x^12 - 985866\*a\*b^7\*x^14 + 45045\*b^8\*x^16))/(a + b\*x^2)^9 + 45045\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/(4128768\*a^(3/2)\*b^(17/2))

**Maple [A]** time = 0.021, size = 124, normalized size = 0.6

$$\frac{1}{(bx^2 + a)^9} \left( -\frac{715 a^7 x}{65536 b^8} - \frac{9295 a^6 x^3}{98304 b^7} - \frac{11869 a^5 x^5}{32768 b^6} - \frac{184327 a^4 x^7}{229376 b^5} - \frac{143 a^3 x^9}{126 b^4} - \frac{241657 a^2 x^{11}}{229376 b^3} - \frac{20899 a x^{13}}{32768 b^2} - \frac{23473 x^{15}}{98304 b} \right) + \frac{715}{65536 ab^8} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^16/(b\*x^2+a)^10,x)

[Out] (-715/65536\*a^7/b^8\*x-9295/98304\*a^6/b^7\*x^3-11869/32768\*a^5/b^6\*x^5-184327/229376\*a^4/b^5\*x^7-143/126\*a^3/b^4\*x^9-241657/229376\*a^2/b^3\*x^11-20899/32768\*a/b^2\*x^13-23473/98304/b\*x^15+715/65536/a\*x^17)/(b\*x^2+a)^9+715/65536/a/b^8/(a\*b)^(1/2)\*arctan(x\*b/(a\*b)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^16/(b\*x^2 + a)^10,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.215625, size = 1, normalized size = 0.01

$$\frac{45045 (b^9 x^{18} + 9 ab^8 x^{16} + 36 a^2 b^7 x^{14} + 84 a^3 b^6 x^{12} + 126 a^4 b^5 x^{10} + 126 a^5 b^4 x^8 + 84 a^6 b^3 x^6 + 36 a^7 b^2 x^4 + 9 a^8 b x^2 + a^9) \log\left(\frac{2 a b x + (b x^2 - a)}{8257536 (ab^{17} x^{18} + 9 a^2 b^{16} x^{16} + 36 a^3 b^{15} x^{14} + 84 a^4 b^{14} x^{12} + 126 a^5 b^{13} x^{10} + 126 a^6 b^{12} x^8 + 84 a^7 b^{11} x^6 + 36 a^8 b^{10} x^4 + 9 a^9 b^9 x^2 + a^{10})}\right)}{8257536 (ab^{17} x^{18} + 9 a^2 b^{16} x^{16} + 36 a^3 b^{15} x^{14} + 84 a^4 b^{14} x^{12} + 126 a^5 b^{13} x^{10} + 126 a^6 b^{12} x^8 + 84 a^7 b^{11} x^6 + 36 a^8 b^{10} x^4 + 9 a^9 b^9 x^2 + a^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^16/(b\*x^2 + a)^10,x, algorithm="fricas")

[Out] [1/8257536\*(45045\*(b^9\*x^18 + 9\*a\*b^8\*x^16 + 36\*a^2\*b^7\*x^14 + 84\*a^3\*b^6\*x^12 + 126\*a^4\*b^5\*x^10 + 126\*a^5\*b^4\*x^8 + 84\*a^6\*b^3\*x^6 + 36\*a^7\*b^2\*x^4 + 9\*a^8\*b\*x^2 + a^9)\*log((2\*a\*b\*x + (b\*x^2 - a))

$$a) \sqrt{-a^*b}) / (b^*x^2 + a)) + 2 * (45045 * b^8 * x^{17} - 985866 * a * b^7 * x^{15} - 2633274 * a^2 * b^6 * x^{13} - 4349826 * a^3 * b^5 * x^{11} - 4685824 * a^4 * b^4 * x^9 - 3317886 * a^5 * b^3 * x^7 - 1495494 * a^6 * b^2 * x^5 - 390390 * a^7 * b * x^3 - 45045 * a^8 * x) * \sqrt{-a^*b}) / ((a^*b^{17} * x^{18} + 9 * a^2 * b^{16} * x^{16} + 36 * a^3 * b^{15} * x^{14} + 84 * a^4 * b^{14} * x^{12} + 126 * a^5 * b^{13} * x^{10} + 126 * a^6 * b^{12} * x^8 + 84 * a^7 * b^{11} * x^6 + 36 * a^8 * b^{10} * x^4 + 9 * a^9 * b^9 * x^2 + a^{10} * b^8) * \sqrt{-a^*b}), 1/4128768 * (45045 * (b^9 * x^{18} + 9 * a * b^8 * x^{16} + 36 * a^2 * b^7 * x^{14} + 84 * a^3 * b^6 * x^{12} + 126 * a^4 * b^5 * x^{10} + 126 * a^5 * b^4 * x^8 + 84 * a^6 * b^3 * x^6 + 36 * a^7 * b^2 * x^4 + 9 * a^8 * b * x^2 + a^9) * \arctan(\sqrt{a^*b}) * x/a + (45045 * b^8 * x^{17} - 985866 * a * b^7 * x^{15} - 2633274 * a^2 * b^6 * x^{13} - 4349826 * a^3 * b^5 * x^{11} - 4685824 * a^4 * b^4 * x^9 - 3317886 * a^5 * b^3 * x^7 - 1495494 * a^6 * b^2 * x^5 - 390390 * a^7 * b * x^3 - 45045 * a^8 * x) * \sqrt{a^*b}) / ((a^*b^{17} * x^{18} + 9 * a^2 * b^{16} * x^{16} + 36 * a^3 * b^{15} * x^{14} + 84 * a^4 * b^{14} * x^{12} + 126 * a^5 * b^{13} * x^{10} + 126 * a^6 * b^{12} * x^8 + 84 * a^7 * b^{11} * x^6 + 36 * a^8 * b^{10} * x^4 + 9 * a^9 * b^9 * x^2 + a^{10} * b^8) * \sqrt{a^*b})]$$

**Sympy [A]** time = 35.5849, size = 289, normalized size = 1.46

$$\frac{715 \sqrt{-\frac{1}{a^3 b^{17}}} \log\left(-a^2 b^8 \sqrt{-\frac{1}{a^3 b^{17}}} + x\right)}{131072} + \frac{715 \sqrt{-\frac{1}{a^3 b^{17}}} \log\left(a^2 b^8 \sqrt{-\frac{1}{a^3 b^{17}}} + x\right)}{131072} + \frac{-45045 a^8 x - 390390 a^7 b x^3 - 1495494 a^6 b^2 x^5 - 3317886 a^5 b^3 x^7 - 4685824 a^4 b^4 x^9 - 4349826 a^3 b^5 x^{11} - 390390 a^2 b^6 x^{13} - 2633274 a b^7 x^{15} - 45045 a^8 x}{4128768 a^{10} b^8 + 37158912 a^9 b^9 x^2 + 148635648 a^8 b^{10} x^4 + 346816512 a^7 b^{11} x^6 + 520224768 a^6 b^{12} x^8 + 520224768 a^5 b^{13} x^{10} + 346816512 a^4 b^{14} x^{12} + 148635648 a^3 b^{15} x^{14} + 37158912 a^2 b^{16} x^{16} + 4128768 a b^{17} x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*16/(b\*x\*\*2+a)\*\*10,x)

[Out] -715 \* sqrt(-1/(a\*\*3\*b\*\*17)) \* log(-a\*\*2\*b\*\*8 \* sqrt(-1/(a\*\*3\*b\*\*17)) + x)/131072 + 715 \* sqrt(-1/(a\*\*3\*b\*\*17)) \* log(a\*\*2\*b\*\*8 \* sqrt(-1/(a\*\*3\*b\*\*17)) + x)/131072 + (-45045 \* a\*\*8 \* x - 390390 \* a\*\*7 \* b \* x\*\*3 - 1495494 \* a\*\*6 \* b\*\*2 \* x\*\*5 - 3317886 \* a\*\*5 \* b\*\*3 \* x\*\*7 - 4685824 \* a\*\*4 \* b\*\*4 \* x\*\*9 - 4349826 \* a\*\*3 \* b\*\*5 \* x\*\*11 - 2633274 \* a\*\*2 \* b\*\*6 \* x\*\*13 - 985866 \* a \* b\*\*7 \* x\*\*15 + 45045 \* b\*\*8 \* x\*\*17) / (4128768 \* a\*\*10 \* b\*\*8 + 37158912 \* a\*\*9 \* b\*\*9 \* x\*\*2 + 148635648 \* a\*\*8 \* b\*\*10 \* x\*\*4 + 346816512 \* a\*\*7 \* b\*\*11 \* x\*\*6 + 520224768 \* a\*\*6 \* b\*\*12 \* x\*\*8 + 520224768 \* a\*\*5 \* b\*\*13 \* x\*\*10 + 346816512 \* a\*\*4 \* b\*\*14 \* x\*\*12 + 148635648 \* a\*\*3 \* b\*\*15 \* x\*\*14 + 37158912 \* a\*\*2 \* b\*\*16 \* x\*\*16 + 4128768 \* a \* b\*\*17 \* x\*\*18)

**GIAC/XCAS [A]** time = 0.208613, size = 173, normalized size = 0.87

$$\frac{715 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{abab^8}} + \frac{45045 b^8 x^{17} - 985866 ab^7 x^{15} - 2633274 a^2 b^6 x^{13} - 4349826 a^3 b^5 x^{11} - 4685824 a^4 b^4 x^9 - 3317886 a^5 b^3 x^7 - 1495494 a^6 b^2 x^5 - 390390 a^7 b x^3 - 45045 a^8 x}{4128768 (bx^2 + a)^9 ab^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^16/(b\*x^2 + a)^10,x, algorithm="giac")

[Out] 715/65536 \* arctan(b\*x/sqrt(a\*b)) / (sqrt(a\*b) \* a\*b^8) + 1/4128768 \* (45045 \* b^8 \* x^{17} - 985866 \* a \* b^7 \* x^{15} - 2633274 \* a^2 \* b^6 \* x^{13} - 4349826 \* a^3 \* b^5 \* x^{11} - 4685824 \* a^4 \* b^4 \* x^9 - 3317886 \* a^5 \* b^3 \* x^7 - 1495494 \* a^6 \* b^2 \* x^5 - 390390 \* a^7 \* b \* x^3 - 45045 \* a^8 \* x) / ((b\*x^2 + a)^9 \* a \* b^8)

$$3.214 \quad \int \frac{x^{14}}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=199

$$\begin{aligned} & \frac{143 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{5/2}b^{15/2}} + \frac{143x}{65536a^2b^7(a+bx^2)} + \frac{143x}{98304ab^7(a+bx^2)^2} \\ & - \frac{143x}{24576b^7(a+bx^2)^3} - \frac{143x^3}{12288b^6(a+bx^2)^4} - \frac{143x^5}{7680b^5(a+bx^2)^5} \\ & - \frac{143x^7}{5376b^4(a+bx^2)^6} - \frac{143x^9}{4032b^3(a+bx^2)^7} - \frac{13x^{11}}{288b^2(a+bx^2)^8} - \frac{x^{13}}{18b(a+bx^2)^9} \end{aligned}$$

[Out]  $-x^{13}/(18*b*(a+b*x^2)^9) - (13*x^{11})/(288*b^2*(a+b*x^2)^8) - (143*x^9)/(4032*b^3*(a+b*x^2)^7) - (143*x^7)/(5376*b^4*(a+b*x^2)^6) - (143*x^5)/(7680*b^5*(a+b*x^2)^5) - (143*x^3)/(12288*b^6*(a+b*x^2)^4) - (143*x)/(24576*b^7*(a+b*x^2)^3) + (143*x)/(98304*a*b^7*(a+b*x^2)^2) + (143*x)/(65536*a^2*b^7*(a+b*x^2)) + (143*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^{(5/2)}*b^{(15/2)})$

**Rubi [A]** time = 0.311144, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\begin{aligned} & \frac{143 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{5/2}b^{15/2}} + \frac{143x}{65536a^2b^7(a+bx^2)} + \frac{143x}{98304ab^7(a+bx^2)^2} \\ & - \frac{143x}{24576b^7(a+bx^2)^3} - \frac{143x^3}{12288b^6(a+bx^2)^4} - \frac{143x^5}{7680b^5(a+bx^2)^5} \\ & - \frac{143x^7}{5376b^4(a+bx^2)^6} - \frac{143x^9}{4032b^3(a+bx^2)^7} - \frac{13x^{11}}{288b^2(a+bx^2)^8} - \frac{x^{13}}{18b(a+bx^2)^9} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^14/(a + b\*x^2)^10, x]

[Out]  $-x^{13}/(18*b*(a+b*x^2)^9) - (13*x^{11})/(288*b^2*(a+b*x^2)^8) - (143*x^9)/(4032*b^3*(a+b*x^2)^7) - (143*x^7)/(5376*b^4*(a+b*x^2)^6) - (143*x^5)/(7680*b^5*(a+b*x^2)^5) - (143*x^3)/(12288*b^6*(a+b*x^2)^4) - (143*x)/(24576*b^7*(a+b*x^2)^3) + (143*x)/(98304*a*b^7*(a+b*x^2)^2) + (143*x)/(65536*a^2*b^7*(a+b*x^2)) + (143*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^{(5/2)}*b^{(15/2)})$

**Rubi in Sympy [A]** time = 49.7464, size = 189, normalized size = 0.95

$$\begin{aligned} & -\frac{x^{13}}{18b(a+bx^2)^9} - \frac{13x^{11}}{288b^2(a+bx^2)^8} - \frac{143x^9}{4032b^3(a+bx^2)^7} - \frac{143x^7}{5376b^4(a+bx^2)^6} \\ & - \frac{143x^5}{7680b^5(a+bx^2)^5} - \frac{143x^3}{12288b^6(a+bx^2)^4} - \frac{143x}{24576b^7(a+bx^2)^3} \\ & + \frac{143x}{98304ab^7(a+bx^2)^2} + \frac{143x}{65536a^2b^7(a+bx^2)} + \frac{143 \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{5/2}b^{15/2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*14/(b\*x\*\*2+a)\*\*10, x)

[Out]  $-x^{13}/(18*b*(a+b*x^2)^9) - 13*x^{11}/(288*b^2*(a+b*x^2)^8) - 143*x^9/(4032*b^3*(a+b*x^2)^7) - 143*x^7/(5376*b^4*(a+b*x^2)^6) - 143*x^5/(7680*b^5*(a+b*x^2)^5) - 143*x^3/(12288*b^6*(a+b*x^2)^4) - 143*x/(24576*b^7*(a+b*x^2)^3) + 143*x/(98304*a*b^7*(a+b*x^2)^2) + 143*x/(65536*a^2*b^7*(a+b*x^2)) + 143*atan(sqrt(b*x)/sqrt(a))/(65536*a^{5/2}*b^{15/2})$

) + 143\*x/(98304\*a\*b\*\*7\*(a + b\*x\*\*2)\*\*2) + 143\*x/(65536\*a\*\*2\*b\*\*7\*(a + b\*x\*\*2)) + 143\*atan(sqrt(b)\*x/sqrt(a))/(65536\*a\*\*(5/2)\*b\*\*(15/2))

**Mathematica [A]** time = 0.122365, size = 138, normalized size = 0.69

$$\frac{\sqrt{a}\sqrt{bx}(-45045a^8-390390a^7bx^2-1495494a^6b^2x^4-3317886a^5b^3x^6-4685824a^4b^4x^8-4349826a^3b^5x^{10}-2633274a^2b^6x^{12}+390390ab^7x^{14}+45045b^8x^{16})}{(a+bx^2)^9} + 45045 \frac{1}{20643840a^{5/2}b^{15/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^14/(a + b\*x^2)^10,x]

[Out] ((Sqrt[a]\*Sqrt[b]\*x\*(-45045\*a^8 - 390390\*a^7\*b\*x^2 - 1495494\*a^6\*b^2\*x^4 - 3317886\*a^5\*b^3\*x^6 - 4685824\*a^4\*b^4\*x^8 - 4349826\*a^3\*b^5\*x^10 - 2633274\*a^2\*b^6\*x^12 + 390390\*a\*b^7\*x^14 + 45045\*b^8\*x^16))/(a + b\*x^2)^9 + 45045\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(20643840\*a^(5/2)\*b^(15/2))

**Maple [A]** time = 0.021, size = 122, normalized size = 0.6

$$\frac{1}{(bx^2 + a)^9} \left( -\frac{143 a^6 x}{65536 b^7} - \frac{1859 a^5 x^3}{98304 b^6} - \frac{11869 a^4 x^5}{163840 b^5} - \frac{184327 a^3 x^7}{1146880 b^4} - \frac{143 a^2 x^9}{630 b^3} - \frac{241657 a x^{11}}{1146880 b^2} - \frac{20899 x^{13}}{163840 b} + \frac{1859 x^{15}}{98304 a} + \frac{143}{65536 a^2 b^7} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^14/(b\*x^2+a)^10,x)

[Out] (-143/65536\*a^6/b^7\*x-1859/98304\*a^5/b^6\*x^3-11869/163840\*a^4/b^5\*x^5-184327/1146880\*a^3/b^4\*x^7-143/630\*a^2/b^3\*x^9-241657/1146880\*a/b^2\*x^11-20899/163840/b\*x^13+1859/98304/a\*x^15+143/65536\*b/a^2\*x^17)/(b\*x^2+a)^9+143/65536/a^2/b^7/(a\*b)^(1/2)\*arctan(x\*b/(a\*b)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14/(b\*x^2 + a)^10,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.217049, size = 1, normalized size = 0.01

$$\left[ \frac{45045 (b^9 x^{18} + 9 a b^8 x^{16} + 36 a^2 b^7 x^{14} + 84 a^3 b^6 x^{12} + 126 a^4 b^5 x^{10} + 126 a^5 b^4 x^8 + 84 a^6 b^3 x^6 + 36 a^7 b^2 x^4 + 9 a^8 b x^2 + a^9) \log}{41287680 (a^2 b^{16} x^{18} + 9 a^3 b^{15} x^{16} + 36 a^4 b^{14} x^{14} + \dots)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14/(b\*x^2 + a)^10,x, algorithm="fricas")

[Out] [1/41287680\*(45045\*(b^9\*x^18 + 9\*a\*b^8\*x^16 + 36\*a^2\*b^7\*x^14 + 84\*a^3\*b^6\*x^12 + 126\*a^4\*b^5\*x^10 + 126\*a^5\*b^4\*x^8 + 84\*a^6\*b^3\*x^6 + 36\*a^7\*b^2\*x^4 + 9\*a^8\*b\*x^2 + a^9)\*log((2\*a\*b\*x + (b\*x^2 - a)\*sqrt(-a\*b))/(b\*x^2 + a)) + 2\*(45045\*b^8\*x^17 + 390390\*a\*b^7\*x^15 - 2633274\*a^2\*b^6\*x^13 - 4349826\*a^3\*b^5\*x^11 - 4685824\*a^4\*b^4\*x^9 - 3317886\*a^5\*b^3\*x^7 - 1495494\*a^6\*b^2\*x^5 - 390390\*a^7\*b\*x^3 - 45045\*a^8\*x)\*sqrt(-a\*b))/((a^2\*b^16\*x^18 + 9\*a^3\*b^15\*x^16 + 36\*a^4\*b^14\*x^14 + 84\*a^5\*b^13\*x^12 + 126\*a^6\*b^12\*x^10 + 126\*a^7\*b^11\*x^8 + 84\*a^8\*b^10\*x^6 + 36\*a^9\*b^9\*x^4 + 9\*a^10\*b^8\*x^2 + a^11\*b^7)\*sqrt(-a\*b)), 1/20643840\*(45045\*(b^9\*x^18 + 9\*a\*b^8\*x^16 + 36\*a^2\*b^7\*x^14 + 84\*a^3\*b^6\*x^12 + 126\*a^4\*b^5\*x^10 + 126\*a^5\*b^4\*x^8 + 84\*a^6\*b^3\*x^6 + 36\*a^7\*b^2\*x^4 + 9\*a^8\*b\*x^2 + a^9)\*arctan(sqrt(a\*b)\*x/a) + (45045\*b^8\*x^17 + 390390\*a\*b^7\*x^15 - 2633274\*a^2\*b^6\*x^13 - 4349826\*a^3\*b^5\*x^11 - 4685824\*a^4\*b^4\*x^9 - 3317886\*a^5\*b^3\*x^7 - 1495494\*a^6\*b^2\*x^5 - 390390\*a^7\*b\*x^3 - 45045\*a^8\*x)\*sqrt(a\*b))/((a^2\*b^16\*x^18 + 9\*a^3\*b^15\*x^16 + 36\*a^4\*b^14\*x^14 + 84\*a^5\*b^13\*x^12 + 126\*a^6\*b^12\*x^10 + 126\*a^7\*b^11\*x^8 + 84\*a^8\*b^10\*x^6 + 36\*a^9\*b^9\*x^4 + 9\*a^10\*b^8\*x^2 + a^11\*b^7)\*sqrt(a\*b))]

**Sympy [A]** time = 35.2115, size = 291, normalized size = 1.46

$$\frac{143\sqrt{-\frac{1}{a^5b^{15}}}\log\left(-a^3b^7\sqrt{-\frac{1}{a^5b^{15}}}+x\right)}{131072} + \frac{143\sqrt{-\frac{1}{a^5b^{15}}}\log\left(a^3b^7\sqrt{-\frac{1}{a^5b^{15}}}+x\right)}{131072} + \frac{-45045a^8x - 390390a^7bx^3 - 1495494a^6b^2x^5 - 3317886a^5b^3x^7 - 4685824a^4b^4x^9 - 4349826a^3b^5x^{11} - 4685824a^2b^6x^{13} + 390390a^10b^8x^2 + 743178240a^9b^9x^4 + 1734082560a^8b^{10}x^6 + 2601123840a^7b^{11}x^8 + 2601123840a^6b^{12}x^{10}}{20643840a^{11}b^7 + 185794560a^{10}b^8x^2 + 743178240a^9b^9x^4 + 1734082560a^8b^{10}x^6 + 2601123840a^7b^{11}x^8 + 2601123840a^6b^{12}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*14/(b\*x\*\*2+a)\*\*10,x)

[Out] -143\*sqrt(-1/(a\*\*5\*b\*\*15))\*log(-a\*\*3\*b\*\*7\*sqrt(-1/(a\*\*5\*b\*\*15)) + x)/131072 + 143\*sqrt(-1/(a\*\*5\*b\*\*15))\*log(a\*\*3\*b\*\*7\*sqrt(-1/(a\*\*5\*b\*\*15)) + x)/131072 + (-45045\*a\*\*8\*x - 390390\*a\*\*7\*b\*x\*\*3 - 1495494\*a\*\*6\*b\*\*2\*x\*\*5 - 3317886\*a\*\*5\*b\*\*3\*x\*\*7 - 4685824\*a\*\*4\*b\*\*4\*x\*\*9 - 4349826\*a\*\*3\*b\*\*5\*x\*\*11 - 2633274\*a\*\*2\*b\*\*6\*x\*\*13 + 390390\*a\*b\*\*7\*x\*\*15 + 45045\*b\*\*8\*x\*\*17)/(20643840\*a\*\*11\*b\*\*7 + 185794560\*a\*\*10\*b\*\*8\*x\*\*2 + 743178240\*a\*\*9\*b\*\*9\*x\*\*4 + 1734082560\*a\*\*8\*b\*\*10\*x\*\*6 + 2601123840\*a\*\*7\*b\*\*11\*x\*\*8 + 2601123840\*a\*\*6\*b\*\*12\*x\*\*10 + 1734082560\*a\*\*5\*b\*\*13\*x\*\*12 + 743178240\*a\*\*4\*b\*\*14\*x\*\*14 + 185794560\*a\*\*3\*b\*\*15\*x\*\*16 + 20643840\*a\*\*2\*b\*\*16\*x\*\*18)

**GIAC/XCAS [A]** time = 0.210483, size = 173, normalized size = 0.87

$$\frac{143 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{aba^2b^7}} + \frac{45045b^8x^{17} + 390390ab^7x^{15} - 2633274a^2b^6x^{13} - 4349826a^3b^5x^{11} - 4685824a^4b^4x^9 - 3317886a^5b^3x^7 - 1495494a^6b^2x^5 - 4349826a^7bx^3 - 45045a^8x}{20643840(bx^2 + a)^9a^2b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14/(b\*x^2 + a)^10,x, algorithm="giac")

[Out] 143/65536\*arctan(b\*x/sqrt(a\*b))/(sqrt(a\*b)\*a^2\*b^7) + 1/20643840\*(45045\*b^8\*x^17 + 390390\*a\*b^7\*x^15 - 2633274\*a^2\*b^6\*x^13 - 4349826\*a^3\*b^5\*x^11 - 4685824\*a^4\*b^4\*x^9 - 3317886\*a^5\*b^3\*x^7 - 1495494\*a^6\*b^2\*x^5 - 390390\*a^7\*b\*x^3 - 45045\*a^8\*x)/((b\*x^2 + a)^9\*a^2\*b^7)

$$3.215 \quad \int \frac{x^{12}}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=200

$$\begin{aligned} & \frac{55 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{7/2}b^{13/2}} + \frac{55x}{65536a^3b^6(a+bx^2)} + \frac{55x}{98304a^2b^6(a+bx^2)^2} \\ & + \frac{11x}{24576ab^6(a+bx^2)^3} - \frac{11x}{4096b^6(a+bx^2)^4} - \frac{11x^3}{1536b^5(a+bx^2)^5} \\ & - \frac{11x^5}{768b^4(a+bx^2)^6} - \frac{11x^7}{448b^3(a+bx^2)^7} - \frac{11x^9}{288b^2(a+bx^2)^8} - \frac{x^{11}}{18b(a+bx^2)^9} \end{aligned}$$

[Out]  $-x^{11}/(18*b*(a+b*x^2)^9) - (11*x^9)/(288*b^2*(a+b*x^2)^8) - (11*x^7)/(448*b^3*(a+b*x^2)^7) - (11*x^5)/(768*b^4*(a+b*x^2)^6) - (11*x^3)/(1536*b^5*(a+b*x^2)^5) - (11*x)/(4096*b^6*(a+b*x^2)^4) + (11*x)/(24576*a*b^6*(a+b*x^2)^3) + (55*x)/(98304*a^2*b^6*(a+b*x^2)^2) + (55*x)/(65536*a^3*b^6*(a+b*x^2)) + (55*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(7/2)*b^(13/2))$

**Rubi [A]** time = 0.305134, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\begin{aligned} & \frac{55 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{7/2}b^{13/2}} + \frac{55x}{65536a^3b^6(a+bx^2)} + \frac{55x}{98304a^2b^6(a+bx^2)^2} \\ & + \frac{11x}{24576ab^6(a+bx^2)^3} - \frac{11x}{4096b^6(a+bx^2)^4} - \frac{11x^3}{1536b^5(a+bx^2)^5} \\ & - \frac{11x^5}{768b^4(a+bx^2)^6} - \frac{11x^7}{448b^3(a+bx^2)^7} - \frac{11x^9}{288b^2(a+bx^2)^8} - \frac{x^{11}}{18b(a+bx^2)^9} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^12/(a + b\*x^2)^10, x]

[Out]  $-x^{11}/(18*b*(a+b*x^2)^9) - (11*x^9)/(288*b^2*(a+b*x^2)^8) - (11*x^7)/(448*b^3*(a+b*x^2)^7) - (11*x^5)/(768*b^4*(a+b*x^2)^6) - (11*x^3)/(1536*b^5*(a+b*x^2)^5) - (11*x)/(4096*b^6*(a+b*x^2)^4) + (11*x)/(24576*a*b^6*(a+b*x^2)^3) + (55*x)/(98304*a^2*b^6*(a+b*x^2)^2) + (55*x)/(65536*a^3*b^6*(a+b*x^2)) + (55*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(7/2)*b^(13/2))$

**Rubi in Sympy [A]** time = 48.5058, size = 190, normalized size = 0.95

$$\begin{aligned} & -\frac{x^{11}}{18b(a+bx^2)^9} - \frac{11x^9}{288b^2(a+bx^2)^8} - \frac{11x^7}{448b^3(a+bx^2)^7} - \frac{11x^5}{768b^4(a+bx^2)^6} \\ & - \frac{11x^3}{1536b^5(a+bx^2)^5} - \frac{11x}{4096b^6(a+bx^2)^4} + \frac{11x}{24576ab^6(a+bx^2)^3} \\ & + \frac{55x}{98304a^2b^6(a+bx^2)^2} + \frac{55x}{65536a^3b^6(a+bx^2)} + \frac{55 \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{7/2}b^{13/2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*12/(b\*x\*\*2+a)\*\*10, x)

[Out]  $-x^{11}/(18*b*(a+b*x^2)^9) - 11*x^9/(288*b^2*(a+b*x^2)^8) - 11*x^7/(448*b^3*(a+b*x^2)^7) - 11*x^5/(768*b^4*(a+b*x^2)^6) - 11*x^3/(1536*b^5*(a+b*x^2)^5) - 11*x/(4096*b^6*(a+b*x^2)^4) + 11*x/(24576*a*b^6*(a+b*x^2)^3) + 55*x/($

$$98304 a^{**2} b^{**6} (a + b x^{**2})^{**2} + 55 x / (65536 a^{**3} b^{**6} (a + b x^{**2})) + 55 \operatorname{atan}(\sqrt{b} x / \sqrt{a}) / (65536 a^{**7/2} b^{**13/2})$$

**Mathematica [A]** time = 0.142483, size = 138, normalized size = 0.69

$$\frac{\sqrt{a}\sqrt{b}x(-3465a^8-30030a^7bx^2-115038a^6b^2x^4-255222a^5b^3x^6-360448a^4b^4x^8-334602a^3b^5x^{10}+115038a^2b^6x^{12}+30030ab^7x^{14}+3465b^8x^{16})}{(a+bx^2)^9} + 3465 \tan^{-1}\left(\frac{y}{x}\right)}{4128768a^{7/2}b^{13/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^12/(a + b\*x^2)^10,x]

[Out] ((Sqrt[a]\*Sqrt[b]\*x\*(-3465\*a^8 - 30030\*a^7\*b\*x^2 - 115038\*a^6\*b^2\*x^4 - 255222\*a^5\*b^3\*x^6 - 360448\*a^4\*b^4\*x^8 - 334602\*a^3\*b^5\*x^10 + 115038\*a^2\*b^6\*x^12 + 30030\*a\*b^7\*x^14 + 3465\*b^8\*x^16))/(a + b\*x^2)^9 + 3465\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(4128768\*a^(7/2)\*b^(13/2))

**Maple [A]** time = 0.02, size = 122, normalized size = 0.6

$$\frac{1}{(bx^2+a)^9} \left( -\frac{55a^5x}{65536b^6} - \frac{715a^4x^3}{98304b^5} - \frac{913a^3x^5}{32768b^4} - \frac{14179a^2x^7}{229376b^3} - \frac{11ax^9}{126b^2} - \frac{18589x^{11}}{229376b} + \frac{913x^{13}}{32768a} + \frac{715bx^{15}}{98304a^2} + \frac{55b^2x^{17}}{65536a^3} \right) + \frac{55}{65536a^3b^6} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^12/(b\*x^2+a)^10,x)

[Out] (-55/65536\*a^5/b^6\*x-715/98304\*a^4/b^5\*x^3-913/32768\*a^3/b^4\*x^5-14179/229376\*a^2/b^3\*x^7-11/126\*a/b^2\*x^9-18589/229376/b\*x^11+913/32768/a\*x^13+715/98304\*b/a^2\*x^15+55/65536\*b^2/a^3\*x^17)/(b\*x^2+a)^9+55/65536/a^3/b^6/(a\*b)^(1/2)\*arctan(x\*b/(a\*b)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12/(b\*x^2 + a)^10,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.213414, size = 1, normalized size = 0.

$$\left[ \frac{3465 (b^9 x^{18} + 9 a b^8 x^{16} + 36 a^2 b^7 x^{14} + 84 a^3 b^6 x^{12} + 126 a^4 b^5 x^{10} + 126 a^5 b^4 x^8 + 84 a^6 b^3 x^6 + 36 a^7 b^2 x^4 + 9 a^8 b x^2 + a^9) \log\left(\frac{y}{x}\right)}{8257536 (a^3 b^{15} x^{18} + 9 a^4 b^{14} x^{16} + 36 a^5 b^{13} x^{14} + \dots)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12/(b\*x^2 + a)^10,x, algorithm="fricas")

[Out] 
$$\frac{1}{8257536} (3465 (b^9 x^{18} + 9 a b^8 x^{16} + 36 a^2 b^7 x^{14} + 84 a^3 b^6 x^{12} + 126 a^4 b^5 x^{10} + 126 a^5 b^4 x^8 + 84 a^6 b^3 x^6 + 36 a^7 b^2 x^4 + 9 a^8 b x^2 + a^9) \log((2 a b x + (b x^2 - a) \sqrt{-a b}) / (b x^2 + a)) + 2 (3465 b^8 x^{17} + 30030 a b^7 x^{15} + 115038 a^2 b^6 x^{13} - 334602 a^3 b^5 x^{11} - 360448 a^4 b^4 x^9 - 255222 a^5 b^3 x^7 - 115038 a^6 b^2 x^5 - 30030 a^7 b x^3 - 3465 a^8 x) \sqrt{-a b}) / ((a^3 b^{15} x^{18} + 9 a^4 b^{14} x^{16} + 36 a^5 b^{13} x^{14} + 84 a^6 b^{12} x^{12} + 126 a^7 b^{11} x^{10} + 126 a^8 b^{10} x^8 + 84 a^9 b^9 x^6 + 36 a^{10} b^8 x^4 + 9 a^{11} b^7 x^2 + a^{12} b^6) \sqrt{-a b}), \frac{1}{4128768} (3465 (b^9 x^{18} + 9 a b^8 x^{16} + 36 a^2 b^7 x^{14} + 84 a^3 b^6 x^{12} + 126 a^4 b^5 x^{10} + 126 a^5 b^4 x^8 + 84 a^6 b^3 x^6 + 36 a^7 b^2 x^4 + 9 a^8 b x^2 + a^9) \arctan(\sqrt{a b} x / a) + (3465 b^8 x^{17} + 30030 a b^7 x^{15} + 115038 a^2 b^6 x^{13} - 334602 a^3 b^5 x^{11} - 360448 a^4 b^4 x^9 - 255222 a^5 b^3 x^7 - 115038 a^6 b^2 x^5 - 30030 a^7 b x^3 - 3465 a^8 x) \sqrt{a b}) / ((a^3 b^{15} x^{18} + 9 a^4 b^{14} x^{16} + 36 a^5 b^{13} x^{14} + 84 a^6 b^{12} x^{12} + 126 a^7 b^{11} x^{10} + 126 a^8 b^{10} x^8 + 84 a^9 b^9 x^6 + 36 a^{10} b^8 x^4 + 9 a^{11} b^7 x^2 + a^{12} b^6) \sqrt{a b})]$$

**Sympy [A]** time = 34.9877, size = 291, normalized size = 1.46

$$\frac{55 \sqrt{-\frac{1}{a^7 b^{13}}} \log\left(-a^4 b^6 \sqrt{-\frac{1}{a^7 b^{13}}} + x\right)}{131072} + \frac{55 \sqrt{-\frac{1}{a^7 b^{13}}} \log\left(a^4 b^6 \sqrt{-\frac{1}{a^7 b^{13}}} + x\right)}{131072} + \frac{-3465 a^8 x - 30030 a^7 b x^3 - 115038 a^6 b^2 x^5 - 255222 a^5 b^3 x^7 - 360448 a^4 b^4 x^9 - 334602 a^3 b^5 x^{11} - 30030 a^2 b^6 x^{13} - 115038 a b^7 x^{15} - 3465 a^8 x^{17}}{4128768 a^{12} b^6 + 37158912 a^{11} b^7 x^2 + 148635648 a^{10} b^8 x^4 + 346816512 a^9 b^9 x^6 + 520224768 a^8 b^{10} x^8 + 520224768 a^7 b^{11} x^{10} + 346816512 a^6 b^{12} x^{12} + 148635648 a^5 b^{13} x^{14} + 30030 a^4 b^{14} x^{16} + 3465 a^3 b^{15} x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*12/(b\*x\*\*2+a)\*\*10,x)

[Out] 
$$-55 \sqrt{-1/(a^{**7} b^{**13})} \log(-a^{**4} b^{**6} \sqrt{-1/(a^{**7} b^{**13})} + x) / 131072 + 55 \sqrt{-1/(a^{**7} b^{**13})} \log(a^{**4} b^{**6} \sqrt{-1/(a^{**7} b^{**13})} + x) / 131072 + (-3465 a^{**8} x - 30030 a^{**7} b x^{**3} - 115038 a^{**6} b^{**2} x^{**5} - 255222 a^{**5} b^{**3} x^{**7} - 360448 a^{**4} b^{**4} x^{**9} - 334602 a^{**3} b^{**5} x^{**11} + 115038 a^{**2} b^{**6} x^{**13} + 30030 a b^{**7} x^{**15} + 3465 b^{**8} x^{**17}) / (4128768 a^{**12} b^{**6} + 37158912 a^{**11} b^{**7} x^{**2} + 148635648 a^{**10} b^{**8} x^{**4} + 346816512 a^{**9} b^{**9} x^{**6} + 520224768 a^{**8} b^{**10} x^{**8} + 520224768 a^{**7} b^{**11} x^{**10} + 346816512 a^{**6} b^{**12} x^{**12} + 148635648 a^{**5} b^{**13} x^{**14} + 37158912 a^{**4} b^{**14} x^{**16} + 4128768 a^{**3} b^{**15} x^{**18})$$

**GIAC/XCAS [A]** time = 0.239501, size = 173, normalized size = 0.86

$$\frac{55 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{aba^3 b^6}} + \frac{3465 b^8 x^{17} + 30030 ab^7 x^{15} + 115038 a^2 b^6 x^{13} - 334602 a^3 b^5 x^{11} - 360448 a^4 b^4 x^9 - 255222 a^5 b^3 x^7 - 115038 a^6 b^2 x^5 - 30030 a^7 b x^3 - 3465 a^8 x}{4128768 (bx^2 + a)^9 a^3 b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12/(b\*x^2 + a)^10,x, algorithm="giac")

[Out] 
$$55/65536 \arctan(bx/\sqrt{ab}) / (\sqrt{ab} a^3 b^6) + 1/4128768 (3465 b^8 x^{17} + 30030 a b^7 x^{15} + 115038 a^2 b^6 x^{13} - 334602 a^3 b^5 x^{11} - 360448 a^4 b^4 x^9 - 255222 a^5 b^3 x^7 - 115038 a^6 b^2 x^5 - 30030 a^7 b x^3 - 3465 a^8 x) / ((b x^2 + a)^9 a^3 b^6)$$



$$3.216 \quad \int \frac{x^{10}}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=201

$$\begin{aligned} & \frac{35 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{9/2}b^{11/2}} + \frac{35x}{65536a^4b^5(a+bx^2)} + \frac{35x}{98304a^3b^5(a+bx^2)^2} \\ & + \frac{7x}{24576a^2b^5(a+bx^2)^3} + \frac{x}{4096ab^5(a+bx^2)^4} - \frac{x}{512b^5(a+bx^2)^5} \\ & - \frac{5x^3}{768b^4(a+bx^2)^6} - \frac{x^5}{64b^3(a+bx^2)^7} - \frac{x^7}{32b^2(a+bx^2)^8} - \frac{x^9}{18b(a+bx^2)^9} \end{aligned}$$

[Out]  $-x^9/(18*b*(a+b*x^2)^9) - x^7/(32*b^2*(a+b*x^2)^8) - x^5/(64*b^3*(a+b*x^2)^7) - (5*x^3)/(768*b^4*(a+b*x^2)^6) - x/(512*b^5*(a+b*x^2)^5) + x/(4096*a*b^5*(a+b*x^2)^4) + (7*x)/(24576*a^2*b^5*(a+b*x^2)^3) + (35*x)/(98304*a^3*b^5*(a+b*x^2)^2) + (35*x)/(65536*a^4*b^5*(a+b*x^2)) + (35*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(9/2)*b^(11/2))$

**Rubi [A]** time = 0.295028, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\begin{aligned} & \frac{35 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{9/2}b^{11/2}} + \frac{35x}{65536a^4b^5(a+bx^2)} + \frac{35x}{98304a^3b^5(a+bx^2)^2} \\ & + \frac{7x}{24576a^2b^5(a+bx^2)^3} + \frac{x}{4096ab^5(a+bx^2)^4} - \frac{x}{512b^5(a+bx^2)^5} \\ & - \frac{5x^3}{768b^4(a+bx^2)^6} - \frac{x^5}{64b^3(a+bx^2)^7} - \frac{x^7}{32b^2(a+bx^2)^8} - \frac{x^9}{18b(a+bx^2)^9} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^10/(a + b\*x^2)^10, x]

[Out]  $-x^9/(18*b*(a+b*x^2)^9) - x^7/(32*b^2*(a+b*x^2)^8) - x^5/(64*b^3*(a+b*x^2)^7) - (5*x^3)/(768*b^4*(a+b*x^2)^6) - x/(512*b^5*(a+b*x^2)^5) + x/(4096*a*b^5*(a+b*x^2)^4) + (7*x)/(24576*a^2*b^5*(a+b*x^2)^3) + (35*x)/(98304*a^3*b^5*(a+b*x^2)^2) + (35*x)/(65536*a^4*b^5*(a+b*x^2)) + (35*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(9/2)*b^(11/2))$

**Rubi in Sympy [A]** time = 47.043, size = 185, normalized size = 0.92

$$\begin{aligned} & -\frac{x^9}{18b(a+bx^2)^9} - \frac{x^7}{32b^2(a+bx^2)^8} - \frac{x^5}{64b^3(a+bx^2)^7} - \frac{5x^3}{768b^4(a+bx^2)^6} \\ & - \frac{x}{512b^5(a+bx^2)^5} + \frac{x}{4096ab^5(a+bx^2)^4} + \frac{7x}{24576a^2b^5(a+bx^2)^3} \\ & + \frac{35x}{98304a^3b^5(a+bx^2)^2} + \frac{35x}{65536a^4b^5(a+bx^2)} + \frac{35 \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{\frac{9}{2}}b^{\frac{11}{2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*10/(b\*x\*\*2+a)\*\*10, x)

[Out]  $-x**9/(18*b*(a+b*x**2)**9) - x**7/(32*b**2*(a+b*x**2)**8) - x**5/(64*b**3*(a+b*x**2)**7) - 5*x**3/(768*b**4*(a+b*x**2)**6) - x/(512*b**5*(a+b*x**2)**5) + x/(4096*a*b**5*(a+b*x**2)**4) + 7*x/(24576*a**2*b**5*(a+b*x**2)**3) + 35*x/(98304*a**3*b**5*$

$$(a + b*x**2)**2 + 35*x/(65536*a**4*b**5*(a + b*x**2)) + 35*atan(\sqrt{b}*x/\sqrt{a})/(65536*a**(9/2)*b**(11/2))$$

**Mathematica [A]** time = 0.133548, size = 138, normalized size = 0.69

$$\frac{\sqrt{a}\sqrt{bx}(-315a^8-2730a^7bx^2-10458a^6b^2x^4-23202a^5b^3x^6-32768a^4b^4x^8+23202a^3b^5x^{10}+10458a^2b^6x^{12}+2730ab^7x^{14}+315b^8x^{16})}{(a+bx^2)^9} + 315 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)$$

$$589824a^{9/2}b^{11/2}$$

Antiderivative was successfully verified.

[In] Integrate[x^10/(a + b\*x^2)^10, x]

[Out] ((Sqrt[a]\*Sqrt[b]\*x\*(-315\*a^8 - 2730\*a^7\*b\*x^2 - 10458\*a^6\*b^2\*x^4 - 23202\*a^5\*b^3\*x^6 - 32768\*a^4\*b^4\*x^8 + 23202\*a^3\*b^5\*x^10 + 10458\*a^2\*b^6\*x^12 + 2730\*a\*b^7\*x^14 + 315\*b^8\*x^16))/(a + b\*x^2)^9 + 315\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(589824\*a^(9/2)\*b^(11/2))

**Maple [A]** time = 0.021, size = 122, normalized size = 0.6

$$\frac{1}{(bx^2 + a)^9} \left( -\frac{35a^4x}{65536b^5} - \frac{455a^3x^3}{98304b^4} - \frac{581a^2x^5}{32768b^3} - \frac{1289ax^7}{32768b^2} - \frac{x^9}{18b} + \frac{1289x^{11}}{32768a} + \frac{581bx^{13}}{32768a^2} + \frac{455b^2x^{15}}{98304a^3} + \frac{35b^3x^{17}}{65536a^4} \right) + \frac{35}{65536a^4b^5} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^10/(b\*x^2+a)^10, x)

[Out] (-35/65536\*a^4\*x/b^5-455/98304\*a^3\*x^3/b^4-581/32768\*a^2\*x^5/b^3-1289/32768\*a\*x^7/b^2-1/18\*x^9/b+1289/32768/a\*x^11+581/32768\*b/a^2\*x^13+455/98304\*b^2/a^3\*x^15+35/65536\*b^3/a^4\*x^17)/(b\*x^2+a)^9+35/65536/a^4/b^5/(a\*b)^(1/2)\*arctan(x\*b/(a\*b)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10/(b\*x^2 + a)^10, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.220914, size = 1, normalized size = 0.

$$\left[ \frac{315(b^9x^{18} + 9ab^8x^{16} + 36a^2b^7x^{14} + 84a^3b^6x^{12} + 126a^4b^5x^{10} + 126a^5b^4x^8 + 84a^6b^3x^6 + 36a^7b^2x^4 + 9a^8bx^2 + a^9) \log\left(\frac{2}{\dots}\right)}{1179648(a^4b^{14}x^{18} + 9a^5b^{13}x^{16} + 36a^6b^{12}x^{14} + 84a^7b^{11}x^{12} + 126a^8b^{10}x^{10} + 126a^9b^9x^8 + 84a^{10}b^8x^6 + 36a^{11}b^7x^4 + 9a^{12}b^6x^2 + a^{13})} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10/(b\*x^2 + a)^10, x, algorithm="fricas")

```
[Out] [1/1179648*(315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(315*b^8*x^17 + 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*b^5*x^11 - 32768*a^4*b^4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 2730*a^7*b*x^3 - 315*a^8*x)*sqrt(-a*b))/((a^4*b^14*x^18 + 9*a^5*b^13*x^16 + 36*a^6*b^12*x^14 + 84*a^7*b^11*x^12 + 126*a^8*b^10*x^10 + 126*a^9*b^9*x^8 + 84*a^10*b^8*x^6 + 36*a^11*b^7*x^4 + 9*a^12*b^6*x^2 + a^13*b^5)*sqrt(-a*b)), 1/589824*(315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*arctan(sqrt(a*b)*x/a) + (315*b^8*x^17 + 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*b^5*x^11 - 32768*a^4*b^4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 2730*a^7*b*x^3 - 315*a^8*x)*sqrt(a*b))/((a^4*b^14*x^18 + 9*a^5*b^13*x^16 + 36*a^6*b^12*x^14 + 84*a^7*b^11*x^12 + 126*a^8*b^10*x^10 + 126*a^9*b^9*x^8 + 84*a^10*b^8*x^6 + 36*a^11*b^7*x^4 + 9*a^12*b^6*x^2 + a^13*b^5)*sqrt(a*b))]
```

---

**Sympy [A]** time = 34.4376, size = 291, normalized size = 1.45

$$\frac{35\sqrt{-\frac{1}{a^9b^{11}}}\log\left(-a^5b^5\sqrt{-\frac{1}{a^9b^{11}}}+x\right)}{131072} + \frac{35\sqrt{-\frac{1}{a^9b^{11}}}\log\left(a^5b^5\sqrt{-\frac{1}{a^9b^{11}}}+x\right)}{131072} + \frac{-315a^8x - 2730a^7bx^3 - 10458a^6b^2x^5 - 23202a^5b^3x^7 - 32768a^4b^4x^9 + 23202a^3b^5x^{11} + 10458a^2b^6x^{13} + 23202a^13b^5 + 5308416a^{12}b^6x^2 + 21233664a^{11}b^7x^4 + 49545216a^{10}b^8x^6 + 74317824a^9b^9x^8 + 74317824a^8b^{10}x^{10} + 49545216a^7b^{11}x^{12} + 21233664a^6b^{12}x^{14} + 5308416a^5b^{13}x^{16} + 589824a^4b^{14}x^{18}}{589824(bx^2 + a)^9a^4b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*10/(b\*x\*\*2+a)\*\*10,x)

```
[Out] -35*sqrt(-1/(a**9*b**11))*log(-a**5*b**5*sqrt(-1/(a**9*b**11))+x)/131072 + 35*sqrt(-1/(a**9*b**11))*log(a**5*b**5*sqrt(-1/(a**9*b**11))+x)/131072 + (-315*a**8*x - 2730*a**7*b*x**3 - 10458*a**6*b**2*x**5 - 23202*a**5*b**3*x**7 - 32768*a**4*b**4*x**9 + 23202*a**3*b**5*x**11 + 10458*a**2*b**6*x**13 + 2730*a*b**7*x**15 + 315*b**8*x**17)/(589824*a**13*b**5 + 5308416*a**12*b**6*x**2 + 21233664*a**11*b**7*x**4 + 49545216*a**10*b**8*x**6 + 74317824*a**9*b**9*x**8 + 74317824*a**8*b**10*x**10 + 49545216*a**7*b**11*x**12 + 21233664*a**6*b**12*x**14 + 5308416*a**5*b**13*x**16 + 589824*a**4*b**14*x**18)
```

---

**GIAC/XCAS [A]** time = 0.238014, size = 173, normalized size = 0.86

$$\frac{35 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{aba^4b^5}} + \frac{315b^8x^{17} + 2730ab^7x^{15} + 10458a^2b^6x^{13} + 23202a^3b^5x^{11} - 32768a^4b^4x^9 - 23202a^5b^3x^7 - 10458a^6b^2x^5 - 2730a^7bx^3 - 315a^8x}{589824(bx^2 + a)^9a^4b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10/(b\*x^2 + a)^10,x, algorithm="giac")

```
[Out] 35/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b^5) + 1/589824*(315*b^8*x^17 + 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*b^5*x^11 - 32768*a^4*b^4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 2730*a^7*b*x^3 - 315*a^8*x)/((b*x^2 + a)^9*a^4*b^5)
```

$$3.217 \quad \int \frac{x^8}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=202

$$\begin{aligned} & \frac{35 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{11/2}b^{9/2}} + \frac{35x}{65536a^5b^4(a+bx^2)} + \frac{35x}{98304a^4b^4(a+bx^2)^2} \\ & + \frac{7x}{24576a^3b^4(a+bx^2)^3} + \frac{x}{4096a^2b^4(a+bx^2)^4} + \frac{x}{4608ab^4(a+bx^2)^5} \\ & - \frac{5x}{2304b^4(a+bx^2)^6} - \frac{5x^3}{576b^3(a+bx^2)^7} - \frac{7x^5}{288b^2(a+bx^2)^8} - \frac{x^7}{18b(a+bx^2)^9} \end{aligned}$$

[Out]  $-x^7/(18*b*(a+b*x^2)^9) - (7*x^5)/(288*b^2*(a+b*x^2)^8) - (5*x^3)/(576*b^3*(a+b*x^2)^7) - (5*x)/(2304*b^4*(a+b*x^2)^6) + x/(4608*a*b^4*(a+b*x^2)^5) + x/(4096*a^2*b^4*(a+b*x^2)^4) + (7*x)/(24576*a^3*b^4*(a+b*x^2)^3) + (35*x)/(98304*a^4*b^4*(a+b*x^2)^2) + (35*x)/(65536*a^5*b^4*(a+b*x^2)) + (35*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(11/2)*b^(9/2))$

**Rubi [A]** time = 0.287041, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\begin{aligned} & \frac{35 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{11/2}b^{9/2}} + \frac{35x}{65536a^5b^4(a+bx^2)} + \frac{35x}{98304a^4b^4(a+bx^2)^2} \\ & + \frac{7x}{24576a^3b^4(a+bx^2)^3} + \frac{x}{4096a^2b^4(a+bx^2)^4} + \frac{x}{4608ab^4(a+bx^2)^5} \\ & - \frac{5x}{2304b^4(a+bx^2)^6} - \frac{5x^3}{576b^3(a+bx^2)^7} - \frac{7x^5}{288b^2(a+bx^2)^8} - \frac{x^7}{18b(a+bx^2)^9} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^8/(a + b\*x^2)^10, x]

[Out]  $-x^7/(18*b*(a+b*x^2)^9) - (7*x^5)/(288*b^2*(a+b*x^2)^8) - (5*x^3)/(576*b^3*(a+b*x^2)^7) - (5*x)/(2304*b^4*(a+b*x^2)^6) + x/(4608*a*b^4*(a+b*x^2)^5) + x/(4096*a^2*b^4*(a+b*x^2)^4) + (7*x)/(24576*a^3*b^4*(a+b*x^2)^3) + (35*x)/(98304*a^4*b^4*(a+b*x^2)^2) + (35*x)/(65536*a^5*b^4*(a+b*x^2)) + (35*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(11/2)*b^(9/2))$

**Rubi in Sympy [A]** time = 44.8905, size = 190, normalized size = 0.94

$$\begin{aligned} & -\frac{x^7}{18b(a+bx^2)^9} - \frac{7x^5}{288b^2(a+bx^2)^8} - \frac{5x^3}{576b^3(a+bx^2)^7} - \frac{5x}{2304b^4(a+bx^2)^6} \\ & + \frac{x}{4608ab^4(a+bx^2)^5} + \frac{x}{4096a^2b^4(a+bx^2)^4} + \frac{7x}{24576a^3b^4(a+bx^2)^3} \\ & + \frac{35x}{98304a^4b^4(a+bx^2)^2} + \frac{35x}{65536a^5b^4(a+bx^2)} + \frac{35 \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{\frac{11}{2}}b^{\frac{9}{2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*8/(b\*x\*\*2+a)\*\*10, x)

[Out]  $-x**7/(18*b*(a+b*x**2)**9) - 7*x**5/(288*b**2*(a+b*x**2)**8) - 5*x**3/(576*b**3*(a+b*x**2)**7) - 5*x/(2304*b**4*(a+b*x**2)**6) + x/(4608*a*b**4*(a+b*x**2)**5) + x/(4096*a**2*b**4*(a+b*x**2)**4) + 7*x/(24576*a**3*b**4*(a+b*x**2)**3) + 35*x/(98304*$

$$a^{**4}b^{**4}(a + b*x^{**2})^{**2} + 35*x/(65536*a^{**5}b^{**4}(a + b*x^{**2})) + 35*atan(sqrt(b)*x/sqrt(a))/(65536*a^{**11/2}b^{**9/2})$$

**Mathematica [A]** time = 0.134277, size = 138, normalized size = 0.68

$$\frac{\sqrt{a}\sqrt{bx}(-315a^8-2730a^7bx^2-10458a^6b^2x^4-23202a^5b^3x^6+32768a^4b^4x^8+23202a^3b^5x^{10}+10458a^2b^6x^{12}+2730ab^7x^{14}+315b^8x^{16})}{(a+bx^2)^9} + 315 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)$$

$$589824a^{11/2}b^{9/2}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(a + b\*x^2)^10, x]

[Out] ((Sqrt[a]\*Sqrt[b]\*x\*(-315\*a^8 - 2730\*a^7\*b\*x^2 - 10458\*a^6\*b^2\*x^4 - 23202\*a^5\*b^3\*x^6 + 32768\*a^4\*b^4\*x^8 + 23202\*a^3\*b^5\*x^10 + 10458\*a^2\*b^6\*x^12 + 2730\*a\*b^7\*x^14 + 315\*b^8\*x^16))/(a + b\*x^2)^9 + 315\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(589824\*a^(11/2)\*b^(9/2))

**Maple [A]** time = 0.02, size = 122, normalized size = 0.6

$$\frac{1}{(bx^2 + a)^9} \left( -\frac{35a^3x}{65536b^4} - \frac{455a^2x^3}{98304b^3} - \frac{581ax^5}{32768b^2} - \frac{1289x^7}{32768b} + \frac{x^9}{18a} + \frac{1289bx^{11}}{32768a^2} + \frac{581b^2x^{13}}{32768a^3} + \frac{455b^3x^{15}}{98304a^4} + \frac{35b^4x^{17}}{65536a^5} \right) + \frac{35}{65536a^5b^4} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(b\*x^2+a)^10, x)

[Out] (-35/65536\*a^3\*x/b^4-455/98304\*a^2\*x^3/b^3-581/32768\*a\*x^5/b^2-1289/32768\*x^7/b+1/18/a\*x^9+1289/32768\*b/a^2\*x^11+581/32768\*b^2/a^3\*x^13+455/98304\*b^3/a^4\*x^15+35/65536\*b^4/a^5\*x^17)/(b\*x^2+a)^9+35/65536/a^5/b^4/(a\*b)^(1/2)\*arctan(x\*b/(a\*b)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b\*x^2 + a)^10, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.218821, size = 1, normalized size = 0.

$$\left[ \frac{315(b^9x^{18} + 9ab^8x^{16} + 36a^2b^7x^{14} + 84a^3b^6x^{12} + 126a^4b^5x^{10} + 126a^5b^4x^8 + 84a^6b^3x^6 + 36a^7b^2x^4 + 9a^8bx^2 + a^9) \log\left(\frac{2}{\dots}\right)}{1179648(a^5b^{13}x^{18} + 9a^6b^{12}x^{16} + 36a^7b^{11}x^{14} + 84a^8b^{10}x^{12} + 36a^9b^9x^{10} + 12a^{10}b^8x^8 + 12a^{11}b^7x^6 + 6a^{12}b^6x^4 + 2a^{13}b^5x^2 + a^{14})} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b\*x^2 + a)^10, x, algorithm="fricas")

[Out] 
$$\frac{1}{1179648} (315 (b^9 x^{18} + 9 a b^8 x^{16} + 36 a^2 b^7 x^{14} + 84 a^3 b^6 x^{12} + 126 a^4 b^5 x^{10} + 126 a^5 b^4 x^8 + 84 a^6 b^3 x^6 + 36 a^7 b^2 x^4 + 9 a^8 b x^2 + a^9) \log((2 a b x + (b x^2 - a) \sqrt{-a b}) / (b x^2 + a)) + 2 (315 b^8 x^{17} + 2730 a b^7 x^{15} + 10458 a^2 b^6 x^{13} + 23202 a^3 b^5 x^{11} + 32768 a^4 b^4 x^9 - 23202 a^5 b^3 x^7 - 10458 a^6 b^2 x^5 - 2730 a^7 b x^3 - 315 a^8 x) \sqrt{-a b}) / ((a^5 b^{13} x^{18} + 9 a^6 b^{12} x^{16} + 36 a^7 b^{11} x^{14} + 84 a^8 b^{10} x^{12} + 126 a^9 b^9 x^{10} + 126 a^{10} b^8 x^8 + 84 a^{11} b^7 x^6 + 36 a^{12} b^6 x^4 + 9 a^{13} b^5 x^2 + a^{14} b^4) \sqrt{-a b})$$
, 
$$\frac{1}{589824} (315 (b^9 x^{18} + 9 a b^8 x^{16} + 36 a^2 b^7 x^{14} + 84 a^3 b^6 x^{12} + 126 a^4 b^5 x^{10} + 126 a^5 b^4 x^8 + 84 a^6 b^3 x^6 + 36 a^7 b^2 x^4 + 9 a^8 b x^2 + a^9) \arctan(\sqrt{a b} x / a) + (315 b^8 x^{17} + 2730 a b^7 x^{15} + 10458 a^2 b^6 x^{13} + 23202 a^3 b^5 x^{11} + 32768 a^4 b^4 x^9 - 23202 a^5 b^3 x^7 - 10458 a^6 b^2 x^5 - 2730 a^7 b x^3 - 315 a^8 x) \sqrt{a b}) / ((a^5 b^{13} x^{18} + 9 a^6 b^{12} x^{16} + 36 a^7 b^{11} x^{14} + 84 a^8 b^{10} x^{12} + 126 a^9 b^9 x^{10} + 126 a^{10} b^8 x^8 + 84 a^{11} b^7 x^6 + 36 a^{12} b^6 x^4 + 9 a^{13} b^5 x^2 + a^{14} b^4) \sqrt{a b})]$$

**Sympy [A]** time = 34.1086, size = 291, normalized size = 1.44

$$\frac{35 \sqrt{-\frac{1}{a^{11} b^9}} \log\left(-a^6 b^4 \sqrt{-\frac{1}{a^{11} b^9}} + x\right)}{131072} + \frac{35 \sqrt{-\frac{1}{a^{11} b^9}} \log\left(a^6 b^4 \sqrt{-\frac{1}{a^{11} b^9}} + x\right)}{131072} + \frac{-315 a^8 x - 2730 a^7 b x^3 - 10458 a^6 b^2 x^5 - 23202 a^5 b^3 x^7 + 32768 a^4 b^4 x^9 + 23202 a^3 b^5 x^{11} + 10458 a^2 b^6 x^{13} + 21233664 a^{12} b^6 x^4 + 49545216 a^{11} b^7 x^6 + 74317824 a^{10} b^8 x^8 + 74317824 a^9 b^9 x^{10} + 49545216 a^8 b^{10} x^{12} + 21233664 a^7 b^{11} x^{14} + 5308416 a^6 b^{12} x^{16} + 589824 a^5 b^{13} x^{18}}{589824 a^{14} b^4 + 5308416 a^{13} b^5 x^2 + 21233664 a^{12} b^6 x^4 + 49545216 a^{11} b^7 x^6 + 74317824 a^{10} b^8 x^8 + 74317824 a^9 b^9 x^{10} + 49545216 a^8 b^{10} x^{12} + 21233664 a^7 b^{11} x^{14} + 5308416 a^6 b^{12} x^{16} + 589824 a^5 b^{13} x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*8/(b\*x\*\*2+a)\*\*10,x)

[Out] 
$$-35 \sqrt{-1/(a^{11} b^9)} \log(-a^6 b^4 \sqrt{-1/(a^{11} b^9)} + x) / 131072 + 35 \sqrt{-1/(a^{11} b^9)} \log(a^6 b^4 \sqrt{-1/(a^{11} b^9)} + x) / 131072 + (-315 a^8 x - 2730 a^7 b x^3 - 10458 a^6 b^2 x^5 - 23202 a^5 b^3 x^7 + 32768 a^4 b^4 x^9 + 23202 a^3 b^5 x^{11} + 10458 a^2 b^6 x^{13} + 2730 a b^7 x^{15} + 315 b^8 x^{17}) / (589824 a^{14} b^4 + 5308416 a^{13} b^5 x^2 + 21233664 a^{12} b^6 x^4 + 49545216 a^{11} b^7 x^6 + 74317824 a^{10} b^8 x^8 + 74317824 a^9 b^9 x^{10} + 49545216 a^8 b^{10} x^{12} + 21233664 a^7 b^{11} x^{14} + 5308416 a^6 b^{12} x^{16} + 589824 a^5 b^{13} x^{18})$$

**GIAC/XCAS [A]** time = 0.214016, size = 173, normalized size = 0.86

$$\frac{35 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab} a^5 b^4} + \frac{315 b^8 x^{17} + 2730 a b^7 x^{15} + 10458 a^2 b^6 x^{13} + 23202 a^3 b^5 x^{11} + 32768 a^4 b^4 x^9 - 23202 a^5 b^3 x^7 - 10458 a^6 b^2 x^5 - 2730 a^7 b x^3 - 315 a^8 x}{589824 (bx^2 + a)^9 a^5 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b\*x^2 + a)^10,x, algorithm="giac")

[Out] 
$$\frac{35}{65536} \arctan(bx/\sqrt{ab}) / (\sqrt{ab} a^5 b^4) + \frac{1}{589824} (315 b^8 x^{17} + 2730 a b^7 x^{15} + 10458 a^2 b^6 x^{13} + 23202 a^3 b^5 x^{11} + 32768 a^4 b^4 x^9 - 23202 a^5 b^3 x^7 - 10458 a^6 b^2 x^5 - 2730 a^7 b x^3 - 315 a^8 x) / ((b x^2 + a)^9 a^5 b^4)$$

$$3.218 \quad \int \frac{x^6}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=203

$$\begin{aligned} & \frac{55 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{13/2}b^{7/2}} + \frac{55x}{65536a^6b^3(a+bx^2)} + \frac{55x}{98304a^5b^3(a+bx^2)^2} \\ & + \frac{11x}{24576a^4b^3(a+bx^2)^3} + \frac{11x}{28672a^3b^3(a+bx^2)^4} + \frac{11x}{32256a^2b^3(a+bx^2)^5} \\ & + \frac{5x}{16128ab^3(a+bx^2)^6} - \frac{5x}{1344b^3(a+bx^2)^7} - \frac{5x^3}{288b^2(a+bx^2)^8} - \frac{x^5}{18b(a+bx^2)^9} \end{aligned}$$

[Out]  $-x^5/(18*b*(a+b*x^2)^9) - (5*x^3)/(288*b^2*(a+b*x^2)^8) - (5*x)/(1344*b^3*(a+b*x^2)^7) + (5*x)/(16128*a*b^3*(a+b*x^2)^6) + (11*x)/(32256*a^2*b^3*(a+b*x^2)^5) + (11*x)/(28672*a^3*b^3*(a+b*x^2)^4) + (11*x)/(24576*a^4*b^3*(a+b*x^2)^3) + (55*x)/(98304*a^5*b^3*(a+b*x^2)^2) + (55*x)/(65536*a^6*b^3*(a+b*x^2)) + (55*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(13/2)*b^(7/2))$

**Rubi [A]** time = 0.273764, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\begin{aligned} & \frac{55 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{13/2}b^{7/2}} + \frac{55x}{65536a^6b^3(a+bx^2)} + \frac{55x}{98304a^5b^3(a+bx^2)^2} \\ & + \frac{11x}{24576a^4b^3(a+bx^2)^3} + \frac{11x}{28672a^3b^3(a+bx^2)^4} + \frac{11x}{32256a^2b^3(a+bx^2)^5} \\ & + \frac{5x}{16128ab^3(a+bx^2)^6} - \frac{5x}{1344b^3(a+bx^2)^7} - \frac{5x^3}{288b^2(a+bx^2)^8} - \frac{x^5}{18b(a+bx^2)^9} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a + b\*x^2)^10, x]

[Out]  $-x^5/(18*b*(a+b*x^2)^9) - (5*x^3)/(288*b^2*(a+b*x^2)^8) - (5*x)/(1344*b^3*(a+b*x^2)^7) + (5*x)/(16128*a*b^3*(a+b*x^2)^6) + (11*x)/(32256*a^2*b^3*(a+b*x^2)^5) + (11*x)/(28672*a^3*b^3*(a+b*x^2)^4) + (11*x)/(24576*a^4*b^3*(a+b*x^2)^3) + (55*x)/(98304*a^5*b^3*(a+b*x^2)^2) + (55*x)/(65536*a^6*b^3*(a+b*x^2)) + (55*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(13/2)*b^(7/2))$

**Rubi in Sympy [A]** time = 41.9993, size = 196, normalized size = 0.97

$$\begin{aligned} & -\frac{x^5}{18b(a+bx^2)^9} - \frac{5x^3}{288b^2(a+bx^2)^8} - \frac{5x}{1344b^3(a+bx^2)^7} + \frac{5x}{16128ab^3(a+bx^2)^6} \\ & + \frac{11x}{32256a^2b^3(a+bx^2)^5} + \frac{11x}{28672a^3b^3(a+bx^2)^4} + \frac{11x}{24576a^4b^3(a+bx^2)^3} \\ & + \frac{55x}{98304a^5b^3(a+bx^2)^2} + \frac{55x}{65536a^6b^3(a+bx^2)} + \frac{55 \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{13/2}b^{7/2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6/(b\*x\*\*2+a)\*\*10, x)

[Out]  $-x**5/(18*b*(a+b*x**2)**9) - 5*x**3/(288*b**2*(a+b*x**2)**8) - 5*x/(1344*b**3*(a+b*x**2)**7) + 5*x/(16128*a*b**3*(a+b*x**2)**6) + 11*x/(32256*a**2*b**3*(a+b*x**2)**5) + 11*x/(28672*a**3*b**3*(a+b*x**2)**4) + 11*x/(24576*a**4*b**3*(a+b*x**2)**3) +$

$$55*x/(98304*a**5*b**3*(a+b*x**2)**2) + 55*x/(65536*a**6*b**3*(a+b*x**2)) + 55*atan(sqrt(b)*x/sqrt(a))/(65536*a**(13/2)*b**(7/2))$$

**Mathematica [A]** time = 0.134103, size = 138, normalized size = 0.68

$$\frac{\sqrt{a}\sqrt{bx}(-3465a^8-30030a^7bx^2-115038a^6b^2x^4+334602a^5b^3x^6+360448a^4b^4x^8+255222a^3b^5x^{10}+115038a^2b^6x^{12}+30030ab^7x^{14}+3465b^8x^{16})}{(a+bx^2)^9} + 3465 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{4128768a^{13/2}b^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a + b\*x^2)^10, x]

[Out] ((Sqrt[a]\*Sqrt[b]\*x\*(-3465\*a^8 - 30030\*a^7\*b\*x^2 - 115038\*a^6\*b^2\*x^4 + 334602\*a^5\*b^3\*x^6 + 360448\*a^4\*b^4\*x^8 + 255222\*a^3\*b^5\*x^10 + 115038\*a^2\*b^6\*x^12 + 30030\*a\*b^7\*x^14 + 3465\*b^8\*x^16))/(a + b\*x^2)^9 + 3465\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(4128768\*a^(13/2)\*b^(7/2))

**Maple [A]** time = 0.02, size = 122, normalized size = 0.6

$$\frac{1}{(bx^2+a)^9} \left( -\frac{55a^2x}{65536b^3} - \frac{715ax^3}{98304b^2} - \frac{913x^5}{32768b} + \frac{18589x^7}{229376a} + \frac{11bx^9}{126a^2} + \frac{14179b^2x^{11}}{229376a^3} + \frac{913b^3x^{13}}{32768a^4} + \frac{715b^4x^{15}}{98304a^5} + \frac{55b^5x^{17}}{65536a^6} \right) + \frac{55}{65536a^6b^3} \arctan\left(bx\frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(b\*x^2+a)^10, x)

[Out] (-55/65536\*a^2\*x/b^3-715/98304\*a\*x^3/b^2-913/32768\*x^5/b+18589/229376/a\*x^7+11/126\*b/a^2\*x^9+14179/229376\*b^2/a^3\*x^11+913/32768\*b^3/a^4\*x^13+715/98304\*b^4/a^5\*x^15+55/65536/a^6\*b^5\*x^17)/(b\*x^2+a)^9+55/65536/a^6/b^3/(a\*b)^(1/2)\*arctan(x\*b/(a\*b)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b\*x^2 + a)^10, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.225893, size = 1, normalized size = 0.

$$\left[ \frac{3465(b^9x^{18} + 9ab^8x^{16} + 36a^2b^7x^{14} + 84a^3b^6x^{12} + 126a^4b^5x^{10} + 126a^5b^4x^8 + 84a^6b^3x^6 + 36a^7b^2x^4 + 9a^8bx^2 + a^9) \log\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8257536(a^6b^{12}x^{18} + 9a^7b^{11}x^{16} + 36a^8b^{10}x^{14})} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b\*x^2 + a)^10, x, algorithm="fricas")



[Out] 
$$\frac{1}{8257536} (3465 (b^9 x^{18} + 9 a b^8 x^{16} + 36 a^2 b^7 x^{14} + 84 a^3 b^6 x^{12} + 126 a^4 b^5 x^{10} + 126 a^5 b^4 x^8 + 84 a^6 b^3 x^6 + 36 a^7 b^2 x^4 + 9 a^8 b x^2 + a^9) \log((2 a b x + (b x^2 - a) \sqrt{-a b}) / (b x^2 + a)) + 2 (3465 b^8 x^{17} + 30030 a b^7 x^{15} + 115038 a^2 b^6 x^{13} + 255222 a^3 b^5 x^{11} + 360448 a^4 b^4 x^9 + 334602 a^5 b^3 x^7 - 115038 a^6 b^2 x^5 - 30030 a^7 b x^3 - 3465 a^8 x) \sqrt{-a b}) / ((a^6 b^{12} x^{18} + 9 a^7 b^{11} x^{16} + 36 a^8 b^{10} x^{14} + 84 a^9 b^9 x^{12} + 126 a^{10} b^8 x^{10} + 126 a^{11} b^7 x^8 + 84 a^{12} b^6 x^6 + 36 a^{13} b^5 x^4 + 9 a^{14} b^4 x^2 + a^{15} b^3) \sqrt{-a b}), \frac{1}{4128768} (3465 (b^9 x^{18} + 9 a b^8 x^{16} + 36 a^2 b^7 x^{14} + 84 a^3 b^6 x^{12} + 126 a^4 b^5 x^{10} + 126 a^5 b^4 x^8 + 84 a^6 b^3 x^6 + 36 a^7 b^2 x^4 + 9 a^8 b x^2 + a^9) \arctan(\sqrt{a b} x / a) + (3465 b^8 x^{17} + 30030 a b^7 x^{15} + 115038 a^2 b^6 x^{13} + 255222 a^3 b^5 x^{11} + 360448 a^4 b^4 x^9 + 334602 a^5 b^3 x^7 - 115038 a^6 b^2 x^5 - 30030 a^7 b x^3 - 3465 a^8 x) \sqrt{a b}) / ((a^6 b^{12} x^{18} + 9 a^7 b^{11} x^{16} + 36 a^8 b^{10} x^{14} + 84 a^9 b^9 x^{12} + 126 a^{10} b^8 x^{10} + 126 a^{11} b^7 x^8 + 84 a^{12} b^6 x^6 + 36 a^{13} b^5 x^4 + 9 a^{14} b^4 x^2 + a^{15} b^3) \sqrt{a b})]$$

**Sympy [A]** time = 34.207, size = 291, normalized size = 1.43

$$\frac{55 \sqrt{-\frac{1}{a^{13} b^7}} \log\left(-a^7 b^3 \sqrt{-\frac{1}{a^{13} b^7}} + x\right)}{131072} + \frac{55 \sqrt{-\frac{1}{a^{13} b^7}} \log\left(a^7 b^3 \sqrt{-\frac{1}{a^{13} b^7}} + x\right)}{131072} + \frac{-3465 a^8 x - 30030 a^7 b x^3 - 115038 a^6 b^2 x^5 + 334602 a^5 b^3 x^7 + 360448 a^4 b^4 x^9 + 255222 a^3 b^5 x^{11} + 360448 a^2 b^6 x^{13} + 115038 a b^7 x^{15} + 3465 a^8 x^{17}}{4128768 a^{15} b^3 + 37158912 a^{14} b^4 x^2 + 148635648 a^{13} b^5 x^4 + 346816512 a^{12} b^6 x^6 + 520224768 a^{11} b^7 x^8 + 520224768 a^{10} b^8 x^{10} + 346816512 a^9 b^9 x^{12} + 148635648 a^8 b^{10} x^{14} + 37158912 a^7 b^{11} x^{16} + 3465 a^8 x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*6/(b\*x\*\*2+a)\*\*10,x)

[Out] 
$$-55 \sqrt{-1/(a^{13} b^7)} \log(-a^{13} b^7 \sqrt{-1/(a^{13} b^7)} + x) / 131072 + 55 \sqrt{-1/(a^{13} b^7)} \log(a^{13} b^7 \sqrt{-1/(a^{13} b^7)} + x) / 131072 + (-3465 a^8 x - 30030 a^7 b x^3 - 115038 a^6 b^2 x^5 + 334602 a^5 b^3 x^7 + 360448 a^4 b^4 x^9 + 255222 a^3 b^5 x^{11} + 115038 a^2 b^6 x^{13} + 30030 a b^7 x^{15} + 3465 a^8 x^{17}) / (4128768 a^{15} b^3 + 37158912 a^{14} b^4 x^2 + 148635648 a^{13} b^5 x^4 + 346816512 a^{12} b^6 x^6 + 520224768 a^{11} b^7 x^8 + 520224768 a^{10} b^8 x^{10} + 346816512 a^9 b^9 x^{12} + 148635648 a^8 b^{10} x^{14} + 37158912 a^7 b^{11} x^{16} + 3465 a^8 x^{17})$$

**GIAC/XCAS [A]** time = 0.210966, size = 173, normalized size = 0.85

$$\frac{55 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab} a^6 b^3} + \frac{3465 b^8 x^{17} + 30030 a b^7 x^{15} + 115038 a^2 b^6 x^{13} + 255222 a^3 b^5 x^{11} + 360448 a^4 b^4 x^9 + 334602 a^5 b^3 x^7 - 115038 a^6 b^2 x^5 - 30030 a^7 b x^3 - 3465 a^8 x}{4128768 (bx^2 + a)^9 a^6 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b\*x^2 + a)^10,x, algorithm="giac")

[Out] 
$$55/65536 \arctan(bx/\sqrt{ab}) / (\sqrt{ab} a^6 b^3) + 1/4128768 (3465 b^8 x^{17} + 30030 a b^7 x^{15} + 115038 a^2 b^6 x^{13} + 255222 a^3 b^5 x^{11} + 360448 a^4 b^4 x^9 + 334602 a^5 b^3 x^7 - 115038 a^6 b^2 x^5 - 30030 a^7 b x^3 - 3465 a^8 x) / ((bx^2 + a)^9 a^6 b^3)$$

$$3.219 \quad \int \frac{x^4}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=204

$$\begin{aligned} & \frac{143 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{15/2}b^{5/2}} + \frac{143x}{65536a^7b^2(a+bx^2)} + \frac{143x}{98304a^6b^2(a+bx^2)^2} \\ & + \frac{143x}{122880a^5b^2(a+bx^2)^3} + \frac{143x}{143360a^4b^2(a+bx^2)^4} + \frac{143x}{161280a^3b^2(a+bx^2)^5} \\ & + \frac{13x}{16128a^2b^2(a+bx^2)^6} + \frac{x}{1344ab^2(a+bx^2)^7} - \frac{x}{96b^2(a+bx^2)^8} - \frac{x^3}{18b(a+bx^2)^9} \end{aligned}$$

[Out]  $-x^3/(18*b*(a+b*x^2)^9) - x/(96*b^2*(a+b*x^2)^8) + x/(1344*a*b^2*(a+b*x^2)^7) + (13*x)/(16128*a^2*b^2*(a+b*x^2)^6) + (143*x)/(161280*a^3*b^2*(a+b*x^2)^5) + (143*x)/(143360*a^4*b^2*(a+b*x^2)^4) + (143*x)/(122880*a^5*b^2*(a+b*x^2)^3) + (143*x)/(98304*a^6*b^2*(a+b*x^2)^2) + (143*x)/(65536*a^7*b^2*(a+b*x^2)) + (143*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(15/2)*b^(5/2))$

**Rubi [A]** time = 0.266309, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\begin{aligned} & \frac{143 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{15/2}b^{5/2}} + \frac{143x}{65536a^7b^2(a+bx^2)} + \frac{143x}{98304a^6b^2(a+bx^2)^2} \\ & + \frac{143x}{122880a^5b^2(a+bx^2)^3} + \frac{143x}{143360a^4b^2(a+bx^2)^4} + \frac{143x}{161280a^3b^2(a+bx^2)^5} \\ & + \frac{13x}{16128a^2b^2(a+bx^2)^6} + \frac{x}{1344ab^2(a+bx^2)^7} - \frac{x}{96b^2(a+bx^2)^8} - \frac{x^3}{18b(a+bx^2)^9} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a + b\*x^2)^10, x]

[Out]  $-x^3/(18*b*(a+b*x^2)^9) - x/(96*b^2*(a+b*x^2)^8) + x/(1344*a*b^2*(a+b*x^2)^7) + (13*x)/(16128*a^2*b^2*(a+b*x^2)^6) + (143*x)/(161280*a^3*b^2*(a+b*x^2)^5) + (143*x)/(143360*a^4*b^2*(a+b*x^2)^4) + (143*x)/(122880*a^5*b^2*(a+b*x^2)^3) + (143*x)/(98304*a^6*b^2*(a+b*x^2)^2) + (143*x)/(65536*a^7*b^2*(a+b*x^2)) + (143*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^(15/2)*b^(5/2))$

**Rubi in Sympy [A]** time = 38.8141, size = 194, normalized size = 0.95

$$\begin{aligned} & -\frac{x^3}{18b(a+bx^2)^9} - \frac{x}{96b^2(a+bx^2)^8} + \frac{x}{1344ab^2(a+bx^2)^7} + \frac{13x}{16128a^2b^2(a+bx^2)^6} \\ & + \frac{143x}{161280a^3b^2(a+bx^2)^5} + \frac{143x}{143360a^4b^2(a+bx^2)^4} + \frac{143x}{122880a^5b^2(a+bx^2)^3} \\ & + \frac{143x}{98304a^6b^2(a+bx^2)^2} + \frac{143x}{65536a^7b^2(a+bx^2)} + \frac{143 \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{15/2}b^{5/2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(b\*x\*\*2+a)\*\*10, x)

[Out]  $-x**3/(18*b*(a+b*x**2)**9) - x/(96*b**2*(a+b*x**2)**8) + x/(1344*a*b**2*(a+b*x**2)**7) + 13*x/(16128*a**2*b**2*(a+b*x**2)**6) + 143*x/(161280*a**3*b**2*(a+b*x**2)**5) + 143*x/(143360*a**4*b**2*(a+b*x**2)**4) + 143*x/(122880*a**5*b**2*(a+b*x**2)**3) + 143*x/(98304*a**6*b**2*(a+b*x**2)**2) + 143*x/(65536*a**7*b**2*(a+b*x**2)) + 143*atan(sqrt(b*x)/sqrt(a))/(65536*a**(15/2)*b**(5/2))$

$$3) + 143*x/(98304*a**6*b**2*(a + b*x**2)**2) + 143*x/(65536*a**7*b**2*(a + b*x**2)) + 143*atan(sqrt(b)*x/sqrt(a))/(65536*a**(15/2)*b**(5/2))$$

**Mathematica [A]** time = 0.130619, size = 138, normalized size = 0.68

$$\frac{\sqrt{a}\sqrt{bx}(-45045a^8-390390a^7bx^2+2633274a^6b^2x^4+4349826a^5b^3x^6+4685824a^4b^4x^8+3317886a^3b^5x^{10}+1495494a^2b^6x^{12}+390390ab^7x^{14}+45045b^8x^{16})}{(a+bx^2)^9} + 45045 \frac{1}{20643840a^{15/2}b^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a + b\*x^2)^10,x]

[Out] ((Sqrt[a]\*Sqrt[b]\*x\*(-45045\*a^8 - 390390\*a^7\*b\*x^2 + 2633274\*a^6\*b^2\*x^4 + 4349826\*a^5\*b^3\*x^6 + 4685824\*a^4\*b^4\*x^8 + 3317886\*a^3\*b^5\*x^10 + 1495494\*a^2\*b^6\*x^12 + 390390\*a\*b^7\*x^14 + 45045\*b^8\*x^16))/(a + b\*x^2)^9 + 45045\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(20643840\*a^(15/2)\*b^(5/2))

**Maple [A]** time = 0.019, size = 122, normalized size = 0.6

$$\frac{1}{(bx^2 + a)^9} \left( -\frac{143ax}{65536b^2} - \frac{1859x^3}{98304b} + \frac{20899x^5}{163840a} + \frac{241657bx^7}{1146880a^2} + \frac{143b^2x^9}{630a^3} + \frac{184327b^3x^{11}}{1146880a^4} + \frac{11869b^4x^{13}}{163840a^5} + \frac{1859b^5x^{15}}{98304a^6} + \frac{143}{65536a^7b^2} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b\*x^2+a)^10,x)

[Out] (-143/65536\*a\*x/b^2-1859/98304\*x^3/b+20899/163840/a\*x^5+241657/1146880\*b/a^2\*x^7+143/630\*b^2/a^3\*x^9+184327/1146880\*b^3/a^4\*x^11+11869/163840\*b^4/a^5\*x^13+1859/98304/a^6\*b^5\*x^15+143/65536/a^7\*b^6\*x^17)/(b\*x^2+a)^9+143/65536/a^7/b^2/(a\*b)^(1/2)\*arctan(x\*b/(a\*b)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b\*x^2 + a)^10,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.217649, size = 1, normalized size = 0.

$$\left[ \frac{45045(b^9x^{18} + 9ab^8x^{16} + 36a^2b^7x^{14} + 84a^3b^6x^{12} + 126a^4b^5x^{10} + 126a^5b^4x^8 + 84a^6b^3x^6 + 36a^7b^2x^4 + 9a^8bx^2 + a^9) \log}{41287680(a^7b^{11}x^{18} + 9a^8b^{10}x^{16} + 36a^9b^9x^{14} + \dots)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b\*x^2 + a)^10,x, algorithm="fricas")

[Out] [1/41287680\*(45045\*(b^9\*x^18 + 9\*a\*b^8\*x^16 + 36\*a^2\*b^7\*x^14 + 84\*a^3\*b^6\*x^12 + 126\*a^4\*b^5\*x^10 + 126\*a^5\*b^4\*x^8 + 84\*a^6\*b^3\*x^6 + 36\*a^7\*b^2\*x^4 + 9\*a^8\*b\*x^2 + a^9)\*log((2\*a\*b\*x + (b\*x^2 - a)\*sqrt(-a\*b))/(b\*x^2 + a)) + 2\*(45045\*b^8\*x^17 + 390390\*a\*b^7\*x^15 + 1495494\*a^2\*b^6\*x^13 + 3317886\*a^3\*b^5\*x^11 + 4685824\*a^4\*b^4\*x^9 + 4349826\*a^5\*b^3\*x^7 + 2633274\*a^6\*b^2\*x^5 - 390390\*a^7\*b\*x^3 - 45045\*a^8\*x)\*sqrt(-a\*b))/((a^7\*b^11\*x^18 + 9\*a^8\*b^10\*x^16 + 36\*a^9\*b^9\*x^14 + 84\*a^10\*b^8\*x^12 + 126\*a^11\*b^7\*x^10 + 126\*a^12\*b^6\*x^8 + 84\*a^13\*b^5\*x^6 + 36\*a^14\*b^4\*x^4 + 9\*a^15\*b^3\*x^2 + a^16\*b^2)\*sqrt(-a\*b)), 1/20643840\*(45045\*(b^9\*x^18 + 9\*a\*b^8\*x^16 + 36\*a^2\*b^7\*x^14 + 84\*a^3\*b^6\*x^12 + 126\*a^4\*b^5\*x^10 + 126\*a^5\*b^4\*x^8 + 84\*a^6\*b^3\*x^6 + 36\*a^7\*b^2\*x^4 + 9\*a^8\*b\*x^2 + a^9)\*arctan(sqrt(a\*b)\*x/a) + (45045\*b^8\*x^17 + 390390\*a\*b^7\*x^15 + 1495494\*a^2\*b^6\*x^13 + 3317886\*a^3\*b^5\*x^11 + 4685824\*a^4\*b^4\*x^9 + 4349826\*a^5\*b^3\*x^7 + 2633274\*a^6\*b^2\*x^5 - 390390\*a^7\*b\*x^3 - 45045\*a^8\*x)\*sqrt(a\*b))/((a^7\*b^11\*x^18 + 9\*a^8\*b^10\*x^16 + 36\*a^9\*b^9\*x^14 + 84\*a^10\*b^8\*x^12 + 126\*a^11\*b^7\*x^10 + 126\*a^12\*b^6\*x^8 + 84\*a^13\*b^5\*x^6 + 36\*a^14\*b^4\*x^4 + 9\*a^15\*b^3\*x^2 + a^16\*b^2)\*sqrt(a\*b))]

**Sympy [A]** time = 34.0249, size = 291, normalized size = 1.43

$$\frac{143\sqrt{-\frac{1}{a^{15}b^5}} \log\left(-a^8b^2\sqrt{-\frac{1}{a^{15}b^5}} + x\right)}{131072} + \frac{143\sqrt{-\frac{1}{a^{15}b^5}} \log\left(a^8b^2\sqrt{-\frac{1}{a^{15}b^5}} + x\right)}{131072} + \frac{-45045a^8x - 390390a^7bx^3 + 2633274a^6b^2x^5 + 4349826a^5b^3x^7 + 4685824a^4b^4x^9 + 3317886a^3b^5x^{11} + 4685824a^4b^4x^9 + 4349826a^5b^3x^7 + 2633274a^6b^2x^5 - 390390a^7b^2x^3 - 45045a^8x}{20643840a^{16}b^2 + 185794560a^{15}b^3x^2 + 743178240a^{14}b^4x^4 + 1734082560a^{13}b^5x^6 + 2601123840a^{12}b^6x^8 + 2601123840a^{11}b^7x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4/(b\*x\*\*2+a)\*\*10,x)

[Out] -143\*sqrt(-1/(a\*\*15\*b\*\*5))\*log(-a\*\*8\*b\*\*2\*sqrt(-1/(a\*\*15\*b\*\*5)) + x)/131072 + 143\*sqrt(-1/(a\*\*15\*b\*\*5))\*log(a\*\*8\*b\*\*2\*sqrt(-1/(a\*\*15\*b\*\*5)) + x)/131072 + (-45045\*a\*\*8\*x - 390390\*a\*\*7\*b\*x\*\*3 + 2633274\*a\*\*6\*b\*\*2\*x\*\*5 + 4349826\*a\*\*5\*b\*\*3\*x\*\*7 + 4685824\*a\*\*4\*b\*\*4\*x\*\*9 + 3317886\*a\*\*3\*b\*\*5\*x\*\*11 + 1495494\*a\*\*2\*b\*\*6\*x\*\*13 + 390390\*a\*b\*\*7\*x\*\*15 + 45045\*b\*\*8\*x\*\*17)/(20643840\*a\*\*16\*b\*\*2 + 185794560\*a\*\*15\*b\*\*3\*x\*\*2 + 743178240\*a\*\*14\*b\*\*4\*x\*\*4 + 1734082560\*a\*\*13\*b\*\*5\*x\*\*6 + 2601123840\*a\*\*12\*b\*\*6\*x\*\*8 + 2601123840\*a\*\*11\*b\*\*7\*x\*\*10 + 1734082560\*a\*\*10\*b\*\*8\*x\*\*12 + 743178240\*a\*\*9\*b\*\*9\*x\*\*14 + 185794560\*a\*\*8\*b\*\*10\*x\*\*16 + 20643840\*a\*\*7\*b\*\*11\*x\*\*18)

**GIAC/XCAS [A]** time = 0.209286, size = 173, normalized size = 0.85

$$\frac{143 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{aba^7b^2}} + \frac{45045b^8x^{17} + 390390ab^7x^{15} + 1495494a^2b^6x^{13} + 3317886a^3b^5x^{11} + 4685824a^4b^4x^9 + 4349826a^5b^3x^7 + 2633274a^6b^2x^5}{20643840(bx^2 + a)^9a^7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b\*x^2 + a)^10,x, algorithm="giac")

[Out] 143/65536\*arctan(b\*x/sqrt(a\*b))/(sqrt(a\*b)\*a^7\*b^2) + 1/20643840\*(45045\*b^8\*x^17 + 390390\*a\*b^7\*x^15 + 1495494\*a^2\*b^6\*x^13 + 3317886\*a^3\*b^5\*x^11 + 4685824\*a^4\*b^4\*x^9 + 4349826\*a^5\*b^3\*x^7 + 2633274\*a^6\*b^2\*x^5 - 390390\*a^7\*b\*x^3 - 45045\*a^8\*x)/((b\*x^2 + a)^9\*a^7\*b^2)

$$3.220 \quad \int \frac{x^2}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=205

$$\begin{aligned} & \frac{715 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{17/2}b^{3/2}} + \frac{715x}{65536a^8b(a+bx^2)} + \frac{715x}{98304a^7b(a+bx^2)^2} \\ & + \frac{143x}{24576a^6b(a+bx^2)^3} + \frac{143x}{28672a^5b(a+bx^2)^4} + \frac{143x}{32256a^4b(a+bx^2)^5} \\ & + \frac{65x}{16128a^3b(a+bx^2)^6} + \frac{5x}{1344a^2b(a+bx^2)^7} + \frac{x}{288ab(a+bx^2)^8} - \frac{x}{18b(a+bx^2)^9} \end{aligned}$$

[Out]  $-x/(18*b*(a+b*x^2)^9) + x/(288*a*b*(a+b*x^2)^8) + (5*x)/(1344*a^2*b*(a+b*x^2)^7) + (65*x)/(16128*a^3*b*(a+b*x^2)^6) + (143*x)/(32256*a^4*b*(a+b*x^2)^5) + (143*x)/(28672*a^5*b*(a+b*x^2)^4) + (143*x)/(24576*a^6*b*(a+b*x^2)^3) + (715*x)/(98304*a^7*b*(a+b*x^2)^2) + (715*x)/(65536*a^8*b*(a+b*x^2)) + (715*ArcTan[(\sqrt{b}*x)/\sqrt{a}])/(65536*a^{(17/2)}*b^{(3/2)})$

**Rubi [A]** time = 0.248221, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\begin{aligned} & \frac{715 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{17/2}b^{3/2}} + \frac{715x}{65536a^8b(a+bx^2)} + \frac{715x}{98304a^7b(a+bx^2)^2} \\ & + \frac{143x}{24576a^6b(a+bx^2)^3} + \frac{143x}{28672a^5b(a+bx^2)^4} + \frac{143x}{32256a^4b(a+bx^2)^5} \\ & + \frac{65x}{16128a^3b(a+bx^2)^6} + \frac{5x}{1344a^2b(a+bx^2)^7} + \frac{x}{288ab(a+bx^2)^8} - \frac{x}{18b(a+bx^2)^9} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*x^2)^10, x]

[Out]  $-x/(18*b*(a+b*x^2)^9) + x/(288*a*b*(a+b*x^2)^8) + (5*x)/(1344*a^2*b*(a+b*x^2)^7) + (65*x)/(16128*a^3*b*(a+b*x^2)^6) + (143*x)/(32256*a^4*b*(a+b*x^2)^5) + (143*x)/(28672*a^5*b*(a+b*x^2)^4) + (143*x)/(24576*a^6*b*(a+b*x^2)^3) + (715*x)/(98304*a^7*b*(a+b*x^2)^2) + (715*x)/(65536*a^8*b*(a+b*x^2)) + (715*ArcTan[(\sqrt{b}*x)/\sqrt{a}])/(65536*a^{(17/2)}*b^{(3/2)})$

**Rubi in Sympy [A]** time = 35.1521, size = 184, normalized size = 0.9

$$\begin{aligned} & -\frac{x}{18b(a+bx^2)^9} + \frac{x}{288ab(a+bx^2)^8} + \frac{5x}{1344a^2b(a+bx^2)^7} + \frac{65x}{16128a^3b(a+bx^2)^6} \\ & + \frac{143x}{32256a^4b(a+bx^2)^5} + \frac{143x}{28672a^5b(a+bx^2)^4} + \frac{143x}{24576a^6b(a+bx^2)^3} \\ & + \frac{715x}{98304a^7b(a+bx^2)^2} + \frac{715x}{65536a^8b(a+bx^2)} + \frac{715 \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{17/2}b^{3/2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(b\*x\*\*2+a)\*\*10, x)

[Out]  $-x/(18*b*(a+b*x**2)**9) + x/(288*a*b*(a+b*x**2)**8) + 5*x/(1344*a**2*b*(a+b*x**2)**7) + 65*x/(16128*a**3*b*(a+b*x**2)**6) + 143*x/(32256*a**4*b*(a+b*x**2)**5) + 143*x/(28672*a**5*b*(a+b*x**2)**4) + 143*x/(24576*a**6*b*(a+b*x**2)**3) + 715*x/(98304*a**7*b*(a+b*x**2)**2) + 715*x/(65536*a**8*b*(a+b*x**2)) + 7$

$15 \cdot \operatorname{atan}(\sqrt{b} \cdot x / \sqrt{a}) / (65536 \cdot a^{17/2} \cdot b^{3/2})$

**Mathematica [A]** time = 0.121624, size = 138, normalized size = 0.67

$$\frac{\sqrt{a}\sqrt{b}x(-45045a^8+985866a^7bx^2+2633274a^6b^2x^4+4349826a^5b^3x^6+4685824a^4b^4x^8+3317886a^3b^5x^{10}+1495494a^2b^6x^{12}+390390ab^7x^{14}+45045b^8x^{16})}{(a+bx^2)^9} + 45045 \frac{4128768a^{17/2}b^{3/2}}{4128768a^{17/2}b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*x^2)^10,x]

[Out] ((Sqrt[a]\*Sqrt[b]\*x\*(-45045\*a^8 + 985866\*a^7\*b\*x^2 + 2633274\*a^6\*b^2\*x^4 + 4349826\*a^5\*b^3\*x^6 + 4685824\*a^4\*b^4\*x^8 + 3317886\*a^3\*b^5\*x^10 + 1495494\*a^2\*b^6\*x^12 + 390390\*a\*b^7\*x^14 + 45045\*b^8\*x^16))/(a + b\*x^2)^9 + 45045\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/(4128768\*a^(17/2)\*b^(3/2))

**Maple [A]** time = 0.019, size = 124, normalized size = 0.6

$$\frac{1}{(bx^2+a)^9} \left( -\frac{715x}{65536b} + \frac{23473x^3}{98304a} + \frac{20899bx^5}{32768a^2} + \frac{241657b^2x^7}{229376a^3} + \frac{143b^3x^9}{126a^4} + \frac{184327b^4x^{11}}{229376a^5} + \frac{11869b^5x^{13}}{32768a^6} + \frac{9295b^6x^{15}}{98304a^7} + \frac{715}{65536a^8b} \arctan\left(bx\frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b\*x^2+a)^10,x)

[Out] (-715/65536\*x/b+23473/98304/a\*x^3+20899/32768\*b/a^2\*x^5+241657/229376\*b^2/a^3\*x^7+143/126\*b^3/a^4\*x^9+184327/229376\*b^4/a^5\*x^11+11869/32768/a^6\*b^5\*x^13+9295/98304/a^7\*b^6\*x^15+715/65536/a^8\*b^7\*x^17)/(b\*x^2+a)^9+715/65536/a^8/b/(a\*b)^(1/2)\*arctan(x\*b/(a\*b)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b\*x^2 + a)^10,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.218748, size = 1, normalized size = 0.

$$\left[ \frac{45045 (b^9 x^{18} + 9 a b^8 x^{16} + 36 a^2 b^7 x^{14} + 84 a^3 b^6 x^{12} + 126 a^4 b^5 x^{10} + 126 a^5 b^4 x^8 + 84 a^6 b^3 x^6 + 36 a^7 b^2 x^4 + 9 a^8 b x^2 + a^9) \log\left(\frac{x^2 + a}{b x^2 + a}\right)}{8257536 (a^8 b^{10} x^{18} + 9 a^9 b^9 x^{16} + 36 a^{10} b^8 x^{14} + \dots)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b\*x^2 + a)^10,x, algorithm="fricas")

[Out] 
$$\frac{1}{8257536} (45045 (b^9 x^{18} + 9 a b^8 x^{16} + 36 a^2 b^7 x^{14} + 84 a^3 b^6 x^{12} + 126 a^4 b^5 x^{10} + 126 a^5 b^4 x^8 + 84 a^6 b^3 x^6 + 36 a^7 b^2 x^4 + 9 a^8 b x^2 + a^9) \log((2 a b x + (b x^2 - a) \sqrt{-a b}) / (b x^2 + a)) + 2 (45045 b^8 x^{17} + 390390 a b^7 x^{15} + 1495494 a^2 b^6 x^{13} + 3317886 a^3 b^5 x^{11} + 4685824 a^4 b^4 x^9 + 4349826 a^5 b^3 x^7 + 2633274 a^6 b^2 x^5 + 985866 a^7 b x^3 - 45045 a^8 x) \sqrt{-a b}) / ((a^8 b^{10} x^{18} + 9 a^9 b^9 x^{16} + 36 a^{10} b^8 x^{14} + 84 a^{11} b^7 x^{12} + 126 a^{12} b^6 x^{10} + 126 a^{13} b^5 x^8 + 84 a^{14} b^4 x^6 + 36 a^{15} b^3 x^4 + 9 a^{16} b^2 x^2 + a^{17} b) \sqrt{-a b}), \frac{1}{4128768} (45045 (b^9 x^{18} + 9 a b^8 x^{16} + 36 a^2 b^7 x^{14} + 84 a^3 b^6 x^{12} + 126 a^4 b^5 x^{10} + 126 a^5 b^4 x^8 + 84 a^6 b^3 x^6 + 36 a^7 b^2 x^4 + 9 a^8 b x^2 + a^9) \arctan(\sqrt{a b} x / a) + (45045 b^8 x^{17} + 390390 a b^7 x^{15} + 1495494 a^2 b^6 x^{13} + 3317886 a^3 b^5 x^{11} + 4685824 a^4 b^4 x^9 + 4349826 a^5 b^3 x^7 + 2633274 a^6 b^2 x^5 + 985866 a^7 b x^3 - 45045 a^8 x) \sqrt{a b}) / ((a^8 b^{10} x^{18} + 9 a^9 b^9 x^{16} + 36 a^{10} b^8 x^{14} + 84 a^{11} b^7 x^{12} + 126 a^{12} b^6 x^{10} + 126 a^{13} b^5 x^8 + 84 a^{14} b^4 x^6 + 36 a^{15} b^3 x^4 + 9 a^{16} b^2 x^2 + a^{17} b) \sqrt{a b})]$$

**Sympy [A]** time = 33.8482, size = 286, normalized size = 1.4

$$\frac{715 \sqrt{-\frac{1}{a^{17} b^3}} \log\left(-a^9 b \sqrt{-\frac{1}{a^{17} b^3}} + x\right)}{131072} + \frac{715 \sqrt{-\frac{1}{a^{17} b^3}} \log\left(a^9 b \sqrt{-\frac{1}{a^{17} b^3}} + x\right)}{131072} + \frac{-45045 a^8 x + 985866 a^7 b x^3 + 2633274 a^6 b^2 x^5 + 4349826 a^5 b^3 x^7 + 4685824 a^4 b^4 x^9 + 3317886 a^3 b^5 x^{11}}{4128768 a^{17} b + 37158912 a^{16} b^2 x^2 + 148635648 a^{15} b^3 x^4 + 346816512 a^{14} b^4 x^6 + 520224768 a^{13} b^5 x^8 + 520224768 a^{12} b^6 x^{10} + 346816512 a^{11} b^7 x^{12} + 148635648 a^{10} b^8 x^{14} + 37158912 a^9 b^9 x^{16} + 4128768 a^8 b^{10} x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(b\*x\*\*2+a)\*\*10,x)

[Out] 
$$\frac{-715 \sqrt{-1/(a^{17} b^3)} \log(-a^9 b \sqrt{-1/(a^{17} b^3)} + x) / 131072 + 715 \sqrt{-1/(a^{17} b^3)} \log(a^9 b \sqrt{-1/(a^{17} b^3)} + x) / 131072 + (-45045 a^8 x + 985866 a^7 b x^3 + 2633274 a^6 b^2 x^5 + 4349826 a^5 b^3 x^7 + 4685824 a^4 b^4 x^9 + 3317886 a^3 b^5 x^{11} + 1495494 a^2 b^6 x^{13} + 390390 a b^7 x^{15} + 45045 b^8 x^{17}) / (4128768 a^{17} b + 37158912 a^{16} b^2 x^2 + 148635648 a^{15} b^3 x^4 + 346816512 a^{14} b^4 x^6 + 520224768 a^{13} b^5 x^8 + 520224768 a^{12} b^6 x^{10} + 346816512 a^{11} b^7 x^{12} + 148635648 a^{10} b^8 x^{14} + 37158912 a^9 b^9 x^{16} + 4128768 a^8 b^{10} x^{18})}{4128768 a^{17} b + 37158912 a^{16} b^2 x^2 + 148635648 a^{15} b^3 x^4 + 346816512 a^{14} b^4 x^6 + 520224768 a^{13} b^5 x^8 + 520224768 a^{12} b^6 x^{10} + 346816512 a^{11} b^7 x^{12} + 148635648 a^{10} b^8 x^{14} + 37158912 a^9 b^9 x^{16} + 4128768 a^8 b^{10} x^{18}}$$

**GIAC/XCAS [A]** time = 0.210571, size = 173, normalized size = 0.84

$$\frac{715 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab} a^8 b} + \frac{45045 b^8 x^{17} + 390390 a b^7 x^{15} + 1495494 a^2 b^6 x^{13} + 3317886 a^3 b^5 x^{11} + 4685824 a^4 b^4 x^9 + 4349826 a^5 b^3 x^7 + 2633274 a^6 b^2 x^5 - 45045 a^8 x}{4128768 (bx^2 + a)^9 a^8 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b\*x^2 + a)^10,x, algorithm="giac")

[Out] 
$$\frac{715}{65536} \arctan(b x / \sqrt{a b}) / (\sqrt{a b} a^8 b) + \frac{1}{4128768} (45045 b^8 x^{17} + 390390 a b^7 x^{15} + 1495494 a^2 b^6 x^{13} + 3317886 a^3 b^5 x^{11} + 4685824 a^4 b^4 x^9 + 4349826 a^5 b^3 x^7 + 2633274 a^6 b^2 x^5 - 45045 a^8 x) / ((b x^2 + a)^9 a^8 b)$$

$$3.221 \quad \int \frac{1}{(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=181

$$\begin{aligned} & \frac{12155 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{19/2}\sqrt{b}} + \frac{12155x}{65536a^9(a+bx^2)} + \frac{12155x}{98304a^8(a+bx^2)^2} \\ & + \frac{2431x}{24576a^7(a+bx^2)^3} + \frac{2431x}{28672a^6(a+bx^2)^4} + \frac{2431x}{32256a^5(a+bx^2)^5} \\ & + \frac{1105x}{16128a^4(a+bx^2)^6} + \frac{85x}{1344a^3(a+bx^2)^7} + \frac{17x}{288a^2(a+bx^2)^8} + \frac{x}{18a(a+bx^2)^9} \end{aligned}$$

[Out] x/(18\*a\*(a+b\*x^2)^9) + (17\*x)/(288\*a^2\*(a+b\*x^2)^8) + (85\*x)/(1344\*a^3\*(a+b\*x^2)^7) + (1105\*x)/(16128\*a^4\*(a+b\*x^2)^6) + (2431\*x)/(32256\*a^5\*(a+b\*x^2)^5) + (2431\*x)/(28672\*a^6\*(a+b\*x^2)^4) + (2431\*x)/(24576\*a^7\*(a+b\*x^2)^3) + (12155\*x)/(98304\*a^8\*(a+b\*x^2)^2) + (12155\*x)/(65536\*a^9\*(a+b\*x^2)) + (12155\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(65536\*a^(19/2)\*Sqrt[b])

**Rubi [A]** time = 0.225243, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\begin{aligned} & \frac{12155 \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{19/2}\sqrt{b}} + \frac{12155x}{65536a^9(a+bx^2)} + \frac{12155x}{98304a^8(a+bx^2)^2} \\ & + \frac{2431x}{24576a^7(a+bx^2)^3} + \frac{2431x}{28672a^6(a+bx^2)^4} + \frac{2431x}{32256a^5(a+bx^2)^5} \\ & + \frac{1105x}{16128a^4(a+bx^2)^6} + \frac{85x}{1344a^3(a+bx^2)^7} + \frac{17x}{288a^2(a+bx^2)^8} + \frac{x}{18a(a+bx^2)^9} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(-10), x]

[Out] x/(18\*a\*(a+b\*x^2)^9) + (17\*x)/(288\*a^2\*(a+b\*x^2)^8) + (85\*x)/(1344\*a^3\*(a+b\*x^2)^7) + (1105\*x)/(16128\*a^4\*(a+b\*x^2)^6) + (2431\*x)/(32256\*a^5\*(a+b\*x^2)^5) + (2431\*x)/(28672\*a^6\*(a+b\*x^2)^4) + (2431\*x)/(24576\*a^7\*(a+b\*x^2)^3) + (12155\*x)/(98304\*a^8\*(a+b\*x^2)^2) + (12155\*x)/(65536\*a^9\*(a+b\*x^2)) + (12155\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(65536\*a^(19/2)\*Sqrt[b])

**Rubi in Sympy [A]** time = 30.7193, size = 173, normalized size = 0.96

$$\begin{aligned} & \frac{x}{18a(a+bx^2)^9} + \frac{17x}{288a^2(a+bx^2)^8} + \frac{85x}{1344a^3(a+bx^2)^7} + \frac{1105x}{16128a^4(a+bx^2)^6} \\ & + \frac{2431x}{32256a^5(a+bx^2)^5} + \frac{2431x}{28672a^6(a+bx^2)^4} + \frac{2431x}{24576a^7(a+bx^2)^3} \\ & + \frac{12155x}{98304a^8(a+bx^2)^2} + \frac{12155x}{65536a^9(a+bx^2)} + \frac{12155 \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{19/2}\sqrt{b}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+a)\*\*10, x)

[Out] x/(18\*a\*(a+b\*x\*\*2)\*\*9) + 17\*x/(288\*a\*\*2\*(a+b\*x\*\*2)\*\*8) + 85\*x/(1344\*a\*\*3\*(a+b\*x\*\*2)\*\*7) + 1105\*x/(16128\*a\*\*4\*(a+b\*x\*\*2)\*\*6) + 2431\*x/(32256\*a\*\*5\*(a+b\*x\*\*2)\*\*5) + 2431\*x/(28672\*a\*\*6\*(a+b\*x\*\*2)\*\*4) + 2431\*x/(24576\*a\*\*7\*(a+b\*x\*\*2)\*\*3) + 12155\*x/(98304\*a\*\*8\*(a+b\*x\*\*2)\*\*2) + 12155\*x/(65536\*a\*\*9\*(a+b\*x\*\*2)) + 12



$155 \cdot \operatorname{atan}(\sqrt{b} \cdot x / \sqrt{a}) / (65536 \cdot a^{19/2} \cdot \sqrt{b})$

**Mathematica [A]** time = 0.194233, size = 131, normalized size = 0.72

$$\frac{765765 \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + \frac{3363003a^8x + 16759722a^7bx^3 + 44765658a^6b^2x^5 + 73947042a^5b^3x^7 + 79659008a^4b^4x^9 + 56404062a^3b^5x^{11} + 25423398a^2b^6x^{13} + 6636630ab^7x^{15}}{a^{19/2}\sqrt{b}}}{4128768}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(-10), x]

[Out] ((3363003\*a^8\*x + 16759722\*a^7\*b\*x^3 + 44765658\*a^6\*b^2\*x^5 + 73947042\*a^5\*b^3\*x^7 + 79659008\*a^4\*b^4\*x^9 + 56404062\*a^3\*b^5\*x^11 + 25423398\*a^2\*b^6\*x^13 + 6636630\*a\*b^7\*x^15 + 765765\*b^8\*x^17)/(a^9\*(a + b\*x^2)^9) + (765765\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(a^(19/2)\*Sqrt[b]))/4128768

**Maple [A]** time = 0.006, size = 156, normalized size = 0.9

$$\begin{aligned} & \frac{x}{18a(bx^2+a)^9} + \frac{17x}{288a^2(bx^2+a)^8} + \frac{85x}{1344a^3(bx^2+a)^7} + \frac{1105x}{16128a^4(bx^2+a)^6} \\ & + \frac{2431x}{32256a^5(bx^2+a)^5} + \frac{2431x}{28672a^6(bx^2+a)^4} + \frac{2431x}{24576a^7(bx^2+a)^3} \\ & + \frac{12155x}{98304a^8(bx^2+a)^2} + \frac{12155x}{65536a^9(bx^2+a)} + \frac{12155}{65536a^9} \arctan\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+a)^10, x)

[Out] 1/18\*x/a/(b\*x^2+a)^9+17/288\*x/a^2/(b\*x^2+a)^8+85/1344\*x/a^3/(b\*x^2+a)^7+1105/16128\*x/a^4/(b\*x^2+a)^6+2431/32256\*x/a^5/(b\*x^2+a)^5+2431/28672\*x/a^6/(b\*x^2+a)^4+2431/24576\*x/a^7/(b\*x^2+a)^3+12155/98304\*x/a^8/(b\*x^2+a)^2+12155/65536\*x/a^9/(b\*x^2+a)+12155/65536/a^9/(a\*b)^(1/2)\*arctan(x\*b/(a\*b)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-10), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.237609, size = 1, normalized size = 0.01

$$\frac{765765 (b^9x^{18} + 9ab^8x^{16} + 36a^2b^7x^{14} + 84a^3b^6x^{12} + 126a^4b^5x^{10} + 126a^5b^4x^8 + 84a^6b^3x^6 + 36a^7b^2x^4 + 9a^8bx^2 + a^9) \log\left(\frac{\dots}{8257536(a^9b^9x^{18} + 9a^{10}b^8x^{16} + \dots)}\right)}{8257536(a^9b^9x^{18} + 9a^{10}b^8x^{16} + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-10),x, algorithm="fricas")

[Out] [1/8257536\*(765765\*(b^9\*x^18 + 9\*a\*b^8\*x^16 + 36\*a^2\*b^7\*x^14 + 84\*a^3\*b^6\*x^12 + 126\*a^4\*b^5\*x^10 + 126\*a^5\*b^4\*x^8 + 84\*a^6\*b^3\*x^6 + 36\*a^7\*b^2\*x^4 + 9\*a^8\*b\*x^2 + a^9)\*log((2\*a\*b\*x + (b\*x^2 - a)\*sqrt(-a\*b))/(b\*x^2 + a)) + 2\*(765765\*b^8\*x^17 + 6636630\*a\*b^7\*x^15 + 25423398\*a^2\*b^6\*x^13 + 56404062\*a^3\*b^5\*x^11 + 79659008\*a^4\*b^4\*x^9 + 73947042\*a^5\*b^3\*x^7 + 44765658\*a^6\*b^2\*x^5 + 16759722\*a^7\*b\*x^3 + 3363003\*a^8\*x)\*sqrt(-a\*b))/((a^9\*b^9\*x^18 + 9\*a^10\*b^8\*x^16 + 36\*a^11\*b^7\*x^14 + 84\*a^12\*b^6\*x^12 + 126\*a^13\*b^5\*x^10 + 126\*a^14\*b^4\*x^8 + 84\*a^15\*b^3\*x^6 + 36\*a^16\*b^2\*x^4 + 9\*a^17\*b\*x^2 + a^18)\*sqrt(-a\*b)), 1/4128768\*(765765\*(b^9\*x^18 + 9\*a\*b^8\*x^16 + 36\*a^2\*b^7\*x^14 + 84\*a^3\*b^6\*x^12 + 126\*a^4\*b^5\*x^10 + 126\*a^5\*b^4\*x^8 + 84\*a^6\*b^3\*x^6 + 36\*a^7\*b^2\*x^4 + 9\*a^8\*b\*x^2 + a^9)\*arctan(sqrt(a\*b)\*x/a) + (765765\*b^8\*x^17 + 6636630\*a\*b^7\*x^15 + 25423398\*a^2\*b^6\*x^13 + 56404062\*a^3\*b^5\*x^11 + 79659008\*a^4\*b^4\*x^9 + 73947042\*a^5\*b^3\*x^7 + 44765658\*a^6\*b^2\*x^5 + 16759722\*a^7\*b\*x^3 + 3363003\*a^8\*x)\*sqrt(a\*b))/((a^9\*b^9\*x^18 + 9\*a^10\*b^8\*x^16 + 36\*a^11\*b^7\*x^14 + 84\*a^12\*b^6\*x^12 + 126\*a^13\*b^5\*x^10 + 126\*a^14\*b^4\*x^8 + 84\*a^15\*b^3\*x^6 + 36\*a^16\*b^2\*x^4 + 9\*a^17\*b\*x^2 + a^18)\*sqrt(a\*b))]

**Sympy [A]** time = 34.0598, size = 272, normalized size = 1.5

$$\frac{12155\sqrt{-\frac{1}{a^{19}b}}\log\left(-a^{10}\sqrt{-\frac{1}{a^{19}b}}+x\right)}{131072} + \frac{12155\sqrt{-\frac{1}{a^{19}b}}\log\left(a^{10}\sqrt{-\frac{1}{a^{19}b}}+x\right)}{131072} + \frac{3363003a^8x + 16759722a^7bx^3 + 44765658a^6b^2x^5 + 73947042a^5b^3x^7 + 79659008a^4b^4x^9 + 56404062a^3b^5x^{11} + 4128768a^{18} + 37158912a^{17}bx^2 + 148635648a^{16}b^2x^4 + 346816512a^{15}b^3x^6 + 520224768a^{14}b^4x^8 + 520224768a^{13}b^5x^{10} + 346816512a^{12}b^6x^{12} + 148635648a^{11}b^7x^{14} + 37158912a^{10}b^8x^{16} + 4128768a^9b^9x^{18}}{4128768a^{18} + 37158912a^{17}bx^2 + 148635648a^{16}b^2x^4 + 346816512a^{15}b^3x^6 + 520224768a^{14}b^4x^8 + 520224768a^{13}b^5x^{10} + 346816512a^{12}b^6x^{12} + 148635648a^{11}b^7x^{14} + 37158912a^{10}b^8x^{16} + 4128768a^9b^9x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*2+a)\*\*10,x)

[Out] -12155\*sqrt(-1/(a\*\*19\*b))\*log(-a\*\*10\*sqrt(-1/(a\*\*19\*b)) + x)/131072 + 12155\*sqrt(-1/(a\*\*19\*b))\*log(a\*\*10\*sqrt(-1/(a\*\*19\*b)) + x)/131072 + (3363003\*a\*\*8\*x + 16759722\*a\*\*7\*b\*x\*\*3 + 44765658\*a\*\*6\*b\*\*2\*x\*\*5 + 73947042\*a\*\*5\*b\*\*3\*x\*\*7 + 79659008\*a\*\*4\*b\*\*4\*x\*\*9 + 56404062\*a\*\*3\*b\*\*5\*x\*\*11 + 25423398\*a\*\*2\*b\*\*6\*x\*\*13 + 6636630\*a\*b\*\*7\*x\*\*15 + 765765\*b\*\*8\*x\*\*17)/(4128768\*a\*\*18 + 37158912\*a\*\*17\*b\*x\*\*2 + 148635648\*a\*\*16\*b\*\*2\*x\*\*4 + 346816512\*a\*\*15\*b\*\*3\*x\*\*6 + 520224768\*a\*\*14\*b\*\*4\*x\*\*8 + 520224768\*a\*\*13\*b\*\*5\*x\*\*10 + 346816512\*a\*\*12\*b\*\*6\*x\*\*12 + 148635648\*a\*\*11\*b\*\*7\*x\*\*14 + 37158912\*a\*\*10\*b\*\*8\*x\*\*16 + 4128768\*a\*\*9\*b\*\*9\*x\*\*18)

**GIAC/XCAS [A]** time = 0.225932, size = 165, normalized size = 0.91

$$\frac{12155 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{aba^9}} + \frac{765765 b^8 x^{17} + 6636630 ab^7 x^{15} + 25423398 a^2 b^6 x^{13} + 56404062 a^3 b^5 x^{11} + 79659008 a^4 b^4 x^9 + 73947042 a^5 b^3 x^7 + 44765658 a^6 b^2 x^5 + 16759722 a^7 b x^3 + 3363003 a^8 x}{4128768 (bx^2 + a)^9 a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-10),x, algorithm="giac")

[Out] 12155/65536\*arctan(b\*x/sqrt(a\*b))/(sqrt(a\*b)\*a^9) + 1/4128768\*(765765\*b^8\*x^17 + 6636630\*a\*b^7\*x^15 + 25423398\*a^2\*b^6\*x^13 + 56404062\*a^3\*b^5\*x^11 + 79659008\*a^4\*b^4\*x^9 + 73947042\*a^5\*b^3\*x^7 + 44765658\*a^6\*b^2\*x^5 + 16759722\*a^7\*b\*x^3 + 3363003\*a^8\*x)/((b\*x^2 + a)^9\*a^9)

$$3.222 \quad \int \frac{1}{x^2(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=209

$$\begin{aligned} & -\frac{230945\sqrt{b}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{21/2}} - \frac{230945}{65536a^{10}x} + \frac{230945}{196608a^9x(a+bx^2)} + \frac{46189}{98304a^8x(a+bx^2)^2} \\ & + \frac{46189}{172032a^7x(a+bx^2)^3} + \frac{46189}{258048a^6x(a+bx^2)^4} + \frac{4199}{32256a^5x(a+bx^2)^5} \\ & + \frac{1615}{16128a^4x(a+bx^2)^6} + \frac{323}{4032a^3x(a+bx^2)^7} + \frac{19}{288a^2x(a+bx^2)^8} + \frac{1}{18ax(a+bx^2)^9} \end{aligned}$$

[Out]  $-230945/(65536*a^{10}*x) + 1/(18*a*x*(a+b*x^2)^9) + 19/(288*a^2*x*(a+b*x^2)^8) + 323/(4032*a^3*x*(a+b*x^2)^7) + 1615/(16128*a^4*x*(a+b*x^2)^6) + 4199/(32256*a^5*x*(a+b*x^2)^5) + 46189/(258048*a^6*x*(a+b*x^2)^4) + 46189/(172032*a^7*x*(a+b*x^2)^3) + 46189/(98304*a^8*x*(a+b*x^2)^2) + 230945/(196608*a^9*x*(a+b*x^2)) - (230945*sqrt[b]*ArcTan[(sqrt[b]*x)/sqrt[a]])/(65536*a^{(21/2)})$

**Rubi [A]** time = 0.318087, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\begin{aligned} & -\frac{230945\sqrt{b}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{21/2}} - \frac{230945}{65536a^{10}x} + \frac{230945}{196608a^9x(a+bx^2)} + \frac{46189}{98304a^8x(a+bx^2)^2} \\ & + \frac{46189}{172032a^7x(a+bx^2)^3} + \frac{46189}{258048a^6x(a+bx^2)^4} + \frac{4199}{32256a^5x(a+bx^2)^5} \\ & + \frac{1615}{16128a^4x(a+bx^2)^6} + \frac{323}{4032a^3x(a+bx^2)^7} + \frac{19}{288a^2x(a+bx^2)^8} + \frac{1}{18ax(a+bx^2)^9} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a+b\*x^2)^10),x]

[Out]  $-230945/(65536*a^{10}*x) + 1/(18*a*x*(a+b*x^2)^9) + 19/(288*a^2*x*(a+b*x^2)^8) + 323/(4032*a^3*x*(a+b*x^2)^7) + 1615/(16128*a^4*x*(a+b*x^2)^6) + 4199/(32256*a^5*x*(a+b*x^2)^5) + 46189/(258048*a^6*x*(a+b*x^2)^4) + 46189/(172032*a^7*x*(a+b*x^2)^3) + 46189/(98304*a^8*x*(a+b*x^2)^2) + 230945/(196608*a^9*x*(a+b*x^2)) - (230945*sqrt[b]*ArcTan[(sqrt[b]*x)/sqrt[a]])/(65536*a^{(21/2)})$

**Rubi in Sympy [A]** time = 54.2078, size = 184, normalized size = 0.88

$$\begin{aligned} & \frac{1}{18ax(a+bx^2)^9} + \frac{19}{288a^2x(a+bx^2)^8} + \frac{323}{4032a^3x(a+bx^2)^7} + \frac{1615}{16128a^4x(a+bx^2)^6} \\ & + \frac{46189}{32256a^5x(a+bx^2)^5} + \frac{46189}{258048a^6x(a+bx^2)^4} + \frac{46189}{172032a^7x(a+bx^2)^3} \\ & + \frac{46189}{98304a^8x(a+bx^2)^2} + \frac{230945}{196608a^9x(a+bx^2)} - \frac{230945}{65536a^{10}x} - \frac{230945\sqrt{b}\operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{21/2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(b\*x\*\*2+a)\*\*10,x)

[Out]  $1/(18*a*x*(a+b*x**2)**9) + 19/(288*a**2*x*(a+b*x**2)**8) + 323/(4032*a**3*x*(a+b*x**2)**7) + 1615/(16128*a**4*x*(a+b*x**2)**6) + 4199/(32256*a**5*x*(a+b*x**2)**5) + 46189/(258048*a**6*x$

$$\begin{aligned} &*(a + b*x**2)**4) + 46189/(172032*a**7*x*(a + b*x**2)**3) + 46189 \\ &/ (98304*a**8*x*(a + b*x**2)**2) + 230945/(196608*a**9*x*(a + b*x** \\ &*2)) - 230945/(65536*a**10*x) - 230945*sqrt(b)*atan(sqrt(b)*x/sqr \\ &t(a))/(65536*a**(21/2)) \end{aligned}$$

**Mathematica [A]** time = 0.185077, size = 147, normalized size = 0.7

$$\frac{\sqrt{a}(4128768a^9+63897057a^8bx^2+318434718a^7b^2x^4+850547502a^6b^3x^6+1404993798a^5b^4x^8+1513521152a^4b^5x^{10}+1071677178a^3b^6x^{12}+483044562a^2b^7x^{14}+126095970ab^8x^{16}+14549535b^9x^{18})}{x(a+bx^2)^9} + \frac{4128768a^{21/2}}{x(a+bx^2)^9}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a + b\*x^2)^10), x]

[Out] 
$$\begin{aligned} &(-((\text{Sqrt}[a]*(4128768*a^9 + 63897057*a^8*b*x^2 + 318434718*a^7*b^2 \\ &*x^4 + 850547502*a^6*b^3*x^6 + 1404993798*a^5*b^4*x^8 + 151352115 \\ &2*a^4*b^5*x^{10} + 1071677178*a^3*b^6*x^{12} + 483044562*a^2*b^7*x^{14} \\ &+ 126095970*a*b^8*x^{16} + 14549535*b^9*x^{18}))/ (x*(a + b*x^2)^9)) \\ &- 14549535*\text{Sqrt}[b]*\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/(4128768*a^{(21/2)} \end{aligned}$$

**Maple [A]** time = 0.027, size = 206, normalized size = 1.

$$\begin{aligned} &-\frac{1}{a^{10}x} - \frac{424415bx}{65536a^2(bx^2+a)^9} - \frac{4042835b^2x^3}{98304a^3(bx^2+a)^9} - \frac{3997865b^3x^5}{32768a^4(bx^2+a)^9} \\ &-\frac{49153835b^4x^7}{229376a^5(bx^2+a)^9} - \frac{30313b^5x^9}{126a^6(bx^2+a)^9} - \frac{40270037b^6x^{11}}{229376a^7(bx^2+a)^9} - \frac{2654039b^7x^{13}}{32768a^8(bx^2+a)^9} \\ &-\frac{2117549b^8x^{15}}{98304a^9(bx^2+a)^9} - \frac{165409b^9x^{17}}{65536a^{10}(bx^2+a)^9} - \frac{230945b}{65536a^{10}} \arctan\left(bx\frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b\*x^2+a)^10, x)

[Out] 
$$\begin{aligned} &-1/a^{10}/x-424415/65536/a^2*b/(b*x^2+a)^9*x-4042835/98304/a^3*b^2/ \\ &(b*x^2+a)^9*x^3-3997865/32768/a^4*b^3/(b*x^2+a)^9*x^5-49153835/22 \\ &9376/a^5*b^4/(b*x^2+a)^9*x^7-30313/126/a^6*b^5/(b*x^2+a)^9*x^9-40 \\ &270037/229376/a^7*b^6/(b*x^2+a)^9*x^{11}-2654039/32768/a^8*b^7/(b*x \\ &^2+a)^9*x^{13}-2117549/98304/a^9*b^8/(b*x^2+a)^9*x^{15}-165409/65536/ \\ &a^{10}*b^9/(b*x^2+a)^9*x^{17}-230945/65536/a^{10}*b/(a*b)^{(1/2)}*\arctan( \\ &x*b/(a*b)^{(1/2)}) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^10\*x^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.244489, size = 1, normalized size = 0.

$$\frac{29099070 b^9 x^{18} + 252191940 ab^8 x^{16} + 966089124 a^2 b^7 x^{14} + 2143354356 a^3 b^6 x^{12} + 3027042304 a^4 b^5 x^{10} + 2809987596 a^5 b^4 x^8 + 1701095004 a^6 b^3 x^6 + 636869436 a^7 b^2 x^4 + 127794114 a^8 b x^2 + 8257536 a^9 - 14549535 (b^9 x^{19} + 9 a b^8 x^{17} + 36 a^2 b^7 x^{15} + 84 a^3 b^6 x^{13} + 126 a^4 b^5 x^{11} + 126 a^5 b^4 x^9 + 84 a^6 b^3 x^7 + 36 a^7 b^2 x^5 + 9 a^8 b x^3 + a^9 x) \sqrt{-b/a} \log((b x^2 - 2 a x \sqrt{-b/a} - a)/(b x^2 + a)) / (a^{10} b^9 x^{19} + 9 a^{11} b^8 x^{17} + 36 a^{12} b^7 x^{15} + 84 a^{13} b^6 x^{13} + 126 a^{14} b^5 x^{11} + 126 a^{15} b^4 x^9 + 84 a^{16} b^3 x^7 + 36 a^{17} b^2 x^5 + 9 a^{18} b x^3 + a^{19} x), -1/4128768 (14549535 b^9 x^{18} + 126095970 ab^8 x^{16} + 483044562 a^2 b^7 x^{14} + 1071677178 a^3 b^6 x^{12} + 1513521152 a^4 b^5 x^{10} + 1404993798 a^5 b^4 x^8 + 850547502 a^6 b^3 x^6 + 318434718 a^7 b^2 x^4 + 63897057 a^8 b x^2 + 4128768 a^9 + 14549535 (b^9 x^{19} + 9 a b^8 x^{17} + 36 a^2 b^7 x^{15} + 84 a^3 b^6 x^{13} + 126 a^4 b^5 x^{11} + 126 a^5 b^4 x^9 + 84 a^6 b^3 x^7 + 36 a^7 b^2 x^5 + 9 a^8 b x^3 + a^9 x) \sqrt{b/a} \arctan(b x / (a \sqrt{b/a}))} / (a^{10} b^9 x^{19} + 9 a^{11} b^8 x^{17} + 36 a^{12} b^7 x^{15} + 84 a^{13} b^6 x^{13} + 126 a^{14} b^5 x^{11} + 126 a^{15} b^4 x^9 + 84 a^{16} b^3 x^7 + 36 a^{17} b^2 x^5 + 9 a^{18} b x^3 + a^{19} x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^10\*x^2),x, algorithm="fricas")

[Out] [-1/8257536\*(29099070\*b^9\*x^18 + 252191940\*a\*b^8\*x^16 + 966089124\*a^2\*b^7\*x^14 + 2143354356\*a^3\*b^6\*x^12 + 3027042304\*a^4\*b^5\*x^10 + 2809987596\*a^5\*b^4\*x^8 + 1701095004\*a^6\*b^3\*x^6 + 636869436\*a^7\*b^2\*x^4 + 127794114\*a^8\*b\*x^2 + 8257536\*a^9 - 14549535\*(b^9\*x^19 + 9\*a\*b^8\*x^17 + 36\*a^2\*b^7\*x^15 + 84\*a^3\*b^6\*x^13 + 126\*a^4\*b^5\*x^11 + 126\*a^5\*b^4\*x^9 + 84\*a^6\*b^3\*x^7 + 36\*a^7\*b^2\*x^5 + 9\*a^8\*b\*x^3 + a^9\*x)\*sqrt(-b/a)\*log((b\*x^2 - 2\*a\*x\*sqrt(-b/a) - a)/(b\*x^2 + a)))/(a^10\*b^9\*x^19 + 9\*a^11\*b^8\*x^17 + 36\*a^12\*b^7\*x^15 + 84\*a^13\*b^6\*x^13 + 126\*a^14\*b^5\*x^11 + 126\*a^15\*b^4\*x^9 + 84\*a^16\*b^3\*x^7 + 36\*a^17\*b^2\*x^5 + 9\*a^18\*b\*x^3 + a^19\*x), -1/4128768\*(14549535\*b^9\*x^18 + 126095970\*a\*b^8\*x^16 + 483044562\*a^2\*b^7\*x^14 + 1071677178\*a^3\*b^6\*x^12 + 1513521152\*a^4\*b^5\*x^10 + 1404993798\*a^5\*b^4\*x^8 + 850547502\*a^6\*b^3\*x^6 + 318434718\*a^7\*b^2\*x^4 + 63897057\*a^8\*b\*x^2 + 4128768\*a^9 + 14549535\*(b^9\*x^19 + 9\*a\*b^8\*x^17 + 36\*a^2\*b^7\*x^15 + 84\*a^3\*b^6\*x^13 + 126\*a^4\*b^5\*x^11 + 126\*a^5\*b^4\*x^9 + 84\*a^6\*b^3\*x^7 + 36\*a^7\*b^2\*x^5 + 9\*a^8\*b\*x^3 + a^9\*x)\*sqrt(b/a)\*arctan(b\*x/(a\*sqrt(b/a)))]/(a^10\*b^9\*x^19 + 9\*a^11\*b^8\*x^17 + 36\*a^12\*b^7\*x^15 + 84\*a^13\*b^6\*x^13 + 126\*a^14\*b^5\*x^11 + 126\*a^15\*b^4\*x^9 + 84\*a^16\*b^3\*x^7 + 36\*a^17\*b^2\*x^5 + 9\*a^18\*b\*x^3 + a^19\*x)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(b\*x\*\*2+a)\*\*10,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.225743, size = 181, normalized size = 0.87

$$\frac{230945 b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab} a^{10}} - \frac{1}{a^{10} x} - \frac{10420767 b^9 x^{17} + 88937058 ab^8 x^{15} + 334408914 a^2 b^7 x^{13} + 724860666 a^3 b^6 x^{11} + 993296384 a^4 b^5 x^9 + 884769030 a^5 b^4 x^7 + 724860666 a^6 b^3 x^5 + 334408914 a^7 b^2 x^3 + 88937058 a^8 b x + 10420767 a^9}{4128768 (bx^2 + a)^9 a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^10\*x^2),x, algorithm="giac")

[Out] -230945/65536\*b\*arctan(b\*x/sqrt(a\*b))/(sqrt(a\*b)\*a^10) - 1/(a^10\*x) - 1/4128768\*(10420767\*b^9\*x^17 + 88937058\*a\*b^8\*x^15 + 334408914\*a^2\*b^7\*x^13 + 724860666\*a^3\*b^6\*x^11 + 993296384\*a^4\*b^5\*x^9 + 884769030\*a^5\*b^4\*x^7 + 724860666\*a^6\*b^3\*x^5 + 334408914\*a^7\*b^2\*x^3 + 88937058\*a^8\*b\*x + 10420767\*a^9)

$$\frac{14a^2b^7x^{13} + 724860666a^3b^6x^{11} + 993296384a^4b^5x^9 + 884769030a^5b^4x^7 + 503730990a^6b^3x^5 + 169799070a^7b^2x^3 + 26738145a^8bx}{(bx^2 + a)^9a^{10}}$$

$$3.223 \quad \int \frac{1}{x^4(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=220

$$\begin{aligned} & \frac{1616615b^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{23/2}} + \frac{1616615b}{65536a^{11}x} - \frac{1616615}{196608a^{10}x^3} + \frac{323323}{65536a^9x^3(a+bx^2)} \\ & + \frac{46189}{32768a^8x^3(a+bx^2)^2} + \frac{46189}{73728a^7x^3(a+bx^2)^3} + \frac{4199}{12288a^6x^3(a+bx^2)^4} + \frac{323}{1536a^5x^3(a+bx^2)^5} \\ & + \frac{323}{2304a^4x^3(a+bx^2)^6} + \frac{19}{192a^3x^3(a+bx^2)^7} + \frac{7}{96a^2x^3(a+bx^2)^8} + \frac{1}{18ax^3(a+bx^2)^9} \end{aligned}$$

[Out]  $-1616615/(196608*a^{10}*x^3) + (1616615*b)/(65536*a^{11}*x) + 1/(18*a*x^3*(a+b*x^2)^9) + 7/(96*a^2*x^3*(a+b*x^2)^8) + 19/(192*a^3*x^3*(a+b*x^2)^7) + 323/(2304*a^4*x^3*(a+b*x^2)^6) + 323/(1536*a^5*x^3*(a+b*x^2)^5) + 4199/(12288*a^6*x^3*(a+b*x^2)^4) + 46189/(73728*a^7*x^3*(a+b*x^2)^3) + 46189/(32768*a^8*x^3*(a+b*x^2)^2) + 323323/(65536*a^9*x^3*(a+b*x^2)) + (1616615*b^{(3/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^{(23/2)})$

**Rubi [A]** time = 0.353685, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\begin{aligned} & \frac{1616615b^{3/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{23/2}} + \frac{1616615b}{65536a^{11}x} - \frac{1616615}{196608a^{10}x^3} + \frac{323323}{65536a^9x^3(a+bx^2)} \\ & + \frac{46189}{32768a^8x^3(a+bx^2)^2} + \frac{46189}{73728a^7x^3(a+bx^2)^3} + \frac{4199}{12288a^6x^3(a+bx^2)^4} + \frac{323}{1536a^5x^3(a+bx^2)^5} \\ & + \frac{323}{2304a^4x^3(a+bx^2)^6} + \frac{19}{192a^3x^3(a+bx^2)^7} + \frac{7}{96a^2x^3(a+bx^2)^8} + \frac{1}{18ax^3(a+bx^2)^9} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a+b\*x^2)^10),x]

[Out]  $-1616615/(196608*a^{10}*x^3) + (1616615*b)/(65536*a^{11}*x) + 1/(18*a*x^3*(a+b*x^2)^9) + 7/(96*a^2*x^3*(a+b*x^2)^8) + 19/(192*a^3*x^3*(a+b*x^2)^7) + 323/(2304*a^4*x^3*(a+b*x^2)^6) + 323/(1536*a^5*x^3*(a+b*x^2)^5) + 4199/(12288*a^6*x^3*(a+b*x^2)^4) + 46189/(73728*a^7*x^3*(a+b*x^2)^3) + 46189/(32768*a^8*x^3*(a+b*x^2)^2) + 323323/(65536*a^9*x^3*(a+b*x^2)) + (1616615*b^{(3/2)}*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(65536*a^{(23/2)})$

**Rubi in Sympy [A]** time = 61.4104, size = 211, normalized size = 0.96

$$\begin{aligned} & \frac{1}{18ax^3(a+bx^2)^9} + \frac{7}{96a^2x^3(a+bx^2)^8} + \frac{19}{192a^3x^3(a+bx^2)^7} + \frac{323}{2304a^4x^3(a+bx^2)^6} \\ & + \frac{323}{1536a^5x^3(a+bx^2)^5} + \frac{4199}{12288a^6x^3(a+bx^2)^4} + \frac{46189}{73728a^7x^3(a+bx^2)^3} + \frac{46189}{32768a^8x^3(a+bx^2)^2} \\ & + \frac{323323}{65536a^9x^3(a+bx^2)} - \frac{1616615}{196608a^{10}x^3} + \frac{1616615b}{65536a^{11}x} + \frac{1616615b^{3/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{23/2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(b\*x\*\*2+a)\*\*10,x)

[Out]  $1/(18*a*x**3*(a+b*x**2)**9) + 7/(96*a**2*x**3*(a+b*x**2)**8) + 19/(192*a**3*x**3*(a+b*x**2)**7) + 323/(2304*a**4*x**3*(a+b*x**2)**6) + 323/(1536*a**5*x**3*(a+b*x**2)**5) + 4199/(12288*a$

$6x^3(a + bx^2)^4 + 46189/(73728a^7x^3(a + bx^2)^3) + 46189/(32768a^8x^3(a + bx^2)^2) + 323323/(65536a^9x^3(a + bx^2)) - 1616615/(196608a^{10}x^3) + 1616615b/(65536a^{11}x) + 1616615b^{3/2} \operatorname{atan}(\sqrt{b}x/\sqrt{a})/(65536a^{23/2})$

**Mathematica [A]** time = 0.168704, size = 157, normalized size = 0.71

$$\frac{\sqrt{a}(-196608a^{10}+4128768a^9bx^2+63897057a^8b^2x^4+318434718a^7b^3x^6+850547502a^6b^4x^8+1404993798a^5b^5x^{10}+1513521152a^4b^6x^{12}+1071677178a^3b^7x^{14}+483044562a^2b^8x^{16}+126095970ab^9x^{18}+14549535b^{10}x^{20})}{x^3(a+bx^2)^9} + \frac{589824a^{23/2}}{x^3(a+bx^2)^9}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a + b\*x^2)^10), x]

[Out] ((Sqrt[a]\*(-196608\*a^10 + 4128768\*a^9\*b\*x^2 + 63897057\*a^8\*b^2\*x^4 + 318434718\*a^7\*b^3\*x^6 + 850547502\*a^6\*b^4\*x^8 + 1404993798\*a^5\*b^5\*x^10 + 1513521152\*a^4\*b^6\*x^12 + 1071677178\*a^3\*b^7\*x^14 + 483044562\*a^2\*b^8\*x^16 + 126095970\*a\*b^9\*x^18 + 14549535\*b^10\*x^20))/(x^3\*(a + b\*x^2)^9) + 14549535\*b^(3/2)\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/(589824\*a^(23/2))

**Maple [A]** time = 0.03, size = 219, normalized size = 1.

$$\begin{aligned} & -\frac{1}{3a^{10}x^3} + 10\frac{b}{a^{11}x} + \frac{1987865b^2x}{65536a^3(bx^2+a)^9} + \frac{20435525b^3x^3}{98304a^4(bx^2+a)^9} + \frac{21103775b^4x^5}{32768a^5(bx^2+a)^9} \\ & + \frac{38143787b^5x^7}{32768a^6(bx^2+a)^9} + \frac{24013b^6x^9}{18a^7(bx^2+a)^9} + \frac{32405717b^7x^{11}}{32768a^8(bx^2+a)^9} + \frac{15137633b^8x^{13}}{32768a^9(bx^2+a)^9} \\ & + \frac{12201403b^9x^{15}}{98304a^{10}(bx^2+a)^9} + \frac{961255b^{10}x^{17}}{65536a^{11}(bx^2+a)^9} + \frac{1616615b^2}{65536a^{11}} \arctan\left(bx\frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b\*x^2+a)^10, x)

[Out] -1/3/a^10/x^3+10\*b/a^11/x+1987865/65536/a^3\*b^2/(b\*x^2+a)^9\*x+20435525/98304/a^4\*b^3/(b\*x^2+a)^9\*x^3+21103775/32768/a^5\*b^4/(b\*x^2+a)^9\*x^5+38143787/32768/a^6\*b^5/(b\*x^2+a)^9\*x^7+24013/18/a^7\*b^6/(b\*x^2+a)^9\*x^9+32405717/32768/a^8\*b^7/(b\*x^2+a)^9\*x^11+15137633/32768/a^9\*b^8/(b\*x^2+a)^9\*x^13+12201403/98304/a^10\*b^9/(b\*x^2+a)^9\*x^15+961255/65536/a^11\*b^10/(b\*x^2+a)^9\*x^17+1616615/65536/a^11\*b^2/(a\*b)^(1/2)\*arctan(x\*b/(a\*b)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^10\*x^4), x, algorithm="maxima")

[Out] Exception raised: ValueError



**Fricas [A]** time = 0.250808, size = 1, normalized size = 0.

$$\frac{29099070 b^{10} x^{20} + 252191940 a b^9 x^{18} + 966089124 a^2 b^8 x^{16} + 2143354356 a^3 b^7 x^{14} + 3027042304 a^4 b^6 x^{12} + 2809987596 a^5 b^5 x^{10} + 1701095004 a^6 b^4 x^8 + 636869436 a^7 b^3 x^6 + 127794114 a^8 b^2 x^4 + 8257536 a^9 b x^2 - 393216 a^{10}}{(b^2 x^2 + a)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^10\*x^4),x, algorithm="fricas")

[Out] [1/1179648\*(29099070\*b^10\*x^20 + 252191940\*a\*b^9\*x^18 + 966089124\*a^2\*b^8\*x^16 + 2143354356\*a^3\*b^7\*x^14 + 3027042304\*a^4\*b^6\*x^12 + 2809987596\*a^5\*b^5\*x^10 + 1701095004\*a^6\*b^4\*x^8 + 636869436\*a^7\*b^3\*x^6 + 127794114\*a^8\*b^2\*x^4 + 8257536\*a^9\*b\*x^2 - 393216\*a^10 + 14549535\*(b^10\*x^21 + 9\*a\*b^9\*x^19 + 36\*a^2\*b^8\*x^17 + 84\*a^3\*b^7\*x^15 + 126\*a^4\*b^6\*x^13 + 126\*a^5\*b^5\*x^11 + 84\*a^6\*b^4\*x^9 + 36\*a^7\*b^3\*x^7 + 9\*a^8\*b^2\*x^5 + a^9\*b\*x^3)\*sqrt(-b/a)\*log((b\*x^2 + 2\*a\*x\*sqrt(-b/a) - a)/(b\*x^2 + a))/(a^11\*b^9\*x^21 + 9\*a^12\*b^8\*x^19 + 36\*a^13\*b^7\*x^17 + 84\*a^14\*b^6\*x^15 + 126\*a^15\*b^5\*x^13 + 126\*a^16\*b^4\*x^11 + 84\*a^17\*b^3\*x^9 + 36\*a^18\*b^2\*x^7 + 9\*a^19\*b\*x^5 + a^20\*x^3), 1/589824\*(14549535\*b^10\*x^20 + 126095970\*a\*b^9\*x^18 + 483044562\*a^2\*b^8\*x^16 + 1071677178\*a^3\*b^7\*x^14 + 1513521152\*a^4\*b^6\*x^12 + 1404993798\*a^5\*b^5\*x^10 + 850547502\*a^6\*b^4\*x^8 + 318434718\*a^7\*b^3\*x^6 + 63897057\*a^8\*b^2\*x^4 + 4128768\*a^9\*b\*x^2 - 196608\*a^10 + 14549535\*(b^10\*x^21 + 9\*a\*b^9\*x^19 + 36\*a^2\*b^8\*x^17 + 84\*a^3\*b^7\*x^15 + 126\*a^4\*b^6\*x^13 + 126\*a^5\*b^5\*x^11 + 84\*a^6\*b^4\*x^9 + 36\*a^7\*b^3\*x^7 + 9\*a^8\*b^2\*x^5 + a^9\*b\*x^3)\*sqrt(b/a)\*arctan(b\*x/(a\*sqrt(b/a)))/(a^11\*b^9\*x^21 + 9\*a^12\*b^8\*x^19 + 36\*a^13\*b^7\*x^17 + 84\*a^14\*b^6\*x^15 + 126\*a^15\*b^5\*x^13 + 126\*a^16\*b^4\*x^11 + 84\*a^17\*b^3\*x^9 + 36\*a^18\*b^2\*x^7 + 9\*a^19\*b\*x^5 + a^20\*x^3)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(b\*x\*\*2+a)\*\*10,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.219334, size = 200, normalized size = 0.91

$$\frac{1616615 b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{ab} a^{11}} + \frac{30 bx^2 - a}{3 a^{11} x^3} + \frac{8651295 b^{10} x^{17} + 73208418 ab^9 x^{15} + 272477394 a^2 b^8 x^{13} + 583302906 a^3 b^7 x^{11} + 786857984 a^4 b^6 x^9 + 686588166 a^5 b^5 x^7 + 379867950 a^6 b^4 x^5 + 122613150 a^7 b^3 x^3 + 17890785 a^8 b^2 x}{589824 (bx^2 + a)^9 a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^10\*x^4),x, algorithm="giac")

[Out] 1616615/65536\*b^2\*arctan(b\*x/sqrt(a\*b))/(sqrt(a\*b)\*a^11) + 1/3\*(30\*b\*x^2 - a)/(a^11\*x^3) + 1/589824\*(8651295\*b^10\*x^17 + 73208418\*a\*b^9\*x^15 + 272477394\*a^2\*b^8\*x^13 + 583302906\*a^3\*b^7\*x^11 + 786857984\*a^4\*b^6\*x^9 + 686588166\*a^5\*b^5\*x^7 + 379867950\*a^6\*b^4\*x^5 + 122613150\*a^7\*b^3\*x^3 + 17890785\*a^8\*b^2\*x)/((b\*x^2 + a)^9\*a^11)

$$3.224 \quad \int \frac{1}{x^6(a+bx^2)^{10}} dx$$

**Optimal.** Leaf size=233

$$\begin{aligned} & -\frac{7436429b^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{25/2}} - \frac{7436429b^2}{65536a^{12}x} + \frac{7436429b}{196608a^{11}x^3} - \frac{7436429}{327680a^{10}x^5} + \frac{1062347}{65536a^9x^5(a+bx^2)} \\ & + \frac{1062347}{294912a^8x^5(a+bx^2)^2} + \frac{96577}{73728a^7x^5(a+bx^2)^3} + \frac{7429}{12288a^6x^5(a+bx^2)^4} + \frac{7429}{23040a^5x^5(a+bx^2)^5} \\ & + \frac{437}{2304a^4x^5(a+bx^2)^6} + \frac{23}{192a^3x^5(a+bx^2)^7} + \frac{23}{288a^2x^5(a+bx^2)^8} + \frac{1}{18ax^5(a+bx^2)^9} \end{aligned}$$

[Out] -7436429/(327680\*a^10\*x^5) + (7436429\*b)/(196608\*a^11\*x^3) - (7436429\*b^2)/(65536\*a^12\*x) + 1/(18\*a\*x^5\*(a+b\*x^2)^9) + 23/(288\*a^2\*x^5\*(a+b\*x^2)^8) + 23/(192\*a^3\*x^5\*(a+b\*x^2)^7) + 437/(2304\*a^4\*x^5\*(a+b\*x^2)^6) + 7429/(23040\*a^5\*x^5\*(a+b\*x^2)^5) + 7429/(12288\*a^6\*x^5\*(a+b\*x^2)^4) + 96577/(73728\*a^7\*x^5\*(a+b\*x^2)^3) + 1062347/(294912\*a^8\*x^5\*(a+b\*x^2)^2) + 1062347/(65536\*a^9\*x^5\*(a+b\*x^2)) - (7436429\*b^(5/2)\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(65536\*a^(25/2))

**Rubi [A]** time = 0.396272, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\begin{aligned} & -\frac{7436429b^{5/2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{25/2}} - \frac{7436429b^2}{65536a^{12}x} + \frac{7436429b}{196608a^{11}x^3} - \frac{7436429}{327680a^{10}x^5} + \frac{1062347}{65536a^9x^5(a+bx^2)} \\ & + \frac{1062347}{294912a^8x^5(a+bx^2)^2} + \frac{96577}{73728a^7x^5(a+bx^2)^3} + \frac{7429}{12288a^6x^5(a+bx^2)^4} + \frac{7429}{23040a^5x^5(a+bx^2)^5} \\ & + \frac{437}{2304a^4x^5(a+bx^2)^6} + \frac{23}{192a^3x^5(a+bx^2)^7} + \frac{23}{288a^2x^5(a+bx^2)^8} + \frac{1}{18ax^5(a+bx^2)^9} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(a+b\*x^2)^10),x]

[Out] -7436429/(327680\*a^10\*x^5) + (7436429\*b)/(196608\*a^11\*x^3) - (7436429\*b^2)/(65536\*a^12\*x) + 1/(18\*a\*x^5\*(a+b\*x^2)^9) + 23/(288\*a^2\*x^5\*(a+b\*x^2)^8) + 23/(192\*a^3\*x^5\*(a+b\*x^2)^7) + 437/(2304\*a^4\*x^5\*(a+b\*x^2)^6) + 7429/(23040\*a^5\*x^5\*(a+b\*x^2)^5) + 7429/(12288\*a^6\*x^5\*(a+b\*x^2)^4) + 96577/(73728\*a^7\*x^5\*(a+b\*x^2)^3) + 1062347/(294912\*a^8\*x^5\*(a+b\*x^2)^2) + 1062347/(65536\*a^9\*x^5\*(a+b\*x^2)) - (7436429\*b^(5/2)\*ArcTan[(Sqrt[b]\*x)/Sqrt[a]])/(65536\*a^(25/2))

**Rubi in Sympy [A]** time = 71.3245, size = 224, normalized size = 0.96

$$\begin{aligned} & \frac{1}{18ax^5(a+bx^2)^9} + \frac{23}{288a^2x^5(a+bx^2)^8} + \frac{23}{192a^3x^5(a+bx^2)^7} + \frac{437}{2304a^4x^5(a+bx^2)^6} \\ & + \frac{7429}{23040a^5x^5(a+bx^2)^5} + \frac{7429}{12288a^6x^5(a+bx^2)^4} + \frac{96577}{73728a^7x^5(a+bx^2)^3} + \frac{1062347}{294912a^8x^5(a+bx^2)^2} \\ & + \frac{1062347}{65536a^9x^5(a+bx^2)} - \frac{7436429}{327680a^{10}x^5} + \frac{7436429b}{196608a^{11}x^3} - \frac{7436429b^2}{65536a^{12}x} - \frac{7436429b^{5/2} \operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{65536a^{25/2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*6/(b\*x\*\*2+a)\*\*10,x)

[Out] 1/(18\*a\*x\*\*5\*(a+b\*x\*\*2)\*\*9) + 23/(288\*a\*\*2\*x\*\*5\*(a+b\*x\*\*2)\*\*8) + 23/(192\*a\*\*3\*x\*\*5\*(a+b\*x\*\*2)\*\*7) + 437/(2304\*a\*\*4\*x\*\*5\*(a +

$$b^6 x^{12} + 7429/(23040 a^5 x^5 (a + b x^2)^5) + 7429/(12288 a^6 x^5 (a + b x^2)^4) + 96577/(73728 a^7 x^5 (a + b x^2)^3) + 1062347/(294912 a^8 x^5 (a + b x^2)^2) + 1062347/(65536 a^9 x^5 (a + b x^2)) - 7436429/(327680 a^{10} x^5) + 7436429 b/(196608 a^{11} x^3) - 7436429 b^2/(65536 a^{12} x) - 7436429 b^{5/2} \operatorname{atan}(\sqrt{b} x/\sqrt{a})/(65536 a^{25/2})$$

**Mathematica [A]** time = 0.178797, size = 169, normalized size = 0.73

$$-334639305 b^{5/2} \tan^{-1}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right) - \frac{\sqrt{a}(589824 a^{11} - 4521984 a^{10} b x^2 + 94961664 a^9 b^2 x^4 + 1469632311 a^8 b^3 x^6 + 7323998514 a^7 b^4 x^8 + 19562592546 a^6 b^5 x^{10} + 32314857354 a^5 b^6 x^{12} + 34810986496 a^4 b^7 x^{14} + 24648575094 a^3 b^8 x^{16} + 11110024926 a^2 b^9 x^{18} + 2900207310 a b^{10} x^{20} + 334639305 b^{11} x^{22})}{x^5 (a + b x^2)^9} - \frac{334639305 b^{5/2} \operatorname{ArcTan}[\sqrt{b} x/\sqrt{a}]}{(2949120 a^{25/2})}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6\*(a + b\*x^2)^10), x]

[Out]  $-\left(\frac{\sqrt{a}(589824 a^{11} - 4521984 a^{10} b x^2 + 94961664 a^9 b^2 x^4 + 1469632311 a^8 b^3 x^6 + 7323998514 a^7 b^4 x^8 + 19562592546 a^6 b^5 x^{10} + 32314857354 a^5 b^6 x^{12} + 34810986496 a^4 b^7 x^{14} + 24648575094 a^3 b^8 x^{16} + 11110024926 a^2 b^9 x^{18} + 2900207310 a b^{10} x^{20} + 334639305 b^{11} x^{22})}{x^5 (a + b x^2)^9} - \frac{334639305 b^{5/2} \operatorname{ArcTan}[\sqrt{b} x/\sqrt{a}]}{(2949120 a^{25/2})}\right)$

**Maple [A]** time = 0.031, size = 230, normalized size = 1.

$$\begin{aligned} &-\frac{1}{5 a^{10} x^5} - 55 \frac{b^2}{a^{12} x} + \frac{10 b}{3 a^{11} x^3} - \frac{6981491 b^3 x}{65536 a^4 (b x^2 + a)^9} - \frac{74539223 b^4 x^3}{98304 a^5 (b x^2 + a)^9} - \frac{394553929 b^5 x^5}{163840 a^6 (b x^2 + a)^9} \\ &-\frac{725918941 b^6 x^7}{163840 a^7 (b x^2 + a)^9} - \frac{463199 b^7 x^9}{90 a^8 (b x^2 + a)^9} - \frac{631790371 b^8 x^{11}}{163840 a^9 (b x^2 + a)^9} - \frac{297702839 b^9 x^{13}}{163840 a^{10} (b x^2 + a)^9} \\ &-\frac{48340777 b^{10} x^{15}}{98304 a^{11} (b x^2 + a)^9} - \frac{3831949 b^{11} x^{17}}{65536 a^{12} (b x^2 + a)^9} - \frac{7436429 b^3}{65536 a^{12}} \arctan\left(\frac{b x}{\sqrt{a b}}\right) \frac{1}{\sqrt{a b}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(b\*x^2+a)^10, x)

[Out]  $-1/5/a^{10}/x^5 - 55*b^2/a^{12}/x + 10/3*b/a^{11}/x^3 - 6981491/65536/a^4*b^3/(b*x^2+a)^9*x - 74539223/98304/a^5*b^4/(b*x^2+a)^9*x^3 - 394553929/163840/a^6*b^5/(b*x^2+a)^9*x^5 - 725918941/163840/a^7*b^6/(b*x^2+a)^9*x^7 - 463199/90/a^8*b^7/(b*x^2+a)^9*x^9 - 631790371/163840/a^9*b^8/(b*x^2+a)^9*x^{11} - 297702839/163840/a^{10}*b^9/(b*x^2+a)^9*x^{13} - 48340777/98304/a^{11}*b^{10}/(b*x^2+a)^9*x^{15} - 3831949/65536/a^{12}*b^{11}/(b*x^2+a)^9*x^{17} - 7436429/65536/a^{12}*b^3/(a*b)^{(1/2)}*\arctan(x*b/(a*b)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^10\*x^6), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.239233, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^10\*x^6),x, algorithm="fricas")

[Out] [-1/5898240\*(669278610\*b^11\*x^22 + 5800414620\*a\*b^10\*x^20 + 22220049852\*a^2\*b^9\*x^18 + 49297150188\*a^3\*b^8\*x^16 + 69621972992\*a^4\*b^7\*x^14 + 64629714708\*a^5\*b^6\*x^12 + 39125185092\*a^6\*b^5\*x^10 + 14647997028\*a^7\*b^4\*x^8 + 2939264622\*a^8\*b^3\*x^6 + 189923328\*a^9\*b^2\*x^4 - 9043968\*a^10\*b\*x^2 + 1179648\*a^11 - 334639305\*(b^11\*x^23 + 9\*a\*b^10\*x^21 + 36\*a^2\*b^9\*x^19 + 84\*a^3\*b^8\*x^17 + 126\*a^4\*b^7\*x^15 + 126\*a^5\*b^6\*x^13 + 84\*a^6\*b^5\*x^11 + 36\*a^7\*b^4\*x^9 + 9\*a^8\*b^3\*x^7 + a^9\*b^2\*x^5)\*sqrt(-b/a)\*log((b\*x^2 - 2\*a\*x\*sqrt(-b/a) - a)/(b\*x^2 + a)))/(a^12\*b^9\*x^23 + 9\*a^13\*b^8\*x^21 + 36\*a^14\*b^7\*x^19 + 84\*a^15\*b^6\*x^17 + 126\*a^16\*b^5\*x^15 + 126\*a^17\*b^4\*x^13 + 84\*a^18\*b^3\*x^11 + 36\*a^19\*b^2\*x^9 + 9\*a^20\*b\*x^7 + a^21\*x^5), -1/2949120\*(334639305\*b^11\*x^22 + 2900207310\*a\*b^10\*x^20 + 11110024926\*a^2\*b^9\*x^18 + 24648575094\*a^3\*b^8\*x^16 + 34810986496\*a^4\*b^7\*x^14 + 32314857354\*a^5\*b^6\*x^12 + 19562592546\*a^6\*b^5\*x^10 + 7323998514\*a^7\*b^4\*x^8 + 1469632311\*a^8\*b^3\*x^6 + 94961664\*a^9\*b^2\*x^4 - 4521984\*a^10\*b\*x^2 + 589824\*a^11 + 334639305\*(b^11\*x^23 + 9\*a\*b^10\*x^21 + 36\*a^2\*b^9\*x^19 + 84\*a^3\*b^8\*x^17 + 126\*a^4\*b^7\*x^15 + 126\*a^5\*b^6\*x^13 + 84\*a^6\*b^5\*x^11 + 36\*a^7\*b^4\*x^9 + 9\*a^8\*b^3\*x^7 + a^9\*b^2\*x^5)\*sqrt(b/a)\*arctan(b\*x/(a\*sqrt(b/a)))]/(a^12\*b^9\*x^23 + 9\*a^13\*b^8\*x^21 + 36\*a^14\*b^7\*x^19 + 84\*a^15\*b^6\*x^17 + 126\*a^16\*b^5\*x^15 + 126\*a^17\*b^4\*x^13 + 84\*a^18\*b^3\*x^11 + 36\*a^19\*b^2\*x^9 + 9\*a^20\*b\*x^7 + a^21\*x^5)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*6/(b\*x\*\*2+a)\*\*10,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.217717, size = 215, normalized size = 0.92

$$\frac{7436429 b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \sqrt{aba}^{12}} - \frac{825 b^2 x^4 - 50 abx^2 + 3 a^2}{15 a^{12} x^5} - \frac{172437705 b^{11} x^{17} + 1450223310 ab^{10} x^{15} + 5358651102 a^2 b^9 x^{13} + 11372226678 a^3 b^8 x^{11} + 15178104832 a^4 b^7 x^9 + 13066540938 a^5 b^6 x^7 + 7101970722 a^6 b^5 x^5 + 2236176690 a^7 b^4 x^3 + 314167095 a^8 b^3 x}{2949120 (bx^2 + a)^9 a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^10\*x^6),x, algorithm="giac")

[Out] -7436429/65536\*b^3\*arctan(b\*x/sqrt(a\*b))/(sqrt(a\*b)\*a^12) - 1/15\*(825\*b^2\*x^4 - 50\*a\*b\*x^2 + 3\*a^2)/(a^12\*x^5) - 1/2949120\*(172437705\*b^11\*x^17 + 1450223310\*a\*b^10\*x^15 + 5358651102\*a^2\*b^9\*x^13 + 11372226678\*a^3\*b^8\*x^11 + 15178104832\*a^4\*b^7\*x^9 + 13066540938\*a^5\*b^6\*x^7 + 7101970722\*a^6\*b^5\*x^5 + 2236176690\*a^7\*b^4\*x^3 + 314167095\*a^8\*b^3\*x)/(b\*x^2 + a)^9\*a^12)

$$3.225 \quad \int \frac{x^3}{a-bx^2} dx$$

**Optimal.** Leaf size=28

$$-\frac{a \log(a - bx^2)}{2b^2} - \frac{x^2}{2b}$$

[Out]  $-x^2/(2*b) - (a*\text{Log}[a - b*x^2])/(2*b^2)$

**Rubi [A]** time = 0.0586737, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\frac{a \log(a - bx^2)}{2b^2} - \frac{x^2}{2b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3/(a - b*x^2), x]$

[Out]  $-x^2/(2*b) - (a*\text{Log}[a - b*x^2])/(2*b^2)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a \log(a - bx^2)}{2b^2} - \int \frac{1}{b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**3}/(-b*x^{**2}+a), x)$

[Out]  $-a*\log(a - b*x^{**2})/(2*b^{**2}) - \text{Integral}(1/b, (x, x^{**2}))/2$

**Mathematica [A]** time = 0.00873298, size = 28, normalized size = 1.

$$-\frac{a \log(a - bx^2)}{2b^2} - \frac{x^2}{2b}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^3/(a - b*x^2), x]$

[Out]  $-x^2/(2*b) - (a*\text{Log}[a - b*x^2])/(2*b^2)$

**Maple [A]** time = 0.004, size = 26, normalized size = 0.9

$$\frac{x^2}{2b} - \frac{a \ln(bx^2 - a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^3/(-b*x^2+a), x)$

[Out]  $-1/2*x^2/b - 1/2*a/b^2*\ln(b*x^2 - a)$

---

**Maxima [A]** time = 1.33983, size = 34, normalized size = 1.21

$$-\frac{x^2}{2b} - \frac{a \log(bx^2 - a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x^3/(b\*x^2 - a), x, algorithm="maxima")

[Out] -1/2\*x^2/b - 1/2\*a\*log(b\*x^2 - a)/b^2

---

**Fricas [A]** time = 0.226322, size = 31, normalized size = 1.11

$$-\frac{bx^2 + a \log(bx^2 - a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x^3/(b\*x^2 - a), x, algorithm="fricas")

[Out] -1/2\*(b\*x^2 + a\*log(b\*x^2 - a))/b^2

---

**Sympy [A]** time = 1.2523, size = 22, normalized size = 0.79

$$-\frac{a \log(-a + bx^2)}{2b^2} - \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(-b\*x\*\*2+a), x)

[Out] -a\*log(-a + b\*x\*\*2)/(2\*b\*\*2) - x\*\*2/(2\*b)

---

**GIAC/XCAS [A]** time = 0.208559, size = 35, normalized size = 1.25

$$-\frac{x^2}{2b} - \frac{a \ln(|bx^2 - a|)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x^3/(b\*x^2 - a), x, algorithm="giac")

[Out] -1/2\*x^2/b - 1/2\*a\*ln(abs(b\*x^2 - a))/b^2

$$3.226 \quad \int \frac{x^2}{a-bx^2} dx$$

**Optimal.** Leaf size=31

$$\frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{3/2}} - \frac{x}{b}$$

[Out]  $-(x/b) + (\text{Sqrt}[a] * \text{ArcTanh}[(\text{Sqrt}[b] * x) / \text{Sqrt}[a]]) / b^{(3/2)}$

**Rubi [A]** time = 0.0396907, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{3/2}} - \frac{x}{b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2/(a - b*x^2), x]$

[Out]  $-(x/b) + (\text{Sqrt}[a] * \text{ArcTanh}[(\text{Sqrt}[b] * x) / \text{Sqrt}[a]]) / b^{(3/2)}$

**Rubi in Sympy [A]** time = 7.39249, size = 26, normalized size = 0.84

$$\frac{\sqrt{a} \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**2}/(-b*x^{**2}+a), x)$

[Out]  $\text{sqrt}(a) * \text{atanh}(\text{sqrt}(b) * x / \text{sqrt}(a)) / b^{**}(3/2) - x/b$

**Mathematica [A]** time = 0.0176442, size = 31, normalized size = 1.

$$\frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{3/2}} - \frac{x}{b}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^2/(a - b*x^2), x]$

[Out]  $-(x/b) + (\text{Sqrt}[a] * \text{ArcTanh}[(\text{Sqrt}[b] * x) / \text{Sqrt}[a]]) / b^{(3/2)}$

**Maple [A]** time = 0.003, size = 27, normalized size = 0.9

$$-\frac{x}{b} + \frac{a}{b} \operatorname{Artanh}\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^2/(-b*x^2+a), x)$

[Out]  $-x/b+a/b/(a*b)^{(1/2)}*\operatorname{arctanh}(x*b/(a*b)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-x^2/(b*x^2 - a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.264197, size = 1, normalized size = 0.03

$$\left[ \frac{\sqrt{\frac{a}{b}} \log\left(\frac{bx^2+2bx\sqrt{\frac{a}{b}}+a}{bx^2-a}\right) - 2x \sqrt{-\frac{a}{b}} \arctan\left(\frac{x}{\sqrt{-\frac{a}{b}}}\right) - x}{2b}, \frac{\sqrt{-\frac{a}{b}} \arctan\left(\frac{x}{\sqrt{-\frac{a}{b}}}\right) - x}{b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-x^2/(b*x^2 - a),x, algorithm="fricas")`

[Out]  $[1/2*(\sqrt{a/b})*\log((b*x^2 + 2*b*x*\sqrt{a/b} + a)/(b*x^2 - a)) - 2*x)/b, (\sqrt{-a/b})*\arctan(x/\sqrt{-a/b}) - x)/b]$

**Sympy [A]** time = 1.24694, size = 49, normalized size = 1.58

$$-\frac{\sqrt{\frac{a}{b^3}} \log\left(-b\sqrt{\frac{a}{b^3}} + x\right)}{2} + \frac{\sqrt{\frac{a}{b^3}} \log\left(b\sqrt{\frac{a}{b^3}} + x\right)}{2} - \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(-b*x**2+a),x)`

[Out]  $-\sqrt{a/b**3}*\log(-b*\sqrt{a/b**3} + x)/2 + \sqrt{a/b**3}*\log(b*\sqrt{a/b**3} + x)/2 - x/b$

**GIAC/XCAS [A]** time = 0.209247, size = 39, normalized size = 1.26

$$-\frac{a \arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-abb}} - \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-x^2/(b*x^2 - a),x, algorithm="giac")`

[Out]  $-a*\arctan(b*x/\sqrt{-a*b})/(\sqrt{-a*b})*b - x/b$



$$3.227 \quad \int \frac{x}{a-bx^2} dx$$

**Optimal.** Leaf size=16

$$-\frac{\log(a-bx^2)}{2b}$$

[Out] -Log[a - b\*x^2]/(2\*b)

**Rubi [A]** time = 0.0107626, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$-\frac{\log(a-bx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Int[x/(a - b\*x^2), x]

[Out] -Log[a - b\*x^2]/(2\*b)

**Rubi in Sympy [A]** time = 2.52453, size = 12, normalized size = 0.75

$$-\frac{\log(a-bx^2)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(-b\*x\*\*2+a), x)

[Out] -log(a - b\*x\*\*2)/(2\*b)

**Mathematica [A]** time = 0.00359437, size = 16, normalized size = 1.

$$-\frac{\log(a-bx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a - b\*x^2), x]

[Out] -Log[a - b\*x^2]/(2\*b)

**Maple [A]** time = 0.001, size = 16, normalized size = 1.

$$-\frac{\ln(bx^2 - a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-b\*x^2+a), x)

[Out] -1/2/b\*ln(b\*x^2-a)

---

**Maxima [A]** time = 1.34938, size = 20, normalized size = 1.25

$$-\frac{\log(bx^2 - a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x/(b\*x^2 - a), x, algorithm="maxima")

[Out] -1/2\*log(b\*x^2 - a)/b

---

**Fricas [A]** time = 0.214308, size = 20, normalized size = 1.25

$$-\frac{\log(bx^2 - a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x/(b\*x^2 - a), x, algorithm="fricas")

[Out] -1/2\*log(b\*x^2 - a)/b

---

**Sympy [A]** time = 0.273871, size = 12, normalized size = 0.75

$$-\frac{\log(-a + bx^2)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-b\*x\*\*2+a), x)

[Out] -log(-a + b\*x\*\*2)/(2\*b)

---

**GIAC/XCAS [A]** time = 0.213447, size = 22, normalized size = 1.38

$$-\frac{\ln(|bx^2 - a|)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x/(b\*x^2 - a), x, algorithm="giac")

[Out] -1/2\*ln(abs(b\*x^2 - a))/b

$$3.228 \quad \int \frac{1}{a-bx^2} dx$$

**Optimal.** Leaf size=24

$$\frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

[Out] ArcTanh[(Sqrt[b]\*x)/Sqrt[a]]/(Sqrt[a]\*Sqrt[b])

**Rubi [A]** time = 0.0216113, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(-1), x]

[Out] ArcTanh[(Sqrt[b]\*x)/Sqrt[a]]/(Sqrt[a]\*Sqrt[b])

**Rubi in Sympy [A]** time = 2.745, size = 22, normalized size = 0.92

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-b\*x\*\*2+a), x)

[Out] atanh(sqrt(b)\*x/sqrt(a))/(sqrt(a)\*sqrt(b))

**Mathematica [A]** time = 0.00769335, size = 24, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(-1), x]

[Out] ArcTanh[(Sqrt[b]\*x)/Sqrt[a]]/(Sqrt[a]\*Sqrt[b])

**Maple [A]** time = 0.002, size = 16, normalized size = 0.7

$$1\operatorname{Artanh}\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^2+a), x)

[Out]  $1/(a*b)^{(1/2)}*\operatorname{arctanh}(x*b/(a*b)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/(b*x^2 - a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.224231, size = 1, normalized size = 0.04

$$\left[ \frac{\log\left(\frac{2abx+(bx^2+a)\sqrt{ab}}{bx^2-a}\right)}{2\sqrt{ab}}, \frac{\operatorname{arctan}\left(\frac{\sqrt{-ab}x}{a}\right)}{\sqrt{-ab}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/(b*x^2 - a),x, algorithm="fricas")`

[Out]  $[1/2*\log((2*a*b*x + (b*x^2 + a)*\sqrt{a*b})/(b*x^2 - a))/\sqrt{a*b}, \operatorname{arctan}(\sqrt{-a*b}*x/a)/\sqrt{-a*b}]$

**Sympy [A]** time = 0.331316, size = 46, normalized size = 1.92

$$-\frac{\sqrt{\frac{1}{ab}} \log\left(-a\sqrt{\frac{1}{ab}} + x\right)}{2} + \frac{\sqrt{\frac{1}{ab}} \log\left(a\sqrt{\frac{1}{ab}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**2+a),x)`

[Out]  $-\sqrt{1/(a*b)}*\log(-a*\sqrt{1/(a*b)} + x)/2 + \sqrt{1/(a*b)}*\log(a*\sqrt{1/(a*b)} + x)/2$

**GIAC/XCAS [A]** time = 0.207205, size = 24, normalized size = 1.

$$-\frac{\operatorname{arctan}\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/(b*x^2 - a),x, algorithm="giac")`

[Out]  $-\operatorname{arctan}(b*x/\sqrt{-a*b})/\sqrt{-a*b}$

$$3.229 \quad \int \frac{1}{x(a-bx^2)} dx$$

**Optimal.** Leaf size=23

$$\frac{\log(x)}{a} - \frac{\log(a-bx^2)}{2a}$$

[Out] Log[x]/a - Log[a - b\*x^2]/(2\*a)

**Rubi [A]** time = 0.0367481, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{\log(x)}{a} - \frac{\log(a-bx^2)}{2a}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(a - b\*x^2)), x]

[Out] Log[x]/a - Log[a - b\*x^2]/(2\*a)

**Rubi in Sympy [A]** time = 6.48221, size = 19, normalized size = 0.83

$$\frac{\log(x^2)}{2a} - \frac{\log(a-bx^2)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(-b\*x\*\*2+a), x)

[Out] log(x\*\*2)/(2\*a) - log(a - b\*x\*\*2)/(2\*a)

**Mathematica [A]** time = 0.0110714, size = 23, normalized size = 1.

$$\frac{\log(x)}{a} - \frac{\log(a-bx^2)}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(a - b\*x^2)), x]

[Out] Log[x]/a - Log[a - b\*x^2]/(2\*a)

**Maple [A]** time = 0.006, size = 23, normalized size = 1.

$$\frac{\ln(x)}{a} - \frac{\ln(bx^2 - a)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-b\*x^2+a), x)

[Out] ln(x)/a-1/2/a\*ln(b\*x^2-a)

---

**Maxima [A]** time = 1.35458, size = 34, normalized size = 1.48

$$-\frac{\log(bx^2 - a)}{2a} + \frac{\log(x^2)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/((b\*x^2 - a)\*x),x, algorithm="maxima")

[Out] -1/2\*log(b\*x^2 - a)/a + 1/2\*log(x^2)/a

---

**Fricas [A]** time = 0.210973, size = 27, normalized size = 1.17

$$\frac{\log(bx^2 - a) - 2 \log(x)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/((b\*x^2 - a)\*x),x, algorithm="fricas")

[Out] -1/2\*(log(b\*x^2 - a) - 2\*log(x))/a

---

**Sympy [A]** time = 0.530493, size = 15, normalized size = 0.65

$$\frac{\log(x)}{a} - \frac{\log(-\frac{a}{b} + x^2)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-b\*x\*\*2+a),x)

[Out] log(x)/a - log(-a/b + x\*\*2)/(2\*a)

---

**GIAC/XCAS [A]** time = 0.209211, size = 35, normalized size = 1.52

$$\frac{\ln(x^2)}{2a} - \frac{\ln(|bx^2 - a|)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/((b\*x^2 - a)\*x),x, algorithm="giac")

[Out] 1/2\*ln(x^2)/a - 1/2\*ln(abs(b\*x^2 - a))/a

$$3.230 \quad \int \frac{1}{x^2(a-bx^2)} dx$$

**Optimal.** Leaf size=33

$$\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{ax}$$

[Out]  $-(1/(a*x)) + (\text{Sqrt}[b]*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/a^{(3/2)}$

**Rubi [A]** time = 0.0375497, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{ax}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^2*(a - b*x^2)), x]`

[Out]  $-(1/(a*x)) + (\text{Sqrt}[b]*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/a^{(3/2)}$

**Rubi in Sympy [A]** time = 7.14497, size = 27, normalized size = 0.82

$$-\frac{1}{ax} + \frac{\sqrt{b} \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**2/(-b*x**2+a), x)`

[Out]  $-1/(a*x) + \text{sqrt}(b)*\text{atanh}(\text{sqrt}(b)*x/\text{sqrt}(a))/a^{(3/2)}$

**Mathematica [A]** time = 0.0206008, size = 33, normalized size = 1.

$$\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{ax}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^2*(a - b*x^2)), x]`

[Out]  $-(1/(a*x)) + (\text{Sqrt}[b]*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/a^{(3/2)}$

**Maple [A]** time = 0.006, size = 29, normalized size = 0.9

$$\frac{b}{a} \operatorname{Artanh}\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(-b*x^2+a), x)`

[Out]  $b/a/(a*b)^{(1/2)}*\operatorname{arctanh}(x*b/(a*b)^{(1/2)})-1/a/x$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((b*x^2 - a)*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.212304, size = 1, normalized size = 0.03

$$\left[ \frac{x\sqrt{\frac{b}{a}} \log\left(\frac{bx^2+2ax\sqrt{\frac{b}{a}}+a}{bx^2-a}\right) - 2x\sqrt{-\frac{b}{a}} \arctan\left(\frac{bx}{a\sqrt{-\frac{b}{a}}}\right) - 1}{2ax}, \frac{x\sqrt{-\frac{b}{a}} \arctan\left(\frac{bx}{a\sqrt{-\frac{b}{a}}}\right) - 1}{ax} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((b*x^2 - a)*x^2),x, algorithm="fricas")`

[Out]  $[1/2*(x*\sqrt{b/a})*\log((b*x^2 + 2*a*x*\sqrt{b/a} + a)/(b*x^2 - a)) - 2)/(a*x), (x*\sqrt{-b/a})*\arctan(b*x/(a*\sqrt{-b/a})) - 1)/(a*x]$

**Sympy [A]** time = 1.32115, size = 58, normalized size = 1.76

$$-\frac{\sqrt{\frac{b}{a^3}} \log\left(-\frac{a^2\sqrt{\frac{b}{a^3}}}{b} + x\right)}{2} + \frac{\sqrt{\frac{b}{a^3}} \log\left(\frac{a^2\sqrt{\frac{b}{a^3}}}{b} + x\right)}{2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(-b*x**2+a),x)`

[Out]  $-\sqrt{b/a^{**3}}*\log(-a^{**2}*\sqrt{b/a^{**3}}/b + x)/2 + \sqrt{b/a^{**3}}*\log(a^{**2}*\sqrt{b/a^{**3}}/b + x)/2 - 1/(a*x)$

**GIAC/XCAS [A]** time = 0.210403, size = 42, normalized size = 1.27

$$-\frac{b \arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-aba}} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((b*x^2 - a)*x^2),x, algorithm="giac")`

[Out]  $-b*\arctan(b*x/\sqrt{-a*b})/(\sqrt{-a*b}*a) - 1/(a*x)$



$$3.231 \quad \int \frac{1}{x^3(a-bx^2)} dx$$

**Optimal.** Leaf size=35

$$-\frac{b \log(a-bx^2)}{2a^2} + \frac{b \log(x)}{a^2} - \frac{1}{2ax^2}$$

[Out]  $-1/(2*a*x^2) + (b*\text{Log}[x])/a^2 - (b*\text{Log}[a - b*x^2])/(2*a^2)$

**Rubi [A]** time = 0.0606195, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\frac{b \log(a-bx^2)}{2a^2} + \frac{b \log(x)}{a^2} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^3*(a - b*x^2)), x]`

[Out]  $-1/(2*a*x^2) + (b*\text{Log}[x])/a^2 - (b*\text{Log}[a - b*x^2])/(2*a^2)$

**Rubi in Sympy [A]** time = 9.48546, size = 34, normalized size = 0.97

$$-\frac{1}{2ax^2} + \frac{b \log(x^2)}{2a^2} - \frac{b \log(a-bx^2)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**3/(-b*x**2+a), x)`

[Out]  $-1/(2*a*x**2) + b*\log(x**2)/(2*a**2) - b*\log(a - b*x**2)/(2*a**2)$

**Mathematica [A]** time = 0.0142872, size = 35, normalized size = 1.

$$-\frac{b \log(a-bx^2)}{2a^2} + \frac{b \log(x)}{a^2} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^3*(a - b*x^2)), x]`

[Out]  $-1/(2*a*x^2) + (b*\text{Log}[x])/a^2 - (b*\text{Log}[a - b*x^2])/(2*a^2)$

**Maple [A]** time = 0.008, size = 33, normalized size = 0.9

$$-\frac{1}{2ax^2} + \frac{b \ln(x)}{a^2} - \frac{b \ln(bx^2 - a)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(-b*x^2+a), x)`

[Out]  $-1/2/a/x^2+b*\ln(x)/a^2-1/2*b/a^2*\ln(b*x^2-a)$

---

**Maxima [A]** time = 1.35136, size = 47, normalized size = 1.34

$$-\frac{b \log (bx^2 - a)}{2 a^2} + \frac{b \log (x^2)}{2 a^2} - \frac{1}{2 ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/((b\*x^2 - a)\*x^3),x, algorithm="maxima")

[Out] -1/2\*b\*log(b\*x^2 - a)/a^2 + 1/2\*b\*log(x^2)/a^2 - 1/2/(a\*x^2)

---

**Fricas [A]** time = 0.206204, size = 45, normalized size = 1.29

$$-\frac{bx^2 \log (bx^2 - a) - 2 bx^2 \log (x) + a}{2 a^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/((b\*x^2 - a)\*x^3),x, algorithm="fricas")

[Out] -1/2\*(b\*x^2\*log(b\*x^2 - a) - 2\*b\*x^2\*log(x) + a)/(a^2\*x^2)

---

**Sympy [A]** time = 1.65492, size = 31, normalized size = 0.89

$$-\frac{1}{2ax^2} + \frac{b \log (x)}{a^2} - \frac{b \log \left(-\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(-b\*x\*\*2+a),x)

[Out] -1/(2\*a\*x\*\*2) + b\*log(x)/a\*\*2 - b\*log(-a/b + x\*\*2)/(2\*a\*\*2)

---

**GIAC/XCAS [A]** time = 0.211988, size = 58, normalized size = 1.66

$$\frac{b \ln (x^2)}{2 a^2} - \frac{b \ln (|bx^2 - a|)}{2 a^2} - \frac{bx^2 + a}{2 a^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/((b\*x^2 - a)\*x^3),x, algorithm="giac")

[Out] 1/2\*b\*ln(x^2)/a^2 - 1/2\*b\*ln(abs(b\*x^2 - a))/a^2 - 1/2\*(b\*x^2 + a)/(a^2\*x^2)

$$3.232 \quad \int \frac{x^3}{(a-bx^2)^2} dx$$

**Optimal.** Leaf size=35

$$\frac{a}{2b^2(a-bx^2)} + \frac{\log(a-bx^2)}{2b^2}$$

[Out]  $a/(2*b^2*(a - b*x^2)) + \text{Log}[a - b*x^2]/(2*b^2)$

**Rubi [A]** time = 0.0669209, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{a}{2b^2(a-bx^2)} + \frac{\log(a-bx^2)}{2b^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3/(a - b*x^2)^2, x]$

[Out]  $a/(2*b^2*(a - b*x^2)) + \text{Log}[a - b*x^2]/(2*b^2)$

**Rubi in Sympy [A]** time = 9.15148, size = 26, normalized size = 0.74

$$\frac{a}{2b^2(a-bx^2)} + \frac{\log(a-bx^2)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**3}/(-b*x^{**2}+a)^{**2}, x)$

[Out]  $a/(2*b^{**2}*(a - b*x^{**2})) + \log(a - b*x^{**2})/(2*b^{**2})$

**Mathematica [A]** time = 0.0215297, size = 29, normalized size = 0.83

$$\frac{\frac{a}{a-bx^2} + \log(a-bx^2)}{2b^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^3/(a - b*x^2)^2, x]$

[Out]  $(a/(a - b*x^2) + \text{Log}[a - b*x^2])/(2*b^2)$

**Maple [A]** time = 0.012, size = 34, normalized size = 1.

$$\frac{\ln(bx^2 - a)}{2b^2} - \frac{a}{2b^2(bx^2 - a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^3/(-b*x^2+a)^2, x)$

[Out]  $1/2/b^2 \ln(bx^2 - a) - 1/2 \cdot a/b^2 / (bx^2 - a)$

**Maxima [A]** time = 1.34635, size = 47, normalized size = 1.34

$$-\frac{a}{2(b^3x^2 - ab^2)} + \frac{\log(bx^2 - a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 - a)^2,x, algorithm="maxima")`

[Out]  $-1/2 \cdot a/(b^3x^2 - a \cdot b^2) + 1/2 \cdot \log(bx^2 - a)/b^2$

**Fricas [A]** time = 0.200784, size = 57, normalized size = 1.63

$$\frac{(bx^2 - a) \log(bx^2 - a) - a}{2(b^3x^2 - ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 - a)^2,x, algorithm="fricas")`

[Out]  $1/2 \cdot ((bx^2 - a) \cdot \log(bx^2 - a) - a)/(b^3x^2 - a \cdot b^2)$

**Sympy [A]** time = 1.37154, size = 29, normalized size = 0.83

$$-\frac{a}{-2ab^2 + 2b^3x^2} + \frac{\log(-a + bx^2)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(-b*x**2+a)**2,x)`

[Out]  $-a/(-2 \cdot a \cdot b^2 + 2 \cdot b^3 \cdot x^2) + \log(-a + b \cdot x^2)/(2 \cdot b^2)$

**GIAC/XCAS [A]** time = 0.212022, size = 72, normalized size = 2.06

$$-\frac{\frac{\ln\left(\frac{|bx^2-a|}{(bx^2-a)^2|b|}\right)}{b} + \frac{a}{(bx^2-a)b}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 - a)^2,x, algorithm="giac")`

[Out]  $-1/2 \cdot (\ln(\text{abs}(bx^2 - a)/((bx^2 - a)^2 \cdot \text{abs}(b))))/b + a/((bx^2 - a) \cdot b)/b$

$$3.233 \quad \int \frac{x^2}{(a-bx^2)^2} dx$$

**Optimal.** Leaf size=46

$$\frac{x}{2b(a-bx^2)} - \frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2\sqrt{ab}^{3/2}}$$

[Out]  $x/(2*b*(a - b*x^2)) - \text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/(2*\text{Sqrt}[a]*b^{(3/2)})$

**Rubi [A]** time = 0.0417818, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{x}{2b(a-bx^2)} - \frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2\sqrt{ab}^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[x^2/(a - b*x^2)^2, x]`

[Out]  $x/(2*b*(a - b*x^2)) - \text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/(2*\text{Sqrt}[a]*b^{(3/2)})$

**Rubi in Sympy [A]** time = 6.73142, size = 36, normalized size = 0.78

$$\frac{x}{2b(a-bx^2)} - \frac{\text{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2\sqrt{ab}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2/(-b*x**2+a)**2, x)`

[Out]  $x/(2*b*(a - b*x**2)) - \text{atanh}(\text{sqrt}(b)*x/\text{sqrt}(a))/(2*\text{sqrt}(a)*b**(3/2))$

**Mathematica [A]** time = 0.0532356, size = 47, normalized size = 1.02

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2\sqrt{ab}^{3/2}} - \frac{x}{2b(bx^2 - a)}$$

Antiderivative was successfully verified.

[In] `Integrate[x^2/(a - b*x^2)^2, x]`

[Out]  $-x/(2*b*(-a + b*x^2)) - \text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/(2*\text{Sqrt}[a]*b^{(3/2)})$

**Maple [A]** time = 0.01, size = 38, normalized size = 0.8

$$-\frac{x}{2b(bx^2 - a)} - \frac{1}{2b} \text{Artanh}\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(-b*x^2+a)^2,x)`

[Out]  $-1/2/b*x/(b*x^2-a)-1/2/b/(a*b)^{(1/2)}*\operatorname{arctanh}(x*b/(a*b)^{(1/2)})$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 - a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.214926, size = 1, normalized size = 0.02

$$\left[ \frac{(bx^2 - a) \log\left(-\frac{2abx - (bx^2 + a)\sqrt{ab}}{bx^2 - a}\right) - 2\sqrt{ab}x}{4(b^2x^2 - ab)\sqrt{ab}}, -\frac{(bx^2 - a) \arctan\left(\frac{\sqrt{-ab}x}{a}\right) + \sqrt{-ab}x}{2(b^2x^2 - ab)\sqrt{-ab}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 - a)^2,x, algorithm="fricas")`

[Out]  $[1/4*((b*x^2 - a)*\log(-(2*a*b*x - (b*x^2 + a)*\sqrt{a*b}))/((b*x^2 - a) - 2*\sqrt{a*b}*x)/((b^2*x^2 - a*b)*\sqrt{a*b}), -1/2*((b*x^2 - a)*\arctan(\sqrt{-a*b}*x/a) + \sqrt{-a*b}*x)/((b^2*x^2 - a*b)*\sqrt{-a*b})]$

**Sympy** [A] time = 1.38893, size = 71, normalized size = 1.54

$$-\frac{x}{-2ab + 2b^2x^2} + \frac{\sqrt{\frac{1}{ab^3}} \log\left(-ab\sqrt{\frac{1}{ab^3} + x}\right)}{4} - \frac{\sqrt{\frac{1}{ab^3}} \log\left(ab\sqrt{\frac{1}{ab^3} + x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(-b*x**2+a)**2,x)`

[Out]  $-x/(-2*a*b + 2*b**2*x**2) + \sqrt{1/(a*b**3)}*\log(-a*b*\sqrt{1/(a*b**3)} + x)/4 - \sqrt{1/(a*b**3)}*\log(a*b*\sqrt{1/(a*b**3)} + x)/4$

**GIAC/XCAS** [A] time = 0.209692, size = 53, normalized size = 1.15

$$\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{2\sqrt{-abb}} - \frac{x}{2(bx^2 - a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 - a)^2,x, algorithm="giac")`

[Out]  $1/2*\arctan(b*x/\sqrt{-a*b})/(\sqrt{-a*b}*b) - 1/2*x/((b*x^2 - a)*b)$

$$3.234 \quad \int \frac{x}{(a-bx^2)^2} dx$$

**Optimal.** Leaf size=17

$$\frac{1}{2b(a-bx^2)}$$

[Out] 1/(2\*b\*(a - b\*x^2))

**Rubi [A]** time = 0.0115491, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{1}{2b(a-bx^2)}$$

Antiderivative was successfully verified.

[In] Int[x/(a - b\*x^2)^2, x]

[Out] 1/(2\*b\*(a - b\*x^2))

**Rubi in Sympy [A]** time = 2.45113, size = 10, normalized size = 0.59

$$\frac{1}{2b(a-bx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(-b\*x\*\*2+a)\*\*2, x)

[Out] 1/(2\*b\*(a - b\*x\*\*2))

**Mathematica [A]** time = 0.00431273, size = 17, normalized size = 1.

$$\frac{1}{2b(a-bx^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a - b\*x^2)^2, x]

[Out] 1/(2\*b\*(a - b\*x^2))

**Maple [A]** time = 0.002, size = 17, normalized size = 1.

$$-\frac{1}{(2bx^2 - 2a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-b\*x^2+a)^2, x)

[Out] -1/2/(b\*x^2-a)/b

---

**Maxima [A]** time = 1.34644, size = 22, normalized size = 1.29

$$-\frac{1}{2(bx^2 - a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 - a)^2,x, algorithm="maxima")`

[Out] `-1/2/((b*x^2 - a)*b)`

---

**Fricas [A]** time = 0.201951, size = 22, normalized size = 1.29

$$-\frac{1}{2(b^2x^2 - ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 - a)^2,x, algorithm="fricas")`

[Out] `-1/2/(b^2*x^2 - a*b)`

---

**Sympy [A]** time = 1.24633, size = 15, normalized size = 0.88

$$-\frac{1}{-2ab + 2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-b*x**2+a)**2,x)`

[Out] `-1/(-2*a*b + 2*b**2*x**2)`

---

**GIAC/XCAS [A]** time = 0.207823, size = 22, normalized size = 1.29

$$-\frac{1}{2(bx^2 - a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 - a)^2,x, algorithm="giac")`

[Out] `-1/2/((b*x^2 - a)*b)`



$$3.235 \quad \int \frac{1}{(a-bx^2)^2} dx$$

**Optimal.** Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{x}{2a(a-bx^2)}$$

[Out] x/(2\*a\*(a - b\*x^2)) + ArcTanh[(Sqrt[b]\*x)/Sqrt[a]]/(2\*a^(3/2)\*Sqrt[b])

**Rubi [A]** time = 0.0353968, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{x}{2a(a-bx^2)}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(-2), x]

[Out] x/(2\*a\*(a - b\*x^2)) + ArcTanh[(Sqrt[b]\*x)/Sqrt[a]]/(2\*a^(3/2)\*Sqrt[b])

**Rubi in Sympy [A]** time = 4.09172, size = 36, normalized size = 0.78

$$\frac{x}{2a(a-bx^2)} + \frac{\operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-b\*x\*\*2+a)\*\*2, x)

[Out] x/(2\*a\*(a - b\*x\*\*2)) + atanh(sqrt(b)\*x/sqrt(a))/(2\*a\*\*(3/2)\*sqrt(b))

**Mathematica [A]** time = 0.0330267, size = 47, normalized size = 1.02

$$\frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} - \frac{x}{2a(bx^2 - a)}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(-2), x]

[Out] -x/(2\*a\*(-a + b\*x^2)) + ArcTanh[(Sqrt[b]\*x)/Sqrt[a]]/(2\*a^(3/2)\*Sqrt[b])

**Maple [A]** time = 0.007, size = 38, normalized size = 0.8

$$-\frac{x}{2a(bx^2 - a)} + \frac{1}{2a} \operatorname{Artanh}\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-b*x^2+a)^2,x)`

[Out]  $-1/2*x/a/(b*x^2-a)+1/2/a/(a*b)^{(1/2)}*\operatorname{arctanh}(x*b/(a*b)^{(1/2)})$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 - a)^(-2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.213178, size = 1, normalized size = 0.02

$$\left[ \frac{(bx^2 - a) \log\left(\frac{2abx + (bx^2 + a)\sqrt{ab}}{bx^2 - a}\right) - 2\sqrt{ab}x}{4(abx^2 - a^2)\sqrt{ab}}, \frac{(bx^2 - a) \operatorname{arctan}\left(\frac{\sqrt{-ab}x}{a}\right) - \sqrt{-ab}x}{2(abx^2 - a^2)\sqrt{-ab}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 - a)^(-2),x, algorithm="fricas")`

[Out]  $[1/4*((b*x^2 - a)*\log((2*a*b*x + (b*x^2 + a)*\sqrt{a*b}))/((b*x^2 - a) - 2*\sqrt{a*b}*x)/((a*b*x^2 - a^2)*\sqrt{a*b}), 1/2*((b*x^2 - a)*\operatorname{arctan}(\sqrt{-a*b}*x/a) - \sqrt{-a*b}*x)/((a*b*x^2 - a^2)*\sqrt{-a*b})]$

**Sympy** [A] time = 1.52893, size = 71, normalized size = 1.54

$$-\frac{x}{-2a^2 + 2abx^2} - \frac{\sqrt{\frac{1}{a^3b}} \log\left(-a^2 \sqrt{\frac{1}{a^3b}} + x\right)}{4} + \frac{\sqrt{\frac{1}{a^3b}} \log\left(a^2 \sqrt{\frac{1}{a^3b}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**2+a)**2,x)`

[Out]  $-x/(-2*a**2 + 2*a*b*x**2) - \sqrt{1/(a**3*b)}*\log(-a**2*\sqrt{1/(a**3*b)} + x)/4 + \sqrt{1/(a**3*b)}*\log(a**2*\sqrt{1/(a**3*b)} + x)/4$

**GIAC/XCAS** [A] time = 0.210708, size = 53, normalized size = 1.15

$$-\frac{\operatorname{arctan}\left(\frac{bx}{\sqrt{-ab}}\right)}{2\sqrt{-aba}} - \frac{x}{2(bx^2 - a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 - a)^(-2),x, algorithm="giac")`

```
[Out] -1/2*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a) - 1/2*x/((b*x^2 - a)*a)
```

$$3.236 \quad \int \frac{1}{x(a-bx^2)^2} dx$$

**Optimal.** Leaf size=40

$$-\frac{\log(a-bx^2)}{2a^2} + \frac{\log(x)}{a^2} + \frac{1}{2a(a-bx^2)}$$

[Out]  $1/(2*a*(a - b*x^2)) + \text{Log}[x]/a^2 - \text{Log}[a - b*x^2]/(2*a^2)$

**Rubi [A]** time = 0.0675177, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\frac{\log(a-bx^2)}{2a^2} + \frac{\log(x)}{a^2} + \frac{1}{2a(a-bx^2)}$$

Antiderivative was successfully verified.

[In] `Int[1/(x*(a - b*x^2)^2), x]`

[Out]  $1/(2*a*(a - b*x^2)) + \text{Log}[x]/a^2 - \text{Log}[a - b*x^2]/(2*a^2)$

**Rubi in Sympy [A]** time = 9.78796, size = 34, normalized size = 0.85

$$\frac{1}{2a(a-bx^2)} + \frac{\log(x^2)}{2a^2} - \frac{\log(a-bx^2)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x/(-b*x**2+a)**2, x)`

[Out]  $1/(2*a*(a - b*x**2)) + \log(x**2)/(2*a**2) - \log(a - b*x**2)/(2*a**2)$

**Mathematica [A]** time = 0.0327199, size = 35, normalized size = 0.88

$$\frac{\frac{a}{a-bx^2} - \log(a-bx^2) + 2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x*(a - b*x^2)^2), x]`

[Out]  $(a/(a - b*x^2) + 2*\text{Log}[x] - \text{Log}[a - b*x^2])/ (2*a^2)$

**Maple [A]** time = 0.017, size = 39, normalized size = 1.

$$\frac{\ln(x)}{a^2} - \frac{\ln(bx^2 - a)}{2a^2} - \frac{1}{2a(bx^2 - a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(-b*x^2+a)^2, x)`

[Out]  $\ln(x)/a^2 - 1/2/a^2 \ln(bx^2 - a) - 1/2/a/(bx^2 - a)$

**Maxima [A]** time = 1.34529, size = 55, normalized size = 1.38

$$-\frac{1}{2(abx^2 - a^2)} - \frac{\log(bx^2 - a)}{2a^2} + \frac{\log(x^2)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 - a)^2*x), x, algorithm="maxima")`

[Out]  $-1/2/(a*b*x^2 - a^2) - 1/2*\log(b*x^2 - a)/a^2 + 1/2*\log(x^2)/a^2$

**Fricas [A]** time = 0.211061, size = 72, normalized size = 1.8

$$-\frac{(bx^2 - a) \log(bx^2 - a) - 2(bx^2 - a) \log(x) + a}{2(a^2bx^2 - a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 - a)^2*x), x, algorithm="fricas")`

[Out]  $-1/2*((b*x^2 - a)*\log(b*x^2 - a) - 2*(b*x^2 - a)*\log(x) + a)/(a^2*b*x^2 - a^3)$

**Sympy [A]** time = 1.71236, size = 34, normalized size = 0.85

$$-\frac{1}{-2a^2 + 2abx^2} + \frac{\log(x)}{a^2} - \frac{\log(-\frac{a}{b} + x^2)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-b*x**2+a)**2, x)`

[Out]  $-1/(-2*a**2 + 2*a*b*x**2) + \log(x)/a**2 - \log(-a/b + x**2)/(2*a**2)$

**GIAC/XCAS [A]** time = 0.218619, size = 69, normalized size = 1.72

$$\frac{\ln(x^2)}{2a^2} - \frac{\ln(|bx^2 - a|)}{2a^2} + \frac{bx^2 - 2a}{2(bx^2 - a)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 - a)^2*x), x, algorithm="giac")`

[Out]  $1/2*\ln(x^2)/a^2 - 1/2*\ln(\text{abs}(b*x^2 - a))/a^2 + 1/2*(b*x^2 - 2*a)/(b*x^2 - a)*a^2$

$$3.237 \quad \int \frac{1}{x^2(a-bx^2)^2} dx$$

**Optimal.** Leaf size=58

$$\frac{3\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{3}{2a^2x} + \frac{1}{2ax(a-bx^2)}$$

[Out]  $-3/(2*a^2*x) + 1/(2*a*x*(a - b*x^2)) + (3*\text{Sqrt}[b]*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/(2*a^{(5/2)})$

**Rubi [A]** time = 0.0568751, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{3\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{3}{2a^2x} + \frac{1}{2ax(a-bx^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a - b\*x^2)^2), x]

[Out]  $-3/(2*a^2*x) + 1/(2*a*x*(a - b*x^2)) + (3*\text{Sqrt}[b]*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/(2*a^{(5/2)})$

**Rubi in Sympy [A]** time = 11.0798, size = 48, normalized size = 0.83

$$\frac{1}{2ax(a-bx^2)} - \frac{3}{2a^2x} + \frac{3\sqrt{b} \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(-b\*x\*\*2+a)\*\*2, x)

[Out]  $1/(2*a*x*(a - b*x^2)) - 3/(2*a^2*x) + 3*\text{sqrt}(b)*\text{atanh}(\text{sqrt}(b)*x/\text{sqrt}(a))/(2*a^{(5/2)})$

**Mathematica [A]** time = 0.0645921, size = 56, normalized size = 0.97

$$\frac{3\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{bx}{2a^2(bx^2 - a)} - \frac{1}{a^2x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a - b\*x^2)^2), x]

[Out]  $-(1/(a^2*x)) - (b*x)/(2*a^2*(-a + b*x^2)) + (3*\text{Sqrt}[b]*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/(2*a^{(5/2)})$

**Maple [A]** time = 0.013, size = 47, normalized size = 0.8

$$-\frac{1}{a^2x} - \frac{b}{a^2} \left( \frac{x}{2bx^2 - 2a} - \frac{3}{2} \operatorname{Artanh}\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(-b*x^2+a)^2,x)`

[Out]  $-1/a^2/x - b/a^2 * (1/2 * x/(b*x^2 - a) - 3/2/(a*b)^{(1/2)} * \operatorname{arctanh}(x*b/(a*b)^{(1/2)}))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 - a)^2*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.218598, size = 1, normalized size = 0.02

$$\left[ \frac{6bx^2 - 3(bx^3 - ax)\sqrt{\frac{b}{a}} \log\left(\frac{bx^2 + 2ax\sqrt{\frac{b}{a}}}{bx^2 - a}\right) - 4a}{4(a^2bx^3 - a^3x)}, \frac{3bx^2 - 3(bx^3 - ax)\sqrt{-\frac{b}{a}} \operatorname{arctan}\left(\frac{bx}{a\sqrt{-\frac{b}{a}}}\right) - 2a}{2(a^2bx^3 - a^3x)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 - a)^2*x^2),x, algorithm="fricas")`

[Out]  $[-1/4 * (6*b*x^2 - 3*(b*x^3 - a*x)*\operatorname{sqrt}(b/a)*\log((b*x^2 + 2*a*x*\operatorname{sqrt}(b/a) + a)/(b*x^2 - a)) - 4*a)/(a^2*b*x^3 - a^3*x), -1/2 * (3*b*x^2 - 3*(b*x^3 - a*x)*\operatorname{sqrt}(-b/a)*\operatorname{arctan}(b*x/(a*\operatorname{sqrt}(-b/a))) - 2*a)/(a^2*b*x^3 - a^3*x)]$

**Sympy [A]** time = 1.8185, size = 83, normalized size = 1.43

$$-\frac{3\sqrt{\frac{b}{a^5}} \log\left(-\frac{a^3\sqrt{\frac{b}{a^5}}}{b} + x\right)}{4} + \frac{3\sqrt{\frac{b}{a^5}} \log\left(\frac{a^3\sqrt{\frac{b}{a^5}}}{b} + x\right)}{4} - \frac{-2a + 3bx^2}{-2a^3x + 2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(-b*x**2+a)**2,x)`

[Out]  $-3*\operatorname{sqrt}(b/a**5)*\log(-a**3*\operatorname{sqrt}(b/a**5)/b + x)/4 + 3*\operatorname{sqrt}(b/a**5)*\log(a**3*\operatorname{sqrt}(b/a**5)/b + x)/4 - (-2*a + 3*b*x**2)/(-2*a**3*x + 2*a**2*b*x**3)$

**GIAC/XCAS [A]** time = 0.221956, size = 68, normalized size = 1.17

$$-\frac{3b \operatorname{arctan}\left(\frac{bx}{\sqrt{-ab}}\right)}{2\sqrt{-ab}a^2} - \frac{3bx^2 - 2a}{2(bx^3 - ax)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^2*x^2),x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^2) - 1/2*(3*b*x^2 - 2*a)/((b*x^3 - a*x)*a^2)
```



$$3.238 \quad \int \frac{1}{x^3(a-bx^2)^2} dx$$

**Optimal.** Leaf size=52

$$-\frac{b \log(a-bx^2)}{a^3} + \frac{2b \log(x)}{a^3} + \frac{b}{2a^2(a-bx^2)} - \frac{1}{2a^2x^2}$$

[Out]  $-1/(2*a^2*x^2) + b/(2*a^2*(a - b*x^2)) + (2*b*Log[x])/a^3 - (b*Log[a - b*x^2])/a^3$

**Rubi [A]** time = 0.0918585, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\frac{b \log(a-bx^2)}{a^3} + \frac{2b \log(x)}{a^3} + \frac{b}{2a^2(a-bx^2)} - \frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(a - b\*x^2)^2), x]

[Out]  $-1/(2*a^2*x^2) + b/(2*a^2*(a - b*x^2)) + (2*b*Log[x])/a^3 - (b*Log[a - b*x^2])/a^3$

**Rubi in Sympy [A]** time = 13.0146, size = 46, normalized size = 0.88

$$\frac{b}{2a^2(a-bx^2)} - \frac{1}{2a^2x^2} + \frac{b \log(x^2)}{a^3} - \frac{b \log(a-bx^2)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(-b\*x\*\*2+a)\*\*2, x)

[Out]  $b/(2*a**2*(a - b*x**2)) - 1/(2*a**2*x**2) + b*log(x**2)/a**3 - b*log(a - b*x**2)/a**3$

**Mathematica [A]** time = 0.0587959, size = 44, normalized size = 0.85

$$\frac{\frac{ab}{a-bx^2} - 2b \log(a-bx^2) - \frac{a}{x^2} + 4b \log(x)}{2a^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a - b\*x^2)^2), x]

[Out]  $(-(a/x^2) + (a*b)/(a - b*x^2) + 4*b*Log[x] - 2*b*Log[a - b*x^2])/ (2*a^3)$

**Maple [A]** time = 0.018, size = 51, normalized size = 1.

$$-\frac{1}{2a^2x^2} + 2\frac{b \ln(x)}{a^3} - \frac{b \ln(bx^2 - a)}{a^3} - \frac{b}{2a^2(bx^2 - a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(-b*x^2+a)^2,x)`

[Out]  $-1/2/a^2/x^2+2*b*\ln(x)/a^3-b/a^3*\ln(b*x^2-a)-1/2*b/a^2/(b*x^2-a)$

**Maxima [A]** time = 1.3998, size = 77, normalized size = 1.48

$$-\frac{2bx^2 - a}{2(a^2bx^4 - a^3x^2)} - \frac{b \log(bx^2 - a)}{a^3} + \frac{b \log(x^2)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 - a)^2*x^3),x, algorithm="maxima")`

[Out]  $-1/2*(2*b*x^2 - a)/(a^2*b*x^4 - a^3*x^2) - b*\log(b*x^2 - a)/a^3 + b*\log(x^2)/a^3$

**Fricas [A]** time = 0.239229, size = 108, normalized size = 2.08

$$-\frac{2abx^2 - a^2 + 2(b^2x^4 - abx^2) \log(bx^2 - a) - 4(b^2x^4 - abx^2) \log(x)}{2(a^3bx^4 - a^4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 - a)^2*x^3),x, algorithm="fricas")`

[Out]  $-1/2*(2*a*b*x^2 - a^2 + 2*(b^2*x^4 - a*b*x^2)*\log(b*x^2 - a) - 4*(b^2*x^4 - a*b*x^2)*\log(x))/(a^3*b*x^4 - a^4*x^2)$

**Sympy [A]** time = 2.17637, size = 49, normalized size = 0.94

$$-\frac{-a + 2bx^2}{-2a^3x^2 + 2a^2bx^4} + \frac{2b \log(x)}{a^3} - \frac{b \log(-\frac{a}{b} + x^2)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(-b*x**2+a)**2,x)`

[Out]  $-(-a + 2*b*x**2)/(-2*a**3*x**2 + 2*a**2*b*x**4) + 2*b*\log(x)/a**3 - b*\log(-a/b + x**2)/a**3$

**GIAC/XCAS [A]** time = 0.20697, size = 76, normalized size = 1.46

$$\frac{b \ln(x^2)}{a^3} - \frac{b \ln(|bx^2 - a|)}{a^3} - \frac{2bx^2 - a}{2(bx^4 - ax^2)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 - a)^2*x^3),x, algorithm="giac")`

[Out]  $b*\ln(x^2)/a^3 - b*\ln(\text{abs}(b*x^2 - a))/a^3 - 1/2*(2*b*x^2 - a)/((b*x^4 - a*x^2)*a^2)$

$$3.239 \quad \int \frac{x^3}{(a-bx^2)^3} dx$$

**Optimal.** Leaf size=20

$$\frac{x^4}{4a(a-bx^2)^2}$$

[Out]  $x^4/(4*a*(a-b*x^2)^2)$

**Rubi [A]** time = 0.0201189, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{x^4}{4a(a-bx^2)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3/(a-b*x^2)^3, x]$

[Out]  $x^4/(4*a*(a-b*x^2)^2)$

**Rubi in Sympy [A]** time = 3.57039, size = 14, normalized size = 0.7

$$\frac{x^4}{4a(a-bx^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**3/(-b*x**2+a)**3, x)$

[Out]  $x**4/(4*a*(a-b*x**2)**2)$

**Mathematica [A]** time = 0.0155355, size = 25, normalized size = 1.25

$$-\frac{a-2bx^2}{4b^2(a-bx^2)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^3/(a-b*x^2)^3, x]$

[Out]  $-(a-2*b*x^2)/(4*b^2*(a-b*x^2)^2)$

**Maple [A]** time = 0.01, size = 35, normalized size = 1.8

$$\frac{a}{4b^2(bx^2-a)^2} + \frac{1}{2b^2(bx^2-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^3/(-b*x^2+a)^3, x)$

[Out]  $1/4 * a/b^2/(b*x^2-a)^2 + 1/2/b^2/(b*x^2-a)$

**Maxima [A]** time = 1.37298, size = 51, normalized size = 2.55

$$\frac{2bx^2 - a}{4(b^4x^4 - 2ab^3x^2 + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-x^3/(b*x^2 - a)^3,x, algorithm="maxima")`

[Out]  $1/4 * (2*b*x^2 - a)/(b^4*x^4 - 2*a*b^3*x^2 + a^2*b^2)$

**Fricas [A]** time = 0.251209, size = 51, normalized size = 2.55

$$\frac{2bx^2 - a}{4(b^4x^4 - 2ab^3x^2 + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-x^3/(b*x^2 - a)^3,x, algorithm="fricas")`

[Out]  $1/4 * (2*b*x^2 - a)/(b^4*x^4 - 2*a*b^3*x^2 + a^2*b^2)$

**Sympy [A]** time = 1.75484, size = 34, normalized size = 1.7

$$\frac{-a + 2bx^2}{4a^2b^2 - 8ab^3x^2 + 4b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(-b*x**2+a)**3,x)`

[Out]  $(-a + 2*b*x^2)/(4*a^2*b^2 - 8*a*b^3*x^2 + 4*b^4*x^4)$

**GIAC/XCAS [A]** time = 0.20949, size = 35, normalized size = 1.75

$$\frac{2bx^2 - a}{4(bx^2 - a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-x^3/(b*x^2 - a)^3,x, algorithm="giac")`

[Out]  $1/4 * (2*b*x^2 - a)/((b*x^2 - a)^2*b^2)$

$$3.240 \quad \int \frac{x^2}{(a-bx^2)^3} dx$$

**Optimal.** Leaf size=67

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}} - \frac{x}{8ab(a-bx^2)} + \frac{x}{4b(a-bx^2)^2}$$

[Out]  $x/(4*b*(a - b*x^2)^2) - x/(8*a*b*(a - b*x^2)) - \text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/(8*a^{(3/2)}*b^{(3/2)})$

**Rubi [A]** time = 0.0619327, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}} - \frac{x}{8ab(a-bx^2)} + \frac{x}{4b(a-bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a - b\*x^2)^3, x]

[Out]  $x/(4*b*(a - b*x^2)^2) - x/(8*a*b*(a - b*x^2)) - \text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/(8*a^{(3/2)}*b^{(3/2)})$

**Rubi in Sympy [A]** time = 8.84361, size = 51, normalized size = 0.76

$$\frac{x}{4b(a-bx^2)^2} - \frac{x}{8ab(a-bx^2)} - \frac{\text{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(-b\*x\*\*2+a)\*\*3, x)

[Out]  $x/(4*b*(a - b*x**2)**2) - x/(8*a*b*(a - b*x**2)) - \text{atanh}(\text{sqrt}(b)*x/\text{sqrt}(a))/(8*a^{(3/2)}*b^{(3/2)})$

**Mathematica [A]** time = 0.0563439, size = 56, normalized size = 0.84

$$\frac{x(a+bx^2)}{8ab(a-bx^2)^2} - \frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a - b\*x^2)^3, x]

[Out]  $(x*(a + b*x^2))/(8*a*b*(a - b*x^2)^2) - \text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/(8*a^{(3/2)}*b^{(3/2)})$

**Maple [A]** time = 0.011, size = 52, normalized size = 0.8

$$-\frac{1}{(bx^2 - a)^2} \left( -\frac{x^3}{8a} - \frac{x}{8b} \right) - \frac{1}{8ab} \text{Artanh} \left( bx \frac{1}{\sqrt{ab}} \right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(-b*x^2+a)^3,x)`

[Out]  $-\left(-\frac{1}{8}/a*x^3-1/8*x/b\right)/(b*x^2-a)^2-1/8/b/a/(a*b)^{(1/2)}*\operatorname{arctanh}(x*b/(a*b)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-x^2/(b*x^2 - a)^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.231819, size = 1, normalized size = 0.01

$$\left[ \frac{(b^2x^4 - 2abx^2 + a^2) \log\left(-\frac{2abx - (bx^2+a)\sqrt{ab}}{bx^2-a}\right) + 2(bx^3 + ax)\sqrt{ab}}{16(ab^3x^4 - 2a^2b^2x^2 + a^3b)\sqrt{ab}}, \right. \\ \left. - \frac{(b^2x^4 - 2abx^2 + a^2) \arctan\left(\frac{\sqrt{-abx}}{a}\right) - (bx^3 + ax)\sqrt{-ab}}{8(ab^3x^4 - 2a^2b^2x^2 + a^3b)\sqrt{-ab}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-x^2/(b*x^2 - a)^3,x, algorithm="fricas")`

[Out]  $[1/16*((b^2*x^4 - 2*a*b*x^2 + a^2)*\log(-(2*a*b*x - (b*x^2 + a)*\operatorname{sqrt}(a*b))/(b*x^2 - a)) + 2*(b*x^3 + a*x)*\operatorname{sqrt}(a*b))/((a*b^3*x^4 - 2*a^2*b^2*x^2 + a^3*b)*\operatorname{sqrt}(a*b)), -1/8*((b^2*x^4 - 2*a*b*x^2 + a^2)*\arctan(\operatorname{sqrt}(-a*b)*x/a) - (b*x^3 + a*x)*\operatorname{sqrt}(-a*b))/((a*b^3*x^4 - 2*a^2*b^2*x^2 + a^3*b)*\operatorname{sqrt}(-a*b))]$

**Sympy [A]** time = 1.85464, size = 104, normalized size = 1.55

$$\frac{\sqrt{\frac{1}{a^3b^3}} \log\left(-a^2b\sqrt{\frac{1}{a^3b^3}} + x\right)}{16} - \frac{\sqrt{\frac{1}{a^3b^3}} \log\left(a^2b\sqrt{\frac{1}{a^3b^3}} + x\right)}{16} + \frac{ax + bx^3}{8a^3b - 16a^2b^2x^2 + 8ab^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(-b*x**2+a)**3,x)`

[Out]  $\operatorname{sqrt}(1/(a**3*b**3))*\log(-a**2*b*\operatorname{sqrt}(1/(a**3*b**3)) + x)/16 - \operatorname{sqrt}(1/(a**3*b**3))*\log(a**2*b*\operatorname{sqrt}(1/(a**3*b**3)) + x)/16 + (a*x + b*x**3)/(8*a**3*b - 16*a**2*b**2*x**2 + 8*a*b**3*x**4)$

**GIAC/XCAS [A]** time = 0.206537, size = 72, normalized size = 1.07

$$\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{8\sqrt{-ab}} + \frac{bx^3 + ax}{8(bx^2 - a)^2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/(b*x^2 - a)^3,x, algorithm="giac")
```

```
[Out] 1/8*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a*b) + 1/8*(b*x^3 + a*x)/(b*x^2 - a)^2*a*b
```

$$3.241 \quad \int \frac{x}{(a-bx^2)^3} dx$$

**Optimal.** Leaf size=17

$$\frac{1}{4b(a-bx^2)^2}$$

[Out] 1/(4\*b\*(a - b\*x^2)^2)

**Rubi [A]** time = 0.0117338, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{1}{4b(a-bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a - b\*x^2)^3, x]

[Out] 1/(4\*b\*(a - b\*x^2)^2)

**Rubi in Sympy [A]** time = 2.46354, size = 12, normalized size = 0.71

$$\frac{1}{4b(a-bx^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(-b\*x\*\*2+a)\*\*3, x)

[Out] 1/(4\*b\*(a - b\*x\*\*2)\*\*2)

**Mathematica [A]** time = 0.0045876, size = 17, normalized size = 1.

$$\frac{1}{4b(a-bx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a - b\*x^2)^3, x]

[Out] 1/(4\*b\*(a - b\*x^2)^2)

**Maple [A]** time = 0., size = 17, normalized size = 1.

$$\frac{1}{4b(bx^2 - a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-b\*x^2+a)^3, x)

[Out] 1/4/b/(b\*x^2-a)^2



---

**Maxima [A]** time = 1.35934, size = 22, normalized size = 1.29

$$\frac{1}{4(bx^2 - a)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x/(b\*x^2 - a)^3,x, algorithm="maxima")

[Out] 1/4/((b\*x^2 - a)^2\*b)

---

**Fricas [A]** time = 0.212751, size = 35, normalized size = 2.06

$$\frac{1}{4(b^3x^4 - 2ab^2x^2 + a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x/(b\*x^2 - a)^3,x, algorithm="fricas")

[Out] 1/4/(b^3\*x^4 - 2\*a\*b^2\*x^2 + a^2\*b)

---

**Sympy [A]** time = 1.62456, size = 26, normalized size = 1.53

$$\frac{1}{4a^2b - 8ab^2x^2 + 4b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-b\*x\*\*2+a)\*\*3,x)

[Out] 1/(4\*a\*\*2\*b - 8\*a\*b\*\*2\*x\*\*2 + 4\*b\*\*3\*x\*\*4)

---

**GIAC/XCAS [A]** time = 0.208122, size = 22, normalized size = 1.29

$$\frac{1}{4(bx^2 - a)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x/(b\*x^2 - a)^3,x, algorithm="giac")

[Out] 1/4/((b\*x^2 - a)^2\*b)

$$3.242 \quad \int \frac{1}{(a-bx^2)^3} dx$$

**Optimal.** Leaf size=64

$$\frac{3 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}} + \frac{3x}{8a^2(a-bx^2)} + \frac{x}{4a(a-bx^2)^2}$$

[Out]  $x/(4*a*(a - b*x^2)^2) + (3*x)/(8*a^2*(a - b*x^2)) + (3*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[b])$

**Rubi [A]** time = 0.0511026, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{3 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}} + \frac{3x}{8a^2(a-bx^2)} + \frac{x}{4a(a-bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(-3), x]

[Out]  $x/(4*a*(a - b*x^2)^2) + (3*x)/(8*a^2*(a - b*x^2)) + (3*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[b])$

**Rubi in Sympy [A]** time = 6.04915, size = 54, normalized size = 0.84

$$\frac{x}{4a(a-bx^2)^2} + \frac{3x}{8a^2(a-bx^2)} + \frac{3 \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-b\*x\*\*2+a)\*\*3, x)

[Out]  $x/(4*a*(a - b*x**2)**2) + 3*x/(8*a**2*(a - b*x**2)) + 3*atanh(sqrt(b)*x/sqrt(a))/(8*a**(5/2)*sqrt(b))$

**Mathematica [A]** time = 0.0741781, size = 56, normalized size = 0.88

$$\frac{3 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}} + \frac{5ax - 3bx^3}{8a^2(a-bx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(-3), x]

[Out]  $(5*a*x - 3*b*x^3)/(8*a^2*(a - b*x^2)^2) + (3*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[b])$

**Maple [A]** time = 0.006, size = 61, normalized size = 1.

$$\frac{x}{4a(bx^2 - a)^2} + \frac{3}{4a} \left( -\frac{x}{2a(bx^2 - a)} + \frac{1}{2a} \operatorname{Artanh}\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-b*x^2+a)^3,x)`

[Out]  $\frac{1}{4} \frac{x}{a} (b x^2 - a)^{-2} + \frac{3}{4} \frac{1}{a} \left( -\frac{1}{2} \frac{x}{a} (b x^2 - a) + \frac{1}{2} \frac{1}{(a b)^{1/2}} \operatorname{arctanh}\left(\frac{x b}{(a b)^{1/2}}\right) \right)$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/(b*x^2 - a)^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.227456, size = 1, normalized size = 0.02

$$\left[ \frac{3(b^2x^4 - 2abx^2 + a^2) \log\left(\frac{2abx + (bx^2 + a)\sqrt{ab}}{bx^2 - a}\right) - 2(3bx^3 - 5ax)\sqrt{ab}}{16(a^2b^2x^4 - 2a^3bx^2 + a^4)\sqrt{ab}}, \frac{3(b^2x^4 - 2abx^2 + a^2) \arctan\left(\frac{\sqrt{-ab}x}{a}\right) - (3bx^3 - 5ax)\sqrt{-ab}}{8(a^2b^2x^4 - 2a^3bx^2 + a^4)\sqrt{-ab}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/(b*x^2 - a)^3,x, algorithm="fricas")`

[Out]  $\left[ \frac{1}{16} (3(b^2x^4 - 2abx^2 + a^2) \log((2abx + (bx^2 + a)\sqrt{ab})/(bx^2 - a)) - 2(3bx^3 - 5ax)\sqrt{ab}) / ((a^2b^2x^4 - 2a^3bx^2 + a^4)\sqrt{ab}), \frac{1}{8} (3(b^2x^4 - 2abx^2 + a^2) \arctan(\sqrt{-ab}x/a) - (3bx^3 - 5ax)\sqrt{-ab}) / ((a^2b^2x^4 - 2a^3bx^2 + a^4)\sqrt{-ab}) \right]$

**Sympy** [A] time = 1.87208, size = 99, normalized size = 1.55

$$-\frac{3\sqrt{\frac{1}{a^5b}} \log\left(-a^3\sqrt{\frac{1}{a^5b}} + x\right)}{16} + \frac{3\sqrt{\frac{1}{a^5b}} \log\left(a^3\sqrt{\frac{1}{a^5b}} + x\right)}{16} - \frac{-5ax + 3bx^3}{8a^4 - 16a^3bx^2 + 8a^2b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**2+a)**3,x)`

[Out]  $-3\sqrt{\frac{1}{a^5b}} \log(-a^3\sqrt{\frac{1}{a^5b}} + x)/16 + 3\sqrt{\frac{1}{a^5b}} \log(a^3\sqrt{\frac{1}{a^5b}} + x)/16 - (-5ax + 3bx^3)/(8a^4 - 16a^3bx^2 + 8a^2b^2x^4)$

**GIAC/XCAS** [A] time = 0.20701, size = 66, normalized size = 1.03

$$-\frac{3 \arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{8\sqrt{-ab}a^2} - \frac{3bx^3 - 5ax}{8(bx^2 - a)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(b*x^2 - a)^3,x, algorithm="giac")
```

```
[Out] -3/8*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^2) - 1/8*(3*b*x^3 - 5*a*x)/((b*x^2 - a)^2*a^2)
```

$$3.243 \quad \int \frac{1}{x(a-bx^2)^3} dx$$

**Optimal.** Leaf size=57

$$-\frac{\log(a-bx^2)}{2a^3} + \frac{\log(x)}{a^3} + \frac{1}{2a^2(a-bx^2)} + \frac{1}{4a(a-bx^2)^2}$$

[Out]  $1/(4*a*(a-b*x^2)^2) + 1/(2*a^2*(a-b*x^2)) + \text{Log}[x]/a^3 - \text{Log}[a-b*x^2]/(2*a^3)$

**Rubi [A]** time = 0.0919974, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\frac{\log(a-bx^2)}{2a^3} + \frac{\log(x)}{a^3} + \frac{1}{2a^2(a-bx^2)} + \frac{1}{4a(a-bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(a-b\*x^2)^3), x]

[Out]  $1/(4*a*(a-b*x^2)^2) + 1/(2*a^2*(a-b*x^2)) + \text{Log}[x]/a^3 - \text{Log}[a-b*x^2]/(2*a^3)$

**Rubi in Sympy [A]** time = 12.7262, size = 49, normalized size = 0.86

$$\frac{1}{4a(a-bx^2)^2} + \frac{1}{2a^2(a-bx^2)} + \frac{\log(x^2)}{2a^3} - \frac{\log(a-bx^2)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(-b\*x\*\*2+a)\*\*3, x)

[Out]  $1/(4*a*(a-b*x**2)**2) + 1/(2*a**2*(a-b*x**2)) + \log(x**2)/(2*a**3) - \log(a-b*x**2)/(2*a**3)$

**Mathematica [A]** time = 0.0621884, size = 45, normalized size = 0.79

$$\frac{\frac{a(3a-2bx^2)}{(a-bx^2)^2} - 2 \log(a-bx^2) + 4 \log(x)}{4a^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(a-b\*x^2)^3), x]

[Out]  $((a*(3*a-2*b*x^2))/(a-b*x^2)^2 + 4*\text{Log}[x] - 2*\text{Log}[a-b*x^2])/(4*a^3)$

**Maple [A]** time = 0.016, size = 55, normalized size = 1.

$$\frac{\ln(x)}{a^3} - \frac{\ln(bx^2-a)}{2a^3} + \frac{1}{4a(bx^2-a)^2} - \frac{1}{2a^2(bx^2-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(-b*x^2+a)^3,x)`

[Out]  $\ln(x)/a^3 - 1/2/a^3 \ln(bx^2 - a) + 1/4/a/(bx^2 - a)^2 - 1/2/a^2/(bx^2 - a)$

**Maxima [A]** time = 1.33124, size = 84, normalized size = 1.47

$$-\frac{2bx^2 - 3a}{4(a^2b^2x^4 - 2a^3bx^2 + a^4)} - \frac{\log(bx^2 - a)}{2a^3} + \frac{\log(x^2)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((b*x^2 - a)^3*x),x, algorithm="maxima")`

[Out]  $-1/4*(2*b*x^2 - 3*a)/(a^2*b^2*x^4 - 2*a^3*b*x^2 + a^4) - 1/2*\log(b*x^2 - a)/a^3 + 1/2*\log(x^2)/a^3$

**Fricas [A]** time = 0.224136, size = 124, normalized size = 2.18

$$\frac{2abx^2 - 3a^2 + 2(b^2x^4 - 2abx^2 + a^2)\log(bx^2 - a) - 4(b^2x^4 - 2abx^2 + a^2)\log(x)}{4(a^3b^2x^4 - 2a^4bx^2 + a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((b*x^2 - a)^3*x),x, algorithm="fricas")`

[Out]  $-1/4*(2*a*b*x^2 - 3*a^2 + 2*(b^2*x^4 - 2*a*b*x^2 + a^2)*\log(b*x^2 - a) - 4*(b^2*x^4 - 2*a*b*x^2 + a^2)*\log(x))/(a^3*b^2*x^4 - 2*a^4*b*x^2 + a^5)$

**Sympy [A]** time = 2.28582, size = 56, normalized size = 0.98

$$-\frac{-3a + 2bx^2}{4a^4 - 8a^3bx^2 + 4a^2b^2x^4} + \frac{\log(x)}{a^3} - \frac{\log(-\frac{a}{b} + x^2)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-b*x**2+a)**3,x)`

[Out]  $-(-3*a + 2*b*x**2)/(4*a**4 - 8*a**3*b*x**2 + 4*a**2*b**2*x**4) + \log(x)/a**3 - \log(-a/b + x**2)/(2*a**3)$

**GIAC/XCAS [A]** time = 0.212794, size = 85, normalized size = 1.49

$$\frac{\ln(x^2)}{2a^3} - \frac{\ln(|bx^2 - a|)}{2a^3} + \frac{3b^2x^4 - 8abx^2 + 6a^2}{4(bx^2 - a)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((b*x^2 - a)^3*x),x, algorithm="giac")`

[Out]  $1/2*\ln(x^2)/a^3 - 1/2*\ln(\text{abs}(b*x^2 - a))/a^3 + 1/4*(3*b^2*x^4 - 8*a*b*x^2 + 6*a^2)/((b*x^2 - a)^2*a^3)$

$$3.244 \quad \int \frac{1}{x^2(a-bx^2)^3} dx$$

**Optimal.** Leaf size=78

$$\frac{15\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{7/2}} - \frac{15}{8a^3x} + \frac{5}{8a^2x(a-bx^2)} + \frac{1}{4ax(a-bx^2)^2}$$

[Out]  $-15/(8*a^3*x) + 1/(4*a*x*(a - b*x^2)^2) + 5/(8*a^2*x*(a - b*x^2)) + (15*\text{Sqrt}[b]*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/(8*a^{(7/2)})$

**Rubi [A]** time = 0.0838416, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{15\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{7/2}} - \frac{15}{8a^3x} + \frac{5}{8a^2x(a-bx^2)} + \frac{1}{4ax(a-bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a - b\*x^2)^3), x]

[Out]  $-15/(8*a^3*x) + 1/(4*a*x*(a - b*x^2)^2) + 5/(8*a^2*x*(a - b*x^2)) + (15*\text{Sqrt}[b]*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/(8*a^{(7/2)})$

**Rubi in Sympy [A]** time = 15.3825, size = 65, normalized size = 0.83

$$\frac{1}{4ax(a-bx^2)^2} + \frac{5}{8a^2x(a-bx^2)} - \frac{15}{8a^3x} + \frac{15\sqrt{b} \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(-b\*x\*\*2+a)\*\*3, x)

[Out]  $1/(4*a*x*(a - b*x**2)**2) + 5/(8*a**2*x*(a - b*x**2)) - 15/(8*a**3*x) + 15*\text{sqrt}(b)*\text{atanh}(\text{sqrt}(b)*x/\text{sqrt}(a))/(8*a**{7/2})$

**Mathematica [A]** time = 0.084513, size = 69, normalized size = 0.88

$$\frac{15\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8a^{7/2}} + \frac{-8a^2 + 25abx^2 - 15b^2x^4}{8a^3x(a-bx^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a - b\*x^2)^3), x]

[Out]  $(-8*a^2 + 25*a*b*x^2 - 15*b^2*x^4)/(8*a^3*x*(a - b*x^2)^2) + (15*\text{Sqrt}[b]*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]])/(8*a^{(7/2)})$

**Maple [A]** time = 0.015, size = 56, normalized size = 0.7

$$-\frac{1}{a^3x} - \frac{b}{a^3} \left( \frac{1}{(bx^2 - a)^2} \left( \frac{7bx^3}{8} - \frac{9ax}{8} \right) - \frac{15}{8} \operatorname{Artanh}\left(bx \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(-b*x^2+a)^3,x)`

[Out]  $-1/a^3/x - 1/a^3*b*((7/8*b*x^3 - 9/8*a*x)/(b*x^2 - a)^2 - 15/8/(a*b)^{(1/2)}) * \operatorname{arctanh}(x*b/(a*b)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((b*x^2 - a)^3*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.229946, size = 1, normalized size = 0.01

$$\left[ \begin{array}{l} \frac{30 b^2 x^4 - 50 a b x^2 - 15 (b^2 x^5 - 2 a b x^3 + a^2 x) \sqrt{\frac{b}{a}} \log\left(\frac{b x^2 + 2 a x \sqrt{\frac{b}{a}} + a}{b x^2 - a}\right) + 16 a^2}{16 (a^3 b^2 x^5 - 2 a^4 b x^3 + a^5 x)}, \\ \frac{15 b^2 x^4 - 25 a b x^2 - 15 (b^2 x^5 - 2 a b x^3 + a^2 x) \sqrt{-\frac{b}{a}} \operatorname{arctan}\left(\frac{b x}{a \sqrt{-\frac{b}{a}}}\right) + 8 a^2}{8 (a^3 b^2 x^5 - 2 a^4 b x^3 + a^5 x)} \end{array} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((b*x^2 - a)^3*x^2),x, algorithm="fricas")`

[Out]  $[-1/16*(30*b^2*x^4 - 50*a*b*x^2 - 15*(b^2*x^5 - 2*a*b*x^3 + a^2*x)*\operatorname{sqrt}(b/a)*\log((b*x^2 + 2*a*x*\operatorname{sqrt}(b/a) + a)/(b*x^2 - a)) + 16*a^2)/(a^3*b^2*x^5 - 2*a^4*b*x^3 + a^5*x), -1/8*(15*b^2*x^4 - 25*a*b*x^2 - 15*(b^2*x^5 - 2*a*b*x^3 + a^2*x)*\operatorname{sqrt}(-b/a)*\operatorname{arctan}(b*x/(a*\operatorname{sqrt}(-b/a))) + 8*a^2)/(a^3*b^2*x^5 - 2*a^4*b*x^3 + a^5*x)]$

**Sympy [A]** time = 2.52647, size = 107, normalized size = 1.37

$$-\frac{15\sqrt{\frac{b}{a^7}} \log\left(-\frac{a^4\sqrt{\frac{b}{a^7}}}{b} + x\right)}{16} + \frac{15\sqrt{\frac{b}{a^7}} \log\left(\frac{a^4\sqrt{\frac{b}{a^7}}}{b} + x\right)}{16} - \frac{8a^2 - 25abx^2 + 15b^2x^4}{8a^5x - 16a^4bx^3 + 8a^3b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(-b*x**2+a)**3,x)`

[Out]  $-15*\operatorname{sqrt}(b/a**7)*\log(-a**4*\operatorname{sqrt}(b/a**7)/b + x)/16 + 15*\operatorname{sqrt}(b/a**7)*\log(a**4*\operatorname{sqrt}(b/a**7)/b + x)/16 - (8*a**2 - 25*a*b*x**2 + 15*b**2*x**4)/(8*a**5*x - 16*a**4*b*x**3 + 8*a**3*b**2*x**5)$



**GIAC/XCAS [A]** time = 0.21321, size = 82, normalized size = 1.05

$$-\frac{15b \arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{8\sqrt{-aba^3}} - \frac{7b^2x^3 - 9abx}{8(bx^2 - a)^2a^3} - \frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/((b\*x^2 - a)^3\*x^2),x, algorithm="giac")

[Out] -15/8\*b\*arctan(b\*x/sqrt(-a\*b))/(sqrt(-a\*b)\*a^3) - 1/8\*(7\*b^2\*x^3 - 9\*a\*b\*x)/((b\*x^2 - a)^2\*a^3) - 1/(a^3\*x)

$$3.245 \quad \int \frac{1}{x^3(a-bx^2)^3} dx$$

**Optimal.** Leaf size=69

$$-\frac{3b \log(a-bx^2)}{2a^4} + \frac{3b \log(x)}{a^4} + \frac{b}{a^3(a-bx^2)} - \frac{1}{2a^3x^2} + \frac{b}{4a^2(a-bx^2)^2}$$

[Out]  $-1/(2*a^3*x^2) + b/(4*a^2*(a - b*x^2)^2) + b/(a^3*(a - b*x^2)) + (3*b*Log[x])/a^4 - (3*b*Log[a - b*x^2])/(2*a^4)$

**Rubi [A]** time = 0.1213, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\frac{3b \log(a-bx^2)}{2a^4} + \frac{3b \log(x)}{a^4} + \frac{b}{a^3(a-bx^2)} - \frac{1}{2a^3x^2} + \frac{b}{4a^2(a-bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(a - b\*x^2)^3), x]

[Out]  $-1/(2*a^3*x^2) + b/(4*a^2*(a - b*x^2)^2) + b/(a^3*(a - b*x^2)) + (3*b*Log[x])/a^4 - (3*b*Log[a - b*x^2])/(2*a^4)$

**Rubi in Sympy [A]** time = 17.7421, size = 66, normalized size = 0.96

$$\frac{b}{4a^2(a-bx^2)^2} + \frac{b}{a^3(a-bx^2)} - \frac{1}{2a^3x^2} + \frac{3b \log(x^2)}{2a^4} - \frac{3b \log(a-bx^2)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(-b\*x\*\*2+a)\*\*3, x)

[Out]  $b/(4*a**2*(a - b*x**2)**2) + b/(a**3*(a - b*x**2)) - 1/(2*a**3*x**2) + 3*b*log(x**2)/(2*a**4) - 3*b*log(a - b*x**2)/(2*a**4)$

**Mathematica [A]** time = 0.10433, size = 60, normalized size = 0.87

$$\frac{\frac{a(-2a^2+9abx^2-6b^2x^4)}{(ax-bx^3)^2} - 6b \log(a-bx^2) + 12b \log(x)}{4a^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a - b\*x^2)^3), x]

[Out]  $((a*(-2*a^2 + 9*a*b*x^2 - 6*b^2*x^4))/(a*x - b*x^3)^2 + 12*b*Log[x] - 6*b*Log[a - b*x^2])/(4*a^4)$

**Maple [A]** time = 0.018, size = 68, normalized size = 1.

$$-\frac{1}{2a^3x^2} + 3\frac{b \ln(x)}{a^4} - \frac{3b \ln(bx^2 - a)}{2a^4} + \frac{b}{4a^2(bx^2 - a)^2} - \frac{b}{a^3(bx^2 - a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(-b*x^2+a)^3,x)`

[Out]  $-1/2/a^3/x^2+3*b*\ln(x)/a^4-3/2*b/a^4*\ln(b*x^2-a)+1/4*b/a^2/(b*x^2-a)^2-b/a^3/(b*x^2-a)$

**Maxima [A]** time = 1.35225, size = 107, normalized size = 1.55

$$-\frac{6b^2x^4 - 9abx^2 + 2a^2}{4(a^3b^2x^6 - 2a^4bx^4 + a^5x^2)} - \frac{3b \log(bx^2 - a)}{2a^4} + \frac{3b \log(x^2)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((b*x^2 - a)^3*x^3),x, algorithm="maxima")`

[Out]  $-1/4*(6*b^2*x^4 - 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 - 2*a^4*b*x^4 + a^5*x^2) - 3/2*b*\log(b*x^2 - a)/a^4 + 3/2*b*\log(x^2)/a^4$

**Fricas [A]** time = 0.221367, size = 163, normalized size = 2.36

$$\frac{6ab^2x^4 - 9a^2bx^2 + 2a^3 + 6(b^3x^6 - 2ab^2x^4 + a^2bx^2) \log(bx^2 - a) - 12(b^3x^6 - 2ab^2x^4 + a^2bx^2) \log(x)}{4(a^4b^2x^6 - 2a^5bx^4 + a^6x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((b*x^2 - a)^3*x^3),x, algorithm="fricas")`

[Out]  $-1/4*(6*a*b^2*x^4 - 9*a^2*b*x^2 + 2*a^3 + 6*(b^3*x^6 - 2*a*b^2*x^4 + a^2*b*x^2)*\log(b*x^2 - a) - 12*(b^3*x^6 - 2*a*b^2*x^4 + a^2*b*x^2)*\log(x))/(a^4*b^2*x^6 - 2*a^5*b*x^4 + a^6*x^2)$

**Sympy [A]** time = 3.0689, size = 78, normalized size = 1.13

$$-\frac{2a^2 - 9abx^2 + 6b^2x^4}{4a^5x^2 - 8a^4bx^4 + 4a^3b^2x^6} + \frac{3b \log(x)}{a^4} - \frac{3b \log(-\frac{a}{b} + x^2)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(-b*x**2+a)**3,x)`

[Out]  $-(2*a**2 - 9*a*b*x**2 + 6*b**2*x**4)/(4*a**5*x**2 - 8*a**4*b*x**4 + 4*a**3*b**2*x**6) + 3*b*\log(x)/a**4 - 3*b*\log(-a/b + x**2)/(2*a**4)$

**GIAC/XCAS [A]** time = 0.209551, size = 113, normalized size = 1.64

$$\frac{3b \ln(x^2)}{2a^4} - \frac{3b \ln(|bx^2 - a|)}{2a^4} + \frac{9b^3x^4 - 22ab^2x^2 + 14a^2b}{4(bx^2 - a)^2a^4} - \frac{3bx^2 + a}{2a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((b*x^2 - a)^3*x^3),x, algorithm="giac")`

[Out]  $3/2*b*\ln(x^2)/a^4 - 3/2*b*\ln(\text{abs}(b*x^2 - a))/a^4 + 1/4*(9*b^3*x^4 - 22*a*b^2*x^2 + 14*a^2*b)/((b*x^2 - a)^2*a^4) - 1/2*(3*b*x^2 + a)/(a^4*x^2)$

$$3.246 \quad \int \frac{x^3}{(a-bx^2)^5} dx$$

**Optimal.** Leaf size=36

$$\frac{a}{8b^2(a-bx^2)^4} - \frac{1}{6b^2(a-bx^2)^3}$$

[Out]  $a/(8*b^2*(a - b*x^2)^4) - 1/(6*b^2*(a - b*x^2)^3)$

**Rubi [A]** time = 0.0770877, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{a}{8b^2(a-bx^2)^4} - \frac{1}{6b^2(a-bx^2)^3}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a - b\*x^2)^5, x]

[Out]  $a/(8*b^2*(a - b*x^2)^4) - 1/(6*b^2*(a - b*x^2)^3)$

**Rubi in Sympy [A]** time = 10.2008, size = 29, normalized size = 0.81

$$\frac{a}{8b^2(a-bx^2)^4} - \frac{1}{6b^2(a-bx^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(-b\*x\*\*2+a)\*\*5, x)

[Out]  $a/(8*b**2*(a - b*x**2)**4) - 1/(6*b**2*(a - b*x**2)**3)$

**Mathematica [A]** time = 0.0173642, size = 25, normalized size = 0.69

$$-\frac{a-4bx^2}{24b^2(a-bx^2)^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a - b\*x^2)^5, x]

[Out]  $-(a - 4*b*x^2)/(24*b^2*(a - b*x^2)^4)$

**Maple [A]** time = 0.011, size = 35, normalized size = 1.

$$\frac{a}{8b^2(bx^2-a)^4} + \frac{1}{6b^2(bx^2-a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(-b\*x^2+a)^5, x)

[Out]  $1/8*a/b^2/(b*x^2-a)^4+1/6/b^2/(b*x^2-a)^3$

---

**Maxima [A]** time = 1.34286, size = 81, normalized size = 2.25

$$\frac{4bx^2 - a}{24(b^6x^8 - 4ab^5x^6 + 6a^2b^4x^4 - 4a^3b^3x^2 + a^4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x^3/(b\*x^2 - a)^5,x, algorithm="maxima")

[Out] 1/24\*(4\*b\*x^2 - a)/(b^6\*x^8 - 4\*a\*b^5\*x^6 + 6\*a^2\*b^4\*x^4 - 4\*a^3\*b^3\*x^2 + a^4\*b^2)

---

**Fricas [A]** time = 0.212867, size = 81, normalized size = 2.25

$$\frac{4bx^2 - a}{24(b^6x^8 - 4ab^5x^6 + 6a^2b^4x^4 - 4a^3b^3x^2 + a^4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x^3/(b\*x^2 - a)^5,x, algorithm="fricas")

[Out] 1/24\*(4\*b\*x^2 - a)/(b^6\*x^8 - 4\*a\*b^5\*x^6 + 6\*a^2\*b^4\*x^4 - 4\*a^3\*b^3\*x^2 + a^4\*b^2)

---

**Sympy [A]** time = 2.82352, size = 58, normalized size = 1.61

$$\frac{-a + 4bx^2}{24a^4b^2 - 96a^3b^3x^2 + 144a^2b^4x^4 - 96ab^5x^6 + 24b^6x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(-b\*x\*\*2+a)\*\*5,x)

[Out] (-a + 4\*b\*x\*\*2)/(24\*a\*\*4\*b\*\*2 - 96\*a\*\*3\*b\*\*3\*x\*\*2 + 144\*a\*\*2\*b\*\*4\*x\*\*4 - 96\*a\*b\*\*5\*x\*\*6 + 24\*b\*\*6\*x\*\*8)

---

**GIAC/XCAS [A]** time = 0.21215, size = 53, normalized size = 1.47

$$\frac{\frac{4}{(bx^2-a)^3b} + \frac{3a}{(bx^2-a)^4b}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x^3/(b\*x^2 - a)^5,x, algorithm="giac")

[Out] 1/24\*(4/((b\*x^2 - a)^3\*b) + 3\*a/((b\*x^2 - a)^4\*b))/b

$$3.247 \quad \int \frac{x^2}{(a-bx^2)^5} dx$$

**Optimal.** Leaf size=109

$$-\frac{5 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}} - \frac{5x}{128a^3b(a-bx^2)} - \frac{5x}{192a^2b(a-bx^2)^2} - \frac{x}{48ab(a-bx^2)^3} + \frac{x}{8b(a-bx^2)^4}$$

[Out]  $x/(8*b*(a - b*x^2)^4) - x/(48*a*b*(a - b*x^2)^3) - (5*x)/(192*a^2*b*(a - b*x^2)^2) - (5*x)/(128*a^3*b*(a - b*x^2)) - (5*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(7/2)*b^(3/2))$

**Rubi [A]** time = 0.108211, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$-\frac{5 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}} - \frac{5x}{128a^3b(a-bx^2)} - \frac{5x}{192a^2b(a-bx^2)^2} - \frac{x}{48ab(a-bx^2)^3} + \frac{x}{8b(a-bx^2)^4}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a - b\*x^2)^5, x]

[Out]  $x/(8*b*(a - b*x^2)^4) - x/(48*a*b*(a - b*x^2)^3) - (5*x)/(192*a^2*b*(a - b*x^2)^2) - (5*x)/(128*a^3*b*(a - b*x^2)) - (5*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(7/2)*b^(3/2))$

**Rubi in Sympy [A]** time = 16.5672, size = 90, normalized size = 0.83

$$\frac{x}{8b(a-bx^2)^4} - \frac{x}{48ab(a-bx^2)^3} - \frac{5x}{192a^2b(a-bx^2)^2} - \frac{5x}{128a^3b(a-bx^2)} - \frac{5 \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(-b\*x\*\*2+a)\*\*5, x)

[Out]  $x/(8*b*(a - b*x**2)**4) - x/(48*a*b*(a - b*x**2)**3) - 5*x/(192*a**2*b*(a - b*x**2)**2) - 5*x/(128*a**3*b*(a - b*x**2)) - 5*atanh(sqrt(b)*x/sqrt(a))/(128*a**(7/2)*b**(3/2))$

**Mathematica [A]** time = 0.106876, size = 81, normalized size = 0.74

$$\frac{15a^3x + 73a^2bx^3 - 55ab^2x^5 + 15b^3x^7}{384a^3b(a-bx^2)^4} - \frac{5 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a - b\*x^2)^5, x]

[Out]  $(15*a^3*x + 73*a^2*b*x^3 - 55*a*b^2*x^5 + 15*b^3*x^7)/(384*a^3*b*(a - b*x^2)^4) - (5*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(7/2)*b^(3/2))$

**Maple [A]** time = 0.012, size = 72, normalized size = 0.7

$$-\frac{1}{(bx^2 - a)^4} \left( -\frac{5b^2x^7}{128a^3} + \frac{55bx^5}{384a^2} - \frac{73x^3}{384a} - \frac{5x}{128b} \right) - \frac{5}{128a^3b} \operatorname{Artanh} \left( bx \frac{1}{\sqrt{ab}} \right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-b\*x^2+a)^5,x)

[Out] -(-5/128\*b^2/a^3\*x^7+55/384\*b/a^2\*x^5-73/384/a\*x^3-5/128\*x/b)/(b\*x^2-a)^4-5/128/a^3/b/(a\*b)^(1/2)\*arctanh(x\*b/(a\*b)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x^2/(b\*x^2 - a)^5,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.226572, size = 1, normalized size = 0.01

$$\left[ \frac{15(b^4x^8 - 4ab^3x^6 + 6a^2b^2x^4 - 4a^3bx^2 + a^4) \log\left(-\frac{2abx - (bx^2+a)\sqrt{ab}}{bx^2-a}\right) + 2(15b^3x^7 - 55ab^2x^5 + 73a^2bx^3 + 15a^3x)\sqrt{ab}}{768(a^3b^5x^8 - 4a^4b^4x^6 + 6a^5b^3x^4 - 4a^6b^2x^2 + a^7b)\sqrt{ab}}, \right. \\ \left. \frac{15(b^4x^8 - 4ab^3x^6 + 6a^2b^2x^4 - 4a^3bx^2 + a^4) \arctan\left(\frac{\sqrt{-abx}}{a}\right) - (15b^3x^7 - 55ab^2x^5 + 73a^2bx^3 + 15a^3x)\sqrt{-ab}}{384(a^3b^5x^8 - 4a^4b^4x^6 + 6a^5b^3x^4 - 4a^6b^2x^2 + a^7b)\sqrt{-ab}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x^2/(b\*x^2 - a)^5,x, algorithm="fricas")

[Out] [1/768\*(15\*(b^4\*x^8 - 4\*a\*b^3\*x^6 + 6\*a^2\*b^2\*x^4 - 4\*a^3\*b\*x^2 + a^4)\*log(-(2\*a\*b\*x - (b\*x^2 + a)\*sqrt(a\*b))/(b\*x^2 - a)) + 2\*(15\*b^3\*x^7 - 55\*a\*b^2\*x^5 + 73\*a^2\*b\*x^3 + 15\*a^3\*x)\*sqrt(a\*b))/((a^3\*b^5\*x^8 - 4\*a^4\*b^4\*x^6 + 6\*a^5\*b^3\*x^4 - 4\*a^6\*b^2\*x^2 + a^7\*b)\*sqrt(a\*b)), -1/384\*(15\*(b^4\*x^8 - 4\*a\*b^3\*x^6 + 6\*a^2\*b^2\*x^4 - 4\*a^3\*b\*x^2 + a^4)\*arctan(sqrt(-a\*b)\*x/a) - (15\*b^3\*x^7 - 55\*a\*b^2\*x^5 + 73\*a^2\*b\*x^3 + 15\*a^3\*x)\*sqrt(-a\*b))/((a^3\*b^5\*x^8 - 4\*a^4\*b^4\*x^6 + 6\*a^5\*b^3\*x^4 - 4\*a^6\*b^2\*x^2 + a^7\*b)\*sqrt(-a\*b))]

**Sympy [A]** time = 3.08027, size = 160, normalized size = 1.47

$$\frac{5\sqrt{\frac{1}{a^7b^3}} \log\left(-a^4b\sqrt{\frac{1}{a^7b^3}} + x\right)}{256} - \frac{5\sqrt{\frac{1}{a^7b^3}} \log\left(a^4b\sqrt{\frac{1}{a^7b^3}} + x\right)}{256} \\ + \frac{15a^3x + 73a^2bx^3 - 55ab^2x^5 + 15b^3x^7}{384a^7b - 1536a^6b^2x^2 + 2304a^5b^3x^4 - 1536a^4b^4x^6 + 384a^3b^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(-b\*x\*\*2+a)\*\*5,x)

```
[Out] 5*sqrt(1/(a**7*b**3))*log(-a**4*b*sqrt(1/(a**7*b**3)) + x)/256 -
5*sqrt(1/(a**7*b**3))*log(a**4*b*sqrt(1/(a**7*b**3)) + x)/256 + (
15*a**3*x + 73*a**2*b*x**3 - 55*a*b**2*x**5 + 15*b**3*x**7)/(384*
a**7*b - 1536*a**6*b**2*x**2 + 2304*a**5*b**3*x**4 - 1536*a**4*b*
*4*x**6 + 384*a**3*b**5*x**8)
```

**GIAC/XCAS [A]** time = 0.215381, size = 104, normalized size = 0.95

$$\frac{5 \arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{128 \sqrt{-ab} a^3 b} + \frac{15 b^3 x^7 - 55 a b^2 x^5 + 73 a^2 b x^3 + 15 a^3 x}{384 (bx^2 - a)^4 a^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/(b*x^2 - a)^5,x, algorithm="giac")
```

```
[Out] 5/128*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^3*b) + 1/384*(15*b^3*x
^7 - 55*a*b^2*x^5 + 73*a^2*b*x^3 + 15*a^3*x)/((b*x^2 - a)^4*a^3*b
)
```



$$3.248 \quad \int \frac{x}{(a-bx^2)^5} dx$$

**Optimal.** Leaf size=17

$$\frac{1}{8b(a-bx^2)^4}$$

[Out] 1/(8\*b\*(a - b\*x^2)^4)

**Rubi [A]** time = 0.0112372, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{1}{8b(a-bx^2)^4}$$

Antiderivative was successfully verified.

[In] Int[x/(a - b\*x^2)^5, x]

[Out] 1/(8\*b\*(a - b\*x^2)^4)

**Rubi in Sympy [A]** time = 2.68558, size = 12, normalized size = 0.71

$$\frac{1}{8b(a-bx^2)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(-b\*x\*\*2+a)\*\*5, x)

[Out] 1/(8\*b\*(a - b\*x\*\*2)\*\*4)

**Mathematica [A]** time = 0.00765399, size = 17, normalized size = 1.

$$\frac{1}{8b(a-bx^2)^4}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a - b\*x^2)^5, x]

[Out] 1/(8\*b\*(a - b\*x^2)^4)

**Maple [A]** time = 0.001, size = 17, normalized size = 1.

$$\frac{1}{8b(bx^2 - a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-b\*x^2+a)^5, x)

[Out] 1/8/b/(b\*x^2-a)^4

---

**Maxima [A]** time = 1.34582, size = 22, normalized size = 1.29

$$\frac{1}{8(bx^2 - a)^4 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x/(b\*x^2 - a)^5,x, algorithm="maxima")

[Out] 1/8/((b\*x^2 - a)^4\*b)

---

**Fricas [A]** time = 0.221801, size = 65, normalized size = 3.82

$$\frac{1}{8(b^5x^8 - 4ab^4x^6 + 6a^2b^3x^4 - 4a^3b^2x^2 + a^4b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x/(b\*x^2 - a)^5,x, algorithm="fricas")

[Out] 1/8/(b^5\*x^8 - 4\*a\*b^4\*x^6 + 6\*a^2\*b^3\*x^4 - 4\*a^3\*b^2\*x^2 + a^4\*b)

---

**Sympy [A]** time = 2.69979, size = 49, normalized size = 2.88

$$\frac{1}{8a^4b - 32a^3b^2x^2 + 48a^2b^3x^4 - 32ab^4x^6 + 8b^5x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-b\*x\*\*2+a)\*\*5,x)

[Out] 1/(8\*a\*\*4\*b - 32\*a\*\*3\*b\*\*2\*x\*\*2 + 48\*a\*\*2\*b\*\*3\*x\*\*4 - 32\*a\*b\*\*4\*x\*\*6 + 8\*b\*\*5\*x\*\*8)

---

**GIAC/XCAS [A]** time = 0.211518, size = 22, normalized size = 1.29

$$\frac{1}{8(bx^2 - a)^4 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x/(b\*x^2 - a)^5,x, algorithm="giac")

[Out] 1/8/((b\*x^2 - a)^4\*b)

$$3.249 \quad \int \frac{1}{(a-bx^2)^5} dx$$

**Optimal.** Leaf size=100

$$\frac{35 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}} + \frac{35x}{128a^4(a-bx^2)} + \frac{35x}{192a^3(a-bx^2)^2} + \frac{7x}{48a^2(a-bx^2)^3} + \frac{x}{8a(a-bx^2)^4}$$

[Out]  $x/(8*a*(a - b*x^2)^4) + (7*x)/(48*a^2*(a - b*x^2)^3) + (35*x)/(192*a^3*(a - b*x^2)^2) + (35*x)/(128*a^4*(a - b*x^2)) + (35*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(9/2)*Sqrt[b])$

**Rubi [A]** time = 0.0899517, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{35 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}} + \frac{35x}{128a^4(a-bx^2)} + \frac{35x}{192a^3(a-bx^2)^2} + \frac{7x}{48a^2(a-bx^2)^3} + \frac{x}{8a(a-bx^2)^4}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(-5), x]

[Out]  $x/(8*a*(a - b*x^2)^4) + (7*x)/(48*a^2*(a - b*x^2)^3) + (35*x)/(192*a^3*(a - b*x^2)^2) + (35*x)/(128*a^4*(a - b*x^2)) + (35*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(9/2)*Sqrt[b])$

**Rubi in Sympy [A]** time = 11.4927, size = 88, normalized size = 0.88

$$\frac{x}{8a(a-bx^2)^4} + \frac{7x}{48a^2(a-bx^2)^3} + \frac{35x}{192a^3(a-bx^2)^2} + \frac{35x}{128a^4(a-bx^2)} + \frac{35 \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-b\*x\*\*2+a)\*\*5, x)

[Out]  $x/(8*a*(a - b*x**2)**4) + 7*x/(48*a**2*(a - b*x**2)**3) + 35*x/(192*a**3*(a - b*x**2)**2) + 35*x/(128*a**4*(a - b*x**2)) + 35*atanh(sqrt(b)*x/sqrt(a))/(128*a**(9/2)*sqrt(b))$

**Mathematica [A]** time = 0.0850246, size = 79, normalized size = 0.79

$$\frac{\sqrt{ax}(279a^3 - 511a^2bx^2 + 385ab^2x^4 - 105b^3x^6)}{(a-bx^2)^4} + \frac{105 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{\sqrt{b}}$$

$$384a^{9/2}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(-5), x]

[Out]  $((Sqrt[a]*x*(279*a^3 - 511*a^2*b*x^2 + 385*a*b^2*x^4 - 105*b^3*x^6))/(a - b*x^2)^4 + (105*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/ (384*a^(9/2))$

**Maple [A]** time = 0.005, size = 107, normalized size = 1.1

$$\frac{x}{8a(bx^2 - a)^4} + \frac{7}{8a} \left( -\frac{x}{6a(bx^2 - a)^3} - \frac{5}{6a} \left( -\frac{x}{4a(bx^2 - a)^2} - \frac{3}{4a} \left( -\frac{x}{2a(bx^2 - a)} + \frac{1}{2a} \operatorname{Artanh} \left( bx \frac{1}{\sqrt{ab}} \right) \frac{1}{\sqrt{ab}} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^2+a)^5, x)

[Out] 1/8\*x/a/(b\*x^2-a)^4+7/8/a\*(-1/6\*x/a/(b\*x^2-a)^3-5/6/a\*(-1/4\*x/a/(b\*x^2-a)^2-3/4/a\*(-1/2\*x/a/(b\*x^2-a)+1/2/a/(a\*b)^(1/2)\*arctanh(x\*b/(a\*b)^(1/2)))))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/(b\*x^2 - a)^5,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.22514, size = 1, normalized size = 0.01

$$\frac{105(b^4x^8 - 4ab^3x^6 + 6a^2b^2x^4 - 4a^3bx^2 + a^4) \log\left(\frac{2abx + (bx^2+a)\sqrt{ab}}{bx^2-a}\right) - 2(105b^3x^7 - 385ab^2x^5 + 511a^2bx^3 - 279a^3x) \sqrt{ab}}{768(a^4b^4x^8 - 4a^5b^3x^6 + 6a^6b^2x^4 - 4a^7bx^2 + a^8)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/(b\*x^2 - a)^5,x, algorithm="fricas")

[Out] [1/768\*(105\*(b^4\*x^8 - 4\*a\*b^3\*x^6 + 6\*a^2\*b^2\*x^4 - 4\*a^3\*b\*x^2 + a^4)\*log((2\*a\*b\*x + (b\*x^2 + a)\*sqrt(a\*b))/(b\*x^2 - a)) - 2\*(105\*b^3\*x^7 - 385\*a\*b^2\*x^5 + 511\*a^2\*b\*x^3 - 279\*a^3\*x)\*sqrt(a\*b))/((a^4\*b^4\*x^8 - 4\*a^5\*b^3\*x^6 + 6\*a^6\*b^2\*x^4 - 4\*a^7\*b\*x^2 + a^8)\*sqrt(a\*b)), 1/384\*(105\*(b^4\*x^8 - 4\*a\*b^3\*x^6 + 6\*a^2\*b^2\*x^4 - 4\*a^3\*b\*x^2 + a^4)\*arctan(sqrt(-a\*b)\*x/a) - (105\*b^3\*x^7 - 385\*a\*b^2\*x^5 + 511\*a^2\*b\*x^3 - 279\*a^3\*x)\*sqrt(-a\*b))/((a^4\*b^4\*x^8 - 4\*a^5\*b^3\*x^6 + 6\*a^6\*b^2\*x^4 - 4\*a^7\*b\*x^2 + a^8)\*sqrt(-a\*b))]

**Sympy [A]** time = 3.13283, size = 146, normalized size = 1.46

$$-\frac{35\sqrt{\frac{1}{a^9b}} \log\left(-a^5\sqrt{\frac{1}{a^9b}} + x\right)}{256} + \frac{35\sqrt{\frac{1}{a^9b}} \log\left(a^5\sqrt{\frac{1}{a^9b}} + x\right)}{256} - \frac{-279a^3x + 511a^2bx^3 - 385ab^2x^5 + 105b^3x^7}{384a^8 - 1536a^7bx^2 + 2304a^6b^2x^4 - 1536a^5b^3x^6 + 384a^4b^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x\*\*2+a)\*\*5, x)

```
[Out] -35*sqrt(1/(a**9*b))*log(-a**5*sqrt(1/(a**9*b)) + x)/256 + 35*sqrt(1/(a**9*b))*log(a**5*sqrt(1/(a**9*b)) + x)/256 - (-279*a**3*x + 511*a**2*b*x**3 - 385*a*b**2*x**5 + 105*b**3*x**7)/(384*a**8 - 1536*a**7*b*x**2 + 2304*a**6*b**2*x**4 - 1536*a**5*b**3*x**6 + 384*a**4*b**4*x**8)
```

**GIAC/XCAS [A]** time = 0.21048, size = 96, normalized size = 0.96

$$-\frac{35 \arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{128 \sqrt{-ab}a^4} - \frac{105b^3x^7 - 385ab^2x^5 + 511a^2bx^3 - 279a^3x}{384(bx^2 - a)^4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(b*x^2 - a)^5,x, algorithm="giac")
```

```
[Out] -35/128*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^4) - 1/384*(105*b^3*x^7 - 385*a*b^2*x^5 + 511*a^2*b*x^3 - 279*a^3*x)/((b*x^2 - a)^4*a^4)
```

$$3.250 \quad \int \frac{1}{x(a-bx^2)^5} dx$$

**Optimal.** Leaf size=91

$$-\frac{\log(a-bx^2)}{2a^5} + \frac{\log(x)}{a^5} + \frac{1}{2a^4(a-bx^2)} + \frac{1}{4a^3(a-bx^2)^2} + \frac{1}{6a^2(a-bx^2)^3} + \frac{1}{8a(a-bx^2)^4}$$

[Out]  $1/(8*a*(a - b*x^2)^4) + 1/(6*a^2*(a - b*x^2)^3) + 1/(4*a^3*(a - b*x^2)^2) + 1/(2*a^4*(a - b*x^2)) + \text{Log}[x]/a^5 - \text{Log}[a - b*x^2]/(2*a^5)$

**Rubi [A]** time = 0.143284, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\frac{\log(a-bx^2)}{2a^5} + \frac{\log(x)}{a^5} + \frac{1}{2a^4(a-bx^2)} + \frac{1}{4a^3(a-bx^2)^2} + \frac{1}{6a^2(a-bx^2)^3} + \frac{1}{8a(a-bx^2)^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(a - b\*x^2)^5), x]

[Out]  $1/(8*a*(a - b*x^2)^4) + 1/(6*a^2*(a - b*x^2)^3) + 1/(4*a^3*(a - b*x^2)^2) + 1/(2*a^4*(a - b*x^2)) + \text{Log}[x]/a^5 - \text{Log}[a - b*x^2]/(2*a^5)$

**Rubi in Sympy [A]** time = 18.9838, size = 80, normalized size = 0.88

$$\frac{1}{8a(a-bx^2)^4} + \frac{1}{6a^2(a-bx^2)^3} + \frac{1}{4a^3(a-bx^2)^2} + \frac{1}{2a^4(a-bx^2)} + \frac{\log(x^2)}{2a^5} - \frac{\log(a-bx^2)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(-b\*x\*\*2+a)\*\*5, x)

[Out]  $1/(8*a*(a - b*x**2)**4) + 1/(6*a**2*(a - b*x**2)**3) + 1/(4*a**3*(a - b*x**2)**2) + 1/(2*a**4*(a - b*x**2)) + \log(x**2)/(2*a**5) - \log(a - b*x**2)/(2*a**5)$

**Mathematica [A]** time = 0.0621618, size = 67, normalized size = 0.74

$$\frac{a(25a^3-52a^2bx^2+42ab^2x^4-12b^3x^6)}{(a-bx^2)^4} - 12 \log(a-bx^2) + 24 \log(x)$$

$$24a^5$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(a - b\*x^2)^5), x]

[Out]  $((a*(25*a^3 - 52*a^2*b*x^2 + 42*a*b^2*x^4 - 12*b^3*x^6))/(a - b*x^2)^4 + 24*\text{Log}[x] - 12*\text{Log}[a - b*x^2])/(24*a^5)$

**Maple [A]** time = 0.02, size = 87, normalized size = 1.

$$\frac{\ln(x)}{a^5} - \frac{\ln(bx^2 - a)}{2a^5} + \frac{1}{4a^3(bx^2 - a)^2} - \frac{1}{2a^4(bx^2 - a)} - \frac{1}{6a^2(bx^2 - a)^3} + \frac{1}{8a(bx^2 - a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(-b*x^2+a)^5,x)`

[Out]  $\ln(x)/a^5 - 1/2/a^5 * \ln(b*x^2 - a) + 1/4/a^3/(b*x^2 - a)^2 - 1/2/a^4/(b*x^2 - a) - 1/6/a^2/(b*x^2 - a)^3 + 1/8/a/(b*x^2 - a)^4$

**Maxima [A]** time = 1.36285, size = 143, normalized size = 1.57

$$-\frac{12b^3x^6 - 42ab^2x^4 + 52a^2bx^2 - 25a^3}{24(a^4b^4x^8 - 4a^5b^3x^6 + 6a^6b^2x^4 - 4a^7bx^2 + a^8)} - \frac{\log(bx^2 - a)}{2a^5} + \frac{\log(x^2)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((b*x^2 - a)^5*x),x, algorithm="maxima")`

[Out]  $-1/24*(12*b^3*x^6 - 42*a*b^2*x^4 + 52*a^2*b*x^2 - 25*a^3)/(a^4*b^4*x^8 - 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 - 4*a^7*b*x^2 + a^8) - 1/2*\log(b*x^2 - a)/a^5 + 1/2*\log(x^2)/a^5$

**Fricas [A]** time = 0.235391, size = 243, normalized size = 2.67

$$\frac{12ab^3x^6 - 42a^2b^2x^4 + 52a^3bx^2 - 25a^4 + 12(b^4x^8 - 4ab^3x^6 + 6a^2b^2x^4 - 4a^3bx^2 + a^4)\log(bx^2 - a) - 24(b^4x^8 - 4ab^3x^6 + 6a^2b^2x^4 - 4a^3bx^2 + a^4)\log(x)}{24(a^5b^4x^8 - 4a^6b^3x^6 + 6a^7b^2x^4 - 4a^8bx^2 + a^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((b*x^2 - a)^5*x),x, algorithm="fricas")`

[Out]  $-1/24*(12*a*b^3*x^6 - 42*a^2*b^2*x^4 + 52*a^3*b*x^2 - 25*a^4 + 12*(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*\log(b*x^2 - a) - 24*(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*\log(x))/(a^5*b^4*x^8 - 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 - 4*a^8*b*x^2 + a^9)$

**Sympy [A]** time = 5.49896, size = 104, normalized size = 1.14

$$-\frac{-25a^3 + 52a^2bx^2 - 42ab^2x^4 + 12b^3x^6}{24a^8 - 96a^7bx^2 + 144a^6b^2x^4 - 96a^5b^3x^6 + 24a^4b^4x^8} + \frac{\log(x)}{a^5} - \frac{\log(-\frac{a}{b} + x^2)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-b*x**2+a)**5,x)`

[Out]  $-(-25*a**3 + 52*a**2*b*x**2 - 42*a*b**2*x**4 + 12*b**3*x**6)/(24*a**8 - 96*a**7*b*x**2 + 144*a**6*b**2*x**4 - 96*a**5*b**3*x**6 + 24*a**4*b**4*x**8) + \log(x)/a**5 - \log(-a/b + x**2)/(2*a**5)$

**GIAC/XCAS [A]** time = 0.215554, size = 115, normalized size = 1.26

$$\frac{\ln(x^2)}{2a^5} - \frac{\ln(|bx^2 - a|)}{2a^5} + \frac{25b^4x^8 - 112ab^3x^6 + 192a^2b^2x^4 - 152a^3bx^2 + 50a^4}{24(bx^2 - a)^4a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 - a)^5*x),x, algorithm="giac")
```

```
[Out] 1/2*ln(x^2)/a^5 - 1/2*ln(abs(b*x^2 - a))/a^5 + 1/24*(25*b^4*x^8 -  
112*a*b^3*x^6 + 192*a^2*b^2*x^4 - 152*a^3*b*x^2 + 50*a^4)/((b*x^  
2 - a)^4*a^5)
```



$$3.251 \quad \int \frac{1}{x^2(a-bx^2)^5} dx$$

**Optimal.** Leaf size=118

$$\frac{315\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{128a^{11/2}} - \frac{315}{128a^5x} + \frac{105}{128a^4x(a-bx^2)} + \frac{21}{64a^3x(a-bx^2)^2} + \frac{3}{16a^2x(a-bx^2)^3} + \frac{1}{8ax(a-bx^2)^4}$$

[Out]  $-315/(128*a^5*x) + 1/(8*a*x*(a-b*x^2)^4) + 3/(16*a^2*x*(a-b*x^2)^3) + 21/(64*a^3*x*(a-b*x^2)^2) + 105/(128*a^4*x*(a-b*x^2)) + (315*sqrt[b]*ArcTanh[(sqrt[b]*x)/sqrt[a]])/(128*a^(11/2))$

**Rubi [A]** time = 0.137384, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{315\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{128a^{11/2}} - \frac{315}{128a^5x} + \frac{105}{128a^4x(a-bx^2)} + \frac{21}{64a^3x(a-bx^2)^2} + \frac{3}{16a^2x(a-bx^2)^3} + \frac{1}{8ax(a-bx^2)^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a-b\*x^2)^5), x]

[Out]  $-315/(128*a^5*x) + 1/(8*a*x*(a-b*x^2)^4) + 3/(16*a^2*x*(a-b*x^2)^3) + 21/(64*a^3*x*(a-b*x^2)^2) + 105/(128*a^4*x*(a-b*x^2)) + (315*sqrt[b]*ArcTanh[(sqrt[b]*x)/sqrt[a]])/(128*a^(11/2))$

**Rubi in Sympy [A]** time = 26.683, size = 99, normalized size = 0.84

$$\frac{1}{8ax(a-bx^2)^4} + \frac{3}{16a^2x(a-bx^2)^3} + \frac{21}{64a^3x(a-bx^2)^2} + \frac{105}{128a^4x(a-bx^2)} - \frac{315}{128a^5x} + \frac{315\sqrt{b} \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{128a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(-b\*x\*\*2+a)\*\*5, x)

[Out]  $1/(8*a*x*(a-b*x**2)**4) + 3/(16*a**2*x*(a-b*x**2)**3) + 21/(64*a**3*x*(a-b*x**2)**2) + 105/(128*a**4*x*(a-b*x**2)) - 315/(128*a**5*x) + 315*sqrt(b)*atanh(sqrt(b)*x/sqrt(a))/(128*a**(11/2))$

**Mathematica [A]** time = 0.104627, size = 92, normalized size = 0.78

$$\frac{\sqrt{a}(-128a^4+837a^3bx^2-1533a^2b^2x^4+1155ab^3x^6-315b^4x^8)}{x(a-bx^2)^4} + 315\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{128a^{11/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a-b\*x^2)^5), x]

[Out]  $((sqrt[a]*(-128*a^4 + 837*a^3*b*x^2 - 1533*a^2*b^2*x^4 + 1155*a*b^3*x^6 - 315*b^4*x^8))/(x*(a-b*x^2)^4) + 315*sqrt[b]*ArcTanh[(sqrt[b]*x)/sqrt[a]])/(128*a^(11/2))$

**Maple [A]** time = 0.018, size = 78, normalized size = 0.7

$$-\frac{1}{a^5x} - \frac{b}{a^5} \left( \frac{1}{(bx^2 - a)^4} \left( \frac{187b^3x^7}{128} - \frac{643ab^2x^5}{128} + \frac{765a^2bx^3}{128} - \frac{325a^3x}{128} \right) - \frac{315}{128} \operatorname{Artanh} \left( bx \frac{1}{\sqrt{ab}} \right) \frac{1}{\sqrt{ab}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(-b\*x^2+a)^5,x)

[Out] -1/a^5/x-1/a^5\*b\*((187/128\*b^3\*x^7-643/128\*a\*b^2\*x^5+765/128\*a^2\*b\*x^3-325/128\*a^3\*x)/(b\*x^2-a)^4-315/128/(a\*b)^(1/2)\*arctanh(x\*b/(a\*b)^(1/2)))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/((b\*x^2 - a)^5\*x^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.233609, size = 1, normalized size = 0.01

$$\left[ \frac{630b^4x^8 - 2310ab^3x^6 + 3066a^2b^2x^4 - 1674a^3bx^2 + 256a^4 - 315(b^4x^9 - 4ab^3x^7 + 6a^2b^2x^5 - 4a^3bx^3 + a^4x)\sqrt{\frac{b}{a}} \log\left(\frac{b^4x^9 - 4ab^3x^7 + 6a^2b^2x^5 - 4a^3bx^3 + a^4x}{256(a^5b^4x^9 - 4a^6b^3x^7 + 6a^7b^2x^5 - 4a^8bx^3 + a^9x)}\right)}{315b^4x^8 - 1155ab^3x^6 + 1533a^2b^2x^4 - 837a^3bx^2 + 128a^4 - 315(b^4x^9 - 4ab^3x^7 + 6a^2b^2x^5 - 4a^3bx^3 + a^4x)\sqrt{-\frac{b}{a}} \arctan\left(\frac{b^4x^9 - 4ab^3x^7 + 6a^2b^2x^5 - 4a^3bx^3 + a^4x}{128(a^5b^4x^9 - 4a^6b^3x^7 + 6a^7b^2x^5 - 4a^8bx^3 + a^9x)}\right)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/((b\*x^2 - a)^5\*x^2),x, algorithm="fricas")

[Out] [-1/256\*(630\*b^4\*x^8 - 2310\*a\*b^3\*x^6 + 3066\*a^2\*b^2\*x^4 - 1674\*a^3\*b\*x^2 + 256\*a^4 - 315\*(b^4\*x^9 - 4\*a\*b^3\*x^7 + 6\*a^2\*b^2\*x^5 - 4\*a^3\*b\*x^3 + a^4\*x)\*sqrt(b/a)\*log((b\*x^2 + 2\*a\*x\*sqrt(b/a) + a)/(b\*x^2 - a)))/(a^5\*b^4\*x^9 - 4\*a^6\*b^3\*x^7 + 6\*a^7\*b^2\*x^5 - 4\*a^8\*b\*x^3 + a^9\*x), -1/128\*(315\*b^4\*x^8 - 1155\*a\*b^3\*x^6 + 1533\*a^2\*b^2\*x^4 - 837\*a^3\*b\*x^2 + 128\*a^4 - 315\*(b^4\*x^9 - 4\*a\*b^3\*x^7 + 6\*a^2\*b^2\*x^5 - 4\*a^3\*b\*x^3 + a^4\*x)\*sqrt(-b/a)\*arctan(b\*x/(a\*sqrt(-b/a)))]/(a^5\*b^4\*x^9 - 4\*a^6\*b^3\*x^7 + 6\*a^7\*b^2\*x^5 - 4\*a^8\*b\*x^3 + a^9\*x)]

**Sympy [A]** time = 7.74925, size = 155, normalized size = 1.31

$$-\frac{315\sqrt{\frac{b}{a^{11}}}\log\left(-\frac{a^6\sqrt{\frac{b}{a^{11}}}}{b}+x\right)}{256} + \frac{315\sqrt{\frac{b}{a^{11}}}\log\left(\frac{a^6\sqrt{\frac{b}{a^{11}}}}{b}+x\right)}{256} - \frac{128a^4 - 837a^3bx^2 + 1533a^2b^2x^4 - 1155ab^3x^6 + 315b^4x^8}{128a^9x - 512a^8bx^3 + 768a^7b^2x^5 - 512a^6b^3x^7 + 128a^5b^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(-b\*x\*\*2+a)\*\*5,x)

[Out] -315\*sqrt(b/a\*\*11)\*log(-a\*\*6\*sqrt(b/a\*\*11)/b + x)/256 + 315\*sqrt(b/a\*\*11)\*log(a\*\*6\*sqrt(b/a\*\*11)/b + x)/256 - (128\*a\*\*4 - 837\*a\*\*3\*b\*x\*\*2 + 1533\*a\*\*2\*b\*\*2\*x\*\*4 - 1155\*a\*b\*\*3\*x\*\*6 + 315\*b\*\*4\*x\*\*8)/(128\*a\*\*9\*x - 512\*a\*\*8\*b\*x\*\*3 + 768\*a\*\*7\*b\*\*2\*x\*\*5 - 512\*a\*\*6\*b\*\*3\*x\*\*7 + 128\*a\*\*5\*b\*\*4\*x\*\*9)

**GIAC/XCAS [A]** time = 0.222759, size = 112, normalized size = 0.95

$$-\frac{315 b \arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{128 \sqrt{-ab} a^5} - \frac{1}{a^5 x} - \frac{187 b^4 x^7 - 643 ab^3 x^5 + 765 a^2 b^2 x^3 - 325 a^3 b x}{128 (bx^2 - a)^4 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/((b\*x^2 - a)^5\*x^2),x, algorithm="giac")

[Out] -315/128\*b\*arctan(b\*x/sqrt(-a\*b))/(sqrt(-a\*b)\*a^5) - 1/(a^5\*x) - 1/128\*(187\*b^4\*x^7 - 643\*a\*b^3\*x^5 + 765\*a^2\*b^2\*x^3 - 325\*a^3\*b\*x)/((b\*x^2 - a)^4\*a^5)

$$3.252 \quad \int \frac{1}{x^3(a-bx^2)^5} dx$$

**Optimal.** Leaf size=106

$$-\frac{5b \log(a-bx^2)}{2a^6} + \frac{5b \log(x)}{a^6} + \frac{2b}{a^5(a-bx^2)} - \frac{1}{2a^5x^2} + \frac{3b}{4a^4(a-bx^2)^2} + \frac{b}{3a^3(a-bx^2)^3} + \frac{b}{8a^2(a-bx^2)^4}$$

[Out]  $-1/(2*a^5*x^2) + b/(8*a^2*(a - b*x^2)^4) + b/(3*a^3*(a - b*x^2)^3) + (3*b)/(4*a^4*(a - b*x^2)^2) + (2*b)/(a^5*(a - b*x^2)) + (5*b*Log[x])/a^6 - (5*b*Log[a - b*x^2])/(2*a^6)$

**Rubi [A]** time = 0.196419, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\frac{5b \log(a-bx^2)}{2a^6} + \frac{5b \log(x)}{a^6} + \frac{2b}{a^5(a-bx^2)} - \frac{1}{2a^5x^2} + \frac{3b}{4a^4(a-bx^2)^2} + \frac{b}{3a^3(a-bx^2)^3} + \frac{b}{8a^2(a-bx^2)^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(a - b\*x^2)^5), x]

[Out]  $-1/(2*a^5*x^2) + b/(8*a^2*(a - b*x^2)^4) + b/(3*a^3*(a - b*x^2)^3) + (3*b)/(4*a^4*(a - b*x^2)^2) + (2*b)/(a^5*(a - b*x^2)) + (5*b*Log[x])/a^6 - (5*b*Log[a - b*x^2])/(2*a^6)$

**Rubi in Sympy [A]** time = 28.7956, size = 100, normalized size = 0.94

$$\frac{b}{8a^2(a-bx^2)^4} + \frac{b}{3a^3(a-bx^2)^3} + \frac{3b}{4a^4(a-bx^2)^2} + \frac{2b}{a^5(a-bx^2)} - \frac{1}{2a^5x^2} + \frac{5b \log(x^2)}{2a^6} - \frac{5b \log(a-bx^2)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(-b\*x\*\*2+a)\*\*5, x)

[Out]  $b/(8*a**2*(a - b*x**2)**4) + b/(3*a**3*(a - b*x**2)**3) + 3*b/(4*a**4*(a - b*x**2)**2) + 2*b/(a**5*(a - b*x**2)) - 1/(2*a**5*x**2) + 5*b*log(x**2)/(2*a**6) - 5*b*log(a - b*x**2)/(2*a**6)$

**Mathematica [A]** time = 0.143626, size = 83, normalized size = 0.78

$$\frac{a(-12a^4+125a^3bx^2-260a^2b^2x^4+210ab^3x^6-60b^4x^8)}{x^2(a-bx^2)^4} - 60b \log(a-bx^2) + 120b \log(x)$$

$$24a^6$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a - b\*x^2)^5), x]

[Out]  $((a*(-12*a^4 + 125*a^3*b*x^2 - 260*a^2*b^2*x^4 + 210*a*b^3*x^6 - 60*b^4*x^8))/(x^2*(a - b*x^2)^4) + 120*b*Log[x] - 60*b*Log[a - b*x^2])/(24*a^6)$

**Maple [A]** time = 0.022, size = 102, normalized size = 1.

$$-\frac{1}{2a^5x^2} + 5\frac{b \ln(x)}{a^6} - \frac{5b \ln(bx^2 - a)}{2a^6} + \frac{3b}{4a^4(bx^2 - a)^2} - 2\frac{b}{a^5(bx^2 - a)} - \frac{b}{3a^3(bx^2 - a)^3} + \frac{b}{8a^2(bx^2 - a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(-b*x^2+a)^5,x)`

[Out] 
$$-1/2/a^5/x^2+5*b*\ln(x)/a^6-5/2*b/a^6*\ln(b*x^2-a)+3/4*b/a^4/(b*x^2-a)^2-2*b/a^5/(b*x^2-a)-1/3*b/a^3/(b*x^2-a)^3+1/8*b/a^2/(b*x^2-a)^4$$

**Maxima [A]** time = 1.36056, size = 166, normalized size = 1.57

$$-\frac{60b^4x^8 - 210ab^3x^6 + 260a^2b^2x^4 - 125a^3bx^2 + 12a^4}{24(a^5b^4x^{10} - 4a^6b^3x^8 + 6a^7b^2x^6 - 4a^8bx^4 + a^9x^2)} - \frac{5b \log(bx^2 - a)}{2a^6} + \frac{5b \log(x^2)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((b*x^2 - a)^5*x^3),x, algorithm="maxima")`

[Out] 
$$-1/24*(60*b^4*x^8 - 210*a*b^3*x^6 + 260*a^2*b^2*x^4 - 125*a^3*b*x^2 + 12*a^4)/(a^5*b^4*x^{10} - 4*a^6*b^3*x^8 + 6*a^7*b^2*x^6 - 4*a^8*b*x^4 + a^9*x^2) - 5/2*b*\log(b*x^2 - a)/a^6 + 5/2*b*\log(x^2)/a^6$$

**Fricas [A]** time = 0.23419, size = 282, normalized size = 2.66

$$-\frac{60ab^4x^8 - 210a^2b^3x^6 + 260a^3b^2x^4 - 125a^4bx^2 + 12a^5 + 60(b^5x^{10} - 4ab^4x^8 + 6a^2b^3x^6 - 4a^3b^2x^4 + a^4bx^2) \log(bx^2 - a)}{24(a^6b^4x^{10} - 4a^7b^3x^8 + 6a^8b^2x^6 - 4a^9bx^4 + a^{10}x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((b*x^2 - a)^5*x^3),x, algorithm="fricas")`

[Out] 
$$-1/24*(60*a*b^4*x^8 - 210*a^2*b^3*x^6 + 260*a^3*b^2*x^4 - 125*a^4*b*x^2 + 12*a^5 + 60*(b^5*x^{10} - 4*a*b^4*x^8 + 6*a^2*b^3*x^6 - 4*a^3*b^2*x^4 + a^4*b*x^2)*\log(b*x^2 - a) - 120*(b^5*x^{10} - 4*a*b^4*x^8 + 6*a^2*b^3*x^6 - 4*a^3*b^2*x^4 + a^4*b*x^2)*\log(x))/(a^6*b^4*x^{10} - 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 - 4*a^9*b*x^4 + a^{10}*x^2)$$

**Sympy [A]** time = 11.5465, size = 126, normalized size = 1.19

$$-\frac{12a^4 - 125a^3bx^2 + 260a^2b^2x^4 - 210ab^3x^6 + 60b^4x^8}{24a^9x^2 - 96a^8bx^4 + 144a^7b^2x^6 - 96a^6b^3x^8 + 24a^5b^4x^{10}} + \frac{5b \log(x)}{a^6} - \frac{5b \log(-\frac{a}{b} + x^2)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(-b*x**2+a)**5,x)`

[Out] 
$$-(12*a^4 - 125*a^3*b*x^2 + 260*a^2*b^2*x^4 - 210*a*b^3*x^6 + 60*b^4*x^8)/(24*a^9*x^2 - 96*a^8*b*x^4 + 144*a^7*b^2*x^6 - 96*a^6*b^3*x^8 + 24*a^5*b^4*x^{10}) + 5*b*\log(x)/a^6 - 5*b*\log(-a/b + x^2)/(2*a^6)$$

**GIAC/XCAS [A]** time = 0.21321, size = 143, normalized size = 1.35

$$\frac{5b \ln(x^2)}{2a^6} - \frac{5b \ln(|bx^2 - a|)}{2a^6} - \frac{5bx^2 + a}{2a^6x^2} + \frac{125b^5x^8 - 548ab^4x^6 + 912a^2b^3x^4 - 688a^3b^2x^2 + 202a^4b}{24(bx^2 - a)^4a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 - a)^5*x^3),x, algorithm="giac")
```

```
[Out] 5/2*b*ln(x^2)/a^6 - 5/2*b*ln(abs(b*x^2 - a))/a^6 - 1/2*(5*b*x^2 +  
a)/(a^6*x^2) + 1/24*(125*b^5*x^8 - 548*a*b^4*x^6 + 912*a^2*b^3*x  
^4 - 688*a^3*b^2*x^2 + 202*a^4*b)/((b*x^2 - a)^4*a^6)
```

$$3.253 \quad \int \frac{1}{x(1+bx^2)} dx$$

**Optimal.** Leaf size=15

$$\log(x) - \frac{1}{2} \log(bx^2 + 1)$$

[Out] Log[x] - Log[1 + b\*x^2]/2

**Rubi [A]** time = 0.0292295, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$

$$\log(x) - \frac{1}{2} \log(bx^2 + 1)$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(1 + b\*x^2)), x]

[Out] Log[x] - Log[1 + b\*x^2]/2

**Rubi in Sympy [A]** time = 5.12321, size = 15, normalized size = 1.

$$\frac{\log(x^2)}{2} - \frac{\log(bx^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(b\*x\*\*2+1), x)

[Out] log(x\*\*2)/2 - log(b\*x\*\*2 + 1)/2

**Mathematica [A]** time = 0.00704091, size = 15, normalized size = 1.

$$\log(x) - \frac{1}{2} \log(bx^2 + 1)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(1 + b\*x^2)), x]

[Out] Log[x] - Log[1 + b\*x^2]/2

**Maple [A]** time = 0.008, size = 14, normalized size = 0.9

$$\ln(x) - \frac{\ln(bx^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b\*x^2+1), x)

[Out] ln(x)-1/2\*ln(b\*x^2+1)

---

**Maxima [A]** time = 1.33948, size = 23, normalized size = 1.53

$$-\frac{1}{2} \log (bx^2 + 1) + \frac{1}{2} \log (x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + 1)*x), x, algorithm="maxima")`

[Out] `-1/2*log(b*x^2 + 1) + 1/2*log(x^2)`

---

**Fricas [A]** time = 0.225082, size = 18, normalized size = 1.2

$$-\frac{1}{2} \log (bx^2 + 1) + \log (x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + 1)*x), x, algorithm="fricas")`

[Out] `-1/2*log(b*x^2 + 1) + log(x)`

---

**Sympy [A]** time = 0.31444, size = 12, normalized size = 0.8

$$\log (x) - \frac{\log \left(x^2 + \frac{1}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x**2+1), x)`

[Out] `log(x) - log(x**2 + 1/b)/2`

---

**GIAC/XCAS [A]** time = 0.209476, size = 24, normalized size = 1.6

$$\frac{1}{2} \ln (x^2) - \frac{1}{2} \ln (|bx^2 + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + 1)*x), x, algorithm="giac")`

[Out] `1/2*ln(x^2) - 1/2*ln(abs(b*x^2 + 1))`



$$3.254 \quad \int \frac{1}{x(-1+bx^2)} dx$$

**Optimal.** Leaf size=18

$$\frac{1}{2} \log(1 - bx^2) - \log(x)$$

[Out] -Log[x] + Log[1 - b\*x^2]/2

**Rubi [A]** time = 0.0313369, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$

$$\frac{1}{2} \log(1 - bx^2) - \log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(-1 + b\*x^2)), x]

[Out] -Log[x] + Log[1 - b\*x^2]/2

**Rubi in Sympy [A]** time = 5.16496, size = 15, normalized size = 0.83

$$-\frac{\log(x^2)}{2} + \frac{\log(-bx^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(b\*x\*\*2-1), x)

[Out] -log(x\*\*2)/2 + log(-b\*x\*\*2 + 1)/2

**Mathematica [A]** time = 0.00802293, size = 18, normalized size = 1.

$$\frac{1}{2} \log(1 - bx^2) - \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(-1 + b\*x^2)), x]

[Out] -Log[x] + Log[1 - b\*x^2]/2

**Maple [A]** time = 0.006, size = 16, normalized size = 0.9

$$-\ln(x) + \frac{\ln(bx^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b\*x^2-1), x)

[Out] -ln(x)+1/2\*ln(b\*x^2-1)

---

**Maxima [A]** time = 1.33542, size = 23, normalized size = 1.28

$$\frac{1}{2} \log (bx^2 - 1) - \frac{1}{2} \log (x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 - 1)*x), x, algorithm="maxima")`

[Out] `1/2*log(b*x^2 - 1) - 1/2*log(x^2)`

---

**Fricas [A]** time = 0.227538, size = 20, normalized size = 1.11

$$\frac{1}{2} \log (bx^2 - 1) - \log (x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 - 1)*x), x, algorithm="fricas")`

[Out] `1/2*log(b*x^2 - 1) - log(x)`

---

**Sympy [A]** time = 0.308755, size = 12, normalized size = 0.67

$$-\log (x) + \frac{\log \left(x^2 - \frac{1}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x**2-1), x)`

[Out] `-log(x) + log(x**2 - 1/b)/2`

---

**GIAC/XCAS [A]** time = 0.219233, size = 24, normalized size = 1.33

$$-\frac{1}{2} \ln (x^2) + \frac{1}{2} \ln (|bx^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 - 1)*x), x, algorithm="giac")`

[Out] `-1/2*ln(x^2) + 1/2*ln(abs(b*x^2 - 1))`

$$3.255 \quad \int \frac{1}{x^3(1+bx^2)} dx$$

**Optimal.** Leaf size=26

$$\frac{1}{2}b \log(bx^2 + 1) - b \log(x) - \frac{1}{2x^2}$$

[Out]  $-1/(2*x^2) - b*\text{Log}[x] + (b*\text{Log}[1 + b*x^2])/2$

**Rubi [A]** time = 0.0447189, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{1}{2}b \log(bx^2 + 1) - b \log(x) - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^3*(1 + b*x^2)), x]`

[Out]  $-1/(2*x^2) - b*\text{Log}[x] + (b*\text{Log}[1 + b*x^2])/2$

**Rubi in Sympy [A]** time = 6.5651, size = 26, normalized size = 1.

$$-\frac{b \log(x^2)}{2} + \frac{b \log(bx^2 + 1)}{2} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**3/(b*x**2+1), x)`

[Out]  $-b*\log(x**2)/2 + b*\log(b*x**2 + 1)/2 - 1/(2*x**2)$

**Mathematica [A]** time = 0.00981676, size = 26, normalized size = 1.

$$\frac{1}{2}b \log(bx^2 + 1) - b \log(x) - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^3*(1 + b*x^2)), x]`

[Out]  $-1/(2*x^2) - b*\text{Log}[x] + (b*\text{Log}[1 + b*x^2])/2$

**Maple [A]** time = 0.008, size = 23, normalized size = 0.9

$$-\frac{1}{2x^2} - b \ln(x) + \frac{b \ln(bx^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(b*x^2+1), x)`

[Out]  $-1/2/x^2 - b*\ln(x) + 1/2*b*\ln(b*x^2+1)$

---

**Maxima [A]** time = 1.45636, size = 32, normalized size = 1.23

$$\frac{1}{2} b \log (bx^2 + 1) - \frac{1}{2} b \log (x^2) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + 1)\*x^3),x, algorithm="maxima")

[Out] 1/2\*b\*log(b\*x^2 + 1) - 1/2\*b\*log(x^2) - 1/2/x^2

---

**Fricas [A]** time = 0.224767, size = 38, normalized size = 1.46

$$\frac{bx^2 \log (bx^2 + 1) - 2 bx^2 \log (x) - 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + 1)\*x^3),x, algorithm="fricas")

[Out] 1/2\*(b\*x^2\*log(b\*x^2 + 1) - 2\*b\*x^2\*log(x) - 1)/x^2

---

**Sympy [A]** time = 1.35317, size = 22, normalized size = 0.85

$$-b \log (x) + \frac{b \log \left(x^2 + \frac{1}{b}\right)}{2} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(b\*x\*\*2+1),x)

[Out] -b\*log(x) + b\*log(x\*\*2 + 1/b)/2 - 1/(2\*x\*\*2)

---

**GIAC/XCAS [A]** time = 0.211711, size = 43, normalized size = 1.65

$$-\frac{1}{2} b \ln (x^2) + \frac{1}{2} b \ln (|bx^2 + 1|) + \frac{bx^2 - 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + 1)\*x^3),x, algorithm="giac")

[Out] -1/2\*b\*ln(x^2) + 1/2\*b\*ln(abs(b\*x^2 + 1)) + 1/2\*(b\*x^2 - 1)/x^2

$$3.256 \quad \int \frac{1}{x^3(-1+bx^2)} dx$$

**Optimal.** Leaf size=27

$$\frac{1}{2}b \log(1 - bx^2) - b \log(x) + \frac{1}{2x^2}$$

[Out]  $1/(2*x^2) - b*\text{Log}[x] + (b*\text{Log}[1 - b*x^2])/2$

**Rubi [A]** time = 0.0475136, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{1}{2}b \log(1 - bx^2) - b \log(x) + \frac{1}{2x^2}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^3*(-1 + b*x^2)), x]`

[Out]  $1/(2*x^2) - b*\text{Log}[x] + (b*\text{Log}[1 - b*x^2])/2$

**Rubi in Sympy [A]** time = 6.6996, size = 26, normalized size = 0.96

$$-\frac{b \log(x^2)}{2} + \frac{b \log(-bx^2 + 1)}{2} + \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubu_integrate(1/x**3/(b*x**2-1), x)`

[Out]  $-b*\log(x**2)/2 + b*\log(-b*x**2 + 1)/2 + 1/(2*x**2)$

**Mathematica [A]** time = 0.00743577, size = 27, normalized size = 1.

$$\frac{1}{2}b \log(1 - bx^2) - b \log(x) + \frac{1}{2x^2}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^3*(-1 + b*x^2)), x]`

[Out]  $1/(2*x^2) - b*\text{Log}[x] + (b*\text{Log}[1 - b*x^2])/2$

**Maple [A]** time = 0.007, size = 23, normalized size = 0.9

$$\frac{1}{2x^2} - b \ln(x) + \frac{b \ln(bx^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(b*x^2-1), x)`

[Out]  $1/2/x^2 - b*\ln(x) + 1/2*b*\ln(b*x^2 - 1)$

---

**Maxima [A]** time = 1.34573, size = 32, normalized size = 1.19

$$\frac{1}{2} b \log (bx^2 - 1) - \frac{1}{2} b \log (x^2) + \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 - 1)\*x^3),x, algorithm="maxima")

[Out] 1/2\*b\*log(b\*x^2 - 1) - 1/2\*b\*log(x^2) + 1/2/x^2

---

**Fricas [A]** time = 0.226036, size = 38, normalized size = 1.41

$$\frac{bx^2 \log (bx^2 - 1) - 2bx^2 \log (x) + 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 - 1)\*x^3),x, algorithm="fricas")

[Out] 1/2\*(b\*x^2\*log(b\*x^2 - 1) - 2\*b\*x^2\*log(x) + 1)/x^2

---

**Sympy [A]** time = 0.547573, size = 22, normalized size = 0.81

$$-b \log (x) + \frac{b \log \left(x^2 - \frac{1}{b}\right)}{2} + \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(b\*x\*\*2-1),x)

[Out] -b\*log(x) + b\*log(x\*\*2 - 1/b)/2 + 1/(2\*x\*\*2)

---

**GIAC/XCAS [A]** time = 0.212017, size = 43, normalized size = 1.59

$$-\frac{1}{2} b \ln (x^2) + \frac{1}{2} b \ln (|bx^2 - 1|) + \frac{bx^2 + 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 - 1)\*x^3),x, algorithm="giac")

[Out] -1/2\*b\*ln(x^2) + 1/2\*b\*ln(abs(b\*x^2 - 1)) + 1/2\*(b\*x^2 + 1)/x^2

$$3.257 \quad \int \frac{1}{-1+ax^2} dx$$

**Optimal.** Leaf size=30

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{1-a}}\right)}{\sqrt{(1-a)a}}$$

[Out] -(ArcTanh[(Sqrt[a]\*x)/Sqrt[1 - a]]/Sqrt[(1 - a)\*a])

**Rubi [A]** time = 0.0575441, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{ax}}{\sqrt{1-a}}\right)}{\sqrt{(1-a)a}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + a + a\*x^2)^(-1), x]

[Out] -(ArcTanh[(Sqrt[a]\*x)/Sqrt[1 - a]]/Sqrt[(1 - a)\*a])

**Rubi in Sympy [A]** time = 3.5803, size = 27, normalized size = 0.9

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{ax}}{\sqrt{-a+1}}\right)}{\sqrt{a}\sqrt{-a+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a\*x\*\*2+a-1), x)

[Out] -atanh(sqrt(a)\*x/sqrt(-a + 1))/(sqrt(a)\*sqrt(-a + 1))

**Mathematica [A]** time = 0.0191679, size = 28, normalized size = 0.93

$$\frac{\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a-1}}\right)}{\sqrt{a-1}\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + a + a\*x^2)^(-1), x]

[Out] ArcTan[(Sqrt[a]\*x)/Sqrt[-1 + a]]/(Sqrt[-1 + a]\*Sqrt[a])

**Maple [A]** time = 0.006, size = 20, normalized size = 0.7

$$1 \operatorname{arctan}\left(ax \frac{1}{\sqrt{(-1+a)a}}\right) \frac{1}{\sqrt{(-1+a)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a*x^2+a-1),x)`

[Out]  $1/((-1+a)*a)^{(1/2)}*\arctan(a*x/((-1+a)*a)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*x^2 + a - 1),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.22842, size = 1, normalized size = 0.03

$$\left[ \frac{\log\left(\frac{2(a^2-a)x+(ax^2-a+1)\sqrt{-a^2+a}}{ax^2+a-1}\right)}{2\sqrt{-a^2+a}}, \frac{\arctan\left(\frac{\sqrt{a^2-ax}}{a-1}\right)}{\sqrt{a^2-a}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*x^2 + a - 1),x, algorithm="fricas")`

[Out]  $[1/2*\log((2*(a^2 - a)*x + (a*x^2 - a + 1)*\sqrt{-a^2 + a})/(a*x^2 + a - 1))/\sqrt{-a^2 + a}, \arctan(\sqrt{a^2 - a}*x/(a - 1))/\sqrt{a^2 - a}]$

**Sympy [A]** time = 0.399604, size = 83, normalized size = 2.77

$$-\frac{\sqrt{-\frac{1}{a(a-1)}} \log\left(-a\sqrt{-\frac{1}{a(a-1)}} + x + \sqrt{-\frac{1}{a(a-1)}}\right)}{2} + \frac{\sqrt{-\frac{1}{a(a-1)}} \log\left(a\sqrt{-\frac{1}{a(a-1)}} + x - \sqrt{-\frac{1}{a(a-1)}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*x**2+a-1),x)`

[Out]  $-\sqrt{-1/(a*(a-1))}*\log(-a*\sqrt{-1/(a*(a-1))} + x + \sqrt{-1/(a*(a-1))})/2 + \sqrt{-1/(a*(a-1))}*\log(a*\sqrt{-1/(a*(a-1))} + x - \sqrt{-1/(a*(a-1))})/2$

**GIAC/XCAS [A]** time = 0.208394, size = 31, normalized size = 1.03

$$\frac{\arctan\left(\frac{ax}{\sqrt{a^2-a}}\right)}{\sqrt{a^2-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*x^2 + a - 1),x, algorithm="giac")`

[Out]  $\arctan(a*x/\sqrt{a^2 - a})/\sqrt{a^2 - a}$



$$3.258 \quad \int \frac{1}{-c-d+(c-d)x^2} dx$$

**Optimal.** Leaf size=37

$$-\frac{\tanh^{-1}\left(\frac{x\sqrt{c-d}}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}}$$

[Out] -(ArcTanh[(Sqrt[c - d]\*x)/Sqrt[c + d]]/(Sqrt[c - d]\*Sqrt[c + d]))

**Rubi [A]** time = 0.0688485, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$-\frac{\tanh^{-1}\left(\frac{x\sqrt{c-d}}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}}$$

Antiderivative was successfully verified.

[In] Int[(-c - d + (c - d)\*x^2)^(-1), x]

[Out] -(ArcTanh[(Sqrt[c - d]\*x)/Sqrt[c + d]]/(Sqrt[c - d]\*Sqrt[c + d]))

**Rubi in Sympy [A]** time = 5.58743, size = 31, normalized size = 0.84

$$-\frac{\operatorname{atanh}\left(\frac{x\sqrt{c-d}}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-c-d+(c-d)\*x\*\*2), x)

[Out] -atanh(x\*sqrt(c - d)/sqrt(c + d))/(sqrt(c - d)\*sqrt(c + d))

**Mathematica [A]** time = 0.0289556, size = 44, normalized size = 1.19

$$\frac{\tan^{-1}\left(\frac{x\sqrt{c-d}}{\sqrt{-c-d}}\right)}{\sqrt{-c-d}\sqrt{c-d}}$$

Antiderivative was successfully verified.

[In] Integrate[(-c - d + (c - d)\*x^2)^(-1), x]

[Out] ArcTan[(Sqrt[c - d]\*x)/Sqrt[-c - d]]/(Sqrt[-c - d]\*Sqrt[c - d])

**Maple [A]** time = 0.008, size = 33, normalized size = 0.9

$$-1 \operatorname{Artanh}\left((c-d)x \frac{1}{\sqrt{(c+d)(c-d)}}\right) \frac{1}{\sqrt{(c+d)(c-d)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-c-d+(c-d)*x^2),x)`

[Out] `-1/((c+d)*(c-d))^(1/2)*arctanh((c-d)*x/((c+d)*(c-d))^(1/2))`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c - d)*x^2 - c - d),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.233139, size = 1, normalized size = 0.03

$$\left[ \frac{\log\left(-\frac{2(c^2-d^2)x - ((c-d)x^2+c+d)\sqrt{c^2-d^2}}{(c-d)x^2-c-d}\right)}{2\sqrt{c^2-d^2}}, -\frac{\arctan\left(\frac{\sqrt{-c^2+d^2}x}{c+d}\right)}{\sqrt{-c^2+d^2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c - d)*x^2 - c - d),x, algorithm="fricas")`

[Out] `[1/2*log(-(2*(c^2 - d^2)*x - ((c - d)*x^2 + c + d)*sqrt(c^2 - d^2)))/((c - d)*x^2 - c - d)/sqrt(c^2 - d^2), -arctan(sqrt(-c^2 + d^2)*x/(c + d))/sqrt(-c^2 + d^2)]`

**Sympy [A]** time = 0.600821, size = 87, normalized size = 2.35

$$\frac{\sqrt{\frac{1}{(c-d)(c+d)}} \log\left(-c\sqrt{\frac{1}{(c-d)(c+d)}} - d\sqrt{\frac{1}{(c-d)(c+d)}} + x\right)}{2} - \frac{\sqrt{\frac{1}{(c-d)(c+d)}} \log\left(c\sqrt{\frac{1}{(c-d)(c+d)}} + d\sqrt{\frac{1}{(c-d)(c+d)}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-c-d+(c-d)*x**2),x)`

[Out] `sqrt(1/((c - d)*(c + d)))*log(-c*sqrt(1/((c - d)*(c + d)))) - d*sqrt(1/((c - d)*(c + d))) + x)/2 - sqrt(1/((c - d)*(c + d)))*log(c*sqrt(1/((c - d)*(c + d))) + d*sqrt(1/((c - d)*(c + d))) + x)/2`

**GIAC/XCAS [A]** time = 0.209535, size = 45, normalized size = 1.22

$$\frac{\arctan\left(\frac{cx-dx}{\sqrt{-c^2+d^2}}\right)}{\sqrt{-c^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c - d)*x^2 - c - d),x, algorithm="giac")`

[Out] `arctan((c*x - d*x)/sqrt(-c^2 + d^2))/sqrt(-c^2 + d^2)`

$$3.259 \quad \int \frac{1}{x(1+bx^2)^2} dx$$

**Optimal.** Leaf size=28

$$\frac{1}{2(bx^2+1)} - \frac{1}{2} \log(bx^2+1) + \log(x)$$

[Out]  $1/(2*(1 + b*x^2)) + \text{Log}[x] - \text{Log}[1 + b*x^2]/2$

**Rubi [A]** time = 0.0482845, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{1}{2(bx^2+1)} - \frac{1}{2} \log(bx^2+1) + \log(x)$$

Antiderivative was successfully verified.

[In] `Int[1/(x*(1 + b*x^2)^2), x]`

[Out]  $1/(2*(1 + b*x^2)) + \text{Log}[x] - \text{Log}[1 + b*x^2]/2$

**Rubi in Sympy [A]** time = 7.10388, size = 26, normalized size = 0.93

$$\frac{\log(x^2)}{2} - \frac{\log(bx^2+1)}{2} + \frac{1}{2(bx^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x/(b*x**2+1)**2, x)`

[Out]  $\log(x**2)/2 - \log(b*x**2 + 1)/2 + 1/(2*(b*x**2 + 1))$

**Mathematica [A]** time = 0.0231959, size = 25, normalized size = 0.89

$$\frac{1}{2bx^2+2} - \frac{1}{2} \log(bx^2+1) + \log(x)$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x*(1 + b*x^2)^2), x]`

[Out]  $(2 + 2*b*x^2)^{-1} + \text{Log}[x] - \text{Log}[1 + b*x^2]/2$

**Maple [A]** time = 0.015, size = 25, normalized size = 0.9

$$\frac{1}{2bx^2+2} + \ln(x) - \frac{\ln(bx^2+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(b*x^2+1)^2, x)`

[Out]  $1/2/(b*x^2+1)+\ln(x)-1/2*\ln(b*x^2+1)$

---

**Maxima [A]** time = 1.35346, size = 38, normalized size = 1.36

$$\frac{1}{2(bx^2 + 1)} - \frac{1}{2} \log(bx^2 + 1) + \frac{1}{2} \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + 1)^2\*x), x, algorithm="maxima")

[Out] 1/2/(b\*x^2 + 1) - 1/2\*log(b\*x^2 + 1) + 1/2\*log(x^2)

---

**Fricas [A]** time = 0.226793, size = 54, normalized size = 1.93

$$\frac{(bx^2 + 1) \log(bx^2 + 1) - 2(bx^2 + 1) \log(x) - 1}{2(bx^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + 1)^2\*x), x, algorithm="fricas")

[Out] -1/2\*((b\*x^2 + 1)\*log(b\*x^2 + 1) - 2\*(b\*x^2 + 1)\*log(x) - 1)/(b\*x^2 + 1)

---

**Sympy [A]** time = 1.3973, size = 22, normalized size = 0.79

$$\log(x) - \frac{\log(x^2 + \frac{1}{b})}{2} + \frac{1}{2bx^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b\*x\*\*2+1)\*\*2, x)

[Out] log(x) - log(x\*\*2 + 1/b)/2 + 1/(2\*b\*x\*\*2 + 2)

---

**GIAC/XCAS [A]** time = 0.213754, size = 49, normalized size = 1.75

$$\frac{bx^2 + 2}{2(bx^2 + 1)} + \frac{1}{2} \ln(x^2) - \frac{1}{2} \ln(|bx^2 + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + 1)^2\*x), x, algorithm="giac")

[Out] 1/2\*(b\*x^2 + 2)/(b\*x^2 + 1) + 1/2\*ln(x^2) - 1/2\*ln(abs(b\*x^2 + 1))

$$3.260 \quad \int \frac{1}{x(-1+bx^2)^2} dx$$

**Optimal.** Leaf size=30

$$\frac{1}{2(1-bx^2)} - \frac{1}{2} \log(1-bx^2) + \log(x)$$

[Out]  $1/(2*(1 - b*x^2)) + \text{Log}[x] - \text{Log}[1 - b*x^2]/2$

**Rubi [A]** time = 0.0523323, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{1}{2(1-bx^2)} - \frac{1}{2} \log(1-bx^2) + \log(x)$$

Antiderivative was successfully verified.

[In] `Int[1/(x*(-1 + b*x^2)^2), x]`

[Out]  $1/(2*(1 - b*x^2)) + \text{Log}[x] - \text{Log}[1 - b*x^2]/2$

**Rubi in Sympy [A]** time = 6.94858, size = 26, normalized size = 0.87

$$\frac{\log(x^2)}{2} - \frac{\log(-bx^2 + 1)}{2} + \frac{1}{2(-bx^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x/(b*x**2-1)**2, x)`

[Out]  $\log(x**2)/2 - \log(-b*x**2 + 1)/2 + 1/(2*(-b*x**2 + 1))$

**Mathematica [A]** time = 0.0240394, size = 26, normalized size = 0.87

$$\frac{1}{2-2bx^2} - \frac{1}{2} \log(1-bx^2) + \log(x)$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x*(-1 + b*x^2)^2), x]`

[Out]  $(2 - 2*b*x^2)^{-1} + \text{Log}[x] - \text{Log}[1 - b*x^2]/2$

**Maple [A]** time = 0.015, size = 25, normalized size = 0.8

$$\ln(x) - \frac{1}{2bx^2 - 2} - \frac{\ln(bx^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(b*x^2-1)^2, x)`

[Out]  $\ln(x) - 1/2/(b*x^2-1) - 1/2*\ln(b*x^2-1)$

---

**Maxima [A]** time = 1.35406, size = 38, normalized size = 1.27

$$-\frac{1}{2(bx^2 - 1)} - \frac{1}{2} \log(bx^2 - 1) + \frac{1}{2} \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 - 1)^2\*x), x, algorithm="maxima")

[Out] -1/2/(b\*x^2 - 1) - 1/2\*log(b\*x^2 - 1) + 1/2\*log(x^2)

---

**Fricas [A]** time = 0.221936, size = 54, normalized size = 1.8

$$\frac{(bx^2 - 1) \log(bx^2 - 1) - 2(bx^2 - 1) \log(x) + 1}{2(bx^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 - 1)^2\*x), x, algorithm="fricas")

[Out] -1/2\*((b\*x^2 - 1)\*log(b\*x^2 - 1) - 2\*(b\*x^2 - 1)\*log(x) + 1)/(b\*x^2 - 1)

---

**Sympy [A]** time = 1.37295, size = 22, normalized size = 0.73

$$\log(x) - \frac{\log(x^2 - \frac{1}{b})}{2} - \frac{1}{2bx^2 - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b\*x\*\*2-1)\*\*2, x)

[Out] log(x) - log(x\*\*2 - 1/b)/2 - 1/(2\*b\*x\*\*2 - 2)

---

**GIAC/XCAS [A]** time = 0.209082, size = 49, normalized size = 1.63

$$\frac{bx^2 - 2}{2(bx^2 - 1)} + \frac{1}{2} \ln(x^2) - \frac{1}{2} \ln(|bx^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 - 1)^2\*x), x, algorithm="giac")

[Out] 1/2\*(b\*x^2 - 2)/(b\*x^2 - 1) + 1/2\*ln(x^2) - 1/2\*ln(abs(b\*x^2 - 1))

$$3.261 \quad \int \frac{1}{a+(b-ac)x^2} dx$$

**Optimal.** Leaf size=34

$$\frac{\tan^{-1}\left(\frac{x\sqrt{b-ac}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

[Out] ArcTan[(Sqrt[b - a\*c]\*x)/Sqrt[a]]/(Sqrt[a]\*Sqrt[b - a\*c])

**Rubi [A]** time = 0.0557801, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{\tan^{-1}\left(\frac{x\sqrt{b-ac}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

Antiderivative was successfully verified.

[In] Int[(a + (b - a\*c)\*x^2)^(-1), x]

[Out] ArcTan[(Sqrt[b - a\*c]\*x)/Sqrt[a]]/(Sqrt[a]\*Sqrt[b - a\*c])

**Rubi in Sympy [A]** time = 4.2146, size = 29, normalized size = 0.85

$$\frac{\text{atan}\left(\frac{x\sqrt{-ac+b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{-ac+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a+(-a\*c+b)\*x\*\*2), x)

[Out] atan(x\*sqrt(-a\*c + b)/sqrt(a))/(sqrt(a)\*sqrt(-a\*c + b))

**Mathematica [A]** time = 0.0277972, size = 36, normalized size = 1.06

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{ac-b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{ac-b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + (b - a\*c)\*x^2)^(-1), x]

[Out] ArcTanh[(Sqrt[-b + a\*c]\*x)/Sqrt[a]]/(Sqrt[a]\*Sqrt[-b + a\*c])

**Maple [A]** time = 0.008, size = 34, normalized size = 1.

$$1\text{Artanh}\left((ac-b)x\frac{1}{\sqrt{a(ac-b)}}\right)\frac{1}{\sqrt{a(ac-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+(-a*c+b)*x^2),x)`

[Out] `1/(a*(a*c-b))^(1/2)*arctanh((a*c-b)*x/(a*(a*c-b))^(1/2))`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((a*c - b)*x^2 - a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.228245, size = 1, normalized size = 0.03

$$\left[ \frac{\log\left(\frac{2(a^2c-ab)x+\sqrt{a^2c-ab}((ac-b)x^2+a)}{(ac-b)x^2-a}\right)}{2\sqrt{a^2c-ab}}, \frac{\arctan\left(\frac{\sqrt{-a^2c+ab}x}{a}\right)}{\sqrt{-a^2c+ab}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((a*c - b)*x^2 - a),x, algorithm="fricas")`

[Out] `[1/2*log((2*(a^2*c - a*b)*x + sqrt(a^2*c - a*b)*((a*c - b)*x^2 + a))/((a*c - b)*x^2 - a))/sqrt(a^2*c - a*b), arctan(sqrt(-a^2*c + a*b)*x/a)/sqrt(-a^2*c + a*b)]`

**Sympy [A]** time = 0.690462, size = 60, normalized size = 1.76

$$-\frac{\sqrt{\frac{1}{a(ac-b)}} \log\left(-a\sqrt{\frac{1}{a(ac-b)}} + x\right)}{2} + \frac{\sqrt{\frac{1}{a(ac-b)}} \log\left(a\sqrt{\frac{1}{a(ac-b)}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+(-a*c+b)*x**2),x)`

[Out] `-sqrt(1/(a*(a*c - b)))*log(-a*sqrt(1/(a*(a*c - b))) + x)/2 + sqrt(1/(a*(a*c - b)))*log(a*sqrt(1/(a*(a*c - b))) + x)/2`

**GIAC/XCAS [A]** time = 0.207112, size = 50, normalized size = 1.47

$$-\frac{\arctan\left(\frac{acx-bx}{\sqrt{-a^2c+ab}}\right)}{\sqrt{-a^2c+ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((a*c - b)*x^2 - a),x, algorithm="giac")`

[Out] `-arctan((a*c*x - b*x)/sqrt(-a^2*c + a*b))/sqrt(-a^2*c + a*b)`



$$3.262 \quad \int \frac{1}{a-(b-ac)x^2} dx$$

**Optimal.** Leaf size=34

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{b-ac}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

[Out] ArcTanh[(Sqrt[b - a\*c]\*x)/Sqrt[a]]/(Sqrt[a]\*Sqrt[b - a\*c])

**Rubi [A]** time = 0.0308608, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{b-ac}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

Antiderivative was successfully verified.

[In] Int[(a - (b - a\*c)\*x^2)^(-1), x]

[Out] ArcTanh[(Sqrt[b - a\*c]\*x)/Sqrt[a]]/(Sqrt[a]\*Sqrt[b - a\*c])

**Rubi in Sympy [A]** time = 4.85595, size = 29, normalized size = 0.85

$$\frac{\operatorname{atanh}\left(\frac{x\sqrt{-ac+b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{-ac+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a-(-a\*c+b)\*x\*\*2), x)

[Out] atanh(x\*sqrt(-a\*c + b)/sqrt(a))/(sqrt(a)\*sqrt(-a\*c + b))

**Mathematica [A]** time = 0.0176621, size = 36, normalized size = 1.06

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ac-b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{ac-b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - (b - a\*c)\*x^2)^(-1), x]

[Out] ArcTan[(Sqrt[-b + a\*c]\*x)/Sqrt[a]]/(Sqrt[a]\*Sqrt[-b + a\*c])

**Maple [A]** time = 0.005, size = 34, normalized size = 1.

$$1 \arctan\left(x(ac-b)\frac{1}{\sqrt{a(ac-b)}}\right)\frac{1}{\sqrt{a(ac-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a-(-a*c+b)*x^2),x)`

[Out] `1/(a*(a*c-b))^(1/2)*arctan((a*c-b)*x/(a*(a*c-b))^(1/2))`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((a*c - b)*x^2 + a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.222538, size = 1, normalized size = 0.03

$$\left[ \frac{\log\left(\frac{2(a^2c-ab)x+\sqrt{-a^2c+ab}((ac-b)x^2-a)}{(ac-b)x^2+a}\right)}{2\sqrt{-a^2c+ab}}, \frac{\arctan\left(\frac{\sqrt{a^2c-ab}x}{a}\right)}{\sqrt{a^2c-ab}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((a*c - b)*x^2 + a),x, algorithm="fricas")`

[Out] `[1/2*log((2*(a^2*c - a*b)*x + sqrt(-a^2*c + a*b))*((a*c - b)*x^2 - a))/((a*c - b)*x^2 + a)/sqrt(-a^2*c + a*b), arctan(sqrt(a^2*c - a*b)*x/a)/sqrt(a^2*c - a*b)]`

**Sympy [A]** time = 0.620811, size = 66, normalized size = 1.94

$$-\frac{\sqrt{-\frac{1}{a(ac-b)}} \log\left(-a\sqrt{-\frac{1}{a(ac-b)}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{a(ac-b)}} \log\left(a\sqrt{-\frac{1}{a(ac-b)}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a-(-a*c+b)*x**2),x)`

[Out] `-sqrt(-1/(a*(a*c - b)))*log(-a*sqrt(-1/(a*(a*c - b)))) + x)/2 + sqrt(-1/(a*(a*c - b)))*log(a*sqrt(-1/(a*(a*c - b)))) + x)/2`

**GIAC/XCAS [A]** time = 0.209454, size = 49, normalized size = 1.44

$$\frac{\arctan\left(\frac{acx-bx}{\sqrt{a^2c-ab}}\right)}{\sqrt{a^2c-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((a*c - b)*x^2 + a),x, algorithm="giac")`

[Out] `arctan((a*c*x - b*x)/sqrt(a^2*c - a*b))/sqrt(a^2*c - a*b)`

$$3.263 \quad \int \frac{1}{c(a-d)-(b-c)x^2} dx$$

**Optimal.** Leaf size=50

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{b-c}}{\sqrt{c}\sqrt{a-d}}\right)}{\sqrt{c}\sqrt{a-d}\sqrt{b-c}}$$

[Out] ArcTanh[(Sqrt[b - c]\*x)/(Sqrt[c]\*Sqrt[a - d])]/(Sqrt[b - c]\*Sqrt[c]\*Sqrt[a - d])

**Rubi [A]** time = 0.104129, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{b-c}}{\sqrt{c}\sqrt{a-d}}\right)}{\sqrt{c}\sqrt{a-d}\sqrt{b-c}}$$

Antiderivative was successfully verified.

[In] Int[(c\*(a - d) - (b - c)\*x^2)^(-1), x]

[Out] ArcTanh[(Sqrt[b - c]\*x)/(Sqrt[c]\*Sqrt[a - d])]/(Sqrt[b - c]\*Sqrt[c]\*Sqrt[a - d])

**Rubi in Sympy [A]** time = 7.6212, size = 39, normalized size = 0.78

$$\frac{\operatorname{atanh}\left(\frac{x\sqrt{b-c}}{\sqrt{c}\sqrt{a-d}}\right)}{\sqrt{c}\sqrt{a-d}\sqrt{b-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*(a-d)-(b-c)\*x\*\*2), x)

[Out] atanh(x\*sqrt(b - c)/(sqrt(c)\*sqrt(a - d)))/(sqrt(c)\*sqrt(a - d)\*sqrt(b - c))

**Mathematica [A]** time = 0.0372223, size = 50, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{x\sqrt{c-b}}{\sqrt{c}\sqrt{a-d}}\right)}{\sqrt{c}\sqrt{a-d}\sqrt{c-b}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*(a - d) - (b - c)\*x^2)^(-1), x]

[Out] ArcTan[(Sqrt[-b + c]\*x)/(Sqrt[c]\*Sqrt[a - d])]/(Sqrt[c]\*Sqrt[-b + c]\*Sqrt[a - d])

**Maple [A]** time = 0.009, size = 38, normalized size = 0.8

$$1\operatorname{Artanh}\left((b-c)x\frac{1}{\sqrt{c(a-d)(b-c)}}\right)\frac{1}{\sqrt{c(a-d)(b-c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*(a-d)-(b-c)*x^2),x)`

[Out]  $1/(c*(a-d)*(b-c))^{1/2}*\operatorname{arctanh}((b-c)*x/(c*(a-d)*(b-c))^{1/2})$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((b-c)*x^2-(a-d)*c),x,algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.232153, size = 1, normalized size = 0.02

$$\left[ \frac{\log\left(\frac{2(abc-ac^2-(bc-c^2)d)x+\sqrt{abc-ac^2-(bc-c^2)d}((b-c)x^2+ac-cd)}{(b-c)x^2-ac+cd}\right)}{2\sqrt{abc-ac^2-(bc-c^2)d}}, -\frac{\operatorname{arctan}\left(-\frac{\sqrt{-abc+ac^2+(bc-c^2)d}x}{ac-cd}\right)}{\sqrt{-abc+ac^2+(bc-c^2)d}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/((b-c)*x^2-(a-d)*c),x,algorithm="fricas")`

[Out]  $[1/2*\log((2*(a*b*c - a*c^2 - (b*c - c^2)*d)*x + \sqrt{a*b*c - a*c^2 - (b*c - c^2)*d}*((b-c)*x^2 + a*c - c*d))/((b-c)*x^2 - a*c + c*d))/\sqrt{a*b*c - a*c^2 - (b*c - c^2)*d}, -\operatorname{arctan}(-\sqrt{-a*b*c + a*c^2 + (b*c - c^2)*d}/(a*c - c*d))/\sqrt{-a*b*c + a*c^2 + (b*c - c^2)*d}]$

**Sympy** [A] time = 1.03231, size = 104, normalized size = 2.08

$$-\frac{\sqrt{\frac{1}{c(a-d)(b-c)}}\log\left(-ac\sqrt{\frac{1}{c(a-d)(b-c)}}+cd\sqrt{\frac{1}{c(a-d)(b-c)}}+x\right)}{2} + \frac{\sqrt{\frac{1}{c(a-d)(b-c)}}\log\left(ac\sqrt{\frac{1}{c(a-d)(b-c)}}-cd\sqrt{\frac{1}{c(a-d)(b-c)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*(a-d)-(b-c)*x**2),x)`

[Out]  $-\sqrt{1/(c*(a-d)*(b-c))}*\log(-a*c*\sqrt{1/(c*(a-d)*(b-c))}+c*d*\sqrt{1/(c*(a-d)*(b-c))}+x)/2 + \sqrt{1/(c*(a-d)*(b-c))}*\log(a*c*\sqrt{1/(c*(a-d)*(b-c))}-c*d*\sqrt{1/(c*(a-d)*(b-c))}+x)/2$

**GIAC/XCAS** [A] time = 0.210557, size = 78, normalized size = 1.56

$$\frac{\operatorname{arctan}\left(\frac{bx-cx}{\sqrt{-abc+ac^2+bcd-c^2d}}\right)}{\sqrt{-abc+ac^2+bcd-c^2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b - c)*x^2 - (a - d)*c),x, algorithm="giac")
```

```
[Out] -arctan((b*x - c*x)/sqrt(-a*b*c + a*c^2 + b*c*d - c^2*d))/sqrt(-a  
*b*c + a*c^2 + b*c*d - c^2*d)
```

### 3.264 $\int x^{7/2} (a + bx^2) dx$

**Optimal.** Leaf size=21

$$\frac{2}{9}ax^{9/2} + \frac{2}{13}bx^{13/2}$$

[Out]  $(2*a*x^{(9/2)})/9 + (2*b*x^{(13/2)})/13$

**Rubi [A]** time = 0.0137314, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{2}{9}ax^{9/2} + \frac{2}{13}bx^{13/2}$$

Antiderivative was successfully verified.

[In] `Int[x^(7/2)*(a + b*x^2), x]`

[Out]  $(2*a*x^{(9/2)})/9 + (2*b*x^{(13/2)})/13$

**Rubi in Sympy [A]** time = 2.9219, size = 19, normalized size = 0.9

$$\frac{2ax^{\frac{9}{2}}}{9} + \frac{2bx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**(7/2)*(b*x**2+a), x)`

[Out]  $2*a*x^{(9/2)}/9 + 2*b*x^{(13/2)}/13$

**Mathematica [A]** time = 0.00804533, size = 21, normalized size = 1.

$$\frac{2}{9}ax^{9/2} + \frac{2}{13}bx^{13/2}$$

Antiderivative was successfully verified.

[In] `Integrate[x^(7/2)*(a + b*x^2), x]`

[Out]  $(2*a*x^{(9/2)})/9 + (2*b*x^{(13/2)})/13$

**Maple [A]** time = 0.004, size = 16, normalized size = 0.8

$$\frac{18bx^2 + 26a}{117}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(7/2)*(b*x^2+a), x)`

[Out]  $2/117*x^{(9/2)}*(9*b*x^2+13*a)$

---

**Maxima [A]** time = 1.34918, size = 18, normalized size = 0.86

$$\frac{2}{13}bx^{\frac{13}{2}} + \frac{2}{9}ax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x^(7/2),x, algorithm="maxima")`

[Out] `2/13*b*x^(13/2) + 2/9*a*x^(9/2)`

---

**Fricas [A]** time = 0.217804, size = 24, normalized size = 1.14

$$\frac{2}{117}(9bx^6 + 13ax^4)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x^(7/2),x, algorithm="fricas")`

[Out] `2/117*(9*b*x^6 + 13*a*x^4)*sqrt(x)`

---

**Sympy [A]** time = 19.8947, size = 19, normalized size = 0.9

$$\frac{2ax^{\frac{9}{2}}}{9} + \frac{2bx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(7/2)*(b*x**2+a),x)`

[Out] `2*a*x**(9/2)/9 + 2*b*x**(13/2)/13`

---

**GIAC/XCAS [A]** time = 0.205132, size = 18, normalized size = 0.86

$$\frac{2}{13}bx^{\frac{13}{2}} + \frac{2}{9}ax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x^(7/2),x, algorithm="giac")`

[Out] `2/13*b*x^(13/2) + 2/9*a*x^(9/2)`

$$3.265 \quad \int x^{5/2} (a + bx^2) dx$$

**Optimal.** Leaf size=21

$$\frac{2}{7}ax^{7/2} + \frac{2}{11}bx^{11/2}$$

[Out]  $(2*a*x^{(7/2)})/7 + (2*b*x^{(11/2)})/11$

**Rubi [A]** time = 0.0140309, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{2}{7}ax^{7/2} + \frac{2}{11}bx^{11/2}$$

Antiderivative was successfully verified.

[In] `Int[x^(5/2)*(a + b*x^2), x]`

[Out]  $(2*a*x^{(7/2)})/7 + (2*b*x^{(11/2)})/11$

**Rubi in Sympy [A]** time = 2.90907, size = 19, normalized size = 0.9

$$\frac{2ax^{\frac{7}{2}}}{7} + \frac{2bx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**(5/2)*(b*x**2+a), x)`

[Out]  $2*a*x^{(7/2)}/7 + 2*b*x^{(11/2)}/11$

**Mathematica [A]** time = 0.00728249, size = 21, normalized size = 1.

$$\frac{2}{7}ax^{7/2} + \frac{2}{11}bx^{11/2}$$

Antiderivative was successfully verified.

[In] `Integrate[x^(5/2)*(a + b*x^2), x]`

[Out]  $(2*a*x^{(7/2)})/7 + (2*b*x^{(11/2)})/11$

**Maple [A]** time = 0.004, size = 16, normalized size = 0.8

$$\frac{14bx^2 + 22a}{77}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(5/2)*(b*x^2+a), x)`

[Out]  $2/77*x^{(7/2)}*(7*b*x^2+11*a)$



---

**Maxima [A]** time = 1.3496, size = 18, normalized size = 0.86

$$\frac{2}{11}bx^{\frac{11}{2}} + \frac{2}{7}ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^(5/2),x, algorithm="maxima")

[Out] 2/11\*b\*x^(11/2) + 2/7\*a\*x^(7/2)

---

**Fricas [A]** time = 0.224887, size = 24, normalized size = 1.14

$$\frac{2}{77}(7bx^5 + 11ax^3)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^(5/2),x, algorithm="fricas")

[Out] 2/77\*(7\*b\*x^5 + 11\*a\*x^3)\*sqrt(x)

---

**Sympy [A]** time = 7.70524, size = 19, normalized size = 0.9

$$\frac{2ax^{\frac{7}{2}}}{7} + \frac{2bx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(5/2)\*(b\*x\*\*2+a),x)

[Out] 2\*a\*x\*\*(7/2)/7 + 2\*b\*x\*\*(11/2)/11

---

**GIAC/XCAS [A]** time = 0.207083, size = 18, normalized size = 0.86

$$\frac{2}{11}bx^{\frac{11}{2}} + \frac{2}{7}ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^(5/2),x, algorithm="giac")

[Out] 2/11\*b\*x^(11/2) + 2/7\*a\*x^(7/2)

### 3.266 $\int x^{3/2} (a + bx^2) dx$

**Optimal.** Leaf size=21

$$\frac{2}{5}ax^{5/2} + \frac{2}{9}bx^{9/2}$$

[Out]  $(2*a*x^{(5/2)})/5 + (2*b*x^{(9/2)})/9$

**Rubi [A]** time = 0.0139708, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{2}{5}ax^{5/2} + \frac{2}{9}bx^{9/2}$$

Antiderivative was successfully verified.

[In] `Int[x^(3/2)*(a + b*x^2), x]`

[Out]  $(2*a*x^{(5/2)})/5 + (2*b*x^{(9/2)})/9$

**Rubi in Sympy [A]** time = 3.0502, size = 19, normalized size = 0.9

$$\frac{2ax^{\frac{5}{2}}}{5} + \frac{2bx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubu_integrate(x**(3/2)*(b*x**2+a), x)`

[Out]  $2*a*x^{(5/2)}/5 + 2*b*x^{(9/2)}/9$

**Mathematica [A]** time = 0.00724314, size = 21, normalized size = 1.

$$\frac{2}{5}ax^{5/2} + \frac{2}{9}bx^{9/2}$$

Antiderivative was successfully verified.

[In] `Integrate[x^(3/2)*(a + b*x^2), x]`

[Out]  $(2*a*x^{(5/2)})/5 + (2*b*x^{(9/2)})/9$

**Maple [A]** time = 0.004, size = 16, normalized size = 0.8

$$\frac{10bx^2 + 18a}{45}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3/2)*(b*x^2+a), x)`

[Out]  $2/45*x^{(5/2)}*(5*b*x^2+9*a)$

---

**Maxima [A]** time = 1.34442, size = 18, normalized size = 0.86

$$\frac{2}{9}bx^{\frac{9}{2}} + \frac{2}{5}ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x^(3/2), x, algorithm="maxima")`

[Out] `2/9*b*x^(9/2) + 2/5*a*x^(5/2)`

---

**Fricas [A]** time = 0.219191, size = 24, normalized size = 1.14

$$\frac{2}{45}(5bx^4 + 9ax^2)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x^(3/2), x, algorithm="fricas")`

[Out] `2/45*(5*b*x^4 + 9*a*x^2)*sqrt(x)`

---

**Sympy [A]** time = 2.7076, size = 19, normalized size = 0.9

$$\frac{2ax^{\frac{5}{2}}}{5} + \frac{2bx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(3/2)*(b*x**2+a), x)`

[Out] `2*a*x**(5/2)/5 + 2*b*x**(9/2)/9`

---

**GIAC/XCAS [A]** time = 0.206735, size = 18, normalized size = 0.86

$$\frac{2}{9}bx^{\frac{9}{2}} + \frac{2}{5}ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*x^(3/2), x, algorithm="giac")`

[Out] `2/9*b*x^(9/2) + 2/5*a*x^(5/2)`

### 3.267 $\int \sqrt{x} (a + bx^2) dx$

**Optimal.** Leaf size=21

$$\frac{2}{3}ax^{3/2} + \frac{2}{7}bx^{7/2}$$

[Out]  $(2*a*x^{(3/2)})/3 + (2*b*x^{(7/2)})/7$

**Rubi [A]** time = 0.0139615, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{2}{3}ax^{3/2} + \frac{2}{7}bx^{7/2}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[x]*(a + b*x^2), x]`

[Out]  $(2*a*x^{(3/2)})/3 + (2*b*x^{(7/2)})/7$

**Rubi in Sympy [A]** time = 2.98048, size = 19, normalized size = 0.9

$$\frac{2ax^{\frac{3}{2}}}{3} + \frac{2bx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)*x**(1/2), x)`

[Out]  $2*a*x^{(3/2)}/3 + 2*b*x^{(7/2)}/7$

**Mathematica [A]** time = 0.00835316, size = 21, normalized size = 1.

$$\frac{2}{3}ax^{3/2} + \frac{2}{7}bx^{7/2}$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[x]*(a + b*x^2), x]`

[Out]  $(2*a*x^{(3/2)})/3 + (2*b*x^{(7/2)})/7$

**Maple [A]** time = 0.003, size = 16, normalized size = 0.8

$$\frac{6bx^2 + 14a}{21}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)*x^(1/2), x)`

[Out]  $2/21*x^{(3/2)}*(3*b*x^2+7*a)$

---

**Maxima [A]** time = 1.3426, size = 18, normalized size = 0.86

$$\frac{2}{7}bx^{\frac{7}{2}} + \frac{2}{3}ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*sqrt(x),x, algorithm="maxima")`

[Out] `2/7*b*x^(7/2) + 2/3*a*x^(3/2)`

---

**Fricas [A]** time = 0.228766, size = 22, normalized size = 1.05

$$\frac{2}{21}(3bx^3 + 7ax)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*sqrt(x),x, algorithm="fricas")`

[Out] `2/21*(3*b*x^3 + 7*a*x)*sqrt(x)`

---

**Sympy [A]** time = 1.48117, size = 19, normalized size = 0.9

$$\frac{2ax^{\frac{3}{2}}}{3} + \frac{2bx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)*x**(1/2),x)`

[Out] `2*a*x**(3/2)/3 + 2*b*x**(7/2)/7`

---

**GIAC/XCAS [A]** time = 0.206018, size = 18, normalized size = 0.86

$$\frac{2}{7}bx^{\frac{7}{2}} + \frac{2}{3}ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)*sqrt(x),x, algorithm="giac")`

[Out] `2/7*b*x^(7/2) + 2/3*a*x^(3/2)`

$$3.268 \quad \int \frac{a+bx^2}{\sqrt{x}} dx$$

**Optimal.** Leaf size=19

$$2a\sqrt{x} + \frac{2}{5}bx^{5/2}$$

[Out]  $2*a*\text{Sqrt}[x] + (2*b*x^{(5/2)})/5$

**Rubi [A]** time = 0.0140092, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$2a\sqrt{x} + \frac{2}{5}bx^{5/2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)/\text{Sqrt}[x], x]$

[Out]  $2*a*\text{Sqrt}[x] + (2*b*x^{(5/2)})/5$

**Rubi in Sympy [A]** time = 2.93951, size = 17, normalized size = 0.89

$$2a\sqrt{x} + \frac{2bx^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x^{**2}+a)/x^{** (1/2)}, x)$

[Out]  $2*a*\text{sqrt}(x) + 2*b*x^{** (5/2)}/5$

**Mathematica [A]** time = 0.00774583, size = 19, normalized size = 1.

$$2a\sqrt{x} + \frac{2}{5}bx^{5/2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b*x^2)/\text{Sqrt}[x], x]$

[Out]  $2*a*\text{Sqrt}[x] + (2*b*x^{(5/2)})/5$

**Maple [A]** time = 0.005, size = 15, normalized size = 0.8

$$\frac{2bx^2 + 10a}{5}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((b*x^2+a)/x^{(1/2)}, x)$

[Out]  $2/5*x^{(1/2)}*(b*x^2+5*a)$

---

**Maxima [A]** time = 1.3384, size = 18, normalized size = 0.95

$$\frac{2}{5}bx^{\frac{5}{2}} + 2a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/sqrt(x), x, algorithm="maxima")`

[Out] `2/5*b*x^(5/2) + 2*a*sqrt(x)`

---

**Fricas [A]** time = 0.2191, size = 19, normalized size = 1.

$$\frac{2}{5}(bx^2 + 5a)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/sqrt(x), x, algorithm="fricas")`

[Out] `2/5*(b*x^2 + 5*a)*sqrt(x)`

---

**Sympy [A]** time = 0.59789, size = 17, normalized size = 0.89

$$2a\sqrt{x} + \frac{2bx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/x**(1/2), x)`

[Out] `2*a*sqrt(x) + 2*b*x**(5/2)/5`

---

**GIAC/XCAS [A]** time = 0.207556, size = 18, normalized size = 0.95

$$\frac{2}{5}bx^{\frac{5}{2}} + 2a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/sqrt(x), x, algorithm="giac")`

[Out] `2/5*b*x^(5/2) + 2*a*sqrt(x)`

$$3.269 \quad \int \frac{a+bx^2}{x^{3/2}} dx$$

**Optimal.** Leaf size=19

$$\frac{2}{3}bx^{3/2} - \frac{2a}{\sqrt{x}}$$

[Out]  $(-2*a)/\text{Sqrt}[x] + (2*b*x^{(3/2)})/3$

**Rubi [A]** time = 0.0137852, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{2}{3}bx^{3/2} - \frac{2a}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)/x^{(3/2)}, x]$

[Out]  $(-2*a)/\text{Sqrt}[x] + (2*b*x^{(3/2)})/3$

**Rubi in Sympy [A]** time = 2.94843, size = 17, normalized size = 0.89

$$-\frac{2a}{\sqrt{x}} + \frac{2bx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x^{**2}+a)/x^{** (3/2)}, x)$

[Out]  $-2*a/\text{sqrt}(x) + 2*b*x^{** (3/2)}/3$

**Mathematica [A]** time = 0.00933006, size = 19, normalized size = 1.

$$\frac{2}{3}bx^{3/2} - \frac{2a}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b*x^2)/x^{(3/2)}, x]$

[Out]  $(-2*a)/\text{Sqrt}[x] + (2*b*x^{(3/2)})/3$

**Maple [A]** time = 0.005, size = 16, normalized size = 0.8

$$-\frac{-2bx^2 + 6a}{3} \frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((b*x^2+a)/x^{(3/2)}, x)$

[Out]  $-2/3*(-b*x^2+3*a)/x^{(1/2)}$



---

**Maxima [A]** time = 1.34411, size = 18, normalized size = 0.95

$$\frac{2}{3}bx^{\frac{3}{2}} - \frac{2a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^(3/2),x, algorithm="maxima")

[Out] 2/3\*b\*x^(3/2) - 2\*a/sqrt(x)

---

**Fricas [A]** time = 0.250493, size = 19, normalized size = 1.

$$\frac{2(bx^2 - 3a)}{3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^(3/2),x, algorithm="fricas")

[Out] 2/3\*(b\*x^2 - 3\*a)/sqrt(x)

---

**Sympy [A]** time = 1.5558, size = 17, normalized size = 0.89

$$-\frac{2a}{\sqrt{x}} + \frac{2bx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/x\*\*(3/2),x)

[Out] -2\*a/sqrt(x) + 2\*b\*x\*\*(3/2)/3

---

**GIAC/XCAS [A]** time = 0.209861, size = 18, normalized size = 0.95

$$\frac{2}{3}bx^{\frac{3}{2}} - \frac{2a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^(3/2),x, algorithm="giac")

[Out] 2/3\*b\*x^(3/2) - 2\*a/sqrt(x)

$$3.270 \quad \int \frac{a+bx^2}{x^{5/2}} dx$$

**Optimal.** Leaf size=19

$$2b\sqrt{x} - \frac{2a}{3x^{3/2}}$$

[Out]  $(-2*a)/(3*x^{(3/2)}) + 2*b*\text{Sqrt}[x]$

**Rubi [A]** time = 0.0143439, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$2b\sqrt{x} - \frac{2a}{3x^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)/x^{(5/2)}, x]$

[Out]  $(-2*a)/(3*x^{(3/2)}) + 2*b*\text{Sqrt}[x]$

**Rubi in Sympy [A]** time = 3.0378, size = 17, normalized size = 0.89

$$-\frac{2a}{3x^{3/2}} + 2b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x^{**2}+a)/x^{** (5/2)}, x)$

[Out]  $-2*a/(3*x^{** (3/2)}) + 2*b*\text{sqrt}(x)$

**Mathematica [A]** time = 0.0097822, size = 19, normalized size = 1.

$$2b\sqrt{x} - \frac{2a}{3x^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b*x^2)/x^{(5/2)}, x]$

[Out]  $(-2*a)/(3*x^{(3/2)}) + 2*b*\text{Sqrt}[x]$

**Maple [A]** time = 0.004, size = 14, normalized size = 0.7

$$-\frac{-6bx^2 + 2a}{3}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((b*x^2+a)/x^{(5/2)}, x)$

[Out]  $-2/3*(-3*b*x^2+a)/x^{(3/2)}$

---

**Maxima [A]** time = 1.34604, size = 18, normalized size = 0.95

$$2b\sqrt{x} - \frac{2a}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^(5/2),x, algorithm="maxima")

[Out] 2\*b\*sqrt(x) - 2/3\*a/x^(3/2)

---

**Fricas [A]** time = 0.252376, size = 20, normalized size = 1.05

$$\frac{2(3bx^2 - a)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^(5/2),x, algorithm="fricas")

[Out] 2/3\*(3\*b\*x^2 - a)/x^(3/2)

---

**Sympy [A]** time = 2.1549, size = 17, normalized size = 0.89

$$-\frac{2a}{3x^{\frac{3}{2}}} + 2b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/x\*\*(5/2),x)

[Out] -2\*a/(3\*x\*\*(3/2)) + 2\*b\*sqrt(x)

---

**GIAC/XCAS [A]** time = 0.211383, size = 18, normalized size = 0.95

$$2b\sqrt{x} - \frac{2a}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)/x^(5/2),x, algorithm="giac")

[Out] 2\*b\*sqrt(x) - 2/3\*a/x^(3/2)

$$3.271 \quad \int \frac{a+bx^2}{x^{7/2}} dx$$

**Optimal.** Leaf size=19

$$-\frac{2a}{5x^{5/2}} - \frac{2b}{\sqrt{x}}$$

[Out]  $(-2*a)/(5*x^(5/2)) - (2*b)/\text{Sqrt}[x]$

**Rubi [A]** time = 0.0141218, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{2a}{5x^{5/2}} - \frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)/x^(7/2), x]$

[Out]  $(-2*a)/(5*x^(5/2)) - (2*b)/\text{Sqrt}[x]$

**Rubi in Sympy [A]** time = 2.92649, size = 19, normalized size = 1.

$$-\frac{2a}{5x^{5/2}} - \frac{2b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x**2+a)/x**(7/2), x)$

[Out]  $-2*a/(5*x**(5/2)) - 2*b/\text{sqrt}(x)$

**Mathematica [A]** time = 0.00816181, size = 19, normalized size = 1.

$$-\frac{2a}{5x^{5/2}} - \frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b*x^2)/x^(7/2), x]$

[Out]  $(-2*a)/(5*x^(5/2)) - (2*b)/\text{Sqrt}[x]$

**Maple [A]** time = 0.005, size = 14, normalized size = 0.7

$$-\frac{10bx^2 + 2a}{5}x^{-5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((b*x^2+a)/x^(7/2), x)$

[Out]  $-2/5*(5*b*x^2+a)/x^(5/2)$

---

**Maxima [A]** time = 1.34618, size = 18, normalized size = 0.95

$$-\frac{2(5bx^2 + a)}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/x^(7/2), x, algorithm="maxima")`

[Out] `-2/5*(5*b*x^2 + a)/x^(5/2)`

---

**Fricas [A]** time = 0.250717, size = 18, normalized size = 0.95

$$-\frac{2(5bx^2 + a)}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/x^(7/2), x, algorithm="fricas")`

[Out] `-2/5*(5*b*x^2 + a)/x^(5/2)`

---

**Sympy [A]** time = 4.70595, size = 19, normalized size = 1.

$$-\frac{2a}{5x^{\frac{5}{2}}} - \frac{2b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/x**(7/2), x)`

[Out] `-2*a/(5*x**(5/2)) - 2*b/sqrt(x)`

---

**GIAC/XCAS [A]** time = 0.205074, size = 18, normalized size = 0.95

$$-\frac{2(5bx^2 + a)}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)/x^(7/2), x, algorithm="giac")`

[Out] `-2/5*(5*b*x^2 + a)/x^(5/2)`

$$3.272 \quad \int x^{7/2} (a + bx^2)^2 dx$$

**Optimal.** Leaf size=36

$$\frac{2}{9}a^2x^{9/2} + \frac{4}{13}abx^{13/2} + \frac{2}{17}b^2x^{17/2}$$

[Out]  $(2*a^2*x^{(9/2)})/9 + (4*a*b*x^{(13/2)})/13 + (2*b^2*x^{(17/2)})/17$

**Rubi [A]** time = 0.0319839, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2}{9}a^2x^{9/2} + \frac{4}{13}abx^{13/2} + \frac{2}{17}b^2x^{17/2}$$

Antiderivative was successfully verified.

[In] Int[x^(7/2)\*(a + b\*x^2)^2, x]

[Out]  $(2*a^2*x^{(9/2)})/9 + (4*a*b*x^{(13/2)})/13 + (2*b^2*x^{(17/2)})/17$

**Rubi in Sympy [A]** time = 5.1147, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{9}{2}}}{9} + \frac{4abx^{\frac{13}{2}}}{13} + \frac{2b^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(7/2)\*(b\*x\*\*2+a)\*\*2, x)

[Out]  $2*a**2*x**(9/2)/9 + 4*a*b*x**(13/2)/13 + 2*b**2*x**(17/2)/17$

**Mathematica [A]** time = 0.0126668, size = 30, normalized size = 0.83

$$\frac{2x^{9/2} (221a^2 + 306abx^2 + 117b^2x^4)}{1989}$$

Antiderivative was successfully verified.

[In] Integrate[x^(7/2)\*(a + b\*x^2)^2, x]

[Out]  $(2*x^{(9/2)}*(221*a^2 + 306*a*b*x^2 + 117*b^2*x^4))/1989$

**Maple [A]** time = 0.007, size = 27, normalized size = 0.8

$$\frac{234b^2x^4 + 612abx^2 + 442a^2}{1989}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)\*(b\*x^2+a)^2, x)

[Out]  $2/1989*x^{(9/2)}*(117*b^2*x^4+306*a*b*x^2+221*a^2)$

---

**Maxima [A]** time = 1.34263, size = 32, normalized size = 0.89

$$\frac{2}{17} b^2 x^{\frac{17}{2}} + \frac{4}{13} abx^{\frac{13}{2}} + \frac{2}{9} a^2 x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2*x^(7/2),x, algorithm="maxima")`

[Out] `2/17*b^2*x^(17/2) + 4/13*a*b*x^(13/2) + 2/9*a^2*x^(9/2)`

---

**Fricas [A]** time = 0.251448, size = 39, normalized size = 1.08

$$\frac{2}{1989} (117 b^2 x^8 + 306 abx^6 + 221 a^2 x^4) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2*x^(7/2),x, algorithm="fricas")`

[Out] `2/1989*(117*b^2*x^8 + 306*a*b*x^6 + 221*a^2*x^4)*sqrt(x)`

---

**Sympy [A]** time = 37.0921, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{9}{2}}}{9} + \frac{4abx^{\frac{13}{2}}}{13} + \frac{2b^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(7/2)*(b*x**2+a)**2,x)`

[Out] `2*a**2*x**(9/2)/9 + 4*a*b*x**(13/2)/13 + 2*b**2*x**(17/2)/17`

---

**GIAC/XCAS [A]** time = 0.205901, size = 32, normalized size = 0.89

$$\frac{2}{17} b^2 x^{\frac{17}{2}} + \frac{4}{13} abx^{\frac{13}{2}} + \frac{2}{9} a^2 x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2*x^(7/2),x, algorithm="giac")`

[Out] `2/17*b^2*x^(17/2) + 4/13*a*b*x^(13/2) + 2/9*a^2*x^(9/2)`

$$3.273 \quad \int x^{5/2} (a + bx^2)^2 dx$$

**Optimal.** Leaf size=36

$$\frac{2}{7}a^2x^{7/2} + \frac{4}{11}abx^{11/2} + \frac{2}{15}b^2x^{15/2}$$

[Out]  $(2*a^2*x^{(7/2)})/7 + (4*a*b*x^{(11/2)})/11 + (2*b^2*x^{(15/2)})/15$

**Rubi [A]** time = 0.0311977, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2}{7}a^2x^{7/2} + \frac{4}{11}abx^{11/2} + \frac{2}{15}b^2x^{15/2}$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)\*(a + b\*x^2)^2, x]

[Out]  $(2*a^2*x^{(7/2)})/7 + (4*a*b*x^{(11/2)})/11 + (2*b^2*x^{(15/2)})/15$

**Rubi in Sympy [A]** time = 5.20149, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{7}{2}}}{7} + \frac{4abx^{\frac{11}{2}}}{11} + \frac{2b^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(5/2)\*(b\*x\*\*2+a)\*\*2, x)

[Out]  $2*a**2*x**(7/2)/7 + 4*a*b*x**(11/2)/11 + 2*b**2*x**(15/2)/15$

**Mathematica [A]** time = 0.0118701, size = 30, normalized size = 0.83

$$\frac{2x^{7/2} (165a^2 + 210abx^2 + 77b^2x^4)}{1155}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)\*(a + b\*x^2)^2, x]

[Out]  $(2*x^{(7/2)}*(165*a^2 + 210*a*b*x^2 + 77*b^2*x^4))/1155$

**Maple [A]** time = 0.007, size = 27, normalized size = 0.8

$$\frac{154b^2x^4 + 420abx^2 + 330a^2}{1155}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)\*(b\*x^2+a)^2, x)

[Out]  $2/1155*x^{(7/2)}*(77*b^2*x^4+210*a*b*x^2+165*a^2)$



---

**Maxima [A]** time = 1.354, size = 32, normalized size = 0.89

$$\frac{2}{15} b^2 x^{\frac{15}{2}} + \frac{4}{11} abx^{\frac{11}{2}} + \frac{2}{7} a^2 x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2*x^(5/2),x, algorithm="maxima")`

[Out] `2/15*b^2*x^(15/2) + 4/11*a*b*x^(11/2) + 2/7*a^2*x^(7/2)`

---

**Fricas [A]** time = 0.248563, size = 39, normalized size = 1.08

$$\frac{2}{1155} (77 b^2 x^7 + 210 abx^5 + 165 a^2 x^3) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2*x^(5/2),x, algorithm="fricas")`

[Out] `2/1155*(77*b^2*x^7 + 210*a*b*x^5 + 165*a^2*x^3)*sqrt(x)`

---

**Sympy [A]** time = 20.0841, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{7}{2}}}{7} + \frac{4abx^{\frac{11}{2}}}{11} + \frac{2b^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(5/2)*(b*x**2+a)**2,x)`

[Out] `2*a**2*x**(7/2)/7 + 4*a*b*x**(11/2)/11 + 2*b**2*x**(15/2)/15`

---

**GIAC/XCAS [A]** time = 0.208476, size = 32, normalized size = 0.89

$$\frac{2}{15} b^2 x^{\frac{15}{2}} + \frac{4}{11} abx^{\frac{11}{2}} + \frac{2}{7} a^2 x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2*x^(5/2),x, algorithm="giac")`

[Out] `2/15*b^2*x^(15/2) + 4/11*a*b*x^(11/2) + 2/7*a^2*x^(7/2)`

$$3.274 \quad \int x^{3/2} (a + bx^2)^2 dx$$

**Optimal.** Leaf size=36

$$\frac{2}{5}a^2x^{5/2} + \frac{4}{9}abx^{9/2} + \frac{2}{13}b^2x^{13/2}$$

[Out]  $(2*a^2*x^{(5/2)})/5 + (4*a*b*x^{(9/2)})/9 + (2*b^2*x^{(13/2)})/13$

**Rubi [A]** time = 0.0299878, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2}{5}a^2x^{5/2} + \frac{4}{9}abx^{9/2} + \frac{2}{13}b^2x^{13/2}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)\*(a + b\*x^2)^2, x]

[Out]  $(2*a^2*x^{(5/2)})/5 + (4*a*b*x^{(9/2)})/9 + (2*b^2*x^{(13/2)})/13$

**Rubi in Sympy [A]** time = 5.20745, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{5}{2}}}{5} + \frac{4abx^{\frac{9}{2}}}{9} + \frac{2b^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(3/2)\*(b\*x\*\*2+a)\*\*2, x)

[Out]  $2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 2*b**2*x**(13/2)/13$

**Mathematica [A]** time = 0.0119834, size = 30, normalized size = 0.83

$$\frac{2}{585}x^{5/2} (117a^2 + 130abx^2 + 45b^2x^4)$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)\*(a + b\*x^2)^2, x]

[Out]  $(2*x^{(5/2)}*(117*a^2 + 130*a*b*x^2 + 45*b^2*x^4))/585$

**Maple [A]** time = 0.006, size = 27, normalized size = 0.8

$$\frac{90b^2x^4 + 260abx^2 + 234a^2}{585}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*(b\*x^2+a)^2, x)

[Out]  $2/585*x^{(5/2)}*(45*b^2*x^4+130*a*b*x^2+117*a^2)$

---

**Maxima [A]** time = 1.34825, size = 32, normalized size = 0.89

$$\frac{2}{13} b^2 x^{\frac{13}{2}} + \frac{4}{9} abx^{\frac{9}{2}} + \frac{2}{5} a^2 x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2\*x^(3/2),x, algorithm="maxima")

[Out] 2/13\*b^2\*x^(13/2) + 4/9\*a\*b\*x^(9/2) + 2/5\*a^2\*x^(5/2)

---

**Fricas [A]** time = 0.209073, size = 39, normalized size = 1.08

$$\frac{2}{585} (45 b^2 x^6 + 130 abx^4 + 117 a^2 x^2) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2\*x^(3/2),x, algorithm="fricas")

[Out] 2/585\*(45\*b^2\*x^6 + 130\*a\*b\*x^4 + 117\*a^2\*x^2)\*sqrt(x)

---

**Sympy [A]** time = 7.93222, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{5}{2}}}{5} + \frac{4abx^{\frac{9}{2}}}{9} + \frac{2b^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(3/2)\*(b\*x\*\*2+a)\*\*2,x)

[Out] 2\*a\*\*2\*x\*\*(5/2)/5 + 4\*a\*b\*x\*\*(9/2)/9 + 2\*b\*\*2\*x\*\*(13/2)/13

---

**GIAC/XCAS [A]** time = 0.207841, size = 32, normalized size = 0.89

$$\frac{2}{13} b^2 x^{\frac{13}{2}} + \frac{4}{9} abx^{\frac{9}{2}} + \frac{2}{5} a^2 x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2\*x^(3/2),x, algorithm="giac")

[Out] 2/13\*b^2\*x^(13/2) + 4/9\*a\*b\*x^(9/2) + 2/5\*a^2\*x^(5/2)

$$3.275 \quad \int \sqrt{x} (a + bx^2)^2 dx$$

**Optimal.** Leaf size=36

$$\frac{2}{3}a^2x^{3/2} + \frac{4}{7}abx^{7/2} + \frac{2}{11}b^2x^{11/2}$$

[Out]  $(2*a^2*x^{(3/2)})/3 + (4*a*b*x^{(7/2)})/7 + (2*b^2*x^{(11/2)})/11$

**Rubi [A]** time = 0.0294525, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2}{3}a^2x^{3/2} + \frac{4}{7}abx^{7/2} + \frac{2}{11}b^2x^{11/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]\*(a + b\*x^2)^2, x]

[Out]  $(2*a^2*x^{(3/2)})/3 + (4*a*b*x^{(7/2)})/7 + (2*b^2*x^{(11/2)})/11$

**Rubi in Sympy [A]** time = 5.36914, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{3}{2}}}{3} + \frac{4abx^{\frac{7}{2}}}{7} + \frac{2b^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*2\*x\*\*(1/2), x)

[Out]  $2*a**2*x**(3/2)/3 + 4*a*b*x**(7/2)/7 + 2*b**2*x**(11/2)/11$

**Mathematica [A]** time = 0.0115895, size = 30, normalized size = 0.83

$$\frac{2}{231}x^{3/2} (77a^2 + 66abx^2 + 21b^2x^4)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]\*(a + b\*x^2)^2, x]

[Out]  $(2*x^{(3/2)}*(77*a^2 + 66*a*b*x^2 + 21*b^2*x^4))/231$

**Maple [A]** time = 0.007, size = 27, normalized size = 0.8

$$\frac{42b^2x^4 + 132abx^2 + 154a^2}{231}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^2\*x^(1/2), x)

[Out]  $2/231*x^{(3/2)}*(21*b^2*x^4+66*a*b*x^2+77*a^2)$

---

**Maxima [A]** time = 1.35683, size = 32, normalized size = 0.89

$$\frac{2}{11} b^2 x^{\frac{11}{2}} + \frac{4}{7} abx^{\frac{7}{2}} + \frac{2}{3} a^2 x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2\*sqrt(x),x, algorithm="maxima")

[Out] 2/11\*b^2\*x^(11/2) + 4/7\*a\*b\*x^(7/2) + 2/3\*a^2\*x^(3/2)

---

**Fricas [A]** time = 0.209899, size = 36, normalized size = 1.

$$\frac{2}{231} (21 b^2 x^5 + 66 abx^3 + 77 a^2 x) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2\*sqrt(x),x, algorithm="fricas")

[Out] 2/231\*(21\*b^2\*x^5 + 66\*a\*b\*x^3 + 77\*a^2\*x)\*sqrt(x)

---

**Sympy [A]** time = 2.4498, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{3}{2}}}{3} + \frac{4abx^{\frac{7}{2}}}{7} + \frac{2b^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*2\*x\*\*(1/2),x)

[Out] 2\*a\*\*2\*x\*\*(3/2)/3 + 4\*a\*b\*x\*\*(7/2)/7 + 2\*b\*\*2\*x\*\*(11/2)/11

---

**GIAC/XCAS [A]** time = 0.207732, size = 32, normalized size = 0.89

$$\frac{2}{11} b^2 x^{\frac{11}{2}} + \frac{4}{7} abx^{\frac{7}{2}} + \frac{2}{3} a^2 x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2\*sqrt(x),x, algorithm="giac")

[Out] 2/11\*b^2\*x^(11/2) + 4/7\*a\*b\*x^(7/2) + 2/3\*a^2\*x^(3/2)

$$3.276 \quad \int \frac{(a+bx^2)^2}{\sqrt{x}} dx$$

**Optimal.** Leaf size=34

$$2a^2\sqrt{x} + \frac{4}{5}abx^{5/2} + \frac{2}{9}b^2x^{9/2}$$

[Out]  $2*a^2*\text{Sqrt}[x] + (4*a*b*x^{(5/2)})/5 + (2*b^2*x^{(9/2)})/9$

**Rubi [A]** time = 0.0294592, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$2a^2\sqrt{x} + \frac{4}{5}abx^{5/2} + \frac{2}{9}b^2x^{9/2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^2/\text{Sqrt}[x], x]$

[Out]  $2*a^2*\text{Sqrt}[x] + (4*a*b*x^{(5/2)})/5 + (2*b^2*x^{(9/2)})/9$

**Rubi in Sympy [A]** time = 5.17326, size = 32, normalized size = 0.94

$$2a^2\sqrt{x} + \frac{4abx^{\frac{5}{2}}}{5} + \frac{2b^2x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x^2+a)^2/x^{(1/2)}, x)$

[Out]  $2*a^2*\text{sqrt}(x) + 4*a*b*x^{(5/2)}/5 + 2*b^2*x^{(9/2)}/9$

**Mathematica [A]** time = 0.012073, size = 30, normalized size = 0.88

$$\frac{2}{45}\sqrt{x}(45a^2 + 18abx^2 + 5b^2x^4)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b*x^2)^2/\text{Sqrt}[x], x]$

[Out]  $(2*\text{Sqrt}[x]*(45*a^2 + 18*a*b*x^2 + 5*b^2*x^4))/45$

**Maple [A]** time = 0.006, size = 27, normalized size = 0.8

$$\frac{10b^2x^4 + 36abx^2 + 90a^2}{45}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((b*x^2+a)^2/x^{(1/2)}, x)$

[Out]  $2/45*x^{(1/2)}*(5*b^2*x^4+18*a*b*x^2+45*a^2)$

---

**Maxima [A]** time = 1.35061, size = 32, normalized size = 0.94

$$\frac{2}{9} b^2 x^{\frac{9}{2}} + \frac{4}{5} a b x^{\frac{5}{2}} + 2 a^2 \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/sqrt(x), x, algorithm="maxima")

[Out] 2/9\*b^2\*x^(9/2) + 4/5\*a\*b\*x^(5/2) + 2\*a^2\*sqrt(x)

---

**Fricas [A]** time = 0.217136, size = 35, normalized size = 1.03

$$\frac{2}{45} (5 b^2 x^4 + 18 a b x^2 + 45 a^2) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/sqrt(x), x, algorithm="fricas")

[Out] 2/45\*(5\*b^2\*x^4 + 18\*a\*b\*x^2 + 45\*a^2)\*sqrt(x)

---

**Sympy [A]** time = 2.24071, size = 32, normalized size = 0.94

$$2 a^2 \sqrt{x} + \frac{4 a b x^{\frac{5}{2}}}{5} + \frac{2 b^2 x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*2/x\*\*(1/2), x)

[Out] 2\*a\*\*2\*sqrt(x) + 4\*a\*b\*x\*\*(5/2)/5 + 2\*b\*\*2\*x\*\*(9/2)/9

---

**GIAC/XCAS [A]** time = 0.206233, size = 32, normalized size = 0.94

$$\frac{2}{9} b^2 x^{\frac{9}{2}} + \frac{4}{5} a b x^{\frac{5}{2}} + 2 a^2 \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/sqrt(x), x, algorithm="giac")

[Out] 2/9\*b^2\*x^(9/2) + 4/5\*a\*b\*x^(5/2) + 2\*a^2\*sqrt(x)

$$3.277 \quad \int \frac{(a+bx^2)^2}{x^{3/2}} dx$$

**Optimal.** Leaf size=34

$$-\frac{2a^2}{\sqrt{x}} + \frac{4}{3}abx^{3/2} + \frac{2}{7}b^2x^{7/2}$$

[Out]  $(-2*a^2)/\text{Sqrt}[x] + (4*a*b*x^{(3/2)})/3 + (2*b^2*x^{(7/2)})/7$

**Rubi [A]** time = 0.0298941, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$-\frac{2a^2}{\sqrt{x}} + \frac{4}{3}abx^{3/2} + \frac{2}{7}b^2x^{7/2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^2/x^(3/2), x]

[Out]  $(-2*a^2)/\text{Sqrt}[x] + (4*a*b*x^{(3/2)})/3 + (2*b^2*x^{(7/2)})/7$

**Rubi in Sympy [A]** time = 5.18576, size = 32, normalized size = 0.94

$$-\frac{2a^2}{\sqrt{x}} + \frac{4abx^{\frac{3}{2}}}{3} + \frac{2b^2x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*2/x\*\*(3/2), x)

[Out]  $-2*a**2/\text{sqrt}(x) + 4*a*b*x**(3/2)/3 + 2*b**2*x**(7/2)/7$

**Mathematica [A]** time = 0.0138034, size = 30, normalized size = 0.88

$$\frac{2(-21a^2 + 14abx^2 + 3b^2x^4)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^2/x^(3/2), x]

[Out]  $(2*(-21*a^2 + 14*a*b*x^2 + 3*b^2*x^4))/(21*\text{Sqrt}[x])$

**Maple [A]** time = 0.006, size = 27, normalized size = 0.8

$$-\frac{-6b^2x^4 - 28abx^2 + 42a^2}{21\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^2/x^(3/2), x)



[Out]  $-2/21 * (-3 * b^2 * x^4 - 14 * a * b * x^2 + 21 * a^2) / x^{(1/2)}$

**Maxima [A]** time = 1.34622, size = 32, normalized size = 0.94

$$\frac{2}{7} b^2 x^{\frac{7}{2}} + \frac{4}{3} a b x^{\frac{3}{2}} - \frac{2 a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2/x^(3/2),x, algorithm="maxima")`

[Out]  $2/7 * b^2 * x^{(7/2)} + 4/3 * a * b * x^{(3/2)} - 2 * a^2 / \text{sqrt}(x)$

**Fricas [A]** time = 0.221594, size = 35, normalized size = 1.03

$$\frac{2 (3 b^2 x^4 + 14 a b x^2 - 21 a^2)}{21 \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2/x^(3/2),x, algorithm="fricas")`

[Out]  $2/21 * (3 * b^2 * x^4 + 14 * a * b * x^2 - 21 * a^2) / \text{sqrt}(x)$

**Sympy [A]** time = 3.50355, size = 32, normalized size = 0.94

$$-\frac{2a^2}{\sqrt{x}} + \frac{4abx^{\frac{3}{2}}}{3} + \frac{2b^2x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**2/x**(3/2),x)`

[Out]  $-2 * a^2 / \text{sqrt}(x) + 4 * a * b * x^{(3/2)} / 3 + 2 * b^2 * x^{(7/2)} / 7$

**GIAC/XCAS [A]** time = 0.205848, size = 32, normalized size = 0.94

$$\frac{2}{7} b^2 x^{\frac{7}{2}} + \frac{4}{3} a b x^{\frac{3}{2}} - \frac{2 a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2/x^(3/2),x, algorithm="giac")`

[Out]  $2/7 * b^2 * x^{(7/2)} + 4/3 * a * b * x^{(3/2)} - 2 * a^2 / \text{sqrt}(x)$

$$3.278 \quad \int \frac{(a+bx^2)^2}{x^{5/2}} dx$$

**Optimal.** Leaf size=34

$$-\frac{2a^2}{3x^{3/2}} + 4ab\sqrt{x} + \frac{2}{5}b^2x^{5/2}$$

[Out]  $(-2*a^2)/(3*x^(3/2)) + 4*a*b*\text{Sqrt}[x] + (2*b^2*x^(5/2))/5$

**Rubi [A]** time = 0.0310483, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$-\frac{2a^2}{3x^{3/2}} + 4ab\sqrt{x} + \frac{2}{5}b^2x^{5/2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^2/x^(5/2), x]$

[Out]  $(-2*a^2)/(3*x^(3/2)) + 4*a*b*\text{Sqrt}[x] + (2*b^2*x^(5/2))/5$

**Rubi in Sympy [A]** time = 5.14268, size = 32, normalized size = 0.94

$$-\frac{2a^2}{3x^{\frac{3}{2}}} + 4ab\sqrt{x} + \frac{2b^2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x^2+a)^2/x^(5/2), x)$

[Out]  $-2*a^2/(3*x^(3/2)) + 4*a*b*\text{sqrt}(x) + 2*b^2*x^(5/2)/5$

**Mathematica [A]** time = 0.0131427, size = 30, normalized size = 0.88

$$\frac{2(-5a^2 + 30abx^2 + 3b^2x^4)}{15x^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b*x^2)^2/x^(5/2), x]$

[Out]  $(2*(-5*a^2 + 30*a*b*x^2 + 3*b^2*x^4))/(15*x^(3/2))$

**Maple [A]** time = 0.007, size = 27, normalized size = 0.8

$$-\frac{-6b^2x^4 - 60abx^2 + 10a^2}{15}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((b*x^2+a)^2/x^(5/2), x)$

[Out]  $-2/15*(-3*b^2*x^4-30*a*b*x^2+5*a^2)/x^(3/2)$

---

**Maxima [A]** time = 1.35009, size = 32, normalized size = 0.94

$$\frac{2}{5} b^2 x^{\frac{5}{2}} + 4 ab \sqrt{x} - \frac{2 a^2}{3 x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^(5/2), x, algorithm="maxima")

[Out] 2/5\*b^2\*x^(5/2) + 4\*a\*b\*sqrt(x) - 2/3\*a^2/x^(3/2)

---

**Fricas [A]** time = 0.214316, size = 35, normalized size = 1.03

$$\frac{2 (3 b^2 x^4 + 30 a b x^2 - 5 a^2)}{15 x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^(5/2), x, algorithm="fricas")

[Out] 2/15\*(3\*b^2\*x^4 + 30\*a\*b\*x^2 - 5\*a^2)/x^(3/2)

---

**Sympy [A]** time = 4.09432, size = 32, normalized size = 0.94

$$-\frac{2a^2}{3x^{\frac{3}{2}}} + 4ab\sqrt{x} + \frac{2b^2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*2/x\*\*(5/2), x)

[Out] -2\*a\*\*2/(3\*x\*\*(3/2)) + 4\*a\*b\*sqrt(x) + 2\*b\*\*2\*x\*\*(5/2)/5

---

**GIAC/XCAS [A]** time = 0.206028, size = 32, normalized size = 0.94

$$\frac{2}{5} b^2 x^{\frac{5}{2}} + 4 ab \sqrt{x} - \frac{2 a^2}{3 x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^2/x^(5/2), x, algorithm="giac")

[Out] 2/5\*b^2\*x^(5/2) + 4\*a\*b\*sqrt(x) - 2/3\*a^2/x^(3/2)

$$3.279 \quad \int \frac{(a+bx^2)^2}{x^{7/2}} dx$$

**Optimal.** Leaf size=34

$$-\frac{2a^2}{5x^{5/2}} - \frac{4ab}{\sqrt{x}} + \frac{2}{3}b^2x^{3/2}$$

[Out]  $(-2*a^2)/(5*x^{(5/2)}) - (4*a*b)/\text{Sqrt}[x] + (2*b^2*x^{(3/2)})/3$

**Rubi [A]** time = 0.0310524, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$-\frac{2a^2}{5x^{5/2}} - \frac{4ab}{\sqrt{x}} + \frac{2}{3}b^2x^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^2/x^(7/2), x]

[Out]  $(-2*a^2)/(5*x^{(5/2)}) - (4*a*b)/\text{Sqrt}[x] + (2*b^2*x^{(3/2)})/3$

**Rubi in Sympy [A]** time = 5.17447, size = 32, normalized size = 0.94

$$-\frac{2a^2}{5x^{5/2}} - \frac{4ab}{\sqrt{x}} + \frac{2b^2x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*2/x\*\*(7/2), x)

[Out]  $-2*a**2/(5*x**(5/2)) - 4*a*b/\text{sqrt}(x) + 2*b**2*x**(3/2)/3$

**Mathematica [A]** time = 0.0140969, size = 30, normalized size = 0.88

$$\frac{2(-3a^2 - 30abx^2 + 5b^2x^4)}{15x^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^2/x^(7/2), x]

[Out]  $(2*(-3*a^2 - 30*a*b*x^2 + 5*b^2*x^4))/(15*x^{(5/2)})$

**Maple [A]** time = 0.007, size = 27, normalized size = 0.8

$$-\frac{-10b^2x^4 + 60abx^2 + 6a^2}{15}x^{-5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^2/x^(7/2), x)

[Out]  $-2/15 * (-5 * b^2 * x^4 + 30 * a * b * x^2 + 3 * a^2) / x^{5/2}$

**Maxima [A]** time = 1.35027, size = 34, normalized size = 1.

$$\frac{2}{3} b^2 x^{\frac{3}{2}} - \frac{2(10 abx^2 + a^2)}{5 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2/x^(7/2),x, algorithm="maxima")`

[Out]  $2/3 * b^2 * x^{3/2} - 2/5 * (10 * a * b * x^2 + a^2) / x^{5/2}$

**Fricas [A]** time = 0.211569, size = 35, normalized size = 1.03

$$\frac{2(5 b^2 x^4 - 30 abx^2 - 3 a^2)}{15 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2/x^(7/2),x, algorithm="fricas")`

[Out]  $2/15 * (5 * b^2 * x^4 - 30 * a * b * x^2 - 3 * a^2) / x^{5/2}$

**Sympy [A]** time = 6.11686, size = 32, normalized size = 0.94

$$-\frac{2a^2}{5x^{\frac{5}{2}}} - \frac{4ab}{\sqrt{x}} + \frac{2b^2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**2/x**(7/2),x)`

[Out]  $-2 * a^{**2} / (5 * x^{(5/2)}) - 4 * a * b / \text{sqrt}(x) + 2 * b^{**2} * x^{(3/2)} / 3$

**GIAC/XCAS [A]** time = 0.208889, size = 34, normalized size = 1.

$$\frac{2}{3} b^2 x^{\frac{3}{2}} - \frac{2(10 abx^2 + a^2)}{5 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2/x^(7/2),x, algorithm="giac")`

[Out]  $2/3 * b^2 * x^{3/2} - 2/5 * (10 * a * b * x^2 + a^2) / x^{5/2}$

$$3.280 \quad \int x^{7/2} (a + bx^2)^3 dx$$

**Optimal.** Leaf size=51

$$\frac{2}{9}a^3x^{9/2} + \frac{6}{13}a^2bx^{13/2} + \frac{6}{17}ab^2x^{17/2} + \frac{2}{21}b^3x^{21/2}$$

[Out]  $(2*a^3*x^{(9/2)})/9 + (6*a^2*b*x^{(13/2)})/13 + (6*a*b^2*x^{(17/2)})/17 + (2*b^3*x^{(21/2)})/21$

**Rubi [A]** time = 0.0425014, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2}{9}a^3x^{9/2} + \frac{6}{13}a^2bx^{13/2} + \frac{6}{17}ab^2x^{17/2} + \frac{2}{21}b^3x^{21/2}$$

Antiderivative was successfully verified.

[In] Int[x^(7/2)\*(a + b\*x^2)^3, x]

[Out]  $(2*a^3*x^{(9/2)})/9 + (6*a^2*b*x^{(13/2)})/13 + (6*a*b^2*x^{(17/2)})/17 + (2*b^3*x^{(21/2)})/21$

**Rubi in Sympy [A]** time = 6.35518, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{9}{2}}}{9} + \frac{6a^2bx^{\frac{13}{2}}}{13} + \frac{6ab^2x^{\frac{17}{2}}}{17} + \frac{2b^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(7/2)\*(b\*x\*\*2+a)\*\*3, x)

[Out]  $2*a**3*x**(9/2)/9 + 6*a**2*b*x**(13/2)/13 + 6*a*b**2*x**(17/2)/17 + 2*b**3*x**(21/2)/21$

**Mathematica [A]** time = 0.0143093, size = 41, normalized size = 0.8

$$\frac{2x^{9/2} (1547a^3 + 3213a^2bx^2 + 2457ab^2x^4 + 663b^3x^6)}{13923}$$

Antiderivative was successfully verified.

[In] Integrate[x^(7/2)\*(a + b\*x^2)^3, x]

[Out]  $(2*x^{(9/2)}*(1547*a^3 + 3213*a^2*b*x^2 + 2457*a*b^2*x^4 + 663*b^3*x^6))/13923$

**Maple [A]** time = 0.008, size = 38, normalized size = 0.8

$$\frac{1326b^3x^6 + 4914ab^2x^4 + 6426a^2bx^2 + 3094a^3}{13923}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)\*(b\*x^2+a)^3, x)

[Out]  $2/13923 * x^{(9/2)} * (663 * b^3 * x^6 + 2457 * a * b^2 * x^4 + 3213 * a^2 * b * x^2 + 1547 * a^3)$

**Maxima [A]** time = 1.34791, size = 47, normalized size = 0.92

$$\frac{2}{21} b^3 x^{\frac{21}{2}} + \frac{6}{17} ab^2 x^{\frac{17}{2}} + \frac{6}{13} a^2 b x^{\frac{13}{2}} + \frac{2}{9} a^3 x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3*x^(7/2),x, algorithm="maxima")`

[Out]  $2/21 * b^3 * x^{(21/2)} + 6/17 * a * b^2 * x^{(17/2)} + 6/13 * a^2 * b * x^{(13/2)} + 2/9 * a^3 * x^{(9/2)}$

**Fricas [A]** time = 0.211878, size = 54, normalized size = 1.06

$$\frac{2}{13923} (663 b^3 x^{10} + 2457 ab^2 x^8 + 3213 a^2 b x^6 + 1547 a^3 x^4) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3*x^(7/2),x, algorithm="fricas")`

[Out]  $2/13923 * (663 * b^3 * x^{10} + 2457 * a * b^2 * x^8 + 3213 * a^2 * b * x^6 + 1547 * a^3 * x^4) * \text{sqrt}(x)$

**Sympy [A]** time = 70.7934, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{9}{2}}}{9} + \frac{6a^2bx^{\frac{13}{2}}}{13} + \frac{6ab^2x^{\frac{17}{2}}}{17} + \frac{2b^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(7/2)*(b*x**2+a)**3,x)`

[Out]  $2 * a ** 3 * x ** (9/2) / 9 + 6 * a ** 2 * b * x ** (13/2) / 13 + 6 * a * b ** 2 * x ** (17/2) / 17 + 2 * b ** 3 * x ** (21/2) / 21$

**GIAC/XCAS [A]** time = 0.206874, size = 47, normalized size = 0.92

$$\frac{2}{21} b^3 x^{\frac{21}{2}} + \frac{6}{17} ab^2 x^{\frac{17}{2}} + \frac{6}{13} a^2 b x^{\frac{13}{2}} + \frac{2}{9} a^3 x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3*x^(7/2),x, algorithm="giac")`

[Out]  $2/21 * b^3 * x^{(21/2)} + 6/17 * a * b^2 * x^{(17/2)} + 6/13 * a^2 * b * x^{(13/2)} + 2/9 * a^3 * x^{(9/2)}$

$$3.281 \quad \int x^{5/2} (a + bx^2)^3 dx$$

**Optimal.** Leaf size=51

$$\frac{2}{7}a^3x^{7/2} + \frac{6}{11}a^2bx^{11/2} + \frac{2}{5}ab^2x^{15/2} + \frac{2}{19}b^3x^{19/2}$$

[Out]  $(2*a^3*x^{(7/2)})/7 + (6*a^2*b*x^{(11/2)})/11 + (2*a*b^2*x^{(15/2)})/5 + (2*b^3*x^{(19/2)})/19$

**Rubi [A]** time = 0.040649, antiderivative size = 51, normalized size of antiderivative = 1., number of rules used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2}{7}a^3x^{7/2} + \frac{6}{11}a^2bx^{11/2} + \frac{2}{5}ab^2x^{15/2} + \frac{2}{19}b^3x^{19/2}$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)\*(a + b\*x^2)^3, x]

[Out]  $(2*a^3*x^{(7/2)})/7 + (6*a^2*b*x^{(11/2)})/11 + (2*a*b^2*x^{(15/2)})/5 + (2*b^3*x^{(19/2)})/19$

**Rubi in Sympy [A]** time = 6.26203, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{7}{2}}}{7} + \frac{6a^2bx^{\frac{11}{2}}}{11} + \frac{2ab^2x^{\frac{15}{2}}}{5} + \frac{2b^3x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(5/2)\*(b\*x\*\*2+a)\*\*3, x)

[Out]  $2*a**3*x**(7/2)/7 + 6*a**2*b*x**(11/2)/11 + 2*a*b**2*x**(15/2)/5 + 2*b**3*x**(19/2)/19$

**Mathematica [A]** time = 0.013639, size = 41, normalized size = 0.8

$$\frac{2x^{7/2} (1045a^3 + 1995a^2bx^2 + 1463ab^2x^4 + 385b^3x^6)}{7315}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)\*(a + b\*x^2)^3, x]

[Out]  $(2*x^{(7/2)}*(1045*a^3 + 1995*a^2*b*x^2 + 1463*a*b^2*x^4 + 385*b^3*x^6))/7315$

**Maple [A]** time = 0.007, size = 38, normalized size = 0.8

$$\frac{770b^3x^6 + 2926ab^2x^4 + 3990a^2bx^2 + 2090a^3}{7315}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)\*(b\*x^2+a)^3, x)



[Out]  $2/7315 * x^{(7/2)} * (385 * b^3 * x^6 + 1463 * a * b^2 * x^4 + 1995 * a^2 * b * x^2 + 1045 * a^3)$

**Maxima [A]** time = 1.34542, size = 47, normalized size = 0.92

$$\frac{2}{19} b^3 x^{\frac{19}{2}} + \frac{2}{5} ab^2 x^{\frac{15}{2}} + \frac{6}{11} a^2 b x^{\frac{11}{2}} + \frac{2}{7} a^3 x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3*x^(5/2),x, algorithm="maxima")`

[Out]  $2/19 * b^3 * x^{(19/2)} + 2/5 * a * b^2 * x^{(15/2)} + 6/11 * a^2 * b * x^{(11/2)} + 2/7 * a^3 * x^{(7/2)}$

**Fricas [A]** time = 0.213923, size = 54, normalized size = 1.06

$$\frac{2}{7315} (385 b^3 x^9 + 1463 a b^2 x^7 + 1995 a^2 b x^5 + 1045 a^3 x^3) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3*x^(5/2),x, algorithm="fricas")`

[Out]  $2/7315 * (385 * b^3 * x^9 + 1463 * a * b^2 * x^7 + 1995 * a^2 * b * x^5 + 1045 * a^3 * x^3) * \text{sqrt}(x)$

**Sympy [A]** time = 37.3172, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{7}{2}}}{7} + \frac{6a^2bx^{\frac{11}{2}}}{11} + \frac{2ab^2x^{\frac{15}{2}}}{5} + \frac{2b^3x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(5/2)*(b*x**2+a)**3,x)`

[Out]  $2 * a ** 3 * x ** (7/2) / 7 + 6 * a ** 2 * b * x ** (11/2) / 11 + 2 * a * b ** 2 * x ** (15/2) / 5 + 2 * b ** 3 * x ** (19/2) / 19$

**GIAC/XCAS [A]** time = 0.207157, size = 47, normalized size = 0.92

$$\frac{2}{19} b^3 x^{\frac{19}{2}} + \frac{2}{5} ab^2 x^{\frac{15}{2}} + \frac{6}{11} a^2 b x^{\frac{11}{2}} + \frac{2}{7} a^3 x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3*x^(5/2),x, algorithm="giac")`

[Out]  $2/19 * b^3 * x^{(19/2)} + 2/5 * a * b^2 * x^{(15/2)} + 6/11 * a^2 * b * x^{(11/2)} + 2/7 * a^3 * x^{(7/2)}$

$$3.282 \quad \int x^{3/2} (a + bx^2)^3 dx$$

**Optimal.** Leaf size=51

$$\frac{2}{5}a^3x^{5/2} + \frac{2}{3}a^2bx^{9/2} + \frac{6}{13}ab^2x^{13/2} + \frac{2}{17}b^3x^{17/2}$$

[Out]  $(2*a^3*x^{(5/2)})/5 + (2*a^2*b*x^{(9/2)})/3 + (6*a*b^2*x^{(13/2)})/13 + (2*b^3*x^{(17/2)})/17$

**Rubi [A]** time = 0.0431283, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2}{5}a^3x^{5/2} + \frac{2}{3}a^2bx^{9/2} + \frac{6}{13}ab^2x^{13/2} + \frac{2}{17}b^3x^{17/2}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)\*(a + b\*x^2)^3, x]

[Out]  $(2*a^3*x^{(5/2)})/5 + (2*a^2*b*x^{(9/2)})/3 + (6*a*b^2*x^{(13/2)})/13 + (2*b^3*x^{(17/2)})/17$

**Rubi in Sympy [A]** time = 6.46993, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{5}{2}}}{5} + \frac{2a^2bx^{\frac{9}{2}}}{3} + \frac{6ab^2x^{\frac{13}{2}}}{13} + \frac{2b^3x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(3/2)\*(b\*x\*\*2+a)\*\*3, x)

[Out]  $2*a**3*x**(5/2)/5 + 2*a**2*b*x**(9/2)/3 + 6*a*b**2*x**(13/2)/13 + 2*b**3*x**(17/2)/17$

**Mathematica [A]** time = 0.0138322, size = 41, normalized size = 0.8

$$\frac{2x^{5/2} (663a^3 + 1105a^2bx^2 + 765ab^2x^4 + 195b^3x^6)}{3315}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)\*(a + b\*x^2)^3, x]

[Out]  $(2*x^{(5/2)}*(663*a^3 + 1105*a^2*b*x^2 + 765*a*b^2*x^4 + 195*b^3*x^6))/3315$

**Maple [A]** time = 0.007, size = 38, normalized size = 0.8

$$\frac{390b^3x^6 + 1530ab^2x^4 + 2210a^2bx^2 + 1326a^3}{3315}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*(b\*x^2+a)^3, x)

[Out]  $2/3315 * x^{(5/2)} * (195 * b^3 * x^6 + 765 * a * b^2 * x^4 + 1105 * a^2 * b * x^2 + 663 * a^3)$

**Maxima [A]** time = 1.37484, size = 47, normalized size = 0.92

$$\frac{2}{17} b^3 x^{\frac{17}{2}} + \frac{6}{13} a b^2 x^{\frac{13}{2}} + \frac{2}{3} a^2 b x^{\frac{9}{2}} + \frac{2}{5} a^3 x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3*x^(3/2),x, algorithm="maxima")`

[Out]  $2/17 * b^3 * x^{(17/2)} + 6/13 * a * b^2 * x^{(13/2)} + 2/3 * a^2 * b * x^{(9/2)} + 2/5 * a^3 * x^{(5/2)}$

**Fricas [A]** time = 0.210796, size = 54, normalized size = 1.06

$$\frac{2}{3315} (195 b^3 x^8 + 765 a b^2 x^6 + 1105 a^2 b x^4 + 663 a^3 x^2) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3*x^(3/2),x, algorithm="fricas")`

[Out]  $2/3315 * (195 * b^3 * x^8 + 765 * a * b^2 * x^6 + 1105 * a^2 * b * x^4 + 663 * a^3 * x^2) * \text{sqrt}(x)$

**Sympy [A]** time = 20.2958, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{5}{2}}}{5} + \frac{2a^2bx^{\frac{9}{2}}}{3} + \frac{6ab^2x^{\frac{13}{2}}}{13} + \frac{2b^3x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(3/2)*(b*x**2+a)**3,x)`

[Out]  $2 * a^3 * x^{(5/2)} / 5 + 2 * a^2 * b * x^{(9/2)} / 3 + 6 * a * b^2 * x^{(13/2)} / 13 + 2 * b^3 * x^{(17/2)} / 17$

**GIAC/XCAS [A]** time = 0.208302, size = 47, normalized size = 0.92

$$\frac{2}{17} b^3 x^{\frac{17}{2}} + \frac{6}{13} a b^2 x^{\frac{13}{2}} + \frac{2}{3} a^2 b x^{\frac{9}{2}} + \frac{2}{5} a^3 x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3*x^(3/2),x, algorithm="giac")`

[Out]  $2/17 * b^3 * x^{(17/2)} + 6/13 * a * b^2 * x^{(13/2)} + 2/3 * a^2 * b * x^{(9/2)} + 2/5 * a^3 * x^{(5/2)}$

### 3.283 $\int \sqrt{x} (a + bx^2)^3 dx$

**Optimal.** Leaf size=51

$$\frac{2}{3}a^3x^{3/2} + \frac{6}{7}a^2bx^{7/2} + \frac{6}{11}ab^2x^{11/2} + \frac{2}{15}b^3x^{15/2}$$

[Out]  $(2*a^3*x^{(3/2)})/3 + (6*a^2*b*x^{(7/2)})/7 + (6*a*b^2*x^{(11/2)})/11 + (2*b^3*x^{(15/2)})/15$

**Rubi [A]** time = 0.041889, antiderivative size = 51, normalized size of antiderivative = 1., number of rules used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2}{3}a^3x^{3/2} + \frac{6}{7}a^2bx^{7/2} + \frac{6}{11}ab^2x^{11/2} + \frac{2}{15}b^3x^{15/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]\*(a + b\*x^2)^3,x]

[Out]  $(2*a^3*x^{(3/2)})/3 + (6*a^2*b*x^{(7/2)})/7 + (6*a*b^2*x^{(11/2)})/11 + (2*b^3*x^{(15/2)})/15$

**Rubi in Sympy [A]** time = 6.38475, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{3}{2}}}{3} + \frac{6a^2bx^{\frac{7}{2}}}{7} + \frac{6ab^2x^{\frac{11}{2}}}{11} + \frac{2b^3x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*3\*x\*\*(1/2),x)

[Out]  $2*a**3*x**(3/2)/3 + 6*a**2*b*x**(7/2)/7 + 6*a*b**2*x**(11/2)/11 + 2*b**3*x**(15/2)/15$

**Mathematica [A]** time = 0.0132643, size = 41, normalized size = 0.8

$$\frac{2x^{3/2} (385a^3 + 495a^2bx^2 + 315ab^2x^4 + 77b^3x^6)}{1155}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]\*(a + b\*x^2)^3,x]

[Out]  $(2*x^{(3/2)}*(385*a^3 + 495*a^2*b*x^2 + 315*a*b^2*x^4 + 77*b^3*x^6))/1155$

**Maple [A]** time = 0.008, size = 38, normalized size = 0.8

$$\frac{154b^3x^6 + 630ab^2x^4 + 990a^2bx^2 + 770a^3}{1155}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^3\*x^(1/2),x)

[Out]  $2/1155 * x^{(3/2)} * (77 * b^3 * x^6 + 315 * a * b^2 * x^4 + 495 * a^2 * b * x^2 + 385 * a^3)$

**Maxima [A]** time = 1.33375, size = 47, normalized size = 0.92

$$\frac{2}{15} b^3 x^{\frac{15}{2}} + \frac{6}{11} a b^2 x^{\frac{11}{2}} + \frac{6}{7} a^2 b x^{\frac{7}{2}} + \frac{2}{3} a^3 x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3*sqrt(x),x, algorithm="maxima")`

[Out]  $2/15 * b^3 * x^{(15/2)} + 6/11 * a * b^2 * x^{(11/2)} + 6/7 * a^2 * b * x^{(7/2)} + 2/3 * a^3 * x^{(3/2)}$

**Fricas [A]** time = 0.215428, size = 51, normalized size = 1.

$$\frac{2}{1155} (77 b^3 x^7 + 315 a b^2 x^5 + 495 a^2 b x^3 + 385 a^3 x) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3*sqrt(x),x, algorithm="fricas")`

[Out]  $2/1155 * (77 * b^3 * x^7 + 315 * a * b^2 * x^5 + 495 * a^2 * b * x^3 + 385 * a^3 * x) * \text{sqrt}(x)$

**Sympy [A]** time = 4.63105, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{3}{2}}}{3} + \frac{6a^2bx^{\frac{7}{2}}}{7} + \frac{6ab^2x^{\frac{11}{2}}}{11} + \frac{2b^3x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**3*x**(1/2),x)`

[Out]  $2 * a^3 * x^{(3/2)} / 3 + 6 * a^2 * b * x^{(7/2)} / 7 + 6 * a * b^2 * x^{(11/2)} / 11 + 2 * b^3 * x^{(15/2)} / 15$

**GIAC/XCAS [A]** time = 0.223662, size = 47, normalized size = 0.92

$$\frac{2}{15} b^3 x^{\frac{15}{2}} + \frac{6}{11} a b^2 x^{\frac{11}{2}} + \frac{6}{7} a^2 b x^{\frac{7}{2}} + \frac{2}{3} a^3 x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3*sqrt(x),x, algorithm="giac")`

[Out]  $2/15 * b^3 * x^{(15/2)} + 6/11 * a * b^2 * x^{(11/2)} + 6/7 * a^2 * b * x^{(7/2)} + 2/3 * a^3 * x^{(3/2)}$

$$3.284 \quad \int \frac{(a+bx^2)^3}{\sqrt{x}} dx$$

**Optimal.** Leaf size=49

$$2a^3\sqrt{x} + \frac{6}{5}a^2bx^{5/2} + \frac{2}{3}ab^2x^{9/2} + \frac{2}{13}b^3x^{13/2}$$

[Out]  $2*a^3*\text{Sqrt}[x] + (6*a^2*b*x^{(5/2)})/5 + (2*a*b^2*x^{(9/2)})/3 + (2*b^3*x^{(13/2)})/13$

**Rubi [A]** time = 0.0400382, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$2a^3\sqrt{x} + \frac{6}{5}a^2bx^{5/2} + \frac{2}{3}ab^2x^{9/2} + \frac{2}{13}b^3x^{13/2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^3/Sqrt[x], x]

[Out]  $2*a^3*\text{Sqrt}[x] + (6*a^2*b*x^{(5/2)})/5 + (2*a*b^2*x^{(9/2)})/3 + (2*b^3*x^{(13/2)})/13$

**Rubi in Sympy [A]** time = 6.32968, size = 48, normalized size = 0.98

$$2a^3\sqrt{x} + \frac{6a^2bx^{\frac{5}{2}}}{5} + \frac{2ab^2x^{\frac{9}{2}}}{3} + \frac{2b^3x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*3/x\*\*(1/2), x)

[Out]  $2*a**3*\text{sqrt}(x) + 6*a**2*b*x**(5/2)/5 + 2*a*b**2*x**(9/2)/3 + 2*b**3*x**(13/2)/13$

**Mathematica [A]** time = 0.0125801, size = 41, normalized size = 0.84

$$\frac{2}{195}\sqrt{x}(195a^3 + 117a^2bx^2 + 65ab^2x^4 + 15b^3x^6)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^3/Sqrt[x], x]

[Out]  $(2*\text{Sqrt}[x]*(195*a^3 + 117*a^2*b*x^2 + 65*a*b^2*x^4 + 15*b^3*x^6))/195$

**Maple [A]** time = 0.006, size = 38, normalized size = 0.8

$$\frac{30b^3x^6 + 130ab^2x^4 + 234a^2bx^2 + 390a^3}{195}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^3/x^(1/2), x)

[Out]  $2/195 * x^{(1/2)} * (15 * b^3 * x^6 + 65 * a * b^2 * x^4 + 117 * a^2 * b * x^2 + 195 * a^3)$

**Maxima [A]** time = 1.34807, size = 47, normalized size = 0.96

$$\frac{2}{13} b^3 x^{\frac{13}{2}} + \frac{2}{3} a b^2 x^{\frac{9}{2}} + \frac{6}{5} a^2 b x^{\frac{5}{2}} + 2 a^3 \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/sqrt(x),x, algorithm="maxima")`

[Out]  $2/13 * b^3 * x^{(13/2)} + 2/3 * a * b^2 * x^{(9/2)} + 6/5 * a^2 * b * x^{(5/2)} + 2 * a^3 * \sqrt{x}$

**Fricas [A]** time = 0.209291, size = 50, normalized size = 1.02

$$\frac{2}{195} (15 b^3 x^6 + 65 a b^2 x^4 + 117 a^2 b x^2 + 195 a^3) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/sqrt(x),x, algorithm="fricas")`

[Out]  $2/195 * (15 * b^3 * x^6 + 65 * a * b^2 * x^4 + 117 * a^2 * b * x^2 + 195 * a^3) * \sqrt{x}$

**Sympy [A]** time = 6.44597, size = 48, normalized size = 0.98

$$2a^3\sqrt{x} + \frac{6a^2bx^{\frac{5}{2}}}{5} + \frac{2ab^2x^{\frac{9}{2}}}{3} + \frac{2b^3x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**3/x**(1/2),x)`

[Out]  $2 * a^3 * \sqrt{x} + 6 * a^2 * b * x^{(5/2)} / 5 + 2 * a * b^2 * x^{(9/2)} / 3 + 2 * b^3 * x^{(13/2)} / 13$

**GIAC/XCAS [A]** time = 0.217871, size = 47, normalized size = 0.96

$$\frac{2}{13} b^3 x^{\frac{13}{2}} + \frac{2}{3} a b^2 x^{\frac{9}{2}} + \frac{6}{5} a^2 b x^{\frac{5}{2}} + 2 a^3 \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/sqrt(x),x, algorithm="giac")`

[Out]  $2/13 * b^3 * x^{(13/2)} + 2/3 * a * b^2 * x^{(9/2)} + 6/5 * a^2 * b * x^{(5/2)} + 2 * a^3 * \sqrt{x}$

$$3.285 \quad \int \frac{(a+bx^2)^3}{x^{3/2}} dx$$

**Optimal.** Leaf size=47

$$-\frac{2a^3}{\sqrt{x}} + 2a^2bx^{3/2} + \frac{6}{7}ab^2x^{7/2} + \frac{2}{11}b^3x^{11/2}$$

[Out]  $(-2*a^3)/\text{Sqrt}[x] + 2*a^2*b*x^{(3/2)} + (6*a*b^2*x^{(7/2)})/7 + (2*b^3*x^{(11/2)})/11$

**Rubi [A]** time = 0.0451163, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$-\frac{2a^3}{\sqrt{x}} + 2a^2bx^{3/2} + \frac{6}{7}ab^2x^{7/2} + \frac{2}{11}b^3x^{11/2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^3/x^(3/2), x]

[Out]  $(-2*a^3)/\text{Sqrt}[x] + 2*a^2*b*x^{(3/2)} + (6*a*b^2*x^{(7/2)})/7 + (2*b^3*x^{(11/2)})/11$

**Rubi in Sympy [A]** time = 6.34066, size = 46, normalized size = 0.98

$$-\frac{2a^3}{\sqrt{x}} + 2a^2bx^{\frac{3}{2}} + \frac{6ab^2x^{\frac{7}{2}}}{7} + \frac{2b^3x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*3/x\*\*(3/2), x)

[Out]  $-2*a**3/\text{sqrt}(x) + 2*a**2*b*x**(3/2) + 6*a*b**2*x**(7/2)/7 + 2*b**3*x**(11/2)/11$

**Mathematica [A]** time = 0.0148207, size = 41, normalized size = 0.87

$$\frac{2(-77a^3 + 77a^2bx^2 + 33ab^2x^4 + 7b^3x^6)}{77\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^3/x^(3/2), x]

[Out]  $(2*(-77*a^3 + 77*a^2*b*x^2 + 33*a*b^2*x^4 + 7*b^3*x^6))/(77*\text{Sqrt}[x])$

**Maple [A]** time = 0.007, size = 38, normalized size = 0.8

$$-\frac{-14b^3x^6 - 66ab^2x^4 - 154a^2bx^2 + 154a^3}{77} \frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((b*x^2+a)^3/x^(3/2),x)`

[Out]  $-2/77*(-7*b^3*x^6-33*a*b^2*x^4-77*a^2*b*x^2+77*a^3)/x^{(1/2)}$

**Maxima [A]** time = 1.3336, size = 47, normalized size = 1.

$$\frac{2}{11}b^3x^{\frac{11}{2}} + \frac{6}{7}ab^2x^{\frac{7}{2}} + 2a^2bx^{\frac{3}{2}} - \frac{2a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^(3/2),x, algorithm="maxima")`

[Out]  $2/11*b^3*x^{(11/2)} + 6/7*a*b^2*x^{(7/2)} + 2*a^2*b*x^{(3/2)} - 2*a^3/sqrt(x)$

**Fricas [A]** time = 0.213242, size = 50, normalized size = 1.06

$$\frac{2(7b^3x^6 + 33ab^2x^4 + 77a^2bx^2 - 77a^3)}{77\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^(3/2),x, algorithm="fricas")`

[Out]  $2/77*(7*b^3*x^6 + 33*a*b^2*x^4 + 77*a^2*b*x^2 - 77*a^3)/sqrt(x)$

**Sympy [A]** time = 8.35432, size = 46, normalized size = 0.98

$$-\frac{2a^3}{\sqrt{x}} + 2a^2bx^{\frac{3}{2}} + \frac{6ab^2x^{\frac{7}{2}}}{7} + \frac{2b^3x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**3/x**(3/2),x)`

[Out]  $-2*a^3/sqrt(x) + 2*a^2*b*x^{(3/2)} + 6*a*b^2*x^{(7/2)}/7 + 2*b^3*x^{(11/2)}/11$

**GIAC/XCAS [A]** time = 0.211612, size = 47, normalized size = 1.

$$\frac{2}{11}b^3x^{\frac{11}{2}} + \frac{6}{7}ab^2x^{\frac{7}{2}} + 2a^2bx^{\frac{3}{2}} - \frac{2a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^(3/2),x, algorithm="giac")`

[Out]  $2/11*b^3*x^{(11/2)} + 6/7*a*b^2*x^{(7/2)} + 2*a^2*b*x^{(3/2)} - 2*a^3/sqrt(x)$

$$3.286 \quad \int \frac{(a+bx^2)^3}{x^{5/2}} dx$$

**Optimal.** Leaf size=49

$$-\frac{2a^3}{3x^{3/2}} + 6a^2b\sqrt{x} + \frac{6}{5}ab^2x^{5/2} + \frac{2}{9}b^3x^{9/2}$$

[Out]  $(-2*a^3)/(3*x^(3/2)) + 6*a^2*b*\text{Sqrt}[x] + (6*a*b^2*x^(5/2))/5 + (2*b^3*x^(9/2))/9$

**Rubi [A]** time = 0.0406058, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$-\frac{2a^3}{3x^{3/2}} + 6a^2b\sqrt{x} + \frac{6}{5}ab^2x^{5/2} + \frac{2}{9}b^3x^{9/2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^3/x^(5/2), x]$

[Out]  $(-2*a^3)/(3*x^(3/2)) + 6*a^2*b*\text{Sqrt}[x] + (6*a*b^2*x^(5/2))/5 + (2*b^3*x^(9/2))/9$

**Rubi in Sympy [A]** time = 6.32281, size = 48, normalized size = 0.98

$$-\frac{2a^3}{3x^{\frac{3}{2}}} + 6a^2b\sqrt{x} + \frac{6ab^2x^{\frac{5}{2}}}{5} + \frac{2b^3x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x**2+a)**3/x**(5/2), x)$

[Out]  $-2*a**3/(3*x**(3/2)) + 6*a**2*b*\text{sqrt}(x) + 6*a*b**2*x**(5/2)/5 + 2*b**3*x**(9/2)/9$

**Mathematica [A]** time = 0.0162183, size = 41, normalized size = 0.84

$$\frac{2(-15a^3 + 135a^2bx^2 + 27ab^2x^4 + 5b^3x^6)}{45x^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b*x^2)^3/x^(5/2), x]$

[Out]  $(2*(-15*a^3 + 135*a^2*b*x^2 + 27*a*b^2*x^4 + 5*b^3*x^6))/(45*x^(3/2))$

**Maple [A]** time = 0.008, size = 38, normalized size = 0.8

$$-\frac{-10b^3x^6 - 54ab^2x^4 - 270a^2bx^2 + 30a^3}{45}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^3/x^(5/2),x)`

[Out]  $-2/45*(-5*b^3*x^6-27*a*b^2*x^4-135*a^2*b*x^2+15*a^3)/x^{3/2}$

**Maxima [A]** time = 1.32591, size = 47, normalized size = 0.96

$$\frac{2}{9}b^3x^{\frac{9}{2}} + \frac{6}{5}ab^2x^{\frac{5}{2}} + 6a^2b\sqrt{x} - \frac{2a^3}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^(5/2),x, algorithm="maxima")`

[Out]  $2/9*b^3*x^{9/2} + 6/5*a*b^2*x^{5/2} + 6*a^2*b*\text{sqrt}(x) - 2/3*a^3/x^{3/2}$

**Fricas [A]** time = 0.208586, size = 50, normalized size = 1.02

$$\frac{2(5b^3x^6 + 27ab^2x^4 + 135a^2bx^2 - 15a^3)}{45x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^(5/2),x, algorithm="fricas")`

[Out]  $2/45*(5*b^3*x^6 + 27*a*b^2*x^4 + 135*a^2*b*x^2 - 15*a^3)/x^{3/2}$

**Sympy [A]** time = 9.91293, size = 48, normalized size = 0.98

$$-\frac{2a^3}{3x^{\frac{3}{2}}} + 6a^2b\sqrt{x} + \frac{6ab^2x^{\frac{5}{2}}}{5} + \frac{2b^3x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**3/x**(5/2),x)`

[Out]  $-2*a**3/(3*x**(3/2)) + 6*a**2*b*\text{sqrt}(x) + 6*a*b**2*x**(5/2)/5 + 2*b**3*x**(9/2)/9$

**GIAC/XCAS [A]** time = 0.222117, size = 47, normalized size = 0.96

$$\frac{2}{9}b^3x^{\frac{9}{2}} + \frac{6}{5}ab^2x^{\frac{5}{2}} + 6a^2b\sqrt{x} - \frac{2a^3}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^(5/2),x, algorithm="giac")`

[Out]  $2/9*b^3*x^{9/2} + 6/5*a*b^2*x^{5/2} + 6*a^2*b*\text{sqrt}(x) - 2/3*a^3/x^{3/2}$

$$3.287 \quad \int \frac{(a+bx^2)^3}{x^{7/2}} dx$$

**Optimal.** Leaf size=47

$$-\frac{2a^3}{5x^{5/2}} - \frac{6a^2b}{\sqrt{x}} + 2ab^2x^{3/2} + \frac{2}{7}b^3x^{7/2}$$

[Out]  $(-2*a^3)/(5*x^(5/2)) - (6*a^2*b)/\text{Sqrt}[x] + 2*a*b^2*x^(3/2) + (2*b^3*x^(7/2))/7$

**Rubi [A]** time = 0.0405316, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$-\frac{2a^3}{5x^{5/2}} - \frac{6a^2b}{\sqrt{x}} + 2ab^2x^{3/2} + \frac{2}{7}b^3x^{7/2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^3/x^(7/2), x]

[Out]  $(-2*a^3)/(5*x^(5/2)) - (6*a^2*b)/\text{Sqrt}[x] + 2*a*b^2*x^(3/2) + (2*b^3*x^(7/2))/7$

**Rubi in Sympy [A]** time = 6.30185, size = 46, normalized size = 0.98

$$-\frac{2a^3}{5x^{\frac{5}{2}}} - \frac{6a^2b}{\sqrt{x}} + 2ab^2x^{\frac{3}{2}} + \frac{2b^3x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*3/x\*\*(7/2), x)

[Out]  $-2*a**3/(5*x**(5/2)) - 6*a**2*b/\text{sqrt}(x) + 2*a*b**2*x**(3/2) + 2*b**3*x**(7/2)/7$

**Mathematica [A]** time = 0.0154513, size = 41, normalized size = 0.87

$$\frac{2(-7a^3 - 105a^2bx^2 + 35ab^2x^4 + 5b^3x^6)}{35x^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^3/x^(7/2), x]

[Out]  $(2*(-7*a^3 - 105*a^2*b*x^2 + 35*a*b^2*x^4 + 5*b^3*x^6))/(35*x^(5/2))$

**Maple [A]** time = 0.007, size = 38, normalized size = 0.8

$$-\frac{-10b^3x^6 - 70ab^2x^4 + 210a^2bx^2 + 14a^3}{35}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^3/x^(7/2),x)`

[Out]  $-2/35*(-5*b^3*x^6-35*a*b^2*x^4+105*a^2*b*x^2+7*a^3)/x^(5/2)$

**Maxima [A]** time = 1.35074, size = 49, normalized size = 1.04

$$\frac{2}{7}b^3x^{\frac{7}{2}} + 2ab^2x^{\frac{3}{2}} - \frac{2(15a^2bx^2 + a^3)}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^(7/2),x, algorithm="maxima")`

[Out]  $2/7*b^3*x^(7/2) + 2*a*b^2*x^(3/2) - 2/5*(15*a^2*b*x^2 + a^3)/x^(5/2)$

**Fricas [A]** time = 0.212577, size = 50, normalized size = 1.06

$$\frac{2(5b^3x^6 + 35ab^2x^4 - 105a^2bx^2 - 7a^3)}{35x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^(7/2),x, algorithm="fricas")`

[Out]  $2/35*(5*b^3*x^6 + 35*a*b^2*x^4 - 105*a^2*b*x^2 - 7*a^3)/x^(5/2)$

**Sympy [A]** time = 15.2386, size = 46, normalized size = 0.98

$$-\frac{2a^3}{5x^{\frac{5}{2}}} - \frac{6a^2b}{\sqrt{x}} + 2ab^2x^{\frac{3}{2}} + \frac{2b^3x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**3/x**(7/2),x)`

[Out]  $-2*a**3/(5*x**(5/2)) - 6*a**2*b/sqrt(x) + 2*a*b**2*x**(3/2) + 2*b**3*x**(7/2)/7$

**GIAC/XCAS [A]** time = 0.208272, size = 49, normalized size = 1.04

$$\frac{2}{7}b^3x^{\frac{7}{2}} + 2ab^2x^{\frac{3}{2}} - \frac{2(15a^2bx^2 + a^3)}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^3/x^(7/2),x, algorithm="giac")`

[Out]  $2/7*b^3*x^(7/2) + 2*a*b^2*x^(3/2) - 2/5*(15*a^2*b*x^2 + a^3)/x^(5/2)$

$$3.288 \quad \int \frac{x^{7/2}}{a+bx^2} dx$$

**Optimal.** Leaf size=215

$$\begin{aligned} & -\frac{a^{5/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}b^{9/4}} + \frac{a^{5/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}b^{9/4}} \\ & -\frac{a^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{9/4}} + \frac{a^{5/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{\sqrt{2}b^{9/4}} - \frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} \end{aligned}$$

[Out]  $(-2*a*\text{Sqrt}[x])/b^2 + (2*x^{(5/2)})/(5*b) - (a^{(5/4)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/( \text{Sqrt}[2]*b^{(9/4)}) + (a^{(5/4)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/( \text{Sqrt}[2]*b^{(9/4)}) - (a^{(5/4)}*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(2*\text{Sqrt}[2]*b^{(9/4)}) + (a^{(5/4)}*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(2*\text{Sqrt}[2]*b^{(9/4)})$

**Rubi [A]** time = 0.480564, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$

$$\begin{aligned} & -\frac{a^{5/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}b^{9/4}} + \frac{a^{5/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}b^{9/4}} \\ & -\frac{a^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{9/4}} + \frac{a^{5/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{\sqrt{2}b^{9/4}} - \frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(7/2)}/(a + b*x^2), x]$

[Out]  $(-2*a*\text{Sqrt}[x])/b^2 + (2*x^{(5/2)})/(5*b) - (a^{(5/4)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/( \text{Sqrt}[2]*b^{(9/4)}) + (a^{(5/4)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/( \text{Sqrt}[2]*b^{(9/4)}) - (a^{(5/4)}*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(2*\text{Sqrt}[2]*b^{(9/4)}) + (a^{(5/4)}*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(2*\text{Sqrt}[2]*b^{(9/4)})$

**Rubi in Sympy [A]** time = 69.7146, size = 204, normalized size = 0.95

$$\begin{aligned} & -\frac{\sqrt{2}a^{5/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{4b^{9/4}} + \frac{\sqrt{2}a^{5/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{4b^{9/4}} \\ & -\frac{\sqrt{2}a^{5/4} \text{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2b^{9/4}} + \frac{\sqrt{2}a^{5/4} \text{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2b^{9/4}} - \frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{(7/2)}/(b*x^2+a), x)$

[Out]  $-\text{sqrt}(2)*a^{(5/4)}*\log(-\text{sqrt}(2)*a^{(1/4)}*b^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(a) + \text{sqrt}(b)*x)/(4*b^{(9/4)}) + \text{sqrt}(2)*a^{(5/4)}*\log(\text{sqrt}(2)*a^{(1/4)}*b^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(a) + \text{sqrt}(b)*x)/(4*b^{(9/4)}) - \text{sqrt}(2)*a^{(5/4)}*\text{atan}(1 - \text{sqrt}(2)*b^{(1/4)}*\text{sqrt}(x)/a^{(1/4)})/(2*b^{(9/4)}) + \text{sqrt}(2)*a^{(5/4)}*\text{atan}(1 + \text{sqrt}(2)*b^{(1/4)}*\text{sqrt}(x)/a^{(1/4)})/(2*b^{(9/4)}) - 2*a*\text{sqrt}(x)/b^2 + 2*x^{(5/2)}/(5*b)$

**Mathematica [A]** time = 0.120423, size = 203, normalized size = 0.94

$$\frac{-5\sqrt{2}a^{5/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) + 5\sqrt{2}a^{5/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) - 10\sqrt{2}a^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) + 10\sqrt{2}a^{5/4}}{20b^{9/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(7/2)/(a + b\*x^2), x]

[Out] (-40\*a\*b^(1/4)\*Sqrt[x] + 8\*b^(5/4)\*x^(5/2) - 10\*Sqrt[2]\*a^(5/4)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)] + 10\*Sqrt[2]\*a^(5/4)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)] - 5\*Sqrt[2]\*a^(5/4)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x] + 5\*Sqrt[2]\*a^(5/4)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(20\*b^(9/4))

**Maple [A]** time = 0.017, size = 152, normalized size = 0.7

$$\frac{2}{5b}x^{\frac{5}{2}} - 2\frac{a\sqrt{x}}{b^2} + \frac{a\sqrt{2}}{4b^2}\sqrt[4]{\frac{a}{b}} \ln\left(1\left(x + \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{a\sqrt{2}}{2b^2}\sqrt[4]{\frac{a}{b}} \arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) + \frac{a\sqrt{2}}{2b^2}\sqrt[4]{\frac{a}{b}} \arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)/(b\*x^2+a), x)

[Out] 2/5\*x^(5/2)/b-2\*a\*x^(1/2)/b^2+1/4\*a/b^2\*(a/b)^(1/4)\*2^(1/2)\*ln((x+(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2)))+1/2\*a/b^2\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)+1)+1/2\*a/b^2\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)-1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(b\*x^2 + a), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.235442, size = 207, normalized size = 0.96

$$\frac{20b^2\left(-\frac{a^5}{b^9}\right)^{\frac{1}{4}} \arctan\left(\frac{b^2\left(-\frac{a^5}{b^9}\right)^{\frac{1}{4}}}{a\sqrt{x}+\sqrt{b^4\sqrt{-\frac{a^5}{b^9}+a^2x}}}\right) - 5b^2\left(-\frac{a^5}{b^9}\right)^{\frac{1}{4}} \log\left(b^2\left(-\frac{a^5}{b^9}\right)^{\frac{1}{4}} + a\sqrt{x}\right) + 5b^2\left(-\frac{a^5}{b^9}\right)^{\frac{1}{4}} \log\left(-b^2\left(-\frac{a^5}{b^9}\right)^{\frac{1}{4}} + a\sqrt{x}\right)}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(b\*x^2 + a), x, algorithm="fricas")

[Out]  $-1/10*(20*b^2*(-a^5/b^9)^{(1/4)}*\arctan(b^2*(-a^5/b^9)^{(1/4)}/(a*\sqrt{x} + \sqrt{b^4*\sqrt{-a^5/b^9} + a^2*x})) - 5*b^2*(-a^5/b^9)^{(1/4)}*\log(b^2*(-a^5/b^9)^{(1/4)} + a*\sqrt{x}) + 5*b^2*(-a^5/b^9)^{(1/4)}*\log(-b^2*(-a^5/b^9)^{(1/4)} + a*\sqrt{x}) - 4*(b*x^2 - 5*a)*\sqrt{x})/b^2$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(7/2)/(b*x**2+a), x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.215668, size = 265, normalized size = 1.23

$$\frac{\sqrt{2}(ab^3)^{\frac{1}{4}}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^3} + \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^3}$$

$$+ \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}a\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4b^3}$$

$$- \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}a\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4b^3} + \frac{2\left(b^4x^{\frac{5}{2}}-5ab^3\sqrt{x}\right)}{5b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(7/2)/(b*x^2 + a), x, algorithm="giac")`

[Out]  $1/2*\sqrt{2}*(a*b^3)^{(1/4)}*a*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(a/b)^{(1/4)} + 2*\sqrt{x})/(a/b)^{(1/4)})/b^3 + 1/2*\sqrt{2}*(a*b^3)^{(1/4)}*a*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(a/b)^{(1/4)} - 2*\sqrt{x})/(a/b)^{(1/4)})/b^3 + 1/4*\sqrt{2}*(a*b^3)^{(1/4)}*a*\ln(\sqrt{2}*\sqrt{x}*(a/b)^{(1/4)} + x + \sqrt{a/b})/b^3 - 1/4*\sqrt{2}*(a*b^3)^{(1/4)}*a*\ln(-\sqrt{2}*\sqrt{x}*(a/b)^{(1/4)} + x + \sqrt{a/b})/b^3 + 2/5*(b^4*x^{(5/2)} - 5*a*b^3*\sqrt{x})/b^5$



$$3.289 \quad \int \frac{x^{5/2}}{a+bx^2} dx$$

**Optimal.** Leaf size=204

$$\begin{aligned} & -\frac{a^{3/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}b^{7/4}} + \frac{a^{3/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}b^{7/4}} \\ & + \frac{a^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}} - \frac{a^{3/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{\sqrt{2}b^{7/4}} + \frac{2x^{3/2}}{3b} \end{aligned}$$

[Out] (2\*x^(3/2))/(3\*b) + (a^(3/4)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(Sqrt[2]\*b^(7/4))) - (a^(3/4)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(Sqrt[2]\*b^(7/4))) - (a^(3/4)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(2\*Sqrt[2]\*b^(7/4)) + (a^(3/4)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(2\*Sqrt[2]\*b^(7/4))

**Rubi [A]** time = 0.410086, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$

$$\begin{aligned} & -\frac{a^{3/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}b^{7/4}} + \frac{a^{3/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}b^{7/4}} \\ & + \frac{a^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}} - \frac{a^{3/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{\sqrt{2}b^{7/4}} + \frac{2x^{3/2}}{3b} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)/(a + b\*x^2), x]

[Out] (2\*x^(3/2))/(3\*b) + (a^(3/4)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(Sqrt[2]\*b^(7/4))) - (a^(3/4)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(Sqrt[2]\*b^(7/4))) - (a^(3/4)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(2\*Sqrt[2]\*b^(7/4)) + (a^(3/4)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(2\*Sqrt[2]\*b^(7/4))

**Rubi in Sympy [A]** time = 65.0618, size = 192, normalized size = 0.94

$$\begin{aligned} & -\frac{\sqrt{2}a^{3/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{4b^{7/4}} + \frac{\sqrt{2}a^{3/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{4b^{7/4}} \\ & + \frac{\sqrt{2}a^{3/4} \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2b^{7/4}} - \frac{\sqrt{2}a^{3/4} \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2b^{7/4}} + \frac{2x^{3/2}}{3b} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(5/2)/(b\*x\*\*2+a), x)

[Out] -sqrt(2)\*a\*\*(3/4)\*log(-sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*sqrt(x) + sqrt(a) + sqrt(b)\*x)/(4\*b\*\*(7/4)) + sqrt(2)\*a\*\*(3/4)\*log(sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*sqrt(x) + sqrt(a) + sqrt(b)\*x)/(4\*b\*\*(7/4)) + sqrt(2)\*a\*\*(3/4)\*atan(1 - sqrt(2)\*b\*\*(1/4)\*sqrt(x)/a\*\*(1/4))/(2\*b\*\*(7/4)) - sqrt(2)\*a\*\*(3/4)\*atan(1 + sqrt(2)\*b\*\*(1/4)\*sqrt(x)/a\*\*(1/4))/(2\*b\*\*(7/4)) + 2\*x\*\*(3/2)/(3\*b)

**Mathematica [A]** time = 0.0540531, size = 190, normalized size = 0.93

$$\frac{-3\sqrt{2}a^{3/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) + 3\sqrt{2}a^{3/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) + 6\sqrt{2}a^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) - 6\sqrt{2}a^{3/4}}{12b^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)/(a + b\*x^2), x]

[Out] (8\*b^(3/4)\*x^(3/2) + 6\*Sqrt[2]\*a^(3/4)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)] - 6\*Sqrt[2]\*a^(3/4)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)] - 3\*Sqrt[2]\*a^(3/4)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x] + 3\*Sqrt[2]\*a^(3/4)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(12\*b^(7/4))

**Maple [A]** time = 0.01, size = 143, normalized size = 0.7

$$\frac{2}{3b}x^{\frac{3}{2}} - \frac{a\sqrt{2}}{4b^2} \ln\left(1\left(x - \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x + \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \frac{1}{\sqrt[4]{\frac{a}{b}}}$$

$$- \frac{a\sqrt{2}}{2b^2} \arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) \frac{1}{\sqrt[4]{\frac{a}{b}}} - \frac{a\sqrt{2}}{2b^2} \arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) \frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)/(b\*x^2+a), x)

[Out] 2/3\*x^(3/2)/b-1/4\*a/b^2/(a/b)^(1/4)\*2^(1/2)\*ln((x-(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2)))-1/2\*a/b^2/(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)+1)-1/2\*a/b^2/(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)-1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(b\*x^2 + a), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.232156, size = 200, normalized size = 0.98

$$\frac{12b\left(-\frac{a^3}{b^7}\right)^{\frac{1}{4}} \arctan\left(\frac{b^5\left(-\frac{a^3}{b^7}\right)^{\frac{3}{4}}}{a^2\sqrt{x} + \sqrt{-a^3b^3\sqrt{-\frac{a^3}{b^7} + a^4x}}}\right) + 3b\left(-\frac{a^3}{b^7}\right)^{\frac{1}{4}} \log\left(b^5\left(-\frac{a^3}{b^7}\right)^{\frac{3}{4}} + a^2\sqrt{x}\right) - 3b\left(-\frac{a^3}{b^7}\right)^{\frac{1}{4}} \log\left(-b^5\left(-\frac{a^3}{b^7}\right)^{\frac{3}{4}} + a^2\sqrt{x}\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(b\*x^2 + a), x, algorithm="fricas")

[Out]  $-1/6*(12*b*(-a^3/b^7)^{(1/4)}*\arctan(b^5*(-a^3/b^7)^{(3/4)}/(a^2*\sqrt{x} + \sqrt{-a^3*b^3*\sqrt{-a^3/b^7} + a^4*x})) + 3*b*(-a^3/b^7)^{(1/4)}*\log(b^5*(-a^3/b^7)^{(3/4)} + a^2*\sqrt{x}) - 3*b*(-a^3/b^7)^{(1/4)}*\log(-b^5*(-a^3/b^7)^{(3/4)} + a^2*\sqrt{x}) - 4*x^{(3/2)}/b$

**Sympy [A]** time = 126.62, size = 180, normalized size = 0.88

$$\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for } a = 0 \\ \frac{2x^{\frac{7}{2}}}{7a} & \text{for } b = 0 \\ \frac{(-1)^{\frac{3}{4}}a^{\frac{3}{4}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}+\sqrt{x}}\right)}{2b^4\left(\frac{1}{b}\right)^{\frac{9}{4}}} - \frac{(-1)^{\frac{3}{4}}a^{\frac{3}{4}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}+\sqrt{x}}\right)}{2b^4\left(\frac{1}{b}\right)^{\frac{9}{4}}} - \frac{(-1)^{\frac{3}{4}}a^{\frac{3}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{b^4\left(\frac{1}{b}\right)^{\frac{9}{4}}} + \frac{2x^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(5/2)/(b*x**2+a), x)`

[Out] `Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*b), Eq(a, 0)), (2*x**(7/2)/(7*a), Eq(b, 0)), ((-1)**(3/4)*a**(3/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**4*(1/b)**(9/4)) - (-1)**(3/4)*a**(3/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**4*(1/b)**(9/4)) - (-1)**(3/4)*a**(3/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(b**4*(1/b)**(9/4)) + 2*x**(3/2)/(3*b), True)`

**GIAC/XCAS [A]** time = 0.226495, size = 240, normalized size = 1.18

$$\begin{aligned} & \frac{2x^{\frac{3}{2}}}{3b} - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^4} - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^4} \\ & + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4b^4} - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4b^4} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(5/2)/(b*x^2 + a), x, algorithm="giac")`

[Out]  $2/3*x^{(3/2)}/b - 1/2*\sqrt{2}*(a*b^3)^{(3/4)}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(a/b)^{(1/4)} + 2*\sqrt{x})/(a/b)^{(1/4)})/b^4 - 1/2*\sqrt{2}*(a*b^3)^{(3/4)}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(a/b)^{(1/4)} - 2*\sqrt{x})/(a/b)^{(1/4)})/b^4 + 1/4*\sqrt{2}*(a*b^3)^{(3/4)}*\ln(\sqrt{2}*\sqrt{x}*(a/b)^{(1/4)} + x + \sqrt{a/b})/b^4 - 1/4*\sqrt{2}*(a*b^3)^{(3/4)}*\ln(-\sqrt{2}*\sqrt{x}*(a/b)^{(1/4)} + x + \sqrt{a/b})/b^4$

$$3.290 \quad \int \frac{x^{3/2}}{a+bx^2} dx$$

**Optimal.** Leaf size=202

$$\frac{\sqrt[4]{a} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}b^{5/4}} - \frac{\sqrt[4]{a} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}b^{5/4}} + \frac{\sqrt[4]{a} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}} - \frac{\sqrt[4]{a} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{\sqrt{2}b^{5/4}} + \frac{2\sqrt{x}}{b}$$

[Out] (2\*Sqrt[x])/b + (a^(1/4)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(Sqrt[2]\*b^(5/4)) - (a^(1/4)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(Sqrt[2]\*b^(5/4))) + (a^(1/4)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(2\*Sqrt[2]\*b^(5/4)) - (a^(1/4)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(2\*Sqrt[2]\*b^(5/4))

**Rubi [A]** time = 0.375243, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$

$$\frac{\sqrt[4]{a} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}b^{5/4}} - \frac{\sqrt[4]{a} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}b^{5/4}} + \frac{\sqrt[4]{a} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}} - \frac{\sqrt[4]{a} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{\sqrt{2}b^{5/4}} + \frac{2\sqrt{x}}{b}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)/(a + b\*x^2), x]

[Out] (2\*Sqrt[x])/b + (a^(1/4)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(Sqrt[2]\*b^(5/4)) - (a^(1/4)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(Sqrt[2]\*b^(5/4))) + (a^(1/4)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(2\*Sqrt[2]\*b^(5/4)) - (a^(1/4)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(2\*Sqrt[2]\*b^(5/4))

**Rubi in Sympy [A]** time = 62.7505, size = 190, normalized size = 0.94

$$\frac{\sqrt{2}\sqrt[4]{a} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{4b^{5/4}} - \frac{\sqrt{2}\sqrt[4]{a} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{4b^{5/4}} + \frac{\sqrt{2}\sqrt[4]{a} \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2b^{5/4}} - \frac{\sqrt{2}\sqrt[4]{a} \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2b^{5/4}} + \frac{2\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(3/2)/(b\*x\*\*2+a), x)

[Out] sqrt(2)\*a\*\*(1/4)\*log(-sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*sqrt(x) + sqrt(a) + sqrt(b)\*x)/(4\*b\*\*(5/4)) - sqrt(2)\*a\*\*(1/4)\*log(sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*sqrt(x) + sqrt(a) + sqrt(b)\*x)/(4\*b\*\*(5/4)) + sqrt(2)\*a\*\*(1/4)\*atan(1 - sqrt(2)\*b\*\*(1/4)\*sqrt(x)/a\*\*(1/4))/(2\*b\*\*(5/4)) - sqrt(2)\*a\*\*(1/4)\*atan(1 + sqrt(2)\*b\*\*(1/4)\*sqrt(x)/a\*\*(1/4))/(2\*b\*\*(5/4)) + 2\*sqrt(x)/b

**Mathematica [A]** time = 0.0574523, size = 189, normalized size = 0.94

$$\frac{\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{bx}\right)-\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{bx}\right)+2\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-2\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4b^{5/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)/(a + b\*x^2), x]

[Out] (8\*b^(1/4)\*Sqrt[x] + 2\*Sqrt[2]\*a^(1/4)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)] - 2\*Sqrt[2]\*a^(1/4)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)] + Sqrt[2]\*a^(1/4)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x] - Sqrt[2]\*a^(1/4)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(4\*b^(5/4))

**Maple [A]** time = 0.01, size = 140, normalized size = 0.7

$$2\frac{\sqrt{x}}{b} - \frac{\sqrt{2}}{4b}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x + \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) - \frac{\sqrt{2}}{2b}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) - \frac{\sqrt{2}}{2b}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(b\*x^2+a), x)

[Out] 2\*x^(1/2)/b-1/4/b\*(a/b)^(1/4)\*2^(1/2)\*ln((x+(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2)))-1/2/b\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)+1)-1/2/b\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)-1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b\*x^2 + a), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.233138, size = 144, normalized size = 0.71

$$\frac{4b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}}{\sqrt{b^2\sqrt{-\frac{a}{b^5}}+x+\sqrt{x}}}\right)-b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)+b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\log\left(-b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)+4\sqrt{x}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b\*x^2 + a), x, algorithm="fricas")

[Out] 1/2\*(4\*b\*(-a/b^5)^(1/4)\*arctan(b\*(-a/b^5)^(1/4)/(sqrt(b^2\*sqrt(-a/b^5)+x)+sqrt(x)))-b\*(-a/b^5)^(1/4)\*log(b\*(-a/b^5)^(1/4)+sqrt(x))+b\*(-a/b^5)^(1/4)\*log(-b\*(-a/b^5)^(1/4)+sqrt(x))+4\*sqrt(x))

$$\frac{\sqrt{x} + b \cdot (-a/b^5)^{1/4} \cdot \log(-b \cdot (-a/b^5)^{1/4} + \sqrt{x}) + 4 \cdot \sqrt{x}}{b}$$

**Sympy [A]** time = 40.1854, size = 177, normalized size = 0.88

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a = 0 \wedge b = 0 \\ \frac{2\sqrt{x}}{b} & \text{for } a = 0 \\ \frac{2x^{5/2}}{5a} & \text{for } b = 0 \\ \frac{\sqrt[4]{-1}\sqrt[4]{a} \log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt{\frac{1}{b}+\sqrt{x}}\right)}{2b^{21}\left(\frac{1}{b}\right)^{79/4}} - \frac{\sqrt[4]{-1}\sqrt[4]{a} \log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt{\frac{1}{b}+\sqrt{x}}\right)}{2b^{21}\left(\frac{1}{b}\right)^{79/4}} + \frac{\sqrt[4]{-1}\sqrt[4]{a} \operatorname{atan}\left(\frac{(-1)^{3/4}\sqrt{x}}{\sqrt[4]{a}\sqrt{\frac{1}{b}}}\right)}{b^{21}\left(\frac{1}{b}\right)^{79/4}} + \frac{2\sqrt{x}}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(3/2)/(b\*x\*\*2+a), x)

[Out] Piecewise((zoo\*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2\*sqrt(x)/b, Eq(a, 0)), (2\*x\*\*(5/2)/(5\*a), Eq(b, 0)), ((-1)\*\*(1/4)\*a\*\*(1/4)\*log((-1)\*\*(1/4)\*a\*\*(1/4)\*(1/b)\*\*(1/4) + sqrt(x))/(2\*b\*\*21\*(1/b)\*\*(79/4)) - (-1)\*\*(1/4)\*a\*\*(1/4)\*log((-1)\*\*(1/4)\*a\*\*(1/4)\*(1/b)\*\*(1/4) + sqrt(x))/(2\*b\*\*21\*(1/b)\*\*(79/4)) + (-1)\*\*(1/4)\*a\*\*(1/4)\*atan((-1)\*\*(3/4)\*sqrt(x)/(a\*\*(1/4)\*(1/b)\*\*(1/4)))/(b\*\*21\*(1/b)\*\*(79/4)) + 2\*sqrt(x)/b, True))

**GIAC/XCAS [A]** time = 0.216249, size = 240, normalized size = 1.19

$$\begin{aligned} & \frac{\sqrt{2}(ab^3)^{1/4} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{1/4}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{1/4}}\right)}{2b^2} - \frac{\sqrt{2}(ab^3)^{1/4} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{1/4}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{1/4}}\right)}{2b^2} \\ & - \frac{\sqrt{2}(ab^3)^{1/4} \ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{1/4}+x+\sqrt{\frac{a}{b}}\right)}{4b^2} + \frac{\sqrt{2}(ab^3)^{1/4} \ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{1/4}+x+\sqrt{\frac{a}{b}}\right)}{4b^2} + \frac{2\sqrt{x}}{b} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b\*x^2 + a), x, algorithm="giac")

[Out] -1/2\*sqrt(2)\*(a\*b^3)^(1/4)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(a/b)^(1/4) + 2\*sqrt(x))/(a/b)^(1/4))/b^2 - 1/2\*sqrt(2)\*(a\*b^3)^(1/4)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(a/b)^(1/4) - 2\*sqrt(x))/(a/b)^(1/4))/b^2 - 1/4\*sqrt(2)\*(a\*b^3)^(1/4)\*ln(sqrt(2)\*sqrt(x)\*(a/b)^(1/4) + x + sqrt(a/b))/b^2 + 1/4\*sqrt(2)\*(a\*b^3)^(1/4)\*ln(-sqrt(2)\*sqrt(x)\*(a/b)^(1/4) + x + sqrt(a/b))/b^2 + 2\*sqrt(x)/b

$$3.291 \quad \int \frac{\sqrt{x}}{a+bx^2} dx$$

**Optimal.** Leaf size=192

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}\sqrt[4]{ab^{3/4}}} - \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}\sqrt[4]{ab^{3/4}}}$$

$$- \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{ab^{3/4}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{\sqrt{2}\sqrt[4]{ab^{3/4}}}$$

[Out] -(ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(Sqrt[2]\*a^(1/4)\*b^(3/4))) + ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(Sqrt[2]\*a^(1/4)\*b^(3/4)) + Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x]/(2\*Sqrt[2]\*a^(1/4)\*b^(3/4)) - Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x]/(2\*Sqrt[2]\*a^(1/4)\*b^(3/4))

**Rubi [A]** time = 0.338891, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}\sqrt[4]{ab^{3/4}}} - \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}\sqrt[4]{ab^{3/4}}}$$

$$- \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{ab^{3/4}}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{\sqrt{2}\sqrt[4]{ab^{3/4}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(a + b\*x^2), x]

[Out] -(ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(Sqrt[2]\*a^(1/4)\*b^(3/4))) + ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(Sqrt[2]\*a^(1/4)\*b^(3/4)) + Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x]/(2\*Sqrt[2]\*a^(1/4)\*b^(3/4)) - Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x]/(2\*Sqrt[2]\*a^(1/4)\*b^(3/4))

**Rubi in Sympy [A]** time = 57.3687, size = 182, normalized size = 0.95

$$\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{4\sqrt[4]{ab^{3/4}}} - \frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{4\sqrt[4]{ab^{3/4}}}$$

$$- \frac{\sqrt{2}\operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{ab^{3/4}}} + \frac{\sqrt{2}\operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{ab^{3/4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(1/2)/(b\*x\*\*2+a), x)

[Out] sqrt(2)\*log(-sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*sqrt(x) + sqrt(a) + sqrt(b)\*x)/(4\*a\*\*(1/4)\*b\*\*(3/4)) - sqrt(2)\*log(sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*sqrt(x) + sqrt(a) + sqrt(b)\*x)/(4\*a\*\*(1/4)\*b\*\*(3/4)) - sqrt(2)\*atan(1 - sqrt(2)\*b\*\*(1/4)\*sqrt(x)/a\*\*(1/4))/(2\*a\*\*(1/4)\*b\*\*(3/4)) + sqrt(2)\*atan(1 + sqrt(2)\*b\*\*(1/4)\*sqrt(x)/a\*\*(1/4))/(2\*a\*\*(1/4)\*b\*\*(3/4))

**Mathematica [A]** time = 0.0343899, size = 146, normalized size = 0.76

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) - \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) - 2 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) + 2 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}\sqrt[4]{ab^3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(a + b\*x^2), x]

[Out] (-2\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)] + 2\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)] + Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x] - Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(2\*Sqrt[2]\*a^(1/4)\*b^(3/4))

**Maple [A]** time = 0.008, size = 132, normalized size = 0.7

$$\frac{\sqrt{2}}{4b} \ln\left(1 \left(x - \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right) \left(x + \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{\sqrt{2}}{2b} \arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) \frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{\sqrt{2}}{2b} \arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) \frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(b\*x^2+a), x)

[Out] 1/4/b/(a/b)^(1/4)\*2^(1/2)\*ln((x-(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2)))+1/2/b/(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)+1)+1/2/b/(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)-1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x)/(b\*x^2 + a), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.230283, size = 159, normalized size = 0.83

$$2 \left(-\frac{1}{ab^3}\right)^{\frac{1}{4}} \arctan\left(\frac{ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}}{\sqrt{-ab\sqrt{-\frac{1}{ab^3}}+x+\sqrt{x}}}\right) + \frac{1}{2} \left(-\frac{1}{ab^3}\right)^{\frac{1}{4}} \log\left(ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}} + \sqrt{x}\right) - \frac{1}{2} \left(-\frac{1}{ab^3}\right)^{\frac{1}{4}} \log\left(-ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}} + \sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(sqrt(x)/(b\*x^2 + a),x, algorithm="fricas")

[Out]  $2 \cdot (-1/(a \cdot b^3))^{1/4} \cdot \arctan(a \cdot b^2 \cdot (-1/(a \cdot b^3))^{3/4} / (\sqrt{-a \cdot b \cdot \sqrt{-1/(a \cdot b^3)} + x} + \sqrt{x})) + 1/2 \cdot (-1/(a \cdot b^3))^{1/4} \cdot \log(a \cdot b^2 \cdot (-1/(a \cdot b^3))^{3/4} + \sqrt{x}) - 1/2 \cdot (-1/(a \cdot b^3))^{1/4} \cdot \log(-a \cdot b^2 \cdot (-1/(a \cdot b^3))^{3/4} + \sqrt{x})$

**Sympy [A]** time = 18.0833, size = 170, normalized size = 0.89

$$\begin{cases} \frac{\infty}{\sqrt{x}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^2}{3a} & \text{for } b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for } a = 0 \\ -\frac{(-1)^{\frac{3}{4}} \log\left(-\sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b} + \sqrt{x}}\right)}{2\sqrt[4]{ab^3} \left(\frac{1}{b}\right)^{\frac{9}{4}}} + \frac{(-1)^{\frac{3}{4}} \log\left(\sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b} + \sqrt{x}}\right)}{2\sqrt[4]{ab^3} \left(\frac{1}{b}\right)^{\frac{9}{4}}} + \frac{(-1)^{\frac{3}{4}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}}\right)}{\sqrt[4]{ab^3} \left(\frac{1}{b}\right)^{\frac{9}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(1/2)/(b\*x\*\*2+a),x)

[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2\*x\*\*(3/2)/(3\*a), Eq(b, 0)), (-2/(b\*sqrt(x)), Eq(a, 0)), ((-1)\*\*(3/4)\*log((-1)\*\*(1/4)\*a\*\*(1/4)\*(1/b)\*\*(1/4)+sqrt(x))/(2\*a\*\*(1/4)\*b\*\*3\*(1/b)\*\*(9/4))+(-1)\*\*(3/4)\*log((-1)\*\*(1/4)\*a\*\*(1/4)\*(1/b)\*\*(1/4)+sqrt(x))/(2\*a\*\*(1/4)\*b\*\*3\*(1/b)\*\*(9/4))+(-1)\*\*(3/4)\*atan((-1)\*\*(3/4)\*sqrt(x)/(a\*\*(1/4)\*(1/b)\*\*(1/4)))/(a\*\*(1/4)\*b\*\*3\*(1/b)\*\*(9/4)), True))

**GIAC/XCAS [A]** time = 0.219896, size = 246, normalized size = 1.28

$$\begin{aligned} & \frac{\sqrt{2} (ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2 ab^3} + \frac{\sqrt{2} (ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2 ab^3} \\ & - \frac{\sqrt{2} (ab^3)^{\frac{3}{4}} \ln\left(\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4 ab^3} + \frac{\sqrt{2} (ab^3)^{\frac{3}{4}} \ln\left(-\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4 ab^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x)/(b\*x^2 + a),x, algorithm="giac")

[Out]  $1/2 \cdot \sqrt{2} \cdot (a \cdot b^3)^{3/4} \cdot \arctan(1/2 \cdot \sqrt{2} \cdot (\sqrt{2} \cdot (a/b)^{1/4} + 2 \cdot \sqrt{x}) / (a/b)^{1/4}) / (a \cdot b^3) + 1/2 \cdot \sqrt{2} \cdot (a \cdot b^3)^{3/4} \cdot \arctan(-1/2 \cdot \sqrt{2} \cdot (\sqrt{2} \cdot (a/b)^{1/4} - 2 \cdot \sqrt{x}) / (a/b)^{1/4}) / (a \cdot b^3) - 1/4 \cdot \sqrt{2} \cdot (a \cdot b^3)^{3/4} \cdot \ln(\sqrt{2} \cdot \sqrt{x} \cdot (a/b)^{1/4} + x + \sqrt{a/b}) / (a \cdot b^3) + 1/4 \cdot \sqrt{2} \cdot (a \cdot b^3)^{3/4} \cdot \ln(-\sqrt{2} \cdot \sqrt{x} \cdot (a/b)^{1/4} + x + \sqrt{a/b}) / (a \cdot b^3)$

$$3.292 \quad \int \frac{1}{\sqrt{x}(a+bx^2)} dx$$

**Optimal.** Leaf size=192

$$\begin{aligned} & -\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} \\ & -\frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}} \end{aligned}$$

[Out] -(ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(Sqrt[2]\*a^(3/4)\*b^(1/4))) + ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(Sqrt[2]\*a^(3/4)\*b^(1/4)) - Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x]/(2\*Sqrt[2]\*a^(3/4)\*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x]/(2\*Sqrt[2]\*a^(3/4)\*b^(1/4))

**Rubi [A]** time = 0.326536, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$\begin{aligned} & -\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} \\ & -\frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]\*(a + b\*x^2)), x]

[Out] -(ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(Sqrt[2]\*a^(3/4)\*b^(1/4))) + ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(Sqrt[2]\*a^(3/4)\*b^(1/4)) - Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x]/(2\*Sqrt[2]\*a^(3/4)\*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x]/(2\*Sqrt[2]\*a^(3/4)\*b^(1/4))

**Rubi in Sympy [A]** time = 56.0813, size = 182, normalized size = 0.95

$$\begin{aligned} & -\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{4a^{3/4}\sqrt[4]{b}} + \frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{4a^{3/4}\sqrt[4]{b}} \\ & -\frac{\sqrt{2}\operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{\sqrt{2}\operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+a)/x\*\*(1/2), x)

[Out] -sqrt(2)\*log(-sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*sqrt(x) + sqrt(a) + sqrt(b)\*x)/(4\*a\*\*(3/4)\*b\*\*(1/4)) + sqrt(2)\*log(sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*sqrt(x) + sqrt(a) + sqrt(b)\*x)/(4\*a\*\*(3/4)\*b\*\*(1/4)) - sqrt(2)\*atan(1 - sqrt(2)\*b\*\*(1/4)\*sqrt(x)/a\*\*(1/4))/(2\*a\*\*(3/4)\*b\*\*(1/4)) + sqrt(2)\*atan(1 + sqrt(2)\*b\*\*(1/4)\*sqrt(x)/a\*\*(1/4))/(2\*a\*\*(3/4)\*b\*\*(1/4))

**Mathematica [A]** time = 0.0370576, size = 146, normalized size = 0.76

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) + \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) - 2 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right) + 2 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]\*(a + b\*x^2)),x]

[Out] (-2\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)] + 2\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x] + Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(2\*Sqrt[2]\*a^(3/4)\*b^(1/4))

**Maple [A]** time = 0.008, size = 132, normalized size = 0.7

$$\frac{\sqrt{2}}{4a}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x + \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{\sqrt{2}}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) + \frac{\sqrt{2}}{2a}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+a)/x^(1/2),x)

[Out] 1/4\*(a/b)^(1/4)/a\*2^(1/2)\*ln((x+(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2)))+1/2\*(a/b)^(1/4)/a\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)+1)+1/2\*(a/b)^(1/4)/a\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)-1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2+ a)\*sqrt(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.225693, size = 147, normalized size = 0.77

$$-2\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\arctan\left(\frac{a\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}}{\sqrt{a^2\sqrt{-\frac{1}{a^3b}}+x+\sqrt{x}}}\right) + \frac{1}{2}\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}+\sqrt{x}\right) - \frac{1}{2}\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(-a\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)\*sqrt(x)),x, algorithm="fricas")

[Out]  $-2 \cdot (-1/(a^3 \cdot b))^{1/4} \cdot \arctan(a \cdot (-1/(a^3 \cdot b))^{1/4} / (\sqrt{a^2 \cdot \sqrt{-1/(a^3 \cdot b)} + x} + \sqrt{x})) + 1/2 \cdot (-1/(a^3 \cdot b))^{1/4} \cdot \log(a \cdot (-1/(a^3 \cdot b))^{1/4} + \sqrt{x}) - 1/2 \cdot (-1/(a^3 \cdot b))^{1/4} \cdot \log(-a \cdot (-1/(a^3 \cdot b))^{1/4} + \sqrt{x})$

**Sympy [A]** time = 32.7332, size = 170, normalized size = 0.89

$$\begin{cases} \frac{\infty}{x^{\frac{3}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for } a = 0 \\ \frac{2\sqrt{x}}{a} & \text{for } b = 0 \\ -\frac{\sqrt[4]{-1} \log\left(-\sqrt[4]{-1} \sqrt[4]{a^4} \sqrt{\frac{1}{b} + \sqrt{x}}\right)}{2a^{\frac{3}{4}} b^4 \left(\frac{1}{b}\right)^{\frac{15}{4}}} + \frac{\sqrt[4]{-1} \log\left(\sqrt[4]{-1} \sqrt[4]{a^4} \sqrt{\frac{1}{b} + \sqrt{x}}\right)}{2a^{\frac{3}{4}} b^4 \left(\frac{1}{b}\right)^{\frac{15}{4}}} - \frac{\sqrt[4]{-1} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a^4} \sqrt{\frac{1}{b}}}\right)}{a^{\frac{3}{4}} b^4 \left(\frac{1}{b}\right)^{\frac{15}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**2+a)/x**(1/2), x)`

[Out] `Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b*x**(3/2)), Eq(a, 0)), (2*sqrt(x)/a, Eq(b, 0)), (-(-1)**(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4)+sqrt(x))/(2*a**(3/4)*b**4*(1/b)**(15/4))+(-1)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4)+sqrt(x))/(2*a**(3/4)*b**4*(1/b)**(15/4))-(-1)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(a**(3/4)*b**4*(1/b)**(15/4)), True)`

**GIAC/XCAS [A]** time = 0.211964, size = 246, normalized size = 1.28

$$\frac{\sqrt{2} (ab^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2ab} + \frac{\sqrt{2} (ab^3)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2ab} + \frac{\sqrt{2} (ab^3)^{\frac{1}{4}} \ln\left(\sqrt{2}\sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4ab} - \frac{\sqrt{2} (ab^3)^{\frac{1}{4}} \ln\left(-\sqrt{2}\sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2+a)*sqrt(x)), x, algorithm="giac")`

[Out]  $1/2 \cdot \sqrt{2} \cdot (a \cdot b^3)^{1/4} \cdot \arctan(1/2 \cdot \sqrt{2} \cdot (\sqrt{2} \cdot (a/b)^{1/4} + 2 \cdot \sqrt{x}) / (a/b)^{1/4}) / (a \cdot b) + 1/2 \cdot \sqrt{2} \cdot (a \cdot b^3)^{1/4} \cdot \arctan(-1/2 \cdot \sqrt{2} \cdot (\sqrt{2} \cdot (a/b)^{1/4} - 2 \cdot \sqrt{x}) / (a/b)^{1/4}) / (a \cdot b) + 1/4 \cdot \sqrt{2} \cdot (a \cdot b^3)^{1/4} \cdot \ln(\sqrt{2} \cdot \sqrt{x} \cdot (a/b)^{1/4} + x + \sqrt{a/b}) / (a \cdot b) - 1/4 \cdot \sqrt{2} \cdot (a \cdot b^3)^{1/4} \cdot \ln(-\sqrt{2} \cdot \sqrt{x} \cdot (a/b)^{1/4} + x + \sqrt{a/b}) / (a \cdot b)$

$$3.293 \quad \int \frac{1}{x^{3/2}(a+bx^2)} dx$$

**Optimal.** Leaf size=202

$$\begin{aligned} & -\frac{\sqrt[4]{b} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}a^{5/4}} + \frac{\sqrt[4]{b} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}a^{5/4}} \\ & + \frac{\sqrt[4]{b} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}} - \frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{\sqrt{2}a^{5/4}} - \frac{2}{a\sqrt{x}} \end{aligned}$$

[Out]  $-2/(a*\text{Sqrt}[x]) + (b^{(1/4)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/( \text{Sqrt}[2]*a^{(5/4)} ) - (b^{(1/4)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/( \text{Sqrt}[2]*a^{(5/4)} ) - (b^{(1/4)}*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(2*\text{Sqrt}[2]*a^{(5/4)}) + (b^{(1/4)}*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(2*\text{Sqrt}[2]*a^{(5/4)})$

**Rubi [A]** time = 0.380668, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$

$$\begin{aligned} & -\frac{\sqrt[4]{b} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}a^{5/4}} + \frac{\sqrt[4]{b} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}a^{5/4}} \\ & + \frac{\sqrt[4]{b} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}} - \frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{\sqrt{2}a^{5/4}} - \frac{2}{a\sqrt{x}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^{(3/2)}*(a + b*x^2)), x]$

[Out]  $-2/(a*\text{Sqrt}[x]) + (b^{(1/4)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/( \text{Sqrt}[2]*a^{(5/4)} ) - (b^{(1/4)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/( \text{Sqrt}[2]*a^{(5/4)} ) - (b^{(1/4)}*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(2*\text{Sqrt}[2]*a^{(5/4)}) + (b^{(1/4)}*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(2*\text{Sqrt}[2]*a^{(5/4)})$

**Rubi in Sympy [A]** time = 63.8511, size = 190, normalized size = 0.94

$$\begin{aligned} & -\frac{2}{a\sqrt{x}} - \frac{\sqrt{2}\sqrt[4]{b} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{4a^{5/4}} + \frac{\sqrt{2}\sqrt[4]{b} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{4a^{5/4}} \\ & + \frac{\sqrt{2}\sqrt[4]{b} \text{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{5/4}} - \frac{\sqrt{2}\sqrt[4]{b} \text{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{5/4}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{(3/2)}/(b*x^2+a), x)$

[Out]  $-2/(a*\text{sqrt}(x)) - \text{sqrt}(2)*b^{(1/4)}*\log(-\text{sqrt}(2)*a^{(1/4)}*b^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(a) + \text{sqrt}(b)*x)/(4*a^{(5/4)}) + \text{sqrt}(2)*b^{(1/4)}*\log(\text{sqrt}(2)*a^{(1/4)}*b^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(a) + \text{sqrt}(b)*x)/(4*a^{(5/4)}) + \text{sqrt}(2)*b^{(1/4)}*\text{atan}(1 - \text{sqrt}(2)*b^{(1/4)}*\text{sqrt}(x)/a^{(1/4)})/(2*a^{(5/4)}) - \text{sqrt}(2)*b^{(1/4)}*\text{atan}(1 + \text{sqrt}(2)*b^{(1/4)}*\text{sqrt}(x)/a^{(1/4)})/(2*a^{(5/4)})$

**Mathematica [A]** time = 0.110088, size = 189, normalized size = 0.94

$$\frac{-\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{bx}\right)+\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{bx}\right)+2\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-2\sqrt{2}\sqrt[4]{b}}{4a^{5/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(3/2)\*(a + b\*x^2)), x]

[Out] ((-8\*a^(1/4))/Sqrt[x] + 2\*Sqrt[2]\*b^(1/4)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)] - 2\*Sqrt[2]\*b^(1/4)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)] - Sqrt[2]\*b^(1/4)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x] + Sqrt[2]\*b^(1/4)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(4\*a^(5/4))

**Maple [A]** time = 0.01, size = 140, normalized size = 0.7

$$-\frac{\sqrt{2}}{4a}\ln\left(1\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

$$-\frac{\sqrt{2}}{2a}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-\frac{\sqrt{2}}{2a}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}-2\frac{1}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(3/2)/(b\*x^2+a), x)

[Out] -1/4/a/(a/b)^(1/4)\*2^(1/2)\*ln((x-(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2)))-1/2/a/(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)+1)-1/2/a/(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)-1)-2/a/x^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)\*x^(3/2)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.229676, size = 180, normalized size = 0.89

$$\frac{4a\sqrt{x}\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\arctan\left(\frac{a^4\left(-\frac{b}{a^5}\right)^{\frac{3}{4}}}{b\sqrt{x}+\sqrt{-a^3b\sqrt{-\frac{b}{a^5}}+b^2x}}\right)+a\sqrt{x}\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\log\left(a^4\left(-\frac{b}{a^5}\right)^{\frac{3}{4}}+b\sqrt{x}\right)-a\sqrt{x}\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\log\left(-a^4\left(-\frac{b}{a^5}\right)^{\frac{3}{4}}+b\sqrt{x}\right)}{2a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)\*x^(3/2)), x, algorithm="fricas")

[Out]  $-1/2 \cdot (4 \cdot a \cdot \sqrt{x}) \cdot (-b/a^5)^{1/4} \cdot \arctan(a^4 \cdot (-b/a^5)^{3/4} / (b \cdot \sqrt{x} + \sqrt{-a^3 \cdot b \cdot \sqrt{-b/a^5} + b^2 \cdot x})) + a \cdot \sqrt{x} \cdot (-b/a^5)^{1/4} \cdot \log(a^4 \cdot (-b/a^5)^{3/4} + b \cdot \sqrt{x}) - a \cdot \sqrt{x} \cdot (-b/a^5)^{1/4} \cdot \log(-a^4 \cdot (-b/a^5)^{3/4} + b \cdot \sqrt{x}) + 4) / (a \cdot \sqrt{x})$

**Sympy [A]** time = 66.4366, size = 180, normalized size = 0.89

$$\begin{cases} \frac{8}{x^{3/2}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{5bx^{5/2}} & \text{for } a = 0 \\ -\frac{2}{a\sqrt{x}} & \text{for } b = 0 \\ -\frac{2}{a\sqrt{x}} + \frac{(-1)^{3/4} \log\left(-\sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b} + \sqrt{x}}\right)}{2a^{5/4} b^{11} \left(\frac{1}{b}\right)^{45/4}} - \frac{(-1)^{3/4} \log\left(\sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b} + \sqrt{x}}\right)}{2a^{5/4} b^{11} \left(\frac{1}{b}\right)^{45/4}} - \frac{(-1)^{3/4} \operatorname{atan}\left(\frac{(-1)^{3/4} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}}\right)}{a^{5/4} b^{11} \left(\frac{1}{b}\right)^{45/4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**(3/2)/(b*x**2+a), x)`

[Out] `Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a, 0)), (-2/(a*sqrt(x)), Eq(b, 0)), (-2/(a*sqrt(x)) + (-1)**(3/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(5/4)*b**11*(1/b)**(45/4)) - (-1)**(3/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(5/4)*b**11*(1/b)**(45/4)) - (-1)**(3/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(a**(5/4)*b**11*(1/b)**(45/4)), True))`

**GIAC/XCAS [A]** time = 0.214661, size = 257, normalized size = 1.27

$$\begin{aligned} & -\frac{2}{a\sqrt{x}} - \frac{\sqrt{2}(ab^3)^{3/4} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{1/4} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{1/4}}\right)}{2a^2b^2} - \frac{\sqrt{2}(ab^3)^{3/4} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{1/4} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{1/4}}\right)}{2a^2b^2} \\ & + \frac{\sqrt{2}(ab^3)^{3/4} \ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{1/4} + x + \sqrt{\frac{a}{b}}\right)}{4a^2b^2} - \frac{\sqrt{2}(ab^3)^{3/4} \ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{1/4} + x + \sqrt{\frac{a}{b}}\right)}{4a^2b^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2+ a)*x^(3/2)),x, algorithm="giac")`

[Out]  $-2/(a \cdot \sqrt{x}) - 1/2 \cdot \sqrt{2} \cdot (a \cdot b^3)^{3/4} \cdot \arctan(1/2 \cdot \sqrt{2} \cdot (\sqrt{2} \cdot (a/b)^{1/4} + 2 \cdot \sqrt{x}) / (a/b)^{1/4}) / (a^2 \cdot b^2) - 1/2 \cdot \sqrt{2} \cdot (a \cdot b^3)^{3/4} \cdot \arctan(-1/2 \cdot \sqrt{2} \cdot (\sqrt{2} \cdot (a/b)^{1/4} - 2 \cdot \sqrt{x}) / (a/b)^{1/4}) / (a^2 \cdot b^2) + 1/4 \cdot \sqrt{2} \cdot (a \cdot b^3)^{3/4} \cdot \ln(\sqrt{2} \cdot \sqrt{x} \cdot (a/b)^{1/4} + x + \sqrt{a/b}) / (a^2 \cdot b^2) - 1/4 \cdot \sqrt{2} \cdot (a \cdot b^3)^{3/4} \cdot \ln(-\sqrt{2} \cdot \sqrt{x} \cdot (a/b)^{1/4} + x + \sqrt{a/b}) / (a^2 \cdot b^2)$

$$3.294 \quad \int \frac{1}{x^{5/2}(a+bx^2)} dx$$

**Optimal.** Leaf size=204

$$\frac{b^{3/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}a^{7/4}} - \frac{b^{3/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}a^{7/4}} \\ + \frac{b^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}} - \frac{b^{3/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{\sqrt{2}a^{7/4}} - \frac{2}{3ax^{3/2}}$$

[Out]  $-2/(3*a*x^{(3/2)}) + (b^{(3/4)}*ArcTan[1 - (Sqrt[2]*b^{(1/4)}*Sqrt[x])/a^{(1/4)}])/(Sqrt[2]*a^{(7/4)}) - (b^{(3/4)}*ArcTan[1 + (Sqrt[2]*b^{(1/4)}*Sqrt[x])/a^{(1/4)}])/(Sqrt[2]*a^{(7/4)}) + (b^{(3/4)}*Log[Sqrt[a] - Sqrt[2]*a^{(1/4)}*b^{(1/4)}*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^{(7/4)}) - (b^{(3/4)}*Log[Sqrt[a] + Sqrt[2]*a^{(1/4)}*b^{(1/4)}*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^{(7/4)})$

**Rubi [A]** time = 0.373613, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$

$$\frac{b^{3/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}a^{7/4}} - \frac{b^{3/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}a^{7/4}} \\ + \frac{b^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}} - \frac{b^{3/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{\sqrt{2}a^{7/4}} - \frac{2}{3ax^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(5/2)\*(a + b\*x^2)), x]

[Out]  $-2/(3*a*x^{(3/2)}) + (b^{(3/4)}*ArcTan[1 - (Sqrt[2]*b^{(1/4)}*Sqrt[x])/a^{(1/4)}])/(Sqrt[2]*a^{(7/4)}) - (b^{(3/4)}*ArcTan[1 + (Sqrt[2]*b^{(1/4)}*Sqrt[x])/a^{(1/4)}])/(Sqrt[2]*a^{(7/4)}) + (b^{(3/4)}*Log[Sqrt[a] - Sqrt[2]*a^{(1/4)}*b^{(1/4)}*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^{(7/4)}) - (b^{(3/4)}*Log[Sqrt[a] + Sqrt[2]*a^{(1/4)}*b^{(1/4)}*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^{(7/4)})$

**Rubi in Sympy [A]** time = 62.9354, size = 192, normalized size = 0.94

$$-\frac{2}{3ax^{3/2}} + \frac{\sqrt{2}b^{3/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{4a^{7/4}} - \frac{\sqrt{2}b^{3/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{4a^{7/4}} \\ + \frac{\sqrt{2}b^{3/4} \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{7/4}} - \frac{\sqrt{2}b^{3/4} \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*(5/2)/(b\*x\*\*2+a), x)

[Out]  $-2/(3*a*x^{(3/2)}) + \sqrt{2}*b^{(3/4)}*\log(-\sqrt{2}*a^{(1/4)}*b^{(1/4)}*\sqrt{x} + \sqrt{a} + \sqrt{b}*x)/(4*a^{(7/4)}) - \sqrt{2}*b^{(3/4)}*\log(\sqrt{2}*a^{(1/4)}*b^{(1/4)}*\sqrt{x} + \sqrt{a} + \sqrt{b}*x)/(4*a^{(7/4)}) + \sqrt{2}*b^{(3/4)}*\operatorname{atan}(1 - \sqrt{2}*b^{(1/4)}*\sqrt{x}/a^{(1/4)})/(2*a^{(7/4)}) - \sqrt{2}*b^{(3/4)}*\operatorname{atan}(1 + \sqrt{2}*b^{(1/4)}*\sqrt{x}/a^{(1/4)})/(2*a^{(7/4)})$



**Mathematica [A]** time = 0.131253, size = 190, normalized size = 0.93

$$\frac{-\frac{8a^{3/4}}{x^{3/2}} + 3\sqrt{2}b^{3/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) - 3\sqrt{2}b^{3/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) + 6\sqrt{2}b^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{12a^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(5/2)\*(a + b\*x^2)), x]

[Out] ((-8\*a^(3/4))/x^(3/2) + 6\*Sqrt[2]\*b^(3/4)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)] - 6\*Sqrt[2]\*b^(3/4)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)] + 3\*Sqrt[2]\*b^(3/4)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x] - 3\*Sqrt[2]\*b^(3/4)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(12\*a^(7/4))

**Maple [A]** time = 0.012, size = 143, normalized size = 0.7

$$-\frac{b\sqrt{2}}{4a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x + \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) - \frac{b\sqrt{2}}{2a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) - \frac{b\sqrt{2}}{2a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) - \frac{2}{3a}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(5/2)/(b\*x^2+a), x)

[Out] -1/4\*b/a^2\*(a/b)^(1/4)\*2^(1/2)\*ln((x+(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2)))-1/2\*b/a^2\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)+1)-1/2\*b/a^2\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)-1)-2/3/a/x^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)\*x^(5/2)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.230928, size = 197, normalized size = 0.97

$$\frac{12ax^{\frac{3}{2}}\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}\arctan\left(\frac{a^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}}{b\sqrt{x}+\sqrt{a^4\sqrt{-\frac{b^3}{a^7}}+b^2x}}\right) - 3ax^{\frac{3}{2}}\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}+b\sqrt{x}\right) + 3ax^{\frac{3}{2}}\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}\log\left(-a^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}+b\sqrt{x}\right)}{6ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)\*x^(5/2)), x, algorithm="fricas")

[Out]  $\frac{1}{6} \cdot (12 \cdot a \cdot x^{3/2} \cdot (-b^3/a^7)^{1/4} \cdot \arctan(a^2 \cdot (-b^3/a^7)^{1/4} / (b \cdot \sqrt{x} + \sqrt{a^4 \sqrt{-b^3/a^7} + b^2 x})) - 3 \cdot a \cdot x^{3/2} \cdot (-b^3/a^7)^{1/4} \cdot \log(a^2 \cdot (-b^3/a^7)^{1/4} + b \cdot \sqrt{x}) + 3 \cdot a \cdot x^{3/2} \cdot (-b^3/a^7)^{1/4} \cdot \log(-a^2 \cdot (-b^3/a^7)^{1/4} + b \cdot \sqrt{x}) - 4) / (a \cdot x^{3/2})$

**Sympy [A]** time = 160.845, size = 184, normalized size = 0.9

$$\left\{ \begin{array}{l} \frac{\tilde{8}}{x^{\frac{7}{2}}} \\ -\frac{2}{7bx^{\frac{7}{2}}} \\ -\frac{2}{3ax^{\frac{3}{2}}} \end{array} \right. \begin{array}{l} \text{for } a = 0 \wedge \\ \text{for } a = 0 \\ \text{for } b = 0 \end{array}$$

$$\left\{ \begin{array}{l} -\frac{2}{3ax^{\frac{3}{2}}} + \frac{\sqrt[4]{-1}b^7\left(\frac{1}{b}\right)^{\frac{25}{4}} \log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}+\sqrt{x}}\right)}{2a^{\frac{7}{4}}} - \frac{\sqrt[4]{-1}b^7\left(\frac{1}{b}\right)^{\frac{25}{4}} \log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}+\sqrt{x}}\right)}{2a^{\frac{7}{4}}} + \frac{\sqrt[4]{-1}b^7\left(\frac{1}{b}\right)^{\frac{25}{4}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{7}{4}}} \end{array} \right. \text{otherwise}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(5/2)/(b\*x\*\*2+a), x)

[Out] Piecewise((zoo/x\*\*(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7\*b\*x\*\*(7/2)), Eq(a, 0)), (-2/(3\*a\*x\*\*(3/2)), Eq(b, 0)), (-2/(3\*a\*x\*\*(3/2)) + (-1)\*\*(1/4)\*b\*\*7\*(1/b)\*\*(25/4)\*log(-(-1)\*\*(1/4)\*a\*\*(1/4)\*(1/b)\*\*(1/4) + sqrt(x))/(2\*a\*\*(7/4)) - (-1)\*\*(1/4)\*b\*\*7\*(1/b)\*\*(25/4)\*log((-1)\*\*(1/4)\*a\*\*(1/4)\*(1/b)\*\*(1/4) + sqrt(x))/(2\*a\*\*(7/4)) + (-1)\*\*(1/4)\*b\*\*7\*(1/b)\*\*(25/4)\*atan((-1)\*\*(3/4)\*sqrt(x)/(a\*\*(1/4)\*(1/b)\*\*(1/4)))/a\*\*(7/4), True))

**GIAC/XCAS [A]** time = 0.224249, size = 240, normalized size = 1.18

$$\frac{\sqrt{2}(ab^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^2} - \frac{\sqrt{2}(ab^3)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^2}$$

$$- \frac{\sqrt{2}(ab^3)^{\frac{1}{4}} \ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^2} + \frac{\sqrt{2}(ab^3)^{\frac{1}{4}} \ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^2} - \frac{2}{3ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)\*x^(5/2)),x, algorithm="giac")

[Out]  $-1/2 \cdot \sqrt{2} \cdot (a \cdot b^3)^{1/4} \cdot \arctan(1/2 \cdot \sqrt{2} \cdot (\sqrt{2} \cdot (a/b)^{1/4} + 2 \cdot \sqrt{x}) / (a/b)^{1/4}) / a^2 - 1/2 \cdot \sqrt{2} \cdot (a \cdot b^3)^{1/4} \cdot \arctan(-1/2 \cdot \sqrt{2} \cdot (\sqrt{2} \cdot (a/b)^{1/4} - 2 \cdot \sqrt{x}) / (a/b)^{1/4}) / a^2 - 1/4 \cdot \sqrt{2} \cdot (a \cdot b^3)^{1/4} \cdot \ln(\sqrt{2} \cdot \sqrt{x} \cdot (a/b)^{1/4} + x + \sqrt{a/b}) / a^2 + 1/4 \cdot \sqrt{2} \cdot (a \cdot b^3)^{1/4} \cdot \ln(-\sqrt{2} \cdot \sqrt{x} \cdot (a/b)^{1/4} + x + \sqrt{a/b}) / a^2 - 2/3 / (a \cdot x^{3/2})$

$$3.295 \quad \int \frac{1}{x^{7/2}(a+bx^2)} dx$$

**Optimal.** Leaf size=215

$$\frac{b^{5/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}a^{9/4}} - \frac{b^{5/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}a^{9/4}} \\ - \frac{b^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}} + \frac{b^{5/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{\sqrt{2}a^{9/4}} + \frac{2b}{a^2\sqrt{x}} - \frac{2}{5ax^{5/2}}$$

[Out]  $-2/(5*a*x^{(5/2)}) + (2*b)/(a^2*\text{Sqrt}[x]) - (b^{(5/4)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/( \text{Sqrt}[2]*a^{(9/4)}) + (b^{(5/4)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/( \text{Sqrt}[2]*a^{(9/4)}) + (b^{(5/4)}*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(2*\text{Sqrt}[2]*a^{(9/4)}) - (b^{(5/4)}*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(2*\text{Sqrt}[2]*a^{(9/4)})$

**Rubi [A]** time = 0.435769, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$

$$\frac{b^{5/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}a^{9/4}} - \frac{b^{5/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{2\sqrt{2}a^{9/4}} \\ - \frac{b^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}} + \frac{b^{5/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{\sqrt{2}a^{9/4}} + \frac{2b}{a^2\sqrt{x}} - \frac{2}{5ax^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(7/2)\*(a + b\*x^2)), x]

[Out]  $-2/(5*a*x^{(5/2)}) + (2*b)/(a^2*\text{Sqrt}[x]) - (b^{(5/4)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/( \text{Sqrt}[2]*a^{(9/4)}) + (b^{(5/4)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/( \text{Sqrt}[2]*a^{(9/4)}) + (b^{(5/4)}*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(2*\text{Sqrt}[2]*a^{(9/4)}) - (b^{(5/4)}*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(2*\text{Sqrt}[2]*a^{(9/4)})$

**Rubi in SymPy [A]** time = 71.1265, size = 204, normalized size = 0.95

$$-\frac{2}{5ax^{5/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{\sqrt{2}b^{5/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{4a^{9/4}} - \frac{\sqrt{2}b^{5/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{4a^{9/4}} \\ - \frac{\sqrt{2}b^{5/4} \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{9/4}} + \frac{\sqrt{2}b^{5/4} \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2a^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*(7/2)/(b\*x\*\*2+a), x)

[Out]  $-2/(5*a*x^{(5/2)}) + 2*b/(a**2*\text{sqrt}(x)) + \text{sqrt}(2)*b^{(5/4)}*\log(-\text{sqrt}(2)*a^{(1/4)}*b^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(a) + \text{sqrt}(b)*x)/(4*a^{(9/4)}) - \text{sqrt}(2)*b^{(5/4)}*\log(\text{sqrt}(2)*a^{(1/4)}*b^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(a) + \text{sqrt}(b)*x)/(4*a^{(9/4)}) - \text{sqrt}(2)*b^{(5/4)}*\operatorname{atan}(1 - \text{sqrt}(2)*b^{(1/4)}*\text{sqrt}(x)/a^{(1/4)})/(2*a^{(9/4)}) + \text{sqrt}(2)*b^{(5/4)}*\operatorname{atan}(1 + \text{sqrt}(2)*b^{(1/4)}*\text{sqrt}(x)/a^{(1/4)})/(2*a^{(9/4)})$

**Mathematica [A]** time = 0.142279, size = 203, normalized size = 0.94

$$\frac{-\frac{8a^{5/4}}{x^{5/2}} + 5\sqrt{2}b^{5/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) - 5\sqrt{2}b^{5/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) - 10\sqrt{2}b^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{20a^{9/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(7/2)\*(a + b\*x^2)), x]

[Out] ((-8\*a^(5/4))/x^(5/2) + (40\*a^(1/4)\*b)/Sqrt[x] - 10\*Sqrt[2]\*b^(5/4)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)] + 10\*Sqrt[2]\*b^(5/4)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)] + 5\*Sqrt[2]\*b^(5/4)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x] - 5\*Sqrt[2]\*b^(5/4)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(20\*a^(9/4))

**Maple [A]** time = 0.015, size = 152, normalized size = 0.7

$$-\frac{2}{5a}x^{-\frac{5}{2}} + 2\frac{b}{a^2\sqrt{x}} + \frac{b\sqrt{2}}{4a^2} \ln\left(1\left(x - \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x + \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{b\sqrt{2}}{2a^2} \arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) \frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{b\sqrt{2}}{2a^2} \arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) \frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(7/2)/(b\*x^2+a), x)

[Out] -2/5/a/x^(5/2)+2\*b/a^2/x^(1/2)+1/4\*b/a^2/(a/b)^(1/4)\*2^(1/2)\*ln((x-(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2)))+1/2\*b/a^2/(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)+1)+1/2\*b/a^2/(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)-1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)\*x^(7/2)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.271443, size = 230, normalized size = 1.07

$$\frac{20a^2x^{\frac{5}{2}}\left(-\frac{b^5}{a^9}\right)^{\frac{1}{4}} \arctan\left(\frac{a^7\left(-\frac{b^5}{a^9}\right)^{\frac{3}{4}}}{b^4\sqrt{x} + \sqrt{-a^5b^5\sqrt{-\frac{b^5}{a^9}} + b^8x}}\right) + 5a^2x^{\frac{5}{2}}\left(-\frac{b^5}{a^9}\right)^{\frac{1}{4}} \log\left(a^7\left(-\frac{b^5}{a^9}\right)^{\frac{3}{4}} + b^4\sqrt{x}\right) - 5a^2x^{\frac{5}{2}}\left(-\frac{b^5}{a^9}\right)^{\frac{1}{4}} \log\left(-a^7\left(-\frac{b^5}{a^9}\right)^{\frac{3}{4}} + b^4\sqrt{x}\right)}{10a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)\*x^(7/2)), x, algorithm="fricas")

[Out]  $1/10 * (20 * a^2 * x^{(5/2)} * (-b^5/a^9)^{(1/4)} * \arctan(a^7 * (-b^5/a^9)^{(3/4)} / (b^4 * \sqrt{x} + \sqrt{-a^5 * b^5 * \sqrt{-b^5/a^9} + b^8 * x})) + 5 * a^2 * x^{(5/2)} * (-b^5/a^9)^{(1/4)} * \log(a^7 * (-b^5/a^9)^{(3/4)} + b^4 * \sqrt{x}) - 5 * a^2 * x^{(5/2)} * (-b^5/a^9)^{(1/4)} * \log(-a^7 * (-b^5/a^9)^{(3/4)} + b^4 * \sqrt{x}) + 20 * b * x^2 - 4 * a) / (a^2 * x^{(5/2)})$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**(7/2)/(b*x**2+a), x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.220611, size = 270, normalized size = 1.26

$$\frac{\sqrt{2} (ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^3b} + \frac{\sqrt{2} (ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^3b} - \frac{\sqrt{2} (ab^3)^{\frac{3}{4}} \ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4a^3b} + \frac{\sqrt{2} (ab^3)^{\frac{3}{4}} \ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4a^3b} + \frac{2(5bx^2 - a)}{5a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)*x^(7/2)), x, algorithm="giac")`

[Out]  $1/2 * \sqrt{2} * (a * b^3)^{(3/4)} * \arctan(1/2 * \sqrt{2} * (\sqrt{2} * (a/b)^{(1/4)} + 2 * \sqrt{x}) / (a/b)^{(1/4)}) / (a^3 * b) + 1/2 * \sqrt{2} * (a * b^3)^{(3/4)} * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} * (a/b)^{(1/4)} - 2 * \sqrt{x}) / (a/b)^{(1/4)}) / (a^3 * b) - 1/4 * \sqrt{2} * (a * b^3)^{(3/4)} * \ln(\sqrt{2} * \sqrt{x} * (a/b)^{(1/4)} + x + \sqrt{a/b}) / (a^3 * b) + 1/4 * \sqrt{2} * (a * b^3)^{(3/4)} * \ln(-\sqrt{2} * \sqrt{x} * (a/b)^{(1/4)} + x + \sqrt{a/b}) / (a^3 * b) + 2/5 * (5 * b * x^2 - a) / (a^2 * x^{(5/2)})$

$$3.296 \quad \int \frac{x^{7/2}}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=230

$$\frac{5\sqrt[4]{a} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}b^{9/4}} - \frac{5\sqrt[4]{a} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}b^{9/4}} \\ + \frac{5\sqrt[4]{a} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{9/4}} - \frac{5\sqrt[4]{a} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}b^{9/4}} - \frac{x^{5/2}}{2b(a+bx^2)} + \frac{5\sqrt{x}}{2b^2}$$

[Out] (5\*Sqrt[x])/(2\*b^2) - x^(5/2)/(2\*b\*(a + b\*x^2)) + (5\*a^(1/4)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(4\*Sqrt[2]\*b^(9/4)) - (5\*a^(1/4)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(4\*Sqrt[2]\*b^(9/4)) + (5\*a^(1/4)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(8\*Sqrt[2]\*b^(9/4)) - (5\*a^(1/4)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(8\*Sqrt[2]\*b^(9/4))

**Rubi [A]** time = 0.425478, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$

$$\frac{5\sqrt[4]{a} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}b^{9/4}} - \frac{5\sqrt[4]{a} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}b^{9/4}} \\ + \frac{5\sqrt[4]{a} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{9/4}} - \frac{5\sqrt[4]{a} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}b^{9/4}} - \frac{x^{5/2}}{2b(a+bx^2)} + \frac{5\sqrt{x}}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[x^(7/2)/(a + b\*x^2)^2, x]

[Out] (5\*Sqrt[x])/(2\*b^2) - x^(5/2)/(2\*b\*(a + b\*x^2)) + (5\*a^(1/4)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(4\*Sqrt[2]\*b^(9/4)) - (5\*a^(1/4)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(4\*Sqrt[2]\*b^(9/4)) + (5\*a^(1/4)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(8\*Sqrt[2]\*b^(9/4)) - (5\*a^(1/4)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(8\*Sqrt[2]\*b^(9/4))

**Rubi in Sympy [A]** time = 70.1972, size = 216, normalized size = 0.94

$$\frac{5\sqrt{2}\sqrt[4]{a} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{16b^{9/4}} - \frac{5\sqrt{2}\sqrt[4]{a} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{16b^{9/4}} \\ + \frac{5\sqrt{2}\sqrt[4]{a} \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8b^{9/4}} - \frac{5\sqrt{2}\sqrt[4]{a} \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8b^{9/4}} - \frac{x^{5/2}}{2b(a+bx^2)} + \frac{5\sqrt{x}}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(7/2)/(b\*x\*\*2+a)\*\*2, x)

[Out] 5\*sqrt(2)\*a\*\*(1/4)\*log(-sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*sqrt(x) + sqrt(a) + sqrt(b)\*x)/(16\*b\*\*(9/4)) - 5\*sqrt(2)\*a\*\*(1/4)\*log(sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*sqrt(x) + sqrt(a) + sqrt(b)\*x)/(16\*b\*\*(9/4)) + 5\*sqrt(2)\*a\*\*(1/4)\*atan(1 - sqrt(2)\*b\*\*(1/4)\*sqrt(x)/a\*\*(1/4))/(8\*b\*\*(9/4)) - 5\*sqrt(2)\*a\*\*(1/4)\*atan(1 + sqrt(2)\*b\*\*(1/4)\*sqrt(x)/a\*\*(1/4))/(8\*b\*\*(9/4)) - x\*\*(5/2)/(2\*b\*(a + b\*x\*\*2)) + 5\*sqrt(x)/(

$2 * b^{** 2}$ )

**Mathematica [A]** time = 0.238733, size = 212, normalized size = 0.92

$$\frac{8a\sqrt[4]{b}\sqrt{x}}{a+bx^2} + 5\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) - 5\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) + 10\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)$$


---


$$16b^{9/4}$$

Antiderivative was successfully verified.

[In] Integrate[x^(7/2)/(a + b\*x^2)^2, x]

[Out] (32\*b^(1/4)\*Sqrt[x] + (8\*a\*b^(1/4)\*Sqrt[x])/(a + b\*x^2) + 10\*Sqrt[2]\*a^(1/4)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)] - 10\*Sqrt[2]\*a^(1/4)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)] + 5\*Sqrt[2]\*a^(1/4)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x] - 5\*Sqrt[2]\*a^(1/4)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(16\*b^(9/4))

**Maple [A]** time = 0.018, size = 158, normalized size = 0.7

$$2\frac{\sqrt{x}}{b^2} + \frac{a}{2b^2(bx^2+a)}\sqrt{x} - \frac{5\sqrt{2}}{16b^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x + \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right)$$

$$- \frac{5\sqrt{2}}{8b^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) - \frac{5\sqrt{2}}{8b^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)/(b\*x^2+a)^2, x)

[Out] 2\*x^(1/2)/b^2+1/2\*a/b^2\*x^(1/2)/(b\*x^2+a)-5/16/b^2\*(a/b)^(1/4)\*2^(1/2)\*ln((x+(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2)))-5/8/b^2\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)+1)-5/8/b^2\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)-1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(b\*x^2 + a)^2, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.288531, size = 239, normalized size = 1.04

$$20(b^3x^2 + ab^2)\left(-\frac{a}{b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{b^2\left(-\frac{a}{b^9}\right)^{\frac{1}{4}}}{\sqrt{b^4\sqrt{-\frac{a}{b^9}}+x+\sqrt{x}}}\right) - 5(b^3x^2 + ab^2)\left(-\frac{a}{b^9}\right)^{\frac{1}{4}}\log\left(5b^2\left(-\frac{a}{b^9}\right)^{\frac{1}{4}} + 5\sqrt{x}\right) + 5(b^3x^2 + ab^2)\left(-\frac{a}{b^9}\right)^{\frac{1}{4}}$$


---


$$8(b^3x^2 + ab^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(b\*x^2 + a)^2,x, algorithm="fricas")

[Out]  $\frac{1}{8} \cdot (20 \cdot (b^3 x^2 + a b^2) \cdot (-a/b^9)^{1/4} \cdot \arctan(b^2 \cdot (-a/b^9)^{1/4} / (\sqrt{b^4 \sqrt{-a/b^9} + x} + \sqrt{x})) - 5 \cdot (b^3 x^2 + a b^2) \cdot (-a/b^9)^{1/4} \cdot \log(5 \cdot b^2 \cdot (-a/b^9)^{1/4} + 5 \cdot \sqrt{x}) + 5 \cdot (b^3 x^2 + a b^2) \cdot (-a/b^9)^{1/4} \cdot \log(-5 \cdot b^2 \cdot (-a/b^9)^{1/4} + 5 \cdot \sqrt{x}) + 4 \cdot (4 \cdot b x^2 + 5 \cdot a) \cdot \sqrt{x}) / (b^3 x^2 + a b^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(7/2)/(b\*x\*\*2+a)\*\*2,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.218843, size = 265, normalized size = 1.15

$$\begin{aligned} & \frac{5 \sqrt{2} (ab^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8 b^3} - \frac{5 \sqrt{2} (ab^3)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8 b^3} \\ & - \frac{5 \sqrt{2} (ab^3)^{\frac{1}{4}} \ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16 b^3} \\ & + \frac{5 \sqrt{2} (ab^3)^{\frac{1}{4}} \ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16 b^3} + \frac{a\sqrt{x}}{2(bx^2+a)b^2} + \frac{2\sqrt{x}}{b^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(b\*x^2 + a)^2,x, algorithm="giac")

[Out]  $-5/8 \cdot \sqrt{2} \cdot (a \cdot b^3)^{1/4} \cdot \arctan(1/2 \cdot \sqrt{2} \cdot (\sqrt{2} \cdot (a/b)^{1/4} + 2 \cdot \sqrt{x}) / (a/b)^{1/4}) / b^3 - 5/8 \cdot \sqrt{2} \cdot (a \cdot b^3)^{1/4} \cdot \arctan(-1/2 \cdot \sqrt{2} \cdot (\sqrt{2} \cdot (a/b)^{1/4} - 2 \cdot \sqrt{x}) / (a/b)^{1/4}) / b^3 - 5/16 \cdot \sqrt{2} \cdot (a \cdot b^3)^{1/4} \cdot \ln(\sqrt{2} \cdot \sqrt{x} \cdot (a/b)^{1/4} + x + \sqrt{a/b}) / b^3 + 5/16 \cdot \sqrt{2} \cdot (a \cdot b^3)^{1/4} \cdot \ln(-\sqrt{2} \cdot \sqrt{x} \cdot (a/b)^{1/4} + x + \sqrt{a/b}) / b^3 + 1/2 \cdot a \cdot \sqrt{x} / ((b \cdot x^2 + a) \cdot b^2) + 2 \cdot \sqrt{x} / b^2$



$$3.297 \quad \int \frac{x^{5/2}}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=218

$$\frac{3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}\sqrt[4]{ab}^{7/4}} - \frac{3 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}\sqrt[4]{ab}^{7/4}} \\ - \frac{3 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{ab}^{7/4}} + \frac{3 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}\sqrt[4]{ab}^{7/4}} - \frac{x^{3/2}}{2b(a+bx^2)}$$

[Out]  $-x^{3/2}/(2*b*(a+b*x^2)) - (3*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{1/4})*\text{Sqrt}[x])/a^{1/4}]/(4*\text{Sqrt}[2]*a^{1/4}*b^{7/4}) + (3*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{1/4})*\text{Sqrt}[x])/a^{1/4}]/(4*\text{Sqrt}[2]*a^{1/4}*b^{7/4}) + (3*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*b^{1/4}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(8*\text{Sqrt}[2]*a^{1/4}*b^{7/4}) - (3*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*b^{1/4}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(8*\text{Sqrt}[2]*a^{1/4}*b^{7/4})$

**Rubi [A]** time = 0.381215, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$

$$\frac{3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}\sqrt[4]{ab}^{7/4}} - \frac{3 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}\sqrt[4]{ab}^{7/4}} \\ - \frac{3 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{ab}^{7/4}} + \frac{3 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}\sqrt[4]{ab}^{7/4}} - \frac{x^{3/2}}{2b(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)/(a + b\*x^2)^2, x]

[Out]  $-x^{3/2}/(2*b*(a+b*x^2)) - (3*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{1/4})*\text{Sqrt}[x])/a^{1/4}]/(4*\text{Sqrt}[2]*a^{1/4}*b^{7/4}) + (3*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{1/4})*\text{Sqrt}[x])/a^{1/4}]/(4*\text{Sqrt}[2]*a^{1/4}*b^{7/4}) + (3*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*b^{1/4}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(8*\text{Sqrt}[2]*a^{1/4}*b^{7/4}) - (3*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*b^{1/4}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(8*\text{Sqrt}[2]*a^{1/4}*b^{7/4})$

**Rubi in Sympy [A]** time = 62.8104, size = 204, normalized size = 0.94

$$-\frac{x^{3/2}}{2b(a+bx^2)} + \frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{16\sqrt[4]{ab}^{7/4}} - \frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{16\sqrt[4]{ab}^{7/4}} \\ - \frac{3\sqrt{2}\operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8\sqrt[4]{ab}^{7/4}} + \frac{3\sqrt{2}\operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8\sqrt[4]{ab}^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(5/2)/(b\*x\*\*2+a)\*\*2, x)

[Out]  $-x^{3/2}/(2*b*(a+b*x^2)) + 3*\text{sqrt}(2)*\log(-\text{sqrt}(2)*a^{1/4}*b^{1/4}*\text{sqrt}(x) + \text{sqrt}(a) + \text{sqrt}(b)*x)/(16*a^{1/4}*b^{7/4}) - 3*\text{sqrt}(2)*\log(\text{sqrt}(2)*a^{1/4}*b^{1/4}*\text{sqrt}(x) + \text{sqrt}(a) + \text{sqrt}(b)*x)/(16*a^{1/4}*b^{7/4}) - 3*\text{sqrt}(2)*\operatorname{atan}(1 - \text{sqrt}(2)*b^{1/4}*\text{sqrt}(x)/a^{1/4})/(8*a^{1/4}*b^{7/4}) + 3*\text{sqrt}(2)*\operatorname{atan}(1 + \text{sqrt}(2)*b^{1/4}*\text{sqrt}(x)/a^{1/4})/(8*a^{1/4}*b^{7/4})$

**Mathematica [A]** time = 0.247874, size = 199, normalized size = 0.91

$$\frac{-\frac{8b^{3/4}x^{3/2}}{a+bx^2} + \frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x+\sqrt{a}+\sqrt{bx}}\right)}{\sqrt[4]{a}} - \frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x+\sqrt{a}+\sqrt{bx}}\right)}{\sqrt[4]{a}} - \frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}} + \frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{a}}}{16b^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)/(a + b\*x^2)^2, x]

[Out]  $\left(\frac{-8b^{3/4}x^{3/2}}{a+bx^2} - \frac{6\sqrt{2}\operatorname{ArcTan}\left[1 - \left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right]}{a^{1/4}} + \frac{6\sqrt{2}\operatorname{ArcTan}\left[1 + \left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right]}{a^{1/4}} + \frac{3\sqrt{2}\operatorname{Log}\left[\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right]}{a^{1/4}} - \frac{3\sqrt{2}\operatorname{Log}\left[\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right]}{a^{1/4}} - \frac{3\sqrt{2}\operatorname{Log}\left[\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right]}{a^{1/4}}\right) / (16b^{7/4})$

**Maple [A]** time = 0.016, size = 149, normalized size = 0.7

$$-\frac{1}{2b(bx^2+a)}x^{\frac{3}{2}} + \frac{3\sqrt{2}}{16b^2}\ln\left(1\left(x - \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x + \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{3\sqrt{2}}{8b^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{3\sqrt{2}}{8b^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)/(b\*x^2+a)^2, x)

[Out]  $\frac{-1/2*x^{3/2}/b/(b*x^2+a)+3/16/b^2/(a/b)^{1/4}*2^{1/2}*ln((x-(a/b)^{1/4}*x^{1/2}*2^{1/2}+(a/b)^{1/2}))/((x+(a/b)^{1/4}*x^{1/2}*2^{1/2}+(a/b)^{1/2}))+3/8/b^2/(a/b)^{1/4}*2^{1/2}*arctan(2^{1/2}/(a/b)^{1/4}*x^{1/2}+1)+3/8/b^2/(a/b)^{1/4}*2^{1/2}*arctan(2^{1/2}/(a/b)^{1/4}*x^{1/2}-1)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(b\*x^2 + a)^2, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.256784, size = 234, normalized size = 1.07

$$\frac{12(b^2x^2+ab)\left(-\frac{1}{ab^7}\right)^{\frac{1}{4}}\arctan\left(\frac{ab^5\left(-\frac{1}{ab^7}\right)^{\frac{3}{4}}}{\sqrt{-ab^3\sqrt{-\frac{1}{ab^7}}+x+\sqrt{x}}}\right)+3(b^2x^2+ab)\left(-\frac{1}{ab^7}\right)^{\frac{1}{4}}\log\left(ab^5\left(-\frac{1}{ab^7}\right)^{\frac{3}{4}}+\sqrt{x}\right)-3(b^2x^2+ab)\left(-\frac{1}{ab^7}\right)^{\frac{1}{4}}\log\left(ab^5\left(-\frac{1}{ab^7}\right)^{\frac{3}{4}}-\sqrt{x}\right)}{8(b^2x^2+ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(b\*x^2 + a)^2,x, algorithm="fricas")

[Out]  $\frac{1}{8} \cdot (12 \cdot (b^2 x^2 + a \cdot b) \cdot (-1/(a \cdot b^7))^{1/4} \cdot \arctan(a \cdot b^5 \cdot (-1/(a \cdot b^7))^{3/4}) / (\sqrt{-a \cdot b^3 \cdot \sqrt{-1/(a \cdot b^7)}} + x) + \sqrt{x}) + 3 \cdot (b^2 x^2 + a \cdot b) \cdot (-1/(a \cdot b^7))^{1/4} \cdot \log(a \cdot b^5 \cdot (-1/(a \cdot b^7))^{3/4} + \sqrt{x}) - 3 \cdot (b^2 x^2 + a \cdot b) \cdot (-1/(a \cdot b^7))^{1/4} \cdot \log(-a \cdot b^5 \cdot (-1/(a \cdot b^7))^{3/4} + \sqrt{x}) - 4 \cdot x^{3/2}) / (b^2 x^2 + a \cdot b)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(5/2)/(b\*x\*\*2+a)\*\*2,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.225979, size = 269, normalized size = 1.23

$$\begin{aligned} & \frac{x^{\frac{3}{2}}}{2(bx^2 + a)b} + \frac{3\sqrt{2}(ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8ab^4} + \frac{3\sqrt{2}(ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8ab^4} \\ & - \frac{3\sqrt{2}(ab^3)^{\frac{3}{4}} \ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16ab^4} + \frac{3\sqrt{2}(ab^3)^{\frac{3}{4}} \ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16ab^4} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(b\*x^2 + a)^2,x, algorithm="giac")

[Out]  $-1/2 \cdot x^{3/2} / ((b \cdot x^2 + a) \cdot b) + 3/8 \cdot \sqrt{2} \cdot (a \cdot b^3)^{3/4} \cdot \arctan(1/2 \cdot \sqrt{2} \cdot (\sqrt{2} \cdot (a/b)^{1/4} + 2 \cdot \sqrt{x}) / (a/b)^{1/4}) / (a \cdot b^4) + 3/8 \cdot \sqrt{2} \cdot (a \cdot b^3)^{3/4} \cdot \arctan(-1/2 \cdot \sqrt{2} \cdot (\sqrt{2} \cdot (a/b)^{1/4} - 2 \cdot \sqrt{x}) / (a/b)^{1/4}) / (a \cdot b^4) - 3/16 \cdot \sqrt{2} \cdot (a \cdot b^3)^{3/4} \cdot \ln(\sqrt{2} \cdot \sqrt{x} \cdot (a/b)^{1/4} + x + \sqrt{a/b}) / (a \cdot b^4) + 3/16 \cdot \sqrt{2} \cdot (a \cdot b^3)^{3/4} \cdot \ln(-\sqrt{2} \cdot \sqrt{x} \cdot (a/b)^{1/4} + x + \sqrt{a/b}) / (a \cdot b^4)$

$$3.298 \quad \int \frac{x^{3/2}}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=218

$$\begin{aligned} & -\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{3/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{3/4}b^{5/4}} \\ & -\frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}a^{3/4}b^{5/4}} - \frac{\sqrt{x}}{2b(a+bx^2)} \end{aligned}$$

[Out] -Sqrt[x]/(2\*b\*(a + b\*x^2)) - ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(4\*Sqrt[2]\*a^(3/4)\*b^(5/4)) + ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(4\*Sqrt[2]\*a^(3/4)\*b^(5/4)) - Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x]/(8\*Sqrt[2]\*a^(3/4)\*b^(5/4)) + Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x]/(8\*Sqrt[2]\*a^(3/4)\*b^(5/4))

**Rubi [A]** time = 0.366003, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$

$$\begin{aligned} & -\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{3/4}b^{5/4}} + \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{3/4}b^{5/4}} \\ & -\frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}a^{3/4}b^{5/4}} - \frac{\sqrt{x}}{2b(a+bx^2)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)/(a + b\*x^2)^2, x]

[Out] -Sqrt[x]/(2\*b\*(a + b\*x^2)) - ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(4\*Sqrt[2]\*a^(3/4)\*b^(5/4)) + ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(4\*Sqrt[2]\*a^(3/4)\*b^(5/4)) - Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x]/(8\*Sqrt[2]\*a^(3/4)\*b^(5/4)) + Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x]/(8\*Sqrt[2]\*a^(3/4)\*b^(5/4))

**Rubi in Sympy [A]** time = 61.9472, size = 197, normalized size = 0.9

$$\begin{aligned} & -\frac{\sqrt{x}}{2b(a+bx^2)} - \frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{16a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{16a^{\frac{3}{4}}b^{\frac{5}{4}}} \\ & -\frac{\sqrt{2}\operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\sqrt{2}\operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{\frac{3}{4}}b^{\frac{5}{4}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(3/2)/(b\*x\*\*2+a)\*\*2, x)

[Out] -sqrt(x)/(2\*b\*(a + b\*x\*\*2)) - sqrt(2)\*log(-sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*sqrt(x) + sqrt(a) + sqrt(b)\*x)/(16\*a\*\*(3/4)\*b\*\*(5/4)) + sqrt(2)\*log(sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*sqrt(x) + sqrt(a) + sqrt(b)\*x)/(16\*a\*\*(3/4)\*b\*\*(5/4)) - sqrt(2)\*atan(1 - sqrt(2)\*b\*\*(1/4)\*sqrt(x)/a\*\*(1/4))/(8\*a\*\*(3/4)\*b\*\*(5/4)) + sqrt(2)\*atan(1 + sqrt(2)\*b\*\*(1/4)\*sqrt(x)/a\*\*(1/4))/(8\*a\*\*(3/4)\*b\*\*(5/4))

**Mathematica [A]** time = 0.290819, size = 198, normalized size = 0.91

$$\frac{-\frac{\sqrt{2} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} \sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{a^{3/4}} + \frac{\sqrt{2} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} \sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{a^{3/4}} - \frac{2\sqrt{2} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} \sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{2\sqrt{2} \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt{x}}{\sqrt[4]{a}} + 1\right)}{a^{3/4}} - \frac{8 \sqrt[4]{b} \sqrt{x}}{a + bx^2}}{16b^{5/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)/(a + b\*x^2)^2, x]

[Out]  $\left(\frac{-8b^{1/4}\sqrt{x}}{(a + b^2x^2)} - \frac{2\sqrt{2}\operatorname{ArcTan}\left[1 - \left(\sqrt{2}b^{1/4}\sqrt{x}\right)/a^{1/4}\right]}{a^{3/4}} + \frac{2\sqrt{2}\operatorname{ArcTan}\left[1 + \left(\sqrt{2}b^{1/4}\sqrt{x}\right)/a^{1/4}\right]}{a^{3/4}} - \frac{\left(\sqrt{2}\operatorname{Log}\left[\sqrt{a} - \sqrt{2}a^{1/4}b^{1/4}\sqrt{x} + \sqrt{bx}\right] + \sqrt{2}\operatorname{Log}\left[\sqrt{a} + \sqrt{2}a^{1/4}b^{1/4}\sqrt{x} + \sqrt{bx}\right]\right)}{a^{3/4}}\right)/(16b^{5/4})$

**Maple [A]** time = 0.016, size = 158, normalized size = 0.7

$$\begin{aligned} & -\frac{1}{2b(bx^2 + a)}\sqrt{x} + \frac{\sqrt{2}}{16ab}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x + \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \\ & + \frac{\sqrt{2}}{8ab}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) + \frac{\sqrt{2}}{8ab}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(b\*x^2+a)^2, x)

[Out]  $\frac{-1/2*x^{1/2}/b/(b*x^2+a)+1/16/b*(a/b)^{1/4}/a^{1/2}*ln((x+(a/b)^{1/4}*x^{1/2})^{1/2}+(a/b)^{1/4}*x^{1/2})/(x-(a/b)^{1/4}*x^{1/2})^{1/2}+(a/b)^{1/4}*x^{1/2})+1/8/b*(a/b)^{1/4}/a^{1/2}*arctan(2^{1/2}/(a/b)^{1/4}*x^{1/2}+1)+1/8/b*(a/b)^{1/4}/a^{1/2}*arctan(2^{1/2}/(a/b)^{1/4}*x^{1/2}-1)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b\*x^2 + a)^2, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.251618, size = 225, normalized size = 1.03

$$\frac{4(b^2x^2 + ab)\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{ab\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}}}{\sqrt{a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x+\sqrt{x}}}\right) - (b^2x^2 + ab)\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}}\log\left(ab\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}} + \sqrt{x}\right) + (b^2x^2 + ab)\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}}\log\left(ab\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}} - \sqrt{x}\right)}{8(b^2x^2 + ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b\*x^2 + a)^2, x, algorithm="fricas")

[Out] 
$$-1/8 * (4 * (b^2 * x^2 + a * b) * (-1/(a^3 * b^5))^{1/4} * \arctan(a * b * (-1/(a^3 * b^5))^{1/4}) / (\sqrt{a^2 * b^2 * \sqrt{-1/(a^3 * b^5)}} + x) + \sqrt{x}) - (b^2 * x^2 + a * b) * (-1/(a^3 * b^5))^{1/4} * \log(a * b * (-1/(a^3 * b^5))^{1/4} + \sqrt{x}) + (b^2 * x^2 + a * b) * (-1/(a^3 * b^5))^{1/4} * \log(-a * b * (-1/(a^3 * b^5))^{1/4} + \sqrt{x}) + 4 * \sqrt{x}) / (b^2 * x^2 + a * b)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(3/2)/(b*x**2+a)**2,x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.225588, size = 269, normalized size = 1.23

$$\frac{\sqrt{2} (ab^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8ab^2} + \frac{\sqrt{2} (ab^3)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8ab^2} + \frac{\sqrt{2} (ab^3)^{\frac{1}{4}} \ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16ab^2} - \frac{\sqrt{2} (ab^3)^{\frac{1}{4}} \ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16ab^2} - \frac{\sqrt{x}}{2(bx^2+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(3/2)/(b*x^2 + a)^2,x, algorithm="giac")`

[Out] 
$$1/8 * \sqrt{2} * (a * b^3)^{1/4} * \arctan(1/2 * \sqrt{2} * (\sqrt{2} * (a/b)^{1/4} + 2 * \sqrt{x}) / (a/b)^{1/4}) / (a * b^2) + 1/8 * \sqrt{2} * (a * b^3)^{1/4} * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} * (a/b)^{1/4} - 2 * \sqrt{x}) / (a/b)^{1/4}) / (a * b^2) + 1/16 * \sqrt{2} * (a * b^3)^{1/4} * \ln(\sqrt{2} * \sqrt{x} * (a/b)^{1/4} + x + \sqrt{a/b}) / (a * b^2) - 1/16 * \sqrt{2} * (a * b^3)^{1/4} * \ln(-\sqrt{2} * \sqrt{x} * (a/b)^{1/4} + x + \sqrt{a/b}) / (a * b^2) - 1/2 * \sqrt{x} / ((b * x^2 + a) * b)$$

$$3.299 \quad \int \frac{\sqrt{x}}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=218

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{5/4}b^{3/4}} - \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{5/4}b^{3/4}} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}a^{5/4}b^{3/4}} + \frac{x^{3/2}}{2a(a+bx^2)}$$

[Out]  $x^{(3/2)}/(2*a*(a + b*x^2)) - \text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}]/(4*\text{Sqrt}[2]*a^{(5/4)}*b^{(3/4)}) + \text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}]/(4*\text{Sqrt}[2]*a^{(5/4)}*b^{(3/4)}) + \text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x]/(8*\text{Sqrt}[2]*a^{(5/4)}*b^{(3/4)}) - \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x]/(8*\text{Sqrt}[2]*a^{(5/4)}*b^{(3/4)})$

**Rubi [A]** time = 0.38278, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{5/4}b^{3/4}} - \frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{5/4}b^{3/4}} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}a^{5/4}b^{3/4}} + \frac{x^{3/2}}{2a(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(a + b\*x^2)^2, x]

[Out]  $x^{(3/2)}/(2*a*(a + b*x^2)) - \text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}]/(4*\text{Sqrt}[2]*a^{(5/4)}*b^{(3/4)}) + \text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}]/(4*\text{Sqrt}[2]*a^{(5/4)}*b^{(3/4)}) + \text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x]/(8*\text{Sqrt}[2]*a^{(5/4)}*b^{(3/4)}) - \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x]/(8*\text{Sqrt}[2]*a^{(5/4)}*b^{(3/4)})$

**Rubi in Sympy [A]** time = 64.5353, size = 197, normalized size = 0.9

$$\frac{x^{3/2}}{2a(a+bx^2)} + \frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{16a^{5/4}b^{3/4}} - \frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{16a^{5/4}b^{3/4}} - \frac{\sqrt{2}\text{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{5/4}b^{3/4}} + \frac{\sqrt{2}\text{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{5/4}b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(1/2)/(b\*x\*\*2+a)\*\*2, x)

[Out]  $x^{(3/2)}/(2*a*(a + b*x^2)) + \text{sqrt}(2)*\log(-\text{sqrt}(2)*a^{(1/4)}*b^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(a) + \text{sqrt}(b)*x)/(16*a^{(5/4)}*b^{(3/4)}) - \text{sqrt}(2)*\log(\text{sqrt}(2)*a^{(1/4)}*b^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(a) + \text{sqrt}(b)*x)/(16*a^{(5/4)}*b^{(3/4)}) - \text{sqrt}(2)*\text{atan}(1 - \text{sqrt}(2)*b^{(1/4)}*\text{sqrt}(x)/a^{(1/4)})/(8*a^{(5/4)}*b^{(3/4)}) + \text{sqrt}(2)*\text{atan}(1 + \text{sqrt}(2)*b^{(1/4)}*\text{sqrt}(x)/a^{(1/4)})/(8*a^{(5/4)}*b^{(3/4)})$

**Mathematica [A]** time = 0.27544, size = 198, normalized size = 0.91

$$\frac{\frac{\sqrt{2} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} \sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{b^{3/4}} - \frac{\sqrt{2} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} \sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{b^{3/4}} - \frac{2\sqrt{2} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} \sqrt{x}}{\sqrt[4]{a}}\right)}{b^{3/4}} + \frac{2\sqrt{2} \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} \sqrt{x}}{\sqrt[4]{a}} + 1\right)}{b^{3/4}} + \frac{8 \sqrt[4]{a} x^{3/2}}{a + bx^2}}{16a^{5/4}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(a + b\*x^2)^2, x]

[Out]  $\left(\frac{8 a^{1/4} x^{3/2}}{a + b x^2} - \frac{2 \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt[4]{b} \sqrt{x}}{\sqrt[4]{a}}\right] b^{1/4} \sqrt{x}}{b^{3/4}} + \frac{2 \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt[4]{b} \sqrt{x}}{\sqrt[4]{a}}\right] b^{1/4} \sqrt{x}}{b^{3/4}} + \frac{\sqrt{2} \operatorname{Log}\left[\frac{\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{b} \sqrt{x} + \sqrt{bx}}{\sqrt{a} + \sqrt{2} \sqrt[4]{a} \sqrt[4]{b} \sqrt{x} + \sqrt{bx}}\right]}{b^{3/4}} - \frac{\sqrt{2} \operatorname{Log}\left[\frac{\sqrt{a} + \sqrt{2} \sqrt[4]{a} \sqrt[4]{b} \sqrt{x} + \sqrt{bx}}{\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{b} \sqrt{x} + \sqrt{bx}}\right]}{b^{3/4}}\right) / (16 a^{5/4})$

**Maple [A]** time = 0.013, size = 158, normalized size = 0.7

$$\frac{1}{2 a (b x^2 + a)} x^{\frac{3}{2}} + \frac{\sqrt{2}}{16 a b} \ln\left(1 \left(x - \sqrt{\frac{a}{b}} \sqrt{x} \sqrt{2} + \sqrt{\frac{a}{b}}\right) \left(x + \sqrt{\frac{a}{b}} \sqrt{x} \sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{\sqrt{2}}{8 a b} \arctan\left(\sqrt{2} \sqrt{x} \frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) \frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{\sqrt{2}}{8 a b} \arctan\left(\sqrt{2} \sqrt{x} \frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) \frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(b\*x^2+a)^2, x)

[Out]  $\frac{1}{2} x^{3/2} / a / (b x^2 + a) + 1/16 a / b / (a/b)^{1/4} 2^{1/2} \ln\left(\frac{(x - (a/b)^{1/4} x^{1/2}) 2^{1/2} + (a/b)^{1/2}}{(x + (a/b)^{1/4} x^{1/2}) 2^{1/2} + (a/b)^{1/2}}\right) + 1/8 a / b / (a/b)^{1/4} 2^{1/2} \arctan\left(\frac{2^{1/2}}{(a/b)^{1/4} x^{1/2} + 1}\right) + 1/8 a / b / (a/b)^{1/4} 2^{1/2} \arctan\left(\frac{2^{1/2}}{(a/b)^{1/4} x^{1/2} - 1}\right)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x)/(b\*x^2 + a)^2, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.251704, size = 235, normalized size = 1.08

$$\frac{4 (a b x^2 + a^2) \left(-\frac{1}{a^5 b^3}\right)^{\frac{1}{4}} \arctan\left(\frac{a^4 b^2 \left(-\frac{1}{a^5 b^3}\right)^{\frac{3}{4}}}{\sqrt{-a^3 b \sqrt{-\frac{1}{a^5 b^3} + x} + \sqrt{x}}}\right) + (a b x^2 + a^2) \left(-\frac{1}{a^5 b^3}\right)^{\frac{1}{4}} \log\left(a^4 b^2 \left(-\frac{1}{a^5 b^3}\right)^{\frac{3}{4}} + \sqrt{x}\right) - (a b x^2 + a^2) \left(-\frac{1}{a^5 b^3}\right)^{\frac{1}{4}} \log\left(a^4 b^2 \left(-\frac{1}{a^5 b^3}\right)^{\frac{3}{4}} - \sqrt{x}\right)}{8 (a b x^2 + a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(sqrt(x)/(b\*x^2 + a)^2,x, algorithm="fricas")

[Out]  $\frac{1}{8} \cdot (4 \cdot (a \cdot b \cdot x^2 + a^2) \cdot (-1/(a^5 \cdot b^3))^{1/4} \cdot \arctan(a^4 \cdot b^2 \cdot (-1/(a^5 \cdot b^3))^{3/4} / (\sqrt{-a^3 \cdot b \cdot \sqrt{-1/(a^5 \cdot b^3)}} + x) + \sqrt{x})) + (a \cdot b \cdot x^2 + a^2) \cdot (-1/(a^5 \cdot b^3))^{1/4} \cdot \log(a^4 \cdot b^2 \cdot (-1/(a^5 \cdot b^3))^{3/4} + \sqrt{x}) - (a \cdot b \cdot x^2 + a^2) \cdot (-1/(a^5 \cdot b^3))^{1/4} \cdot \log(-a^4 \cdot b^2 \cdot (-1/(a^5 \cdot b^3))^{3/4} + \sqrt{x}) + 4 \cdot x^{3/2}) / (a \cdot b \cdot x^2 + a^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(1/2)/(b\*x\*\*2+a)\*\*2,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.216638, size = 269, normalized size = 1.23

$$\frac{x^{\frac{3}{2}}}{2(bx^2 + a)a} + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2b^3} + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2b^3} - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}} \ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16a^2b^3} + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}} \ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x)/(b\*x^2 + a)^2,x, algorithm="giac")

[Out]  $\frac{1}{2} \cdot x^{3/2} / ((b \cdot x^2 + a) \cdot a) + \frac{1}{8} \cdot \sqrt{2} \cdot (a \cdot b^3)^{3/4} \cdot \arctan(1/2 \cdot \sqrt{2} \cdot (\sqrt{2} \cdot (a/b)^{1/4} + 2 \cdot \sqrt{x}) / (a/b)^{1/4}) / (a^2 \cdot b^3) + \frac{1}{8} \cdot \sqrt{2} \cdot (a \cdot b^3)^{3/4} \cdot \arctan(-1/2 \cdot \sqrt{2} \cdot (\sqrt{2} \cdot (a/b)^{1/4} - 2 \cdot \sqrt{x}) / (a/b)^{1/4}) / (a^2 \cdot b^3) - \frac{1}{16} \cdot \sqrt{2} \cdot (a \cdot b^3)^{3/4} \cdot \ln(\sqrt{2} \cdot \sqrt{x} \cdot (a/b)^{1/4} + x + \sqrt{a/b}) / (a^2 \cdot b^3) + \frac{1}{16} \cdot \sqrt{2} \cdot (a \cdot b^3)^{3/4} \cdot \ln(-\sqrt{2} \cdot \sqrt{x} \cdot (a/b)^{1/4} + x + \sqrt{a/b}) / (a^2 \cdot b^3)$

$$3.300 \quad \int \frac{1}{\sqrt{x}(a+bx^2)^2} dx$$

**Optimal.** Leaf size=218

$$\begin{aligned} & -\frac{3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} \\ & -\frac{3 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{\sqrt{x}}{2a(a+bx^2)} \end{aligned}$$

[Out] Sqrt[x]/(2\*a\*(a + b\*x^2)) - (3\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(4\*Sqrt[2]\*a^(7/4)\*b^(1/4)) + (3\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(4\*Sqrt[2]\*a^(7/4)\*b^(1/4)) - (3\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(8\*Sqrt[2]\*a^(7/4)\*b^(1/4)) + (3\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(8\*Sqrt[2]\*a^(7/4)\*b^(1/4))

**Rubi [A]** time = 0.365604, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$

$$\begin{aligned} & -\frac{3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} \\ & -\frac{3 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{\sqrt{x}}{2a(a+bx^2)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]\*(a + b\*x^2)^2), x]

[Out] Sqrt[x]/(2\*a\*(a + b\*x^2)) - (3\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(4\*Sqrt[2]\*a^(7/4)\*b^(1/4)) + (3\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(4\*Sqrt[2]\*a^(7/4)\*b^(1/4)) - (3\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(8\*Sqrt[2]\*a^(7/4)\*b^(1/4)) + (3\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(8\*Sqrt[2]\*a^(7/4)\*b^(1/4))

**Rubi in Sympy [A]** time = 63.3291, size = 204, normalized size = 0.94

$$\begin{aligned} & \frac{\sqrt{x}}{2a(a+bx^2)} - \frac{3\sqrt{2} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{16a^{7/4}\sqrt[4]{b}} + \frac{3\sqrt{2} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{16a^{7/4}\sqrt[4]{b}} \\ & - \frac{3\sqrt{2} \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}} + \frac{3\sqrt{2} \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+a)\*\*2/x\*\*(1/2), x)

[Out] sqrt(x)/(2\*a\*(a + b\*x\*\*2)) - 3\*sqrt(2)\*log(-sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*sqrt(x) + sqrt(a) + sqrt(b)\*x)/(16\*a\*\*(7/4)\*b\*\*(1/4)) + 3\*sqrt(2)\*log(sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*sqrt(x) + sqrt(a) + sqrt(b)\*x)/(16\*a\*\*(7/4)\*b\*\*(1/4)) - 3\*sqrt(2)\*atan(1 - sqrt(2)\*b\*\*(1/4)\*sqrt(x)/a\*\*(1/4))/(8\*a\*\*(7/4)\*b\*\*(1/4)) + 3\*sqrt(2)\*atan(1 + sqrt(2)\*b\*\*(1/4)\*sqrt(x)/a\*\*(1/4))/(8\*a\*\*(7/4)\*b\*\*(1/4))

**Mathematica [A]** time = 0.279785, size = 199, normalized size = 0.91

$$\frac{\frac{8a^{3/4}\sqrt{x}}{a+bx^2} - \frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x+\sqrt{a}+\sqrt{bx}}\right)}{\sqrt[4]{b}} + \frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x+\sqrt{a}+\sqrt{bx}}\right)}{\sqrt[4]{b}} - \frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} + \frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{b}}}{16a^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]\*(a + b\*x^2)^2), x]

[Out] ((8\*a^(3/4)\*Sqrt[x])/(a + b\*x^2) - (6\*Sqrt[2]\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)])/b^(1/4) + (6\*Sqrt[2]\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)])/b^(1/4) - (3\*Sqrt[2]\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/b^(1/4) + (3\*Sqrt[2]\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/b^(1/4))/(16\*a^(7/4))

**Maple [A]** time = 0.012, size = 149, normalized size = 0.7

$$\frac{1}{2a(bx^2+a)}\sqrt{x} + \frac{3\sqrt{2}}{16a^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x + \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{3\sqrt{2}}{8a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) + \frac{3\sqrt{2}}{8a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+a)^2/x^(1/2), x)

[Out] 1/2\*x^(1/2)/a/(b\*x^2+a)+3/16/a^2\*(a/b)^(1/4)\*2^(1/2)\*Ln((x+(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2)))+3/8/a^2\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)+1)+3/8/a^2\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)-1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^2\*sqrt(x)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.251762, size = 221, normalized size = 1.01

$$\frac{12(abx^2 + a^2)\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}\arctan\left(\frac{a^2\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}}{\sqrt{a^4\sqrt{-\frac{1}{a^7b}}+x+\sqrt{x}}}\right) - 3(abx^2 + a^2)\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}} + \sqrt{x}\right) + 3(abx^2 + a^2)\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}}{8(abx^2 + a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^2\*sqrt(x)),x, algorithm="fricas")

[Out] 
$$-1/8*(12*(a*b*x^2 + a^2)*(-1/(a^7*b))^{1/4}*\arctan(a^2*(-1/(a^7*b))^{1/4}/(\sqrt{a^4*\sqrt{-1/(a^7*b)} + x} + \sqrt{x})) - 3*(a*b*x^2 + a^2)*(-1/(a^7*b))^{1/4}*\log(a^2*(-1/(a^7*b))^{1/4} + \sqrt{x}) + 3*(a*b*x^2 + a^2)*(-1/(a^7*b))^{1/4}*\log(-a^2*(-1/(a^7*b))^{1/4} + \sqrt{x}) - 4*\sqrt{x})/(a*b*x^2 + a^2)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*2+a)\*\*2/x\*\*(1/2),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.219346, size = 269, normalized size = 1.23

$$\frac{3\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2b} + \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2b} + \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^2b} - \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^2b} + \frac{\sqrt{x}}{2(bx^2+a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^2\*sqrt(x)),x, algorithm="giac")

[Out] 
$$3/8*\sqrt{2}*(a*b^3)^{1/4}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(a/b)^{1/4} + 2*\sqrt{x})/(a/b)^{1/4})/(a^2*b) + 3/8*\sqrt{2}*(a*b^3)^{1/4}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(a/b)^{1/4} - 2*\sqrt{x})/(a/b)^{1/4})/(a^2*b) + 3/16*\sqrt{2}*(a*b^3)^{1/4}*\ln(\sqrt{2}*\sqrt{x}*(a/b)^{1/4} + x + \sqrt{a/b})/(a^2*b) - 3/16*\sqrt{2}*(a*b^3)^{1/4}*\ln(-\sqrt{2}*\sqrt{x}*(a/b)^{1/4} + x + \sqrt{a/b})/(a^2*b) + 1/2*\sqrt{x}/((b*x^2 + a)*a)$$

$$3.301 \quad \int \frac{1}{x^{3/2}(a+bx^2)^2} dx$$

**Optimal.** Leaf size=230

$$\begin{aligned} & -\frac{5\sqrt[4]{b} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{9/4}} + \frac{5\sqrt[4]{b} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{9/4}} \\ & + \frac{5\sqrt[4]{b} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}} - \frac{5\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}a^{9/4}} - \frac{5}{2a^2\sqrt{x}} + \frac{1}{2a\sqrt{x}(a+bx^2)} \end{aligned}$$

[Out]  $-5/(2*a^2*\text{Sqrt}[x]) + 1/(2*a*\text{Sqrt}[x]*(a + b*x^2)) + (5*b^{(1/4)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/(4*\text{Sqrt}[2]*a^{(9/4)}) - (5*b^{(1/4)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/(4*\text{Sqrt}[2]*a^{(9/4)}) - (5*b^{(1/4)}*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(8*\text{Sqrt}[2]*a^{(9/4)}) + (5*b^{(1/4)}*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(8*\text{Sqrt}[2]*a^{(9/4)})$

**Rubi [A]** time = 0.425311, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$

$$\begin{aligned} & -\frac{5\sqrt[4]{b} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{9/4}} + \frac{5\sqrt[4]{b} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{9/4}} \\ & + \frac{5\sqrt[4]{b} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}} - \frac{5\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}a^{9/4}} - \frac{5}{2a^2\sqrt{x}} + \frac{1}{2a\sqrt{x}(a+bx^2)} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^{(3/2)}*(a + b*x^2)^2), x]$

[Out]  $-5/(2*a^2*\text{Sqrt}[x]) + 1/(2*a*\text{Sqrt}[x]*(a + b*x^2)) + (5*b^{(1/4)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/(4*\text{Sqrt}[2]*a^{(9/4)}) - (5*b^{(1/4)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/(4*\text{Sqrt}[2]*a^{(9/4)}) - (5*b^{(1/4)}*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(8*\text{Sqrt}[2]*a^{(9/4)}) + (5*b^{(1/4)}*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(8*\text{Sqrt}[2]*a^{(9/4)})$

**Rubi in Sympy [A]** time = 69.1633, size = 218, normalized size = 0.95

$$\begin{aligned} & \frac{1}{2a\sqrt{x}(a+bx^2)} - \frac{5}{2a^2\sqrt{x}} - \frac{5\sqrt{2}\sqrt[4]{b} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{16a^{9/4}} \\ & + \frac{5\sqrt{2}\sqrt[4]{b} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{16a^{9/4}} + \frac{5\sqrt{2}\sqrt[4]{b} \text{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{9/4}} - \frac{5\sqrt{2}\sqrt[4]{b} \text{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{9/4}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{(3/2)}/(b*x^2+a)^2, x)$

[Out]  $1/(2*a*\text{sqrt}(x)*(a + b*x^2)) - 5/(2*a^2*\text{sqrt}(x)) - 5*\text{sqrt}(2)*b^{(1/4)}*\log(-\text{sqrt}(2)*a^{(1/4)}*b^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(a) + \text{sqrt}(b)*x)/(16*a^{(9/4)}) + 5*\text{sqrt}(2)*b^{(1/4)}*\log(\text{sqrt}(2)*a^{(1/4)}*b^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(a) + \text{sqrt}(b)*x)/(16*a^{(9/4)}) + 5*\text{sqrt}(2)*b^{(1/4)}*\text{atan}(1 - \text{sqrt}(2)*b^{(1/4)}*\text{sqrt}(x)/a^{(1/4)})/(8*a^{(9/4)}) - 5*\text{sqrt}(2)*b^{(1/4)}*\text{atan}(1 + \text{sqrt}(2)*b^{(1/4)}*\text{sqrt}(x)/a^{(1/4)})/(8*a^{(9/4)})$

\*\* (9/4))

**Mathematica [A]** time = 0.305119, size = 212, normalized size = 0.92

$$\frac{-\frac{8\sqrt[4]{ab}x^{3/2}}{a+bx^2} - 5\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) + 5\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) + 10\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right)}{16a^{9/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(3/2)\*(a + b\*x^2)^2), x]

[Out] ((-32\*a^(1/4))/Sqrt[x] - (8\*a^(1/4)\*b\*x^(3/2))/(a + b\*x^2) + 10\*Sqrt[2]\*b^(1/4)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)] - 10\*Sqrt[2]\*b^(1/4)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)] - 5\*Sqrt[2]\*b^(1/4)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x] + 5\*Sqrt[2]\*b^(1/4)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(16\*a^(9/4))

**Maple [A]** time = 0.02, size = 158, normalized size = 0.7

$$-2\frac{1}{a^2\sqrt{x}} - \frac{b}{2a^2(bx^2 + a)}x^{\frac{3}{2}} - \frac{5\sqrt{2}}{16a^2}\ln\left(1\left(x - \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x + \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

$$- \frac{5\sqrt{2}}{8a^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} - \frac{5\sqrt{2}}{8a^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(3/2)/(b\*x^2+a)^2, x)

[Out] -2/a^2/x^(1/2) - 1/2\*b/a^2\*x^(3/2)/(b\*x^2+a) - 5/16/a^2/(a/b)^(1/4)\*2^(1/2)\*ln((x - (a/b)^(1/4)\*x^(1/2)\*2^(1/2) + (a/b)^(1/2))/(x + (a/b)^(1/4)\*x^(1/2)\*2^(1/2) + (a/b)^(1/2))) - 5/8/a^2/(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2) + 1) - 5/8/a^2/(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2) - 1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^2\*x^(3/2)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.25931, size = 259, normalized size = 1.13

$$20bx^2 + 20(a^2bx^2 + a^3)\sqrt{x}\left(-\frac{b}{a^9}\right)^{\frac{1}{4}}\arctan\left(\frac{125a^7\left(-\frac{b}{a^9}\right)^{\frac{3}{4}}}{125b\sqrt{x} + \sqrt{-15625a^5b\sqrt{-\frac{b}{a^9}} + 15625b^2x}}\right) + 5(a^2bx^2 + a^3)\sqrt{x}\left(-\frac{b}{a^9}\right)^{\frac{1}{4}}\log\left(125a^7\left(-\frac{b}{a^9}\right)^{\frac{3}{4}}\sqrt{-15625a^5b\sqrt{-\frac{b}{a^9}} + 15625b^2x}\right)$$

$$\frac{1}{8(a^2bx^2 + a^3)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^2\*x^(3/2)),x, algorithm="fricas")

[Out] 
$$-1/8*(20*b*x^2 + 20*(a^2*b*x^2 + a^3)*\sqrt{x}*(-b/a^9)^{1/4}*\arctan(125*a^7*(-b/a^9)^{3/4}/(125*b*\sqrt{x} + \sqrt{-15625*a^5*b*\sqrt{-b/a^9} + 15625*b^2*x})) + 5*(a^2*b*x^2 + a^3)*\sqrt{x}*(-b/a^9)^{1/4}*\log(125*a^7*(-b/a^9)^{3/4} + 125*b*\sqrt{x}) - 5*(a^2*b*x^2 + a^3)*\sqrt{x}*(-b/a^9)^{1/4}*\log(-125*a^7*(-b/a^9)^{3/4} + 125*b*\sqrt{x}) + 16*a)/((a^2*b*x^2 + a^3)*\sqrt{x})$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(3/2)/(b\*x\*\*2+a)\*\*2,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.224339, size = 284, normalized size = 1.23

$$\begin{aligned} & \frac{5\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{\frac{a}{b}}\right)^{\frac{1}{4}}+2\sqrt{x}}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(bx^{\frac{5}{2}}+a\sqrt{x}\right)a^2} - \frac{5\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{\frac{a}{b}}\right)^{\frac{1}{4}}-2\sqrt{x}}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^3b^2} \\ & - \frac{5\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{\frac{a}{b}}\right)^{\frac{1}{4}}-2\sqrt{x}}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^3b^2} + \frac{5\sqrt{2}(ab^3)^{\frac{3}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^3b^2} \\ & - \frac{5\sqrt{2}(ab^3)^{\frac{3}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^3b^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^2\*x^(3/2)),x, algorithm="giac")

[Out] 
$$-1/2*(5*b*x^2 + 4*a)/((b*x^{5/2} + a*\sqrt{x})*a^2) - 5/8*\sqrt{2}*(a*b^3)^{3/4}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(a/b)^{1/4} + 2*\sqrt{x})/(a/b)^{1/4})/(a^3*b^2) - 5/8*\sqrt{2}*(a*b^3)^{3/4}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(a/b)^{1/4} - 2*\sqrt{x})/(a/b)^{1/4})/(a^3*b^2) + 5/16*\sqrt{2}*(a*b^3)^{3/4}*\ln(\sqrt{2}*\sqrt{x}*(a/b)^{1/4} + x + \sqrt{a/b})/(a^3*b^2) - 5/16*\sqrt{2}*(a*b^3)^{3/4}*\ln(-\sqrt{2}*\sqrt{x}*(a/b)^{1/4} + x + \sqrt{a/b})/(a^3*b^2)$$

$$3.302 \quad \int \frac{1}{x^{5/2}(a+bx^2)^2} dx$$

**Optimal.** Leaf size=230

$$\frac{7b^{3/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{11/4}} - \frac{7b^{3/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{11/4}} \\ + \frac{7b^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}} - \frac{7b^{3/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}a^{11/4}} - \frac{7}{6a^2x^{3/2}} + \frac{1}{2ax^{3/2}(a+bx^2)}$$

[Out]  $-7/(6*a^2*x^{3/2}) + 1/(2*a*x^{3/2}*(a+b*x^2)) + (7*b^{3/4}*ArcTan[1 - (Sqrt[2]*b^{1/4}*Sqrt[x])/a^{1/4}])/(4*Sqrt[2]*a^{11/4}) - (7*b^{3/4}*ArcTan[1 + (Sqrt[2]*b^{1/4}*Sqrt[x])/a^{1/4}])/(4*Sqrt[2]*a^{11/4}) + (7*b^{3/4}*Log[Sqrt[a] - Sqrt[2]*a^{1/4}*b^{1/4}*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^{11/4}) - (7*b^{3/4}*Log[Sqrt[a] + Sqrt[2]*a^{1/4}*b^{1/4}*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^{11/4})$

**Rubi [A]** time = 0.421474, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$

$$\frac{7b^{3/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{11/4}} - \frac{7b^{3/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{11/4}} \\ + \frac{7b^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}} - \frac{7b^{3/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}a^{11/4}} - \frac{7}{6a^2x^{3/2}} + \frac{1}{2ax^{3/2}(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(5/2)\*(a + b\*x^2)^2), x]

[Out]  $-7/(6*a^2*x^{3/2}) + 1/(2*a*x^{3/2}*(a+b*x^2)) + (7*b^{3/4}*ArcTan[1 - (Sqrt[2]*b^{1/4}*Sqrt[x])/a^{1/4}])/(4*Sqrt[2]*a^{11/4}) - (7*b^{3/4}*ArcTan[1 + (Sqrt[2]*b^{1/4}*Sqrt[x])/a^{1/4}])/(4*Sqrt[2]*a^{11/4}) + (7*b^{3/4}*Log[Sqrt[a] - Sqrt[2]*a^{1/4}*b^{1/4}*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^{11/4}) - (7*b^{3/4}*Log[Sqrt[a] + Sqrt[2]*a^{1/4}*b^{1/4}*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^{11/4})$

**Rubi in Sympy [A]** time = 68.1911, size = 218, normalized size = 0.95

$$\frac{1}{2ax^{3/2}(a+bx^2)} - \frac{7}{6a^2x^{3/2}} + \frac{7\sqrt{2}b^{3/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{16a^{11/4}} \\ - \frac{7\sqrt{2}b^{3/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{16a^{11/4}} + \frac{7\sqrt{2}b^{3/4} \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{11/4}} - \frac{7\sqrt{2}b^{3/4} \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*(5/2)/(b\*x\*\*2+a)\*\*2, x)

[Out]  $1/(2*a*x^{3/2}*(a+b*x^2)) - 7/(6*a^2*x^{3/2}) + 7*\sqrt{2}*b^{3/4}*\log(-\sqrt{2}*b^{1/4}*sqrt(x) + sqrt(a) + sqrt(b)*x)/(16*a^{11/4}) - 7*\sqrt{2}*b^{3/4}*\log(\sqrt{2}*b^{1/4}*sqrt(x) + sqrt(a) + sqrt(b)*x)/(16*a^{11/4}) + 7*\sqrt{2}*b^{3/4}*\operatorname{atan}(1 - \sqrt{2}*b^{1/4}*sqrt(x)/a^{1/4})/(8*a^{11/4}) - 7*\sqrt{2}*b^{3/4}*\operatorname{atan}(1 + \sqrt{2}*b^{1/4}*sqrt(x)/a^{1/4})/(8*a^{11/4})$



$/(8 * a^{11/4})$

**Mathematica [A]** time = 0.337295, size = 212, normalized size = 0.92

$$\frac{-\frac{24a^{3/4}b\sqrt{x}}{a+bx^2} - \frac{32a^{3/4}}{x^{3/2}} + 21\sqrt{2}b^{3/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) - 21\sqrt{2}b^{3/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) + 42\sqrt{2}b^{3/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}}\right)}{48a^{11/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(5/2)\*(a + b\*x^2)^2), x]

[Out]  $((-32 * a^{(3/4)})/x^{(3/2)} - (24 * a^{(3/4)} * b * \text{Sqrt}[x])/(a + b * x^2) + 42 * \text{Sqrt}[2] * b^{(3/4)} * \text{ArcTan}[1 - (\text{Sqrt}[2] * b^{(1/4)} * \text{Sqrt}[x])/a^{(1/4)}] - 42 * \text{Sqrt}[2] * b^{(3/4)} * \text{ArcTan}[1 + (\text{Sqrt}[2] * b^{(1/4)} * \text{Sqrt}[x])/a^{(1/4)}] + 21 * \text{Sqrt}[2] * b^{(3/4)} * \text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2] * a^{(1/4)} * b^{(1/4)} * \text{Sqrt}[x] + \text{Sqrt}[b] * x] - 21 * \text{Sqrt}[2] * b^{(3/4)} * \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2] * a^{(1/4)} * b^{(1/4)} * \text{Sqrt}[x] + \text{Sqrt}[b] * x])/(48 * a^{(11/4)})$

**Maple [A]** time = 0.02, size = 161, normalized size = 0.7

$$\begin{aligned} &-\frac{2}{3a^2}x^{-\frac{3}{2}} - \frac{b}{2a^2(bx^2+a)}\sqrt{x} - \frac{7b\sqrt{2}}{16a^3}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x + \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \\ &-\frac{7b\sqrt{2}}{8a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) - \frac{7b\sqrt{2}}{8a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(5/2)/(b\*x^2+a)^2, x)

[Out]  $-2/3/a^2/x^{(3/2)} - 1/2*b/a^2*x^{(1/2)}/(b*x^2+a) - 7/16*b/a^3*(a/b)^{(1/4)}*2^{(1/2)}*\ln((x+(a/b)^{(1/4)}*x^{(1/2)}*2^{(1/2)}+(a/b)^{(1/2)})/(x-(a/b)^{(1/4)}*x^{(1/2)}*2^{(1/2)}+(a/b)^{(1/2)})) - 7/8*b/a^3*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x^{(1/2)}+1) - 7/8*b/a^3*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x^{(1/2)}-1)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^2\*x^(5/2)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.25439, size = 282, normalized size = 1.23

$$\frac{28bx^2 - 84(a^2bx^3 + a^3x)\sqrt{x}\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}\arctan\left(\frac{a^3\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}}{b\sqrt{x} + \sqrt{a^6\sqrt{-\frac{b^3}{a^{11}}+b^2x}}}\right) + 21(a^2bx^3 + a^3x)\sqrt{x}\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}\log\left(7a^3\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}} + 7\sqrt{a^6\sqrt{-\frac{b^3}{a^{11}}+b^2x}}\right)}{24(a^2bx^3 + a^3x)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^2\*x^(5/2)),x, algorithm="fricas")

[Out] 
$$-1/24*(28*b*x^2 - 84*(a^2*b*x^3 + a^3*x)*\sqrt{x}*(-b^3/a^{11})^{1/4})*\arctan(a^3*(-b^3/a^{11})^{1/4}/(b*\sqrt{x} + \sqrt{a^6*\sqrt{-b^3/a^{11}} + b^2*x})) + 21*(a^2*b*x^3 + a^3*x)*\sqrt{x}*(-b^3/a^{11})^{1/4}*\log(7*a^3*(-b^3/a^{11})^{1/4} + 7*b*\sqrt{x}) - 21*(a^2*b*x^3 + a^3*x)*\sqrt{x}*(-b^3/a^{11})^{1/4}*\log(-7*a^3*(-b^3/a^{11})^{1/4} + 7*b*\sqrt{x}) + 16*a)/((a^2*b*x^3 + a^3*x)*\sqrt{x})$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(5/2)/(b\*x\*\*2+a)\*\*2,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.216723, size = 265, normalized size = 1.15

$$\begin{aligned} & \frac{7\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^3} - \frac{7\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^3} \\ & - \frac{7\sqrt{2}(ab^3)^{\frac{1}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^3} \\ & + \frac{7\sqrt{2}(ab^3)^{\frac{1}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^3} - \frac{b\sqrt{x}}{2(bx^2+a)a^2} - \frac{2}{3a^2x^{\frac{3}{2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^2\*x^(5/2)),x, algorithm="giac")

[Out] 
$$-7/8*\sqrt{2}*(a*b^3)^{(1/4)}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(a/b)^{(1/4)} + 2*\sqrt{x}))/a^3 - 7/8*\sqrt{2}*(a*b^3)^{(1/4)}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(a/b)^{(1/4)} - 2*\sqrt{x}))/a^3 - 7/16*\sqrt{2}*(a*b^3)^{(1/4)}*\ln(\sqrt{2}*\sqrt{x}*(a/b)^{(1/4)} + x + \sqrt{a/b})/a^3 + 7/16*\sqrt{2}*(a*b^3)^{(1/4)}*\ln(-\sqrt{2}*\sqrt{x}*(a/b)^{(1/4)} + x + \sqrt{a/b})/a^3 - 1/2*b*\sqrt{x}/((b*x^2 + a)*a^2) - 2/3/(a^2*x^{(3/2)})$$

$$3.303 \quad \int \frac{1}{x^{7/2}(a+bx^2)^2} dx$$

**Optimal.** Leaf size=243

$$\frac{9b^{5/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{13/4}} - \frac{9b^{5/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{13/4}}$$

$$- \frac{9b^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}} + \frac{9b^{5/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}a^{13/4}} + \frac{9b}{2a^3\sqrt{x}} - \frac{9}{10a^2x^{5/2}} + \frac{1}{2ax^{5/2}(a+bx^2)}$$

[Out]  $-9/(10*a^2*x^{(5/2)}) + (9*b)/(2*a^3*\text{Sqrt}[x]) + 1/(2*a*x^{(5/2)}*(a + b*x^2)) - (9*b^{(5/4)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/ (4*\text{Sqrt}[2]*a^{(13/4)}) + (9*b^{(5/4)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/ (4*\text{Sqrt}[2]*a^{(13/4)}) + (9*b^{(5/4)}*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/ (8*\text{Sqrt}[2]*a^{(13/4)}) - (9*b^{(5/4)}*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/ (8*\text{Sqrt}[2]*a^{(13/4)})$

**Rubi [A]** time = 0.485036, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$

$$\frac{9b^{5/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{13/4}} - \frac{9b^{5/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{8\sqrt{2}a^{13/4}}$$

$$- \frac{9b^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}} + \frac{9b^{5/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}a^{13/4}} + \frac{9b}{2a^3\sqrt{x}} - \frac{9}{10a^2x^{5/2}} + \frac{1}{2ax^{5/2}(a+bx^2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^{(7/2)}*(a + b*x^2)^2), x]$

[Out]  $-9/(10*a^2*x^{(5/2)}) + (9*b)/(2*a^3*\text{Sqrt}[x]) + 1/(2*a*x^{(5/2)}*(a + b*x^2)) - (9*b^{(5/4)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/ (4*\text{Sqrt}[2]*a^{(13/4)}) + (9*b^{(5/4)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/ (4*\text{Sqrt}[2]*a^{(13/4)}) + (9*b^{(5/4)}*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/ (8*\text{Sqrt}[2]*a^{(13/4)}) - (9*b^{(5/4)}*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/ (8*\text{Sqrt}[2]*a^{(13/4)})$

**Rubi in Sympy [A]** time = 78.6238, size = 231, normalized size = 0.95

$$\frac{1}{2ax^{5/2}(a+bx^2)} - \frac{9}{10a^2x^{5/2}} + \frac{9b}{2a^3\sqrt{x}} + \frac{9\sqrt{2}b^{5/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{16a^{13/4}}$$

$$- \frac{9\sqrt{2}b^{5/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{16a^{13/4}} - \frac{9\sqrt{2}b^{5/4} \text{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{13/4}} + \frac{9\sqrt{2}b^{5/4} \text{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{8a^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{(7/2)}/(b*x^2+a)^2, x)$

[Out]  $1/(2*a*x^{(5/2)}*(a + b*x^2)) - 9/(10*a^2*x^{(5/2)}) + 9*b/(2*a^3*\text{sqrt}(x)) + 9*\text{sqrt}(2)*b^{(5/4)}*\log(-\text{sqrt}(2)*a^{(1/4)}*b^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(a) + \text{sqrt}(b)*x)/(16*a^{(13/4)}) - 9*\text{sqrt}(2)*b^{(5/4)}*\log(\text{sqrt}(2)*a^{(1/4)}*b^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(a) + \text{sqrt}(b)*x)/(16*a^{(13/4)}) - 9*\text{sqrt}(2)*b^{(5/4)}*\text{atan}(1 - \text{sqrt}(2)*b^{(1/4)}*\text{sqrt}(x)/a^{(1/4)})/(8*a^{(13/4)}) + 9*\text{sqrt}(2)*b^{(5/4)}*\text{atan}(1 + \text{sqrt}(2)*b^{(1/4)}*\text{sqrt}(x)/a^{(1/4)})/(8*a^{(13/4)})$

$$*(1/4)*\sqrt{x}/a^{1/4}/(8*a^{13/4})$$

**Mathematica [A]** time = 0.503332, size = 227, normalized size = 0.93

$$\frac{-\frac{32a^{5/4}}{x^{5/2}} + 45\sqrt{2}b^{5/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) - 45\sqrt{2}b^{5/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) - 90\sqrt{2}b^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a} + \sqrt{bx}}\right)}{80a^{13/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(7/2)\*(a + b\*x^2)^2), x]

[Out]  $((-32*a^{5/4})/x^{5/2} + (320*a^{1/4}*b)/\sqrt{x} + (40*a^{1/4}*b^2*x^{3/2})/(a + b*x^2) - 90*\sqrt{2}*b^{5/4}*\text{ArcTan}[1 - (\sqrt{2}*b^{1/4}*\sqrt{x})/a^{1/4}] + 90*\sqrt{2}*b^{5/4}*\text{ArcTan}[1 + (\sqrt{2}*b^{1/4}*\sqrt{x})/a^{1/4}] + 45*\sqrt{2}*b^{5/4}*\text{Log}[\sqrt{a} - \sqrt{2}*a^{1/4}*b^{1/4}*\sqrt{x} + \sqrt{b}*x] - 45*\sqrt{2}*b^{5/4}*\text{Log}[\sqrt{a} + \sqrt{2}*a^{1/4}*b^{1/4}*\sqrt{x} + \sqrt{b}*x])/(80*a^{13/4})$

**Maple [A]** time = 0.021, size = 172, normalized size = 0.7

$$\begin{aligned} &-\frac{2}{5a^2}x^{-\frac{5}{2}} + 4\frac{b}{a^3\sqrt{x}} + \frac{b^2}{2a^3(bx^2+a)}x^{\frac{3}{2}} \\ &+ \frac{9b\sqrt{2}}{16a^3} \ln\left(1\left(x - \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x + \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \frac{1}{\sqrt[4]{\frac{a}{b}}} \\ &+ \frac{9b\sqrt{2}}{8a^3} \arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) \frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{9b\sqrt{2}}{8a^3} \arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) \frac{1}{\sqrt[4]{\frac{a}{b}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(7/2)/(b\*x^2+a)^2, x)

[Out]  $-2/5/a^2/x^{5/2} + 4*b/a^3/x^{1/2} + 1/2*b^2/a^3*x^{3/2}/(b*x^2+a) + 9/16*b/a^3/(a/b)^{1/4}*2^{1/2}*ln((x - (a/b)^{1/4}*x^{1/2})*2^{1/2} + (a/b)^{1/2})/(x + (a/b)^{1/4}*x^{1/2})*2^{1/2} + (a/b)^{1/2}) + 9/8*b/a^3/(a/b)^{1/4}*2^{1/2}*arctan(2^{1/2}/(a/b)^{1/4}*x^{1/2} + 1) + 9/8*b/a^3/(a/b)^{1/4}*2^{1/2}*arctan(2^{1/2}/(a/b)^{1/4}*x^{1/2} - 1)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^2\*x^(7/2)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.258302, size = 325, normalized size = 1.34

$$180 b^2 x^4 + 144 a b x^2 + 180 (a^3 b x^4 + a^4 x^2) \sqrt{x} \left(-\frac{b^5}{a^{13}}\right)^{\frac{1}{4}} \arctan\left(\frac{729 a^{10} \left(-\frac{b^5}{a^{13}}\right)^{\frac{3}{4}}}{729 b^4 \sqrt{x} + \sqrt{-531441 a^7 b^5 \sqrt{-\frac{b^5}{a^{13}} + 531441 b^8 x}}}\right) + 45 (a^3 b x^4 + a^4 x^2) \sqrt{x}$$


---


$$40 (a^3 b x^4 + a^4 x^2) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^2\*x^(7/2)),x, algorithm="fricas")

[Out] 1/40\*(180\*b^2\*x^4 + 144\*a\*b\*x^2 + 180\*(a^3\*b\*x^4 + a^4\*x^2)\*sqrt(x)\*(-b^5/a^13)^(1/4)\*arctan(729\*a^10\*(-b^5/a^13)^(3/4)/(729\*b^4\*sqrt(x) + sqrt(-531441\*a^7\*b^5\*sqrt(-b^5/a^13) + 531441\*b^8\*x))) + 45\*(a^3\*b\*x^4 + a^4\*x^2)\*sqrt(x)\*(-b^5/a^13)^(1/4)\*log(729\*a^10\*(-b^5/a^13)^(3/4) + 729\*b^4\*sqrt(x)) - 45\*(a^3\*b\*x^4 + a^4\*x^2)\*sqrt(x)\*(-b^5/a^13)^(1/4)\*log(-729\*a^10\*(-b^5/a^13)^(3/4) + 729\*b^4\*sqrt(x)) - 16\*a^2)/((a^3\*b\*x^4 + a^4\*x^2)\*sqrt(x))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(7/2)/(b\*x\*\*2+a)\*\*2,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.216768, size = 297, normalized size = 1.22

$$\frac{b^2 x^{\frac{3}{2}}}{2(bx^2 + a)a^3} + \frac{9\sqrt{2}(ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^4b}$$

$$+ \frac{9\sqrt{2}(ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^4b} - \frac{9\sqrt{2}(ab^3)^{\frac{3}{4}} \ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16a^4b}$$

$$+ \frac{9\sqrt{2}(ab^3)^{\frac{3}{4}} \ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16a^4b} + \frac{2(10bx^2 - a)}{5a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^2\*x^(7/2)),x, algorithm="giac")

[Out] 1/2\*b^2\*x^(3/2)/((b\*x^2 + a)\*a^3) + 9/8\*sqrt(2)\*(a\*b^3)^(3/4)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(a/b)^(1/4) + 2\*sqrt(x))/(a/b)^(1/4))/(a^4\*b) + 9/8\*sqrt(2)\*(a\*b^3)^(3/4)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(a/b)^(1/4) - 2\*sqrt(x))/(a/b)^(1/4))/(a^4\*b) - 9/16\*sqrt(2)\*(a\*b^3)^(3/4)\*ln(sqrt(2)\*sqrt(x)\*(a/b)^(1/4) + x + sqrt(a/b))/(a^4\*b) + 9/16\*sqrt(2)\*(a\*b^3)^(3/4)\*ln(-sqrt(2)\*sqrt(x)\*(a/b)^(1/4) + x + sqrt(a/b))/(a^4\*b) + 2/5\*(10\*b\*x^2 - a)/(a^3\*x^(5/2))

$$3.304 \quad \int \frac{x^{7/2}}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=239

$$\begin{aligned} & -\frac{5 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{3/4}b^{9/4}} + \frac{5 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{3/4}b^{9/4}} \\ & -\frac{5 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{3/4}b^{9/4}} + \frac{5 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{32\sqrt{2}a^{3/4}b^{9/4}} - \frac{5\sqrt{x}}{16b^2(a+bx^2)} - \frac{x^{5/2}}{4b(a+bx^2)^2} \end{aligned}$$

[Out]  $-x^{5/2}/(4*b*(a+b*x^2)^2) - (5*\text{Sqrt}[x])/(16*b^2*(a+b*x^2)) - (5*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{1/4}*\text{Sqrt}[x])/a^{1/4}])/(32*\text{Sqrt}[2]*a^{3/4}*b^{9/4}) + (5*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{1/4}*\text{Sqrt}[x])/a^{1/4}])/(32*\text{Sqrt}[2]*a^{3/4}*b^{9/4}) - (5*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*b^{1/4}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(64*\text{Sqrt}[2]*a^{3/4}*b^{9/4}) + (5*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*b^{1/4}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(64*\text{Sqrt}[2]*a^{3/4}*b^{9/4})$

**Rubi [A]** time = 0.430823, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$

$$\begin{aligned} & -\frac{5 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{3/4}b^{9/4}} + \frac{5 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{3/4}b^{9/4}} \\ & -\frac{5 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{3/4}b^{9/4}} + \frac{5 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{32\sqrt{2}a^{3/4}b^{9/4}} - \frac{5\sqrt{x}}{16b^2(a+bx^2)} - \frac{x^{5/2}}{4b(a+bx^2)^2} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{7/2}/(a+b*x^2)^3, x]$

[Out]  $-x^{5/2}/(4*b*(a+b*x^2)^2) - (5*\text{Sqrt}[x])/(16*b^2*(a+b*x^2)) - (5*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{1/4}*\text{Sqrt}[x])/a^{1/4}])/(32*\text{Sqrt}[2]*a^{3/4}*b^{9/4}) + (5*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{1/4}*\text{Sqrt}[x])/a^{1/4}])/(32*\text{Sqrt}[2]*a^{3/4}*b^{9/4}) - (5*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*b^{1/4}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(64*\text{Sqrt}[2]*a^{3/4}*b^{9/4}) + (5*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*b^{1/4}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(64*\text{Sqrt}[2]*a^{3/4}*b^{9/4})$

**Rubi in Sympy [A]** time = 71.4885, size = 224, normalized size = 0.94

$$\begin{aligned} & \frac{x^{5/2}}{4b(a+bx^2)^2} - \frac{5\sqrt{x}}{16b^2(a+bx^2)} - \frac{5\sqrt{2} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{128a^{3/4}b^{9/4}} \\ & + \frac{5\sqrt{2} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{128a^{3/4}b^{9/4}} - \frac{5\sqrt{2} \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{9/4}} + \frac{5\sqrt{2} \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{9/4}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{7/2}/(b*x^2+a)^3, x)$

[Out]  $-x^{5/2}/(4*b*(a+b*x^2)^2) - 5*\text{sqrt}(x)/(16*b^2*(a+b*x^2)) - 5*\text{sqrt}(2)*\log(-\text{sqrt}(2)*a^{1/4}*b^{1/4}*\text{sqrt}(x) + \text{sqrt}(a) + \text{sqrt}(b)*x)/(128*a^{3/4}*b^{9/4}) + 5*\text{sqrt}(2)*\log(\text{sqrt}(2)*a^{1/4}*b^{1/4}*\text{sqrt}(x) + \text{sqrt}(a) + \text{sqrt}(b)*x)/(128*a^{3/4}*b^{9/4}) - 5*\text{sqrt}(2)*\operatorname{atan}(1 - \text{sqrt}(2)*b^{1/4}*\text{sqrt}(x)/a^{1/4})/(64*a^{3/4}*b^{9/4}) + 5*\text{sqrt}(2)*\operatorname{atan}(1 + \text{sqrt}(2)*b^{1/4}*\text{sqrt}(x)/a^{1/4})/(64*a^{3/4}*b^{9/4})$

$$(1/4)/(64*a^{3/4}*b^{9/4})$$

**Mathematica [A]** time = 0.198904, size = 221, normalized size = 0.92

$$\frac{-\frac{5\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x+\sqrt{a}+\sqrt{bx}}\right)}{a^{3/4}} + \frac{5\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x+\sqrt{a}+\sqrt{bx}}\right)}{a^{3/4}} - \frac{10\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{10\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}} + \frac{32a\sqrt[4]{b}\sqrt{x}}{(a+bx^2)^2} - \frac{72}{a}}{128b^{9/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(7/2)/(a + b\*x^2)^3, x]

[Out] ((32\*a\*b^(1/4)\*Sqrt[x])/(a + b\*x^2)^2 - (72\*b^(1/4)\*Sqrt[x])/(a + b\*x^2) - (10\*Sqrt[2]\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)])/a^(3/4) + (10\*Sqrt[2]\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)])/a^(3/4) - (5\*Sqrt[2]\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/a^(3/4) + (5\*Sqrt[2]\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/a^(3/4))/(128\*b^(9/4))

**Maple [A]** time = 0.021, size = 170, normalized size = 0.7

$$2\frac{1}{(bx^2+a)^2}\left(-\frac{9x^{5/2}}{32b}-\frac{5a\sqrt{x}}{32b^2}\right) + \frac{5\sqrt{2}}{128ab^2}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x+\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left(x-\sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{5\sqrt{2}}{64ab^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right) + \frac{5\sqrt{2}}{64ab^2}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)/(b\*x^2+a)^3, x)

[Out] 2\*(-9/32\*x^(5/2)/b-5/32\*a\*x^(1/2)/b^2)/(b\*x^2+a)^2+5/128/b^2\*(a/b)^(1/4)/a\*2^(1/2)\*ln((x+(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2)))+5/64/b^2\*(a/b)^(1/4)/a\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)+1)+5/64/b^2\*(a/b)^(1/4)/a\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)-1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(b\*x^2 + a)^3, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.256293, size = 319, normalized size = 1.33

$$20(b^4x^4 + 2ab^3x^2 + a^2b^2)\left(-\frac{1}{a^3b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{ab^2\left(-\frac{1}{a^3b^9}\right)^{\frac{1}{4}}}{\sqrt{a^2b^4\sqrt{-\frac{1}{a^3b^9}+x+\sqrt{x}}}}\right) - 5(b^4x^4 + 2ab^3x^2 + a^2b^2)\left(-\frac{1}{a^3b^9}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{1}{a^3b^9}\right)^{\frac{1}{4}}\right)$$

$$64(b^4x^4 + 2ab^3x^2 + a^2b^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(b\*x^2 + a)^3,x, algorithm="fricas")

[Out] 
$$-1/64*(20*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-1/(a^3*b^9))^{1/4}*\arctan(a*b^2*(-1/(a^3*b^9))^{1/4}/(\sqrt{a^2*b^4*\sqrt{-1/(a^3*b^9)}} + x) + \sqrt{x})) - 5*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-1/(a^3*b^9))^{1/4}*\log(a*b^2*(-1/(a^3*b^9))^{1/4} + \sqrt{x}) + 5*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-1/(a^3*b^9))^{1/4}*\log(-a*b^2*(-1/(a^3*b^9))^{1/4} + \sqrt{x}) + 4*(9*b*x^2 + 5*a)*\sqrt{x})/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)$$

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(7/2)/(b\*x\*\*2+a)\*\*3,x)

[Out] Timed out

**GIAC/XCAS** [A] time = 0.21977, size = 282, normalized size = 1.18

$$\frac{5\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64ab^3} + \frac{5\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64ab^3} + \frac{5\sqrt{2}(ab^3)^{\frac{1}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128ab^3} - \frac{5\sqrt{2}(ab^3)^{\frac{1}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128ab^3} - \frac{9bx^{\frac{5}{2}}+5a\sqrt{x}}{16(bx^2+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(b\*x^2 + a)^3,x, algorithm="giac")

[Out] 
$$5/64*\sqrt{2}*(a*b^3)^{1/4}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(a/b)^{1/4} + 2*\sqrt{x})/(a/b)^{1/4})/(a*b^3) + 5/64*\sqrt{2}*(a*b^3)^{1/4}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(a/b)^{1/4} - 2*\sqrt{x})/(a/b)^{1/4})/(a*b^3) + 5/128*\sqrt{2}*(a*b^3)^{1/4}*\ln(\sqrt{2}*\sqrt{x}*(a/b)^{1/4} + x + \sqrt{a/b})/(a*b^3) - 5/128*\sqrt{2}*(a*b^3)^{1/4}*\ln(-\sqrt{2}*\sqrt{x}*(a/b)^{1/4} + x + \sqrt{a/b})/(a*b^3) - 1/16*(9*b*x^{5/2} + 5*a*\sqrt{x})/((b*x^2 + a)^2*b^2)$$



$$3.305 \quad \int \frac{x^{5/2}}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=242

$$\frac{3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{5/4}b^{7/4}} - \frac{3 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{5/4}b^{7/4}} - \frac{3 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}} + \frac{3 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{32\sqrt{2}a^{5/4}b^{7/4}} + \frac{3x^{3/2}}{16ab(a+bx^2)} - \frac{x^{3/2}}{4b(a+bx^2)^2}$$

[Out]  $-x^{3/2}/(4*b*(a+b*x^2)^2) + (3*x^{3/2})/(16*a*b*(a+b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^{1/4}*Sqrt[x])/a^{1/4}])/(32*Sqrt[2]*a^{5/4}*b^{7/4}) + (3*ArcTan[1 + (Sqrt[2]*b^{1/4}*Sqrt[x])/a^{1/4}])/(32*Sqrt[2]*a^{5/4}*b^{7/4}) + (3*Log[Sqrt[a] - Sqrt[2]*a^{1/4}*b^{1/4}*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^{5/4}*b^{7/4}) - (3*Log[Sqrt[a] + Sqrt[2]*a^{1/4}*b^{1/4}*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^{5/4}*b^{7/4})$

**Rubi [A]** time = 0.433877, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$

$$\frac{3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{5/4}b^{7/4}} - \frac{3 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{5/4}b^{7/4}} - \frac{3 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}} + \frac{3 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{32\sqrt{2}a^{5/4}b^{7/4}} + \frac{3x^{3/2}}{16ab(a+bx^2)} - \frac{x^{3/2}}{4b(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)/(a + b\*x^2)^3, x]

[Out]  $-x^{3/2}/(4*b*(a+b*x^2)^2) + (3*x^{3/2})/(16*a*b*(a+b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^{1/4}*Sqrt[x])/a^{1/4}])/(32*Sqrt[2]*a^{5/4}*b^{7/4}) + (3*ArcTan[1 + (Sqrt[2]*b^{1/4}*Sqrt[x])/a^{1/4}])/(32*Sqrt[2]*a^{5/4}*b^{7/4}) + (3*Log[Sqrt[a] - Sqrt[2]*a^{1/4}*b^{1/4}*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^{5/4}*b^{7/4}) - (3*Log[Sqrt[a] + Sqrt[2]*a^{1/4}*b^{1/4}*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^{5/4}*b^{7/4})$

**Rubi in Sympy [A]** time = 72.4117, size = 224, normalized size = 0.93

$$-\frac{x^{3/2}}{4b(a+bx^2)^2} + \frac{3x^{3/2}}{16ab(a+bx^2)} + \frac{3\sqrt{2} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{128a^{5/4}b^{7/4}} - \frac{3\sqrt{2} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{128a^{5/4}b^{7/4}} - \frac{3\sqrt{2} \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{7/4}} + \frac{3\sqrt{2} \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(5/2)/(b\*x\*\*2+a)\*\*3, x)

[Out]  $-x^{3/2}/(4*b*(a+b*x^2)^2) + 3*x^{3/2}/(16*a*b*(a+b*x^2)) + 3*\sqrt{2}*\log(-\sqrt{2}*a^{1/4}*b^{1/4}*\sqrt{x} + \sqrt{a} + \sqrt{b*x})/(128*a^{5/4}*b^{7/4}) - 3*\sqrt{2}*\log(\sqrt{2}*a^{1/4}*b^{1/4}*\sqrt{x} + \sqrt{a} + \sqrt{b*x})/(128*a^{5/4}*b^{7/4}) - 3*\sqrt{2}*\operatorname{atan}(1 - \sqrt{2}*b^{1/4}*\sqrt{x}/a^{1/4})/(64*a^{5/4}*b^{7/4}) + 3*\sqrt{2}*\operatorname{atan}(1 + \sqrt{2}*b^{1/4}*\sqrt{x}/a^{1/4})/(64*a^{5/4}*b^{7/4})$

$$(1/4))/(64*a**(5/4)*b**(7/4))$$

**Mathematica [A]** time = 0.213542, size = 223, normalized size = 0.92

$$\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x+\sqrt{a}+\sqrt{bx}}\right)}{a^{5/4}} - \frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x+\sqrt{a}+\sqrt{bx}}\right)}{a^{5/4}} - \frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt[4]{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{5/4}} + \frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{5/4}} + \frac{24b^{3/4}x^{3/2}}{a^2+abx^2} - \frac{32b^{3/4}}{(a+bx)}$$

$$128b^{7/4}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)/(a + b\*x^2)^3, x]

[Out] 
$$\frac{(-32*b^{3/4}*x^{3/2})/(a + b*x^2)^2 + (24*b^{3/4}*x^{3/2})/(a^2 + a*b*x^2) - (6*\text{Sqrt}[2]*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{1/4}*\text{Sqrt}[x])/a^{1/4}])/a^{5/4} + (6*\text{Sqrt}[2]*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{1/4}*\text{Sqrt}[x])/a^{1/4}])/a^{5/4} + (3*\text{Sqrt}[2]*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*b^{1/4}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/a^{5/4} - (3*\text{Sqrt}[2]*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*b^{1/4}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/a^{5/4}}{128*b^{7/4}}$$

**Maple [A]** time = 0.021, size = 169, normalized size = 0.7

$$2 \frac{1}{(bx^2 + a)^2} \left( \frac{3x^{7/2}}{32a} - \frac{1}{32} \frac{x^{3/2}}{b} \right) + \frac{3\sqrt{2}}{128ab^2} \ln \left( 1 \left( x - \sqrt[4]{\frac{a}{b}} \sqrt{x} \sqrt{2} + \sqrt{\frac{a}{b}} \right) \left( x + \sqrt[4]{\frac{a}{b}} \sqrt{x} \sqrt{2} + \sqrt{\frac{a}{b}} \right)^{-1} \right) \frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{3\sqrt{2}}{64ab^2} \arctan \left( \sqrt{2} \sqrt{x} \frac{1}{\sqrt[4]{\frac{a}{b}}} + 1 \right) \frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{3\sqrt{2}}{64ab^2} \arctan \left( \sqrt{2} \sqrt{x} \frac{1}{\sqrt[4]{\frac{a}{b}}} - 1 \right) \frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)/(b\*x^2+a)^3, x)

[Out] 
$$2*(3/32/a*x^{7/2}-1/32*x^{3/2}/b)/(b*x^2+a)^2+3/128/b^2/a/(a/b)^{(1/4)}*2^{(1/2)}*\ln((x-(a/b)^{(1/4)}*x^{(1/2)}*2^{(1/2)}+(a/b)^{(1/2)})/(x+(a/b)^{(1/4)}*x^{(1/2)}*2^{(1/2)}+(a/b)^{(1/2)}))+3/64/b^2/a/(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x^{(1/2)}+1)+3/64/b^2/a/(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x^{(1/2)}-1)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(b\*x^2 + a)^3, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.258834, size = 335, normalized size = 1.38

$$12 (ab^3x^4 + 2a^2b^2x^2 + a^3b) \left(-\frac{1}{a^5b^7}\right)^{\frac{1}{4}} \arctan\left(\frac{a^4b^5\left(-\frac{1}{a^5b^7}\right)^{\frac{3}{4}}}{\sqrt{-a^3b^3\sqrt{-\frac{1}{a^5b^7}+x+\sqrt{x}}}}\right) + 3 (ab^3x^4 + 2a^2b^2x^2 + a^3b) \left(-\frac{1}{a^5b^7}\right)^{\frac{1}{4}} \log\left(a^4b^5\left(-\frac{1}{a^5b^7}\right)^{\frac{3}{4}}\right)$$


---


$$64(ab^3x^4 + 2a^2b^2x^2 + a^3b)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(b\*x^2 + a)^3,x, algorithm="fricas")

[Out] 1/64\*(12\*(a\*b^3\*x^4 + 2\*a^2\*b^2\*x^2 + a^3\*b)\*(-1/(a^5\*b^7))^(1/4)\*arctan(a^4\*b^5\*(-1/(a^5\*b^7))^(3/4)/(sqrt(-a^3\*b^3\*sqrt(-1/(a^5\*b^7)) + x) + sqrt(x))) + 3\*(a\*b^3\*x^4 + 2\*a^2\*b^2\*x^2 + a^3\*b)\*(-1/(a^5\*b^7))^(1/4)\*log(a^4\*b^5\*(-1/(a^5\*b^7))^(3/4) + sqrt(x)) - 3\*(a\*b^3\*x^4 + 2\*a^2\*b^2\*x^2 + a^3\*b)\*(-1/(a^5\*b^7))^(1/4)\*log(-a^4\*b^5\*(-1/(a^5\*b^7))^(3/4) + sqrt(x)) + 4\*(3\*b\*x^3 - a\*x)\*sqrt(x))/(a\*b^3\*x^4 + 2\*a^2\*b^2\*x^2 + a^3\*b)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(5/2)/(b\*x\*\*2+a)\*\*3,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.222816, size = 286, normalized size = 1.18

$$\frac{3bx^{\frac{7}{2}} - ax^{\frac{3}{2}}}{16(bx^2 + a)^2ab} + \frac{3\sqrt{2}(ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{\frac{a}{b}}\right)^{\frac{1}{4}} + 2\sqrt{x}}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^2b^4} + \frac{3\sqrt{2}(ab^3)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{\frac{a}{b}}\right)^{\frac{1}{4}} - 2\sqrt{x}}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^2b^4}$$

$$- \frac{3\sqrt{2}(ab^3)^{\frac{3}{4}} \ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128a^2b^4} + \frac{3\sqrt{2}(ab^3)^{\frac{3}{4}} \ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(b\*x^2 + a)^3,x, algorithm="giac")

[Out] 1/16\*(3\*b\*x^(7/2) - a\*x^(3/2))/((b\*x^2 + a)^2\*a\*b) + 3/64\*sqrt(2)\*(a\*b^3)^(3/4)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(a/b)^(1/4) + 2\*sqrt(x)))/(a/b)^(1/4))/(a^2\*b^4) + 3/64\*sqrt(2)\*(a\*b^3)^(3/4)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(a/b)^(1/4) - 2\*sqrt(x))/(a/b)^(1/4))/(a^2\*b^4) - 3/128\*sqrt(2)\*(a\*b^3)^(3/4)\*ln(sqrt(2)\*sqrt(x)\*(a/b)^(1/4) + x + sqrt(a/b))/(a^2\*b^4) + 3/128\*sqrt(2)\*(a\*b^3)^(3/4)\*ln(-sqrt(2)\*sqrt(x)\*(a/b)^(1/4) + x + sqrt(a/b))/(a^2\*b^4)

$$3.306 \quad \int \frac{x^{3/2}}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=242

$$\begin{aligned} & -\frac{3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{7/4}b^{5/4}} + \frac{3 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{7/4}b^{5/4}} \\ & -\frac{3 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}} + \frac{3 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{32\sqrt{2}a^{7/4}b^{5/4}} + \frac{\sqrt{x}}{16ab(a+bx^2)} - \frac{\sqrt{x}}{4b(a+bx^2)^2} \end{aligned}$$

[Out]  $-\text{Sqrt}[x]/(4*b*(a + b*x^2)^2) + \text{Sqrt}[x]/(16*a*b*(a + b*x^2)) - (3*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/(32*\text{Sqrt}[2]*a^{(7/4)}*b^{(5/4)}) + (3*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/(32*\text{Sqrt}[2]*a^{(7/4)}*b^{(5/4)}) - (3*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(64*\text{Sqrt}[2]*a^{(7/4)}*b^{(5/4)}) + (3*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(64*\text{Sqrt}[2]*a^{(7/4)}*b^{(5/4)})$

**Rubi [A]** time = 0.420875, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$

$$\begin{aligned} & -\frac{3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{7/4}b^{5/4}} + \frac{3 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{7/4}b^{5/4}} \\ & -\frac{3 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}} + \frac{3 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{32\sqrt{2}a^{7/4}b^{5/4}} + \frac{\sqrt{x}}{16ab(a+bx^2)} - \frac{\sqrt{x}}{4b(a+bx^2)^2} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(3/2)}/(a + b*x^2)^3, x]$

[Out]  $-\text{Sqrt}[x]/(4*b*(a + b*x^2)^2) + \text{Sqrt}[x]/(16*a*b*(a + b*x^2)) - (3*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/(32*\text{Sqrt}[2]*a^{(7/4)}*b^{(5/4)}) + (3*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/(32*\text{Sqrt}[2]*a^{(7/4)}*b^{(5/4)}) - (3*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(64*\text{Sqrt}[2]*a^{(7/4)}*b^{(5/4)}) + (3*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(64*\text{Sqrt}[2]*a^{(7/4)}*b^{(5/4)})$

**Rubi in Sympy [A]** time = 71.0417, size = 223, normalized size = 0.92

$$\begin{aligned} & -\frac{\sqrt{x}}{4b(a+bx^2)^2} + \frac{\sqrt{x}}{16ab(a+bx^2)} - \frac{3\sqrt{2} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{128a^{7/4}b^{5/4}} \\ & + \frac{3\sqrt{2} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{128a^{7/4}b^{5/4}} - \frac{3\sqrt{2} \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{7/4}b^{5/4}} + \frac{3\sqrt{2} \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{7/4}b^{5/4}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{(3/2)}/(b*x^2+a)^3, x)$

[Out]  $-\text{sqrt}(x)/(4*b*(a + b*x^2)^2) + \text{sqrt}(x)/(16*a*b*(a + b*x^2)) - 3*\text{sqrt}(2)*\log(-\text{sqrt}(2)*a^{(1/4)}*b^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(a) + \text{sqrt}(b)*x)/(128*a^{(7/4)}*b^{(5/4)}) + 3*\text{sqrt}(2)*\log(\text{sqrt}(2)*a^{(1/4)}*b^{(1/4)}*\text{sqrt}(x) + \text{sqrt}(a) + \text{sqrt}(b)*x)/(128*a^{(7/4)}*b^{(5/4)}) - 3*\text{sqrt}(2)*\operatorname{atan}(1 - \text{sqrt}(2)*b^{(1/4)}*\text{sqrt}(x)/a^{(1/4)})/(64*a^{(7/4)}*b^{(5/4)}) + 3*\text{sqrt}(2)*\operatorname{atan}(1 + \text{sqrt}(2)*b^{(1/4)}*\text{sqrt}(x)/a^{(1/4)})/(64*a^{(7/4)}*b^{(5/4)})$

)) / (64 \* a \*\* (7/4) \* b \*\* (5/4))

**Mathematica [A]** time = 0.23846, size = 223, normalized size = 0.92

$$\frac{-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x+\sqrt{a}+\sqrt{bx}}\right)}{a^{7/4}} + \frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x+\sqrt{a}+\sqrt{bx}}\right)}{a^{7/4}} - \frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}} + \frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{7/4}} + \frac{8\sqrt[4]{b}\sqrt{x}}{a^2+abx^2} - \frac{32\sqrt[4]{b}}{(a+bx^2)}}{128b^{5/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)/(a + b\*x^2)^3, x]

[Out] ((-32\*b^(1/4)\*Sqrt[x])/(a + b\*x^2)^2 + (8\*b^(1/4)\*Sqrt[x])/(a^2 + a\*b\*x^2) - (6\*Sqrt[2]\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]) / a^(7/4) + (6\*Sqrt[2]\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]) / a^(7/4) - (3\*Sqrt[2]\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x]) / a^(7/4) + (3\*Sqrt[2]\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x]) / a^(7/4)) / (128\*b^(5/4))

**Maple [A]** time = 0.02, size = 169, normalized size = 0.7

$$2 \frac{1}{(bx^2 + a)^2} \left( \frac{1}{32} \frac{x^{5/2}}{a} - \frac{3\sqrt{x}}{32b} \right) + \frac{3\sqrt{2}}{128a^2b} \sqrt[4]{\frac{a}{b}} \ln \left( 1 \left( x + \sqrt[4]{\frac{a}{b}} \sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}} \right) \left( x - \sqrt[4]{\frac{a}{b}} \sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}} \right)^{-1} \right) + \frac{3\sqrt{2}}{64a^2b} \sqrt[4]{\frac{a}{b}} \arctan \left( \sqrt{2}\sqrt{x} \frac{1}{\sqrt[4]{\frac{a}{b}}} + 1 \right) + \frac{3\sqrt{2}}{64a^2b} \sqrt[4]{\frac{a}{b}} \arctan \left( \sqrt{2}\sqrt{x} \frac{1}{\sqrt[4]{\frac{a}{b}}} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(b\*x^2+a)^3, x)

[Out] 2\*(1/32/a\*x^(5/2)-3/32\*x^(1/2)/b)/(b\*x^2+a)^2+3/128/b/a^2\*(a/b)^(1/4)\*2^(1/2)\*ln((x+(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2)))+3/64/b/a^2\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)+1)+3/64/b/a^2\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)-1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b\*x^2 + a)^3, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.258096, size = 323, normalized size = 1.33

$$12(ab^3x^4 + 2a^2b^2x^2 + a^3b) \left(-\frac{1}{a^7b^5}\right)^{\frac{1}{4}} \arctan\left(\frac{a^2b\left(-\frac{1}{a^7b^5}\right)^{\frac{1}{4}}}{\sqrt{a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x+\sqrt{x}}}\right) - 3(ab^3x^4 + 2a^2b^2x^2 + a^3b) \left(-\frac{1}{a^7b^5}\right)^{\frac{1}{4}} \log\left(a^2b\left(-\frac{1}{a^7b^5}\right)^{\frac{1}{4}}\right)$$

64(ab^3x^4 + 2a^2b^2x^2 + a^3b)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b\*x^2 + a)^3,x, algorithm="fricas")

[Out] 
$$-1/64*(12*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a^7*b^5))^{1/4}) * \arctan(a^2*b*(-1/(a^7*b^5))^{1/4}/(\sqrt{a^4*b^2*\sqrt{-1/(a^7*b^5)} + x) + \sqrt{x})) - 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a^7*b^5))^{1/4} * \log(a^2*b*(-1/(a^7*b^5))^{1/4} + \sqrt{x}) + 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a^7*b^5))^{1/4} * \log(-a^2*b*(-1/(a^7*b^5))^{1/4} + \sqrt{x}) - 4*(b*x^2 - 3*a)*\sqrt{x}/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(3/2)/(b\*x\*\*2+a)\*\*3,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.222119, size = 285, normalized size = 1.18

$$\frac{3\sqrt{2}(ab^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^2b^2} + \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^2b^2} + \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}} \ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128a^2b^2} - \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}} \ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128a^2b^2} + \frac{bx^{\frac{5}{2}} - 3a\sqrt{x}}{16(bx^2 + a)^2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(b\*x^2 + a)^3,x, algorithm="giac")

[Out] 
$$3/64*\sqrt{2}*(a*b^3)^{(1/4)}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(a/b)^{(1/4)} + 2*\sqrt{x})/(a/b)^{(1/4)})/(a^2*b^2) + 3/64*\sqrt{2}*(a*b^3)^{(1/4)}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(a/b)^{(1/4)} - 2*\sqrt{x})/(a/b)^{(1/4)})/(a^2*b^2) + 3/128*\sqrt{2}*(a*b^3)^{(1/4)}*\ln(\sqrt{2}*\sqrt{x}*(a/b)^{(1/4)} + x + \sqrt{a/b})/(a^2*b^2) - 3/128*\sqrt{2}*(a*b^3)^{(1/4)}*\ln(-\sqrt{2}*\sqrt{x}*(a/b)^{(1/4)} + x + \sqrt{a/b})/(a^2*b^2) + 1/16*(b*x^{(5/2)} - 3*a*\sqrt{x})/((b*x^2 + a)^2*a*b)$$

$$3.307 \quad \int \frac{\sqrt{x}}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=239

$$\frac{5 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{9/4}b^{3/4}} - \frac{5 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{9/4}b^{3/4}} \\ - \frac{5 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}} + \frac{5 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}} + 1\right)}{32\sqrt{2}a^{9/4}b^{3/4}} + \frac{5x^{3/2}}{16a^2(a+bx^2)} + \frac{x^{3/2}}{4a(a+bx^2)^2}$$

[Out]  $x^{(3/2)}/(4*a*(a+b*x^2)^2) + (5*x^{(3/2)})/(16*a^2*(a+b*x^2)) - (5*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(3/4)) + (5*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(3/4)) + (5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(9/4)*b^(3/4)) - (5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(9/4)*b^(3/4))$

**Rubi [A]** time = 0.43039, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$

$$\frac{5 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{9/4}b^{3/4}} - \frac{5 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{9/4}b^{3/4}} \\ - \frac{5 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}} + \frac{5 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}} + 1\right)}{32\sqrt{2}a^{9/4}b^{3/4}} + \frac{5x^{3/2}}{16a^2(a+bx^2)} + \frac{x^{3/2}}{4a(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(a + b\*x^2)^3, x]

[Out]  $x^{(3/2)}/(4*a*(a+b*x^2)^2) + (5*x^{(3/2)})/(16*a^2*(a+b*x^2)) - (5*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(3/4)) + (5*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(3/4)) + (5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(9/4)*b^(3/4)) - (5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(9/4)*b^(3/4))$

**Rubi in Sympy [A]** time = 71.2657, size = 224, normalized size = 0.94

$$\frac{x^{\frac{3}{2}}}{4a(a+bx^2)^2} + \frac{5x^{\frac{3}{2}}}{16a^2(a+bx^2)} + \frac{5\sqrt{2} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{128a^{\frac{9}{4}}b^{\frac{3}{4}}} \\ - \frac{5\sqrt{2} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{128a^{\frac{9}{4}}b^{\frac{3}{4}}} - \frac{5\sqrt{2} \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}}\right)}{64a^{\frac{9}{4}}b^{\frac{3}{4}}} + \frac{5\sqrt{2} \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}}\right)}{64a^{\frac{9}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(1/2)/(b\*x\*\*2+a)\*\*3, x)

[Out]  $x^{(3/2)}/(4*a*(a+b*x^2)^2) + 5*x^{(3/2)}/(16*a^2*(a+b*x^2)) + 5*\sqrt{2}*\log(-\sqrt{2}*a^{(1/4)}*b^{(1/4)}*\sqrt{x} + \sqrt{a} + \sqrt{b*x})/(128*a^{(9/4)}*b^{(3/4)}) - 5*\sqrt{2}*\log(\sqrt{2}*a^{(1/4)}*b^{(1/4)}*\sqrt{x} + \sqrt{a} + \sqrt{b*x})/(128*a^{(9/4)}*b^{(3/4)}) - 5*\sqrt{2}*\operatorname{atan}(1 - \sqrt{2}*b^{(1/4)}*\sqrt{x}/a^{(1/4)})/(64*a^{(9/4)}*b^{(3/4)}) + 5*\sqrt{2}*\operatorname{atan}(1 + \sqrt{2}*b^{(1/4)}*\sqrt{x}/a^{(1/4)})/(64*a^{(9/4)}*b^{(3/4)})$

$$(1/4))/(64*a**(9/4)*b**(3/4))$$

**Mathematica [A]** time = 0.218929, size = 220, normalized size = 0.92

$$\frac{\frac{32a^{5/4}x^{3/2}}{(a+bx^2)^2} + \frac{5\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x+\sqrt{a+\sqrt{bx}}}\right)}{b^{3/4}} - \frac{5\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x+\sqrt{a+\sqrt{bx}}}\right)}{b^{3/4}} - \frac{10\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{b^{3/4}} + \frac{10\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{b^{3/4}} + \frac{40\sqrt[4]{a}}{a+bx^2}}{128a^{9/4}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(a + b\*x^2)^3, x]

[Out] ((32\*a^(5/4)\*x^(3/2))/(a + b\*x^2)^2 + (40\*a^(1/4)\*x^(3/2))/(a + b\*x^2) - (10\*Sqrt[2]\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)])/b^(3/4) + (10\*Sqrt[2]\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)])/b^(3/4) + (5\*Sqrt[2]\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/b^(3/4) - (5\*Sqrt[2]\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/b^(3/4))/(128\*a^(9/4))

**Maple [A]** time = 0.011, size = 175, normalized size = 0.7

$$\frac{1}{4a(bx^2+a)^2}x^{\frac{3}{2}} + \frac{5}{16a^2(bx^2+a)}x^{\frac{3}{2}} + \frac{5\sqrt{2}}{128a^2b}\ln\left(1\left(x - \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x + \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{5\sqrt{2}}{64a^2b}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{5\sqrt{2}}{64a^2b}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(b\*x^2+a)^3, x)

[Out] 1/4\*x^(3/2)/a/(b\*x^2+a)^2+5/16\*x^(3/2)/a^2/(b\*x^2+a)+5/128/a^2/b/(a/b)^(1/4)\*2^(1/2)\*ln((x-(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2)))+5/64/a^2/b/(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)+1)+5/64/a^2/b/(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)-1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x)/(b\*x^2 + a)^3, x, algorithm="maxima")

[Out] Exception raised: ValueError



**Fricas [A]** time = 0.255096, size = 321, normalized size = 1.34

$$20 (a^2 b^2 x^4 + 2 a^3 b x^2 + a^4) \left(-\frac{1}{a^9 b^3}\right)^{\frac{1}{4}} \arctan\left(\frac{a^7 b^2 \left(-\frac{1}{a^9 b^3}\right)^{\frac{3}{4}}}{\sqrt{-a^5 b \sqrt{-\frac{1}{a^9 b^3} + x} + \sqrt{x}}}\right) + 5 (a^2 b^2 x^4 + 2 a^3 b x^2 + a^4) \left(-\frac{1}{a^9 b^3}\right)^{\frac{1}{4}} \log\left(a^7 b^2 \left(-\frac{1}{a^9 b^3}\right)\right)$$


---


$$64 (a^2 b^2 x^4 + 2 a^3 b x^2 + a^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x)/(b\*x^2 + a)^3,x, algorithm="fricas")

[Out] 1/64\*(20\*(a^2\*b^2\*x^4 + 2\*a^3\*b\*x^2 + a^4)\*(-1/(a^9\*b^3))^(1/4)\*arctan(a^7\*b^2\*(-1/(a^9\*b^3))^(3/4)/(sqrt(-a^5\*b\*sqrt(-1/(a^9\*b^3)) + x) + sqrt(x))) + 5\*(a^2\*b^2\*x^4 + 2\*a^3\*b\*x^2 + a^4)\*(-1/(a^9\*b^3))^(1/4)\*log(a^7\*b^2\*(-1/(a^9\*b^3))^(3/4) + sqrt(x)) - 5\*(a^2\*b^2\*x^4 + 2\*a^3\*b\*x^2 + a^4)\*(-1/(a^9\*b^3))^(1/4)\*log(-a^7\*b^2\*(-1/(a^9\*b^3))^(3/4) + sqrt(x)) + 4\*(5\*b\*x^3 + 9\*a\*x)\*sqrt(x))/(a^2\*b^2\*x^4 + 2\*a^3\*b\*x^2 + a^4)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(1/2)/(b\*x\*\*2+a)\*\*3,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.219675, size = 282, normalized size = 1.18

$$\frac{5 b x^{\frac{7}{2}} + 9 a x^{\frac{3}{2}}}{16 (b x^2 + a)^2 a^2} + \frac{5 \sqrt{2} (a b^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} + 2 \sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64 a^3 b^3} + \frac{5 \sqrt{2} (a b^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} - 2 \sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64 a^3 b^3}$$

$$- \frac{5 \sqrt{2} (a b^3)^{\frac{3}{4}} \ln\left(\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128 a^3 b^3} + \frac{5 \sqrt{2} (a b^3)^{\frac{3}{4}} \ln\left(-\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128 a^3 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x)/(b\*x^2 + a)^3,x, algorithm="giac")

[Out] 1/16\*(5\*b\*x^(7/2) + 9\*a\*x^(3/2))/((b\*x^2 + a)^2\*a^2) + 5/64\*sqrt(2)\*(a\*b^3)^(3/4)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(a/b)^(1/4) + 2\*sqrt(x))/(a/b)^(1/4))/(a^3\*b^3) + 5/64\*sqrt(2)\*(a\*b^3)^(3/4)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(a/b)^(1/4) - 2\*sqrt(x))/(a/b)^(1/4))/(a^3\*b^3) - 5/128\*sqrt(2)\*(a\*b^3)^(3/4)\*ln(sqrt(2)\*sqrt(x)\*(a/b)^(1/4) + x + sqrt(a/b))/(a^3\*b^3) + 5/128\*sqrt(2)\*(a\*b^3)^(3/4)\*ln(-sqrt(2)\*sqrt(x)\*(a/b)^(1/4) + x + sqrt(a/b))/(a^3\*b^3)

$$3.308 \quad \int \frac{1}{\sqrt{x}(a+bx^2)^3} dx$$

**Optimal.** Leaf size=239

$$\begin{aligned} & -\frac{21 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} \\ & -\frac{21 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{7\sqrt{x}}{16a^2(a+bx^2)} + \frac{\sqrt{x}}{4a(a+bx^2)^2} \end{aligned}$$

[Out] Sqrt[x]/(4\*a\*(a + b\*x^2)^2) + (7\*Sqrt[x])/(16\*a^2\*(a + b\*x^2)) - (21\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(32\*Sqrt[2]\*a^(11/4)\*b^(1/4))) + (21\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(32\*Sqrt[2]\*a^(11/4)\*b^(1/4))) - (21\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(64\*Sqrt[2]\*a^(11/4)\*b^(1/4)) + (21\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(64\*Sqrt[2]\*a^(11/4)\*b^(1/4))

**Rubi [A]** time = 0.41684, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$

$$\begin{aligned} & -\frac{21 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} \\ & -\frac{21 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{7\sqrt{x}}{16a^2(a+bx^2)} + \frac{\sqrt{x}}{4a(a+bx^2)^2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]\*(a + b\*x^2)^3), x]

[Out] Sqrt[x]/(4\*a\*(a + b\*x^2)^2) + (7\*Sqrt[x])/(16\*a^2\*(a + b\*x^2)) - (21\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(32\*Sqrt[2]\*a^(11/4)\*b^(1/4))) + (21\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)]/(32\*Sqrt[2]\*a^(11/4)\*b^(1/4))) - (21\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(64\*Sqrt[2]\*a^(11/4)\*b^(1/4)) + (21\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/(64\*Sqrt[2]\*a^(11/4)\*b^(1/4))

**Rubi in Sympy [A]** time = 70.2678, size = 224, normalized size = 0.94

$$\begin{aligned} & \frac{\sqrt{x}}{4a(a+bx^2)^2} + \frac{7\sqrt{x}}{16a^2(a+bx^2)} - \frac{21\sqrt{2} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{128a^{11/4}\sqrt[4]{b}} \\ & + \frac{21\sqrt{2} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{128a^{11/4}\sqrt[4]{b}} - \frac{21\sqrt{2} \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}} + \frac{21\sqrt{2} \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+a)\*\*3/x\*\*(1/2), x)

[Out] sqrt(x)/(4\*a\*(a + b\*x\*\*2)\*\*2) + 7\*sqrt(x)/(16\*a\*\*2\*(a + b\*x\*\*2)) - 21\*sqrt(2)\*log(-sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*sqrt(x) + sqrt(a) + sqrt(b)\*x)/(128\*a\*\*(11/4)\*b\*\*(1/4)) + 21\*sqrt(2)\*log(sqrt(2)\*a\*\*(1/4)\*b\*\*(1/4)\*sqrt(x) + sqrt(a) + sqrt(b)\*x)/(128\*a\*\*(11/4)\*b\*\*(1/4)) - 21\*sqrt(2)\*atan(1 - sqrt(2)\*b\*\*(1/4)\*sqrt(x)/a\*\*(1/4))/(64\*a\*\*(11/4)\*b\*\*(1/4)) + 21\*sqrt(2)\*atan(1 + sqrt(2)\*b\*\*(1/4)\*sqrt(x)/a\*\*(1/4))/(64\*a\*\*(11/4)\*b\*\*(1/4))

)/a\*\*(1/4))/(64\*a\*\*(11/4)\*b\*\*(1/4))

**Mathematica [A]** time = 0.183656, size = 220, normalized size = 0.92

$$\frac{\frac{32a^{7/4}\sqrt{x}}{(a+bx^2)^2} + \frac{56a^{3/4}\sqrt{x}}{a+bx^2} - \frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x+\sqrt{a}+\sqrt{bx}}\right)}{\sqrt[4]{b}} + \frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x+\sqrt{a}+\sqrt{bx}}\right)}{\sqrt[4]{b}} - \frac{42\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} + \frac{42\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}}{128a^{11/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]\*(a + b\*x^2)^3), x]

[Out] ((32\*a^(7/4)\*Sqrt[x])/(a + b\*x^2)^2 + (56\*a^(3/4)\*Sqrt[x])/(a + b\*x^2) - (42\*Sqrt[2]\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)])/b^(1/4) + (42\*Sqrt[2]\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[x])/a^(1/4)])/b^(1/4) - (21\*Sqrt[2]\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/b^(1/4) + (21\*Sqrt[2]\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*Sqrt[x] + Sqrt[b]\*x])/b^(1/4))/(128\*a^(11/4))

**Maple [A]** time = 0.01, size = 166, normalized size = 0.7

$$\frac{1}{4a(bx^2+a)^2}\sqrt{x} + \frac{7}{16a^2(bx^2+a)}\sqrt{x} + \frac{21\sqrt{2}}{128a^3}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x + \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) + \frac{21\sqrt{2}}{64a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) + \frac{21\sqrt{2}}{64a^3}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+a)^3/x^(1/2), x)

[Out] 1/4\*x^(1/2)/a/(b\*x^2+a)^2+7/16\*x^(1/2)/a^2/(b\*x^2+a)+21/128/a^3\*(a/b)^(1/4)\*2^(1/2)\*ln((x+(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)\*x^(1/2)\*2^(1/2)+(a/b)^(1/2)))+21/64/a^3\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)+1)+21/64/a^3\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x^(1/2)-1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^3\*sqrt(x)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.254237, size = 305, normalized size = 1.28

$$84(a^2b^2x^4 + 2a^3bx^2 + a^4)\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}\arctan\left(\frac{a^3\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}}{\sqrt{a^6\sqrt{-\frac{1}{a^{11}b}}+x+\sqrt{x}}}\right) - 21(a^2b^2x^4 + 2a^3bx^2 + a^4)\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}\log\left(a^3\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}\right)$$

64(a^2b^2x^4 + 2a^3bx^2 + a^4)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^3\*sqrt(x)),x, algorithm="fricas")

[Out] 
$$-1/64*(84*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^{11}*b))^{1/4}*a \operatorname{rctan}(a^3*(-1/(a^{11}*b))^{1/4}/(\sqrt{a^6*\sqrt{-1/(a^{11}*b))} + x) + \sqrt{x})) - 21*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^{11}*b))^{1/4}*\log(a^3*(-1/(a^{11}*b))^{1/4} + \sqrt{x}) + 21*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^{11}*b))^{1/4}*\log(-a^3*(-1/(a^{11}*b))^{1/4} + \sqrt{x}) - 4*(7*b*x^2 + 11*a)*\sqrt{x})/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)$$

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*2+a)\*\*3/x\*\*(1/2),x)

[Out] Timed out

**GIAC/XCAS** [A] time = 0.215292, size = 282, normalized size = 1.18

$$\frac{21\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^3b} + \frac{21\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^3b} + \frac{21\sqrt{2}(ab^3)^{\frac{1}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128a^3b} - \frac{21\sqrt{2}(ab^3)^{\frac{1}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128a^3b} + \frac{7bx^{\frac{5}{2}}+11a\sqrt{x}}{16(bx^2+a)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^3\*sqrt(x)),x, algorithm="giac")

[Out] 
$$21/64*\sqrt{2}*(a*b^3)^{1/4}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(a/b)^{1/4} + 2*\sqrt{x})/(a/b)^{1/4})/(a^3*b) + 21/64*\sqrt{2}*(a*b^3)^{1/4}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(a/b)^{1/4} - 2*\sqrt{x})/(a/b)^{1/4})/(a^3*b) + 21/128*\sqrt{2}*(a*b^3)^{1/4}*\ln(\sqrt{2}*\sqrt{x}*(a/b)^{1/4} + x + \sqrt{a/b})/(a^3*b) - 21/128*\sqrt{2}*(a*b^3)^{1/4}*\ln(-\sqrt{2}*\sqrt{x}*(a/b)^{1/4} + x + \sqrt{a/b})/(a^3*b) + 1/16*(7*b*x^{5/2} + 11*a*\sqrt{x})/((b*x^2 + a)^2*a^2)$$

$$3.309 \quad \int \frac{1}{x^{3/2}(a+bx^2)^3} dx$$

**Optimal.** Leaf size=251

$$\begin{aligned} & -\frac{45\sqrt[4]{b} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{13/4}} + \frac{45\sqrt[4]{b} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{13/4}} \\ & + \frac{45\sqrt[4]{b} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{13/4}} - \frac{45\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{32\sqrt{2}a^{13/4}} \\ & - \frac{45}{16a^3\sqrt{x}} + \frac{9}{16a^2\sqrt{x}(a+bx^2)} + \frac{1}{4a\sqrt{x}(a+bx^2)^2} \end{aligned}$$

[Out]  $-45/(16*a^3*\text{Sqrt}[x]) + 1/(4*a*\text{Sqrt}[x]*(a + b*x^2)^2) + 9/(16*a^2*\text{Sqrt}[x]*(a + b*x^2)) + (45*b^{(1/4)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/(32*\text{Sqrt}[2]*a^{(13/4)}) - (45*b^{(1/4)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/(32*\text{Sqrt}[2]*a^{(13/4)}) - (45*b^{(1/4)}*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(64*\text{Sqrt}[2]*a^{(13/4)}) + (45*b^{(1/4)}*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(64*\text{Sqrt}[2]*a^{(13/4)})$

**Rubi [A]** time = 0.478548, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$

$$\begin{aligned} & -\frac{45\sqrt[4]{b} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{13/4}} + \frac{45\sqrt[4]{b} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{13/4}} \\ & + \frac{45\sqrt[4]{b} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{13/4}} - \frac{45\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{32\sqrt{2}a^{13/4}} \\ & - \frac{45}{16a^3\sqrt{x}} + \frac{9}{16a^2\sqrt{x}(a+bx^2)} + \frac{1}{4a\sqrt{x}(a+bx^2)^2} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^{(3/2)}*(a + b*x^2)^3), x]$

[Out]  $-45/(16*a^3*\text{Sqrt}[x]) + 1/(4*a*\text{Sqrt}[x]*(a + b*x^2)^2) + 9/(16*a^2*\text{Sqrt}[x]*(a + b*x^2)) + (45*b^{(1/4)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/(32*\text{Sqrt}[2]*a^{(13/4)}) - (45*b^{(1/4)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/(32*\text{Sqrt}[2]*a^{(13/4)}) - (45*b^{(1/4)}*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(64*\text{Sqrt}[2]*a^{(13/4)}) + (45*b^{(1/4)}*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(64*\text{Sqrt}[2]*a^{(13/4)})$

**Rubi in Sympy [A]** time = 79.031, size = 238, normalized size = 0.95

$$\begin{aligned} & \frac{1}{4a\sqrt{x}(a+bx^2)^2} + \frac{9}{16a^2\sqrt{x}(a+bx^2)} - \frac{45}{16a^3\sqrt{x}} \\ & - \frac{45\sqrt{2}\sqrt[4]{b} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{128a^{13/4}} + \frac{45\sqrt{2}\sqrt[4]{b} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{128a^{13/4}} \\ & + \frac{45\sqrt{2}\sqrt[4]{b} \text{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{13/4}} - \frac{45\sqrt{2}\sqrt[4]{b} \text{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{13/4}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{(3/2)}/(b*x^{(2+a)}), x)$

[Out]  $1/(4*a*\sqrt{x}*(a+b*x^2)^2) + 9/(16*a^2*\sqrt{x}*(a+b*x^2)) - 45/(16*a^3*\sqrt{x}) - 45*\sqrt{2}*b^{1/4}*\log(-\sqrt{2}*a^{1/4}*(1/4)*b^{1/4}*\sqrt{x} + \sqrt{a} + \sqrt{b}*x)/(128*a^{13/4}) + 45*\sqrt{2}*b^{1/4}*\log(\sqrt{2}*a^{1/4}*(1/4)*b^{1/4}*\sqrt{x} + \sqrt{a} + \sqrt{b}*x)/(128*a^{13/4}) + 45*\sqrt{2}*b^{1/4}*\operatorname{atan}(1 - \sqrt{2}*b^{1/4}*\sqrt{x}/a^{1/4})/(64*a^{13/4}) - 45*\sqrt{2}*b^{1/4}*\operatorname{atan}(1 + \sqrt{2}*b^{1/4}*\sqrt{x}/a^{1/4})/(64*a^{13/4})$

**Mathematica [A]** time = 0.208313, size = 234, normalized size = 0.93

$$\frac{-\frac{32a^{5/4}bx^{3/2}}{(a+bx^2)^2} - \frac{104\sqrt[4]{ab}bx^{3/2}}{a+bx^2} - 45\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) + 45\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) + 90\sqrt{2}\sqrt[4]{b}}{128a^{13/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(3/2)\*(a+b\*x^2)^3),x]

[Out]  $((-256*a^{1/4})/\operatorname{Sqrt}[x] - (32*a^{5/4}*b*x^{3/2})/(a+b*x^2)^2 - (104*a^{1/4}*b*x^{3/2})/(a+b*x^2) + 90*\operatorname{Sqrt}[2]*b^{1/4}*\operatorname{ArcTan}[1 - (\operatorname{Sqrt}[2]*b^{1/4}*\operatorname{Sqrt}[x])/a^{1/4}] - 90*\operatorname{Sqrt}[2]*b^{1/4}*\operatorname{ArcTan}[1 + (\operatorname{Sqrt}[2]*b^{1/4}*\operatorname{Sqrt}[x])/a^{1/4}] - 45*\operatorname{Sqrt}[2]*b^{1/4}*\operatorname{Log}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[2]*a^{1/4}*b^{1/4}*\operatorname{Sqrt}[x] + \operatorname{Sqrt}[b]*x] + 45*\operatorname{Sqrt}[2]*b^{1/4}*\operatorname{Log}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[2]*a^{1/4}*b^{1/4}*\operatorname{Sqrt}[x] + \operatorname{Sqrt}[b]*x])/(128*a^{13/4})$

**Maple [A]** time = 0.024, size = 178, normalized size = 0.7

$$\begin{aligned} & -2\frac{1}{a^3\sqrt{x}} - \frac{13b^2}{16a^3(bx^2+a)^2}x^{\frac{7}{2}} - \frac{17b}{16a^2(bx^2+a)^2}x^{\frac{3}{2}} \\ & - \frac{45\sqrt{2}}{128a^3}\ln\left(1\left(x - \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x + \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} \\ & - \frac{45\sqrt{2}}{64a^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} - \frac{45\sqrt{2}}{64a^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(3/2)/(b\*x^2+a)^3,x)

[Out]  $-2/a^3/x^{1/2} - 13/16*b^2/a^3/(b*x^2+a)^2*x^{7/2} - 17/16*b/a^2/(b*x^2+a)^2*x^{3/2} - 45/128/a^3/(a/b)^{1/4}*2^{1/2}*\ln((x-(a/b)^{1/4})*x^{1/2}*2^{1/2}+(a/b)^{1/4})/(x+(a/b)^{1/4}*x^{1/2}*2^{1/2}+(a/b)^{1/4}) - 45/64/a^3/(a/b)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(a/b)^{1/4})*x^{1/2}+1 - 45/64/a^3/(a/b)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(a/b)^{1/4})*x^{1/2}-1$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^3\*x^(3/2)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.26403, size = 333, normalized size = 1.33

$$180 b^2 x^4 + 324 a b x^2 + 180 (a^3 b^2 x^4 + 2 a^4 b x^2 + a^5) \sqrt{x} \left(-\frac{b}{a^{13}}\right)^{\frac{1}{4}} \arctan\left(\frac{91125 a^{10} \left(-\frac{b}{a^{13}}\right)^{\frac{3}{4}}}{91125 b \sqrt{x} + \sqrt{-8303765625 a^7 b \sqrt{-\frac{b}{a^{13}} + 8303765625 b^2 x}}}\right) + 45$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^3\*x^(3/2)),x, algorithm="fricas")

[Out] -1/64\*(180\*b^2\*x^4 + 324\*a\*b\*x^2 + 180\*(a^3\*b^2\*x^4 + 2\*a^4\*b\*x^2 + a^5)\*sqrt(x)\*(-b/a^13)^(1/4)\*arctan(91125\*a^10\*(-b/a^13)^(3/4)/(91125\*b\*sqrt(x) + sqrt(-8303765625\*a^7\*b\*sqrt(-b/a^13) + 8303765625\*b^2\*x))) + 45\*(a^3\*b^2\*x^4 + 2\*a^4\*b\*x^2 + a^5)\*sqrt(x)\*(-b/a^13)^(1/4)\*log(91125\*a^10\*(-b/a^13)^(3/4) + 91125\*b\*sqrt(x)) - 45\*(a^3\*b^2\*x^4 + 2\*a^4\*b\*x^2 + a^5)\*sqrt(x)\*(-b/a^13)^(1/4)\*log(-91125\*a^10\*(-b/a^13)^(3/4) + 91125\*b\*sqrt(x)) + 128\*a^2/((a^3\*b^2\*x^4 + 2\*a^4\*b\*x^2 + a^5)\*sqrt(x))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(3/2)/(b\*x\*\*2+a)\*\*3,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.217048, size = 297, normalized size = 1.18

$$\frac{2}{a^3 \sqrt{x}} - \frac{45 \sqrt{2} (ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} + 2 \sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64 a^4 b^2} - \frac{45 \sqrt{2} (ab^3)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} - 2 \sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64 a^4 b^2} + \frac{45 \sqrt{2} (ab^3)^{\frac{3}{4}} \ln\left(\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128 a^4 b^2} - \frac{45 \sqrt{2} (ab^3)^{\frac{3}{4}} \ln\left(-\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128 a^4 b^2} - \frac{13 b^2 x^{\frac{7}{2}} + 17 a b x^{\frac{3}{2}}}{16 (bx^2 + a)^2 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^3\*x^(3/2)),x, algorithm="giac")

[Out] -2/(a^3\*sqrt(x)) - 45/64\*sqrt(2)\*(a\*b^3)^(3/4)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(a/b)^(1/4) + 2\*sqrt(x))/(a/b)^(1/4))/(a^4\*b^2) - 45/64\*sqrt(2)\*(a\*b^3)^(3/4)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(a/b)^(1/4) - 2\*sqrt(x))/(a/b)^(1/4))/(a^4\*b^2) + 45/128\*sqrt(2)\*(a\*b^3)^(3/4)\*ln(sqrt(2)\*sqrt(x)\*(a/b)^(1/4) + x + sqrt(a/b))/(a^4\*b^2) - 45/128\*sqrt(2)\*(a\*b^3)^(3/4)\*ln(-sqrt(2)\*sqrt(x)\*(a/b)^(1/4) + x + sqrt(a/b))/(a^4\*b^2) - 1/16\*(13\*b^2\*x^(7/2) + 17\*a\*b\*x^(3/2))/((b\*x^2 + a)^2\*a^3)

$$3.310 \quad \int \frac{1}{x^{5/2}(a+bx^2)^3} dx$$

**Optimal.** Leaf size=251

$$\frac{77b^{3/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{15/4}} - \frac{77b^{3/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{15/4}} \\ + \frac{77b^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{15/4}} - \frac{77b^{3/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{32\sqrt{2}a^{15/4}} \\ - \frac{77}{48a^3x^{3/2}} + \frac{11}{16a^2x^{3/2}(a+bx^2)} + \frac{1}{4ax^{3/2}(a+bx^2)^2}$$

[Out]  $-77/(48*a^3*x^{3/2}) + 1/(4*a*x^{3/2}*(a+b*x^2)^2) + 11/(16*a^2*x^{3/2}*(a+b*x^2)) + (77*b^{3/4}*ArcTan[1 - (Sqrt[2]*b^{1/4}*Sqrt[x])/a^{1/4}])/(32*Sqrt[2]*a^{15/4}) - (77*b^{3/4}*ArcTan[1 + (Sqrt[2]*b^{1/4}*Sqrt[x])/a^{1/4}])/(32*Sqrt[2]*a^{15/4}) + (77*b^{3/4}*Log[Sqrt[a] - Sqrt[2]*a^{1/4}*b^{1/4}*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^{15/4}) - (77*b^{3/4}*Log[Sqrt[a] + Sqrt[2]*a^{1/4}*b^{1/4}*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^{15/4})$

**Rubi [A]** time = 0.478062, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$

$$\frac{77b^{3/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{15/4}} - \frac{77b^{3/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{15/4}} \\ + \frac{77b^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{15/4}} - \frac{77b^{3/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{32\sqrt{2}a^{15/4}} \\ - \frac{77}{48a^3x^{3/2}} + \frac{11}{16a^2x^{3/2}(a+bx^2)} + \frac{1}{4ax^{3/2}(a+bx^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(5/2)\*(a+b\*x^2)^3),x]

[Out]  $-77/(48*a^3*x^{3/2}) + 1/(4*a*x^{3/2}*(a+b*x^2)^2) + 11/(16*a^2*x^{3/2}*(a+b*x^2)) + (77*b^{3/4}*ArcTan[1 - (Sqrt[2]*b^{1/4}*Sqrt[x])/a^{1/4}])/(32*Sqrt[2]*a^{15/4}) - (77*b^{3/4}*ArcTan[1 + (Sqrt[2]*b^{1/4}*Sqrt[x])/a^{1/4}])/(32*Sqrt[2]*a^{15/4}) + (77*b^{3/4}*Log[Sqrt[a] - Sqrt[2]*a^{1/4}*b^{1/4}*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^{15/4}) - (77*b^{3/4}*Log[Sqrt[a] + Sqrt[2]*a^{1/4}*b^{1/4}*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^{15/4})$

**Rubi in Sympy [A]** time = 76.1596, size = 238, normalized size = 0.95

$$\frac{1}{4ax^{3/2}(a+bx^2)^2} + \frac{11}{16a^2x^{3/2}(a+bx^2)} - \frac{77}{48a^3x^{3/2}} + \frac{77\sqrt{2}b^{3/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{128a^{15/4}} \\ - \frac{77\sqrt{2}b^{3/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{128a^{15/4}} \\ + \frac{77\sqrt{2}b^{3/4} \operatorname{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{15/4}} - \frac{77\sqrt{2}b^{3/4} \operatorname{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*(5/2)/(b\*x\*\*2+a)\*\*3,x)



[Out]  $1/(4*a*x^{(3/2)}*(a+b*x^2)^2) + 11/(16*a^2*x^{(3/2)}*(a+b*x^2)) - 77/(48*a^3*x^{(3/2)}) + 77*\sqrt{2}*b^{(3/4)}*\log(-\sqrt{2}*a^{(1/4)}*b^{(1/4)}*\sqrt{x} + \sqrt{a} + \sqrt{b}*x)/(128*a^{(15/4)}) - 77*\sqrt{2}*b^{(3/4)}*\log(\sqrt{2}*a^{(1/4)}*b^{(1/4)}*\sqrt{x} + \sqrt{a} + \sqrt{b}*x)/(128*a^{(15/4)}) + 77*\sqrt{2}*b^{(3/4)}*\operatorname{atan}(1 - \sqrt{2}*b^{(1/4)}*\sqrt{x}/a^{(1/4)})/(64*a^{(15/4)}) - 77*\sqrt{2}*b^{(3/4)}*\operatorname{atan}(1 + \sqrt{2}*b^{(1/4)}*\sqrt{x}/a^{(1/4)})/(64*a^{(15/4)})$

---

**Mathematica [A]** time = 0.236379, size = 234, normalized size = 0.93

$$\frac{-\frac{96a^{7/4}b\sqrt{x}}{(a+bx^2)^2} - \frac{360a^{3/4}b\sqrt{x}}{a+bx^2} - \frac{256a^{3/4}}{x^{3/2}} + 231\sqrt{2}b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) - 231\sqrt{2}b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{384a^{15/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(5/2)\*(a + b\*x^2)^3), x]

[Out]  $((-256*a^{(3/4)})/x^{(3/2)} - (96*a^{(7/4)}*b*\operatorname{Sqrt}[x])/(a + b*x^2)^2 - (360*a^{(3/4)}*b*\operatorname{Sqrt}[x])/(a + b*x^2) + 462*\operatorname{Sqrt}[2]*b^{(3/4)}*\operatorname{ArcTan}[1 - (\operatorname{Sqrt}[2]*b^{(1/4)}*\operatorname{Sqrt}[x])/a^{(1/4)}] - 462*\operatorname{Sqrt}[2]*b^{(3/4)}*\operatorname{ArcTan}[1 + (\operatorname{Sqrt}[2]*b^{(1/4)}*\operatorname{Sqrt}[x])/a^{(1/4)}] + 231*\operatorname{Sqrt}[2]*b^{(3/4)}*\operatorname{Log}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\operatorname{Sqrt}[x] + \operatorname{Sqrt}[b]*x] - 231*\operatorname{Sqrt}[2]*b^{(3/4)}*\operatorname{Log}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\operatorname{Sqrt}[x] + \operatorname{Sqrt}[b]*x])/(384*a^{(15/4)})$

---

**Maple [A]** time = 0.025, size = 181, normalized size = 0.7

$$\begin{aligned} &-\frac{2}{3a^3}x^{-\frac{3}{2}} - \frac{15b^2}{16a^3(bx^2+a)^2}x^{\frac{5}{2}} - \frac{19b}{16a^2(bx^2+a)^2}\sqrt{x} \\ &- \frac{77b\sqrt{2}}{128a^4}\sqrt[4]{\frac{a}{b}}\ln\left(1\left(x + \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x - \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right) \\ &- \frac{77b\sqrt{2}}{64a^4}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right) - \frac{77b\sqrt{2}}{64a^4}\sqrt[4]{\frac{a}{b}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(5/2)/(b\*x^2+a)^3, x)

[Out]  $-2/3/a^3/x^{(3/2)} - 15/16/a^3*b^2/(b*x^2+a)^2*x^{(5/2)} - 19/16/a^2*b/(b*x^2+a)^2*x^{(1/2)} - 77/128/a^4*b*(a/b)^{(1/4)}*2^{(1/2)}*\ln((x+(a/b)^{(1/4)}*x^{(1/2)}*2^{(1/2)}+(a/b)^{(1/2)})/(x-(a/b)^{(1/4)}*x^{(1/2)}*2^{(1/2)}+(a/b)^{(1/2)})) - 77/64/a^4*b*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x^{(1/2)}+1) - 77/64/a^4*b*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x^{(1/2)}-1)$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^3\*x^(5/2)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.260707, size = 356, normalized size = 1.42

$$308 b^2 x^4 + 484 a b x^2 - 924 (a^3 b^2 x^5 + 2 a^4 b x^3 + a^5 x) \sqrt{x} \left(-\frac{b^3}{a^{15}}\right)^{\frac{1}{4}} \arctan\left(\frac{a^4 \left(-\frac{b^3}{a^{15}}\right)^{\frac{1}{4}}}{b \sqrt{x} + \sqrt{a^8 \sqrt{-\frac{b^3}{a^{15}} + b^2 x}}}\right) + 231 (a^3 b^2 x^5 + 2 a^4 b x^3 + a^5 x)$$

192 (a^3 b^2 x^5 + 2 a^4 b x^3 + a^5 x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^3\*x^(5/2)),x, algorithm="fricas")

[Out] -1/192\*(308\*b^2\*x^4 + 484\*a\*b\*x^2 - 924\*(a^3\*b^2\*x^5 + 2\*a^4\*b\*x^3 + a^5\*x)\*sqrt(x)\*(-b^3/a^15)^(1/4)\*arctan(a^4\*(-b^3/a^15)^(1/4)/(b\*sqrt(x) + sqrt(a^8\*sqrt(-b^3/a^15) + b^2\*x))) + 231\*(a^3\*b^2\*x^5 + 2\*a^4\*b\*x^3 + a^5\*x)\*sqrt(x)\*(-b^3/a^15)^(1/4)\*log(77\*a^4\*(-b^3/a^15)^(1/4) + 77\*b\*sqrt(x)) - 231\*(a^3\*b^2\*x^5 + 2\*a^4\*b\*x^3 + a^5\*x)\*sqrt(x)\*(-b^3/a^15)^(1/4)\*log(-77\*a^4\*(-b^3/a^15)^(1/4) + 77\*b\*sqrt(x)) + 128\*a^2/((a^3\*b^2\*x^5 + 2\*a^4\*b\*x^3 + a^5\*x)\*sqrt(x))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(5/2)/(b\*x\*\*2+a)\*\*3,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.213639, size = 281, normalized size = 1.12

$$\frac{77 \sqrt{2} (ab^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64 a^4} - \frac{77 \sqrt{2} (ab^3)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64 a^4} - \frac{77 \sqrt{2} (ab^3)^{\frac{1}{4}} \ln\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128 a^4} + \frac{77 \sqrt{2} (ab^3)^{\frac{1}{4}} \ln\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128 a^4} - \frac{15 b^2 x^{\frac{5}{2}} + 19 ab \sqrt{x}}{16 (bx^2 + a)^2 a^3} - \frac{2}{3 a^3 x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^3\*x^(5/2)),x, algorithm="giac")

[Out] -77/64\*sqrt(2)\*(a\*b^3)^(1/4)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(a/b)^(1/4) + 2\*sqrt(x))/(a/b)^(1/4))/a^4 - 77/64\*sqrt(2)\*(a\*b^3)^(1/4)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(a/b)^(1/4) - 2\*sqrt(x))/(a/b)^(1/4))/a^4 - 77/128\*sqrt(2)\*(a\*b^3)^(1/4)\*ln(sqrt(2)\*sqrt(x)\*(a/b)^(1/4) + x + sqrt(a/b))/a^4 + 77/128\*sqrt(2)\*(a\*b^3)^(1/4)\*ln(-sqrt(2)\*sqrt(x)\*(a/b)^(1/4) + x + sqrt(a/b))/a^4 - 1/16\*(15\*b^2\*x^(5/2) + 19\*a\*b\*sqrt(x))/((b\*x^2 + a)^2\*a^3) - 2/3/(a^3\*x^(3/2))

$$3.311 \quad \int \frac{1}{x^{7/2}(a+bx^2)^3} dx$$

**Optimal.** Leaf size=264

$$\begin{aligned} & \frac{117b^{5/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{17/4}} - \frac{117b^{5/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{17/4}} \\ & - \frac{117b^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{17/4}} + \frac{117b^{5/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{32\sqrt{2}a^{17/4}} \\ & + \frac{117b}{16a^4\sqrt{x}} - \frac{117}{80a^3x^{5/2}} + \frac{13}{16a^2x^{5/2}(a+bx^2)} + \frac{1}{4ax^{5/2}(a+bx^2)^2} \end{aligned}$$

[Out]  $-117/(80*a^3*x^{(5/2)}) + (117*b)/(16*a^4*\text{Sqrt}[x]) + 1/(4*a*x^{(5/2)}*(a + b*x^2)^2) + 13/(16*a^2*x^{(5/2)}*(a + b*x^2)) - (117*b^{(5/4)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/(32*\text{Sqrt}[2]*a^{(17/4)}) + (117*b^{(5/4)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/(32*\text{Sqrt}[2]*a^{(17/4)}) + (117*b^{(5/4)}*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(64*\text{Sqrt}[2]*a^{(17/4)}) - (117*b^{(5/4)}*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(64*\text{Sqrt}[2]*a^{(17/4)})$

**Rubi [A]** time = 0.549861, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$

$$\begin{aligned} & \frac{117b^{5/4} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{17/4}} - \frac{117b^{5/4} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{64\sqrt{2}a^{17/4}} \\ & - \frac{117b^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{17/4}} + \frac{117b^{5/4} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}} + 1\right)}{32\sqrt{2}a^{17/4}} \\ & + \frac{117b}{16a^4\sqrt{x}} - \frac{117}{80a^3x^{5/2}} + \frac{13}{16a^2x^{5/2}(a+bx^2)} + \frac{1}{4ax^{5/2}(a+bx^2)^2} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^{(7/2)}*(a + b*x^2)^3), x]$

[Out]  $-117/(80*a^3*x^{(5/2)}) + (117*b)/(16*a^4*\text{Sqrt}[x]) + 1/(4*a*x^{(5/2)}*(a + b*x^2)^2) + 13/(16*a^2*x^{(5/2)}*(a + b*x^2)) - (117*b^{(5/4)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/(32*\text{Sqrt}[2]*a^{(17/4)}) + (117*b^{(5/4)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[x])/a^{(1/4)}])/(32*\text{Sqrt}[2]*a^{(17/4)}) + (117*b^{(5/4)}*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(64*\text{Sqrt}[2]*a^{(17/4)}) - (117*b^{(5/4)}*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[b]*x])/(64*\text{Sqrt}[2]*a^{(17/4)})$

**Rubi in Sympy [A]** time = 85.171, size = 252, normalized size = 0.95

$$\begin{aligned} & \frac{1}{4ax^{\frac{5}{2}}(a+bx^2)^2} + \frac{13}{16a^2x^{\frac{5}{2}}(a+bx^2)} - \frac{117}{80a^3x^{\frac{5}{2}}} + \frac{117b}{16a^4\sqrt{x}} \\ & + \frac{117\sqrt{2}b^{\frac{5}{4}} \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{128a^{\frac{17}{4}}} - \frac{117\sqrt{2}b^{\frac{5}{4}} \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right)}{128a^{\frac{17}{4}}} \\ & - \frac{117\sqrt{2}b^{\frac{5}{4}} \text{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{\frac{17}{4}}} + \frac{117\sqrt{2}b^{\frac{5}{4}} \text{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{64a^{\frac{17}{4}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**(7/2)/(b*x**2+a)**3,x)`

[Out]  $1/(4*a*x^{5/2}*(a+b*x^2)^2) + 13/(16*a^2*x^{5/2}*(a+b*x^2)) - 117/(80*a^3*x^{5/2}) + 117*b/(16*a^4*\sqrt{x}) + 117*\sqrt{2}*b^{5/4}*\log(-\sqrt{2}*a^{1/4}*b^{1/4}*\sqrt{x} + \sqrt{a} + \sqrt{b}*x)/(128*a^{17/4}) - 117*\sqrt{2}*b^{5/4}*\log(\sqrt{2}*a^{1/4}*b^{1/4}*\sqrt{x} + \sqrt{a} + \sqrt{b}*x)/(128*a^{17/4}) - 117*\sqrt{2}*b^{5/4}*\operatorname{atan}(1 - \sqrt{2}*b^{1/4}*\sqrt{x}/a^{1/4})/(64*a^{17/4}) + 117*\sqrt{2}*b^{5/4}*\operatorname{atan}(1 + \sqrt{2}*b^{1/4}*\sqrt{x}/a^{1/4})/(64*a^{17/4})$

**Mathematica [A]** time = 0.242638, size = 251, normalized size = 0.95

$$\frac{160a^{5/4}b^2x^{3/2}}{(a+bx^2)^2} - \frac{256a^{5/4}}{x^{5/2}} + 585\sqrt{2}b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) - 585\sqrt{2}b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x} + \sqrt{a} + \sqrt{bx}\right) - 1170\sqrt{2}b^{5/4}$$

640a<sup>17/4</sup>

Antiderivative was successfully verified.

[In] `Integrate[1/(x^(7/2)*(a+b*x^2)^3),x]`

[Out]  $((-256*a^{5/4})/x^{5/2} + (3840*a^{1/4}*b)/\operatorname{Sqrt}[x] + (160*a^{5/4}*b^2*x^{3/2})/(a+b*x^2)^2 + (840*a^{1/4}*b^2*x^{3/2})/(a+b*x^2) - 1170*\operatorname{Sqrt}[2]*b^{5/4}*\operatorname{ArcTan}[1 - (\operatorname{Sqrt}[2]*b^{1/4}*\operatorname{Sqrt}[x])/a^{1/4}] + 1170*\operatorname{Sqrt}[2]*b^{5/4}*\operatorname{ArcTan}[1 + (\operatorname{Sqrt}[2]*b^{1/4}*\operatorname{Sqrt}[x])/a^{1/4}] + 585*\operatorname{Sqrt}[2]*b^{5/4}*\operatorname{Log}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[2]*a^{1/4}*b^{1/4}*\operatorname{Sqrt}[x] + \operatorname{Sqrt}[b]*x] - 585*\operatorname{Sqrt}[2]*b^{5/4}*\operatorname{Log}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[2]*a^{1/4}*b^{1/4}*\operatorname{Sqrt}[x] + \operatorname{Sqrt}[b]*x])/(640*a^{17/4})$

**Maple [A]** time = 0.027, size = 192, normalized size = 0.7

$$-\frac{2}{5a^3}x^{-5/2} + 6\frac{b}{a^4\sqrt{x}} + \frac{21b^3}{16a^4(bx^2+a)^2}x^{7/2} + \frac{25b^2}{16a^3(bx^2+a)^2}x^{3/2} + \frac{117b\sqrt{2}}{128a^4}\ln\left(1\left(x - \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)\left(x + \sqrt[4]{\frac{a}{b}}\sqrt{x}\sqrt{2} + \sqrt{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{117b\sqrt{2}}{64a^4}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} + 1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{117b\sqrt{2}}{64a^4}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}} - 1\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^(7/2)/(b*x^2+a)^3,x)`

[Out]  $-2/5/a^3/x^{5/2} + 6*b/a^4/x^{1/2} + 21/16*b^3/a^4/(b*x^2+a)^2*x^{7/2} + 25/16*b^2/a^3/(b*x^2+a)^2*x^{3/2} + 117/128*b/a^4/(a/b)^{1/4}*2^{1/2}*(1/2)*\ln((x - (a/b)^{1/4}*x^{1/2})^{1/2} + (a/b)^{1/2})/(x + (a/b)^{1/4}*x^{1/2})^{1/2} + (a/b)^{1/2}) + 117/64*b/a^4/(a/b)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(a/b)^{1/4}*x^{1/2} + 1) + 117/64*b/a^4/(a/b)^{1/4}*2^{1/2}*\arctan(2^{1/2}/(a/b)^{1/4}*x^{1/2} - 1)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^3\*x^(7/2)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.259916, size = 400, normalized size = 1.52

$$2340 b^3 x^6 + 4212 ab^2 x^4 + 1664 a^2 b x^2 - 128 a^3 + 2340 (a^4 b^2 x^6 + 2 a^5 b x^4 + a^6 x^2) \sqrt{x} \left(-\frac{b^5}{a^{17}}\right)^{\frac{1}{4}} \arctan\left(\frac{1601613 b^4 \sqrt{x} + \sqrt{-2565164201769 a^9 b^5 \sqrt{-b^5/a^{17}} + 2565164201769 b^8 x}}{1601613 b^4 \sqrt{x} + \sqrt{-2565164201769 a^9 b^5 \sqrt{-b^5/a^{17}} + 2565164201769 b^8 x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^3\*x^(7/2)),x, algorithm="fricas")

[Out]  $\frac{1}{320} (2340 b^3 x^6 + 4212 a b^2 x^4 + 1664 a^2 b x^2 - 128 a^3 + 2340 (a^4 b^2 x^6 + 2 a^5 b x^4 + a^6 x^2) \sqrt{x} (-b^5/a^{17})^{1/4} \arctan(1601613 a^{13} (-b^5/a^{17})^{3/4} / (1601613 b^4 \sqrt{x} + \sqrt{-2565164201769 a^9 b^5 \sqrt{-b^5/a^{17}} + 2565164201769 b^8 x})) + 585 (a^4 b^2 x^6 + 2 a^5 b x^4 + a^6 x^2) \sqrt{x} (-b^5/a^{17})^{1/4} \log(1601613 a^{13} (-b^5/a^{17})^{3/4} + 1601613 b^4 \sqrt{x}) - 585 (a^4 b^2 x^6 + 2 a^5 b x^4 + a^6 x^2) \sqrt{x} (-b^5/a^{17})^{1/4} \log(-1601613 a^{13} (-b^5/a^{17})^{3/4} + 1601613 b^4 \sqrt{x})) / ((a^4 b^2 x^6 + 2 a^5 b x^4 + a^6 x^2) \sqrt{x})$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(7/2)/(b\*x\*\*2+a)\*\*3,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.216641, size = 313, normalized size = 1.19

$$\frac{117 \sqrt{2} (ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} + 2 \sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64 a^5 b} + \frac{117 \sqrt{2} (ab^3)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \left(\frac{a}{b}\right)^{\frac{1}{4}} - 2 \sqrt{x}\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64 a^5 b} - \frac{117 \sqrt{2} (ab^3)^{\frac{3}{4}} \ln\left(\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128 a^5 b} + \frac{117 \sqrt{2} (ab^3)^{\frac{3}{4}} \ln\left(-\sqrt{2} \sqrt{x} \left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128 a^5 b} + \frac{21 b^3 x^{\frac{7}{2}} + 25 ab^2 x^{\frac{3}{2}}}{16 (bx^2 + a)^2 a^4} + \frac{2 (15 bx^2 - a)}{5 a^4 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^3\*x^(7/2)),x, algorithm="giac")

[Out]  $\frac{117}{64} \sqrt{2} (a^3 b)^{3/4} \arctan(1/2 \sqrt{2} (\sqrt{2} (a/b)^{1/4} + 2 \sqrt{x}) / (a/b)^{1/4}) / (a^5 b) + \frac{117}{64} \sqrt{2} (a^3 b)^{3/4} \arctan(-1/2 \sqrt{2} (\sqrt{2} (a/b)^{1/4} - 2 \sqrt{x}) / (a/b)^{1/4}) / (a^5 b) - \frac{117}{128} \sqrt{2} (a^3 b)^{3/4} \ln(\sqrt{2} \sqrt{x} (a/b)^{1/4} + x + \sqrt{a/b}) / (a^5 b) + \frac{117}{128} \sqrt{2} (a^3 b)^{3/4} \ln(-\sqrt{2} \sqrt{x} (a/b)^{1/4} + x + \sqrt{a/b}) / (a^5 b) + \frac{21 b^3 x^{7/2} + 25 ab^2 x^{3/2}}{16 (bx^2 + a)^2 a^4} + \frac{2 (15 bx^2 - a)}{5 a^4 x^{5/2}}$

$$\begin{aligned} & /4) * \ln(-\sqrt{2}) * \sqrt{x} * (a/b)^{(1/4)} + x + \sqrt{a/b}) / (a^5 * b) + 1/ \\ & 16 * (21 * b^3 * x^{(7/2)} + 25 * a * b^2 * x^{(3/2)}) / ((b * x^2 + a)^2 * a^4) + 2/5 * \\ & (15 * b * x^2 - a) / (a^4 * x^{(5/2)}) \end{aligned}$$

$$3.312 \quad \int \frac{\sqrt{x}}{a-bx^2} dx$$

**Optimal.** Leaf size=58

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{ab^{3/4}}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{ab^{3/4}}}$$

[Out]  $-(\text{ArcTan}[(b^{(1/4)} * \text{Sqrt}[x])/a^{(1/4)}]/(a^{(1/4)} * b^{(3/4)})) + \text{ArcTanh}[(b^{(1/4)} * \text{Sqrt}[x])/a^{(1/4)}]/(a^{(1/4)} * b^{(3/4)})$

**Rubi [A]** time = 0.0895296, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{ab^{3/4}}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{ab^{3/4}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(a - b\*x^2), x]

[Out]  $-(\text{ArcTan}[(b^{(1/4)} * \text{Sqrt}[x])/a^{(1/4)}]/(a^{(1/4)} * b^{(3/4)})) + \text{ArcTanh}[(b^{(1/4)} * \text{Sqrt}[x])/a^{(1/4)}]/(a^{(1/4)} * b^{(3/4)})$

**Rubi in Sympy [A]** time = 17.6884, size = 53, normalized size = 0.91

$$-\frac{\text{atan}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{ab^{3/4}}} + \frac{\text{atanh}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{ab^{3/4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(1/2)/(-b\*x\*\*2+a), x)

[Out]  $-\text{atan}(b^{(1/4)} * \text{sqrt}(x)/a^{(1/4)})/(a^{(1/4)} * b^{(3/4)}) + \text{atanh}(b^{(1/4)} * \text{sqrt}(x)/a^{(1/4)})/(a^{(1/4)} * b^{(3/4)})$

**Mathematica [A]** time = 0.0323244, size = 73, normalized size = 1.26

$$\frac{\log\left(\sqrt[4]{a} - \sqrt[4]{b}\sqrt{x}\right) - \log\left(\sqrt[4]{a} + \sqrt[4]{b}\sqrt{x}\right) + 2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{ab^{3/4}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(a - b\*x^2), x]

[Out]  $-(2 * \text{ArcTan}[(b^{(1/4)} * \text{Sqrt}[x])/a^{(1/4)}] + \text{Log}[a^{(1/4)} - b^{(1/4)} * \text{Sqrt}[x]] - \text{Log}[a^{(1/4)} + b^{(1/4)} * \text{Sqrt}[x]])/(2 * a^{(1/4)} * b^{(3/4)})$

**Maple [A]** time = 0.012, size = 66, normalized size = 1.1

$$-\frac{1}{b} \arctan\left(1\sqrt{x}\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}} + \frac{1}{2b} \ln\left(1\left(\sqrt{x} + \sqrt[4]{\frac{a}{b}}\right)\left(\sqrt{x} - \sqrt[4]{\frac{a}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{a}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(-b\*x^2+a), x)

[Out] -1/b/(a/b)^(1/4)\*arctan(x^(1/2)/(a/b)^(1/4))+1/2/b/(a/b)^(1/4)\*ln((x^(1/2)+(a/b)^(1/4))/(x^(1/2)-(a/b)^(1/4)))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sqrt(x)/(b\*x^2 - a), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.255073, size = 149, normalized size = 2.57

$$2\left(\frac{1}{ab^3}\right)^{\frac{1}{4}} \arctan\left(\frac{ab^2\left(\frac{1}{ab^3}\right)^{\frac{3}{4}}}{\sqrt{ab\sqrt{\frac{1}{ab^3}}+x+\sqrt{x}}}\right) + \frac{1}{2}\left(\frac{1}{ab^3}\right)^{\frac{1}{4}} \log\left(ab^2\left(\frac{1}{ab^3}\right)^{\frac{3}{4}} + \sqrt{x}\right) - \frac{1}{2}\left(\frac{1}{ab^3}\right)^{\frac{1}{4}} \log\left(-ab^2\left(\frac{1}{ab^3}\right)^{\frac{3}{4}} + \sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sqrt(x)/(b\*x^2 - a), x, algorithm="fricas")

[Out] 2\*(1/(a\*b^3))^(1/4)\*arctan(a\*b^2\*(1/(a\*b^3))^(3/4)/(sqrt(a\*b\*sqrt(1/(a\*b^3))+x)+sqrt(x)))+1/2\*(1/(a\*b^3))^(1/4)\*log(a\*b^2\*(1/(a\*b^3))^(3/4)+sqrt(x))-1/2\*(1/(a\*b^3))^(1/4)\*log(-a\*b^2\*(1/(a\*b^3))^(3/4)+sqrt(x))

**Sympy [A]** time = 17.5813, size = 128, normalized size = 2.21

$$\begin{cases} \frac{\infty}{\sqrt{x}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{for } b = 0 \\ \frac{2}{b\sqrt{x}} & \text{for } a = 0 \\ -\frac{\log\left(-\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2\sqrt[4]{ab^3}\left(\frac{1}{b}\right)^{\frac{9}{4}}} + \frac{\log\left(\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2\sqrt[4]{ab^3}\left(\frac{1}{b}\right)^{\frac{9}{4}}} - \frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{\sqrt[4]{ab^3}\left(\frac{1}{b}\right)^{\frac{9}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(x\*\*(1/2)/(-b\*x\*\*2+a),x)

[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2\*x\*\*(3/2)/(3\*a), Eq(b, 0)), (2/(b\*sqrt(x)), Eq(a, 0)), (-log(-a\*\*(1/4)\*(1/b)\*\*(1/4) + sqrt(x))/(2\*a\*\*(1/4)\*b\*\*3\*(1/b)\*\*(9/4)) + log(a\*\*(1/4)\*(1/b)\*\*(1/4) + sqrt(x))/(2\*a\*\*(1/4)\*b\*\*3\*(1/b)\*\*(9/4)) - atan(sqrt(x)/(a\*\*(1/4)\*(1/b)\*\*(1/4)))/(a\*\*(1/4)\*b\*\*3\*(1/b)\*\*(9/4)), True))

**GIAC/XCAS [A]** time = 0.218923, size = 262, normalized size = 4.52

$$\frac{\sqrt{2}(-ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2ab^3} + \frac{\sqrt{2}(-ab^3)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2ab^3}$$

$$- \frac{\sqrt{2}(-ab^3)^{\frac{3}{4}} \ln\left(\sqrt{2}\sqrt{x}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{-\frac{a}{b}}\right)}{4ab^3} + \frac{\sqrt{2}(-ab^3)^{\frac{3}{4}} \ln\left(-\sqrt{2}\sqrt{x}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{-\frac{a}{b}}\right)}{4ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sqrt(x)/(b\*x^2 - a),x, algorithm="giac")

[Out] 1/2\*sqrt(2)\*(-a\*b^3)^(3/4)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(-a/b)^(1/4) + 2\*sqrt(x))/(-a/b)^(1/4))/(a\*b^3) + 1/2\*sqrt(2)\*(-a\*b^3)^(3/4)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(-a/b)^(1/4) - 2\*sqrt(x))/(-a/b)^(1/4))/(a\*b^3) - 1/4\*sqrt(2)\*(-a\*b^3)^(3/4)\*ln(sqrt(2)\*sqrt(x)\*(-a/b)^(1/4) + x + sqrt(-a/b))/(a\*b^3) + 1/4\*sqrt(2)\*(-a\*b^3)^(3/4)\*ln(-sqrt(2)\*sqrt(x)\*(-a/b)^(1/4) + x + sqrt(-a/b))/(a\*b^3)

### 3.313 $\int \frac{x^{7/2}}{1+x^2} dx$

**Optimal.** Leaf size=108

$$\frac{2x^{5/2}}{5} - 2\sqrt{x} - \frac{\log(x - \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} + \frac{\log(x + \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} - \frac{\tan^{-1}(1 - \sqrt{2}\sqrt{x})}{\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{\sqrt{2}}$$

[Out]  $-2*\text{Sqrt}[x] + (2*x^{(5/2)})/5 - \text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]]/\text{Sqrt}[2]$   
 $+ \text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]]/\text{Sqrt}[2] - \text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[x] +$   
 $x]/(2*\text{Sqrt}[2]) + \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x]/(2*\text{Sqrt}[2])$

**Rubi [A]** time = 0.145155, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$

$$\frac{2x^{5/2}}{5} - 2\sqrt{x} - \frac{\log(x - \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} + \frac{\log(x + \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} - \frac{\tan^{-1}(1 - \sqrt{2}\sqrt{x})}{\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(7/2)}/(1 + x^2), x]$

[Out]  $-2*\text{Sqrt}[x] + (2*x^{(5/2)})/5 - \text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]]/\text{Sqrt}[2]$   
 $+ \text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]]/\text{Sqrt}[2] - \text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[x] +$   
 $x]/(2*\text{Sqrt}[2]) + \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x]/(2*\text{Sqrt}[2])$

**Rubi in Sympy [A]** time = 19.6065, size = 99, normalized size = 0.92

$$\frac{2x^{5/2}}{5} - 2\sqrt{x} - \frac{\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{4} + \frac{\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{4}$$

$$+ \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{2} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{(7/2)}/(x^2+1), x)$

[Out]  $2*x^{(5/2)}/5 - 2*\text{sqrt}(x) - \text{sqrt}(2)*\log(-\text{sqrt}(2)*\text{sqrt}(x) + x + 1)/$   
 $4 + \text{sqrt}(2)*\log(\text{sqrt}(2)*\text{sqrt}(x) + x + 1)/4 + \text{sqrt}(2)*\operatorname{atan}(\text{sqrt}(2)$   
 $*\text{sqrt}(x) - 1)/2 + \text{sqrt}(2)*\operatorname{atan}(\text{sqrt}(2)*\text{sqrt}(x) + 1)/2$

**Mathematica [A]** time = 0.0558188, size = 107, normalized size = 0.99

$$\frac{1}{20} \left( 8x^{5/2} - 40\sqrt{x} - 5\sqrt{2} \log(x - \sqrt{2}\sqrt{x} + 1) + 5\sqrt{2} \log(x + \sqrt{2}\sqrt{x} + 1) \right.$$

$$\left. - 10\sqrt{2} \tan^{-1}(1 - \sqrt{2}\sqrt{x}) + 10\sqrt{2} \tan^{-1}(\sqrt{2}\sqrt{x} + 1) \right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^{(7/2)}/(1 + x^2), x]$

[Out]  $(-40*\text{Sqrt}[x] + 8*x^{(5/2)} - 10*\text{Sqrt}[2]*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]]$   
 $+ 10*\text{Sqrt}[2]*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]] - 5*\text{Sqrt}[2]*\text{Log}[1 - \text{Sqr}$   
 $t[2]*\text{Sqrt}[x] + x] + 5*\text{Sqrt}[2]*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x])/20$

---

**Maple [A]** time = 0.011, size = 72, normalized size = 0.7

$$\frac{2}{5}x^{\frac{5}{2}} - 2\sqrt{x} + \frac{\sqrt{2}}{2}\arctan\left(1 + \sqrt{2}\sqrt{x}\right) + \frac{\sqrt{2}}{2}\arctan\left(\sqrt{2}\sqrt{x} - 1\right) + \frac{\sqrt{2}}{4}\ln\left(1\left(1 + x + \sqrt{2}\sqrt{x}\right)\left(1 + x - \sqrt{2}\sqrt{x}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)/(x^2+1), x)

[Out] 2/5\*x^(5/2)-2\*x^(1/2)+1/2\*arctan(1+2^(1/2)\*x^(1/2))\*2^(1/2)+1/2\*arctan(2^(1/2)\*x^(1/2)-1)\*2^(1/2)+1/4\*2^(1/2)\*ln((1+x+2^(1/2)\*x^(1/2))/(1+x-2^(1/2)\*x^(1/2)))

---

**Maxima [A]** time = 1.4949, size = 113, normalized size = 1.05

$$\frac{2}{5}x^{\frac{5}{2}} + \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) + \frac{1}{4}\sqrt{2}\log(\sqrt{2}\sqrt{x} + x + 1) - \frac{1}{4}\sqrt{2}\log(-\sqrt{2}\sqrt{x} + x + 1) - 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(x^2 + 1), x, algorithm="maxima")

[Out] 2/5\*x^(5/2) + 1/2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) + 1/2\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) + 1/4\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1) - 1/4\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1) - 2\*sqrt(x)

---

**Fricas [A]** time = 0.254332, size = 153, normalized size = 1.42

$$\frac{2}{5}(x^2 - 5)\sqrt{x} - \sqrt{2}\arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{2}\sqrt{2}\sqrt{x} + 2x + 2 + 1}\right) - \sqrt{2}\arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{-2}\sqrt{2}\sqrt{x} + 2x + 2 - 1}\right) + \frac{1}{4}\sqrt{2}\log(2\sqrt{2}\sqrt{x} + 2x + 2) - \frac{1}{4}\sqrt{2}\log(-2\sqrt{2}\sqrt{x} + 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(x^2 + 1), x, algorithm="fricas")

[Out] 2/5\*(x^2 - 5)\*sqrt(x) - sqrt(2)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(2)\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 1)) - sqrt(2)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(-2)\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 1)) + 1/4\*sqrt(2)\*log(2\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 1/4\*sqrt(2)\*log(-2\*sqrt(2)\*sqrt(x) + 2\*x + 2)

---

**Sympy [A]** time = 22.1255, size = 105, normalized size = 0.97

$$\frac{2x^{\frac{5}{2}}}{5} - 2\sqrt{x} - \frac{\sqrt{2} \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{4} + \frac{\sqrt{2} \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{4} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{2} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(7/2)/(x\*\*2+1), x)

[Out] 2\*x\*\*(5/2)/5 - 2\*sqrt(x) - sqrt(2)\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/4 + sqrt(2)\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/4 + sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/2 + sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/2

**GIAC/XCAS [A]** time = 0.209826, size = 113, normalized size = 1.05

$$\frac{2}{5}x^{\frac{5}{2}} + \frac{1}{2}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) + \frac{1}{2}\sqrt{2} \arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) + \frac{1}{4}\sqrt{2} \ln(\sqrt{2}\sqrt{x} + x + 1) - \frac{1}{4}\sqrt{2} \ln(-\sqrt{2}\sqrt{x} + x + 1) - 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(x^2 + 1), x, algorithm="giac")

[Out] 2/5\*x^(5/2) + 1/2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) + 1/2\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) + 1/4\*sqrt(2)\*ln(sqrt(2)\*sqrt(x) + x + 1) - 1/4\*sqrt(2)\*ln(-sqrt(2)\*sqrt(x) + x + 1) - 2\*sqrt(x)

$$3.314 \quad \int \frac{x^{5/2}}{1+x^2} dx$$

**Optimal.** Leaf size=101

$$\frac{2x^{3/2}}{3} - \frac{\log(x - \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} + \frac{\log(x + \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} + \frac{\tan^{-1}(1 - \sqrt{2}\sqrt{x})}{\sqrt{2}} - \frac{\tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{\sqrt{2}}$$

[Out] (2\*x^(3/2))/3 + ArcTan[1 - Sqrt[2]\*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]\*Sqrt[x]]/Sqrt[2] - Log[1 - Sqrt[2]\*Sqrt[x] + x]/(2\*Sqrt[2]) + Log[1 + Sqrt[2]\*Sqrt[x] + x]/(2\*Sqrt[2])

**Rubi [A]** time = 0.136844, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$

$$\frac{2x^{3/2}}{3} - \frac{\log(x - \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} + \frac{\log(x + \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} + \frac{\tan^{-1}(1 - \sqrt{2}\sqrt{x})}{\sqrt{2}} - \frac{\tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)/(1 + x^2), x]

[Out] (2\*x^(3/2))/3 + ArcTan[1 - Sqrt[2]\*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]\*Sqrt[x]]/Sqrt[2] - Log[1 - Sqrt[2]\*Sqrt[x] + x]/(2\*Sqrt[2]) + Log[1 + Sqrt[2]\*Sqrt[x] + x]/(2\*Sqrt[2])

**Rubi in Sympy [A]** time = 19.4782, size = 92, normalized size = 0.91

$$\frac{2x^{3/2}}{3} - \frac{\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{4} + \frac{\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{4} - \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{2} - \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(5/2)/(x\*\*2+1), x)

[Out] 2\*x\*\*(3/2)/3 - sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1)/4 + sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1)/4 - sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/2 - sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/2

**Mathematica [A]** time = 0.034067, size = 100, normalized size = 0.99

$$\frac{1}{12} \left( 8x^{3/2} - 3\sqrt{2} \log(x - \sqrt{2}\sqrt{x} + 1) + 3\sqrt{2} \log(x + \sqrt{2}\sqrt{x} + 1) + 6\sqrt{2} \tan^{-1}(1 - \sqrt{2}\sqrt{x}) - 6\sqrt{2} \tan^{-1}(\sqrt{2}\sqrt{x} + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)/(1 + x^2), x]

[Out] (8\*x^(3/2) + 6\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[x]] - 6\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[x]] - 3\*Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[x] + x] + 3\*Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/12

**Maple [A]** time = 0.009, size = 67, normalized size = 0.7

$$\frac{2}{3}x^{\frac{3}{2}} - \frac{\sqrt{2}}{2} \arctan\left(1 + \sqrt{2}\sqrt{x}\right) - \frac{\sqrt{2}}{2} \arctan\left(\sqrt{2}\sqrt{x} - 1\right) - \frac{\sqrt{2}}{4} \ln\left(1\left(1 + x - \sqrt{2}\sqrt{x}\right)\left(1 + x + \sqrt{2}\sqrt{x}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)/(x^2+1), x)

[Out] 2/3\*x^(3/2)-1/2\*arctan(1+2^(1/2)\*x^(1/2))\*2^(1/2)-1/2\*arctan(2^(1/2)\*x^(1/2)-1)\*2^(1/2)-1/4\*2^(1/2)\*ln((1+x-2^(1/2)\*x^(1/2))/(1+x+2^(1/2)\*x^(1/2)))

**Maxima [A]** time = 1.49609, size = 107, normalized size = 1.06

$$\begin{aligned} & \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{2}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) - \frac{1}{2}\sqrt{2} \arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) \\ & + \frac{1}{4}\sqrt{2} \log\left(\sqrt{2}\sqrt{x} + x + 1\right) - \frac{1}{4}\sqrt{2} \log\left(-\sqrt{2}\sqrt{x} + x + 1\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(x^2 + 1), x, algorithm="maxima")

[Out] 2/3\*x^(3/2) - 1/2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) - 1/2\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) + 1/4\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1) - 1/4\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1)

**Fricas [A]** time = 0.247336, size = 143, normalized size = 1.42

$$\begin{aligned} & \frac{2}{3}x^{\frac{3}{2}} + \sqrt{2} \arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{2}\sqrt{2}\sqrt{x} + 2x + 2 + 1}\right) + \sqrt{2} \arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{-2}\sqrt{2}\sqrt{x} + 2x + 2 - 1}\right) \\ & + \frac{1}{4}\sqrt{2} \log\left(2\sqrt{2}\sqrt{x} + 2x + 2\right) - \frac{1}{4}\sqrt{2} \log\left(-2\sqrt{2}\sqrt{x} + 2x + 2\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(x^2 + 1), x, algorithm="fricas")

[Out] 2/3\*x^(3/2) + sqrt(2)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(2\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 1)) + sqrt(2)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(-2\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 1)) + 1/4\*sqrt(2)\*log(2\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 1/4\*sqrt(2)\*log(-2\*sqrt(2)\*sqrt(x) + 2\*x + 2)

**Sympy [A]** time = 7.76015, size = 99, normalized size = 0.98

$$\begin{aligned} & \frac{2x^{\frac{3}{2}}}{3} - \frac{\sqrt{2} \log\left(-4\sqrt{2}\sqrt{x} + 4x + 4\right)}{4} + \frac{\sqrt{2} \log\left(4\sqrt{2}\sqrt{x} + 4x + 4\right)}{4} \\ & - \frac{\sqrt{2} \operatorname{atan}\left(\sqrt{2}\sqrt{x} - 1\right)}{2} - \frac{\sqrt{2} \operatorname{atan}\left(\sqrt{2}\sqrt{x} + 1\right)}{2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(5/2)/(x\*\*2+1),x)

[Out] 2\*x\*\*(3/2)/3 - sqrt(2)\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/4 + sqrt(2)\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/4 - sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/2 - sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/2

**GIAC/XCAS [A]** time = 0.21496, size = 107, normalized size = 1.06

$$\frac{2}{3} x^{\frac{3}{2}} - \frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) - \frac{1}{2} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) + \frac{1}{4} \sqrt{2} \ln(\sqrt{2}\sqrt{x} + x + 1) - \frac{1}{4} \sqrt{2} \ln(-\sqrt{2}\sqrt{x} + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(x^2 + 1),x, algorithm="giac")

[Out] 2/3\*x^(3/2) - 1/2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) - 1/2\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) + 1/4\*sqrt(2)\*ln(sqrt(2)\*sqrt(x) + x + 1) - 1/4\*sqrt(2)\*ln(-sqrt(2)\*sqrt(x) + x + 1)

### 3.315 $\int \frac{x^{3/2}}{1+x^2} dx$

**Optimal.** Leaf size=99

$$2\sqrt{x} + \frac{\log(x - \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} - \frac{\log(x + \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} + \frac{\tan^{-1}(1 - \sqrt{2}\sqrt{x})}{\sqrt{2}} - \frac{\tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{\sqrt{2}}$$

[Out] 2\*Sqrt[x] + ArcTan[1 - Sqrt[2]\*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]\*Sqrt[x]]/Sqrt[2] + Log[1 - Sqrt[2]\*Sqrt[x] + x]/(2\*Sqrt[2]) - Log[1 + Sqrt[2]\*Sqrt[x] + x]/(2\*Sqrt[2])

**Rubi [A]** time = 0.130425, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$

$$2\sqrt{x} + \frac{\log(x - \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} - \frac{\log(x + \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} + \frac{\tan^{-1}(1 - \sqrt{2}\sqrt{x})}{\sqrt{2}} - \frac{\tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)/(1 + x^2), x]

[Out] 2\*Sqrt[x] + ArcTan[1 - Sqrt[2]\*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]\*Sqrt[x]]/Sqrt[2] + Log[1 - Sqrt[2]\*Sqrt[x] + x]/(2\*Sqrt[2]) - Log[1 + Sqrt[2]\*Sqrt[x] + x]/(2\*Sqrt[2])

**Rubi in Sympy [A]** time = 18.0023, size = 90, normalized size = 0.91

$$2\sqrt{x} + \frac{\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{4} - \frac{\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{4} - \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{2} - \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(3/2)/(x\*\*2+1), x)

[Out] 2\*sqrt(x) + sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1)/4 - sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1)/4 - sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/2 - sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/2

**Mathematica [A]** time = 0.0335189, size = 99, normalized size = 1.

$$\frac{1}{4} \left( 8\sqrt{x} + \sqrt{2} \log(x - \sqrt{2}\sqrt{x} + 1) - \sqrt{2} \log(x + \sqrt{2}\sqrt{x} + 1) + 2\sqrt{2} \tan^{-1}(1 - \sqrt{2}\sqrt{x}) - 2\sqrt{2} \tan^{-1}(\sqrt{2}\sqrt{x} + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)/(1 + x^2), x]

[Out] (8\*Sqrt[x] + 2\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[x]] - 2\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[x]] + Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[x] + x] - Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/4



**Maple [A]** time = 0.008, size = 67, normalized size = 0.7

$$2\sqrt{x} - \frac{\sqrt{2}}{2} \arctan\left(\sqrt{2}\sqrt{x} - 1\right) - \frac{\sqrt{2}}{4} \ln\left(1\left(1 + x + \sqrt{2}\sqrt{x}\right)\left(1 + x - \sqrt{2}\sqrt{x}\right)^{-1}\right) - \frac{\sqrt{2}}{2} \arctan\left(1 + \sqrt{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(x^2+1), x)

[Out] 2\*x^(1/2)-1/2\*arctan(2^(1/2)\*x^(1/2)-1)\*2^(1/2)-1/4\*2^(1/2)\*ln((1+x+2^(1/2)\*x^(1/2))/(1+x-2^(1/2)\*x^(1/2)))-1/2\*arctan(1+2^(1/2)\*x^(1/2))\*2^(1/2)

**Maxima [A]** time = 1.50559, size = 107, normalized size = 1.08

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(x^2 + 1), x, algorithm="maxima")

[Out] -1/2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) - 1/2\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) - 1/4\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1) + 1/4\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1) + 2\*sqrt(x)

**Fricas [A]** time = 0.252073, size = 143, normalized size = 1.44

$$\sqrt{2}\arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{2}\sqrt{2}\sqrt{x} + 2x + 2 + 1}\right) + \sqrt{2}\arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{-2}\sqrt{2}\sqrt{x} + 2x + 2 - 1}\right) - \frac{1}{4}\sqrt{2}\log\left(2\sqrt{2}\sqrt{x} + 2x + 2\right) + \frac{1}{4}\sqrt{2}\log\left(-2\sqrt{2}\sqrt{x} + 2x + 2\right) + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(x^2 + 1), x, algorithm="fricas")

[Out] sqrt(2)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(2\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 1)) + sqrt(2)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(-2\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 1)) - 1/4\*sqrt(2)\*log(2\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 1/4\*sqrt(2)\*log(-2\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 2\*sqrt(x)

**Sympy [A]** time = 3.34918, size = 97, normalized size = 0.98

$$2\sqrt{x} + \frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x} + 4x + 4\right)}{4} - \frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x} + 4x + 4\right)}{4} - \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x} - 1\right)}{2} - \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x} + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(3/2)/(x\*\*2+1),x)

[Out] 2\*sqrt(x) + sqrt(2)\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/4 - sqrt(2)\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/4 - sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/2 - sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/2

**GIAC/XCAS [A]** time = 0.209112, size = 107, normalized size = 1.08

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)-\frac{1}{4}\sqrt{2}\ln\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\ln\left(-\sqrt{2}\sqrt{x}+x+1\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(x^2 + 1),x, algorithm="giac")

[Out] -1/2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) - 1/2\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) - 1/4\*sqrt(2)\*ln(sqrt(2)\*sqrt(x) + x + 1) + 1/4\*sqrt(2)\*ln(-sqrt(2)\*sqrt(x) + x + 1) + 2\*sqrt(x)

$$3.316 \quad \int \frac{\sqrt{x}}{1+x^2} dx$$

**Optimal.** Leaf size=92

$$\frac{\log(x - \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} - \frac{\log(x + \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} - \frac{\tan^{-1}(1 - \sqrt{2}\sqrt{x})}{\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{\sqrt{2}}$$

[Out] -(ArcTan[1 - Sqrt[2]\*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]\*Sqrt[x]]/Sqrt[2] + Log[1 - Sqrt[2]\*Sqrt[x] + x]/(2\*Sqrt[2]) - Log[1 + Sqrt[2]\*Sqrt[x] + x]/(2\*Sqrt[2])

**Rubi [A]** time = 0.127897, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$

$$\frac{\log(x - \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} - \frac{\log(x + \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} - \frac{\tan^{-1}(1 - \sqrt{2}\sqrt{x})}{\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(1 + x^2), x]

[Out] -(ArcTan[1 - Sqrt[2]\*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]\*Sqrt[x]]/Sqrt[2] + Log[1 - Sqrt[2]\*Sqrt[x] + x]/(2\*Sqrt[2]) - Log[1 + Sqrt[2]\*Sqrt[x] + x]/(2\*Sqrt[2])

**Rubi in Sympy [A]** time = 17.3342, size = 83, normalized size = 0.9

$$\frac{\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{4} - \frac{\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{4} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{2} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(1/2)/(x\*\*2+1), x)

[Out] sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1)/4 - sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1)/4 + sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/2 + sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/2

**Mathematica [A]** time = 0.026588, size = 76, normalized size = 0.83

$$\frac{\log(x - \sqrt{2}\sqrt{x} + 1) - \log(x + \sqrt{2}\sqrt{x} + 1) - 2 \tan^{-1}(1 - \sqrt{2}\sqrt{x}) + 2 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(1 + x^2), x]

[Out] (-2\*ArcTan[1 - Sqrt[2]\*Sqrt[x]] + 2\*ArcTan[1 + Sqrt[2]\*Sqrt[x]] + Log[1 - Sqrt[2]\*Sqrt[x] + x] - Log[1 + Sqrt[2]\*Sqrt[x] + x])/(2\*Sqrt[2])

**Maple [A]** time = 0.007, size = 62, normalized size = 0.7

$$\frac{\sqrt{2}}{2} \arctan(\sqrt{2}\sqrt{x} - 1) + \frac{\sqrt{2}}{4} \ln\left(1 \left(1 + x - \sqrt{2}\sqrt{x}\right) \left(1 + x + \sqrt{2}\sqrt{x}\right)^{-1}\right) + \frac{\sqrt{2}}{2} \arctan(1 + \sqrt{2}\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x^2+1), x)

[Out] 1/2\*arctan(2^(1/2)\*x^(1/2)-1)\*2^(1/2)+1/4\*2^(1/2)\*ln((1+x-2^(1/2)\*x^(1/2))/(1+x+2^(1/2)\*x^(1/2)))+1/2\*arctan(1+2^(1/2)\*x^(1/2))\*2^(1/2)

**Maxima [A]** time = 1.48703, size = 100, normalized size = 1.09

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) + \frac{1}{2} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) - \frac{1}{4} \sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1) + \frac{1}{4} \sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x)/(x^2 + 1), x, algorithm="maxima")

[Out] 1/2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) + 1/2\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) - 1/4\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1) + 1/4\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1)

**Fricas [A]** time = 0.252344, size = 139, normalized size = 1.51

$$-\sqrt{2} \arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{2\sqrt{2}\sqrt{x} + 2x + 2 + 1}}\right) - \sqrt{2} \arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{-2\sqrt{2}\sqrt{x} + 2x + 2 - 1}}\right) - \frac{1}{4} \sqrt{2} \log(2\sqrt{2}\sqrt{x} + 2x + 2) + \frac{1}{4} \sqrt{2} \log(-2\sqrt{2}\sqrt{x} + 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x)/(x^2 + 1), x, algorithm="fricas")

[Out] -sqrt(2)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(2\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 1)) - sqrt(2)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(-2\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 1)) - 1/4\*sqrt(2)\*log(2\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 1/4\*sqrt(2)\*log(-2\*sqrt(2)\*sqrt(x) + 2\*x + 2)

**Sympy [A]** time = 2.00071, size = 90, normalized size = 0.98

$$\frac{\sqrt{2} \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{4} - \frac{\sqrt{2} \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{4} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{2} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(1/2)/(x\*\*2+1), x)

[Out] sqrt(2)\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/4 - sqrt(2)\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/4 + sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/2 + s

$\text{qrt}(2) * \text{atan}(\text{sqrt}(2) * \text{sqrt}(x) + 1) / 2$

---

**GIAC/XCAS [A]** time = 0.214758, size = 100, normalized size = 1.09

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} (\sqrt{2} + 2 \sqrt{x})\right) + \frac{1}{2} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2} (\sqrt{2} - 2 \sqrt{x})\right) - \frac{1}{4} \sqrt{2} \ln(\sqrt{2} \sqrt{x} + x + 1) + \frac{1}{4} \sqrt{2} \ln(-\sqrt{2} \sqrt{x} + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x)/(x^2 + 1),x, algorithm="giac")

[Out] 1/2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) + 1/2\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) - 1/4\*sqrt(2)\*ln(sqrt(2)\*sqrt(x) + x + 1) + 1/4\*sqrt(2)\*ln(-sqrt(2)\*sqrt(x) + x + 1)

$$3.317 \quad \int \frac{1}{\sqrt{x}(1+x^2)} dx$$

**Optimal.** Leaf size=92

$$-\frac{\log(x - \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} + \frac{\log(x + \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} - \frac{\tan^{-1}(1 - \sqrt{2}\sqrt{x})}{\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{\sqrt{2}}$$

[Out] -(ArcTan[1 - Sqrt[2]\*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]\*Sqrt[x]]/Sqrt[2] - Log[1 - Sqrt[2]\*Sqrt[x] + x]/(2\*Sqrt[2]) + Log[1 + Sqrt[2]\*Sqrt[x] + x]/(2\*Sqrt[2])

**Rubi [A]** time = 0.123013, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$

$$-\frac{\log(x - \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} + \frac{\log(x + \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} - \frac{\tan^{-1}(1 - \sqrt{2}\sqrt{x})}{\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]\*(1 + x^2)), x]

[Out] -(ArcTan[1 - Sqrt[2]\*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]\*Sqrt[x]]/Sqrt[2] - Log[1 - Sqrt[2]\*Sqrt[x] + x]/(2\*Sqrt[2]) + Log[1 + Sqrt[2]\*Sqrt[x] + x]/(2\*Sqrt[2])

**Rubi in Sympy [A]** time = 16.2786, size = 83, normalized size = 0.9

$$-\frac{\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{4} + \frac{\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{4} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{2} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(x\*\*2+1)/x\*\*(1/2), x)

[Out] -sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1)/4 + sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1)/4 + sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/2 + sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/2

**Mathematica [A]** time = 0.0220481, size = 76, normalized size = 0.83

$$\frac{-\log(x - \sqrt{2}\sqrt{x} + 1) + \log(x + \sqrt{2}\sqrt{x} + 1) - 2 \tan^{-1}(1 - \sqrt{2}\sqrt{x}) + 2 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]\*(1 + x^2)), x]

[Out] (-2\*ArcTan[1 - Sqrt[2]\*Sqrt[x]] + 2\*ArcTan[1 + Sqrt[2]\*Sqrt[x]] - Log[1 - Sqrt[2]\*Sqrt[x] + x] + Log[1 + Sqrt[2]\*Sqrt[x] + x])/(2\*Sqrt[2])

**Maple [A]** time = 0.007, size = 62, normalized size = 0.7

$$\frac{\sqrt{2}}{2} \arctan(\sqrt{2}\sqrt{x} - 1) + \frac{\sqrt{2}}{4} \ln\left(1 \left(1 + x + \sqrt{2}\sqrt{x}\right) \left(1 + x - \sqrt{2}\sqrt{x}\right)^{-1}\right) + \frac{\sqrt{2}}{2} \arctan\left(1 + \sqrt{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2+1)/x^(1/2), x)`

[Out] `1/2*arctan(2^(1/2)*x^(1/2)-1)*2^(1/2)+1/4*2^(1/2)*ln((1+x+2^(1/2)*x^(1/2))/(1+x-2^(1/2)*x^(1/2)))+1/2*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)`

**Maxima [A]** time = 1.48905, size = 100, normalized size = 1.09

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) + \frac{1}{2} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) + \frac{1}{4} \sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1) - \frac{1}{4} \sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 + 1)*sqrt(x)), x, algorithm="maxima")`

[Out] `1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)`

**Fricas [A]** time = 0.250424, size = 139, normalized size = 1.51

$$-\sqrt{2} \arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{2\sqrt{2}\sqrt{x} + 2x + 2 + 1}}\right) - \sqrt{2} \arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{-2\sqrt{2}\sqrt{x} + 2x + 2 - 1}}\right) + \frac{1}{4} \sqrt{2} \log(2\sqrt{2}\sqrt{x} + 2x + 2) - \frac{1}{4} \sqrt{2} \log(-2\sqrt{2}\sqrt{x} + 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 + 1)*sqrt(x)), x, algorithm="fricas")`

[Out] `-sqrt(2)*arctan(1/(sqrt(2)*sqrt(x) + sqrt(2*sqrt(2)*sqrt(x) + 2*x + 2) + 1)) - sqrt(2)*arctan(1/(sqrt(2)*sqrt(x) + sqrt(-2*sqrt(2)*sqrt(x) + 2*x + 2) - 1)) + 1/4*sqrt(2)*log(2*sqrt(2)*sqrt(x) + 2*x + 2) - 1/4*sqrt(2)*log(-2*sqrt(2)*sqrt(x) + 2*x + 2)`

**Sympy [A]** time = 2.85345, size = 90, normalized size = 0.98

$$-\frac{\sqrt{2} \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{4} + \frac{\sqrt{2} \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{4} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{2} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2+1)/x**(1/2), x)`

[Out] `-sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 +`

$\sqrt{2} \operatorname{atan}(\sqrt{2} \sqrt{x} + 1) / 2$

---

**GIAC/XCAS [A]** time = 0.208833, size = 100, normalized size = 1.09

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} (\sqrt{2} + 2 \sqrt{x})\right) + \frac{1}{2} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2} (\sqrt{2} - 2 \sqrt{x})\right) \\ + \frac{1}{4} \sqrt{2} \ln(\sqrt{2} \sqrt{x} + x + 1) - \frac{1}{4} \sqrt{2} \ln(-\sqrt{2} \sqrt{x} + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)\*sqrt(x)),x, algorithm="giac")

[Out] 1/2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) + 1/2\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) + 1/4\*sqrt(2)\*ln(sqrt(2)\*sqrt(x) + x + 1) - 1/4\*sqrt(2)\*ln(-sqrt(2)\*sqrt(x) + x + 1)



$$3.318 \quad \int \frac{1}{x^{3/2}(1+x^2)} dx$$

**Optimal.** Leaf size=99

$$-\frac{2}{\sqrt{x}} - \frac{\log(x - \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} + \frac{\log(x + \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} + \frac{\tan^{-1}(1 - \sqrt{2}\sqrt{x})}{\sqrt{2}} - \frac{\tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{\sqrt{2}}$$

[Out] -2/Sqrt[x] + ArcTan[1 - Sqrt[2]\*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]\*Sqrt[x]]/Sqrt[2] - Log[1 - Sqrt[2]\*Sqrt[x] + x]/(2\*Sqrt[2]) + Log[1 + Sqrt[2]\*Sqrt[x] + x]/(2\*Sqrt[2])

**Rubi [A]** time = 0.133273, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$

$$-\frac{2}{\sqrt{x}} - \frac{\log(x - \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} + \frac{\log(x + \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} + \frac{\tan^{-1}(1 - \sqrt{2}\sqrt{x})}{\sqrt{2}} - \frac{\tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(3/2)\*(1 + x^2)), x]

[Out] -2/Sqrt[x] + ArcTan[1 - Sqrt[2]\*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]\*Sqrt[x]]/Sqrt[2] - Log[1 - Sqrt[2]\*Sqrt[x] + x]/(2\*Sqrt[2]) + Log[1 + Sqrt[2]\*Sqrt[x] + x]/(2\*Sqrt[2])

**Rubi in Sympy [A]** time = 18.9603, size = 90, normalized size = 0.91

$$-\frac{\sqrt{2}\log(-\sqrt{2}\sqrt{x} + x + 1)}{4} + \frac{\sqrt{2}\log(\sqrt{2}\sqrt{x} + x + 1)}{4} - \frac{\sqrt{2}\operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{2} - \frac{\sqrt{2}\operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{2} - \frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*(3/2)/(x\*\*2+1), x)

[Out] -sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1)/4 + sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1)/4 - sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/2 - sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/2 - 2/sqrt(x)

**Mathematica [A]** time = 0.0554511, size = 99, normalized size = 1.

$$\frac{1}{4} \left( -\frac{8}{\sqrt{x}} - \sqrt{2}\log(x - \sqrt{2}\sqrt{x} + 1) + \sqrt{2}\log(x + \sqrt{2}\sqrt{x} + 1) + 2\sqrt{2}\tan^{-1}(1 - \sqrt{2}\sqrt{x}) - 2\sqrt{2}\tan^{-1}(\sqrt{2}\sqrt{x} + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(3/2)\*(1 + x^2)), x]

[Out] (-8/Sqrt[x] + 2\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[x]] - 2\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[x]] - Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[x] + x] + Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/4

**Maple [A]** time = 0.01, size = 67, normalized size = 0.7

$$-\frac{\sqrt{2}}{2} \arctan\left(1 + \sqrt{2}\sqrt{x}\right) - \frac{\sqrt{2}}{2} \arctan\left(\sqrt{2}\sqrt{x} - 1\right) - \frac{\sqrt{2}}{4} \ln\left(1 \left(1 + x - \sqrt{2}\sqrt{x}\right) \left(1 + x + \sqrt{2}\sqrt{x}\right)^{-1}\right) - 2 \frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(3/2)/(x^2+1), x)

[Out] -1/2\*arctan(1+2^(1/2)\*x^(1/2))\*2^(1/2)-1/2\*arctan(2^(1/2)\*x^(1/2)-1)\*2^(1/2)-1/4\*2^(1/2)\*ln((1+x-2^(1/2)\*x^(1/2))/(1+x+2^(1/2)\*x^(1/2)))-2/x^(1/2)

**Maxima [A]** time = 1.51559, size = 107, normalized size = 1.08

$$-\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) - \frac{1}{2} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) + \frac{1}{4} \sqrt{2} \log\left(\sqrt{2}\sqrt{x} + x + 1\right) - \frac{1}{4} \sqrt{2} \log\left(-\sqrt{2}\sqrt{x} + x + 1\right) - \frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)\*x^(3/2)), x, algorithm="maxima")

[Out] -1/2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) - 1/2\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) + 1/4\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1) - 1/4\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1) - 2/sqrt(x)

**Fricas [A]** time = 0.247709, size = 157, normalized size = 1.59

$$4\sqrt{2}x \arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{2}\sqrt{2}\sqrt{x} + 2x + 2 + 1}\right) + 4\sqrt{2}x \arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{-2}\sqrt{2}\sqrt{x} + 2x + 2 - 1}\right) + \sqrt{2}x \log\left(2\sqrt{2}\sqrt{x} + 2x + 2\right) - \sqrt{2}x \log\left(-2\sqrt{2}\sqrt{x} + 2x + 2\right) - \frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)\*x^(3/2)), x, algorithm="fricas")

[Out] 1/4\*(4\*sqrt(2)\*x\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(2\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 1)) + 4\*sqrt(2)\*x\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(-2\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 1)) + sqrt(2)\*x\*log(2\*sqrt(2)\*sqrt(x) + 2\*x + 2) - sqrt(2)\*x\*log(-2\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 8\*sqrt(x))/x

**Sympy [A]** time = 6.21941, size = 97, normalized size = 0.98

$$\frac{\sqrt{2} \log\left(-4\sqrt{2}\sqrt{x} + 4x + 4\right)}{4} + \frac{\sqrt{2} \log\left(4\sqrt{2}\sqrt{x} + 4x + 4\right)}{4} - \frac{\sqrt{2} \operatorname{atan}\left(\sqrt{2}\sqrt{x} - 1\right)}{2} - \frac{\sqrt{2} \operatorname{atan}\left(\sqrt{2}\sqrt{x} + 1\right)}{2} - \frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(3/2)/(x\*\*2+1), x)

```
[Out] -sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/2 - 2/sqrt(x)
```

**GIAC/XCAS [A]** time = 0.211222, size = 107, normalized size = 1.08

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2}+2\sqrt{x})\right) - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2}-2\sqrt{x})\right) + \frac{1}{4}\sqrt{2}\ln(\sqrt{2}\sqrt{x}+x+1) - \frac{1}{4}\sqrt{2}\ln(-\sqrt{2}\sqrt{x}+x+1) - \frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^2 + 1)*x^(3/2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*ln(sqrt(2)*sqrt(x) + x + 1) - 1/4*sqrt(2)*ln(-sqrt(2)*sqrt(x) + x + 1) - 2/sqrt(x)
```

$$3.319 \quad \int \frac{1}{x^{5/2}(1+x^2)} dx$$

**Optimal.** Leaf size=101

$$-\frac{2}{3x^{3/2}} + \frac{\log(x - \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} - \frac{\log(x + \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} + \frac{\tan^{-1}(1 - \sqrt{2}\sqrt{x})}{\sqrt{2}} - \frac{\tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{\sqrt{2}}$$

[Out]  $-2/(3*x^{(3/2)}) + \text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]]/\text{Sqrt}[2] - \text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]]/\text{Sqrt}[2] + \text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[x] + x]/(2*\text{Sqrt}[2]) - \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x]/(2*\text{Sqrt}[2])$

**Rubi [A]** time = 0.128431, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$

$$-\frac{2}{3x^{3/2}} + \frac{\log(x - \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} - \frac{\log(x + \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} + \frac{\tan^{-1}(1 - \sqrt{2}\sqrt{x})}{\sqrt{2}} - \frac{\tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(5/2)\*(1 + x^2)), x]

[Out]  $-2/(3*x^{(3/2)}) + \text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]]/\text{Sqrt}[2] - \text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]]/\text{Sqrt}[2] + \text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[x] + x]/(2*\text{Sqrt}[2]) - \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x]/(2*\text{Sqrt}[2])$

**Rubi in Sympy [A]** time = 17.9805, size = 92, normalized size = 0.91

$$\frac{\sqrt{2}\log(-\sqrt{2}\sqrt{x} + x + 1)}{4} - \frac{\sqrt{2}\log(\sqrt{2}\sqrt{x} + x + 1)}{4} - \frac{\sqrt{2}\text{atan}(\sqrt{2}\sqrt{x} - 1)}{2} - \frac{\sqrt{2}\text{atan}(\sqrt{2}\sqrt{x} + 1)}{2} - \frac{2}{3x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*(5/2)/(x\*\*2+1), x)

[Out]  $\text{sqrt}(2)*\log(-\text{sqrt}(2)*\text{sqrt}(x) + x + 1)/4 - \text{sqrt}(2)*\log(\text{sqrt}(2)*\text{sqrt}(x) + x + 1)/4 - \text{sqrt}(2)*\text{atan}(\text{sqrt}(2)*\text{sqrt}(x) - 1)/2 - \text{sqrt}(2)*\text{atan}(\text{sqrt}(2)*\text{sqrt}(x) + 1)/2 - 2/(3*x^{(3/2)})$

**Mathematica [A]** time = 0.0656346, size = 100, normalized size = 0.99

$$\frac{1}{12} \left( -\frac{8}{x^{3/2}} + 3\sqrt{2}\log(x - \sqrt{2}\sqrt{x} + 1) - 3\sqrt{2}\log(x + \sqrt{2}\sqrt{x} + 1) + 6\sqrt{2}\tan^{-1}(1 - \sqrt{2}\sqrt{x}) - 6\sqrt{2}\tan^{-1}(\sqrt{2}\sqrt{x} + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(5/2)\*(1 + x^2)), x]

[Out]  $(-8/x^{(3/2)} + 6*\text{Sqrt}[2]*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]] - 6*\text{Sqrt}[2]*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]] + 3*\text{Sqrt}[2]*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[x] + x] - 3*\text{Sqrt}[2]*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x])/12$

**Maple [A]** time = 0.01, size = 67, normalized size = 0.7

$$-\frac{\sqrt{2}}{2} \arctan\left(1 + \sqrt{2}\sqrt{x}\right) - \frac{\sqrt{2}}{2} \arctan\left(\sqrt{2}\sqrt{x} - 1\right) - \frac{\sqrt{2}}{4} \ln\left(1 \left(1 + x + \sqrt{2}\sqrt{x}\right) \left(1 + x - \sqrt{2}\sqrt{x}\right)^{-1}\right) - \frac{2}{3}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(5/2)/(x^2+1), x)

[Out] -1/2\*arctan(1+2^(1/2)\*x^(1/2))\*2^(1/2)-1/2\*arctan(2^(1/2)\*x^(1/2)-1)\*2^(1/2)-1/4\*2^(1/2)\*ln((1+x+2^(1/2)\*x^(1/2))/(1+x-2^(1/2)\*x^(1/2)))-2/3/x^(3/2)

**Maxima [A]** time = 1.50241, size = 107, normalized size = 1.06

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) - \frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)\*x^(5/2)), x, algorithm="maxima")

[Out] -1/2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) - 1/2\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) - 1/4\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1) + 1/4\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1) - 2/3/x^(3/2)

**Fricas [A]** time = 0.250619, size = 169, normalized size = 1.67

$$\frac{12\sqrt{2}x^2 \arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{2}\sqrt{2}\sqrt{x} + 2x + 2}\right) + 12\sqrt{2}x^2 \arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{-2}\sqrt{2}\sqrt{x} + 2x + 2}\right) - 3\sqrt{2}x^2 \log\left(2\sqrt{2}\sqrt{x} + 2x + 2\right) + 3\sqrt{2}}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)\*x^(5/2)), x, algorithm="fricas")

[Out] 1/12\*(12\*sqrt(2)\*x^2\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(2\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 1)) + 12\*sqrt(2)\*x^2\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(-2\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 1)) - 3\*sqrt(2)\*x^2\*log(2\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 3\*sqrt(2)\*x^2\*log(-2\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 8\*sqrt(x))/x^2

**Sympy [A]** time = 12.3369, size = 99, normalized size = 0.98

$$\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x} + 4x + 4\right)}{4} - \frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x} + 4x + 4\right)}{4} - \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x} - 1\right)}{2} - \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x} + 1\right)}{2} - \frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(5/2)/(x\*\*2+1), x)

```
[Out] sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sqrt(
2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - s
qrt(2)*atan(sqrt(2)*sqrt(x) + 1)/2 - 2/(3*x**(3/2))
```

---

**GIAC/XCAS [A]** time = 0.208802, size = 107, normalized size = 1.06

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2}+2\sqrt{x})\right) - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2}-2\sqrt{x})\right) \\ - \frac{1}{4}\sqrt{2}\ln\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{1}{4}\sqrt{2}\ln\left(-\sqrt{2}\sqrt{x}+x+1\right) - \frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^2 + 1)*x^(5/2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*ln(s
qrt(2)*sqrt(x) + x + 1) + 1/4*sqrt(2)*ln(-sqrt(2)*sqrt(x) + x + 1
) - 2/3/x^(3/2)
```

$$3.320 \quad \int \frac{1}{x^{7/2}(1+x^2)} dx$$

**Optimal.** Leaf size=108

$$-\frac{2}{5x^{5/2}} + \frac{2}{\sqrt{x}} + \frac{\log(x - \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} - \frac{\log(x + \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} - \frac{\tan^{-1}(1 - \sqrt{2}\sqrt{x})}{\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{\sqrt{2}}$$

[Out]  $-2/(5*x^{(5/2)}) + 2/\text{Sqrt}[x] - \text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]]/\text{Sqrt}[2]$   
 $+ \text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]]/\text{Sqrt}[2] + \text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[x] +$   
 $x]/(2*\text{Sqrt}[2]) - \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x]/(2*\text{Sqrt}[2])$

**Rubi [A]** time = 0.140206, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$

$$-\frac{2}{5x^{5/2}} + \frac{2}{\sqrt{x}} + \frac{\log(x - \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} - \frac{\log(x + \sqrt{2}\sqrt{x} + 1)}{2\sqrt{2}} - \frac{\tan^{-1}(1 - \sqrt{2}\sqrt{x})}{\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^{(7/2)}*(1 + x^2)), x]$

[Out]  $-2/(5*x^{(5/2)}) + 2/\text{Sqrt}[x] - \text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]]/\text{Sqrt}[2]$   
 $+ \text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]]/\text{Sqrt}[2] + \text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[x] +$   
 $x]/(2*\text{Sqrt}[2]) - \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x]/(2*\text{Sqrt}[2])$

**Rubi in Sympy [A]** time = 20.9247, size = 99, normalized size = 0.92

$$\frac{\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{4} - \frac{\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{4}$$

$$+ \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{2} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{2} + \frac{2}{\sqrt{x}} - \frac{2}{5x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{(7/2)}/(x^{(2+1)}), x)$

[Out]  $\text{sqrt}(2)*\log(-\text{sqrt}(2)*\text{sqrt}(x) + x + 1)/4 - \text{sqrt}(2)*\log(\text{sqrt}(2)*\text{sqrt}(x) + x + 1)/4$   
 $+ \text{sqrt}(2)*\operatorname{atan}(\text{sqrt}(2)*\text{sqrt}(x) - 1)/2 + \text{sqrt}(2)*\operatorname{atan}(\text{sqrt}(2)*\text{sqrt}(x) + 1)/2$   
 $+ 2/\text{sqrt}(x) - 2/(5*x^{(5/2)})$

**Mathematica [A]** time = 0.0763655, size = 107, normalized size = 0.99

$$\frac{1}{20} \left( -\frac{8}{x^{5/2}} + \frac{40}{\sqrt{x}} + 5\sqrt{2} \log(x - \sqrt{2}\sqrt{x} + 1) - 5\sqrt{2} \log(x + \sqrt{2}\sqrt{x} + 1) \right.$$

$$\left. - 10\sqrt{2} \tan^{-1}(1 - \sqrt{2}\sqrt{x}) + 10\sqrt{2} \tan^{-1}(\sqrt{2}\sqrt{x} + 1) \right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/(x^{(7/2)}*(1 + x^2)), x]$

[Out]  $(-8/x^{(5/2)} + 40/\text{Sqrt}[x] - 10*\text{Sqrt}[2]*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]]$   
 $+ 10*\text{Sqrt}[2]*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]] + 5*\text{Sqrt}[2]*\text{Log}[1 - \text{Sqr$

$t[2]*\text{Sqrt}[x] + x] - 5*\text{Sqrt}[2]*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x])/20$

**Maple [A]** time = 0.012, size = 72, normalized size = 0.7

$$-\frac{2}{5}x^{-\frac{5}{2}} + 2\frac{1}{\sqrt{x}} + \frac{\sqrt{2}}{2}\arctan\left(\sqrt{2}\sqrt{x} - 1\right) + \frac{\sqrt{2}}{4}\ln\left(1\left(1+x-\sqrt{2}\sqrt{x}\right)\left(1+x+\sqrt{2}\sqrt{x}\right)^{-1}\right) + \frac{\sqrt{2}}{2}\arctan\left(1+\sqrt{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^(7/2)/(x^2+1), x)`

[Out]  $-2/5/x^{5/2}+2/x^{1/2}+1/2*\arctan(2^{1/2}*x^{1/2}-1)*2^{1/2}+1/4*2^{1/2}*\ln((1+x-2^{1/2}*x^{1/2})*(1+x+2^{1/2}*x^{1/2}))+1/2*\arctan(1+2^{1/2}*x^{1/2})*2^{1/2}$

**Maxima [A]** time = 1.52781, size = 116, normalized size = 1.07

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2}+2\sqrt{x})\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2}-2\sqrt{x})\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{2(5x^2-1)}{5x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 + 1)*x^(7/2)), x, algorithm="maxima")`

[Out]  $1/2*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*( \text{sqrt}(2) + 2*\text{sqrt}(x))) + 1/2*\text{sqrt}(2)*\arctan(-1/2*\text{sqrt}(2)*( \text{sqrt}(2) - 2*\text{sqrt}(x))) - 1/4*\text{sqrt}(2)*\log(\text{sqrt}(2)*\text{sqrt}(x) + x + 1) + 1/4*\text{sqrt}(2)*\log(-\text{sqrt}(2)*\text{sqrt}(x) + x + 1) + 2/5*(5*x^2 - 1)/x^{5/2}$

**Fricas [A]** time = 0.251762, size = 178, normalized size = 1.65

$$\frac{20\sqrt{2}x^3\arctan\left(\frac{1}{\sqrt{2}\sqrt{x}+\sqrt{2}\sqrt{2}\sqrt{x}+2x+2+1}}\right) + 20\sqrt{2}x^3\arctan\left(\frac{1}{\sqrt{2}\sqrt{x}+\sqrt{-2}\sqrt{2}\sqrt{x}+2x+2-1}}\right) + 5\sqrt{2}x^3\log\left(2\sqrt{2}\sqrt{x}+2x+2\right) - 5\sqrt{2}x^3\log\left(-2\sqrt{2}\sqrt{x}+2x+2\right)}{20x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 + 1)*x^(7/2)), x, algorithm="fricas")`

[Out]  $-1/20*(20*\text{sqrt}(2)*x^3*\arctan(1/(\text{sqrt}(2)*\text{sqrt}(x) + \text{sqrt}(2*\text{sqrt}(2)*\text{sqrt}(x) + 2*x + 2) + 1)) + 20*\text{sqrt}(2)*x^3*\arctan(1/(\text{sqrt}(2)*\text{sqrt}(x) + \text{sqrt}(-2*\text{sqrt}(2)*\text{sqrt}(x) + 2*x + 2) - 1)) + 5*\text{sqrt}(2)*x^3*\log(2*\text{sqrt}(2)*\text{sqrt}(x) + 2*x + 2) - 5*\text{sqrt}(2)*x^3*\log(-2*\text{sqrt}(2)*\text{sqrt}(x) + 2*x + 2) - 8*(5*x^2 - 1)*\text{sqrt}(x))/x^3$

**Sympy [A]** time = 32.9248, size = 105, normalized size = 0.97

$$\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4} - \frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4} + \frac{\sqrt{2}\text{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2} + \frac{\sqrt{2}\text{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2} + \frac{2}{\sqrt{x}} - \frac{2}{5x^{5/2}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(7/2)/(x\*\*2+1),x)

[Out] sqrt(2)\*log(-4\*sqrt(2)\*sqrt(x)+4\*x+4)/4 - sqrt(2)\*log(4\*sqrt(2)\*sqrt(x)+4\*x+4)/4 + sqrt(2)\*atan(sqrt(2)\*sqrt(x)-1)/2 + sqrt(2)\*atan(sqrt(2)\*sqrt(x)+1)/2 + 2/sqrt(x) - 2/(5\*x\*\*(5/2))

**GIAC/XCAS [A]** time = 0.21297, size = 116, normalized size = 1.07

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2}+2\sqrt{x})\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2}-2\sqrt{x})\right) - \frac{1}{4}\sqrt{2}\ln(\sqrt{2}\sqrt{x}+x+1) + \frac{1}{4}\sqrt{2}\ln(-\sqrt{2}\sqrt{x}+x+1) + \frac{2(5x^2-1)}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)\*x^(7/2)),x, algorithm="giac")

[Out] 1/2\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2)+2\*sqrt(x))) + 1/2\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)-2\*sqrt(x))) - 1/4\*sqrt(2)\*ln(sqrt(2)\*sqrt(x)+x+1) + 1/4\*sqrt(2)\*ln(-sqrt(2)\*sqrt(x)+x+1) + 2/5\*(5\*x^2-1)/x^(5/2)

$$3.321 \quad \int \frac{x^{7/2}}{(1+x^2)^2} dx$$

**Optimal.** Leaf size=122

$$\begin{aligned} & -\frac{x^{5/2}}{2(x^2+1)} + \frac{5\sqrt{x}}{2} + \frac{5 \log(x - \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} - \frac{5 \log(x + \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} \\ & + \frac{5 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{4\sqrt{2}} - \frac{5 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{4\sqrt{2}} \end{aligned}$$

[Out] (5\*Sqrt[x])/2 - x^(5/2)/(2\*(1+x^2)) + (5\*ArcTan[1 - Sqrt[2]\*Sqrt[x]]/(4\*Sqrt[2]) - (5\*ArcTan[1 + Sqrt[2]\*Sqrt[x]]/(4\*Sqrt[2]) + (5\*Log[1 - Sqrt[2]\*Sqrt[x] + x])/(8\*Sqrt[2]) - (5\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/(8\*Sqrt[2]))

**Rubi [A]** time = 0.146124, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$

$$\begin{aligned} & -\frac{x^{5/2}}{2(x^2+1)} + \frac{5\sqrt{x}}{2} + \frac{5 \log(x - \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} - \frac{5 \log(x + \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} \\ & + \frac{5 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{4\sqrt{2}} - \frac{5 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{4\sqrt{2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^(7/2)/(1+x^2)^2,x]

[Out] (5\*Sqrt[x])/2 - x^(5/2)/(2\*(1+x^2)) + (5\*ArcTan[1 - Sqrt[2]\*Sqrt[x]]/(4\*Sqrt[2]) - (5\*ArcTan[1 + Sqrt[2]\*Sqrt[x]]/(4\*Sqrt[2]) + (5\*Log[1 - Sqrt[2]\*Sqrt[x] + x])/(8\*Sqrt[2]) - (5\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/(8\*Sqrt[2]))

**Rubi in Sympy [A]** time = 20.1913, size = 110, normalized size = 0.9

$$\begin{aligned} & -\frac{x^{5/2}}{2(x^2+1)} + \frac{5\sqrt{x}}{2} + \frac{5\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{16} - \frac{5\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{16} \\ & - \frac{5\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{8} - \frac{5\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{8} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(7/2)/(x\*\*2+1)\*\*2,x)

[Out] -x\*\*(5/2)/(2\*(x\*\*2+1)) + 5\*sqrt(x)/2 + 5\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1)/16 - 5\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1)/16 - 5\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/8 - 5\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/8

**Mathematica [A]** time = 0.132696, size = 114, normalized size = 0.93

$$\begin{aligned} & \frac{1}{16} \left( \frac{8\sqrt{x}}{x^2+1} + 32\sqrt{x} + 5\sqrt{2} \log(x - \sqrt{2}\sqrt{x} + 1) - 5\sqrt{2} \log(x + \sqrt{2}\sqrt{x} + 1) \right. \\ & \left. + 10\sqrt{2} \tan^{-1}(1 - \sqrt{2}\sqrt{x}) - 10\sqrt{2} \tan^{-1}(\sqrt{2}\sqrt{x} + 1) \right) \end{aligned}$$

Antiderivative was successfully verified.

[In] Integrate[x^(7/2)/(1 + x^2)^2, x]

[Out] (32\*sqrt(x) + (8\*sqrt(x)))/(1 + x^2) + 10\*sqrt(2)\*ArcTan[1 - Sqrt[2]\*sqrt(x)] - 10\*sqrt(2)\*ArcTan[1 + Sqrt[2]\*sqrt(x)] + 5\*sqrt(2)\*Log[1 - Sqrt[2]\*sqrt(x) + x] - 5\*sqrt(2)\*Log[1 + Sqrt[2]\*sqrt(x) + x])/16

**Maple [A]** time = 0.015, size = 79, normalized size = 0.7

$$2\sqrt{x} + \frac{1}{2x^2+2}\sqrt{x} - \frac{5\sqrt{2}}{8}\arctan\left(1 + \sqrt{2}\sqrt{x}\right) - \frac{5\sqrt{2}}{8}\arctan\left(\sqrt{2}\sqrt{x} - 1\right) - \frac{5\sqrt{2}}{16}\ln\left(1\left(1+x+\sqrt{2}\sqrt{x}\right)\left(1+x-\sqrt{2}\sqrt{x}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)/(x^2+1)^2, x)

[Out] 2\*x^(1/2)+1/2\*x^(1/2)/(x^2+1)-5/8\*arctan(1+2^(1/2)\*x^(1/2))\*2^(1/2)-5/8\*arctan(2^(1/2)\*x^(1/2)-1)\*2^(1/2)-5/16\*2^(1/2)\*ln((1+x+2^(1/2)\*x^(1/2))/(1+x-2^(1/2)\*x^(1/2)))

**Maxima [A]** time = 1.49682, size = 123, normalized size = 1.01

$$-\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) - \frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{5}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{5}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + 2\sqrt{x} + \frac{\sqrt{x}}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(x^2 + 1)^2, x, algorithm="maxima")

[Out] -5/8\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) - 5/8\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) - 5/16\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1) + 5/16\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1) + 2\*sqrt(x) + 1/2\*sqrt(x)/(x^2 + 1)

**Fricas [A]** time = 0.250109, size = 194, normalized size = 1.59

$$\frac{20\sqrt{2}(x^2+1)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x}+\sqrt{2}\sqrt{x}+2x+2+1}}\right) + 20\sqrt{2}(x^2+1)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x}+\sqrt{-2}\sqrt{2}\sqrt{x}+2x+2-1}}\right) - 5\sqrt{2}(x^2+1)\log\left(2\sqrt{2}\sqrt{x}\right)}{16(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(x^2 + 1)^2, x, algorithm="fricas")

[Out] 1/16\*(20\*sqrt(2)\*(x^2 + 1)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(2)\*sqrt(x) + 2\*x + 2) + 1) + 20\*sqrt(2)\*(x^2 + 1)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(-2)\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 1)) - 5\*sqrt(2)\*(x^2 + 1)\*log(2\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 5\*sqrt(2)\*(x^2 + 1)\*log(-2\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 8\*(4\*x^2 + 5)\*sqrt(x))/(x^2 + 1)

**Sympy [A]** time = 62.2104, size = 277, normalized size = 2.27

$$\frac{32x^{\frac{5}{2}}}{16x^2 + 16} + \frac{40\sqrt{x}}{16x^2 + 16} + \frac{5\sqrt{2}x^2 \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{16x^2 + 16} - \frac{5\sqrt{2}x^2 \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{16x^2 + 16}$$

$$- \frac{10\sqrt{2}x^2 \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{16x^2 + 16} - \frac{10\sqrt{2}x^2 \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{16x^2 + 16} + \frac{5\sqrt{2} \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{16x^2 + 16}$$

$$- \frac{5\sqrt{2} \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{16x^2 + 16} - \frac{10\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{16x^2 + 16} - \frac{10\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{16x^2 + 16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(7/2)/(x\*\*2+1)\*\*2, x)

[Out] 32\*x\*\*(5/2)/(16\*x\*\*2 + 16) + 40\*sqrt(x)/(16\*x\*\*2 + 16) + 5\*sqrt(2)\*x\*\*2\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(16\*x\*\*2 + 16) - 5\*sqrt(2)\*x\*\*2\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(16\*x\*\*2 + 16) - 10\*sqrt(2)\*x\*\*2\*atan(sqrt(2)\*sqrt(x) - 1)/(16\*x\*\*2 + 16) - 10\*sqrt(2)\*x\*\*2\*atan(sqrt(2)\*sqrt(x) + 1)/(16\*x\*\*2 + 16) + 5\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(16\*x\*\*2 + 16) - 5\*sqrt(2)\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(16\*x\*\*2 + 16) - 10\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/(16\*x\*\*2 + 16) - 10\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/(16\*x\*\*2 + 16)

**GIAC/XCAS [A]** time = 0.209961, size = 123, normalized size = 1.01

$$-\frac{5}{8}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) - \frac{5}{8}\sqrt{2} \arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2} - 2\sqrt{x})\right)$$

$$- \frac{5}{16}\sqrt{2} \ln(\sqrt{2}\sqrt{x} + x + 1) + \frac{5}{16}\sqrt{2} \ln(-\sqrt{2}\sqrt{x} + x + 1) + 2\sqrt{x} + \frac{\sqrt{x}}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(x^2 + 1)^2, x, algorithm="giac")

[Out] -5/8\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) - 5/8\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) - 5/16\*sqrt(2)\*ln(sqrt(2)\*sqrt(x) + x + 1) + 5/16\*sqrt(2)\*ln(-sqrt(2)\*sqrt(x) + x + 1) + 2\*sqrt(x) + 1/2\*sqrt(x)/(x^2 + 1)

$$3.322 \quad \int \frac{x^{5/2}}{(1+x^2)^2} dx$$

**Optimal.** Leaf size=113

$$-\frac{x^{3/2}}{2(x^2+1)} + \frac{3 \log(x - \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} - \frac{3 \log(x + \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} - \frac{3 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{4\sqrt{2}} + \frac{3 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{4\sqrt{2}}$$

[Out]  $-x^{3/2}/(2*(1+x^2)) - (3*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]])/(4*\text{Sqrt}[2]) + (3*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]])/(4*\text{Sqrt}[2]) + (3*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[x] + x])/(8*\text{Sqrt}[2]) - (3*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x])/(8*\text{Sqrt}[2])$

**Rubi [A]** time = 0.146961, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$

$$-\frac{x^{3/2}}{2(x^2+1)} + \frac{3 \log(x - \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} - \frac{3 \log(x + \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} - \frac{3 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{4\sqrt{2}} + \frac{3 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{5/2}/(1+x^2)^2, x]$

[Out]  $-x^{3/2}/(2*(1+x^2)) - (3*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]])/(4*\text{Sqrt}[2]) + (3*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]])/(4*\text{Sqrt}[2]) + (3*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[x] + x])/(8*\text{Sqrt}[2]) - (3*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x])/(8*\text{Sqrt}[2])$

**Rubi in Sympy [A]** time = 19.5482, size = 102, normalized size = 0.9

$$-\frac{x^{3/2}}{2(x^2+1)} + \frac{3\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{16} - \frac{3\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{16} + \frac{3\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{8} + \frac{3\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{5/2}/(x^2+1)^2, x)$

[Out]  $-x^{3/2}/(2*(x^2+1)) + 3*\text{sqrt}(2)*\log(-\text{sqrt}(2)*\text{sqrt}(x) + x + 1)/16 - 3*\text{sqrt}(2)*\log(\text{sqrt}(2)*\text{sqrt}(x) + x + 1)/16 + 3*\text{sqrt}(2)*\operatorname{atan}(\text{sqrt}(2)*\text{sqrt}(x) - 1)/8 + 3*\text{sqrt}(2)*\operatorname{atan}(\text{sqrt}(2)*\text{sqrt}(x) + 1)/8$

**Mathematica [A]** time = 0.14101, size = 107, normalized size = 0.95

$$\frac{1}{16} \left( -\frac{8x^{3/2}}{x^2+1} + 3\sqrt{2} \log(x - \sqrt{2}\sqrt{x} + 1) - 3\sqrt{2} \log(x + \sqrt{2}\sqrt{x} + 1) - 6\sqrt{2} \tan^{-1}(1 - \sqrt{2}\sqrt{x}) + 6\sqrt{2} \tan^{-1}(\sqrt{2}\sqrt{x} + 1) \right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^{5/2}/(1+x^2)^2, x]$

[Out]  $((-8*x^{(3/2)})/(1 + x^2) - 6*\text{Sqrt}[2]*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]] + 6*\text{Sqrt}[2]*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]] + 3*\text{Sqrt}[2]*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[x] + x] - 3*\text{Sqrt}[2]*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x])/16$

**Maple [A]** time = 0.014, size = 74, normalized size = 0.7

$$-\frac{1}{2x^2+2}x^{\frac{3}{2}} + \frac{3\sqrt{2}}{8}\arctan\left(\sqrt{2}\sqrt{x}-1\right) + \frac{3\sqrt{2}}{16}\ln\left(1\left(1+x-\sqrt{2}\sqrt{x}\right)\left(1+x+\sqrt{2}\sqrt{x}\right)^{-1}\right) + \frac{3\sqrt{2}}{8}\arctan\left(1+\sqrt{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(5/2)/(x^2+1)^2,x)`

[Out]  $-1/2*x^{(3/2)}/(x^2+1)+3/8*\arctan(2^{(1/2)}*x^{(1/2)}-1)*2^{(1/2)}+3/16*2^{(1/2)}*\ln((1+x-2^{(1/2)}*x^{(1/2)})/(1+x+2^{(1/2)}*x^{(1/2)}))+3/8*\arctan(1+2^{(1/2)}*x^{(1/2)})*2^{(1/2)}$

**Maxima [A]** time = 1.50652, size = 116, normalized size = 1.03

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{3}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) - \frac{x^{\frac{3}{2}}}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(5/2)/(x^2 + 1)^2,x, algorithm="maxima")`

[Out]  $3/8*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*( \text{sqrt}(2) + 2*\text{sqrt}(x))) + 3/8*\text{sqrt}(2)*\arctan(-1/2*\text{sqrt}(2)*( \text{sqrt}(2) - 2*\text{sqrt}(x))) - 3/16*\text{sqrt}(2)*\log(\text{sqrt}(2)*\text{sqrt}(x) + x + 1) + 3/16*\text{sqrt}(2)*\log(-\text{sqrt}(2)*\text{sqrt}(x) + x + 1) - 1/2*x^{(3/2)}/(x^2 + 1)$

**Fricas [A]** time = 0.251427, size = 185, normalized size = 1.64

$$\frac{12\sqrt{2}(x^2+1)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x}+\sqrt{2}\sqrt{2x+2x+1}}\right) + 12\sqrt{2}(x^2+1)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x}+\sqrt{-2}\sqrt{2x+2x+1}}\right) + 3\sqrt{2}(x^2+1)\log\left(2\sqrt{2}\sqrt{x}\right)}{16(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(5/2)/(x^2 + 1)^2,x, algorithm="fricas")`

[Out]  $-1/16*(12*\text{sqrt}(2)*(x^2 + 1)*\arctan(1/(\text{sqrt}(2)*\text{sqrt}(x) + \text{sqrt}(2*\text{sqrt}(2)*\text{sqrt}(x) + 2*x + 2) + 1)) + 12*\text{sqrt}(2)*(x^2 + 1)*\arctan(1/(\text{sqrt}(2)*\text{sqrt}(x) + \text{sqrt}(-2*\text{sqrt}(2)*\text{sqrt}(x) + 2*x + 2) - 1)) + 3*\text{sqrt}(2)*(x^2 + 1)*\log(2*\text{sqrt}(2)*\text{sqrt}(x) + 2*x + 2) - 3*\text{sqrt}(2)*(x^2 + 1)*\log(-2*\text{sqrt}(2)*\text{sqrt}(x) + 2*x + 2) + 8*x^{(3/2)})/(x^2 + 1)$

**Sympy [A]** time = 35.1342, size = 264, normalized size = 2.34

$$\begin{aligned}
 & -\frac{8x^{\frac{3}{2}}}{16x^2 + 16} + \frac{3\sqrt{2}x^2 \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{16x^2 + 16} - \frac{3\sqrt{2}x^2 \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{16x^2 + 16} \\
 & + \frac{6\sqrt{2}x^2 \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{16x^2 + 16} + \frac{6\sqrt{2}x^2 \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{16x^2 + 16} + \frac{3\sqrt{2} \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{16x^2 + 16} \\
 & - \frac{3\sqrt{2} \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{16x^2 + 16} + \frac{6\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{16x^2 + 16} + \frac{6\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{16x^2 + 16}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(5/2)/(x\*\*2+1)\*\*2,x)

[Out]  $-8*x^{3/2}/(16*x^2 + 16) + 3*\sqrt{2}*x^2*\log(-4*\sqrt{2}*\sqrt{x} + 4*x + 4)/(16*x^2 + 16) - 3*\sqrt{2}*x^2*\log(4*\sqrt{2}*\sqrt{x} + 4*x + 4)/(16*x^2 + 16) + 6*\sqrt{2}*x^2*\operatorname{atan}(\sqrt{2}*\sqrt{x} - 1)/(16*x^2 + 16) + 6*\sqrt{2}*x^2*\operatorname{atan}(\sqrt{2}*\sqrt{x} + 1)/(16*x^2 + 16) + 3*\sqrt{2}*\log(-4*\sqrt{2}*\sqrt{x} + 4*x + 4)/(16*x^2 + 16) - 3*\sqrt{2}*\log(4*\sqrt{2}*\sqrt{x} + 4*x + 4)/(16*x^2 + 16) + 6*\sqrt{2}*\operatorname{atan}(\sqrt{2}*\sqrt{x} - 1)/(16*x^2 + 16) + 6*\sqrt{2}*\operatorname{atan}(\sqrt{2}*\sqrt{x} + 1)/(16*x^2 + 16)$

**GIAC/XCAS [A]** time = 0.215082, size = 116, normalized size = 1.03

$$\begin{aligned}
 & \frac{3}{8} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) + \frac{3}{8} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) \\
 & - \frac{3}{16} \sqrt{2} \ln(\sqrt{2}\sqrt{x} + x + 1) + \frac{3}{16} \sqrt{2} \ln(-\sqrt{2}\sqrt{x} + x + 1) - \frac{x^{\frac{3}{2}}}{2(x^2 + 1)}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(x^2 + 1)^2,x, algorithm="giac")

[Out]  $3/8*\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2*\sqrt{x})) + 3/8*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2*\sqrt{x})) - 3/16*\sqrt{2}*\ln(\sqrt{2}*\sqrt{x} + x + 1) + 3/16*\sqrt{2}*\ln(-\sqrt{2}*\sqrt{x} + x + 1) - 1/2*x^{3/2}/(x^2 + 1)$

$$3.323 \quad \int \frac{x^{3/2}}{(1+x^2)^2} dx$$

**Optimal.** Leaf size=113

$$-\frac{\sqrt{x}}{2(x^2+1)} - \frac{\log(x - \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} + \frac{\log(x + \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} - \frac{\tan^{-1}(1 - \sqrt{2}\sqrt{x})}{4\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{4\sqrt{2}}$$

[Out] -Sqrt[x]/(2\*(1+x^2)) - ArcTan[1 - Sqrt[2]\*Sqrt[x]]/(4\*Sqrt[2]) + ArcTan[1 + Sqrt[2]\*Sqrt[x]]/(4\*Sqrt[2]) - Log[1 - Sqrt[2]\*Sqrt[x] + x]/(8\*Sqrt[2]) + Log[1 + Sqrt[2]\*Sqrt[x] + x]/(8\*Sqrt[2])

**Rubi [A]** time = 0.141027, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$

$$-\frac{\sqrt{x}}{2(x^2+1)} - \frac{\log(x - \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} + \frac{\log(x + \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} - \frac{\tan^{-1}(1 - \sqrt{2}\sqrt{x})}{4\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)/(1+x^2)^2,x]

[Out] -Sqrt[x]/(2\*(1+x^2)) - ArcTan[1 - Sqrt[2]\*Sqrt[x]]/(4\*Sqrt[2]) + ArcTan[1 + Sqrt[2]\*Sqrt[x]]/(4\*Sqrt[2]) - Log[1 - Sqrt[2]\*Sqrt[x] + x]/(8\*Sqrt[2]) + Log[1 + Sqrt[2]\*Sqrt[x] + x]/(8\*Sqrt[2])

**Rubi in Sympy [A]** time = 18.4613, size = 95, normalized size = 0.84

$$-\frac{\sqrt{x}}{2(x^2+1)} - \frac{\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{16} + \frac{\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{16} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{8} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(3/2)/(x\*\*2+1)\*\*2,x)

[Out] -sqrt(x)/(2\*(x\*\*2+1)) - sqrt(2)\*log(-sqrt(2)\*sqrt(x)+x+1)/16 + sqrt(2)\*log(sqrt(2)\*sqrt(x)+x+1)/16 + sqrt(2)\*atan(sqrt(2)\*sqrt(x)-1)/8 + sqrt(2)\*atan(sqrt(2)\*sqrt(x)+1)/8

**Mathematica [A]** time = 0.115291, size = 106, normalized size = 0.94

$$\frac{1}{16} \left( -\frac{8\sqrt{x}}{x^2+1} - \sqrt{2} \log(x - \sqrt{2}\sqrt{x} + 1) + \sqrt{2} \log(x + \sqrt{2}\sqrt{x} + 1) - 2\sqrt{2} \tan^{-1}(1 - \sqrt{2}\sqrt{x}) + 2\sqrt{2} \tan^{-1}(\sqrt{2}\sqrt{x} + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)/(1+x^2)^2,x]

[Out] ((-8\*Sqrt[x])/(1+x^2) - 2\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[x]] + 2\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[x]] - Sqrt[2]\*Log[1 - Sqrt[2]\*



$\text{Sqrt}[x] + x] + \text{Sqrt}[2] * \text{Log}[1 + \text{Sqrt}[2] * \text{Sqrt}[x] + x] / 16$

**Maple [A]** time = 0.013, size = 74, normalized size = 0.7

$$-\frac{1}{2x^2+2}\sqrt{x} + \frac{\sqrt{2}}{8}\arctan\left(\sqrt{2}\sqrt{x}-1\right) + \frac{\sqrt{2}}{16}\ln\left(1\left(1+x+\sqrt{2}\sqrt{x}\right)\left(1+x-\sqrt{2}\sqrt{x}\right)^{-1}\right) + \frac{\sqrt{2}}{8}\arctan\left(1+\sqrt{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3/2)/(x^2+1)^2,x)`

[Out] `-1/2*x^(1/2)/(x^2+1)+1/8*arctan(2^(1/2)*x^(1/2)-1)*2^(1/2)+1/16*2^(1/2)*ln((1+x+2^(1/2)*x^(1/2))/(1+x-2^(1/2)*x^(1/2)))+1/8*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)`

**Maxima [A]** time = 1.51413, size = 116, normalized size = 1.03

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) + \frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{1}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) - \frac{\sqrt{x}}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(3/2)/(x^2+1)^2,x, algorithm="maxima")`

[Out] `1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)+2*sqrt(x)))+1/8*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)-2*sqrt(x)))+1/16*sqrt(2)*log(sqrt(2)*sqrt(x)+x+1)-1/16*sqrt(2)*log(-sqrt(2)*sqrt(x)+x+1)-1/2*sqrt(x)/(x^2+1)`

**Fricas [A]** time = 0.249379, size = 184, normalized size = 1.63

$$4\sqrt{2}(x^2+1)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x}+\sqrt{2}\sqrt{2}\sqrt{x}+2x+2+1}}\right) + 4\sqrt{2}(x^2+1)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x}+\sqrt{-2}\sqrt{2}\sqrt{x}+2x+2-1}}\right) - \sqrt{2}(x^2+1)\log\left(2\sqrt{2}\sqrt{x} + \dots\right)$$

$16(x^2+1)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(3/2)/(x^2+1)^2,x, algorithm="fricas")`

[Out] `-1/16*(4*sqrt(2)*(x^2+1)*arctan(1/(sqrt(2)*sqrt(x)+sqrt(2*sqrt(2)*sqrt(x)+2*x+2)+1))+4*sqrt(2)*(x^2+1)*arctan(1/(sqrt(2)*sqrt(x)+sqrt(-2*sqrt(2)*sqrt(x)+2*x+2)-1))-sqrt(2)*(x^2+1)*log(2*sqrt(2)*sqrt(x)+2*x+2)+sqrt(2)*(x^2+1)*log(-2*sqrt(2)*sqrt(x)+2*x+2)+8*sqrt(x))/(x^2+1)`

**Sympy [A]** time = 20.4515, size = 257, normalized size = 2.27

$$\begin{aligned} & \frac{8\sqrt{x}}{16x^2 + 16} - \frac{\sqrt{2}x^2 \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{16x^2 + 16} + \frac{\sqrt{2}x^2 \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{16x^2 + 16} \\ & + \frac{2\sqrt{2}x^2 \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{16x^2 + 16} + \frac{2\sqrt{2}x^2 \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{16x^2 + 16} - \frac{\sqrt{2} \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{16x^2 + 16} \\ & + \frac{\sqrt{2} \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{16x^2 + 16} + \frac{2\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{16x^2 + 16} + \frac{2\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{16x^2 + 16} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(3/2)/(x\*\*2+1)\*\*2, x)

[Out] -8\*sqrt(x)/(16\*x\*\*2 + 16) - sqrt(2)\*x\*\*2\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(16\*x\*\*2 + 16) + sqrt(2)\*x\*\*2\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(16\*x\*\*2 + 16) + 2\*sqrt(2)\*x\*\*2\*atan(sqrt(2)\*sqrt(x) - 1)/(16\*x\*\*2 + 16) + 2\*sqrt(2)\*x\*\*2\*atan(sqrt(2)\*sqrt(x) + 1)/(16\*x\*\*2 + 16) - sqrt(2)\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(16\*x\*\*2 + 16) + sqrt(2)\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(16\*x\*\*2 + 16) + 2\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/(16\*x\*\*2 + 16) + 2\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/(16\*x\*\*2 + 16)

**GIAC/XCAS [A]** time = 0.2281, size = 116, normalized size = 1.03

$$\begin{aligned} & \frac{1}{8} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) + \frac{1}{8} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) \\ & + \frac{1}{16} \sqrt{2} \ln(\sqrt{2}\sqrt{x} + x + 1) - \frac{1}{16} \sqrt{2} \ln(-\sqrt{2}\sqrt{x} + x + 1) - \frac{\sqrt{x}}{2(x^2 + 1)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(x^2 + 1)^2, x, algorithm="giac")

[Out] 1/8\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) + 1/8\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) + 1/16\*sqrt(2)\*ln(sqrt(2)\*sqrt(x) + x + 1) - 1/16\*sqrt(2)\*ln(-sqrt(2)\*sqrt(x) + x + 1) - 1/2\*sqrt(x)/(x^2 + 1)

$$3.324 \quad \int \frac{\sqrt{x}}{(1+x^2)^2} dx$$

**Optimal.** Leaf size=113

$$\frac{x^{3/2}}{2(x^2+1)} + \frac{\log(x - \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} - \frac{\log(x + \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} - \frac{\tan^{-1}(1 - \sqrt{2}\sqrt{x})}{4\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{4\sqrt{2}}$$

[Out]  $x^{(3/2)}/(2*(1+x^2)) - \text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]]/(4*\text{Sqrt}[2]) + \text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]]/(4*\text{Sqrt}[2]) + \text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[x] + x]/(8*\text{Sqrt}[2]) - \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x]/(8*\text{Sqrt}[2])$

**Rubi [A]** time = 0.145266, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$

$$\frac{x^{3/2}}{2(x^2+1)} + \frac{\log(x - \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} - \frac{\log(x + \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} - \frac{\tan^{-1}(1 - \sqrt{2}\sqrt{x})}{4\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[x]/(1+x^2)^2,x]`

[Out]  $x^{(3/2)}/(2*(1+x^2)) - \text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]]/(4*\text{Sqrt}[2]) + \text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]]/(4*\text{Sqrt}[2]) + \text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[x] + x]/(8*\text{Sqrt}[2]) - \text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x]/(8*\text{Sqrt}[2])$

**Rubi in Sympy [A]** time = 19.3458, size = 95, normalized size = 0.84

$$\frac{x^{3/2}}{2(x^2+1)} + \frac{\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{16} - \frac{\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{16} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{8} + \frac{\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**(1/2)/(x**2+1)**2,x)`

[Out]  $x^{(3/2)}/(2*(x^2+1)) + \text{sqrt}(2)*\log(-\text{sqrt}(2)*\text{sqrt}(x) + x + 1)/16 - \text{sqrt}(2)*\log(\text{sqrt}(2)*\text{sqrt}(x) + x + 1)/16 + \text{sqrt}(2)*\operatorname{atan}(\text{sqrt}(2)*\text{sqrt}(x) - 1)/8 + \text{sqrt}(2)*\operatorname{atan}(\text{sqrt}(2)*\text{sqrt}(x) + 1)/8$

**Mathematica [A]** time = 0.0991947, size = 106, normalized size = 0.94

$$\frac{1}{16} \left( \frac{8x^{3/2}}{x^2+1} + \sqrt{2} \log(x - \sqrt{2}\sqrt{x} + 1) - \sqrt{2} \log(x + \sqrt{2}\sqrt{x} + 1) - 2\sqrt{2} \tan^{-1}(1 - \sqrt{2}\sqrt{x}) + 2\sqrt{2} \tan^{-1}(\sqrt{2}\sqrt{x} + 1) \right)$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[x]/(1+x^2)^2,x]`

[Out]  $((8*x^{(3/2)})/(1+x^2) - 2*\text{Sqrt}[2]*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]] + 2*\text{Sqrt}[2]*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]] + \text{Sqrt}[2]*\text{Log}[1 - \text{Sqrt}[2]*\text{S}$

$\text{qrt}[x] + x] - \text{Sqrt}[2] * \text{Log}[1 + \text{Sqrt}[2] * \text{Sqrt}[x] + x] / 16$

**Maple [A]** time = 0.011, size = 74, normalized size = 0.7

$$\frac{1}{2x^2+2}x^{\frac{3}{2}} + \frac{\sqrt{2}}{8}\arctan\left(1 + \sqrt{2}\sqrt{x}\right) + \frac{\sqrt{2}}{8}\arctan\left(\sqrt{2}\sqrt{x} - 1\right) + \frac{\sqrt{2}}{16}\ln\left(1\left(1+x - \sqrt{2}\sqrt{x}\right)\left(1+x + \sqrt{2}\sqrt{x}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)/(x^2+1)^2,x)`

[Out] `1/2*x^(3/2)/(x^2+1)+1/8*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+1/8*arctan(2^(1/2)*x^(1/2)-1)*2^(1/2)+1/16*2^(1/2)*ln((1+x-2^(1/2)*x^(1/2))/(1+x+2^(1/2)*x^(1/2)))`

**Maxima [A]** time = 1.50476, size = 116, normalized size = 1.03

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + 2\sqrt{x}\right)\right) + \frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - 2\sqrt{x}\right)\right) - \frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x} + x + 1\right) + \frac{1}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x} + x + 1\right) + \frac{x^{\frac{3}{2}}}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(x)/(x^2 + 1)^2,x, algorithm="maxima")`

[Out] `1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) + x + 1) + 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*x^(3/2)/(x^2 + 1)`

**Fricas [A]** time = 0.248485, size = 184, normalized size = 1.63

$$\frac{4\sqrt{2}(x^2+1)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x}+\sqrt{2}\sqrt{x+2x+1}}\right) + 4\sqrt{2}(x^2+1)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x}+\sqrt{-2\sqrt{x}+2x+2-1}}\right) + \sqrt{2}(x^2+1)\log\left(2\sqrt{2}\sqrt{x} + \dots\right)}{16(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(x)/(x^2 + 1)^2,x, algorithm="fricas")`

[Out] `-1/16*(4*sqrt(2)*(x^2 + 1)*arctan(1/(sqrt(2)*sqrt(x) + sqrt(2*sqrt(2)*sqrt(x) + 2*x + 2) + 1)) + 4*sqrt(2)*(x^2 + 1)*arctan(1/(sqrt(2)*sqrt(x) + sqrt(-2*sqrt(2)*sqrt(x) + 2*x + 2) - 1)) + sqrt(2)*(x^2 + 1)*log(2*sqrt(2)*sqrt(x) + 2*x + 2) - sqrt(2)*(x^2 + 1)*log(-2*sqrt(2)*sqrt(x) + 2*x + 2) - 8*x^(3/2))/(x^2 + 1)`

**Sympy [A]** time = 12.1386, size = 257, normalized size = 2.27

$$\begin{aligned} & \frac{8x^{\frac{3}{2}}}{16x^2 + 16} + \frac{\sqrt{2}x^2 \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{16x^2 + 16} - \frac{\sqrt{2}x^2 \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{16x^2 + 16} \\ & + \frac{2\sqrt{2}x^2 \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{16x^2 + 16} + \frac{2\sqrt{2}x^2 \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{16x^2 + 16} + \frac{\sqrt{2} \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{16x^2 + 16} \\ & - \frac{\sqrt{2} \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{16x^2 + 16} + \frac{2\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{16x^2 + 16} + \frac{2\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{16x^2 + 16} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(1/2)/(x\*\*2+1)\*\*2, x)

[Out] 8\*x\*\*(3/2)/(16\*x\*\*2 + 16) + sqrt(2)\*x\*\*2\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(16\*x\*\*2 + 16) - sqrt(2)\*x\*\*2\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(16\*x\*\*2 + 16) + 2\*sqrt(2)\*x\*\*2\*atan(sqrt(2)\*sqrt(x) - 1)/(16\*x\*\*2 + 16) + 2\*sqrt(2)\*x\*\*2\*atan(sqrt(2)\*sqrt(x) + 1)/(16\*x\*\*2 + 16) + sqrt(2)\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(16\*x\*\*2 + 16) - sqrt(2)\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(16\*x\*\*2 + 16) + 2\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/(16\*x\*\*2 + 16) + 2\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/(16\*x\*\*2 + 16)

**GIAC/XCAS [A]** time = 0.21371, size = 116, normalized size = 1.03

$$\begin{aligned} & \frac{1}{8} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) + \frac{1}{8} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) \\ & - \frac{1}{16} \sqrt{2} \ln(\sqrt{2}\sqrt{x} + x + 1) + \frac{1}{16} \sqrt{2} \ln(-\sqrt{2}\sqrt{x} + x + 1) + \frac{x^{\frac{3}{2}}}{2(x^2 + 1)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x)/(x^2 + 1)^2, x, algorithm="giac")

[Out] 1/8\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) + 1/8\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) - 1/16\*sqrt(2)\*ln(sqrt(2)\*sqrt(x) + x + 1) + 1/16\*sqrt(2)\*ln(-sqrt(2)\*sqrt(x) + x + 1) + 1/2\*x^(3/2)/(x^2 + 1)

$$3.325 \quad \int \frac{1}{\sqrt{x}(1+x^2)^2} dx$$

**Optimal.** Leaf size=113

$$\frac{\sqrt{x}}{2(x^2+1)} - \frac{3 \log(x - \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} + \frac{3 \log(x + \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} - \frac{3 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{4\sqrt{2}} + \frac{3 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{4\sqrt{2}}$$

[Out] Sqrt[x]/(2\*(1 + x^2)) - (3\*ArcTan[1 - Sqrt[2]\*Sqrt[x]])/(4\*Sqrt[2]) + (3\*ArcTan[1 + Sqrt[2]\*Sqrt[x]])/(4\*Sqrt[2]) - (3\*Log[1 - Sqrt[2]\*Sqrt[x] + x])/(8\*Sqrt[2]) + (3\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/(8\*Sqrt[2])

**Rubi [A]** time = 0.140837, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$

$$\frac{\sqrt{x}}{2(x^2+1)} - \frac{3 \log(x - \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} + \frac{3 \log(x + \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} - \frac{3 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{4\sqrt{2}} + \frac{3 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]\*(1 + x^2)^2), x]

[Out] Sqrt[x]/(2\*(1 + x^2)) - (3\*ArcTan[1 - Sqrt[2]\*Sqrt[x]])/(4\*Sqrt[2]) + (3\*ArcTan[1 + Sqrt[2]\*Sqrt[x]])/(4\*Sqrt[2]) - (3\*Log[1 - Sqrt[2]\*Sqrt[x] + x])/(8\*Sqrt[2]) + (3\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/(8\*Sqrt[2])

**Rubi in SymPy [A]** time = 18.3902, size = 102, normalized size = 0.9

$$\frac{\sqrt{x}}{2(x^2+1)} - \frac{3\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{16} + \frac{3\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{16} + \frac{3\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{8} + \frac{3\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(x\*\*2+1)\*\*2/x\*\*(1/2), x)

[Out] sqrt(x)/(2\*(x\*\*2 + 1)) - 3\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1)/16 + 3\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1)/16 + 3\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/8 + 3\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/8

**Mathematica [A]** time = 0.112097, size = 107, normalized size = 0.95

$$\frac{1}{16} \left( \frac{8\sqrt{x}}{x^2+1} - 3\sqrt{2} \log(x - \sqrt{2}\sqrt{x} + 1) + 3\sqrt{2} \log(x + \sqrt{2}\sqrt{x} + 1) - 6\sqrt{2} \tan^{-1}(1 - \sqrt{2}\sqrt{x}) + 6\sqrt{2} \tan^{-1}(\sqrt{2}\sqrt{x} + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]\*(1 + x^2)^2), x]

[Out]  $((8*\text{Sqrt}[x])/(1 + x^2) - 6*\text{Sqrt}[2]*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]] + 6*\text{Sqrt}[2]*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]] - 3*\text{Sqrt}[2]*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[x] + x] + 3*\text{Sqrt}[2]*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x])/16$

**Maple [A]** time = 0.009, size = 74, normalized size = 0.7

$$\frac{1}{2x^2 + 2}\sqrt{x} + \frac{3\sqrt{2}}{8}\arctan\left(1 + \sqrt{2}\sqrt{x}\right) + \frac{3\sqrt{2}}{8}\arctan\left(\sqrt{2}\sqrt{x} - 1\right) + \frac{3\sqrt{2}}{16}\ln\left(1\left(1 + x + \sqrt{2}\sqrt{x}\right)\left(1 + x - \sqrt{2}\sqrt{x}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2+1)^2/x^(1/2), x)`

[Out]  $1/2*x^{(1/2)}/(x^2+1)+3/8*\arctan(1+2^{(1/2)}*x^{(1/2)})*2^{(1/2)}+3/8*\arctan(2^{(1/2)}*x^{(1/2)}-1)*2^{(1/2)}+3/16*2^{(1/2)}*\ln((1+x+2^{(1/2)}*x^{(1/2)})/(1+x-2^{(1/2)}*x^{(1/2)}))$

**Maxima [A]** time = 1.50774, size = 116, normalized size = 1.03

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) + \frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) + \frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x} + x + 1\right) - \frac{3}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x} + x + 1\right) + \frac{\sqrt{x}}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 + 1)^2*sqrt(x)), x, algorithm="maxima")`

[Out]  $3/8*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*( \text{sqrt}(2) + 2*\text{sqrt}(x))) + 3/8*\text{sqrt}(2)*\arctan(-1/2*\text{sqrt}(2)*( \text{sqrt}(2) - 2*\text{sqrt}(x))) + 3/16*\text{sqrt}(2)*\log(\text{sqrt}(2)*\text{sqrt}(x) + x + 1) - 3/16*\text{sqrt}(2)*\log(-\text{sqrt}(2)*\text{sqrt}(x) + x + 1) + 1/2*\text{sqrt}(x)/(x^2 + 1)$

**Fricas [A]** time = 0.24791, size = 185, normalized size = 1.64

$$\frac{12\sqrt{2}(x^2 + 1)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{2}\sqrt{2}\sqrt{x} + 2x + 2}\right) + 12\sqrt{2}(x^2 + 1)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{-2}\sqrt{2}\sqrt{x} + 2x + 2}\right) - 3\sqrt{2}(x^2 + 1)\log\left(2\sqrt{2}\sqrt{x} + 2x + 2\right)}{16(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 + 1)^2*sqrt(x)), x, algorithm="fricas")`

[Out]  $-1/16*(12*\text{sqrt}(2)*(x^2 + 1)*\arctan(1/(\text{sqrt}(2)*\text{sqrt}(x) + \text{sqrt}(2)*\text{sqrt}(x) + 2*x + 2) + 12*\text{sqrt}(2)*(x^2 + 1)*\arctan(1/(\text{sqrt}(2)*\text{sqrt}(x) + \text{sqrt}(-2*\text{sqrt}(2)*\text{sqrt}(x) + 2*x + 2) - 1)) - 3*\text{sqrt}(2)*(x^2 + 1)*\log(2*\text{sqrt}(2)*\text{sqrt}(x) + 2*x + 2) + 3*\text{sqrt}(2)*(x^2 + 1)*\log(-2*\text{sqrt}(2)*\text{sqrt}(x) + 2*x + 2) - 8*\text{sqrt}(x))/(x^2 + 1)$

**Sympy [A]** time = 16.5569, size = 264, normalized size = 2.34

$$\begin{aligned} & \frac{8\sqrt{x}}{16x^2 + 16} - \frac{3\sqrt{2}x^2 \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{16x^2 + 16} + \frac{3\sqrt{2}x^2 \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{16x^2 + 16} \\ & + \frac{6\sqrt{2}x^2 \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{16x^2 + 16} + \frac{6\sqrt{2}x^2 \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{16x^2 + 16} - \frac{3\sqrt{2} \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{16x^2 + 16} \\ & + \frac{3\sqrt{2} \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{16x^2 + 16} + \frac{6\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{16x^2 + 16} + \frac{6\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{16x^2 + 16} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x\*\*2+1)\*\*2/x\*\*(1/2), x)

[Out] 8\*sqrt(x)/(16\*x\*\*2 + 16) - 3\*sqrt(2)\*x\*\*2\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(16\*x\*\*2 + 16) + 3\*sqrt(2)\*x\*\*2\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(16\*x\*\*2 + 16) + 6\*sqrt(2)\*x\*\*2\*atan(sqrt(2)\*sqrt(x) - 1)/(16\*x\*\*2 + 16) + 6\*sqrt(2)\*x\*\*2\*atan(sqrt(2)\*sqrt(x) + 1)/(16\*x\*\*2 + 16) - 3\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(16\*x\*\*2 + 16) + 3\*sqrt(2)\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(16\*x\*\*2 + 16) + 6\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/(16\*x\*\*2 + 16) + 6\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/(16\*x\*\*2 + 16)

**GIAC/XCAS [A]** time = 0.209126, size = 116, normalized size = 1.03

$$\begin{aligned} & \frac{3}{8} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) + \frac{3}{8} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) \\ & + \frac{3}{16} \sqrt{2} \ln(\sqrt{2}\sqrt{x} + x + 1) - \frac{3}{16} \sqrt{2} \ln(-\sqrt{2}\sqrt{x} + x + 1) + \frac{\sqrt{x}}{2(x^2 + 1)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^2\*sqrt(x)), x, algorithm="giac")

[Out] 3/8\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) + 3/8\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) + 3/16\*sqrt(2)\*ln(sqrt(2)\*sqrt(x) + x + 1) - 3/16\*sqrt(2)\*ln(-sqrt(2)\*sqrt(x) + x + 1) + 1/2\*sqrt(x)/(x^2 + 1)



$$3.326 \quad \int \frac{1}{x^{3/2}(1+x^2)^2} dx$$

**Optimal.** Leaf size=122

$$\frac{1}{2\sqrt{x}(x^2+1)} - \frac{5}{2\sqrt{x}} - \frac{5 \log(x - \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} + \frac{5 \log(x + \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} \\ + \frac{5 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{4\sqrt{2}} - \frac{5 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{4\sqrt{2}}$$

[Out]  $-5/(2*\text{Sqrt}[x]) + 1/(2*\text{Sqrt}[x]*(1+x^2)) + (5*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]])/(4*\text{Sqrt}[2]) - (5*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]])/(4*\text{Sqrt}[2]) - (5*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[x] + x])/(8*\text{Sqrt}[2]) + (5*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x])/(8*\text{Sqrt}[2])$

**Rubi [A]** time = 0.155009, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$

$$\frac{1}{2\sqrt{x}(x^2+1)} - \frac{5}{2\sqrt{x}} - \frac{5 \log(x - \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} + \frac{5 \log(x + \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} \\ + \frac{5 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{4\sqrt{2}} - \frac{5 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^{(3/2)}*(1+x^2)^2), x]$

[Out]  $-5/(2*\text{Sqrt}[x]) + 1/(2*\text{Sqrt}[x]*(1+x^2)) + (5*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]])/(4*\text{Sqrt}[2]) - (5*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]])/(4*\text{Sqrt}[2]) - (5*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[x] + x])/(8*\text{Sqrt}[2]) + (5*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x])/(8*\text{Sqrt}[2])$

**Rubi in Sympy [A]** time = 21.6616, size = 112, normalized size = 0.92

$$-\frac{5\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{16} + \frac{5\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{16} \\ - \frac{5\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{8} - \frac{5\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{8} - \frac{5}{2\sqrt{x}} + \frac{1}{2\sqrt{x}(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{(3/2)}/(x^{*2}+1)^{*2}, x)$

[Out]  $-5*\text{sqrt}(2)*\log(-\text{sqrt}(2)*\text{sqrt}(x) + x + 1)/16 + 5*\text{sqrt}(2)*\log(\text{sqrt}(2)*\text{sqrt}(x) + x + 1)/16 - 5*\text{sqrt}(2)*\operatorname{atan}(\text{sqrt}(2)*\text{sqrt}(x) - 1)/8 - 5*\text{sqrt}(2)*\operatorname{atan}(\text{sqrt}(2)*\text{sqrt}(x) + 1)/8 - 5/(2*\text{sqrt}(x)) + 1/(2*\text{sqrt}(x)*(x^{*2} + 1))$

**Mathematica [A]** time = 0.150101, size = 114, normalized size = 0.93

$$\frac{1}{16} \left( -\frac{8x^{3/2}}{x^2+1} - \frac{32}{\sqrt{x}} - 5\sqrt{2} \log(x - \sqrt{2}\sqrt{x} + 1) + 5\sqrt{2} \log(x + \sqrt{2}\sqrt{x} + 1) \right. \\ \left. + 10\sqrt{2} \tan^{-1}(1 - \sqrt{2}\sqrt{x}) - 10\sqrt{2} \tan^{-1}(\sqrt{2}\sqrt{x} + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(3/2)\*(1 + x^2)^2),x]

[Out] (-32/Sqrt[x] - (8\*x^(3/2)))/(1 + x^2) + 10\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[x]] - 10\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[x]] - 5\*Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[x] + x] + 5\*Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/16

**Maple [A]** time = 0.016, size = 79, normalized size = 0.7

$$-2 \frac{1}{\sqrt{x}} - \frac{1}{2x^2 + 2} x^{\frac{3}{2}} - \frac{5\sqrt{2}}{8} \arctan\left(1 + \sqrt{2}\sqrt{x}\right) - \frac{5\sqrt{2}}{8} \arctan\left(\sqrt{2}\sqrt{x} - 1\right) - \frac{5\sqrt{2}}{16} \ln\left(1 \left(1 + x - \sqrt{2}\sqrt{x}\right) \left(1 + x + \sqrt{2}\sqrt{x}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(3/2)/(x^2+1)^2,x)

[Out] -2/x^(1/2)-1/2\*x^(3/2)/(x^2+1)-5/8\*arctan(1+2^(1/2)\*x^(1/2))\*2^(1/2)-5/8\*arctan(2^(1/2)\*x^(1/2)-1)\*2^(1/2)-5/16\*2^(1/2)\*ln((1+x-2^(1/2)\*x^(1/2))/(1+x+2^(1/2)\*x^(1/2)))

**Maxima [A]** time = 1.52433, size = 124, normalized size = 1.02

$$-\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) - \frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) + \frac{5}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{5}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) - \frac{5x^2+4}{2\left(x^{\frac{5}{2}}+\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^2\*x^(3/2)),x, algorithm="maxima")

[Out] -5/8\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) - 5/8\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) + 5/16\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1) - 5/16\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1) - 1/2\*(5\*x^2 + 4)/(x^(5/2) + sqrt(x))

**Fricas [A]** time = 0.24612, size = 194, normalized size = 1.59

$$\frac{20\sqrt{2}(x^3+x)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x}+\sqrt{2\sqrt{2}\sqrt{x}+2x+2+1}}\right) + 20\sqrt{2}(x^3+x)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x}+\sqrt{-2\sqrt{2}\sqrt{x}+2x+2-1}}\right) + 5\sqrt{2}(x^3+x)\log\left(2\sqrt{2}\sqrt{x}\right)}{16(x^3+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^2\*x^(3/2)),x, algorithm="fricas")

[Out] 1/16\*(20\*sqrt(2)\*(x^3 + x)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(2)\*sqrt(x) + 2\*x + 2) + 1) + 20\*sqrt(2)\*(x^3 + x)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(-2\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 1)) + 5\*sqrt(2)\*(x^3 + x)\*log(2\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 5\*sqrt(2)\*(x^3 + x)\*log(-2\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 8\*(5\*x^2 + 4)\*sqrt(x))/(x^3 + x)

**Sympy [A]** time = 28.7005, size = 366, normalized size = 3.

$$\begin{aligned} & -\frac{5\sqrt{2}x^{\frac{5}{2}}\log(-4\sqrt{2}\sqrt{x}+4x+4)}{16x^{\frac{5}{2}}+16\sqrt{x}} + \frac{5\sqrt{2}x^{\frac{5}{2}}\log(4\sqrt{2}\sqrt{x}+4x+4)}{16x^{\frac{5}{2}}+16\sqrt{x}} - \frac{10\sqrt{2}x^{\frac{5}{2}}\operatorname{atan}(\sqrt{2}\sqrt{x}-1)}{16x^{\frac{5}{2}}+16\sqrt{x}} \\ & - \frac{10\sqrt{2}x^{\frac{5}{2}}\operatorname{atan}(\sqrt{2}\sqrt{x}+1)}{16x^{\frac{5}{2}}+16\sqrt{x}} - \frac{5\sqrt{2}\sqrt{x}\log(-4\sqrt{2}\sqrt{x}+4x+4)}{16x^{\frac{5}{2}}+16\sqrt{x}} + \frac{5\sqrt{2}\sqrt{x}\log(4\sqrt{2}\sqrt{x}+4x+4)}{16x^{\frac{5}{2}}+16\sqrt{x}} \\ & - \frac{10\sqrt{2}\sqrt{x}\operatorname{atan}(\sqrt{2}\sqrt{x}-1)}{16x^{\frac{5}{2}}+16\sqrt{x}} - \frac{10\sqrt{2}\sqrt{x}\operatorname{atan}(\sqrt{2}\sqrt{x}+1)}{16x^{\frac{5}{2}}+16\sqrt{x}} - \frac{40x^2}{16x^{\frac{5}{2}}+16\sqrt{x}} - \frac{32}{16x^{\frac{5}{2}}+16\sqrt{x}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(3/2)/(x\*\*2+1)\*\*2,x)

[Out]  $-5*\sqrt{2}*x^{(5/2)}*\log(-4*\sqrt{2}*\sqrt{x}+4*x+4)/(16*x^{(5/2)}+16*\sqrt{x}) + 5*\sqrt{2}*x^{(5/2)}*\log(4*\sqrt{2}*\sqrt{x}+4*x+4)/(16*x^{(5/2)}+16*\sqrt{x}) - 10*\sqrt{2}*x^{(5/2)}*\operatorname{atan}(\sqrt{2}*\sqrt{x}-1)/(16*x^{(5/2)}+16*\sqrt{x}) - 10*\sqrt{2}*x^{(5/2)}*\operatorname{atan}(\sqrt{2}*\sqrt{x}+1)/(16*x^{(5/2)}+16*\sqrt{x}) - 5*\sqrt{2}*x^{(5/2)}*\sqrt{x}*\log(-4*\sqrt{2}*\sqrt{x}+4*x+4)/(16*x^{(5/2)}+16*\sqrt{x}) + 5*\sqrt{2}*x^{(5/2)}*\sqrt{x}*\log(4*\sqrt{2}*\sqrt{x}+4*x+4)/(16*x^{(5/2)}+16*\sqrt{x}) - 10*\sqrt{2}*x^{(5/2)}*\sqrt{x}*\operatorname{atan}(\sqrt{2}*\sqrt{x}-1)/(16*x^{(5/2)}+16*\sqrt{x}) - 10*\sqrt{2}*x^{(5/2)}*\sqrt{x}*\operatorname{atan}(\sqrt{2}*\sqrt{x}+1)/(16*x^{(5/2)}+16*\sqrt{x}) - 40*x^2/(16*x^{(5/2)}+16*\sqrt{x}) - 32/(16*x^{(5/2)}+16*\sqrt{x})$

**GIAC/XCAS [A]** time = 0.210406, size = 124, normalized size = 1.02

$$\begin{aligned} & -\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2}+2\sqrt{x})\right) - \frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2}-2\sqrt{x})\right) \\ & + \frac{5}{16}\sqrt{2}\ln(\sqrt{2}\sqrt{x}+x+1) - \frac{5}{16}\sqrt{2}\ln(-\sqrt{2}\sqrt{x}+x+1) - \frac{5x^2+4}{2(x^{\frac{5}{2}}+\sqrt{x})} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^2\*x^(3/2)),x, algorithm="giac")

[Out]  $-5/8*\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2}+2*\sqrt{x})) - 5/8*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}-2*\sqrt{x})) + 5/16*\sqrt{2}*\ln(\sqrt{2}*\sqrt{x}+x+1) - 5/16*\sqrt{2}*\ln(-\sqrt{2}*\sqrt{x}+x+1) - 1/2*(5*x^2+4)/(x^{(5/2)}+\sqrt{x})$

$$3.327 \quad \int \frac{1}{x^{5/2}(1+x^2)^2} dx$$

**Optimal.** Leaf size=122

$$\begin{aligned} & -\frac{7}{6x^{3/2}} + \frac{1}{2x^{3/2}(x^2+1)} + \frac{7 \log(x - \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} - \frac{7 \log(x + \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} \\ & + \frac{7 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{4\sqrt{2}} - \frac{7 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{4\sqrt{2}} \end{aligned}$$

[Out]  $-7/(6*x^{3/2}) + 1/(2*x^{3/2}*(1+x^2)) + (7*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) - (7*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (7*Log[1 - Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2]) - (7*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2])$

**Rubi [A]** time = 0.149198, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$

$$\begin{aligned} & -\frac{7}{6x^{3/2}} + \frac{1}{2x^{3/2}(x^2+1)} + \frac{7 \log(x - \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} - \frac{7 \log(x + \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} \\ & + \frac{7 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{4\sqrt{2}} - \frac{7 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{4\sqrt{2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(5/2)\*(1+x^2)^2),x]

[Out]  $-7/(6*x^{3/2}) + 1/(2*x^{3/2}*(1+x^2)) + (7*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) - (7*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(4*Sqrt[2]) + (7*Log[1 - Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2]) - (7*Log[1 + Sqrt[2]*Sqrt[x] + x])/(8*Sqrt[2])$

**Rubi in Sympy [A]** time = 20.0951, size = 112, normalized size = 0.92

$$\begin{aligned} & \frac{7\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{16} - \frac{7\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{16} \\ & - \frac{7\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{8} - \frac{7\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{8} - \frac{7}{6x^{3/2}} + \frac{1}{2x^{3/2}(x^2+1)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*(5/2)/(x\*\*2+1)\*\*2,x)

[Out]  $7*\sqrt{2}*\log(-\sqrt{2}*\sqrt{x} + x + 1)/16 - 7*\sqrt{2}*\log(\sqrt{2}*\sqrt{x} + x + 1)/16 - 7*\sqrt{2}*\operatorname{atan}(\sqrt{2}*\sqrt{x} - 1)/8 - 7*\sqrt{2}*\operatorname{atan}(\sqrt{2}*\sqrt{x} + 1)/8 - 7/(6*x^{3/2}) + 1/(2*x^{3/2}*(x^2 + 1))$

**Mathematica [A]** time = 0.161299, size = 114, normalized size = 0.93

$$\begin{aligned} & \frac{1}{48} \left( -\frac{32}{x^{3/2}} - \frac{24\sqrt{x}}{x^2+1} + 21\sqrt{2} \log(x - \sqrt{2}\sqrt{x} + 1) - 21\sqrt{2} \log(x + \sqrt{2}\sqrt{x} + 1) \right. \\ & \left. + 42\sqrt{2} \tan^{-1}(1 - \sqrt{2}\sqrt{x}) - 42\sqrt{2} \tan^{-1}(\sqrt{2}\sqrt{x} + 1) \right) \end{aligned}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(5/2)\*(1 + x^2)^2),x]

[Out]  $(-32/x^{3/2} - (24*\sqrt{x})/(1 + x^2) + 42*\sqrt{2}*\text{ArcTan}[1 - \sqrt{2}*\sqrt{x}] - 42*\sqrt{2}*\text{ArcTan}[1 + \sqrt{2}*\sqrt{x}] + 21*\sqrt{2}*\text{Log}[1 - \sqrt{2}*\sqrt{x} + x] - 21*\sqrt{2}*\text{Log}[1 + \sqrt{2}*\sqrt{x} + x])/48$

**Maple [A]** time = 0.015, size = 79, normalized size = 0.7

$$-\frac{2}{3}x^{-\frac{3}{2}} - \frac{1}{2x^2 + 2}\sqrt{x} - \frac{7\sqrt{2}}{8}\arctan(\sqrt{2}\sqrt{x} - 1) - \frac{7\sqrt{2}}{16}\ln\left(1\left(1 + x + \sqrt{2}\sqrt{x}\right)\left(1 + x - \sqrt{2}\sqrt{x}\right)^{-1}\right) - \frac{7\sqrt{2}}{8}\arctan\left(1 + \sqrt{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(5/2)/(x^2+1)^2,x)

[Out]  $-2/3/x^{3/2} - 1/2*x^{1/2}/(x^2+1) - 7/8*\arctan(2^{1/2}*x^{1/2}-1)*2^{1/2} - 7/16*2^{1/2}*\ln((1+x+2^{1/2}*x^{1/2})/(1+x-2^{1/2}*x^{1/2})) - 7/8*\arctan(1+2^{1/2}*x^{1/2})*2^{1/2}$

**Maxima [A]** time = 1.51589, size = 124, normalized size = 1.02

$$-\frac{7}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) - \frac{7}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) - \frac{7}{16}\sqrt{2}\log(\sqrt{2}\sqrt{x} + x + 1) + \frac{7}{16}\sqrt{2}\log(-\sqrt{2}\sqrt{x} + x + 1) - \frac{7x^2 + 4}{6(x^{\frac{7}{2}} + x^{\frac{3}{2}})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^2\*x^(5/2)),x, algorithm="maxima")

[Out]  $-7/8*\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2*\sqrt{x})) - 7/8*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2*\sqrt{x})) - 7/16*\sqrt{2}*\log(\sqrt{2}*\sqrt{x} + x + 1) + 7/16*\sqrt{2}*\log(-\sqrt{2}*\sqrt{x} + x + 1) - 1/6*(7*x^2 + 4)/(x^{7/2} + x^{3/2})$

**Fricas [A]** time = 0.249191, size = 208, normalized size = 1.7

$$\frac{84\sqrt{2}(x^4 + x^2)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{2}\sqrt{x} + 2x + 2}\right) + 84\sqrt{2}(x^4 + x^2)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{-2}\sqrt{2}\sqrt{x} + 2x + 2}\right) - 21\sqrt{2}(x^4 + x^2)\log(2\sqrt{x} + x + 1)}{48(x^4 + x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^2\*x^(5/2)),x, algorithm="fricas")

[Out]  $1/48*(84*\sqrt{2}*(x^4 + x^2)*\arctan(1/(\sqrt{2}*\sqrt{x} + \sqrt{2}*\sqrt{x} + 2*x + 2)) + 84*\sqrt{2}*(x^4 + x^2)*\arctan(1/(\sqrt{2}*\sqrt{x} + \sqrt{-2}*\sqrt{2}*\sqrt{x} + 2*x + 2) - 1)) - 21*\sqrt{2}*(x^4 + x^2)*\log(2*\sqrt{2}*\sqrt{x} + 2*x + 2) + 21*\sqrt{2}*(x^4 + x^2)*\log(-2*\sqrt{2}*\sqrt{x} + 2*x + 2) - 8*(7*x^2 + 4)*\sqrt{2}*\sqrt{x})/(x^4 + x^2)$

**Sympy [A]** time = 62.7663, size = 366, normalized size = 3.

$$\frac{21\sqrt{2}x^{\frac{7}{2}} \log\left(-4\sqrt{2}\sqrt{x} + 4x + 4\right)}{48x^{\frac{7}{2}} + 48x^{\frac{3}{2}}} - \frac{21\sqrt{2}x^{\frac{7}{2}} \log\left(4\sqrt{2}\sqrt{x} + 4x + 4\right)}{48x^{\frac{7}{2}} + 48x^{\frac{3}{2}}} - \frac{42\sqrt{2}x^{\frac{7}{2}} \operatorname{atan}\left(\sqrt{2}\sqrt{x} - 1\right)}{48x^{\frac{7}{2}} + 48x^{\frac{3}{2}}}$$

$$- \frac{42\sqrt{2}x^{\frac{7}{2}} \operatorname{atan}\left(\sqrt{2}\sqrt{x} + 1\right)}{48x^{\frac{7}{2}} + 48x^{\frac{3}{2}}} + \frac{21\sqrt{2}x^{\frac{3}{2}} \log\left(-4\sqrt{2}\sqrt{x} + 4x + 4\right)}{48x^{\frac{7}{2}} + 48x^{\frac{3}{2}}} - \frac{21\sqrt{2}x^{\frac{3}{2}} \log\left(4\sqrt{2}\sqrt{x} + 4x + 4\right)}{48x^{\frac{7}{2}} + 48x^{\frac{3}{2}}}$$

$$- \frac{42\sqrt{2}x^{\frac{3}{2}} \operatorname{atan}\left(\sqrt{2}\sqrt{x} - 1\right)}{48x^{\frac{7}{2}} + 48x^{\frac{3}{2}}} - \frac{42\sqrt{2}x^{\frac{3}{2}} \operatorname{atan}\left(\sqrt{2}\sqrt{x} + 1\right)}{48x^{\frac{7}{2}} + 48x^{\frac{3}{2}}} - \frac{56x^2}{48x^{\frac{7}{2}} + 48x^{\frac{3}{2}}} - \frac{32}{48x^{\frac{7}{2}} + 48x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(5/2)/(x\*\*2+1)\*\*2, x)

[Out] 21\*sqrt(2)\*x\*\*(7/2)\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(48\*x\*\*(7/2) + 48\*x\*\*(3/2)) - 21\*sqrt(2)\*x\*\*(7/2)\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(48\*x\*\*(7/2) + 48\*x\*\*(3/2)) - 42\*sqrt(2)\*x\*\*(7/2)\*atan(sqrt(2)\*sqrt(x) - 1)/(48\*x\*\*(7/2) + 48\*x\*\*(3/2)) - 42\*sqrt(2)\*x\*\*(7/2)\*atan(sqrt(2)\*sqrt(x) + 1)/(48\*x\*\*(7/2) + 48\*x\*\*(3/2)) + 21\*sqrt(2)\*x\*\*(3/2)\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(48\*x\*\*(7/2) + 48\*x\*\*(3/2)) - 21\*sqrt(2)\*x\*\*(3/2)\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(48\*x\*\*(7/2) + 48\*x\*\*(3/2)) - 42\*sqrt(2)\*x\*\*(3/2)\*atan(sqrt(2)\*sqrt(x) - 1)/(48\*x\*\*(7/2) + 48\*x\*\*(3/2)) - 42\*sqrt(2)\*x\*\*(3/2)\*atan(sqrt(2)\*sqrt(x) + 1)/(48\*x\*\*(7/2) + 48\*x\*\*(3/2)) - 56\*x\*\*2/(48\*x\*\*(7/2) + 48\*x\*\*(3/2)) - 32/(48\*x\*\*(7/2) + 48\*x\*\*(3/2))

**GIAC/XCAS [A]** time = 0.223687, size = 123, normalized size = 1.01

$$-\frac{7}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + 2\sqrt{x}\right)\right) - \frac{7}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - 2\sqrt{x}\right)\right)$$

$$- \frac{7}{16}\sqrt{2}\ln\left(\sqrt{2}\sqrt{x} + x + 1\right) + \frac{7}{16}\sqrt{2}\ln\left(-\sqrt{2}\sqrt{x} + x + 1\right) - \frac{\sqrt{x}}{2(x^2 + 1)} - \frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^2\*x^(5/2)), x, algorithm="giac")

[Out] -7/8\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) - 7/8\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) - 7/16\*sqrt(2)\*ln(sqrt(2)\*sqrt(x) + x + 1) + 7/16\*sqrt(2)\*ln(-sqrt(2)\*sqrt(x) + x + 1) - 1/2\*sqrt(x)/(x^2 + 1) - 2/3/x^(3/2)

$$3.328 \quad \int \frac{1}{x^{7/2}(1+x^2)^2} dx$$

**Optimal.** Leaf size=131

$$\begin{aligned} & -\frac{9}{10x^{5/2}} + \frac{1}{2x^{5/2}(x^2+1)} + \frac{9}{2\sqrt{x}} + \frac{9 \log(x - \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} \\ & - \frac{9 \log(x + \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} - \frac{9 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{4\sqrt{2}} + \frac{9 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{4\sqrt{2}} \end{aligned}$$

[Out]  $-9/(10*x^{(5/2)}) + 9/(2*\text{Sqrt}[x]) + 1/(2*x^{(5/2)}*(1 + x^2)) - (9*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]])/(4*\text{Sqrt}[2]) + (9*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]])/(4*\text{Sqrt}[2]) + (9*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[x] + x])/(8*\text{Sqrt}[2]) - (9*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x])/(8*\text{Sqrt}[2])$

**Rubi [A]** time = 0.162669, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$

$$\begin{aligned} & -\frac{9}{10x^{5/2}} + \frac{1}{2x^{5/2}(x^2+1)} + \frac{9}{2\sqrt{x}} + \frac{9 \log(x - \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} \\ & - \frac{9 \log(x + \sqrt{2}\sqrt{x} + 1)}{8\sqrt{2}} - \frac{9 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{4\sqrt{2}} + \frac{9 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{4\sqrt{2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^{(7/2)}*(1 + x^2)^2), x]$

[Out]  $-9/(10*x^{(5/2)}) + 9/(2*\text{Sqrt}[x]) + 1/(2*x^{(5/2)}*(1 + x^2)) - (9*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]])/(4*\text{Sqrt}[2]) + (9*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]])/(4*\text{Sqrt}[2]) + (9*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[x] + x])/(8*\text{Sqrt}[2]) - (9*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x])/(8*\text{Sqrt}[2])$

**Rubi in Sympy [A]** time = 24.5787, size = 121, normalized size = 0.92

$$\begin{aligned} & \frac{9\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{16} - \frac{9\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{16} + \frac{9\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{8} \\ & + \frac{9\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{8} + \frac{9}{2\sqrt{x}} - \frac{9}{10x^{5/2}} + \frac{1}{2x^{5/2}(x^2+1)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{(7/2)}/(x^{(2+1)})^{(2)}, x)$

[Out]  $9*\text{sqrt}(2)*\log(-\text{sqrt}(2)*\text{sqrt}(x) + x + 1)/16 - 9*\text{sqrt}(2)*\log(\text{sqrt}(2)*\text{sqrt}(x) + x + 1)/16 + 9*\text{sqrt}(2)*\operatorname{atan}(\text{sqrt}(2)*\text{sqrt}(x) - 1)/8 + 9*\text{sqrt}(2)*\operatorname{atan}(\text{sqrt}(2)*\text{sqrt}(x) + 1)/8 + 9/(2*\text{sqrt}(x)) - 9/(10*x^{(5/2)}) + 1/(2*x^{(5/2)}*(x^{(2)} + 1))$

**Mathematica [A]** time = 0.191439, size = 121, normalized size = 0.92

$$\begin{aligned} & \frac{1}{80} \left( -\frac{32}{x^{5/2}} + \frac{40x^{3/2}}{x^2+1} + \frac{320}{\sqrt{x}} \right. \\ & \left. + 45\sqrt{2} \log(x - \sqrt{2}\sqrt{x} + 1) - 45\sqrt{2} \log(x + \sqrt{2}\sqrt{x} + 1) - 90\sqrt{2} \tan^{-1}(1 - \sqrt{2}\sqrt{x}) + 90\sqrt{2} \tan^{-1}(\sqrt{2}\sqrt{x} + 1) \right) \end{aligned}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(7/2)\*(1+x^2)^2),x]

[Out] (-32/x^(5/2) + 320/Sqrt[x] + (40\*x^(3/2))/(1+x^2) - 90\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[x]] + 90\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[x]] + 45\*Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[x] + x] - 45\*Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/80

**Maple [A]** time = 0.02, size = 84, normalized size = 0.6

$$-\frac{2}{5}x^{-\frac{5}{2}} + 4\frac{1}{\sqrt{x}} + \frac{1}{2x^2+2}x^{\frac{3}{2}} + \frac{9\sqrt{2}}{8}\arctan\left(1 + \sqrt{2}\sqrt{x}\right) + \frac{9\sqrt{2}}{8}\arctan\left(\sqrt{2}\sqrt{x} - 1\right) + \frac{9\sqrt{2}}{16}\ln\left(1\left(1+x - \sqrt{2}\sqrt{x}\right)\left(1+x + \sqrt{2}\sqrt{x}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(7/2)/(x^2+1)^2,x)

[Out] -2/5/x^(5/2)+4/x^(1/2)+1/2\*x^(3/2)/(x^2+1)+9/8\*arctan(1+2^(1/2)\*x^(1/2))\*2^(1/2)+9/8\*arctan(2^(1/2)\*x^(1/2)-1)\*2^(1/2)+9/16\*2^(1/2)\*ln((1+x-2^(1/2)\*x^(1/2))/(1+x+2^(1/2)\*x^(1/2)))

**Maxima [A]** time = 1.51088, size = 131, normalized size = 1.

$$\frac{9}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{9}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{9}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{9}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{45x^4+36x^2-4}{10\left(x^{\frac{9}{2}}+x^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^2\*x^(7/2)),x, algorithm="maxima")

[Out] 9/8\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) + 9/8\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) - 9/16\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1) + 9/16\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1) + 1/10\*(45\*x^4 + 36\*x^2 - 4)/(x^(9/2) + x^(5/2))

**Fricas [A]** time = 0.248315, size = 215, normalized size = 1.64

$$\frac{180\sqrt{2}(x^5+x^3)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x}+\sqrt{2}\sqrt{2}\sqrt{x}+2x+2}\right) + 180\sqrt{2}(x^5+x^3)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x}+\sqrt{-2}\sqrt{2}\sqrt{x}+2x+2}\right) + 45\sqrt{2}(x^5+x^3)\log\left(\frac{1}{\sqrt{2}\sqrt{x}+\sqrt{2}\sqrt{2}\sqrt{x}+2x+2}\right) + 45\sqrt{2}(x^5+x^3)\log\left(\frac{1}{\sqrt{2}\sqrt{x}+\sqrt{-2}\sqrt{2}\sqrt{x}+2x+2}\right)}{80(x^5+x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^2\*x^(7/2)),x, algorithm="fricas")

[Out] -1/80\*(180\*sqrt(2)\*(x^5 + x^3)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(2)\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 1)) + 180\*sqrt(2)\*(x^5 + x^3)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(-2)\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 1)) + 45\*sqrt(2)\*(x^5 + x^3)\*log(2\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 45\*sqrt(2)\*(x^5 + x^3)\*log(-2\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 8\*(45\*x^4 + 36\*x^2 - 4)\*sqrt(x)/(x^5 + x^3)



**Sympy [A]** time = 164.549, size = 384, normalized size = 2.93

$$\begin{aligned} & \frac{45\sqrt{2}x^{\frac{9}{2}} \log\left(-4\sqrt{2}\sqrt{x} + 4x + 4\right)}{80x^{\frac{9}{2}} + 80x^{\frac{5}{2}}} - \frac{45\sqrt{2}x^{\frac{9}{2}} \log\left(4\sqrt{2}\sqrt{x} + 4x + 4\right)}{80x^{\frac{9}{2}} + 80x^{\frac{5}{2}}} \\ & + \frac{90\sqrt{2}x^{\frac{9}{2}} \operatorname{atan}\left(\sqrt{2}\sqrt{x} - 1\right)}{80x^{\frac{9}{2}} + 80x^{\frac{5}{2}}} + \frac{90\sqrt{2}x^{\frac{9}{2}} \operatorname{atan}\left(\sqrt{2}\sqrt{x} + 1\right)}{80x^{\frac{9}{2}} + 80x^{\frac{5}{2}}} + \frac{45\sqrt{2}x^{\frac{5}{2}} \log\left(-4\sqrt{2}\sqrt{x} + 4x + 4\right)}{80x^{\frac{9}{2}} + 80x^{\frac{5}{2}}} \\ & - \frac{45\sqrt{2}x^{\frac{5}{2}} \log\left(4\sqrt{2}\sqrt{x} + 4x + 4\right)}{80x^{\frac{9}{2}} + 80x^{\frac{5}{2}}} + \frac{90\sqrt{2}x^{\frac{5}{2}} \operatorname{atan}\left(\sqrt{2}\sqrt{x} - 1\right)}{80x^{\frac{9}{2}} + 80x^{\frac{5}{2}}} \\ & + \frac{90\sqrt{2}x^{\frac{5}{2}} \operatorname{atan}\left(\sqrt{2}\sqrt{x} + 1\right)}{80x^{\frac{9}{2}} + 80x^{\frac{5}{2}}} + \frac{360x^4}{80x^{\frac{9}{2}} + 80x^{\frac{5}{2}}} + \frac{288x^2}{80x^{\frac{9}{2}} + 80x^{\frac{5}{2}}} - \frac{32}{80x^{\frac{9}{2}} + 80x^{\frac{5}{2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(7/2)/(x\*\*2+1)\*\*2, x)

[Out] 45\*sqrt(2)\*x\*\*(9/2)\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(80\*x\*\*(9/2) + 80\*x\*\*(5/2)) - 45\*sqrt(2)\*x\*\*(9/2)\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(80\*x\*\*(9/2) + 80\*x\*\*(5/2)) + 90\*sqrt(2)\*x\*\*(9/2)\*atan(sqrt(2)\*sqrt(x) - 1)/(80\*x\*\*(9/2) + 80\*x\*\*(5/2)) + 90\*sqrt(2)\*x\*\*(9/2)\*atan(sqrt(2)\*sqrt(x) + 1)/(80\*x\*\*(9/2) + 80\*x\*\*(5/2)) + 45\*sqrt(2)\*x\*\*(5/2)\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(80\*x\*\*(9/2) + 80\*x\*\*(5/2)) - 45\*sqrt(2)\*x\*\*(5/2)\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(80\*x\*\*(9/2) + 80\*x\*\*(5/2)) + 90\*sqrt(2)\*x\*\*(5/2)\*atan(sqrt(2)\*sqrt(x) - 1)/(80\*x\*\*(9/2) + 80\*x\*\*(5/2)) + 90\*sqrt(2)\*x\*\*(5/2)\*atan(sqrt(2)\*sqrt(x) + 1)/(80\*x\*\*(9/2) + 80\*x\*\*(5/2)) + 360\*x\*\*4/(80\*x\*\*(9/2) + 80\*x\*\*(5/2)) + 288\*x\*\*2/(80\*x\*\*(9/2) + 80\*x\*\*(5/2)) - 32/(80\*x\*\*(9/2) + 80\*x\*\*(5/2))

**GIAC/XCAS [A]** time = 0.209794, size = 132, normalized size = 1.01

$$\begin{aligned} & \frac{9}{8} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) + \frac{9}{8} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) \\ & - \frac{9}{16} \sqrt{2} \ln\left(\sqrt{2}\sqrt{x} + x + 1\right) + \frac{9}{16} \sqrt{2} \ln\left(-\sqrt{2}\sqrt{x} + x + 1\right) + \frac{x^{\frac{3}{2}}}{2(x^2 + 1)} + \frac{2(10x^2 - 1)}{5x^{\frac{5}{2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^2\*x^(7/2)), x, algorithm="giac")

[Out] 9/8\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) + 9/8\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) - 9/16\*sqrt(2)\*ln(sqrt(2)\*sqrt(x) + x + 1) + 9/16\*sqrt(2)\*ln(-sqrt(2)\*sqrt(x) + x + 1) + 1/2\*x^(3/2)/(x^2 + 1) + 2/5\*(10\*x^2 - 1)/x^(5/2)

$$3.329 \quad \int \frac{x^{7/2}}{(1+x^2)^3} dx$$

**Optimal.** Leaf size=129

$$\begin{aligned} & -\frac{5\sqrt{x}}{16(x^2+1)} - \frac{x^{5/2}}{4(x^2+1)^2} - \frac{5\log(x-\sqrt{2}\sqrt{x}+1)}{64\sqrt{2}} \\ & + \frac{5\log(x+\sqrt{2}\sqrt{x}+1)}{64\sqrt{2}} - \frac{5\tan^{-1}(1-\sqrt{2}\sqrt{x})}{32\sqrt{2}} + \frac{5\tan^{-1}(\sqrt{2}\sqrt{x}+1)}{32\sqrt{2}} \end{aligned}$$

[Out]  $-x^{5/2}/(4*(1+x^2)^2) - (5*\text{Sqrt}[x])/(16*(1+x^2)) - (5*\text{ArcTan}[1-\text{Sqrt}[2]*\text{Sqrt}[x]])/(32*\text{Sqrt}[2]) + (5*\text{ArcTan}[1+\text{Sqrt}[2]*\text{Sqrt}[x]])/(32*\text{Sqrt}[2]) - (5*\text{Log}[1-\text{Sqrt}[2]*\text{Sqrt}[x]+x])/(64*\text{Sqrt}[2]) + (5*\text{Log}[1+\text{Sqrt}[2]*\text{Sqrt}[x]+x])/(64*\text{Sqrt}[2])$

**Rubi [A]** time = 0.161231, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$

$$\begin{aligned} & -\frac{5\sqrt{x}}{16(x^2+1)} - \frac{x^{5/2}}{4(x^2+1)^2} - \frac{5\log(x-\sqrt{2}\sqrt{x}+1)}{64\sqrt{2}} \\ & + \frac{5\log(x+\sqrt{2}\sqrt{x}+1)}{64\sqrt{2}} - \frac{5\tan^{-1}(1-\sqrt{2}\sqrt{x})}{32\sqrt{2}} + \frac{5\tan^{-1}(\sqrt{2}\sqrt{x}+1)}{32\sqrt{2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^(7/2)/(1+x^2)^3,x]

[Out]  $-x^{5/2}/(4*(1+x^2)^2) - (5*\text{Sqrt}[x])/(16*(1+x^2)) - (5*\text{ArcTan}[1-\text{Sqrt}[2]*\text{Sqrt}[x]])/(32*\text{Sqrt}[2]) + (5*\text{ArcTan}[1+\text{Sqrt}[2]*\text{Sqrt}[x]])/(32*\text{Sqrt}[2]) - (5*\text{Log}[1-\text{Sqrt}[2]*\text{Sqrt}[x]+x])/(64*\text{Sqrt}[2]) + (5*\text{Log}[1+\text{Sqrt}[2]*\text{Sqrt}[x]+x])/(64*\text{Sqrt}[2])$

**Rubi in Sympy [A]** time = 20.6587, size = 117, normalized size = 0.91

$$\begin{aligned} & -\frac{x^{5/2}}{4(x^2+1)^2} - \frac{5\sqrt{x}}{16(x^2+1)} - \frac{5\sqrt{2}\log(-\sqrt{2}\sqrt{x}+x+1)}{128} \\ & + \frac{5\sqrt{2}\log(\sqrt{2}\sqrt{x}+x+1)}{128} + \frac{5\sqrt{2}\text{atan}(\sqrt{2}\sqrt{x}-1)}{64} + \frac{5\sqrt{2}\text{atan}(\sqrt{2}\sqrt{x}+1)}{64} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(7/2)/(x\*\*2+1)\*\*3,x)

[Out]  $-x^{5/2}/(4*(x^2+1)^2) - 5*\text{sqrt}(x)/(16*(x^2+1)) - 5*\text{sqrt}(2)*\log(-\text{sqrt}(2)*\text{sqrt}(x)+x+1)/128 + 5*\text{sqrt}(2)*\log(\text{sqrt}(2)*\text{sqrt}(x)+x+1)/128 + 5*\text{sqrt}(2)*\text{atan}(\text{sqrt}(2)*\text{sqrt}(x)-1)/64 + 5*\text{sqrt}(2)*\text{atan}(\text{sqrt}(2)*\text{sqrt}(x)+1)/64$

**Mathematica [A]** time = 0.0899152, size = 121, normalized size = 0.94

$$\begin{aligned} & \frac{1}{128} \left( -\frac{72\sqrt{x}}{x^2+1} + \frac{32\sqrt{x}}{(x^2+1)^2} - 5\sqrt{2}\log(x-\sqrt{2}\sqrt{x}+1) + 5\sqrt{2}\log(x+\sqrt{2}\sqrt{x}+1) \right. \\ & \left. - 10\sqrt{2}\tan^{-1}(1-\sqrt{2}\sqrt{x}) + 10\sqrt{2}\tan^{-1}(\sqrt{2}\sqrt{x}+1) \right) \end{aligned}$$

Antiderivative was successfully verified.

[In] Integrate[x^(7/2)/(1 + x^2)^3, x]

[Out] ((32\*sqrt[x])/(1 + x^2)^2 - (72\*sqrt[x])/(1 + x^2) - 10\*sqrt[2]\*ArcTan[1 - sqrt[2]\*sqrt[x]] + 10\*sqrt[2]\*ArcTan[1 + sqrt[2]\*sqrt[x]]) - 5\*sqrt[2]\*Log[1 - sqrt[2]\*sqrt[x] + x] + 5\*sqrt[2]\*Log[1 + sqrt[2]\*sqrt[x] + x])/128

**Maple [A]** time = 0.015, size = 82, normalized size = 0.6

$$2 \frac{1}{(x^2 + 1)^2} \left( -\frac{9x^{5/2}}{32} - \frac{5\sqrt{x}}{32} \right) + \frac{5\sqrt{2}}{64} \arctan(1 + \sqrt{2}\sqrt{x}) + \frac{5\sqrt{2}}{64} \arctan(\sqrt{2}\sqrt{x} - 1) + \frac{5\sqrt{2}}{128} \ln\left(1 \left(1 + x + \sqrt{2}\sqrt{x}\right) \left(1 + x - \sqrt{2}\sqrt{x}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(7/2)/(x^2+1)^3, x)

[Out] 2\*(-9/32\*x^(5/2)-5/32\*x^(1/2))/(x^2+1)^2+5/64\*arctan(1+2^(1/2)\*x^(1/2))\*2^(1/2)+5/64\*arctan(2^(1/2)\*x^(1/2)-1)\*2^(1/2)+5/128\*2^(1/2)\*ln((1+x+2^(1/2)\*x^(1/2))/(1+x-2^(1/2)\*x^(1/2)))

**Maxima [A]** time = 1.50123, size = 134, normalized size = 1.04

$$\frac{5}{64} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) + \frac{5}{64} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) + \frac{5}{128} \sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1) - \frac{5}{128} \sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1) - \frac{9x^{5/2} + 5\sqrt{x}}{16(x^4 + 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(x^2 + 1)^3, x, algorithm="maxima")

[Out] 5/64\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) + 5/64\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) + 5/128\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1) - 5/128\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1) - 1/16\*(9\*x^(5/2) + 5\*sqrt(x))/(x^4 + 2\*x^2 + 1)

**Fricas [A]** time = 0.250353, size = 228, normalized size = 1.77

$$\frac{20\sqrt{2}(x^4 + 2x^2 + 1) \arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{2}\sqrt{2}\sqrt{x} + 2x + 2 + 1}\right) + 20\sqrt{2}(x^4 + 2x^2 + 1) \arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{-2}\sqrt{2}\sqrt{x} + 2x + 2 - 1}\right) - 5\sqrt{2}(x^4 + 2x^2 + 1)}{128(x^4 + 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(x^2 + 1)^3, x, algorithm="fricas")

[Out] -1/128\*(20\*sqrt(2)\*(x^4 + 2\*x^2 + 1)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(2)\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 1)) + 20\*sqrt(2)\*(x^4 + 2\*x^2 + 1)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(-2)\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 1)) - 5\*sqrt(2)\*(x^4 + 2\*x^2 + 1)\*log(2\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 5\*sqrt(2)\*(x^4 + 2\*x^2 + 1)\*log(-2\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 8\*(9\*x^2 + 5)\*sqrt(x))/(x^4 + 2\*x^2 + 1)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(7/2)/(x\*\*2+1)\*\*3,x)

[Out] Timed out

---

**GIAC/XCAS [A]** time = 0.210421, size = 127, normalized size = 0.98

$$\frac{5}{64} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) + \frac{5}{64} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) + \frac{5}{128} \sqrt{2} \ln(\sqrt{2}\sqrt{x} + x + 1) - \frac{5}{128} \sqrt{2} \ln(-\sqrt{2}\sqrt{x} + x + 1) - \frac{9x^{\frac{5}{2}} + 5\sqrt{x}}{16(x^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(7/2)/(x^2 + 1)^3,x, algorithm="giac")

[Out] 5/64\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) + 5/64\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) + 5/128\*sqrt(2)\*ln(sqrt(2)\*sqrt(x) + x + 1) - 5/128\*sqrt(2)\*ln(-sqrt(2)\*sqrt(x) + x + 1) - 1/16\*(9\*x^(5/2) + 5\*sqrt(x))/(x^2 + 1)^2

$$3.330 \quad \int \frac{x^{5/2}}{(1+x^2)^3} dx$$

**Optimal.** Leaf size=129

$$\frac{3x^{3/2}}{16(x^2+1)} - \frac{x^{3/2}}{4(x^2+1)^2} + \frac{3 \log(x - \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} - \frac{3 \log(x + \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} - \frac{3 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{32\sqrt{2}} + \frac{3 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{32\sqrt{2}}$$

[Out]  $-x^{3/2}/(4*(1+x^2)^2) + (3*x^{3/2})/(16*(1+x^2)) - (3*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]])/(32*\text{Sqrt}[2]) + (3*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]])/(32*\text{Sqrt}[2]) + (3*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[x] + x])/(64*\text{Sqrt}[2]) - (3*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x])/(64*\text{Sqrt}[2])$

**Rubi [A]** time = 0.166945, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$

$$\frac{3x^{3/2}}{16(x^2+1)} - \frac{x^{3/2}}{4(x^2+1)^2} + \frac{3 \log(x - \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} - \frac{3 \log(x + \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} - \frac{3 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{32\sqrt{2}} + \frac{3 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)/(1+x^2)^3,x]

[Out]  $-x^{3/2}/(4*(1+x^2)^2) + (3*x^{3/2})/(16*(1+x^2)) - (3*\text{ArcTan}[1 - \text{Sqrt}[2]*\text{Sqrt}[x]])/(32*\text{Sqrt}[2]) + (3*\text{ArcTan}[1 + \text{Sqrt}[2]*\text{Sqrt}[x]])/(32*\text{Sqrt}[2]) + (3*\text{Log}[1 - \text{Sqrt}[2]*\text{Sqrt}[x] + x])/(64*\text{Sqrt}[2]) - (3*\text{Log}[1 + \text{Sqrt}[2]*\text{Sqrt}[x] + x])/(64*\text{Sqrt}[2])$

**Rubi in Sympy [A]** time = 21.4979, size = 117, normalized size = 0.91

$$\frac{3x^{3/2}}{16(x^2+1)} - \frac{x^{3/2}}{4(x^2+1)^2} + \frac{3\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{128} - \frac{3\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{128} + \frac{3\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{64} + \frac{3\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(5/2)/(x\*\*2+1)\*\*3,x)

[Out]  $3*x^{3/2}/(16*(x^2+1)) - x^{3/2}/(4*(x^2+1)^2) + 3*\text{sqrt}(2)*\log(-\text{sqrt}(2)*\text{sqrt}(x) + x + 1)/128 - 3*\text{sqrt}(2)*\log(\text{sqrt}(2)*\text{sqrt}(x) + x + 1)/128 + 3*\text{sqrt}(2)*\operatorname{atan}(\text{sqrt}(2)*\text{sqrt}(x) - 1)/64 + 3*\text{sqrt}(2)*\operatorname{atan}(\text{sqrt}(2)*\text{sqrt}(x) + 1)/64$

**Mathematica [A]** time = 0.0867551, size = 121, normalized size = 0.94

$$\frac{1}{128} \left( \frac{24x^{3/2}}{x^2+1} - \frac{32x^{3/2}}{(x^2+1)^2} + 3\sqrt{2} \log(x - \sqrt{2}\sqrt{x} + 1) - 3\sqrt{2} \log(x + \sqrt{2}\sqrt{x} + 1) - 6\sqrt{2} \tan^{-1}(1 - \sqrt{2}\sqrt{x}) + 6\sqrt{2} \tan^{-1}(\sqrt{2}\sqrt{x} + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)/(1 + x^2)^3, x]

[Out]  $((-32x^{3/2})/(1 + x^2)^2 + (24x^{3/2})/(1 + x^2) - 6\sqrt{2}\operatorname{ArcTan}[1 - \sqrt{2}\sqrt{x}] + 6\sqrt{2}\operatorname{ArcTan}[1 + \sqrt{2}\sqrt{x}] + 3\sqrt{2}\operatorname{Log}[1 - \sqrt{2}\sqrt{x} + x] - 3\sqrt{2}\operatorname{Log}[1 + \sqrt{2}\sqrt{x}])/128$

**Maple [A]** time = 0.014, size = 82, normalized size = 0.6

$$2 \frac{1}{(x^2 + 1)^2} \left( \frac{3x^{7/2}}{32} - \frac{1}{32}x^{3/2} \right) + \frac{3\sqrt{2}}{64} \arctan(\sqrt{2}\sqrt{x} - 1) + \frac{3\sqrt{2}}{128} \ln\left(1(1 + x - \sqrt{2}\sqrt{x})(1 + x + \sqrt{2}\sqrt{x})^{-1}\right) + \frac{3\sqrt{2}}{64} \arctan(1 + \sqrt{2}\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)/(x^2+1)^3, x)

[Out]  $2*(3/32*x^{7/2}-1/32*x^{3/2})/(x^2+1)^2+3/64*\arctan(2^{1/2}*x^{1/2}-1)*2^{1/2}+3/128*2^{1/2}*\ln((1+x-2^{1/2}*x^{1/2})/(1+x+2^{1/2}*x^{1/2}))+3/64*\arctan(1+2^{1/2}*x^{1/2})*2^{1/2}$

**Maxima [A]** time = 1.50065, size = 134, normalized size = 1.04

$$\frac{3}{64} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) + \frac{3}{64} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) - \frac{3}{128} \sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1) + \frac{3}{128} \sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1) + \frac{3x^{7/2} - x^{3/2}}{16(x^4 + 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(x^2 + 1)^3, x, algorithm="maxima")

[Out]  $3/64*\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2*\sqrt{x})) + 3/64*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2*\sqrt{x})) - 3/128*\sqrt{2}*\log(\sqrt{2}*\sqrt{x} + x + 1) + 3/128*\sqrt{2}*\log(-\sqrt{2}*\sqrt{x} + x + 1) + 1/16*(3*x^{7/2} - x^{3/2})/(x^4 + 2*x^2 + 1)$

**Fricas [A]** time = 0.247294, size = 231, normalized size = 1.79

$$\frac{12\sqrt{2}(x^4 + 2x^2 + 1) \arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{2}\sqrt{2}\sqrt{x} + 2x + 2}\right) + 12\sqrt{2}(x^4 + 2x^2 + 1) \arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{-2}\sqrt{2}\sqrt{x} + 2x + 2}\right) + 3\sqrt{2}(x^4 + 2x^2 + 1)}{128(x^4 + 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(x^2 + 1)^3, x, algorithm="fricas")

[Out]  $-1/128*(12*\sqrt{2}*(x^4 + 2*x^2 + 1)*\arctan(1/(\sqrt{2}*\sqrt{x} + \sqrt{2}*\sqrt{2}*\sqrt{x} + 2*x + 2)) + 12*\sqrt{2}*(x^4 + 2*x^2 + 1)*\arctan(1/(\sqrt{2}*\sqrt{x} + \sqrt{-2}*\sqrt{2}*\sqrt{x} + 2*x + 2)) - 1) + 3*\sqrt{2}*(x^4 + 2*x^2 + 1)*\log(2*\sqrt{2}*\sqrt{x} + 2*x + 2) - 3*\sqrt{2}*(x^4 + 2*x^2 + 1)*\log(-2*\sqrt{2}*\sqrt{x} + 2*x + 2) - 8*(3*x^3 - x)*\sqrt{2}*\sqrt{x})/(x^4 + 2*x^2 + 1)$

**Sympy [A]** time = 131.675, size = 481, normalized size = 3.73

$$\begin{aligned} & \frac{24x^{\frac{7}{2}}}{128x^4 + 256x^2 + 128} - \frac{8x^{\frac{3}{2}}}{128x^4 + 256x^2 + 128} + \frac{3\sqrt{2}x^4 \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} \\ & - \frac{3\sqrt{2}x^4 \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} + \frac{6\sqrt{2}x^4 \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{128x^4 + 256x^2 + 128} + \frac{6\sqrt{2}x^4 \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{128x^4 + 256x^2 + 128} \\ & + \frac{6\sqrt{2}x^2 \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} - \frac{6\sqrt{2}x^2 \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} \\ & + \frac{12\sqrt{2}x^2 \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{128x^4 + 256x^2 + 128} + \frac{12\sqrt{2}x^2 \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{128x^4 + 256x^2 + 128} + \frac{3\sqrt{2} \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} \\ & - \frac{3\sqrt{2} \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} + \frac{6\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{128x^4 + 256x^2 + 128} + \frac{6\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{128x^4 + 256x^2 + 128} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(5/2)/(x\*\*2+1)\*\*3,x)

[Out]  $24*x^{(7/2)}/(128*x^{*4} + 256*x^{*2} + 128) - 8*x^{(3/2)}/(128*x^{*4} + 256*x^{*2} + 128) + 3*\sqrt{2}*x^{*4}*\log(-4*\sqrt{2}*\sqrt{x} + 4*x + 4)/(128*x^{*4} + 256*x^{*2} + 128) - 3*\sqrt{2}*x^{*4}*\log(4*\sqrt{2}*\sqrt{x} + 4*x + 4)/(128*x^{*4} + 256*x^{*2} + 128) + 6*\sqrt{2}*x^{*4}*\operatorname{atan}(\sqrt{2}*\sqrt{x} - 1)/(128*x^{*4} + 256*x^{*2} + 128) + 6*\sqrt{2}*x^{*4}*\operatorname{atan}(\sqrt{2}*\sqrt{x} + 1)/(128*x^{*4} + 256*x^{*2} + 128) + 6*\sqrt{2}*x^{*2}*\log(-4*\sqrt{2}*\sqrt{x} + 4*x + 4)/(128*x^{*4} + 256*x^{*2} + 128) - 6*\sqrt{2}*x^{*2}*\log(4*\sqrt{2}*\sqrt{x} + 4*x + 4)/(128*x^{*4} + 256*x^{*2} + 128) + 12*\sqrt{2}*x^{*2}*\operatorname{atan}(\sqrt{2}*\sqrt{x} - 1)/(128*x^{*4} + 256*x^{*2} + 128) + 12*\sqrt{2}*x^{*2}*\operatorname{atan}(\sqrt{2}*\sqrt{x} + 1)/(128*x^{*4} + 256*x^{*2} + 128) + 3*\sqrt{2}*\log(-4*\sqrt{2}*\sqrt{x} + 4*x + 4)/(128*x^{*4} + 256*x^{*2} + 128) - 3*\sqrt{2}*\log(4*\sqrt{2}*\sqrt{x} + 4*x + 4)/(128*x^{*4} + 256*x^{*2} + 128) + 6*\sqrt{2}*\operatorname{atan}(\sqrt{2}*\sqrt{x} - 1)/(128*x^{*4} + 256*x^{*2} + 128) + 6*\sqrt{2}*\operatorname{atan}(\sqrt{2}*\sqrt{x} + 1)/(128*x^{*4} + 256*x^{*2} + 128)$

**GIAC/XCAS [A]** time = 0.211297, size = 127, normalized size = 0.98

$$\begin{aligned} & \frac{3}{64} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) + \frac{3}{64} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) \\ & - \frac{3}{128} \sqrt{2} \ln(\sqrt{2}\sqrt{x} + x + 1) + \frac{3}{128} \sqrt{2} \ln(-\sqrt{2}\sqrt{x} + x + 1) + \frac{3x^{\frac{7}{2}} - x^{\frac{3}{2}}}{16(x^2 + 1)^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/(x^2 + 1)^3,x, algorithm="giac")

[Out]  $3/64*\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2*\sqrt{x})) + 3/64*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2*\sqrt{x})) - 3/128*\sqrt{2}* \ln(\sqrt{2}*\sqrt{x} + x + 1) + 3/128*\sqrt{2}* \ln(-\sqrt{2}*\sqrt{x} + x + 1) + 1/16*(3*x^{(7/2)} - x^{(3/2)})/(x^2 + 1)^2$

$$3.331 \quad \int \frac{x^{3/2}}{(1+x^2)^3} dx$$

**Optimal.** Leaf size=129

$$\frac{\sqrt{x}}{16(x^2+1)} - \frac{\sqrt{x}}{4(x^2+1)^2} - \frac{3 \log(x - \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} + \frac{3 \log(x + \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} \\ - \frac{3 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{32\sqrt{2}} + \frac{3 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{32\sqrt{2}}$$

[Out] -Sqrt[x]/(4\*(1+x^2)^2) + Sqrt[x]/(16\*(1+x^2)) - (3\*ArcTan[1 - Sqrt[2]\*Sqrt[x]]/(32\*Sqrt[2])) + (3\*ArcTan[1 + Sqrt[2]\*Sqrt[x]]/(32\*Sqrt[2])) - (3\*Log[1 - Sqrt[2]\*Sqrt[x] + x]/(64\*Sqrt[2])) + (3\*Log[1 + Sqrt[2]\*Sqrt[x] + x]/(64\*Sqrt[2]))

**Rubi [A]** time = 0.159877, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$

$$\frac{\sqrt{x}}{16(x^2+1)} - \frac{\sqrt{x}}{4(x^2+1)^2} - \frac{3 \log(x - \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} + \frac{3 \log(x + \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} \\ - \frac{3 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{32\sqrt{2}} + \frac{3 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)/(1+x^2)^3,x]

[Out] -Sqrt[x]/(4\*(1+x^2)^2) + Sqrt[x]/(16\*(1+x^2)) - (3\*ArcTan[1 - Sqrt[2]\*Sqrt[x]]/(32\*Sqrt[2])) + (3\*ArcTan[1 + Sqrt[2]\*Sqrt[x]]/(32\*Sqrt[2])) - (3\*Log[1 - Sqrt[2]\*Sqrt[x] + x]/(64\*Sqrt[2])) + (3\*Log[1 + Sqrt[2]\*Sqrt[x] + x]/(64\*Sqrt[2]))

**Rubi in Sympy [A]** time = 20.5745, size = 116, normalized size = 0.9

$$\frac{\sqrt{x}}{16(x^2+1)} - \frac{\sqrt{x}}{4(x^2+1)^2} - \frac{3\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{128} \\ + \frac{3\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{128} + \frac{3\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{64} + \frac{3\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(3/2)/(x\*\*2+1)\*\*3,x)

[Out] sqrt(x)/(16\*(x\*\*2+1)) - sqrt(x)/(4\*(x\*\*2+1)\*\*2) - 3\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1)/128 + 3\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1)/128 + 3\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/64 + 3\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/64

**Mathematica [A]** time = 0.0800409, size = 121, normalized size = 0.94

$$\frac{1}{128} \left( \frac{8\sqrt{x}}{x^2+1} - \frac{32\sqrt{x}}{(x^2+1)^2} - 3\sqrt{2} \log(x - \sqrt{2}\sqrt{x} + 1) + 3\sqrt{2} \log(x + \sqrt{2}\sqrt{x} + 1) \right. \\ \left. - 6\sqrt{2} \tan^{-1}(1 - \sqrt{2}\sqrt{x}) + 6\sqrt{2} \tan^{-1}(\sqrt{2}\sqrt{x} + 1) \right)$$



Antiderivative was successfully verified.

[In] Integrate[x^(3/2)/(1 + x^2)^3, x]

[Out] ((-32\*Sqrt[x])/(1 + x^2)^2 + (8\*Sqrt[x])/(1 + x^2) - 6\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[x]] + 6\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[x]] - 3\*Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[x] + x] + 3\*Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/128

**Maple [A]** time = 0.015, size = 82, normalized size = 0.6

$$2 \frac{1}{(x^2 + 1)^2} \left( \frac{1}{32} x^{5/2} - \frac{3\sqrt{x}}{32} \right) + \frac{3\sqrt{2}}{64} \arctan(\sqrt{2}\sqrt{x} - 1) + \frac{3\sqrt{2}}{128} \ln \left( 1 \left( 1 + x + \sqrt{2}\sqrt{x} \right) \left( 1 + x - \sqrt{2}\sqrt{x} \right)^{-1} \right) + \frac{3\sqrt{2}}{64} \arctan(1 + \sqrt{2}\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(x^2+1)^3, x)

[Out] 2\*(1/32\*x^(5/2)-3/32\*x^(1/2))/(x^2+1)^2+3/64\*arctan(2^(1/2)\*x^(1/2)-1)\*2^(1/2)+3/128\*2^(1/2)\*ln((1+x+2^(1/2)\*x^(1/2))/(1+x-2^(1/2)\*x^(1/2)))+3/64\*arctan(1+2^(1/2)\*x^(1/2))\*2^(1/2)

**Maxima [A]** time = 1.48985, size = 131, normalized size = 1.02

$$\frac{3}{64} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) + \frac{3}{64} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) + \frac{3}{128} \sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1) - \frac{3}{128} \sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1) + \frac{x^{5/2} - 3\sqrt{x}}{16(x^4 + 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(x^2 + 1)^3, x, algorithm="maxima")

[Out] 3/64\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) + 3/64\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) + 3/128\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1) - 3/128\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1) + 1/16\*(x^(5/2) - 3\*sqrt(x))/(x^4 + 2\*x^2 + 1)

**Fricas [A]** time = 0.245325, size = 225, normalized size = 1.74

$$\frac{12\sqrt{2}(x^4 + 2x^2 + 1) \arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{2}\sqrt{2}\sqrt{x} + 2x + 2 + 1}\right) + 12\sqrt{2}(x^4 + 2x^2 + 1) \arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{-2}\sqrt{2}\sqrt{x} + 2x + 2 - 1}\right) - 3\sqrt{2}(x^4 + 2x^2 + 1)}{128(x^4 + 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(x^2 + 1)^3, x, algorithm="fricas")

[Out] -1/128\*(12\*sqrt(2)\*(x^4 + 2\*x^2 + 1)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(2)\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 1)) + 12\*sqrt(2)\*(x^4 + 2\*x^2 + 1)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(-2)\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 1)) - 3\*sqrt(2)\*(x^4 + 2\*x^2 + 1)\*log(2\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 3\*sqrt(2)\*(x^4 + 2\*x^2 + 1)\*log(-2\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 8\*(x^2 - 3)\*sqrt(x)/(x^4 + 2\*x^2 + 1)

**Sympy [A]** time = 83.8601, size = 481, normalized size = 3.73

$$\begin{aligned} & \frac{8x^{\frac{5}{2}}}{128x^4 + 256x^2 + 128} - \frac{24\sqrt{x}}{128x^4 + 256x^2 + 128} - \frac{3\sqrt{2}x^4 \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} \\ & + \frac{3\sqrt{2}x^4 \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} + \frac{6\sqrt{2}x^4 \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{128x^4 + 256x^2 + 128} + \frac{6\sqrt{2}x^4 \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{128x^4 + 256x^2 + 128} \\ & - \frac{6\sqrt{2}x^2 \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} + \frac{6\sqrt{2}x^2 \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} \\ & + \frac{12\sqrt{2}x^2 \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{128x^4 + 256x^2 + 128} + \frac{12\sqrt{2}x^2 \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{128x^4 + 256x^2 + 128} - \frac{3\sqrt{2} \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} \\ & + \frac{3\sqrt{2} \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} + \frac{6\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{128x^4 + 256x^2 + 128} + \frac{6\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{128x^4 + 256x^2 + 128} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(3/2)/(x\*\*2+1)\*\*3,x)

[Out] 8\*x\*\*(5/2)/(128\*x\*\*4 + 256\*x\*\*2 + 128) - 24\*sqrt(x)/(128\*x\*\*4 + 256\*x\*\*2 + 128) - 3\*sqrt(2)\*x\*\*4\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 3\*sqrt(2)\*x\*\*4\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 6\*sqrt(2)\*x\*\*4\*atan(sqrt(2)\*sqrt(x) - 1)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 6\*sqrt(2)\*x\*\*4\*atan(sqrt(2)\*sqrt(x) + 1)/(128\*x\*\*4 + 256\*x\*\*2 + 128) - 6\*sqrt(2)\*x\*\*2\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 6\*sqrt(2)\*x\*\*2\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 12\*sqrt(2)\*x\*\*2\*atan(sqrt(2)\*sqrt(x) - 1)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 12\*sqrt(2)\*x\*\*2\*atan(sqrt(2)\*sqrt(x) + 1)/(128\*x\*\*4 + 256\*x\*\*2 + 128) - 3\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 3\*sqrt(2)\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 6\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 6\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/(128\*x\*\*4 + 256\*x\*\*2 + 128)

**GIAC/XCAS [A]** time = 0.211146, size = 124, normalized size = 0.96

$$\begin{aligned} & \frac{3}{64} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) + \frac{3}{64} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) \\ & + \frac{3}{128} \sqrt{2} \ln(\sqrt{2}\sqrt{x} + x + 1) - \frac{3}{128} \sqrt{2} \ln(-\sqrt{2}\sqrt{x} + x + 1) + \frac{x^{\frac{5}{2}} - 3\sqrt{x}}{16(x^2 + 1)^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(x^2 + 1)^3,x, algorithm="giac")

[Out] 3/64\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) + 3/64\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) + 3/128\*sqrt(2)\*ln(sqrt(2)\*sqrt(x) + x + 1) - 3/128\*sqrt(2)\*ln(-sqrt(2)\*sqrt(x) + x + 1) + 1/16\*(x^(5/2) - 3\*sqrt(x))/(x^2 + 1)^2

$$3.332 \quad \int \frac{\sqrt{x}}{(1+x^2)^3} dx$$

**Optimal.** Leaf size=129

$$\frac{5x^{3/2}}{16(x^2+1)} + \frac{x^{3/2}}{4(x^2+1)^2} + \frac{5 \log(x - \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} - \frac{5 \log(x + \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} - \frac{5 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{32\sqrt{2}} + \frac{5 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{32\sqrt{2}}$$

[Out]  $x^{(3/2)}/(4*(1+x^2)^2) + (5*x^{(3/2)})/(16*(1+x^2)) - (5*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) + (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) + (5*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) - (5*Log[1 + Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2])$

**Rubi [A]** time = 0.168566, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$

$$\frac{5x^{3/2}}{16(x^2+1)} + \frac{x^{3/2}}{4(x^2+1)^2} + \frac{5 \log(x - \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} - \frac{5 \log(x + \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} - \frac{5 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{32\sqrt{2}} + \frac{5 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(1+x^2)^3,x]

[Out]  $x^{(3/2)}/(4*(1+x^2)^2) + (5*x^{(3/2)})/(16*(1+x^2)) - (5*ArcTan[1 - Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) + (5*ArcTan[1 + Sqrt[2]*Sqrt[x]])/(32*Sqrt[2]) + (5*Log[1 - Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2]) - (5*Log[1 + Sqrt[2]*Sqrt[x] + x])/(64*Sqrt[2])$

**Rubi in Sympy [A]** time = 21.5784, size = 117, normalized size = 0.91

$$\frac{5x^{3/2}}{16(x^2+1)} + \frac{x^{3/2}}{4(x^2+1)^2} + \frac{5\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{128} - \frac{5\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{128} + \frac{5\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{64} + \frac{5\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(1/2)/(x\*\*2+1)\*\*3,x)

[Out]  $5*x^{(3/2)}/(16*(x^2+1)) + x^{(3/2)}/(4*(x^2+1)**2) + 5*\sqrt{2}*\log(-\sqrt{2}*\sqrt{x} + x + 1)/128 - 5*\sqrt{2}*\log(\sqrt{2}*\sqrt{x} + x + 1)/128 + 5*\sqrt{2}*\operatorname{atan}(\sqrt{2}*\sqrt{x} - 1)/64 + 5*\sqrt{2}*\operatorname{atan}(\sqrt{2}*\sqrt{x} + 1)/64$

**Mathematica [A]** time = 0.0829428, size = 121, normalized size = 0.94

$$\frac{1}{128} \left( \frac{40x^{3/2}}{x^2+1} + \frac{32x^{3/2}}{(x^2+1)^2} + 5\sqrt{2} \log(x - \sqrt{2}\sqrt{x} + 1) - 5\sqrt{2} \log(x + \sqrt{2}\sqrt{x} + 1) - 10\sqrt{2} \tan^{-1}(1 - \sqrt{2}\sqrt{x}) + 10\sqrt{2} \tan^{-1}(\sqrt{2}\sqrt{x} + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(1 + x^2)^3, x]

[Out] ((32\*x^(3/2))/(1 + x^2)^2 + (40\*x^(3/2))/(1 + x^2) - 10\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[x]] + 10\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[x]] + 5\*Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[x] + x] - 5\*Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/128

**Maple [A]** time = 0.01, size = 86, normalized size = 0.7

$$\frac{1}{4(x^2+1)^2}x^{\frac{3}{2}} + \frac{5}{16x^2+16}x^{\frac{3}{2}} + \frac{5\sqrt{2}}{64}\arctan\left(1 + \sqrt{2}\sqrt{x}\right) + \frac{5\sqrt{2}}{64}\arctan\left(\sqrt{2}\sqrt{x} - 1\right) + \frac{5\sqrt{2}}{128}\ln\left(1\left(1+x - \sqrt{2}\sqrt{x}\right)\left(1+x + \sqrt{2}\sqrt{x}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x^2+1)^3, x)

[Out] 1/4\*x^(3/2)/(x^2+1)^2+5/16\*x^(3/2)/(x^2+1)+5/64\*arctan(1+2^(1/2)\*x^(1/2))\*2^(1/2)+5/64\*arctan(2^(1/2)\*x^(1/2)-1)\*2^(1/2)+5/128\*2^(1/2)\*ln((1+x-2^(1/2)\*x^(1/2))/(1+x+2^(1/2)\*x^(1/2)))

**Maxima [A]** time = 1.49829, size = 134, normalized size = 1.04

$$\frac{5}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{5}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{5}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{5}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{5x^{\frac{7}{2}}+9x^{\frac{3}{2}}}{16(x^4+2x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x)/(x^2 + 1)^3, x, algorithm="maxima")

[Out] 5/64\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) + 5/64\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) - 5/128\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1) + 5/128\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1) + 1/16\*(5\*x^(7/2) + 9\*x^(3/2))/(x^4 + 2\*x^2 + 1)

**Fricas [A]** time = 0.249519, size = 231, normalized size = 1.79

$$\frac{20\sqrt{2}(x^4+2x^2+1)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x}+\sqrt{2}\sqrt{2}\sqrt{x}+2x+2+1}}\right) + 20\sqrt{2}(x^4+2x^2+1)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x}+\sqrt{-2}\sqrt{2}\sqrt{x}+2x+2-1}}\right) + 5\sqrt{2}(x^4+2x^2+1)}{128(x^4+2x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x)/(x^2 + 1)^3, x, algorithm="fricas")

[Out] -1/128\*(20\*sqrt(2)\*(x^4 + 2\*x^2 + 1)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(2)\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 1)) + 20\*sqrt(2)\*(x^4 + 2\*x^2 + 1)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(-2)\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 1)) + 5\*sqrt(2)\*(x^4 + 2\*x^2 + 1)\*log(2\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 5\*sqrt(2)\*(x^4 + 2\*x^2 + 1)\*log(-2\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 8\*(5\*x^3 + 9\*x)\*sqrt(x)/(x^4 + 2\*x^2 + 1)

**Sympy [A]** time = 46.9091, size = 481, normalized size = 3.73

$$\begin{aligned} & \frac{40x^{\frac{7}{2}}}{128x^4 + 256x^2 + 128} + \frac{72x^{\frac{3}{2}}}{128x^4 + 256x^2 + 128} + \frac{5\sqrt{2}x^4 \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} \\ & - \frac{5\sqrt{2}x^4 \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} + \frac{10\sqrt{2}x^4 \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{128x^4 + 256x^2 + 128} + \frac{10\sqrt{2}x^4 \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{128x^4 + 256x^2 + 128} \\ & + \frac{10\sqrt{2}x^2 \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} - \frac{10\sqrt{2}x^2 \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} \\ & + \frac{20\sqrt{2}x^2 \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{128x^4 + 256x^2 + 128} + \frac{20\sqrt{2}x^2 \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{128x^4 + 256x^2 + 128} + \frac{5\sqrt{2} \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} \\ & - \frac{5\sqrt{2} \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} + \frac{10\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{128x^4 + 256x^2 + 128} + \frac{10\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{128x^4 + 256x^2 + 128} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(1/2)/(x\*\*2+1)\*\*3,x)

[Out] 40\*x\*\*(7/2)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 72\*x\*\*(3/2)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 5\*sqrt(2)\*x\*\*4\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(128\*x\*\*4 + 256\*x\*\*2 + 128) - 5\*sqrt(2)\*x\*\*4\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 10\*sqrt(2)\*x\*\*4\*atan(sqrt(2)\*sqrt(x) - 1)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 10\*sqrt(2)\*x\*\*4\*atan(sqrt(2)\*sqrt(x) + 1)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 10\*sqrt(2)\*x\*\*2\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(128\*x\*\*4 + 256\*x\*\*2 + 128) - 10\*sqrt(2)\*x\*\*2\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 20\*sqrt(2)\*x\*\*2\*atan(sqrt(2)\*sqrt(x) - 1)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 20\*sqrt(2)\*x\*\*2\*atan(sqrt(2)\*sqrt(x) + 1)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 5\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(128\*x\*\*4 + 256\*x\*\*2 + 128) - 5\*sqrt(2)\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 10\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 10\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/(128\*x\*\*4 + 256\*x\*\*2 + 128)

**GIAC/XCAS [A]** time = 0.211035, size = 127, normalized size = 0.98

$$\begin{aligned} & \frac{5}{64} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) + \frac{5}{64} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) \\ & - \frac{5}{128} \sqrt{2} \ln(\sqrt{2}\sqrt{x} + x + 1) + \frac{5}{128} \sqrt{2} \ln(-\sqrt{2}\sqrt{x} + x + 1) + \frac{5x^{\frac{7}{2}} + 9x^{\frac{3}{2}}}{16(x^2 + 1)^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x)/(x^2 + 1)^3,x, algorithm="giac")

[Out] 5/64\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) + 5/64\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) - 5/128\*sqrt(2)\*ln(sqrt(2)\*sqrt(x) + x + 1) + 5/128\*sqrt(2)\*ln(-sqrt(2)\*sqrt(x) + x + 1) + 1/16\*(5\*x^(7/2) + 9\*x^(3/2))/(x^2 + 1)^2

$$3.333 \quad \int \frac{1}{\sqrt{x}(1+x^2)^3} dx$$

**Optimal.** Leaf size=129

$$\frac{7\sqrt{x}}{16(x^2+1)} + \frac{\sqrt{x}}{4(x^2+1)^2} - \frac{21 \log(x - \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} + \frac{21 \log(x + \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} - \frac{21 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{32\sqrt{2}} + \frac{21 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{32\sqrt{2}}$$

[Out] Sqrt[x]/(4\*(1+x^2)^2) + (7\*Sqrt[x])/(16\*(1+x^2)) - (21\*ArcTan[1 - Sqrt[2]\*Sqrt[x]])/(32\*Sqrt[2]) + (21\*ArcTan[1 + Sqrt[2]\*Sqrt[x]])/(32\*Sqrt[2]) - (21\*Log[1 - Sqrt[2]\*Sqrt[x] + x])/(64\*Sqrt[2]) + (21\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/(64\*Sqrt[2])

**Rubi [A]** time = 0.162733, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$

$$\frac{7\sqrt{x}}{16(x^2+1)} + \frac{\sqrt{x}}{4(x^2+1)^2} - \frac{21 \log(x - \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} + \frac{21 \log(x + \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} - \frac{21 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{32\sqrt{2}} + \frac{21 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]\*(1+x^2)^3),x]

[Out] Sqrt[x]/(4\*(1+x^2)^2) + (7\*Sqrt[x])/(16\*(1+x^2)) - (21\*ArcTan[1 - Sqrt[2]\*Sqrt[x]])/(32\*Sqrt[2]) + (21\*ArcTan[1 + Sqrt[2]\*Sqrt[x]])/(32\*Sqrt[2]) - (21\*Log[1 - Sqrt[2]\*Sqrt[x] + x])/(64\*Sqrt[2]) + (21\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/(64\*Sqrt[2])

**Rubi in Sympy [A]** time = 20.4869, size = 117, normalized size = 0.91

$$\frac{7\sqrt{x}}{16(x^2+1)} + \frac{\sqrt{x}}{4(x^2+1)^2} - \frac{21\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{128} + \frac{21\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{128} + \frac{21\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{64} + \frac{21\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(x\*\*2+1)\*\*3/x\*\*(1/2),x)

[Out] 7\*sqrt(x)/(16\*(x\*\*2+1)) + sqrt(x)/(4\*(x\*\*2+1)\*\*2) - 21\*sqrt(2)\*log(-sqrt(2)\*sqrt(x)+x+1)/128 + 21\*sqrt(2)\*log(sqrt(2)\*sqrt(x)+x+1)/128 + 21\*sqrt(2)\*atan(sqrt(2)\*sqrt(x)-1)/64 + 21\*sqrt(2)\*atan(sqrt(2)\*sqrt(x)+1)/64

**Mathematica [A]** time = 0.083033, size = 121, normalized size = 0.94

$$\frac{1}{128} \left( \frac{56\sqrt{x}}{x^2+1} + \frac{32\sqrt{x}}{(x^2+1)^2} - 21\sqrt{2} \log(x - \sqrt{2}\sqrt{x} + 1) + 21\sqrt{2} \log(x + \sqrt{2}\sqrt{x} + 1) - 42\sqrt{2} \tan^{-1}(1 - \sqrt{2}\sqrt{x}) + 42\sqrt{2} \tan^{-1}(\sqrt{2}\sqrt{x} + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]\*(1 + x^2)^3),x]

[Out] ((32\*Sqrt[x])/(1 + x^2)^2 + (56\*Sqrt[x])/(1 + x^2) - 42\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[x]] + 42\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[x]] - 21\*Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[x] + x] + 21\*Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/128

**Maple [A]** time = 0.01, size = 86, normalized size = 0.7

$$\frac{1}{4(x^2+1)^2}\sqrt{x} + \frac{7}{16x^2+16}\sqrt{x} + \frac{21\sqrt{2}}{64}\arctan\left(\sqrt{2}\sqrt{x}-1\right) + \frac{21\sqrt{2}}{128}\ln\left(1\left(1+x+\sqrt{2}\sqrt{x}\right)\left(1+x-\sqrt{2}\sqrt{x}\right)^{-1}\right) + \frac{21\sqrt{2}}{64}\arctan\left(1+\sqrt{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+1)^3/x^(1/2),x)

[Out] 1/4\*x^(1/2)/(x^2+1)^2+7/16\*x^(1/2)/(x^2+1)+21/64\*arctan(2^(1/2)\*x^(1/2)-1)\*2^(1/2)+21/128\*2^(1/2)\*ln((1+x+2^(1/2)\*x^(1/2))/(1+x-2^(1/2)\*x^(1/2)))+21/64\*arctan(1+2^(1/2)\*x^(1/2))\*2^(1/2)

**Maxima [A]** time = 1.56018, size = 134, normalized size = 1.04

$$\frac{21}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{21}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) + \frac{21}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{21}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{7x^{\frac{5}{2}}+11\sqrt{x}}{16(x^4+2x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^3\*sqrt(x)),x, algorithm="maxima")

[Out] 21/64\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) + 21/64\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) + 21/128\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1) - 21/128\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1) + 1/16\*(7\*x^(5/2) + 11\*sqrt(x))/(x^4 + 2\*x^2 + 1)

**Fricas [A]** time = 0.258218, size = 228, normalized size = 1.77

$$\frac{84\sqrt{2}(x^4+2x^2+1)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x}+\sqrt{2}\sqrt{2}\sqrt{x}+2x+2+1}\right) + 84\sqrt{2}(x^4+2x^2+1)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x}+\sqrt{-2}\sqrt{2}\sqrt{x}+2x+2-1}\right) - 21\sqrt{2}(x^4+2x^2+1)}{128(x^4+2x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^3\*sqrt(x)),x, algorithm="fricas")

[Out] -1/128\*(84\*sqrt(2)\*(x^4 + 2\*x^2 + 1)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(2)\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 1)) + 84\*sqrt(2)\*(x^4 + 2\*x^2 + 1)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(-2)\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 1)) - 21\*sqrt(2)\*(x^4 + 2\*x^2 + 1)\*log(2\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 21\*sqrt(2)\*(x^4 + 2\*x^2 + 1)\*log(-2\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 8\*(7\*x^2 + 11)\*sqrt(x))/(x^4 + 2\*x^2 + 1)

**Sympy [A]** time = 64.7262, size = 481, normalized size = 3.73

$$\begin{aligned} & \frac{56x^{\frac{5}{2}}}{128x^4 + 256x^2 + 128} + \frac{88\sqrt{x}}{128x^4 + 256x^2 + 128} - \frac{21\sqrt{2}x^4 \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} \\ & + \frac{21\sqrt{2}x^4 \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} + \frac{42\sqrt{2}x^4 \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{128x^4 + 256x^2 + 128} + \frac{42\sqrt{2}x^4 \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{128x^4 + 256x^2 + 128} \\ & - \frac{42\sqrt{2}x^2 \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} + \frac{42\sqrt{2}x^2 \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} \\ & + \frac{84\sqrt{2}x^2 \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{128x^4 + 256x^2 + 128} + \frac{84\sqrt{2}x^2 \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{128x^4 + 256x^2 + 128} - \frac{21\sqrt{2} \log(-4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} \\ & + \frac{21\sqrt{2} \log(4\sqrt{2}\sqrt{x} + 4x + 4)}{128x^4 + 256x^2 + 128} + \frac{42\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{128x^4 + 256x^2 + 128} + \frac{42\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{128x^4 + 256x^2 + 128} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x\*\*2+1)\*\*3/x\*\*(1/2), x)

[Out] 56\*x\*\*(5/2)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 88\*sqrt(x)/(128\*x\*\*4 + 256\*x\*\*2 + 128) - 21\*sqrt(2)\*x\*\*4\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 21\*sqrt(2)\*x\*\*4\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 42\*sqrt(2)\*x\*\*4\*atan(sqrt(2)\*sqrt(x) - 1)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 42\*sqrt(2)\*x\*\*4\*atan(sqrt(2)\*sqrt(x) + 1)/(128\*x\*\*4 + 256\*x\*\*2 + 128) - 42\*sqrt(2)\*x\*\*2\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 42\*sqrt(2)\*x\*\*2\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 84\*sqrt(2)\*x\*\*2\*atan(sqrt(2)\*sqrt(x) - 1)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 84\*sqrt(2)\*x\*\*2\*atan(sqrt(2)\*sqrt(x) + 1)/(128\*x\*\*4 + 256\*x\*\*2 + 128) - 21\*sqrt(2)\*log(-4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 21\*sqrt(2)\*log(4\*sqrt(2)\*sqrt(x) + 4\*x + 4)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 42\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/(128\*x\*\*4 + 256\*x\*\*2 + 128) + 42\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/(128\*x\*\*4 + 256\*x\*\*2 + 128)

**GIAC/XCAS [A]** time = 0.210828, size = 127, normalized size = 0.98

$$\begin{aligned} & \frac{21}{64} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) + \frac{21}{64} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) \\ & + \frac{21}{128} \sqrt{2} \ln(\sqrt{2}\sqrt{x} + x + 1) - \frac{21}{128} \sqrt{2} \ln(-\sqrt{2}\sqrt{x} + x + 1) + \frac{7x^{\frac{5}{2}} + 11\sqrt{x}}{16(x^2 + 1)^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^3\*sqrt(x)), x, algorithm="giac")

[Out] 21/64\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) + 21/64\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) + 21/128\*sqrt(2)\*ln(sqrt(2)\*sqrt(x) + x + 1) - 21/128\*sqrt(2)\*ln(-sqrt(2)\*sqrt(x) + x + 1) + 1/16\*(7\*x^(5/2) + 11\*sqrt(x))/(x^2 + 1)^2



$$3.334 \quad \int \frac{1}{x^{3/2}(1+x^2)^3} dx$$

**Optimal.** Leaf size=138

$$\frac{9}{16\sqrt{x}(x^2+1)} + \frac{1}{4\sqrt{x}(x^2+1)^2} - \frac{45}{16\sqrt{x}} - \frac{45 \log(x - \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}}$$

$$+ \frac{45 \log(x + \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} + \frac{45 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{32\sqrt{2}} - \frac{45 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{32\sqrt{2}}$$

[Out] -45/(16\*Sqrt[x]) + 1/(4\*Sqrt[x]\*(1 + x^2)^2) + 9/(16\*Sqrt[x]\*(1 + x^2)) + (45\*ArcTan[1 - Sqrt[2]\*Sqrt[x]])/(32\*Sqrt[2]) - (45\*ArcTan[1 + Sqrt[2]\*Sqrt[x]])/(32\*Sqrt[2]) - (45\*Log[1 - Sqrt[2]\*Sqrt[x] + x])/(64\*Sqrt[2]) + (45\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/(64\*Sqrt[2])

**Rubi [A]** time = 0.180288, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$

$$\frac{9}{16\sqrt{x}(x^2+1)} + \frac{1}{4\sqrt{x}(x^2+1)^2} - \frac{45}{16\sqrt{x}} - \frac{45 \log(x - \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}}$$

$$+ \frac{45 \log(x + \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} + \frac{45 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{32\sqrt{2}} - \frac{45 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(3/2)\*(1 + x^2)^3), x]

[Out] -45/(16\*Sqrt[x]) + 1/(4\*Sqrt[x]\*(1 + x^2)^2) + 9/(16\*Sqrt[x]\*(1 + x^2)) + (45\*ArcTan[1 - Sqrt[2]\*Sqrt[x]])/(32\*Sqrt[2]) - (45\*ArcTan[1 + Sqrt[2]\*Sqrt[x]])/(32\*Sqrt[2]) - (45\*Log[1 - Sqrt[2]\*Sqrt[x] + x])/(64\*Sqrt[2]) + (45\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/(64\*Sqrt[2])

**Rubi in Sympy [A]** time = 23.3514, size = 128, normalized size = 0.93

$$-\frac{45\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{128} + \frac{45\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{128} - \frac{45\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{64}$$

$$- \frac{45\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{64} - \frac{45}{16\sqrt{x}} + \frac{9}{16\sqrt{x}(x^2+1)} + \frac{1}{4\sqrt{x}(x^2+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*(3/2)/(x\*\*2+1)\*\*3, x)

[Out] -45\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1)/128 + 45\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1)/128 - 45\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/64 - 45\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/64 - 45/(16\*sqrt(x)) + 9/(16\*sqrt(x)\*(x\*\*2 + 1)) + 1/(4\*sqrt(x)\*(x\*\*2 + 1)\*\*2)

**Mathematica [A]** time = 0.10592, size = 128, normalized size = 0.93

$$\frac{1}{128} \left( -\frac{104x^{3/2}}{x^2+1} - \frac{32x^{3/2}}{(x^2+1)^2} - \frac{256}{\sqrt{x}} \right)$$

$$- 45\sqrt{2} \log(x - \sqrt{2}\sqrt{x} + 1) + 45\sqrt{2} \log(x + \sqrt{2}\sqrt{x} + 1) + 90\sqrt{2} \tan^{-1}(1 - \sqrt{2}\sqrt{x}) - 90\sqrt{2} \tan^{-1}(\sqrt{2}\sqrt{x} + 1)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(3/2)\*(1 + x^2)^3),x]

[Out] (-256/Sqrt[x] - (32\*x^(3/2))/(1 + x^2)^2 - (104\*x^(3/2))/(1 + x^2) + 90\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[x]] - 90\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[x]] - 45\*Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[x] + x] + 45\*Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/128

**Maple [A]** time = 0.018, size = 87, normalized size = 0.6

$$-2 \frac{1}{\sqrt{x}} - 2 \frac{1}{(x^2 + 1)^2} \left( \frac{13x^{7/2}}{32} + \frac{17x^{3/2}}{32} \right) - \frac{45\sqrt{2}}{64} \arctan(1 + \sqrt{2}\sqrt{x}) - \frac{45\sqrt{2}}{64} \arctan(\sqrt{2}\sqrt{x} - 1) - \frac{45\sqrt{2}}{128} \ln\left(1(1 + x - \sqrt{2}\sqrt{x})(1 + x + \sqrt{2}\sqrt{x})^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(3/2)/(x^2+1)^3,x)

[Out] -2/x^(1/2)-2\*(13/32\*x^(7/2)+17/32\*x^(3/2))/(x^2+1)^2-45/64\*arctan(1+2^(1/2)\*x^(1/2))\*2^(1/2)-45/64\*arctan(2^(1/2)\*x^(1/2)-1)\*2^(1/2)-45/128\*2^(1/2)\*ln((1+x-2^(1/2)\*x^(1/2))/(1+x+2^(1/2)\*x^(1/2)))

**Maxima [A]** time = 1.49316, size = 138, normalized size = 1.

$$-\frac{45}{64} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) - \frac{45}{64} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) + \frac{45}{128} \sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1) - \frac{45}{128} \sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1) - \frac{45x^4 + 81x^2 + 32}{16(x^{\frac{9}{2}} + 2x^{\frac{5}{2}} + \sqrt{x})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^3\*x^(3/2)),x, algorithm="maxima")

[Out] -45/64\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) - 45/64\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) + 45/128\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1) - 45/128\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1) - 1/16\*(45\*x^4 + 81\*x^2 + 32)/(x^(9/2) + 2\*x^(5/2) + sqrt(x))

**Fricas [A]** time = 0.254989, size = 235, normalized size = 1.7

$$180 \sqrt{2}(x^5 + 2x^3 + x) \arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{2}\sqrt{2}\sqrt{x} + 2x + 1}\right) + 180 \sqrt{2}(x^5 + 2x^3 + x) \arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{-2}\sqrt{2}\sqrt{x} + 2x + 2 - 1}\right) + 45 \sqrt{2}(x^5 + 2x^3 + x) \log\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{2}\sqrt{2}\sqrt{x} + 2x + 1}\right) + 45 \sqrt{2}(x^5 + 2x^3 + x) \log\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{-2}\sqrt{2}\sqrt{x} + 2x + 2 - 1}\right) + 45 \sqrt{2}(x^5 + 2x^3 + x) \log(2\sqrt{2}\sqrt{x} + 2x + 2) - 45 \sqrt{2}(x^5 + 2x^3 + x) \log(-2\sqrt{2}\sqrt{x} + 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^3\*x^(3/2)),x, algorithm="fricas")

[Out] 1/128\*(180\*sqrt(2)\*(x^5 + 2\*x^3 + x)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(2)\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 1) + 180\*sqrt(2)\*(x^5 + 2\*x^3 + x)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(-2)\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 1) + 45\*sqrt(2)\*(x^5 + 2\*x^3 + x)\*log(2\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 45\*sqrt(2)\*(x^5 + 2\*x^3 + x)\*log(-2\*sqrt(2)\*sqrt(x) + 2\*x + 2)

$$2^*x + 2) - 8^*(45^*x^4 + 81^*x^2 + 32)^*\text{sqrt}(x))/(x^5 + 2^*x^3 + x)$$

**Sympy [A]** time = 111.671, size = 653, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(3/2)/(x\*\*2+1)\*\*3,x)

[Out] 
$$\begin{aligned} & -45*\text{sqrt}(2)*x^{(9/2)}*\log(-4*\text{sqrt}(2)*\text{sqrt}(x) + 4*x + 4)/(128*x^{(9/2)} + 256*x^{(5/2)} + 128*\text{sqrt}(x)) + 45*\text{sqrt}(2)*x^{(9/2)}*\log(4*\text{sqrt}(2)*\text{sqrt}(x) + 4*x + 4)/(128*x^{(9/2)} + 256*x^{(5/2)} + 128*\text{sqrt}(x)) \\ & - 90*\text{sqrt}(2)*x^{(9/2)}*\text{atan}(\text{sqrt}(2)*\text{sqrt}(x) - 1)/(128*x^{(9/2)} + 256*x^{(5/2)} + 128*\text{sqrt}(x)) - 90*\text{sqrt}(2)*x^{(9/2)}*\text{atan}(\text{sqrt}(2)*\text{sqrt}(x) + 1)/(128*x^{(9/2)} + 256*x^{(5/2)} + 128*\text{sqrt}(x)) \\ & - 90*\text{sqrt}(2)*x^{(5/2)}*\log(-4*\text{sqrt}(2)*\text{sqrt}(x) + 4*x + 4)/(128*x^{(9/2)} + 256*x^{(5/2)} + 128*\text{sqrt}(x)) + 90*\text{sqrt}(2)*x^{(5/2)}*\log(4*\text{sqrt}(2)*\text{sqrt}(x) + 4*x + 4)/(128*x^{(9/2)} + 256*x^{(5/2)} + 128*\text{sqrt}(x)) \\ & - 180*\text{sqrt}(2)*x^{(5/2)}*\text{atan}(\text{sqrt}(2)*\text{sqrt}(x) - 1)/(128*x^{(9/2)} + 256*x^{(5/2)} + 128*\text{sqrt}(x)) - 180*\text{sqrt}(2)*x^{(5/2)}*\text{atan}(\text{sqrt}(2)*\text{sqrt}(x) + 1)/(128*x^{(9/2)} + 256*x^{(5/2)} + 128*\text{sqrt}(x)) \\ & - 45*\text{sqrt}(2)*\text{sqrt}(x)*\log(-4*\text{sqrt}(2)*\text{sqrt}(x) + 4*x + 4)/(128*x^{(9/2)} + 256*x^{(5/2)} + 128*\text{sqrt}(x)) + 45*\text{sqrt}(2)*\text{sqrt}(x)*\log(4*\text{sqrt}(2)*\text{sqrt}(x) + 4*x + 4)/(128*x^{(9/2)} + 256*x^{(5/2)} + 128*\text{sqrt}(x)) \\ & - 90*\text{sqrt}(2)*\text{sqrt}(x)*\text{atan}(\text{sqrt}(2)*\text{sqrt}(x) - 1)/(128*x^{(9/2)} + 256*x^{(5/2)} + 128*\text{sqrt}(x)) - 90*\text{sqrt}(2)*\text{sqrt}(x)*\text{atan}(\text{sqrt}(2)*\text{sqrt}(x) + 1)/(128*x^{(9/2)} + 256*x^{(5/2)} + 128*\text{sqrt}(x)) \\ & - 360*x^4/(128*x^{(9/2)} + 256*x^{(5/2)} + 128*\text{sqrt}(x)) - 648*x^2/(128*x^{(9/2)} + 256*x^{(5/2)} + 128*\text{sqrt}(x)) - 256/(128*x^{(9/2)} + 256*x^{(5/2)} + 128*\text{sqrt}(x)) \end{aligned}$$

**GIAC/XCAS [A]** time = 0.211938, size = 134, normalized size = 0.97

$$\begin{aligned} & -\frac{45}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2}+2\sqrt{x})\right) - \frac{45}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2}-2\sqrt{x})\right) \\ & + \frac{45}{128}\sqrt{2}\ln(\sqrt{2}\sqrt{x}+x+1) - \frac{45}{128}\sqrt{2}\ln(-\sqrt{2}\sqrt{x}+x+1) - \frac{2}{\sqrt{x}} - \frac{13x^{\frac{7}{2}}+17x^{\frac{3}{2}}}{16(x^2+1)^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^3\*x^(3/2)),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -45/64*\text{sqrt}(2)*\arctan(1/2*\text{sqrt}(2)*(\text{sqrt}(2) + 2*\text{sqrt}(x))) - 45/64* \\ & \text{sqrt}(2)*\arctan(-1/2*\text{sqrt}(2)*(\text{sqrt}(2) - 2*\text{sqrt}(x))) + 45/128*\text{sqrt}(2)* \\ & \ln(\text{sqrt}(2)*\text{sqrt}(x) + x + 1) - 45/128*\text{sqrt}(2)*\ln(-\text{sqrt}(2)*\text{sqrt}(x) \\ & + x + 1) - 2/\text{sqrt}(x) - 1/16*(13*x^{(7/2)} + 17*x^{(3/2)})/(x^2 + 1)^2 \end{aligned}$$

$$3.335 \quad \int \frac{1}{x^{5/2}(1+x^2)^3} dx$$

**Optimal.** Leaf size=138

$$\begin{aligned} & -\frac{77}{48x^{3/2}} + \frac{11}{16x^{3/2}(x^2+1)} + \frac{1}{4x^{3/2}(x^2+1)^2} + \frac{77 \log(x - \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} \\ & - \frac{77 \log(x + \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} + \frac{77 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{32\sqrt{2}} - \frac{77 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{32\sqrt{2}} \end{aligned}$$

[Out] -77/(48\*x^(3/2)) + 1/(4\*x^(3/2)\*(1 + x^2)^2) + 11/(16\*x^(3/2)\*(1 + x^2)) + (77\*ArcTan[1 - Sqrt[2]\*Sqrt[x]])/(32\*Sqrt[2]) - (77\*ArcTan[1 + Sqrt[2]\*Sqrt[x]])/(32\*Sqrt[2]) + (77\*Log[1 - Sqrt[2]\*Sqrt[x] + x])/(64\*Sqrt[2]) - (77\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/(64\*Sqrt[2])

**Rubi [A]** time = 0.170225, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$

$$\begin{aligned} & -\frac{77}{48x^{3/2}} + \frac{11}{16x^{3/2}(x^2+1)} + \frac{1}{4x^{3/2}(x^2+1)^2} + \frac{77 \log(x - \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} \\ & - \frac{77 \log(x + \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} + \frac{77 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{32\sqrt{2}} - \frac{77 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{32\sqrt{2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(5/2)\*(1 + x^2)^3), x]

[Out] -77/(48\*x^(3/2)) + 1/(4\*x^(3/2)\*(1 + x^2)^2) + 11/(16\*x^(3/2)\*(1 + x^2)) + (77\*ArcTan[1 - Sqrt[2]\*Sqrt[x]])/(32\*Sqrt[2]) - (77\*ArcTan[1 + Sqrt[2]\*Sqrt[x]])/(32\*Sqrt[2]) + (77\*Log[1 - Sqrt[2]\*Sqrt[x] + x])/(64\*Sqrt[2]) - (77\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/(64\*Sqrt[2])

**Rubi in Sympy [A]** time = 22.2736, size = 128, normalized size = 0.93

$$\begin{aligned} & \frac{77\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{128} - \frac{77\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{128} - \frac{77\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{64} \\ & - \frac{77\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{64} - \frac{77}{48x^{3/2}} + \frac{11}{16x^{3/2}(x^2+1)} + \frac{1}{4x^{3/2}(x^2+1)^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*(5/2)/(x\*\*2+1)\*\*3, x)

[Out] 77\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1)/128 - 77\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1)/128 - 77\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/64 - 77\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/64 - 77/(48\*x\*\*(3/2)) + 11/(16\*x\*\*(3/2)\*(x\*\*2 + 1)) + 1/(4\*x\*\*(3/2)\*(x\*\*2 + 1)\*\*2)

**Mathematica [A]** time = 0.105269, size = 128, normalized size = 0.93

$$\begin{aligned} & \frac{1}{384} \left( -\frac{256}{x^{3/2}} - \frac{360\sqrt{x}}{x^2+1} - \frac{96\sqrt{x}}{(x^2+1)^2} + 231\sqrt{2} \log(x - \sqrt{2}\sqrt{x} + 1) \right. \\ & \left. - 231\sqrt{2} \log(x + \sqrt{2}\sqrt{x} + 1) + 462\sqrt{2} \tan^{-1}(1 - \sqrt{2}\sqrt{x}) - 462\sqrt{2} \tan^{-1}(\sqrt{2}\sqrt{x} + 1) \right) \end{aligned}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(5/2)\*(1 + x^2)^3),x]

[Out] (-256/x^(3/2) - (96\*sqrt[x])/(1 + x^2)^2 - (360\*sqrt[x])/(1 + x^2) + 462\*sqrt[2]\*ArcTan[1 - sqrt[2]\*sqrt[x]] - 462\*sqrt[2]\*ArcTan[1 + sqrt[2]\*sqrt[x]] + 231\*sqrt[2]\*Log[1 - sqrt[2]\*sqrt[x] + x] - 231\*sqrt[2]\*Log[1 + sqrt[2]\*sqrt[x] + x])/384

**Maple [A]** time = 0.016, size = 87, normalized size = 0.6

$$-\frac{2}{3}x^{-\frac{3}{2}} - 2 \frac{1}{(x^2 + 1)^2} \left( \frac{15x^{5/2}}{32} + \frac{19\sqrt{x}}{32} \right) - \frac{77\sqrt{2}}{64} \arctan(\sqrt{2}\sqrt{x} - 1) - \frac{77\sqrt{2}}{128} \ln\left(1 \left(1 + x + \sqrt{2}\sqrt{x}\right) \left(1 + x - \sqrt{2}\sqrt{x}\right)^{-1}\right) - \frac{77\sqrt{2}}{64} \arctan\left(1 + \sqrt{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(5/2)/(x^2+1)^3,x)

[Out] -2/3/x^(3/2)-2\*(15/32\*x^(5/2)+19/32\*x^(1/2))/(x^2+1)^2-77/64\*arctan(2^(1/2)\*x^(1/2)-1)\*2^(1/2)-77/128\*2^(1/2)\*ln((1+x+2^(1/2)\*x^(1/2))/(1+x-2^(1/2)\*x^(1/2)))-77/64\*arctan(1+2^(1/2)\*x^(1/2))\*2^(1/2)

**Maxima [A]** time = 1.50314, size = 138, normalized size = 1.

$$-\frac{77}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2}+2\sqrt{x})\right) - \frac{77}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2}-2\sqrt{x})\right) - \frac{77}{128}\sqrt{2}\log(\sqrt{2}\sqrt{x}+x+1) + \frac{77}{128}\sqrt{2}\log(-\sqrt{2}\sqrt{x}+x+1) - \frac{77x^4+121x^2+32}{48\left(x^{\frac{11}{2}}+2x^{\frac{7}{2}}+x^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^3\*x^(5/2)),x, algorithm="maxima")

[Out] -77/64\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) - 77/64\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) - 77/128\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1) + 77/128\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1) - 1/48\*(77\*x^4 + 121\*x^2 + 32)/(x^(11/2) + 2\*x^(7/2) + x^(3/2))

**Fricas [A]** time = 0.256738, size = 248, normalized size = 1.8

$$924\sqrt{2}(x^6 + 2x^4 + x^2)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{2}\sqrt{2}\sqrt{x} + 2x + 2 + 1}\right) + 924\sqrt{2}(x^6 + 2x^4 + x^2)\arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{-2}\sqrt{2}\sqrt{x} + 2x + 2 - 1}\right) - 231\sqrt{2}(x^6 + 2x^4 + x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^3\*x^(5/2)),x, algorithm="fricas")

[Out] 1/384\*(924\*sqrt(2)\*(x^6 + 2\*x^4 + x^2)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(2)\*sqrt(2)\*sqrt(x) + 2\*x + 2) + 1)) + 924\*sqrt(2)\*(x^6 + 2\*x^4 + x^2)\*arctan(1/(sqrt(2)\*sqrt(x) + sqrt(-2)\*sqrt(2)\*sqrt(x) + 2\*x + 2) - 1)) - 231\*sqrt(2)\*(x^6 + 2\*x^4 + x^2)\*log(2\*sqrt(2)\*sqrt(x) + 1) - 231\*sqrt(2)\*(x^6 + 2\*x^4 + x^2)\*log(2\*sqrt(2)\*sqrt(x) - 1)

$$\frac{\sqrt{2} \sqrt{x+2} + 231 \sqrt{2} (x^6 + 2x^4 + x^2) \log(-2\sqrt{2} \sqrt{x+2}) - 8(77x^4 + 121x^2 + 32)\sqrt{x}}{(x^6 + 2x^4 + x^2)}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(5/2)/(x\*\*2+1)\*\*3,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.212246, size = 134, normalized size = 0.97

$$-\frac{77}{64} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} (\sqrt{2} + 2\sqrt{x})\right) - \frac{77}{64} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2} (\sqrt{2} - 2\sqrt{x})\right) - \frac{77}{128} \sqrt{2} \ln(\sqrt{2}\sqrt{x} + x + 1) + \frac{77}{128} \sqrt{2} \ln(-\sqrt{2}\sqrt{x} + x + 1) - \frac{15x^{\frac{5}{2}} + 19\sqrt{x}}{16(x^2 + 1)^2} - \frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^3\*x^(5/2)),x, algorithm="giac")

[Out] 
$$-\frac{77}{64} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} (\sqrt{2} + 2\sqrt{x})\right) - \frac{77}{64} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2} (\sqrt{2} - 2\sqrt{x})\right) - \frac{77}{128} \sqrt{2} \ln(\sqrt{2}\sqrt{x} + x + 1) + \frac{77}{128} \sqrt{2} \ln(-\sqrt{2}\sqrt{x} + x + 1) - \frac{1}{16} (15x^{5/2} + 19\sqrt{x}) / (x^2 + 1)^2 - \frac{2}{3x^{3/2}}$$

$$3.336 \quad \int \frac{1}{x^{7/2}(1+x^2)^3} dx$$

**Optimal.** Leaf size=147

$$\begin{aligned} & -\frac{117}{80x^{5/2}} + \frac{13}{16x^{5/2}(x^2+1)} + \frac{1}{4x^{5/2}(x^2+1)^2} + \frac{117}{16\sqrt{x}} + \frac{117 \log(x - \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} \\ & - \frac{117 \log(x + \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} - \frac{117 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{32\sqrt{2}} + \frac{117 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{32\sqrt{2}} \end{aligned}$$

[Out] -117/(80\*x^(5/2)) + 117/(16\*Sqrt[x]) + 1/(4\*x^(5/2)\*(1 + x^2)^2) + 13/(16\*x^(5/2)\*(1 + x^2)) - (117\*ArcTan[1 - Sqrt[2]\*Sqrt[x]])/(32\*Sqrt[2]) + (117\*ArcTan[1 + Sqrt[2]\*Sqrt[x]])/(32\*Sqrt[2]) + (117\*Log[1 - Sqrt[2]\*Sqrt[x] + x])/(64\*Sqrt[2]) - (117\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/(64\*Sqrt[2])

**Rubi [A]** time = 0.185712, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$

$$\begin{aligned} & -\frac{117}{80x^{5/2}} + \frac{13}{16x^{5/2}(x^2+1)} + \frac{1}{4x^{5/2}(x^2+1)^2} + \frac{117}{16\sqrt{x}} + \frac{117 \log(x - \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} \\ & - \frac{117 \log(x + \sqrt{2}\sqrt{x} + 1)}{64\sqrt{2}} - \frac{117 \tan^{-1}(1 - \sqrt{2}\sqrt{x})}{32\sqrt{2}} + \frac{117 \tan^{-1}(\sqrt{2}\sqrt{x} + 1)}{32\sqrt{2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(7/2)\*(1 + x^2)^3), x]

[Out] -117/(80\*x^(5/2)) + 117/(16\*Sqrt[x]) + 1/(4\*x^(5/2)\*(1 + x^2)^2) + 13/(16\*x^(5/2)\*(1 + x^2)) - (117\*ArcTan[1 - Sqrt[2]\*Sqrt[x]])/(32\*Sqrt[2]) + (117\*ArcTan[1 + Sqrt[2]\*Sqrt[x]])/(32\*Sqrt[2]) + (117\*Log[1 - Sqrt[2]\*Sqrt[x] + x])/(64\*Sqrt[2]) - (117\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/(64\*Sqrt[2])

**Rubi in Sympy [A]** time = 24.9316, size = 136, normalized size = 0.93

$$\begin{aligned} & \frac{117\sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1)}{128} - \frac{117\sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1)}{128} + \frac{117\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1)}{64} \\ & + \frac{117\sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)}{64} + \frac{117}{16\sqrt{x}} - \frac{117}{80x^{5/2}} + \frac{13}{16x^{5/2}(x^2+1)} + \frac{1}{4x^{5/2}(x^2+1)^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*(7/2)/(x\*\*2+1)\*\*3, x)

[Out] 117\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1)/128 - 117\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1)/128 + 117\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) - 1)/64 + 117\*sqrt(2)\*atan(sqrt(2)\*sqrt(x) + 1)/64 + 117/(16\*sqrt(x)) - 117/(80\*x\*\*(5/2)) + 13/(16\*x\*\*(5/2)\*(x\*\*2 + 1)) + 1/(4\*x\*\*(5/2)\*(x\*\*2 + 1)\*\*2)

**Mathematica [A]** time = 0.112855, size = 135, normalized size = 0.92

$$\frac{1}{640} \left( -\frac{256}{x^{5/2}} + \frac{840x^{3/2}}{x^2 + 1} + \frac{160x^{3/2}}{(x^2 + 1)^2} + \frac{3840}{\sqrt{x}} \right) + 585\sqrt{2} \log(x - \sqrt{2}\sqrt{x} + 1) - 585\sqrt{2} \log(x + \sqrt{2}\sqrt{x} + 1) - 1170\sqrt{2} \tan^{-1}(1 - \sqrt{2}\sqrt{x}) + 1170\sqrt{2} \tan^{-1}(\sqrt{2}\sqrt{x} + 1)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(7/2)\*(1 + x^2)^3), x]

[Out] (-256/x^(5/2) + 3840/Sqrt[x] + (160\*x^(3/2))/(1 + x^2)^2 + (840\*x^(3/2))/(1 + x^2) - 1170\*Sqrt[2]\*ArcTan[1 - Sqrt[2]\*Sqrt[x]] + 1170\*Sqrt[2]\*ArcTan[1 + Sqrt[2]\*Sqrt[x]] + 585\*Sqrt[2]\*Log[1 - Sqrt[2]\*Sqrt[x] + x] - 585\*Sqrt[2]\*Log[1 + Sqrt[2]\*Sqrt[x] + x])/640

**Maple [A]** time = 0.02, size = 92, normalized size = 0.6

$$-\frac{2}{5}x^{-5/2} + 6\frac{1}{\sqrt{x}} + 2\frac{1}{(x^2 + 1)^2} \left( \frac{21x^{7/2}}{32} + \frac{25x^{3/2}}{32} \right) + \frac{117\sqrt{2}}{64} \arctan(\sqrt{2}\sqrt{x} - 1) + \frac{117\sqrt{2}}{128} \ln\left(1(1 + x - \sqrt{2}\sqrt{x})(1 + x + \sqrt{2}\sqrt{x})^{-1}\right) + \frac{117\sqrt{2}}{64} \arctan(1 + \sqrt{2}\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(7/2)/(x^2+1)^3, x)

[Out] -2/5/x^(5/2)+6/x^(1/2)+2\*(21/32\*x^(7/2)+25/32\*x^(3/2))/(x^2+1)^2+117/64\*arctan(2^(1/2)\*x^(1/2)-1)\*2^(1/2)+117/128\*2^(1/2)\*ln((1+x-2^(1/2)\*x^(1/2))/(1+x+2^(1/2)\*x^(1/2)))+117/64\*arctan(1+2^(1/2)\*x^(1/2))\*2^(1/2)

**Maxima [A]** time = 1.50189, size = 144, normalized size = 0.98

$$\frac{117}{64} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) + \frac{117}{64} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) - \frac{117}{128} \sqrt{2} \log(\sqrt{2}\sqrt{x} + x + 1) + \frac{117}{128} \sqrt{2} \log(-\sqrt{2}\sqrt{x} + x + 1) + \frac{585x^6 + 1053x^4 + 416x^2 - 32}{80(x^{13/2} + 2x^{9/2} + x^{5/2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^3\*x^(7/2)), x, algorithm="maxima")

[Out] 117/64\*sqrt(2)\*arctan(1/2\*sqrt(2)\*(sqrt(2) + 2\*sqrt(x))) + 117/64\*sqrt(2)\*arctan(-1/2\*sqrt(2)\*(sqrt(2) - 2\*sqrt(x))) - 117/128\*sqrt(2)\*log(sqrt(2)\*sqrt(x) + x + 1) + 117/128\*sqrt(2)\*log(-sqrt(2)\*sqrt(x) + x + 1) + 1/80\*(585\*x^6 + 1053\*x^4 + 416\*x^2 - 32)/(x^(13/2) + 2\*x^(9/2) + x^(5/2))

**Fricas [A]** time = 0.255998, size = 255, normalized size = 1.73

$$2340\sqrt{2}(x^7 + 2x^5 + x^3) \arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{2}\sqrt{2}\sqrt{x} + 2x + 2 + 1}\right) + 2340\sqrt{2}(x^7 + 2x^5 + x^3) \arctan\left(\frac{1}{\sqrt{2}\sqrt{x} + \sqrt{-2}\sqrt{2}\sqrt{x} + 2x + 2 - 1}\right) + 585$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(1/((x^2 + 1)^3\*x^(7/2)),x, algorithm="fricas")

[Out] 
$$-1/640*(2340*\sqrt{2}*(x^7 + 2*x^5 + x^3)*\arctan(1/(\sqrt{2}*\sqrt{x}) + \sqrt{2*\sqrt{2}*\sqrt{x} + 2*x + 2} + 1)) + 2340*\sqrt{2}*(x^7 + 2*x^5 + x^3)*\arctan(1/(\sqrt{2}*\sqrt{x} + \sqrt{-2*\sqrt{2}*\sqrt{x} + 2*x + 2} - 1)) + 585*\sqrt{2}*(x^7 + 2*x^5 + x^3)*\log(2*\sqrt{2}*\sqrt{x} + 2*x + 2) - 585*\sqrt{2}*(x^7 + 2*x^5 + x^3)*\log(-2*\sqrt{2}*\sqrt{x} + 2*x + 2) - 8*(585*x^6 + 1053*x^4 + 416*x^2 - 32)*\sqrt{x})/(x^7 + 2*x^5 + x^3)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(7/2)/(x\*\*2+1)\*\*3,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.210788, size = 143, normalized size = 0.97

$$\begin{aligned} & \frac{117}{64} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) + \frac{117}{64} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}(\sqrt{2} - 2\sqrt{x})\right) \\ & - \frac{117}{128} \sqrt{2} \ln(\sqrt{2}\sqrt{x} + x + 1) + \frac{117}{128} \sqrt{2} \ln(-\sqrt{2}\sqrt{x} + x + 1) + \frac{21x^{\frac{7}{2}} + 25x^{\frac{3}{2}}}{16(x^2 + 1)^2} + \frac{2(15x^2 - 1)}{5x^{\frac{5}{2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 + 1)^3\*x^(7/2)),x, algorithm="giac")

[Out] 
$$117/64*\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2*\sqrt{x})) + 117/64*\sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2*\sqrt{x})) - 117/128*\sqrt{2}*\ln(\sqrt{2}*\sqrt{x} + x + 1) + 117/128*\sqrt{2}*\ln(-\sqrt{2}*\sqrt{x} + x + 1) + 1/16*(21*x^(7/2) + 25*x^(3/2))/(x^2 + 1)^2 + 2/5*(15*x^2 - 1)/x^(5/2)$$

$$3.337 \quad \int \frac{\sqrt{x}}{1-x^2} dx$$

**Optimal.** Leaf size=15

$$\tanh^{-1}(\sqrt{x}) - \tan^{-1}(\sqrt{x})$$

[Out] -ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]

**Rubi [A]** time = 0.0282065, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\tanh^{-1}(\sqrt{x}) - \tan^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(1 - x^2), x]

[Out] -ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]

**Rubi in Sympy [A]** time = 5.73564, size = 12, normalized size = 0.8

$$-\operatorname{atan}(\sqrt{x}) + \operatorname{atanh}(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(1/2)/(-x\*\*2+1), x)

[Out] -atan(sqrt(x)) + atanh(sqrt(x))

**Mathematica [B]** time = 0.00975884, size = 35, normalized size = 2.33

$$-\frac{1}{2} \log(1 - \sqrt{x}) + \frac{1}{2} \log(\sqrt{x} + 1) - \tan^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(1 - x^2), x]

[Out] -ArcTan[Sqrt[x]] - Log[1 - Sqrt[x]]/2 + Log[1 + Sqrt[x]]/2

**Maple [B]** time = 0.012, size = 24, normalized size = 1.6

$$-\frac{1}{2} \ln(\sqrt{x} - 1) + \frac{1}{2} \ln(1 + \sqrt{x}) - \arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(-x^2+1), x)

[Out] -1/2\*ln(x^(1/2)-1)+1/2\*ln(1+x^(1/2))-arctan(x^(1/2))

**Maxima [A]** time = 1.47876, size = 31, normalized size = 2.07

$$-\arctan(\sqrt{x}) + \frac{1}{2} \log(\sqrt{x} + 1) - \frac{1}{2} \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sqrt(x)/(x^2 - 1),x, algorithm="maxima")

[Out] -arctan(sqrt(x)) + 1/2\*log(sqrt(x) + 1) - 1/2\*log(sqrt(x) - 1)

---

**Fricas [A]** time = 0.24534, size = 31, normalized size = 2.07

$$-\arctan(\sqrt{x}) + \frac{1}{2} \log(\sqrt{x} + 1) - \frac{1}{2} \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sqrt(x)/(x^2 - 1),x, algorithm="fricas")

[Out] -arctan(sqrt(x)) + 1/2\*log(sqrt(x) + 1) - 1/2\*log(sqrt(x) - 1)

---

**Sympy [A]** time = 0.912869, size = 26, normalized size = 1.73

$$-\frac{\log(\sqrt{x} - 1)}{2} + \frac{\log(\sqrt{x} + 1)}{2} - \operatorname{atan}(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(1/2)/(-x\*\*2+1),x)

[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 - atan(sqrt(x))

---

**GIAC/XCAS [A]** time = 0.212205, size = 32, normalized size = 2.13

$$-\arctan(\sqrt{x}) + \frac{1}{2} \ln(\sqrt{x} + 1) - \frac{1}{2} \ln(|\sqrt{x} - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sqrt(x)/(x^2 - 1),x, algorithm="giac")

[Out] -arctan(sqrt(x)) + 1/2\*ln(sqrt(x) + 1) - 1/2\*ln(abs(sqrt(x) - 1))

$$3.338 \quad \int \frac{x^{2/3}}{1+x^2} dx$$

**Optimal.** Leaf size=73

$$-\frac{1}{2}\sqrt{3} \tanh^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x}}{x^{2/3}+1}\right) - \frac{1}{2} \tan^{-1}\left(\sqrt{3}-2\sqrt[3]{x}\right) + \frac{1}{2} \tan^{-1}\left(2\sqrt[3]{x}+\sqrt{3}\right) + \tan^{-1}\left(\sqrt[3]{x}\right)$$

[Out] -ArcTan[Sqrt[3] - 2\*x^(1/3)]/2 + ArcTan[Sqrt[3] + 2\*x^(1/3)]/2 + ArcTan[x^(1/3)] - (Sqrt[3]\*ArcTanh[(Sqrt[3]\*x^(1/3))/(1 + x^(2/3))])/2

**Rubi [A]** time = 0.598668, antiderivative size = 100, normalized size of antiderivative = 1.37, number of steps used = 11, number of rules used = 7, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$

$$\frac{1}{4}\sqrt{3} \log\left(x^{2/3}-\sqrt{3}\sqrt[3]{x}+1\right) - \frac{1}{4}\sqrt{3} \log\left(x^{2/3}+\sqrt{3}\sqrt[3]{x}+1\right) - \frac{1}{2} \tan^{-1}\left(\sqrt{3}-2\sqrt[3]{x}\right) + \frac{1}{2} \tan^{-1}\left(2\sqrt[3]{x}+\sqrt{3}\right) + \tan^{-1}\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

[In] Int[x^(2/3)/(1 + x^2), x]

[Out] -ArcTan[Sqrt[3] - 2\*x^(1/3)]/2 + ArcTan[Sqrt[3] + 2\*x^(1/3)]/2 + ArcTan[x^(1/3)] + (Sqrt[3]\*Log[1 - Sqrt[3]\*x^(1/3) + x^(2/3)])/4 - (Sqrt[3]\*Log[1 + Sqrt[3]\*x^(1/3) + x^(2/3)])/4

**Rubi in Sympy [A]** time = 158.3, size = 87, normalized size = 1.19

$$\frac{\sqrt{3} \log\left(x^{2/3}-\sqrt{3}\sqrt[3]{x}+1\right)}{4} - \frac{\sqrt{3} \log\left(x^{2/3}+\sqrt{3}\sqrt[3]{x}+1\right)}{4} + \operatorname{atan}\left(\sqrt[3]{x}\right) + \frac{\operatorname{atan}\left(2\sqrt[3]{x}-\sqrt{3}\right)}{2} + \frac{\operatorname{atan}\left(2\sqrt[3]{x}+\sqrt{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(2/3)/(x\*\*2+1), x)

[Out] sqrt(3)\*log(x\*\*(2/3) - sqrt(3)\*x\*\*(1/3) + 1)/4 - sqrt(3)\*log(x\*\*(2/3) + sqrt(3)\*x\*\*(1/3) + 1)/4 + atan(x\*\*(1/3)) + atan(2\*x\*\*(1/3) - sqrt(3))/2 + atan(2\*x\*\*(1/3) + sqrt(3))/2

**Mathematica [A]** time = 0.038518, size = 97, normalized size = 1.33

$$\frac{1}{4}\left(\sqrt{3} \log\left(x^{2/3}-\sqrt{3}\sqrt[3]{x}+1\right) - \sqrt{3} \log\left(x^{2/3}+\sqrt{3}\sqrt[3]{x}+1\right) - 2 \tan^{-1}\left(\sqrt{3}-2\sqrt[3]{x}\right) + 2 \tan^{-1}\left(2\sqrt[3]{x}+\sqrt{3}\right) + 4 \tan^{-1}\left(\sqrt[3]{x}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(2/3)/(1 + x^2), x]

[Out] (-2\*ArcTan[Sqrt[3] - 2\*x^(1/3)] + 2\*ArcTan[Sqrt[3] + 2\*x^(1/3)] + 4\*ArcTan[x^(1/3)] + Sqrt[3]\*Log[1 - Sqrt[3]\*x^(1/3) + x^(2/3)] - Sqrt[3]\*Log[1 + Sqrt[3]\*x^(1/3) + x^(2/3)])/4

**Maple [A]** time = 0.102, size = 69, normalized size = 1.

$$\arctan(\sqrt[3]{x}) + \frac{1}{2} \arctan(2\sqrt[3]{x} - \sqrt{3}) + \frac{1}{2} \arctan(2\sqrt[3]{x} + \sqrt{3}) \\ + \frac{\sqrt{3}}{4} \ln(1 + x^{\frac{2}{3}} - \sqrt[3]{x}\sqrt{3}) - \frac{\sqrt{3}}{4} \ln(1 + x^{\frac{2}{3}} + \sqrt[3]{x}\sqrt{3})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2/3)/(x^2+1), x)

[Out] arctan(x^(1/3))+1/2\*arctan(2\*x^(1/3)-3^(1/2))+1/2\*arctan(2\*x^(1/3)+3^(1/2))+1/4\*ln(1+x^(2/3)-x^(1/3)\*3^(1/2))\*3^(1/2)-1/4\*ln(1+x^(2/3)+x^(1/3)\*3^(1/2))\*3^(1/2)

**Maxima [A]** time = 1.51675, size = 92, normalized size = 1.26

$$-\frac{1}{4} \sqrt{3} \log(\sqrt{3}x^{\frac{1}{3}} + x^{\frac{2}{3}} + 1) + \frac{1}{4} \sqrt{3} \log(-\sqrt{3}x^{\frac{1}{3}} + x^{\frac{2}{3}} + 1) \\ + \frac{1}{2} \arctan(\sqrt{3} + 2x^{\frac{1}{3}}) + \frac{1}{2} \arctan(-\sqrt{3} + 2x^{\frac{1}{3}}) + \arctan(x^{\frac{1}{3}})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2/3)/(x^2 + 1), x, algorithm="maxima")

[Out] -1/4\*sqrt(3)\*log(sqrt(3)\*x^(1/3) + x^(2/3) + 1) + 1/4\*sqrt(3)\*log(-sqrt(3)\*x^(1/3) + x^(2/3) + 1) + 1/2\*arctan(sqrt(3) + 2\*x^(1/3)) + 1/2\*arctan(-sqrt(3) + 2\*x^(1/3)) + arctan(x^(1/3))

**Fricas [A]** time = 0.262391, size = 184, normalized size = 2.52

$$-\frac{1}{4} \sqrt{3} \log(8\sqrt{3}x^{\frac{1}{3}} + 8x^{\frac{2}{3}} + 8) + \frac{1}{4} \sqrt{3} \log(-8\sqrt{3}x^{\frac{1}{3}} + 8x^{\frac{2}{3}} + 8) \\ + \arctan(x^{\frac{1}{3}}) - \arctan\left(\frac{\sqrt{2}}{\sqrt{3}\sqrt{2} + 2\sqrt{2}\sqrt{\sqrt{3}x^{\frac{1}{3}} + x^{\frac{2}{3}} + 1} + 2\sqrt{2}x^{\frac{1}{3}}}\right) \\ - \arctan\left(-\frac{\sqrt{2}}{\sqrt{3}\sqrt{2} - 2\sqrt{2}x^{\frac{1}{3}} - \sqrt{-8\sqrt{3}x^{\frac{1}{3}} + 8x^{\frac{2}{3}} + 8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2/3)/(x^2 + 1), x, algorithm="fricas")

[Out] -1/4\*sqrt(3)\*log(8\*sqrt(3)\*x^(1/3) + 8\*x^(2/3) + 8) + 1/4\*sqrt(3)\*log(-8\*sqrt(3)\*x^(1/3) + 8\*x^(2/3) + 8) + arctan(x^(1/3)) - arctan(sqrt(2)/(sqrt(3)\*sqrt(2) + 2\*sqrt(2)\*sqrt(sqrt(3)\*x^(1/3) + x^(2/3) + 1) + 2\*sqrt(2)\*x^(1/3))) - arctan(-sqrt(2)/(sqrt(3)\*sqrt(2) - 2\*sqrt(2)\*x^(1/3) - sqrt(-8\*sqrt(3)\*x^(1/3) + 8\*x^(2/3) + 8)))

**Sympy [A]** time = 10.9957, size = 94, normalized size = 1.29

$$\frac{\sqrt{3} \log(4x^{\frac{2}{3}} - 4\sqrt{3}\sqrt[3]{x} + 4)}{4} - \frac{\sqrt{3} \log(4x^{\frac{2}{3}} + 4\sqrt{3}\sqrt[3]{x} + 4)}{4} \\ + \operatorname{atan}(\sqrt[3]{x}) + \frac{\operatorname{atan}(2\sqrt[3]{x} - \sqrt{3})}{2} + \frac{\operatorname{atan}(2\sqrt[3]{x} + \sqrt{3})}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(2/3)/(x\*\*2+1),x)

[Out] sqrt(3)\*log(4\*x\*\*(2/3) - 4\*sqrt(3)\*x\*\*(1/3) + 4)/4 - sqrt(3)\*log(4\*x\*\*(2/3) + 4\*sqrt(3)\*x\*\*(1/3) + 4)/4 + atan(x\*\*(1/3)) + atan(2\*x\*\*(1/3) - sqrt(3))/2 + atan(2\*x\*\*(1/3) + sqrt(3))/2

**GIAC/XCAS [A]** time = 0.208677, size = 92, normalized size = 1.26

$$-\frac{1}{4}\sqrt{3}\ln\left(\sqrt{3}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{1}{4}\sqrt{3}\ln\left(-\sqrt{3}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right) \\ +\frac{1}{2}\arctan\left(\sqrt{3}+2x^{\frac{1}{3}}\right)+\frac{1}{2}\arctan\left(-\sqrt{3}+2x^{\frac{1}{3}}\right)+\arctan\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2/3)/(x^2 + 1),x, algorithm="giac")

[Out] -1/4\*sqrt(3)\*ln(sqrt(3)\*x^(1/3) + x^(2/3) + 1) + 1/4\*sqrt(3)\*ln(-sqrt(3)\*x^(1/3) + x^(2/3) + 1) + 1/2\*arctan(sqrt(3) + 2\*x^(1/3)) + 1/2\*arctan(-sqrt(3) + 2\*x^(1/3)) + arctan(x^(1/3))

### 3.339 $\int x^m (a + bx^2)^5 dx$

**Optimal.** Leaf size=97

$$\frac{a^5 x^{m+1}}{m+1} + \frac{5a^4 b x^{m+3}}{m+3} + \frac{10a^3 b^2 x^{m+5}}{m+5} + \frac{10a^2 b^3 x^{m+7}}{m+7} + \frac{5ab^4 x^{m+9}}{m+9} + \frac{b^5 x^{m+11}}{m+11}$$

[Out]  $(a^5 x^{(1+m)})/(1+m) + (5 a^4 b x^{(3+m)})/(3+m) + (10 a^3 b^2 x^{(5+m)})/(5+m) + (10 a^2 b^3 x^{(7+m)})/(7+m) + (5 a b^4 x^{(9+m)})/(9+m) + (b^5 x^{(11+m)})/(11+m)$

**Rubi [A]** time = 0.11396, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{a^5 x^{m+1}}{m+1} + \frac{5a^4 b x^{m+3}}{m+3} + \frac{10a^3 b^2 x^{m+5}}{m+5} + \frac{10a^2 b^3 x^{m+7}}{m+7} + \frac{5ab^4 x^{m+9}}{m+9} + \frac{b^5 x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

[In] Int[x^m\*(a + b\*x^2)^5, x]

[Out]  $(a^5 x^{(1+m)})/(1+m) + (5 a^4 b x^{(3+m)})/(3+m) + (10 a^3 b^2 x^{(5+m)})/(5+m) + (10 a^2 b^3 x^{(7+m)})/(7+m) + (5 a b^4 x^{(9+m)})/(9+m) + (b^5 x^{(11+m)})/(11+m)$

**Rubi in Sympy [A]** time = 17.008, size = 87, normalized size = 0.9

$$\frac{a^5 x^{m+1}}{m+1} + \frac{5a^4 b x^{m+3}}{m+3} + \frac{10a^3 b^2 x^{m+5}}{m+5} + \frac{10a^2 b^3 x^{m+7}}{m+7} + \frac{5ab^4 x^{m+9}}{m+9} + \frac{b^5 x^{m+11}}{m+11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*m\*(b\*x\*\*2+a)\*\*5, x)

[Out]  $a**5*x**(m+1)/(m+1) + 5*a**4*b*x**(m+3)/(m+3) + 10*a**3*b**2*x**(m+5)/(m+5) + 10*a**2*b**3*x**(m+7)/(m+7) + 5*a*b**4*x**(m+9)/(m+9) + b**5*x**(m+11)/(m+11)$

**Mathematica [A]** time = 0.0561545, size = 87, normalized size = 0.9

$$x^m \left( \frac{a^5 x}{m+1} + \frac{5a^4 b x^3}{m+3} + \frac{10a^3 b^2 x^5}{m+5} + \frac{10a^2 b^3 x^7}{m+7} + \frac{5ab^4 x^9}{m+9} + \frac{b^5 x^{11}}{m+11} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*(a + b\*x^2)^5, x]

[Out]  $x^m*((a^5*x)/(1+m) + (5*a^4*b*x^3)/(3+m) + (10*a^3*b^2*x^5)/(5+m) + (10*a^2*b^3*x^7)/(7+m) + (5*a*b^4*x^9)/(9+m) + (b^5*x^11)/(11+m))$

**Maple [B]** time = 0.01, size = 432, normalized size = 4.5

$$x^{1+m} (b^5 m^5 x^{10} + 25 b^5 m^4 x^{10} + 5 ab^4 m^5 x^8 + 230 b^5 m^3 x^{10} + 135 ab^4 m^4 x^8 + 950 b^5 m^2 x^{10} + 10 a^2 b^3 m^5 x^6 + 1310 ab^4 m^3 x^8 + 16$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*(b*x^2+a)^5,x)`

[Out]  $x^{(1+m)} \cdot (b^5 m^5 x^{10} + 25 b^5 m^4 x^{10} + 5 a b^4 m^5 x^8 + 230 b^5 m^3 x^{10} + 135 a b^4 m^4 x^8 + 950 b^5 m^2 x^{10} + 10 a^2 b^3 m^5 x^6 + 1310 a b^4 m^3 x^8 + 1689 b^5 m x^{10} + 290 a^2 b^3 m^4 x^6 + 5610 a b^4 m^2 x^8 + 945 b^5 x^{10} + 10 a^3 b^2 m^5 x^4 + 3020 a^2 b^3 m^3 x^6 + 10205 a b^4 m x^8 + 310 a^3 b^2 m^4 x^4 + 13660 a^2 b^3 m^2 x^6 + 5775 a b^4 x^8 + 5 a^4 b m^5 x^2 + 3500 a^3 b^2 m^3 x^4 + 25770 a^2 b^3 m x^6 + 165 a^4 b m^4 x^2 + 17300 a^3 b^2 m^2 x^4 + 14850 a^2 b^3 x^6 + a^5 m^5 + 2030 a^4 b m^3 x^2 + 34890 a^3 b^2 m x^4 + 35 a^5 m^4 + 11310 a^4 b m^2 x^2 + 20790 a^3 b^2 x^4 + 470 a^5 m^3 + 26765 a^4 b m x^2 + 3010 a^5 m^2 + 17325 a^4 b x^2 + 9129 a^5 m + 10395 a^5) / ((11+m) / (9+m) / (7+m) / (5+m) / (3+m) / (1+m))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^m,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.249803, size = 495, normalized size = 5.1

$$\frac{((b^5 m^5 + 25 b^5 m^4 + 230 b^5 m^3 + 950 b^5 m^2 + 1689 b^5 m + 945 b^5) x^{11} + 5 (ab^4 m^5 + 27 ab^4 m^4 + 262 ab^4 m^3 + 1122 ab^4 m^2 + 204$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^5*x^m,x, algorithm="fricas")`

[Out]  $((b^5 m^5 + 25 b^5 m^4 + 230 b^5 m^3 + 950 b^5 m^2 + 1689 b^5 m + 945 b^5) x^{11} + 5 (a b^4 m^5 + 27 a b^4 m^4 + 262 a b^4 m^3 + 1122 a b^4 m^2 + 2041 a b^4 m + 1155 a b^4) x^9 + 10 (a^2 b^3 m^5 + 29 a^2 b^3 m^4 + 302 a^2 b^3 m^3 + 1366 a^2 b^3 m^2 + 2577 a^2 b^3 m + 1485 a^2 b^3) x^7 + 10 (a^3 b^2 m^5 + 31 a^3 b^2 m^4 + 350 a^3 b^2 m^3 + 1730 a^3 b^2 m^2 + 3489 a^3 b^2 m + 2079 a^3 b^2) x^5 + 5 (a^4 b m^5 + 33 a^4 b m^4 + 406 a^4 b m^3 + 2262 a^4 b m^2 + 5353 a^4 b m + 3465 a^4 b) x^3 + (a^5 m^5 + 35 a^5 m^4 + 470 a^5 m^3 + 3010 a^5 m^2 + 9129 a^5 m + 10395 a^5) x) x^m / (m^6 + 36 m^5 + 505 m^4 + 3480 m^3 + 12139 m^2 + 19524 m + 10395)$

**Sympy [A]** time = 13.4555, size = 1999, normalized size = 20.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*(b*x**2+a)**5,x)`

[Out]  $\text{Piecewise}((-a^{**5}/(10*x^{**10}) - 5*a^{**4}*b/(8*x^{**8}) - 5*a^{**3}*b^{**2}/(3*x^{**6}) - 5*a^{**2}*b^{**3}/(2*x^{**4}) - 5*a*b^{**4}/(2*x^{**2}) + b^{**5}*\log(x), \text{Eq}(m, -11)), (-a^{**5}/(8*x^{**8}) - 5*a^{**4}*b/(6*x^{**6}) - 5*a^{**3}*b^{**2}/(2*x^{**4}) - 5*a^{**2}*b^{**3}/x^{**2} + 5*a*b^{**4}*\log(x) + b^{**5}*x^{**2}/2, \text{Eq}(m, -9)), (-a^{**5}/(6*x^{**6}) - 5*a^{**4}*b/(4*x^{**4}) - 5*a^{**3}*b^{**2}/x^{**2} + 10*$



```

a**2*b**3*log(x) + 5*a*b**4*x**2/2 + b**5*x**4/4, Eq(m, -7)), (-a
**5/(4*x**4) - 5*a**4*b/(2*x**2) + 10*a**3*b**2*log(x) + 5*a**2*b
**3*x**2 + 5*a*b**4*x**4/4 + b**5*x**6/6, Eq(m, -5)), (-a**5/(2*x
**2) + 5*a**4*b*log(x) + 5*a**3*b**2*x**2 + 5*a**2*b**3*x**4/2 +
5*a*b**4*x**6/6 + b**5*x**8/8, Eq(m, -3)), (a**5*log(x) + 5*a**4*
b*x**2/2 + 5*a**3*b**2*x**4/2 + 5*a**2*b**3*x**6/3 + 5*a*b**4*x**
8/8 + b**5*x**10/10, Eq(m, -1)), (a**5*m**5*x*x**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 35*a**
5*m**4*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2
+ 19524*m + 10395) + 470*a**5*m**3*x*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3010*a**5*m**2
*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 195
24*m + 10395) + 9129*a**5*m*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3
480*m**3 + 12139*m**2 + 19524*m + 10395) + 10395*a**5*x*x**m/(m**
6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395
) + 5*a**4*b*m**5*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**
3 + 12139*m**2 + 19524*m + 10395) + 165*a**4*b*m**4*x**3*x**m/(m*
**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1039
5) + 2030*a**4*b*m**3*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480
*m**3 + 12139*m**2 + 19524*m + 10395) + 11310*a**4*b*m**2*x**3*x*
**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
+ 10395) + 26765*a**4*b*m*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 +
3480*m**3 + 12139*m**2 + 19524*m + 10395) + 17325*a**4*b*x**3*x**
m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m +
10395) + 10*a**3*b**2*m**5*x**5*x**m/(m**6 + 36*m**5 + 505*m**4
+ 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 310*a**3*b**2*m**4*
x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 1
9524*m + 10395) + 3500*a**3*b**2*m**3*x**5*x**m/(m**6 + 36*m**5 +
505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 17300*a**
3*b**2*m**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12
139*m**2 + 19524*m + 10395) + 34890*a**3*b**2*m*x**5*x**m/(m**6 +
36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
20790*a**3*b**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
+ 12139*m**2 + 19524*m + 10395) + 10*a**2*b**3*m**5*x**7*x**m/(m
**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 103
95) + 290*a**2*b**3*m**4*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3
480*m**3 + 12139*m**2 + 19524*m + 10395) + 3020*a**2*b**3*m**3*x*
**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 195
24*m + 10395) + 13660*a**2*b**3*m**2*x**7*x**m/(m**6 + 36*m**5 +
505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 25770*a**2
*b**3*m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*
m**2 + 19524*m + 10395) + 14850*a**2*b**3*x**7*x**m/(m**6 + 36*m*
**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 5*a*b
**4*m**5*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139
*m**2 + 19524*m + 10395) + 135*a*b**4*m**4*x**9*x**m/(m**6 + 36*m
**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1310
*a*b**4*m**3*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1
2139*m**2 + 19524*m + 10395) + 5610*a*b**4*m**2*x**9*x**m/(m**6 +
36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
10205*a*b**4*m*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
+ 12139*m**2 + 19524*m + 10395) + 5775*a*b**4*x**9*x**m/(m**6 + 3
6*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + b
**5*m**5*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1213
9*m**2 + 19524*m + 10395) + 25*b**5*m**4*x**11*x**m/(m**6 + 36*m*
**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 230*b
**5*m**3*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1213
9*m**2 + 19524*m + 10395) + 950*b**5*m**2*x**11*x**m/(m**6 + 36*m
**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1689
*b**5*m*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139
*m**2 + 19524*m + 10395) + 945*b**5*x**11*x**m/(m**6 + 36*m**5 +
505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395), True))

```

---

**GIAC/XCAS [A]** time = 0.217624, size = 826, normalized size = 8.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^5\*x^m,x, algorithm="giac")

[Out]  $(b^5 m^5 x^{11} e^{(m \ln(x))} + 25 b^5 m^4 x^{11} e^{(m \ln(x))} + 5 a b^4 m^5 x^9 e^{(m \ln(x))} + 230 b^5 m^3 x^{11} e^{(m \ln(x))} + 135 a b^4 m^4 x^9 e^{(m \ln(x))} + 950 b^5 m^2 x^{11} e^{(m \ln(x))} + 10 a^2 b^3 m^5 x^7 e^{(m \ln(x))} + 1310 a b^4 m^3 x^9 e^{(m \ln(x))} + 1689 b^5 m^2 x^{11} e^{(m \ln(x))} + 290 a^2 b^3 m^4 x^7 e^{(m \ln(x))} + 5610 a b^4 m^2 x^9 e^{(m \ln(x))} + 945 b^5 x^{11} e^{(m \ln(x))} + 10 a^3 b^2 m^5 x^5 e^{(m \ln(x))} + 3020 a^2 b^3 m^3 x^7 e^{(m \ln(x))} + 10205 a b^4 m^2 x^9 e^{(m \ln(x))} + 310 a^3 b^2 m^4 x^5 e^{(m \ln(x))} + 13660 a^2 b^3 m^2 x^7 e^{(m \ln(x))} + 5775 a b^4 x^9 e^{(m \ln(x))} + 5 a^4 b m^5 x^3 e^{(m \ln(x))} + 3500 a^3 b^2 m^3 x^5 e^{(m \ln(x))} + 25770 a^2 b^3 m^2 x^7 e^{(m \ln(x))} + 165 a^4 b m^4 x^3 e^{(m \ln(x))} + 17300 a^3 b^2 m^2 x^5 e^{(m \ln(x))} + 14850 a^2 b^3 x^7 e^{(m \ln(x))} + a^5 m^5 x e^{(m \ln(x))} + 2030 a^4 b m^3 x^3 e^{(m \ln(x))} + 34890 a^3 b^2 m^2 x^5 e^{(m \ln(x))} + 35 a^5 m^4 x e^{(m \ln(x))} + 11310 a^4 b m^2 x^3 e^{(m \ln(x))} + 20790 a^3 b^2 x^5 e^{(m \ln(x))} + 470 a^5 m^3 x e^{(m \ln(x))} + 26765 a^4 b m^2 x^3 e^{(m \ln(x))} + 3010 a^5 m^2 x e^{(m \ln(x))} + 17325 a^4 b x^3 e^{(m \ln(x))} + 9129 a^5 m^2 x e^{(m \ln(x))} + 10395 a^5 x e^{(m \ln(x))}) / (m^6 + 36 m^5 + 505 m^4 + 3480 m^3 + 12139 m^2 + 19524 m + 10395)$

### 3.340 $\int x^m (a + bx^2)^4 dx$

**Optimal.** Leaf size=79

$$\frac{a^4 x^{m+1}}{m+1} + \frac{4a^3 b x^{m+3}}{m+3} + \frac{6a^2 b^2 x^{m+5}}{m+5} + \frac{4ab^3 x^{m+7}}{m+7} + \frac{b^4 x^{m+9}}{m+9}$$

[Out]  $(a^4 x^{m+1})/(m+1) + (4a^3 b x^{m+3})/(m+3) + (6a^2 b^2 x^{m+5})/(m+5) + (4ab^3 x^{m+7})/(m+7) + (b^4 x^{m+9})/(m+9)$

**Rubi [A]** time = 0.0862479, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{a^4 x^{m+1}}{m+1} + \frac{4a^3 b x^{m+3}}{m+3} + \frac{6a^2 b^2 x^{m+5}}{m+5} + \frac{4ab^3 x^{m+7}}{m+7} + \frac{b^4 x^{m+9}}{m+9}$$

Antiderivative was successfully verified.

[In] Int[x^m\*(a + b\*x^2)^4, x]

[Out]  $(a^4 x^{m+1})/(m+1) + (4a^3 b x^{m+3})/(m+3) + (6a^2 b^2 x^{m+5})/(m+5) + (4ab^3 x^{m+7})/(m+7) + (b^4 x^{m+9})/(m+9)$

**Rubi in Sympy [A]** time = 13.8013, size = 70, normalized size = 0.89

$$\frac{a^4 x^{m+1}}{m+1} + \frac{4a^3 b x^{m+3}}{m+3} + \frac{6a^2 b^2 x^{m+5}}{m+5} + \frac{4ab^3 x^{m+7}}{m+7} + \frac{b^4 x^{m+9}}{m+9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*m\*(b\*x\*\*2+a)\*\*4, x)

[Out]  $a^4 x^{m+1}/(m+1) + 4a^3 b x^{m+3}/(m+3) + 6a^2 b^2 x^{m+5}/(m+5) + 4ab^3 x^{m+7}/(m+7) + b^4 x^{m+9}/(m+9)$

**Mathematica [A]** time = 0.0421853, size = 71, normalized size = 0.9

$$x^m \left( \frac{a^4 x}{m+1} + \frac{4a^3 b x^3}{m+3} + \frac{6a^2 b^2 x^5}{m+5} + \frac{4ab^3 x^7}{m+7} + \frac{b^4 x^9}{m+9} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*(a + b\*x^2)^4, x]

[Out]  $x^m \left( \frac{a^4 x}{m+1} + \frac{4a^3 b x^3}{m+3} + \frac{6a^2 b^2 x^5}{m+5} + \frac{4ab^3 x^7}{m+7} + \frac{b^4 x^9}{m+9} \right)$

**Maple [B]** time = 0.009, size = 291, normalized size = 3.7

$$x^{1+m} (b^4 m^4 x^8 + 16 b^4 m^3 x^8 + 4 ab^3 m^4 x^6 + 86 b^4 m^2 x^8 + 72 ab^3 m^3 x^6 + 176 b^4 m x^8 + 6 a^2 b^2 m^4 x^4 + 416 ab^3 m^2 x^6 + 105 b^4 x^8 + \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*(b*x^2+a)^4,x)`

[Out]  $x^{(1+m)} \cdot (b^4 m^4 x^8 + 16 b^4 m^3 x^8 + 4 a b^3 m^4 x^6 + 86 b^4 m^2 x^8 + 72 a b^3 m^3 x^6 + 176 b^4 m x^8 + 6 a^2 b^2 m^4 x^4 + 416 a b^3 m^2 x^6 + 105 b^4 x^8 + 120 a^2 b^2 m^3 x^4 + 888 a b^3 m x^6 + 4 a^3 b m^4 x^2 + 780 a^2 b^2 m^2 x^4 + 540 a b^3 x^6 + 88 a^3 b m^3 x^2 + 1800 a^2 b^2 m x^4 + a^4 m^4 + 656 a^3 b m^2 x^2 + 1134 a^2 b^2 x^4 + 24 a^4 m^3 + 1832 a^3 b m x^2 + 206 a^4 m^2 + 1260 a^3 b x^2 + 744 a^4 m + 945 a^4) / (9+m) / (7+m) / (5+m) / (3+m) / (1+m)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^4*x^m,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.249321, size = 339, normalized size = 4.29

$$\frac{((b^4 m^4 + 16 b^4 m^3 + 86 b^4 m^2 + 176 b^4 m + 105 b^4) x^9 + 4 (a b^3 m^4 + 18 a b^3 m^3 + 104 a b^3 m^2 + 222 a b^3 m + 135 a b^3) x^7 + 6 (a^2 b^2 m^4 + 18 a^2 b^2 m^3 + 104 a^2 b^2 m^2 + 222 a^2 b^2 m + 135 a^2 b^2) x^5 + 4 (a^3 b m^4 + 22 a^3 b m^3 + 164 a^3 b m^2 + 458 a^3 b m + 315 a^3 b) x^3 + (a^4 m^4 + 24 a^4 m^3 + 206 a^4 m^2 + 744 a^4 m + 945 a^4) x) x^m}{(m^5 + 25 m^4 + 230 m^3 + 950 m^2 + 1689 m + 945)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^4*x^m,x, algorithm="fricas")`

[Out]  $((b^4 m^4 + 16 b^4 m^3 + 86 b^4 m^2 + 176 b^4 m + 105 b^4) x^9 + 4 (a b^3 m^4 + 18 a b^3 m^3 + 104 a b^3 m^2 + 222 a b^3 m + 135 a b^3) x^7 + 6 (a^2 b^2 m^4 + 18 a^2 b^2 m^3 + 104 a^2 b^2 m^2 + 222 a^2 b^2 m + 135 a^2 b^2) x^5 + 4 (a^3 b m^4 + 22 a^3 b m^3 + 164 a^3 b m^2 + 458 a^3 b m + 315 a^3 b) x^3 + (a^4 m^4 + 24 a^4 m^3 + 206 a^4 m^2 + 744 a^4 m + 945 a^4) x) x^m / (m^5 + 25 m^4 + 230 m^3 + 950 m^2 + 1689 m + 945)$

**Sympy [A]** time = 8.15085, size = 1221, normalized size = 15.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*(b*x**2+a)**4,x)`

[Out]  $\text{Piecewise}((-a^{**4}/(8*x^{**8}) - 2*a^{**3}*b/(3*x^{**6}) - 3*a^{**2}*b^{**2}/(2*x^{**4}) - 2*a*b^{**3}/x^{**2} + b^{**4}*\log(x), \text{Eq}(m, -9)), (-a^{**4}/(6*x^{**6}) - a^{**3}*b/x^{**4} - 3*a^{**2}*b^{**2}/x^{**2} + 4*a*b^{**3}*\log(x) + b^{**4}*x^{**2}/2, \text{Eq}(m, -7)), (-a^{**4}/(4*x^{**4}) - 2*a^{**3}*b/x^{**2} + 6*a^{**2}*b^{**2}*\log(x) + 2*a*b^{**3}*x^{**2} + b^{**4}*x^{**4}/4, \text{Eq}(m, -5)), (-a^{**4}/(2*x^{**2}) + 4*a^{**3}*b*\log(x) + 3*a^{**2}*b^{**2}*x^{**2} + a*b^{**3}*x^{**4} + b^{**4}*x^{**6}/6, \text{Eq}(m, -3)), (a^{**4}*\log(x) + 2*a^{**3}*b*x^{**2} + 3*a^{**2}*b^{**2}*x^{**4}/2 + 2*a*b^{**3}*x^{**6}/3 + b^{**4}*x^{**8}/8, \text{Eq}(m, -1)), (a^{**4}*m^{**4}*x*x**m/(m^{**5} + 25*m^{**4} + 230*m^{**3} + 950*m^{**2} + 1689*m + 945) + 24*a^{**4}*m^{**3}*x*x**m/(m^{**5} + 25*m^{**4} + 230*m^{**3} + 950*m^{**2} + 1689*m + 945) + 206*a^{**4}*m^{**2}*x*x**m/(m^{**5} + 25*m^{**4} + 230*m^{**3} + 950*m^{**2} + 1689*m + 945) + 744*a^{**4}*m*x*x**m/(m^{**5} + 25*m^{**4} + 230*m^{**3} + 950*m^{**2} + 1689$

```

*m + 945) + 945*a**4*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
+ 1689*m + 945) + 4*a**3*b*m**4*x**3*x**m/(m**5 + 25*m**4 + 230*
m**3 + 950*m**2 + 1689*m + 945) + 88*a**3*b*m**3*x**3*x**m/(m**5
+ 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 656*a**3*b*m**2
*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945)
+ 1832*a**3*b*m*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
1689*m + 945) + 1260*a**3*b*x**3*x**m/(m**5 + 25*m**4 + 230*m**3
+ 950*m**2 + 1689*m + 945) + 6*a**2*b**2*m**4*x**5*x**m/(m**5 +
25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 120*a**2*b**2*m**
3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945)
+ 780*a**2*b**2*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*
m**2 + 1689*m + 945) + 1800*a**2*b**2*m*x**5*x**m/(m**5 + 25*m**4
+ 230*m**3 + 950*m**2 + 1689*m + 945) + 1134*a**2*b**2*x**5*x**m
/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 4*a*b**3
*m**4*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m +
945) + 72*a*b**3*m**3*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*
m**2 + 1689*m + 945) + 416*a*b**3*m**2*x**7*x**m/(m**5 + 25*m**4
+ 230*m**3 + 950*m**2 + 1689*m + 945) + 888*a*b**3*m*x**7*x**m/(m
**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 540*a*b**3*
x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
b**4*m**4*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689
*m + 945) + 16*b**4*m**3*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 9
50*m**2 + 1689*m + 945) + 86*b**4*m**2*x**9*x**m/(m**5 + 25*m**4
+ 230*m**3 + 950*m**2 + 1689*m + 945) + 176*b**4*m*x**9*x**m/(m**
5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 105*b**4*x**9
*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945), True
))

```

---

**GIAC/XCAS [A]** time = 0.214517, size = 560, normalized size = 7.09

$$b^4 m^4 x^9 e^{(m \ln(x))} + 16 b^4 m^3 x^9 e^{(m \ln(x))} + 4 a b^3 m^4 x^7 e^{(m \ln(x))} + 86 b^4 m^2 x^9 e^{(m \ln(x))} + 72 a b^3 m^3 x^7 e^{(m \ln(x))} + 176 b^4 m x^9 e^{(m \ln(x))} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^4*x^m,x, algorithm="giac")
```

```

[Out] (b^4*m^4*x^9*e^(m*ln(x)) + 16*b^4*m^3*x^9*e^(m*ln(x)) + 4*a*b^3*m
^4*x^7*e^(m*ln(x)) + 86*b^4*m^2*x^9*e^(m*ln(x)) + 72*a*b^3*m^3*x^
7*e^(m*ln(x)) + 176*b^4*m*x^9*e^(m*ln(x)) + 6*a^2*b^2*m^4*x^5*e^(
m*ln(x)) + 416*a*b^3*m^2*x^7*e^(m*ln(x)) + 105*b^4*x^9*e^(m*ln(x)
) + 120*a^2*b^2*m^3*x^5*e^(m*ln(x)) + 888*a*b^3*m*x^7*e^(m*ln(x))
+ 4*a^3*b*m^4*x^3*e^(m*ln(x)) + 780*a^2*b^2*m^2*x^5*e^(m*ln(x))
+ 540*a*b^3*x^7*e^(m*ln(x)) + 88*a^3*b*m^3*x^3*e^(m*ln(x)) + 1800
*a^2*b^2*m*x^5*e^(m*ln(x)) + a^4*m^4*x*e^(m*ln(x)) + 656*a^3*b*m^
2*x^3*e^(m*ln(x)) + 1134*a^2*b^2*x^5*e^(m*ln(x)) + 24*a^4*m^3*x^e
^(m*ln(x)) + 1832*a^3*b*m*x^3*e^(m*ln(x)) + 206*a^4*m^2*x^e^(m*ln
(x)) + 1260*a^3*b*x^3*e^(m*ln(x)) + 744*a^4*m*x^e^(m*ln(x)) + 945
*a^4*x^e^(m*ln(x)))/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m +
945)

```

### 3.341 $\int x^m (a + bx^2)^3 dx$

**Optimal.** Leaf size=61

$$\frac{a^3 x^{m+1}}{m+1} + \frac{3a^2 b x^{m+3}}{m+3} + \frac{3ab^2 x^{m+5}}{m+5} + \frac{b^3 x^{m+7}}{m+7}$$

[Out]  $(a^3 x^{m+1})/(m+1) + (3 a^2 b x^{m+3})/(m+3) + (3 a b^2 x^{m+5})/(m+5) + (b^3 x^{m+7})/(m+7)$

**Rubi [A]** time = 0.0631842, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{a^3 x^{m+1}}{m+1} + \frac{3a^2 b x^{m+3}}{m+3} + \frac{3ab^2 x^{m+5}}{m+5} + \frac{b^3 x^{m+7}}{m+7}$$

Antiderivative was successfully verified.

[In] Int[x^m\*(a + b\*x^2)^3, x]

[Out]  $(a^3 x^{m+1})/(m+1) + (3 a^2 b x^{m+3})/(m+3) + (3 a b^2 x^{m+5})/(m+5) + (b^3 x^{m+7})/(m+7)$

**Rubi in Sympy [A]** time = 10.6474, size = 53, normalized size = 0.87

$$\frac{a^3 x^{m+1}}{m+1} + \frac{3a^2 b x^{m+3}}{m+3} + \frac{3ab^2 x^{m+5}}{m+5} + \frac{b^3 x^{m+7}}{m+7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*m\*(b\*x\*\*2+a)\*\*3, x)

[Out]  $a**3*x**(m+1)/(m+1) + 3*a**2*b*x**(m+3)/(m+3) + 3*a*b**2*x**(m+5)/(m+5) + b**3*x**(m+7)/(m+7)$

**Mathematica [A]** time = 0.0411165, size = 55, normalized size = 0.9

$$x^m \left( \frac{a^3 x}{m+1} + \frac{3a^2 b x^3}{m+3} + \frac{3ab^2 x^5}{m+5} + \frac{b^3 x^7}{m+7} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*(a + b\*x^2)^3, x]

[Out]  $x^m*((a^3*x)/(m+1) + (3*a^2*b*x^3)/(m+3) + (3*a*b^2*x^5)/(m+5) + (b^3*x^7)/(m+7))$

**Maple [B]** time = 0.008, size = 178, normalized size = 2.9

$$\frac{x^{1+m} (b^3 m^3 x^6 + 9 b^3 m^2 x^6 + 3 a b^2 m^3 x^4 + 23 b^3 m x^6 + 33 a b^2 m^2 x^4 + 15 b^3 x^6 + 3 a^2 b m^3 x^2 + 93 a b^2 m x^4 + 39 a^2 b m^2 x^2 + 63 a b^2 m x^2 + 3 a^3 x^2)}{(7+m)(5+m)(3+m)(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*(b\*x^2+a)^3, x)

[Out]  $x^{(1+m)} \cdot (b^3 m^3 x^6 + 9 b^3 m^2 x^6 + 3 a b^2 m^3 x^4 + 23 b^3 m x^6 + 3 a^3 m^2 x^4 + 15 b^3 x^6 + 3 a^2 b m^3 x^2 + 93 a b^2 m x^4 + 39 a^2 b m^2 x^2 + 63 a b^2 x^4 + a^3 m^3 + 141 a^2 b m x^2 + 15 a^3 m^2 + 105 a^2 b x^2 + 71 a^3 m + 105 a^3) / (7+m) / (5+m) / (3+m) / (1+m)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3\*x^m,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.246305, size = 212, normalized size = 3.48

$$\frac{(b^3 m^3 + 9 b^3 m^2 + 23 b^3 m + 15 b^3) x^7 + 3 (a b^2 m^3 + 11 a b^2 m^2 + 31 a b^2 m + 21 a b^2) x^5 + 3 (a^2 b m^3 + 13 a^2 b m^2 + 47 a^2 b m + 35 a^2 b) x^3 + (a^3 m^3 + 15 a^3 m^2 + 71 a^3 m + 105 a^3) x}{m^4 + 16 m^3 + 86 m^2 + 176 m + 105}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^3\*x^m,x, algorithm="fricas")

[Out]  $((b^3 m^3 + 9 b^3 m^2 + 23 b^3 m + 15 b^3) x^7 + 3 (a b^2 m^3 + 11 a b^2 m^2 + 31 a b^2 m + 21 a b^2) x^5 + 3 (a^2 b m^3 + 13 a^2 b m^2 + 47 a^2 b m + 35 a^2 b) x^3 + (a^3 m^3 + 15 a^3 m^2 + 71 a^3 m + 105 a^3) x) / (m^4 + 16 m^3 + 86 m^2 + 176 m + 105)$

**Sympy [A]** time = 4.91485, size = 683, normalized size = 11.2

$$\left\{ \begin{array}{l} -\frac{a^3}{6x^6} - \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} + b^3 \log(x) \\ -\frac{a^3}{4x^4} - \frac{3a^2b}{2x^2} + 3ab^2 \log(x) + \frac{b^3x^2}{2} \\ -\frac{a^3}{2x^2} + 3a^2b \log(x) + \frac{3ab^2x^2}{2} + \frac{b^3x^4}{4} \\ a^3 \log(x) + \frac{3a^2bx^2}{2} + \frac{3ab^2x^4}{4} + \frac{b^3x^6}{6} \end{array} \right. / \frac{a^3 m^3 x x^m}{m^4 + 16 m^3 + 86 m^2 + 176 m + 105} + \frac{15 a^3 m^2 x x^m}{m^4 + 16 m^3 + 86 m^2 + 176 m + 105} + \frac{71 a^3 m x x^m}{m^4 + 16 m^3 + 86 m^2 + 176 m + 105} + \frac{105 a^3 x x^m}{m^4 + 16 m^3 + 86 m^2 + 176 m + 105} + \frac{3 a^2 b m^3 x^3 x^m}{m^4 + 16 m^3 + 86 m^2 + 176 m + 105}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*(b\*x\*\*2+a)\*\*3,x)

[Out] Piecewise((-a\*\*3/(6\*x\*\*6) - 3\*a\*\*2\*b/(4\*x\*\*4) - 3\*a\*b\*\*2/(2\*x\*\*2) + b\*\*3\*log(x), Eq(m, -7)), (-a\*\*3/(4\*x\*\*4) - 3\*a\*\*2\*b/(2\*x\*\*2) + 3\*a\*b\*\*2\*log(x) + b\*\*3\*x\*\*2/2, Eq(m, -5)), (-a\*\*3/(2\*x\*\*2) + 3\*a\*\*2\*b\*log(x) + 3\*a\*b\*\*2\*x\*\*2/2 + b\*\*3\*x\*\*4/4, Eq(m, -3)), (a\*\*3\*log(x) + 3\*a\*\*2\*b\*x\*\*2/2 + 3\*a\*b\*\*2\*x\*\*4/4 + b\*\*3\*x\*\*6/6, Eq(m, -1)), (a\*\*3\*m\*\*3\*x\*x\*\*m/(m\*\*4 + 16\*m\*\*3 + 86\*m\*\*2 + 176\*m + 105) + 15\*a\*\*3\*m\*\*2\*x\*x\*\*m/(m\*\*4 + 16\*m\*\*3 + 86\*m\*\*2 + 176\*m + 105) + 71\*a\*\*3\*m\*x\*x\*\*m/(m\*\*4 + 16\*m\*\*3 + 86\*m\*\*2 + 176\*m + 105) + 105\*a\*\*3\*x\*x\*\*m/(m\*\*4 + 16\*m\*\*3 + 86\*m\*\*2 + 176\*m + 105) + 3\*a\*\*2\*b\*m\*\*3\*x\*\*3\*x\*\*m/(m\*\*4 + 16\*m\*\*3 + 86\*m\*\*2 + 176\*m + 105) + 39\*a\*\*2\*b\*m\*\*2\*x\*\*3\*x\*\*m/(m\*\*4 + 16\*m\*\*3 + 86\*m\*\*2 + 176\*m + 105) + 141\*a\*\*2\*b\*m\*x\*\*3\*x\*\*m/(m\*\*4 + 16\*m\*\*3 + 86\*m\*\*2 + 176\*m + 105) + 105\*a\*\*2\*b\*x\*\*3\*x\*\*m/(m\*\*4 + 16\*m\*\*3 + 86\*m\*\*2 + 176\*m + 105) + 3\*a\*b\*\*2\*m\*\*3\*x\*\*5\*x\*\*m/(m\*\*4 + 16\*m\*\*3 + 86\*m\*\*2 + 176\*m + 105) + 33\*a\*\*2\*m\*\*2\*x\*\*5\*x\*\*m/(m\*\*4 + 16\*m\*\*3 + 86\*m\*\*2 + 176\*m + 105) + 93\*a\*b\*\*2\*m\*\*2\*x\*\*5\*x\*\*m/(m\*\*4 + 16\*m\*\*3 + 86\*m\*\*2 + 176\*m + 105) + 63\*a\*b\*\*2\*x\*\*5\*x\*\*m/(m\*\*4 + 16\*m\*\*3 + 86\*m\*\*2 + 176\*m + 105) + b\*\*3\*m\*\*3\*x\*\*7\*x\*\*m/(m\*\*4 + 16\*m\*\*3 + 86\*m\*\*2 + 176\*m + 105) + 9\*b\*\*3\*

```
m**2*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 23*b**3
*m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*b**3*x
**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105), True))
```

**GIAC/XCAS [A]** time = 0.211535, size = 346, normalized size = 5.67

$$b^3 m^3 x^7 e^{(m \ln(x))} + 9 b^3 m^2 x^7 e^{(m \ln(x))} + 3 a b^2 m^3 x^5 e^{(m \ln(x))} + 23 b^3 m x^7 e^{(m \ln(x))} + 33 a b^2 m^2 x^5 e^{(m \ln(x))} + 15 b^3 x^7 e^{(m \ln(x))} + 3 a^2 b m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^3*x^m,x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^7*e^(m*ln(x)) + 9*b^3*m^2*x^7*e^(m*ln(x)) + 3*a*b^2*m^
3*x^5*e^(m*ln(x)) + 23*b^3*m*x^7*e^(m*ln(x)) + 33*a*b^2*m^2*x^5*e
^(m*ln(x)) + 15*b^3*x^7*e^(m*ln(x)) + 3*a^2*b*m^3*x^3*e^(m*ln(x))
+ 93*a*b^2*m*x^5*e^(m*ln(x)) + 39*a^2*b*m^2*x^3*e^(m*ln(x)) + 63
*a*b^2*x^5*e^(m*ln(x)) + a^3*m^3*x*e^(m*ln(x)) + 141*a^2*b*m*x^3*
e^(m*ln(x)) + 15*a^3*m^2*x*e^(m*ln(x)) + 105*a^2*b*x^3*e^(m*ln(x))
) + 71*a^3*m*x*e^(m*ln(x)) + 105*a^3*x*e^(m*ln(x)))/(m^4 + 16*m^3
+ 86*m^2 + 176*m + 105)
```



### 3.342 $\int x^m (a + bx^2)^2 dx$

**Optimal.** Leaf size=43

$$\frac{a^2 x^{m+1}}{m+1} + \frac{2abx^{m+3}}{m+3} + \frac{b^2 x^{m+5}}{m+5}$$

[Out]  $(a^2 x^{(1+m)})/(1+m) + (2*a*b*x^{(3+m)})/(3+m) + (b^2*x^{(5+m)})/(5+m)$

**Rubi [A]** time = 0.0461377, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{a^2 x^{m+1}}{m+1} + \frac{2abx^{m+3}}{m+3} + \frac{b^2 x^{m+5}}{m+5}$$

Antiderivative was successfully verified.

[In] Int[x^m\*(a + b\*x^2)^2, x]

[Out]  $(a^2*x^{(1+m)})/(1+m) + (2*a*b*x^{(3+m)})/(3+m) + (b^2*x^{(5+m)})/(5+m)$

**Rubi in Sympy [A]** time = 7.77866, size = 36, normalized size = 0.84

$$\frac{a^2 x^{m+1}}{m+1} + \frac{2abx^{m+3}}{m+3} + \frac{b^2 x^{m+5}}{m+5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*m\*(b\*x\*\*2+a)\*\*2, x)

[Out]  $a**2*x**(m+1)/(m+1) + 2*a*b*x**(m+3)/(m+3) + b**2*x**(m+5)/(m+5)$

**Mathematica [A]** time = 0.0286375, size = 39, normalized size = 0.91

$$x^m \left( \frac{a^2 x}{m+1} + \frac{2abx^3}{m+3} + \frac{b^2 x^5}{m+5} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*(a + b\*x^2)^2, x]

[Out]  $x^m*((a^2*x)/(1+m) + (2*a*b*x^3)/(3+m) + (b^2*x^5)/(5+m))$

**Maple [B]** time = 0.007, size = 93, normalized size = 2.2

$$\frac{x^{1+m} (b^2 m^2 x^4 + 4 b^2 m x^4 + 2 a b m^2 x^2 + 3 b^2 x^4 + 12 a b m x^2 + a^2 m^2 + 10 a b x^2 + 8 a^2 m + 15 a^2)}{(5+m)(3+m)(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*(b\*x^2+a)^2, x)

[Out]  $x^{(1+m)} \cdot (b^2 m^2 x^4 + 4 b^2 m x^4 + 2 a b m^2 x^2 + 3 b^2 x^4 + 12 a b m x^2 + a^2 m^2 + 10 a b x^2 + 8 a^2 m + 15 a^2) / (5+m) / (3+m) / (1+m)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2*x^m,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.248165, size = 115, normalized size = 2.67

$$\frac{((b^2 m^2 + 4 b^2 m + 3 b^2) x^5 + 2 (a b m^2 + 6 a b m + 5 a b) x^3 + (a^2 m^2 + 8 a^2 m + 15 a^2) x) x^m}{m^3 + 9 m^2 + 23 m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^2*x^m,x, algorithm="fricas")`

[Out]  $((b^2 m^2 + 4 b^2 m + 3 b^2) x^5 + 2 (a b m^2 + 6 a b m + 5 a b) x^3 + (a^2 m^2 + 8 a^2 m + 15 a^2) x) x^m / (m^3 + 9 m^2 + 23 m + 15)$

**Sympy [A]** time = 2.59933, size = 306, normalized size = 7.12

$$\left\{ \begin{array}{l} -\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \log(x) \\ -\frac{a^2}{2x^2} + 2ab \log(x) + \frac{b^2 x^2}{2} \\ a^2 \log(x) + abx^2 + \frac{b^2 x^4}{4} \\ \frac{a^2 m^2 x x^m}{m^3 + 9m^2 + 23m + 15} + \frac{8a^2 m x x^m}{m^3 + 9m^2 + 23m + 15} + \frac{15a^2 x x^m}{m^3 + 9m^2 + 23m + 15} + \frac{2abm^2 x^3 x^m}{m^3 + 9m^2 + 23m + 15} + \frac{12abmx^3 x^m}{m^3 + 9m^2 + 23m + 15} + \frac{10abx^3 x^m}{m^3 + 9m^2 + 23m + 15} + \frac{b^2 m^2 x^5 x^m}{m^3 + 9m^2 + 23m + 15} + \dots \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*(b*x**2+a)**2,x)`

[Out] `Piecewise((-a**2/(4*x**4) - a*b/x**2 + b**2*log(x), Eq(m, -5)), (-a**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2, Eq(m, -3)), (a**2*log(x) + a*b*x**2 + b**2*x**4/4, Eq(m, -1)), (a**2*m**2*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 8*a**2*m*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 15*a**2*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 2*a*b*m**2*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 12*a*b*m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 10*a*b*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + b**2*m**2*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 4*b**2*m*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 3*b**2*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15), True))`

**GIAC/XCAS [A]** time = 0.210923, size = 182, normalized size = 4.23

$$\frac{b^2 m^2 x^5 e^{(m \ln(x))} + 4 b^2 m x^5 e^{(m \ln(x))} + 2 a b m^2 x^3 e^{(m \ln(x))} + 3 b^2 x^5 e^{(m \ln(x))} + 12 a b m x^3 e^{(m \ln(x))} + a^2 m^2 x e^{(m \ln(x))} + 10 a b x^3 e^{(m \ln(x))}}{m^3 + 9 m^2 + 23 m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2*x^m,x, algorithm="giac")
```

```
[Out] (b^2*m^2*x^5*e^(m*ln(x)) + 4*b^2*m*x^5*e^(m*ln(x)) + 2*a*b*m^2*x^3*e^(m*ln(x)) + 3*b^2*x^5*e^(m*ln(x)) + 12*a*b*m*x^3*e^(m*ln(x)) + a^2*m^2*x*e^(m*ln(x)) + 10*a*b*x^3*e^(m*ln(x)) + 8*a^2*m*x*e^(m*ln(x)) + 15*a^2*x*e^(m*ln(x)))/(m^3 + 9*m^2 + 23*m + 15)
```

### 3.343 $\int x^m (a + bx^2) dx$

**Optimal.** Leaf size=25

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{m+3}}{m+3}$$

[Out]  $(a*x^{(1+m)})/(1+m) + (b*x^{(3+m)})/(3+m)$

**Rubi [A]** time = 0.0230493, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{m+3}}{m+3}$$

Antiderivative was successfully verified.

[In] Int[x^m\*(a + b\*x^2), x]

[Out]  $(a*x^{(1+m)})/(1+m) + (b*x^{(3+m)})/(3+m)$

**Rubi in Sympy [A]** time = 4.0615, size = 19, normalized size = 0.76

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{m+3}}{m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*m\*(b\*x\*\*2+a), x)

[Out]  $a*x^{(m+1)}/(m+1) + b*x^{(m+3)}/(m+3)$

**Mathematica [A]** time = 0.0242669, size = 23, normalized size = 0.92

$$x^m \left( \frac{ax}{m+1} + \frac{bx^3}{m+3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*(a + b\*x^2), x]

[Out]  $x^m*((a*x)/(1+m) + (b*x^3)/(3+m))$

**Maple [A]** time = 0.003, size = 35, normalized size = 1.4

$$\frac{x^{1+m} (bmx^2 + bx^2 + am + 3a)}{(3+m)(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*(b\*x^2+a), x)

[Out]  $x^{(1+m)}*(b*m*x^2+b*x^2+a*m+3*a)/(3+m)/(1+m)$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^m,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.244869, size = 45, normalized size = 1.8

$$\frac{((bm + b)x^3 + (am + 3a)x)x^m}{m^2 + 4m + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^m,x, algorithm="fricas")

[Out] ((b\*m + b)\*x^3 + (a\*m + 3\*a)\*x)\*x^m/(m^2 + 4\*m + 3)

---

**Sympy [A]** time = 1.10704, size = 94, normalized size = 3.76

$$\begin{cases} -\frac{a}{2x^2} + b \log(x) & \text{for } m = -3 \\ a \log(x) + \frac{bx^2}{2} & \text{for } m = -1 \\ \frac{amxx^m}{m^2+4m+3} + \frac{3axx^m}{m^2+4m+3} + \frac{bmx^3x^m}{m^2+4m+3} + \frac{bx^3x^m}{m^2+4m+3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*(b\*x\*\*2+a), x)

[Out] Piecewise((-a/(2\*x\*\*2) + b\*log(x), Eq(m, -3)), (a\*log(x) + b\*x\*\*2/2, Eq(m, -1)), (a\*m\*x\*x\*\*m/(m\*\*2 + 4\*m + 3) + 3\*a\*x\*x\*\*m/(m\*\*2 + 4\*m + 3) + b\*m\*x\*\*3\*x\*\*m/(m\*\*2 + 4\*m + 3) + b\*x\*\*3\*x\*\*m/(m\*\*2 + 4\*m + 3), True))

---

**GIAC/XCAS [A]** time = 0.205902, size = 69, normalized size = 2.76

$$\frac{bmx^3e^{m\ln(x)} + bx^3e^{m\ln(x)} + amxe^{m\ln(x)} + 3axe^{m\ln(x)}}{m^2 + 4m + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)\*x^m,x, algorithm="giac")

[Out] (b\*m\*x^3\*e^(m\*ln(x)) + b\*x^3\*e^(m\*ln(x)) + a\*m\*x\*e^(m\*ln(x)) + 3\*a\*x\*e^(m\*ln(x)))/(m^2 + 4\*m + 3)

$$3.344 \quad \int \frac{x^m}{a+bx^2} dx$$

**Optimal.** Leaf size=39

$$\frac{x^{m+1} {}_2F_1\left(1, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{a(m+1)}$$

[Out] (x^(1 + m)\*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b\*x^2)/a)])/ (a\*(1 + m))

**Rubi [A]** time = 0.0317599, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{x^{m+1} {}_2F_1\left(1, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

[In] Int[x^m/(a + b\*x^2), x]

[Out] (x^(1 + m)\*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b\*x^2)/a)])/ (a\*(1 + m))

**Rubi in Sympy [A]** time = 4.98728, size = 29, normalized size = 0.74

$$\frac{x^{m+1} {}_2F_1\left(1, \frac{m}{2} + \frac{1}{2} \middle| \frac{m}{2} + \frac{3}{2}; -\frac{bx^2}{a}\right)}{a(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*m/(b\*x\*\*2+a), x)

[Out] x\*\*(m + 1)\*hyper((1, m/2 + 1/2), (m/2 + 3/2, ), -b\*x\*\*2/a)/(a\*(m + 1))

**Mathematica [A]** time = 0.0265096, size = 41, normalized size = 1.05

$$\frac{x^{m+1} {}_2F_1\left(1, \frac{m+1}{2}; \frac{m+1}{2} + 1; -\frac{bx^2}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^m/(a + b\*x^2), x]

[Out] (x^(1 + m)\*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -((b\*x^2)/a)])/ (a\*(1 + m))

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int \frac{x^m}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m/(b*x^2+a), x)`

[Out] `int(x^m/(b*x^2+a), x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/(b*x^2 + a), x, algorithm="maxima")`

[Out] `integrate(x^m/(b*x^2 + a), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/(b*x^2 + a), x, algorithm="fricas")`

[Out] `integral(x^m/(b*x^2 + a), x)`

**Sympy** [A] time = 4.48813, size = 88, normalized size = 2.26

$$\frac{mx^m \left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right) \left(\frac{m}{2} + \frac{1}{2}\right)}{4a \left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{xx^m \left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right) \left(\frac{m}{2} + \frac{1}{2}\right)}{4a \left(\frac{m}{2} + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m/(b*x**2+a), x)`

[Out] `m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2))`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/(b*x^2 + a), x, algorithm="giac")`

[Out] `integrate(x^m/(b*x^2 + a), x)`

$$3.345 \quad \int \frac{x^m}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=39

$$\frac{x^{m+1} {}_2F_1\left(2, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{a^2(m+1)}$$

[Out] (x^(1 + m)\*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b\*x^2)/a)])/(a^2\*(1 + m))

**Rubi [A]** time = 0.0303773, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{x^{m+1} {}_2F_1\left(2, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

[In] Int[x^m/(a + b\*x^2)^2, x]

[Out] (x^(1 + m)\*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b\*x^2)/a)])/(a^2\*(1 + m))

**Rubi in Sympy [A]** time = 4.45326, size = 31, normalized size = 0.79

$$\frac{x^{m+1} {}_2F_1\left(2, \frac{m}{2} + \frac{1}{2} \middle| \frac{m}{2} + \frac{3}{2}; -\frac{bx^2}{a}\right)}{a^2(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*m/(b\*x\*\*2+a)\*\*2, x)

[Out] x\*\*(m + 1)\*hyper((2, m/2 + 1/2), (m/2 + 3/2, ), -b\*x\*\*2/a)/(a\*\*2\*(m + 1))

**Mathematica [A]** time = 0.0298717, size = 41, normalized size = 1.05

$$\frac{x^{m+1} {}_2F_1\left(2, \frac{m+1}{2}; \frac{m+1}{2} + 1; -\frac{bx^2}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^m/(a + b\*x^2)^2, x]

[Out] (x^(1 + m)\*Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/2, -((b\*x^2)/a)])/(a^2\*(1 + m))

**Maple [F]** time = 0.053, size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2 + a)^2} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m/(b*x^2+a)^2,x)`

[Out] `int(x^m/(b*x^2+a)^2,x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/(b*x^2 + a)^2,x, algorithm="maxima")`

[Out] `integrate(x^m/(b*x^2 + a)^2, x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{b^2x^4 + 2abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/(b*x^2 + a)^2,x, algorithm="fricas")`

[Out] `integral(x^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)`

**Sympy** [A] time = 27.6192, size = 374, normalized size = 9.59

$$\begin{aligned} & -\frac{am^2xx^m\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right)\left(\frac{m}{2} + \frac{1}{2}\right)}{8a^3\left(\frac{m}{2} + \frac{3}{2}\right) + 8a^2bx^2\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{2amxx^m\left(\frac{m}{2} + \frac{1}{2}\right)}{8a^3\left(\frac{m}{2} + \frac{3}{2}\right) + 8a^2bx^2\left(\frac{m}{2} + \frac{3}{2}\right)} \\ & + \frac{axx^m\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right)\left(\frac{m}{2} + \frac{1}{2}\right)}{8a^3\left(\frac{m}{2} + \frac{3}{2}\right) + 8a^2bx^2\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{2axx^m\left(\frac{m}{2} + \frac{1}{2}\right)}{8a^3\left(\frac{m}{2} + \frac{3}{2}\right) + 8a^2bx^2\left(\frac{m}{2} + \frac{3}{2}\right)} \\ & - \frac{bm^2x^3x^m\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right)\left(\frac{m}{2} + \frac{1}{2}\right)}{8a^3\left(\frac{m}{2} + \frac{3}{2}\right) + 8a^2bx^2\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{bx^3x^m\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right)\left(\frac{m}{2} + \frac{1}{2}\right)}{8a^3\left(\frac{m}{2} + \frac{3}{2}\right) + 8a^2bx^2\left(\frac{m}{2} + \frac{3}{2}\right)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m/(b*x**2+a)**2,x)`

[Out] `-a*m**2*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + 2*a*m*x*x**m*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + a*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + 2*a*x*x**m*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) - b*m**2*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + b*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x^2 + a)^2, x)
```

$$3.346 \quad \int \frac{x^m}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=39

$$\frac{x^{m+1} {}_2F_1\left(3, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{a^3(m+1)}$$

[Out] (x^(1 + m)\*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b\*x^2)/a)])/(a^3\*(1 + m))

**Rubi [A]** time = 0.0297888, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{x^{m+1} {}_2F_1\left(3, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{a^3(m+1)}$$

Antiderivative was successfully verified.

[In] Int[x^m/(a + b\*x^2)^3, x]

[Out] (x^(1 + m)\*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b\*x^2)/a)])/(a^3\*(1 + m))

**Rubi in Sympy [A]** time = 4.45092, size = 31, normalized size = 0.79

$$\frac{x^{m+1} {}_2F_1\left(3, \frac{m}{2} + \frac{1}{2}; \frac{m}{2} + \frac{3}{2}; -\frac{bx^2}{a}\right)}{a^3(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*m/(b\*x\*\*2+a)\*\*3, x)

[Out] x\*\*(m + 1)\*hyper((3, m/2 + 1/2), (m/2 + 3/2, ), -b\*x\*\*2/a)/(a\*\*3\*(m + 1))

**Mathematica [A]** time = 0.031982, size = 41, normalized size = 1.05

$$\frac{x^{m+1} {}_2F_1\left(3, \frac{m+1}{2}; \frac{m+1}{2} + 1; -\frac{bx^2}{a}\right)}{a^3(m+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^m/(a + b\*x^2)^3, x]

[Out] (x^(1 + m)\*Hypergeometric2F1[3, (1 + m)/2, 1 + (1 + m)/2, -((b\*x^2)/a)])/(a^3\*(1 + m))

**Maple [F]** time = 0.072, size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2 + a)^3} dx$$



$$\begin{aligned}
& 3/2)) - 2*a*b*m**2*x**3*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 \\
& + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma \\
& a(m/2 + 3/2)) - 2*a*b*m*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi) \\
& /a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64 \\
& *a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2 \\
& )) + 4*a*b*m*x**3*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) \\
& + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 \\
& + 3/2)) + 6*a*b*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m \\
& /2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b* \\
& x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 6*a \\
& *b*x**3*x**m*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4 \\
& *b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + \\
& b**2*m**3*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1 \\
& /2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*g \\
& amma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 3*b**2*m* \\
& **2*x**5*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gam \\
& ma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/ \\
& 2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - b**2*m*x**5*x**m \\
& *lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2 \\
& )/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 3 \\
& 2*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 3*b**2*x**5*x**m*lerchphi(b* \\
& x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*g \\
& amma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2* \\
& x**4*gamma(m/2 + 3/2))
\end{aligned}$$

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/(b\*x^2 + a)^3,x, algorithm="giac")

[Out] integrate(x^m/(b\*x^2 + a)^3, x)

$$3.347 \quad \int \frac{(cx)^{1+m}}{a+bx^2} dx$$

**Optimal.** Leaf size=44

$$\frac{(cx)^{m+2} {}_2F_1\left(1, \frac{m+2}{2}; \frac{m+4}{2}; -\frac{bx^2}{a}\right)}{ac(m+2)}$$

[Out]  $((c*x)^{(2+m)} * \text{Hypergeometric2F1}[1, (2+m)/2, (4+m)/2, -((b*x^2)/a)]) / (a*c*(2+m))$

**Rubi [A]** time = 0.0423558, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{(cx)^{m+2} {}_2F_1\left(1, \frac{m+2}{2}; \frac{m+4}{2}; -\frac{bx^2}{a}\right)}{ac(m+2)}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(1+m)/(a+b\*x^2),x]

[Out]  $((c*x)^{(2+m)} * \text{Hypergeometric2F1}[1, (2+m)/2, (4+m)/2, -((b*x^2)/a)]) / (a*c*(2+m))$

**Rubi in Sympy [A]** time = 5.18764, size = 29, normalized size = 0.66

$$\frac{(cx)^{m+2} {}_2F_1\left(1, \frac{m}{2} + 1; \frac{m}{2} + 2; -\frac{bx^2}{a}\right)}{ac(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(1+m)/(b\*x\*\*2+a),x)

[Out]  $(c*x)**(m+2) * \text{hyper}((1, m/2 + 1), (m/2 + 2, ), -b*x**2/a) / (a*c*(m+2))$

**Mathematica [A]** time = 0.0383368, size = 45, normalized size = 1.02

$$\frac{cx^2(cx)^m {}_2F_1\left(1, \frac{m+2}{2}; \frac{m+2}{2} + 1; -\frac{bx^2}{a}\right)}{a(m+2)}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(1+m)/(a+b\*x^2),x]

[Out]  $(c*x^2*(c*x)^m * \text{Hypergeometric2F1}[1, (2+m)/2, 1 + (2+m)/2, -((b*x^2)/a)]) / (a*(2+m))$

**Maple [F]** time = 0.044, size = 0, normalized size = 0.

$$\int \frac{(cx)^{1+m}}{bx^2+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(1+m)/(b*x^2+a), x)`

[Out] `int((c*x)^(1+m)/(b*x^2+a), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{m+1}}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(m + 1)/(b*x^2 + a), x, algorithm="maxima")`

[Out] `integrate((c*x)^(m + 1)/(b*x^2 + a), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^{m+1}}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(m + 1)/(b*x^2 + a), x, algorithm="fricas")`

[Out] `integral((c*x)^(m + 1)/(b*x^2 + a), x)`

**Sympy [A]** time = 19.4873, size = 92, normalized size = 2.09

$$\frac{cc^m mx^2 x^m \left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + 1\right) \left(\frac{m}{2} + 1\right)}{4a \left(\frac{m}{2} + 2\right)} + \frac{cc^m x^2 x^m \left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + 1\right) \left(\frac{m}{2} + 1\right)}{2a \left(\frac{m}{2} + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(1+m)/(b*x**2+a), x)`

[Out] `c*c**m*x**2*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2 + 1)/(4*a*gamma(m/2 + 2)) + c*c**m*x**2*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2 + 1)/(2*a*gamma(m/2 + 2))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{m+1}}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(m + 1)/(b*x^2 + a), x, algorithm="giac")`

[Out] `integrate((c*x)^(m + 1)/(b*x^2 + a), x)`

$$3.348 \quad \int \frac{(cx)^m}{a+bx^2} dx$$

**Optimal.** Leaf size=44

$$\frac{(cx)^{m+1} {}_2F_1\left(1, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{ac(m+1)}$$

[Out] ((c\*x)^(1 + m)\*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b\*x^2)/a)]/(a\*c\*(1 + m))

**Rubi [A]** time = 0.0384325, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{(cx)^{m+1} {}_2F_1\left(1, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{ac(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^m/(a + b\*x^2), x]

[Out] ((c\*x)^(1 + m)\*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b\*x^2)/a)]/(a\*c\*(1 + m))

**Rubi in Sympy [A]** time = 5.1492, size = 32, normalized size = 0.73

$$\frac{(cx)^{m+1} {}_2F_1\left(1, \frac{m}{2} + \frac{1}{2}; \frac{m}{2} + \frac{3}{2}; -\frac{bx^2}{a}\right)}{ac(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*m/(b\*x\*\*2+a), x)

[Out] (c\*x)\*\*(m + 1)\*hyper((1, m/2 + 1/2), (m/2 + 3/2, ), -b\*x\*\*2/a)/(a\*c\*(m + 1))

**Mathematica [A]** time = 0.0203663, size = 42, normalized size = 0.95

$$\frac{x(cx)^m {}_2F_1\left(1, \frac{m+1}{2}; \frac{m+1}{2} + 1; -\frac{bx^2}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^m/(a + b\*x^2), x]

[Out] (x\*(c\*x)^m\*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -((b\*x^2)/a)]/(a\*(1 + m))

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{bx^2 + a} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^m/(b*x^2+a),x)`

[Out] `int((c*x)^m/(b*x^2+a),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^m/(b*x^2 + a),x, algorithm="maxima")`

[Out] `integrate((c*x)^m/(b*x^2 + a), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^m}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^m/(b*x^2 + a),x, algorithm="fricas")`

[Out] `integral((c*x)^m/(b*x^2 + a), x)`

**Sympy [A]** time = 4.5323, size = 95, normalized size = 2.16

$$\frac{c^m m x x^m \left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right) \left(\frac{m}{2} + \frac{1}{2}\right)}{4a \left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{c^m x x^m \left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right) \left(\frac{m}{2} + \frac{1}{2}\right)}{4a \left(\frac{m}{2} + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**m/(b*x**2+a),x)`

[Out] `c**m*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + c**m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^m/(b*x^2 + a),x, algorithm="giac")`

[Out] `integrate((c*x)^m/(b*x^2 + a), x)`

$$3.349 \quad \int \frac{(cx)^{-1+m}}{a+bx^2} dx$$

**Optimal.** Leaf size=38

$$\frac{(cx)^m {}_2F_1\left(1, \frac{m}{2}; \frac{m+2}{2}; -\frac{bx^2}{a}\right)}{acm}$$

[Out]  $((c*x)^m \text{Hypergeometric2F1}[1, m/2, (2 + m)/2, -((b*x^2)/a)]) / (a*c*m)$

**Rubi [A]** time = 0.0353322, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{(cx)^m {}_2F_1\left(1, \frac{m}{2}; \frac{m+2}{2}; -\frac{bx^2}{a}\right)}{acm}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(-1 + m)/(a + b\*x^2), x]

[Out]  $((c*x)^m \text{Hypergeometric2F1}[1, m/2, (2 + m)/2, -((b*x^2)/a)]) / (a*c*m)$

**Rubi in Sympy [A]** time = 4.99646, size = 24, normalized size = 0.63

$$\frac{(cx)^m {}_2F_1\left(1, \frac{m}{2} \middle| -\frac{bx^2}{a}\right)}{acm}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(-1+m)/(b\*x\*\*2+a), x)

[Out]  $(c*x)**m \text{hyper}((1, m/2), (m/2 + 1, ), -b*x**2/a) / (a*c*m)$

**Mathematica [A]** time = 0.0500373, size = 58, normalized size = 1.53

$$\frac{(cx)^m \left( a(m+2) - bmx^2 {}_2F_1\left(1, \frac{m}{2} + 1; \frac{m}{2} + 2; -\frac{bx^2}{a}\right) \right)}{a^2 cm(m+2)}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(-1 + m)/(a + b\*x^2), x]

[Out]  $((c*x)^m (a*(2 + m) - b*m*x^2 \text{Hypergeometric2F1}[1, 1 + m/2, 2 + m/2, -((b*x^2)/a)])) / (a^2*c*m*(2 + m))$

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int \frac{(cx)^{-1+m}}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(-1+m)/(b*x^2+a),x)`

[Out] `int((c*x)^(-1+m)/(b*x^2+a),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{m-1}}{bx^2+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(m-1)/(b*x^2+a),x, algorithm="maxima")`

[Out] `integrate((c*x)^(m-1)/(b*x^2+a),x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^{m-1}}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(m-1)/(b*x^2+a),x, algorithm="fricas")`

[Out] `integral((c*x)^(m-1)/(b*x^2+a),x)`

**Sympy [A]** time = 67.745, size = 39, normalized size = 1.03

$$\frac{c^m m x^m \left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2}\right) \left(\frac{m}{2}\right)}{4ac \left(\frac{m}{2} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(-1+m)/(b*x**2+a),x)`

[Out] `c**m*m*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2)*gamma(m/2)/(4*a*c*gamma(m/2+1))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{m-1}}{bx^2+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(m-1)/(b*x^2+a),x, algorithm="giac")`

[Out] `integrate((c*x)^(m-1)/(b*x^2+a),x)`

$$3.350 \quad \int \frac{(cx)^{-2+m}}{a+bx^2} dx$$

**Optimal.** Leaf size=47

$$-\frac{(cx)^{m-1} {}_2F_1\left(1, \frac{m-1}{2}; \frac{m+1}{2}; -\frac{bx^2}{a}\right)}{ac(1-m)}$$

[Out] -(((c\*x)^(-1 + m)\*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -((b\*x^2)/a)])/(a\*c\*(1 - m)))

**Rubi [A]** time = 0.0487984, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$-\frac{(cx)^{m-1} {}_2F_1\left(1, \frac{m-1}{2}; \frac{m+1}{2}; -\frac{bx^2}{a}\right)}{ac(1-m)}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(-2 + m)/(a + b\*x^2), x]

[Out] -(((c\*x)^(-1 + m)\*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -((b\*x^2)/a)])/(a\*c\*(1 - m)))

**Rubi in Sympy [A]** time = 5.22391, size = 34, normalized size = 0.72

$$-\frac{(cx)^{m-1} {}_2F_1\left(1, \frac{m}{2} - \frac{1}{2}; \frac{m}{2} + \frac{1}{2}; -\frac{bx^2}{a}\right)}{ac(-m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(-2+m)/(b\*x\*\*2+a), x)

[Out] -(c\*x)\*\*(m - 1)\*hyper((1, m/2 - 1/2), (m/2 + 1/2, ), -b\*x\*\*2/a)/(a\*c\*(-m + 1))

**Mathematica [A]** time = 0.075732, size = 59, normalized size = 1.26

$$\frac{x(cx)^{m-2} \left( a(m+1) - b(m-1)x^2 {}_2F_1\left(1, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right) \right)}{a^2(m^2-1)}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(-2 + m)/(a + b\*x^2), x]

[Out] (x\*(c\*x)^(-2 + m)\*(a\*(1 + m) - b\*(-1 + m)\*x^2\*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b\*x^2)/a)]))/(a^2\*(-1 + m^2))

**Maple [F]** time = 0.044, size = 0, normalized size = 0.

$$\int \frac{(cx)^{-2+m}}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(-2+m)/(b*x^2+a), x)`

[Out] `int((c*x)^(-2+m)/(b*x^2+a), x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{m-2}}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(m - 2)/(b*x^2 + a), x, algorithm="maxima")`

[Out] `integrate((c*x)^(m - 2)/(b*x^2 + a), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^{m-2}}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(m - 2)/(b*x^2 + a), x, algorithm="fricas")`

[Out] `integral((c*x)^(m - 2)/(b*x^2 + a), x)`

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(-2+m)/(b*x**2+a), x)`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{m-2}}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(m - 2)/(b*x^2 + a), x, algorithm="giac")`

[Out] `integrate((c*x)^(m - 2)/(b*x^2 + a), x)`

$$3.351 \quad \int \frac{(cx)^{-3+m}}{a+bx^2} dx$$

**Optimal.** Leaf size=45

$$\frac{(cx)^{m-2} {}_2F_1\left(1, \frac{m-2}{2}; \frac{m}{2}; -\frac{bx^2}{a}\right)}{ac(2-m)}$$

[Out] -(((c\*x)^(-2 + m)\*Hypergeometric2F1[1, (-2 + m)/2, m/2, -(b\*x^2/a)])/(a\*c\*(2 - m)))

**Rubi [A]** time = 0.0476106, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{(cx)^{m-2} {}_2F_1\left(1, \frac{m-2}{2}; \frac{m}{2}; -\frac{bx^2}{a}\right)}{ac(2-m)}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(-3 + m)/(a + b\*x^2), x]

[Out] -(((c\*x)^(-2 + m)\*Hypergeometric2F1[1, (-2 + m)/2, m/2, -(b\*x^2/a)])/(a\*c\*(2 - m)))

**Rubi in Sympy [A]** time = 5.28221, size = 29, normalized size = 0.64

$$\frac{(cx)^{m-2} {}_2F_1\left(1, \frac{m}{2} - 1; \frac{m}{2}; -\frac{bx^2}{a}\right)}{ac(-m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(-3+m)/(b\*x\*\*2+a), x)

[Out] -(c\*x)\*\*(m - 2)\*hyper((1, m/2 - 1), (m/2, ), -b\*x\*\*2/a)/(a\*c\*(-m + 2))

**Mathematica [A]** time = 0.0955517, size = 80, normalized size = 1.78

$$\frac{(cx)^m \left( b^2(m-2)mx^4 {}_2F_1\left(1, \frac{m}{2} + 1; \frac{m}{2} + 2; -\frac{bx^2}{a}\right) + a(m+2)(am - b(m-2)x^2) \right)}{a^3 c^3 m (m^2 - 4) x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(-3 + m)/(a + b\*x^2), x]

[Out] (((c\*x)^m\*(a\*(2 + m)\*(a\*m - b\*(-2 + m)\*x^2) + b^2\*(-2 + m)\*m\*x^4\*Hypergeometric2F1[1, 1 + m/2, 2 + m/2, -(b\*x^2/a)]))/(a^3\*c^3\*m\*(-4 + m^2)\*x^2)

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int \frac{(cx)^{-3+m}}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(-3+m)/(b*x^2+a), x)`

[Out] `int((c*x)^(-3+m)/(b*x^2+a), x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{m-3}}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(m - 3)/(b*x^2 + a), x, algorithm="maxima")`

[Out] `integrate((c*x)^(m - 3)/(b*x^2 + a), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^{m-3}}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(m - 3)/(b*x^2 + a), x, algorithm="fricas")`

[Out] `integral((c*x)^(m - 3)/(b*x^2 + a), x)`

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(-3+m)/(b*x**2+a), x)`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{m-3}}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(m - 3)/(b*x^2 + a), x, algorithm="giac")`

[Out] `integrate((c*x)^(m - 3)/(b*x^2 + a), x)`

$$3.352 \quad \int \frac{x^m}{\left(1 + \frac{ax^2}{b}\right)^2} dx$$

**Optimal.** Leaf size=36

$$\frac{x^{m+1} {}_2F_1\left(2, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{ax^2}{b}\right)}{m+1}$$

[Out] (x^(1 + m)\*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((a\*x^2)/b)])/ (1 + m)

**Rubi [A]** time = 0.02968, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{x^{m+1} {}_2F_1\left(2, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{ax^2}{b}\right)}{m+1}$$

Antiderivative was successfully verified.

[In] Int[x^m/(1 + (a\*x^2)/b)^2, x]

[Out] (x^(1 + m)\*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((a\*x^2)/b)])/ (1 + m)

**Rubi in Sympy [A]** time = 4.49914, size = 27, normalized size = 0.75

$$\frac{x^{m+1} {}_2F_1\left(2, \frac{m}{2} + \frac{1}{2}; \frac{m}{2} + \frac{3}{2}; -\frac{ax^2}{b}\right)}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*m/(1+a\*x\*\*2/b)\*\*2, x)

[Out] x\*\*(m + 1)\*hyper((2, m/2 + 1/2), (m/2 + 3/2, ), -a\*x\*\*2/b)/(m + 1)

**Mathematica [A]** time = 0.0298579, size = 38, normalized size = 1.06

$$\frac{x^{m+1} {}_2F_1\left(2, \frac{m+1}{2}; \frac{m+1}{2} + 1; -\frac{ax^2}{b}\right)}{m+1}$$

Antiderivative was successfully verified.

[In] Integrate[x^m/(1 + (a\*x^2)/b)^2, x]

[Out] (x^(1 + m)\*Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/2, -((a\*x^2)/b)])/ (1 + m)

**Maple [C]** time = 0.097, size = 92, normalized size = 2.6

$$\frac{1}{2} \left(\frac{a}{b}\right)^{-\frac{1}{2}-\frac{m}{2}} \left(2x^{1+m} \left(\frac{a}{b}\right)^{1/2+m/2} \left(2\frac{ax^2}{b} + 2\right)^{-1} + 2\frac{x^{1+m}(-1/4m^2 + 1/4)}{1+m} \left(\frac{a}{b}\right)^{1/2+m/2} \text{LerchPhi}\left(-\frac{ax^2}{b}, 1, 1/2 + m/2\right)\right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m/(1+a*x^2/b)^2,x)`

[Out]  $\frac{1}{2} \left( \frac{a}{b} \right)^{-1/2-1/2*m} \left( 2*x^{(1+m)} \left( \frac{a}{b} \right)^{(1/2+1/2*m)} / (2*a*x^2/b+2) + 2/(1+m)*x^{(1+m)} \left( \frac{a}{b} \right)^{(1/2+1/2*m)} \right) (-1/4*m^2+1/4)*\text{LerchPhi}(-a*x^2/b, 1, 1/2+1/2*m)$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{ax^2}{b} + 1\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/(a*x^2/b + 1)^2,x, algorithm="maxima")`

[Out] `integrate(x^m/(a*x^2/b + 1)^2, x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^m}{a^2x^4 + 2abx^2 + b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/(a*x^2/b + 1)^2,x, algorithm="fricas")`

[Out] `integral(b^2*x^m/(a^2*x^4 + 2*a*b*x^2 + b^2), x)`

**Sympy** [A] time = 28.1612, size = 343, normalized size = 9.53

$$\begin{aligned} & \frac{am^2x^3x^m \left(\frac{ax^2e^{i\pi}}{b}, 1, \frac{m}{2} + \frac{1}{2}\right) \left(\frac{m}{2} + \frac{1}{2}\right)}{8ax^2 \left(\frac{m}{2} + \frac{3}{2}\right) + 8b \left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{ax^3x^m \left(\frac{ax^2e^{i\pi}}{b}, 1, \frac{m}{2} + \frac{1}{2}\right) \left(\frac{m}{2} + \frac{1}{2}\right)}{8ax^2 \left(\frac{m}{2} + \frac{3}{2}\right) + 8b \left(\frac{m}{2} + \frac{3}{2}\right)} \\ & - \frac{bm^2xx^m \left(\frac{ax^2e^{i\pi}}{b}, 1, \frac{m}{2} + \frac{1}{2}\right) \left(\frac{m}{2} + \frac{1}{2}\right)}{8ax^2 \left(\frac{m}{2} + \frac{3}{2}\right) + 8b \left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{2bmx^m \left(\frac{m}{2} + \frac{1}{2}\right)}{8ax^2 \left(\frac{m}{2} + \frac{3}{2}\right) + 8b \left(\frac{m}{2} + \frac{3}{2}\right)} \\ & + \frac{bxx^m \left(\frac{ax^2e^{i\pi}}{b}, 1, \frac{m}{2} + \frac{1}{2}\right) \left(\frac{m}{2} + \frac{1}{2}\right)}{8ax^2 \left(\frac{m}{2} + \frac{3}{2}\right) + 8b \left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{2bxx^m \left(\frac{m}{2} + \frac{1}{2}\right)}{8ax^2 \left(\frac{m}{2} + \frac{3}{2}\right) + 8b \left(\frac{m}{2} + \frac{3}{2}\right)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m/(1+a*x**2/b)**2,x)`

[Out]  $-a*m**2*x**3*x**m*\text{lerchphi}(a*x**2*\text{exp\_polar}(I*\text{pi})/b, 1, m/2 + 1/2)*\text{gamma}(m/2 + 1/2)/(8*a*x**2*\text{gamma}(m/2 + 3/2) + 8*b*\text{gamma}(m/2 + 3/2)) + a*x**3*x**m*\text{lerchphi}(a*x**2*\text{exp\_polar}(I*\text{pi})/b, 1, m/2 + 1/2)*\text{gamma}(m/2 + 1/2)/(8*a*x**2*\text{gamma}(m/2 + 3/2) + 8*b*\text{gamma}(m/2 + 3/2)) - b*m**2*x*x**m*\text{lerchphi}(a*x**2*\text{exp\_polar}(I*\text{pi})/b, 1, m/2 + 1/2)*\text{gamma}(m/2 + 1/2)/(8*a*x**2*\text{gamma}(m/2 + 3/2) + 8*b*\text{gamma}(m/2 + 3/2)) + 2*b*m*x*x**m*\text{gamma}(m/2 + 1/2)/(8*a*x**2*\text{gamma}(m/2 + 3/2) + 8*b*\text{gamma}(m/2 + 3/2)) + b*x*x**m*\text{lerchphi}(a*x**2*\text{exp\_polar}(I*\text{pi})/b, 1, m/2 + 1/2)*\text{gamma}(m/2 + 1/2)/(8*a*x**2*\text{gamma}(m/2 + 3/2) + 8*b*\text{gamma}(m/2 + 3/2)) + 2*b*x*x**m*\text{gamma}(m/2 + 1/2)/(8*a*x**2*\text{gamma}(m/2 + 3/2) + 8*b*\text{gamma}(m/2 + 3/2))$

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\frac{ax^2}{b} + 1\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/(a*x^2/b + 1)^2,x, algorithm="giac")`

[Out] `integrate(x^m/(a*x^2/b + 1)^2, x)`

### 3.353 $\int x^7 \sqrt{a + bx^2} dx$

**Optimal.** Leaf size=80

$$-\frac{a^3 (a + bx^2)^{3/2}}{3b^4} + \frac{3a^2 (a + bx^2)^{5/2}}{5b^4} + \frac{(a + bx^2)^{9/2}}{9b^4} - \frac{3a (a + bx^2)^{7/2}}{7b^4}$$

[Out]  $-(a^3*(a + b*x^2)^(3/2))/(3*b^4) + (3*a^2*(a + b*x^2)^(5/2))/(5*b^4) - (3*a*(a + b*x^2)^(7/2))/(7*b^4) + (a + b*x^2)^(9/2)/(9*b^4)$

**Rubi [A]** time = 0.122207, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{a^3 (a + bx^2)^{3/2}}{3b^4} + \frac{3a^2 (a + bx^2)^{5/2}}{5b^4} + \frac{(a + bx^2)^{9/2}}{9b^4} - \frac{3a (a + bx^2)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

[In] Int[x^7\*sqrt[a + b\*x^2], x]

[Out]  $-(a^3*(a + b*x^2)^(3/2))/(3*b^4) + (3*a^2*(a + b*x^2)^(5/2))/(5*b^4) - (3*a*(a + b*x^2)^(7/2))/(7*b^4) + (a + b*x^2)^(9/2)/(9*b^4)$

**Rubi in Sympy [A]** time = 15.6719, size = 71, normalized size = 0.89

$$-\frac{a^3 (a + bx^2)^{\frac{3}{2}}}{3b^4} + \frac{3a^2 (a + bx^2)^{\frac{5}{2}}}{5b^4} - \frac{3a (a + bx^2)^{\frac{7}{2}}}{7b^4} + \frac{(a + bx^2)^{\frac{9}{2}}}{9b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*7\*(b\*x\*\*2+a)\*\*(1/2), x)

[Out]  $-a**3*(a + b*x**2)**(3/2)/(3*b**4) + 3*a**2*(a + b*x**2)**(5/2)/(5*b**4) - 3*a*(a + b*x**2)**(7/2)/(7*b**4) + (a + b*x**2)**(9/2)/(9*b**4)$

**Mathematica [A]** time = 0.029153, size = 61, normalized size = 0.76

$$\frac{\sqrt{a + bx^2} (-16a^4 + 8a^3bx^2 - 6a^2b^2x^4 + 5ab^3x^6 + 35b^4x^8)}{315b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^7\*sqrt[a + b\*x^2], x]

[Out]  $(\text{sqrt}[a + b*x^2]*(-16*a^4 + 8*a^3*b*x^2 - 6*a^2*b^2*x^4 + 5*a*b^3*x^6 + 35*b^4*x^8))/(315*b^4)$

**Maple [A]** time = 0.008, size = 47, normalized size = 0.6

$$-\frac{-35b^3x^6 + 30ab^2x^4 - 24a^2bx^2 + 16a^3}{315b^4} (bx^2 + a)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(b*x^2+a)^(1/2),x)`

[Out]  $-1/315*(b*x^2+a)^{(3/2)}*(-35*b^3*x^6+30*a*b^2*x^4-24*a^2*b*x^2+16*a^3)/b^4$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*x^7,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.23268, size = 77, normalized size = 0.96

$$\frac{(35 b^4 x^8 + 5 a b^3 x^6 - 6 a^2 b^2 x^4 + 8 a^3 b x^2 - 16 a^4) \sqrt{b x^2 + a}}{315 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*x^7,x, algorithm="fricas")`

[Out]  $1/315*(35*b^4*x^8 + 5*a*b^3*x^6 - 6*a^2*b^2*x^4 + 8*a^3*b*x^2 - 16*a^4)*\sqrt{b*x^2 + a}/b^4$

**Sympy [A]** time = 3.77464, size = 110, normalized size = 1.38

$$\begin{cases} -\frac{16a^4\sqrt{a+bx^2}}{315b^4} + \frac{8a^3x^2\sqrt{a+bx^2}}{315b^3} - \frac{2a^2x^4\sqrt{a+bx^2}}{105b^2} + \frac{ax^6\sqrt{a+bx^2}}{63b} + \frac{x^8\sqrt{a+bx^2}}{9} & \text{for } b \neq 0 \\ \frac{\sqrt{ax^8}}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(b*x**2+a)**(1/2),x)`

[Out] `Piecewise((-16*a**4*sqrt(a + b*x**2)/(315*b**4) + 8*a**3*x**2*sqrt(a + b*x**2)/(315*b**3) - 2*a**2*x**4*sqrt(a + b*x**2)/(105*b**2) + a*x**6*sqrt(a + b*x**2)/(63*b) + x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (sqrt(a)*x**8/8, True))`

**GIAC/XCAS [A]** time = 0.206812, size = 77, normalized size = 0.96

$$\frac{35 (bx^2 + a)^{\frac{9}{2}} - 135 (bx^2 + a)^{\frac{7}{2}} a + 189 (bx^2 + a)^{\frac{5}{2}} a^2 - 105 (bx^2 + a)^{\frac{3}{2}} a^3}{315 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*x^7,x, algorithm="giac")`

[Out]  $1/315*(35*(b*x^2 + a)^{(9/2)} - 135*(b*x^2 + a)^{(7/2)}*a + 189*(b*x^2 + a)^{(5/2)}*a^2 - 105*(b*x^2 + a)^{(3/2)}*a^3)/b^4$

### 3.354 $\int x^5 \sqrt{a + bx^2} dx$

**Optimal.** Leaf size=59

$$\frac{a^2 (a + bx^2)^{3/2}}{3b^3} + \frac{(a + bx^2)^{7/2}}{7b^3} - \frac{2a (a + bx^2)^{5/2}}{5b^3}$$

[Out]  $(a^2 * (a + b * x^2)^{(3/2)}) / (3 * b^3) - (2 * a * (a + b * x^2)^{(5/2)}) / (5 * b^3) + (a + b * x^2)^{(7/2)} / (7 * b^3)$

**Rubi [A]** time = 0.0954688, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{a^2 (a + bx^2)^{3/2}}{3b^3} + \frac{(a + bx^2)^{7/2}}{7b^3} - \frac{2a (a + bx^2)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5 \* Sqrt[a + b \* x^2], x]

[Out]  $(a^2 * (a + b * x^2)^{(3/2)}) / (3 * b^3) - (2 * a * (a + b * x^2)^{(5/2)}) / (5 * b^3) + (a + b * x^2)^{(7/2)} / (7 * b^3)$

**Rubi in Sympy [A]** time = 11.843, size = 51, normalized size = 0.86

$$\frac{a^2 (a + bx^2)^{\frac{3}{2}}}{3b^3} - \frac{2a (a + bx^2)^{\frac{5}{2}}}{5b^3} + \frac{(a + bx^2)^{\frac{7}{2}}}{7b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(b\*x\*\*2+a)\*\*(1/2), x)

[Out]  $a**2*(a + b*x**2)**(3/2)/(3*b**3) - 2*a*(a + b*x**2)**(5/2)/(5*b**3) + (a + b*x**2)**(7/2)/(7*b**3)$

**Mathematica [A]** time = 0.0222199, size = 50, normalized size = 0.85

$$\frac{\sqrt{a + bx^2} (8a^3 - 4a^2bx^2 + 3ab^2x^4 + 15b^3x^6)}{105b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*Sqrt[a + b\*x^2], x]

[Out]  $(\text{Sqrt}[a + b * x^2] * (8 * a^3 - 4 * a^2 * b * x^2 + 3 * a * b^2 * x^4 + 15 * b^3 * x^6)) / (105 * b^3)$

**Maple [A]** time = 0.008, size = 36, normalized size = 0.6

$$\frac{15 b^2 x^4 - 12 a b x^2 + 8 a^2}{105 b^3} (b x^2 + a)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(b*x^2+a)^(1/2),x)`

[Out]  $1/105*(b*x^2+a)^{(3/2)}*(15*b^2*x^4-12*a*b*x^2+8*a^2)/b^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*x^5,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.22806, size = 62, normalized size = 1.05

$$\frac{(15b^3x^6 + 3ab^2x^4 - 4a^2bx^2 + 8a^3)\sqrt{bx^2 + a}}{105b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*x^5,x, algorithm="fricas")`

[Out]  $1/105*(15*b^3*x^6 + 3*a*b^2*x^4 - 4*a^2*b*x^2 + 8*a^3)*\sqrt{b*x^2 + a}/b^3$

**Sympy [A]** time = 1.87953, size = 87, normalized size = 1.47

$$\begin{cases} \frac{8a^3\sqrt{a+bx^2}}{105b^3} - \frac{4a^2x^2\sqrt{a+bx^2}}{105b^2} + \frac{ax^4\sqrt{a+bx^2}}{35b} + \frac{x^6\sqrt{a+bx^2}}{7} & \text{for } b \neq 0 \\ \frac{\sqrt{ax^6}}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(b*x**2+a)**(1/2),x)`

[Out] `Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**2)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, Ne(b, 0)), (sqrt(a)*x**6/6, True))`

**GIAC/XCAS [A]** time = 0.207517, size = 58, normalized size = 0.98

$$\frac{15(bx^2 + a)^{\frac{7}{2}} - 42(bx^2 + a)^{\frac{5}{2}}a + 35(bx^2 + a)^{\frac{3}{2}}a^2}{105b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*x^5,x, algorithm="giac")`

[Out]  $1/105*(15*(b*x^2 + a)^{(7/2)} - 42*(b*x^2 + a)^{(5/2)}*a + 35*(b*x^2 + a)^{(3/2)}*a^2)/b^3$

### 3.355 $\int x^3 \sqrt{a + bx^2} dx$

**Optimal.** Leaf size=38

$$\frac{(a + bx^2)^{5/2}}{5b^2} - \frac{a(a + bx^2)^{3/2}}{3b^2}$$

[Out]  $-(a*(a + b*x^2)^(3/2))/(3*b^2) + (a + b*x^2)^(5/2)/(5*b^2)$

**Rubi [A]** time = 0.0663795, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{(a + bx^2)^{5/2}}{5b^2} - \frac{a(a + bx^2)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

[In] `Int[x^3*Sqrt[a + b*x^2], x]`

[Out]  $-(a*(a + b*x^2)^(3/2))/(3*b^2) + (a + b*x^2)^(5/2)/(5*b^2)$

**Rubi in Sympy [A]** time = 7.90249, size = 31, normalized size = 0.82

$$-\frac{a(a + bx^2)^{3/2}}{3b^2} + \frac{(a + bx^2)^{5/2}}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3*(b*x**2+a)**(1/2), x)`

[Out]  $-a*(a + b*x**2)**(3/2)/(3*b**2) + (a + b*x**2)**(5/2)/(5*b**2)$

**Mathematica [A]** time = 0.0183357, size = 38, normalized size = 1.

$$\frac{\sqrt{a + bx^2}(-2a^2 + abx^2 + 3b^2x^4)}{15b^2}$$

Antiderivative was successfully verified.

[In] `Integrate[x^3*Sqrt[a + b*x^2], x]`

[Out]  $(\text{Sqrt}[a + b*x^2]*(-2*a^2 + a*b*x^2 + 3*b^2*x^4))/(15*b^2)$

**Maple [A]** time = 0.007, size = 25, normalized size = 0.7

$$-\frac{-3bx^2 + 2a}{15b^2} (bx^2 + a)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(b*x^2+a)^(1/2), x)`

[Out]  $-1/15*(b*x^2+a)^(3/2)*(-3*b*x^2+2*a)/b^2$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b\*x^2 + a)\*x^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.226704, size = 46, normalized size = 1.21

$$\frac{(3b^2x^4 + abx^2 - 2a^2)\sqrt{bx^2 + a}}{15b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b\*x^2 + a)\*x^3,x, algorithm="fricas")

[Out] 1/15\*(3\*b^2\*x^4 + a\*b\*x^2 - 2\*a^2)\*sqrt(b\*x^2 + a)/b^2

---

**Sympy [A]** time = 0.849566, size = 63, normalized size = 1.66

$$\begin{cases} -\frac{2a^2\sqrt{a+bx^2}}{15b^2} + \frac{ax^2\sqrt{a+bx^2}}{15b} + \frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(b\*x\*\*2+a)\*\*(1/2), x)

[Out] Piecewise((-2\*a\*\*2\*sqrt(a + b\*x\*\*2)/(15\*b\*\*2) + a\*x\*\*2\*sqrt(a + b\*x\*\*2)/(15\*b) + x\*\*4\*sqrt(a + b\*x\*\*2)/5, Ne(b, 0)), (sqrt(a)\*x\*\*4/4, True))

---

**GIAC/XCAS [A]** time = 0.205889, size = 39, normalized size = 1.03

$$\frac{3(bx^2 + a)^{\frac{5}{2}} - 5(bx^2 + a)^{\frac{3}{2}}a}{15b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b\*x^2 + a)\*x^3,x, algorithm="giac")

[Out] 1/15\*(3\*(b\*x^2 + a)^(5/2) - 5\*(b\*x^2 + a)^(3/2)\*a)/b^2



$$3.356 \quad \int x\sqrt{a+bx^2} dx$$

**Optimal.** Leaf size=18

$$\frac{(a+bx^2)^{3/2}}{3b}$$

[Out] (a + b\*x^2)^(3/2)/(3\*b)

**Rubi [A]** time = 0.0112509, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{(a+bx^2)^{3/2}}{3b}$$

Antiderivative was successfully verified.

[In] Int[x\*Sqrt[a + b\*x^2], x]

[Out] (a + b\*x^2)^(3/2)/(3\*b)

**Rubi in Sympy [A]** time = 2.15105, size = 12, normalized size = 0.67

$$\frac{(a+bx^2)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(b\*x\*\*2+a)\*\*(1/2), x)

[Out] (a + b\*x\*\*2)\*\*(3/2)/(3\*b)

**Mathematica [A]** time = 0.00573346, size = 18, normalized size = 1.

$$\frac{(a+bx^2)^{3/2}}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Sqrt[a + b\*x^2], x]

[Out] (a + b\*x^2)^(3/2)/(3\*b)

**Maple [A]** time = 0.003, size = 15, normalized size = 0.8

$$\frac{1}{3b} (bx^2 + a)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(b\*x^2+a)^(1/2), x)

[Out] 1/3\*(b\*x^2+a)^(3/2)/b

---

**Maxima [A]** time = 1.34866, size = 19, normalized size = 1.06

$$\frac{(bx^2 + a)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*x,x, algorithm="maxima")`

[Out] `1/3*(b*x^2 + a)^(3/2)/b`

---

**Fricas [A]** time = 0.229414, size = 19, normalized size = 1.06

$$\frac{(bx^2 + a)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*x,x, algorithm="fricas")`

[Out] `1/3*(b*x^2 + a)^(3/2)/b`

---

**Sympy [A]** time = 0.400421, size = 39, normalized size = 2.17

$$\begin{cases} \frac{a\sqrt{a+bx^2}}{3b} + \frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b \neq 0 \\ \frac{\sqrt{ax^2}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x**2+a)**(1/2),x)`

[Out] `Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True))`

---

**GIAC/XCAS [A]** time = 0.208481, size = 19, normalized size = 1.06

$$\frac{(bx^2 + a)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*x,x, algorithm="giac")`

[Out] `1/3*(b*x^2 + a)^(3/2)/b`

$$3.357 \quad \int \frac{\sqrt{a+bx^2}}{x} dx$$

**Optimal.** Leaf size=37

$$\sqrt{a+bx^2} - \sqrt{a} \tanh^{-1} \left( \frac{\sqrt{a+bx^2}}{\sqrt{a}} \right)$$

[Out] Sqrt[a + b\*x^2] - Sqrt[a]\*ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]]

**Rubi [A]** time = 0.0741292, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\sqrt{a+bx^2} - \sqrt{a} \tanh^{-1} \left( \frac{\sqrt{a+bx^2}}{\sqrt{a}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x^2]/x, x]

[Out] Sqrt[a + b\*x^2] - Sqrt[a]\*ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]]

**Rubi in Sympy [A]** time = 7.52066, size = 31, normalized size = 0.84

$$-\sqrt{a} \operatorname{atanh} \left( \frac{\sqrt{a+bx^2}}{\sqrt{a}} \right) + \sqrt{a+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/2)/x, x)

[Out] -sqrt(a)\*atanh(sqrt(a + b\*x\*\*2)/sqrt(a)) + sqrt(a + b\*x\*\*2)

**Mathematica [A]** time = 0.029536, size = 47, normalized size = 1.27

$$\sqrt{a+bx^2} - \sqrt{a} \log \left( \sqrt{a}\sqrt{a+bx^2} + a \right) + \sqrt{a} \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x^2]/x, x]

[Out] Sqrt[a + b\*x^2] + Sqrt[a]\*Log[x] - Sqrt[a]\*Log[a + Sqrt[a]\*Sqrt[a + b\*x^2]]

**Maple [A]** time = 0.006, size = 39, normalized size = 1.1

$$\sqrt{bx^2+a} - \sqrt{a} \ln \left( \frac{1}{x} \left( 2a + 2\sqrt{a}\sqrt{bx^2+a} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(1/2)/x, x)

[Out]  $(b*x^2+a)^{(1/2)}-a^{(1/2)}*\ln((2*a+2*a^{(1/2)}*(b*x^2+a)^{(1/2)})/x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/x,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.246157, size = 1, normalized size = 0.03

$$\left[ \frac{1}{2} \sqrt{a} \log \left( -\frac{bx^2 - 2\sqrt{bx^2 + a}\sqrt{a} + 2a}{x^2} \right) + \sqrt{bx^2 + a}, -\sqrt{-a} \arctan \left( \frac{a}{\sqrt{bx^2 + a}\sqrt{-a}} \right) + \sqrt{bx^2 + a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/x,x, algorithm="fricas")`

[Out]  $[1/2*\sqrt{a}*\log(-(b*x^2 - 2*\sqrt{b*x^2 + a})*\sqrt{a} + 2*a)/x^2) + \sqrt{b*x^2 + a}, -\sqrt{-a}*\arctan(a/(\sqrt{b*x^2 + a}*\sqrt{-a})) + \sqrt{b*x^2 + a}]$

**Sympy [A]** time = 4.84016, size = 56, normalized size = 1.51

$$-\sqrt{a} \operatorname{asinh} \left( \frac{\sqrt{a}}{\sqrt{bx}} \right) + \frac{a}{\sqrt{bx}\sqrt{\frac{a}{bx^2} + 1}} + \frac{\sqrt{bx}}{\sqrt{\frac{a}{bx^2} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/2)/x,x)`

[Out]  $-\sqrt{a}*\operatorname{asinh}(\sqrt{a}/(\sqrt{b}*x)) + a/(\sqrt{b}*x*\sqrt{a/(b*x**2) + 1}) + \sqrt{b}*x/\sqrt{a/(b*x**2) + 1}$

**GIAC/XCAS [A]** time = 0.209539, size = 45, normalized size = 1.22

$$\frac{a \arctan \left( \frac{\sqrt{bx^2+a}}{\sqrt{-a}} \right)}{\sqrt{-a}} + \sqrt{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/x,x, algorithm="giac")`

[Out]  $a*\arctan(\sqrt{b*x^2 + a}/\sqrt{-a})/\sqrt{-a} + \sqrt{b*x^2 + a}$

$$3.358 \quad \int \frac{\sqrt{a+bx^2}}{x^3} dx$$

**Optimal.** Leaf size=47

$$-\frac{\sqrt{a+bx^2}}{2x^2} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

[Out]  $-\text{Sqrt}[a + b*x^2]/(2*x^2) - (b*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(2*\text{Sqrt}[a])$

**Rubi [A]** time = 0.078623, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{\sqrt{a+bx^2}}{2x^2} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[a + b*x^2]/x^3, x]$

[Out]  $-\text{Sqrt}[a + b*x^2]/(2*x^2) - (b*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(2*\text{Sqrt}[a])$

**Rubi in Sympy [A]** time = 7.78796, size = 41, normalized size = 0.87

$$-\frac{\sqrt{a+bx^2}}{2x^2} - \frac{b \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x**2+a)**(1/2)/x**3, x)$

[Out]  $-\text{sqrt}(a + b*x**2)/(2*x**2) - b*\operatorname{atanh}(\text{sqrt}(a + b*x**2)/\text{sqrt}(a))/(2*\text{sqrt}(a))$

**Mathematica [A]** time = 0.0459745, size = 58, normalized size = 1.23

$$\frac{1}{2} \left( -\frac{\sqrt{a+bx^2}}{x^2} - \frac{b \log\left(\sqrt{a}\sqrt{a+bx^2} + a\right)}{\sqrt{a}} + \frac{b \log(x)}{\sqrt{a}} \right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Sqrt}[a + b*x^2]/x^3, x]$

[Out]  $(-(\text{Sqrt}[a + b*x^2]/x^2) + (b*\text{Log}[x])/ \text{Sqrt}[a] - (b*\text{Log}[a + \text{Sqrt}[a]*\text{Sqrt}[a + b*x^2]])/\text{Sqrt}[a])/2$

**Maple [A]** time = 0.006, size = 63, normalized size = 1.3

$$-\frac{1}{2ax^2} (bx^2 + a)^{\frac{3}{2}} - \frac{b}{2} \ln\left(\frac{1}{x} (2a + 2\sqrt{a}\sqrt{bx^2 + a})\right) \frac{1}{\sqrt{a}} + \frac{b}{2a} \sqrt{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/2)/x^3,x)`

[Out]  $-1/2/a/x^2*(b*x^2+a)^{3/2}-1/2*b/a^{1/2}*\ln((2*a+2*a^{1/2}*(b*x^2+a)^{1/2})/x)+1/2*b/a*(b*x^2+a)^{1/2}$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/x^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.236683, size = 1, normalized size = 0.02

$$\left[ \frac{bx^2 \log\left(-\frac{(bx^2+2a)\sqrt{a}-2\sqrt{bx^2+aa}}{x^2}\right) - 2\sqrt{bx^2+a}\sqrt{a}}{4\sqrt{ax^2}}, -\frac{bx^2 \arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) + \sqrt{bx^2+a}\sqrt{-a}}{2\sqrt{-ax^2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/x^3,x, algorithm="fricas")`

[Out]  $[1/4*(b*x^2*\log(-((b*x^2 + 2*a)*\sqrt{a} - 2*\sqrt{b*x^2 + a})*a)/x^2) - 2*\sqrt{b*x^2 + a}*\sqrt{a})/(\sqrt{a}*x^2), -1/2*(b*x^2*\arctan(\sqrt{-a}/\sqrt{b*x^2 + a}) + \sqrt{b*x^2 + a}*\sqrt{-a})/(\sqrt{-a}*x^2)]$

**Sympy** [A] time = 6.45371, size = 42, normalized size = 0.89

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2} + 1}}{2x} - \frac{b \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/2)/x**3,x)`

[Out]  $-\sqrt{b}*\sqrt{a/(b*x^2) + 1}/(2*x) - b*\operatorname{asinh}(\sqrt{a}/(\sqrt{b}*x))/(2*\sqrt{a})$

**GIAC/XCAS** [A] time = 0.208675, size = 58, normalized size = 1.23

$$\frac{1}{2}b\left(\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{bx^2+a}}{bx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b*(arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x^2 + a)
)/(b*x^2)
```

$$3.359 \quad \int \frac{\sqrt{a+bx^2}}{x^5} dx$$

**Optimal.** Leaf size=71

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}} - \frac{b\sqrt{a+bx^2}}{8ax^2} - \frac{\sqrt{a+bx^2}}{4x^4}$$

[Out] -Sqrt[a + b\*x^2]/(4\*x^4) - (b\*Sqrt[a + b\*x^2])/(8\*a\*x^2) + (b^2\*ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]])/(8\*a^(3/2))

**Rubi [A]** time = 0.113865, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}} - \frac{b\sqrt{a+bx^2}}{8ax^2} - \frac{\sqrt{a+bx^2}}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x^2]/x^5, x]

[Out] -Sqrt[a + b\*x^2]/(4\*x^4) - (b\*Sqrt[a + b\*x^2])/(8\*a\*x^2) + (b^2\*ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]])/(8\*a^(3/2))

**Rubi in Sympy [A]** time = 10.562, size = 60, normalized size = 0.85

$$-\frac{\sqrt{a+bx^2}}{4x^4} - \frac{b\sqrt{a+bx^2}}{8ax^2} + \frac{b^2 \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/2)/x\*\*5, x)

[Out] -sqrt(a + b\*x\*\*2)/(4\*x\*\*4) - b\*sqrt(a + b\*x\*\*2)/(8\*a\*x\*\*2) + b\*\*2\*atanh(sqrt(a + b\*x\*\*2)/sqrt(a))/(8\*a\*\*(3/2))

**Mathematica [A]** time = 0.0590909, size = 78, normalized size = 1.1

$$\frac{b^2 \log\left(\sqrt{a}\sqrt{a+bx^2} + a\right)}{8a^{3/2}} - \frac{b^2 \log(x)}{8a^{3/2}} + \left(-\frac{b}{8ax^2} - \frac{1}{4x^4}\right) \sqrt{a+bx^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x^2]/x^5, x]

[Out] (-1/(4\*x^4) - b/(8\*a\*x^2))\*Sqrt[a + b\*x^2] - (b^2\*Log[x])/(8\*a^(3/2)) + (b^2\*Log[a + Sqrt[a]\*Sqrt[a + b\*x^2]])/(8\*a^(3/2))

**Maple [A]** time = 0.009, size = 85, normalized size = 1.2

$$-\frac{1}{4ax^4} (bx^2 + a)^{\frac{3}{2}} + \frac{b}{8a^2x^2} (bx^2 + a)^{\frac{3}{2}} + \frac{b^2}{8} \ln\left(\frac{1}{x} (2a + 2\sqrt{a}\sqrt{bx^2 + a})\right) a^{-\frac{3}{2}} - \frac{b^2}{8a^2} \sqrt{bx^2 + a}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/2)/x^5,x)`

[Out] 
$$-1/4/a/x^4*(b*x^2+a)^{3/2}+1/8*b/a^2/x^2*(b*x^2+a)^{3/2}+1/8*b^2/a^{3/2}*\ln((2*a+2*a^{1/2}*(b*x^2+a)^{1/2})/x)-1/8*b^2/a^2*(b*x^2+a)^{1/2}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/x^5,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.245542, size = 1, normalized size = 0.01

$$\left[ \frac{b^2 x^4 \log\left(-\frac{(bx^2+2a)\sqrt{a+2\sqrt{bx^2+aa}}}{x^2}\right) - 2(bx^2+2a)\sqrt{bx^2+a}\sqrt{a}}{16 a^{\frac{3}{2}} x^4}, \frac{b^2 x^4 \arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) - (bx^2+2a)\sqrt{bx^2+a}\sqrt{-a}}{8 \sqrt{-a} x^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/x^5,x, algorithm="fricas")`

[Out] 
$$\left[ \frac{1}{16}*(b^2*x^4*\log(-((b*x^2 + 2*a)*\sqrt{a} + 2*\sqrt{b*x^2 + a})*a)/x^2) - 2*(b*x^2 + 2*a)*\sqrt{b*x^2 + a}*\sqrt{a})/(a^{3/2}*x^4), \frac{1}{8}*(b^2*x^4*\arctan(\sqrt{-a}/\sqrt{b*x^2 + a}) - (b*x^2 + 2*a)*\sqrt{b*x^2 + a}*\sqrt{-a})/(\sqrt{-a}*a*x^4) \right]$$

**Sympy [A]** time = 12.1276, size = 92, normalized size = 1.3

$$-\frac{a}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{b^{\frac{3}{2}}}{8ax\sqrt{\frac{a}{bx^2}+1}} + \frac{b^2 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{8a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/2)/x**5,x)`

[Out] 
$$-a/(4*\sqrt{b}*x^5*\sqrt{a/(b*x^2) + 1}) - 3*\sqrt{b}/(8*x^3*\sqrt{a/(b*x^2) + 1}) - b^{3/2}/(8*a*x*\sqrt{a/(b*x^2) + 1}) + b^2*\operatorname{asinh}(\sqrt{a}/(\sqrt{b}*x))/(8*a^{3/2})$$

**GIAC/XCAS [A]** time = 0.214014, size = 84, normalized size = 1.18

$$-\frac{1}{8}b^2\left(\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-aa}} + \frac{(bx^2+a)^{\frac{3}{2}} + \sqrt{bx^2+aa}}{ab^2x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)/x^5,x, algorithm="giac")
```

```
[Out] -1/8*b^2*(arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + ((b*x^2 + a)^(3/2) + sqrt(b*x^2 + a)*a)/(a*b^2*x^4))
```

$$3.360 \quad \int \frac{\sqrt{a+bx^2}}{x^7} dx$$

**Optimal.** Leaf size=95

$$-\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}} + \frac{b^2\sqrt{a+bx^2}}{16a^2x^2} - \frac{\sqrt{a+bx^2}}{6x^6} - \frac{b\sqrt{a+bx^2}}{24ax^4}$$

[Out] -Sqrt[a + b\*x^2]/(6\*x^6) - (b\*Sqrt[a + b\*x^2])/(24\*a\*x^4) + (b^2\*Sqrt[a + b\*x^2])/(16\*a^2\*x^2) - (b^3\*ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]])/(16\*a^(5/2))

**Rubi [A]** time = 0.152887, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}} + \frac{b^2\sqrt{a+bx^2}}{16a^2x^2} - \frac{\sqrt{a+bx^2}}{6x^6} - \frac{b\sqrt{a+bx^2}}{24ax^4}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x^2]/x^7, x]

[Out] -Sqrt[a + b\*x^2]/(6\*x^6) - (b\*Sqrt[a + b\*x^2])/(24\*a\*x^4) + (b^2\*Sqrt[a + b\*x^2])/(16\*a^2\*x^2) - (b^3\*ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]])/(16\*a^(5/2))

**Rubi in Sympy [A]** time = 14.2184, size = 82, normalized size = 0.86

$$-\frac{\sqrt{a+bx^2}}{6x^6} - \frac{b\sqrt{a+bx^2}}{24ax^4} + \frac{b^2\sqrt{a+bx^2}}{16a^2x^2} - \frac{b^3 \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/2)/x\*\*7, x)

[Out] -sqrt(a + b\*x\*\*2)/(6\*x\*\*6) - b\*sqrt(a + b\*x\*\*2)/(24\*a\*x\*\*4) + b\*\*2\*sqrt(a + b\*x\*\*2)/(16\*a\*\*2\*x\*\*2) - b\*\*3\*atanh(sqrt(a + b\*x\*\*2)/sqrt(a))/(16\*a\*\*(5/2))

**Mathematica [A]** time = 0.0742489, size = 91, normalized size = 0.96

$$-\frac{b^3 \log\left(\sqrt{a}\sqrt{a+bx^2} + a\right)}{16a^{5/2}} + \frac{b^3 \log(x)}{16a^{5/2}} + \left(\frac{b^2}{16a^2x^2} - \frac{b}{24ax^4} - \frac{1}{6x^6}\right) \sqrt{a+bx^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x^2]/x^7, x]

[Out] (-1/(6\*x^6) - b/(24\*a\*x^4) + b^2/(16\*a^2\*x^2))\*Sqrt[a + b\*x^2] + (b^3\*Log[x])/(16\*a^(5/2)) - (b^3\*Log[a + Sqrt[a]\*Sqrt[a + b\*x^2]])/(16\*a^(5/2))

**Maple [A]** time = 0.011, size = 105, normalized size = 1.1

$$-\frac{1}{6ax^6}(bx^2+a)^{\frac{3}{2}} + \frac{b}{8a^2x^4}(bx^2+a)^{\frac{3}{2}} - \frac{b^2}{16a^3x^2}(bx^2+a)^{\frac{3}{2}} - \frac{b^3}{16} \ln\left(\frac{1}{x}(2a+2\sqrt{a}\sqrt{bx^2+a})\right) a^{-\frac{5}{2}} + \frac{b^3}{16a^3}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(1/2)/x^7, x)

[Out] -1/6/a/x^6\*(b\*x^2+a)^(3/2)+1/8\*b/a^2/x^4\*(b\*x^2+a)^(3/2)-1/16\*b^2/a^3/x^2\*(b\*x^2+a)^(3/2)-1/16\*b^3/a^(5/2)\*ln((2\*a+2\*a^(1/2)\*(b\*x^2+a)^(1/2))/x)+1/16\*b^3/a^3\*(b\*x^2+a)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b\*x^2 + a)/x^7, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.25732, size = 1, normalized size = 0.01

$$\left[ \frac{3b^3x^6 \log\left(-\frac{(bx^2+2a)\sqrt{a-2\sqrt{bx^2+aa}}}{x^2}\right) + 2(3b^2x^4 - 2abx^2 - 8a^2)\sqrt{bx^2+a}\sqrt{a}}{96a^{\frac{5}{2}}x^6}, \right. \\ \left. -\frac{3b^3x^6 \arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) - (3b^2x^4 - 2abx^2 - 8a^2)\sqrt{bx^2+a}\sqrt{-a}}{48\sqrt{-aa^2}x^6} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b\*x^2 + a)/x^7, x, algorithm="fricas")

[Out] [1/96\*(3\*b^3\*x^6\*log(-((b\*x^2 + 2\*a)\*sqrt(a) - 2\*sqrt(b\*x^2 + a)\*a)/x^2) + 2\*(3\*b^2\*x^4 - 2\*a\*b\*x^2 - 8\*a^2)\*sqrt(b\*x^2 + a)\*sqrt(a))/(a^(5/2)\*x^6), -1/48\*(3\*b^3\*x^6\*arctan(sqrt(-a)/sqrt(b\*x^2 + a)) - (3\*b^2\*x^4 - 2\*a\*b\*x^2 - 8\*a^2)\*sqrt(b\*x^2 + a)\*sqrt(-a))/(sqrt(-a)\*a^2\*x^6)]

**Sympy [A]** time = 19.3802, size = 117, normalized size = 1.23

$$-\frac{a}{6\sqrt{bx^7}\sqrt{\frac{a}{bx^2}+1}} - \frac{5\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{b^{\frac{3}{2}}}{48ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{b^{\frac{5}{2}}}{16a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{b^3 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{16a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/2)/x\*\*7, x)

```
[Out] -a/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 5*sqrt(b)/(24*x**5*sqrt(a/(b*x**2) + 1)) + b**(3/2)/(48*a*x**3*sqrt(a/(b*x**2) + 1)) + b**(5/2)/(16*a**2*x*sqrt(a/(b*x**2) + 1)) - b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(5/2))
```

**GIAC/XCAS [A]** time = 0.210707, size = 108, normalized size = 1.14

$$\frac{1}{48} b^3 \left( \frac{3 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-aa^2}} + \frac{3 (bx^2 + a)^{\frac{5}{2}} - 8 (bx^2 + a)^{\frac{3}{2}} a - 3 \sqrt{bx^2 + aa^2}}{a^2 b^3 x^6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*b^3*(3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 + a)^(5/2) - 8*(b*x^2 + a)^(3/2)*a - 3*sqrt(b*x^2 + a)*a^2)/(a^2*b^3*x^6))
```

### 3.361 $\int x^4 \sqrt{a + bx^2} dx$

**Optimal.** Leaf size=94

$$\frac{a^3 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{16b^{5/2}} - \frac{a^2 x \sqrt{a+bx^2}}{16b^2} + \frac{1}{6} x^5 \sqrt{a+bx^2} + \frac{ax^3 \sqrt{a+bx^2}}{24b}$$

[Out]  $-(a^2 x \sqrt{a + b x^2}) / (16 b^2) + (a x^3 \sqrt{a + b x^2}) / (24 b) + (x^5 \sqrt{a + b x^2}) / 6 + (a^3 \operatorname{ArcTanh}[(\sqrt{b} x) / \sqrt{a + b x^2}]) / (16 b^{5/2})$

**Rubi [A]** time = 0.110948, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{a^3 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{16b^{5/2}} - \frac{a^2 x \sqrt{a+bx^2}}{16b^2} + \frac{1}{6} x^5 \sqrt{a+bx^2} + \frac{ax^3 \sqrt{a+bx^2}}{24b}$$

Antiderivative was successfully verified.

[In] Int[x^4\*Sqrt[a + b\*x^2], x]

[Out]  $-(a^2 x \sqrt{a + b x^2}) / (16 b^2) + (a x^3 \sqrt{a + b x^2}) / (24 b) + (x^5 \sqrt{a + b x^2}) / 6 + (a^3 \operatorname{ArcTanh}[(\sqrt{b} x) / \sqrt{a + b x^2}]) / (16 b^{5/2})$

**Rubi in Sympy [A]** time = 12.984, size = 82, normalized size = 0.87

$$\frac{a^3 \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{16b^{5/2}} - \frac{a^2 x \sqrt{a+bx^2}}{16b^2} + \frac{ax^3 \sqrt{a+bx^2}}{24b} + \frac{x^5 \sqrt{a+bx^2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(b\*x\*\*2+a)\*\*(1/2), x)

[Out]  $a^3 \operatorname{atanh}(\sqrt{b} x / \sqrt{a + b x^2}) / (16 b^{5/2}) - a^2 x \sqrt{a + b x^2} / (16 b^2) + a x^3 \sqrt{a + b x^2} / (24 b) + x^5 \sqrt{a + b x^2} / 6$

**Mathematica [A]** time = 0.0586014, size = 77, normalized size = 0.82

$$\frac{a^3 \log\left(\sqrt{b} \sqrt{a + bx^2} + bx\right)}{16b^{5/2}} + \sqrt{a + bx^2} \left(-\frac{a^2 x}{16b^2} + \frac{ax^3}{24b} + \frac{x^5}{6}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*Sqrt[a + b\*x^2], x]

[Out]  $\sqrt{a + b x^2} \left(-\frac{a^2 x}{16 b^2} + \frac{a x^3}{24 b} + \frac{x^5}{6}\right) + a^3 \operatorname{Log}[b x + \sqrt{b} \sqrt{a + b x^2}] / (16 b^{5/2})$

**Maple [A]** time = 0.009, size = 77, normalized size = 0.8

$$\frac{x^3}{6b} (bx^2 + a)^{3/2} - \frac{ax}{8b^2} (bx^2 + a)^{3/2} + \frac{a^2 x}{16b^2} \sqrt{bx^2 + a} + \frac{a^3}{16} \ln(x\sqrt{b} + \sqrt{bx^2 + a}) b^{-5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(b*x^2+a)^(1/2),x)`

[Out]  $\frac{1}{6}x^3(bx^2+a)^{3/2}/b - \frac{1}{8}a/b^2x(bx^2+a)^{3/2} + \frac{1}{16}a^2x(bx^2+a)^{1/2}/b^2 + \frac{1}{16}a^3/b^{5/2} \ln(xb^{1/2} + (bx^2+a)^{1/2})$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*x^4,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.249886, size = 1, normalized size = 0.01

$$\left[ \frac{3a^3 \log\left(-2\sqrt{bx^2+ax} - (2bx^2+a)\sqrt{b}\right) + 2(8b^2x^5 + 2abx^3 - 3a^2x)\sqrt{bx^2+a}\sqrt{b}}{96b^{5/2}}, \frac{3a^3 \arctan\left(\frac{\sqrt{-bx}}{\sqrt{bx^2+a}}\right) + (8b^2x^5 + 2abx^3 - 3a^2x)\sqrt{-b}}{48\sqrt{-b}b^{5/2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*x^4,x, algorithm="fricas")`

[Out]  $\left[ \frac{1}{96} \left( 3a^3 \log(-2\sqrt{bx^2+a}bx - (2bx^2+a)\sqrt{b}) + 2(8b^2x^5 + 2abx^3 - 3a^2x)\sqrt{bx^2+a}\sqrt{b} \right) / b^{5/2}, \frac{1}{48} \left( 3a^3 \arctan(\sqrt{-b}x/\sqrt{bx^2+a}) + (8b^2x^5 + 2abx^3 - 3a^2x)\sqrt{-b} \right) / (\sqrt{-b}b^2) \right]$

**Sympy** [A] time = 18.3848, size = 117, normalized size = 1.24

$$-\frac{a^{5/2}x}{16b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{a^{3/2}x^3}{48b\sqrt{1+\frac{bx^2}{a}}} + \frac{5\sqrt{a}x^5}{24\sqrt{1+\frac{bx^2}{a}}} + \frac{a^3 \operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{16b^{5/2}} + \frac{bx^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(b*x**2+a)**(1/2),x)`

[Out]  $-a^{5/2}x/(16b^2\sqrt{1+bx^2/a}) - a^{3/2}x^3/(48b\sqrt{1+bx^2/a}) + 5\sqrt{a}x^5/(24\sqrt{1+bx^2/a}) + a^3 \operatorname{asinh}(\sqrt{b}x/\sqrt{a})/(16b^{5/2}) + bx^7/(6\sqrt{a}\sqrt{1+bx^2/a})$

**GIAC/XCAS** [A] time = 0.211553, size = 86, normalized size = 0.91

$$\frac{1}{48} \left( 2 \left( 4x^2 + \frac{a}{b} \right) x^2 - \frac{3a^2}{b^2} \right) \sqrt{bx^2+ax} - \frac{a^3 \ln\left(\left| -\sqrt{bx} + \sqrt{bx^2+a} \right| \right)}{16b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)*x^4,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*x^2 + a/b)*x^2 - 3*a^2/b^2)*sqrt(b*x^2 + a)*x - 1/16*a  
^3*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```



### 3.362 $\int x^2 \sqrt{a + bx^2} dx$

**Optimal.** Leaf size=70

$$-\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{8b^{3/2}} + \frac{ax\sqrt{a+bx^2}}{8b} + \frac{1}{4}x^3\sqrt{a+bx^2}$$

[Out] (a\*x\*Sqrt[a + b\*x^2])/(8\*b) + (x^3\*Sqrt[a + b\*x^2])/4 - (a^2\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(8\*b^(3/2))

**Rubi [A]** time = 0.0727705, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{8b^{3/2}} + \frac{ax\sqrt{a+bx^2}}{8b} + \frac{1}{4}x^3\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

[In] Int[x^2\*Sqrt[a + b\*x^2], x]

[Out] (a\*x\*Sqrt[a + b\*x^2])/(8\*b) + (x^3\*Sqrt[a + b\*x^2])/4 - (a^2\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(8\*b^(3/2))

**Rubi in Sympy [A]** time = 9.28896, size = 60, normalized size = 0.86

$$-\frac{a^2 \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{8b^{\frac{3}{2}}} + \frac{ax\sqrt{a+bx^2}}{8b} + \frac{x^3\sqrt{a+bx^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(b\*x\*\*2+a)\*\*(1/2), x)

[Out] -a\*\*2\*atanh(sqrt(b)\*x/sqrt(a + b\*x\*\*2))/(8\*b\*\*(3/2)) + a\*x\*sqrt(a + b\*x\*\*2)/(8\*b) + x\*\*3\*sqrt(a + b\*x\*\*2)/4

**Mathematica [A]** time = 0.0377583, size = 64, normalized size = 0.91

$$\sqrt{a+bx^2} \left( \frac{ax}{8b} + \frac{x^3}{4} \right) - \frac{a^2 \log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right)}{8b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Sqrt[a + b\*x^2], x]

[Out] Sqrt[a + b\*x^2]\*((a\*x)/(8\*b) + x^3/4) - (a^2\*Log[b\*x + Sqrt[b]\*Sqrt[a + b\*x^2]])/(8\*b^(3/2))

**Maple [A]** time = 0.008, size = 57, normalized size = 0.8

$$\frac{x}{4b} (bx^2 + a)^{\frac{3}{2}} - \frac{ax}{8b} \sqrt{bx^2 + a} - \frac{a^2}{8} \ln\left(x\sqrt{b} + \sqrt{bx^2 + a}\right) b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^2+a)^(1/2),x)`

[Out]  $\frac{1}{4}x(bx^2+a)^{3/2}/b - \frac{1}{8}ax(bx^2+a)^{1/2}/b - \frac{1}{8}a^2/b^{3/2} \ln(xb^{1/2} + (bx^2+a)^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*x^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.246921, size = 1, normalized size = 0.01

$$\left[ \frac{a^2 \log\left(2\sqrt{bx^2+ax} - (2bx^2+a)\sqrt{b}\right) + 2(2bx^3+ax)\sqrt{bx^2+a}\sqrt{b}}{16b^{3/2}}, \frac{a^2 \arctan\left(\frac{\sqrt{-bx}}{\sqrt{bx^2+a}}\right) - (2bx^3+ax)\sqrt{bx^2+a}\sqrt{-b}}{8\sqrt{-bb}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*x^2,x, algorithm="fricas")`

[Out]  $[1/16*(a^2*\log(2*\sqrt{bx^2+a}*bx - (2*b*x^2+a)*\sqrt{b})) + 2*(2*b*x^3+a*x)*\sqrt{bx^2+a}*\sqrt{b})/b^{3/2}, -1/8*(a^2*\arctan(\sqrt{-b}*x/\sqrt{bx^2+a}) - (2*b*x^3+a*x)*\sqrt{bx^2+a}*\sqrt{-b})/(\sqrt{-b}*b)]$

**Sympy [A]** time = 11.5336, size = 92, normalized size = 1.31

$$\frac{a^{3/2}x}{8b\sqrt{1+\frac{bx^2}{a}}} + \frac{3\sqrt{a}x^3}{8\sqrt{1+\frac{bx^2}{a}}} - \frac{a^2 \operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8b^{3/2}} + \frac{bx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**2+a)**(1/2),x)`

[Out]  $a^{3/2}x/(8*b*\sqrt{1+b*x^2/a}) + 3*\sqrt{a}*x^3/(8*\sqrt{1+b*x^2/a}) - a^2*2*\operatorname{asinh}(\sqrt{b}*x/\sqrt{a})/(8*b^{3/2}) + b*x^5/(4*\sqrt{a}*\sqrt{1+b*x^2/a})$

**GIAC/XCAS [A]** time = 0.212077, size = 68, normalized size = 0.97

$$\frac{1}{8}\sqrt{bx^2+a}\left(2x^2+\frac{a}{b}\right)x + \frac{a^2\ln\left(\left|-\sqrt{bx}+\sqrt{bx^2+a}\right|\right)}{8b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)*x^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*x^2 + a/b)*x + 1/8*a^2*ln(abs(-sqrt(b)*x +  
sqrt(b*x^2 + a)))/b^(3/2)
```

### 3.363 $\int \sqrt{a + bx^2} dx$

**Optimal.** Leaf size=46

$$\frac{1}{2}x\sqrt{a + bx^2} + \frac{a \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

[Out] (x\*Sqrt[a + b\*x^2])/2 + (a\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(2\*Sqrt[b])

**Rubi [A]** time = 0.0317778, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{1}{2}x\sqrt{a + bx^2} + \frac{a \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x^2], x]

[Out] (x\*Sqrt[a + b\*x^2])/2 + (a\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(2\*Sqrt[b])

**Rubi in Sympy [A]** time = 3.12733, size = 39, normalized size = 0.85

$$\frac{a \operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}} + \frac{x\sqrt{a + bx^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/2), x)

[Out] a\*atanh(sqrt(b)\*x/sqrt(a + b\*x\*\*2))/(2\*sqrt(b)) + x\*sqrt(a + b\*x\*\*2)/2

**Mathematica [A]** time = 0.0268882, size = 49, normalized size = 1.07

$$\frac{1}{2}x\sqrt{a + bx^2} + \frac{a \log\left(\sqrt{b}\sqrt{a + bx^2} + bx\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x^2], x]

[Out] (x\*Sqrt[a + b\*x^2])/2 + (a\*Log[b\*x + Sqrt[b]\*Sqrt[a + b\*x^2]])/(2\*Sqrt[b])

**Maple [A]** time = 0.004, size = 36, normalized size = 0.8

$$\frac{x}{2}\sqrt{bx^2 + a} + \frac{a}{2} \ln\left(x\sqrt{b} + \sqrt{bx^2 + a}\right) \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/2),x)`

[Out] `1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.246303, size = 1, normalized size = 0.02

$$\left[ \frac{2\sqrt{bx^2+a}\sqrt{bx} + a \log\left(-2\sqrt{bx^2+abx} - (2bx^2+a)\sqrt{b}\right)}{4\sqrt{b}}, \frac{\sqrt{bx^2+a}\sqrt{-bx} + a \arctan\left(\frac{\sqrt{-bx}}{\sqrt{bx^2+a}}\right)}{2\sqrt{-b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a),x, algorithm="fricas")`

[Out] `[1/4*(2*sqrt(b*x^2 + a)*sqrt(b)*x + a*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/sqrt(b), 1/2*(sqrt(b*x^2 + a)*sqrt(-b)*x + a*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/sqrt(-b)]`

**Sympy [A]** time = 6.30513, size = 41, normalized size = 0.89

$$\frac{\sqrt{ax}\sqrt{1+\frac{bx^2}{a}}}{2} + \frac{a \operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/2),x)`

[Out] `sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))`

**GIAC/XCAS [A]** time = 0.213936, size = 50, normalized size = 1.09

$$\frac{1}{2}\sqrt{bx^2+ax} - \frac{a \ln\left(\left|-\sqrt{bx} + \sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a),x, algorithm="giac")`

[Out] `1/2*sqrt(b*x^2 + a)*x - 1/2*a*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)`

$$3.364 \quad \int \frac{\sqrt{a+bx^2}}{x^2} dx$$

**Optimal.** Leaf size=42

$$\sqrt{b} \tanh^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a+bx^2}} \right) - \frac{\sqrt{a+bx^2}}{x}$$

[Out] -(Sqrt[a + b\*x^2]/x) + Sqrt[b]\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]]

**Rubi [A]** time = 0.0392507, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\sqrt{b} \tanh^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a+bx^2}} \right) - \frac{\sqrt{a+bx^2}}{x}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x^2]/x^2, x]

[Out] -(Sqrt[a + b\*x^2]/x) + Sqrt[b]\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]]

**Rubi in Sympy [A]** time = 5.208, size = 34, normalized size = 0.81

$$\sqrt{b} \operatorname{atanh} \left( \frac{\sqrt{bx}}{\sqrt{a+bx^2}} \right) - \frac{\sqrt{a+bx^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/2)/x\*\*2, x)

[Out] sqrt(b)\*atanh(sqrt(b)\*x/sqrt(a + b\*x\*\*2)) - sqrt(a + b\*x\*\*2)/x

**Mathematica [A]** time = 0.02174, size = 45, normalized size = 1.07

$$\sqrt{b} \log \left( \sqrt{b} \sqrt{a+bx^2} + bx \right) - \frac{\sqrt{a+bx^2}}{x}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x^2]/x^2, x]

[Out] -(Sqrt[a + b\*x^2]/x) + Sqrt[b]\*Log[b\*x + Sqrt[b]\*Sqrt[a + b\*x^2]]

**Maple [A]** time = 0.006, size = 54, normalized size = 1.3

$$-\frac{1}{ax} (bx^2 + a)^{\frac{3}{2}} + \frac{bx}{a} \sqrt{bx^2 + a} + \sqrt{b} \ln \left( x\sqrt{b} + \sqrt{bx^2 + a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(1/2)/x^2, x)

[Out]  $-1/a/x*(b*x^2+a)^{(3/2)}+b/a*x*(b*x^2+a)^{(1/2)}+b^{(1/2)}*\ln(x*b^{(1/2)}+(b*x^2+a)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/x^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.247891, size = 1, normalized size = 0.02

$$\left[ \frac{\sqrt{bx} \log(-2bx^2 - 2\sqrt{bx^2+a}\sqrt{bx} - a) - 2\sqrt{bx^2+a} \sqrt{-bx} \arctan\left(\frac{bx}{\sqrt{bx^2+a}\sqrt{-b}}\right) - \sqrt{bx^2+a}}{2x}, \frac{\sqrt{-bx} \arctan\left(\frac{bx}{\sqrt{bx^2+a}\sqrt{-b}}\right) - \sqrt{bx^2+a}}{x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/x^2,x, algorithm="fricas")`

[Out]  $[1/2*(\sqrt{b}*x*\log(-2*b*x^2 - 2*\sqrt{b*x^2 + a}*\sqrt{b}*x - a) - 2*\sqrt{b*x^2 + a})/x, (\sqrt{-b}*x*\arctan(b*x/(\sqrt{b*x^2 + a}*\sqrt{-b})) - \sqrt{b*x^2 + a})/x]$

**Sympy [A]** time = 4.87381, size = 56, normalized size = 1.33

$$-\frac{\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}} + \sqrt{b} \operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) - \frac{bx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/2)/x**2,x)`

[Out]  $-\sqrt{a}/(x*\sqrt{1+b*x**2/a}) + \sqrt{b}*\operatorname{asinh}(\sqrt{b}*x/\sqrt{a}) - b*x/(\sqrt{a}*\sqrt{1+b*x**2/a})$

**GIAC/XCAS [A]** time = 0.214872, size = 77, normalized size = 1.83

$$-\frac{1}{2}\sqrt{b}\ln\left(\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^2\right) + \frac{2a\sqrt{b}}{\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/x^2,x, algorithm="giac")`

[Out]  $-1/2*\sqrt{b}*\ln((\sqrt{b}*x - \sqrt{b*x^2 + a})^2) + 2*a*\sqrt{b}/((\sqrt{b}*x - \sqrt{b*x^2 + a})^2 - a)$

$$3.365 \quad \int \frac{\sqrt{a+bx^2}}{x^4} dx$$

**Optimal.** Leaf size=21

$$-\frac{(a+bx^2)^{3/2}}{3ax^3}$$

[Out]  $-(a + b*x^2)^{(3/2)}/(3*a*x^3)$

**Rubi [A]** time = 0.0219531, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$-\frac{(a+bx^2)^{3/2}}{3ax^3}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + b*x^2]/x^4, x]`

[Out]  $-(a + b*x^2)^{(3/2)}/(3*a*x^3)$

**Rubi in Sympy [A]** time = 3.1768, size = 17, normalized size = 0.81

$$-\frac{(a+bx^2)^{\frac{3}{2}}}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(1/2)/x**4, x)`

[Out]  $-(a + b*x**2)**(3/2)/(3*a*x**3)$

**Mathematica [A]** time = 0.0151685, size = 21, normalized size = 1.

$$-\frac{(a+bx^2)^{3/2}}{3ax^3}$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[a + b*x^2]/x^4, x]`

[Out]  $-(a + b*x^2)^{(3/2)}/(3*a*x^3)$

**Maple [A]** time = 0.006, size = 18, normalized size = 0.9

$$-\frac{1}{3ax^3} (bx^2 + a)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/2)/x^4, x)`

[Out]  $-1/3*(b*x^2+a)^{(3/2)}/a/x^3$



---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b\*x^2 + a)/x^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.234115, size = 23, normalized size = 1.1

$$-\frac{(bx^2 + a)^{\frac{3}{2}}}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b\*x^2 + a)/x^4,x, algorithm="fricas")

[Out] -1/3\*(b\*x^2 + a)^(3/2)/(a\*x^3)

---

**Sympy [A]** time = 2.0858, size = 42, normalized size = 2.

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2} + 1}}{3x^2} - \frac{b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2} + 1}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/2)/x\*\*4,x)

[Out] -sqrt(b)\*sqrt(a/(b\*x\*\*2) + 1)/(3\*x\*\*2) - b\*\*(3/2)\*sqrt(a/(b\*x\*\*2) + 1)/(3\*a)

---

**GIAC/XCAS [A]** time = 0.211832, size = 80, normalized size = 3.81

$$\frac{2 \left( 3 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^4 b^{\frac{3}{2}} + a^2 b^{\frac{3}{2}} \right)}{3 \left( \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 - a \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b\*x^2 + a)/x^4,x, algorithm="giac")

[Out] 2/3\*(3\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^4\*b^(3/2) + a^2\*b^(3/2))/((sqrt(b)\*x - sqrt(b\*x^2 + a))^2 - a)^3

$$3.366 \quad \int \frac{\sqrt{a+bx^2}}{x^6} dx$$

**Optimal.** Leaf size=44

$$\frac{2b(a+bx^2)^{3/2}}{15a^2x^3} - \frac{(a+bx^2)^{3/2}}{5ax^5}$$

[Out]  $-(a + b*x^2)^{(3/2)}/(5*a*x^5) + (2*b*(a + b*x^2)^{(3/2)})/(15*a^2*x^3)$

**Rubi [A]** time = 0.044526, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{2b(a+bx^2)^{3/2}}{15a^2x^3} - \frac{(a+bx^2)^{3/2}}{5ax^5}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x^2]/x^6, x]

[Out]  $-(a + b*x^2)^{(3/2)}/(5*a*x^5) + (2*b*(a + b*x^2)^{(3/2)})/(15*a^2*x^3)$

**Rubi in Sympy [A]** time = 5.1883, size = 37, normalized size = 0.84

$$-\frac{(a+bx^2)^{\frac{3}{2}}}{5ax^5} + \frac{2b(a+bx^2)^{\frac{3}{2}}}{15a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/2)/x\*\*6, x)

[Out]  $-(a + b*x^2)^{(3/2)}/(5*a*x^5) + 2*b*(a + b*x^2)^{(3/2)}/(15*a^2*x^3)$

**Mathematica [A]** time = 0.0193084, size = 41, normalized size = 0.93

$$-\frac{\sqrt{a+bx^2}(3a^2+abx^2-2b^2x^4)}{15a^2x^5}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x^2]/x^6, x]

[Out]  $-(\text{Sqrt}[a + b*x^2]*(3*a^2 + a*b*x^2 - 2*b^2*x^4))/(15*a^2*x^5)$

**Maple [A]** time = 0.006, size = 28, normalized size = 0.6

$$-\frac{-2bx^2+3a}{15a^2x^5}(bx^2+a)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(1/2)/x^6, x)

[Out]  $-1/15 * (b * x^2 + a)^{(3/2)} * (-2 * b * x^2 + 3 * a) / a^2 / x^5$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/x^6,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.240028, size = 51, normalized size = 1.16

$$\frac{(2b^2x^4 - abx^2 - 3a^2)\sqrt{bx^2 + a}}{15a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/x^6,x, algorithm="fricas")`

[Out]  $1/15 * (2 * b^2 * x^4 - a * b * x^2 - 3 * a^2) * \text{sqrt}(b * x^2 + a) / (a^2 * x^5)$

**Sympy [A]** time = 2.91928, size = 68, normalized size = 1.55

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2} + 1}}{5x^4} - \frac{b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2} + 1}}{15ax^2} + \frac{2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2} + 1}}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/2)/x**6,x)`

[Out]  $-\text{sqrt}(b) * \text{sqrt}(a / (b * x^2) + 1) / (5 * x^4) - b^{(3/2)} * \text{sqrt}(a / (b * x^2) + 1) / (15 * a * x^2) + 2 * b^{(5/2)} * \text{sqrt}(a / (b * x^2) + 1) / (15 * a^2)$

**GIAC/XCAS [A]** time = 0.21318, size = 151, normalized size = 3.43

$$\frac{4 \left( 15 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^6 b^{\frac{5}{2}} + 5 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^4 ab^{\frac{5}{2}} + 5 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 a^2 b^{\frac{5}{2}} - a^3 b^{\frac{5}{2}} \right)}{15 \left( \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 - a \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/x^6,x, algorithm="giac")`

[Out]  $4/15 * (15 * (\text{sqrt}(b) * x - \text{sqrt}(b * x^2 + a))^6 * b^{(5/2)} + 5 * (\text{sqrt}(b) * x - \text{sqrt}(b * x^2 + a))^4 * a * b^{(5/2)} + 5 * (\text{sqrt}(b) * x - \text{sqrt}(b * x^2 + a))^2 * a^2 * b^{(5/2)} - a^3 * b^{(5/2)}) / ((\text{sqrt}(b) * x - \text{sqrt}(b * x^2 + a))^2 - a)^5$

$$3.367 \quad \int \frac{\sqrt{a+bx^2}}{x^8} dx$$

**Optimal.** Leaf size=68

$$-\frac{8b^2(a+bx^2)^{3/2}}{105a^3x^3} + \frac{4b(a+bx^2)^{3/2}}{35a^2x^5} - \frac{(a+bx^2)^{3/2}}{7ax^7}$$

[Out]  $-(a + b*x^2)^{(3/2)}/(7*a*x^7) + (4*b*(a + b*x^2)^{(3/2)})/(35*a^2*x^5) - (8*b^2*(a + b*x^2)^{(3/2)})/(105*a^3*x^3)$

**Rubi [A]** time = 0.0714653, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{8b^2(a+bx^2)^{3/2}}{105a^3x^3} + \frac{4b(a+bx^2)^{3/2}}{35a^2x^5} - \frac{(a+bx^2)^{3/2}}{7ax^7}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x^2]/x^8, x]

[Out]  $-(a + b*x^2)^{(3/2)}/(7*a*x^7) + (4*b*(a + b*x^2)^{(3/2)})/(35*a^2*x^5) - (8*b^2*(a + b*x^2)^{(3/2)})/(105*a^3*x^3)$

**Rubi in Sympy [A]** time = 8.10911, size = 61, normalized size = 0.9

$$-\frac{(a+bx^2)^{\frac{3}{2}}}{7ax^7} + \frac{4b(a+bx^2)^{\frac{3}{2}}}{35a^2x^5} - \frac{8b^2(a+bx^2)^{\frac{3}{2}}}{105a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/2)/x\*\*8, x)

[Out]  $-(a + b*x^2)^{(3/2)}/(7*a*x^7) + 4*b*(a + b*x^2)^{(3/2)}/(35*a^2*x^5) - 8*b^2*(a + b*x^2)^{(3/2)}/(105*a^3*x^3)$

**Mathematica [A]** time = 0.0243929, size = 53, normalized size = 0.78

$$-\frac{\sqrt{a+bx^2}(15a^3 + 3a^2bx^2 - 4ab^2x^4 + 8b^3x^6)}{105a^3x^7}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x^2]/x^8, x]

[Out]  $-(\text{Sqrt}[a + b*x^2]*(15*a^3 + 3*a^2*b*x^2 - 4*a*b^2*x^4 + 8*b^3*x^6))/(105*a^3*x^7)$

**Maple [A]** time = 0.007, size = 39, normalized size = 0.6

$$-\frac{8b^2x^4 - 12abx^2 + 15a^2}{105a^3x^7} (bx^2 + a)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/2)/x^8,x)`

[Out]  $-1/105*(b*x^2+a)^{(3/2)}*(8*b^2*x^4-12*a*b*x^2+15*a^2)/a^3/x^7$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/x^8,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.244603, size = 66, normalized size = 0.97

$$\frac{(8b^3x^6 - 4ab^2x^4 + 3a^2bx^2 + 15a^3)\sqrt{bx^2 + a}}{105a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/x^8,x, algorithm="fricas")`

[Out]  $-1/105*(8*b^3*x^6 - 4*a*b^2*x^4 + 3*a^2*b*x^2 + 15*a^3)*\sqrt{b*x^2 + a}/(a^3*x^7)$

**Sympy [A]** time = 4.49782, size = 359, normalized size = 5.28

$$\begin{aligned} & \frac{15a^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2} + 1}}{105a^5b^4x^6 + 210a^4b^5x^8 + 105a^3b^6x^{10}} - \frac{33a^4b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2} + 1}}{105a^5b^4x^6 + 210a^4b^5x^8 + 105a^3b^6x^{10}} \\ & - \frac{17a^3b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2} + 1}}{105a^5b^4x^6 + 210a^4b^5x^8 + 105a^3b^6x^{10}} - \frac{3a^2b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2} + 1}}{105a^5b^4x^6 + 210a^4b^5x^8 + 105a^3b^6x^{10}} \\ & - \frac{12ab^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2} + 1}}{105a^5b^4x^6 + 210a^4b^5x^8 + 105a^3b^6x^{10}} - \frac{8b^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2} + 1}}{105a^5b^4x^6 + 210a^4b^5x^8 + 105a^3b^6x^{10}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/2)/x**8,x)`

[Out]  $-15*a**5*b**(9/2)*\sqrt{a/(b*x**2) + 1}/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 33*a**4*b**(11/2)*x**2*\sqrt{a/(b*x**2) + 1}/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*a**3*b**(13/2)*x**4*\sqrt{a/(b*x**2) + 1}/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 3*a**2*b**(15/2)*x**6*\sqrt{a/(b*x**2) + 1}/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*a*b**(17/2)*x**8*\sqrt{a/(b*x**2) + 1}/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 8*b**(19/2)*x**10*\sqrt{a/(b*x**2) + 1}/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10)$

**GIAC/XCAS [A]** time = 0.214277, size = 186, normalized size = 2.74

$$\frac{16\left(70\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^8 b^{\frac{7}{2}} + 35\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^6 ab^{\frac{7}{2}} + 21\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^4 a^2 b^{\frac{7}{2}} - 7\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^2 a^3 b^{\frac{7}{2}} + a^4\right)}{105\left(\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^2 - a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/x^8,x, algorithm="giac")`

[Out] 
$$\frac{16}{105} \cdot (70 \cdot (\sqrt{b} \cdot x - \sqrt{b \cdot x^2 + a})^8 \cdot b^{7/2} + 35 \cdot (\sqrt{b} \cdot x - \sqrt{b \cdot x^2 + a})^6 \cdot a \cdot b^{7/2} + 21 \cdot (\sqrt{b} \cdot x - \sqrt{b \cdot x^2 + a})^4 \cdot a^2 \cdot b^{7/2} - 7 \cdot (\sqrt{b} \cdot x - \sqrt{b \cdot x^2 + a})^2 \cdot a^3 \cdot b^{7/2} + a^4 \cdot b^{7/2}) / ((\sqrt{b} \cdot x - \sqrt{b \cdot x^2 + a})^2 - a)^7$$

$$3.368 \quad \int \frac{\sqrt{a+bx^2}}{x^{10}} dx$$

**Optimal.** Leaf size=92

$$\frac{16b^3 (a+bx^2)^{3/2}}{315a^4x^3} - \frac{8b^2 (a+bx^2)^{3/2}}{105a^3x^5} + \frac{2b (a+bx^2)^{3/2}}{21a^2x^7} - \frac{(a+bx^2)^{3/2}}{9ax^9}$$

[Out]  $-(a + b*x^2)^{(3/2)}/(9*a*x^9) + (2*b*(a + b*x^2)^{(3/2)})/(21*a^2*x^7) - (8*b^2*(a + b*x^2)^{(3/2)})/(105*a^3*x^5) + (16*b^3*(a + b*x^2)^{(3/2)})/(315*a^4*x^3)$

**Rubi [A]** time = 0.101193, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{16b^3 (a+bx^2)^{3/2}}{315a^4x^3} - \frac{8b^2 (a+bx^2)^{3/2}}{105a^3x^5} + \frac{2b (a+bx^2)^{3/2}}{21a^2x^7} - \frac{(a+bx^2)^{3/2}}{9ax^9}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x^2]/x^10, x]

[Out]  $-(a + b*x^2)^{(3/2)}/(9*a*x^9) + (2*b*(a + b*x^2)^{(3/2)})/(21*a^2*x^7) - (8*b^2*(a + b*x^2)^{(3/2)})/(105*a^3*x^5) + (16*b^3*(a + b*x^2)^{(3/2)})/(315*a^4*x^3)$

**Rubi in Sympy [A]** time = 11.6256, size = 85, normalized size = 0.92

$$-\frac{(a+bx^2)^{\frac{3}{2}}}{9ax^9} + \frac{2b(a+bx^2)^{\frac{3}{2}}}{21a^2x^7} - \frac{8b^2(a+bx^2)^{\frac{3}{2}}}{105a^3x^5} + \frac{16b^3(a+bx^2)^{\frac{3}{2}}}{315a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/2)/x\*\*10, x)

[Out]  $-(a + b*x^2)^{(3/2)}/(9*a*x^9) + 2*b*(a + b*x^2)^{(3/2)}/(21*a^2*x^7) - 8*b^2*(a + b*x^2)^{(3/2)}/(105*a^3*x^5) + 16*b^3*(a + b*x^2)^{(3/2)}/(315*a^4*x^3)$

**Mathematica [A]** time = 0.0310576, size = 64, normalized size = 0.7

$$-\frac{\sqrt{a+bx^2} (35a^4 + 5a^3bx^2 - 6a^2b^2x^4 + 8ab^3x^6 - 16b^4x^8)}{315a^4x^9}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x^2]/x^10, x]

[Out]  $-(\text{Sqrt}[a + b*x^2]*(35*a^4 + 5*a^3*b*x^2 - 6*a^2*b^2*x^4 + 8*a*b^3*x^6 - 16*b^4*x^8))/(315*a^4*x^9)$

**Maple [A]** time = 0.007, size = 50, normalized size = 0.5

$$-\frac{-16b^3x^6 + 24ab^2x^4 - 30a^2bx^2 + 35a^3}{315x^9a^4} (bx^2 + a)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/2)/x^10,x)`

[Out]  $-1/315 * (b*x^2+a)^{(3/2)} * (-16*b^3*x^6+24*a*b^2*x^4-30*a^2*b*x^2+35*a^3)/x^9/a^4$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/x^10,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.265322, size = 81, normalized size = 0.88

$$\frac{(16b^4x^8 - 8ab^3x^6 + 6a^2b^2x^4 - 5a^3bx^2 - 35a^4)\sqrt{bx^2 + a}}{315a^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/x^10,x, algorithm="fricas")`

[Out]  $1/315*(16*b^4*x^8 - 8*a*b^3*x^6 + 6*a^2*b^2*x^4 - 5*a^3*b*x^2 - 35*a^4)*\sqrt{(b*x^2 + a)/(a^4*x^9)}$

**Sympy [A]** time = 6.61436, size = 575, normalized size = 6.25

$$\begin{aligned} & \frac{35a^7b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2} + 1}}{315a^7b^9x^8 + 945a^6b^{10}x^{10} + 945a^5b^{11}x^{12} + 315a^4b^{12}x^{14}} \\ & - \frac{110a^6b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2} + 1}}{315a^7b^9x^8 + 945a^6b^{10}x^{10} + 945a^5b^{11}x^{12} + 315a^4b^{12}x^{14}} \\ & - \frac{114a^5b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2} + 1}}{315a^7b^9x^8 + 945a^6b^{10}x^{10} + 945a^5b^{11}x^{12} + 315a^4b^{12}x^{14}} \\ & - \frac{40a^4b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2} + 1}}{315a^7b^9x^8 + 945a^6b^{10}x^{10} + 945a^5b^{11}x^{12} + 315a^4b^{12}x^{14}} \\ & + \frac{5a^3b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2} + 1}}{315a^7b^9x^8 + 945a^6b^{10}x^{10} + 945a^5b^{11}x^{12} + 315a^4b^{12}x^{14}} \\ & + \frac{30a^2b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2} + 1}}{315a^7b^9x^8 + 945a^6b^{10}x^{10} + 945a^5b^{11}x^{12} + 315a^4b^{12}x^{14}} \\ & + \frac{40ab^{\frac{31}{2}}x^{12}\sqrt{\frac{a}{bx^2} + 1}}{315a^7b^9x^8 + 945a^6b^{10}x^{10} + 945a^5b^{11}x^{12} + 315a^4b^{12}x^{14}} \\ & + \frac{16b^{\frac{33}{2}}x^{14}\sqrt{\frac{a}{bx^2} + 1}}{315a^7b^9x^8 + 945a^6b^{10}x^{10} + 945a^5b^{11}x^{12} + 315a^4b^{12}x^{14}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/2)/x**10,x)`



```
[Out] -35*a**7*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945
*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14)
- 110*a**6*b**(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**
8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*
x**14) - 114*a**5*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(315*a**7*b
**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4
*b**12*x**14) - 40*a**4*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(315*
a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 31
5*a**4*b**12*x**14) + 5*a**3*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/
(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12
+ 315*a**4*b**12*x**14) + 30*a**2*b**(29/2)*x**10*sqrt(a/(b*x**2
) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**1
1*x**12 + 315*a**4*b**12*x**14) + 40*a*b**(31/2)*x**12*sqrt(a/(b*
x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*
b**11*x**12 + 315*a**4*b**12*x**14) + 16*b**(33/2)*x**14*sqrt(a/(
b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**
5*b**11*x**12 + 315*a**4*b**12*x**14)
```

**GIAC/XCAS [A]** time = 0.211792, size = 224, normalized size = 2.43

$$\frac{32 \left( 315 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{10} b^{\frac{9}{2}} + 189 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^8 ab^{\frac{9}{2}} + 84 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^6 a^2 b^{\frac{9}{2}} - 36 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^4 a^3 b^{\frac{9}{2}} \right)}{315 \left( \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 - a \right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)/x^10,x, algorithm="giac")
```

```
[Out] 32/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(9/2) + 189*(sqrt(
b)*x - sqrt(b*x^2 + a))^8*a*b^(9/2) + 84*(sqrt(b)*x - sqrt(b*x^2
+ a))^6*a^2*b^(9/2) - 36*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(9
/2) + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(9/2) - a^5*b^(9/2)
)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

$$3.369 \quad \int x^7 (a + bx^2)^{3/2} dx$$

**Optimal.** Leaf size=80

$$-\frac{a^3 (a + bx^2)^{5/2}}{5b^4} + \frac{3a^2 (a + bx^2)^{7/2}}{7b^4} + \frac{(a + bx^2)^{11/2}}{11b^4} - \frac{a (a + bx^2)^{9/2}}{3b^4}$$

[Out]  $-(a^3 (a + b*x^2)^{(5/2)})/(5*b^4) + (3*a^2*(a + b*x^2)^{(7/2)})/(7*b^4) - (a*(a + b*x^2)^{(9/2)})/(3*b^4) + (a + b*x^2)^{(11/2)}/(11*b^4)$

**Rubi [A]** time = 0.11816, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{a^3 (a + bx^2)^{5/2}}{5b^4} + \frac{3a^2 (a + bx^2)^{7/2}}{7b^4} + \frac{(a + bx^2)^{11/2}}{11b^4} - \frac{a (a + bx^2)^{9/2}}{3b^4}$$

Antiderivative was successfully verified.

[In] Int[x^7\*(a + b\*x^2)^(3/2), x]

[Out]  $-(a^3 (a + b*x^2)^{(5/2)})/(5*b^4) + (3*a^2*(a + b*x^2)^{(7/2)})/(7*b^4) - (a*(a + b*x^2)^{(9/2)})/(3*b^4) + (a + b*x^2)^{(11/2)}/(11*b^4)$

**Rubi in Sympy [A]** time = 15.6366, size = 70, normalized size = 0.88

$$-\frac{a^3 (a + bx^2)^{5/2}}{5b^4} + \frac{3a^2 (a + bx^2)^{7/2}}{7b^4} - \frac{a (a + bx^2)^{9/2}}{3b^4} + \frac{(a + bx^2)^{11/2}}{11b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*7\*(b\*x\*\*2+a)\*\*(3/2), x)

[Out]  $-a**3*(a + b*x**2)**(5/2)/(5*b**4) + 3*a**2*(a + b*x**2)**(7/2)/(7*b**4) - a*(a + b*x**2)**(9/2)/(3*b**4) + (a + b*x**2)**(11/2)/(11*b**4)$

**Mathematica [A]** time = 0.0446056, size = 50, normalized size = 0.62

$$\frac{(a + bx^2)^{5/2} (-16a^3 + 40a^2bx^2 - 70ab^2x^4 + 105b^3x^6)}{1155b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^7\*(a + b\*x^2)^(3/2), x]

[Out]  $((a + b*x^2)^{(5/2)}*(-16*a^3 + 40*a^2*b*x^2 - 70*a*b^2*x^4 + 105*b^3*x^6))/(1155*b^4)$

**Maple [A]** time = 0.009, size = 47, normalized size = 0.6

$$-\frac{-105 b^3 x^6 + 70 a b^2 x^4 - 40 a^2 b x^2 + 16 a^3}{1155 b^4} (bx^2 + a)^{5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(b*x^2+a)^(3/2),x)`

[Out]  $-1/1155*(b*x^2+a)^{(5/2)}*(-105*b^3*x^6+70*a*b^2*x^4-40*a^2*b*x^2+16*a^3)/b^4$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)*x^7,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.237386, size = 92, normalized size = 1.15

$$\frac{(105 b^5 x^{10} + 140 a b^4 x^8 + 5 a^2 b^3 x^6 - 6 a^3 b^2 x^4 + 8 a^4 b x^2 - 16 a^5) \sqrt{b x^2 + a}}{1155 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)*x^7,x, algorithm="fricas")`

[Out]  $1/1155*(105*b^5*x^{10} + 140*a*b^4*x^8 + 5*a^2*b^3*x^6 - 6*a^3*b^2*x^4 + 8*a^4*b*x^2 - 16*a^5)*\text{sqrt}(b*x^2 + a)/b^4$

**Sympy [A]** time = 11.7839, size = 133, normalized size = 1.66

$$\begin{cases} -\frac{16a^5\sqrt{a+bx^2}}{1155b^4} + \frac{8a^4x^2\sqrt{a+bx^2}}{1155b^3} - \frac{2a^3x^4\sqrt{a+bx^2}}{385b^2} + \frac{a^2x^6\sqrt{a+bx^2}}{231b} + \frac{4ax^8\sqrt{a+bx^2}}{33} + \frac{bx^{10}\sqrt{a+bx^2}}{11} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(b*x**2+a)**(3/2),x)`

[Out] `Piecewise((-16*a**5*sqrt(a + b*x**2)/(1155*b**4) + 8*a**4*x**2*sqrt(a + b*x**2)/(1155*b**3) - 2*a**3*x**4*sqrt(a + b*x**2)/(385*b**2) + a**2*x**6*sqrt(a + b*x**2)/(231*b) + 4*a*x**8*sqrt(a + b*x**2)/33 + b*x**10*sqrt(a + b*x**2)/11, Ne(b, 0)), (a**(3/2)*x**8/8, True))`

**GIAC/XCAS [A]** time = 0.209992, size = 181, normalized size = 2.26

$$\frac{11 \left( 35 (bx^2+a)^{\frac{9}{2}} - 135 (bx^2+a)^{\frac{7}{2}} a + 189 (bx^2+a)^{\frac{5}{2}} a^2 - 105 (bx^2+a)^{\frac{3}{2}} a^3 \right) a}{b^3} + \frac{315 (bx^2+a)^{\frac{11}{2}} - 1540 (bx^2+a)^{\frac{9}{2}} a + 2970 (bx^2+a)^{\frac{7}{2}} a^2 - 2772 (bx^2+a)^{\frac{5}{2}} a^3 + 1155 (bx^2+a)^{\frac{3}{2}} a^4}{b^3}$$

$3465 b$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)*x^7,x, algorithm="giac")`

[Out]  $1/3465*(11*(35*(b*x^2 + a)^{(9/2)} - 135*(b*x^2 + a)^{(7/2)}*a + 189*(b*x^2 + a)^{(5/2)}*a^2 - 105*(b*x^2 + a)^{(3/2)}*a^3)*a/b^3 + (315*($

$$\frac{b^2 x^2 + a)^{11/2} - 1540 (b^2 x^2 + a)^{9/2} a + 2970 (b^2 x^2 + a)^{7/2} a^2 - 2772 (b^2 x^2 + a)^{5/2} a^3 + 1155 (b^2 x^2 + a)^{3/2} a^4}{b^3}$$

$$3.370 \quad \int x^5 (a + bx^2)^{3/2} dx$$

**Optimal.** Leaf size=59

$$\frac{a^2 (a + bx^2)^{5/2}}{5b^3} + \frac{(a + bx^2)^{9/2}}{9b^3} - \frac{2a (a + bx^2)^{7/2}}{7b^3}$$

[Out]  $(a^2 (a + b x^2)^{(5/2)}) / (5 b^3) - (2 a (a + b x^2)^{(7/2)}) / (7 b^3) + (a + b x^2)^{(9/2)} / (9 b^3)$

**Rubi [A]** time = 0.0935262, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{a^2 (a + bx^2)^{5/2}}{5b^3} + \frac{(a + bx^2)^{9/2}}{9b^3} - \frac{2a (a + bx^2)^{7/2}}{7b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(a + b\*x^2)^(3/2), x]

[Out]  $(a^2 (a + b x^2)^{(5/2)}) / (5 b^3) - (2 a (a + b x^2)^{(7/2)}) / (7 b^3) + (a + b x^2)^{(9/2)} / (9 b^3)$

**Rubi in Sympy [A]** time = 11.7695, size = 51, normalized size = 0.86

$$\frac{a^2 (a + bx^2)^{\frac{5}{2}}}{5b^3} - \frac{2a (a + bx^2)^{\frac{7}{2}}}{7b^3} + \frac{(a + bx^2)^{\frac{9}{2}}}{9b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(b\*x\*\*2+a)\*\*(3/2), x)

[Out]  $a**2*(a + b*x**2)**(5/2)/(5*b**3) - 2*a*(a + b*x**2)**(7/2)/(7*b**3) + (a + b*x**2)**(9/2)/(9*b**3)$

**Mathematica [A]** time = 0.0355322, size = 39, normalized size = 0.66

$$\frac{(a + bx^2)^{5/2} (8a^2 - 20abx^2 + 35b^2x^4)}{315b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*x^2)^(3/2), x]

[Out]  $((a + b x^2)^{(5/2)} (8 a^2 - 20 a b x^2 + 35 b^2 x^4)) / (315 b^3)$

**Maple [A]** time = 0.006, size = 36, normalized size = 0.6

$$\frac{35 b^2 x^4 - 20 a b x^2 + 8 a^2}{315 b^3} (b x^2 + a)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(b*x^2+a)^(3/2),x)`

[Out]  $1/315*(b*x^2+a)^{(5/2)}*(35*b^2*x^4-20*a*b*x^2+8*a^2)/b^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)*x^5,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.238168, size = 77, normalized size = 1.31

$$\frac{(35 b^4 x^8 + 50 a b^3 x^6 + 3 a^2 b^2 x^4 - 4 a^3 b x^2 + 8 a^4) \sqrt{b x^2 + a}}{315 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)*x^5,x, algorithm="fricas")`

[Out]  $1/315*(35*b^4*x^8 + 50*a*b^3*x^6 + 3*a^2*b^2*x^4 - 4*a^3*b*x^2 + 8*a^4)*\text{sqrt}(b*x^2 + a)/b^3$

**Sympy [A]** time = 6.34492, size = 109, normalized size = 1.85

$$\begin{cases} \frac{8a^4\sqrt{a+bx^2}}{315b^3} - \frac{4a^3x^2\sqrt{a+bx^2}}{315b^2} + \frac{a^2x^4\sqrt{a+bx^2}}{105b} + \frac{10ax^6\sqrt{a+bx^2}}{63} + \frac{bx^8\sqrt{a+bx^2}}{9} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(b*x**2+a)**(3/2),x)`

[Out] `Piecewise((8*a**4*sqrt(a + b*x**2)/(315*b**3) - 4*a**3*x**2*sqrt(a + b*x**2)/(315*b**2) + a**2*x**4*sqrt(a + b*x**2)/(105*b) + 10*a*x**6*sqrt(a + b*x**2)/63 + b*x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (a**(3/2)*x**6/6, True))`

**GIAC/XCAS [A]** time = 0.206692, size = 143, normalized size = 2.42

$$\frac{3 \left( 15 (bx^2+a)^{\frac{7}{2}} - 42 (bx^2+a)^{\frac{5}{2}} a + 35 (bx^2+a)^{\frac{3}{2}} a^2 \right) a}{b^2} + \frac{35 (bx^2+a)^{\frac{9}{2}} - 135 (bx^2+a)^{\frac{7}{2}} a + 189 (bx^2+a)^{\frac{5}{2}} a^2 - 105 (bx^2+a)^{\frac{3}{2}} a^3}{b^2}$$

315 b

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)*x^5,x, algorithm="giac")`

[Out]  $1/315*(3*(15*(b*x^2 + a)^{(7/2)} - 42*(b*x^2 + a)^{(5/2)}*a + 35*(b*x^2 + a)^{(3/2)}*a^2)*a/b^2 + (35*(b*x^2 + a)^{(9/2)} - 135*(b*x^2 + a)^{(7/2)}*a + 189*(b*x^2 + a)^{(5/2)}*a^2 - 105*(b*x^2 + a)^{(3/2)}*a^3)/b^2)/b$

$$3.371 \quad \int x^3 (a + bx^2)^{3/2} dx$$

**Optimal.** Leaf size=38

$$\frac{(a + bx^2)^{7/2}}{7b^2} - \frac{a(a + bx^2)^{5/2}}{5b^2}$$

[Out]  $-(a*(a + b*x^2)^(5/2))/(5*b^2) + (a + b*x^2)^(7/2)/(7*b^2)$

**Rubi [A]** time = 0.0657783, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{(a + bx^2)^{7/2}}{7b^2} - \frac{a(a + bx^2)^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*x^2)^(3/2), x]

[Out]  $-(a*(a + b*x^2)^(5/2))/(5*b^2) + (a + b*x^2)^(7/2)/(7*b^2)$

**Rubi in Sympy [A]** time = 7.91357, size = 31, normalized size = 0.82

$$-\frac{a(a + bx^2)^{5/2}}{5b^2} + \frac{(a + bx^2)^{7/2}}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(b\*x\*\*2+a)\*\*(3/2), x)

[Out]  $-a*(a + b*x**2)**(5/2)/(5*b**2) + (a + b*x**2)**(7/2)/(7*b**2)$

**Mathematica [A]** time = 0.0316473, size = 28, normalized size = 0.74

$$\frac{(a + bx^2)^{5/2} (5bx^2 - 2a)}{35b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*x^2)^(3/2), x]

[Out]  $((a + b*x^2)^(5/2)*(-2*a + 5*b*x^2))/(35*b^2)$

**Maple [A]** time = 0.008, size = 25, normalized size = 0.7

$$-\frac{-5bx^2 + 2a}{35b^2} (bx^2 + a)^{5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(b\*x^2+a)^(3/2), x)

[Out]  $-1/35*(b*x^2+a)^(5/2)*(-5*b*x^2+2*a)/b^2$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)\*x^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.234097, size = 61, normalized size = 1.61

$$\frac{(5b^3x^6 + 8ab^2x^4 + a^2bx^2 - 2a^3)\sqrt{bx^2 + a}}{35b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)\*x^3,x, algorithm="fricas")

[Out] 1/35\*(5\*b^3\*x^6 + 8\*a\*b^2\*x^4 + a^2\*b\*x^2 - 2\*a^3)\*sqrt(b\*x^2 + a)/b^2

---

**Sympy [A]** time = 3.44814, size = 85, normalized size = 2.24

$$\begin{cases} -\frac{2a^3\sqrt{a+bx^2}}{35b^2} + \frac{a^2x^2\sqrt{a+bx^2}}{35b} + \frac{8ax^4\sqrt{a+bx^2}}{35} + \frac{bx^6\sqrt{a+bx^2}}{7} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(b\*x\*\*2+a)\*\*(3/2),x)

[Out] Piecewise((-2\*a\*\*3\*sqrt(a + b\*x\*\*2)/(35\*b\*\*2) + a\*\*2\*x\*\*2\*sqrt(a + b\*x\*\*2)/(35\*b) + 8\*a\*x\*\*4\*sqrt(a + b\*x\*\*2)/35 + b\*x\*\*6\*sqrt(a + b\*x\*\*2)/7, Ne(b, 0)), (a\*\*(3/2)\*x\*\*4/4, True))

---

**GIAC/XCAS [A]** time = 0.212545, size = 105, normalized size = 2.76

$$\frac{7\left(3(bx^2+a)^{\frac{5}{2}}-5(bx^2+a)^{\frac{3}{2}}\right)a}{b} + \frac{15(bx^2+a)^{\frac{7}{2}}-42(bx^2+a)^{\frac{5}{2}}a+35(bx^2+a)^{\frac{3}{2}}a^2}{105b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)\*x^3,x, algorithm="giac")

[Out] 1/105\*(7\*(3\*(b\*x^2 + a)^(5/2) - 5\*(b\*x^2 + a)^(3/2)\*a)\*a/b + (15\*(b\*x^2 + a)^(7/2) - 42\*(b\*x^2 + a)^(5/2)\*a + 35\*(b\*x^2 + a)^(3/2)\*a^2)/b/b



$$3.372 \quad \int x (a + bx^2)^{3/2} dx$$

**Optimal.** Leaf size=18

$$\frac{(a + bx^2)^{5/2}}{5b}$$

[Out]  $(a + b*x^2)^{(5/2)}/(5*b)$

**Rubi [A]** time = 0.0113552, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{(a + bx^2)^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In] `Int[x*(a + b*x^2)^(3/2), x]`

[Out]  $(a + b*x^2)^{(5/2)}/(5*b)$

**Rubi in Sympy [A]** time = 2.22883, size = 12, normalized size = 0.67

$$\frac{(a + bx^2)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x*(b*x**2+a)**(3/2), x)`

[Out]  $(a + b*x**2)**(5/2)/(5*b)$

**Mathematica [A]** time = 0.00598816, size = 18, normalized size = 1.

$$\frac{(a + bx^2)^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In] `Integrate[x*(a + b*x^2)^(3/2), x]`

[Out]  $(a + b*x^2)^{(5/2)}/(5*b)$

**Maple [A]** time = 0.004, size = 15, normalized size = 0.8

$$\frac{1}{5b} (bx^2 + a)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x^2+a)^(3/2), x)`

[Out]  $1/5*(b*x^2+a)^{(5/2)}/b$

---

**Maxima [A]** time = 1.34003, size = 19, normalized size = 1.06

$$\frac{(bx^2 + a)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)*x,x, algorithm="maxima")`

[Out] `1/5*(b*x^2 + a)^(5/2)/b`

---

**Fricas [A]** time = 0.241192, size = 43, normalized size = 2.39

$$\frac{(b^2x^4 + 2abx^2 + a^2)\sqrt{bx^2 + a}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)*x,x, algorithm="fricas")`

[Out] `1/5*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)/b`

---

**Sympy [A]** time = 1.62562, size = 61, normalized size = 3.39

$$\begin{cases} \frac{a^2\sqrt{a+bx^2}}{5b} + \frac{2ax^2\sqrt{a+bx^2}}{5} + \frac{bx^4\sqrt{a+bx^2}}{5} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x**2+a)**(3/2),x)`

[Out] `Piecewise((a**2*sqrt(a + b*x**2)/(5*b) + 2*a*x**2*sqrt(a + b*x**2)/5 + b*x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (a**(3/2)*x**2/2, True))`

---

**GIAC/XCAS [A]** time = 0.208045, size = 19, normalized size = 1.06

$$\frac{(bx^2 + a)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)*x,x, algorithm="giac")`

[Out] `1/5*(b*x^2 + a)^(5/2)/b`

$$3.373 \quad \int \frac{(a+bx^2)^{3/2}}{x} dx$$

**Optimal.** Leaf size=54

$$a^{3/2} \left( -\tanh^{-1} \left( \frac{\sqrt{a+bx^2}}{\sqrt{a}} \right) \right) + a\sqrt{a+bx^2} + \frac{1}{3} (a+bx^2)^{3/2}$$

[Out] a\*Sqrt[a + b\*x^2] + (a + b\*x^2)^(3/2)/3 - a^(3/2)\*ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]]

**Rubi [A]** time = 0.0991253, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$a^{3/2} \left( -\tanh^{-1} \left( \frac{\sqrt{a+bx^2}}{\sqrt{a}} \right) \right) + a\sqrt{a+bx^2} + \frac{1}{3} (a+bx^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(3/2)/x, x]

[Out] a\*Sqrt[a + b\*x^2] + (a + b\*x^2)^(3/2)/3 - a^(3/2)\*ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]]

**Rubi in Sympy [A]** time = 9.44854, size = 44, normalized size = 0.81

$$-a^{3/2} \operatorname{atanh} \left( \frac{\sqrt{a+bx^2}}{\sqrt{a}} \right) + a\sqrt{a+bx^2} + \frac{(a+bx^2)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(3/2)/x, x)

[Out] -a\*\*(3/2)\*atanh(sqrt(a + b\*x\*\*2)/sqrt(a)) + a\*sqrt(a + b\*x\*\*2) + (a + b\*x\*\*2)\*\*(3/2)/3

**Mathematica [A]** time = 0.0603539, size = 62, normalized size = 1.15

$$-a^{3/2} \log \left( \sqrt{a}\sqrt{a+bx^2} + a \right) + a^{3/2} \log(x) + \left( \frac{4a}{3} + \frac{bx^2}{3} \right) \sqrt{a+bx^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(3/2)/x, x]

[Out] ((4\*a)/3 + (b\*x^2)/3)\*Sqrt[a + b\*x^2] + a^(3/2)\*Log[x] - a^(3/2)\*Log[a + Sqrt[a]\*Sqrt[a + b\*x^2]]

**Maple [A]** time = 0.007, size = 52, normalized size = 1.

$$\frac{1}{3} (bx^2 + a)^{3/2} - a^{3/2} \ln \left( \frac{1}{x} \left( 2a + 2\sqrt{a}\sqrt{bx^2 + a} \right) \right) + a\sqrt{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(3/2)/x,x)`

[Out]  $\frac{1}{3} (b x^2 + a)^{3/2} - a^{3/2} \ln\left(\frac{(2 a + 2 a^{1/2}) (b x^2 + a)^{1/2}}{x} + a (b x^2 + a)^{1/2}\right)$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.247842, size = 1, normalized size = 0.02

$$\left[ \frac{1}{2} a^{\frac{3}{2}} \log\left(-\frac{bx^2 - 2\sqrt{bx^2 + a}\sqrt{a} + 2a}{x^2}\right) + \frac{1}{3} (bx^2 + 4a)\sqrt{bx^2 + a}, \right. \\ \left. -\sqrt{-aa} \arctan\left(\frac{a}{\sqrt{bx^2 + a}\sqrt{-a}}\right) + \frac{1}{3} (bx^2 + 4a)\sqrt{bx^2 + a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x,x, algorithm="fricas")`

[Out]  $\left[\frac{1}{2} a^{3/2} \log\left(-\frac{b x^2 - 2 \sqrt{b x^2 + a} \sqrt{a} + 2 a}{x^2}\right) + \frac{1}{3} (b x^2 + 4 a) \sqrt{b x^2 + a}, -\sqrt{-a} a \arctan\left(\frac{a}{\sqrt{b x^2 + a} \sqrt{-a}}\right) + \frac{1}{3} (b x^2 + 4 a) \sqrt{b x^2 + a}\right]$

**Sympy** [A] time = 6.93548, size = 78, normalized size = 1.44

$$\frac{4a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}{3} + \frac{a^{\frac{3}{2}}\log\left(\frac{bx^2}{a}\right)}{2} - a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right) + \frac{\sqrt{a}bx^2\sqrt{1+\frac{bx^2}{a}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(3/2)/x,x)`

[Out]  $4 a^{3/2} \sqrt{1 + b x^2/a} / 3 + a^{3/2} \log(b x^2/a) / 2 - a^{3/2} (\log(\sqrt{1 + b x^2/a} + 1) + \sqrt{a} b x^2 \sqrt{1 + b x^2/a}) / 3$

**GIAC/XCAS** [A] time = 0.208983, size = 65, normalized size = 1.2

$$\frac{a^2 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{1}{3} (bx^2 + a)^{\frac{3}{2}} + \sqrt{bx^2 + aa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/3*(b*x^2 + a)^(3/2) + sqrt(b*x^2 + a)*a
```

$$3.374 \quad \int \frac{(a+bx^2)^{3/2}}{x^3} dx$$

**Optimal.** Leaf size=63

$$-\frac{(a+bx^2)^{3/2}}{2x^2} + \frac{3}{2}b\sqrt{a+bx^2} - \frac{3}{2}\sqrt{ab} \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

[Out] (3\*b\*Sqrt[a + b\*x^2])/2 - (a + b\*x^2)^(3/2)/(2\*x^2) - (3\*Sqrt[a] \* b\*ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]])/2

**Rubi [A]** time = 0.102388, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{(a+bx^2)^{3/2}}{2x^2} + \frac{3}{2}b\sqrt{a+bx^2} - \frac{3}{2}\sqrt{ab} \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(3/2)/x^3, x]

[Out] (3\*b\*Sqrt[a + b\*x^2])/2 - (a + b\*x^2)^(3/2)/(2\*x^2) - (3\*Sqrt[a] \* b\*ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]])/2

**Rubi in Sympy [A]** time = 9.71469, size = 56, normalized size = 0.89

$$-\frac{3\sqrt{ab} \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2} + \frac{3b\sqrt{a+bx^2}}{2} - \frac{(a+bx^2)^{\frac{3}{2}}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(3/2)/x\*\*3, x)

[Out] -3\*sqrt(a)\*b\*atanh(sqrt(a + b\*x\*\*2)/sqrt(a))/2 + 3\*b\*sqrt(a + b\*x\*\*2)/2 - (a + b\*x\*\*2)\*\*(3/2)/(2\*x\*\*2)

**Mathematica [A]** time = 0.0748293, size = 65, normalized size = 1.03

$$\sqrt{a+bx^2} \left(b - \frac{a}{2x^2}\right) - \frac{3}{2}\sqrt{ab} \log\left(\sqrt{a}\sqrt{a+bx^2} + a\right) + \frac{3}{2}\sqrt{ab} \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(3/2)/x^3, x]

[Out] (b - a/(2\*x^2))\*Sqrt[a + b\*x^2] + (3\*Sqrt[a]\*b\*Log[x])/2 - (3\*Sqrt[a]\*b\*Log[a + Sqrt[a]\*Sqrt[a + b\*x^2]])/2

**Maple [A]** time = 0.006, size = 75, normalized size = 1.2

$$-\frac{1}{2ax^2} (bx^2 + a)^{\frac{5}{2}} + \frac{b}{2a} (bx^2 + a)^{\frac{3}{2}} - \frac{3b}{2}\sqrt{a} \ln\left(\frac{1}{x} (2a + 2\sqrt{a}\sqrt{bx^2 + a})\right) + \frac{3b}{2}\sqrt{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(3/2)/x^3,x)`

[Out]  $-1/2/a/x^2*(b*x^2+a)^{(5/2)}+1/2*b/a*(b*x^2+a)^{(3/2)}-3/2*b*a^{(1/2)}*\ln((2*a+2*a^{(1/2)}*(b*x^2+a)^{(1/2)})/x)+3/2*b*(b*x^2+a)^{(1/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.252361, size = 1, normalized size = 0.02

$$\left[ \frac{3\sqrt{ab}x^2 \log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a+2a}}{x^2}\right) + 2(2bx^2-a)\sqrt{bx^2+a}}{4x^2}, \right. \\ \left. -\frac{3\sqrt{-ab}x^2 \arctan\left(\frac{a}{\sqrt{bx^2+a}\sqrt{-a}}\right) - (2bx^2-a)\sqrt{bx^2+a}}{2x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x^3,x, algorithm="fricas")`

[Out]  $[1/4*(3*\sqrt{a}*b*x^2*\log(-(b*x^2 - 2*\sqrt{b*x^2 + a})*\sqrt{a} + 2*a)/x^2) + 2*(2*b*x^2 - a)*\sqrt{b*x^2 + a})/x^2, -1/2*(3*\sqrt{-a})*b*x^2*\arctan(a/(\sqrt{b*x^2 + a})*\sqrt{-a})) - (2*b*x^2 - a)*\sqrt{b*x^2 + a})/x^2]$

**Sympy [A]** time = 8.5297, size = 88, normalized size = 1.4

$$-\frac{3\sqrt{ab} \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} - \frac{a^2}{2\sqrt{bx^3}\sqrt{\frac{a}{bx^2} + 1}} + \frac{a\sqrt{b}}{2x\sqrt{\frac{a}{bx^2} + 1}} + \frac{b^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(3/2)/x**3,x)`

[Out]  $-3*\sqrt{a}*b*\operatorname{asinh}(\sqrt{a}/(\sqrt{b}*x))/2 - a**2/(2*\sqrt{b}*x**3*\sqrt{a/(b*x**2) + 1}) + a*\sqrt{b}/(2*x*\sqrt{a/(b*x**2) + 1}) + b*(3/2)*x/\sqrt{a/(b*x**2) + 1}$

**GIAC/XCAS [A]** time = 0.211175, size = 77, normalized size = 1.22

$$\frac{1}{2} \left( \frac{3a \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{bx^2+a} - \frac{\sqrt{bx^2+aa}}{bx^2} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*a*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x^2 + a) - sqrt(b*x^2 + a)*a/(b*x^2))*b
```



$$3.375 \quad \int \frac{(a+bx^2)^{3/2}}{x^5} dx$$

**Optimal.** Leaf size=68

$$-\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}} - \frac{3b\sqrt{a+bx^2}}{8x^2} - \frac{(a+bx^2)^{3/2}}{4x^4}$$

[Out]  $(-3*b*\text{Sqrt}[a + b*x^2])/(8*x^2) - (a + b*x^2)^{(3/2)}/(4*x^4) - (3*b^{^2}*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(8*\text{Sqrt}[a])$

**Rubi [A]** time = 0.106895, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}} - \frac{3b\sqrt{a+bx^2}}{8x^2} - \frac{(a+bx^2)^{3/2}}{4x^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^{(3/2)}/x^5, x]$

[Out]  $(-3*b*\text{Sqrt}[a + b*x^2])/(8*x^2) - (a + b*x^2)^{(3/2)}/(4*x^4) - (3*b^{^2}*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(8*\text{Sqrt}[a])$

**Rubi in Sympy [A]** time = 10.3809, size = 63, normalized size = 0.93

$$-\frac{3b\sqrt{a+bx^2}}{8x^2} - \frac{(a+bx^2)^{\frac{3}{2}}}{4x^4} - \frac{3b^2 \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x^{**2}+a)^{(3/2)}/x^{**5}, x)$

[Out]  $-3*b*\text{sqrt}(a + b*x^{**2})/(8*x^{**2}) - (a + b*x^{**2})^{(3/2)}/(4*x^{**4}) - 3*b^{**2}*\text{atanh}(\text{sqrt}(a + b*x^{**2})/\text{sqrt}(a))/(8*\text{sqrt}(a))$

**Mathematica [A]** time = 0.0922687, size = 73, normalized size = 1.07

$$\frac{1}{8} \left( -\frac{3b^2 \log\left(\sqrt{a}\sqrt{a+bx^2} + a\right)}{\sqrt{a}} + \frac{3b^2 \log(x)}{\sqrt{a}} - \frac{\sqrt{a+bx^2}(2a+5bx^2)}{x^4} \right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b*x^2)^{(3/2)}/x^5, x]$

[Out]  $(-((\text{Sqrt}[a + b*x^2])*(2*a + 5*b*x^2))/x^4) + (3*b^2*\text{Log}[x])/\text{Sqrt}[a] - (3*b^2*\text{Log}[a + \text{Sqrt}[a]*\text{Sqrt}[a + b*x^2]])/\text{Sqrt}[a])/8$

**Maple [A]** time = 0.009, size = 102, normalized size = 1.5

$$-\frac{1}{4ax^4} (bx^2 + a)^{\frac{5}{2}} - \frac{b}{8a^2x^2} (bx^2 + a)^{\frac{5}{2}} + \frac{b^2}{8a^2} (bx^2 + a)^{\frac{3}{2}} - \frac{3b^2}{8} \ln\left(\frac{1}{x} (2a + 2\sqrt{a}\sqrt{bx^2 + a})\right) \frac{1}{\sqrt{a}} + \frac{3b^2}{8a} \sqrt{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(3/2)/x^5,x)`

[Out] 
$$-1/4/a/x^4*(b*x^2+a)^{(5/2)}-1/8*b/a^2/x^2*(b*x^2+a)^{(5/2)}+1/8*b^2/a^2*(b*x^2+a)^{(3/2)}-3/8*b^2/a^{(1/2)}*\ln((2*a+2*a^{(1/2)}*(b*x^2+a)^{(1/2)})/x)+3/8*b^2/a*(b*x^2+a)^{(1/2)}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x^5,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.251275, size = 1, normalized size = 0.01

$$\left[ \frac{3b^2x^4 \log\left(-\frac{(bx^2+2a)\sqrt{a-2\sqrt{bx^2+aa}}}{x^2}\right) - 2(5bx^2+2a)\sqrt{bx^2+a}\sqrt{a}}{16\sqrt{ax^4}}, \right. \\ \left. -\frac{3b^2x^4 \arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) + (5bx^2+2a)\sqrt{bx^2+a}\sqrt{-a}}{8\sqrt{-ax^4}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x^5,x, algorithm="fricas")`

[Out] 
$$\left[ \frac{1}{16} * (3 * b^2 * x^4 * \log(-((b * x^2 + 2 * a) * \sqrt{a} - 2 * \sqrt{b * x^2 + a} * a) / x^2) - 2 * (5 * b * x^2 + 2 * a) * \sqrt{b * x^2 + a} * \sqrt{a}) / (\sqrt{a} * x^4), -1/8 * (3 * b^2 * x^4 * \arctan(\sqrt{-a} / \sqrt{b * x^2 + a}) + (5 * b * x^2 + 2 * a) * \sqrt{b * x^2 + a} * \sqrt{-a}) / (\sqrt{-a} * x^4) \right]$$

**Sympy [A]** time = 10.8545, size = 71, normalized size = 1.04

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{4x^3} - \frac{5b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{8x} - \frac{3b^2 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(3/2)/x**5,x)`

[Out] 
$$-a*\sqrt{b}*\sqrt{a/(b*x**2)+1}/(4*x**3) - 5*b**(3/2)*\sqrt{a/(b*x**2)+1}/(8*x) - 3*b**2*\operatorname{asinh}(\sqrt{a}/(\sqrt{b}*x))/(8*\sqrt{a})$$

**GIAC/XCAS [A]** time = 0.211108, size = 82, normalized size = 1.21

$$\frac{1}{8}b^2\left(\frac{3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{5(bx^2+a)^{\frac{3}{2}} - 3\sqrt{bx^2+aa}}{b^2x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*b^2*(3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - (5*(b*x^2 + a)^(3/2) - 3*sqrt(b*x^2 + a)*a)/(b^2*x^4))
```

$$3.376 \quad \int \frac{(a+bx^2)^{3/2}}{x^7} dx$$

**Optimal.** Leaf size=92

$$\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}} - \frac{b^2 \sqrt{a+bx^2}}{16ax^2} - \frac{(a+bx^2)^{3/2}}{6x^6} - \frac{b\sqrt{a+bx^2}}{8x^4}$$

[Out]  $-(b*\text{Sqrt}[a + b*x^2])/(8*x^4) - (b^2*\text{Sqrt}[a + b*x^2])/(16*a*x^2) - (a + b*x^2)^{(3/2)}/(6*x^6) + (b^3*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(16*a^{(3/2)})$

**Rubi [A]** time = 0.142741, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}} - \frac{b^2 \sqrt{a+bx^2}}{16ax^2} - \frac{(a+bx^2)^{3/2}}{6x^6} - \frac{b\sqrt{a+bx^2}}{8x^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(3/2)/x^7, x]

[Out]  $-(b*\text{Sqrt}[a + b*x^2])/(8*x^4) - (b^2*\text{Sqrt}[a + b*x^2])/(16*a*x^2) - (a + b*x^2)^{(3/2)}/(6*x^6) + (b^3*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(16*a^{(3/2)})$

**Rubi in Sympy [A]** time = 13.8014, size = 78, normalized size = 0.85

$$-\frac{b\sqrt{a+bx^2}}{8x^4} - \frac{(a+bx^2)^{\frac{3}{2}}}{6x^6} - \frac{b^2\sqrt{a+bx^2}}{16ax^2} + \frac{b^3 \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(3/2)/x\*\*7, x)

[Out]  $-b*\text{sqrt}(a + b*x**2)/(8*x**4) - (a + b*x**2)**(3/2)/(6*x**6) - b**2*\text{sqrt}(a + b*x**2)/(16*a*x**2) + b**3*\text{atanh}(\text{sqrt}(a + b*x**2)/\text{sqrt}(a))/(16*a**(3/2))$

**Mathematica [A]** time = 0.0797174, size = 89, normalized size = 0.97

$$\frac{b^3 \log\left(\sqrt{a}\sqrt{a+bx^2} + a\right)}{16a^{3/2}} - \frac{b^3 \log(x)}{16a^{3/2}} + \left(-\frac{b^2}{16ax^2} - \frac{a}{6x^6} - \frac{7b}{24x^4}\right) \sqrt{a+bx^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(3/2)/x^7, x]

[Out]  $(-a/(6*x^6) - (7*b)/(24*x^4) - b^2/(16*a*x^2))*\text{Sqrt}[a + b*x^2] - (b^3*\text{Log}[x])/(16*a^{(3/2)}) + (b^3*\text{Log}[a + \text{Sqrt}[a]*\text{Sqrt}[a + b*x^2]])/(16*a^{(3/2)})$

**Maple [A]** time = 0.012, size = 122, normalized size = 1.3

$$-\frac{1}{6ax^6}(bx^2+a)^{\frac{5}{2}} + \frac{b}{24a^2x^4}(bx^2+a)^{\frac{5}{2}} + \frac{b^2}{48a^3x^2}(bx^2+a)^{\frac{5}{2}} - \frac{b^3}{48a^3}(bx^2+a)^{\frac{3}{2}} + \frac{b^3}{16} \ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right) a^{-\frac{3}{2}} - \frac{b^3}{16a^2}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(3/2)/x^7,x)

[Out] -1/6/a/x^6\*(b\*x^2+a)^(5/2)+1/24\*b/a^2/x^4\*(b\*x^2+a)^(5/2)+1/48\*b^2/a^3/x^2\*(b\*x^2+a)^(5/2)-1/48\*b^3/a^3\*(b\*x^2+a)^(3/2)+1/16\*b^3/a^(3/2)\*ln((2\*a+2\*a^(1/2)\*(b\*x^2+a)^(1/2))/x)-1/16\*b^3/a^2\*(b\*x^2+a)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/x^7,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.257037, size = 1, normalized size = 0.01

$$\left[ \frac{3b^3x^6 \log\left(-\frac{(bx^2+2a)\sqrt{a+2}\sqrt{bx^2+aa}}{x^2}\right) - 2(3b^2x^4 + 14abx^2 + 8a^2)\sqrt{bx^2+a}\sqrt{a} - 3b^3x^6 \arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) - (3b^2x^4 + 14abx^2)}{96a^{\frac{3}{2}}x^6}, \frac{3b^3x^6 \arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) - (3b^2x^4 + 14abx^2)}{48\sqrt{-a}ax^6} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/x^7,x, algorithm="fricas")

[Out] [1/96\*(3\*b^3\*x^6\*log(-((b\*x^2 + 2\*a)\*sqrt(a) + 2\*sqrt(b\*x^2 + a)\*a)/x^2) - 2\*(3\*b^2\*x^4 + 14\*a\*b\*x^2 + 8\*a^2)\*sqrt(b\*x^2 + a)\*sqrt(a))/(a^(3/2)\*x^6), 1/48\*(3\*b^3\*x^6\*arctan(sqrt(-a)/sqrt(b\*x^2 + a)) - (3\*b^2\*x^4 + 14\*a\*b\*x^2 + 8\*a^2)\*sqrt(b\*x^2 + a)\*sqrt(-a))/(sqrt(-a)\*a\*x^6)]

**Sympy [A]** time = 18.6225, size = 119, normalized size = 1.29

$$-\frac{a^2}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{11a\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{17b^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{b^{\frac{5}{2}}}{16ax\sqrt{\frac{a}{bx^2}+1}} + \frac{b^3 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{16a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(3/2)/x\*\*7,x)

[Out] -a\*\*2/(6\*sqrt(b)\*x\*\*7\*sqrt(a/(b\*x\*\*2) + 1)) - 11\*a\*sqrt(b)/(24\*x\*\*5\*sqrt(a/(b\*x\*\*2) + 1)) - 17\*b\*\*(3/2)/(48\*x\*\*3\*sqrt(a/(b\*x\*\*2) + 1)) - b\*\*(5/2)/(16\*a\*x\*sqrt(a/(b\*x\*\*2) + 1)) + b\*\*3\*asinh(sqrt(a/

)/(sqrt(b)\*x))/(16\*a\*\*(3/2))

---

**GIAC/XCAS [A]** time = 0.209722, size = 108, normalized size = 1.17

$$-\frac{1}{48} b^3 \left( \frac{3 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-aa}} + \frac{3 (bx^2 + a)^{\frac{5}{2}} + 8 (bx^2 + a)^{\frac{3}{2}} a - 3 \sqrt{bx^2 + aa^2}}{ab^3 x^6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/x^7,x, algorithm="giac")

[Out] -1/48\*b^3\*(3\*arctan(sqrt(b\*x^2 + a)/sqrt(-a))/(sqrt(-a)\*a) + (3\*(b\*x^2 + a)^(5/2) + 8\*(b\*x^2 + a)^(3/2)\*a - 3\*sqrt(b\*x^2 + a)\*a^2)/(a\*b^3\*x^6))

$$3.377 \quad \int \frac{(a+bx^2)^{3/2}}{x^9} dx$$

**Optimal.** Leaf size=116

$$-\frac{3b^4 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}} + \frac{3b^3\sqrt{a+bx^2}}{128a^2x^2} - \frac{b^2\sqrt{a+bx^2}}{64ax^4} - \frac{(a+bx^2)^{3/2}}{8x^8} - \frac{b\sqrt{a+bx^2}}{16x^6}$$

[Out]  $-(b*\text{Sqrt}[a + b*x^2])/(16*x^6) - (b^2*\text{Sqrt}[a + b*x^2])/(64*a*x^4) + (3*b^3*\text{Sqrt}[a + b*x^2])/(128*a^2*x^2) - (a + b*x^2)^{(3/2)}/(8*x^8) - (3*b^4*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(128*a^{(5/2)})$

**Rubi [A]** time = 0.18348, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{3b^4 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}} + \frac{3b^3\sqrt{a+bx^2}}{128a^2x^2} - \frac{b^2\sqrt{a+bx^2}}{64ax^4} - \frac{(a+bx^2)^{3/2}}{8x^8} - \frac{b\sqrt{a+bx^2}}{16x^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(3/2)/x^9, x]

[Out]  $-(b*\text{Sqrt}[a + b*x^2])/(16*x^6) - (b^2*\text{Sqrt}[a + b*x^2])/(64*a*x^4) + (3*b^3*\text{Sqrt}[a + b*x^2])/(128*a^2*x^2) - (a + b*x^2)^{(3/2)}/(8*x^8) - (3*b^4*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(128*a^{(5/2)})$

**Rubi in Sympy [A]** time = 17.9348, size = 104, normalized size = 0.9

$$-\frac{b\sqrt{a+bx^2}}{16x^6} - \frac{(a+bx^2)^{3/2}}{8x^8} - \frac{b^2\sqrt{a+bx^2}}{64ax^4} + \frac{3b^3\sqrt{a+bx^2}}{128a^2x^2} - \frac{3b^4 \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(3/2)/x\*\*9, x)

[Out]  $-b*\text{sqrt}(a + b*x**2)/(16*x**6) - (a + b*x**2)**(3/2)/(8*x**8) - b**2*\text{sqrt}(a + b*x**2)/(64*a*x**4) + 3*b**3*\text{sqrt}(a + b*x**2)/(128*a**2*x**2) - 3*b**4*\text{atanh}(\text{sqrt}(a + b*x**2)/\text{sqrt}(a))/(128*a**(5/2))$

**Mathematica [A]** time = 0.128904, size = 102, normalized size = 0.88

$$-\frac{3b^4 \log\left(\sqrt{a}\sqrt{a+bx^2} + a\right)}{128a^{5/2}} + \frac{3b^4 \log(x)}{128a^{5/2}} + \left(\frac{3b^3}{128a^2x^2} - \frac{b^2}{64ax^4} - \frac{a}{8x^8} - \frac{3b}{16x^6}\right) \sqrt{a+bx^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(3/2)/x^9, x]

[Out]  $(-a/(8*x^8) - (3*b)/(16*x^6) - b^2/(64*a*x^4) + (3*b^3)/(128*a^2*x^2))*\text{Sqrt}[a + b*x^2] + (3*b^4*\text{Log}[x])/(128*a^{(5/2)}) - (3*b^4*\text{Log}[a + \text{Sqrt}[a]*\text{Sqrt}[a + b*x^2]])/(128*a^{(5/2)})$

**Maple [A]** time = 0.019, size = 142, normalized size = 1.2

$$-\frac{1}{8ax^8}(bx^2+a)^{\frac{5}{2}} + \frac{b}{16a^2x^6}(bx^2+a)^{\frac{5}{2}} - \frac{b^2}{64a^3x^4}(bx^2+a)^{\frac{5}{2}} - \frac{b^3}{128a^4x^2}(bx^2+a)^{\frac{5}{2}} \\ + \frac{b^4}{128a^4}(bx^2+a)^{\frac{3}{2}} - \frac{3b^4}{128} \ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}} + \frac{3b^4}{128a^3}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(3/2)/x^9,x)

[Out] -1/8/a/x^8\*(b\*x^2+a)^(5/2)+1/16\*b/a^2/x^6\*(b\*x^2+a)^(5/2)-1/64\*b^2/a^3/x^4\*(b\*x^2+a)^(5/2)-1/128\*b^3/a^4/x^2\*(b\*x^2+a)^(5/2)+1/128\*b^4/a^4\*(b\*x^2+a)^(3/2)-3/128\*b^4/a^(5/2)\*ln((2\*a+2\*a^(1/2)\*(b\*x^2+a)^(1/2))/x)+3/128\*b^4/a^3\*(b\*x^2+a)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/x^9,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.271716, size = 1, normalized size = 0.01

$$\left[ \frac{3b^4x^8 \log\left(-\frac{(bx^2+2a)\sqrt{a-2\sqrt{bx^2+aa}}}{x^2}\right) + 2(3b^3x^6 - 2ab^2x^4 - 24a^2bx^2 - 16a^3)\sqrt{bx^2+a}\sqrt{a}}{256a^{\frac{5}{2}}x^8}, \right. \\ \left. -\frac{3b^4x^8 \arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) - (3b^3x^6 - 2ab^2x^4 - 24a^2bx^2 - 16a^3)\sqrt{bx^2+a}\sqrt{-a}}{128\sqrt{-aa^2}x^8} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/x^9,x, algorithm="fricas")

[Out] [1/256\*(3\*b^4\*x^8\*log(-((b\*x^2 + 2\*a)\*sqrt(a) - 2\*sqrt(b\*x^2 + a)\*a)/x^2) + 2\*(3\*b^3\*x^6 - 2\*a\*b^2\*x^4 - 24\*a^2\*b\*x^2 - 16\*a^3)\*sqrt(b\*x^2 + a)\*sqrt(a))/(a^(5/2)\*x^8), -1/128\*(3\*b^4\*x^8\*arctan(sqrt(-a)/sqrt(b\*x^2 + a)) - (3\*b^3\*x^6 - 2\*a\*b^2\*x^4 - 24\*a^2\*b\*x^2 - 16\*a^3)\*sqrt(b\*x^2 + a)\*sqrt(-a))/(sqrt(-a)\*a^2\*x^8)]

**Sympy [A]** time = 28.5302, size = 148, normalized size = 1.28

$$-\frac{a^2}{8\sqrt{bx^9}\sqrt{\frac{a}{bx^2}+1}} - \frac{5a\sqrt{b}}{16x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{13b^{\frac{3}{2}}}{64x^5\sqrt{\frac{a}{bx^2}+1}} \\ + \frac{b^{\frac{5}{2}}}{128ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{3b^{\frac{7}{2}}}{128a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{3b^4 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{128a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((b\*x\*\*2+a)\*\*(3/2)/x\*\*9,x)

[Out]  $-a^{**2}/(8*\text{sqrt}(b)*x^{**9}*\text{sqrt}(a/(b*x^{**2}) + 1)) - 5*a*\text{sqrt}(b)/(16*x^{**7}*\text{sqrt}(a/(b*x^{**2}) + 1)) - 13*b^{**}(3/2)/(64*x^{**5}*\text{sqrt}(a/(b*x^{**2}) + 1)) + b^{**}(5/2)/(128*a*x^{**3}*\text{sqrt}(a/(b*x^{**2}) + 1)) + 3*b^{**}(7/2)/(128*a^{**2}*x*\text{sqrt}(a/(b*x^{**2}) + 1)) - 3*b^{**4}*\text{asinh}(\text{sqrt}(a)/(\text{sqrt}(b)*x))/(128*a^{**}(5/2))$

**GIAC/XCAS [A]** time = 0.211433, size = 127, normalized size = 1.09

$$\frac{1}{128} b^4 \left( \frac{3 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-aa^2}} + \frac{3 (bx^2 + a)^{\frac{7}{2}} - 11 (bx^2 + a)^{\frac{5}{2}} a - 11 (bx^2 + a)^{\frac{3}{2}} a^2 + 3 \sqrt{bx^2 + aa^3}}{a^2 b^4 x^8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/x^9,x, algorithm="giac")

[Out]  $1/128*b^4*(3*\arctan(\text{sqrt}(b*x^2 + a)/\text{sqrt}(-a))/(\text{sqrt}(-a)*a^2) + (3*(b*x^2 + a)^{(7/2)} - 11*(b*x^2 + a)^{(5/2)}*a - 11*(b*x^2 + a)^{(3/2)}*a^2 + 3*\text{sqrt}(b*x^2 + a)*a^3)/(a^2*b^4*x^8)$

$$3.378 \quad \int x^4 (a + bx^2)^{3/2} dx$$

**Optimal.** Leaf size=115

$$\frac{3a^4 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{128b^{5/2}} - \frac{3a^3x\sqrt{a+bx^2}}{128b^2} + \frac{a^2x^3\sqrt{a+bx^2}}{64b} + \frac{1}{8}x^5(a+bx^2)^{3/2} + \frac{1}{16}ax^5\sqrt{a+bx^2}$$

[Out]  $(-3*a^3*x*\text{Sqrt}[a + b*x^2])/((128*b^2) + (a^2*x^3*\text{Sqrt}[a + b*x^2])/(64*b) + (a*x^5*\text{Sqrt}[a + b*x^2])/16 + (x^5*(a + b*x^2)^(3/2))/8 + (3*a^4*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]])/(128*b^(5/2)))$

**Rubi [A]** time = 0.136401, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{3a^4 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{128b^{5/2}} - \frac{3a^3x\sqrt{a+bx^2}}{128b^2} + \frac{a^2x^3\sqrt{a+bx^2}}{64b} + \frac{1}{8}x^5(a+bx^2)^{3/2} + \frac{1}{16}ax^5\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a + b\*x^2)^(3/2), x]

[Out]  $(-3*a^3*x*\text{Sqrt}[a + b*x^2])/((128*b^2) + (a^2*x^3*\text{Sqrt}[a + b*x^2])/(64*b) + (a*x^5*\text{Sqrt}[a + b*x^2])/16 + (x^5*(a + b*x^2)^(3/2))/8 + (3*a^4*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]])/(128*b^(5/2)))$

**Rubi in Sympy [A]** time = 16.9259, size = 104, normalized size = 0.9

$$\frac{3a^4 \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{128b^{5/2}} - \frac{3a^3x\sqrt{a+bx^2}}{128b^2} + \frac{a^2x^3\sqrt{a+bx^2}}{64b} + \frac{ax^5\sqrt{a+bx^2}}{16} + \frac{x^5(a+bx^2)^{3/2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(b\*x\*\*2+a)\*\*(3/2), x)

[Out]  $3*a**4*\operatorname{atanh}(\operatorname{sqrt}(b)*x/\operatorname{sqrt}(a + b*x**2))/((128*b**(5/2)) - 3*a**3*x*\operatorname{sqrt}(a + b*x**2)/(128*b**2) + a**2*x**3*\operatorname{sqrt}(a + b*x**2)/(64*b) + a*x**5*\operatorname{sqrt}(a + b*x**2)/16 + x**5*(a + b*x**2)**(3/2)/8$

**Mathematica [A]** time = 0.0731491, size = 88, normalized size = 0.77

$$\frac{3a^4 \log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right)}{128b^{5/2}} + \sqrt{a+bx^2} \left(-\frac{3a^3x}{128b^2} + \frac{a^2x^3}{64b} + \frac{3ax^5}{16} + \frac{bx^7}{8}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a + b\*x^2)^(3/2), x]

[Out]  $\text{Sqrt}[a + b*x^2]*((-3*a^3*x)/(128*b^2) + (a^2*x^3)/(64*b) + (3*a*x^5)/16 + (b*x^7)/8) + (3*a^4*\text{Log}[b*x + \text{Sqrt}[b]*\text{Sqrt}[a + b*x^2]])/(128*b^(5/2))$

**Maple [A]** time = 0.01, size = 95, normalized size = 0.8

$$\frac{x^3}{8b} (bx^2 + a)^{\frac{5}{2}} - \frac{ax}{16b^2} (bx^2 + a)^{\frac{5}{2}} + \frac{a^2x}{64b^2} (bx^2 + a)^{\frac{3}{2}} + \frac{3a^3x}{128b^2} \sqrt{bx^2 + a} + \frac{3a^4}{128} \ln(x\sqrt{b} + \sqrt{bx^2 + a}) b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(b\*x^2+a)^(3/2), x)

[Out] 1/8\*x^3\*(b\*x^2+a)^(5/2)/b-1/16\*a/b^2\*x\*(b\*x^2+a)^(5/2)+1/64\*a^2/b^2\*x\*(b\*x^2+a)^(3/2)+3/128\*a^3\*x\*(b\*x^2+a)^(1/2)/b^2+3/128\*a^4/b^2\*(5/2)\*ln(x\*b^(1/2)+(b\*x^2+a)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)\*x^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.265435, size = 1, normalized size = 0.01

$$\left[ \frac{3a^4 \log(-2\sqrt{bx^2+abx} - (2bx^2+a)\sqrt{b}) + 2(16b^3x^7 + 24ab^2x^5 + 2a^2bx^3 - 3a^3x)\sqrt{bx^2+a}\sqrt{b} - 3a^4 \arctan\left(\frac{\sqrt{-bx}}{\sqrt{bx^2+a}}\right)}{256b^{\frac{5}{2}}}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)\*x^4,x, algorithm="fricas")

[Out] [1/256\*(3\*a^4\*log(-2\*sqrt(b\*x^2 + a)\*b\*x - (2\*b\*x^2 + a)\*sqrt(b)) + 2\*(16\*b^3\*x^7 + 24\*a\*b^2\*x^5 + 2\*a^2\*b\*x^3 - 3\*a^3\*x)\*sqrt(b\*x^2 + a)\*sqrt(b))/b^(5/2), 1/128\*(3\*a^4\*arctan(sqrt(-b)\*x/sqrt(b\*x^2 + a)) + (16\*b^3\*x^7 + 24\*a\*b^2\*x^5 + 2\*a^2\*b\*x^3 - 3\*a^3\*x)\*sqrt(b\*x^2 + a)\*sqrt(-b))/(sqrt(-b)\*b^2)]

**Sympy [A]** time = 26.197, size = 148, normalized size = 1.29

$$-\frac{3a^{\frac{7}{2}}x}{128b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{a^{\frac{5}{2}}x^3}{128b\sqrt{1+\frac{bx^2}{a}}} + \frac{13a^{\frac{3}{2}}x^5}{64\sqrt{1+\frac{bx^2}{a}}} + \frac{5\sqrt{ab}x^7}{16\sqrt{1+\frac{bx^2}{a}}} + \frac{3a^4 \operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}} + \frac{b^2x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(b\*x\*\*2+a)\*\*(3/2), x)

[Out] -3\*a\*\*(7/2)\*x/(128\*b\*\*2\*sqrt(1 + b\*x\*\*2/a)) - a\*\*(5/2)\*x\*\*3/(128\*b\*sqrt(1 + b\*x\*\*2/a)) + 13\*a\*\*(3/2)\*x\*\*5/(64\*sqrt(1 + b\*x\*\*2/a)) + 5\*sqrt(a)\*b\*x\*\*7/(16\*sqrt(1 + b\*x\*\*2/a)) + 3\*a\*\*4\*asinh(sqrt(b)\*x/sqrt(a))/(128\*b\*\*(5/2)) + b\*\*2\*x\*\*9/(8\*sqrt(a)\*sqrt(1 + b\*x\*\*2/a))

**GIAC/XCAS [A]** time = 0.215324, size = 103, normalized size = 0.9

$$\frac{1}{128} \left( 2 \left( 4 \left( 2bx^2 + 3a \right) x^2 + \frac{a^2}{b} \right) x^2 - \frac{3a^3}{b^2} \right) \sqrt{bx^2 + ax} - \frac{3a^4 \ln \left( \left| -\sqrt{b}x + \sqrt{bx^2 + a} \right| \right)}{128 b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)\*x^4,x, algorithm="giac")

[Out] 1/128\*(2\*(4\*(2\*b\*x^2 + 3\*a)\*x^2 + a^2/b)\*x^2 - 3\*a^3/b^2)\*sqrt(b\*x^2 + a)\*x - 3/128\*a^4\*ln(abs(-sqrt(b)\*x + sqrt(b\*x^2 + a)))/b^(5/2)

$$3.379 \quad \int x^2 (a + bx^2)^{3/2} dx$$

**Optimal.** Leaf size=91

$$-\frac{a^3 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{16b^{3/2}} + \frac{a^2 x \sqrt{a+bx^2}}{16b} + \frac{1}{8} a x^3 \sqrt{a+bx^2} + \frac{1}{6} x^3 (a+bx^2)^{3/2}$$

[Out] (a^2\*x\*Sqrt[a + b\*x^2])/(16\*b) + (a\*x^3\*Sqrt[a + b\*x^2])/8 + (x^3\*(a + b\*x^2)^(3/2))/6 - (a^3\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(16\*b^(3/2))

**Rubi [A]** time = 0.101538, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{a^3 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{16b^{3/2}} + \frac{a^2 x \sqrt{a+bx^2}}{16b} + \frac{1}{8} a x^3 \sqrt{a+bx^2} + \frac{1}{6} x^3 (a+bx^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*x^2)^(3/2), x]

[Out] (a^2\*x\*Sqrt[a + b\*x^2])/(16\*b) + (a\*x^3\*Sqrt[a + b\*x^2])/8 + (x^3\*(a + b\*x^2)^(3/2))/6 - (a^3\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(16\*b^(3/2))

**Rubi in Sympy [A]** time = 12.8823, size = 78, normalized size = 0.86

$$-\frac{a^3 \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{16b^{3/2}} + \frac{a^2 x \sqrt{a+bx^2}}{16b} + \frac{a x^3 \sqrt{a+bx^2}}{8} + \frac{x^3 (a+bx^2)^{3/2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(b\*x\*\*2+a)\*\*(3/2), x)

[Out] -a\*\*3\*atanh(sqrt(b)\*x/sqrt(a + b\*x\*\*2))/(16\*b\*\*(3/2)) + a\*\*2\*x\*sqrt(a + b\*x\*\*2)/(16\*b) + a\*x\*\*3\*sqrt(a + b\*x\*\*2)/8 + x\*\*3\*(a + b\*x\*\*2)\*\*(3/2)/6

**Mathematica [A]** time = 0.0564863, size = 75, normalized size = 0.82

$$\sqrt{a+bx^2} \left( \frac{a^2 x}{16b} + \frac{7ax^3}{24} + \frac{bx^5}{6} \right) - \frac{a^3 \log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right)}{16b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*x^2)^(3/2), x]

[Out] Sqrt[a + b\*x^2]\*((a^2\*x)/(16\*b) + (7\*a\*x^3)/24 + (b\*x^5)/6) - (a^3\*Log[b\*x + Sqrt[b]\*Sqrt[a + b\*x^2]])/(16\*b^(3/2))

**Maple [A]** time = 0.008, size = 75, normalized size = 0.8

$$\frac{x}{6b} (bx^2 + a)^{5/2} - \frac{ax}{24b} (bx^2 + a)^{3/2} - \frac{a^2 x}{16b} \sqrt{bx^2 + a} - \frac{a^3}{16} \ln(x\sqrt{b} + \sqrt{bx^2 + a}) b^{-3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^2+a)^(3/2),x)`

[Out]  $\frac{1}{6}x(bx^2+a)^{5/2}/b - \frac{1}{24}a/bx(bx^2+a)^{3/2} - \frac{1}{16}a^2x(bx^2+a)^{1/2}/b - \frac{1}{16}a^3/b^{3/2} \ln(xb^{1/2} + (bx^2+a)^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)*x^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.255876, size = 1, normalized size = 0.01

$$\left[ \frac{3a^3 \log\left(2\sqrt{bx^2+ax} - (2bx^2+a)\sqrt{b}\right) + 2(8b^2x^5 + 14abx^3 + 3a^2x)\sqrt{bx^2+a}\sqrt{b}}{96b^{3/2}}, \right. \\ \left. - \frac{3a^3 \arctan\left(\frac{\sqrt{-bx}}{\sqrt{bx^2+a}}\right) - (8b^2x^5 + 14abx^3 + 3a^2x)\sqrt{bx^2+a}\sqrt{-b}}{48\sqrt{-bb}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)*x^2,x, algorithm="fricas")`

[Out]  $\left[ \frac{1}{96} \left( 3a^3 \log(2\sqrt{bx^2+ax})\sqrt{bx^2+a} - (2bx^2+a)\sqrt{b} \right) + 2(8b^2x^5 + 14abx^3 + 3a^2x)\sqrt{bx^2+a}\sqrt{b} \right] / b^{3/2}, -\frac{1}{48} \left( 3a^3 \arctan(\sqrt{-bx}/\sqrt{bx^2+a}) - (8b^2x^5 + 14abx^3 + 3a^2x)\sqrt{bx^2+a}\sqrt{-b} \right) / (\sqrt{-b})\sqrt{b} \right]$

**Sympy [A]** time = 17.1265, size = 119, normalized size = 1.31

$$\frac{a^{5/2}x}{16b\sqrt{1+\frac{bx^2}{a}}} + \frac{17a^{3/2}x^3}{48\sqrt{1+\frac{bx^2}{a}}} + \frac{11\sqrt{ab}x^5}{24\sqrt{1+\frac{bx^2}{a}}} - \frac{a^3 \operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{16b^{3/2}} + \frac{b^2x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**2+a)**(3/2),x)`

[Out]  $a^{5/2}x/(16b\sqrt{1+b*x^2/a}) + 17*a^{3/2}*x^3/(48*\sqrt{1+b*x^2/a}) + 11*\sqrt{a}*b*x^5/(24*\sqrt{1+b*x^2/a}) - a^3*\operatorname{asinh}(\sqrt{b}*x/\sqrt{a})/(16*b^{3/2}) + b^2*x^7/(6*\sqrt{a}*\sqrt{1+b*x^2/a})$

**GIAC/XCAS [A]** time = 0.21102, size = 85, normalized size = 0.93

$$\frac{1}{48} \left( 2(4bx^2 + 7a)x^2 + \frac{3a^2}{b} \right) \sqrt{bx^2 + ax} + \frac{a^3 \ln\left(\left| -\sqrt{bx} + \sqrt{bx^2 + a} \right| \right)}{16b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b*x^2 + 7*a)*x^2 + 3*a^2/b)*sqrt(b*x^2 + a)*x + 1/16*a  
^3*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

### 3.380 $\int (a + bx^2)^{3/2} dx$

**Optimal.** Leaf size=65

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}} + \frac{3}{8}ax\sqrt{a+bx^2} + \frac{1}{4}x(a+bx^2)^{3/2}$$

[Out] (3\*a\*x\*Sqrt[a + b\*x^2])/8 + (x\*(a + b\*x^2)^(3/2))/4 + (3\*a^2\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(8\*Sqrt[b])

**Rubi [A]** time = 0.0453195, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}} + \frac{3}{8}ax\sqrt{a+bx^2} + \frac{1}{4}x(a+bx^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(3/2), x]

[Out] (3\*a\*x\*Sqrt[a + b\*x^2])/8 + (x\*(a + b\*x^2)^(3/2))/4 + (3\*a^2\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(8\*Sqrt[b])

**Rubi in Sympy [A]** time = 4.35898, size = 60, normalized size = 0.92

$$\frac{3a^2 \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}} + \frac{3ax\sqrt{a+bx^2}}{8} + \frac{x(a+bx^2)^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(3/2), x)

[Out] 3\*a\*\*2\*atanh(sqrt(b)\*x/sqrt(a + b\*x\*\*2))/(8\*sqrt(b)) + 3\*a\*x\*sqrt(a + b\*x\*\*2)/8 + x\*(a + b\*x\*\*2)\*\*(3/2)/4

**Mathematica [A]** time = 0.058801, size = 62, normalized size = 0.95

$$\frac{3a^2 \log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right)}{8\sqrt{b}} + \sqrt{a+bx^2} \left(\frac{5ax}{8} + \frac{bx^3}{4}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(3/2), x]

[Out] Sqrt[a + b\*x^2]\*((5\*a\*x)/8 + (b\*x^3)/4) + (3\*a^2\*Log[b\*x + Sqrt[b]\*Sqrt[a + b\*x^2]])/(8\*Sqrt[b])

**Maple [A]** time = 0.003, size = 51, normalized size = 0.8

$$\frac{x}{4}(bx^2 + a)^{\frac{3}{2}} + \frac{3ax}{8}\sqrt{bx^2 + a} + \frac{3a^2}{8} \ln\left(x\sqrt{b} + \sqrt{bx^2 + a}\right) \frac{1}{\sqrt{b}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(3/2),x)`

[Out]  $\frac{1}{4}x(bx^2+a)^{3/2} + \frac{3}{8}a^2x(bx^2+a)^{1/2} + \frac{3}{8}a^2/b^{1/2} \ln(xb^{1/2} + (bx^2+a)^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.24433, size = 1, normalized size = 0.02

$$\left[ \frac{3a^2 \log\left(-2\sqrt{bx^2+abx} - (2bx^2+a)\sqrt{b}\right) + 2(2bx^3+5ax)\sqrt{bx^2+a}\sqrt{b}}{16\sqrt{b}}, \frac{3a^2 \arctan\left(\frac{\sqrt{-bx}}{\sqrt{bx^2+a}}\right) + (2bx^3+5ax)\sqrt{bx^2+a}}{8\sqrt{-b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2),x, algorithm="fricas")`

[Out]  $\left[ \frac{1}{16} \left( 3a^2 \log(-2\sqrt{bx^2+a}bx - (2bx^2+a)\sqrt{b}) + 2(2bx^3+5ax)\sqrt{bx^2+a}\sqrt{b} \right) / \sqrt{b}, \frac{1}{8} \left( 3a^2 \arctan(\sqrt{-b}x/\sqrt{bx^2+a}) + (2bx^3+5ax)\sqrt{bx^2+a}\sqrt{-b} \right) / \sqrt{-b} \right]$

**Sympy [A]** time = 9.89968, size = 70, normalized size = 1.08

$$\frac{5a^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{8} + \frac{\sqrt{ab}x^3\sqrt{1+\frac{bx^2}{a}}}{4} + \frac{3a^2 \operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(3/2),x)`

[Out]  $\frac{5a^{3/2}x\sqrt{1+bx^2/a}}{8} + \sqrt{a}bx^{3/2}\sqrt{1+bx^2/a}/4 + 3a^{3/2}\operatorname{asinh}(\sqrt{b}x/\sqrt{a})/(8\sqrt{b})$

**GIAC/XCAS [A]** time = 0.212433, size = 66, normalized size = 1.02

$$\frac{1}{8} (2bx^2+5a)\sqrt{bx^2+ax} - \frac{3a^2 \ln\left(\left|-\sqrt{bx} + \sqrt{bx^2+a}\right|\right)}{8\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2),x, algorithm="giac")`

```
[Out] 1/8*(2*b*x^2 + 5*a)*sqrt(b*x^2 + a)*x - 3/8*a^2*ln(abs(-sqrt(b)*x  
+ sqrt(b*x^2 + a)))/sqrt(b)
```

$$3.381 \quad \int \frac{(a+bx^2)^{3/2}}{x^2} dx$$

**Optimal.** Leaf size=63

$$-\frac{(a+bx^2)^{3/2}}{x} + \frac{3}{2}bx\sqrt{a+bx^2} + \frac{3}{2}a\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)$$

[Out] (3\*b\*x\*Sqrt[a + b\*x^2])/2 - (a + b\*x^2)^(3/2)/x + (3\*a\*Sqrt[b]\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/2

**Rubi [A]** time = 0.0534704, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{(a+bx^2)^{3/2}}{x} + \frac{3}{2}bx\sqrt{a+bx^2} + \frac{3}{2}a\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(3/2)/x^2, x]

[Out] (3\*b\*x\*Sqrt[a + b\*x^2])/2 - (a + b\*x^2)^(3/2)/x + (3\*a\*Sqrt[b]\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/2

**Rubi in Sympy [A]** time = 6.09879, size = 56, normalized size = 0.89

$$\frac{3a\sqrt{b} \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{2} + \frac{3bx\sqrt{a+bx^2}}{2} - \frac{(a+bx^2)^{\frac{3}{2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(3/2)/x\*\*2, x)

[Out] 3\*a\*sqrt(b)\*atanh(sqrt(b)\*x/sqrt(a + b\*x\*\*2))/2 + 3\*b\*x\*sqrt(a + b\*x\*\*2)/2 - (a + b\*x\*\*2)\*\*(3/2)/x

**Mathematica [A]** time = 0.0449941, size = 58, normalized size = 0.92

$$\sqrt{a+bx^2} \left( \frac{bx}{2} - \frac{a}{x} \right) + \frac{3}{2}a\sqrt{b} \log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(3/2)/x^2, x]

[Out] (-(a/x) + (b\*x)/2)\*Sqrt[a + b\*x^2] + (3\*a\*Sqrt[b]\*Log[b\*x + Sqrt[b]\*Sqrt[a + b\*x^2]])/2

**Maple [A]** time = 0.006, size = 69, normalized size = 1.1

$$-\frac{1}{ax} (bx^2 + a)^{\frac{5}{2}} + \frac{bx}{a} (bx^2 + a)^{\frac{3}{2}} + \frac{3bx}{2} \sqrt{bx^2 + a} + \frac{3a}{2} \sqrt{b} \ln(x\sqrt{b} + \sqrt{bx^2 + a})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(3/2)/x^2,x)`

[Out]  $-1/a/x*(b*x^2+a)^{5/2}+b/a*x*(b*x^2+a)^{3/2}+3/2*b*x*(b*x^2+a)^{1/2}+3/2*b^{1/2}*a*\ln(x*b^{1/2}+(b*x^2+a)^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.247236, size = 1, normalized size = 0.02

$$\left[ \frac{3a\sqrt{bx} \log\left(-2bx^2 - 2\sqrt{bx^2+a}\sqrt{bx} - a\right) + 2\sqrt{bx^2+a}(bx^2 - 2a)}{4x}, \frac{3a\sqrt{-bx} \arctan\left(\frac{bx}{\sqrt{bx^2+a}\sqrt{-b}}\right) + \sqrt{bx^2+a}(bx^2 - 2a)}{2x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x^2,x, algorithm="fricas")`

[Out]  $[1/4*(3*a*\sqrt{b}*x*\log(-2*b*x^2 - 2*\sqrt{b*x^2 + a}*\sqrt{b}*x - a) + 2*\sqrt{b*x^2 + a}*(b*x^2 - 2*a))/x, 1/2*(3*a*\sqrt{-b}*x*\arctan(b*x/(\sqrt{b*x^2 + a}*\sqrt{-b})) + \sqrt{b*x^2 + a}*(b*x^2 - 2*a))/x]$

**Sympy [A]** time = 8.38128, size = 88, normalized size = 1.4

$$-\frac{a^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{\sqrt{abx}}{2\sqrt{1+\frac{bx^2}{a}}} + \frac{3a\sqrt{b} \operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} + \frac{b^2x^3}{2\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(3/2)/x**2,x)`

[Out]  $-a^{3/2}/(x*\sqrt{1+b*x^2/a}) - \sqrt{a}*b*x/(2*\sqrt{1+b*x^2/a}) + 3*a*\sqrt{b}*asinh(\sqrt{b}*x/\sqrt{a})/2 + b^2*x^3/(2*\sqrt{a}*\sqrt{1+b*x^2/a})$

**GIAC/XCAS [A]** time = 0.217496, size = 99, normalized size = 1.57

$$\frac{1}{2}\sqrt{bx^2+abx} - \frac{3}{4}a\sqrt{b}\ln\left(\left(\sqrt{bx} - \sqrt{bx^2+a}\right)^2\right) + \frac{2a^2\sqrt{b}}{\left(\sqrt{bx} - \sqrt{bx^2+a}\right)^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x^2,x, algorithm="giac")`

```
[Out] 1/2*sqrt(b*x^2 + a)*b*x - 3/4*a*sqrt(b)*ln((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2*a^2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

$$3.382 \quad \int \frac{(a+bx^2)^{3/2}}{x^4} dx$$

**Optimal.** Leaf size=61

$$b^{3/2} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right) - \frac{b\sqrt{a+bx^2}}{x} - \frac{(a+bx^2)^{3/2}}{3x^3}$$

[Out]  $-\left(\frac{b\sqrt{a+bx^2}}{x}\right) - \frac{(a+bx^2)^{3/2}}{3x^3} + b^{3/2} \operatorname{Arctanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)$

**Rubi [A]** time = 0.0627503, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$b^{3/2} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right) - \frac{b\sqrt{a+bx^2}}{x} - \frac{(a+bx^2)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(3/2)/x^4, x]

[Out]  $-\left(\frac{b\sqrt{a+bx^2}}{x}\right) - \frac{(a+bx^2)^{3/2}}{3x^3} + b^{3/2} \operatorname{Arctanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)$

**Rubi in Sympy [A]** time = 7.93087, size = 51, normalized size = 0.84

$$b^{3/2} \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right) - \frac{b\sqrt{a+bx^2}}{x} - \frac{(a+bx^2)^{3/2}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(3/2)/x\*\*4, x)

[Out]  $b^{3/2} \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right) - \frac{b\sqrt{a+bx^2}}{x} - \frac{(a+bx^2)^{3/2}}{3x^3}$

**Mathematica [A]** time = 0.0477523, size = 55, normalized size = 0.9

$$b^{3/2} \log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right) - \frac{\sqrt{a+bx^2}(a+4bx^2)}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(3/2)/x^4, x]

[Out]  $-\frac{\sqrt{a+bx^2}(a+4bx^2)}{3x^3} + b^{3/2} \operatorname{Log}\left[bx + \sqrt{bx^2+a}\right]$

**Maple [A]** time = 0.007, size = 92, normalized size = 1.5

$$-\frac{1}{3ax^3} (bx^2+a)^{5/2} - \frac{2b}{3a^2x} (bx^2+a)^{5/2} + \frac{2b^2x}{3a^2} (bx^2+a)^{3/2} + \frac{b^2x}{a} \sqrt{bx^2+a} + b^{3/2} \ln\left(x\sqrt{b} + \sqrt{bx^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(3/2)/x^4,x)`

[Out] 
$$-1/3/a/x^3*(b*x^2+a)^{(5/2)}-2/3*b/a^2/x*(b*x^2+a)^{(5/2)}+2/3*b^2/a^2*x*(b*x^2+a)^{(3/2)}+b^2/a*x*(b*x^2+a)^{(1/2)}+b^{(3/2)}*\ln(x*b^{(1/2)}+(b*x^2+a)^{(1/2)})$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x^4,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.246801, size = 1, normalized size = 0.02

$$\left[ \frac{3b^{\frac{3}{2}}x^3 \log(-2bx^2 - 2\sqrt{bx^2+a}\sqrt{bx-a}) - 2(4bx^2+a)\sqrt{bx^2+a} - 3\sqrt{-b}bx^3 \arctan\left(\frac{bx}{\sqrt{bx^2+a}\sqrt{-b}}\right) - (4bx^2+a)\sqrt{bx^2+a}}{6x^3}, \frac{3\sqrt{-b}bx^3 \arctan\left(\frac{bx}{\sqrt{bx^2+a}\sqrt{-b}}\right) - (4bx^2+a)\sqrt{bx^2+a}}{3x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x^4,x, algorithm="fricas")`

[Out] 
$$\left[ \frac{1}{6}*(3*b^{(3/2)}*x^3*\log(-2*b*x^2 - 2*\sqrt{b*x^2 + a}*\sqrt{b}*x - a) - 2*(4*b*x^2 + a)*\sqrt{b*x^2 + a})/x^3, \frac{1}{3}*(3*\sqrt{-b}*b*x^3*\arctan(b*x/(\sqrt{b*x^2 + a}*\sqrt{-b})) - (4*b*x^2 + a)*\sqrt{b*x^2 + a})/x^3 \right]$$

**Sympy [A]** time = 7.38371, size = 78, normalized size = 1.28

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3} - \frac{b^{\frac{3}{2}}\log\left(\frac{a}{bx^2}\right)}{2} + b^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(3/2)/x**4,x)`

[Out] 
$$-a*\sqrt{b}*\sqrt{a/(b*x**2) + 1}/(3*x**2) - 4*b**(3/2)*\sqrt{a/(b*x**2) + 1}/3 - b**(3/2)*\log(a/(b*x**2))/2 + b**(3/2)*\log(\sqrt{a/(b*x**2) + 1} + 1)$$

**GIAC/XCAS [A]** time = 0.216687, size = 154, normalized size = 2.52

$$-\frac{1}{2}b^{\frac{3}{2}}\ln\left(\left(\sqrt{bx} - \sqrt{bx^2+a}\right)^2\right) + \frac{4\left(3\left(\sqrt{bx} - \sqrt{bx^2+a}\right)^4 ab^{\frac{3}{2}} - 3\left(\sqrt{bx} - \sqrt{bx^2+a}\right)^2 a^2 b^{\frac{3}{2}} + 2a^3 b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{bx} - \sqrt{bx^2+a}\right)^2 - a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/2*b^(3/2)*ln((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 4/3*(3*(sqrt(b)
)*x - sqrt(b*x^2 + a))^4*a*b^(3/2) - 3*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*a^2*b^(3/2) + 2*a^3*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))
^2 - a)^3
```



$$3.383 \quad \int \frac{(a+bx^2)^{3/2}}{x^6} dx$$

**Optimal.** Leaf size=21

$$-\frac{(a+bx^2)^{5/2}}{5ax^5}$$

[Out]  $-(a + b*x^2)^{(5/2)/(5*a*x^5)}$

**Rubi [A]** time = 0.0214373, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$-\frac{(a+bx^2)^{5/2}}{5ax^5}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^{(3/2)/x^6}, x]$

[Out]  $-(a + b*x^2)^{(5/2)/(5*a*x^5)}$

**Rubi in Sympy [A]** time = 3.20305, size = 17, normalized size = 0.81

$$-\frac{(a+bx^2)^{\frac{5}{2}}}{5ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x^{**2}+a)^{(3/2)/x^{**6}}, x)$

[Out]  $-(a + b*x^{**2})^{(5/2)/(5*a*x^{**5})}$

**Mathematica [A]** time = 0.0260236, size = 21, normalized size = 1.

$$-\frac{(a+bx^2)^{5/2}}{5ax^5}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b*x^2)^{(3/2)/x^6}, x]$

[Out]  $-(a + b*x^2)^{(5/2)/(5*a*x^5)}$

**Maple [A]** time = 0.006, size = 18, normalized size = 0.9

$$-\frac{1}{5ax^5} (bx^2 + a)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((b*x^2+a)^{(3/2)/x^6}, x)$

[Out]  $-1/5 * (b * x^2 + a)^{5/2} / a / x^5$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x^6,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.233553, size = 47, normalized size = 2.24

$$-\frac{(b^2x^4 + 2abx^2 + a^2)\sqrt{bx^2 + a}}{5ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x^6,x, algorithm="fricas")`

[Out]  $-1/5 * (b^2 * x^4 + 2 * a * b * x^2 + a^2) * \text{sqrt}(b * x^2 + a) / (a * x^5)$

**Sympy [A]** time = 3.77138, size = 68, normalized size = 3.24

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx^2} + 1}}{5x^4} - \frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2} + 1}}{5x^2} - \frac{b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2} + 1}}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(3/2)/x**6,x)`

[Out]  $-a * \text{sqrt}(b) * \text{sqrt}(a / (b * x^2) + 1) / (5 * x^4) - 2 * b^{3/2} * \text{sqrt}(a / (b * x^2) + 1) / (5 * x^2) - b^{5/2} * \text{sqrt}(a / (b * x^2) + 1) / (5 * a)$

**GIAC/XCAS [A]** time = 0.213246, size = 116, normalized size = 5.52

$$\frac{2 \left( 5 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^8 b^{\frac{5}{2}} + 10 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^4 a^2 b^{\frac{5}{2}} + a^4 b^{\frac{5}{2}} \right)}{5 \left( \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 - a \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x^6,x, algorithm="giac")`

[Out]  $2/5 * (5 * (\text{sqrt}(b) * x - \text{sqrt}(b * x^2 + a))^8 * b^{5/2} + 10 * (\text{sqrt}(b) * x - \text{sqrt}(b * x^2 + a))^4 * a^2 * b^{5/2} + a^4 * b^{5/2}) / ((\text{sqrt}(b) * x - \text{sqrt}(b * x^2 + a))^2 - a)^5$

$$3.384 \quad \int \frac{(a+bx^2)^{3/2}}{x^8} dx$$

**Optimal.** Leaf size=44

$$\frac{2b(a+bx^2)^{5/2}}{35a^2x^5} - \frac{(a+bx^2)^{5/2}}{7ax^7}$$

[Out]  $-(a + b*x^2)^{(5/2)}/(7*a*x^7) + (2*b*(a + b*x^2)^{(5/2)})/(35*a^2*x^5)$

**Rubi [A]** time = 0.0443042, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{2b(a+bx^2)^{5/2}}{35a^2x^5} - \frac{(a+bx^2)^{5/2}}{7ax^7}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(3/2)/x^8, x]

[Out]  $-(a + b*x^2)^{(5/2)}/(7*a*x^7) + (2*b*(a + b*x^2)^{(5/2)})/(35*a^2*x^5)$

**Rubi in Sympy [A]** time = 5.16875, size = 37, normalized size = 0.84

$$-\frac{(a+bx^2)^{\frac{5}{2}}}{7ax^7} + \frac{2b(a+bx^2)^{\frac{5}{2}}}{35a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(3/2)/x\*\*8, x)

[Out]  $-(a + b*x**2)**(5/2)/(7*a*x**7) + 2*b*(a + b*x**2)**(5/2)/(35*a**2*x**5)$

**Mathematica [A]** time = 0.033564, size = 31, normalized size = 0.7

$$\frac{(a+bx^2)^{5/2}(2bx^2-5a)}{35a^2x^7}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(3/2)/x^8, x]

[Out]  $((a + b*x^2)^{(5/2)}*(-5*a + 2*b*x^2))/(35*a^2*x^7)$

**Maple [A]** time = 0.006, size = 28, normalized size = 0.6

$$-\frac{-2bx^2 + 5a}{35x^7a^2} (bx^2 + a)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(3/2)/x^8,x)`

[Out]  $-1/35*(b*x^2+a)^{(5/2)*(-2*b*x^2+5*a)/x^7/a^2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x^8,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.242443, size = 66, normalized size = 1.5

$$\frac{(2b^3x^6 - ab^2x^4 - 8a^2bx^2 - 5a^3)\sqrt{bx^2 + a}}{35a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x^8,x, algorithm="fricas")`

[Out]  $1/35*(2*b^3*x^6 - a*b^2*x^4 - 8*a^2*b*x^2 - 5*a^3)*\text{sqrt}(b*x^2 + a)/(a^2*x^7)$

**Sympy [A]** time = 5.13333, size = 94, normalized size = 2.14

$$-\frac{a\sqrt{b}\sqrt{\frac{a}{bx^2} + 1}}{7x^6} - \frac{8b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2} + 1}}{35x^4} - \frac{b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2} + 1}}{35ax^2} + \frac{2b^{\frac{7}{2}}\sqrt{\frac{a}{bx^2} + 1}}{35a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(3/2)/x**8,x)`

[Out]  $-a*\text{sqrt}(b)*\text{sqrt}(a/(b*x**2) + 1)/(7*x**6) - 8*b**(3/2)*\text{sqrt}(a/(b*x**2) + 1)/(35*x**4) - b**(5/2)*\text{sqrt}(a/(b*x**2) + 1)/(35*a*x**2) + 2*b**(7/2)*\text{sqrt}(a/(b*x**2) + 1)/(35*a**2)$

**GIAC/XCAS [A]** time = 0.216848, size = 224, normalized size = 5.09

$$\frac{4\left(35\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^{10} b^{\frac{7}{2}} + 35\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^8 ab^{\frac{7}{2}} + 70\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^6 a^2 b^{\frac{7}{2}} + 14\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^4 a^3 b^{\frac{7}{2}} + 7\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^2 a^4 b^{\frac{7}{2}} - a^5 b^{\frac{7}{2}}\right)}{35\left(\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^2 - a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x^8,x, algorithm="giac")`

[Out]  $4/35*(35*(\text{sqrt}(b)*x - \text{sqrt}(b*x^2 + a))^{10}*b^{(7/2)} + 35*(\text{sqrt}(b)*x - \text{sqrt}(b*x^2 + a))^{8}*a*b^{(7/2)} + 70*(\text{sqrt}(b)*x - \text{sqrt}(b*x^2 + a))^{6}*a^2*b^{(7/2)} + 14*(\text{sqrt}(b)*x - \text{sqrt}(b*x^2 + a))^{4}*a^3*b^{(7/2)} + 7*(\text{sqrt}(b)*x - \text{sqrt}(b*x^2 + a))^{2}*a^4*b^{(7/2)} - a^5*b^{(7/2)})/((\text{sqrt}(b)*x - \text{sqrt}(b*x^2 + a))^2 - a)^7$

$$3.385 \quad \int \frac{(a+bx^2)^{3/2}}{x^{10}} dx$$

**Optimal.** Leaf size=68

$$-\frac{8b^2(a+bx^2)^{5/2}}{315a^3x^5} + \frac{4b(a+bx^2)^{5/2}}{63a^2x^7} - \frac{(a+bx^2)^{5/2}}{9ax^9}$$

[Out]  $-(a + b*x^2)^{(5/2)}/(9*a*x^9) + (4*b*(a + b*x^2)^{(5/2)})/(63*a^2*x^7) - (8*b^2*(a + b*x^2)^{(5/2)})/(315*a^3*x^5)$

**Rubi [A]** time = 0.0702974, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{8b^2(a+bx^2)^{5/2}}{315a^3x^5} + \frac{4b(a+bx^2)^{5/2}}{63a^2x^7} - \frac{(a+bx^2)^{5/2}}{9ax^9}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(3/2)/x^10, x]

[Out]  $-(a + b*x^2)^{(5/2)}/(9*a*x^9) + (4*b*(a + b*x^2)^{(5/2)})/(63*a^2*x^7) - (8*b^2*(a + b*x^2)^{(5/2)})/(315*a^3*x^5)$

**Rubi in Sympy [A]** time = 8.05282, size = 61, normalized size = 0.9

$$-\frac{(a+bx^2)^{5/2}}{9ax^9} + \frac{4b(a+bx^2)^{5/2}}{63a^2x^7} - \frac{8b^2(a+bx^2)^{5/2}}{315a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(3/2)/x\*\*10, x)

[Out]  $-(a + b*x^2)^{(5/2)}/(9*a*x^9) + 4*b*(a + b*x^2)^{(5/2)}/(63*a^2*x^7) - 8*b^2*(a + b*x^2)^{(5/2)}/(315*a^3*x^5)$

**Mathematica [A]** time = 0.0442504, size = 42, normalized size = 0.62

$$-\frac{(a+bx^2)^{5/2}(35a^2 - 20abx^2 + 8b^2x^4)}{315a^3x^9}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(3/2)/x^10, x]

[Out]  $-((a + b*x^2)^{(5/2)}*(35*a^2 - 20*a*b*x^2 + 8*b^2*x^4))/(315*a^3*x^9)$

**Maple [A]** time = 0.006, size = 39, normalized size = 0.6

$$-\frac{8b^2x^4 - 20abx^2 + 35a^2}{315x^9a^3} (bx^2 + a)^{5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(3/2)/x^10,x)`

[Out]  $-1/315*(b*x^2+a)^{(5/2)}*(8*b^2*x^4-20*a*b*x^2+35*a^2)/x^9/a^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x^10,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.260837, size = 81, normalized size = 1.19

$$-\frac{(8b^4x^8 - 4ab^3x^6 + 3a^2b^2x^4 + 50a^3bx^2 + 35a^4)\sqrt{bx^2 + a}}{315a^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x^10,x, algorithm="fricas")`

[Out]  $-1/315*(8*b^4*x^8 - 4*a*b^3*x^6 + 3*a^2*b^2*x^4 + 50*a^3*b*x^2 + 35*a^4)*\text{sqrt}(b*x^2 + a)/(a^3*x^9)$

**Sympy [A]** time = 7.90687, size = 420, normalized size = 6.18

$$\begin{aligned} &-\frac{35a^6b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2} + 1}}{315a^5b^4x^8 + 630a^4b^5x^{10} + 315a^3b^6x^{12}} - \frac{120a^5b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2} + 1}}{315a^5b^4x^8 + 630a^4b^5x^{10} + 315a^3b^6x^{12}} \\ &-\frac{138a^4b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2} + 1}}{315a^5b^4x^8 + 630a^4b^5x^{10} + 315a^3b^6x^{12}} - \frac{52a^3b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2} + 1}}{315a^5b^4x^8 + 630a^4b^5x^{10} + 315a^3b^6x^{12}} \\ &-\frac{3a^2b^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2} + 1}}{315a^5b^4x^8 + 630a^4b^5x^{10} + 315a^3b^6x^{12}} - \frac{12ab^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2} + 1}}{315a^5b^4x^8 + 630a^4b^5x^{10} + 315a^3b^6x^{12}} \\ &-\frac{8b^{\frac{21}{2}}x^{12}\sqrt{\frac{a}{bx^2} + 1}}{315a^5b^4x^8 + 630a^4b^5x^{10} + 315a^3b^6x^{12}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(3/2)/x**10,x)`

[Out]  $-35*a**6*b**(9/2)*\text{sqrt}(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 120*a**5*b**(11/2)*x**2*\text{sqrt}(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 138*a**4*b**(13/2)*x**4*\text{sqrt}(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 52*a**3*b**(15/2)*x**6*\text{sqrt}(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 3*a**2*b**(17/2)*x**8*\text{sqrt}(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 12*a*b**(19/2)*x**10*\text{sqrt}(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 8*b**(21/2)*x**12*\text{sqrt}(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12)$

**GIAC/XCAS [A]** time = 0.215708, size = 259, normalized size = 3.81

$$\frac{16 \left( 210 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{12} b^{\frac{9}{2}} + 315 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{10} ab^{\frac{9}{2}} + 441 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^8 a^2 b^{\frac{9}{2}} + 126 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^6 a^3 b^{\frac{9}{2}} + 36 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^4 a^4 b^{\frac{9}{2}} - 9 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 a^5 b^{\frac{9}{2}} + a^6 b^{\frac{9}{2}} \right)}{315 \left( \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 - a \right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/x^10,x, algorithm="giac")

[Out] 16/315\*(210\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^12\*b^(9/2) + 315\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^10\*a\*b^(9/2) + 441\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^8\*a^2\*b^(9/2) + 126\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^6\*a^3\*b^(9/2) + 36\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^4\*a^4\*b^(9/2) - 9\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^2\*a^5\*b^(9/2) + a^6\*b^(9/2))/((sqrt(b)\*x - sqrt(b\*x^2 + a))^2 - a)^9

$$3.386 \quad \int \frac{(a+bx^2)^{3/2}}{x^{12}} dx$$

**Optimal.** Leaf size=92

$$\frac{16b^3 (a+bx^2)^{5/2}}{1155a^4x^5} - \frac{8b^2 (a+bx^2)^{5/2}}{231a^3x^7} + \frac{2b (a+bx^2)^{5/2}}{33a^2x^9} - \frac{(a+bx^2)^{5/2}}{11ax^{11}}$$

[Out]  $-(a + b*x^2)^{(5/2)}/(11*a*x^{11}) + (2*b*(a + b*x^2)^{(5/2)})/(33*a^2*x^9) - (8*b^2*(a + b*x^2)^{(5/2)})/(231*a^3*x^7) + (16*b^3*(a + b*x^2)^{(5/2)})/(1155*a^4*x^5)$

**Rubi [A]** time = 0.100224, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{16b^3 (a+bx^2)^{5/2}}{1155a^4x^5} - \frac{8b^2 (a+bx^2)^{5/2}}{231a^3x^7} + \frac{2b (a+bx^2)^{5/2}}{33a^2x^9} - \frac{(a+bx^2)^{5/2}}{11ax^{11}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(3/2)/x^12, x]

[Out]  $-(a + b*x^2)^{(5/2)}/(11*a*x^{11}) + (2*b*(a + b*x^2)^{(5/2)})/(33*a^2*x^9) - (8*b^2*(a + b*x^2)^{(5/2)})/(231*a^3*x^7) + (16*b^3*(a + b*x^2)^{(5/2)})/(1155*a^4*x^5)$

**Rubi in Sympy [A]** time = 11.6192, size = 85, normalized size = 0.92

$$-\frac{(a+bx^2)^{\frac{5}{2}}}{11ax^{11}} + \frac{2b(a+bx^2)^{\frac{5}{2}}}{33a^2x^9} - \frac{8b^2(a+bx^2)^{\frac{5}{2}}}{231a^3x^7} + \frac{16b^3(a+bx^2)^{\frac{5}{2}}}{1155a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(3/2)/x\*\*12, x)

[Out]  $-(a + b*x^2)^{(5/2)}/(11*a*x^{11}) + 2*b*(a + b*x^2)^{(5/2)}/(33*a^2*x^9) - 8*b^2*(a + b*x^2)^{(5/2)}/(231*a^3*x^7) + 16*b^3*(a + b*x^2)^{(5/2)}/(1155*a^4*x^5)$

**Mathematica [A]** time = 0.0474804, size = 53, normalized size = 0.58

$$\frac{(a+bx^2)^{5/2}(-105a^3 + 70a^2bx^2 - 40ab^2x^4 + 16b^3x^6)}{1155a^4x^{11}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(3/2)/x^12, x]

[Out]  $((a + b*x^2)^{(5/2)}*(-105*a^3 + 70*a^2*b*x^2 - 40*a*b^2*x^4 + 16*b^3*x^6))/(1155*a^4*x^{11})$

**Maple [A]** time = 0.008, size = 50, normalized size = 0.5

$$-\frac{-16b^3x^6 + 40ab^2x^4 - 70a^2bx^2 + 105a^3}{1155x^{11}a^4}(bx^2 + a)^{\frac{5}{2}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(3/2)/x^12,x)`

[Out]  $-1/1155*(b*x^2+a)^{(5/2)}*(-16*b^3*x^6+40*a*b^2*x^4-70*a^2*b*x^2+105*a^3)/x^{11}/a^4$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x^12,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.303392, size = 96, normalized size = 1.04

$$\frac{(16b^5x^{10} - 8ab^4x^8 + 6a^2b^3x^6 - 5a^3b^2x^4 - 140a^4bx^2 - 105a^5)\sqrt{bx^2 + a}}{1155a^4x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/x^12,x, algorithm="fricas")`

[Out]  $1/1155*(16*b^5*x^{10} - 8*a*b^4*x^8 + 6*a^2*b^3*x^6 - 5*a^3*b^2*x^4 - 140*a^4*b*x^2 - 105*a^5)*\text{sqrt}(b*x^2 + a)/(a^4*x^{11})$

**Sympy** [A] time = 12.2473, size = 648, normalized size = 7.04

$$\begin{aligned} & \frac{105a^8b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2} + 1}}{1155a^7b^9x^{10} + 3465a^6b^{10}x^{12} + 3465a^5b^{11}x^{14} + 1155a^4b^{12}x^{16}} \\ & - \frac{455a^7b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2} + 1}}{1155a^7b^9x^{10} + 3465a^6b^{10}x^{12} + 3465a^5b^{11}x^{14} + 1155a^4b^{12}x^{16}} \\ & - \frac{740a^6b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2} + 1}}{1155a^7b^9x^{10} + 3465a^6b^{10}x^{12} + 3465a^5b^{11}x^{14} + 1155a^4b^{12}x^{16}} \\ & - \frac{534a^5b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2} + 1}}{1155a^7b^9x^{10} + 3465a^6b^{10}x^{12} + 3465a^5b^{11}x^{14} + 1155a^4b^{12}x^{16}} \\ & - \frac{145a^4b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2} + 1}}{1155a^7b^9x^{10} + 3465a^6b^{10}x^{12} + 3465a^5b^{11}x^{14} + 1155a^4b^{12}x^{16}} \\ & - \frac{5a^3b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2} + 1}}{1155a^7b^9x^{10} + 3465a^6b^{10}x^{12} + 3465a^5b^{11}x^{14} + 1155a^4b^{12}x^{16}} \\ & + \frac{30a^2b^{\frac{31}{2}}x^{12}\sqrt{\frac{a}{bx^2} + 1}}{1155a^7b^9x^{10} + 3465a^6b^{10}x^{12} + 3465a^5b^{11}x^{14} + 1155a^4b^{12}x^{16}} \\ & + \frac{40ab^{\frac{33}{2}}x^{14}\sqrt{\frac{a}{bx^2} + 1}}{1155a^7b^9x^{10} + 3465a^6b^{10}x^{12} + 3465a^5b^{11}x^{14} + 1155a^4b^{12}x^{16}} \\ & + \frac{16b^{\frac{35}{2}}x^{16}\sqrt{\frac{a}{bx^2} + 1}}{1155a^7b^9x^{10} + 3465a^6b^{10}x^{12} + 3465a^5b^{11}x^{14} + 1155a^4b^{12}x^{16}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(3/2)/x\*\*12,x)

[Out] 
$$\begin{aligned} & -105*a**8*b**(19/2)*\sqrt{a/(b*x**2) + 1}/(1155*a**7*b**9*x**10 + \\ & 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) - 455*a**7*b** \\ & (21/2)*x**2*\sqrt{a/(b*x**2) + 1}/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155* \\ & a**4*b**12*x**16) - 740*a**6*b**(23/2)*x**4*\sqrt{a/(b*x**2) + 1}/ \\ & (1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) - 534*a**5*b** \\ & (25/2)*x**6*\sqrt{a/(b*x**2) + 1}/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) - 145*a**4*b** \\ & (27/2)*x**8*\sqrt{a/(b*x**2) + 1}/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) + 5*a**3*b** \\ & (29/2)*x**10*\sqrt{a/(b*x**2) + 1}/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) \\ & + 30*a**2*b**(31/2)*x**12*\sqrt{a/(b*x**2) + 1}/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) + 40*a*b** \\ & (33/2)*x**14*\sqrt{a/(b*x**2) + 1}/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) + 16*b** \\ & (35/2)*x**16*\sqrt{a/(b*x**2) + 1}/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) \end{aligned}$$

**GIAC/XCAS [A]** time = 0.217041, size = 297, normalized size = 3.23

$$32 \left( 1155 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{14} b^{\frac{11}{2}} + 2079 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{12} ab^{\frac{11}{2}} + 2541 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{10} a^2 b^{\frac{11}{2}} + 825 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^8 a^3 b^{\frac{11}{2}} + 165 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^6 a^4 b^{\frac{11}{2}} - 55 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^4 a^5 b^{\frac{11}{2}} + 11 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 a^6 b^{\frac{11}{2}} - a^7 b^{\frac{11}{2}} \right) / \left( \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 - a \right)^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/x^12,x, algorithm="giac")

[Out] 
$$\begin{aligned} & 32/1155*(1155*(\sqrt{b}*x - \sqrt{b*x^2 + a})^{14}*b^{(11/2)} + 2079*(\sqrt{b}*x - \sqrt{b*x^2 + a})^{12}*a*b^{(11/2)} + 2541*(\sqrt{b}*x - \sqrt{b*x^2 + a})^{10}*a^2*b^{(11/2)} + 825*(\sqrt{b}*x - \sqrt{b*x^2 + a})^8*a^3*b^{(11/2)} + 165*(\sqrt{b}*x - \sqrt{b*x^2 + a})^6*a^4*b^{(11/2)} - 55*(\sqrt{b}*x - \sqrt{b*x^2 + a})^4*a^5*b^{(11/2)} + 11*(\sqrt{b}*x - \sqrt{b*x^2 + a})^2*a^6*b^{(11/2)} - a^7*b^{(11/2)})/((\sqrt{b}*x - \sqrt{b*x^2 + a})^2 - a)^{11} \end{aligned}$$

$$3.387 \quad \int x^7 (a + bx^2)^{5/2} dx$$

**Optimal.** Leaf size=80

$$-\frac{a^3 (a + bx^2)^{7/2}}{7b^4} + \frac{a^2 (a + bx^2)^{9/2}}{3b^4} + \frac{(a + bx^2)^{13/2}}{13b^4} - \frac{3a (a + bx^2)^{11/2}}{11b^4}$$

[Out]  $-(a^3 (a + b*x^2)^{(7/2)})/(7*b^4) + (a^2*(a + b*x^2)^{(9/2)})/(3*b^4) - (3*a*(a + b*x^2)^{(11/2)})/(11*b^4) + (a + b*x^2)^{(13/2)}/(13*b^4)$

**Rubi [A]** time = 0.11995, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{a^3 (a + bx^2)^{7/2}}{7b^4} + \frac{a^2 (a + bx^2)^{9/2}}{3b^4} + \frac{(a + bx^2)^{13/2}}{13b^4} - \frac{3a (a + bx^2)^{11/2}}{11b^4}$$

Antiderivative was successfully verified.

[In] Int[x^7\*(a + b\*x^2)^(5/2), x]

[Out]  $-(a^3*(a + b*x^2)^{(7/2)})/(7*b^4) + (a^2*(a + b*x^2)^{(9/2)})/(3*b^4) - (3*a*(a + b*x^2)^{(11/2)})/(11*b^4) + (a + b*x^2)^{(13/2)}/(13*b^4)$

**Rubi in Sympy [A]** time = 15.7985, size = 70, normalized size = 0.88

$$-\frac{a^3 (a + bx^2)^{\frac{7}{2}}}{7b^4} + \frac{a^2 (a + bx^2)^{\frac{9}{2}}}{3b^4} - \frac{3a (a + bx^2)^{\frac{11}{2}}}{11b^4} + \frac{(a + bx^2)^{\frac{13}{2}}}{13b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*7\*(b\*x\*\*2+a)\*\*(5/2), x)

[Out]  $-a**3*(a + b*x**2)**(7/2)/(7*b**4) + a**2*(a + b*x**2)**(9/2)/(3*b**4) - 3*a*(a + b*x**2)**(11/2)/(11*b**4) + (a + b*x**2)**(13/2)/(13*b**4)$

**Mathematica [A]** time = 0.0526711, size = 50, normalized size = 0.62

$$\frac{(a + bx^2)^{7/2} (-16a^3 + 56a^2bx^2 - 126ab^2x^4 + 231b^3x^6)}{3003b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^7\*(a + b\*x^2)^(5/2), x]

[Out]  $((a + b*x^2)^{(7/2)}*(-16*a^3 + 56*a^2*b*x^2 - 126*a*b^2*x^4 + 231*b^3*x^6))/(3003*b^4)$

**Maple [A]** time = 0.008, size = 47, normalized size = 0.6

$$-\frac{-231b^3x^6 + 126ab^2x^4 - 56a^2bx^2 + 16a^3}{3003b^4} (bx^2 + a)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(b*x^2+a)^(5/2),x)`

[Out]  $-1/3003*(b*x^2+a)^{(7/2)}*(-231*b^3*x^6+126*a*b^2*x^4-56*a^2*b*x^2+16*a^3)/b^4$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)*x^7,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.238294, size = 107, normalized size = 1.34

$$\frac{(231 b^6 x^{12} + 567 a b^5 x^{10} + 371 a^2 b^4 x^8 + 5 a^3 b^3 x^6 - 6 a^4 b^2 x^4 + 8 a^5 b x^2 - 16 a^6) \sqrt{b x^2 + a}}{3003 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)*x^7,x, algorithm="fricas")`

[Out]  $1/3003*(231*b^6*x^{12} + 567*a*b^5*x^{10} + 371*a^2*b^4*x^8 + 5*a^3*b^3*x^6 - 6*a^4*b^2*x^4 + 8*a^5*b*x^2 - 16*a^6)*\text{sqrt}(b*x^2 + a)/b^4$

**Sympy [A]** time = 29.3681, size = 158, normalized size = 1.98

$$\begin{cases} -\frac{16a^6\sqrt{a+bx^2}}{3003b^4} + \frac{8a^5x^2\sqrt{a+bx^2}}{3003b^3} - \frac{2a^4x^4\sqrt{a+bx^2}}{1001b^2} + \frac{5a^3x^6\sqrt{a+bx^2}}{3003b} + \frac{53a^2x^8\sqrt{a+bx^2}}{429} + \frac{27abx^{10}\sqrt{a+bx^2}}{143} + \frac{b^2x^{12}\sqrt{a+bx^2}}{13} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{2}}x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(b*x**2+a)**(5/2),x)`

[Out] `Piecewise((-16*a**6*sqrt(a + b*x**2)/(3003*b**4) + 8*a**5*x**2*sqrt(a + b*x**2)/(3003*b**3) - 2*a**4*x**4*sqrt(a + b*x**2)/(1001*b**2) + 5*a**3*x**6*sqrt(a + b*x**2)/(3003*b) + 53*a**2*x**8*sqrt(a + b*x**2)/429 + 27*a*b*x**10*sqrt(a + b*x**2)/143 + b**2*x**12*sqrt(a + b*x**2)/13, Ne(b, 0)), (a**(5/2)*x**8/8, True))`

**GIAC/XCAS [A]** time = 0.210306, size = 301, normalized size = 3.76

$$\frac{143 \left( 35 (bx^2+a)^{\frac{9}{2}} - 135 (bx^2+a)^{\frac{7}{2}} a + 189 (bx^2+a)^{\frac{5}{2}} a^2 - 105 (bx^2+a)^{\frac{3}{2}} a^3 \right) a^2}{b^3} + \frac{26 \left( 315 (bx^2+a)^{\frac{11}{2}} - 1540 (bx^2+a)^{\frac{9}{2}} a + 2970 (bx^2+a)^{\frac{7}{2}} a^2 - 2772 (bx^2+a)^{\frac{5}{2}} a^3 + 1188 (bx^2+a)^{\frac{3}{2}} a^4 \right) a^2}{b^3}$$

45045 b

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)*x^7,x, algorithm="giac")`

[Out]  $\frac{1}{45045} (143 (35 (b^2 x^2 + a)^{9/2} - 135 (b^2 x^2 + a)^{7/2} a + 189 (b^2 x^2 + a)^{5/2} a^2 - 105 (b^2 x^2 + a)^{3/2} a^3) a^2 / b^3 + 26 (315 (b^2 x^2 + a)^{11/2} - 1540 (b^2 x^2 + a)^{9/2} a + 2970 (b^2 x^2 + a)^{7/2} a^2 - 2772 (b^2 x^2 + a)^{5/2} a^3 + 1155 (b^2 x^2 + a)^{3/2} a^4) a / b^3 + 5 (693 (b^2 x^2 + a)^{13/2} - 4095 (b^2 x^2 + a)^{11/2} a + 10010 (b^2 x^2 + a)^{9/2} a^2 - 12870 (b^2 x^2 + a)^{7/2} a^3 + 9009 (b^2 x^2 + a)^{5/2} a^4 - 3003 (b^2 x^2 + a)^{3/2} a^5) / b^3$

$$3.388 \quad \int x^5 (a + bx^2)^{5/2} dx$$

**Optimal.** Leaf size=59

$$\frac{a^2 (a + bx^2)^{7/2}}{7b^3} + \frac{(a + bx^2)^{11/2}}{11b^3} - \frac{2a (a + bx^2)^{9/2}}{9b^3}$$

[Out]  $(a^2 (a + b x^2)^{(7/2)}) / (7 b^3) - (2 a (a + b x^2)^{(9/2)}) / (9 b^3) + (a + b x^2)^{(11/2)} / (11 b^3)$

**Rubi [A]** time = 0.0920095, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{a^2 (a + bx^2)^{7/2}}{7b^3} + \frac{(a + bx^2)^{11/2}}{11b^3} - \frac{2a (a + bx^2)^{9/2}}{9b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(a + b\*x^2)^(5/2), x]

[Out]  $(a^2 (a + b x^2)^{(7/2)}) / (7 b^3) - (2 a (a + b x^2)^{(9/2)}) / (9 b^3) + (a + b x^2)^{(11/2)} / (11 b^3)$

**Rubi in Sympy [A]** time = 11.7736, size = 51, normalized size = 0.86

$$\frac{a^2 (a + bx^2)^{7/2}}{7b^3} - \frac{2a (a + bx^2)^{9/2}}{9b^3} + \frac{(a + bx^2)^{11/2}}{11b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(b\*x\*\*2+a)\*\*(5/2), x)

[Out]  $a^2 (a + b x^2)^{(7/2)} / (7 b^3) - 2 a (a + b x^2)^{(9/2)} / (9 b^3) + (a + b x^2)^{(11/2)} / (11 b^3)$

**Mathematica [A]** time = 0.0411706, size = 39, normalized size = 0.66

$$\frac{(a + bx^2)^{7/2} (8a^2 - 28abx^2 + 63b^2x^4)}{693b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*x^2)^(5/2), x]

[Out]  $((a + b x^2)^{(7/2)} (8 a^2 - 28 a b x^2 + 63 b^2 x^4)) / (693 b^3)$

**Maple [A]** time = 0.007, size = 36, normalized size = 0.6

$$\frac{63 b^2 x^4 - 28 a b x^2 + 8 a^2}{693 b^3} (b x^2 + a)^{7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(b*x^2+a)^(5/2),x)`

[Out]  $1/693*(b*x^2+a)^{(7/2)}*(63*b^2*x^4-28*a*b*x^2+8*a^2)/b^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)*x^5,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.235479, size = 92, normalized size = 1.56

$$\frac{(63 b^5 x^{10} + 161 a b^4 x^8 + 113 a^2 b^3 x^6 + 3 a^3 b^2 x^4 - 4 a^4 b x^2 + 8 a^5) \sqrt{b x^2 + a}}{693 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)*x^5,x, algorithm="fricas")`

[Out]  $1/693*(63*b^5*x^{10} + 161*a*b^4*x^8 + 113*a^2*b^3*x^6 + 3*a^3*b^2*x^4 - 4*a^4*b*x^2 + 8*a^5)*\text{sqrt}(b*x^2 + a)/b^3$

**Sympy [A]** time = 18.5332, size = 133, normalized size = 2.25

$$\begin{cases} \frac{8a^5\sqrt{a+bx^2}}{693b^3} - \frac{4a^4x^2\sqrt{a+bx^2}}{693b^2} + \frac{a^3x^4\sqrt{a+bx^2}}{231b} + \frac{113a^2x^6\sqrt{a+bx^2}}{693} + \frac{23abx^8\sqrt{a+bx^2}}{99} + \frac{b^2x^{10}\sqrt{a+bx^2}}{11} & \text{for } b \neq 0 \\ \frac{a^5x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(b*x**2+a)**(5/2),x)`

[Out] `Piecewise((8*a**5*sqrt(a + b*x**2)/(693*b**3) - 4*a**4*x**2*sqrt(a + b*x**2)/(693*b**2) + a**3*x**4*sqrt(a + b*x**2)/(231*b) + 113*a**2*x**6*sqrt(a + b*x**2)/693 + 23*a*b*x**8*sqrt(a + b*x**2)/99 + b**2*x**10*sqrt(a + b*x**2)/11, Ne(b, 0)), (a**(5/2)*x**6/6, True))`

**GIAC/XCAS [A]** time = 0.208322, size = 243, normalized size = 4.12

$$\frac{33 \left( 15 (bx^2+a)^{\frac{7}{2}} - 42 (bx^2+a)^{\frac{5}{2}} a + 35 (bx^2+a)^{\frac{3}{2}} a^2 \right) a^2}{b^2} + \frac{22 \left( 35 (bx^2+a)^{\frac{9}{2}} - 135 (bx^2+a)^{\frac{7}{2}} a + 189 (bx^2+a)^{\frac{5}{2}} a^2 - 105 (bx^2+a)^{\frac{3}{2}} a^3 \right) a}{b^2} + \frac{315 (bx^2+a)^{\frac{11}{2}} - 1540 (bx^2+a)^{\frac{9}{2}} a + 1575 (bx^2+a)^{\frac{7}{2}} a^2 - 1050 (bx^2+a)^{\frac{5}{2}} a^3 + 315 (bx^2+a)^{\frac{3}{2}} a^4}{3465 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)*x^5,x, algorithm="giac")`

[Out]  $1/3465*(33*(15*(b*x^2 + a)^{(7/2)} - 42*(b*x^2 + a)^{(5/2)}*a + 35*(b*x^2 + a)^{(3/2)}*a^2)*a^2/b^2 + 22*(35*(b*x^2 + a)^{(9/2)} - 135*(b*x^2 + a)^{(7/2)}*a + 189*(b*x^2 + a)^{(5/2)}*a^2 - 105*(b*x^2 + a)^{(3/2)}*a^3 + 315*(b*x^2 + a)^{(1/2)}*a^4)/3465*b$

$$\begin{aligned} & x^2 + a)^{(7/2)} * a + 189 * (b * x^2 + a)^{(5/2)} * a^2 - 105 * (b * x^2 + a)^{(3/2)} * a^3) * a / b^2 + (315 * (b * x^2 + a)^{(11/2)} - 1540 * (b * x^2 + a)^{(9/2)} \\ & * a + 2970 * (b * x^2 + a)^{(7/2)} * a^2 - 2772 * (b * x^2 + a)^{(5/2)} * a^3 + 1155 * (b * x^2 + a)^{(3/2)} * a^4) / b^2) / b \end{aligned}$$



$$3.389 \quad \int x^3 (a + bx^2)^{5/2} dx$$

**Optimal.** Leaf size=38

$$\frac{(a + bx^2)^{9/2}}{9b^2} - \frac{a(a + bx^2)^{7/2}}{7b^2}$$

[Out]  $-(a*(a + b*x^2)^(7/2))/(7*b^2) + (a + b*x^2)^(9/2)/(9*b^2)$

**Rubi [A]** time = 0.0647921, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{(a + bx^2)^{9/2}}{9b^2} - \frac{a(a + bx^2)^{7/2}}{7b^2}$$

Antiderivative was successfully verified.

[In] `Int[x^3*(a + b*x^2)^(5/2), x]`

[Out]  $-(a*(a + b*x^2)^(7/2))/(7*b^2) + (a + b*x^2)^(9/2)/(9*b^2)$

**Rubi in Sympy [A]** time = 7.99259, size = 31, normalized size = 0.82

$$-\frac{a(a + bx^2)^{7/2}}{7b^2} + \frac{(a + bx^2)^{9/2}}{9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3*(b*x**2+a)**(5/2), x)`

[Out]  $-a*(a + b*x**2)**(7/2)/(7*b**2) + (a + b*x**2)**(9/2)/(9*b**2)$

**Mathematica [A]** time = 0.0372764, size = 28, normalized size = 0.74

$$\frac{(a + bx^2)^{7/2} (7bx^2 - 2a)}{63b^2}$$

Antiderivative was successfully verified.

[In] `Integrate[x^3*(a + b*x^2)^(5/2), x]`

[Out]  $((a + b*x^2)^(7/2)*(-2*a + 7*b*x^2))/(63*b^2)$

**Maple [A]** time = 0.007, size = 25, normalized size = 0.7

$$-\frac{-7bx^2 + 2a}{63b^2} (bx^2 + a)^{7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(b*x^2+a)^(5/2), x)`

[Out]  $-1/63*(b*x^2+a)^(7/2)*(-7*b*x^2+2*a)/b^2$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)\*x^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.236804, size = 76, normalized size = 2.

$$\frac{(7b^4x^8 + 19ab^3x^6 + 15a^2b^2x^4 + a^3bx^2 - 2a^4)\sqrt{bx^2 + a}}{63b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)\*x^3,x, algorithm="fricas")

[Out] 1/63\*(7\*b^4\*x^8 + 19\*a\*b^3\*x^6 + 15\*a^2\*b^2\*x^4 + a^3\*b\*x^2 - 2\*a^4)\*sqrt(b\*x^2 + a)/b^2

---

**Sympy [A]** time = 11.427, size = 109, normalized size = 2.87

$$\begin{cases} \frac{-2a^4\sqrt{a+bx^2}}{63b^2} + \frac{a^3x^2\sqrt{a+bx^2}}{63b} + \frac{5a^2x^4\sqrt{a+bx^2}}{21} + \frac{19abx^6\sqrt{a+bx^2}}{63} + \frac{b^2x^8\sqrt{a+bx^2}}{9} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(b\*x\*\*2+a)\*\*(5/2),x)

[Out] Piecewise((-2\*a\*\*4\*sqrt(a + b\*x\*\*2)/(63\*b\*\*2) + a\*\*3\*x\*\*2\*sqrt(a + b\*x\*\*2)/(63\*b) + 5\*a\*\*2\*x\*\*4\*sqrt(a + b\*x\*\*2)/21 + 19\*a\*b\*x\*\*6\*sqrt(a + b\*x\*\*2)/63 + b\*\*2\*x\*\*8\*sqrt(a + b\*x\*\*2)/9, Ne(b, 0)), (a\*\*(5/2)\*x\*\*4/4, True))

---

**GIAC/XCAS [A]** time = 0.20921, size = 186, normalized size = 4.89

$$\frac{21 \left( 3 (bx^2+a)^{\frac{5}{2}} - 5 (bx^2+a)^{\frac{3}{2}} \right) a^2}{b} + \frac{6 \left( 15 (bx^2+a)^{\frac{7}{2}} - 42 (bx^2+a)^{\frac{5}{2}} a + 35 (bx^2+a)^{\frac{3}{2}} a^2 \right) a}{b} + \frac{35 (bx^2+a)^{\frac{9}{2}} - 135 (bx^2+a)^{\frac{7}{2}} a + 189 (bx^2+a)^{\frac{5}{2}} a^2 - 105 (bx^2+a)^{\frac{3}{2}} a^3}{b}$$

315 b

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)\*x^3,x, algorithm="giac")

[Out] 1/315\*(21\*(3\*(b\*x^2 + a)^(5/2) - 5\*(b\*x^2 + a)^(3/2)\*a)\*a^2/b + 6\*(15\*(b\*x^2 + a)^(7/2) - 42\*(b\*x^2 + a)^(5/2)\*a + 35\*(b\*x^2 + a)^(3/2)\*a^2)\*a/b + (35\*(b\*x^2 + a)^(9/2) - 135\*(b\*x^2 + a)^(7/2)\*a + 189\*(b\*x^2 + a)^(5/2)\*a^2 - 105\*(b\*x^2 + a)^(3/2)\*a^3)/b)/b

$$3.390 \quad \int x (a + bx^2)^{5/2} dx$$

**Optimal.** Leaf size=18

$$\frac{(a + bx^2)^{7/2}}{7b}$$

[Out]  $(a + b*x^2)^{(7/2)}/(7*b)$

**Rubi [A]** time = 0.0115575, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{(a + bx^2)^{7/2}}{7b}$$

Antiderivative was successfully verified.

[In] `Int[x*(a + b*x^2)^(5/2), x]`

[Out]  $(a + b*x^2)^{(7/2)}/(7*b)$

**Rubi in Sympy [A]** time = 2.13901, size = 12, normalized size = 0.67

$$\frac{(a + bx^2)^{\frac{7}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x*(b*x**2+a)**(5/2), x)`

[Out]  $(a + b*x**2)**(7/2)/(7*b)$

**Mathematica [A]** time = 0.00778807, size = 18, normalized size = 1.

$$\frac{(a + bx^2)^{7/2}}{7b}$$

Antiderivative was successfully verified.

[In] `Integrate[x*(a + b*x^2)^(5/2), x]`

[Out]  $(a + b*x^2)^{(7/2)}/(7*b)$

**Maple [A]** time = 0.004, size = 15, normalized size = 0.8

$$\frac{1}{7b} (bx^2 + a)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x^2+a)^(5/2), x)`

[Out]  $1/7*(b*x^2+a)^{(7/2)}/b$

---

**Maxima [A]** time = 1.34018, size = 19, normalized size = 1.06

$$\frac{(bx^2 + a)^{\frac{7}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)*x,x, algorithm="maxima")`

[Out] `1/7*(b*x^2 + a)^(7/2)/b`

---

**Fricas [A]** time = 0.242635, size = 58, normalized size = 3.22

$$\frac{(b^3x^6 + 3ab^2x^4 + 3a^2bx^2 + a^3)\sqrt{bx^2 + a}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)*x,x, algorithm="fricas")`

[Out] `1/7*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(b*x^2 + a)/b`

---

**Sympy [A]** time = 6.00327, size = 85, normalized size = 4.72

$$\begin{cases} \frac{a^3\sqrt{a+bx^2}}{7b} + \frac{3a^2x^2\sqrt{a+bx^2}}{7} + \frac{3abx^4\sqrt{a+bx^2}}{7} + \frac{b^2x^6\sqrt{a+bx^2}}{7} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x**2+a)**(5/2),x)`

[Out] `Piecewise((a**3*sqrt(a + b*x**2)/(7*b) + 3*a**2*x**2*sqrt(a + b*x**2)/7 + 3*a*b*x**4*sqrt(a + b*x**2)/7 + b**2*x**6*sqrt(a + b*x**2)/7, Ne(b, 0)), (a**(5/2)*x**2/2, True))`

---

**GIAC/XCAS [A]** time = 0.207265, size = 95, normalized size = 5.28

$$\frac{15(bx^2 + a)^{\frac{7}{2}} - 42(bx^2 + a)^{\frac{5}{2}}a + 70(bx^2 + a)^{\frac{3}{2}}a^2 + 14\left(3(bx^2 + a)^{\frac{5}{2}} - 5(bx^2 + a)^{\frac{3}{2}}a\right)a}{105b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)*x,x, algorithm="giac")`

[Out] `1/105*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 70*(b*x^2 + a)^(3/2)*a^2 + 14*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)*a)*a)/b`

$$3.391 \quad \int \frac{(a+bx^2)^{5/2}}{x} dx$$

**Optimal.** Leaf size=72

$$a^{5/2} \left( -\tanh^{-1} \left( \frac{\sqrt{a+bx^2}}{\sqrt{a}} \right) \right) + a^2 \sqrt{a+bx^2} + \frac{1}{3} a (a+bx^2)^{3/2} + \frac{1}{5} (a+bx^2)^{5/2}$$

[Out]  $a^2 \sqrt{a+bx^2} + (a^2 (a+bx^2)^{3/2})/3 + (a+bx^2)^{5/2}/5 - a^{5/2} \operatorname{ArcTanh}[\sqrt{a+bx^2}/\sqrt{a}]$

**Rubi [A]** time = 0.126076, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$a^{5/2} \left( -\tanh^{-1} \left( \frac{\sqrt{a+bx^2}}{\sqrt{a}} \right) \right) + a^2 \sqrt{a+bx^2} + \frac{1}{3} a (a+bx^2)^{3/2} + \frac{1}{5} (a+bx^2)^{5/2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(5/2)/x, x]

[Out]  $a^2 \sqrt{a+bx^2} + (a^2 (a+bx^2)^{3/2})/3 + (a+bx^2)^{5/2}/5 - a^{5/2} \operatorname{ArcTanh}[\sqrt{a+bx^2}/\sqrt{a}]$

**Rubi in Sympy [A]** time = 11.9501, size = 60, normalized size = 0.83

$$-a^{5/2} \operatorname{atanh} \left( \frac{\sqrt{a+bx^2}}{\sqrt{a}} \right) + a^2 \sqrt{a+bx^2} + \frac{a (a+bx^2)^{3/2}}{3} + \frac{(a+bx^2)^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(5/2)/x, x)

[Out]  $-a^{5/2} \operatorname{atanh}(\sqrt{a+bx^2}/\sqrt{a}) + a^2 \sqrt{a+bx^2} + \frac{a (a+bx^2)^{3/2}}{3} + \frac{(a+bx^2)^{5/2}}{5}$

**Mathematica [A]** time = 0.074612, size = 72, normalized size = 1.

$$-a^{5/2} \log \left( \sqrt{a} \sqrt{a+bx^2} + a \right) + a^{5/2} \log(x) + \frac{1}{15} \sqrt{a+bx^2} (23a^2 + 11abx^2 + 3b^2x^4)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(5/2)/x, x]

[Out]  $(\sqrt{a+bx^2} (23a^2 + 11a^2bx^2 + 3b^2x^4))/15 + a^{5/2} \operatorname{Log}[x] - a^{5/2} \operatorname{Log}[a + \sqrt{a} \sqrt{a+bx^2}]$

**Maple [A]** time = 0.007, size = 66, normalized size = 0.9

$$\frac{1}{5} (bx^2 + a)^{5/2} + \frac{a}{3} (bx^2 + a)^{3/2} - a^{5/2} \ln \left( \frac{1}{x} (2a + 2\sqrt{a}\sqrt{bx^2 + a}) \right) + a^2 \sqrt{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(5/2)/x,x)`

[Out]  $\frac{1}{5} (b x^2 + a)^{5/2} + \frac{1}{3} a (b x^2 + a)^{3/2} - a^{5/2} \ln\left(\frac{2 a + 2 a^{1/2} (b x^2 + a)^{1/2}}{x}\right) + a^2 (b x^2 + a)^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.24829, size = 1, normalized size = 0.01

$$\left[ \frac{1}{2} a^{\frac{5}{2}} \log\left(-\frac{bx^2 - 2\sqrt{bx^2 + a}\sqrt{a} + 2a}{x^2}\right) + \frac{1}{15} (3b^2x^4 + 11abx^2 + 23a^2)\sqrt{bx^2 + a}, \right. \\ \left. -\sqrt{-aa^2} \arctan\left(\frac{a}{\sqrt{bx^2 + a}\sqrt{-a}}\right) + \frac{1}{15} (3b^2x^4 + 11abx^2 + 23a^2)\sqrt{bx^2 + a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x,x, algorithm="fricas")`

[Out]  $\left[ \frac{1}{2} a^{5/2} \log\left(-\frac{bx^2 - 2\sqrt{bx^2 + a}\sqrt{a} + 2a}{x^2}\right) + \frac{1}{15} (3b^2x^4 + 11abx^2 + 23a^2)\sqrt{bx^2 + a}, -\sqrt{-a} a^2 \arctan\left(\frac{a}{\sqrt{bx^2 + a}\sqrt{-a}}\right) + \frac{1}{15} (3b^2x^4 + 11abx^2 + 23a^2)\sqrt{bx^2 + a} \right]$

**Sympy [A]** time = 11.8058, size = 105, normalized size = 1.46

$$\frac{23a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}}{15} + \frac{a^{\frac{5}{2}}\log\left(\frac{bx^2}{a}\right)}{2} - a^{\frac{5}{2}}\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right) + \frac{11a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}{15} + \frac{\sqrt{ab^2x^4}\sqrt{1+\frac{bx^2}{a}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(5/2)/x,x)`

[Out]  $23 a^{5/2} \sqrt{1 + b x^2 / a} / 15 + a^{5/2} \log(b x^2 / a) / 2 - a^{5/2} \log(\sqrt{1 + b x^2 / a} + 1) + 11 a^{3/2} b x^2 \sqrt{1 + b x^2 / a} / 15 + \sqrt{a} b^2 x^4 \sqrt{1 + b x^2 / a} / 5$

**GIAC/XCAS [A]** time = 0.210943, size = 84, normalized size = 1.17

$$\frac{a^3 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{1}{5} (bx^2 + a)^{\frac{5}{2}} + \frac{1}{3} (bx^2 + a)^{\frac{3}{2}} a + \sqrt{bx^2 + aa^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)/x,x, algorithm="giac")
```

```
[Out] a^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/5*(b*x^2 + a)^(5/2) + 1/3*(b*x^2 + a)^(3/2)*a + sqrt(b*x^2 + a)*a^2
```

$$3.392 \quad \int \frac{(a+bx^2)^{5/2}}{x^3} dx$$

**Optimal.** Leaf size=80

$$-\frac{5}{2}a^{3/2}b \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{(a+bx^2)^{5/2}}{2x^2} + \frac{5}{6}b(a+bx^2)^{3/2} + \frac{5}{2}ab\sqrt{a+bx^2}$$

[Out] (5\*a\*b\*Sqrt[a + b\*x^2])/2 + (5\*b\*(a + b\*x^2)^(3/2))/6 - (a + b\*x^2)^(5/2)/(2\*x^2) - (5\*a^(3/2)\*b\*ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]])/2

**Rubi [A]** time = 0.130733, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{5}{2}a^{3/2}b \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{(a+bx^2)^{5/2}}{2x^2} + \frac{5}{6}b(a+bx^2)^{3/2} + \frac{5}{2}ab\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(5/2)/x^3, x]

[Out] (5\*a\*b\*Sqrt[a + b\*x^2])/2 + (5\*b\*(a + b\*x^2)^(3/2))/6 - (a + b\*x^2)^(5/2)/(2\*x^2) - (5\*a^(3/2)\*b\*ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]])/2

**Rubi in Sympy [A]** time = 11.9321, size = 73, normalized size = 0.91

$$-\frac{5a^{3/2}b \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2} + \frac{5ab\sqrt{a+bx^2}}{2} + \frac{5b(a+bx^2)^{3/2}}{6} - \frac{(a+bx^2)^{5/2}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(5/2)/x\*\*3, x)

[Out] -5\*a\*\*(3/2)\*b\*atanh(sqrt(a + b\*x\*\*2)/sqrt(a))/2 + 5\*a\*b\*sqrt(a + b\*x\*\*2)/2 + 5\*b\*(a + b\*x\*\*2)\*\*(3/2)/6 - (a + b\*x\*\*2)\*\*(5/2)/(2\*x\*\*2)

**Mathematica [A]** time = 0.108891, size = 76, normalized size = 0.95

$$\frac{1}{6} \left( -15a^{3/2}b \log\left(\sqrt{a}\sqrt{a+bx^2} + a\right) + 15a^{3/2}b \log(x) + \sqrt{a+bx^2} \left( -\frac{3a^2}{x^2} + 14ab + 2b^2x^2 \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(5/2)/x^3, x]

[Out] (Sqrt[a + b\*x^2]\*(14\*a\*b - (3\*a^2)/x^2 + 2\*b^2\*x^2) + 15\*a^(3/2)\*b\*Log[x] - 15\*a^(3/2)\*b\*Log[a + Sqrt[a]\*Sqrt[a + b\*x^2]])/6

**Maple [A]** time = 0.008, size = 88, normalized size = 1.1

$$-\frac{1}{2ax^2}(bx^2+a)^{7/2} + \frac{b}{2a}(bx^2+a)^{5/2} + \frac{5b}{6}(bx^2+a)^{3/2} - \frac{5b}{2}a^{3/2} \ln\left(\frac{1}{x}\left(2a + 2\sqrt{a}\sqrt{bx^2+a}\right)\right) + \frac{5ab}{2}\sqrt{bx^2+a}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(5/2)/x^3,x)`

[Out] 
$$-1/2/a/x^2*(b*x^2+a)^{(7/2)}+1/2*b/a*(b*x^2+a)^{(5/2)}+5/6*b*(b*x^2+a)^{(3/2)}-5/2*b*a^{(3/2)}*\ln((2*a+2*a^{(1/2)}*(b*x^2+a)^{(1/2)})/x)+5/2*a*b*(b*x^2+a)^{(1/2)}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.250088, size = 1, normalized size = 0.01

$$\left[ \frac{15 a^{\frac{3}{2}} b x^2 \log\left(-\frac{b x^2 - 2 \sqrt{b x^2 + a} \sqrt{a} + 2 a}{x^2}\right) + 2 (2 b^2 x^4 + 14 a b x^2 - 3 a^2) \sqrt{b x^2 + a}}{12 x^2}, \right. \\ \left. - \frac{15 \sqrt{-a} a b x^2 \arctan\left(\frac{a}{\sqrt{b x^2 + a} \sqrt{-a}}\right) - (2 b^2 x^4 + 14 a b x^2 - 3 a^2) \sqrt{b x^2 + a}}{6 x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^3,x, algorithm="fricas")`

[Out] 
$$\left[ \frac{1}{12} * (15 * a^{(3/2)} * b * x^2 * \log(- (b * x^2 - 2 * \sqrt{b * x^2 + a}) * \sqrt{a} + 2 * a) / x^2) + 2 * (2 * b^2 * x^4 + 14 * a * b * x^2 - 3 * a^2) * \sqrt{b * x^2 + a} \right] / x^2, -1/6 * (15 * \sqrt{-a} * a * b * x^2 * \arctan(a / (\sqrt{b * x^2 + a}) * \sqrt{-a})) - (2 * b^2 * x^4 + 14 * a * b * x^2 - 3 * a^2) * \sqrt{b * x^2 + a} / x^2 ]$$

**Sympy [A]** time = 11.4189, size = 112, normalized size = 1.4

$$-\frac{a^{\frac{5}{2}} \sqrt{1 + \frac{b x^2}{a}}}{2 x^2} + \frac{7 a^{\frac{3}{2}} b \sqrt{1 + \frac{b x^2}{a}}}{3} + \frac{5 a^{\frac{3}{2}} b \log\left(\frac{b x^2}{a}\right)}{4} - \frac{5 a^{\frac{3}{2}} b \log\left(\sqrt{1 + \frac{b x^2}{a}} + 1\right)}{2} + \frac{\sqrt{a} b^2 x^2 \sqrt{1 + \frac{b x^2}{a}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(5/2)/x**3,x)`

[Out] 
$$-a^{(5/2)}*\sqrt{1+b*x**2/a}/(2*x**2)+7*a^{(3/2)}*b*\sqrt{1+b*x**2/a}/3+5*a^{(3/2)}*b*\log(b*x**2/a)/4-5*a^{(3/2)}*b*\log(\sqrt{1+b*x**2/a}+1)/2+\sqrt{a}*b**2*x**2*\sqrt{1+b*x**2/a}/3$$

**GIAC/XCAS [A]** time = 0.211039, size = 99, normalized size = 1.24

$$\frac{1}{6} \left( \frac{15 a^2 \arctan\left(\frac{\sqrt{b x^2 + a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2 (b x^2 + a)^{\frac{3}{2}} + 12 \sqrt{b x^2 + a} a - \frac{3 \sqrt{b x^2 + a} a^2}{b x^2} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*a^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 2*(b*x^2 + a)^(3/2) + 12*sqrt(b*x^2 + a)*a - 3*sqrt(b*x^2 + a)*a^2/(b*x^2))*b
```

$$3.393 \quad \int \frac{(a+bx^2)^{5/2}}{x^5} dx$$

**Optimal.** Leaf size=86

$$\frac{15}{8}b^2\sqrt{a+bx^2} - \frac{15}{8}\sqrt{ab^2} \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{5b(a+bx^2)^{3/2}}{8x^2} - \frac{(a+bx^2)^{5/2}}{4x^4}$$

[Out] (15\*b^2\*Sqrt[a + b\*x^2])/8 - (5\*b\*(a + b\*x^2)^(3/2))/(8\*x^2) - (a + b\*x^2)^(5/2)/(4\*x^4) - (15\*Sqrt[a]\*b^2\*ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]])/8

**Rubi [A]** time = 0.13347, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{15}{8}b^2\sqrt{a+bx^2} - \frac{15}{8}\sqrt{ab^2} \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{5b(a+bx^2)^{3/2}}{8x^2} - \frac{(a+bx^2)^{5/2}}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(5/2)/x^5, x]

[Out] (15\*b^2\*Sqrt[a + b\*x^2])/8 - (5\*b\*(a + b\*x^2)^(3/2))/(8\*x^2) - (a + b\*x^2)^(5/2)/(4\*x^4) - (15\*Sqrt[a]\*b^2\*ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]])/8

**Rubi in Sympy [A]** time = 12.9067, size = 78, normalized size = 0.91

$$-\frac{15\sqrt{ab^2} \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8} + \frac{15b^2\sqrt{a+bx^2}}{8} - \frac{5b(a+bx^2)^{3/2}}{8x^2} - \frac{(a+bx^2)^{5/2}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(5/2)/x\*\*5, x)

[Out] -15\*sqrt(a)\*b\*\*2\*atanh(sqrt(a + b\*x\*\*2)/sqrt(a))/8 + 15\*b\*\*2\*sqrt(a + b\*x\*\*2)/8 - 5\*b\*(a + b\*x\*\*2)\*\*(3/2)/(8\*x\*\*2) - (a + b\*x\*\*2)\*\*(5/2)/(4\*x\*\*4)

**Mathematica [A]** time = 0.118451, size = 82, normalized size = 0.95

$$\left(-\frac{a^2}{4x^4} - \frac{9ab}{8x^2} + b^2\right)\sqrt{a+bx^2} - \frac{15}{8}\sqrt{ab^2} \log\left(\sqrt{a}\sqrt{a+bx^2} + a\right) + \frac{15}{8}\sqrt{ab^2} \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(5/2)/x^5, x]

[Out] (b^2 - a^2/(4\*x^4) - (9\*a\*b)/(8\*x^2))\*Sqrt[a + b\*x^2] + (15\*Sqrt[a]\*b^2\*Log[x])/8 - (15\*Sqrt[a]\*b^2\*Log[a + Sqrt[a]\*Sqrt[a + b\*x^2]])/8

**Maple [A]** time = 0.008, size = 116, normalized size = 1.4

$$-\frac{1}{4ax^4}(bx^2+a)^{\frac{7}{2}} - \frac{3b}{8a^2x^2}(bx^2+a)^{\frac{7}{2}} + \frac{3b^2}{8a^2}(bx^2+a)^{\frac{5}{2}} + \frac{5b^2}{8a}(bx^2+a)^{\frac{3}{2}} - \frac{15b^2}{8}\sqrt{a}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right) + \frac{15b^2}{8}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(5/2)/x^5,x)`

[Out] `-1/4/a/x^4*(b*x^2+a)^(7/2)-3/8*b/a^2/x^2*(b*x^2+a)^(7/2)+3/8*b^2/a^2*(b*x^2+a)^(5/2)+5/8*b^2/a*(b*x^2+a)^(3/2)-15/8*b^2*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+15/8*b^2*(b*x^2+a)^(1/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^5,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.257592, size = 1, normalized size = 0.01

$$\left[ \frac{15\sqrt{ab^2x^4}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a+2a}}{x^2}\right) + 2(8b^2x^4 - 9abx^2 - 2a^2)\sqrt{bx^2+a}}{16x^4}, \frac{15\sqrt{-ab^2x^4}\arctan\left(\frac{a}{\sqrt{bx^2+a}\sqrt{-a}}\right) - (8b^2x^4 - 9abx^2 - 2a^2)\sqrt{bx^2+a}}{8x^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^5,x, algorithm="fricas")`

[Out] `[1/16*(15*sqrt(a)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a))*sqrt(a) + 2*a)/x^2) + 2*(8*b^2*x^4 - 9*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/x^4, -1/8*(15*sqrt(-a)*b^2*x^4*arctan(a/(sqrt(b*x^2 + a))*sqrt(-a))) - (8*b^2*x^4 - 9*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/x^4]`

**Sympy [A]** time = 13.1975, size = 117, normalized size = 1.36

$$-\frac{15\sqrt{ab^2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{8} - \frac{a^3}{4\sqrt{bx^5}\sqrt{\frac{a}{bx^2}+1}} - \frac{11a^2\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{ab^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^2}+1}} + \frac{b^{\frac{5}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(5/2)/x**5,x)`

[Out] `-15*sqrt(a)*b**2*asinh(sqrt(a)/(sqrt(b)*x))/8 - a**3/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 11*a**2*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) - ab**(3/2)/(8*x*sqrt(a/(b*x**2) + 1)) + b**(5/2)*x/sqrt(a/(b*x**2) + 1)`

2) + 1)) - a\*b\*\*(3/2)/(8\*x\*sqrt(a/(b\*x\*\*2) + 1)) + b\*\*(5/2)\*x/sqrt(a/(b\*x\*\*2) + 1)

---

**GIAC/XCAS [A]** time = 0.213009, size = 103, normalized size = 1.2

$$\frac{1}{8} \left( \frac{15 a \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 8 \sqrt{bx^2+a} - \frac{9 (bx^2+a)^{\frac{3}{2}} a - 7 \sqrt{bx^2+aa^2}}{b^2 x^4} \right) b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)/x^5,x, algorithm="giac")

[Out] 1/8\*(15\*a\*arctan(sqrt(b\*x^2 + a)/sqrt(-a))/sqrt(-a) + 8\*sqrt(b\*x^2 + a) - (9\*(b\*x^2 + a)^(3/2)\*a - 7\*sqrt(b\*x^2 + a)\*a^2)/(b^2\*x^4)) \* b^2

$$3.394 \quad \int \frac{(a+bx^2)^{5/2}}{x^7} dx$$

**Optimal.** Leaf size=89

$$-\frac{5b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}} - \frac{5b^2\sqrt{a+bx^2}}{16x^2} - \frac{(a+bx^2)^{5/2}}{6x^6} - \frac{5b(a+bx^2)^{3/2}}{24x^4}$$

[Out]  $(-5*b^2*\text{Sqrt}[a + b*x^2])/(16*x^2) - (5*b*(a + b*x^2)^(3/2))/(24*x^4) - (a + b*x^2)^(5/2)/(6*x^6) - (5*b^3*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(16*\text{Sqrt}[a])$

**Rubi [A]** time = 0.141389, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{5b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}} - \frac{5b^2\sqrt{a+bx^2}}{16x^2} - \frac{(a+bx^2)^{5/2}}{6x^6} - \frac{5b(a+bx^2)^{3/2}}{24x^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(5/2)/x^7, x]

[Out]  $(-5*b^2*\text{Sqrt}[a + b*x^2])/(16*x^2) - (5*b*(a + b*x^2)^(3/2))/(24*x^4) - (a + b*x^2)^(5/2)/(6*x^6) - (5*b^3*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(16*\text{Sqrt}[a])$

**Rubi in Sympy [A]** time = 13.261, size = 83, normalized size = 0.93

$$-\frac{5b^2\sqrt{a+bx^2}}{16x^2} - \frac{5b(a+bx^2)^{3/2}}{24x^4} - \frac{(a+bx^2)^{5/2}}{6x^6} - \frac{5b^3 \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(5/2)/x\*\*7, x)

[Out]  $-5*b**2*\text{sqrt}(a + b*x**2)/(16*x**2) - 5*b*(a + b*x**2)**(3/2)/(24*x**4) - (a + b*x**2)**(5/2)/(6*x**6) - 5*b**3*\operatorname{atanh}(\text{sqrt}(a + b*x**2)/\text{sqrt}(a))/(16*\text{sqrt}(a))$

**Mathematica [A]** time = 0.11405, size = 84, normalized size = 0.94

$$\frac{1}{48} \left( -\frac{\sqrt{a+bx^2}(8a^2+26abx^2+33b^2x^4)}{x^6} - \frac{15b^3 \log(\sqrt{a}\sqrt{a+bx^2}+a)}{\sqrt{a}} + \frac{15b^3 \log(x)}{\sqrt{a}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(5/2)/x^7, x]

[Out]  $(-((\text{Sqrt}[a + b*x^2]*(8*a^2 + 26*a*b*x^2 + 33*b^2*x^4))/x^6) + (15*b^3*\text{Log}[x])/ \text{Sqrt}[a] - (15*b^3*\text{Log}[a + \text{Sqrt}[a]*\text{Sqrt}[a + b*x^2]])/ \text{Sqrt}[a])/48$

**Maple [A]** time = 0.014, size = 139, normalized size = 1.6

$$-\frac{1}{6ax^6}(bx^2+a)^{\frac{7}{2}} - \frac{b}{24a^2x^4}(bx^2+a)^{\frac{7}{2}} - \frac{b^2}{16a^3x^2}(bx^2+a)^{\frac{7}{2}} + \frac{b^3}{16a^3}(bx^2+a)^{\frac{5}{2}} + \frac{5b^3}{48a^2}(bx^2+a)^{\frac{3}{2}} - \frac{5b^3}{16} \ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right) \frac{1}{\sqrt{a}} + \frac{5b^3}{16a}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(5/2)/x^7, x)

[Out] -1/6/a/x^6\*(b\*x^2+a)^(7/2)-1/24\*b/a^2/x^4\*(b\*x^2+a)^(7/2)-1/16\*b^2/a^3/x^2\*(b\*x^2+a)^(7/2)+1/16\*b^3/a^3\*(b\*x^2+a)^(5/2)+5/48\*b^3/a^2\*(b\*x^2+a)^(3/2)-5/16\*b^3/a^(1/2)\*ln((2\*a+2\*a^(1/2)\*(b\*x^2+a)^(1/2))/x)+5/16\*b^3/a\*(b\*x^2+a)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)/x^7, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.260486, size = 1, normalized size = 0.01

$$\left[ \frac{15b^3x^6 \log\left(-\frac{(bx^2+2a)\sqrt{a-2\sqrt{bx^2+aa}}}{x^2}\right) - 2(33b^2x^4 + 26abx^2 + 8a^2)\sqrt{bx^2+a}\sqrt{a}}{96\sqrt{ax^6}}, \right. \\ \left. - \frac{15b^3x^6 \arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) + (33b^2x^4 + 26abx^2 + 8a^2)\sqrt{bx^2+a}\sqrt{-a}}{48\sqrt{-ax^6}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)/x^7, x, algorithm="fricas")

[Out] [1/96\*(15\*b^3\*x^6\*log(-((b\*x^2 + 2\*a)\*sqrt(a) - 2\*sqrt(b\*x^2 + a)\*a)/x^2) - 2\*(33\*b^2\*x^4 + 26\*a\*b\*x^2 + 8\*a^2)\*sqrt(b\*x^2 + a)\*sqrt(a))/(sqrt(a)\*x^6), -1/48\*(15\*b^3\*x^6\*arctan(sqrt(-a)/sqrt(b\*x^2 + a)) + (33\*b^2\*x^4 + 26\*a\*b\*x^2 + 8\*a^2)\*sqrt(b\*x^2 + a)\*sqrt(-a))/(sqrt(-a)\*x^6)]

**Sympy [A]** time = 16.6, size = 99, normalized size = 1.11

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{6x^5} - \frac{13ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{24x^3} - \frac{11b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{16x} - \frac{5b^3 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{16\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(5/2)/x\*\*7, x)

[Out]  $-a^{**2} \sqrt{b} \sqrt{a/(b*x^{**2}) + 1}/(6*x^{**5}) - 13*a*b^{**3/2} \sqrt{a/(b*x^{**2}) + 1}/(24*x^{**3}) - 11*b^{**5/2} \sqrt{a/(b*x^{**2}) + 1}/(16*x) - 5*b^{**3} \operatorname{asinh}(\sqrt{a}/(\sqrt{b}*x))/(16*\sqrt{a})$

**GIAC/XCAS [A]** time = 0.209696, size = 101, normalized size = 1.13

$$\frac{1}{48} b^3 \left( \frac{15 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{33 (bx^2 + a)^{\frac{5}{2}} - 40 (bx^2 + a)^{\frac{3}{2}} a + 15 \sqrt{bx^2 + aa^2}}{b^3 x^6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^7,x, algorithm="giac")`

[Out]  $1/48*b^3*(15*\arctan(\sqrt{b*x^2 + a}/\sqrt{-a})/\sqrt{-a} - (33*(b*x^2 + a)^{5/2} - 40*(b*x^2 + a)^{3/2}*a + 15*\sqrt{b*x^2 + a}*a^2)/(b^3*x^6))$



$$3.395 \quad \int \frac{(a+bx^2)^{5/2}}{x^9} dx$$

**Optimal.** Leaf size=113

$$\frac{5b^4 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}} - \frac{5b^3\sqrt{a+bx^2}}{128ax^2} - \frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{(a+bx^2)^{5/2}}{8x^8} - \frac{5b(a+bx^2)^{3/2}}{48x^6}$$

[Out]  $(-5*b^2*sqrt[a + b*x^2])/(64*x^4) - (5*b^3*sqrt[a + b*x^2])/(128*a*x^2) - (5*b*(a + b*x^2)^(3/2))/(48*x^6) - (a + b*x^2)^(5/2)/(8*x^8) + (5*b^4*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(3/2))$

**Rubi [A]** time = 0.181825, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{5b^4 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}} - \frac{5b^3\sqrt{a+bx^2}}{128ax^2} - \frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{(a+bx^2)^{5/2}}{8x^8} - \frac{5b(a+bx^2)^{3/2}}{48x^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(5/2)/x^9, x]

[Out]  $(-5*b^2*sqrt[a + b*x^2])/(64*x^4) - (5*b^3*sqrt[a + b*x^2])/(128*a*x^2) - (5*b*(a + b*x^2)^(3/2))/(48*x^6) - (a + b*x^2)^(5/2)/(8*x^8) + (5*b^4*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(3/2))$

**Rubi in Sympy [A]** time = 17.2222, size = 104, normalized size = 0.92

$$-\frac{5b^2\sqrt{a+bx^2}}{64x^4} - \frac{5b(a+bx^2)^{\frac{3}{2}}}{48x^6} - \frac{(a+bx^2)^{\frac{5}{2}}}{8x^8} - \frac{5b^3\sqrt{a+bx^2}}{128ax^2} + \frac{5b^4 \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(5/2)/x\*\*9, x)

[Out]  $-5*b**2*sqrt(a + b*x**2)/(64*x**4) - 5*b*(a + b*x**2)**(3/2)/(48*x**6) - (a + b*x**2)**(5/2)/(8*x**8) - 5*b**3*sqrt(a + b*x**2)/(128*a*x**2) + 5*b**4*atanh(sqrt(a + b*x**2)/sqrt(a))/(128*a**(3/2))$

**Mathematica [A]** time = 0.129644, size = 102, normalized size = 0.9

$$\frac{5b^4 \log\left(\sqrt{a}\sqrt{a+bx^2} + a\right)}{128a^{3/2}} - \frac{5b^4 \log(x)}{128a^{3/2}} + \left(-\frac{a^2}{8x^8} - \frac{5b^3}{128ax^2} - \frac{17ab}{48x^6} - \frac{59b^2}{192x^4}\right) \sqrt{a+bx^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(5/2)/x^9, x]

[Out]  $(-a^2/(8*x^8) - (17*a*b)/(48*x^6) - (59*b^2)/(192*x^4) - (5*b^3)/(128*a*x^2))*sqrt[a + b*x^2] - (5*b^4*Log[x])/(128*a^(3/2)) + (5*b^4*Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/(128*a^(3/2))$

**Maple [A]** time = 0.023, size = 159, normalized size = 1.4

$$-\frac{1}{8ax^8}(bx^2+a)^{\frac{7}{2}} + \frac{b}{48a^2x^6}(bx^2+a)^{\frac{7}{2}} + \frac{b^2}{192a^3x^4}(bx^2+a)^{\frac{7}{2}} + \frac{b^3}{128a^4x^2}(bx^2+a)^{\frac{7}{2}} \\ - \frac{b^4}{128a^4}(bx^2+a)^{\frac{5}{2}} - \frac{5b^4}{384a^3}(bx^2+a)^{\frac{3}{2}} + \frac{5b^4}{128} \ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right) a^{-\frac{3}{2}} - \frac{5b^4}{128a^2}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(5/2)/x^9, x)

[Out] -1/8/a/x^8\*(b\*x^2+a)^(7/2)+1/48\*b/a^2/x^6\*(b\*x^2+a)^(7/2)+1/192\*b^2/a^3/x^4\*(b\*x^2+a)^(7/2)+1/128\*b^3/a^4/x^2\*(b\*x^2+a)^(7/2)-1/128\*b^4/a^4\*(b\*x^2+a)^(5/2)-5/384\*b^4/a^3\*(b\*x^2+a)^(3/2)+5/128\*b^4/a^(3/2)\*ln((2\*a+2\*a^(1/2)\*(b\*x^2+a)^(1/2))/x)-5/128\*b^4/a^2\*(b\*x^2+a)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)/x^9, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.275334, size = 1, normalized size = 0.01

$$\left[ \frac{15b^4x^8 \log\left(-\frac{(bx^2+2a)\sqrt{a+2\sqrt{bx^2+aa}}}{x^2}\right) - 2(15b^3x^6 + 118ab^2x^4 + 136a^2bx^2 + 48a^3)\sqrt{bx^2+a}\sqrt{a} - 15b^4x^8 \arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{768a^{\frac{3}{2}}x^8}, \frac{15b^4x^8 \arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{768a^{\frac{3}{2}}x^8} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)/x^9, x, algorithm="fricas")

[Out] [1/768\*(15\*b^4\*x^8\*log(-((b\*x^2 + 2\*a)\*sqrt(a) + 2\*sqrt(b\*x^2 + a)\*a)/x^2) - 2\*(15\*b^3\*x^6 + 118\*a\*b^2\*x^4 + 136\*a^2\*b\*x^2 + 48\*a^3)\*sqrt(b\*x^2 + a)\*sqrt(a))/(a^(3/2)\*x^8), 1/384\*(15\*b^4\*x^8\*arctan(sqrt(-a)/sqrt(b\*x^2 + a)) - (15\*b^3\*x^6 + 118\*a\*b^2\*x^4 + 136\*a^2\*b\*x^2 + 48\*a^3)\*sqrt(b\*x^2 + a)\*sqrt(-a))/(sqrt(-a)\*a\*x^8)]

**Sympy [A]** time = 27.9867, size = 150, normalized size = 1.33

$$-\frac{a^3}{8\sqrt{bx^9}\sqrt{\frac{a}{bx^2}+1}} - \frac{23a^2\sqrt{b}}{48x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{127ab^{\frac{3}{2}}}{192x^5\sqrt{\frac{a}{bx^2}+1}} \\ - \frac{133b^{\frac{5}{2}}}{384x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{5b^{\frac{7}{2}}}{128ax\sqrt{\frac{a}{bx^2}+1}} + \frac{5b^4 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{128a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(5/2)/x\*\*9, x)

```
[Out] -a**3/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 23*a**2*sqrt(b)/(48
*x**7*sqrt(a/(b*x**2) + 1)) - 127*a*b**(3/2)/(192*x**5*sqrt(a/(b*
x**2) + 1)) - 133*b**(5/2)/(384*x**3*sqrt(a/(b*x**2) + 1)) - 5*b*
*(7/2)/(128*a*x*sqrt(a/(b*x**2) + 1)) + 5*b**4*asinh(sqrt(a)/(sqr
t(b*x)))/(128*a**(3/2))
```

**GIAC/XCAS [A]** time = 0.213009, size = 127, normalized size = 1.12

$$-\frac{1}{384}b^4\left(\frac{15\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-aa}} + \frac{15(bx^2+a)^{\frac{7}{2}} + 73(bx^2+a)^{\frac{5}{2}}a - 55(bx^2+a)^{\frac{3}{2}}a^2 + 15\sqrt{bx^2+aa^3}}{ab^4x^8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/384*b^4*(15*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + (1
5*(b*x^2 + a)^(7/2) + 73*(b*x^2 + a)^(5/2)*a - 55*(b*x^2 + a)^(3/
2)*a^2 + 15*sqrt(b*x^2 + a)*a^3)/(a*b^4*x^8))
```

$$3.396 \quad \int \frac{(a+bx^2)^{5/2}}{x^{11}} dx$$

**Optimal.** Leaf size=137

$$-\frac{3b^5 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}} + \frac{3b^4\sqrt{a+bx^2}}{256a^2x^2} - \frac{b^3\sqrt{a+bx^2}}{128ax^4} - \frac{b^2\sqrt{a+bx^2}}{32x^6} - \frac{(a+bx^2)^{5/2}}{10x^{10}} - \frac{b(a+bx^2)^{3/2}}{16x^8}$$

[Out]  $-(b^2*\text{Sqrt}[a + b*x^2])/(32*x^6) - (b^3*\text{Sqrt}[a + b*x^2])/(128*a*x^4) + (3*b^4*\text{Sqrt}[a + b*x^2])/(256*a^2*x^2) - (b*(a + b*x^2)^(3/2))/(16*x^8) - (a + b*x^2)^(5/2)/(10*x^10) - (3*b^5*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(256*a^(5/2))$

**Rubi [A]** time = 0.224676, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{3b^5 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}} + \frac{3b^4\sqrt{a+bx^2}}{256a^2x^2} - \frac{b^3\sqrt{a+bx^2}}{128ax^4} - \frac{b^2\sqrt{a+bx^2}}{32x^6} - \frac{(a+bx^2)^{5/2}}{10x^{10}} - \frac{b(a+bx^2)^{3/2}}{16x^8}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(5/2)/x^11, x]

[Out]  $-(b^2*\text{Sqrt}[a + b*x^2])/(32*x^6) - (b^3*\text{Sqrt}[a + b*x^2])/(128*a*x^4) + (3*b^4*\text{Sqrt}[a + b*x^2])/(256*a^2*x^2) - (b*(a + b*x^2)^(3/2))/(16*x^8) - (a + b*x^2)^(5/2)/(10*x^10) - (3*b^5*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(256*a^(5/2))$

**Rubi in Sympy [A]** time = 22.0654, size = 122, normalized size = 0.89

$$-\frac{b^2\sqrt{a+bx^2}}{32x^6} - \frac{b(a+bx^2)^{3/2}}{16x^8} - \frac{(a+bx^2)^{5/2}}{10x^{10}} - \frac{b^3\sqrt{a+bx^2}}{128ax^4} + \frac{3b^4\sqrt{a+bx^2}}{256a^2x^2} - \frac{3b^5 \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(5/2)/x\*\*11, x)

[Out]  $-b**2*\text{sqrt}(a + b*x**2)/(32*x**6) - b*(a + b*x**2)**(3/2)/(16*x**8) - (a + b*x**2)**(5/2)/(10*x**10) - b**3*\text{sqrt}(a + b*x**2)/(128*a*x**4) + 3*b**4*\text{sqrt}(a + b*x**2)/(256*a**2*x**2) - 3*b**5*\text{atanh}(\text{sqrt}(a + b*x**2)/\text{sqrt}(a))/(256*a**(5/2))$

**Mathematica [A]** time = 0.113661, size = 112, normalized size = 0.82

$$\frac{-\sqrt{a}\sqrt{a+bx^2}(128a^4 + 336a^3bx^2 + 248a^2b^2x^4 + 10ab^3x^6 - 15b^4x^8) - 15b^5x^{10} \log(\sqrt{a}\sqrt{a+bx^2} + a) + 15b^5x^{10} \log(x)}{1280a^{5/2}x^{10}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(5/2)/x^11, x]

[Out]  $(-\text{Sqrt}[a]*\text{Sqrt}[a + b*x^2]*(128*a^4 + 336*a^3*b*x^2 + 248*a^2*b^2*x^4 + 10*a*b^3*x^6 - 15*b^4*x^8)) + 15*b^5*x^{10}*\text{Log}[x] - 15*b^5*x^{10}*\text{Log}[a + \text{Sqrt}[a]*\text{Sqrt}[a + b*x^2]]/(1280*a^(5/2)*x^{10})$

**Maple [A]** time = 0.046, size = 179, normalized size = 1.3

$$\begin{aligned}
 & -\frac{1}{10ax^{10}}(bx^2+a)^{\frac{7}{2}} + \frac{3b}{80a^2x^8}(bx^2+a)^{\frac{7}{2}} - \frac{b^2}{160a^3x^6}(bx^2+a)^{\frac{7}{2}} \\
 & - \frac{b^3}{640a^4x^4}(bx^2+a)^{\frac{7}{2}} - \frac{3b^4}{1280a^5x^2}(bx^2+a)^{\frac{7}{2}} + \frac{3b^5}{1280a^5}(bx^2+a)^{\frac{5}{2}} \\
 & + \frac{b^5}{256a^4}(bx^2+a)^{\frac{3}{2}} - \frac{3b^5}{256} \ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{5}{2}} + \frac{3b^5}{256a^3}\sqrt{bx^2+a}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(5/2)/x^11,x)

[Out] -1/10/a/x^10\*(b\*x^2+a)^(7/2)+3/80\*b/a^2/x^8\*(b\*x^2+a)^(7/2)-1/160\*b^2/a^3/x^6\*(b\*x^2+a)^(7/2)-1/640\*b^3/a^4/x^4\*(b\*x^2+a)^(7/2)-3/1280\*b^4/a^5/x^2\*(b\*x^2+a)^(7/2)+3/1280\*b^5/a^5\*(b\*x^2+a)^(5/2)+1/256\*b^5/a^4\*(b\*x^2+a)^(3/2)-3/256\*b^5/a^4\*(5/2)\*ln((2\*a+2\*a^(1/2)\*(b\*x^2+a)^(1/2))/x)+3/256\*b^5/a^3\*(b\*x^2+a)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)/x^11,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.302067, size = 1, normalized size = 0.01

$$\left[ \frac{15b^5x^{10} \log\left(-\frac{(bx^2+2a)\sqrt{a-2\sqrt{bx^2+aa}}}{x^2}\right) + 2(15b^4x^8 - 10ab^3x^6 - 248a^2b^2x^4 - 336a^3bx^2 - 128a^4)\sqrt{bx^2+a}\sqrt{a}}{2560a^{\frac{5}{2}}x^{10}}, \frac{15b^5x^{10} \arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) - (15b^4x^8 - 10ab^3x^6 - 248a^2b^2x^4 - 336a^3bx^2 - 128a^4)\sqrt{bx^2+a}\sqrt{-a}}{1280\sqrt{-aa^2}x^{10}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)/x^11,x, algorithm="fricas")

[Out] [1/2560\*(15\*b^5\*x^10\*log(-((b\*x^2 + 2\*a)\*sqrt(a) - 2\*sqrt(b\*x^2 + a)\*a)/x^2) + 2\*(15\*b^4\*x^8 - 10\*a\*b^3\*x^6 - 248\*a^2\*b^2\*x^4 - 336\*a^3\*b\*x^2 - 128\*a^4)\*sqrt(b\*x^2 + a)\*sqrt(a))/(a^(5/2)\*x^10), -1/1280\*(15\*b^5\*x^10\*arctan(sqrt(-a)/sqrt(b\*x^2 + a)) - (15\*b^4\*x^8 - 10\*a\*b^3\*x^6 - 248\*a^2\*b^2\*x^4 - 336\*a^3\*b\*x^2 - 128\*a^4)\*sqrt(b\*x^2 + a)\*sqrt(-a))/(sqrt(-a)\*a^2\*x^10)]

**Sympy [A]** time = 42.7264, size = 175, normalized size = 1.28

$$\begin{aligned}
 & -\frac{a^3}{10\sqrt{b}x^{11}\sqrt{\frac{a}{bx^2}+1}} - \frac{29a^2\sqrt{b}}{80x^9\sqrt{\frac{a}{bx^2}+1}} - \frac{73ab^{\frac{3}{2}}}{160x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{129b^{\frac{5}{2}}}{640x^5\sqrt{\frac{a}{bx^2}+1}} \\
 & + \frac{b^{\frac{7}{2}}}{256ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{3b^{\frac{9}{2}}}{256a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{3b^5 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{256a^{\frac{5}{2}}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(5/2)/x\*\*11,x)

[Out]  $-a^{3/2}/(10\sqrt{b}x^{11}\sqrt{a/(bx^2)+1}) - 29a^{5/2}\sqrt{b}/(80x^9\sqrt{a/(bx^2)+1}) - 73a^{3/2}b^{3/2}/(160x^7\sqrt{a/(bx^2)+1}) - 129b^{5/2}/(640x^5\sqrt{a/(bx^2)+1}) + b^{7/2}/(256ax^3\sqrt{a/(bx^2)+1}) + 3b^{9/2}/(256a^2x\sqrt{a/(bx^2)+1}) - 3b^5 \operatorname{asinh}(\sqrt{a}/(\sqrt{b}x))/(256a^{5/2})$

**GIAC/XCAS [A]** time = 0.210335, size = 146, normalized size = 1.07

$$\frac{1}{1280} b^5 \left( \frac{15 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-aa^2}} + \frac{15 (bx^2+a)^{\frac{9}{2}} - 70 (bx^2+a)^{\frac{7}{2}} a - 128 (bx^2+a)^{\frac{5}{2}} a^2 + 70 (bx^2+a)^{\frac{3}{2}} a^3 - 15 \sqrt{bx^2+aa^4}}{a^2 b^5 x^{10}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)/x^11,x, algorithm="giac")

[Out]  $1/1280*b^5*(15*\arctan(\sqrt{b*x^2+a}/\sqrt{-a})/(\sqrt{-a}*a^2) + (15*(b*x^2+a)^{(9/2)} - 70*(b*x^2+a)^{(7/2)}*a - 128*(b*x^2+a)^{(5/2)}*a^2 + 70*(b*x^2+a)^{(3/2)}*a^3 - 15*\sqrt{b*x^2+aa^4})/(a^2*b^5*x^{10}))$

$$3.397 \quad \int x^4 (a + bx^2)^{5/2} dx$$

**Optimal.** Leaf size=136

$$\frac{3a^5 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{256b^{5/2}} - \frac{3a^4x\sqrt{a+bx^2}}{256b^2} + \frac{a^3x^3\sqrt{a+bx^2}}{128b} + \frac{1}{32}a^2x^5\sqrt{a+bx^2} + \frac{1}{16}ax^5(a+bx^2)^{3/2} + \frac{1}{10}x^5(a+bx^2)^{5/2}$$

[Out]  $(-3*a^4*x*\text{Sqrt}[a + b*x^2])/(256*b^2) + (a^3*x^3*\text{Sqrt}[a + b*x^2])/(128*b) + (a^2*x^5*\text{Sqrt}[a + b*x^2])/32 + (a*x^5*(a + b*x^2)^(3/2))/16 + (x^5*(a + b*x^2)^(5/2))/10 + (3*a^5*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]])/(256*b^(5/2))$

**Rubi [A]** time = 0.174438, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{3a^5 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{256b^{5/2}} - \frac{3a^4x\sqrt{a+bx^2}}{256b^2} + \frac{a^3x^3\sqrt{a+bx^2}}{128b} + \frac{1}{32}a^2x^5\sqrt{a+bx^2} + \frac{1}{16}ax^5(a+bx^2)^{3/2} + \frac{1}{10}x^5(a+bx^2)^{5/2}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a + b\*x^2)^(5/2), x]

[Out]  $(-3*a^4*x*\text{Sqrt}[a + b*x^2])/(256*b^2) + (a^3*x^3*\text{Sqrt}[a + b*x^2])/(128*b) + (a^2*x^5*\text{Sqrt}[a + b*x^2])/32 + (a*x^5*(a + b*x^2)^(3/2))/16 + (x^5*(a + b*x^2)^(5/2))/10 + (3*a^5*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]])/(256*b^(5/2))$

**Rubi in Sympy [A]** time = 21.3923, size = 122, normalized size = 0.9

$$\frac{3a^5 \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{256b^{5/2}} - \frac{3a^4x\sqrt{a+bx^2}}{256b^2} + \frac{a^3x^3\sqrt{a+bx^2}}{128b} + \frac{a^2x^5\sqrt{a+bx^2}}{32} + \frac{ax^5(a+bx^2)^{3/2}}{16} + \frac{x^5(a+bx^2)^{5/2}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(b\*x\*\*2+a)\*\*(5/2), x)

[Out]  $3*a**5*\operatorname{atanh}(\operatorname{sqrt}(b)*x/\operatorname{sqrt}(a + b*x**2))/(256*b**(5/2)) - 3*a**4*x*\operatorname{sqrt}(a + b*x**2)/(256*b**2) + a**3*x**3*\operatorname{sqrt}(a + b*x**2)/(128*b) + a**2*x**5*\operatorname{sqrt}(a + b*x**2)/32 + a*x**5*(a + b*x**2)**(3/2)/16 + x**5*(a + b*x**2)**(5/2)/10$

**Mathematica [A]** time = 0.0812738, size = 98, normalized size = 0.72

$$\frac{15a^5 \log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right) + \sqrt{bx}\sqrt{a+bx^2}(-15a^4 + 10a^3bx^2 + 248a^2b^2x^4 + 336ab^3x^6 + 128b^4x^8)}{1280b^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a + b\*x^2)^(5/2), x]

[Out]  $(\text{Sqrt}[b]*x*\text{Sqrt}[a + b*x^2]*(-15*a^4 + 10*a^3*b*x^2 + 248*a^2*b^2*x^4 + 336*a*b^3*x^6 + 128*b^4*x^8) + 15*a^5*\text{Log}[b*x + \text{Sqrt}[b]*\text{Sqrt}[a + b*x^2]])/(1280*b^(5/2))$

$t[a + b \cdot x^2]) / (1280 \cdot b^{5/2})$

**Maple [A]** time = 0.01, size = 113, normalized size = 0.8

$$\frac{x^3}{10b} (bx^2 + a)^{\frac{7}{2}} - \frac{3ax}{80b^2} (bx^2 + a)^{\frac{7}{2}} + \frac{a^2x}{160b^2} (bx^2 + a)^{\frac{5}{2}} + \frac{a^3x}{128b^2} (bx^2 + a)^{\frac{3}{2}} + \frac{3a^4x}{256b^2} \sqrt{bx^2 + a} + \frac{3a^5}{256} \ln(x\sqrt{b} + \sqrt{bx^2 + a}) b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(b*x^2+a)^(5/2),x)`

[Out]  $1/10 \cdot x^3 \cdot (b \cdot x^2 + a)^{7/2} / b - 3/80 \cdot a / b^2 \cdot x \cdot (b \cdot x^2 + a)^{7/2} + 1/160 \cdot a^2 / b^2 \cdot x \cdot (b \cdot x^2 + a)^{5/2} + 1/128 \cdot a^3 / b^2 \cdot x \cdot (b \cdot x^2 + a)^{3/2} + 3/256 \cdot a^4 \cdot x \cdot (b \cdot x^2 + a)^{1/2} / b^2 + 3/256 \cdot a^5 / b^{5/2} \cdot \ln(x \cdot b^{1/2} + (b \cdot x^2 + a)^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)*x^4,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.306724, size = 1, normalized size = 0.01

$$\frac{15a^5 \log(-2\sqrt{bx^2 + a}bx - (2bx^2 + a)\sqrt{b}) + 2(128b^4x^9 + 336ab^3x^7 + 248a^2b^2x^5 + 10a^3bx^3 - 15a^4x)\sqrt{bx^2 + a}\sqrt{b}}{2560b^{\frac{5}{2}}},$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)*x^4,x, algorithm="fricas")`

[Out]  $[1/2560 \cdot (15 \cdot a^5 \cdot \log(-2 \cdot \sqrt{b \cdot x^2 + a}) \cdot b \cdot x - (2 \cdot b \cdot x^2 + a) \cdot \sqrt{b})) + 2 \cdot (128 \cdot b^4 \cdot x^9 + 336 \cdot a \cdot b^3 \cdot x^7 + 248 \cdot a^2 \cdot b^2 \cdot x^5 + 10 \cdot a^3 \cdot b \cdot x^3 - 15 \cdot a^4 \cdot x) \cdot \sqrt{b \cdot x^2 + a} \cdot \sqrt{b}) / b^{5/2}, 1/1280 \cdot (15 \cdot a^5 \cdot \arctan(\sqrt{-b} \cdot x / \sqrt{b \cdot x^2 + a}) + (128 \cdot b^4 \cdot x^9 + 336 \cdot a \cdot b^3 \cdot x^7 + 248 \cdot a^2 \cdot b^2 \cdot x^5 + 10 \cdot a^3 \cdot b \cdot x^3 - 15 \cdot a^4 \cdot x) \cdot \sqrt{b \cdot x^2 + a}) \cdot \sqrt{-b}) / (\sqrt{-b} \cdot b^2)]$

**Sympy [A]** time = 36.5915, size = 175, normalized size = 1.29

$$-\frac{3a^{\frac{9}{2}}x}{256b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{a^{\frac{7}{2}}x^3}{256b\sqrt{1+\frac{bx^2}{a}}} + \frac{129a^{\frac{5}{2}}x^5}{640\sqrt{1+\frac{bx^2}{a}}} + \frac{73a^{\frac{3}{2}}bx^7}{160\sqrt{1+\frac{bx^2}{a}}} + \frac{29\sqrt{a}b^2x^9}{80\sqrt{1+\frac{bx^2}{a}}} + \frac{3a^5 \operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{256b^{\frac{5}{2}}} + \frac{b^3x^{11}}{10\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(x\*\*4\*(b\*x\*\*2+a)\*\*(5/2),x)

[Out]  $-3*a^{9/2}*x/(256*b^{5/2}*sqrt(1 + b*x^{2}/a)) - a^{7/2}*x^{3}/(256*b*sqrt(1 + b*x^{2}/a)) + 129*a^{5/2}*x^{5}/(640*sqrt(1 + b*x^{2}/a)) + 73*a^{3/2}*b*x^{7}/(160*sqrt(1 + b*x^{2}/a)) + 29*sqrt(a)*b^{5/2}*x^{9}/(80*sqrt(1 + b*x^{2}/a)) + 3*a^{5/2}*asinh(sqrt(b)*x/sqrt(a))/(256*b^{5/2}) + b^{3/2}*x^{11}/(10*sqrt(a)*sqrt(1 + b*x^{2}/a))$

**GIAC/XCAS [A]** time = 0.213653, size = 123, normalized size = 0.9

$$-\frac{3a^5 \ln\left(\left|-\sqrt{bx} + \sqrt{bx^2 + a}\right|\right)}{256b^{5/2}} + \frac{1}{1280} \left(2 \left(4(2(8b^2x^2 + 21ab)x^2 + 31a^2)x^2 + \frac{5a^3}{b}\right)x^2 - \frac{15a^4}{b^2}\right) \sqrt{bx^2 + ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)\*x^4,x, algorithm="giac")

[Out]  $-3/256*a^5*\ln(\text{abs}(-\text{sqrt}(b)*x + \text{sqrt}(b*x^2 + a)))/b^{5/2} + 1/1280*(2*(4*(2*(8*b^2*x^2 + 21*a*b)*x^2 + 31*a^2)*x^2 + 5*a^3/b)*x^2 - 15*a^4/b^2)*\text{sqrt}(b*x^2 + a)*x$

$$3.398 \quad \int x^2 (a + bx^2)^{5/2} dx$$

**Optimal.** Leaf size=112

$$-\frac{5a^4 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{128b^{3/2}} + \frac{5a^3x\sqrt{a+bx^2}}{128b} + \frac{5}{64}a^2x^3\sqrt{a+bx^2} + \frac{5}{48}ax^3(a+bx^2)^{3/2} + \frac{1}{8}x^3(a+bx^2)^{5/2}$$

[Out]  $(5*a^3*x*\text{Sqrt}[a + b*x^2])/(128*b) + (5*a^2*x^3*\text{Sqrt}[a + b*x^2])/64 + (5*a*x^3*(a + b*x^2)^(3/2))/48 + (x^3*(a + b*x^2)^(5/2))/8 - (5*a^4*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]])/(128*b^(3/2))$

**Rubi [A]** time = 0.131637, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{5a^4 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{128b^{3/2}} + \frac{5a^3x\sqrt{a+bx^2}}{128b} + \frac{5}{64}a^2x^3\sqrt{a+bx^2} + \frac{5}{48}ax^3(a+bx^2)^{3/2} + \frac{1}{8}x^3(a+bx^2)^{5/2}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*x^2)^(5/2), x]

[Out]  $(5*a^3*x*\text{Sqrt}[a + b*x^2])/(128*b) + (5*a^2*x^3*\text{Sqrt}[a + b*x^2])/64 + (5*a*x^3*(a + b*x^2)^(3/2))/48 + (x^3*(a + b*x^2)^(5/2))/8 - (5*a^4*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]])/(128*b^(3/2))$

**Rubi in Sympy [A]** time = 16.9391, size = 104, normalized size = 0.93

$$-\frac{5a^4 \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{128b^{3/2}} + \frac{5a^3x\sqrt{a+bx^2}}{128b} + \frac{5a^2x^3\sqrt{a+bx^2}}{64} + \frac{5ax^3(a+bx^2)^{3/2}}{48} + \frac{x^3(a+bx^2)^{5/2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(b\*x\*\*2+a)\*\*(5/2), x)

[Out]  $-5*a**4*\operatorname{atanh}(\operatorname{sqrt}(b)*x/\operatorname{sqrt}(a + b*x**2))/(128*b**(3/2)) + 5*a**3*x*\operatorname{sqrt}(a + b*x**2)/(128*b) + 5*a**2*x**3*\operatorname{sqrt}(a + b*x**2)/64 + 5*a*x**3*(a + b*x**2)**(3/2)/48 + x**3*(a + b*x**2)**(5/2)/8$

**Mathematica [A]** time = 0.063069, size = 88, normalized size = 0.79

$$\sqrt{a+bx^2} \left( \frac{5a^3x}{128b} + \frac{59a^2x^3}{192} + \frac{17}{48}abx^5 + \frac{b^2x^7}{8} \right) - \frac{5a^4 \log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right)}{128b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*x^2)^(5/2), x]

[Out]  $\text{Sqrt}[a + b*x^2]*((5*a^3*x)/(128*b) + (59*a^2*x^3)/192 + (17*a*b*x^5)/48 + (b^2*x^7)/8) - (5*a^4*\text{Log}[b*x + \text{Sqrt}[b]*\text{Sqrt}[a + b*x^2]])/(128*b^(3/2))$

**Maple [A]** time = 0.007, size = 93, normalized size = 0.8

$$\frac{x}{8b} (bx^2 + a)^{\frac{7}{2}} - \frac{ax}{48b} (bx^2 + a)^{\frac{5}{2}} - \frac{5a^2x}{192b} (bx^2 + a)^{\frac{3}{2}} - \frac{5a^3x}{128b} \sqrt{bx^2 + a} - \frac{5a^4}{128} \ln(x\sqrt{b} + \sqrt{bx^2 + a}) b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(b\*x^2+a)^(5/2), x)

[Out] 1/8\*x\*(b\*x^2+a)^(7/2)/b-1/48\*a/b\*x\*(b\*x^2+a)^(5/2)-5/192\*a^2/b\*x\*(b\*x^2+a)^(3/2)-5/128\*a^3\*x\*(b\*x^2+a)^(1/2)/b-5/128\*a^4/b^(3/2)\*ln(x\*b^(1/2)+(b\*x^2+a)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)\*x^2, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.269395, size = 1, normalized size = 0.01

$$\left[ \frac{15a^4 \log\left(2\sqrt{bx^2 + a}bx - (2bx^2 + a)\sqrt{b}\right) + 2(48b^3x^7 + 136ab^2x^5 + 118a^2bx^3 + 15a^3x)\sqrt{bx^2 + a}\sqrt{b}}{768b^{\frac{3}{2}}}, \frac{15a^4 \arctan\left(\frac{\sqrt{-bx}}{\sqrt{bx^2 + a}}\right) - (48b^3x^7 + 136ab^2x^5 + 118a^2bx^3 + 15a^3x)\sqrt{bx^2 + a}\sqrt{-b}}{384\sqrt{-bb}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)\*x^2, x, algorithm="fricas")

[Out] [1/768\*(15\*a^4\*log(2\*sqrt(b\*x^2 + a)\*b\*x - (2\*b\*x^2 + a)\*sqrt(b)) + 2\*(48\*b^3\*x^7 + 136\*a\*b^2\*x^5 + 118\*a^2\*b\*x^3 + 15\*a^3\*x)\*sqrt(b\*x^2 + a)\*sqrt(b))/b^(3/2), -1/384\*(15\*a^4\*arctan(sqrt(-b)\*x/sqrt(b\*x^2 + a)) - (48\*b^3\*x^7 + 136\*a\*b^2\*x^5 + 118\*a^2\*b\*x^3 + 15\*a^3\*x)\*sqrt(b\*x^2 + a)\*sqrt(-b))/(sqrt(-b)\*b)]

**Sympy [A]** time = 24.5863, size = 150, normalized size = 1.34

$$\frac{5a^{\frac{7}{2}}x}{128b\sqrt{1 + \frac{bx^2}{a}}} + \frac{133a^{\frac{5}{2}}x^3}{384\sqrt{1 + \frac{bx^2}{a}}} + \frac{127a^{\frac{3}{2}}bx^5}{192\sqrt{1 + \frac{bx^2}{a}}} + \frac{23\sqrt{ab^2}x^7}{48\sqrt{1 + \frac{bx^2}{a}}} - \frac{5a^4 \operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}} + \frac{b^3x^9}{8\sqrt{a}\sqrt{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(b\*x\*\*2+a)\*\*(5/2), x)

[Out] 5\*a\*\*(7/2)\*x/(128\*b\*sqrt(1 + b\*x\*\*2/a)) + 133\*a\*\*(5/2)\*x\*\*3/(384\*sqrt(1 + b\*x\*\*2/a)) + 127\*a\*\*(3/2)\*b\*x\*\*5/(192\*sqrt(1 + b\*x\*\*2/a)) + 23\*sqrt(a)\*b\*\*2\*x\*\*7/(48\*sqrt(1 + b\*x\*\*2/a)) - 5\*a\*\*4\*asinh(s

$\sqrt[3]{b}x/\sqrt{a}/(128b^{3/2}) + b^3x^9/(8\sqrt{a})\sqrt{1 + b^2x^2/a}$

**GIAC/XCAS [A]** time = 0.211582, size = 104, normalized size = 0.93

$$\frac{5a^4 \ln\left(\left|-\sqrt{bx} + \sqrt{bx^2 + a}\right|\right)}{128b^{\frac{3}{2}}} + \frac{1}{384} \left(2(4(6b^2x^2 + 17ab)x^2 + 59a^2)x^2 + \frac{15a^3}{b}\right) \sqrt{bx^2 + ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)\*x^2,x, algorithm="giac")

[Out] 5/128\*a^4\*ln(abs(-sqrt(b)\*x + sqrt(b\*x^2 + a)))/b^(3/2) + 1/384\*(2\*(4\*(6\*b^2\*x^2 + 17\*a\*b)\*x^2 + 59\*a^2)\*x^2 + 15\*a^3/b)\*sqrt(b\*x^2 + a)\*x

### 3.399 $\int (a + bx^2)^{5/2} dx$

**Optimal.** Leaf size=84

$$\frac{5a^3 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}} + \frac{5}{16}a^2x\sqrt{a+bx^2} + \frac{5}{24}ax(a+bx^2)^{3/2} + \frac{1}{6}x(a+bx^2)^{5/2}$$

[Out] (5\*a^2\*x\*Sqrt[a + b\*x^2])/16 + (5\*a\*x\*(a + b\*x^2)^(3/2))/24 + (x\*(a + b\*x^2)^(5/2))/6 + (5\*a^3\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(16\*Sqrt[b])

**Rubi [A]** time = 0.0609324, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{5a^3 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}} + \frac{5}{16}a^2x\sqrt{a+bx^2} + \frac{5}{24}ax(a+bx^2)^{3/2} + \frac{1}{6}x(a+bx^2)^{5/2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(5/2), x]

[Out] (5\*a^2\*x\*Sqrt[a + b\*x^2])/16 + (5\*a\*x\*(a + b\*x^2)^(3/2))/24 + (x\*(a + b\*x^2)^(5/2))/6 + (5\*a^3\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(16\*Sqrt[b])

**Rubi in Sympy [A]** time = 5.75486, size = 78, normalized size = 0.93

$$\frac{5a^3 \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}} + \frac{5a^2x\sqrt{a+bx^2}}{16} + \frac{5ax(a+bx^2)^{3/2}}{24} + \frac{x(a+bx^2)^{5/2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(5/2), x)

[Out] 5\*a\*\*3\*atanh(sqrt(b)\*x/sqrt(a + b\*x\*\*2))/(16\*sqrt(b)) + 5\*a\*\*2\*x\*sqrt(a + b\*x\*\*2)/16 + 5\*a\*x\*(a + b\*x\*\*2)\*\*(3/2)/24 + x\*(a + b\*x\*\*2)\*\*(5/2)/6

**Mathematica [A]** time = 0.0768996, size = 71, normalized size = 0.85

$$\frac{1}{48} \left( \frac{15a^3 \log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right)}{\sqrt{b}} + x\sqrt{a+bx^2} (33a^2 + 26abx^2 + 8b^2x^4) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(5/2), x]

[Out] (x\*Sqrt[a + b\*x^2]\*(33\*a^2 + 26\*a\*b\*x^2 + 8\*b^2\*x^4) + (15\*a^3\*Log[b\*x + Sqrt[b]\*Sqrt[a + b\*x^2]]))/Sqrt[b])/48

**Maple [A]** time = 0.004, size = 66, normalized size = 0.8

$$\frac{x}{6} (bx^2 + a)^{\frac{5}{2}} + \frac{5ax}{24} (bx^2 + a)^{\frac{3}{2}} + \frac{5a^2x}{16} \sqrt{bx^2 + a} + \frac{5a^3}{16} \ln(x\sqrt{b} + \sqrt{bx^2 + a}) \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(5/2), x)

[Out] 1/6\*x\*(b\*x^2+a)^(5/2)+5/24\*a\*x\*(b\*x^2+a)^(3/2)+5/16\*a^2\*x\*(b\*x^2+a)^(1/2)+5/16\*a^3/b^(1/2)\*ln(x\*b^(1/2)+(b\*x^2+a)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.257092, size = 1, normalized size = 0.01

$$\left[ \frac{15a^3 \log(-2\sqrt{bx^2+abx} - (2bx^2+a)\sqrt{b}) + 2(8b^2x^5 + 26abx^3 + 33a^2x)\sqrt{bx^2+a}\sqrt{b}}{96\sqrt{b}}, \frac{15a^3 \arctan\left(\frac{\sqrt{-bx}}{\sqrt{bx^2+a}}\right) + (8b^2x^5}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2), x, algorithm="fricas")

[Out] [1/96\*(15\*a^3\*log(-2\*sqrt(b\*x^2 + a)\*b\*x - (2\*b\*x^2 + a)\*sqrt(b)) + 2\*(8\*b^2\*x^5 + 26\*a\*b\*x^3 + 33\*a^2\*x)\*sqrt(b\*x^2 + a)\*sqrt(b))/sqrt(b), 1/48\*(15\*a^3\*arctan(sqrt(-b)\*x/sqrt(b\*x^2 + a)) + (8\*b^2\*x^5 + 26\*a\*b\*x^3 + 33\*a^2\*x)\*sqrt(b\*x^2 + a)\*sqrt(-b))/sqrt(-b)]

**Sympy [A]** time = 14.4441, size = 97, normalized size = 1.15

$$\frac{11a^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{16} + \frac{13a^{\frac{3}{2}}bx^3\sqrt{1+\frac{bx^2}{a}}}{24} + \frac{\sqrt{ab^2x^5}\sqrt{1+\frac{bx^2}{a}}}{6} + \frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{16\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(5/2), x)

[Out] 11\*a\*\*(5/2)\*x\*sqrt(1 + b\*x\*\*2/a)/16 + 13\*a\*\*(3/2)\*b\*x\*\*3\*sqrt(1 + b\*x\*\*2/a)/24 + sqrt(a)\*b\*\*2\*x\*\*5\*sqrt(1 + b\*x\*\*2/a)/6 + 5\*a\*\*3\*a\*sinh(sqrt(b)\*x/sqrt(a))/(16\*sqrt(b))

**GIAC/XCAS [A]** time = 0.212144, size = 85, normalized size = 1.01

$$-\frac{5a^3\ln\left(\left|-\sqrt{bx} + \sqrt{bx^2 + a}\right|\right)}{16\sqrt{b}} + \frac{1}{48} (2(4b^2x^2 + 13ab)x^2 + 33a^2)\sqrt{bx^2 + ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/16*a^3*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/48*(2  
*(4*b^2*x^2 + 13*a*b)*x^2 + 33*a^2)*sqrt(b*x^2 + a)*x
```

$$3.400 \quad \int \frac{(a+bx^2)^{5/2}}{x^2} dx$$

**Optimal.** Leaf size=83

$$\frac{15}{8}a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right) - \frac{(a+bx^2)^{5/2}}{x} + \frac{5}{4}bx(a+bx^2)^{3/2} + \frac{15}{8}abx\sqrt{a+bx^2}$$

[Out] (15\*a\*b\*x\*Sqrt[a + b\*x^2])/8 + (5\*b\*x\*(a + b\*x^2)^(3/2))/4 - (a + b\*x^2)^(5/2)/x + (15\*a^2\*Sqrt[b]\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/8

**Rubi [A]** time = 0.073047, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{15}{8}a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right) - \frac{(a+bx^2)^{5/2}}{x} + \frac{5}{4}bx(a+bx^2)^{3/2} + \frac{15}{8}abx\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(5/2)/x^2, x]

[Out] (15\*a\*b\*x\*Sqrt[a + b\*x^2])/8 + (5\*b\*x\*(a + b\*x^2)^(3/2))/4 - (a + b\*x^2)^(5/2)/x + (15\*a^2\*Sqrt[b]\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/8

**Rubi in Sympy [A]** time = 7.74218, size = 76, normalized size = 0.92

$$\frac{15a^2\sqrt{b}\operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{8} + \frac{15abx\sqrt{a+bx^2}}{8} + \frac{5bx(a+bx^2)^{\frac{3}{2}}}{4} - \frac{(a+bx^2)^{\frac{5}{2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(5/2)/x\*\*2, x)

[Out] 15\*a\*\*2\*sqrt(b)\*atanh(sqrt(b)\*x/sqrt(a + b\*x\*\*2))/8 + 15\*a\*b\*x\*sqrt(a + b\*x\*\*2)/8 + 5\*b\*x\*(a + b\*x\*\*2)\*\*(3/2)/4 - (a + b\*x\*\*2)\*\*(5/2)/x

**Mathematica [A]** time = 0.0600864, size = 73, normalized size = 0.88

$$\sqrt{a+bx^2}\left(-\frac{a^2}{x} + \frac{9abx}{8} + \frac{b^2x^3}{4}\right) + \frac{15}{8}a^2\sqrt{b}\log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(5/2)/x^2, x]

[Out] Sqrt[a + b\*x^2]\*(-(a^2/x) + (9\*a\*b\*x)/8 + (b^2\*x^3)/4) + (15\*a^2\*Sqrt[b]\*Log[b\*x + Sqrt[b]\*Sqrt[a + b\*x^2]])/8

**Maple [A]** time = 0.007, size = 85, normalized size = 1.

$$-\frac{1}{ax}(bx^2+a)^{\frac{7}{2}} + \frac{bx}{a}(bx^2+a)^{\frac{5}{2}} + \frac{5bx}{4}(bx^2+a)^{\frac{3}{2}} + \frac{15abx}{8}\sqrt{bx^2+a} + \frac{15a^2}{8}\sqrt{b}\ln(x\sqrt{b} + \sqrt{bx^2+a})$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(5/2)/x^2,x)`

[Out]  $-1/a/x*(b*x^2+a)^{7/2}+b/a*x*(b*x^2+a)^{5/2}+5/4*b*x*(b*x^2+a)^{3/2}+15/8*a*b*x*(b*x^2+a)^{1/2}+15/8*b^{1/2}*a^2*\ln(x*b^{1/2}+(b*x^2+a)^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.255858, size = 1, normalized size = 0.01

$$\left[ \frac{15 a^2 \sqrt{b x} \log \left( -2 b x^2 - 2 \sqrt{b x^2 + a} \sqrt{b x} - a \right) + 2 \left( 2 b^2 x^4 + 9 a b x^2 - 8 a^2 \right) \sqrt{b x^2 + a}}{16 x}, \frac{15 a^2 \sqrt{-b x} \arctan \left( \frac{b x}{\sqrt{b x^2 + a} \sqrt{-b}} \right) + (2 b^2 x^4 + 9 a b x^2 - 8 a^2) \sqrt{-b x}}{8 x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^2,x, algorithm="fricas")`

[Out]  $[1/16*(15*a^2*\sqrt{b}*x*\log(-2*b*x^2 - 2*\sqrt{b*x^2 + a})*\sqrt{b}*x - a) + 2*(2*b^2*x^4 + 9*a*b*x^2 - 8*a^2)*\sqrt{b*x^2 + a})/x, 1/8*(15*a^2*\sqrt{-b}*x*\arctan(b*x/(\sqrt{b*x^2 + a})*\sqrt{-b})) + (2*b^2*x^4 + 9*a*b*x^2 - 8*a^2)*\sqrt{-b*x})/x]$

**Sympy [A]** time = 13.0899, size = 117, normalized size = 1.41

$$-\frac{a^{5/2}}{x\sqrt{1+\frac{bx^2}{a}}} + \frac{a^{3/2}bx}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{11\sqrt{a}b^2x^3}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{15a^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8} + \frac{b^3x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(5/2)/x**2,x)`

[Out]  $-a^{5/2}/(x*\sqrt{1+b*x^2/a})+a^{3/2}*b*x/(8*\sqrt{1+b*x^2/a})+11*\sqrt{a}*b^2*x^3/(8*\sqrt{1+b*x^2/a})+15*a^2*\sqrt{b}*x*\operatorname{asinh}(\sqrt{b}*x/\sqrt{a})/8+b^3*x^5/(4*\sqrt{a}*\sqrt{1+b*x^2/a})$

**GIAC/XCAS [A]** time = 0.215292, size = 117, normalized size = 1.41

$$-\frac{15}{16}a^2\sqrt{b}\ln\left(\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^2\right)+\frac{2a^3\sqrt{b}}{\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{8}(2b^2x^2+9ab)\sqrt{bx^2+ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] -15/16*a^2*sqrt(b)*ln((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2*a^3*sq  
rt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/8*(2*b^2*x^2 + 9*  
a*b)*sqrt(b*x^2 + a)*x
```

$$3.401 \quad \int \frac{(a+bx^2)^{5/2}}{x^4} dx$$

**Optimal.** Leaf size=86

$$\frac{5}{2}ab^{3/2} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right) + \frac{5}{2}b^2x\sqrt{a+bx^2} - \frac{5b(a+bx^2)^{3/2}}{3x} - \frac{(a+bx^2)^{5/2}}{3x^3}$$

[Out] (5\*b^2\*x\*Sqrt[a + b\*x^2])/2 - (5\*b\*(a + b\*x^2)^(3/2))/(3\*x) - (a + b\*x^2)^(5/2)/(3\*x^3) + (5\*a\*b^(3/2)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/2

**Rubi [A]** time = 0.079175, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{5}{2}ab^{3/2} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right) + \frac{5}{2}b^2x\sqrt{a+bx^2} - \frac{5b(a+bx^2)^{3/2}}{3x} - \frac{(a+bx^2)^{5/2}}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(5/2)/x^4, x]

[Out] (5\*b^2\*x\*Sqrt[a + b\*x^2])/2 - (5\*b\*(a + b\*x^2)^(3/2))/(3\*x) - (a + b\*x^2)^(5/2)/(3\*x^3) + (5\*a\*b^(3/2)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/2

**Rubi in Sympy [A]** time = 9.21725, size = 78, normalized size = 0.91

$$\frac{5ab^{\frac{3}{2}} \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{2} + \frac{5b^2x\sqrt{a+bx^2}}{2} - \frac{5b(a+bx^2)^{\frac{3}{2}}}{3x} - \frac{(a+bx^2)^{\frac{5}{2}}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(5/2)/x\*\*4, x)

[Out] 5\*a\*b\*\*(3/2)\*atanh(sqrt(b)\*x/sqrt(a + b\*x\*\*2))/2 + 5\*b\*\*2\*x\*sqrt(a + b\*x\*\*2)/2 - 5\*b\*(a + b\*x\*\*2)\*\*(3/2)/(3\*x) - (a + b\*x\*\*2)\*\*(5/2)/(3\*x\*\*3)

**Mathematica [A]** time = 0.0683628, size = 73, normalized size = 0.85

$$\left(-\frac{a^2}{3x^3} - \frac{7ab}{3x} + \frac{b^2x}{2}\right)\sqrt{a+bx^2} + \frac{5}{2}ab^{3/2}\log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(5/2)/x^4, x]

[Out] (-a^2/(3\*x^3) - (7\*a\*b)/(3\*x) + (b^2\*x)/2)\*Sqrt[a + b\*x^2] + (5\*a\*b^(3/2)\*Log[b\*x + Sqrt[b]\*Sqrt[a + b\*x^2]])/2

**Maple [A]** time = 0.007, size = 110, normalized size = 1.3

$$-\frac{1}{3ax^3}(bx^2+a)^{\frac{7}{2}} - \frac{4b}{3a^2x}(bx^2+a)^{\frac{7}{2}} + \frac{4b^2x}{3a^2}(bx^2+a)^{\frac{5}{2}} + \frac{5b^2x}{3a}(bx^2+a)^{\frac{3}{2}} + \frac{5b^2x}{2}\sqrt{bx^2+a} + \frac{5a}{2}b^{\frac{3}{2}}\ln\left(x\sqrt{b} + \sqrt{bx^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(5/2)/x^4, x)

[Out]  $-1/3/a/x^3*(b*x^2+a)^{(7/2)} - 4/3*b/a^2/x*(b*x^2+a)^{(7/2)} + 4/3*b^2/a^2*x*(b*x^2+a)^{(5/2)} + 5/3*b^2/a*x*(b*x^2+a)^{(3/2)} + 5/2*b^2*x*(b*x^2+a)^{(1/2)} + 5/2*b^{(3/2)}*a*\ln(x*b^{(1/2)}+(b*x^2+a)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)/x^4, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.249989, size = 1, normalized size = 0.01

$$\left[ \frac{15ab^{\frac{3}{2}}x^3 \log\left(-2bx^2 - 2\sqrt{bx^2+a}\sqrt{bx-a}\right) + 2(3b^2x^4 - 14abx^2 - 2a^2)\sqrt{bx^2+a} + 15a\sqrt{-bbx^3} \arctan\left(\frac{bx}{\sqrt{bx^2+a}\sqrt{-b}}\right)}{12x^3}, \frac{15a\sqrt{-bbx^3} \arctan\left(\frac{bx}{\sqrt{bx^2+a}\sqrt{-b}}\right)}{6x^3} \right] + (\dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)/x^4, x, algorithm="fricas")

[Out]  $[1/12*(15*a*b^{(3/2)}*x^3*\log(-2*b*x^2 - 2*\sqrt{b*x^2 + a})*\sqrt{b}*x - a) + 2*(3*b^2*x^4 - 14*a*b*x^2 - 2*a^2)*\sqrt{b*x^2 + a})/x^3, 1/6*(15*a*\sqrt{-b}*b*x^3*\arctan(b*x/(\sqrt{b*x^2 + a})*\sqrt{-b})) + (3*b^2*x^4 - 14*a*b*x^2 - 2*a^2)*\sqrt{b*x^2 + a})/x^3]$

**Sympy [A]** time = 11.4392, size = 112, normalized size = 1.3

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{7ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3} - \frac{5ab^{\frac{3}{2}}\log\left(\frac{a}{bx^2}\right)}{4} + \frac{5ab^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)}{2} + \frac{b^{\frac{5}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(5/2)/x\*\*4, x)

[Out]  $-a**2*\sqrt{b}*\sqrt{a/(b*x**2) + 1}/(3*x**2) - 7*a*b**(3/2)*\sqrt{a/(b*x**2) + 1}/3 - 5*a*b**(3/2)*\log(a/(b*x**2))/4 + 5*a*b**(3/2)*\log(\sqrt{a/(b*x**2) + 1} + 1)/2 + b**(5/2)*x**2*\sqrt{a/(b*x**2) + 1}/2$

**GIAC/XCAS [A]** time = 0.219529, size = 178, normalized size = 2.07

$$\frac{\frac{1}{2}\sqrt{bx^2+ab^2}x - \frac{5}{4}ab^{\frac{3}{2}}\ln\left(\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^2\right)}{2\left(9\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^4a^2b^{\frac{3}{2}} - 12\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^2a^3b^{\frac{3}{2}} + 7a^4b^{\frac{3}{2}}\right)} + \frac{3\left(\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^2 - a\right)^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)/x^4,x, algorithm="giac")

[Out] 1/2\*sqrt(b\*x^2 + a)\*b^2\*x - 5/4\*a\*b^(3/2)\*ln((sqrt(b)\*x - sqrt(b\*x^2 + a))^2) + 2/3\*(9\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^4\*a^2\*b^(3/2) - 12\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^2\*a^3\*b^(3/2) + 7\*a^4\*b^(3/2))/((sqrt(b)\*x - sqrt(b\*x^2 + a))^2 - a)^3

$$3.402 \quad \int \frac{(a+bx^2)^{5/2}}{x^6} dx$$

**Optimal.** Leaf size=82

$$b^{5/2} \tanh^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a+bx^2}} \right) - \frac{b^2 \sqrt{a+bx^2}}{x} - \frac{(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{3/2}}{3x^3}$$

[Out] -((b^2\*Sqrt[a + b\*x^2])/x) - (b\*(a + b\*x^2)^(3/2))/(3\*x^3) - (a + b\*x^2)^(5/2)/(5\*x^5) + b^(5/2)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]]

**Rubi [A]** time = 0.0885073, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$b^{5/2} \tanh^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a+bx^2}} \right) - \frac{b^2 \sqrt{a+bx^2}}{x} - \frac{(a+bx^2)^{5/2}}{5x^5} - \frac{b(a+bx^2)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(5/2)/x^6, x]

[Out] -((b^2\*Sqrt[a + b\*x^2])/x) - (b\*(a + b\*x^2)^(3/2))/(3\*x^3) - (a + b\*x^2)^(5/2)/(5\*x^5) + b^(5/2)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]]

**Rubi in Sympy [A]** time = 10.9482, size = 70, normalized size = 0.85

$$b^{5/2} \operatorname{atanh} \left( \frac{\sqrt{bx}}{\sqrt{a+bx^2}} \right) - \frac{b^2 \sqrt{a+bx^2}}{x} - \frac{b(a+bx^2)^{3/2}}{3x^3} - \frac{(a+bx^2)^{5/2}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(5/2)/x\*\*6, x)

[Out] b\*\*(5/2)\*atanh(sqrt(b)\*x/sqrt(a + b\*x\*\*2)) - b\*\*2\*sqrt(a + b\*x\*\*2)/x - b\*(a + b\*x\*\*2)\*\*(3/2)/(3\*x\*\*3) - (a + b\*x\*\*2)\*\*(5/2)/(5\*x\*\*5)

**Mathematica [A]** time = 0.0561669, size = 68, normalized size = 0.83

$$b^{5/2} \log \left( \sqrt{b} \sqrt{a+bx^2} + bx \right) - \frac{\sqrt{a+bx^2} (3a^2 + 11abx^2 + 23b^2x^4)}{15x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(5/2)/x^6, x]

[Out] -(Sqrt[a + b\*x^2]\*(3\*a^2 + 11\*a\*b\*x^2 + 23\*b^2\*x^4))/(15\*x^5) + b^(5/2)\*Log[b\*x + Sqrt[b]\*Sqrt[a + b\*x^2]]

**Maple [A]** time = 0.012, size = 130, normalized size = 1.6

$$-\frac{1}{5ax^5}(bx^2+a)^{\frac{7}{2}} - \frac{2b}{15a^2x^3}(bx^2+a)^{\frac{7}{2}} - \frac{8b^2}{15a^3x}(bx^2+a)^{\frac{7}{2}} + \frac{8b^3x}{15a^3}(bx^2+a)^{\frac{5}{2}} + \frac{2b^3x}{3a^2}(bx^2+a)^{\frac{3}{2}} + \frac{b^3x}{a}\sqrt{bx^2+a} + b^{\frac{5}{2}}\ln(x\sqrt{b} + \sqrt{bx^2+a})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(5/2)/x^6,x)`

[Out] `-1/5/a/x^5*(b*x^2+a)^(7/2)-2/15*b/a^2/x^3*(b*x^2+a)^(7/2)-8/15*b^2/a^3/x*(b*x^2+a)^(7/2)+8/15*b^3/a^3*x*(b*x^2+a)^(5/2)+2/3*b^3/a^2*x*(b*x^2+a)^(3/2)+b^3/a*x*(b*x^2+a)^(1/2)+b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^6,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.247394, size = 1, normalized size = 0.01

$$\left[ \frac{15b^{\frac{5}{2}}x^5 \log(-2bx^2 - 2\sqrt{bx^2+a}\sqrt{bx-a}) - 2(23b^2x^4 + 11abx^2 + 3a^2)\sqrt{bx^2+a} - 15\sqrt{-b}b^2x^5 \arctan\left(\frac{bx}{\sqrt{bx^2+a}\sqrt{-b}}\right)}{30x^5}, \frac{15\sqrt{-b}b^2x^5 \arctan\left(\frac{bx}{\sqrt{bx^2+a}\sqrt{-b}}\right)}{15x^5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^6,x, algorithm="fricas")`

[Out] `[1/30*(15*b^(5/2)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(23*b^2*x^4 + 11*a*b*x^2 + 3*a^2)*sqrt(b*x^2 + a))/x^5, 1/15*(15*sqrt(-b)*b^2*x^5*arctan(b*x/(sqrt(b*x^2 + a)*sqrt(-b))) - (23*b^2*x^4 + 11*a*b*x^2 + 3*a^2)*sqrt(b*x^2 + a))/x^5]`

**Sympy [A]** time = 12.9194, size = 105, normalized size = 1.28

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{11ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15x^2} - \frac{23b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15} - \frac{b^{\frac{5}{2}}\log\left(\frac{a}{bx^2}\right)}{2} + b^{\frac{5}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(5/2)/x**6,x)`

[Out] `-a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 11*a*b**(3/2)*sqrt(a/(b*x**2) + 1)/(15*x**2) - 23*b**(5/2)*sqrt(a/(b*x**2) + 1)/15 - b**(5/2)*log(a/(b*x**2))/2 + b**(5/2)*log(sqrt(a/(b*x**2) + 1) + 1)`

**GIAC/XCAS [A]** time = 0.226094, size = 227, normalized size = 2.77

$$-\frac{1}{2} b^{\frac{5}{2}} \ln \left( \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 \right) + \frac{2 \left( 45 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^8 a b^{\frac{5}{2}} - 90 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^6 a^2 b^{\frac{5}{2}} + 140 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^4 a^3 b^{\frac{5}{2}} - 70 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 a^4 b^{\frac{5}{2}} \right)}{15 \left( \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 - a \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)/x^6,x, algorithm="giac")

[Out] -1/2\*b^(5/2)\*ln((sqrt(b)\*x - sqrt(b\*x^2 + a))^2) + 2/15\*(45\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^8\*a\*b^(5/2) - 90\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^6\*a^2\*b^(5/2) + 140\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^4\*a^3\*b^(5/2) - 70\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^2\*a^4\*b^(5/2) + 23\*a^5\*b^(5/2))/((sqrt(b)\*x - sqrt(b\*x^2 + a))^2 - a)^5



$$3.403 \quad \int \frac{(a+bx^2)^{5/2}}{x^8} dx$$

**Optimal.** Leaf size=21

$$-\frac{(a+bx^2)^{7/2}}{7ax^7}$$

[Out]  $-(a + b*x^2)^{(7/2)}/(7*a*x^7)$

**Rubi [A]** time = 0.0222983, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$-\frac{(a+bx^2)^{7/2}}{7ax^7}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^{(5/2)}/x^8, x]$

[Out]  $-(a + b*x^2)^{(7/2)}/(7*a*x^7)$

**Rubi in Sympy [A]** time = 3.19426, size = 17, normalized size = 0.81

$$-\frac{(a+bx^2)^{\frac{7}{2}}}{7ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x^{**2}+a)^{(5/2)}/x^{**8}, x)$

[Out]  $-(a + b*x^{**2})^{(7/2)}/(7*a*x^{**7})$

**Mathematica [A]** time = 0.0328821, size = 21, normalized size = 1.

$$-\frac{(a+bx^2)^{7/2}}{7ax^7}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b*x^2)^{(5/2)}/x^8, x]$

[Out]  $-(a + b*x^2)^{(7/2)}/(7*a*x^7)$

**Maple [A]** time = 0.006, size = 18, normalized size = 0.9

$$-\frac{1}{7ax^7} (bx^2 + a)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((b*x^2+a)^{(5/2)}/x^8, x)$

[Out]  $-1/7 * (b * x^2 + a)^{(7/2)} / a / x^7$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^8,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.236962, size = 62, normalized size = 2.95

$$\frac{(b^3x^6 + 3ab^2x^4 + 3a^2bx^2 + a^3)\sqrt{bx^2 + a}}{7ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^8,x, algorithm="fricas")`

[Out]  $-1/7 * (b^3 * x^6 + 3 * a * b^2 * x^4 + 3 * a^2 * b * x^2 + a^3) * \text{sqrt}(b * x^2 + a) / (a * x^7)$

**Sympy [A]** time = 6.4575, size = 95, normalized size = 4.52

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2} + 1}}{7x^6} - \frac{3ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2} + 1}}{7x^4} - \frac{3b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2} + 1}}{7x^2} - \frac{b^{\frac{7}{2}}\sqrt{\frac{a}{bx^2} + 1}}{7a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(5/2)/x**8,x)`

[Out]  $-a^{**2} * \text{sqrt}(b) * \text{sqrt}(a / (b * x^{**2}) + 1) / (7 * x^{**6}) - 3 * a * b^{** (3/2)} * \text{sqrt}(a / (b * x^{**2}) + 1) / (7 * x^{**4}) - 3 * b^{** (5/2)} * \text{sqrt}(a / (b * x^{**2}) + 1) / (7 * x^{**2}) - b^{** (7/2)} * \text{sqrt}(a / (b * x^{**2}) + 1) / (7 * a)$

**GIAC/XCAS [A]** time = 0.216156, size = 153, normalized size = 7.29

$$\frac{2 \left( 7 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{12} b^{\frac{7}{2}} + 35 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^8 a^2 b^{\frac{7}{2}} + 21 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^4 a^4 b^{\frac{7}{2}} + a^6 b^{\frac{7}{2}} \right)}{7 \left( \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 - a \right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^8,x, algorithm="giac")`

[Out]  $2/7 * (7 * (\text{sqrt}(b) * x - \text{sqrt}(b * x^2 + a))^{12} * b^{(7/2)} + 35 * (\text{sqrt}(b) * x - \text{sqrt}(b * x^2 + a))^{8} * a^2 * b^{(7/2)} + 21 * (\text{sqrt}(b) * x - \text{sqrt}(b * x^2 + a))^{4} * a^4 * b^{(7/2)} + a^6 * b^{(7/2)}) / ((\text{sqrt}(b) * x - \text{sqrt}(b * x^2 + a))^{2} - a)^7$

$$3.404 \quad \int \frac{(a+bx^2)^{5/2}}{x^{10}} dx$$

**Optimal.** Leaf size=44

$$\frac{2b(a+bx^2)^{7/2}}{63a^2x^7} - \frac{(a+bx^2)^{7/2}}{9ax^9}$$

[Out]  $-(a + b*x^2)^{(7/2)}/(9*a*x^9) + (2*b*(a + b*x^2)^{(7/2)})/(63*a^2*x^7)$

**Rubi [A]** time = 0.0452174, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{2b(a+bx^2)^{7/2}}{63a^2x^7} - \frac{(a+bx^2)^{7/2}}{9ax^9}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(5/2)/x^10, x]

[Out]  $-(a + b*x^2)^{(7/2)}/(9*a*x^9) + (2*b*(a + b*x^2)^{(7/2)})/(63*a^2*x^7)$

**Rubi in Sympy [A]** time = 5.20474, size = 37, normalized size = 0.84

$$-\frac{(a+bx^2)^{\frac{7}{2}}}{9ax^9} + \frac{2b(a+bx^2)^{\frac{7}{2}}}{63a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(5/2)/x\*\*10, x)

[Out]  $-(a + b*x**2)**(7/2)/(9*a*x**9) + 2*b*(a + b*x**2)**(7/2)/(63*a**2*x**7)$

**Mathematica [A]** time = 0.041712, size = 31, normalized size = 0.7

$$\frac{(a+bx^2)^{7/2}(2bx^2-7a)}{63a^2x^9}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(5/2)/x^10, x]

[Out]  $((a + b*x^2)^{(7/2)}*(-7*a + 2*b*x^2))/(63*a^2*x^9)$

**Maple [A]** time = 0.007, size = 28, normalized size = 0.6

$$-\frac{-2bx^2+7a}{63x^9a^2}(bx^2+a)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(5/2)/x^10,x)`

[Out]  $-1/63*(b*x^2+a)^{(7/2)*(-2*b*x^2+7*a)}/x^9/a^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^10,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.255372, size = 81, normalized size = 1.84

$$\frac{(2b^4x^8 - ab^3x^6 - 15a^2b^2x^4 - 19a^3bx^2 - 7a^4)\sqrt{bx^2 + a}}{63a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^10,x, algorithm="fricas")`

[Out]  $1/63*(2*b^4*x^8 - a*b^3*x^6 - 15*a^2*b^2*x^4 - 19*a^3*b*x^2 - 7*a^4)*\text{sqrt}(b*x^2 + a)/(a^2*x^9)$

**Sympy [A]** time = 10.2129, size = 121, normalized size = 2.75

$$-\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2} + 1}}{9x^8} - \frac{19ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2} + 1}}{63x^6} - \frac{5b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2} + 1}}{21x^4} - \frac{b^{\frac{7}{2}}\sqrt{\frac{a}{bx^2} + 1}}{63ax^2} + \frac{2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2} + 1}}{63a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(5/2)/x**10,x)`

[Out]  $-a^{**2}*\text{sqrt}(b)*\text{sqrt}(a/(b*x^{**2}) + 1)/(9*x^{**8}) - 19*a*b^{**}(3/2)*\text{sqrt}(a/(b*x^{**2}) + 1)/(63*x^{**6}) - 5*b^{**}(5/2)*\text{sqrt}(a/(b*x^{**2}) + 1)/(21*x^{**4}) - b^{**}(7/2)*\text{sqrt}(a/(b*x^{**2}) + 1)/(63*a*x^{**2}) + 2*b^{**}(9/2)*\text{sqrt}(a/(b*x^{**2}) + 1)/(63*a^{**2})$

**GIAC/XCAS [A]** time = 0.215922, size = 297, normalized size = 6.75

$$\frac{4\left(63\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^{14} b^{\frac{9}{2}} + 105\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^{12} ab^{\frac{9}{2}} + 315\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^{10} a^2 b^{\frac{9}{2}} + 189\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^8 a^3 b^{\frac{9}{2}}\right)}{63\left(\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^2 - \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^10,x, algorithm="giac")`

[Out]  $4/63*(63*(\text{sqrt}(b)*x - \text{sqrt}(b*x^2 + a))^{14}*b^{(9/2)} + 105*(\text{sqrt}(b)*x - \text{sqrt}(b*x^2 + a))^{12}*a*b^{(9/2)} + 315*(\text{sqrt}(b)*x - \text{sqrt}(b*x^2 +$

$$\frac{a^{10} a^2 b^{9/2} + 189 (\sqrt{b} x - \sqrt{b x^2 + a})^8 a^3 b^{9/2} + 189 (\sqrt{b} x - \sqrt{b x^2 + a})^6 a^4 b^{9/2} + 27 (\sqrt{b} x - \sqrt{b x^2 + a})^4 a^5 b^{9/2} + 9 (\sqrt{b} x - \sqrt{b x^2 + a})^2 a^6 b^{9/2} - a^7 b^{9/2}}{(\sqrt{b} x - \sqrt{b x^2 + a})^2 - a^9}$$

$$3.405 \quad \int \frac{(a+bx^2)^{5/2}}{x^{12}} dx$$

**Optimal.** Leaf size=68

$$-\frac{8b^2(a+bx^2)^{7/2}}{693a^3x^7} + \frac{4b(a+bx^2)^{7/2}}{99a^2x^9} - \frac{(a+bx^2)^{7/2}}{11ax^{11}}$$

[Out]  $-(a + b*x^2)^{(7/2)}/(11*a*x^{11}) + (4*b*(a + b*x^2)^{(7/2)})/(99*a^2*x^9) - (8*b^2*(a + b*x^2)^{(7/2)})/(693*a^3*x^7)$

**Rubi [A]** time = 0.0706926, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{8b^2(a+bx^2)^{7/2}}{693a^3x^7} + \frac{4b(a+bx^2)^{7/2}}{99a^2x^9} - \frac{(a+bx^2)^{7/2}}{11ax^{11}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(5/2)/x^12, x]

[Out]  $-(a + b*x^2)^{(7/2)}/(11*a*x^{11}) + (4*b*(a + b*x^2)^{(7/2)})/(99*a^2*x^9) - (8*b^2*(a + b*x^2)^{(7/2)})/(693*a^3*x^7)$

**Rubi in Sympy [A]** time = 8.04231, size = 61, normalized size = 0.9

$$-\frac{(a+bx^2)^{\frac{7}{2}}}{11ax^{11}} + \frac{4b(a+bx^2)^{\frac{7}{2}}}{99a^2x^9} - \frac{8b^2(a+bx^2)^{\frac{7}{2}}}{693a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(5/2)/x\*\*12, x)

[Out]  $-(a + b*x^2)^{(7/2)}/(11*a*x^{11}) + 4*b*(a + b*x^2)^{(7/2)}/(99*a^2*x^9) - 8*b^2*(a + b*x^2)^{(7/2)}/(693*a^3*x^7)$

**Mathematica [A]** time = 0.0512606, size = 42, normalized size = 0.62

$$-\frac{(a+bx^2)^{7/2}(63a^2 - 28abx^2 + 8b^2x^4)}{693a^3x^{11}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(5/2)/x^12, x]

[Out]  $-((a + b*x^2)^{(7/2)}*(63*a^2 - 28*a*b*x^2 + 8*b^2*x^4))/(693*a^3*x^{11})$

**Maple [A]** time = 0.007, size = 39, normalized size = 0.6

$$-\frac{8b^2x^4 - 28abx^2 + 63a^2}{693x^{11}a^3} (bx^2 + a)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(5/2)/x^12,x)`

[Out]  $-1/693*(b*x^2+a)^{(7/2)}*(8*b^2*x^4-28*a*b*x^2+63*a^2)/x^{11}/a^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^12,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.312848, size = 96, normalized size = 1.41

$$\frac{(8b^5x^{10} - 4ab^4x^8 + 3a^2b^3x^6 + 113a^3b^2x^4 + 161a^4bx^2 + 63a^5)\sqrt{bx^2 + a}}{693a^3x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^12,x, algorithm="fricas")`

[Out]  $-1/693*(8*b^5*x^{10} - 4*a*b^4*x^8 + 3*a^2*b^3*x^6 + 113*a^3*b^2*x^4 + 161*a^4*b*x^2 + 63*a^5)*\text{sqrt}(b*x^2 + a)/(a^3*x^{11})$

**Sympy [A]** time = 15.4823, size = 481, normalized size = 7.07

$$\begin{aligned} & \frac{63a^7b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2} + 1}}{x^2(693a^5b^4x^8 + 1386a^4b^5x^{10} + 693a^3b^6x^{12})} - \frac{287a^6b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2} + 1}}{693a^5b^4x^8 + 1386a^4b^5x^{10} + 693a^3b^6x^{12}} \\ & - \frac{498a^5b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx^2} + 1}}{693a^5b^4x^8 + 1386a^4b^5x^{10} + 693a^3b^6x^{12}} - \frac{390a^4b^{\frac{15}{2}}x^4\sqrt{\frac{a}{bx^2} + 1}}{693a^5b^4x^8 + 1386a^4b^5x^{10} + 693a^3b^6x^{12}} \\ & - \frac{115a^3b^{\frac{17}{2}}x^6\sqrt{\frac{a}{bx^2} + 1}}{693a^5b^4x^8 + 1386a^4b^5x^{10} + 693a^3b^6x^{12}} - \frac{3a^2b^{\frac{19}{2}}x^8\sqrt{\frac{a}{bx^2} + 1}}{693a^5b^4x^8 + 1386a^4b^5x^{10} + 693a^3b^6x^{12}} \\ & - \frac{12ab^{\frac{21}{2}}x^{10}\sqrt{\frac{a}{bx^2} + 1}}{693a^5b^4x^8 + 1386a^4b^5x^{10} + 693a^3b^6x^{12}} - \frac{8b^{\frac{23}{2}}x^{12}\sqrt{\frac{a}{bx^2} + 1}}{693a^5b^4x^8 + 1386a^4b^5x^{10} + 693a^3b^6x^{12}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(5/2)/x**12,x)`

[Out]  $-63*a^{**7}*b^{**9/2}*\text{sqrt}(a/(b*x^{**2}) + 1)/(x^{**2}*(693*a^{**5}*b^{**4}*x^{**8} + 1386*a^{**4}*b^{**5}*x^{**10} + 693*a^{**3}*b^{**6}*x^{**12})) - 287*a^{**6}*b^{**11/2}*(11/2)*\text{sqrt}(a/(b*x^{**2}) + 1)/(693*a^{**5}*b^{**4}*x^{**8} + 1386*a^{**4}*b^{**5}*x^{**10} + 693*a^{**3}*b^{**6}*x^{**12}) - 498*a^{**5}*b^{**13/2}*x^2*\text{sqrt}(a/(b*x^{**2}) + 1)/(693*a^{**5}*b^{**4}*x^{**8} + 1386*a^{**4}*b^{**5}*x^{**10} + 693*a^{**3}*b^{**6}*x^{**12}) - 390*a^{**4}*b^{**15/2}*x^4*\text{sqrt}(a/(b*x^{**2}) + 1)/(693*a^{**5}*b^{**4}*x^{**8} + 1386*a^{**4}*b^{**5}*x^{**10} + 693*a^{**3}*b^{**6}*x^{**12}) - 115*a^{**3}*b^{**17/2}*x^6*\text{sqrt}(a/(b*x^{**2}) + 1)/(693*a^{**5}*b^{**4}*x^{**8} + 1386*a^{**4}*b^{**5}*x^{**10} + 693*a^{**3}*b^{**6}*x^{**12}) - 3*a^{**2}*b^{**19/2}*x^8*\text{sqrt}(a/(b*x^{**2}) + 1)/(693*a^{**5}*b^{**4}*x^{**8} + 1386*a^{**4}*b^{**5}*x^{**10} + 693*a^{**3}*b^{**6}*x^{**12}) - 12*a*b^{**21/2}*x^{10}*\text{sqrt}(a/(b*x^{**2}) + 1)/(693*a^{**5}*b^{**4}*x^{**8} + 1386*a^{**4}*b^{**5}*x^{**10} + 693*a^{**3}*b^{**6}*x^{**12}) - 8*b^{**23/2}*x^{12}*\text{sqrt}(a/(b*x^{**2}) + 1)/(693*a^{**5}*b^{**4}*x^{**8} + 1386*a^{**4}*b^{**5}*x^{**10} + 693*a^{**3}*b^{**6}*x^{**12})$

---

**GIAC/XCAS [A]** time = 0.215204, size = 332, normalized size = 4.88

$$16 \left( 462 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{16} b^{\frac{11}{2}} + 1155 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{14} ab^{\frac{11}{2}} + 2541 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{12} a^2 b^{\frac{11}{2}} + 2079 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{10} a^3 b^{\frac{11}{2}} + 1485 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^8 a^4 b^{\frac{11}{2}} + 297 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^6 a^5 b^{\frac{11}{2}} + 55 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^4 a^6 b^{\frac{11}{2}} - 11 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 a^7 b^{\frac{11}{2}} + a^8 b^{\frac{11}{2}} \right) / \left( \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 - a \right)^{11}$$


---

693

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)/x^12,x, algorithm="giac")

[Out] 16/693\*(462\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^16\*b^(11/2) + 1155\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^14\*a\*b^(11/2) + 2541\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^12\*a^2\*b^(11/2) + 2079\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^10\*a^3\*b^(11/2) + 1485\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^8\*a^4\*b^(11/2) + 297\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^6\*a^5\*b^(11/2) + 55\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^4\*a^6\*b^(11/2) - 11\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^2\*a^7\*b^(11/2) + a^8\*b^(11/2))/((sqrt(b)\*x - sqrt(b\*x^2 + a))^2 - a)^11



$$3.406 \quad \int \frac{(a+bx^2)^{5/2}}{x^{14}} dx$$

**Optimal.** Leaf size=92

$$\frac{16b^3 (a+bx^2)^{7/2}}{3003a^4x^7} - \frac{8b^2 (a+bx^2)^{7/2}}{429a^3x^9} + \frac{6b (a+bx^2)^{7/2}}{143a^2x^{11}} - \frac{(a+bx^2)^{7/2}}{13ax^{13}}$$

[Out]  $-(a + b*x^2)^{(7/2)}/(13*a*x^{13}) + (6*b*(a + b*x^2)^{(7/2)})/(143*a^2*x^{11}) - (8*b^2*(a + b*x^2)^{(7/2)})/(429*a^3*x^9) + (16*b^3*(a + b*x^2)^{(7/2)})/(3003*a^4*x^7)$

**Rubi [A]** time = 0.102532, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{16b^3 (a+bx^2)^{7/2}}{3003a^4x^7} - \frac{8b^2 (a+bx^2)^{7/2}}{429a^3x^9} + \frac{6b (a+bx^2)^{7/2}}{143a^2x^{11}} - \frac{(a+bx^2)^{7/2}}{13ax^{13}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(5/2)/x^14, x]

[Out]  $-(a + b*x^2)^{(7/2)}/(13*a*x^{13}) + (6*b*(a + b*x^2)^{(7/2)})/(143*a^2*x^{11}) - (8*b^2*(a + b*x^2)^{(7/2)})/(429*a^3*x^9) + (16*b^3*(a + b*x^2)^{(7/2)})/(3003*a^4*x^7)$

**Rubi in Sympy [A]** time = 11.7493, size = 85, normalized size = 0.92

$$-\frac{(a+bx^2)^{\frac{7}{2}}}{13ax^{13}} + \frac{6b(a+bx^2)^{\frac{7}{2}}}{143a^2x^{11}} - \frac{8b^2(a+bx^2)^{\frac{7}{2}}}{429a^3x^9} + \frac{16b^3(a+bx^2)^{\frac{7}{2}}}{3003a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(5/2)/x\*\*14, x)

[Out]  $-(a + b*x**2)**(7/2)/(13*a*x**13) + 6*b*(a + b*x**2)**(7/2)/(143*a**2*x**11) - 8*b**2*(a + b*x**2)**(7/2)/(429*a**3*x**9) + 16*b**3*(a + b*x**2)**(7/2)/(3003*a**4*x**7)$

**Mathematica [A]** time = 0.0547021, size = 53, normalized size = 0.58

$$\frac{(a+bx^2)^{7/2} (-231a^3 + 126a^2bx^2 - 56ab^2x^4 + 16b^3x^6)}{3003a^4x^{13}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(5/2)/x^14, x]

[Out]  $((a + b*x^2)^{(7/2)}*(-231*a^3 + 126*a^2*b*x^2 - 56*a*b^2*x^4 + 16*b^3*x^6))/(3003*a^4*x^{13})$

**Maple [A]** time = 0.007, size = 50, normalized size = 0.5

$$-\frac{-16b^3x^6 + 56ab^2x^4 - 126a^2bx^2 + 231a^3}{3003x^{13}a^4} (bx^2 + a)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(5/2)/x^14,x)`

[Out]  $-1/3003*(b*x^2+a)^{(7/2)}*(-16*b^3*x^6+56*a*b^2*x^4-126*a^2*b*x^2+231*a^3)/x^{13}/a^4$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^14,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.387877, size = 111, normalized size = 1.21

$$\frac{(16b^6x^{12} - 8ab^5x^{10} + 6a^2b^4x^8 - 5a^3b^3x^6 - 371a^4b^2x^4 - 567a^5bx^2 - 231a^6)\sqrt{bx^2 + a}}{3003a^4x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^14,x, algorithm="fricas")`

[Out]  $1/3003*(16*b^6*x^{12} - 8*a*b^5*x^{10} + 6*a^2*b^4*x^8 - 5*a^3*b^3*x^6 - 371*a^4*b^2*x^4 - 567*a^5*b*x^2 - 231*a^6)*\text{sqrt}(b*x^2 + a)/(a^4*x^{13})$

**Sympy [A]** time = 22.6919, size = 721, normalized size = 7.84

$$\begin{aligned} & \frac{231a^9b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2} + 1}}{3003a^7b^9x^{12} + 9009a^6b^{10}x^{14} + 9009a^5b^{11}x^{16} + 3003a^4b^{12}x^{18}} \\ & - \frac{1260a^8b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2} + 1}}{3003a^7b^9x^{12} + 9009a^6b^{10}x^{14} + 9009a^5b^{11}x^{16} + 3003a^4b^{12}x^{18}} \\ & - \frac{2765a^7b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2} + 1}}{3003a^7b^9x^{12} + 9009a^6b^{10}x^{14} + 9009a^5b^{11}x^{16} + 3003a^4b^{12}x^{18}} \\ & - \frac{3050a^6b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2} + 1}}{3003a^7b^9x^{12} + 9009a^6b^{10}x^{14} + 9009a^5b^{11}x^{16} + 3003a^4b^{12}x^{18}} \\ & - \frac{1689a^5b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2} + 1}}{3003a^7b^9x^{12} + 9009a^6b^{10}x^{14} + 9009a^5b^{11}x^{16} + 3003a^4b^{12}x^{18}} \\ & - \frac{376a^4b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2} + 1}}{3003a^7b^9x^{12} + 9009a^6b^{10}x^{14} + 9009a^5b^{11}x^{16} + 3003a^4b^{12}x^{18}} \\ & + \frac{5a^3b^{\frac{31}{2}}x^{12}\sqrt{\frac{a}{bx^2} + 1}}{3003a^7b^9x^{12} + 9009a^6b^{10}x^{14} + 9009a^5b^{11}x^{16} + 3003a^4b^{12}x^{18}} \\ & + \frac{30a^2b^{\frac{33}{2}}x^{14}\sqrt{\frac{a}{bx^2} + 1}}{3003a^7b^9x^{12} + 9009a^6b^{10}x^{14} + 9009a^5b^{11}x^{16} + 3003a^4b^{12}x^{18}} \\ & + \frac{40ab^{\frac{35}{2}}x^{16}\sqrt{\frac{a}{bx^2} + 1}}{3003a^7b^9x^{12} + 9009a^6b^{10}x^{14} + 9009a^5b^{11}x^{16} + 3003a^4b^{12}x^{18}} \\ & + \frac{16b^{\frac{37}{2}}x^{18}\sqrt{\frac{a}{bx^2} + 1}}{3003a^7b^9x^{12} + 9009a^6b^{10}x^{14} + 9009a^5b^{11}x^{16} + 3003a^4b^{12}x^{18}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(5/2)/x\*\*14,x)

[Out] 
$$\begin{aligned} & -231*a**9*b**(19/2)*\sqrt{a/(b*x**2) + 1}/(3003*a**7*b**9*x**12 + \\ & 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 1260*a**8*b**(21/2)*x**2*\sqrt{a/(b*x**2) + 1}/(3003*a**7* \\ & b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003 \\ & *a**4*b**12*x**18) - 2765*a**7*b**(23/2)*x**4*\sqrt{a/(b*x**2) + 1} \\ & )/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11 \\ & *x**16 + 3003*a**4*b**12*x**18) - 3050*a**6*b**(25/2)*x**6*\sqrt{a \\ & /(b*x**2) + 1}/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 90 \\ & 09*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 1689*a**5*b**(27/2) \\ & )*x**8*\sqrt{a/(b*x**2) + 1}/(3003*a**7*b**9*x**12 + 9009*a**6*b** \\ & 10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 376*a \\ & **4*b**(29/2)*x**10*\sqrt{a/(b*x**2) + 1}/(3003*a**7*b**9*x**12 + \\ & 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x** \\ & **18) + 5*a**3*b**(31/2)*x**12*\sqrt{a/(b*x**2) + 1}/(3003*a**7*b** \\ & **9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a \\ & **4*b**12*x**18) + 30*a**2*b**(33/2)*x**14*\sqrt{a/(b*x**2) + 1}/( \\ & 3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x* \\ & **16 + 3003*a**4*b**12*x**18) + 40*a*b**(35/2)*x**16*\sqrt{a/(b*x** \\ & 2) + 1}/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5 \\ & *b**11*x**16 + 3003*a**4*b**12*x**18) + 16*b**(37/2)*x**18*\sqrt{a \\ & /(b*x**2) + 1}/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 90 \\ & 09*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) \end{aligned}$$

---

**GIAC/XCAS [A]** time = 0.215857, size = 370, normalized size = 4.02

$$32 \left( 3003 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{18} b^{\frac{13}{2}} + 9009 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{16} ab^{\frac{13}{2}} + 18018 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{14} a^2 b^{\frac{13}{2}} + 16302 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{12} a^3 b^{\frac{13}{2}} + 10296 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{10} a^4 b^{\frac{13}{2}} + 2288 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^8 a^5 b^{\frac{13}{2}} + 286 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^6 a^6 b^{\frac{13}{2}} - 78 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^4 a^7 b^{\frac{13}{2}} + 13 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 a^8 b^{\frac{13}{2}} - a^9 b^{\frac{13}{2}} \right) / \left( \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 - a \right)^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)/x^14,x, algorithm="giac")

[Out] 
$$\begin{aligned} & 32/3003*(3003*(\sqrt{b}*x - \sqrt{b*x^2 + a})^{18}*b^{(13/2)} + 9009*(\sqrt{b}*x - \sqrt{b*x^2 + a})^{16}*a*b^{(13/2)} + 18018*(\sqrt{b}*x - \sqrt{b*x^2 + a})^{14}*a^2*b^{(13/2)} + 16302*(\sqrt{b}*x - \sqrt{b*x^2 + a})^{12}*a^3*b^{(13/2)} + 10296*(\sqrt{b}*x - \sqrt{b*x^2 + a})^{10}*a^4*b^{(13/2)} + 2288*(\sqrt{b}*x - \sqrt{b*x^2 + a})^8*a^5*b^{(13/2)} + 286*(\sqrt{b}*x - \sqrt{b*x^2 + a})^6*a^6*b^{(13/2)} - 78*(\sqrt{b}*x - \sqrt{b*x^2 + a})^4*a^7*b^{(13/2)} + 13*(\sqrt{b}*x - \sqrt{b*x^2 + a})^2*a^8*b^{(13/2)} - a^9*b^{(13/2)})/((\sqrt{b}*x - \sqrt{b*x^2 + a})^2 - a)^{13} \end{aligned}$$

$$3.407 \quad \int \frac{(a+bx^2)^{5/2}}{x^{16}} dx$$

**Optimal.** Leaf size=116

$$-\frac{128b^4(a+bx^2)^{7/2}}{45045a^5x^7} + \frac{64b^3(a+bx^2)^{7/2}}{6435a^4x^9} - \frac{16b^2(a+bx^2)^{7/2}}{715a^3x^{11}} + \frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}} - \frac{(a+bx^2)^{7/2}}{15ax^{15}}$$

[Out]  $-(a + b*x^2)^{(7/2)}/(15*a*x^{15}) + (8*b*(a + b*x^2)^{(7/2)})/(195*a^2*x^{13}) - (16*b^2*(a + b*x^2)^{(7/2)})/(715*a^3*x^{11}) + (64*b^3*(a + b*x^2)^{(7/2)})/(6435*a^4*x^9) - (128*b^4*(a + b*x^2)^{(7/2)})/(45045*a^5*x^7)$

**Rubi [A]** time = 0.134523, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{128b^4(a+bx^2)^{7/2}}{45045a^5x^7} + \frac{64b^3(a+bx^2)^{7/2}}{6435a^4x^9} - \frac{16b^2(a+bx^2)^{7/2}}{715a^3x^{11}} + \frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}} - \frac{(a+bx^2)^{7/2}}{15ax^{15}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(5/2)/x^16, x]

[Out]  $-(a + b*x^2)^{(7/2)}/(15*a*x^{15}) + (8*b*(a + b*x^2)^{(7/2)})/(195*a^2*x^{13}) - (16*b^2*(a + b*x^2)^{(7/2)})/(715*a^3*x^{11}) + (64*b^3*(a + b*x^2)^{(7/2)})/(6435*a^4*x^9) - (128*b^4*(a + b*x^2)^{(7/2)})/(45045*a^5*x^7)$

**Rubi in Sympy [A]** time = 15.819, size = 109, normalized size = 0.94

$$-\frac{(a+bx^2)^{7/2}}{15ax^{15}} + \frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}} - \frac{16b^2(a+bx^2)^{7/2}}{715a^3x^{11}} + \frac{64b^3(a+bx^2)^{7/2}}{6435a^4x^9} - \frac{128b^4(a+bx^2)^{7/2}}{45045a^5x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(5/2)/x\*\*16, x)

[Out]  $-(a + b*x**2)**(7/2)/(15*a*x**15) + 8*b*(a + b*x**2)**(7/2)/(195*a**2*x**13) - 16*b**2*(a + b*x**2)**(7/2)/(715*a**3*x**11) + 64*b**3*(a + b*x**2)**(7/2)/(6435*a**4*x**9) - 128*b**4*(a + b*x**2)**(7/2)/(45045*a**5*x**7)$

**Mathematica [A]** time = 0.0635435, size = 64, normalized size = 0.55

$$\frac{(a+bx^2)^{7/2}(3003a^4 - 1848a^3bx^2 + 1008a^2b^2x^4 - 448ab^3x^6 + 128b^4x^8)}{45045a^5x^{15}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(5/2)/x^16, x]

[Out]  $-\frac{(a + b*x^2)^{(7/2)}*(3003*a^4 - 1848*a^3*b*x^2 + 1008*a^2*b^2*x^4 - 448*a*b^3*x^6 + 128*b^4*x^8)}{45045*a^5*x^{15}}$

**Maple [A]** time = 0.009, size = 61, normalized size = 0.5

$$\frac{128 b^4 x^8 - 448 b^3 x^6 a + 1008 b^2 x^4 a^2 - 1848 b x^2 a^3 + 3003 a^4}{45045 x^{15} a^5} (b x^2 + a)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(5/2)/x^16,x)

[Out] -1/45045\*(b\*x^2+a)^(7/2)\*(128\*b^4\*x^8-448\*a\*b^3\*x^6+1008\*a^2\*b^2\*x^4-1848\*a^3\*b\*x^2+3003\*a^4)/x^15/a^5

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)/x^16,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.54867, size = 126, normalized size = 1.09

$$\frac{(128 b^7 x^{14} - 64 a b^6 x^{12} + 48 a^2 b^5 x^{10} - 40 a^3 b^4 x^8 + 35 a^4 b^3 x^6 + 4473 a^5 b^2 x^4 + 7161 a^6 b x^2 + 3003 a^7) \sqrt{b x^2 + a}}{45045 a^5 x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)/x^16,x, algorithm="fricas")

[Out] -1/45045\*(128\*b^7\*x^14 - 64\*a\*b^6\*x^12 + 48\*a^2\*b^5\*x^10 - 40\*a^3\*b^4\*x^8 + 35\*a^4\*b^3\*x^6 + 4473\*a^5\*b^2\*x^4 + 7161\*a^6\*b\*x^2 + 3003\*a^7)\*sqrt(b\*x^2 + a)/(a^5\*x^15)

**Sympy [A]** time = 32.1489, size = 1012, normalized size = 8.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(5/2)/x\*\*16,x)

[Out] -3003\*a\*\*11\*b\*\*(33/2)\*sqrt(a/(b\*x\*\*2) + 1)/(45045\*a\*\*9\*b\*\*16\*x\*\*14 + 180180\*a\*\*8\*b\*\*17\*x\*\*16 + 270270\*a\*\*7\*b\*\*18\*x\*\*18 + 180180\*a\*\*6\*b\*\*19\*x\*\*20 + 45045\*a\*\*5\*b\*\*20\*x\*\*22) - 19173\*a\*\*10\*b\*\*(35/2)\*x\*\*2\*sqrt(a/(b\*x\*\*2) + 1)/(45045\*a\*\*9\*b\*\*16\*x\*\*14 + 180180\*a\*\*8\*b\*\*17\*x\*\*16 + 270270\*a\*\*7\*b\*\*18\*x\*\*18 + 180180\*a\*\*6\*b\*\*19\*x\*\*20 + 45045\*a\*\*5\*b\*\*20\*x\*\*22) - 51135\*a\*\*9\*b\*\*(37/2)\*x\*\*4\*sqrt(a/(b\*x\*\*2) + 1)/(45045\*a\*\*9\*b\*\*16\*x\*\*14 + 180180\*a\*\*8\*b\*\*17\*x\*\*16 + 270270\*a\*\*7\*b\*\*18\*x\*\*18 + 180180\*a\*\*6\*b\*\*19\*x\*\*20 + 45045\*a\*\*5\*b\*\*20\*x\*\*22) - 72905\*a\*\*8\*b\*\*(39/2)\*x\*\*6\*sqrt(a/(b\*x\*\*2) + 1)/(45045\*a\*\*9\*b\*\*16\*x\*\*14 + 180180\*a\*\*8\*b\*\*17\*x\*\*16 + 270270\*a\*\*7\*b\*\*18\*x\*\*18 + 180180\*a\*\*6\*b\*\*19\*x\*\*20 + 45045\*a\*\*5\*b\*\*20\*x\*\*22) - 58585\*a\*\*7\*b\*\*(41/2)\*x\*\*8\*sqrt(a/(b\*x\*\*2) + 1)/(45045\*a\*\*9\*b\*\*16\*x\*\*14 + 180180\*a\*\*8\*b\*\*17\*x\*\*16 + 270270\*a\*\*7\*b\*\*18\*x\*\*18 + 180180\*a\*\*6\*b\*\*19\*x\*\*20 + 45045\*a\*\*5\*b\*\*20\*x\*\*22) - 25151\*a\*\*6\*b\*\*(43/2)\*x\*\*10\*sqrt(a/(b\*x\*\*2) + 1)/(45045\*a\*\*9\*b\*\*16\*x\*\*14 + 180180\*a\*\*8\*b\*\*17\*x\*\*16 + 270270\*a\*\*7\*b\*\*18\*x\*\*18 + 180180\*a\*\*6\*b\*\*19\*x\*\*20 + 45045\*a\*\*5\*b\*\*20\*x\*\*22)

```

**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*
a**5*b**20*x**22) - 4501*a**5*b** (45/2)*x**12*sqrt(a/(b*x**2) + 1
)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7
*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22)
- 35*a**4*b** (47/2)*x**14*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*
x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 18018
0*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 280*a**3*b** (49/2)
*x**16*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8
*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20
+ 45045*a**5*b**20*x**22) - 560*a**2*b** (51/2)*x**18*sqrt(a/(b*x*
**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 2702
70*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*
x**22) - 448*a*b** (53/2)*x**20*sqrt(a/(b*x**2) + 1)/(45045*a**9*b
**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 +
180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 128*b** (55/2)
*x**22*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8
*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20
+ 45045*a**5*b**20*x**22)

```

---

**GIAC/XCAS [A]** time = 0.212954, size = 405, normalized size = 3.49

$$256 \left( 18018 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{20} b^{\frac{15}{2}} + 60060 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{18} ab^{\frac{15}{2}} + 115830 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{16} a^2 b^{\frac{15}{2}} + 109395 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{14} a^3 b^{\frac{15}{2}} + 65065 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{12} a^4 b^{\frac{15}{2}} + 15015 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{10} a^5 b^{\frac{15}{2}} + 1365 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^8 a^6 b^{\frac{15}{2}} - 455 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^6 a^7 b^{\frac{15}{2}} + 105 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^4 a^8 b^{\frac{15}{2}} - 15 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 a^9 b^{\frac{15}{2}} + a^{10} b^{\frac{15}{2}} \right) / \left( \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 - a \right)^{15}$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)/x^16,x, algorithm="giac")

[Out] 256/45045\*(18018\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^20\*b^(15/2) + 60060\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^18\*a\*b^(15/2) + 115830\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^16\*a^2\*b^(15/2) + 109395\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^14\*a^3\*b^(15/2) + 65065\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^12\*a^4\*b^(15/2) + 15015\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^10\*a^5\*b^(15/2) + 1365\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^8\*a^6\*b^(15/2) - 455\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^6\*a^7\*b^(15/2) + 105\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^4\*a^8\*b^(15/2) - 15\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^2\*a^9\*b^(15/2) + a^10\*b^(15/2))/((sqrt(b)\*x - sqrt(b\*x^2 + a))^2 - a)^15

$$3.408 \quad \int \frac{(a+bx^2)^{5/2}}{x^{18}} dx$$

**Optimal.** Leaf size=140

$$\frac{256b^5 (a+bx^2)^{7/2}}{153153a^6x^7} - \frac{128b^4 (a+bx^2)^{7/2}}{21879a^5x^9} + \frac{32b^3 (a+bx^2)^{7/2}}{2431a^4x^{11}} - \frac{16b^2 (a+bx^2)^{7/2}}{663a^3x^{13}} + \frac{2b (a+bx^2)^{7/2}}{51a^2x^{15}} - \frac{(a+bx^2)^{7/2}}{17ax^{17}}$$

[Out]  $-(a + b*x^2)^{(7/2)}/(17*a*x^{17}) + (2*b*(a + b*x^2)^{(7/2)})/(51*a^2*x^{15}) - (16*b^2*(a + b*x^2)^{(7/2)})/(663*a^3*x^{13}) + (32*b^3*(a + b*x^2)^{(7/2)})/(2431*a^4*x^{11}) - (128*b^4*(a + b*x^2)^{(7/2)})/(21879*a^5*x^9) + (256*b^5*(a + b*x^2)^{(7/2)})/(153153*a^6*x^7)$

**Rubi [A]** time = 0.171818, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{256b^5 (a+bx^2)^{7/2}}{153153a^6x^7} - \frac{128b^4 (a+bx^2)^{7/2}}{21879a^5x^9} + \frac{32b^3 (a+bx^2)^{7/2}}{2431a^4x^{11}} - \frac{16b^2 (a+bx^2)^{7/2}}{663a^3x^{13}} + \frac{2b (a+bx^2)^{7/2}}{51a^2x^{15}} - \frac{(a+bx^2)^{7/2}}{17ax^{17}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(5/2)/x^18, x]

[Out]  $-(a + b*x^2)^{(7/2)}/(17*a*x^{17}) + (2*b*(a + b*x^2)^{(7/2)})/(51*a^2*x^{15}) - (16*b^2*(a + b*x^2)^{(7/2)})/(663*a^3*x^{13}) + (32*b^3*(a + b*x^2)^{(7/2)})/(2431*a^4*x^{11}) - (128*b^4*(a + b*x^2)^{(7/2)})/(21879*a^5*x^9) + (256*b^5*(a + b*x^2)^{(7/2)})/(153153*a^6*x^7)$

**Rubi in Sympy [A]** time = 20.9123, size = 133, normalized size = 0.95

$$-\frac{(a+bx^2)^{\frac{7}{2}}}{17ax^{17}} + \frac{2b(a+bx^2)^{\frac{7}{2}}}{51a^2x^{15}} - \frac{16b^2(a+bx^2)^{\frac{7}{2}}}{663a^3x^{13}} + \frac{32b^3(a+bx^2)^{\frac{7}{2}}}{2431a^4x^{11}} - \frac{128b^4(a+bx^2)^{\frac{7}{2}}}{21879a^5x^9} + \frac{256b^5(a+bx^2)^{\frac{7}{2}}}{153153a^6x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(5/2)/x\*\*18, x)

[Out]  $-(a + b*x**2)**(7/2)/(17*a*x**17) + 2*b*(a + b*x**2)**(7/2)/(51*a**2*x**15) - 16*b**2*(a + b*x**2)**(7/2)/(663*a**3*x**13) + 32*b**3*(a + b*x**2)**(7/2)/(2431*a**4*x**11) - 128*b**4*(a + b*x**2)**(7/2)/(21879*a**5*x**9) + 256*b**5*(a + b*x**2)**(7/2)/(153153*a**6*x**7)$

**Mathematica [A]** time = 0.0690831, size = 75, normalized size = 0.54

$$\frac{(a+bx^2)^{7/2}(-9009a^5 + 6006a^4bx^2 - 3696a^3b^2x^4 + 2016a^2b^3x^6 - 896ab^4x^8 + 256b^5x^{10})}{153153a^6x^{17}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(5/2)/x^18, x]

[Out]  $((a + b^2x^2)^{7/2}(-9009a^5 + 6006a^4b^2x^2 - 3696a^3b^2x^4 + 2016a^2b^3x^6 - 896ab^4x^8 + 256b^5x^{10})) / (153153a^6x^{17})$

**Maple [A]** time = 0.009, size = 72, normalized size = 0.5

$$\frac{-256b^5x^{10} + 896ab^4x^8 - 2016a^2b^3x^6 + 3696a^3b^2x^4 - 6006a^4bx^2 + 9009a^5}{153153x^{17}a^6} (bx^2 + a)^{7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(5/2)/x^18,x)`

[Out]  $-1/153153(b^2x^2+a)^{7/2}(-256b^5x^{10}+896a^2b^4x^8-2016a^2b^3x^6+3696a^3b^2x^4-6006a^4bx^2+9009a^5)/x^{17}/a^6$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^18,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.767374, size = 140, normalized size = 1.

$$\frac{(256b^8x^{16} - 128ab^7x^{14} + 96a^2b^6x^{12} - 80a^3b^5x^{10} + 70a^4b^4x^8 - 63a^5b^3x^6 - 12705a^6b^2x^4 - 21021a^7bx^2 - 9009a^8)\sqrt{bx^2}}{153153a^6x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/2)/x^18,x, algorithm="fricas")`

[Out]  $1/153153(256b^8x^{16} - 128a^2b^7x^{14} + 96a^2b^6x^{12} - 80a^3b^5x^{10} + 70a^4b^4x^8 - 63a^5b^3x^6 - 12705a^6b^2x^4 - 21021a^7b^2x^2 - 9009a^8)\sqrt{bx^2 + a}/(a^6x^{17})$

**Sympy [A]** time = 39.243, size = 1346, normalized size = 9.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(5/2)/x**18,x)`

[Out]  $-9009a^{13}b^{51/2}\sqrt{a/(b^2x^2) + 1}/(153153a^{11}b^{25}x^{16} + 765765a^{10}b^{26}x^{18} + 1531530a^9b^{27}x^{20} + 1531530a^8b^{28}x^{22} + 765765a^7b^{29}x^{24} + 153153a^6b^{30}x^{26}) - 66066a^{12}b^{53/2}x^2\sqrt{a/(b^2x^2) + 1}/(153153a^{11}b^{25}x^{16} + 765765a^{10}b^{26}x^{18} + 1531530a^9b^{27}x^{20} + 1531530a^8b^{28}x^{22} + 765765a^7b^{29}x^{24} + 153153a^6b^{30}x^{26}) - 207900a^{11}b^{55/2}x^4\sqrt{a/(b^2x^2) + 1}/(153153a^{11}b^{25}x^{16} + 765765a^{10}b^{26}x^{18} + 1531530a^9b^{27}x^{20} + 1531530a^8b^{28}x^{22} + 765765a^7b^{29}x^{24} + 153153a^6b^{30}x^{26})$



```

531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*
b**29*x**24 + 153153*a**6*b**30*x**26) - 363888*a**10*b**(57/2)*x
**6*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10
*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**2
2 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) - 382550*a
**9*b**(59/2)*x**8*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16
+ 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*
a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x*
**26) - 241524*a**8*b**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(153153*a
**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*
x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 1531
53*a**6*b**30*x**26) - 84780*a**7*b**(63/2)*x**12*sqrt(a/(b*x**2)
+ 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531
530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**
29*x**24 + 153153*a**6*b**30*x**26) - 12768*a**6*b**(65/2)*x**14*
sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**
26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 +
765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) + 63*a**5*b**
(67/2)*x**16*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 7657
65*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b*
**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) +
630*a**4*b**(69/2)*x**18*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25
*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 15
31530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b*
**30*x**26) + 1680*a**3*b**(71/2)*x**20*sqrt(a/(b*x**2) + 1)/(1531
53*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b*
**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 +
153153*a**6*b**30*x**26) + 2016*a**2*b**(73/2)*x**22*sqrt(a/(b*x*
**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1
531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*
b**29*x**24 + 153153*a**6*b**30*x**26) + 1152*a*b**(75/2)*x**24*s
qrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**2
6*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 7
65765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) + 256*b**(77/2)
*x**26*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a*
**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x
**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26)

```

---

**GIAC/XCAS [A]** time = 0.214715, size = 443, normalized size = 3.16

$$512 \left( 102102 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{22} b^{\frac{17}{2}} + 364650 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{20} ab^{\frac{17}{2}} + 692835 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{18} a^2 b^{\frac{17}{2}} + 668525 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{16} a^3 b^{\frac{17}{2}} + 384098 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{14} a^4 b^{\frac{17}{2}} + 89726 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{12} a^5 b^{\frac{17}{2}} + 6188 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{10} a^6 b^{\frac{17}{2}} - 2380 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^8 a^7 b^{\frac{17}{2}} + 680 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^6 a^8 b^{\frac{17}{2}} - 136 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^4 a^9 b^{\frac{17}{2}} + 17 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 a^{10} b^{\frac{17}{2}} - a^{11} b^{\frac{17}{2}} \right) / \left( \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 - a \right)^{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(5/2)/x^18,x, algorithm="giac")

[Out] 512/153153\*(102102\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^22\*b^(17/2) + 364650\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^20\*a\*b^(17/2) + 692835\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^18\*a^2\*b^(17/2) + 668525\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^16\*a^3\*b^(17/2) + 384098\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^14\*a^4\*b^(17/2) + 89726\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^12\*a^5\*b^(17/2) + 6188\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^10\*a^6\*b^(17/2) - 2380\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^8\*a^7\*b^(17/2) + 680\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^6\*a^8\*b^(17/2) - 136\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^4\*a^9\*b^(17/2) + 17\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^2\*a^10\*b^(17/2) - a^11\*b^(17/2))/((sqrt(b)\*x - sqrt(b\*x^2 + a))^2 - a)^17

### 3.409 $\int x^{15} (a + bx^2)^{9/2} dx$

**Optimal.** Leaf size=161

$$\begin{aligned} & -\frac{a^7 (a + bx^2)^{11/2}}{11b^8} + \frac{7a^6 (a + bx^2)^{13/2}}{13b^8} - \frac{7a^5 (a + bx^2)^{15/2}}{5b^8} + \frac{35a^4 (a + bx^2)^{17/2}}{17b^8} \\ & - \frac{35a^3 (a + bx^2)^{19/2}}{19b^8} + \frac{a^2 (a + bx^2)^{21/2}}{b^8} + \frac{(a + bx^2)^{25/2}}{25b^8} - \frac{7a (a + bx^2)^{23/2}}{23b^8} \end{aligned}$$

[Out]  $-(a^7 (a + b^*x^2)^{(11/2)})/(11*b^8) + (7*a^6 (a + b^*x^2)^{(13/2)})/(13*b^8) - (7*a^5 (a + b^*x^2)^{(15/2)})/(5*b^8) + (35*a^4 (a + b^*x^2)^{(17/2)})/(17*b^8) - (35*a^3 (a + b^*x^2)^{(19/2)})/(19*b^8) + (a^2 (a + b^*x^2)^{(21/2)})/b^8 - (7*a (a + b^*x^2)^{(23/2)})/(23*b^8) + (a + b^*x^2)^{(25/2)}/(25*b^8)$

**Rubi [A]** time = 0.231865, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\begin{aligned} & -\frac{a^7 (a + bx^2)^{11/2}}{11b^8} + \frac{7a^6 (a + bx^2)^{13/2}}{13b^8} - \frac{7a^5 (a + bx^2)^{15/2}}{5b^8} + \frac{35a^4 (a + bx^2)^{17/2}}{17b^8} \\ & - \frac{35a^3 (a + bx^2)^{19/2}}{19b^8} + \frac{a^2 (a + bx^2)^{21/2}}{b^8} + \frac{(a + bx^2)^{25/2}}{25b^8} - \frac{7a (a + bx^2)^{23/2}}{23b^8} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^15\*(a + b\*x^2)^(9/2),x]

[Out]  $-(a^7 (a + b^*x^2)^{(11/2)})/(11*b^8) + (7*a^6 (a + b^*x^2)^{(13/2)})/(13*b^8) - (7*a^5 (a + b^*x^2)^{(15/2)})/(5*b^8) + (35*a^4 (a + b^*x^2)^{(17/2)})/(17*b^8) - (35*a^3 (a + b^*x^2)^{(19/2)})/(19*b^8) + (a^2 (a + b^*x^2)^{(21/2)})/b^8 - (7*a (a + b^*x^2)^{(23/2)})/(23*b^8) + (a + b^*x^2)^{(25/2)}/(25*b^8)$

**Rubi in Sympy [A]** time = 32.2356, size = 150, normalized size = 0.93

$$\begin{aligned} & -\frac{a^7 (a + bx^2)^{\frac{11}{2}}}{11b^8} + \frac{7a^6 (a + bx^2)^{\frac{13}{2}}}{13b^8} - \frac{7a^5 (a + bx^2)^{\frac{15}{2}}}{5b^8} + \frac{35a^4 (a + bx^2)^{\frac{17}{2}}}{17b^8} \\ & - \frac{35a^3 (a + bx^2)^{\frac{19}{2}}}{19b^8} + \frac{a^2 (a + bx^2)^{\frac{21}{2}}}{b^8} - \frac{7a (a + bx^2)^{\frac{23}{2}}}{23b^8} + \frac{(a + bx^2)^{\frac{25}{2}}}{25b^8} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*15\*(b\*x\*\*2+a)\*\*(9/2),x)

[Out]  $-a**7*(a + b*x**2)**(11/2)/(11*b**8) + 7*a**6*(a + b*x**2)**(13/2)/(13*b**8) - 7*a**5*(a + b*x**2)**(15/2)/(5*b**8) + 35*a**4*(a + b*x**2)**(17/2)/(17*b**8) - 35*a**3*(a + b*x**2)**(19/2)/(19*b**8) + a**2*(a + b*x**2)**(21/2)/b**8 - 7*a*(a + b*x**2)**(23/2)/(23*b**8) + (a + b*x**2)**(25/2)/(25*b**8)$

**Mathematica [A]** time = 0.089002, size = 94, normalized size = 0.58

$$\frac{(a + bx^2)^{11/2} (-2048a^7 + 11264a^6bx^2 - 36608a^5b^2x^4 + 91520a^4b^3x^6 - 194480a^3b^4x^8 + 369512a^2b^5x^{10} - 646646ab^6x^{12} + 26558675b^8)}{26558675b^8}$$

Antiderivative was successfully verified.

[In] Integrate[x^15\*(a + b\*x^2)^(9/2),x]

[Out]  $((a + b*x^2)^{(11/2)} * (-2048*a^7 + 11264*a^6*b*x^2 - 36608*a^5*b^2*x^4 + 91520*a^4*b^3*x^6 - 194480*a^3*b^4*x^8 + 369512*a^2*b^5*x^{10} - 646646*a*b^6*x^{12} + 1062347*b^7*x^{14})) / (26558675*b^8)$

**Maple [A]** time = 0.012, size = 91, normalized size = 0.6

$$\frac{-1062347 x^{14} b^7 + 646646 a x^{12} b^6 - 369512 a^2 x^{10} b^5 + 194480 a^3 x^8 b^4 - 91520 a^4 x^6 b^3 + 36608 a^5 x^4 b^2 - 11264 a^6 x^2 b + 2048 a^7}{26558675 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^15\*(b\*x^2+a)^(9/2),x)

[Out]  $-1/26558675 * (b*x^2+a)^{(11/2)} * (-1062347*b^7*x^{14}+646646*a*b^6*x^{12}-369512*a^2*b^5*x^{10}+194480*a^3*b^4*x^8-91520*a^4*b^3*x^6+36608*a^5*b^2*x^4-11264*a^6*b*x^2+2048*a^7)/b^8$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)\*x^15,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.259314, size = 196, normalized size = 1.22

$$\frac{(1062347 b^{12} x^{24} + 4665089 a b^{11} x^{22} + 7759752 a^2 b^{10} x^{20} + 5810090 a^3 b^9 x^{18} + 1659515 a^4 b^8 x^{16} + 429 a^5 b^7 x^{14} - 462 a^6 b^6 x^{12} + 2048 a^7 b^5 x^{10} - 11264 a^8 b^4 x^8 + 640 a^9 b^3 x^6 - 768 a^{10} b^2 x^4 + 1024 a^{11} b x^2 - 2048 a^{12}) \sqrt{b x^2 + a}}{26558675 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)\*x^15,x, algorithm="fricas")

[Out]  $1/26558675 * (1062347*b^{12}*x^{24} + 4665089*a*b^{11}*x^{22} + 7759752*a^2*b^{10}*x^{20} + 5810090*a^3*b^9*x^{18} + 1659515*a^4*b^8*x^{16} + 429*a^5*b^7*x^{14} - 462*a^6*b^6*x^{12} + 504*a^7*b^5*x^{10} - 560*a^8*b^4*x^8 + 640*a^9*b^3*x^6 - 768*a^{10}*b^2*x^4 + 1024*a^{11}*b*x^2 - 2048*a^{12}) * \sqrt{b*x^2 + a} / b^8$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*15\*(b\*x\*\*2+a)\*\*(9/2),x)

[Out] Timed out

---

**GIAC/XCAS [A]** time = 0.224003, size = 1, normalized size = 0.01

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)*x^15,x, algorithm="giac")`

[Out] Done

$$3.410 \quad \int x^{13} (a + bx^2)^{9/2} dx$$

**Optimal.** Leaf size=140

$$\frac{a^6 (a + bx^2)^{11/2}}{11b^7} - \frac{6a^5 (a + bx^2)^{13/2}}{13b^7} + \frac{a^4 (a + bx^2)^{15/2}}{b^7} - \frac{20a^3 (a + bx^2)^{17/2}}{17b^7} \\ + \frac{15a^2 (a + bx^2)^{19/2}}{19b^7} + \frac{(a + bx^2)^{23/2}}{23b^7} - \frac{2a (a + bx^2)^{21/2}}{7b^7}$$

[Out]  $(a^6 (a + b x^2)^{(11/2)}) / (11 b^7) - (6 a^5 (a + b x^2)^{(13/2)}) / (13 b^7) + (a^4 (a + b x^2)^{(15/2)}) / b^7 - (20 a^3 (a + b x^2)^{(17/2)}) / (17 b^7) + (15 a^2 (a + b x^2)^{(19/2)}) / (19 b^7) - (2 a (a + b x^2)^{(21/2)}) / (7 b^7) + (a + b x^2)^{(23/2)} / (23 b^7)$

**Rubi [A]** time = 0.202669, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{a^6 (a + bx^2)^{11/2}}{11b^7} - \frac{6a^5 (a + bx^2)^{13/2}}{13b^7} + \frac{a^4 (a + bx^2)^{15/2}}{b^7} - \frac{20a^3 (a + bx^2)^{17/2}}{17b^7} \\ + \frac{15a^2 (a + bx^2)^{19/2}}{19b^7} + \frac{(a + bx^2)^{23/2}}{23b^7} - \frac{2a (a + bx^2)^{21/2}}{7b^7}$$

Antiderivative was successfully verified.

[In] Int[x^13\*(a + b\*x^2)^(9/2), x]

[Out]  $(a^6 (a + b x^2)^{(11/2)}) / (11 b^7) - (6 a^5 (a + b x^2)^{(13/2)}) / (13 b^7) + (a^4 (a + b x^2)^{(15/2)}) / b^7 - (20 a^3 (a + b x^2)^{(17/2)}) / (17 b^7) + (15 a^2 (a + b x^2)^{(19/2)}) / (19 b^7) - (2 a (a + b x^2)^{(21/2)}) / (7 b^7) + (a + b x^2)^{(23/2)} / (23 b^7)$

**Rubi in Sympy [A]** time = 27.5785, size = 129, normalized size = 0.92

$$\frac{a^6 (a + bx^2)^{\frac{11}{2}}}{11b^7} - \frac{6a^5 (a + bx^2)^{\frac{13}{2}}}{13b^7} + \frac{a^4 (a + bx^2)^{\frac{15}{2}}}{b^7} - \frac{20a^3 (a + bx^2)^{\frac{17}{2}}}{17b^7} \\ + \frac{15a^2 (a + bx^2)^{\frac{19}{2}}}{19b^7} - \frac{2a (a + bx^2)^{\frac{21}{2}}}{7b^7} + \frac{(a + bx^2)^{\frac{23}{2}}}{23b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*13\*(b\*x\*\*2+a)\*\*(9/2), x)

[Out]  $a^{**6} (a + b x^{**2})^{** (11/2)} / (11 b^{**7}) - 6 a^{**5} (a + b x^{**2})^{** (13/2)} / (13 b^{**7}) + a^{**4} (a + b x^{**2})^{** (15/2)} / b^{**7} - 20 a^{**3} (a + b x^{**2})^{** (17/2)} / (17 b^{**7}) + 15 a^{**2} (a + b x^{**2})^{** (19/2)} / (19 b^{**7}) - 2 a^{**1} (a + b x^{**2})^{** (21/2)} / (7 b^{**7}) + (a + b x^{**2})^{** (23/2)} / (23 b^{**7})$

**Mathematica [A]** time = 0.0669657, size = 83, normalized size = 0.59

$$\frac{(a + bx^2)^{11/2} (1024a^6 - 5632a^5bx^2 + 18304a^4b^2x^4 - 45760a^3b^3x^6 + 97240a^2b^4x^8 - 184756ab^5x^{10} + 323323b^6x^{12})}{7436429b^7}$$

Antiderivative was successfully verified.

[In] Integrate[x^13\*(a + b\*x^2)^(9/2), x]

[Out]  $((a + b x^2)^{(11/2)} * (1024 a^6 - 5632 a^5 b x^2 + 18304 a^4 b^2 x^4 - 45760 a^3 b^3 x^6 + 97240 a^2 b^4 x^8 - 184756 a b^5 x^{10} + 323323 b^6 x^{12})) / (7436429 b^7)$

**Maple [A]** time = 0.01, size = 80, normalized size = 0.6

$$\frac{323323 x^{12} b^6 - 184756 a x^{10} b^5 + 97240 a^2 x^8 b^4 - 45760 a^3 x^6 b^3 + 18304 a^4 x^4 b^2 - 5632 a^5 x^2 b + 1024 a^6}{7436429 b^7} (b x^2 + a)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^13*(b*x^2+a)^(9/2),x)`

[Out]  $1/7436429 * (b x^2 + a)^{(11/2)} * (323323 b^6 x^{12} - 184756 a b^5 x^{10} + 97240 a^2 b^4 x^8 - 45760 a^3 b^3 x^6 + 18304 a^4 b^2 x^4 - 5632 a^5 b x^2 + 1024 a^6) / b^7$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)*x^13,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.2542, size = 181, normalized size = 1.29

$$\frac{(323323 b^{11} x^{22} + 1431859 a b^{10} x^{20} + 2406690 a^2 b^9 x^{18} + 1826110 a^3 b^8 x^{16} + 530959 a^4 b^7 x^{14} + 231 a^5 b^6 x^{12} - 252 a^6 b^5 x^{10} + 280 a^7 b^4 x^8 - 320 a^8 b^3 x^6 + 384 a^9 b^2 x^4 - 512 a^{10} b x^2 + 1024 a^{11}) * \sqrt{b x^2 + a}}{7436429 b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)*x^13,x, algorithm="fricas")`

[Out]  $1/7436429 * (323323 b^{11} x^{22} + 1431859 a b^{10} x^{20} + 2406690 a^2 b^9 x^{18} + 1826110 a^3 b^8 x^{16} + 530959 a^4 b^7 x^{14} + 231 a^5 b^6 x^{12} - 252 a^6 b^5 x^{10} + 280 a^7 b^4 x^8 - 320 a^8 b^3 x^6 + 384 a^9 b^2 x^4 - 512 a^{10} b x^2 + 1024 a^{11}) * \sqrt{b x^2 + a} / b^7$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**13*(b*x**2+a)**(9/2),x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.224934, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(9/2)*x^13,x, algorithm="giac")
```

```
[Out] Done
```

$$3.411 \quad \int x^{11} (a + bx^2)^{9/2} dx$$

**Optimal.** Leaf size=122

$$\begin{aligned} & -\frac{a^5 (a + bx^2)^{11/2}}{11b^6} + \frac{5a^4 (a + bx^2)^{13/2}}{13b^6} - \frac{2a^3 (a + bx^2)^{15/2}}{3b^6} \\ & + \frac{10a^2 (a + bx^2)^{17/2}}{17b^6} + \frac{(a + bx^2)^{21/2}}{21b^6} - \frac{5a (a + bx^2)^{19/2}}{19b^6} \end{aligned}$$

[Out]  $-(a^5 (a + b*x^2)^{(11/2)})/(11*b^6) + (5*a^4*(a + b*x^2)^{(13/2)})/(13*b^6) - (2*a^3*(a + b*x^2)^{(15/2)})/(3*b^6) + (10*a^2*(a + b*x^2)^{(17/2)})/(17*b^6) - (5*a*(a + b*x^2)^{(19/2)})/(19*b^6) + (a + b*x^2)^{(21/2)}/(21*b^6)$

**Rubi [A]** time = 0.174919, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\begin{aligned} & -\frac{a^5 (a + bx^2)^{11/2}}{11b^6} + \frac{5a^4 (a + bx^2)^{13/2}}{13b^6} - \frac{2a^3 (a + bx^2)^{15/2}}{3b^6} \\ & + \frac{10a^2 (a + bx^2)^{17/2}}{17b^6} + \frac{(a + bx^2)^{21/2}}{21b^6} - \frac{5a (a + bx^2)^{19/2}}{19b^6} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^11\*(a + b\*x^2)^(9/2), x]

[Out]  $-(a^5 (a + b*x^2)^{(11/2)})/(11*b^6) + (5*a^4*(a + b*x^2)^{(13/2)})/(13*b^6) - (2*a^3*(a + b*x^2)^{(15/2)})/(3*b^6) + (10*a^2*(a + b*x^2)^{(17/2)})/(17*b^6) - (5*a*(a + b*x^2)^{(19/2)})/(19*b^6) + (a + b*x^2)^{(21/2)}/(21*b^6)$

**Rubi in Sympy [A]** time = 24.0092, size = 112, normalized size = 0.92

$$-\frac{a^5 (a + bx^2)^{\frac{11}{2}}}{11b^6} + \frac{5a^4 (a + bx^2)^{\frac{13}{2}}}{13b^6} - \frac{2a^3 (a + bx^2)^{\frac{15}{2}}}{3b^6} + \frac{10a^2 (a + bx^2)^{\frac{17}{2}}}{17b^6} - \frac{5a (a + bx^2)^{\frac{19}{2}}}{19b^6} + \frac{(a + bx^2)^{\frac{21}{2}}}{21b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*11\*(b\*x\*\*2+a)\*\*(9/2), x)

[Out]  $-a**5*(a + b*x**2)**(11/2)/(11*b**6) + 5*a**4*(a + b*x**2)**(13/2)/(13*b**6) - 2*a**3*(a + b*x**2)**(15/2)/(3*b**6) + 10*a**2*(a + b*x**2)**(17/2)/(17*b**6) - 5*a*(a + b*x**2)**(19/2)/(19*b**6) + (a + b*x**2)**(21/2)/(21*b**6)$

**Mathematica [A]** time = 0.0746111, size = 72, normalized size = 0.59

$$\frac{(a + bx^2)^{11/2} (-256a^5 + 1408a^4bx^2 - 4576a^3b^2x^4 + 11440a^2b^3x^6 - 24310ab^4x^8 + 46189b^5x^{10})}{969969b^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^11\*(a + b\*x^2)^(9/2), x]

[Out]  $((a + b*x^2)^{(11/2)}*(-256*a^5 + 1408*a^4*b*x^2 - 4576*a^3*b^2*x^4 + 11440*a^2*b^3*x^6 - 24310*a*b^4*x^8 + 46189*b^5*x^{10}))/969969$



\*b^6)

**Maple [A]** time = 0.009, size = 69, normalized size = 0.6

$$\frac{-46189 b^5 x^{10} + 24310 a b^4 x^8 - 11440 a^2 b^3 x^6 + 4576 a^3 b^2 x^4 - 1408 a^4 b x^2 + 256 a^5}{969969 b^6} (bx^2 + a)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11\*(b\*x^2+a)^(9/2),x)

[Out] -1/969969\*(b\*x^2+a)^(11/2)\*(-46189\*b^5\*x^10+24310\*a\*b^4\*x^8-11440\*a^2\*b^3\*x^6+4576\*a^3\*b^2\*x^4-1408\*a^4\*b\*x^2+256\*a^5)/b^6

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)\*x^11,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.252354, size = 166, normalized size = 1.36

$$\frac{(46189 b^{10} x^{20} + 206635 a b^9 x^{18} + 351780 a^2 b^8 x^{16} + 271414 a^3 b^7 x^{14} + 80773 a^4 b^6 x^{12} + 63 a^5 b^5 x^{10} - 70 a^6 b^4 x^8 + 80 a^7 b^3 x^6 - 256 a^8 b^2 x^4 + 128 a^9 b x^2 - 256 a^{10}) \sqrt{b x^2 + a}}{969969 b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)\*x^11,x, algorithm="fricas")

[Out] 1/969969\*(46189\*b^10\*x^20 + 206635\*a\*b^9\*x^18 + 351780\*a^2\*b^8\*x^16 + 271414\*a^3\*b^7\*x^14 + 80773\*a^4\*b^6\*x^12 + 63\*a^5\*b^5\*x^10 - 70\*a^6\*b^4\*x^8 + 80\*a^7\*b^3\*x^6 - 256\*a^8\*b^2\*x^4 + 128\*a^9\*b\*x^2 - 256\*a^10)\*sqrt(b\*x^2 + a)/b^6

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*11\*(b\*x\*\*2+a)\*\*(9/2),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.218227, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(9/2)*x^11,x, algorithm="giac")
```

```
[Out] Done
```

### 3.412 $\int x^9 (a + bx^2)^{9/2} dx$

**Optimal.** Leaf size=101

$$\frac{a^4 (a + bx^2)^{11/2}}{11b^5} - \frac{4a^3 (a + bx^2)^{13/2}}{13b^5} + \frac{2a^2 (a + bx^2)^{15/2}}{5b^5} + \frac{(a + bx^2)^{19/2}}{19b^5} - \frac{4a (a + bx^2)^{17/2}}{17b^5}$$

[Out]  $(a^4 (a + b x^2)^{(11/2)}) / (11 b^5) - (4 a^3 (a + b x^2)^{(13/2)}) / (13 b^5) + (2 a^2 (a + b x^2)^{(15/2)}) / (5 b^5) - (4 a (a + b x^2)^{(17/2)}) / (17 b^5) + (a + b x^2)^{(19/2)} / (19 b^5)$

**Rubi [A]** time = 0.147018, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{a^4 (a + bx^2)^{11/2}}{11b^5} - \frac{4a^3 (a + bx^2)^{13/2}}{13b^5} + \frac{2a^2 (a + bx^2)^{15/2}}{5b^5} + \frac{(a + bx^2)^{19/2}}{19b^5} - \frac{4a (a + bx^2)^{17/2}}{17b^5}$$

Antiderivative was successfully verified.

[In] Int[x^9\*(a + b\*x^2)^(9/2), x]

[Out]  $(a^4 (a + b x^2)^{(11/2)}) / (11 b^5) - (4 a^3 (a + b x^2)^{(13/2)}) / (13 b^5) + (2 a^2 (a + b x^2)^{(15/2)}) / (5 b^5) - (4 a (a + b x^2)^{(17/2)}) / (17 b^5) + (a + b x^2)^{(19/2)} / (19 b^5)$

**Rubi in Sympy [A]** time = 19.5078, size = 92, normalized size = 0.91

$$\frac{a^4 (a + bx^2)^{\frac{11}{2}}}{11b^5} - \frac{4a^3 (a + bx^2)^{\frac{13}{2}}}{13b^5} + \frac{2a^2 (a + bx^2)^{\frac{15}{2}}}{5b^5} - \frac{4a (a + bx^2)^{\frac{17}{2}}}{17b^5} + \frac{(a + bx^2)^{\frac{19}{2}}}{19b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*9\*(b\*x\*\*2+a)\*\*(9/2), x)

[Out]  $a^4 (a + b x^2)^{(11/2)} / (11 b^5) - 4 a^3 (a + b x^2)^{(13/2)} / (13 b^5) + 2 a^2 (a + b x^2)^{(15/2)} / (5 b^5) - 4 a (a + b x^2)^{(17/2)} / (17 b^5) + (a + b x^2)^{(19/2)} / (19 b^5)$

**Mathematica [A]** time = 0.0593421, size = 61, normalized size = 0.6

$$\frac{(a + bx^2)^{11/2} (128a^4 - 704a^3bx^2 + 2288a^2b^2x^4 - 5720ab^3x^6 + 12155b^4x^8)}{230945b^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^9\*(a + b\*x^2)^(9/2), x]

[Out]  $((a + b x^2)^{(11/2)} (128 a^4 - 704 a^3 b x^2 + 2288 a^2 b^2 x^4 - 5720 a b^3 x^6 + 12155 b^4 x^8)) / (230945 b^5)$

**Maple [A]** time = 0.007, size = 58, normalized size = 0.6

$$\frac{12155 x^8 b^4 - 5720 a x^6 b^3 + 2288 a^2 x^4 b^2 - 704 a^3 x^2 b + 128 a^4}{230945 b^5} (bx^2 + a)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9*(b*x^2+a)^(9/2),x)`

[Out]  $\frac{1}{230945} (b^2 x^2 + a)^{11/2} (12155 b^4 x^8 - 5720 a b^3 x^6 + 2288 a^2 b^2 x^4 - 704 a^3 b x^2 + 128 a^4) / b^5$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)*x^9,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.244616, size = 151, normalized size = 1.5

$$\frac{(12155 b^9 x^{18} + 55055 a b^8 x^{16} + 95238 a^2 b^7 x^{14} + 75086 a^3 b^6 x^{12} + 23063 a^4 b^5 x^{10} + 35 a^5 b^4 x^8 - 40 a^6 b^3 x^6 + 48 a^7 b^2 x^4 - 64 a^8 b x^2 + 128 a^9) \sqrt{b x^2 + a}}{230945 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)*x^9,x, algorithm="fricas")`

[Out]  $\frac{1}{230945} (12155 b^9 x^{18} + 55055 a b^8 x^{16} + 95238 a^2 b^7 x^{14} + 75086 a^3 b^6 x^{12} + 23063 a^4 b^5 x^{10} + 35 a^5 b^4 x^8 - 40 a^6 b^3 x^6 + 48 a^7 b^2 x^4 - 64 a^8 b x^2 + 128 a^9) \sqrt{b x^2 + a} / b^5$

**Sympy** [A] time = 155.645, size = 230, normalized size = 2.28

$$\left\{ \frac{128 a^9 \sqrt{a + b x^2}}{230945 b^5} - \frac{64 a^8 x^2 \sqrt{a + b x^2}}{230945 b^4} + \frac{48 a^7 x^4 \sqrt{a + b x^2}}{230945 b^3} - \frac{8 a^6 x^6 \sqrt{a + b x^2}}{46189 b^2} + \frac{7 a^5 x^8 \sqrt{a + b x^2}}{46189 b} + \frac{23063 a^4 x^{10} \sqrt{a + b x^2}}{230945} + \frac{6826 a^3 b x^{12} \sqrt{a + b x^2}}{20995} + \frac{666 a^2 b^2 x^{14} \sqrt{a + b x^2}}{1615} + \frac{a^2 x^{10}}{10} \right\}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9*(b*x**2+a)**(9/2),x)`

[Out] `Piecewise(((128*a**9*sqrt(a + b*x**2)/(230945*b**5) - 64*a**8*x**2*sqrt(a + b*x**2)/(230945*b**4) + 48*a**7*x**4*sqrt(a + b*x**2)/(230945*b**3) - 8*a**6*x**6*sqrt(a + b*x**2)/(46189*b**2) + 7*a**5*x**8*sqrt(a + b*x**2)/(46189*b) + 23063*a**4*x**10*sqrt(a + b*x**2)/230945 + 6826*a**3*b*x**12*sqrt(a + b*x**2)/20995 + 666*a**2*b**2*x**14*sqrt(a + b*x**2)/1615 + 77*a*b**3*x**16*sqrt(a + b*x**2)/323 + b**4*x**18*sqrt(a + b*x**2)/19, Ne(b, 0)), (a**(9/2)*x**10/10, True))`

**GIAC/XCAS** [A] time = 0.212408, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(9/2)*x^9,x, algorithm="giac")
```

```
[Out] Done
```

$$3.413 \quad \int x^7 (a + bx^2)^{9/2} dx$$

**Optimal.** Leaf size=80

$$-\frac{a^3 (a + bx^2)^{11/2}}{11b^4} + \frac{3a^2 (a + bx^2)^{13/2}}{13b^4} + \frac{(a + bx^2)^{17/2}}{17b^4} - \frac{a (a + bx^2)^{15/2}}{5b^4}$$

[Out]  $-(a^3*(a + b*x^2)^(11/2))/(11*b^4) + (3*a^2*(a + b*x^2)^(13/2))/(13*b^4) - (a*(a + b*x^2)^(15/2))/(5*b^4) + (a + b*x^2)^(17/2)/(17*b^4)$

**Rubi [A]** time = 0.122435, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{a^3 (a + bx^2)^{11/2}}{11b^4} + \frac{3a^2 (a + bx^2)^{13/2}}{13b^4} + \frac{(a + bx^2)^{17/2}}{17b^4} - \frac{a (a + bx^2)^{15/2}}{5b^4}$$

Antiderivative was successfully verified.

[In] Int[x^7\*(a + b\*x^2)^(9/2), x]

[Out]  $-(a^3*(a + b*x^2)^(11/2))/(11*b^4) + (3*a^2*(a + b*x^2)^(13/2))/(13*b^4) - (a*(a + b*x^2)^(15/2))/(5*b^4) + (a + b*x^2)^(17/2)/(17*b^4)$

**Rubi in Sympy [A]** time = 15.9908, size = 70, normalized size = 0.88

$$-\frac{a^3 (a + bx^2)^{\frac{11}{2}}}{11b^4} + \frac{3a^2 (a + bx^2)^{\frac{13}{2}}}{13b^4} - \frac{a (a + bx^2)^{\frac{15}{2}}}{5b^4} + \frac{(a + bx^2)^{\frac{17}{2}}}{17b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*7\*(b\*x\*\*2+a)\*\*(9/2), x)

[Out]  $-a**3*(a + b*x**2)**(11/2)/(11*b**4) + 3*a**2*(a + b*x**2)**(13/2)/(13*b**4) - a*(a + b*x**2)**(15/2)/(5*b**4) + (a + b*x**2)**(17/2)/(17*b**4)$

**Mathematica [A]** time = 0.0621042, size = 50, normalized size = 0.62

$$\frac{(a + bx^2)^{11/2} (-16a^3 + 88a^2bx^2 - 286ab^2x^4 + 715b^3x^6)}{12155b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^7\*(a + b\*x^2)^(9/2), x]

[Out]  $((a + b*x^2)^(11/2)*(-16*a^3 + 88*a^2*b*x^2 - 286*a*b^2*x^4 + 715*b^3*x^6))/(12155*b^4)$

**Maple [A]** time = 0.009, size = 47, normalized size = 0.6

$$-\frac{-715b^3x^6 + 286ab^2x^4 - 88a^2bx^2 + 16a^3}{12155b^4} (bx^2 + a)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(b*x^2+a)^(9/2),x)`

[Out]  $-1/12155*(b*x^2+a)^{(11/2)}*(-715*b^3*x^6+286*a*b^2*x^4-88*a^2*b*x^2+16*a^3)/b^4$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)*x^7,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.240204, size = 136, normalized size = 1.7

$$\frac{(715 b^8 x^{16} + 3289 a b^7 x^{14} + 5808 a^2 b^6 x^{12} + 4714 a^3 b^5 x^{10} + 1515 a^4 b^4 x^8 + 5 a^5 b^3 x^6 - 6 a^6 b^2 x^4 + 8 a^7 b x^2 - 16 a^8) \sqrt{b x^2 + a}}{12155 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)*x^7,x, algorithm="fricas")`

[Out]  $1/12155*(715*b^8*x^{16} + 3289*a*b^7*x^{14} + 5808*a^2*b^6*x^{12} + 4714*a^3*b^5*x^{10} + 1515*a^4*b^4*x^8 + 5*a^5*b^3*x^6 - 6*a^6*b^2*x^4 + 8*a^7*b*x^2 - 16*a^8)*\text{sqrt}(b*x^2 + a)/b^4$

**Sympy [A]** time = 116.136, size = 204, normalized size = 2.55

$$\left\{ \begin{array}{l} -\frac{16a^8\sqrt{a+bx^2}}{12155b^4} + \frac{8a^7x^2\sqrt{a+bx^2}}{12155b^3} - \frac{6a^6x^4\sqrt{a+bx^2}}{12155b^2} + \frac{a^5x^6\sqrt{a+bx^2}}{2431b} + \frac{303a^4x^8\sqrt{a+bx^2}}{2431} + \frac{4714a^3bx^{10}\sqrt{a+bx^2}}{12155} + \frac{528a^2b^2x^{12}\sqrt{a+bx^2}}{1105} + \frac{23ab^3x^{14}\sqrt{a+bx^2}}{85} \\ \frac{a^9x^8}{8} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(b*x**2+a)**(9/2),x)`

[Out] `Piecewise((-16*a**8*sqrt(a + b*x**2)/(12155*b**4) + 8*a**7*x**2*sqrt(a + b*x**2)/(12155*b**3) - 6*a**6*x**4*sqrt(a + b*x**2)/(12155*b**2) + a**5*x**6*sqrt(a + b*x**2)/(2431*b) + 303*a**4*x**8*sqrt(a + b*x**2)/2431 + 4714*a**3*b*x**10*sqrt(a + b*x**2)/12155 + 528*a**2*b**2*x**12*sqrt(a + b*x**2)/1105 + 23*a*b**3*x**14*sqrt(a + b*x**2)/85 + b**4*x**16*sqrt(a + b*x**2)/17, Ne(b, 0)), (a**(9/2)*x**8/8, True))`

**GIAC/XCAS [A]** time = 0.224453, size = 595, normalized size = 7.44

$$\frac{2431 \left( 35 (bx^2+a)^{\frac{9}{2}} - 135 (bx^2+a)^{\frac{7}{2}} a + 189 (bx^2+a)^{\frac{5}{2}} a^2 - 105 (bx^2+a)^{\frac{3}{2}} a^3 \right) a^4}{b^3} + \frac{884 \left( 315 (bx^2+a)^{\frac{11}{2}} - 1540 (bx^2+a)^{\frac{9}{2}} a + 2970 (bx^2+a)^{\frac{7}{2}} a^2 - 2772 (bx^2+a)^{\frac{5}{2}} a^3 \right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)\*x^7,x, algorithm="giac")

[Out]  $\frac{1}{765765} \cdot (2431 \cdot (35 \cdot (b \cdot x^2 + a)^{9/2}) - 135 \cdot (b \cdot x^2 + a)^{7/2} \cdot a + 189 \cdot (b \cdot x^2 + a)^{5/2} \cdot a^2 - 105 \cdot (b \cdot x^2 + a)^{3/2} \cdot a^3) \cdot a^4 / b^3 + 884 \cdot (315 \cdot (b \cdot x^2 + a)^{11/2}) - 1540 \cdot (b \cdot x^2 + a)^{9/2} \cdot a + 2970 \cdot (b \cdot x^2 + a)^{7/2} \cdot a^2 - 2772 \cdot (b \cdot x^2 + a)^{5/2} \cdot a^3 + 1155 \cdot (b \cdot x^2 + a)^{3/2} \cdot a^4) \cdot a^3 / b^3 + 510 \cdot (693 \cdot (b \cdot x^2 + a)^{13/2}) - 4095 \cdot (b \cdot x^2 + a)^{11/2} \cdot a + 10010 \cdot (b \cdot x^2 + a)^{9/2} \cdot a^2 - 12870 \cdot (b \cdot x^2 + a)^{7/2} \cdot a^3 + 9009 \cdot (b \cdot x^2 + a)^{5/2} \cdot a^4 - 3003 \cdot (b \cdot x^2 + a)^{3/2} \cdot a^5) \cdot a^2 / b^3 + 68 \cdot (3003 \cdot (b \cdot x^2 + a)^{15/2}) - 20790 \cdot (b \cdot x^2 + a)^{13/2} \cdot a + 61425 \cdot (b \cdot x^2 + a)^{11/2} \cdot a^2 - 100100 \cdot (b \cdot x^2 + a)^{9/2} \cdot a^3 + 96525 \cdot (b \cdot x^2 + a)^{7/2} \cdot a^4 - 54054 \cdot (b \cdot x^2 + a)^{5/2} \cdot a^5 + 15015 \cdot (b \cdot x^2 + a)^{3/2} \cdot a^6) \cdot a / b^3 + 7 \cdot (6435 \cdot (b \cdot x^2 + a)^{17/2}) - 51051 \cdot (b \cdot x^2 + a)^{15/2} \cdot a + 176715 \cdot (b \cdot x^2 + a)^{13/2} \cdot a^2 - 348075 \cdot (b \cdot x^2 + a)^{11/2} \cdot a^3 + 425425 \cdot (b \cdot x^2 + a)^{9/2} \cdot a^4 - 328185 \cdot (b \cdot x^2 + a)^{7/2} \cdot a^5 + 153153 \cdot (b \cdot x^2 + a)^{5/2} \cdot a^6 - 36465 \cdot (b \cdot x^2 + a)^{3/2} \cdot a^7) / b^3 / b$



$$3.414 \quad \int x^5 (a + bx^2)^{9/2} dx$$

**Optimal.** Leaf size=59

$$\frac{a^2 (a + bx^2)^{11/2}}{11b^3} + \frac{(a + bx^2)^{15/2}}{15b^3} - \frac{2a (a + bx^2)^{13/2}}{13b^3}$$

[Out]  $(a^2 * (a + b * x^2)^{(11/2)}) / (11 * b^3) - (2 * a * (a + b * x^2)^{(13/2)}) / (13 * b^3) + (a + b * x^2)^{(15/2)} / (15 * b^3)$

**Rubi [A]** time = 0.0953802, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{a^2 (a + bx^2)^{11/2}}{11b^3} + \frac{(a + bx^2)^{15/2}}{15b^3} - \frac{2a (a + bx^2)^{13/2}}{13b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(a + b\*x^2)^(9/2), x]

[Out]  $(a^2 * (a + b * x^2)^{(11/2)}) / (11 * b^3) - (2 * a * (a + b * x^2)^{(13/2)}) / (13 * b^3) + (a + b * x^2)^{(15/2)} / (15 * b^3)$

**Rubi in Sympy [A]** time = 11.8901, size = 51, normalized size = 0.86

$$\frac{a^2 (a + bx^2)^{\frac{11}{2}}}{11b^3} - \frac{2a (a + bx^2)^{\frac{13}{2}}}{13b^3} + \frac{(a + bx^2)^{\frac{15}{2}}}{15b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(b\*x\*\*2+a)\*\*(9/2), x)

[Out]  $a**2*(a + b*x**2)**(11/2)/(11*b**3) - 2*a*(a + b*x**2)**(13/2)/(13*b**3) + (a + b*x**2)**(15/2)/(15*b**3)$

**Mathematica [A]** time = 0.0510744, size = 39, normalized size = 0.66

$$\frac{(a + bx^2)^{11/2} (8a^2 - 44abx^2 + 143b^2x^4)}{2145b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*x^2)^(9/2), x]

[Out]  $((a + b * x^2)^{(11/2)} * (8 * a^2 - 44 * a * b * x^2 + 143 * b^2 * x^4)) / (2145 * b^3)$

**Maple [A]** time = 0.007, size = 36, normalized size = 0.6

$$\frac{143 b^2 x^4 - 44 a b x^2 + 8 a^2}{2145 b^3} (bx^2 + a)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(b*x^2+a)^(9/2),x)`

[Out]  $1/2145*(b*x^2+a)^{(11/2)}*(143*b^2*x^4-44*a*b*x^2+8*a^2)/b^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)*x^5,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.237039, size = 122, normalized size = 2.07

$$\frac{(143 b^7 x^{14} + 671 a b^6 x^{12} + 1218 a^2 b^5 x^{10} + 1030 a^3 b^4 x^8 + 355 a^4 b^3 x^6 + 3 a^5 b^2 x^4 - 4 a^6 b x^2 + 8 a^7) \sqrt{b x^2 + a}}{2145 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)*x^5,x, algorithm="fricas")`

[Out]  $1/2145*(143*b^7*x^{14} + 671*a*b^6*x^{12} + 1218*a^2*b^5*x^{10} + 1030*a^3*b^4*x^8 + 355*a^4*b^3*x^6 + 3*a^5*b^2*x^4 - 4*a^6*b*x^2 + 8*a^7)*\text{sqrt}(b*x^2 + a)/b^3$

**Sympy [A]** time = 85.2284, size = 180, normalized size = 3.05

$$\left\{ \begin{array}{l} \frac{8a^7\sqrt{a+bx^2}}{2145b^3} - \frac{4a^6x^2\sqrt{a+bx^2}}{2145b^2} + \frac{a^5x^4\sqrt{a+bx^2}}{715b} + \frac{71a^4x^6\sqrt{a+bx^2}}{429} + \frac{206a^3bx^8\sqrt{a+bx^2}}{429} + \frac{406a^2b^2x^{10}\sqrt{a+bx^2}}{715} + \frac{61ab^3x^{12}\sqrt{a+bx^2}}{195} + \frac{b^4x^{14}\sqrt{a+bx^2}}{15} \\ \frac{a^2x^6}{6} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(b*x**2+a)**(9/2),x)`

[Out] `Piecewise((8*a**7*sqrt(a + b*x**2)/(2145*b**3) - 4*a**6*x**2*sqrt(a + b*x**2)/(2145*b**2) + a**5*x**4*sqrt(a + b*x**2)/(715*b) + 71*a**4*x**6*sqrt(a + b*x**2)/429 + 206*a**3*b*x**8*sqrt(a + b*x**2)/429 + 406*a**2*b**2*x**10*sqrt(a + b*x**2)/715 + 61*a*b**3*x**12*sqrt(a + b*x**2)/195 + b**4*x**14*sqrt(a + b*x**2)/15, Ne(b, 0)), (a**(9/2)*x**6/6, True))`

**GIAC/XCAS [A]** time = 0.217601, size = 500, normalized size = 8.47

$$\frac{429 \left( 15 (bx^2+a)^{\frac{7}{2}} - 42 (bx^2+a)^{\frac{5}{2}} a + 35 (bx^2+a)^{\frac{3}{2}} a^2 \right) a^4}{b^2} + \frac{572 \left( 35 (bx^2+a)^{\frac{9}{2}} - 135 (bx^2+a)^{\frac{7}{2}} a + 189 (bx^2+a)^{\frac{5}{2}} a^2 - 105 (bx^2+a)^{\frac{3}{2}} a^3 \right) a^3}{b^2} + \frac{78 \left( 315 (bx^2+a)^{\frac{11}{2}} - 105 (bx^2+a)^{\frac{9}{2}} a + 105 (bx^2+a)^{\frac{7}{2}} a^2 - 35 (bx^2+a)^{\frac{5}{2}} a^3 \right) a^2}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)*x^5,x, algorithm="giac")`

[Out]  $\frac{1}{45045} \cdot (429 \cdot (15 \cdot (b^2 x^2 + a)^{7/2} - 42 \cdot (b^2 x^2 + a)^{5/2} \cdot a + 35 \cdot (b^2 x^2 + a)^{3/2} \cdot a^2) \cdot a^4 / b^2 + 572 \cdot (35 \cdot (b^2 x^2 + a)^{9/2} - 135 \cdot (b^2 x^2 + a)^{7/2} \cdot a + 189 \cdot (b^2 x^2 + a)^{5/2} \cdot a^2 - 105 \cdot (b^2 x^2 + a)^{3/2} \cdot a^3) \cdot a^3 / b^2 + 78 \cdot (315 \cdot (b^2 x^2 + a)^{11/2} - 1540 \cdot (b^2 x^2 + a)^{9/2} \cdot a + 2970 \cdot (b^2 x^2 + a)^{7/2} \cdot a^2 - 2772 \cdot (b^2 x^2 + a)^{5/2} \cdot a^3 + 1155 \cdot (b^2 x^2 + a)^{3/2} \cdot a^4) \cdot a^2 / b^2 + 20 \cdot (693 \cdot (b^2 x^2 + a)^{13/2} - 4095 \cdot (b^2 x^2 + a)^{11/2} \cdot a + 10010 \cdot (b^2 x^2 + a)^{9/2} \cdot a^2 - 12870 \cdot (b^2 x^2 + a)^{7/2} \cdot a^3 + 9009 \cdot (b^2 x^2 + a)^{5/2} \cdot a^4 - 3003 \cdot (b^2 x^2 + a)^{3/2} \cdot a^5) \cdot a / b^2 + (3003 \cdot (b^2 x^2 + a)^{15/2} - 20790 \cdot (b^2 x^2 + a)^{13/2} \cdot a + 61425 \cdot (b^2 x^2 + a)^{11/2} \cdot a^2 - 100100 \cdot (b^2 x^2 + a)^{9/2} \cdot a^3 + 96525 \cdot (b^2 x^2 + a)^{7/2} \cdot a^4 - 54054 \cdot (b^2 x^2 + a)^{5/2} \cdot a^5 + 15015 \cdot (b^2 x^2 + a)^{3/2} \cdot a^6) / b^2) / b$

$$3.415 \quad \int x^3 (a + bx^2)^{9/2} dx$$

**Optimal.** Leaf size=38

$$\frac{(a + bx^2)^{13/2}}{13b^2} - \frac{a(a + bx^2)^{11/2}}{11b^2}$$

[Out]  $-(a*(a + b*x^2)^(11/2))/(11*b^2) + (a + b*x^2)^(13/2)/(13*b^2)$

**Rubi [A]** time = 0.0671408, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{(a + bx^2)^{13/2}}{13b^2} - \frac{a(a + bx^2)^{11/2}}{11b^2}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*x^2)^(9/2), x]

[Out]  $-(a*(a + b*x^2)^(11/2))/(11*b^2) + (a + b*x^2)^(13/2)/(13*b^2)$

**Rubi in Sympy [A]** time = 8.07922, size = 31, normalized size = 0.82

$$-\frac{a(a + bx^2)^{11/2}}{11b^2} + \frac{(a + bx^2)^{13/2}}{13b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(b\*x\*\*2+a)\*\*(9/2), x)

[Out]  $-a*(a + b*x**2)**(11/2)/(11*b**2) + (a + b*x**2)**(13/2)/(13*b**2)$

**Mathematica [A]** time = 0.0504133, size = 28, normalized size = 0.74

$$\frac{(a + bx^2)^{11/2} (11bx^2 - 2a)}{143b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*x^2)^(9/2), x]

[Out]  $((a + b*x^2)^(11/2)*(-2*a + 11*b*x^2))/(143*b^2)$

**Maple [A]** time = 0.006, size = 25, normalized size = 0.7

$$-\frac{-11bx^2 + 2a}{143b^2} (bx^2 + a)^{11/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(b\*x^2+a)^(9/2), x)

[Out]  $-1/143 * (b * x^2 + a)^{(11/2)} * (-11 * b * x^2 + 2 * a) / b^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)*x^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.240407, size = 105, normalized size = 2.76

$$\frac{(11b^6x^{12} + 53ab^5x^{10} + 100a^2b^4x^8 + 90a^3b^3x^6 + 35a^4b^2x^4 + a^5bx^2 - 2a^6)\sqrt{bx^2 + a}}{143b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)*x^3,x, algorithm="fricas")`

[Out]  $1/143 * (11 * b^6 * x^{12} + 53 * a * b^5 * x^{10} + 100 * a^2 * b^4 * x^8 + 90 * a^3 * b^3 * x^6 + 35 * a^4 * b^2 * x^4 + a^5 * b * x^2 - 2 * a^6) * \text{sqrt}(b * x^2 + a) / b^2$

**Sympy [A]** time = 62.1553, size = 156, normalized size = 4.11

$$\begin{cases} -\frac{2a^6\sqrt{a+bx^2}}{143b^2} + \frac{a^5x^2\sqrt{a+bx^2}}{143b} + \frac{35a^4x^4\sqrt{a+bx^2}}{143} + \frac{90a^3bx^6\sqrt{a+bx^2}}{143} + \frac{100a^2b^2x^8\sqrt{a+bx^2}}{143} + \frac{53ab^3x^{10}\sqrt{a+bx^2}}{143} + \frac{b^4x^{12}\sqrt{a+bx^2}}{13} & \text{for } b \neq 0 \\ \frac{a^2x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**2+a)**(9/2),x)`

[Out] `Piecewise((-2*a**6*sqrt(a + b*x**2)/(143*b**2) + a**5*x**2*sqrt(a + b*x**2)/(143*b) + 35*a**4*x**4*sqrt(a + b*x**2)/143 + 90*a**3*b*x**6*sqrt(a + b*x**2)/143 + 100*a**2*b**2*x**8*sqrt(a + b*x**2)/143 + 53*a*b**3*x**10*sqrt(a + b*x**2)/143 + b**4*x**12*sqrt(a + b*x**2)/13, Ne(b, 0)), (a**(9/2)*x**4/4, True))`

**GIAC/XCAS [A]** time = 0.210361, size = 406, normalized size = 10.68

$$\frac{3003 \left( 3 (bx^2+a)^{\frac{5}{2}} - 5 (bx^2+a)^{\frac{3}{2}} \right) a^4}{b} + \frac{1716 \left( 15 (bx^2+a)^{\frac{7}{2}} - 42 (bx^2+a)^{\frac{5}{2}} a + 35 (bx^2+a)^{\frac{3}{2}} a^2 \right) a^3}{b} + \frac{858 \left( 35 (bx^2+a)^{\frac{9}{2}} - 135 (bx^2+a)^{\frac{7}{2}} a + 189 (bx^2+a)^{\frac{5}{2}} a^2 - 105 (bx^2+a)^{\frac{3}{2}} a^3 \right) a^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)*x^3,x, algorithm="giac")`

[Out]  $1/45045 * (3003 * (3 * (b * x^2 + a)^{(5/2)} - 5 * (b * x^2 + a)^{(3/2)} * a) * a^4 / b + 1716 * (15 * (b * x^2 + a)^{(7/2)} - 42 * (b * x^2 + a)^{(5/2)} * a + 35 * (b * x^2 + a)^{(3/2)} * a^2) * a^3 / b + 858 * (35 * (b * x^2 + a)^{(9/2)} - 135 * (b * x^2 + a)^{(7/2)} * a + 189 * (b * x^2 + a)^{(5/2)} * a^2 - 105 * (b * x^2 + a)^{(3/2)} * a^3) * a^2 / b + 52 * (315 * (b * x^2 + a)^{(11/2)} - 1540 * (b * x^2 + a)^{(9/2)} * a) * a^2 / b)$

$$\begin{aligned} & a + 2970 \cdot (b \cdot x^2 + a)^{7/2} \cdot a^2 - 2772 \cdot (b \cdot x^2 + a)^{5/2} \cdot a^3 + 115 \\ & 5 \cdot (b \cdot x^2 + a)^{3/2} \cdot a^4 \cdot a/b + 5 \cdot (693 \cdot (b \cdot x^2 + a)^{13/2} - 4095 \cdot ( \\ & b \cdot x^2 + a)^{11/2} \cdot a + 10010 \cdot (b \cdot x^2 + a)^{9/2} \cdot a^2 - 12870 \cdot (b \cdot x^2 \\ & + a)^{7/2} \cdot a^3 + 9009 \cdot (b \cdot x^2 + a)^{5/2} \cdot a^4 - 3003 \cdot (b \cdot x^2 + a)^{3 \\ & /2} \cdot a^5)/b)/b \end{aligned}$$

$$3.416 \quad \int x (a + bx^2)^{9/2} dx$$

**Optimal.** Leaf size=18

$$\frac{(a + bx^2)^{11/2}}{11b}$$

[Out] (a + b\*x^2)^(11/2)/(11\*b)

**Rubi [A]** time = 0.0112103, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{(a + bx^2)^{11/2}}{11b}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*x^2)^(9/2), x]

[Out] (a + b\*x^2)^(11/2)/(11\*b)

**Rubi in Sympy [A]** time = 2.1749, size = 12, normalized size = 0.67

$$\frac{(a + bx^2)^{\frac{11}{2}}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(b\*x\*\*2+a)\*\*(9/2), x)

[Out] (a + b\*x\*\*2)\*\*(11/2)/(11\*b)

**Mathematica [A]** time = 0.0135474, size = 18, normalized size = 1.

$$\frac{(a + bx^2)^{11/2}}{11b}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*x^2)^(9/2), x]

[Out] (a + b\*x^2)^(11/2)/(11\*b)

**Maple [A]** time = 0.005, size = 15, normalized size = 0.8

$$\frac{1}{11b} (bx^2 + a)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(b\*x^2+a)^(9/2), x)

[Out] 1/11\*(b\*x^2+a)^(11/2)/b

---

**Maxima [A]** time = 1.32404, size = 19, normalized size = 1.06

$$\frac{(bx^2 + a)^{\frac{11}{2}}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)\*x,x, algorithm="maxima")

[Out] 1/11\*(b\*x^2 + a)^(11/2)/b

---

**Fricas [A]** time = 0.25506, size = 88, normalized size = 4.89

$$\frac{(b^5x^{10} + 5ab^4x^8 + 10a^2b^3x^6 + 10a^3b^2x^4 + 5a^4bx^2 + a^5)\sqrt{bx^2 + a}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)\*x,x, algorithm="fricas")

[Out] 1/11\*(b^5\*x^10 + 5\*a\*b^4\*x^8 + 10\*a^2\*b^3\*x^6 + 10\*a^3\*b^2\*x^4 + 5\*a^4\*b\*x^2 + a^5)\*sqrt(b\*x^2 + a)/b

---

**Sympy [A]** time = 43.375, size = 133, normalized size = 7.39

$$\begin{cases} \frac{a^5\sqrt{a+bx^2}}{11b} + \frac{5a^4x^2\sqrt{a+bx^2}}{11} + \frac{10a^3bx^4\sqrt{a+bx^2}}{11} + \frac{10a^2b^2x^6\sqrt{a+bx^2}}{11} + \frac{5ab^3x^8\sqrt{a+bx^2}}{11} + \frac{b^4x^{10}\sqrt{a+bx^2}}{11} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x\*\*2+a)\*\*(9/2),x)

[Out] Piecewise(((a\*\*5\*sqrt(a + b\*x\*\*2))/(11\*b) + 5\*a\*\*4\*x\*\*2\*sqrt(a + b\*x\*\*2)/11 + 10\*a\*\*3\*b\*x\*\*4\*sqrt(a + b\*x\*\*2)/11 + 10\*a\*\*2\*b\*\*2\*x\*\*6\*sqrt(a + b\*x\*\*2)/11 + 5\*a\*b\*\*3\*x\*\*8\*sqrt(a + b\*x\*\*2)/11 + b\*\*4\*x\*\*10\*sqrt(a + b\*x\*\*2)/11, Ne(b, 0)), (a\*\*(9/2)\*x\*\*2/2, True))

---

**GIAC/XCAS [A]** time = 0.208728, size = 267, normalized size = 14.83

$$315 (bx^2 + a)^{\frac{11}{2}} - 1540 (bx^2 + a)^{\frac{9}{2}}a + 2970 (bx^2 + a)^{\frac{7}{2}}a^2 - 2772 (bx^2 + a)^{\frac{5}{2}}a^3 + 2310 (bx^2 + a)^{\frac{3}{2}}a^4 + 924 \left( 3 (bx^2 + a)^{\frac{5}{2}} - 5 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)\*x,x, algorithm="giac")

[Out] 1/3465\*(315\*(b\*x^2 + a)^(11/2) - 1540\*(b\*x^2 + a)^(9/2)\*a + 2970\*(b\*x^2 + a)^(7/2)\*a^2 - 2772\*(b\*x^2 + a)^(5/2)\*a^3 + 2310\*(b\*x^2 + a)^(3/2)\*a^4 + 924\*(3\*(b\*x^2 + a)^(5/2) - 5\*(b\*x^2 + a)^(3/2))\*a + 198\*(15\*(b\*x^2 + a)^(7/2) - 42\*(b\*x^2 + a)^(5/2))\*a + 35\*(b\*x^2 + a)^(3/2)\*a^2 + 44\*(35\*(b\*x^2 + a)^(9/2) - 135\*(b\*x^2 + a)^(7/2))\*a + 189\*(b\*x^2 + a)^(5/2)\*a^2 - 105\*(b\*x^2 + a)^(3/2)\*a^3)/b



$$3.417 \quad \int \frac{(a+bx^2)^{9/2}}{x} dx$$

**Optimal.** Leaf size=108

$$a^{9/2} \left( -\tanh^{-1} \left( \frac{\sqrt{a+bx^2}}{\sqrt{a}} \right) \right) + a^4 \sqrt{a+bx^2} + \frac{1}{3} a^3 (a+bx^2)^{3/2} \\ + \frac{1}{5} a^2 (a+bx^2)^{5/2} + \frac{1}{7} a (a+bx^2)^{7/2} + \frac{1}{9} (a+bx^2)^{9/2}$$

[Out]  $a^4 \sqrt{a+bx^2} + (a^3 (a+bx^2)^{3/2})/3 + (a^2 (a+bx^2)^{5/2})/5 + (a (a+bx^2)^{7/2})/7 + (a+bx^2)^{9/2}/9 - a^{9/2} \operatorname{ArcTanh}[\sqrt{a+bx^2}/\sqrt{a}]$

**Rubi [A]** time = 0.191432, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$a^{9/2} \left( -\tanh^{-1} \left( \frac{\sqrt{a+bx^2}}{\sqrt{a}} \right) \right) + a^4 \sqrt{a+bx^2} + \frac{1}{3} a^3 (a+bx^2)^{3/2} \\ + \frac{1}{5} a^2 (a+bx^2)^{5/2} + \frac{1}{7} a (a+bx^2)^{7/2} + \frac{1}{9} (a+bx^2)^{9/2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(9/2)/x, x]

[Out]  $a^4 \sqrt{a+bx^2} + (a^3 (a+bx^2)^{3/2})/3 + (a^2 (a+bx^2)^{5/2})/5 + (a (a+bx^2)^{7/2})/7 + (a+bx^2)^{9/2}/9 - a^{9/2} \operatorname{ArcTanh}[\sqrt{a+bx^2}/\sqrt{a}]$

**Rubi in Sympy [A]** time = 17.796, size = 90, normalized size = 0.83

$$-a^{9/2} \operatorname{atanh} \left( \frac{\sqrt{a+bx^2}}{\sqrt{a}} \right) + a^4 \sqrt{a+bx^2} + \frac{a^3 (a+bx^2)^{3/2}}{3} + \frac{a^2 (a+bx^2)^{5/2}}{5} + \frac{a (a+bx^2)^{7/2}}{7} + \frac{(a+bx^2)^{9/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(9/2)/x, x)

[Out]  $-a^{9/2} \operatorname{atanh}(\sqrt{a+bx^2}/\sqrt{a}) + a^4 \sqrt{a+bx^2} + a^3 (a+bx^2)^{3/2}/3 + a^2 (a+bx^2)^{5/2}/5 + a (a+bx^2)^{7/2}/7 + (a+bx^2)^{9/2}/9$

**Mathematica [A]** time = 0.111024, size = 94, normalized size = 0.87

$$-a^{9/2} \log \left( \sqrt{a} \sqrt{a+bx^2} + a \right) \\ + a^{9/2} \log(x) + \frac{1}{315} \sqrt{a+bx^2} (563a^4 + 506a^3bx^2 + 408a^2b^2x^4 + 185ab^3x^6 + 35b^4x^8)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(9/2)/x, x]

[Out]  $(\sqrt{a+bx^2} (563a^4 + 506a^3bx^2 + 408a^2b^2x^4 + 185ab^3x^6 + 35b^4x^8))/315 + a^{9/2} \operatorname{Log}[x] - a^{9/2} \operatorname{Log}[a +$

Sqrt[a]\*Sqrt[a + b\*x^2]]

**Maple [A]** time = 0.007, size = 94, normalized size = 0.9

$$\frac{1}{9} (bx^2 + a)^{\frac{9}{2}} + \frac{a}{7} (bx^2 + a)^{\frac{7}{2}} + \frac{a^2}{5} (bx^2 + a)^{\frac{5}{2}} + \frac{a^3}{3} (bx^2 + a)^{\frac{3}{2}} - a^{\frac{9}{2}} \ln\left(\frac{1}{x} (2a + 2\sqrt{a}\sqrt{bx^2 + a})\right) + a^4 \sqrt{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(9/2)/x,x)

[Out] 1/9\*(b\*x^2+a)^(9/2)+1/7\*a\*(b\*x^2+a)^(7/2)+1/5\*a^2\*(b\*x^2+a)^(5/2)+1/3\*a^3\*(b\*x^2+a)^(3/2)-a^(9/2)\*ln((2\*a+2\*a^(1/2)\*(b\*x^2+a)^(1/2))/x)+a^4\*(b\*x^2+a)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.263718, size = 1, normalized size = 0.01

$$\left[ \frac{1}{2} a^{\frac{9}{2}} \log\left(-\frac{bx^2 - 2\sqrt{bx^2 + a}\sqrt{a} + 2a}{x^2}\right) + \frac{1}{315} (35b^4x^8 + 185ab^3x^6 + 408a^2b^2x^4 + 506a^3bx^2 + 563a^4)\sqrt{bx^2 + a}, -\sqrt{-aa^4} \arctan\left(\frac{a}{\sqrt{bx^2 + a}\sqrt{-a}}\right) + \frac{1}{315} (35b^4x^8 + 185ab^3x^6 + 408a^2b^2x^4 + 506a^3bx^2 + 563a^4)\sqrt{bx^2 + a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x,x, algorithm="fricas")

[Out] [1/2\*a^(9/2)\*log(-(b\*x^2 - 2\*sqrt(b\*x^2 + a)\*sqrt(a) + 2\*a)/x^2) + 1/315\*(35\*b^4\*x^8 + 185\*a\*b^3\*x^6 + 408\*a^2\*b^2\*x^4 + 506\*a^3\*b\*x^2 + 563\*a^4)\*sqrt(b\*x^2 + a), -sqrt(-a)\*a^4\*arctan(a/(sqrt(b\*x^2 + a)\*sqrt(-a))) + 1/315\*(35\*b^4\*x^8 + 185\*a\*b^3\*x^6 + 408\*a^2\*b^2\*x^4 + 506\*a^3\*b\*x^2 + 563\*a^4)\*sqrt(b\*x^2 + a)]

**Sympy [A]** time = 36.8011, size = 160, normalized size = 1.48

$$\frac{563a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}}{315} + \frac{a^{\frac{9}{2}}\log\left(\frac{bx^2}{a}\right)}{2} - a^{\frac{9}{2}}\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right) + \frac{506a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}{315} + \frac{136a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}}{105} + \frac{37a^{\frac{3}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}{63} + \frac{\sqrt{ab^4}x^8\sqrt{1+\frac{bx^2}{a}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(9/2)/x,x)

[Out] 563\*a\*\*(9/2)\*sqrt(1 + b\*x\*\*2/a)/315 + a\*\*(9/2)\*log(b\*x\*\*2/a)/2 - a\*\*(9/2)\*log(sqrt(1 + b\*x\*\*2/a) + 1) + 506\*a\*\*(7/2)\*b\*x\*\*2\*sqrt(1 + b\*x\*\*2/a)/315 + 136\*a\*\*(5/2)\*b\*\*2\*x\*\*4\*sqrt(1 + b\*x\*\*2/a)/105 + 37\*a\*\*(3/2)\*b\*\*3\*x\*\*6\*sqrt(1 + b\*x\*\*2/a)/63 + sqrt(a)\*b\*\*4\*x\*\*8\*sqrt(1 + b\*x\*\*2/a)/9

**GIAC/XCAS [A]** time = 0.210249, size = 122, normalized size = 1.13

$$\frac{a^5 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{1}{9} (bx^2 + a)^{\frac{9}{2}} + \frac{1}{7} (bx^2 + a)^{\frac{7}{2}} a + \frac{1}{5} (bx^2 + a)^{\frac{5}{2}} a^2 + \frac{1}{3} (bx^2 + a)^{\frac{3}{2}} a^3 + \sqrt{bx^2 + a} a^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x,x, algorithm="giac")

[Out] a^5\*arctan(sqrt(b\*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/9\*(b\*x^2 + a)^(9/2) + 1/7\*(b\*x^2 + a)^(7/2)\*a + 1/5\*(b\*x^2 + a)^(5/2)\*a^2 + 1/3\*(b\*x^2 + a)^(3/2)\*a^3 + sqrt(b\*x^2 + a)\*a^4

$$3.418 \quad \int \frac{(a+bx^2)^{9/2}}{x^3} dx$$

**Optimal.** Leaf size=118

$$-\frac{9}{2}a^{7/2}b \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{9}{2}a^3b\sqrt{a+bx^2} + \frac{3}{2}a^2b(a+bx^2)^{3/2} \\ - \frac{(a+bx^2)^{9/2}}{2x^2} + \frac{9}{14}b(a+bx^2)^{7/2} + \frac{9}{10}ab(a+bx^2)^{5/2}$$

[Out]  $(9*a^3*b*\text{Sqrt}[a + b*x^2])/2 + (3*a^2*b*(a + b*x^2)^(3/2))/2 + (9*a*b*(a + b*x^2)^(5/2))/10 + (9*b*(a + b*x^2)^(7/2))/14 - (a + b*x^2)^(9/2)/(2*x^2) - (9*a^(7/2)*b*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/2$

**Rubi [A]** time = 0.192969, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{9}{2}a^{7/2}b \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{9}{2}a^3b\sqrt{a+bx^2} + \frac{3}{2}a^2b(a+bx^2)^{3/2} \\ - \frac{(a+bx^2)^{9/2}}{2x^2} + \frac{9}{14}b(a+bx^2)^{7/2} + \frac{9}{10}ab(a+bx^2)^{5/2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(9/2)/x^3, x]

[Out]  $(9*a^3*b*\text{Sqrt}[a + b*x^2])/2 + (3*a^2*b*(a + b*x^2)^(3/2))/2 + (9*a*b*(a + b*x^2)^(5/2))/10 + (9*b*(a + b*x^2)^(7/2))/14 - (a + b*x^2)^(9/2)/(2*x^2) - (9*a^(7/2)*b*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/2$

**Rubi in Sympy [A]** time = 18.0144, size = 110, normalized size = 0.93

$$-\frac{9a^{7/2}b \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2} + \frac{9a^3b\sqrt{a+bx^2}}{2} + \frac{3a^2b(a+bx^2)^{3/2}}{2} + \frac{9ab(a+bx^2)^{5/2}}{10} + \frac{9b(a+bx^2)^{7/2}}{14} - \frac{(a+bx^2)^{9/2}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*3, x)

[Out]  $-9*a**(7/2)*b*\operatorname{atanh}(\operatorname{sqrt}(a + b*x**2)/\operatorname{sqrt}(a))/2 + 9*a**3*b*\operatorname{sqrt}(a + b*x**2)/2 + 3*a**2*b*(a + b*x**2)**(3/2)/2 + 9*a*b*(a + b*x**2)**(5/2)/10 + 9*b*(a + b*x**2)**(7/2)/14 - (a + b*x**2)**(9/2)/(2*x**2)$

**Mathematica [A]** time = 0.14314, size = 104, normalized size = 0.88

$$-\frac{9}{2}a^{7/2}b \log\left(\sqrt{a}\sqrt{a+bx^2} + a\right) \\ + \frac{9}{2}a^{7/2}b \log(x) + \frac{\sqrt{a+bx^2}(-35a^4 + 388a^3bx^2 + 156a^2b^2x^4 + 58ab^3x^6 + 10b^4x^8)}{70x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(9/2)/x^3, x]

[Out]  $(\text{Sqrt}[a + b*x^2] * (-35*a^4 + 388*a^3*b*x^2 + 156*a^2*b^2*x^4 + 58*a*b^3*x^6 + 10*b^4*x^8)) / (70*x^2) + (9*a^{(7/2)}*b*\text{Log}[x]) / 2 - (9*a^{(7/2)}*b*\text{Log}[a + \text{Sqrt}[a]*\text{Sqrt}[a + b*x^2]]) / 2$

**Maple [A]** time = 0.008, size = 118, normalized size = 1.

$$-\frac{1}{2ax^2} (bx^2 + a)^{\frac{11}{2}} + \frac{b}{2a} (bx^2 + a)^{\frac{9}{2}} + \frac{9b}{14} (bx^2 + a)^{\frac{7}{2}} + \frac{9ab}{10} (bx^2 + a)^{\frac{5}{2}} + \frac{3a^2b}{2} (bx^2 + a)^{\frac{3}{2}} - \frac{9b}{2} a^{\frac{7}{2}} \ln\left(\frac{1}{x} (2a + 2\sqrt{a}\sqrt{bx^2 + a})\right) + \frac{9a^3b}{2} \sqrt{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(9/2)/x^3,x)`

[Out]  $-1/2/a/x^2 * (b*x^2+a)^{(11/2)} + 1/2*b/a * (b*x^2+a)^{(9/2)} + 9/14*b * (b*x^2+a)^{(7/2)} + 9/10*a*b * (b*x^2+a)^{(5/2)} + 3/2*a^2*b * (b*x^2+a)^{(3/2)} - 9/2*b*a^{(7/2)} * \ln((2*a+2*a^{(1/2)}*(b*x^2+a)^{(1/2)})/x) + 9/2*a^3*b * (b*x^2+a)^{(1/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)/x^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.259711, size = 1, normalized size = 0.01

$$\left[ \frac{315 a^{\frac{7}{2}} b x^2 \log\left(-\frac{b x^2 - 2 \sqrt{b x^2 + a} \sqrt{a + 2 a}}{x^2}\right) + 2 (10 b^4 x^8 + 58 a b^3 x^6 + 156 a^2 b^2 x^4 + 388 a^3 b x^2 - 35 a^4) \sqrt{b x^2 + a}}{140 x^2}, \frac{315 \sqrt{-a} a^3 b x^2 \arctan\left(\frac{a}{\sqrt{b x^2 + a} \sqrt{-a}}\right) - (10 b^4 x^8 + 58 a b^3 x^6 + 156 a^2 b^2 x^4 + 388 a^3 b x^2 - 35 a^4) \sqrt{b x^2 + a}}{70 x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)/x^3,x, algorithm="fricas")`

[Out]  $[1/140 * (315*a^{(7/2)}*b*x^2*\log(-(b*x^2 - 2*\text{sqrt}(b*x^2 + a))*\text{sqrt}(a) + 2*a)/x^2) + 2*(10*b^4*x^8 + 58*a*b^3*x^6 + 156*a^2*b^2*x^4 + 388*a^3*b*x^2 - 35*a^4)*\text{sqrt}(b*x^2 + a)/x^2, -1/70*(315*\text{sqrt}(-a)*a^3*b*x^2*\arctan(a/(\text{sqrt}(b*x^2 + a)*\text{sqrt}(-a))) - (10*b^4*x^8 + 58*a*b^3*x^6 + 156*a^2*b^2*x^4 + 388*a^3*b*x^2 - 35*a^4)*\text{sqrt}(b*x^2 + a))/x^2]$

**Sympy [A]** time = 37.8905, size = 167, normalized size = 1.42

$$-\frac{a^{\frac{9}{2}} \sqrt{1 + \frac{bx^2}{a}}}{2x^2} + \frac{194a^{\frac{7}{2}}b\sqrt{1 + \frac{bx^2}{a}}}{35} + \frac{9a^{\frac{7}{2}}b \log\left(\frac{bx^2}{a}\right)}{4} - \frac{9a^{\frac{7}{2}}b \log\left(\sqrt{1 + \frac{bx^2}{a}} + 1\right)}{2} + \frac{78a^{\frac{5}{2}}b^2x^2\sqrt{1 + \frac{bx^2}{a}}}{35} + \frac{29a^{\frac{3}{2}}b^3x^4\sqrt{1 + \frac{bx^2}{a}}}{35} + \frac{\sqrt{ab^4}x^6\sqrt{1 + \frac{bx^2}{a}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*3,x)

[Out]  $-a^{9/2} \sqrt{1 + b x^2/a} / (2 x^2) + 194 a^{7/2} b \sqrt{1 + b x^2/a} / 35 + 9 a^{7/2} b \log(b x^2/a) / 4 - 9 a^{7/2} b \log(\sqrt{1 + b x^2/a} + 1) / 2 + 78 a^{5/2} b^2 x^2 \sqrt{1 + b x^2/a} / 35 + 29 a^{3/2} b^3 x^4 \sqrt{1 + b x^2/a} / 35 + \sqrt{a} b^4 x^6 \sqrt{1 + b x^2/a} / 7$

**GIAC/XCAS [A]** time = 0.211968, size = 136, normalized size = 1.15

$$\frac{1}{70} \left( \frac{315 a^4 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 10 (bx^2 + a)^{7/2} + 28 (bx^2 + a)^{5/2} a + 70 (bx^2 + a)^{3/2} a^2 + 280 \sqrt{bx^2 + a} a^3 - \frac{35 \sqrt{bx^2 + a} a^4}{bx^2} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^3,x, algorithm="giac")

[Out]  $1/70 * (315 * a^4 * \arctan(\sqrt{b * x^2 + a} / \sqrt{-a}) / \sqrt{-a} + 10 * (b * x^2 + a)^{7/2} + 28 * (b * x^2 + a)^{5/2} * a + 70 * (b * x^2 + a)^{3/2} * a^2 + 280 * \sqrt{b * x^2 + a} * a^3 - 35 * \sqrt{b * x^2 + a} * a^4 / (b * x^2)) * b$

$$3.419 \quad \int \frac{(a+bx^2)^{9/2}}{x^5} dx$$

**Optimal.** Leaf size=126

$$-\frac{63}{8}a^{5/2}b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{63}{8}a^2b^2\sqrt{a+bx^2} + \frac{63}{40}b^2(a+bx^2)^{5/2} \\ + \frac{21}{8}ab^2(a+bx^2)^{3/2} - \frac{9b(a+bx^2)^{7/2}}{8x^2} - \frac{(a+bx^2)^{9/2}}{4x^4}$$

[Out] (63\*a^2\*b^2\*Sqrt[a + b\*x^2])/8 + (21\*a\*b^2\*(a + b\*x^2)^(3/2))/8 + (63\*b^2\*(a + b\*x^2)^(5/2))/40 - (9\*b\*(a + b\*x^2)^(7/2))/(8\*x^2) - (a + b\*x^2)^(9/2)/(4\*x^4) - (63\*a^(5/2)\*b^2\*ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]])/8

**Rubi [A]** time = 0.204302, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{63}{8}a^{5/2}b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{63}{8}a^2b^2\sqrt{a+bx^2} + \frac{63}{40}b^2(a+bx^2)^{5/2} \\ + \frac{21}{8}ab^2(a+bx^2)^{3/2} - \frac{9b(a+bx^2)^{7/2}}{8x^2} - \frac{(a+bx^2)^{9/2}}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(9/2)/x^5, x]

[Out] (63\*a^2\*b^2\*Sqrt[a + b\*x^2])/8 + (21\*a\*b^2\*(a + b\*x^2)^(3/2))/8 + (63\*b^2\*(a + b\*x^2)^(5/2))/40 - (9\*b\*(a + b\*x^2)^(7/2))/(8\*x^2) - (a + b\*x^2)^(9/2)/(4\*x^4) - (63\*a^(5/2)\*b^2\*ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]])/8

**Rubi in Sympy [A]** time = 19.1461, size = 117, normalized size = 0.93

$$-\frac{63a^{5/2}b^2 \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8} + \frac{63a^2b^2\sqrt{a+bx^2}}{8} + \frac{21ab^2(a+bx^2)^{3/2}}{8} \\ + \frac{63b^2(a+bx^2)^{5/2}}{40} - \frac{9b(a+bx^2)^{7/2}}{8x^2} - \frac{(a+bx^2)^{9/2}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*5, x)

[Out] -63\*a\*\*(5/2)\*b\*\*2\*atanh(sqrt(a + b\*x\*\*2)/sqrt(a))/8 + 63\*a\*\*2\*b\*\*2\*sqrt(a + b\*x\*\*2)/8 + 21\*a\*b\*\*2\*(a + b\*x\*\*2)\*\*(3/2)/8 + 63\*b\*\*2\*(a + b\*x\*\*2)\*\*(5/2)/40 - 9\*b\*(a + b\*x\*\*2)\*\*(7/2)/(8\*x\*\*2) - (a + b\*x\*\*2)\*\*(9/2)/(4\*x\*\*4)

**Mathematica [A]** time = 0.153942, size = 105, normalized size = 0.83

$$\frac{1}{40} \left( -315a^{5/2}b^2 \log\left(\sqrt{a}\sqrt{a+bx^2} + a\right) \right. \\ \left. + 315a^{5/2}b^2 \log(x) + \frac{\sqrt{a+bx^2}(-10a^4 - 85a^3bx^2 + 288a^2b^2x^4 + 56ab^3x^6 + 8b^4x^8)}{x^4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(9/2)/x^5, x]

[Out] ((Sqrt[a + b\*x^2]\*(-10\*a^4 - 85\*a^3\*b\*x^2 + 288\*a^2\*b^2\*x^4 + 56\*a\*b^3\*x^6 + 8\*b^4\*x^8))/x^4 + 315\*a^(5/2)\*b^2\*Log[x] - 315\*a^(5/2)\*b^2\*Log[a + Sqrt[a]\*Sqrt[a + b\*x^2]])/40

**Maple [A]** time = 0.01, size = 148, normalized size = 1.2

$$-\frac{1}{4ax^4}(bx^2+a)^{\frac{11}{2}} - \frac{7b}{8a^2x^2}(bx^2+a)^{\frac{11}{2}} + \frac{7b^2}{8a^2}(bx^2+a)^{\frac{9}{2}} + \frac{9b^2}{8a}(bx^2+a)^{\frac{7}{2}} + \frac{63b^2}{40}(bx^2+a)^{\frac{5}{2}} + \frac{21ab^2}{8}(bx^2+a)^{\frac{3}{2}} - \frac{63b^2}{8}a^{\frac{5}{2}}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right) + \frac{63a^2b^2}{8}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(9/2)/x^5, x)

[Out] -1/4/a/x^4\*(b\*x^2+a)^(11/2)-7/8\*b/a^2/x^2\*(b\*x^2+a)^(11/2)+7/8\*b^2/a^2\*(b\*x^2+a)^(9/2)+9/8\*b^2/a\*(b\*x^2+a)^(7/2)+63/40\*b^2\*(b\*x^2+a)^(5/2)+21/8\*a\*b^2\*(b\*x^2+a)^(3/2)-63/8\*b^2\*a^(5/2)\*ln((2\*a+2\*a^(1/2)\*(b\*x^2+a)^(1/2))/x)+63/8\*a^2\*b^2\*(b\*x^2+a)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^5, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.257097, size = 1, normalized size = 0.01

$$\left[ \frac{315 a^{\frac{5}{2}} b^2 x^4 \log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a+2a}}{x^2}\right) + 2(8b^4x^8 + 56ab^3x^6 + 288a^2b^2x^4 - 85a^3bx^2 - 10a^4)\sqrt{bx^2+a}}{80x^4}, \frac{315\sqrt{-aa}b^2x^4 \arctan\left(\frac{a}{\sqrt{bx^2+a}\sqrt{-a}}\right) - (8b^4x^8 + 56ab^3x^6 + 288a^2b^2x^4 - 85a^3bx^2 - 10a^4)\sqrt{bx^2+a}}{40x^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^5, x, algorithm="fricas")

[Out] [1/80\*(315\*a^(5/2)\*b^2\*x^4\*log(-(b\*x^2 - 2\*sqrt(b\*x^2 + a)\*sqrt(a) + 2\*a)/x^2) + 2\*(8\*b^4\*x^8 + 56\*a\*b^3\*x^6 + 288\*a^2\*b^2\*x^4 - 85\*a^3\*b\*x^2 - 10\*a^4)\*sqrt(b\*x^2 + a))/x^4, -1/40\*(315\*sqrt(-a)\*a^2\*b^2\*x^4\*arctan(a/(sqrt(b\*x^2 + a)\*sqrt(-a))) - (8\*b^4\*x^8 + 56\*a\*b^3\*x^6 + 288\*a^2\*b^2\*x^4 - 85\*a^3\*b\*x^2 - 10\*a^4)\*sqrt(b\*x^2 + a))/x^4]



**Sympy [A]** time = 36.1965, size = 175, normalized size = 1.39

$$\frac{63a^{\frac{5}{2}}b^2 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{8} - \frac{a^5}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{19a^4\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}}$$

$$+ \frac{203a^3b^{\frac{3}{2}}}{40x\sqrt{\frac{a}{bx^2}+1}} + \frac{43a^2b^{\frac{5}{2}}x}{5\sqrt{\frac{a}{bx^2}+1}} + \frac{8ab^{\frac{7}{2}}x^3}{5\sqrt{\frac{a}{bx^2}+1}} + \frac{b^{\frac{9}{2}}x^5}{5\sqrt{\frac{a}{bx^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*5,x)

[Out] -63\*a\*\*(5/2)\*b\*\*2\*asinh(sqrt(a)/(sqrt(b)\*x))/8 - a\*\*5/(4\*sqrt(b)\*x\*\*5\*sqrt(a/(b\*x\*\*2)+1)) - 19\*a\*\*4\*sqrt(b)/(8\*x\*\*3\*sqrt(a/(b\*x\*\*2)+1)) + 203\*a\*\*3\*b\*\*(3/2)/(40\*x\*sqrt(a/(b\*x\*\*2)+1)) + 43\*a\*\*2\*b\*\*(5/2)\*x/(5\*sqrt(a/(b\*x\*\*2)+1)) + 8\*a\*b\*\*(7/2)\*x\*\*3/(5\*sqrt(a/(b\*x\*\*2)+1)) + b\*\*(9/2)\*x\*\*5/(5\*sqrt(a/(b\*x\*\*2)+1))

**GIAC/XCAS [A]** time = 0.212422, size = 143, normalized size = 1.13

$$\frac{1}{40} \left( \frac{315 a^3 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 8 (bx^2 + a)^{\frac{5}{2}} + 40 (bx^2 + a)^{\frac{3}{2}} a + 240 \sqrt{bx^2 + a} a^2 - \frac{5 \left( 17 (bx^2 + a)^{\frac{3}{2}} a^3 - 15 \sqrt{bx^2 + a} a^4 \right)}{b^2 x^4} \right) b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^5,x, algorithm="giac")

[Out] 1/40\*(315\*a^3\*arctan(sqrt(b\*x^2 + a)/sqrt(-a))/sqrt(-a) + 8\*(b\*x^2 + a)^(5/2) + 40\*(b\*x^2 + a)^(3/2)\*a + 240\*sqrt(b\*x^2 + a)\*a^2 - 5\*(17\*(b\*x^2 + a)^(3/2)\*a^3 - 15\*sqrt(b\*x^2 + a)\*a^4)/(b^2\*x^4)\*b^2

$$3.420 \quad \int \frac{(a+bx^2)^{9/2}}{x^7} dx$$

**Optimal.** Leaf size=126

$$-\frac{105}{16}a^{3/2}b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{35}{16}b^3(a+bx^2)^{3/2} + \frac{105}{16}ab^3\sqrt{a+bx^2} \\ - \frac{21b^2(a+bx^2)^{5/2}}{16x^2} - \frac{(a+bx^2)^{9/2}}{6x^6} - \frac{3b(a+bx^2)^{7/2}}{8x^4}$$

[Out] (105\*a\*b^3\*Sqrt[a + b\*x^2])/16 + (35\*b^3\*(a + b\*x^2)^(3/2))/16 - (21\*b^2\*(a + b\*x^2)^(5/2))/(16\*x^2) - (3\*b\*(a + b\*x^2)^(7/2))/(8\*x^4) - (a + b\*x^2)^(9/2)/(6\*x^6) - (105\*a^(3/2)\*b^3\*ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]])/16

**Rubi [A]** time = 0.203708, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{105}{16}a^{3/2}b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{35}{16}b^3(a+bx^2)^{3/2} + \frac{105}{16}ab^3\sqrt{a+bx^2} \\ - \frac{21b^2(a+bx^2)^{5/2}}{16x^2} - \frac{(a+bx^2)^{9/2}}{6x^6} - \frac{3b(a+bx^2)^{7/2}}{8x^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(9/2)/x^7, x]

[Out] (105\*a\*b^3\*Sqrt[a + b\*x^2])/16 + (35\*b^3\*(a + b\*x^2)^(3/2))/16 - (21\*b^2\*(a + b\*x^2)^(5/2))/(16\*x^2) - (3\*b\*(a + b\*x^2)^(7/2))/(8\*x^4) - (a + b\*x^2)^(9/2)/(6\*x^6) - (105\*a^(3/2)\*b^3\*ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]])/16

**Rubi in Sympy [A]** time = 19.117, size = 117, normalized size = 0.93

$$-\frac{105a^{\frac{3}{2}}b^3 \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16} + \frac{105ab^3\sqrt{a+bx^2}}{16} + \frac{35b^3(a+bx^2)^{\frac{3}{2}}}{16} \\ - \frac{21b^2(a+bx^2)^{\frac{5}{2}}}{16x^2} - \frac{3b(a+bx^2)^{\frac{7}{2}}}{8x^4} - \frac{(a+bx^2)^{\frac{9}{2}}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*7, x)

[Out] -105\*a\*\*(3/2)\*b\*\*3\*atanh(sqrt(a + b\*x\*\*2)/sqrt(a))/16 + 105\*a\*b\*\*3\*sqrt(a + b\*x\*\*2)/16 + 35\*b\*\*3\*(a + b\*x\*\*2)\*\*(3/2)/16 - 21\*b\*\*2\*(a + b\*x\*\*2)\*\*(5/2)/(16\*x\*\*2) - 3\*b\*(a + b\*x\*\*2)\*\*(7/2)/(8\*x\*\*4) - (a + b\*x\*\*2)\*\*(9/2)/(6\*x\*\*6)

**Mathematica [A]** time = 0.154709, size = 105, normalized size = 0.83

$$\frac{1}{48} \left( -315a^{3/2}b^3 \log\left(\sqrt{a}\sqrt{a+bx^2} + a\right) + 315a^{3/2}b^3 \log(x) + \frac{\sqrt{a+bx^2}(-8a^4 - 50a^3bx^2 - 165a^2b^2x^4 + 208ab^3x^6 + 16b^4x^8)}{x^6} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(9/2)/x^7, x]

[Out] ((Sqrt[a + b\*x^2]\*(-8\*a^4 - 50\*a^3\*b\*x^2 - 165\*a^2\*b^2\*x^4 + 208\*a\*b^3\*x^6 + 16\*b^4\*x^8))/x^6 + 315\*a^(3/2)\*b^3\*Log[x] - 315\*a^(3/2)\*b^3\*Log[a + Sqrt[a]\*Sqrt[a + b\*x^2]])/48

**Maple [A]** time = 0.018, size = 168, normalized size = 1.3

$$-\frac{1}{6ax^6}(bx^2+a)^{\frac{11}{2}} - \frac{5b}{24a^2x^4}(bx^2+a)^{\frac{11}{2}} - \frac{35b^2}{48a^3x^2}(bx^2+a)^{\frac{11}{2}} + \frac{35b^3}{48a^3}(bx^2+a)^{\frac{9}{2}} + \frac{15b^3}{16a^2}(bx^2+a)^{\frac{7}{2}} + \frac{21b^3}{16a}(bx^2+a)^{\frac{5}{2}} + \frac{35b^3}{16}(bx^2+a)^{\frac{3}{2}} - \frac{105b^3}{16}a^{\frac{3}{2}}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right) + \frac{105ab^3}{16}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(9/2)/x^7, x)

[Out] -1/6/a/x^6\*(b\*x^2+a)^(11/2)-5/24\*b/a^2/x^4\*(b\*x^2+a)^(11/2)-35/48\*b^2/a^3/x^2\*(b\*x^2+a)^(11/2)+35/48\*b^3/a^3\*(b\*x^2+a)^(9/2)+15/16\*b^3/a^2\*(b\*x^2+a)^(7/2)+21/16\*b^3/a\*(b\*x^2+a)^(5/2)+35/16\*b^3\*(b\*x^2+a)^(3/2)-105/16\*b^3\*a^(3/2)\*ln((2\*a+2\*a^(1/2)\*(b\*x^2+a)^(1/2))/x)+105/16\*a\*b^3\*(b\*x^2+a)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^7, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.251017, size = 1, normalized size = 0.01

$$\left[ \frac{315 a^{\frac{3}{2}} b^3 x^6 \log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a+2a}}{x^2}\right) + 2(16b^4x^8 + 208ab^3x^6 - 165a^2b^2x^4 - 50a^3bx^2 - 8a^4)\sqrt{bx^2+a}}{96x^6}, \frac{315\sqrt{-a}ab^3x^6 \arctan\left(\frac{a}{\sqrt{bx^2+a}\sqrt{-a}}\right) - (16b^4x^8 + 208ab^3x^6 - 165a^2b^2x^4 - 50a^3bx^2 - 8a^4)\sqrt{bx^2+a}}{48x^6} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^7, x, algorithm="fricas")

[Out] [1/96\*(315\*a^(3/2)\*b^3\*x^6\*log(-(b\*x^2 - 2\*sqrt(b\*x^2 + a)\*sqrt(a) + 2\*a)/x^2) + 2\*(16\*b^4\*x^8 + 208\*a\*b^3\*x^6 - 165\*a^2\*b^2\*x^4 - 50\*a^3\*b\*x^2 - 8\*a^4)\*sqrt(b\*x^2 + a))/x^6, -1/48\*(315\*sqrt(-a)\*a\*b^3\*x^6\*arctan(a/(sqrt(b\*x^2 + a)\*sqrt(-a))) - (16\*b^4\*x^8 + 208\*a\*b^3\*x^6 - 165\*a^2\*b^2\*x^4 - 50\*a^3\*b\*x^2 - 8\*a^4)\*sqrt(b\*x^2 + a))/x^6]

**Sympy [A]** time = 33.4413, size = 175, normalized size = 1.39

$$\begin{aligned} & -\frac{105a^{\frac{3}{2}}b^3 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{16} - \frac{a^5}{6\sqrt{bx^7}\sqrt{\frac{a}{bx^2}+1}} - \frac{29a^4\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}} \\ & - \frac{215a^3b^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}} + \frac{43a^2b^{\frac{5}{2}}}{48x\sqrt{\frac{a}{bx^2}+1}} + \frac{14ab^{\frac{7}{2}}x}{3\sqrt{\frac{a}{bx^2}+1}} + \frac{b^{\frac{9}{2}}x^3}{3\sqrt{\frac{a}{bx^2}+1}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*7,x)

[Out] -105\*a\*\*(3/2)\*b\*\*3\*asinh(sqrt(a)/(sqrt(b)\*x))/16 - a\*\*5/(6\*sqrt(b)\*x\*\*7\*sqrt(a/(b\*x\*\*2)+1)) - 29\*a\*\*4\*sqrt(b)/(24\*x\*\*5\*sqrt(a/(b\*x\*\*2)+1)) - 215\*a\*\*3\*b\*\*(3/2)/(48\*x\*\*3\*sqrt(a/(b\*x\*\*2)+1)) + 43\*a\*\*2\*b\*\*(5/2)/(48\*x\*sqrt(a/(b\*x\*\*2)+1)) + 14\*a\*b\*\*(7/2)\*x/(3\*sqrt(a/(b\*x\*\*2)+1)) + b\*\*(9/2)\*x\*\*3/(3\*sqrt(a/(b\*x\*\*2)+1))

**GIAC/XCAS [A]** time = 0.212559, size = 143, normalized size = 1.13

$$\frac{1}{48} \left( \frac{315 a^2 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 16 (bx^2 + a)^{\frac{3}{2}} + 192 \sqrt{bx^2 + aa} - \frac{165 (bx^2 + a)^{\frac{5}{2}} a^2 - 280 (bx^2 + a)^{\frac{3}{2}} a^3 + 123 \sqrt{bx^2 + aa^4}}{b^3 x^6} \right) b^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^7,x, algorithm="giac")

[Out] 1/48\*(315\*a^2\*arctan(sqrt(b\*x^2 + a)/sqrt(-a))/sqrt(-a) + 16\*(b\*x^2 + a)^(3/2) + 192\*sqrt(b\*x^2 + a)\*a - (165\*(b\*x^2 + a)^(5/2)\*a^2 - 280\*(b\*x^2 + a)^(3/2)\*a^3 + 123\*sqrt(b\*x^2 + a)\*a^4)/(b^3\*x^6)\*b^3

$$3.421 \quad \int \frac{(a+bx^2)^{9/2}}{x^9} dx$$

**Optimal.** Leaf size=128

$$\frac{315}{128}b^4\sqrt{a+bx^2} - \frac{315}{128}\sqrt{ab^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{105b^3(a+bx^2)^{3/2}}{128x^2} \\ - \frac{21b^2(a+bx^2)^{5/2}}{64x^4} - \frac{(a+bx^2)^{9/2}}{8x^8} - \frac{3b(a+bx^2)^{7/2}}{16x^6}$$

[Out] (315\*b^4\*Sqrt[a + b\*x^2])/128 - (105\*b^3\*(a + b\*x^2)^(3/2))/(128\*x^2) - (21\*b^2\*(a + b\*x^2)^(5/2))/(64\*x^4) - (3\*b\*(a + b\*x^2)^(7/2))/(16\*x^6) - (a + b\*x^2)^(9/2)/(8\*x^8) - (315\*Sqrt[a]\*b^4\*ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]])/128

**Rubi [A]** time = 0.20584, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{315}{128}b^4\sqrt{a+bx^2} - \frac{315}{128}\sqrt{ab^4}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{105b^3(a+bx^2)^{3/2}}{128x^2} \\ - \frac{21b^2(a+bx^2)^{5/2}}{64x^4} - \frac{(a+bx^2)^{9/2}}{8x^8} - \frac{3b(a+bx^2)^{7/2}}{16x^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(9/2)/x^9, x]

[Out] (315\*b^4\*Sqrt[a + b\*x^2])/128 - (105\*b^3\*(a + b\*x^2)^(3/2))/(128\*x^2) - (21\*b^2\*(a + b\*x^2)^(5/2))/(64\*x^4) - (3\*b\*(a + b\*x^2)^(7/2))/(16\*x^6) - (a + b\*x^2)^(9/2)/(8\*x^8) - (315\*Sqrt[a]\*b^4\*ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]])/128

**Rubi in Sympy [A]** time = 19.7408, size = 119, normalized size = 0.93

$$-\frac{315\sqrt{ab^4}\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128} + \frac{315b^4\sqrt{a+bx^2}}{128} - \frac{105b^3(a+bx^2)^{\frac{3}{2}}}{128x^2} \\ - \frac{21b^2(a+bx^2)^{\frac{5}{2}}}{64x^4} - \frac{3b(a+bx^2)^{\frac{7}{2}}}{16x^6} - \frac{(a+bx^2)^{\frac{9}{2}}}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*9, x)

[Out] -315\*sqrt(a)\*b\*\*4\*atanh(sqrt(a + b\*x\*\*2)/sqrt(a))/128 + 315\*b\*\*4\*sqrt(a + b\*x\*\*2)/128 - 105\*b\*\*3\*(a + b\*x\*\*2)\*\*(3/2)/(128\*x\*\*2) - 21\*b\*\*2\*(a + b\*x\*\*2)\*\*(5/2)/(64\*x\*\*4) - 3\*b\*(a + b\*x\*\*2)\*\*(7/2)/(16\*x\*\*6) - (a + b\*x\*\*2)\*\*(9/2)/(8\*x\*\*8)

**Mathematica [A]** time = 0.149809, size = 108, normalized size = 0.84

$$\left(-\frac{a^4}{8x^8} - \frac{11a^3b}{16x^6} - \frac{105a^2b^2}{64x^4} - \frac{325ab^3}{128x^2} + b^4\right)\sqrt{a+bx^2} - \frac{315}{128}\sqrt{ab^4}\log\left(\sqrt{a}\sqrt{a+bx^2} + a\right) + \frac{315}{128}\sqrt{ab^4}\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(9/2)/x^9, x]

[Out] (b^4 - a^4/(8\*x^8) - (11\*a^3\*b)/(16\*x^6) - (105\*a^2\*b^2)/(64\*x^4) - (325\*a\*b^3)/(128\*x^2))\*Sqrt[a + b\*x^2] + (315\*Sqrt[a]\*b^4\*Log[x])/128 - (315\*Sqrt[a]\*b^4\*Log[a + Sqrt[a]\*Sqrt[a + b\*x^2]])/128

**Maple [A]** time = 0.031, size = 190, normalized size = 1.5

$$\begin{aligned} & -\frac{1}{8ax^8}(bx^2+a)^{\frac{11}{2}} - \frac{b}{16a^2x^6}(bx^2+a)^{\frac{11}{2}} - \frac{5b^2}{64a^3x^4}(bx^2+a)^{\frac{11}{2}} \\ & - \frac{35b^3}{128a^4x^2}(bx^2+a)^{\frac{11}{2}} + \frac{35b^4}{128a^4}(bx^2+a)^{\frac{9}{2}} + \frac{45b^4}{128a^3}(bx^2+a)^{\frac{7}{2}} + \frac{63b^4}{128a^2}(bx^2+a)^{\frac{5}{2}} \\ & + \frac{105b^4}{128a}(bx^2+a)^{\frac{3}{2}} - \frac{315b^4}{128}\sqrt{a}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right) + \frac{315b^4}{128}\sqrt{bx^2+a} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(9/2)/x^9, x)

[Out] -1/8/a/x^8\*(b\*x^2+a)^(11/2)-1/16\*b/a^2/x^6\*(b\*x^2+a)^(11/2)-5/64\*b^2/a^3/x^4\*(b\*x^2+a)^(11/2)-35/128\*b^3/a^4/x^2\*(b\*x^2+a)^(11/2)+35/128\*b^4/a^4\*(b\*x^2+a)^(9/2)+45/128\*b^4/a^3\*(b\*x^2+a)^(7/2)+63/128\*b^4/a^2\*(b\*x^2+a)^(5/2)+105/128\*b^4/a\*(b\*x^2+a)^(3/2)-315/128\*b^4\*a^(1/2)\*ln((2\*a+2\*a^(1/2)\*(b\*x^2+a)^(1/2))/x)+315/128\*b^4\*(b\*x^2+a)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^9, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.252632, size = 1, normalized size = 0.01

$$\left[ \frac{315\sqrt{ab^4x^8}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a+2a}}{x^2}\right) + 2(128b^4x^8 - 325ab^3x^6 - 210a^2b^2x^4 - 88a^3bx^2 - 16a^4)\sqrt{bx^2+a}}{256x^8}, \frac{315\sqrt{-ab^4x^8}\arctan\left(\frac{a}{\sqrt{bx^2+a}\sqrt{-a}}\right) - (128b^4x^8 - 325ab^3x^6 - 210a^2b^2x^4 - 88a^3bx^2 - 16a^4)\sqrt{bx^2+a}}{128x^8} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^9, x, algorithm="fricas")

[Out] [1/256\*(315\*sqrt(a)\*b^4\*x^8\*log(-(b\*x^2 - 2\*sqrt(b\*x^2 + a)\*sqrt(a) + 2\*a)/x^2) + 2\*(128\*b^4\*x^8 - 325\*a\*b^3\*x^6 - 210\*a^2\*b^2\*x^4 - 88\*a^3\*b\*x^2 - 16\*a^4)\*sqrt(b\*x^2 + a))/x^8, -1/128\*(315\*sqrt(-a)\*b^4\*x^8\*arctan(a/(sqrt(b\*x^2 + a)\*sqrt(-a))) - (128\*b^4\*x^8 - 325\*a\*b^3\*x^6 - 210\*a^2\*b^2\*x^4 - 88\*a^3\*b\*x^2 - 16\*a^4)\*sqrt(b\*x^2 + a))/x^8]

**Sympy [A]** time = 35.7254, size = 173, normalized size = 1.35

$$\frac{315\sqrt{ab^4} \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{128} - \frac{a^5}{8\sqrt{bx^9} \sqrt{\frac{a}{bx^2} + 1}} - \frac{13a^4\sqrt{b}}{16x^7 \sqrt{\frac{a}{bx^2} + 1}}$$

$$- \frac{149a^3b^{\frac{3}{2}}}{64x^5 \sqrt{\frac{a}{bx^2} + 1}} - \frac{535a^2b^{\frac{5}{2}}}{128x^3 \sqrt{\frac{a}{bx^2} + 1}} - \frac{197ab^{\frac{7}{2}}}{128x \sqrt{\frac{a}{bx^2} + 1}} + \frac{b^{\frac{9}{2}}x}{\sqrt{\frac{a}{bx^2} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*9, x)

[Out] -315\*sqrt(a)\*b\*\*4\*asinh(sqrt(a)/(sqrt(b)\*x))/128 - a\*\*5/(8\*sqrt(b)\*x\*\*9\*sqrt(a/(b\*x\*\*2) + 1)) - 13\*a\*\*4\*sqrt(b)/(16\*x\*\*7\*sqrt(a/(b\*x\*\*2) + 1)) - 149\*a\*\*3\*b\*\*(3/2)/(64\*x\*\*5\*sqrt(a/(b\*x\*\*2) + 1)) - 535\*a\*\*2\*b\*\*(5/2)/(128\*x\*\*3\*sqrt(a/(b\*x\*\*2) + 1)) - 197\*a\*b\*\*(7/2)/(128\*x\*sqrt(a/(b\*x\*\*2) + 1)) + b\*\*(9/2)\*x/sqrt(a/(b\*x\*\*2) + 1)

**GIAC/XCAS [A]** time = 0.211431, size = 140, normalized size = 1.09

$$\frac{1}{128} \left( \frac{315 a \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 128 \sqrt{bx^2+a} - \frac{325 (bx^2+a)^{\frac{7}{2}} a - 765 (bx^2+a)^{\frac{5}{2}} a^2 + 643 (bx^2+a)^{\frac{3}{2}} a^3 - 187 \sqrt{bx^2+aa^4}}{b^4 x^8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^9, x, algorithm="giac")

[Out] 1/128\*(315\*a\*arctan(sqrt(b\*x^2 + a)/sqrt(-a))/sqrt(-a) + 128\*sqrt(b\*x^2 + a) - (325\*(b\*x^2 + a)^(7/2)\*a - 765\*(b\*x^2 + a)^(5/2)\*a^2 + 643\*(b\*x^2 + a)^(3/2)\*a^3 - 187\*sqrt(b\*x^2 + a)\*a^4)/(b^4\*x^8))

$$3.422 \quad \int \frac{(a+bx^2)^{9/2}}{x^{11}} dx$$

**Optimal.** Leaf size=131

$$\frac{63b^5 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256\sqrt{a}} - \frac{63b^4\sqrt{a+bx^2}}{256x^2} - \frac{21b^3(a+bx^2)^{3/2}}{128x^4} - \frac{21b^2(a+bx^2)^{5/2}}{160x^6} - \frac{(a+bx^2)^{9/2}}{10x^{10}} - \frac{9b(a+bx^2)^{7/2}}{80x^8}$$

[Out]  $(-63*b^4*\text{Sqrt}[a + b*x^2])/(256*x^2) - (21*b^3*(a + b*x^2)^(3/2))/(128*x^4) - (21*b^2*(a + b*x^2)^(5/2))/(160*x^6) - (9*b*(a + b*x^2)^(7/2))/(80*x^8) - (a + b*x^2)^(9/2)/(10*x^{10}) - (63*b^5*\text{ArcTan}[h[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]]]/(256*\text{Sqrt}[a]))$

**Rubi [A]** time = 0.213947, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{63b^5 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256\sqrt{a}} - \frac{63b^4\sqrt{a+bx^2}}{256x^2} - \frac{21b^3(a+bx^2)^{3/2}}{128x^4} - \frac{21b^2(a+bx^2)^{5/2}}{160x^6} - \frac{(a+bx^2)^{9/2}}{10x^{10}} - \frac{9b(a+bx^2)^{7/2}}{80x^8}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(9/2)/x^11, x]

[Out]  $(-63*b^4*\text{Sqrt}[a + b*x^2])/(256*x^2) - (21*b^3*(a + b*x^2)^(3/2))/(128*x^4) - (21*b^2*(a + b*x^2)^(5/2))/(160*x^6) - (9*b*(a + b*x^2)^(7/2))/(80*x^8) - (a + b*x^2)^(9/2)/(10*x^{10}) - (63*b^5*\text{ArcTan}[h[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]]]/(256*\text{Sqrt}[a]))$

**Rubi in Sympy [A]** time = 20.1436, size = 124, normalized size = 0.95

$$\frac{63b^4\sqrt{a+bx^2}}{256x^2} - \frac{21b^3(a+bx^2)^{3/2}}{128x^4} - \frac{21b^2(a+bx^2)^{5/2}}{160x^6} - \frac{9b(a+bx^2)^{7/2}}{80x^8} - \frac{(a+bx^2)^{9/2}}{10x^{10}} - \frac{63b^5 \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*11, x)

[Out]  $-63*b^4*\text{sqrt}(a + b*x^2)/(256*x^2) - 21*b^3*(a + b*x^2)^(3/2)/(128*x^4) - 21*b^2*(a + b*x^2)^(5/2)/(160*x^6) - 9*b*(a + b*x^2)^(7/2)/(80*x^8) - (a + b*x^2)^(9/2)/(10*x^{10}) - 63*b^5*\operatorname{atanh}(\text{sqrt}(a + b*x^2)/\text{sqrt}(a))/(256*\text{sqrt}(a))$

**Mathematica [A]** time = 0.173875, size = 106, normalized size = 0.81

$$\frac{\sqrt{a+bx^2}(128a^4+656a^3bx^2+1368a^2b^2x^4+1490ab^3x^6+965b^4x^8)}{x^{10}} - \frac{315b^5 \log(\sqrt{a}\sqrt{a+bx^2}+a)}{\sqrt{a}} + \frac{315b^5 \log(x)}{\sqrt{a}}$$

1280

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(9/2)/x^11, x]



[Out] 
$$\begin{aligned} & -\left(\frac{\sqrt{a + b^2 x^2} (128 a^4 + 656 a^3 b x^2 + 1368 a^2 b^2 x^4 + 1490 a b^3 x^6 + 965 b^4 x^8)}{x^{10}} + \frac{315 b^5 \operatorname{Log}[x]}{\sqrt{a}}\right) / \sqrt{a} \\ & - \frac{315 b^5 \operatorname{Log}[a + \sqrt{a} \sqrt{a + b^2 x^2}]}{\sqrt{a}} / 1280 \end{aligned}$$

**Maple [B]** time = 0.069, size = 213, normalized size = 1.6

$$\begin{aligned} & -\frac{1}{10 a x^{10}} (b x^2 + a)^{\frac{11}{2}} - \frac{b}{80 a^2 x^8} (b x^2 + a)^{\frac{11}{2}} - \frac{b^2}{160 a^3 x^6} (b x^2 + a)^{\frac{11}{2}} - \frac{b^3}{128 a^4 x^4} (b x^2 + a)^{\frac{11}{2}} \\ & - \frac{7 b^4}{256 a^5 x^2} (b x^2 + a)^{\frac{11}{2}} + \frac{7 b^5}{256 a^5} (b x^2 + a)^{\frac{9}{2}} + \frac{9 b^5}{256 a^4} (b x^2 + a)^{\frac{7}{2}} + \frac{63 b^5}{1280 a^3} (b x^2 + a)^{\frac{5}{2}} \\ & + \frac{21 b^5}{256 a^2} (b x^2 + a)^{\frac{3}{2}} - \frac{63 b^5}{256} \ln\left(\frac{1}{x} (2 a + 2 \sqrt{a} \sqrt{b x^2 + a})\right) \frac{1}{\sqrt{a}} + \frac{63 b^5}{256 a} \sqrt{b x^2 + a} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(9/2)/x^11,x)`

[Out] 
$$\begin{aligned} & -1/10/a/x^{10} * (b*x^2+a)^{(11/2)} - 1/80*b/a^2/x^8 * (b*x^2+a)^{(11/2)} - 1/160*b^2/a^3/x^6 * (b*x^2+a)^{(11/2)} \\ & - 1/128*b^3/a^4/x^4 * (b*x^2+a)^{(11/2)} - 7/256*b^4/a^5/x^2 * (b*x^2+a)^{(11/2)} + 7/256*b^5/a^5 * (b*x^2+a)^{(9/2)} \\ & + 9/256*b^5/a^4 * (b*x^2+a)^{(7/2)} + 63/1280*b^5/a^3 * (b*x^2+a)^{(5/2)} + 21/256*b^5/a^2 * (b*x^2+a)^{(3/2)} \\ & - 63/256*b^5/a^{(1/2)} * \ln((2*a+2*\sqrt{a}\sqrt{bx^2+a})/x) + 63/256*b^5/a * (b*x^2+a)^{(1/2)} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)/x^11,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.313055, size = 1, normalized size = 0.01

$$\left[ \frac{315 b^5 x^{10} \log\left(-\frac{(b x^2 + 2 a) \sqrt{-a} - 2 \sqrt{b x^2 + a a}}{x^2}\right) - 2 (965 b^4 x^8 + 1490 a b^3 x^6 + 1368 a^2 b^2 x^4 + 656 a^3 b x^2 + 128 a^4) \sqrt{b x^2 + a} \sqrt{-a}}{2560 \sqrt{a} x^{10}}, \frac{315 b^5 x^{10} \arctan\left(\frac{\sqrt{-a}}{\sqrt{b x^2 + a}}\right) + (965 b^4 x^8 + 1490 a b^3 x^6 + 1368 a^2 b^2 x^4 + 656 a^3 b x^2 + 128 a^4) \sqrt{b x^2 + a} \sqrt{-a}}{1280 \sqrt{-a} x^{10}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)/x^11,x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & [1/2560 * (315 * b^5 * x^{10} * \log(-((b * x^2 + 2 * a) * \operatorname{sqrt}(a) - 2 * \operatorname{sqrt}(b * x^2 + a) * a) / x^2) - 2 * (965 * b^4 * x^8 + 1490 * a * b^3 * x^6 + 1368 * a^2 * b^2 * x^4 + 656 * a^3 * b * x^2 + 128 * a^4) * \operatorname{sqrt}(b * x^2 + a) * \operatorname{sqrt}(a)) / (\operatorname{sqrt}(a) * x^{10}), \\ & -1/1280 * (315 * b^5 * x^{10} * \operatorname{arctan}(\operatorname{sqrt}(-a) / \operatorname{sqrt}(b * x^2 + a)) + (965 * b^4 * x^8 + 1490 * a * b^3 * x^6 + 1368 * a^2 * b^2 * x^4 + 656 * a^3 * b * x^2 + 128 * a^4) * \operatorname{sqrt}(b * x^2 + a) * \operatorname{sqrt}(-a)) / (\operatorname{sqrt}(-a) * x^{10})] \end{aligned}$$

**Sympy [A]** time = 45.2475, size = 153, normalized size = 1.17

$$\frac{a^4 \sqrt{b} \sqrt{\frac{a}{bx^2} + 1}}{10x^9} - \frac{41a^3 b^{\frac{3}{2}} \sqrt{\frac{a}{bx^2} + 1}}{80x^7} - \frac{171a^2 b^{\frac{5}{2}} \sqrt{\frac{a}{bx^2} + 1}}{160x^5} - \frac{149ab^{\frac{7}{2}} \sqrt{\frac{a}{bx^2} + 1}}{128x^3} - \frac{193b^{\frac{9}{2}} \sqrt{\frac{a}{bx^2} + 1}}{256x} - \frac{63b^5 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{256\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*11,x)

[Out] -a\*\*4\*sqrt(b)\*sqrt(a/(b\*x\*\*2) + 1)/(10\*x\*\*9) - 41\*a\*\*3\*b\*\*(3/2)\*sqrt(a/(b\*x\*\*2) + 1)/(80\*x\*\*7) - 171\*a\*\*2\*b\*\*(5/2)\*sqrt(a/(b\*x\*\*2) + 1)/(160\*x\*\*5) - 149\*a\*b\*\*(7/2)\*sqrt(a/(b\*x\*\*2) + 1)/(128\*x\*\*3) - 193\*b\*\*(9/2)\*sqrt(a/(b\*x\*\*2) + 1)/(256\*x) - 63\*b\*\*5\*asinh(sqrt(a)/(sqrt(b)\*x))/(256\*sqrt(a))

**GIAC/XCAS [A]** time = 0.224892, size = 139, normalized size = 1.06

$$\frac{1}{1280} b^5 \left( \frac{315 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{965 (bx^2 + a)^{\frac{9}{2}} - 2370 (bx^2 + a)^{\frac{7}{2}} a + 2688 (bx^2 + a)^{\frac{5}{2}} a^2 - 1470 (bx^2 + a)^{\frac{3}{2}} a^3 + 315 \sqrt{bx^2+a}}{b^5 x^{10}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^11,x, algorithm="giac")

[Out] 1/1280\*b^5\*(315\*arctan(sqrt(b\*x^2 + a)/sqrt(-a))/sqrt(-a) - (965\*(b\*x^2 + a)^(9/2) - 2370\*(b\*x^2 + a)^(7/2)\*a + 2688\*(b\*x^2 + a)^(5/2)\*a^2 - 1470\*(b\*x^2 + a)^(3/2)\*a^3 + 315\*sqrt(b\*x^2 + a)\*a^4)/(b^5\*x^10)

$$3.423 \quad \int \frac{(a+bx^2)^{9/2}}{x^{13}} dx$$

**Optimal.** Leaf size=155

$$\frac{21b^6 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{1024a^{3/2}} - \frac{21b^5\sqrt{a+bx^2}}{1024ax^2} - \frac{21b^4\sqrt{a+bx^2}}{512x^4} - \frac{7b^3(a+bx^2)^{3/2}}{128x^6} \\ - \frac{21b^2(a+bx^2)^{5/2}}{320x^8} - \frac{(a+bx^2)^{9/2}}{12x^{12}} - \frac{3b(a+bx^2)^{7/2}}{40x^{10}}$$

[Out]  $(-21*b^4*\text{Sqrt}[a + b*x^2])/(512*x^4) - (21*b^5*\text{Sqrt}[a + b*x^2])/(1024*a*x^2) - (7*b^3*(a + b*x^2)^(3/2))/(128*x^6) - (21*b^2*(a + b*x^2)^(5/2))/(320*x^8) - (3*b*(a + b*x^2)^(7/2))/(40*x^10) - (a + b*x^2)^(9/2)/(12*x^12) + (21*b^6*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(1024*a^(3/2))$

**Rubi [A]** time = 0.264059, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{21b^6 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{1024a^{3/2}} - \frac{21b^5\sqrt{a+bx^2}}{1024ax^2} - \frac{21b^4\sqrt{a+bx^2}}{512x^4} - \frac{7b^3(a+bx^2)^{3/2}}{128x^6} \\ - \frac{21b^2(a+bx^2)^{5/2}}{320x^8} - \frac{(a+bx^2)^{9/2}}{12x^{12}} - \frac{3b(a+bx^2)^{7/2}}{40x^{10}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(9/2)/x^13, x]

[Out]  $(-21*b^4*\text{Sqrt}[a + b*x^2])/(512*x^4) - (21*b^5*\text{Sqrt}[a + b*x^2])/(1024*a*x^2) - (7*b^3*(a + b*x^2)^(3/2))/(128*x^6) - (21*b^2*(a + b*x^2)^(5/2))/(320*x^8) - (3*b*(a + b*x^2)^(7/2))/(40*x^10) - (a + b*x^2)^(9/2)/(12*x^12) + (21*b^6*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(1024*a^(3/2))$

**Rubi in Sympy [A]** time = 24.957, size = 144, normalized size = 0.93

$$-\frac{21b^4\sqrt{a+bx^2}}{512x^4} - \frac{7b^3(a+bx^2)^{3/2}}{128x^6} - \frac{21b^2(a+bx^2)^{5/2}}{320x^8} - \frac{3b(a+bx^2)^{7/2}}{40x^{10}} \\ - \frac{(a+bx^2)^{9/2}}{12x^{12}} - \frac{21b^5\sqrt{a+bx^2}}{1024ax^2} + \frac{21b^6 \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{1024a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*13, x)

[Out]  $-21*b^4*\text{sqrt}(a + b*x^2)/(512*x^4) - 7*b^3*(a + b*x^2)^(3/2)/(128*x^6) - 21*b^2*(a + b*x^2)^(5/2)/(320*x^8) - 3*b*(a + b*x^2)^(7/2)/(40*x^10) - (a + b*x^2)^(9/2)/(12*x^12) - 21*b^5*\text{sqrt}(a + b*x^2)/(1024*a*x^2) + 21*b^6*\text{atanh}(\text{sqrt}(a + b*x^2)/\text{sqrt}(a))/(1024*a^(3/2))$

**Mathematica [A]** time = 0.141899, size = 123, normalized size = 0.79

$$-\sqrt{a}\sqrt{a+bx^2}(1280a^5 + 6272a^4bx^2 + 12144a^3b^2x^4 + 11432a^2b^3x^6 + 4910ab^4x^8 + 315b^5x^{10}) + 315b^6x^{12} \log\left(\sqrt{a}\sqrt{a+bx^2} + \sqrt{a}\sqrt{a+bx^2}\right) \\ 15360a^{3/2}x^{12}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(9/2)/x^13, x]

[Out] 
$$\frac{-(\sqrt{a} \sqrt{a + b x^2}) (1280 a^5 + 6272 a^4 b x^2 + 12144 a^3 b^2 x^4 + 11432 a^2 b^3 x^6 + 4910 a b^4 x^8 + 315 b^5 x^{10}) - 315 b^6 x^{12} \operatorname{Log}[x] + 315 b^6 x^{12} \operatorname{Log}[a + \sqrt{a} \sqrt{a + b x^2}]}{(15360 a^{3/2} x^{12})}$$

**Maple [A]** time = 0.17, size = 233, normalized size = 1.5

$$\begin{aligned} & -\frac{1}{12 a x^{12}} (b x^2 + a)^{\frac{11}{2}} + \frac{b}{120 a^2 x^{10}} (b x^2 + a)^{\frac{11}{2}} + \frac{b^2}{960 a^3 x^8} (b x^2 + a)^{\frac{11}{2}} \\ & + \frac{b^3}{1920 a^4 x^6} (b x^2 + a)^{\frac{11}{2}} + \frac{b^4}{1536 a^5 x^4} (b x^2 + a)^{\frac{11}{2}} + \frac{7 b^5}{3072 a^6 x^2} (b x^2 + a)^{\frac{11}{2}} \\ & - \frac{7 b^6}{3072 a^6} (b x^2 + a)^{\frac{9}{2}} - \frac{3 b^6}{1024 a^5} (b x^2 + a)^{\frac{7}{2}} - \frac{21 b^6}{5120 a^4} (b x^2 + a)^{\frac{5}{2}} \\ & - \frac{7 b^6}{1024 a^3} (b x^2 + a)^{\frac{3}{2}} + \frac{21 b^6}{1024} \ln\left(\frac{1}{x} \left(2 a + 2 \sqrt{a} \sqrt{b x^2 + a}\right)\right) a^{-\frac{3}{2}} - \frac{21 b^6}{1024 a^2} \sqrt{b x^2 + a} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(9/2)/x^13, x)

[Out] 
$$\begin{aligned} & -1/12/a/x^{12} * (b*x^2+a)^{(11/2)} + 1/120*b/a^2/x^{10} * (b*x^2+a)^{(11/2)} + 1/960*b^2/a^3/x^8 * (b*x^2+a)^{(11/2)} + 1/1920*b^3/a^4/x^6 * (b*x^2+a)^{(11/2)} \\ & + 1/1536*b^4/a^5/x^4 * (b*x^2+a)^{(11/2)} + 7/3072*b^5/a^6/x^2 * (b*x^2+a)^{(11/2)} - 7/3072*b^6/a^6 * (b*x^2+a)^{(9/2)} - 3/1024*b^6/a^5 * (b*x^2+a)^{(7/2)} \\ & - 21/5120*b^6/a^4 * (b*x^2+a)^{(5/2)} - 7/1024*b^6/a^3 * (b*x^2+a)^{(3/2)} + 21/1024*b^6/a^{(3/2)} * \ln((2*a+2*a^{(1/2)}*(b*x^2+a)^{(1/2)})/x) - 21/1024*b^6/a^2 * (b*x^2+a)^{(1/2)} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^13, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.361881, size = 1, normalized size = 0.01

$$\left[ \frac{315 b^6 x^{12} \log\left(-\frac{(b x^2 + 2 a) \sqrt{a} + 2 \sqrt{b x^2 + a a}}{x^2}\right) - 2 (315 b^5 x^{10} + 4910 a b^4 x^8 + 11432 a^2 b^3 x^6 + 12144 a^3 b^2 x^4 + 6272 a^4 b x^2 + 1280 a^5)}{30720 a^{\frac{3}{2}} x^{12}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^13, x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [1/30720 * (315 * b^6 * x^{12} * \log(-((b * x^2 + 2 * a) * \operatorname{sqrt}(a) + 2 * \operatorname{sqrt}(b * x^2 + a) * a) / x^2) - 2 * (315 * b^5 * x^{10} + 4910 * a * b^4 * x^8 + 11432 * a^2 * b^3 * x^6 + 12144 * a^3 * b^2 * x^4 + 6272 * a^4 * b * x^2 + 1280 * a^5) * \operatorname{sqrt}(b * x^2 + a) * \operatorname{sqrt}(a)) / (a^{(3/2)} * x^{12}), 1/15360 * (315 * b^6 * x^{12} * \operatorname{arctan}(\operatorname{sqrt}(-a) / \operatorname{sqrt}(b * x^2 + a)) - (315 * b^5 * x^{10} + 4910 * a * b^4 * x^8 + 11432 * a^2 * b^3 * x^6 + 12144 * a^3 * b^2 * x^4 + 6272 * a^4 * b * x^2 + 1280 * a^5) * \operatorname{sqrt}(b * x^2 + a) * \operatorname{sqrt}(a)) / (a^{(3/2)} * x^{12})] \end{aligned}$$

$$^3x^6 + 12144a^3b^2x^4 + 6272a^4bx^2 + 1280a^5) \sqrt{bx^2 + a} \sqrt{-a}) / (\sqrt{-a} a x^{12})]$$

---

**Sympy [A]** time = 64.9683, size = 204, normalized size = 1.32

$$\begin{aligned} & -\frac{a^5}{12\sqrt{b}x^{13}\sqrt{\frac{a}{bx^2}+1}} - \frac{59a^4\sqrt{b}}{120x^{11}\sqrt{\frac{a}{bx^2}+1}} - \frac{1151a^3b^{\frac{3}{2}}}{960x^9\sqrt{\frac{a}{bx^2}+1}} - \frac{2947a^2b^{\frac{5}{2}}}{1920x^7\sqrt{\frac{a}{bx^2}+1}} \\ & - \frac{8171ab^{\frac{7}{2}}}{7680x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{1045b^{\frac{9}{2}}}{3072x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{21b^{\frac{11}{2}}}{1024ax\sqrt{\frac{a}{bx^2}+1}} + \frac{21b^6 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{1024a^{\frac{3}{2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*13,x)

[Out] -a\*\*5/(12\*sqrt(b)\*x\*\*13\*sqrt(a/(b\*x\*\*2)+1)) - 59\*a\*\*4\*sqrt(b)/(120\*x\*\*11\*sqrt(a/(b\*x\*\*2)+1)) - 1151\*a\*\*3\*b\*\*(3/2)/(960\*x\*\*9\*sqrt(a/(b\*x\*\*2)+1)) - 2947\*a\*\*2\*b\*\*(5/2)/(1920\*x\*\*7\*sqrt(a/(b\*x\*\*2)+1)) - 8171\*a\*b\*\*(7/2)/(7680\*x\*\*5\*sqrt(a/(b\*x\*\*2)+1)) - 1045\*b\*\*(9/2)/(3072\*x\*\*3\*sqrt(a/(b\*x\*\*2)+1)) - 21\*b\*\*(11/2)/(1024\*a\*x\*sqrt(a/(b\*x\*\*2)+1)) + 21\*b\*\*6\*asinh(sqrt(a)/(sqrt(b)\*x))/(1024\*a\*\*(3/2))

---

**GIAC/XCAS [A]** time = 0.213854, size = 165, normalized size = 1.06

$$-\frac{1}{15360} b^6 \left( \frac{315 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-aa}} + \frac{315 (bx^2+a)^{\frac{11}{2}} + 3335 (bx^2+a)^{\frac{9}{2}} a - 5058 (bx^2+a)^{\frac{7}{2}} a^2 + 4158 (bx^2+a)^{\frac{5}{2}} a^3 - 1785 (bx^2+a)^{\frac{3}{2}} a^4 + 315 \sqrt{bx^2+a} a^5}{ab^6x^{12}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^13,x, algorithm="giac")

[Out] -1/15360\*b^6\*(315\*arctan(sqrt(b\*x^2 + a)/sqrt(-a))/(sqrt(-a)\*a) + (315\*(b\*x^2 + a)^(11/2) + 3335\*(b\*x^2 + a)^(9/2)\*a - 5058\*(b\*x^2 + a)^(7/2)\*a^2 + 4158\*(b\*x^2 + a)^(5/2)\*a^3 - 1785\*(b\*x^2 + a)^(3/2)\*a^4 + 315\*sqrt(b\*x^2 + a)\*a^5)/(a\*b^6\*x^12)

$$3.424 \quad \int \frac{(a+bx^2)^{9/2}}{x^{15}} dx$$

**Optimal.** Leaf size=179

$$\begin{aligned} & -\frac{9b^7 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2048a^{5/2}} + \frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2} - \frac{3b^5\sqrt{a+bx^2}}{1024ax^4} - \frac{3b^4\sqrt{a+bx^2}}{256x^6} \\ & - \frac{3b^3(a+bx^2)^{3/2}}{128x^8} - \frac{3b^2(a+bx^2)^{5/2}}{80x^{10}} - \frac{(a+bx^2)^{9/2}}{14x^{14}} - \frac{3b(a+bx^2)^{7/2}}{56x^{12}} \end{aligned}$$

[Out]  $(-3*b^4*\text{Sqrt}[a + b*x^2])/(256*x^6) - (3*b^5*\text{Sqrt}[a + b*x^2])/(1024*a*x^4) + (9*b^6*\text{Sqrt}[a + b*x^2])/(2048*a^2*x^2) - (3*b^3*(a + b*x^2)^{(3/2)})/(128*x^8) - (3*b^2*(a + b*x^2)^{(5/2)})/(80*x^{10}) - (3*b*(a + b*x^2)^{(7/2)})/(56*x^{12}) - (a + b*x^2)^{(9/2)}/(14*x^{14}) - (9*b^7*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(2048*a^{(5/2)})$

**Rubi [A]** time = 0.313151, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\begin{aligned} & -\frac{9b^7 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2048a^{5/2}} + \frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2} - \frac{3b^5\sqrt{a+bx^2}}{1024ax^4} - \frac{3b^4\sqrt{a+bx^2}}{256x^6} \\ & - \frac{3b^3(a+bx^2)^{3/2}}{128x^8} - \frac{3b^2(a+bx^2)^{5/2}}{80x^{10}} - \frac{(a+bx^2)^{9/2}}{14x^{14}} - \frac{3b(a+bx^2)^{7/2}}{56x^{12}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(9/2)/x^15, x]

[Out]  $(-3*b^4*\text{Sqrt}[a + b*x^2])/(256*x^6) - (3*b^5*\text{Sqrt}[a + b*x^2])/(1024*a*x^4) + (9*b^6*\text{Sqrt}[a + b*x^2])/(2048*a^2*x^2) - (3*b^3*(a + b*x^2)^{(3/2)})/(128*x^8) - (3*b^2*(a + b*x^2)^{(5/2)})/(80*x^{10}) - (3*b*(a + b*x^2)^{(7/2)})/(56*x^{12}) - (a + b*x^2)^{(9/2)}/(14*x^{14}) - (9*b^7*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(2048*a^{(5/2)})$

**Rubi in Sympy [A]** time = 31.688, size = 168, normalized size = 0.94

$$\begin{aligned} & -\frac{3b^4\sqrt{a+bx^2}}{256x^6} - \frac{3b^3(a+bx^2)^{\frac{3}{2}}}{128x^8} - \frac{3b^2(a+bx^2)^{\frac{5}{2}}}{80x^{10}} - \frac{3b(a+bx^2)^{\frac{7}{2}}}{56x^{12}} \\ & - \frac{(a+bx^2)^{\frac{9}{2}}}{14x^{14}} - \frac{3b^5\sqrt{a+bx^2}}{1024ax^4} + \frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2} - \frac{9b^7 \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2048a^{\frac{5}{2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*15, x)

[Out]  $-3*b**4*\text{sqrt}(a + b*x**2)/(256*x**6) - 3*b**3*(a + b*x**2)**(3/2)/(128*x**8) - 3*b**2*(a + b*x**2)**(5/2)/(80*x**10) - 3*b*(a + b*x**2)**(7/2)/(56*x**12) - (a + b*x**2)**(9/2)/(14*x**14) - 3*b**5*\text{sqrt}(a + b*x**2)/(1024*a*x**4) + 9*b**6*\text{sqrt}(a + b*x**2)/(2048*a**2*x**2) - 9*b**7*\text{atanh}(\text{sqrt}(a + b*x**2)/\text{sqrt}(a))/(2048*a**(5/2))$

**Mathematica [A]** time = 0.201484, size = 134, normalized size = 0.75

$$-\sqrt{a}\sqrt{a+bx^2}(5120a^6 + 24320a^5bx^2 + 44928a^4b^2x^4 + 39056a^3b^3x^6 + 14168a^2b^4x^8 + 210ab^5x^{10} - 315b^6x^{12}) - 315b^7x^{14} \log\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{315b^7x^{14}}{71680a^{5/2}x^{14}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(9/2)/x^15, x]

[Out] 
$$\frac{-(\sqrt{a} \sqrt{a + b x^2}) (5120 a^6 + 24320 a^5 b x^2 + 44928 a^4 b^2 x^4 + 39056 a^3 b^3 x^6 + 14168 a^2 b^4 x^8 + 210 a b^5 x^{10} - 315 b^6 x^{12}) + 315 b^7 x^{14} \operatorname{Log}[x] - 315 b^7 x^{14} \operatorname{Log}[\sqrt{a} \sqrt{a + b x^2}]}{(71680 a^{5/2}) x^{14}}$$

**Maple [A]** time = 0.437, size = 253, normalized size = 1.4

$$\begin{aligned} & -\frac{1}{14 a x^{14}} (b x^2 + a)^{\frac{11}{2}} + \frac{b}{56 a^2 x^{12}} (b x^2 + a)^{\frac{11}{2}} - \frac{b^2}{560 a^3 x^{10}} (b x^2 + a)^{\frac{11}{2}} \\ & - \frac{b^3}{4480 a^4 x^8} (b x^2 + a)^{\frac{11}{2}} - \frac{b^4}{8960 a^5 x^6} (b x^2 + a)^{\frac{11}{2}} - \frac{b^5}{7168 a^6 x^4} (b x^2 + a)^{\frac{11}{2}} \\ & - \frac{b^6}{2048 a^7 x^2} (b x^2 + a)^{\frac{11}{2}} + \frac{b^7}{2048 a^7} (b x^2 + a)^{\frac{9}{2}} + \frac{9 b^7}{14336 a^6} (b x^2 + a)^{\frac{7}{2}} + \frac{9 b^7}{10240 a^5} (b x^2 + a)^{\frac{5}{2}} \\ & + \frac{3 b^7}{2048 a^4} (b x^2 + a)^{\frac{3}{2}} - \frac{9 b^7}{2048} \ln\left(\frac{1}{x} \left(2 a + 2 \sqrt{a} \sqrt{b x^2 + a}\right)\right) a^{-\frac{5}{2}} + \frac{9 b^7}{2048 a^3} \sqrt{b x^2 + a} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(9/2)/x^15, x)

[Out] 
$$\frac{-1/14/a/x^{14} * (b*x^2+a)^{(11/2)} + 1/56*b/a^2/x^{12} * (b*x^2+a)^{(11/2)} - 1/560*b^2/a^3/x^{10} * (b*x^2+a)^{(11/2)} - 1/4480*b^3/a^4/x^8 * (b*x^2+a)^{(11/2)} - 1/8960*b^4/a^5/x^6 * (b*x^2+a)^{(11/2)} - 1/7168*b^5/a^6/x^4 * (b*x^2+a)^{(11/2)} - 1/2048*b^6/a^7/x^2 * (b*x^2+a)^{(11/2)} + 1/2048*b^7/a^7 * (b*x^2+a)^{(9/2)} + 9/14336*b^7/a^6 * (b*x^2+a)^{(7/2)} + 9/10240*b^7/a^5 * (b*x^2+a)^{(5/2)} + 3/2048*b^7/a^4 * (b*x^2+a)^{(3/2)} - 9/2048*b^7/a^{(5/2)} * \ln\left(\frac{(2*a+2*a^{(1/2)} * (b*x^2+a)^{(1/2)})}{x}\right) + 9/2048*b^7/a^3 * (b*x^2+a)^{(1/2)}}{1}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^15, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.492759, size = 1, normalized size = 0.01

$$\frac{315 b^7 x^{14} \log\left(-\frac{(b x^2 + 2 a) \sqrt{-a} - 2 \sqrt{b x^2 + a a}}{x^2}\right) + 2 (315 b^6 x^{12} - 210 a b^5 x^{10} - 14168 a^2 b^4 x^8 - 39056 a^3 b^3 x^6 - 44928 a^4 b^2 x^4 - 24320 a^5 b x^2 - 24320 a^6)}{143360 a^{\frac{5}{2}} x^{14}} - \frac{315 b^7 x^{14} \arctan\left(\frac{\sqrt{-a}}{\sqrt{b x^2 + a}}\right) - (315 b^6 x^{12} - 210 a b^5 x^{10} - 14168 a^2 b^4 x^8 - 39056 a^3 b^3 x^6 - 44928 a^4 b^2 x^4 - 24320 a^5 b x^2 - 24320 a^6)}{71680 \sqrt{-a} x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^15, x, algorithm="fricas")

[Out]  $[1/143360*(315*b^7*x^{14}*\log(-(b*x^2 + 2*a)*\sqrt{a}) - 2*\sqrt{b*x^2 + a})/x^2) + 2*(315*b^6*x^{12} - 210*a*b^5*x^{10} - 14168*a^2*b^4*x^8 - 39056*a^3*b^3*x^6 - 44928*a^4*b^2*x^4 - 24320*a^5*b*x^2 - 5120*a^6)*\sqrt{b*x^2 + a}*\sqrt{a})/(a^{(5/2)}*x^{14}), -1/71680*(315*b^7*x^{14}*\arctan(\sqrt{-a}/\sqrt{b*x^2 + a}) - (315*b^6*x^{12} - 210*a*b^5*x^{10} - 14168*a^2*b^4*x^8 - 39056*a^3*b^3*x^6 - 44928*a^4*b^2*x^4 - 24320*a^5*b*x^2 - 5120*a^6)*\sqrt{b*x^2 + a}*\sqrt{-a})/(\sqrt{-a}*a^2*x^{14})]$

**Sympy [A]** time = 92.3171, size = 231, normalized size = 1.29

$$\begin{aligned} & -\frac{a^5}{14\sqrt{b}x^{15}\sqrt{\frac{a}{bx^2}+1}} - \frac{23a^4\sqrt{b}}{56x^{13}\sqrt{\frac{a}{bx^2}+1}} - \frac{541a^3b^{\frac{3}{2}}}{560x^{11}\sqrt{\frac{a}{bx^2}+1}} - \frac{5249a^2b^{\frac{5}{2}}}{4480x^9\sqrt{\frac{a}{bx^2}+1}} - \frac{6653ab^{\frac{7}{2}}}{8960x^7\sqrt{\frac{a}{bx^2}+1}} \\ & - \frac{1027b^{\frac{9}{2}}}{5120x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{3b^{\frac{11}{2}}}{2048ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{9b^{\frac{13}{2}}}{2048a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{9b^7\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2048a^{\frac{5}{2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(9/2)/x**15,x)`

[Out]  $-a^{5/2}/(14*\sqrt{b}*x^{15}*\sqrt{a/(b*x^2) + 1}) - 23*a^{4/2}*\sqrt{b}/(56*x^{13}*\sqrt{a/(b*x^2) + 1}) - 541*a^{3/2}*b^{3/2}/(560*x^{11}*\sqrt{a/(b*x^2) + 1}) - 5249*a^{2/2}*b^{5/2}/(4480*x^9*\sqrt{a/(b*x^2) + 1}) - 6653*a^{1/2}*b^{7/2}/(8960*x^7*\sqrt{a/(b*x^2) + 1}) - 1027*b^{9/2}/(5120*x^5*\sqrt{a/(b*x^2) + 1}) + 3*b^{11/2}/(2048*a*x^3*\sqrt{a/(b*x^2) + 1}) + 9*b^{13/2}/(2048*a^2*x*\sqrt{a/(b*x^2) + 1}) - 9*b^7*\operatorname{asinh}(\sqrt{a}/(\sqrt{b}*x))/ (2048*a^{5/2})$

**GIAC/XCAS [A]** time = 0.213348, size = 184, normalized size = 1.03

$$\frac{1}{71680} b^7 \left( \frac{315 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-aa^2}} + \frac{315 (bx^2 + a)^{\frac{13}{2}} - 2100 (bx^2 + a)^{\frac{11}{2}} a - 8393 (bx^2 + a)^{\frac{9}{2}} a^2 + 9216 (bx^2 + a)^{\frac{7}{2}} a^3 - 5943 (bx^2 + a)^{\frac{5}{2}} a^4 + 2100 (bx^2 + a)^{\frac{3}{2}} a^5 - 315 \sqrt{bx^2 + a} a^6}{a^2 b^7 x^{14}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)/x^15,x, algorithm="giac")`

[Out]  $1/71680*b^7*(315*\arctan(\sqrt{b*x^2 + a}/\sqrt{-a})/(\sqrt{-a}*a^2) + (315*(b*x^2 + a)^{(13/2)} - 2100*(b*x^2 + a)^{(11/2)}*a - 8393*(b*x^2 + a)^{(9/2)}*a^2 + 9216*(b*x^2 + a)^{(7/2)}*a^3 - 5943*(b*x^2 + a)^{(5/2)}*a^4 + 2100*(b*x^2 + a)^{(3/2)}*a^5 - 315*\sqrt{b*x^2 + a}*a^6)/(a^2*b^7*x^{14}))$



$$3.425 \quad \int x^6 (a + bx^2)^{9/2} dx$$

**Optimal.** Leaf size=202

$$\begin{aligned} & -\frac{45a^8 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{32768b^{7/2}} + \frac{45a^7x\sqrt{a+bx^2}}{32768b^3} - \frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2} + \frac{3a^5x^5\sqrt{a+bx^2}}{4096b} + \frac{9a^4x^7\sqrt{a+bx^2}}{2048} \\ & + \frac{3}{256}a^3x^7(a+bx^2)^{3/2} + \frac{3}{128}a^2x^7(a+bx^2)^{5/2} + \frac{9}{224}ax^7(a+bx^2)^{7/2} + \frac{1}{16}x^7(a+bx^2)^{9/2} \end{aligned}$$

[Out] (45\*a^7\*x\*Sqrt[a + b\*x^2])/(32768\*b^3) - (15\*a^6\*x^3\*Sqrt[a + b\*x^2])/(16384\*b^2) + (3\*a^5\*x^5\*Sqrt[a + b\*x^2])/(4096\*b) + (9\*a^4\*x^7\*Sqrt[a + b\*x^2])/2048 + (3\*a^3\*x^7\*(a + b\*x^2)^(3/2))/256 + (3\*a^2\*x^7\*(a + b\*x^2)^(5/2))/128 + (9\*a\*x^7\*(a + b\*x^2)^(7/2))/224 + (x^7\*(a + b\*x^2)^(9/2))/16 - (45\*a^8\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(32768\*b^(7/2))

**Rubi [A]** time = 0.318463, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\begin{aligned} & -\frac{45a^8 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{32768b^{7/2}} + \frac{45a^7x\sqrt{a+bx^2}}{32768b^3} - \frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2} + \frac{3a^5x^5\sqrt{a+bx^2}}{4096b} + \frac{9a^4x^7\sqrt{a+bx^2}}{2048} \\ & + \frac{3}{256}a^3x^7(a+bx^2)^{3/2} + \frac{3}{128}a^2x^7(a+bx^2)^{5/2} + \frac{9}{224}ax^7(a+bx^2)^{7/2} + \frac{1}{16}x^7(a+bx^2)^{9/2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^6\*(a + b\*x^2)^(9/2), x]

[Out] (45\*a^7\*x\*Sqrt[a + b\*x^2])/(32768\*b^3) - (15\*a^6\*x^3\*Sqrt[a + b\*x^2])/(16384\*b^2) + (3\*a^5\*x^5\*Sqrt[a + b\*x^2])/(4096\*b) + (9\*a^4\*x^7\*Sqrt[a + b\*x^2])/2048 + (3\*a^3\*x^7\*(a + b\*x^2)^(3/2))/256 + (3\*a^2\*x^7\*(a + b\*x^2)^(5/2))/128 + (9\*a\*x^7\*(a + b\*x^2)^(7/2))/224 + (x^7\*(a + b\*x^2)^(9/2))/16 - (45\*a^8\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(32768\*b^(7/2))

**Rubi in Sympy [A]** time = 38.3372, size = 192, normalized size = 0.95

$$\begin{aligned} & -\frac{45a^8 \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{32768b^{7/2}} + \frac{45a^7x\sqrt{a+bx^2}}{32768b^3} - \frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2} + \frac{3a^5x^5\sqrt{a+bx^2}}{4096b} \\ & + \frac{9a^4x^7\sqrt{a+bx^2}}{2048} + \frac{3a^3x^7(a+bx^2)^{3/2}}{256} + \frac{3a^2x^7(a+bx^2)^{5/2}}{128} + \frac{9ax^7(a+bx^2)^{7/2}}{224} + \frac{x^7(a+bx^2)^{9/2}}{16} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6\*(b\*x\*\*2+a)\*\*(9/2), x)

[Out] -45\*a\*\*8\*atanh(sqrt(b)\*x/sqrt(a + b\*x\*\*2))/(32768\*b\*\*(7/2)) + 45\*a\*\*7\*x\*sqrt(a + b\*x\*\*2)/(32768\*b\*\*3) - 15\*a\*\*6\*x\*\*3\*sqrt(a + b\*x\*\*2)/(16384\*b\*\*2) + 3\*a\*\*5\*x\*\*5\*sqrt(a + b\*x\*\*2)/(4096\*b) + 9\*a\*\*4\*x\*\*7\*sqrt(a + b\*x\*\*2)/2048 + 3\*a\*\*3\*x\*\*7\*(a + b\*x\*\*2)\*\*(3/2)/256 + 3\*a\*\*2\*x\*\*7\*(a + b\*x\*\*2)\*\*(5/2)/128 + 9\*a\*x\*\*7\*(a + b\*x\*\*2)\*\*(7/2)/224 + x\*\*7\*(a + b\*x\*\*2)\*\*(9/2)/16

**Mathematica [A]** time = 0.125762, size = 131, normalized size = 0.65

$$\sqrt{bx}\sqrt{a+bx^2}(315a^7 - 210a^6bx^2 + 168a^5b^2x^4 + 32624a^4b^3x^6 + 98432a^3b^4x^8 + 119040a^2b^5x^{10} + 66560ab^6x^{12} + 14336b^7x^{14}) + 229376b^{7/2}$$

Antiderivative was successfully verified.

[In] Integrate[x^6\*(a + b\*x^2)^(9/2), x]

[Out] (Sqrt[b]\*x\*Sqrt[a + b\*x^2]\*(315\*a^7 - 210\*a^6\*b\*x^2 + 168\*a^5\*b^2\*x^4 + 32624\*a^4\*b^3\*x^6 + 98432\*a^3\*b^4\*x^8 + 119040\*a^2\*b^5\*x^10 + 66560\*a\*b^6\*x^12 + 14336\*b^7\*x^14) - 315\*a^8\*Log[b\*x + Sqrt[b]\*Sqrt[a + b\*x^2]])/(229376\*b^(7/2))

**Maple [A]** time = 0.016, size = 169, normalized size = 0.8

$$\begin{aligned} & \frac{x^5}{16b} (bx^2 + a)^{\frac{11}{2}} - \frac{5ax^3}{224b^2} (bx^2 + a)^{\frac{11}{2}} + \frac{5a^2x}{896b^3} (bx^2 + a)^{\frac{11}{2}} - \frac{a^3x}{1792b^3} (bx^2 + a)^{\frac{9}{2}} \\ & - \frac{9a^4x}{14336b^3} (bx^2 + a)^{\frac{7}{2}} - \frac{3a^5x}{4096b^3} (bx^2 + a)^{\frac{5}{2}} - \frac{15a^6x}{16384b^3} (bx^2 + a)^{\frac{3}{2}} \\ & - \frac{45a^7x}{32768b^3} \sqrt{bx^2 + a} - \frac{45a^8}{32768} \ln(x\sqrt{b} + \sqrt{bx^2 + a}) b^{-\frac{7}{2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6\*(b\*x^2+a)^(9/2), x)

[Out] 1/16\*x^5\*(b\*x^2+a)^(11/2)/b-5/224\*a/b^2\*x^3\*(b\*x^2+a)^(11/2)+5/896\*a^2/b^3\*x\*(b\*x^2+a)^(11/2)-1/1792\*a^3/b^3\*x\*(b\*x^2+a)^(9/2)-9/14336\*a^4/b^3\*x\*(b\*x^2+a)^(7/2)-3/4096\*a^5/b^3\*x\*(b\*x^2+a)^(5/2)-15/16384\*a^6/b^3\*x\*(b\*x^2+a)^(3/2)-45/32768\*a^7\*x\*(b\*x^2+a)^(1/2)/b^3-45/32768\*a^8/b^(7/2)\*ln(x\*b^(1/2)+(b\*x^2+a)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)\*x^6,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.666697, size = 1, normalized size = 0.

$$\frac{315a^8 \log\left(2\sqrt{bx^2+ax} - (2bx^2+a)\sqrt{b}\right) + 2(14336b^7x^{15} + 66560ab^6x^{13} + 119040a^2b^5x^{11} + 98432a^3b^4x^9 + 32624a^4b^3x^7 + 168a^5b^2x^5 - 229376a^6b^2x^3 + 14336a^7x)}{458752b^{\frac{7}{2}}}$$

$$- \frac{315a^8 \arctan\left(\frac{\sqrt{-bx}}{\sqrt{bx^2+a}}\right) - (14336b^7x^{15} + 66560ab^6x^{13} + 119040a^2b^5x^{11} + 98432a^3b^4x^9 + 32624a^4b^3x^7 + 168a^5b^2x^5 - 229376a^6b^2x^3 + 14336a^7x)}{229376\sqrt{-bb^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)\*x^6,x, algorithm="fricas")

[Out] [1/458752\*(315\*a^8\*log(2\*sqrt(b\*x^2 + a)\*b\*x - (2\*b\*x^2 + a)\*sqrt(b)) + 2\*(14336\*b^7\*x^15 + 66560\*a\*b^6\*x^13 + 119040\*a^2\*b^5\*x^11 + 98432\*a^3\*b^4\*x^9 + 32624\*a^4\*b^3\*x^7 + 168\*a^5\*b^2\*x^5 - 210\*a^6\*b\*x^3 + 315\*a^7\*x)\*sqrt(b\*x^2 + a)\*sqrt(b))/b^(7/2), -1/229376\*(315\*a^8\*arctan(sqrt(-b)\*x/sqrt(b\*x^2 + a)) - (14336\*b^7\*x^15 + 66560\*a\*b^6\*x^13 + 119040\*a^2\*b^5\*x^11 + 98432\*a^3\*b^4\*x^9 + 32624\*a^4\*b^3\*x^7 + 168\*a^5\*b^2\*x^5 - 210\*a^6\*b\*x^3 + 315\*a^7\*x))]

$$24*a^4*b^3*x^7 + 168*a^5*b^2*x^5 - 210*a^6*b*x^3 + 315*a^7*x) * \text{sqrt}(b*x^2 + a) * \text{sqrt}(-b) / (\text{sqrt}(-b) * b^3)]$$

**Sympy [A]** time = 100.404, size = 258, normalized size = 1.28

$$\frac{45a^{\frac{15}{2}}x}{32768b^3\sqrt{1+\frac{bx^2}{a}}} + \frac{15a^{\frac{13}{2}}x^3}{32768b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a^{\frac{11}{2}}x^5}{16384b\sqrt{1+\frac{bx^2}{a}}} + \frac{4099a^{\frac{9}{2}}x^7}{28672\sqrt{1+\frac{bx^2}{a}}} + \frac{8191a^{\frac{7}{2}}bx^9}{14336\sqrt{1+\frac{bx^2}{a}}} + \frac{1699a^{\frac{5}{2}}b^2x^{11}}{1792\sqrt{1+\frac{bx^2}{a}}} + \frac{725a^{\frac{3}{2}}b^3x^{13}}{896\sqrt{1+\frac{bx^2}{a}}} + \frac{79\sqrt{ab^4}x^{15}}{224\sqrt{1+\frac{bx^2}{a}}} - \frac{45a^8 \operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{32768b^{\frac{7}{2}}} + \frac{b^5x^{17}}{16\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*6\*(b\*x\*\*2+a)\*\*(9/2),x)

[Out]  $45*a^{(15/2)}*x/(32768*b^{**3}*\text{sqrt}(1 + b*x^{**2}/a)) + 15*a^{(13/2)}*x^{**3}/(32768*b^{**2}*\text{sqrt}(1 + b*x^{**2}/a)) - 3*a^{(11/2)}*x^{**5}/(16384*b*\text{sqrt}(1 + b*x^{**2}/a)) + 4099*a^{(9/2)}*x^{**7}/(28672*\text{sqrt}(1 + b*x^{**2}/a)) + 8191*a^{(7/2)}*b*x^{**9}/(14336*\text{sqrt}(1 + b*x^{**2}/a)) + 1699*a^{(5/2)}*b^{**2}*x^{**11}/(1792*\text{sqrt}(1 + b*x^{**2}/a)) + 725*a^{(3/2)}*b^{**3}*x^{**13}/(896*\text{sqrt}(1 + b*x^{**2}/a)) + 79*\text{sqrt}(a)*b^{**4}*x^{**15}/(224*\text{sqrt}(1 + b*x^{**2}/a)) - 45*a^{**8}*\text{asinh}(\text{sqrt}(b)*x/\text{sqrt}(a))/(32768*b^{(7/2)}) + b^{**5}*x^{**17}/(16*\text{sqrt}(a)*\text{sqrt}(1 + b*x^{**2}/a))$

**GIAC/XCAS [A]** time = 0.214166, size = 180, normalized size = 0.89

$$\frac{45a^8 \ln\left(\left|-\sqrt{bx} + \sqrt{bx^2 + a}\right|\right)}{32768b^{\frac{7}{2}}} + \frac{1}{229376} \left( \frac{315a^7}{b^3} - 2 \left( \frac{105a^6}{b^2} - 4 \left( \frac{21a^5}{b} + 2(2039a^4 + 8(769a^3b + 2(465a^2b^2 + 4(14b^4x^2 + 65ab^3)x^2)x^2)x^2 \right) x^2 \right) x^2 \right) x^2 \right) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)\*x^6,x, algorithm="giac")

[Out]  $45/32768*a^8*\ln(\text{abs}(-\text{sqrt}(b)*x + \text{sqrt}(b*x^2 + a)))/b^{(7/2)} + 1/229376*(315*a^7/b^3 - 2*(105*a^6/b^2 - 4*(21*a^5/b + 2*(2039*a^4 + 8*(769*a^3*b + 2*(465*a^2*b^2 + 4*(14*b^4*x^2 + 65*a*b^3)*x^2)*x^2)*x^2)*x^2)*x^2)*\text{sqrt}(b*x^2 + a)*x$

### 3.426 $\int x^4 (a + bx^2)^{9/2} dx$

**Optimal.** Leaf size=178

$$\frac{9a^7 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{2048b^{5/2}} - \frac{9a^6x\sqrt{a+bx^2}}{2048b^2} + \frac{3a^5x^3\sqrt{a+bx^2}}{1024b} + \frac{3}{256}a^4x^5\sqrt{a+bx^2} + \frac{3}{128}a^3x^5(a+bx^2)^{3/2} + \frac{3}{80}a^2x^5(a+bx^2)^{5/2} + \frac{3}{56}ax^5(a+bx^2)^{7/2} + \frac{1}{14}x^5(a+bx^2)^{9/2}$$

[Out]  $(-9*a^6*x*\text{Sqrt}[a + b*x^2])/(2048*b^2) + (3*a^5*x^3*\text{Sqrt}[a + b*x^2])/(1024*b) + (3*a^4*x^5*\text{Sqrt}[a + b*x^2])/256 + (3*a^3*x^5*(a + b*x^2)^{(3/2)})/128 + (3*a^2*x^5*(a + b*x^2)^{(5/2)})/80 + (3*a*x^5*(a + b*x^2)^{(7/2)})/56 + (x^5*(a + b*x^2)^{(9/2)})/14 + (9*a^7*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]])/(2048*b^{(5/2)})$

**Rubi [A]** time = 0.25589, antiderivative size = 178, normalized size of antiderivative = 1., number of rules used = 9, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{9a^7 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{2048b^{5/2}} - \frac{9a^6x\sqrt{a+bx^2}}{2048b^2} + \frac{3a^5x^3\sqrt{a+bx^2}}{1024b} + \frac{3}{256}a^4x^5\sqrt{a+bx^2} + \frac{3}{128}a^3x^5(a+bx^2)^{3/2} + \frac{3}{80}a^2x^5(a+bx^2)^{5/2} + \frac{3}{56}ax^5(a+bx^2)^{7/2} + \frac{1}{14}x^5(a+bx^2)^{9/2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^4*(a + b*x^2)^{(9/2)}, x]$

[Out]  $(-9*a^6*x*\text{Sqrt}[a + b*x^2])/(2048*b^2) + (3*a^5*x^3*\text{Sqrt}[a + b*x^2])/(1024*b) + (3*a^4*x^5*\text{Sqrt}[a + b*x^2])/256 + (3*a^3*x^5*(a + b*x^2)^{(3/2)})/128 + (3*a^2*x^5*(a + b*x^2)^{(5/2)})/80 + (3*a*x^5*(a + b*x^2)^{(7/2)})/56 + (x^5*(a + b*x^2)^{(9/2)})/14 + (9*a^7*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]])/(2048*b^{(5/2)})$

**Rubi in Sympy [A]** time = 31.0129, size = 168, normalized size = 0.94

$$\frac{9a^7 \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{2048b^{5/2}} - \frac{9a^6x\sqrt{a+bx^2}}{2048b^2} + \frac{3a^5x^3\sqrt{a+bx^2}}{1024b} + \frac{3a^4x^5\sqrt{a+bx^2}}{256} + \frac{3a^3x^5(a+bx^2)^{3/2}}{128} + \frac{3a^2x^5(a+bx^2)^{5/2}}{80} + \frac{3ax^5(a+bx^2)^{7/2}}{56} + \frac{x^5(a+bx^2)^{9/2}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**4*(b*x**2+a)**(9/2), x)$

[Out]  $9*a**7*\operatorname{atanh}(\text{sqrt}(b)*x/\text{sqrt}(a + b*x**2))/(2048*b**(5/2)) - 9*a**6*x*\text{sqrt}(a + b*x**2)/(2048*b**2) + 3*a**5*x**3*\text{sqrt}(a + b*x**2)/(1024*b) + 3*a**4*x**5*\text{sqrt}(a + b*x**2)/256 + 3*a**3*x**5*(a + b*x**2)**(3/2)/128 + 3*a**2*x**5*(a + b*x**2)**(5/2)/80 + 3*a*x**5*(a + b*x**2)**(7/2)/56 + x**5*(a + b*x**2)**(9/2)/14$

**Mathematica [A]** time = 0.0982156, size = 120, normalized size = 0.67

$$315a^7 \log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right) + \sqrt{bx}\sqrt{a+bx^2}(-315a^6 + 210a^5bx^2 + 14168a^4b^2x^4 + 39056a^3b^3x^6 + 44928a^2b^4x^8 + 24320ab^5x^{10}) + 71680b^{5/2}x^5(a+bx^2)^{9/2}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a + b\*x^2)^(9/2),x]

[Out] (Sqrt[b]\*x\*Sqrt[a + b\*x^2]\*(-315\*a^6 + 210\*a^5\*b\*x^2 + 14168\*a^4\*b^2\*x^4 + 39056\*a^3\*b^3\*x^6 + 44928\*a^2\*b^4\*x^8 + 24320\*a\*b^5\*x^10 + 5120\*b^6\*x^12) + 315\*a^7\*Log[b\*x + Sqrt[b]\*Sqrt[a + b\*x^2]])/(71680\*b^(5/2))

**Maple [A]** time = 0.012, size = 149, normalized size = 0.8

$$\frac{x^3}{14b} (bx^2 + a)^{\frac{11}{2}} - \frac{ax}{56b^2} (bx^2 + a)^{\frac{11}{2}} + \frac{a^2x}{560b^2} (bx^2 + a)^{\frac{9}{2}} + \frac{9a^3x}{4480b^2} (bx^2 + a)^{\frac{7}{2}} + \frac{3a^4x}{1280b^2} (bx^2 + a)^{\frac{5}{2}} + \frac{3a^5x}{1024b^2} (bx^2 + a)^{\frac{3}{2}} + \frac{9a^6x}{2048b^2} \sqrt{bx^2 + a} + \frac{9a^7}{2048} \ln(x\sqrt{b} + \sqrt{bx^2 + a}) b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(b\*x^2+a)^(9/2),x)

[Out] 1/14\*x^3\*(b\*x^2+a)^(11/2)/b-1/56\*a/b^2\*x\*(b\*x^2+a)^(11/2)+1/560\*a^2/b^2\*x\*(b\*x^2+a)^(9/2)+9/4480\*a^3/b^2\*x\*(b\*x^2+a)^(7/2)+3/1280\*a^4/b^2\*x\*(b\*x^2+a)^(5/2)+3/1024\*a^5/b^2\*x\*(b\*x^2+a)^(3/2)+9/2048\*a^6\*x\*(b\*x^2+a)^(1/2)/b^2+9/2048\*a^7/b^(5/2)\*ln(x\*b^(1/2)+(b\*x^2+a)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)\*x^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.490239, size = 1, normalized size = 0.01

$$\frac{315 a^7 \log(-2 \sqrt{bx^2 + a} bx - (2 bx^2 + a) \sqrt{b}) + 2 (5120 b^6 x^{13} + 24320 ab^5 x^{11} + 44928 a^2 b^4 x^9 + 39056 a^3 b^3 x^7 + 14168 a^4 b^2 x^5 + 315 a^5 b x^3 - 315 a^6 x) \sqrt{bx^2 + a} \sqrt{b}}{143360 b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)\*x^4,x, algorithm="fricas")

[Out] [1/143360\*(315\*a^7\*log(-2\*sqrt(b\*x^2 + a)\*b\*x - (2\*b\*x^2 + a)\*sqrt(b)) + 2\*(5120\*b^6\*x^13 + 24320\*a\*b^5\*x^11 + 44928\*a^2\*b^4\*x^9 + 39056\*a^3\*b^3\*x^7 + 14168\*a^4\*b^2\*x^5 + 210\*a^5\*b\*x^3 - 315\*a^6\*x)\*sqrt(b\*x^2 + a)\*sqrt(b))/b^(5/2), 1/71680\*(315\*a^7\*arctan(sqrt(-b)\*x/sqrt(b\*x^2 + a)) + (5120\*b^6\*x^13 + 24320\*a\*b^5\*x^11 + 44928\*a^2\*b^4\*x^9 + 39056\*a^3\*b^3\*x^7 + 14168\*a^4\*b^2\*x^5 + 210\*a^5\*b\*x^3 - 315\*a^6\*x)\*sqrt(b\*x^2 + a)\*sqrt(-b))/(sqrt(-b)\*b^2)]

**Sympy [A]** time = 74.3441, size = 231, normalized size = 1.3

$$\begin{aligned} & -\frac{9a^{\frac{13}{2}}x}{2048b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a^{\frac{11}{2}}x^3}{2048b\sqrt{1+\frac{bx^2}{a}}} + \frac{1027a^{\frac{9}{2}}x^5}{5120\sqrt{1+\frac{bx^2}{a}}} + \frac{6653a^{\frac{7}{2}}bx^7}{8960\sqrt{1+\frac{bx^2}{a}}} \\ & + \frac{5249a^{\frac{5}{2}}b^2x^9}{4480\sqrt{1+\frac{bx^2}{a}}} + \frac{541a^{\frac{3}{2}}b^3x^{11}}{560\sqrt{1+\frac{bx^2}{a}}} + \frac{23\sqrt{ab^4}x^{13}}{56\sqrt{1+\frac{bx^2}{a}}} + \frac{9a^7 \operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2048b^{\frac{5}{2}}} + \frac{b^5x^{15}}{14\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(b\*x\*\*2+a)\*\*(9/2),x)

[Out]  $-9*a^{13/2}*x/(2048*b^{5/2}*sqrt(1+b*x^2/a)) - 3*a^{11/2}*x^3/(2048*b*sqrt(1+b*x^2/a)) + 1027*a^{9/2}*x^5/(5120*sqrt(1+b*x^2/a)) + 6653*a^{7/2}*b*x^7/(8960*sqrt(1+b*x^2/a)) + 5249*a^{5/2}*b^2*x^9/(4480*sqrt(1+b*x^2/a)) + 541*a^{3/2}*b^3*x^{11}/(560*sqrt(1+b*x^2/a)) + 23*sqrt(a)*b^4*x^{13}/(56*sqrt(1+b*x^2/a)) + 9*a^7*asinh(sqrt(b)*x/sqrt(a))/(2048*b^{5/2}) + b^5*x^{15}/(14*sqrt(a)*sqrt(1+b*x^2/a))$

**GIAC/XCAS [A]** time = 0.213462, size = 161, normalized size = 0.9

$$\begin{aligned} & -\frac{9a^7 \ln\left(\left|-\sqrt{bx} + \sqrt{bx^2 + a}\right|\right)}{2048b^{\frac{5}{2}}} \\ & -\frac{1}{71680} \left( \frac{315a^6}{b^2} - 2 \left( \frac{105a^5}{b} + 4(1771a^4 + 2(2441a^3b + 8(351a^2b^2 + 10(4b^4x^2 + 19ab^3)x^2)x^2)x^2)x^2 \right) \sqrt{bx^2 + ax} \right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)\*x^4,x, algorithm="giac")

[Out]  $-9/2048*a^7*\ln(\operatorname{abs}(-\operatorname{sqrt}(b)*x + \operatorname{sqrt}(b*x^2 + a)))/b^{5/2} - 1/71680*(315*a^6/b^2 - 2*(105*a^5/b + 4*(1771*a^4 + 2*(2441*a^3*b + 8*(351*a^2*b^2 + 10*(4*b^4*x^2 + 19*a*b^3)*x^2)*x^2)*x^2)*\operatorname{sqrt}(b*x^2 + a)*x$

$$3.427 \quad \int x^2 (a + bx^2)^{9/2} dx$$

**Optimal.** Leaf size=154

$$-\frac{21a^6 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{1024b^{3/2}} + \frac{21a^5 x \sqrt{a+bx^2}}{1024b} + \frac{21}{512} a^4 x^3 \sqrt{a+bx^2} + \frac{7}{128} a^3 x^3 (a+bx^2)^{3/2} + \frac{21}{320} a^2 x^3 (a+bx^2)^{5/2} + \frac{3}{40} a x^3 (a+bx^2)^{7/2} + \frac{1}{12} x^3 (a+bx^2)^{9/2}$$

[Out] (21\*a^5\*x\*Sqrt[a + b\*x^2])/(1024\*b) + (21\*a^4\*x^3\*Sqrt[a + b\*x^2])/512 + (7\*a^3\*x^3\*(a + b\*x^2)^(3/2))/128 + (21\*a^2\*x^3\*(a + b\*x^2)^(5/2))/320 + (3\*a\*x^3\*(a + b\*x^2)^(7/2))/40 + (x^3\*(a + b\*x^2)^(9/2))/12 - (21\*a^6\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(1024\*b^(3/2))

**Rubi [A]** time = 0.203663, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{21a^6 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{1024b^{3/2}} + \frac{21a^5 x \sqrt{a+bx^2}}{1024b} + \frac{21}{512} a^4 x^3 \sqrt{a+bx^2} + \frac{7}{128} a^3 x^3 (a+bx^2)^{3/2} + \frac{21}{320} a^2 x^3 (a+bx^2)^{5/2} + \frac{3}{40} a x^3 (a+bx^2)^{7/2} + \frac{1}{12} x^3 (a+bx^2)^{9/2}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*x^2)^(9/2), x]

[Out] (21\*a^5\*x\*Sqrt[a + b\*x^2])/(1024\*b) + (21\*a^4\*x^3\*Sqrt[a + b\*x^2])/512 + (7\*a^3\*x^3\*(a + b\*x^2)^(3/2))/128 + (21\*a^2\*x^3\*(a + b\*x^2)^(5/2))/320 + (3\*a\*x^3\*(a + b\*x^2)^(7/2))/40 + (x^3\*(a + b\*x^2)^(9/2))/12 - (21\*a^6\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(1024\*b^(3/2))

**Rubi in Sympy [A]** time = 25.7598, size = 144, normalized size = 0.94

$$-\frac{21a^6 \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{1024b^{\frac{3}{2}}} + \frac{21a^5 x \sqrt{a+bx^2}}{1024b} + \frac{21a^4 x^3 \sqrt{a+bx^2}}{512} + \frac{7a^3 x^3 (a+bx^2)^{\frac{3}{2}}}{128} + \frac{21a^2 x^3 (a+bx^2)^{\frac{5}{2}}}{320} + \frac{3ax^3 (a+bx^2)^{\frac{7}{2}}}{40} + \frac{x^3 (a+bx^2)^{\frac{9}{2}}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(b\*x\*\*2+a)\*\*(9/2), x)

[Out] -21\*a\*\*6\*atanh(sqrt(b)\*x/sqrt(a + b\*x\*\*2))/(1024\*b\*\*(3/2)) + 21\*a\*\*5\*x\*sqrt(a + b\*x\*\*2)/(1024\*b) + 21\*a\*\*4\*x\*\*3\*sqrt(a + b\*x\*\*2)/512 + 7\*a\*\*3\*x\*\*3\*(a + b\*x\*\*2)\*\*(3/2)/128 + 21\*a\*\*2\*x\*\*3\*(a + b\*x\*\*2)\*\*(5/2)/320 + 3\*a\*x\*\*3\*(a + b\*x\*\*2)\*\*(7/2)/40 + x\*\*3\*(a + b\*x\*\*2)\*\*(9/2)/12

**Mathematica [A]** time = 0.091718, size = 109, normalized size = 0.71

$$\frac{\sqrt{bx}\sqrt{a+bx^2} (315a^5 + 4910a^4bx^2 + 11432a^3b^2x^4 + 12144a^2b^3x^6 + 6272ab^4x^8 + 1280b^5x^{10}) - 315a^6 \log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right)}{15360b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*x^2)^(9/2),x]

[Out] (Sqrt[b]\*x\*Sqrt[a + b\*x^2]\*(315\*a^5 + 4910\*a^4\*b\*x^2 + 11432\*a^3\*b^2\*x^4 + 12144\*a^2\*b^3\*x^6 + 6272\*a\*b^4\*x^8 + 1280\*b^5\*x^10) - 3\*15\*a^6\*Log[b\*x + Sqrt[b]\*Sqrt[a + b\*x^2]])/(15360\*b^(3/2))

**Maple [A]** time = 0.009, size = 129, normalized size = 0.8

$$\frac{x}{12b} (bx^2 + a)^{\frac{11}{2}} - \frac{ax}{120b} (bx^2 + a)^{\frac{9}{2}} - \frac{3a^2x}{320b} (bx^2 + a)^{\frac{7}{2}} - \frac{7a^3x}{640b} (bx^2 + a)^{\frac{5}{2}} - \frac{7a^4x}{512b} (bx^2 + a)^{\frac{3}{2}} - \frac{21a^5x}{1024b} \sqrt{bx^2 + a} - \frac{21a^6}{1024} \ln(x\sqrt{b} + \sqrt{bx^2 + a}) b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(b\*x^2+a)^(9/2),x)

[Out] 1/12\*x\*(b\*x^2+a)^(11/2)/b-1/120\*a/b\*x\*(b\*x^2+a)^(9/2)-3/320\*a^2/b\*x\*(b\*x^2+a)^(7/2)-7/640\*a^3/b\*x\*(b\*x^2+a)^(5/2)-7/512\*a^4/b\*x\*(b\*x^2+a)^(3/2)-21/1024\*a^5\*x\*(b\*x^2+a)^(1/2)/b-21/1024\*a^6/b^(3/2)\*ln(x\*b^(1/2)+(b\*x^2+a)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)\*x^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.381089, size = 1, normalized size = 0.01

$$\frac{315 a^6 \log\left(2 \sqrt{bx^2 + abx} - (2bx^2 + a)\sqrt{b}\right) + 2(1280 b^5 x^{11} + 6272 ab^4 x^9 + 12144 a^2 b^3 x^7 + 11432 a^3 b^2 x^5 + 4910 a^4 b x^3 + 315 a^5 x)}{30720 b^{\frac{3}{2}}} - \frac{315 a^6 \arctan\left(\frac{\sqrt{-bx}}{\sqrt{bx^2+a}}\right) - (1280 b^5 x^{11} + 6272 ab^4 x^9 + 12144 a^2 b^3 x^7 + 11432 a^3 b^2 x^5 + 4910 a^4 b x^3 + 315 a^5 x) \sqrt{bx^2 + a} \sqrt{-b}}{15360 \sqrt{-bb}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)\*x^2,x, algorithm="fricas")

[Out] [1/30720\*(315\*a^6\*log(2\*sqrt(b\*x^2 + a)\*b\*x - (2\*b\*x^2 + a)\*sqrt(b)) + 2\*(1280\*b^5\*x^11 + 6272\*a\*b^4\*x^9 + 12144\*a^2\*b^3\*x^7 + 11432\*a^3\*b^2\*x^5 + 4910\*a^4\*b\*x^3 + 315\*a^5\*x)\*sqrt(b\*x^2 + a)\*sqrt(b))/b^(3/2), -1/15360\*(315\*a^6\*arctan(sqrt(-b)\*x/sqrt(b\*x^2 + a)) - (1280\*b^5\*x^11 + 6272\*a\*b^4\*x^9 + 12144\*a^2\*b^3\*x^7 + 11432\*a^3\*b^2\*x^5 + 4910\*a^4\*b\*x^3 + 315\*a^5\*x)\*sqrt(b\*x^2 + a)\*sqrt(-b))/(sqrt(-b)\*b)]



**Sympy [A]** time = 54.2865, size = 204, normalized size = 1.32

$$\frac{21a^{\frac{11}{2}}x}{1024b\sqrt{1+\frac{bx^2}{a}}} + \frac{1045a^{\frac{9}{2}}x^3}{3072\sqrt{1+\frac{bx^2}{a}}} + \frac{8171a^{\frac{7}{2}}bx^5}{7680\sqrt{1+\frac{bx^2}{a}}} + \frac{2947a^{\frac{5}{2}}b^2x^7}{1920\sqrt{1+\frac{bx^2}{a}}} + \frac{1151a^{\frac{3}{2}}b^3x^9}{960\sqrt{1+\frac{bx^2}{a}}} + \frac{59\sqrt{ab^4}x^{11}}{120\sqrt{1+\frac{bx^2}{a}}} - \frac{21a^6 \operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{1024b^{\frac{3}{2}}} + \frac{b^5x^{13}}{12\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(b\*x\*\*2+a)\*\*(9/2),x)

[Out] 21\*a\*\*(11/2)\*x/(1024\*b\*sqrt(1 + b\*x\*\*2/a)) + 1045\*a\*\*(9/2)\*x\*\*3/(3072\*sqrt(1 + b\*x\*\*2/a)) + 8171\*a\*\*(7/2)\*b\*x\*\*5/(7680\*sqrt(1 + b\*x\*\*2/a)) + 2947\*a\*\*(5/2)\*b\*\*2\*x\*\*7/(1920\*sqrt(1 + b\*x\*\*2/a)) + 1151\*a\*\*(3/2)\*b\*\*3\*x\*\*9/(960\*sqrt(1 + b\*x\*\*2/a)) + 59\*sqrt(a)\*b\*\*4\*x\*\*11/(120\*sqrt(1 + b\*x\*\*2/a)) - 21\*a\*\*6\*asinh(sqrt(b)\*x/sqrt(a))/(1024\*b\*\*(3/2)) + b\*\*5\*x\*\*13/(12\*sqrt(a)\*sqrt(1 + b\*x\*\*2/a))

**GIAC/XCAS [A]** time = 0.217041, size = 142, normalized size = 0.92

$$\frac{21a^6 \ln\left(\left|-\sqrt{bx} + \sqrt{bx^2 + a}\right|\right)}{1024b^{\frac{3}{2}}} + \frac{1}{15360} \left( \frac{315a^5}{b} + 2(2455a^4 + 4(1429a^3b + 2(759a^2b^2 + 8(10b^4x^2 + 49ab^3)x^2)x^2)x^2) \right) \sqrt{bx^2 + ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)\*x^2,x, algorithm="giac")

[Out] 21/1024\*a^6\*ln(abs(-sqrt(b)\*x + sqrt(b\*x^2 + a)))/b^(3/2) + 1/15360\*(315\*a^5/b + 2\*(2455\*a^4 + 4\*(1429\*a^3\*b + 2\*(759\*a^2\*b^2 + 8\*(10\*b^4\*x^2 + 49\*a\*b^3)\*x^2)\*x^2)\*x^2)\*sqrt(b\*x^2 + a)\*x

### 3.428 $\int (a + bx^2)^{9/2} dx$

**Optimal.** Leaf size=122

$$\frac{63a^5 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{256\sqrt{b}} + \frac{63}{256}a^4x\sqrt{a+bx^2} + \frac{21}{128}a^3x(a+bx^2)^{3/2} + \frac{21}{160}a^2x(a+bx^2)^{5/2} + \frac{9}{80}ax(a+bx^2)^{7/2} + \frac{1}{10}x(a+bx^2)^{9/2}$$

[Out] (63\*a^4\*x\*Sqrt[a + b\*x^2])/256 + (21\*a^3\*x\*(a + b\*x^2)^(3/2))/128 + (21\*a^2\*x\*(a + b\*x^2)^(5/2))/160 + (9\*a\*x\*(a + b\*x^2)^(7/2))/80 + (x\*(a + b\*x^2)^(9/2))/10 + (63\*a^5\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(256\*Sqrt[b])

**Rubi [A]** time = 0.101756, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{63a^5 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{256\sqrt{b}} + \frac{63}{256}a^4x\sqrt{a+bx^2} + \frac{21}{128}a^3x(a+bx^2)^{3/2} + \frac{21}{160}a^2x(a+bx^2)^{5/2} + \frac{9}{80}ax(a+bx^2)^{7/2} + \frac{1}{10}x(a+bx^2)^{9/2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(9/2), x]

[Out] (63\*a^4\*x\*Sqrt[a + b\*x^2])/256 + (21\*a^3\*x\*(a + b\*x^2)^(3/2))/128 + (21\*a^2\*x\*(a + b\*x^2)^(5/2))/160 + (9\*a\*x\*(a + b\*x^2)^(7/2))/80 + (x\*(a + b\*x^2)^(9/2))/10 + (63\*a^5\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/(256\*Sqrt[b])

**Rubi in Sympy [A]** time = 9.68275, size = 116, normalized size = 0.95

$$\frac{63a^5 \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{256\sqrt{b}} + \frac{63a^4x\sqrt{a+bx^2}}{256} + \frac{21a^3x(a+bx^2)^{\frac{3}{2}}}{128} + \frac{21a^2x(a+bx^2)^{\frac{5}{2}}}{160} + \frac{9ax(a+bx^2)^{\frac{7}{2}}}{80} + \frac{x(a+bx^2)^{\frac{9}{2}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(9/2), x)

[Out] 63\*a\*\*5\*atanh(sqrt(b)\*x/sqrt(a + b\*x\*\*2))/(256\*sqrt(b)) + 63\*a\*\*4\*x\*sqrt(a + b\*x\*\*2)/256 + 21\*a\*\*3\*x\*(a + b\*x\*\*2)\*\*(3/2)/128 + 21\*a\*\*2\*x\*(a + b\*x\*\*2)\*\*(5/2)/160 + 9\*a\*x\*(a + b\*x\*\*2)\*\*(7/2)/80 + x\*(a + b\*x\*\*2)\*\*(9/2)/10

**Mathematica [A]** time = 0.0880513, size = 93, normalized size = 0.76

$$\frac{315a^5 \log\left(\frac{\sqrt{b}\sqrt{a+bx^2}+bx}{\sqrt{b}}\right)}{1280} + x\sqrt{a+bx^2} (965a^4 + 1490a^3bx^2 + 1368a^2b^2x^4 + 656ab^3x^6 + 128b^4x^8)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(9/2), x]

[Out] (x\*Sqrt[a + b\*x^2]\*(965\*a^4 + 1490\*a^3\*b\*x^2 + 1368\*a^2\*b^2\*x^4 + 656\*a\*b^3\*x^6 + 128\*b^4\*x^8) + (315\*a^5\*Log[b\*x + Sqrt[b]\*Sqrt[a + b\*x^2]]))/Sqrt[b]/1280

**Maple [A]** time = 0.005, size = 96, normalized size = 0.8

$$\frac{x}{10} (bx^2 + a)^{\frac{9}{2}} + \frac{9ax}{80} (bx^2 + a)^{\frac{7}{2}} + \frac{21a^2x}{160} (bx^2 + a)^{\frac{5}{2}} + \frac{21a^3x}{128} (bx^2 + a)^{\frac{3}{2}} + \frac{63a^4x}{256} \sqrt{bx^2 + a} + \frac{63a^5}{256} \ln\left(x\sqrt{b} + \sqrt{bx^2 + a}\right) \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(9/2), x)

[Out] 1/10\*x\*(b\*x^2+a)^(9/2)+9/80\*a\*x\*(b\*x^2+a)^(7/2)+21/160\*a^2\*x\*(b\*x^2+a)^(5/2)+21/128\*a^3\*x\*(b\*x^2+a)^(3/2)+63/256\*a^4\*x\*(b\*x^2+a)^(1/2)+63/256\*a^5/b^(1/2)\*ln(x\*b^(1/2)+(b\*x^2+a)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.299741, size = 1, normalized size = 0.01

$$\frac{315a^5 \log\left(-2\sqrt{bx^2 + abx} - (2bx^2 + a)\sqrt{b}\right) + 2(128b^4x^9 + 656ab^3x^7 + 1368a^2b^2x^5 + 1490a^3bx^3 + 965a^4x)\sqrt{bx^2 + a}}{2560\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2), x, algorithm="fricas")

[Out] [1/2560\*(315\*a^5\*log(-2\*sqrt(b\*x^2 + a)\*b\*x - (2\*b\*x^2 + a)\*sqrt(b)) + 2\*(128\*b^4\*x^9 + 656\*a\*b^3\*x^7 + 1368\*a^2\*b^2\*x^5 + 1490\*a^3\*b\*x^3 + 965\*a^4\*x)\*sqrt(b\*x^2 + a)\*sqrt(b))/sqrt(b), 1/1280\*(315\*a^5\*arctan(sqrt(-b)\*x/sqrt(b\*x^2 + a)) + (128\*b^4\*x^9 + 656\*a\*b^3\*x^7 + 1368\*a^2\*b^2\*x^5 + 1490\*a^3\*b\*x^3 + 965\*a^4\*x)\*sqrt(b\*x^2 + a)\*sqrt(-b))/sqrt(-b)]

**Sympy [A]** time = 35.8613, size = 151, normalized size = 1.24

$$\frac{193a^{\frac{9}{2}}x\sqrt{1 + \frac{bx^2}{a}}}{256} + \frac{149a^{\frac{7}{2}}bx^3\sqrt{1 + \frac{bx^2}{a}}}{128} + \frac{171a^{\frac{5}{2}}b^2x^5\sqrt{1 + \frac{bx^2}{a}}}{160} + \frac{41a^{\frac{3}{2}}b^3x^7\sqrt{1 + \frac{bx^2}{a}}}{80} + \frac{\sqrt{ab^4}x^9\sqrt{1 + \frac{bx^2}{a}}}{10} + \frac{63a^5 \operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{256\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(9/2),x)

[Out]  $193*a^{9/2}*x*\sqrt{1+b*x^2/a}/256 + 149*a^{7/2}*b*x^3*\sqrt{1+b*x^2/a}/128 + 171*a^{5/2}*b^2*x^5*\sqrt{1+b*x^2/a}/160 + 41*a^{3/2}*b^3*x^7*\sqrt{1+b*x^2/a}/80 + \sqrt{a}*b^4*x^9*\sqrt{1+b*x^2/a}/10 + 63*a^5*\operatorname{asinh}(\sqrt{b}*x/\sqrt{a})/(256*\sqrt{b})$

**GIAC/XCAS [A]** time = 0.219057, size = 123, normalized size = 1.01

$$-\frac{63 a^5 \ln \left( \left| -\sqrt{b} x + \sqrt{b x^2 + a} \right| \right)}{256 \sqrt{b}} + \frac{1}{1280} (965 a^4 + 2 (745 a^3 b + 4 (171 a^2 b^2 + 2 (8 b^4 x^2 + 41 a b^3) x^2) x^2) \sqrt{b x^2 + a} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2),x, algorithm="giac")

[Out]  $-63/256*a^5*\ln(\operatorname{abs}(-\sqrt{b}*x + \sqrt{b*x^2 + a}))/\sqrt{b} + 1/1280*(965*a^4 + 2*(745*a^3*b + 4*(171*a^2*b^2 + 2*(8*b^4*x^2 + 41*a*b^3)*x^2)*x^2)*\sqrt{b*x^2 + a}*x$

$$3.429 \quad \int \frac{(a+bx^2)^{9/2}}{x^2} dx$$

**Optimal.** Leaf size=123

$$\begin{aligned} & \frac{315}{128} a^4 \sqrt{b} \tanh^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a+bx^2}} \right) + \frac{315}{128} a^3 bx \sqrt{a+bx^2} + \frac{105}{64} a^2 bx (a+bx^2)^{3/2} \\ & - \frac{(a+bx^2)^{9/2}}{x} + \frac{9}{8} bx (a+bx^2)^{7/2} + \frac{21}{16} abx (a+bx^2)^{5/2} \end{aligned}$$

[Out] (315\*a^3\*b\*x\*Sqrt[a + b\*x^2])/128 + (105\*a^2\*b\*x\*(a + b\*x^2)^(3/2))/64 + (21\*a\*b\*x\*(a + b\*x^2)^(5/2))/16 + (9\*b\*x\*(a + b\*x^2)^(7/2))/8 - (a + b\*x^2)^(9/2)/x + (315\*a^4\*Sqrt[b]\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/128

**Rubi [A]** time = 0.113565, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\begin{aligned} & \frac{315}{128} a^4 \sqrt{b} \tanh^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a+bx^2}} \right) + \frac{315}{128} a^3 bx \sqrt{a+bx^2} + \frac{105}{64} a^2 bx (a+bx^2)^{3/2} \\ & - \frac{(a+bx^2)^{9/2}}{x} + \frac{9}{8} bx (a+bx^2)^{7/2} + \frac{21}{16} abx (a+bx^2)^{5/2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(9/2)/x^2, x]

[Out] (315\*a^3\*b\*x\*Sqrt[a + b\*x^2])/128 + (105\*a^2\*b\*x\*(a + b\*x^2)^(3/2))/64 + (21\*a\*b\*x\*(a + b\*x^2)^(5/2))/16 + (9\*b\*x\*(a + b\*x^2)^(7/2))/8 - (a + b\*x^2)^(9/2)/x + (315\*a^4\*Sqrt[b]\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/128

**Rubi in Sympy [A]** time = 11.6248, size = 117, normalized size = 0.95

$$\begin{aligned} & \frac{315a^4\sqrt{b} \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{128} + \frac{315a^3bx\sqrt{a+bx^2}}{128} + \frac{105a^2bx(a+bx^2)^{\frac{3}{2}}}{64} \\ & + \frac{21abx(a+bx^2)^{\frac{5}{2}}}{16} + \frac{9bx(a+bx^2)^{\frac{7}{2}}}{8} - \frac{(a+bx^2)^{\frac{9}{2}}}{x} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*2, x)

[Out] 315\*a\*\*4\*sqrt(b)\*atanh(sqrt(b)\*x/sqrt(a + b\*x\*\*2))/128 + 315\*a\*\*3\*b\*x\*sqrt(a + b\*x\*\*2)/128 + 105\*a\*\*2\*b\*x\*(a + b\*x\*\*2)\*\*(3/2)/64 + 21\*a\*b\*x\*(a + b\*x\*\*2)\*\*(5/2)/16 + 9\*b\*x\*(a + b\*x\*\*2)\*\*(7/2)/8 - (a + b\*x\*\*2)\*\*(9/2)/x

**Mathematica [A]** time = 0.0734393, size = 96, normalized size = 0.78

$$\frac{315a^4\sqrt{bx} \log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right) + \sqrt{a+bx^2}(-128a^4 + 325a^3bx^2 + 210a^2b^2x^4 + 88ab^3x^6 + 16b^4x^8)}{128x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(9/2)/x^2, x]

[Out] (Sqrt[a + b\*x^2]\*(-128\*a^4 + 325\*a^3\*b\*x^2 + 210\*a^2\*b^2\*x^4 + 88\*a\*b^3\*x^6 + 16\*b^4\*x^8) + 315\*a^4\*Sqrt[b]\*x\*Log[b\*x + Sqrt[b]\*Sqrt[a + b\*x^2]])/(128\*x)

**Maple [A]** time = 0.007, size = 117, normalized size = 1.

$$-\frac{1}{ax} (bx^2 + a)^{\frac{11}{2}} + \frac{bx}{a} (bx^2 + a)^{\frac{9}{2}} + \frac{9bx}{8} (bx^2 + a)^{\frac{7}{2}} + \frac{21abx}{16} (bx^2 + a)^{\frac{5}{2}} + \frac{105a^2bx}{64} (bx^2 + a)^{\frac{3}{2}} + \frac{315a^3bx}{128} \sqrt{bx^2 + a} + \frac{315a^4}{128} \sqrt{b} \ln(x\sqrt{b} + \sqrt{bx^2 + a})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(9/2)/x^2, x)

[Out] -1/a/x\*(b\*x^2+a)^(11/2)+b/a\*x\*(b\*x^2+a)^(9/2)+9/8\*b\*x\*(b\*x^2+a)^(7/2)+21/16\*a\*b\*x\*(b\*x^2+a)^(5/2)+105/64\*a^2\*b\*x\*(b\*x^2+a)^(3/2)+315/128\*a^3\*b\*x\*(b\*x^2+a)^(1/2)+315/128\*b^(1/2)\*a^4\*ln(x\*b^(1/2)+(b\*x^2+a)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^2, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.268672, size = 1, normalized size = 0.01

$$\left[ \frac{315a^4\sqrt{bx} \log(-2bx^2 - 2\sqrt{bx^2 + a}\sqrt{bx} - a) + 2(16b^4x^8 + 88ab^3x^6 + 210a^2b^2x^4 + 325a^3bx^2 - 128a^4)\sqrt{bx^2 + a}}{256x}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^2, x, algorithm="fricas")

[Out] [1/256\*(315\*a^4\*sqrt(b)\*x\*log(-2\*b\*x^2 - 2\*sqrt(b\*x^2 + a)\*sqrt(b)\*x - a) + 2\*(16\*b^4\*x^8 + 88\*a\*b^3\*x^6 + 210\*a^2\*b^2\*x^4 + 325\*a^3\*b\*x^2 - 128\*a^4)\*sqrt(b\*x^2 + a))/x, 1/128\*(315\*a^4\*sqrt(-b)\*x\*arctan(b\*x/(sqrt(b\*x^2 + a)\*sqrt(-b))) + (16\*b^4\*x^8 + 88\*a\*b^3\*x^6 + 210\*a^2\*b^2\*x^4 + 325\*a^3\*b\*x^2 - 128\*a^4)\*sqrt(b\*x^2 + a))/x]

**Sympy [A]** time = 35.0736, size = 173, normalized size = 1.41

$$-\frac{a^{\frac{9}{2}}}{x\sqrt{1 + \frac{bx^2}{a}}} + \frac{197a^{\frac{7}{2}}bx}{128\sqrt{1 + \frac{bx^2}{a}}} + \frac{535a^{\frac{5}{2}}b^2x^3}{128\sqrt{1 + \frac{bx^2}{a}}} + \frac{149a^{\frac{3}{2}}b^3x^5}{64\sqrt{1 + \frac{bx^2}{a}}} + \frac{13\sqrt{ab^4}x^7}{16\sqrt{1 + \frac{bx^2}{a}}} + \frac{315a^4\sqrt{b} \operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{128} + \frac{b^5x^9}{8\sqrt{a}\sqrt{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*2,x)

[Out]  $-a^{9/2}/(x\sqrt{1+b*x^2/a}) + 197*a^{7/2}*b*x/(128*\sqrt{1+b*x^2/a}) + 535*a^{5/2}*b^2*x^3/(128*\sqrt{1+b*x^2/a}) + 149*a^{3/2}*b^3*x^5/(64*\sqrt{1+b*x^2/a}) + 13*\sqrt{a}*b^4*x^7/(16*\sqrt{1+b*x^2/a}) + 315*a^4*\sqrt{b}*asinh(\sqrt{b}*x/\sqrt{a})/128 + b^5*x^9/(8*\sqrt{a}*\sqrt{1+b*x^2/a})$

**GIAC/XCAS [A]** time = 0.212991, size = 155, normalized size = 1.26

$$-\frac{315}{256}a^4\sqrt{b}\ln\left(\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^2\right)+\frac{2a^5\sqrt{b}}{\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{128}\left(325a^3b+2\left(105a^2b^2+4\left(2b^4x^2+11ab^3\right)x^2\right)x^2\right)\sqrt{bx^2+ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^2,x, algorithm="giac")

[Out]  $-315/256*a^4*\sqrt{b}*\ln((\sqrt{b}*x - \sqrt{b*x^2 + a})^2) + 2*a^5*\sqrt{b}/((\sqrt{b}*x - \sqrt{b*x^2 + a})^2 - a) + 1/128*(325*a^3*b + 2*(105*a^2*b^2 + 4*(2*b^4*x^2 + 11*a*b^3)*x^2)*x^2)*\sqrt{b*x^2 + a}*x$

$$3.430 \quad \int \frac{(a+bx^2)^{9/2}}{x^4} dx$$

**Optimal.** Leaf size=128

$$\begin{aligned} & \frac{105}{16} a^3 b^{3/2} \tanh^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a+bx^2}} \right) + \frac{105}{16} a^2 b^2 x \sqrt{a+bx^2} + \frac{7}{2} b^2 x (a+bx^2)^{5/2} \\ & + \frac{35}{8} ab^2 x (a+bx^2)^{3/2} - \frac{3b(a+bx^2)^{7/2}}{x} - \frac{(a+bx^2)^{9/2}}{3x^3} \end{aligned}$$

[Out] (105\*a^2\*b^2\*x\*Sqrt[a + b\*x^2])/16 + (35\*a\*b^2\*x\*(a + b\*x^2)^(3/2))/8 + (7\*b^2\*x\*(a + b\*x^2)^(5/2))/2 - (3\*b\*(a + b\*x^2)^(7/2))/x - (a + b\*x^2)^(9/2)/(3\*x^3) + (105\*a^3\*b^(3/2)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/16

**Rubi [A]** time = 0.126791, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\begin{aligned} & \frac{105}{16} a^3 b^{3/2} \tanh^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a+bx^2}} \right) + \frac{105}{16} a^2 b^2 x \sqrt{a+bx^2} + \frac{7}{2} b^2 x (a+bx^2)^{5/2} \\ & + \frac{35}{8} ab^2 x (a+bx^2)^{3/2} - \frac{3b(a+bx^2)^{7/2}}{x} - \frac{(a+bx^2)^{9/2}}{3x^3} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(9/2)/x^4, x]

[Out] (105\*a^2\*b^2\*x\*Sqrt[a + b\*x^2])/16 + (35\*a\*b^2\*x\*(a + b\*x^2)^(3/2))/8 + (7\*b^2\*x\*(a + b\*x^2)^(5/2))/2 - (3\*b\*(a + b\*x^2)^(7/2))/x - (a + b\*x^2)^(9/2)/(3\*x^3) + (105\*a^3\*b^(3/2)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/16

**Rubi in Sympy [A]** time = 13.7662, size = 121, normalized size = 0.95

$$\begin{aligned} & \frac{105a^3b^{\frac{3}{2}} \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{16} + \frac{105a^2b^2x\sqrt{a+bx^2}}{16} + \frac{35ab^2x(a+bx^2)^{\frac{3}{2}}}{8} \\ & + \frac{7b^2x(a+bx^2)^{\frac{5}{2}}}{2} - \frac{3b(a+bx^2)^{\frac{7}{2}}}{x} - \frac{(a+bx^2)^{\frac{9}{2}}}{3x^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*4, x)

[Out] 105\*a\*\*3\*b\*\*(3/2)\*atanh(sqrt(b)\*x/sqrt(a + b\*x\*\*2))/16 + 105\*a\*\*2\*b\*\*2\*x\*sqrt(a + b\*x\*\*2)/16 + 35\*a\*b\*\*2\*x\*(a + b\*x\*\*2)\*\*(3/2)/8 + 7\*b\*\*2\*x\*(a + b\*x\*\*2)\*\*(5/2)/2 - 3\*b\*(a + b\*x\*\*2)\*\*(7/2)/x - (a + b\*x\*\*2)\*\*(9/2)/(3\*x\*\*3)

**Mathematica [A]** time = 0.103198, size = 95, normalized size = 0.74

$$\frac{1}{48} \left( 315a^3b^{3/2} \log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right) + \frac{\sqrt{a+bx^2}(-16a^4 - 208a^3bx^2 + 165a^2b^2x^4 + 50ab^3x^6 + 8b^4x^8)}{x^3} \right)$$

Antiderivative was successfully verified.



[In] Integrate[(a + b\*x^2)^(9/2)/x^4, x]

[Out] ((Sqrt[a + b\*x^2]\*(-16\*a^4 - 208\*a^3\*b\*x^2 + 165\*a^2\*b^2\*x^4 + 50\*a\*b^3\*x^6 + 8\*b^4\*x^8))/x^3 + 315\*a^3\*b^(3/2)\*Log[b\*x + Sqrt[b]\*Sqrt[a + b\*x^2]])/48

**Maple [A]** time = 0.008, size = 146, normalized size = 1.1

$$-\frac{1}{3ax^3}(bx^2+a)^{\frac{11}{2}} - \frac{8b}{3a^2x}(bx^2+a)^{\frac{11}{2}} + \frac{8b^2x}{3a^2}(bx^2+a)^{\frac{9}{2}} + 3\frac{b^2x(bx^2+a)^{7/2}}{a} + \frac{7b^2x}{2}(bx^2+a)^{\frac{5}{2}} + \frac{35ab^2x}{8}(bx^2+a)^{\frac{3}{2}} + \frac{105a^2b^2x}{16}\sqrt{bx^2+a} + \frac{105a^3}{16}b^{\frac{3}{2}}\ln(x\sqrt{b} + \sqrt{bx^2+a})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(9/2)/x^4, x)

[Out] -1/3/a/x^3\*(b\*x^2+a)^(11/2)-8/3\*b/a^2/x\*(b\*x^2+a)^(11/2)+8/3\*b^2/a^2\*x\*(b\*x^2+a)^(9/2)+3\*b^2/a\*x\*(b\*x^2+a)^(7/2)+7/2\*b^2\*x\*(b\*x^2+a)^(5/2)+35/8\*a\*b^2\*x\*(b\*x^2+a)^(3/2)+105/16\*a^2\*b^2\*x\*(b\*x^2+a)^(1/2)+105/16\*b^(3/2)\*a^3\*ln(x\*b^(1/2)+(b\*x^2+a)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^4, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.271192, size = 1, normalized size = 0.01

$$\left[ \frac{315 a^3 b^{\frac{3}{2}} x^3 \log\left(-2 b x^2 - 2 \sqrt{b x^2 + a} \sqrt{b x} - a\right) + 2\left(8 b^4 x^8 + 50 a b^3 x^6 + 165 a^2 b^2 x^4 - 208 a^3 b x^2 - 16 a^4\right) \sqrt{b x^2 + a}}{96 x^3}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^4, x, algorithm="fricas")

[Out] [1/96\*(315\*a^3\*b^(3/2)\*x^3\*log(-2\*b\*x^2 - 2\*sqrt(b\*x^2 + a)\*sqrt(b)\*x - a) + 2\*(8\*b^4\*x^8 + 50\*a\*b^3\*x^6 + 165\*a^2\*b^2\*x^4 - 208\*a^3\*b\*x^2 - 16\*a^4)\*sqrt(b\*x^2 + a))/x^3, 1/48\*(315\*a^3\*sqrt(-b)\*b\*x^3\*arctan(b\*x/(sqrt(b\*x^2 + a)\*sqrt(-b))) + (8\*b^4\*x^8 + 50\*a\*b^3\*x^6 + 165\*a^2\*b^2\*x^4 - 208\*a^3\*b\*x^2 - 16\*a^4)\*sqrt(b\*x^2 + a))/x^3]

**Sympy [A]** time = 33.113, size = 175, normalized size = 1.37

$$-\frac{a^{\frac{9}{2}}}{3x^3\sqrt{1+\frac{bx^2}{a}}} - \frac{14a^{\frac{7}{2}}b}{3x\sqrt{1+\frac{bx^2}{a}}} - \frac{43a^{\frac{5}{2}}b^2x}{48\sqrt{1+\frac{bx^2}{a}}} + \frac{215a^{\frac{3}{2}}b^3x^3}{48\sqrt{1+\frac{bx^2}{a}}} + \frac{29\sqrt{ab^4}x^5}{24\sqrt{1+\frac{bx^2}{a}}} + \frac{105a^3b^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{16} + \frac{b^5x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*4,x)

[Out]  $-a^{9/2}/(3x^3\sqrt{1+bx^2/a}) - 14a^{7/2}b/(3x\sqrt{1+bx^2/a}) - 43a^{5/2}b^2x/(48\sqrt{1+bx^2/a}) + 215a^{3/2}b^3x^3/(48\sqrt{1+bx^2/a}) + 29\sqrt{a}b^4x^5/(24\sqrt{1+bx^2/a}) + 105a^3b^{3/2}\operatorname{asinh}(\sqrt{b}x/\sqrt{a})/16 + b^5x^7/(6\sqrt{a}\sqrt{1+bx^2/a})$

**GIAC/XCAS [A]** time = 0.212968, size = 216, normalized size = 1.69

$$-\frac{105}{32}a^3b^{\frac{3}{2}}\ln\left(\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^2\right)+\frac{1}{48}\left(165a^2b^2+2\left(4b^4x^2+25ab^3\right)x^2\right)\sqrt{bx^2+ax}+2\frac{\left(15\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^4a^4b^{\frac{3}{2}}-24\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^2a^5b^{\frac{3}{2}}+13a^6b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^4,x, algorithm="giac")

[Out]  $-105/32*a^3*b^{3/2}*ln((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 1/48*(165*a^2*b^2 + 2*(4*b^4*x^2 + 25*a*b^3)*x^2)*sqrt(b*x^2 + a)*x + 2/3*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^{3/2} - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^{3/2} + 13*a^6*b^{3/2})/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3$

$$3.431 \quad \int \frac{(a+bx^2)^{9/2}}{x^6} dx$$

**Optimal.** Leaf size=129

$$\frac{63}{8}a^2b^{5/2} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right) + \frac{21}{4}b^3x(a+bx^2)^{3/2} + \frac{63}{8}ab^3x\sqrt{a+bx^2} - \frac{21b^2(a+bx^2)^{5/2}}{5x} - \frac{(a+bx^2)^{9/2}}{5x^5} - \frac{3b(a+bx^2)^{7/2}}{5x^3}$$

[Out] (63\*a\*b^3\*x\*Sqrt[a + b\*x^2])/8 + (21\*b^3\*x\*(a + b\*x^2)^(3/2))/4 - (21\*b^2\*(a + b\*x^2)^(5/2))/(5\*x) - (3\*b\*(a + b\*x^2)^(7/2))/(5\*x^3) - (a + b\*x^2)^(9/2)/(5\*x^5) + (63\*a^2\*b^(5/2)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/8

**Rubi [A]** time = 0.136732, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{63}{8}a^2b^{5/2} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right) + \frac{21}{4}b^3x(a+bx^2)^{3/2} + \frac{63}{8}ab^3x\sqrt{a+bx^2} - \frac{21b^2(a+bx^2)^{5/2}}{5x} - \frac{(a+bx^2)^{9/2}}{5x^5} - \frac{3b(a+bx^2)^{7/2}}{5x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(9/2)/x^6, x]

[Out] (63\*a\*b^3\*x\*Sqrt[a + b\*x^2])/8 + (21\*b^3\*x\*(a + b\*x^2)^(3/2))/4 - (21\*b^2\*(a + b\*x^2)^(5/2))/(5\*x) - (3\*b\*(a + b\*x^2)^(7/2))/(5\*x^3) - (a + b\*x^2)^(9/2)/(5\*x^5) + (63\*a^2\*b^(5/2)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]])/8

**Rubi in Sympy [A]** time = 15.4578, size = 121, normalized size = 0.94

$$\frac{63a^2b^{\frac{5}{2}} \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{8} + \frac{63ab^3x\sqrt{a+bx^2}}{8} + \frac{21b^3x(a+bx^2)^{\frac{3}{2}}}{4} - \frac{21b^2(a+bx^2)^{\frac{5}{2}}}{5x} - \frac{3b(a+bx^2)^{\frac{7}{2}}}{5x^3} - \frac{(a+bx^2)^{\frac{9}{2}}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*6, x)

[Out] 63\*a\*\*2\*b\*\*(5/2)\*atanh(sqrt(b)\*x/sqrt(a + b\*x\*\*2))/8 + 63\*a\*b\*\*3\*x\*sqrt(a + b\*x\*\*2)/8 + 21\*b\*\*3\*x\*(a + b\*x\*\*2)\*\*(3/2)/4 - 21\*b\*\*2\*(a + b\*x\*\*2)\*\*(5/2)/(5\*x) - 3\*b\*(a + b\*x\*\*2)\*\*(7/2)/(5\*x\*\*3) - (a + b\*x\*\*2)\*\*(9/2)/(5\*x\*\*5)

**Mathematica [A]** time = 0.103078, size = 96, normalized size = 0.74

$$\frac{63}{8}a^2b^{5/2} \log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right) - \frac{\sqrt{a+bx^2}(8a^4 + 56a^3bx^2 + 288a^2b^2x^4 - 85ab^3x^6 - 10b^4x^8)}{40x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(9/2)/x^6, x]

[Out]  $-(\text{Sqrt}[a + b*x^2]*(8*a^4 + 56*a^3*b*x^2 + 288*a^2*b^2*x^4 - 85*a*b^3*x^6 - 10*b^4*x^8))/(40*x^5) + (63*a^2*b^{(5/2)}*\text{Log}[b*x + \text{Sqrt}[b]*\text{Sqrt}[a + b*x^2]])/8$

**Maple [A]** time = 0.013, size = 166, normalized size = 1.3

$$-\frac{1}{5ax^5}(bx^2+a)^{\frac{11}{2}} - \frac{2b}{5a^2x^3}(bx^2+a)^{\frac{11}{2}} - \frac{16b^2}{5a^3x}(bx^2+a)^{\frac{11}{2}} + \frac{16b^3x}{5a^3}(bx^2+a)^{\frac{9}{2}} + \frac{18b^3x}{5a^2}(bx^2+a)^{\frac{7}{2}} + \frac{21b^3x}{5a}(bx^2+a)^{\frac{5}{2}} + \frac{21b^3x}{4}(bx^2+a)^{\frac{3}{2}} + \frac{63ab^3x}{8}\sqrt{bx^2+a} + \frac{63a^2}{8}b^{\frac{5}{2}}\ln(x\sqrt{b} + \sqrt{bx^2+a})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(9/2)/x^6, x)

[Out]  $-1/5/a/x^5*(b*x^2+a)^{(11/2)} - 2/5*b/a^2/x^3*(b*x^2+a)^{(11/2)} - 16/5*b^2/a^3/x*(b*x^2+a)^{(11/2)} + 16/5*b^3/a^3*x*(b*x^2+a)^{(9/2)} + 18/5*b^3/a^2*x*(b*x^2+a)^{(7/2)} + 21/5*b^3/a*x*(b*x^2+a)^{(5/2)} + 21/4*b^3*x*(b*x^2+a)^{(3/2)} + 63/8*a*b^3*x*(b*x^2+a)^{(1/2)} + 63/8*b^{(5/2)}*a^2*\ln(x*b^{(1/2)} + (b*x^2+a)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^6, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.274783, size = 1, normalized size = 0.01

$$\left[ \frac{315 a^2 b^{\frac{5}{2}} x^5 \log\left(-2 b x^2 - 2 \sqrt{b x^2 + a} \sqrt{b} x - a\right) + 2\left(10 b^4 x^8 + 85 a b^3 x^6 - 288 a^2 b^2 x^4 - 56 a^3 b x^2 - 8 a^4\right) \sqrt{b x^2 + a}}{80 x^5}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^6, x, algorithm="fricas")

[Out]  $[1/80*(315*a^2*b^{(5/2)}*x^5*\log(-2*b*x^2 - 2*\text{sqrt}(b*x^2 + a)*\text{sqrt}(b)*x - a) + 2*(10*b^4*x^8 + 85*a*b^3*x^6 - 288*a^2*b^2*x^4 - 56*a^3*b*x^2 - 8*a^4)*\text{sqrt}(b*x^2 + a))/x^5, 1/40*(315*a^2*\text{sqrt}(-b)*b^2*x^5*\text{arctan}(b*x/(\text{sqrt}(b*x^2 + a)*\text{sqrt}(-b))) + (10*b^4*x^8 + 85*a*b^3*x^6 - 288*a^2*b^2*x^4 - 56*a^3*b*x^2 - 8*a^4)*\text{sqrt}(b*x^2 + a))/x^5]$

**Sympy [A]** time = 35.2797, size = 175, normalized size = 1.36

$$-\frac{a^{\frac{9}{2}}}{5x^5\sqrt{1+\frac{bx^2}{a}}}-\frac{8a^{\frac{7}{2}}b}{5x^3\sqrt{1+\frac{bx^2}{a}}}-\frac{43a^{\frac{5}{2}}b^2}{5x\sqrt{1+\frac{bx^2}{a}}}-\frac{203a^{\frac{3}{2}}b^3x}{40\sqrt{1+\frac{bx^2}{a}}}+\frac{19\sqrt{ab^4}x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{63a^2b^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8}+\frac{b^5x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*6,x)

[Out] -a\*\*(9/2)/(5\*x\*\*5\*sqrt(1+b\*x\*\*2/a))-8\*a\*\*(7/2)\*b/(5\*x\*\*3\*sqrt(1+b\*x\*\*2/a))-43\*a\*\*(5/2)\*b\*\*2/(5\*x\*sqrt(1+b\*x\*\*2/a))-203\*a\*\*(3/2)\*b\*\*3\*x/(40\*sqrt(1+b\*x\*\*2/a))+19\*sqrt(a)\*b\*\*4\*x\*\*3/(8\*sqrt(1+b\*x\*\*2/a))+63\*a\*\*2\*b\*\*(5/2)\*asinh(sqrt(b)\*x/sqrt(a))/8+b\*\*5\*x\*\*5/(4\*sqrt(a)\*sqrt(1+b\*x\*\*2/a))

**GIAC/XCAS [A]** time = 0.230327, size = 270, normalized size = 2.09

$$-\frac{63}{16}a^2b^{\frac{5}{2}}\ln\left(\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^2\right)+\frac{1}{8}(2b^4x^2+17ab^3)\sqrt{bx^2+ax}+4\left(25\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^8a^3b^{\frac{5}{2}}-75\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^6a^4b^{\frac{5}{2}}+105\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^4a^5b^{\frac{5}{2}}-65\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^2a^6b^{\frac{5}{2}}\right)+\frac{5\left(\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^2-a\right)^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^6,x, algorithm="giac")

[Out] -63/16\*a^2\*b^(5/2)\*ln((sqrt(b)\*x - sqrt(b\*x^2 + a))^2) + 1/8\*(2\*b^4\*x^2 + 17\*a\*b^3)\*sqrt(b\*x^2 + a)\*x + 4/5\*(25\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^8\*a^3\*b^(5/2) - 75\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^6\*a^4\*b^(5/2) + 105\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^4\*a^5\*b^(5/2) - 65\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^2\*a^6\*b^(5/2) + 18\*a^7\*b^(5/2))/((sqrt(b)\*x - sqrt(b\*x^2 + a))^2 - a)^5

$$3.432 \quad \int \frac{(a+bx^2)^{9/2}}{x^8} dx$$

**Optimal.** Leaf size=126

$$\frac{9}{2}ab^{7/2} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right) + \frac{9}{2}b^4x\sqrt{a+bx^2} - \frac{3b^3(a+bx^2)^{3/2}}{x} - \frac{3b^2(a+bx^2)^{5/2}}{5x^3} - \frac{(a+bx^2)^{9/2}}{7x^7} - \frac{9b(a+bx^2)^{7/2}}{35x^5}$$

[Out]  $(9*b^4*x*\text{Sqrt}[a + b*x^2])/2 - (3*b^3*(a + b*x^2)^(3/2))/x - (3*b^2*(a + b*x^2)^(5/2))/(5*x^3) - (9*b*(a + b*x^2)^(7/2))/(35*x^5) - (a + b*x^2)^(9/2)/(7*x^7) + (9*a*b^(7/2)*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]])/2$

**Rubi [A]** time = 0.143521, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{9}{2}ab^{7/2} \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right) + \frac{9}{2}b^4x\sqrt{a+bx^2} - \frac{3b^3(a+bx^2)^{3/2}}{x} - \frac{3b^2(a+bx^2)^{5/2}}{5x^3} - \frac{(a+bx^2)^{9/2}}{7x^7} - \frac{9b(a+bx^2)^{7/2}}{35x^5}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(9/2)/x^8, x]

[Out]  $(9*b^4*x*\text{Sqrt}[a + b*x^2])/2 - (3*b^3*(a + b*x^2)^(3/2))/x - (3*b^2*(a + b*x^2)^(5/2))/(5*x^3) - (9*b*(a + b*x^2)^(7/2))/(35*x^5) - (a + b*x^2)^(9/2)/(7*x^7) + (9*a*b^(7/2)*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]])/2$

**Rubi in Sympy [A]** time = 16.4963, size = 117, normalized size = 0.93

$$\frac{9ab^{7/2} \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{2} + \frac{9b^4x\sqrt{a+bx^2}}{2} - \frac{3b^3(a+bx^2)^{3/2}}{x} - \frac{3b^2(a+bx^2)^{5/2}}{5x^3} - \frac{9b(a+bx^2)^{7/2}}{35x^5} - \frac{(a+bx^2)^{9/2}}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*8, x)

[Out]  $9*a*b**(7/2)*\operatorname{atanh}(\operatorname{sqrt}(b)*x/\operatorname{sqrt}(a + b*x**2))/2 + 9*b**4*x*\operatorname{sqrt}(a + b*x**2)/2 - 3*b**3*(a + b*x**2)**(3/2)/x - 3*b**2*(a + b*x**2)**(5/2)/(5*x**3) - 9*b*(a + b*x**2)**(7/2)/(35*x**5) - (a + b*x**2)**(9/2)/(7*x**7)$

**Mathematica [A]** time = 0.0850259, size = 94, normalized size = 0.75

$$\frac{9}{2}ab^{7/2} \log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right) - \frac{\sqrt{a+bx^2}(10a^4 + 58a^3bx^2 + 156a^2b^2x^4 + 388ab^3x^6 - 35b^4x^8)}{70x^7}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(9/2)/x^8, x]

[Out]  $-(\text{Sqrt}[a + b*x^2] * (10*a^4 + 58*a^3*b*x^2 + 156*a^2*b^2*x^4 + 388*a*b^3*x^6 - 35*b^4*x^8)) / (70*x^7) + (9*a*b^{(7/2)} * \text{Log}[b*x + \text{Sqrt}[b] * \text{Sqrt}[a + b*x^2]]) / 2$

**Maple [A]** time = 0.022, size = 186, normalized size = 1.5

$$-\frac{1}{7ax^7}(bx^2+a)^{\frac{11}{2}} - \frac{4b}{35a^2x^5}(bx^2+a)^{\frac{11}{2}} - \frac{8b^2}{35a^3x^3}(bx^2+a)^{\frac{11}{2}} - \frac{64b^3}{35a^4x}(bx^2+a)^{\frac{11}{2}} \\ + \frac{64b^4x}{35a^4}(bx^2+a)^{\frac{9}{2}} + \frac{72b^4x}{35a^3}(bx^2+a)^{\frac{7}{2}} + \frac{12b^4x}{5a^2}(bx^2+a)^{\frac{5}{2}} \\ + 3\frac{b^4x(bx^2+a)^{3/2}}{a} + \frac{9b^4x}{2}\sqrt{bx^2+a} + \frac{9a}{2}b^{\frac{7}{2}}\ln(x\sqrt{b} + \sqrt{bx^2+a})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(9/2)/x^8,x)`

[Out]  $-1/7/a/x^7*(b*x^2+a)^{(11/2)} - 4/35*b/a^2/x^5*(b*x^2+a)^{(11/2)} - 8/35*b^2/a^3/x^3*(b*x^2+a)^{(11/2)} - 64/35*b^3/a^4/x*(b*x^2+a)^{(11/2)} + 64/35*b^4/a^4*x*(b*x^2+a)^{(9/2)} + 72/35*b^4/a^3*x*(b*x^2+a)^{(7/2)} + 12/5*b^4/a^2*x*(b*x^2+a)^{(5/2)} + 3*b^4/a*x*(b*x^2+a)^{(3/2)} + 9/2*b^4*x*(b*x^2+a)^{(1/2)} + 9/2*b^{(7/2)}*a*\ln(x*b^{(1/2)}+(b*x^2+a)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)/x^8,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.270387, size = 1, normalized size = 0.01

$$\left[ \frac{315ab^{\frac{7}{2}}x^7 \log\left(-2bx^2 - 2\sqrt{bx^2+a}\sqrt{bx-a}\right) + 2(35b^4x^8 - 388ab^3x^6 - 156a^2b^2x^4 - 58a^3bx^2 - 10a^4)\sqrt{bx^2+a}}{140x^7}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)/x^8,x, algorithm="fricas")`

[Out]  $[1/140*(315*a*b^{(7/2)}*x^7*\log(-2*b*x^2 - 2*\text{sqrt}(b*x^2 + a)*\text{sqrt}(b*x - a) + 2*(35*b^4*x^8 - 388*a*b^3*x^6 - 156*a^2*b^2*x^4 - 58*a^3*b*x^2 - 10*a^4)*\text{sqrt}(b*x^2 + a)))/x^7, 1/70*(315*a*\text{sqrt}(-b)*b^3*x^7*\arctan(b*x/(\text{sqrt}(b*x^2 + a)*\text{sqrt}(-b))) + (35*b^4*x^8 - 388*a*b^3*x^6 - 156*a^2*b^2*x^4 - 58*a^3*b*x^2 - 10*a^4)*\text{sqrt}(b*x^2 + a))/x^7]$

**Sympy [A]** time = 37.6491, size = 167, normalized size = 1.33

$$-\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{7x^6} - \frac{29a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{35x^4} - \frac{78a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{35x^2} - \frac{194ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{35} \\ - \frac{9ab^{\frac{7}{2}}\log\left(\frac{a}{bx^2}\right)}{4} + \frac{9ab^{\frac{7}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)}{2} + \frac{b^{\frac{9}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*8,x)

[Out]  $-a^{**4} \sqrt{b} \sqrt{a/(b*x^{**2}) + 1}/(7*x^{**6}) - 29*a^{**3}b^{**}(3/2)*\sqrt{a/(b*x^{**2}) + 1}/(35*x^{**4}) - 78*a^{**2}b^{**}(5/2)*\sqrt{a/(b*x^{**2}) + 1}/(35*x^{**2}) - 194*a*b^{**}(7/2)*\sqrt{a/(b*x^{**2}) + 1}/35 - 9*a*b^{**}(7/2)*\log(a/(b*x^{**2}))/4 + 9*a*b^{**}(7/2)*\log(\sqrt{a/(b*x^{**2}) + 1}) + 1)/2 + b^{**}(9/2)*x^{**2}*\sqrt{a/(b*x^{**2}) + 1}/2$

**GIAC/XCAS [A]** time = 0.216359, size = 324, normalized size = 2.57

$$\frac{\frac{1}{2} \sqrt{bx^2 + a} b^4 x - \frac{9}{4} a b^{\frac{7}{2}} \ln\left(\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^2\right) + 4 \left(175 \left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^{12} a^2 b^{\frac{7}{2}} - 700 \left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^{10} a^3 b^{\frac{7}{2}} + 1575 \left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^8 a^4 b^{\frac{7}{2}} - 1820 \left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^6 a^5 b^{\frac{7}{2}} + 1337 \left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^4 a^6 b^{\frac{7}{2}} - 504 \left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^2 a^7 b^{\frac{7}{2}} + 97 a^8 b^{\frac{7}{2}}\right)}{35 \left(\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^2 - a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^8,x, algorithm="giac")

[Out]  $1/2*\sqrt{b*x^2 + a}*b^4*x - 9/4*a*b^{(7/2)}*\ln((\sqrt{b}*x - \sqrt{b*x^2 + a})^2) + 4/35*(175*(\sqrt{b}*x - \sqrt{b*x^2 + a})^{12}*a^2*b^{(7/2)} - 700*(\sqrt{b}*x - \sqrt{b*x^2 + a})^{10}*a^3*b^{(7/2)} + 1575*(\sqrt{b}*x - \sqrt{b*x^2 + a})^8*a^4*b^{(7/2)} - 1820*(\sqrt{b}*x - \sqrt{b*x^2 + a})^6*a^5*b^{(7/2)} + 1337*(\sqrt{b}*x - \sqrt{b*x^2 + a})^4*a^6*b^{(7/2)} - 504*(\sqrt{b}*x - \sqrt{b*x^2 + a})^2*a^7*b^{(7/2)} + 97*a^8*b^{(7/2)})/((\sqrt{b}*x - \sqrt{b*x^2 + a})^2 - a)^7$



$$3.433 \quad \int \frac{(a+bx^2)^{9/2}}{x^{10}} dx$$

**Optimal.** Leaf size=124

$$b^{9/2} \tanh^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a+bx^2}} \right) - \frac{b^4 \sqrt{a+bx^2}}{x} - \frac{b^3 (a+bx^2)^{3/2}}{3x^3} - \frac{b^2 (a+bx^2)^{5/2}}{5x^5} - \frac{(a+bx^2)^{9/2}}{9x^9} - \frac{b (a+bx^2)^{7/2}}{7x^7}$$

[Out] -((b^4\*Sqrt[a + b\*x^2])/x) - (b^3\*(a + b\*x^2)^(3/2))/(3\*x^3) - (b^2\*(a + b\*x^2)^(5/2))/(5\*x^5) - (b\*(a + b\*x^2)^(7/2))/(7\*x^7) - (a + b\*x^2)^(9/2)/(9\*x^9) + b^(9/2)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]]

**Rubi [A]** time = 0.150345, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$b^{9/2} \tanh^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a+bx^2}} \right) - \frac{b^4 \sqrt{a+bx^2}}{x} - \frac{b^3 (a+bx^2)^{3/2}}{3x^3} - \frac{b^2 (a+bx^2)^{5/2}}{5x^5} - \frac{(a+bx^2)^{9/2}}{9x^9} - \frac{b (a+bx^2)^{7/2}}{7x^7}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(9/2)/x^10, x]

[Out] -((b^4\*Sqrt[a + b\*x^2])/x) - (b^3\*(a + b\*x^2)^(3/2))/(3\*x^3) - (b^2\*(a + b\*x^2)^(5/2))/(5\*x^5) - (b\*(a + b\*x^2)^(7/2))/(7\*x^7) - (a + b\*x^2)^(9/2)/(9\*x^9) + b^(9/2)\*ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]]

**Rubi in Sympy [A]** time = 18.2268, size = 107, normalized size = 0.86

$$b^{9/2} \operatorname{atanh} \left( \frac{\sqrt{bx}}{\sqrt{a+bx^2}} \right) - \frac{b^4 \sqrt{a+bx^2}}{x} - \frac{b^3 (a+bx^2)^{3/2}}{3x^3} - \frac{b^2 (a+bx^2)^{5/2}}{5x^5} - \frac{b (a+bx^2)^{7/2}}{7x^7} - \frac{(a+bx^2)^{9/2}}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*10, x)

[Out] b\*\*(9/2)\*atanh(sqrt(b)\*x/sqrt(a + b\*x\*\*2)) - b\*\*4\*sqrt(a + b\*x\*\*2)/x - b\*\*3\*(a + b\*x\*\*2)\*\*(3/2)/(3\*x\*\*3) - b\*\*2\*(a + b\*x\*\*2)\*\*(5/2)/(5\*x\*\*5) - b\*(a + b\*x\*\*2)\*\*(7/2)/(7\*x\*\*7) - (a + b\*x\*\*2)\*\*(9/2)/(9\*x\*\*9)

**Mathematica [A]** time = 0.0839914, size = 90, normalized size = 0.73

$$b^{9/2} \log \left( \sqrt{b} \sqrt{a+bx^2} + bx \right) - \frac{\sqrt{a+bx^2} (35a^4 + 185a^3bx^2 + 408a^2b^2x^4 + 506ab^3x^6 + 563b^4x^8)}{315x^9}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(9/2)/x^10, x]

[Out] -(Sqrt[a + b\*x^2]\*(35\*a^4 + 185\*a^3\*b\*x^2 + 408\*a^2\*b^2\*x^4 + 506\*a\*b^3\*x^6 + 563\*b^4\*x^8))/(315\*x^9) + b^(9/2)\*Log[b\*x + Sqrt[b]\*Sqrt[a + b\*x^2]]

**Maple [B]** time = 0.047, size = 206, normalized size = 1.7

$$\begin{aligned}
 & -\frac{1}{9ax^9}(bx^2+a)^{\frac{11}{2}} - \frac{2b}{63a^2x^7}(bx^2+a)^{\frac{11}{2}} - \frac{8b^2}{315a^3x^5}(bx^2+a)^{\frac{11}{2}} - \frac{16b^3}{315a^4x^3}(bx^2+a)^{\frac{11}{2}} \\
 & - \frac{128b^4}{315a^5x}(bx^2+a)^{\frac{11}{2}} + \frac{128b^5x}{315a^5}(bx^2+a)^{\frac{9}{2}} + \frac{16b^5x}{35a^4}(bx^2+a)^{\frac{7}{2}} \\
 & + \frac{8b^5x}{15a^3}(bx^2+a)^{\frac{5}{2}} + \frac{2b^5x}{3a^2}(bx^2+a)^{\frac{3}{2}} + \frac{b^5x}{a}\sqrt{bx^2+a} + b^{\frac{9}{2}}\ln(x\sqrt{b} + \sqrt{bx^2+a})
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(9/2)/x^10,x)

[Out] -1/9/a/x^9\*(b\*x^2+a)^(11/2)-2/63\*b/a^2/x^7\*(b\*x^2+a)^(11/2)-8/315\*b^2/a^3/x^5\*(b\*x^2+a)^(11/2)-16/315\*b^3/a^4/x^3\*(b\*x^2+a)^(11/2)-128/315\*b^4/a^5/x\*(b\*x^2+a)^(11/2)+128/315\*b^5/a^5\*x\*(b\*x^2+a)^(9/2)+16/35\*b^5/a^4\*x\*(b\*x^2+a)^(7/2)+8/15\*b^5/a^3\*x\*(b\*x^2+a)^(5/2)+2/3\*b^5/a^2\*x\*(b\*x^2+a)^(3/2)+b^5/a\*x\*(b\*x^2+a)^(1/2)+b^(9/2)\*ln(x\*b^(1/2)+(b\*x^2+a)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^10,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.272093, size = 1, normalized size = 0.01

$$\left[ \frac{315 b^{\frac{9}{2}} x^9 \log\left(-2 b x^2 - 2 \sqrt{b x^2 + a} \sqrt{b x} - a\right) - 2\left(563 b^4 x^8 + 506 a b^3 x^6 + 408 a^2 b^2 x^4 + 185 a^3 b x^2 + 35 a^4\right) \sqrt{b x^2 + a}}{630 x^9}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^10,x, algorithm="fricas")

[Out] [1/630\*(315\*b^(9/2)\*x^9\*log(-2\*b\*x^2 - 2\*sqrt(b\*x^2 + a)\*sqrt(b)\*x - a) - 2\*(563\*b^4\*x^8 + 506\*a\*b^3\*x^6 + 408\*a^2\*b^2\*x^4 + 185\*a^3\*b\*x^2 + 35\*a^4)\*sqrt(b\*x^2 + a))/x^9, 1/315\*(315\*sqrt(-b)\*b^4\*x^9\*arctan(b\*x/(sqrt(b\*x^2 + a)\*sqrt(-b))) - (563\*b^4\*x^8 + 506\*a\*b^3\*x^6 + 408\*a^2\*b^2\*x^4 + 185\*a^3\*b\*x^2 + 35\*a^4)\*sqrt(b\*x^2 + a))/x^9]

**Sympy [A]** time = 42.9649, size = 160, normalized size = 1.29

$$\begin{aligned}
 & \frac{a^4 \sqrt{b} \sqrt{\frac{a}{bx^2} + 1}}{9x^8} - \frac{37a^3 b^{\frac{3}{2}} \sqrt{\frac{a}{bx^2} + 1}}{63x^6} - \frac{136a^2 b^{\frac{5}{2}} \sqrt{\frac{a}{bx^2} + 1}}{105x^4} - \frac{506ab^{\frac{7}{2}} \sqrt{\frac{a}{bx^2} + 1}}{315x^2} \\
 & - \frac{563b^{\frac{9}{2}} \sqrt{\frac{a}{bx^2} + 1}}{315} - \frac{b^{\frac{9}{2}} \log\left(\frac{a}{bx^2}\right)}{2} + b^{\frac{9}{2}} \log\left(\sqrt{\frac{a}{bx^2} + 1} + 1\right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*10,x)

[Out]  $-a^{**4} \sqrt{b} \sqrt{a/(b*x^{**2}) + 1}/(9*x^{**8}) - 37*a^{**3}*b^{**}(3/2)*\sqrt{a/(b*x^{**2}) + 1}/(63*x^{**6}) - 136*a^{**2}*b^{**}(5/2)*\sqrt{a/(b*x^{**2}) + 1}/(105*x^{**4}) - 506*a*b^{**}(7/2)*\sqrt{a/(b*x^{**2}) + 1}/(315*x^{**2}) - 563*b^{**}(9/2)*\sqrt{a/(b*x^{**2}) + 1}/315 - b^{**}(9/2)*\log(a/(b*x^{**2})/2 + b^{**}(9/2)*\log(\sqrt{a/(b*x^{**2}) + 1} + 1))$

**GIAC/XCAS [A]** time = 0.21991, size = 373, normalized size = 3.01

$$-\frac{1}{2} b^{\frac{9}{2}} \ln \left( \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 \right) + 2 \left( 1575 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{16} ab^{\frac{9}{2}} - 6300 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{14} a^2 b^{\frac{9}{2}} + 21000 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{12} a^3 b^{\frac{9}{2}} - 31500 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{10} a^4 b^{\frac{9}{2}} + 39438 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^8 a^5 b^{\frac{9}{2}} - 26292 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^6 a^6 b^{\frac{9}{2}} + 13968 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^4 a^7 b^{\frac{9}{2}} - 3492 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 a^8 b^{\frac{9}{2}} + 563 a^9 b^{\frac{9}{2}} \right) / \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 - a^9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^10,x, algorithm="giac")

[Out]  $-1/2*b^{(9/2)}*ln((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/315*(1575*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a*b^{(9/2)} - 6300*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^2*b^{(9/2)} + 21000*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3*b^{(9/2)} - 31500*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^4*b^{(9/2)} + 39438*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^5*b^{(9/2)} - 26292*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^6*b^{(9/2)} + 13968*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^7*b^{(9/2)} - 3492*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^8*b^{(9/2)} + 563*a^9*b^{(9/2)})/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a^9)$

$$3.434 \quad \int \frac{(a+bx^2)^{9/2}}{x^{12}} dx$$

**Optimal.** Leaf size=21

$$-\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

[Out]  $-(a + b*x^2)^{(11/2)}/(11*a*x^{11})$

**Rubi [A]** time = 0.022698, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$-\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^{(9/2)}/x^{12}, x]$

[Out]  $-(a + b*x^2)^{(11/2)}/(11*a*x^{11})$

**Rubi in Sympy [A]** time = 3.15369, size = 17, normalized size = 0.81

$$-\frac{(a+bx^2)^{\frac{11}{2}}}{11ax^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x^{**2}+a)**(9/2)/x^{**12}, x)$

[Out]  $-(a + b*x^{**2})^{**}(11/2)/(11*a*x^{**11})$

**Mathematica [A]** time = 0.0476743, size = 21, normalized size = 1.

$$-\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b*x^2)^{(9/2)}/x^{12}, x]$

[Out]  $-(a + b*x^2)^{(11/2)}/(11*a*x^{11})$

**Maple [A]** time = 0.005, size = 18, normalized size = 0.9

$$-\frac{1}{11ax^{11}}(bx^2+a)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((b*x^2+a)^{(9/2)}/x^{12}, x)$

[Out]  $-1/11*(b*x^2+a)^{(11/2)}/a/x^{11}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)/x^12,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.294274, size = 92, normalized size = 4.38

$$\frac{(b^5x^{10} + 5ab^4x^8 + 10a^2b^3x^6 + 10a^3b^2x^4 + 5a^4bx^2 + a^5)\sqrt{bx^2 + a}}{11ax^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)/x^12,x, algorithm="fricas")`

[Out]  $-1/11*(b^5*x^{10} + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*\text{sqrt}(b*x^2 + a)/(a*x^{11})$

**Sympy [A]** time = 28.865, size = 150, normalized size = 7.14

$$\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2} + 1}}{11x^{10}} - \frac{5a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2} + 1}}{11x^8} - \frac{10a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2} + 1}}{11x^6} - \frac{10ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2} + 1}}{11x^4} - \frac{5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2} + 1}}{11x^2} - \frac{b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2} + 1}}{11a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(9/2)/x**12,x)`

[Out]  $-a^{**4}*\text{sqrt}(b)*\text{sqrt}(a/(b*x^{**2}) + 1)/(11*x^{**10}) - 5*a^{**3}*b^{**}(3/2)*\text{sqrt}(a/(b*x^{**2}) + 1)/(11*x^{**8}) - 10*a^{**2}*b^{**}(5/2)*\text{sqrt}(a/(b*x^{**2}) + 1)/(11*x^{**6}) - 10*a*b^{**}(7/2)*\text{sqrt}(a/(b*x^{**2}) + 1)/(11*x^{**4}) - 5*b^{**}(9/2)*\text{sqrt}(a/(b*x^{**2}) + 1)/(11*x^{**2}) - b^{**}(11/2)*\text{sqrt}(a/(b*x^{**2}) + 1)/(11*a)$

**GIAC/XCAS [A]** time = 0.215937, size = 225, normalized size = 10.71

$$\frac{2\left(11\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^{20} b^{\frac{11}{2}} + 165\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^{16} a^2 b^{\frac{11}{2}} + 462\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^{12} a^4 b^{\frac{11}{2}} + 330\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^8 a^6 b^{\frac{11}{2}}\right)}{11\left(\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^2 - a\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)/x^12,x, algorithm="giac")`

[Out]  $2/11*(11*(\text{sqrt}(b)*x - \text{sqrt}(b*x^2 + a))^{20}*b^{(11/2)} + 165*(\text{sqrt}(b)*x - \text{sqrt}(b*x^2 + a))^{16}*a^2*b^{(11/2)} + 462*(\text{sqrt}(b)*x - \text{sqrt}(b*x^2 + a))^{12}*a^4*b^{(11/2)} + 330*(\text{sqrt}(b)*x - \text{sqrt}(b*x^2 + a))^{8}*a^6*b^{(11/2)} + 55*(\text{sqrt}(b)*x - \text{sqrt}(b*x^2 + a))^{4}*a^8*b^{(11/2)} + a^10*b^{(11/2)})/((\text{sqrt}(b)*x - \text{sqrt}(b*x^2 + a))^2 - a)^{11}$

$$3.435 \quad \int \frac{(a+bx^2)^{9/2}}{x^{14}} dx$$

**Optimal.** Leaf size=44

$$\frac{2b(a+bx^2)^{11/2}}{143a^2x^{11}} - \frac{(a+bx^2)^{11/2}}{13ax^{13}}$$

[Out]  $-(a + b*x^2)^{(11/2)}/(13*a*x^{13}) + (2*b*(a + b*x^2)^{(11/2)})/(143*a^2*x^{11})$

**Rubi [A]** time = 0.0451653, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{2b(a+bx^2)^{11/2}}{143a^2x^{11}} - \frac{(a+bx^2)^{11/2}}{13ax^{13}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(9/2)/x^14, x]

[Out]  $-(a + b*x^2)^{(11/2)}/(13*a*x^{13}) + (2*b*(a + b*x^2)^{(11/2)})/(143*a^2*x^{11})$

**Rubi in Sympy [A]** time = 5.16331, size = 37, normalized size = 0.84

$$-\frac{(a+bx^2)^{\frac{11}{2}}}{13ax^{13}} + \frac{2b(a+bx^2)^{\frac{11}{2}}}{143a^2x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*14, x)

[Out]  $-(a + b*x**2)**(11/2)/(13*a*x**13) + 2*b*(a + b*x**2)**(11/2)/(143*a**2*x**11)$

**Mathematica [A]** time = 0.0555558, size = 31, normalized size = 0.7

$$\frac{(a+bx^2)^{11/2}(2bx^2-11a)}{143a^2x^{13}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(9/2)/x^14, x]

[Out]  $((a + b*x^2)^{(11/2)}*(-11*a + 2*b*x^2))/(143*a^2*x^{13})$

**Maple [A]** time = 0.006, size = 28, normalized size = 0.6

$$-\frac{-2bx^2 + 11a}{143x^{13}a^2} (bx^2 + a)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(9/2)/x^14,x)`

[Out]  $-1/143*(b*x^2+a)^{(11/2)*(-2*b*x^2+11*a)}/x^{13}/a^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)/x^14,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.370995, size = 111, normalized size = 2.52

$$\frac{(2b^6x^{12} - ab^5x^{10} - 35a^2b^4x^8 - 90a^3b^3x^6 - 100a^4b^2x^4 - 53a^5bx^2 - 11a^6)\sqrt{bx^2 + a}}{143a^2x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)/x^14,x, algorithm="fricas")`

[Out]  $1/143*(2*b^6*x^{12} - a*b^5*x^{10} - 35*a^2*b^4*x^8 - 90*a^3*b^3*x^6 - 100*a^4*b^2*x^4 - 53*a^5*b*x^2 - 11*a^6)*\text{sqrt}(b*x^2 + a)/(a^2*x^{13})$

**Sympy [A]** time = 34.6716, size = 175, normalized size = 3.98

$$\begin{aligned} & \frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2} + 1}}{13x^{12}} - \frac{53a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2} + 1}}{143x^{10}} - \frac{100a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2} + 1}}{143x^8} \\ & - \frac{90ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2} + 1}}{143x^6} - \frac{35b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2} + 1}}{143x^4} - \frac{b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2} + 1}}{143ax^2} + \frac{2b^{\frac{13}{2}}\sqrt{\frac{a}{bx^2} + 1}}{143a^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(9/2)/x**14,x)`

[Out]  $-a^{*4}\text{sqrt}(b)*\text{sqrt}(a/(b*x^{*2}) + 1)/(13*x^{*12}) - 53*a^{*3}*b^{*(3/2)}*\text{sqrt}(a/(b*x^{*2}) + 1)/(143*x^{*10}) - 100*a^{*2}*b^{*(5/2)}*\text{sqrt}(a/(b*x^{*2}) + 1)/(143*x^{*8}) - 90*a*b^{*(7/2)}*\text{sqrt}(a/(b*x^{*2}) + 1)/(143*x^{*6}) - 35*b^{*(9/2)}*\text{sqrt}(a/(b*x^{*2}) + 1)/(143*x^{*4}) - b^{*(11/2)}*\text{sqrt}(a/(b*x^{*2}) + 1)/(143*a*x^{*2}) + 2*b^{*(13/2)}*\text{sqrt}(a/(b*x^{*2}) + 1)/(143*a^{*2})$

**GIAC/XCAS [A]** time = 0.217859, size = 443, normalized size = 10.07

$$4 \left( 143 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{22} b^{\frac{13}{2}} + 429 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{20} ab^{\frac{13}{2}} + 2145 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{18} a^2 b^{\frac{13}{2}} + 3003 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{16} a^3 b^{\frac{13}{2}} + 2145 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{14} a^4 b^{\frac{13}{2}} + 858 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{12} a^5 b^{\frac{13}{2}} + 214 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{10} a^6 b^{\frac{13}{2}} + 33 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^8 a^7 b^{\frac{13}{2}} + 3 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^6 a^8 b^{\frac{13}{2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^14,x, algorithm="giac")

[Out] 
$$\frac{4}{143} \cdot (143 \cdot (\sqrt{b}x - \sqrt{bx^2 + a})^{22} b^{13/2} + 429 \cdot (\sqrt{b}x - \sqrt{bx^2 + a})^{20} a b^{13/2} + 2145 \cdot (\sqrt{b}x - \sqrt{bx^2 + a})^{18} a^2 b^{13/2} + 3003 \cdot (\sqrt{b}x - \sqrt{bx^2 + a})^{16} a^3 b^{13/2} + 6006 \cdot (\sqrt{b}x - \sqrt{bx^2 + a})^{14} a^4 b^{13/2} + 4290 \cdot (\sqrt{b}x - \sqrt{bx^2 + a})^{12} a^5 b^{13/2} + 4290 \cdot (\sqrt{b}x - \sqrt{bx^2 + a})^{10} a^6 b^{13/2} + 1430 \cdot (\sqrt{b}x - \sqrt{bx^2 + a})^8 a^7 b^{13/2} + 715 \cdot (\sqrt{b}x - \sqrt{bx^2 + a})^6 a^8 b^{13/2} + 65 \cdot (\sqrt{b}x - \sqrt{bx^2 + a})^4 a^9 b^{13/2} + 13 \cdot (\sqrt{b}x - \sqrt{bx^2 + a})^2 a^{10} b^{13/2} - a^{11} b^{13/2}) / ((\sqrt{b}x - \sqrt{bx^2 + a})^2 - a)^{13}$$



$$3.436 \quad \int \frac{(a+bx^2)^{9/2}}{x^{16}} dx$$

**Optimal.** Leaf size=68

$$-\frac{8b^2(a+bx^2)^{11/2}}{2145a^3x^{11}} + \frac{4b(a+bx^2)^{11/2}}{195a^2x^{13}} - \frac{(a+bx^2)^{11/2}}{15ax^{15}}$$

[Out]  $-(a + b*x^2)^{(11/2)}/(15*a*x^{15}) + (4*b*(a + b*x^2)^{(11/2)})/(195*a^2*x^{13}) - (8*b^2*(a + b*x^2)^{(11/2)})/(2145*a^3*x^{11})$

**Rubi [A]** time = 0.0723616, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{8b^2(a+bx^2)^{11/2}}{2145a^3x^{11}} + \frac{4b(a+bx^2)^{11/2}}{195a^2x^{13}} - \frac{(a+bx^2)^{11/2}}{15ax^{15}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(9/2)/x^16, x]

[Out]  $-(a + b*x^2)^{(11/2)}/(15*a*x^{15}) + (4*b*(a + b*x^2)^{(11/2)})/(195*a^2*x^{13}) - (8*b^2*(a + b*x^2)^{(11/2)})/(2145*a^3*x^{11})$

**Rubi in Sympy [A]** time = 8.13084, size = 61, normalized size = 0.9

$$-\frac{(a+bx^2)^{\frac{11}{2}}}{15ax^{15}} + \frac{4b(a+bx^2)^{\frac{11}{2}}}{195a^2x^{13}} - \frac{8b^2(a+bx^2)^{\frac{11}{2}}}{2145a^3x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*16, x)

[Out]  $-(a + b*x^2)^{(11/2)}/(15*a*x^{15}) + 4*b*(a + b*x^2)^{(11/2)}/(195*a^2*x^{13}) - 8*b^2*(a + b*x^2)^{(11/2)}/(2145*a^3*x^{11})$

**Mathematica [A]** time = 0.0674275, size = 42, normalized size = 0.62

$$-\frac{(a+bx^2)^{11/2}(143a^2 - 44abx^2 + 8b^2x^4)}{2145a^3x^{15}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(9/2)/x^16, x]

[Out]  $-((a + b*x^2)^{(11/2)}*(143*a^2 - 44*a*b*x^2 + 8*b^2*x^4))/(2145*a^3*x^{15})$

**Maple [A]** time = 0.007, size = 39, normalized size = 0.6

$$-\frac{8b^2x^4 - 44abx^2 + 143a^2}{2145x^{15}a^3} (bx^2 + a)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(9/2)/x^16,x)`

[Out]  $-1/2145*(b*x^2+a)^{(11/2)}*(8*b^2*x^4-44*a*b*x^2+143*a^2)/x^{15}/a^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)/x^16,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.545779, size = 126, normalized size = 1.85

$$\frac{(8b^7x^{14} - 4ab^6x^{12} + 3a^2b^5x^{10} + 355a^3b^4x^8 + 1030a^4b^3x^6 + 1218a^5b^2x^4 + 671a^6bx^2 + 143a^7)\sqrt{bx^2 + a}}{2145a^3x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)/x^16,x, algorithm="fricas")`

[Out]  $-1/2145*(8*b^7*x^{14} - 4*a*b^6*x^{12} + 3*a^2*b^5*x^{10} + 355*a^3*b^4*x^8 + 1030*a^4*b^3*x^6 + 1218*a^5*b^2*x^4 + 671*a^6*b*x^2 + 143*a^7)*\text{sqrt}(b*x^2 + a)/(a^3*x^{15})$

**Sympy [A]** time = 62.1568, size = 604, normalized size = 8.88

$$\begin{aligned} & \frac{143a^9b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2} + 1}}{x^6(2145a^5b^4x^8 + 4290a^4b^5x^{10} + 2145a^3b^6x^{12})} \\ & - \frac{957a^8b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2} + 1}}{x^4(2145a^5b^4x^8 + 4290a^4b^5x^{10} + 2145a^3b^6x^{12})} \\ & - \frac{2703a^7b^{\frac{13}{2}}\sqrt{\frac{a}{bx^2} + 1}}{x^2(2145a^5b^4x^8 + 4290a^4b^5x^{10} + 2145a^3b^6x^{12})} - \frac{4137a^6b^{\frac{15}{2}}\sqrt{\frac{a}{bx^2} + 1}}{2145a^5b^4x^8 + 4290a^4b^5x^{10} + 2145a^3b^6x^{12}} \\ & - \frac{3633a^5b^{\frac{17}{2}}x^2\sqrt{\frac{a}{bx^2} + 1}}{2145a^5b^4x^8 + 4290a^4b^5x^{10} + 2145a^3b^6x^{12}} - \frac{1743a^4b^{\frac{19}{2}}x^4\sqrt{\frac{a}{bx^2} + 1}}{2145a^5b^4x^8 + 4290a^4b^5x^{10} + 2145a^3b^6x^{12}} \\ & - \frac{357a^3b^{\frac{21}{2}}x^6\sqrt{\frac{a}{bx^2} + 1}}{2145a^5b^4x^8 + 4290a^4b^5x^{10} + 2145a^3b^6x^{12}} - \frac{3a^2b^{\frac{23}{2}}x^8\sqrt{\frac{a}{bx^2} + 1}}{2145a^5b^4x^8 + 4290a^4b^5x^{10} + 2145a^3b^6x^{12}} \\ & - \frac{12ab^{\frac{25}{2}}x^{10}\sqrt{\frac{a}{bx^2} + 1}}{2145a^5b^4x^8 + 4290a^4b^5x^{10} + 2145a^3b^6x^{12}} - \frac{8b^{\frac{27}{2}}x^{12}\sqrt{\frac{a}{bx^2} + 1}}{2145a^5b^4x^8 + 4290a^4b^5x^{10} + 2145a^3b^6x^{12}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(9/2)/x**16,x)`

[Out]  $-143*a**9*b**(9/2)*\text{sqrt}(a/(b*x**2) + 1)/(x**6*(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12)) - 957*a**8*b**(11/2)*\text{sqrt}(a/(b*x**2) + 1)/(x**4*(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12)) - 2703*a**7*b**(13/2)*\text{sqrt}(a/(b*x**2) + 1)/(x**2*(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12)) - 4137*a**6*b**(15/2)*\text{sqrt}(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12)$

```

**12) - 3633*a**5*b**(17/2)*x**2*sqrt(a/(b*x**2) + 1)/(2145*a**5*
b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12) - 1743*a
**4*b**(19/2)*x**4*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 42
90*a**4*b**5*x**10 + 2145*a**3*b**6*x**12) - 357*a**3*b**(21/2)*x
**6*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x
**10 + 2145*a**3*b**6*x**12) - 3*a**2*b**(23/2)*x**8*sqrt(a/(b*x**
2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b
**6*x**12) - 12*a*b**(25/2)*x**10*sqrt(a/(b*x**2) + 1)/(2145*a**5
*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12) - 8*b**
(27/2)*x**12*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**
4*b**5*x**10 + 2145*a**3*b**6*x**12)

```

**GIAC/XCAS [A]** time = 0.219985, size = 478, normalized size = 7.03

$$16 \left( 1430 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{24} b^{\frac{15}{2}} + 6435 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{22} ab^{\frac{15}{2}} + 24453 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{20} a^2 b^{\frac{15}{2}} + 45045 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{18} a^3 b^{\frac{15}{2}} + 70785 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{16} a^4 b^{\frac{15}{2}} + 64350 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{14} a^5 b^{\frac{15}{2}} + 50050 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{12} a^6 b^{\frac{15}{2}} + 21450 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{10} a^7 b^{\frac{15}{2}} + 7800 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^8 a^8 b^{\frac{15}{2}} + 975 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^6 a^9 b^{\frac{15}{2}} + 105 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^4 a^{10} b^{\frac{15}{2}} - 15 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 a^{11} b^{\frac{15}{2}} + a^{12} b^{\frac{15}{2}} \right) / \left( \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 - a \right)^{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(9/2)/x^16,x, algorithm="giac")
```

```
[Out] 16/2145*(1430*(sqrt(b)*x - sqrt(b*x^2 + a))^24*b^(15/2) + 6435*(s
qrt(b)*x - sqrt(b*x^2 + a))^22*a*b^(15/2) + 24453*(sqrt(b)*x - sq
rt(b*x^2 + a))^20*a^2*b^(15/2) + 45045*(sqrt(b)*x - sqrt(b*x^2 +
a))^18*a^3*b^(15/2) + 70785*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^4*
b^(15/2) + 64350*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^5*b^(15/2) +
50050*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^6*b^(15/2) + 21450*(sqrt
(b)*x - sqrt(b*x^2 + a))^10*a^7*b^(15/2) + 7800*(sqrt(b)*x - sqrt
(b*x^2 + a))^8*a^8*b^(15/2) + 975*(sqrt(b)*x - sqrt(b*x^2 + a))^6
*a^9*b^(15/2) + 105*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^10*b^(15/2)
- 15*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^11*b^(15/2) + a^12*b^(15/
2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^15

```

$$3.437 \quad \int \frac{(a+bx^2)^{9/2}}{x^{18}} dx$$

**Optimal.** Leaf size=92

$$\frac{16b^3(a+bx^2)^{11/2}}{12155a^4x^{11}} - \frac{8b^2(a+bx^2)^{11/2}}{1105a^3x^{13}} + \frac{2b(a+bx^2)^{11/2}}{85a^2x^{15}} - \frac{(a+bx^2)^{11/2}}{17ax^{17}}$$

[Out]  $-(a + b*x^2)^{(11/2)}/(17*a*x^{17}) + (2*b*(a + b*x^2)^{(11/2)})/(85*a^2*x^{15}) - (8*b^2*(a + b*x^2)^{(11/2)})/(1105*a^3*x^{13}) + (16*b^3*(a + b*x^2)^{(11/2)})/(12155*a^4*x^{11})$

**Rubi [A]** time = 0.102357, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{16b^3(a+bx^2)^{11/2}}{12155a^4x^{11}} - \frac{8b^2(a+bx^2)^{11/2}}{1105a^3x^{13}} + \frac{2b(a+bx^2)^{11/2}}{85a^2x^{15}} - \frac{(a+bx^2)^{11/2}}{17ax^{17}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(9/2)/x^18, x]

[Out]  $-(a + b*x^2)^{(11/2)}/(17*a*x^{17}) + (2*b*(a + b*x^2)^{(11/2)})/(85*a^2*x^{15}) - (8*b^2*(a + b*x^2)^{(11/2)})/(1105*a^3*x^{13}) + (16*b^3*(a + b*x^2)^{(11/2)})/(12155*a^4*x^{11})$

**Rubi in Sympy [A]** time = 11.5613, size = 85, normalized size = 0.92

$$-\frac{(a+bx^2)^{\frac{11}{2}}}{17ax^{17}} + \frac{2b(a+bx^2)^{\frac{11}{2}}}{85a^2x^{15}} - \frac{8b^2(a+bx^2)^{\frac{11}{2}}}{1105a^3x^{13}} + \frac{16b^3(a+bx^2)^{\frac{11}{2}}}{12155a^4x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*18, x)

[Out]  $-(a + b*x^2)^{(11/2)}/(17*a*x^{17}) + 2*b*(a + b*x^2)^{(11/2)}/(85*a^2*x^{15}) - 8*b^2*(a + b*x^2)^{(11/2)}/(1105*a^3*x^{13}) + 16*b^3*(a + b*x^2)^{(11/2)}/(12155*a^4*x^{11})$

**Mathematica [A]** time = 0.0690104, size = 53, normalized size = 0.58

$$\frac{(a+bx^2)^{11/2}(-715a^3 + 286a^2bx^2 - 88ab^2x^4 + 16b^3x^6)}{12155a^4x^{17}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(9/2)/x^18, x]

[Out]  $((a + b*x^2)^{(11/2)}*(-715*a^3 + 286*a^2*b*x^2 - 88*a*b^2*x^4 + 16*b^3*x^6))/(12155*a^4*x^{17})$

**Maple [A]** time = 0.008, size = 50, normalized size = 0.5

$$-\frac{-16b^3x^6 + 88ab^2x^4 - 286a^2bx^2 + 715a^3}{12155x^{17}a^4}(bx^2 + a)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(9/2)/x^18,x)`

[Out] 
$$\frac{-1/12155*(b*x^2+a)^{(11/2)}*(-16*b^3*x^6+88*a*b^2*x^4-286*a^2*b*x^2+715*a^3)}{x^{17}/a^4}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)/x^18,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.760418, size = 140, normalized size = 1.52

$$\frac{(16b^8x^{16} - 8ab^7x^{14} + 6a^2b^6x^{12} - 5a^3b^5x^{10} - 1515a^4b^4x^8 - 4714a^5b^3x^6 - 5808a^6b^2x^4 - 3289a^7bx^2 - 715a^8)\sqrt{bx^2 + a}}{12155a^4x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)/x^18,x, algorithm="fricas")`

[Out] 
$$\frac{1/12155*(16*b^8*x^{16} - 8*a*b^7*x^{14} + 6*a^2*b^6*x^{12} - 5*a^3*b^5*x^{10} - 1515*a^4*b^4*x^8 - 4714*a^5*b^3*x^6 - 5808*a^6*b^2*x^4 - 3289*a^7*b*x^2 - 715*a^8)*\text{sqrt}(b*x^2 + a)}{(a^4*x^{17})}$$

**Sympy [A]** time = 83.0811, size = 867, normalized size = 9.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(9/2)/x**18,x)`

[Out] 
$$\begin{aligned} & -715*a^{11}*b^{19/2}*\text{sqrt}(a/(b*x^2) + 1)/(12155*a^{7*b^9*x^{16}} \\ & + 36465*a^{6*b^{10}*x^{18}} + 36465*a^{5*b^{11}*x^{20}} + 12155*a^{4*b^{12}*x^{22}} - 5434*a^{10*b^{21/2}}*x^{22}*\text{sqrt}(a/(b*x^2) + 1)/(1215 \\ & 5*a^{7*b^9*x^{16}} + 36465*a^{6*b^{10}*x^{18}} + 36465*a^{5*b^{11}*x^{20}} + 12155*a^{4*b^{12}*x^{22}} - 17820*a^{9*b^{23/2}}*x^{24}*\text{sqrt}(a/( \\ & b*x^2) + 1)/(12155*a^{7*b^9*x^{16}} + 36465*a^{6*b^{10}*x^{18}} + 36 \\ & 465*a^{5*b^{11}*x^{20}} + 12155*a^{4*b^{12}*x^{22}} - 32720*a^{8*b^{25/2}}*x^{26}*\text{sqrt}(a/(b*x^2) + 1)/(12155*a^{7*b^9*x^{16}} + 36465*a^{6 \\ & 6*b^{10}*x^{18}} + 36465*a^{5*b^{11}*x^{20}} + 12155*a^{4*b^{12}*x^{22}}) \\ & - 36370*a^{7*b^{27/2}}*x^{28}*\text{sqrt}(a/(b*x^2) + 1)/(12155*a^{7*b^9*x^{16}} \\ & *x^{16} + 36465*a^{6*b^{10}*x^{18}} + 36465*a^{5*b^{11}*x^{20}} + 12155*a^{4*b^{12}*x^{22}} - 24500*a^{6*b^{29/2}}*x^{30}*\text{sqrt}(a/(b*x^2) + \\ & 1)/(12155*a^{7*b^9*x^{16}} + 36465*a^{6*b^{10}*x^{18}} + 36465*a^{5*b^{11}*x^{20}} + 12155*a^{4*b^{12}*x^{22}}) - 9268*a^{5*b^{31/2}}*x^{32} \\ & \text{sqrt}(a/(b*x^2) + 1)/(12155*a^{7*b^9*x^{16}} + 36465*a^{6*b^{10}*x^{18}} + 36465*a^{5*b^{11}*x^{20}} + 12155*a^{4*b^{12}*x^{22}}) \\ & - 1520*a^{4*b^{33/2}}*x^{34}*\text{sqrt}(a/(b*x^2) + 1)/(12155*a^{7*b^9*x^{16}} + 3 \\ & 6465*a^{6*b^{10}*x^{18}} + 36465*a^{5*b^{11}*x^{20}} + 12155*a^{4*b^{12}*x^{22}} \\ & *x^{22}) + 5*a^{3*b^{35/2}}*x^{36}*\text{sqrt}(a/(b*x^2) + 1)/(12155*a^{7 \\ & *b^9*x^{16}} + 36465*a^{6*b^{10}*x^{18}} + 36465*a^{5*b^{11}*x^{20}} + 1 \\ & 2155*a^{4*b^{12}*x^{22}}) + 30*a^{2*b^{37/2}}*x^{38}*\text{sqrt}(a/(b*x^2) \end{aligned}$$

$$+ 1)/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) + 40*a*b**(39/2)*x**20*\sqrt{a/(b*x**2) + 1}/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) + 16*b**(41/2)*x**22*\sqrt{a/(b*x**2) + 1}/(12155*a**7*b**9*x**16 + 36465*a**6*b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22)$$

**GIAC/XCAS [A]** time = 0.216754, size = 516, normalized size = 5.61

$$32 \left( 12155 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{26} b^{\frac{17}{2}} + 65637 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{24} ab^{\frac{17}{2}} + 233376 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{22} a^2 b^{\frac{17}{2}} + 466752 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{20} a^3 b^{\frac{17}{2}} + 692835 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{18} a^4 b^{\frac{17}{2}} + 668525 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{16} a^5 b^{\frac{17}{2}} + 486200 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{14} a^6 b^{\frac{17}{2}} + 221000 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{12} a^7 b^{\frac{17}{2}} + 71825 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{10} a^8 b^{\frac{17}{2}} + 9775 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^8 a^9 b^{\frac{17}{2}} + 680 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^6 a^{10} b^{\frac{17}{2}} - 136 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^4 a^{11} b^{\frac{17}{2}} + 17 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 a^{12} b^{\frac{17}{2}} - a^{13} b^{\frac{17}{2}} \right) / \left( \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 - a \right)^{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^18,x, algorithm="giac")

[Out] 32/12155\*(12155\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^26\*b^(17/2) + 65637\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^24\*a\*b^(17/2) + 233376\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^22\*a^2\*b^(17/2) + 466752\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^20\*a^3\*b^(17/2) + 692835\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^18\*a^4\*b^(17/2) + 668525\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^16\*a^5\*b^(17/2) + 486200\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^14\*a^6\*b^(17/2) + 221000\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^12\*a^7\*b^(17/2) + 71825\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^10\*a^8\*b^(17/2) + 9775\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^8\*a^9\*b^(17/2) + 680\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^6\*a^10\*b^(17/2) - 136\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^4\*a^11\*b^(17/2) + 17\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^2\*a^12\*b^(17/2) - a^13\*b^(17/2))/((sqrt(b)\*x - sqrt(b\*x^2 + a))^2 - a)^17

$$3.438 \quad \int \frac{(a+bx^2)^{9/2}}{x^{20}} dx$$

**Optimal.** Leaf size=116

$$-\frac{128b^4(a+bx^2)^{11/2}}{230945a^5x^{11}} + \frac{64b^3(a+bx^2)^{11/2}}{20995a^4x^{13}} - \frac{16b^2(a+bx^2)^{11/2}}{1615a^3x^{15}} + \frac{8b(a+bx^2)^{11/2}}{323a^2x^{17}} - \frac{(a+bx^2)^{11/2}}{19ax^{19}}$$

[Out]  $-(a + b*x^2)^{(11/2)}/(19*a*x^{19}) + (8*b*(a + b*x^2)^{(11/2)})/(323*a^2*x^{17}) - (16*b^2*(a + b*x^2)^{(11/2)})/(1615*a^3*x^{15}) + (64*b^3*(a + b*x^2)^{(11/2)})/(20995*a^4*x^{13}) - (128*b^4*(a + b*x^2)^{(11/2)})/(230945*a^5*x^{11})$

**Rubi [A]** time = 0.135212, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{128b^4(a+bx^2)^{11/2}}{230945a^5x^{11}} + \frac{64b^3(a+bx^2)^{11/2}}{20995a^4x^{13}} - \frac{16b^2(a+bx^2)^{11/2}}{1615a^3x^{15}} + \frac{8b(a+bx^2)^{11/2}}{323a^2x^{17}} - \frac{(a+bx^2)^{11/2}}{19ax^{19}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(9/2)/x^20, x]

[Out]  $-(a + b*x^2)^{(11/2)}/(19*a*x^{19}) + (8*b*(a + b*x^2)^{(11/2)})/(323*a^2*x^{17}) - (16*b^2*(a + b*x^2)^{(11/2)})/(1615*a^3*x^{15}) + (64*b^3*(a + b*x^2)^{(11/2)})/(20995*a^4*x^{13}) - (128*b^4*(a + b*x^2)^{(11/2)})/(230945*a^5*x^{11})$

**Rubi in Sympy [A]** time = 15.8904, size = 109, normalized size = 0.94

$$-\frac{(a+bx^2)^{\frac{11}{2}}}{19ax^{19}} + \frac{8b(a+bx^2)^{\frac{11}{2}}}{323a^2x^{17}} - \frac{16b^2(a+bx^2)^{\frac{11}{2}}}{1615a^3x^{15}} + \frac{64b^3(a+bx^2)^{\frac{11}{2}}}{20995a^4x^{13}} - \frac{128b^4(a+bx^2)^{\frac{11}{2}}}{230945a^5x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*20, x)

[Out]  $-(a + b*x**2)**(11/2)/(19*a*x**19) + 8*b*(a + b*x**2)**(11/2)/(323*a**2*x**17) - 16*b**2*(a + b*x**2)**(11/2)/(1615*a**3*x**15) + 64*b**3*(a + b*x**2)**(11/2)/(20995*a**4*x**13) - 128*b**4*(a + b*x**2)**(11/2)/(230945*a**5*x**11)$

**Mathematica [A]** time = 0.0823812, size = 64, normalized size = 0.55

$$-\frac{(a+bx^2)^{11/2}(12155a^4 - 5720a^3bx^2 + 2288a^2b^2x^4 - 704ab^3x^6 + 128b^4x^8)}{230945a^5x^{19}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(9/2)/x^20, x]

[Out]  $-((a + b*x^2)^{(11/2)}*(12155*a^4 - 5720*a^3*b*x^2 + 2288*a^2*b^2*x^4 - 704*a*b^3*x^6 + 128*b^4*x^8))/(230945*a^5*x^{19})$

**Maple [A]** time = 0.009, size = 61, normalized size = 0.5

$$\frac{128 b^4 x^8 - 704 b^3 x^6 a + 2288 b^2 x^4 a^2 - 5720 b x^2 a^3 + 12155 a^4}{230945 x^{19} a^5} (b x^2 + a)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(9/2)/x^20,x)`

[Out] `-1/230945*(b*x^2+a)^(11/2)*(128*b^4*x^8-704*a*b^3*x^6+2288*a^2*b^2*x^4-5720*a^3*b*x^2+12155*a^4)/x^19/a^5`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)/x^20,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.084, size = 155, normalized size = 1.34

$$\frac{(128 b^9 x^{18} - 64 a b^8 x^{16} + 48 a^2 b^7 x^{14} - 40 a^3 b^6 x^{12} + 35 a^4 b^5 x^{10} + 23063 a^5 b^4 x^8 + 75086 a^6 b^3 x^6 + 95238 a^7 b^2 x^4 + 55055 a^8 b x^2 + 12155 a^9) \sqrt{b x^2 + a}}{230945 a^5 x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(9/2)/x^20,x, algorithm="fricas")`

[Out] `-1/230945*(128*b^9*x^18 - 64*a*b^8*x^16 + 48*a^2*b^7*x^14 - 40*a^3*b^6*x^12 + 35*a^4*b^5*x^10 + 23063*a^5*b^4*x^8 + 75086*a^6*b^3*x^6 + 95238*a^7*b^2*x^4 + 55055*a^8*b*x^2 + 12155*a^9)*sqrt(b*x^2 + a)/(a^5*x^19)`

**Sympy [A]** time = 111.166, size = 1182, normalized size = 10.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(9/2)/x**20,x)`

[Out] `-12155*a**13*b**(33/2)*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 103675*a**12*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 388388*a**11*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 834988*a**10*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 1127210*a**9*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 97881`



$$\begin{aligned}
& 0 \cdot a^{**8} \cdot b^{** (43/2)} \cdot x^{**10} \cdot \sqrt{a/(b \cdot x^{**2}) + 1} / (230945 \cdot a^{**9} \cdot b^{**16} \cdot x^{**18} + 923780 \cdot a^{**8} \cdot b^{**17} \cdot x^{**20} + 1385670 \cdot a^{**7} \cdot b^{**18} \cdot x^{**22} + 923780 \cdot a^{**6} \cdot b^{**19} \cdot x^{**24} + 230945 \cdot a^{**5} \cdot b^{**20} \cdot x^{**26}) - 534060 \cdot a^{**7} \cdot b^{** (45/2)} \cdot x^{**12} \cdot \sqrt{a/(b \cdot x^{**2}) + 1} / (230945 \cdot a^{**9} \cdot b^{**16} \cdot x^{**18} + 923780 \cdot a^{**8} \cdot b^{**17} \cdot x^{**20} + 1385670 \cdot a^{**7} \cdot b^{**18} \cdot x^{**22} + 923780 \cdot a^{**6} \cdot b^{**19} \cdot x^{**24} + 230945 \cdot a^{**5} \cdot b^{**20} \cdot x^{**26}) - 167436 \cdot a^{**6} \cdot b^{** (47/2)} \cdot x^{**14} \cdot \sqrt{a/(b \cdot x^{**2}) + 1} / (230945 \cdot a^{**9} \cdot b^{**16} \cdot x^{**18} + 923780 \cdot a^{**8} \cdot b^{**17} \cdot x^{**20} + 1385670 \cdot a^{**7} \cdot b^{**18} \cdot x^{**22} + 923780 \cdot a^{**6} \cdot b^{**19} \cdot x^{**24} + 230945 \cdot a^{**5} \cdot b^{**20} \cdot x^{**26}) - 23091 \cdot a^{**5} \cdot b^{** (49/2)} \cdot x^{**16} \cdot \sqrt{a/(b \cdot x^{**2}) + 1} / (230945 \cdot a^{**9} \cdot b^{**16} \cdot x^{**18} + 923780 \cdot a^{**8} \cdot b^{**17} \cdot x^{**20} + 1385670 \cdot a^{**7} \cdot b^{**18} \cdot x^{**22} + 923780 \cdot a^{**6} \cdot b^{**19} \cdot x^{**24} + 230945 \cdot a^{**5} \cdot b^{**20} \cdot x^{**26}) - 35 \cdot a^{**4} \cdot b^{** (51/2)} \cdot x^{**18} \cdot \sqrt{a/(b \cdot x^{**2}) + 1} / (230945 \cdot a^{**9} \cdot b^{**16} \cdot x^{**18} + 923780 \cdot a^{**8} \cdot b^{**17} \cdot x^{**20} + 1385670 \cdot a^{**7} \cdot b^{**18} \cdot x^{**22} + 923780 \cdot a^{**6} \cdot b^{**19} \cdot x^{**24} + 230945 \cdot a^{**5} \cdot b^{**20} \cdot x^{**26}) - 280 \cdot a^{**3} \cdot b^{** (53/2)} \cdot x^{**20} \cdot \sqrt{a/(b \cdot x^{**2}) + 1} / (230945 \cdot a^{**9} \cdot b^{**16} \cdot x^{**18} + 923780 \cdot a^{**8} \cdot b^{**17} \cdot x^{**20} + 1385670 \cdot a^{**7} \cdot b^{**18} \cdot x^{**22} + 923780 \cdot a^{**6} \cdot b^{**19} \cdot x^{**24} + 230945 \cdot a^{**5} \cdot b^{**20} \cdot x^{**26}) - 560 \cdot a^{**2} \cdot b^{** (55/2)} \cdot x^{**22} \cdot \sqrt{a/(b \cdot x^{**2}) + 1} / (230945 \cdot a^{**9} \cdot b^{**16} \cdot x^{**18} + 923780 \cdot a^{**8} \cdot b^{**17} \cdot x^{**20} + 1385670 \cdot a^{**7} \cdot b^{**18} \cdot x^{**22} + 923780 \cdot a^{**6} \cdot b^{**19} \cdot x^{**24} + 230945 \cdot a^{**5} \cdot b^{**20} \cdot x^{**26}) - 448 \cdot a \cdot b^{** (57/2)} \cdot x^{**24} \cdot \sqrt{a/(b \cdot x^{**2}) + 1} / (230945 \cdot a^{**9} \cdot b^{**16} \cdot x^{**18} + 923780 \cdot a^{**8} \cdot b^{**17} \cdot x^{**20} + 1385670 \cdot a^{**7} \cdot b^{**18} \cdot x^{**22} + 923780 \cdot a^{**6} \cdot b^{**19} \cdot x^{**24} + 230945 \cdot a^{**5} \cdot b^{**20} \cdot x^{**26}) - 128 \cdot b^{** (59/2)} \cdot x^{**26} \cdot \sqrt{a/(b \cdot x^{**2}) + 1} / (230945 \cdot a^{**9} \cdot b^{**16} \cdot x^{**18} + 923780 \cdot a^{**8} \cdot b^{**17} \cdot x^{**20} + 1385670 \cdot a^{**7} \cdot b^{**18} \cdot x^{**22} + 923780 \cdot a^{**6} \cdot b^{**19} \cdot x^{**24} + 230945 \cdot a^{**5} \cdot b^{**20} \cdot x^{**26})
\end{aligned}$$


---

**GIAC/XCAS [A]** time = 0.221567, size = 551, normalized size = 4.75

$$256 \left( 92378 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{28} b^{\frac{19}{2}} + 554268 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{26} ab^{\frac{19}{2}} + 1939938 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{24} a^2 b^{\frac{19}{2}} + 4018443 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{22} a^3 b^{\frac{19}{2}} + 5866003 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{20} a^4 b^{\frac{19}{2}} + 5773625 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{18} a^5 b^{\frac{19}{2}} + 4094025 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{16} a^6 b^{\frac{19}{2}} + 1889550 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{14} a^7 b^{\frac{19}{2}} + 581400 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{12} a^8 b^{\frac{19}{2}} + 80750 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{10} a^9 b^{\frac{19}{2}} + 3876 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^8 a^{10} b^{\frac{19}{2}} - 969 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^6 a^{11} b^{\frac{19}{2}} + 171 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^4 a^{12} b^{\frac{19}{2}} - 19 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 a^{13} b^{\frac{19}{2}} + a^{14} b^{\frac{19}{2}} \right) / ((\sqrt{bx} - \sqrt{bx^2 + a})^2 - a)^{19}$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^20,x, algorithm="giac")

[Out] 256/230945\*(92378\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^28\*b^(19/2) + 554268\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^26\*a\*b^(19/2) + 1939938\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^24\*a^2\*b^(19/2) + 4018443\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^22\*a^3\*b^(19/2) + 5866003\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^20\*a^4\*b^(19/2) + 5773625\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^18\*a^5\*b^(19/2) + 4094025\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^16\*a^6\*b^(19/2) + 1889550\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^14\*a^7\*b^(19/2) + 581400\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^12\*a^8\*b^(19/2) + 80750\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^10\*a^9\*b^(19/2) + 3876\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^8\*a^10\*b^(19/2) - 969\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^6\*a^11\*b^(19/2) + 171\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^4\*a^12\*b^(19/2) - 19\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^2\*a^13\*b^(19/2) + a^14\*b^(19/2))/((sqrt(b)\*x - sqrt(b\*x^2 + a))^2 - a)^19

$$3.439 \quad \int \frac{(a+bx^2)^{9/2}}{x^{22}} dx$$

**Optimal.** Leaf size=140

$$\frac{256b^5 (a+bx^2)^{11/2}}{969969a^6x^{11}} - \frac{128b^4 (a+bx^2)^{11/2}}{88179a^5x^{13}} + \frac{32b^3 (a+bx^2)^{11/2}}{6783a^4x^{15}} - \frac{80b^2 (a+bx^2)^{11/2}}{6783a^3x^{17}} + \frac{10b (a+bx^2)^{11/2}}{399a^2x^{19}} - \frac{(a+bx^2)^{11/2}}{21ax^{21}}$$

[Out]  $-(a + b*x^2)^{(11/2)}/(21*a*x^{21}) + (10*b*(a + b*x^2)^{(11/2)})/(399*a^2*x^{19}) - (80*b^2*(a + b*x^2)^{(11/2)})/(6783*a^3*x^{17}) + (32*b^3*(a + b*x^2)^{(11/2)})/(6783*a^4*x^{15}) - (128*b^4*(a + b*x^2)^{(11/2)})/(88179*a^5*x^{13}) + (256*b^5*(a + b*x^2)^{(11/2)})/(969969*a^6*x^{11})$

**Rubi [A]** time = 0.173934, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{256b^5 (a+bx^2)^{11/2}}{969969a^6x^{11}} - \frac{128b^4 (a+bx^2)^{11/2}}{88179a^5x^{13}} + \frac{32b^3 (a+bx^2)^{11/2}}{6783a^4x^{15}} - \frac{80b^2 (a+bx^2)^{11/2}}{6783a^3x^{17}} + \frac{10b (a+bx^2)^{11/2}}{399a^2x^{19}} - \frac{(a+bx^2)^{11/2}}{21ax^{21}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(9/2)/x^22, x]

[Out]  $-(a + b*x^2)^{(11/2)}/(21*a*x^{21}) + (10*b*(a + b*x^2)^{(11/2)})/(399*a^2*x^{19}) - (80*b^2*(a + b*x^2)^{(11/2)})/(6783*a^3*x^{17}) + (32*b^3*(a + b*x^2)^{(11/2)})/(6783*a^4*x^{15}) - (128*b^4*(a + b*x^2)^{(11/2)})/(88179*a^5*x^{13}) + (256*b^5*(a + b*x^2)^{(11/2)})/(969969*a^6*x^{11})$

**Rubi in Sympy [A]** time = 20.6245, size = 133, normalized size = 0.95

$$-\frac{(a+bx^2)^{\frac{11}{2}}}{21ax^{21}} + \frac{10b(a+bx^2)^{\frac{11}{2}}}{399a^2x^{19}} - \frac{80b^2(a+bx^2)^{\frac{11}{2}}}{6783a^3x^{17}} + \frac{32b^3(a+bx^2)^{\frac{11}{2}}}{6783a^4x^{15}} - \frac{128b^4(a+bx^2)^{\frac{11}{2}}}{88179a^5x^{13}} + \frac{256b^5(a+bx^2)^{\frac{11}{2}}}{969969a^6x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*22, x)

[Out]  $-(a + b*x^2)^{(11/2)}/(21*a*x^{21}) + 10*b*(a + b*x^2)^{(11/2)}/(399*a^2*x^{19}) - 80*b^2*(a + b*x^2)^{(11/2)}/(6783*a^3*x^{17}) + 32*b^3*(a + b*x^2)^{(11/2)}/(6783*a^4*x^{15}) - 128*b^4*(a + b*x^2)^{(11/2)}/(88179*a^5*x^{13}) + 256*b^5*(a + b*x^2)^{(11/2)}/(969969*a^6*x^{11})$

**Mathematica [A]** time = 0.0832919, size = 75, normalized size = 0.54

$$\frac{(a+bx^2)^{11/2} (-46189a^5 + 24310a^4bx^2 - 11440a^3b^2x^4 + 4576a^2b^3x^6 - 1408ab^4x^8 + 256b^5x^{10})}{969969a^6x^{21}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(9/2)/x^22, x]

[Out] ((a + b\*x^2)^(11/2) \* (-46189\*a^5 + 24310\*a^4\*b\*x^2 - 11440\*a^3\*b^2\*x^4 + 4576\*a^2\*b^3\*x^6 - 1408\*a\*b^4\*x^8 + 256\*b^5\*x^10))/(969969\*a^6\*x^21)

**Maple [A]** time = 0.01, size = 72, normalized size = 0.5

$$\frac{-256 b^5 x^{10} + 1408 a b^4 x^8 - 4576 a^2 b^3 x^6 + 11440 a^3 b^2 x^4 - 24310 a^4 b x^2 + 46189 a^5}{969969 x^{21} a^6} (b x^2 + a)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(9/2)/x^22, x)

[Out] -1/969969\*(b\*x^2+a)^(11/2)\*(-256\*b^5\*x^10+1408\*a\*b^4\*x^8-4576\*a^2\*b^3\*x^6+11440\*a^3\*b^2\*x^4-24310\*a^4\*b\*x^2+46189\*a^5)/x^21/a^6

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^22, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.56772, size = 170, normalized size = 1.21

$$\frac{(256 b^{10} x^{20} - 128 a b^9 x^{18} + 96 a^2 b^8 x^{16} - 80 a^3 b^7 x^{14} + 70 a^4 b^6 x^{12} - 63 a^5 b^5 x^{10} - 80773 a^6 b^4 x^8 - 271414 a^7 b^3 x^6 - 351780 a^8 b^2 x^4 - 206635 a^9 b x^2 - 46189 a^{10}) \sqrt{b x^2 + a}}{969969 a^6 x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^22, x, algorithm="fricas")

[Out] 1/969969\*(256\*b^10\*x^20 - 128\*a\*b^9\*x^18 + 96\*a^2\*b^8\*x^16 - 80\*a^3\*b^7\*x^14 + 70\*a^4\*b^6\*x^12 - 63\*a^5\*b^5\*x^10 - 80773\*a^6\*b^4\*x^8 - 271414\*a^7\*b^3\*x^6 - 351780\*a^8\*b^2\*x^4 - 206635\*a^9\*b\*x^2 - 46189\*a^10)\*sqrt(b\*x^2 + a)/(a^6\*x^21)

**Sympy [A]** time = 114.539, size = 1540, normalized size = 11.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*22, x)

[Out] -46189\*a\*\*15\*b\*\*(51/2)\*sqrt(a/(b\*x\*\*2) + 1)/(969969\*a\*\*11\*b\*\*25\*x\*\*20 + 4849845\*a\*\*10\*b\*\*26\*x\*\*22 + 9699690\*a\*\*9\*b\*\*27\*x\*\*24 + 9699690\*a\*\*8\*b\*\*28\*x\*\*26 + 4849845\*a\*\*7\*b\*\*29\*x\*\*28 + 969969\*a\*\*6\*b\*\*30\*x\*\*30) - 437580\*a\*\*14\*b\*\*(53/2)\*x\*\*2\*sqrt(a/(b\*x\*\*2) + 1)/(96

```

9969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9
*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**2
8 + 969969*a**6*b**30*x**30) - 1846845*a**13*b**(55/2)*x**4*sqrt(
a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x
**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849
845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) - 4558554*a**12*b
**(57/2)*x**6*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 48
49845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8
*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30
) - 7252938*a**11*b**(59/2)*x**8*sqrt(a/(b*x**2) + 1)/(969969*a**
11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x
**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 9699
69*a**6*b**30*x**30) - 7715232*a**10*b**(61/2)*x**10*sqrt(a/(b*x*
**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 +
9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**
7*b**29*x**28 + 969969*a**6*b**30*x**30) - 5487650*a**9*b**(63/2)
*x**12*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a
**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*
x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) - 251
6940*a**8*b**(65/2)*x**14*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**2
5*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 +
9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6
*b**30*x**30) - 675513*a**7*b**(67/2)*x**16*sqrt(a/(b*x**2) + 1)/
(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a
**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x
**28 + 969969*a**6*b**30*x**30) - 80836*a**6*b**(69/2)*x**18*sqrt
(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*
x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 484
9845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) + 63*a**5*b**(71
/2)*x**20*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 484984
5*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**
28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) +
630*a**4*b**(73/2)*x**22*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25
*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9
699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*
b**30*x**30) + 1680*a**3*b**(75/2)*x**24*sqrt(a/(b*x**2) + 1)/(96
9969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9
*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**2
8 + 969969*a**6*b**30*x**30) + 2016*a**2*b**(77/2)*x**26*sqrt(a/(
b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**2
2 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845
*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) + 1152*a*b**(79/2)*x
**28*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**
10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x*
**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) + 256*b
**(81/2)*x**30*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4
849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**
8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**3
0)

```

---

**GIAC/XCAS [A]** time = 0.219935, size = 589, normalized size = 4.21

$$512 \left( 646646 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{30} b^{\frac{21}{2}} + 4157010 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{28} ab^{\frac{21}{2}} + 14549535 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{26} a^2 b^{\frac{21}{2}} + 3071568 \right)$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^22,x, algorithm="giac")

[Out] 512/969969\*(646646\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^30\*b^(21/2) + 4157010\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^28\*a\*b^(21/2) + 14549535\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^26\*a^2\*b^(21/2) + 30715685\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^24\*a^3\*b^(21/2) + 44618574\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^22\*a^4\*b^(21/2) + 44265858\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^20\*a^5\*b^(21/2) + 31009615\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^18\*a^6\*b^(21/2) + 14346045\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^16\*a^7\*b^(21/2) + 4273290\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^14\*a^8\*b^(21/2) + 59238

$$\begin{aligned}
& 2 * (\text{sqrt}(b) * x - \text{sqrt}(b * x^2 + a))^{12} * a^9 * b^{(21/2)} + 20349 * (\text{sqrt}(b) * \\
& x - \text{sqrt}(b * x^2 + a))^{10} * a^{10} * b^{(21/2)} - 5985 * (\text{sqrt}(b) * x - \text{sqrt}(b * \\
& x^2 + a))^{8} * a^{11} * b^{(21/2)} + 1330 * (\text{sqrt}(b) * x - \text{sqrt}(b * x^2 + a))^{6} * \\
& a^{12} * b^{(21/2)} - 210 * (\text{sqrt}(b) * x - \text{sqrt}(b * x^2 + a))^{4} * a^{13} * b^{(21/2)} \\
& + 21 * (\text{sqrt}(b) * x - \text{sqrt}(b * x^2 + a))^{2} * a^{14} * b^{(21/2)} - a^{15} * b^{(21/2)} \\
& ) / ((\text{sqrt}(b) * x - \text{sqrt}(b * x^2 + a))^2 - a)^{21}
\end{aligned}$$

$$3.440 \quad \int \frac{(a+bx^2)^{9/2}}{x^{24}} dx$$

**Optimal.** Leaf size=164

$$\begin{aligned} & -\frac{1024b^6(a+bx^2)^{11/2}}{7436429a^7x^{11}} + \frac{512b^5(a+bx^2)^{11/2}}{676039a^6x^{13}} - \frac{128b^4(a+bx^2)^{11/2}}{52003a^5x^{15}} \\ & + \frac{320b^3(a+bx^2)^{11/2}}{52003a^4x^{17}} - \frac{40b^2(a+bx^2)^{11/2}}{3059a^3x^{19}} + \frac{4b(a+bx^2)^{11/2}}{161a^2x^{21}} - \frac{(a+bx^2)^{11/2}}{23ax^{23}} \end{aligned}$$

[Out]  $-(a + b*x^2)^{(11/2)}/(23*a*x^{23}) + (4*b*(a + b*x^2)^{(11/2)})/(161*a^2*x^{21}) - (40*b^2*(a + b*x^2)^{(11/2)})/(3059*a^3*x^{19}) + (320*b^3*(a + b*x^2)^{(11/2)})/(52003*a^4*x^{17}) - (128*b^4*(a + b*x^2)^{(11/2)})/(52003*a^5*x^{15}) + (512*b^5*(a + b*x^2)^{(11/2)})/(676039*a^6*x^{13}) - (1024*b^6*(a + b*x^2)^{(11/2)})/(7436429*a^7*x^{11})$

**Rubi [A]** time = 0.213991, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\begin{aligned} & -\frac{1024b^6(a+bx^2)^{11/2}}{7436429a^7x^{11}} + \frac{512b^5(a+bx^2)^{11/2}}{676039a^6x^{13}} - \frac{128b^4(a+bx^2)^{11/2}}{52003a^5x^{15}} \\ & + \frac{320b^3(a+bx^2)^{11/2}}{52003a^4x^{17}} - \frac{40b^2(a+bx^2)^{11/2}}{3059a^3x^{19}} + \frac{4b(a+bx^2)^{11/2}}{161a^2x^{21}} - \frac{(a+bx^2)^{11/2}}{23ax^{23}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(9/2)/x^24, x]

[Out]  $-(a + b*x^2)^{(11/2)}/(23*a*x^{23}) + (4*b*(a + b*x^2)^{(11/2)})/(161*a^2*x^{21}) - (40*b^2*(a + b*x^2)^{(11/2)})/(3059*a^3*x^{19}) + (320*b^3*(a + b*x^2)^{(11/2)})/(52003*a^4*x^{17}) - (128*b^4*(a + b*x^2)^{(11/2)})/(52003*a^5*x^{15}) + (512*b^5*(a + b*x^2)^{(11/2)})/(676039*a^6*x^{13}) - (1024*b^6*(a + b*x^2)^{(11/2)})/(7436429*a^7*x^{11})$

**Rubi in Sympy [A]** time = 26.5061, size = 156, normalized size = 0.95

$$\begin{aligned} & -\frac{(a+bx^2)^{\frac{11}{2}}}{23ax^{23}} + \frac{4b(a+bx^2)^{\frac{11}{2}}}{161a^2x^{21}} - \frac{40b^2(a+bx^2)^{\frac{11}{2}}}{3059a^3x^{19}} + \frac{320b^3(a+bx^2)^{\frac{11}{2}}}{52003a^4x^{17}} \\ & - \frac{128b^4(a+bx^2)^{\frac{11}{2}}}{52003a^5x^{15}} + \frac{512b^5(a+bx^2)^{\frac{11}{2}}}{676039a^6x^{13}} - \frac{1024b^6(a+bx^2)^{\frac{11}{2}}}{7436429a^7x^{11}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*24, x)

[Out]  $-(a + b*x^2)^{(11/2)}/(23*a*x^{23}) + 4*b*(a + b*x^2)^{(11/2)}/(161*a^2*x^{21}) - 40*b^2*(a + b*x^2)^{(11/2)}/(3059*a^3*x^{19}) + 320*b^3*(a + b*x^2)^{(11/2)}/(52003*a^4*x^{17}) - 128*b^4*(a + b*x^2)^{(11/2)}/(52003*a^5*x^{15}) + 512*b^5*(a + b*x^2)^{(11/2)}/(676039*a^6*x^{13}) - 1024*b^6*(a + b*x^2)^{(11/2)}/(7436429*a^7*x^{11})$

**Mathematica [A]** time = 0.0935969, size = 86, normalized size = 0.52

$$\frac{(a+bx^2)^{11/2}(323323a^6 - 184756a^5bx^2 + 97240a^4b^2x^4 - 45760a^3b^3x^6 + 18304a^2b^4x^8 - 5632ab^5x^{10} + 1024b^6x^{12})}{7436429a^7x^{23}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(9/2)/x^24, x]

[Out]  $-\frac{((a + b^2x^2)^{11/2} (323323a^6 - 184756a^5bx^2 + 97240a^4b^2x^4 - 45760a^3b^3x^6 + 18304a^2b^4x^8 - 5632ab^5x^{10} + 1024b^6x^{12}))}{7436429a^7x^{23}}$

**Maple [A]** time = 0.011, size = 83, normalized size = 0.5

$$\frac{1024b^6x^{12} - 5632b^5x^{10}a + 18304b^4x^8a^2 - 45760b^3x^6a^3 + 97240b^2x^4a^4 - 184756bx^2a^5 + 323323a^6}{7436429x^{23}a^7} (bx^2 + a)^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(9/2)/x^24, x)

[Out]  $-\frac{1}{7436429} (bx^2+a)^{11/2} (1024b^6x^{12} - 5632ab^5x^{10} + 18304a^2b^4x^8 - 45760a^3b^3x^6 + 97240a^4b^2x^4 - 184756a^5bx^2 + 323323a^6) / x^{23} / a^7$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^24, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.35318, size = 185, normalized size = 1.13

$$\frac{(1024b^{11}x^{22} - 512ab^{10}x^{20} + 384a^2b^9x^{18} - 320a^3b^8x^{16} + 280a^4b^7x^{14} - 252a^5b^6x^{12} + 231a^6b^5x^{10} + 530959a^7b^4x^8 + 1826110a^8b^3x^6 + 2406690a^9b^2x^4 + 1431859a^{10}bx^2 + 323323a^{11}) \sqrt{bx^2 + a}}{7436429a^7x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^24, x, algorithm="fricas")

[Out]  $-\frac{1}{7436429} (1024b^{11}x^{22} - 512a^3b^{10}x^{20} + 384a^2b^9x^{18} - 320a^4b^8x^{16} + 280a^5b^7x^{14} - 252a^6b^6x^{12} + 231a^7b^5x^{10} + 530959a^8b^4x^8 + 1826110a^9b^3x^6 + 2406690a^{10}b^2x^4 + 1431859a^{11}bx^2 + 323323a^{11}) \sqrt{bx^2 + a} / (a^7x^{23})$

**Sympy [A]** time = 146.819, size = 1950, normalized size = 11.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(9/2)/x\*\*24, x)

[Out]  $-323323a^{17}b^{73/2} \sqrt{a/(bx^2) + 1} / (7436429a^{13}b^{36}x^{22} + 44618574a^{12}b^{37}x^{24} + 111546435a^{11}b^{38}x^{26})$





**GIAC/XCAS [A]** time = 0.222338, size = 624, normalized size = 3.8

$$2048 \left( 4249388 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{32} b^{\frac{23}{2}} + 28683369 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{30} ab^{\frac{23}{2}} + 100922965 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{28} a^2 b^{\frac{23}{2}} + 215656441 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{26} a^3 b^{\frac{23}{2}} + 313006057 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{24} a^4 b^{\frac{23}{2}} + 311653979 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{22} a^5 b^{\frac{23}{2}} + 216800507 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{20} a^6 b^{\frac{23}{2}} + 100105775 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{18} a^7 b^{\frac{23}{2}} + 29173683 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{16} a^8 b^{\frac{23}{2}} + 4004231 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{14} a^9 b^{\frac{23}{2}} + 100947 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{12} a^{10} b^{\frac{23}{2}} - 33649 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^{10} a^{11} b^{\frac{23}{2}} + 8855 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^8 a^{12} b^{\frac{23}{2}} - 1771 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^6 a^{13} b^{\frac{23}{2}} + 253 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^4 a^{14} b^{\frac{23}{2}} - 23 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 a^{15} b^{\frac{23}{2}} + a^{16} b^{\frac{23}{2}} \right) / \left( \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 - a \right)^{23}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(9/2)/x^24,x, algorithm="giac")

[Out] 2048/7436429\*(4249388\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^32\*b^(23/2) + 28683369\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^30\*a\*b^(23/2) + 100922965\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^28\*a^2\*b^(23/2) + 215656441\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^26\*a^3\*b^(23/2) + 313006057\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^24\*a^4\*b^(23/2) + 311653979\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^22\*a^5\*b^(23/2) + 216800507\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^20\*a^6\*b^(23/2) + 100105775\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^18\*a^7\*b^(23/2) + 29173683\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^16\*a^8\*b^(23/2) + 4004231\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^14\*a^9\*b^(23/2) + 100947\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^12\*a^10\*b^(23/2) - 33649\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^10\*a^11\*b^(23/2) + 8855\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^8\*a^12\*b^(23/2) - 1771\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^6\*a^13\*b^(23/2) + 253\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^4\*a^14\*b^(23/2) - 23\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^2\*a^15\*b^(23/2) + a^16\*b^(23/2))/((sqrt(b)\*x - sqrt(b\*x^2 + a))^2 - a)^23

### 3.441 $\int x^5 \sqrt{9 + 4x^2} dx$

**Optimal.** Leaf size=46

$$\frac{1}{448} (4x^2 + 9)^{7/2} - \frac{9}{160} (4x^2 + 9)^{5/2} + \frac{27}{64} (4x^2 + 9)^{3/2}$$

[Out]  $(27*(9 + 4*x^2)^(3/2))/64 - (9*(9 + 4*x^2)^(5/2))/160 + (9 + 4*x^2)^(7/2)/448$

**Rubi [A]** time = 0.0587316, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{1}{448} (4x^2 + 9)^{7/2} - \frac{9}{160} (4x^2 + 9)^{5/2} + \frac{27}{64} (4x^2 + 9)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x^5\*sqrt[9 + 4\*x^2], x]

[Out]  $(27*(9 + 4*x^2)^(3/2))/64 - (9*(9 + 4*x^2)^(5/2))/160 + (9 + 4*x^2)^(7/2)/448$

**Rubi in Sympy [A]** time = 6.62534, size = 37, normalized size = 0.8

$$\frac{(4x^2 + 9)^{7/2}}{448} - \frac{9(4x^2 + 9)^{5/2}}{160} + \frac{27(4x^2 + 9)^{3/2}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(4\*x\*\*2+9)\*\*(1/2), x)

[Out]  $(4*x**2 + 9)**(7/2)/448 - 9*(4*x**2 + 9)**(5/2)/160 + 27*(4*x**2 + 9)**(3/2)/64$

**Mathematica [A]** time = 0.016122, size = 27, normalized size = 0.59

$$\frac{1}{280} (4x^2 + 9)^{3/2} (10x^4 - 18x^2 + 27)$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*sqrt[9 + 4\*x^2], x]

[Out]  $((9 + 4*x^2)^(3/2)*(27 - 18*x^2 + 10*x^4))/280$

**Maple [A]** time = 0.006, size = 24, normalized size = 0.5

$$\frac{10x^4 - 18x^2 + 27}{280} (4x^2 + 9)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(4\*x^2+9)^(1/2), x)

[Out]  $1/280 * (4 * x^2 + 9)^{(3/2)} * (10 * x^4 - 18 * x^2 + 27)$

**Maxima [A]** time = 1.5017, size = 54, normalized size = 1.17

$$\frac{1}{28} (4x^2 + 9)^{\frac{3}{2}} x^4 - \frac{9}{140} (4x^2 + 9)^{\frac{3}{2}} x^2 + \frac{27}{280} (4x^2 + 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 + 9)*x^5,x, algorithm="maxima")`

[Out]  $1/28 * (4 * x^2 + 9)^{(3/2)} * x^4 - 9/140 * (4 * x^2 + 9)^{(3/2)} * x^2 + 27/280 * (4 * x^2 + 9)^{(3/2)}$

**Fricas [A]** time = 0.221754, size = 180, normalized size = 3.91

$$\frac{655360x^{14} + 3612672x^{12} + 5999616x^{10} + 4844448x^8 + 14451696x^6 + 28934010x^4 + 17360406x^2 - 2(163840x^{13} + 7188848x^{11} + 794880x^9 + 655128x^7 + 3031182x^5 + 4133430x^3 + 1240029x) \sqrt{4x^2 + 9} + 1594323}{280(8192x^7 + 32256x^5 + 36288x^3 - (4096x^6 + 11520x^4 + 7776x^2 + 729) \sqrt{4x^2 + 9} + 10206x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 + 9)*x^5,x, algorithm="fricas")`

[Out]  $-1/280 * (655360 * x^{14} + 3612672 * x^{12} + 5999616 * x^{10} + 4844448 * x^8 + 14451696 * x^6 + 28934010 * x^4 + 17360406 * x^2 - 2 * (163840 * x^{13} + 7188848 * x^{11} + 794880 * x^9 + 655128 * x^7 + 3031182 * x^5 + 4133430 * x^3 + 1240029 * x) * \text{sqrt}(4 * x^2 + 9) + 1594323) / (8192 * x^7 + 32256 * x^5 + 36288 * x^3 - (4096 * x^6 + 11520 * x^4 + 7776 * x^2 + 729) * \text{sqrt}(4 * x^2 + 9) + 10206 * x)$

**Sympy [A]** time = 5.94183, size = 61, normalized size = 1.33

$$\frac{x^6 \sqrt{4x^2 + 9}}{7} + \frac{9x^4 \sqrt{4x^2 + 9}}{140} - \frac{27x^2 \sqrt{4x^2 + 9}}{140} + \frac{243 \sqrt{4x^2 + 9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(4*x**2+9)**(1/2),x)`

[Out]  $x^{**6} * \text{sqrt}(4 * x^{**2} + 9) / 7 + 9 * x^{**4} * \text{sqrt}(4 * x^{**2} + 9) / 140 - 27 * x^{**2} * \text{sqrt}(4 * x^{**2} + 9) / 140 + 243 * \text{sqrt}(4 * x^{**2} + 9) / 280$

**GIAC/XCAS [A]** time = 0.203657, size = 46, normalized size = 1.

$$\frac{1}{448} (4x^2 + 9)^{\frac{7}{2}} - \frac{9}{160} (4x^2 + 9)^{\frac{5}{2}} + \frac{27}{64} (4x^2 + 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 + 9)*x^5,x, algorithm="giac")`

[Out]  $1/448 * (4 * x^2 + 9)^{(7/2)} - 9/160 * (4 * x^2 + 9)^{(5/2)} + 27/64 * (4 * x^2 + 9)^{(3/2)}$

$$3.442 \quad \int x^4 \sqrt{9 + 4x^2} dx$$

**Optimal.** Leaf size=63

$$-\frac{81}{256} \sqrt{4x^2 + 9} + \frac{1}{6} \sqrt{4x^2 + 9} x^5 + \frac{3}{32} \sqrt{4x^2 + 9} x^3 + \frac{729}{512} \sinh^{-1} \left( \frac{2x}{3} \right)$$

[Out]  $(-81*x*\text{Sqrt}[9 + 4*x^2])/256 + (3*x^3*\text{Sqrt}[9 + 4*x^2])/32 + (x^5*\text{Sqrt}[9 + 4*x^2])/6 + (729*\text{ArcSinh}[(2*x)/3])/512$

**Rubi [A]** time = 0.0590177, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{81}{256} \sqrt{4x^2 + 9} + \frac{1}{6} \sqrt{4x^2 + 9} x^5 + \frac{3}{32} \sqrt{4x^2 + 9} x^3 + \frac{729}{512} \sinh^{-1} \left( \frac{2x}{3} \right)$$

Antiderivative was successfully verified.

[In] Int[x^4\*Sqrt[9 + 4\*x^2], x]

[Out]  $(-81*x*\text{Sqrt}[9 + 4*x^2])/256 + (3*x^3*\text{Sqrt}[9 + 4*x^2])/32 + (x^5*\text{Sqrt}[9 + 4*x^2])/6 + (729*\text{ArcSinh}[(2*x)/3])/512$

**Rubi in Sympy [A]** time = 7.55554, size = 56, normalized size = 0.89

$$\frac{x^5 \sqrt{4x^2 + 9}}{6} + \frac{3x^3 \sqrt{4x^2 + 9}}{32} - \frac{81x \sqrt{4x^2 + 9}}{256} + \frac{729 \operatorname{asinh}\left(\frac{2x}{3}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(4\*x\*\*2+9)\*\*(1/2), x)

[Out]  $x**5*\text{sqrt}(4*x**2 + 9)/6 + 3*x**3*\text{sqrt}(4*x**2 + 9)/32 - 81*x*\text{sqrt}(4*x**2 + 9)/256 + 729*\text{asinh}(2*x/3)/512$

**Mathematica [A]** time = 0.022538, size = 39, normalized size = 0.62

$$\frac{1}{768} x \sqrt{4x^2 + 9} (128x^4 + 72x^2 - 243) + \frac{729}{512} \sinh^{-1} \left( \frac{2x}{3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*Sqrt[9 + 4\*x^2], x]

[Out]  $(x*\text{Sqrt}[9 + 4*x^2]*(-243 + 72*x^2 + 128*x^4))/768 + (729*\text{ArcSinh}[(2*x)/3])/512$

**Maple [A]** time = 0.008, size = 46, normalized size = 0.7

$$\frac{x^3}{24} (4x^2 + 9)^{\frac{3}{2}} - \frac{9x}{128} (4x^2 + 9)^{\frac{3}{2}} + \frac{81x}{256} \sqrt{4x^2 + 9} + \frac{729}{512} \operatorname{Arcsinh} \left( \frac{2x}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(sqrt(4*x^2 + 9)*x^4,x, algorithm="giac")
```

```
[Out] 1/768*(8*(16*x^2 + 9)*x^2 - 243)*sqrt(4*x^2 + 9)*x - 729/512*ln(-  
2*x + sqrt(4*x^2 + 9))
```

$$3.443 \quad \int x^3 \sqrt{9 + 4x^2} dx$$

**Optimal.** Leaf size=31

$$\frac{1}{80} (4x^2 + 9)^{5/2} - \frac{3}{16} (4x^2 + 9)^{3/2}$$

[Out]  $(-3*(9 + 4*x^2)^(3/2))/16 + (9 + 4*x^2)^(5/2)/80$

**Rubi [A]** time = 0.0427033, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{1}{80} (4x^2 + 9)^{5/2} - \frac{3}{16} (4x^2 + 9)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x^3\*Sqrt[9 + 4\*x^2], x]

[Out]  $(-3*(9 + 4*x^2)^(3/2))/16 + (9 + 4*x^2)^(5/2)/80$

**Rubi in Sympy [A]** time = 5.538, size = 24, normalized size = 0.77

$$\frac{(4x^2 + 9)^{\frac{5}{2}}}{80} - \frac{3(4x^2 + 9)^{\frac{3}{2}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(4\*x\*\*2+9)\*\*(1/2), x)

[Out]  $(4*x**2 + 9)**(5/2)/80 - 3*(4*x**2 + 9)**(3/2)/16$

**Mathematica [A]** time = 0.00925391, size = 22, normalized size = 0.71

$$\frac{1}{40} (2x^2 - 3) (4x^2 + 9)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Sqrt[9 + 4\*x^2], x]

[Out]  $((-3 + 2*x^2)*(9 + 4*x^2)^(3/2))/40$

**Maple [A]** time = 0.005, size = 19, normalized size = 0.6

$$\frac{2x^2 - 3}{40} (4x^2 + 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(4\*x^2+9)^(1/2), x)

[Out]  $1/40*(4*x^2+9)^(3/2)*(2*x^2-3)$

---

**Maxima [A]** time = 1.49697, size = 35, normalized size = 1.13

$$\frac{1}{20} (4x^2 + 9)^{\frac{3}{2}} x^2 - \frac{3}{40} (4x^2 + 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 + 9)\*x^3,x, algorithm="maxima")

[Out] 1/20\*(4\*x^2 + 9)^(3/2)\*x^2 - 3/40\*(4\*x^2 + 9)^(3/2)

---

**Fricas [A]** time = 0.226146, size = 139, normalized size = 4.48

$$\frac{8192x^{10} + 38400x^8 + 30240x^6 - 77760x^4 - 109350x^2 - 2(2048x^9 + 7296x^7 + 648x^5 - 17010x^3 - 10935x)\sqrt{4x^2 + 9} - 40(512x^5 + 1440x^3 - (256x^4 + 432x^2 + 81)\sqrt{4x^2 + 9} + 810x)}{40(512x^5 + 1440x^3 - (256x^4 + 432x^2 + 81)\sqrt{4x^2 + 9} + 810x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 + 9)\*x^3,x, algorithm="fricas")

[Out] -1/40\*(8192\*x^10 + 38400\*x^8 + 30240\*x^6 - 77760\*x^4 - 109350\*x^2 - 2\*(2048\*x^9 + 7296\*x^7 + 648\*x^5 - 17010\*x^3 - 10935\*x)\*sqrt(4\*x^2 + 9) - 19683)/(512\*x^5 + 1440\*x^3 - (256\*x^4 + 432\*x^2 + 81)\*sqrt(4\*x^2 + 9) + 810\*x)

---

**Sympy [A]** time = 1.76944, size = 44, normalized size = 1.42

$$\frac{x^4\sqrt{4x^2 + 9}}{5} + \frac{3x^2\sqrt{4x^2 + 9}}{20} - \frac{27\sqrt{4x^2 + 9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(4\*x\*\*2+9)\*\*(1/2),x)

[Out] x\*\*4\*sqrt(4\*x\*\*2 + 9)/5 + 3\*x\*\*2\*sqrt(4\*x\*\*2 + 9)/20 - 27\*sqrt(4\*x\*\*2 + 9)/40

---

**GIAC/XCAS [A]** time = 0.202521, size = 31, normalized size = 1.

$$\frac{1}{80} (4x^2 + 9)^{\frac{5}{2}} - \frac{3}{16} (4x^2 + 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 + 9)\*x^3,x, algorithm="giac")

[Out] 1/80\*(4\*x^2 + 9)^(5/2) - 3/16\*(4\*x^2 + 9)^(3/2)



$$3.444 \quad \int x^2 \sqrt{9 + 4x^2} dx$$

**Optimal.** Leaf size=45

$$\frac{9}{32} \sqrt{4x^2 + 9} + \frac{1}{4} \sqrt{4x^2 + 9} x^3 - \frac{81}{64} \sinh^{-1} \left( \frac{2x}{3} \right)$$

[Out] (9\*x\*Sqrt[9 + 4\*x^2])/32 + (x^3\*Sqrt[9 + 4\*x^2])/4 - (81\*ArcSinh[(2\*x)/3])/64

**Rubi [A]** time = 0.0392728, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{9}{32} \sqrt{4x^2 + 9} + \frac{1}{4} \sqrt{4x^2 + 9} x^3 - \frac{81}{64} \sinh^{-1} \left( \frac{2x}{3} \right)$$

Antiderivative was successfully verified.

[In] Int[x^2\*Sqrt[9 + 4\*x^2], x]

[Out] (9\*x\*Sqrt[9 + 4\*x^2])/32 + (x^3\*Sqrt[9 + 4\*x^2])/4 - (81\*ArcSinh[(2\*x)/3])/64

**Rubi in Sympy [A]** time = 5.87698, size = 39, normalized size = 0.87

$$\frac{x^3 \sqrt{4x^2 + 9}}{4} + \frac{9x \sqrt{4x^2 + 9}}{32} - \frac{81 \operatorname{asinh} \left( \frac{2x}{3} \right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(4\*x\*\*2+9)\*\*(1/2), x)

[Out] x\*\*3\*sqrt(4\*x\*\*2 + 9)/4 + 9\*x\*sqrt(4\*x\*\*2 + 9)/32 - 81\*asinh(2\*x/3)/64

**Mathematica [A]** time = 0.025997, size = 36, normalized size = 0.8

$$\sqrt{4x^2 + 9} \left( \frac{x^3}{4} + \frac{9x}{32} \right) - \frac{81}{64} \sinh^{-1} \left( \frac{2x}{3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Sqrt[9 + 4\*x^2], x]

[Out] Sqrt[9 + 4\*x^2]\*((9\*x)/32 + x^3/4) - (81\*ArcSinh[(2\*x)/3])/64

**Maple [A]** time = 0.007, size = 32, normalized size = 0.7

$$\frac{x}{16} (4x^2 + 9)^{\frac{3}{2}} - \frac{9x}{32} \sqrt{4x^2 + 9} - \frac{81}{64} \operatorname{Arcsinh} \left( \frac{2x}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(4\*x^2+9)^(1/2), x)

[Out]  $1/16 * x * (4 * x^2 + 9)^{(3/2)} - 9/32 * x * (4 * x^2 + 9)^{(1/2)} - 81/64 * \operatorname{arcsinh}(2/3 * x)$

**Maxima [A]** time = 1.49525, size = 42, normalized size = 0.93

$$\frac{1}{16} (4x^2 + 9)^{\frac{3}{2}} x - \frac{9}{32} \sqrt{4x^2 + 9} x - \frac{81}{64} \operatorname{arsinh}\left(\frac{2}{3} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 + 9)*x^2,x, algorithm="maxima")`

[Out]  $1/16 * (4 * x^2 + 9)^{(3/2)} * x - 9/32 * \operatorname{sqrt}(4 * x^2 + 9) * x - 81/64 * \operatorname{arcsinh}(2/3 * x)$

**Fricas [A]** time = 0.225716, size = 182, normalized size = 4.04

$$\frac{4096x^8 + 18432x^6 + 25920x^4 + 11664x^2 - 81 \left(128x^4 + 288x^2 - 8(8x^3 + 9x)\sqrt{4x^2 + 9} + 81\right) \log\left(-2x + \sqrt{4x^2 + 9}\right) - 81 \left(128x^4 + 288x^2 - 8(8x^3 + 9x)\sqrt{4x^2 + 9} + 81\right)}{64 \left(128x^4 + 288x^2 - 8(8x^3 + 9x)\sqrt{4x^2 + 9} + 81\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 + 9)*x^2,x, algorithm="fricas")`

[Out]  $-1/64 * (4096 * x^8 + 18432 * x^6 + 25920 * x^4 + 11664 * x^2 - 81 * (128 * x^4 + 288 * x^2 - 8 * (8 * x^3 + 9 * x) * \operatorname{sqrt}(4 * x^2 + 9) + 81) * \log(-2 * x + \operatorname{sqrt}(4 * x^2 + 9)) - 2 * (1024 * x^7 + 3456 * x^5 + 3240 * x^3 + 729 * x) * \operatorname{sqrt}(4 * x^2 + 9)) / (128 * x^4 + 288 * x^2 - 8 * (8 * x^3 + 9 * x) * \operatorname{sqrt}(4 * x^2 + 9) + 81)$

**Sympy [A]** time = 8.95295, size = 54, normalized size = 1.2

$$\frac{x^5}{\sqrt{4x^2 + 9}} + \frac{27x^3}{8\sqrt{4x^2 + 9}} + \frac{81x}{32\sqrt{4x^2 + 9}} - \frac{81 \operatorname{asinh}\left(\frac{2x}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(4*x**2+9)**(1/2),x)`

[Out]  $x^5/\operatorname{sqrt}(4 * x^2 + 9) + 27 * x^3 / (8 * \operatorname{sqrt}(4 * x^2 + 9)) + 81 * x / (32 * \operatorname{sqrt}(4 * x^2 + 9)) - 81 * \operatorname{asinh}(2 * x / 3) / 64$

**GIAC/XCAS [A]** time = 0.205831, size = 49, normalized size = 1.09

$$\frac{1}{32} (8x^2 + 9) \sqrt{4x^2 + 9} x + \frac{81}{64} \ln\left(-2x + \sqrt{4x^2 + 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 + 9)*x^2,x, algorithm="giac")`

[Out]  $1/32 * (8 * x^2 + 9) * \operatorname{sqrt}(4 * x^2 + 9) * x + 81/64 * \ln(-2 * x + \operatorname{sqrt}(4 * x^2 + 9))$

$$3.445 \quad \int x\sqrt{9 + 4x^2} dx$$

**Optimal.** Leaf size=15

$$\frac{1}{12} (4x^2 + 9)^{3/2}$$

[Out] (9 + 4\*x^2)^(3/2)/12

**Rubi [A]** time = 0.0086213, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{1}{12} (4x^2 + 9)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x\*Sqrt[9 + 4\*x^2], x]

[Out] (9 + 4\*x^2)^(3/2)/12

**Rubi in Sympy [A]** time = 1.93062, size = 10, normalized size = 0.67

$$\frac{(4x^2 + 9)^{3/2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(4\*x\*\*2+9)\*\*(1/2), x)

[Out] (4\*x\*\*2 + 9)\*\*(3/2)/12

**Mathematica [A]** time = 0.00316399, size = 15, normalized size = 1.

$$\frac{1}{12} (4x^2 + 9)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Sqrt[9 + 4\*x^2], x]

[Out] (9 + 4\*x^2)^(3/2)/12

**Maple [A]** time = 0.004, size = 12, normalized size = 0.8

$$\frac{1}{12} (4x^2 + 9)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(4\*x^2+9)^(1/2), x)

[Out] 1/12\*(4\*x^2+9)^(3/2)

---

**Maxima [A]** time = 1.34162, size = 15, normalized size = 1.

$$\frac{1}{12} (4x^2 + 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 + 9)\*x,x, algorithm="maxima")

[Out] 1/12\*(4\*x^2 + 9)^(3/2)

---

**Fricas [A]** time = 0.2247, size = 99, normalized size = 6.6

$$\frac{256x^6 + 1296x^4 + 1944x^2 - 2(64x^5 + 252x^3 + 243x)\sqrt{4x^2 + 9} + 729}{12(32x^3 - (16x^2 + 9)\sqrt{4x^2 + 9} + 54x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 + 9)\*x,x, algorithm="fricas")

[Out] -1/12\*(256\*x^6 + 1296\*x^4 + 1944\*x^2 - 2\*(64\*x^5 + 252\*x^3 + 243\*x)\*sqrt(4\*x^2 + 9) + 729)/(32\*x^3 - (16\*x^2 + 9)\*sqrt(4\*x^2 + 9) + 54\*x)

---

**Sympy [A]** time = 0.460088, size = 27, normalized size = 1.8

$$\frac{x^2\sqrt{4x^2 + 9}}{3} + \frac{3\sqrt{4x^2 + 9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(4\*x\*\*2+9)\*\*(1/2),x)

[Out] x\*\*2\*sqrt(4\*x\*\*2 + 9)/3 + 3\*sqrt(4\*x\*\*2 + 9)/4

---

**GIAC/XCAS [A]** time = 0.204219, size = 15, normalized size = 1.

$$\frac{1}{12} (4x^2 + 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 + 9)\*x,x, algorithm="giac")

[Out] 1/12\*(4\*x^2 + 9)^(3/2)

$$3.446 \quad \int \sqrt{9 + 4x^2} dx$$

**Optimal.** Leaf size=27

$$\frac{1}{2}\sqrt{4x^2 + 9x} + \frac{9}{4} \sinh^{-1}\left(\frac{2x}{3}\right)$$

[Out] (x\*Sqrt[9 + 4\*x^2])/2 + (9\*ArcSinh[(2\*x)/3])/4

**Rubi [A]** time = 0.0136354, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{1}{2}\sqrt{4x^2 + 9x} + \frac{9}{4} \sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[9 + 4\*x^2], x]

[Out] (x\*Sqrt[9 + 4\*x^2])/2 + (9\*ArcSinh[(2\*x)/3])/4

**Rubi in Sympy [A]** time = 1.27449, size = 22, normalized size = 0.81

$$\frac{x\sqrt{4x^2 + 9}}{2} + \frac{9 \operatorname{asinh}\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((4\*x\*\*2+9)\*\*(1/2), x)

[Out] x\*sqrt(4\*x\*\*2 + 9)/2 + 9\*asinh(2\*x/3)/4

**Mathematica [A]** time = 0.0108145, size = 27, normalized size = 1.

$$\frac{1}{2}\sqrt{4x^2 + 9x} + \frac{9}{4} \sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[9 + 4\*x^2], x]

[Out] (x\*Sqrt[9 + 4\*x^2])/2 + (9\*ArcSinh[(2\*x)/3])/4

**Maple [A]** time = 0.003, size = 20, normalized size = 0.7

$$\frac{9}{4} \operatorname{Arcsinh}\left(\frac{2x}{3}\right) + \frac{x}{2} \sqrt{4x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^2+9)^(1/2), x)

[Out] 9/4\*arcsinh(2/3\*x)+1/2\*x\*(4\*x^2+9)^(1/2)

---

**Maxima [A]** time = 1.49607, size = 26, normalized size = 0.96

$$\frac{1}{2} \sqrt{4x^2 + 9x} + \frac{9}{4} \operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 + 9), x, algorithm="maxima")

[Out] 1/2\*sqrt(4\*x^2 + 9)\*x + 9/4\*arcsinh(2/3\*x)

---

**Fricas [A]** time = 0.225448, size = 120, normalized size = 4.44

$$\frac{32x^4 + 72x^2 + 9 \left(8x^2 - 4\sqrt{4x^2 + 9x} + 9\right) \log\left(-2x + \sqrt{4x^2 + 9}\right) - 2(8x^3 + 9x)\sqrt{4x^2 + 9}}{4 \left(8x^2 - 4\sqrt{4x^2 + 9x} + 9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 + 9), x, algorithm="fricas")

[Out] -1/4\*(32\*x^4 + 72\*x^2 + 9\*(8\*x^2 - 4\*sqrt(4\*x^2 + 9)\*x + 9)\*log(-2\*x + sqrt(4\*x^2 + 9)) - 2\*(8\*x^3 + 9\*x)\*sqrt(4\*x^2 + 9))/(8\*x^2 - 4\*sqrt(4\*x^2 + 9)\*x + 9)

---

**Sympy [A]** time = 0.485953, size = 22, normalized size = 0.81

$$\frac{x\sqrt{4x^2 + 9}}{2} + \frac{9 \operatorname{asinh}\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x\*\*2+9)\*\*(1/2), x)

[Out] x\*sqrt(4\*x\*\*2 + 9)/2 + 9\*asinh(2\*x/3)/4

---

**GIAC/XCAS [A]** time = 0.204013, size = 39, normalized size = 1.44

$$\frac{1}{2} \sqrt{4x^2 + 9x} - \frac{9}{4} \ln\left(-2x + \sqrt{4x^2 + 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 + 9), x, algorithm="giac")

[Out] 1/2\*sqrt(4\*x^2 + 9)\*x - 9/4\*ln(-2\*x + sqrt(4\*x^2 + 9))

$$3.447 \quad \int \frac{\sqrt{9+4x^2}}{x} dx$$

**Optimal.** Leaf size=30

$$\sqrt{4x^2 + 9} - 3 \tanh^{-1} \left( \frac{1}{3} \sqrt{4x^2 + 9} \right)$$

[Out] Sqrt[9 + 4\*x^2] - 3\*ArcTanh[Sqrt[9 + 4\*x^2]/3]

**Rubi [A]** time = 0.0478208, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\sqrt{4x^2 + 9} - 3 \tanh^{-1} \left( \frac{1}{3} \sqrt{4x^2 + 9} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[9 + 4\*x^2]/x, x]

[Out] Sqrt[9 + 4\*x^2] - 3\*ArcTanh[Sqrt[9 + 4\*x^2]/3]

**Rubi in Sympy [A]** time = 5.65849, size = 24, normalized size = 0.8

$$\sqrt{4x^2 + 9} - 3 \operatorname{atanh} \left( \frac{\sqrt{4x^2 + 9}}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((4\*x\*\*2+9)\*\*(1/2)/x, x)

[Out] sqrt(4\*x\*\*2 + 9) - 3\*atanh(sqrt(4\*x\*\*2 + 9)/3)

**Mathematica [A]** time = 0.0126364, size = 32, normalized size = 1.07

$$\sqrt{4x^2 + 9} - 3 \log \left( \sqrt{4x^2 + 9} + 3 \right) + 3 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[9 + 4\*x^2]/x, x]

[Out] Sqrt[9 + 4\*x^2] + 3\*Log[x] - 3\*Log[3 + Sqrt[9 + 4\*x^2]]

**Maple [A]** time = 0.005, size = 25, normalized size = 0.8

$$\sqrt{4x^2 + 9} - 3 \operatorname{Artanh} \left( 3 \frac{1}{\sqrt{4x^2 + 9}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^2+9)^(1/2)/x, x)

[Out] (4\*x^2+9)^(1/2)-3\*arctanh(3/(4\*x^2+9)^(1/2))

---

**Maxima [A]** time = 1.49876, size = 26, normalized size = 0.87

$$\sqrt{4x^2 + 9} - 3 \operatorname{arsinh}\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 + 9)/x,x, algorithm="maxima")

[Out] sqrt(4\*x^2 + 9) - 3\*arcsinh(3/2/abs(x))

---

**Fricas [A]** time = 0.231239, size = 138, normalized size = 4.6

$$\frac{4x^2 + 3 \left(2x - \sqrt{4x^2 + 9}\right) \log\left(-2x + \sqrt{4x^2 + 9} + 3\right) - 3 \left(2x - \sqrt{4x^2 + 9}\right) \log\left(-2x + \sqrt{4x^2 + 9} - 3\right) - 2\sqrt{4x^2 + 9}x + 9}{2x - \sqrt{4x^2 + 9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 + 9)/x,x, algorithm="fricas")

[Out] -(4\*x^2 + 3\*(2\*x - sqrt(4\*x^2 + 9))\*log(-2\*x + sqrt(4\*x^2 + 9) + 3) - 3\*(2\*x - sqrt(4\*x^2 + 9))\*log(-2\*x + sqrt(4\*x^2 + 9) - 3) - 2\*sqrt(4\*x^2 + 9)\*x + 9)/(2\*x - sqrt(4\*x^2 + 9))

---

**Sympy [A]** time = 4.27456, size = 39, normalized size = 1.3

$$\frac{2x}{\sqrt{1 + \frac{9}{4x^2}}} - 3 \operatorname{asinh}\left(\frac{3}{2x}\right) + \frac{9}{2x\sqrt{1 + \frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x\*\*2+9)\*\*(1/2)/x,x)

[Out] 2\*x/sqrt(1 + 9/(4\*x\*\*2)) - 3\*asinh(3/(2\*x)) + 9/(2\*x\*sqrt(1 + 9/(4\*x\*\*2)))

---

**GIAC/XCAS [A]** time = 0.206703, size = 51, normalized size = 1.7

$$\sqrt{4x^2 + 9} - \frac{3}{2} \ln\left(\sqrt{4x^2 + 9} + 3\right) + \frac{3}{2} \ln\left(\sqrt{4x^2 + 9} - 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 + 9)/x,x, algorithm="giac")

[Out] sqrt(4\*x^2 + 9) - 3/2\*ln(sqrt(4\*x^2 + 9) + 3) + 3/2\*ln(sqrt(4\*x^2 + 9) - 3)



$$3.448 \quad \int \frac{\sqrt{9+4x^2}}{x^2} dx$$

**Optimal.** Leaf size=25

$$2 \sinh^{-1} \left( \frac{2x}{3} \right) - \frac{\sqrt{4x^2 + 9}}{x}$$

[Out]  $-(\text{Sqrt}[9 + 4*x^2]/x) + 2*\text{ArcSinh}[(2*x)/3]$

**Rubi [A]** time = 0.0219201, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$2 \sinh^{-1} \left( \frac{2x}{3} \right) - \frac{\sqrt{4x^2 + 9}}{x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[9 + 4*x^2]/x^2, x]$

[Out]  $-(\text{Sqrt}[9 + 4*x^2]/x) + 2*\text{ArcSinh}[(2*x)/3]$

**Rubi in Sympy [A]** time = 3.29761, size = 19, normalized size = 0.76

$$2 \operatorname{asinh} \left( \frac{2x}{3} \right) - \frac{\sqrt{4x^2 + 9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((4*x**2+9)**(1/2)/x**2, x)$

[Out]  $2*\operatorname{asinh}(2*x/3) - \text{sqrt}(4*x**2 + 9)/x$

**Mathematica [A]** time = 0.0136399, size = 25, normalized size = 1.

$$2 \sinh^{-1} \left( \frac{2x}{3} \right) - \frac{\sqrt{4x^2 + 9}}{x}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Sqrt}[9 + 4*x^2]/x^2, x]$

[Out]  $-(\text{Sqrt}[9 + 4*x^2]/x) + 2*\text{ArcSinh}[(2*x)/3]$

**Maple [A]** time = 0.006, size = 34, normalized size = 1.4

$$-\frac{1}{9x} (4x^2 + 9)^{\frac{3}{2}} + \frac{4x}{9} \sqrt{4x^2 + 9} + 2 \operatorname{Arcsinh}(2/3 x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((4*x^2+9)^(1/2)/x^2, x)$

[Out]  $-1/9/x*(4*x^2+9)^{(3/2)}+4/9*x*(4*x^2+9)^{(1/2)}+2*\operatorname{arcsinh}(2/3*x)$

**Maxima [A]** time = 1.49106, size = 28, normalized size = 1.12

$$-\frac{\sqrt{4x^2+9}}{x} + 2 \operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 + 9)/x^2,x, algorithm="maxima")`

[Out]  $-\operatorname{sqrt}(4*x^2 + 9)/x + 2*\operatorname{arcsinh}(2/3*x)$

**Fricas [A]** time = 0.231843, size = 78, normalized size = 3.12

$$\frac{2\left(2x^2 - \sqrt{4x^2 + 9}x\right) \log\left(-2x + \sqrt{4x^2 + 9}\right) - 9}{2x^2 - \sqrt{4x^2 + 9}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 + 9)/x^2,x, algorithm="fricas")`

[Out]  $-(2*(2*x^2 - \operatorname{sqrt}(4*x^2 + 9)*x)*\log(-2*x + \operatorname{sqrt}(4*x^2 + 9)) - 9)/(2*x^2 - \operatorname{sqrt}(4*x^2 + 9)*x)$

**Sympy [A]** time = 0.662192, size = 19, normalized size = 0.76

$$2 \operatorname{asinh}\left(\frac{2x}{3}\right) - \frac{\sqrt{4x^2+9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2+9)**(1/2)/x**2,x)`

[Out]  $2*\operatorname{asinh}(2*x/3) - \operatorname{sqrt}(4*x**2 + 9)/x$

**GIAC/XCAS [A]** time = 0.205073, size = 54, normalized size = 2.16

$$\frac{36}{\left(2x - \sqrt{4x^2 + 9}\right)^2 - 9} - 2 \ln\left(-2x + \sqrt{4x^2 + 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 + 9)/x^2,x, algorithm="giac")`

[Out]  $36/((2*x - \operatorname{sqrt}(4*x^2 + 9))^2 - 9) - 2*\ln(-2*x + \operatorname{sqrt}(4*x^2 + 9))$

$$3.449 \quad \int \frac{\sqrt{9+4x^2}}{x^3} dx$$

**Optimal.** Leaf size=39

$$-\frac{\sqrt{4x^2+9}}{2x^2} - \frac{2}{3} \tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

[Out] -Sqrt[9 + 4\*x^2]/(2\*x^2) - (2\*ArcTanh[Sqrt[9 + 4\*x^2]/3])/3

**Rubi [A]** time = 0.0501769, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{\sqrt{4x^2+9}}{2x^2} - \frac{2}{3} \tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[9 + 4\*x^2]/x^3, x]

[Out] -Sqrt[9 + 4\*x^2]/(2\*x^2) - (2\*ArcTanh[Sqrt[9 + 4\*x^2]/3])/3

**Rubi in Sympy [A]** time = 5.88493, size = 32, normalized size = 0.82

$$-\frac{2 \operatorname{atanh}\left(\frac{\sqrt{4x^2+9}}{3}\right)}{3} - \frac{\sqrt{4x^2+9}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((4\*x\*\*2+9)\*\*(1/2)/x\*\*3, x)

[Out] -2\*atanh(sqrt(4\*x\*\*2 + 9)/3)/3 - sqrt(4\*x\*\*2 + 9)/(2\*x\*\*2)

**Mathematica [A]** time = 0.0222228, size = 43, normalized size = 1.1

$$-\frac{\sqrt{4x^2+9}}{2x^2} - \frac{2}{3} \log\left(\sqrt{4x^2+9}+3\right) + \frac{2 \log(x)}{3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[9 + 4\*x^2]/x^3, x]

[Out] -Sqrt[9 + 4\*x^2]/(2\*x^2) + (2\*Log[x])/3 - (2\*Log[3 + Sqrt[9 + 4\*x^2]])/3

**Maple [A]** time = 0.006, size = 41, normalized size = 1.1

$$-\frac{1}{18x^2} (4x^2+9)^{\frac{3}{2}} + \frac{2}{9}\sqrt{4x^2+9} - \frac{2}{3} \operatorname{Artanh}\left(3 \frac{1}{\sqrt{4x^2+9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^2+9)^(1/2)/x^3, x)

[Out]  $-1/18/x^2*(4*x^2+9)^{(3/2)}+2/9*(4*x^2+9)^{(1/2)}-2/3*\operatorname{arctanh}(3/(4*x^2+9)^{(1/2)})$

**Maxima [A]** time = 1.4986, size = 47, normalized size = 1.21

$$\frac{2}{9}\sqrt{4x^2+9}-\frac{(4x^2+9)^{\frac{3}{2}}}{18x^2}-\frac{2}{3}\operatorname{arsinh}\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 + 9)/x^3,x, algorithm="maxima")`

[Out]  $2/9*\sqrt{4*x^2 + 9} - 1/18*(4*x^2 + 9)^{(3/2)}/x^2 - 2/3*\operatorname{arcsinh}(3/2/\operatorname{abs}(x))$

**Fricas [A]** time = 0.231006, size = 189, normalized size = 4.85

$$\frac{48x^3 - 4\left(8x^4 - 4\sqrt{4x^2 + 9}x^3 + 9x^2\right)\log\left(-2x + \sqrt{4x^2 + 9} + 3\right) + 4\left(8x^4 - 4\sqrt{4x^2 + 9}x^3 + 9x^2\right)\log\left(-2x + \sqrt{4x^2 + 9} - 3\right)}{6\left(8x^4 - 4\sqrt{4x^2 + 9}x^3 + 9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 + 9)/x^3,x, algorithm="fricas")`

[Out]  $1/6*(48*x^3 - 4*(8*x^4 - 4*\sqrt{4*x^2 + 9}*x^3 + 9*x^2)*\log(-2*x + \sqrt{4*x^2 + 9} + 3) + 4*(8*x^4 - 4*\sqrt{4*x^2 + 9}*x^3 + 9*x^2)*\log(-2*x + \sqrt{4*x^2 + 9} - 3) - 3*(8*x^2 + 9)*\sqrt{4*x^2 + 9} + 108*x)/(8*x^4 - 4*\sqrt{4*x^2 + 9}*x^3 + 9*x^2)$

**Sympy [A]** time = 5.92481, size = 24, normalized size = 0.62

$$-\frac{2\operatorname{asinh}\left(\frac{3}{2x}\right)}{3}-\frac{\sqrt{1+\frac{9}{4x^2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2+9)**(1/2)/x**3,x)`

[Out]  $-2*\operatorname{asinh}(3/(2*x))/3 - \sqrt{1 + 9/(4*x**2)}/x$

**GIAC/XCAS [A]** time = 0.206722, size = 58, normalized size = 1.49

$$-\frac{\sqrt{4x^2+9}}{2x^2}-\frac{1}{3}\ln\left(\sqrt{4x^2+9}+3\right)+\frac{1}{3}\ln\left(\sqrt{4x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 + 9)/x^3,x, algorithm="giac")`

[Out]  $-1/2*\sqrt{4*x^2 + 9}/x^2 - 1/3*\ln(\sqrt{4*x^2 + 9} + 3) + 1/3*\ln(\sqrt{4*x^2 + 9} - 3)$

$$3.450 \quad \int \frac{\sqrt{9+4x^2}}{x^4} dx$$

**Optimal.** Leaf size=18

$$-\frac{(4x^2 + 9)^{3/2}}{27x^3}$$

[Out]  $-(9 + 4x^2)^{3/2}/(27x^3)$

**Rubi [A]** time = 0.0155205, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$-\frac{(4x^2 + 9)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[9 + 4\*x^2]/x^4, x]

[Out]  $-(9 + 4x^2)^{3/2}/(27x^3)$

**Rubi in Sympy [A]** time = 2.94644, size = 15, normalized size = 0.83

$$-\frac{(4x^2 + 9)^{\frac{3}{2}}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((4\*x\*\*2+9)\*\*(1/2)/x\*\*4, x)

[Out]  $-(4x^2 + 9)^{3/2}/(27x^3)$

**Mathematica [A]** time = 0.00916559, size = 18, normalized size = 1.

$$-\frac{(4x^2 + 9)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[9 + 4\*x^2]/x^4, x]

[Out]  $-(9 + 4x^2)^{3/2}/(27x^3)$

**Maple [A]** time = 0.005, size = 15, normalized size = 0.8

$$-\frac{1}{27x^3} (4x^2 + 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^2+9)^(1/2)/x^4, x)

[Out]  $-1/27*(4x^2+9)^{3/2}/x^3$

---

**Maxima [A]** time = 1.49686, size = 19, normalized size = 1.06

$$-\frac{(4x^2 + 9)^{\frac{3}{2}}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 + 9)/x^4,x, algorithm="maxima")

[Out] -1/27\*(4\*x^2 + 9)^(3/2)/x^3

---

**Fricas [A]** time = 0.224494, size = 92, normalized size = 5.11

$$\frac{32x^4 + 72x^2 - 2(8x^3 + 9x)\sqrt{4x^2 + 9} + 27}{32x^6 + 54x^4 - (16x^5 + 9x^3)\sqrt{4x^2 + 9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 + 9)/x^4,x, algorithm="fricas")

[Out] (32\*x^4 + 72\*x^2 - 2\*(8\*x^3 + 9\*x)\*sqrt(4\*x^2 + 9) + 27)/(32\*x^6 + 54\*x^4 - (16\*x^5 + 9\*x^3)\*sqrt(4\*x^2 + 9))

---

**Sympy [A]** time = 3.67633, size = 34, normalized size = 1.89

$$-\frac{8\sqrt{1 + \frac{9}{4x^2}}}{27} - \frac{2\sqrt{1 + \frac{9}{4x^2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x\*\*2+9)\*\*(1/2)/x\*\*4,x)

[Out] -8\*sqrt(1 + 9/(4\*x\*\*2))/27 - 2\*sqrt(1 + 9/(4\*x\*\*2))/(3\*x\*\*2)

---

**GIAC/XCAS [A]** time = 0.205939, size = 57, normalized size = 3.17

$$\frac{16 \left( (2x - \sqrt{4x^2 + 9})^4 + 27 \right)}{\left( (2x - \sqrt{4x^2 + 9})^2 - 9 \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 + 9)/x^4,x, algorithm="giac")

[Out] 16\*((2\*x - sqrt(4\*x^2 + 9))^4 + 27)/((2\*x - sqrt(4\*x^2 + 9))^2 - 9)^3

$$3.451 \quad \int \frac{\sqrt{9+4x^2}}{x^5} dx$$

**Optimal.** Leaf size=57

$$-\frac{\sqrt{4x^2+9}}{18x^2} + \frac{2}{27} \tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right) - \frac{\sqrt{4x^2+9}}{4x^4}$$

[Out] -Sqrt[9 + 4\*x^2]/(4\*x^4) - Sqrt[9 + 4\*x^2]/(18\*x^2) + (2\*ArcTanh[Sqrt[9 + 4\*x^2]/3])/27

**Rubi [A]** time = 0.0657639, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{\sqrt{4x^2+9}}{18x^2} + \frac{2}{27} \tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right) - \frac{\sqrt{4x^2+9}}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[9 + 4\*x^2]/x^5, x]

[Out] -Sqrt[9 + 4\*x^2]/(4\*x^4) - Sqrt[9 + 4\*x^2]/(18\*x^2) + (2\*ArcTanh[Sqrt[9 + 4\*x^2]/3])/27

**Rubi in Sympy [A]** time = 6.68454, size = 46, normalized size = 0.81

$$\frac{2 \operatorname{atanh}\left(\frac{\sqrt{4x^2+9}}{3}\right)}{27} - \frac{\sqrt{4x^2+9}}{18x^2} - \frac{\sqrt{4x^2+9}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((4\*x\*\*2+9)\*\*(1/2)/x\*\*5, x)

[Out] 2\*atanh(sqrt(4\*x\*\*2 + 9)/3)/27 - sqrt(4\*x\*\*2 + 9)/(18\*x\*\*2) - sqrt(4\*x\*\*2 + 9)/(4\*x\*\*4)

**Mathematica [A]** time = 0.0408519, size = 48, normalized size = 0.84

$$\frac{1}{108} \left( 8 \log(\sqrt{4x^2+9}+3) - \frac{3\sqrt{4x^2+9}(2x^2+9)}{x^4} - 8 \log(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[9 + 4\*x^2]/x^5, x]

[Out] ((-3\*(9 + 2\*x^2)\*Sqrt[9 + 4\*x^2])/x^4 - 8\*Log[x] + 8\*Log[3 + Sqrt[9 + 4\*x^2]])/108

**Maple [A]** time = 0.008, size = 55, normalized size = 1.

$$-\frac{1}{36x^4} (4x^2+9)^{\frac{3}{2}} + \frac{1}{162x^2} (4x^2+9)^{\frac{3}{2}} - \frac{2}{81} \sqrt{4x^2+9} + \frac{2}{27} \operatorname{Artanh}\left(3 \frac{1}{\sqrt{4x^2+9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^2+9)^(1/2)/x^5,x)`

[Out]  $-1/36/x^4*(4*x^2+9)^(3/2)+1/162/x^2*(4*x^2+9)^(3/2)-2/81*(4*x^2+9)^(1/2)+2/27*\operatorname{arctanh}(3/(4*x^2+9)^(1/2))$

**Maxima [A]** time = 1.49798, size = 66, normalized size = 1.16

$$-\frac{2}{81}\sqrt{4x^2+9} + \frac{(4x^2+9)^{\frac{3}{2}}}{162x^2} - \frac{(4x^2+9)^{\frac{3}{2}}}{36x^4} + \frac{2}{27}\operatorname{arsinh}\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 + 9)/x^5,x, algorithm="maxima")`

[Out]  $-2/81*\sqrt{4*x^2 + 9} + 1/162*(4*x^2 + 9)^(3/2)/x^2 - 1/36*(4*x^2 + 9)^(3/2)/x^4 + 2/27*\operatorname{arcsinh}(3/2/\operatorname{abs}(x))$

**Fricas [A]** time = 0.22987, size = 269, normalized size = 4.72

$$\frac{1536x^7 + 12096x^5 + 27216x^3 + 8(128x^8 + 288x^6 + 81x^4 - 8(8x^7 + 9x^5)\sqrt{4x^2 + 9})\log(-2x + \sqrt{4x^2 + 9} + 3) - 8(128x^8 + 288x^6 + 81x^4 - 8(8x^7 + 9x^5)\sqrt{4x^2 + 9})\log(-2x + \sqrt{4x^2 + 9} - 3) - 3(256x^6 + 1728x^4 + 2754x^2 + 729)\sqrt{4x^2 + 9} + 17496x}{108(128x^8 + 288x^6 + 81x^4 - 8(8x^7 + 9x^5)\sqrt{4x^2 + 9})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 + 9)/x^5,x, algorithm="fricas")`

[Out]  $1/108*(1536*x^7 + 12096*x^5 + 27216*x^3 + 8*(128*x^8 + 288*x^6 + 81*x^4 - 8*(8*x^7 + 9*x^5)*\sqrt{4*x^2 + 9})*\log(-2*x + \sqrt{4*x^2 + 9} + 3) - 8*(128*x^8 + 288*x^6 + 81*x^4 - 8*(8*x^7 + 9*x^5)*\sqrt{4*x^2 + 9})*\log(-2*x + \sqrt{4*x^2 + 9} - 3) - 3*(256*x^6 + 1728*x^4 + 2754*x^2 + 729)*\sqrt{4*x^2 + 9} + 17496*x)/(128*x^8 + 288*x^6 + 81*x^4 - 8*(8*x^7 + 9*x^5)*\sqrt{4*x^2 + 9})$

**Sympy [A]** time = 12.4241, size = 63, normalized size = 1.11

$$\frac{2\operatorname{asinh}\left(\frac{3}{2x}\right)}{27} - \frac{1}{9x\sqrt{1+\frac{9}{4x^2}}} - \frac{3}{4x^3\sqrt{1+\frac{9}{4x^2}}} - \frac{9}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2+9)**(1/2)/x**5,x)`

[Out]  $2*\operatorname{asinh}(3/(2*x))/27 - 1/(9*x*\sqrt{1+9/(4*x**2)}) - 3/(4*x**3*\sqrt{1+9/(4*x**2)}) - 9/(8*x**5*\sqrt{1+9/(4*x**2)})$

**GIAC/XCAS [A]** time = 0.205551, size = 74, normalized size = 1.3

$$-\frac{(4x^2+9)^{\frac{3}{2}}+9\sqrt{4x^2+9}}{72x^4} + \frac{1}{27}\ln\left(\sqrt{4x^2+9}+3\right) - \frac{1}{27}\ln\left(\sqrt{4x^2+9}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(sqrt(4*x^2 + 9)/x^5,x, algorithm="giac")
```

```
[Out] -1/72*((4*x^2 + 9)^(3/2) + 9*sqrt(4*x^2 + 9))/x^4 + 1/27*ln(sqrt(4*x^2 + 9) + 3) - 1/27*ln(sqrt(4*x^2 + 9) - 3)
```

### 3.452 $\int x^5 \sqrt{9 - 4x^2} dx$

**Optimal.** Leaf size=46

$$-\frac{1}{448} (9 - 4x^2)^{7/2} + \frac{9}{160} (9 - 4x^2)^{5/2} - \frac{27}{64} (9 - 4x^2)^{3/2}$$

[Out]  $(-27*(9 - 4*x^2)^(3/2))/64 + (9*(9 - 4*x^2)^(5/2))/160 - (9 - 4*x^2)^(7/2)/448$

**Rubi [A]** time = 0.0576222, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{1}{448} (9 - 4x^2)^{7/2} + \frac{9}{160} (9 - 4x^2)^{5/2} - \frac{27}{64} (9 - 4x^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x^5\*Sqrt[9 - 4\*x^2], x]

[Out]  $(-27*(9 - 4*x^2)^(3/2))/64 + (9*(9 - 4*x^2)^(5/2))/160 - (9 - 4*x^2)^(7/2)/448$

**Rubi in Sympy [A]** time = 6.90961, size = 37, normalized size = 0.8

$$-\frac{(-4x^2 + 9)^{7/2}}{448} + \frac{9(-4x^2 + 9)^{5/2}}{160} - \frac{27(-4x^2 + 9)^{3/2}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(-4\*x\*\*2+9)\*\*(1/2), x)

[Out]  $-(-4*x**2 + 9)**(7/2)/448 + 9*(-4*x**2 + 9)**(5/2)/160 - 27*(-4*x**2 + 9)**(3/2)/64$

**Mathematica [A]** time = 0.020182, size = 27, normalized size = 0.59

$$-\frac{1}{280} (9 - 4x^2)^{3/2} (10x^4 + 18x^2 + 27)$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*Sqrt[9 - 4\*x^2], x]

[Out]  $-((9 - 4*x^2)^(3/2)*(27 + 18*x^2 + 10*x^4))/280$

**Maple [A]** time = 0.007, size = 34, normalized size = 0.7

$$\frac{(2x - 3)(2x + 3)(10x^4 + 18x^2 + 27)\sqrt{-4x^2 + 9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(-4\*x^2+9)^(1/2), x)

[Out]  $1/280 * (2 * x - 3) * (2 * x + 3) * (10 * x^4 + 18 * x^2 + 27) * (-4 * x^2 + 9)^{(1/2)}$

**Maxima [A]** time = 1.49631, size = 54, normalized size = 1.17

$$-\frac{1}{28} (-4x^2 + 9)^{\frac{3}{2}} x^4 - \frac{9}{140} (-4x^2 + 9)^{\frac{3}{2}} x^2 - \frac{27}{280} (-4x^2 + 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 + 9)*x^5,x, algorithm="maxima")`

[Out]  $-1/28 * (-4 * x^2 + 9)^{(3/2)} * x^4 - 9/140 * (-4 * x^2 + 9)^{(3/2)} * x^2 - 27/280 * (-4 * x^2 + 9)^{(3/2)}$

**Fricas [A]** time = 0.227392, size = 142, normalized size = 3.09

$$\frac{80x^{14} - 4536x^{12} + 44037x^{10} - 144585x^8 + 153090x^6 + 21(20x^{12} - 369x^{10} + 1755x^8 - 2430x^6)\sqrt{-4x^2 + 9}}{140(21x^6 - 378x^4 + 1701x^2 - (x^6 - 54x^4 + 405x^2 - 729)\sqrt{-4x^2 + 9} - 2187)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 + 9)*x^5,x, algorithm="fricas")`

[Out]  $1/140 * (80 * x^{14} - 4536 * x^{12} + 44037 * x^{10} - 144585 * x^8 + 153090 * x^6 + 21 * (20 * x^{12} - 369 * x^{10} + 1755 * x^8 - 2430 * x^6) * \text{sqrt}(-4 * x^2 + 9)) / (21 * x^6 - 378 * x^4 + 1701 * x^2 - (x^6 - 54 * x^4 + 405 * x^2 - 729) * \text{sqrt}(-4 * x^2 + 9) - 2187)$

**Sympy [A]** time = 5.95734, size = 61, normalized size = 1.33

$$\frac{x^6 \sqrt{-4x^2 + 9}}{7} - \frac{9x^4 \sqrt{-4x^2 + 9}}{140} - \frac{27x^2 \sqrt{-4x^2 + 9}}{140} - \frac{243 \sqrt{-4x^2 + 9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(-4*x**2+9)**(1/2),x)`

[Out]  $x^{**6} * \text{sqrt}(-4 * x^{**2} + 9) / 7 - 9 * x^{**4} * \text{sqrt}(-4 * x^{**2} + 9) / 140 - 27 * x^{**2} * \text{sqrt}(-4 * x^{**2} + 9) / 140 - 243 * \text{sqrt}(-4 * x^{**2} + 9) / 280$

**GIAC/XCAS [A]** time = 0.202115, size = 70, normalized size = 1.52

$$\frac{1}{448} (4x^2 - 9)^3 \sqrt{-4x^2 + 9} + \frac{9}{160} (4x^2 - 9)^2 \sqrt{-4x^2 + 9} - \frac{27}{64} (-4x^2 + 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 + 9)*x^5,x, algorithm="giac")`

[Out]  $1/448 * (4 * x^2 - 9)^3 * \text{sqrt}(-4 * x^2 + 9) + 9/160 * (4 * x^2 - 9)^2 * \text{sqrt}(-4 * x^2 + 9) - 27/64 * (-4 * x^2 + 9)^{(3/2)}$

### 3.453 $\int x^4 \sqrt{9 - 4x^2} dx$

**Optimal.** Leaf size=63

$$-\frac{81}{256} \sqrt{9 - 4x^2} x + \frac{1}{6} \sqrt{9 - 4x^2} x^5 - \frac{3}{32} \sqrt{9 - 4x^2} x^3 + \frac{729}{512} \sin^{-1} \left( \frac{2x}{3} \right)$$

[Out]  $(-81*x*\text{Sqrt}[9 - 4*x^2])/256 - (3*x^3*\text{Sqrt}[9 - 4*x^2])/32 + (x^5*\text{Sqrt}[9 - 4*x^2])/6 + (729*\text{ArcSin}[(2*x)/3])/512$

**Rubi [A]** time = 0.0590932, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{81}{256} \sqrt{9 - 4x^2} x + \frac{1}{6} \sqrt{9 - 4x^2} x^5 - \frac{3}{32} \sqrt{9 - 4x^2} x^3 + \frac{729}{512} \sin^{-1} \left( \frac{2x}{3} \right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^4*\text{Sqrt}[9 - 4*x^2], x]$

[Out]  $(-81*x*\text{Sqrt}[9 - 4*x^2])/256 - (3*x^3*\text{Sqrt}[9 - 4*x^2])/32 + (x^5*\text{Sqrt}[9 - 4*x^2])/6 + (729*\text{ArcSin}[(2*x)/3])/512$

**Rubi in Sympy [A]** time = 7.72144, size = 56, normalized size = 0.89

$$\frac{x^5 \sqrt{-4x^2 + 9}}{6} - \frac{3x^3 \sqrt{-4x^2 + 9}}{32} - \frac{81x \sqrt{-4x^2 + 9}}{256} + \frac{729 \operatorname{asin}\left(\frac{2x}{3}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**4}*(-4*x^{**2}+9)**(1/2), x)$

[Out]  $x^{**5}*\text{sqrt}(-4*x^{**2} + 9)/6 - 3*x^{**3}*\text{sqrt}(-4*x^{**2} + 9)/32 - 81*x*\text{sqrt}(-4*x^{**2} + 9)/256 + 729*\text{asin}(2*x/3)/512$

**Mathematica [A]** time = 0.0232509, size = 39, normalized size = 0.62

$$\frac{1}{768} x \sqrt{9 - 4x^2} (128x^4 - 72x^2 - 243) + \frac{729}{512} \sin^{-1} \left( \frac{2x}{3} \right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^4*\text{Sqrt}[9 - 4*x^2], x]$

[Out]  $(x*\text{Sqrt}[9 - 4*x^2]*(-243 - 72*x^2 + 128*x^4))/768 + (729*\text{ArcSin}[(2*x)/3])/512$

**Maple [A]** time = 0.009, size = 46, normalized size = 0.7

$$-\frac{x^3}{24} (-4x^2 + 9)^{\frac{3}{2}} - \frac{9x}{128} (-4x^2 + 9)^{\frac{3}{2}} + \frac{81x}{256} \sqrt{-4x^2 + 9} + \frac{729}{512} \arcsin \left( \frac{2x}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(-4*x^2+9)^(1/2),x)`

[Out]  $-1/24*x^3*(-4*x^2+9)^(3/2)-9/128*x*(-4*x^2+9)^(3/2)+81/256*x*(-4*x^2+9)^(1/2)+729/512*\arcsin(2/3*x)$

**Maxima [A]** time = 1.49836, size = 61, normalized size = 0.97

$$-\frac{1}{24}(-4x^2+9)^{\frac{3}{2}}x^3 - \frac{9}{128}(-4x^2+9)^{\frac{3}{2}}x + \frac{81}{256}\sqrt{-4x^2+9}x + \frac{729}{512}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 + 9)*x^4,x, algorithm="maxima")`

[Out]  $-1/24*(-4*x^2 + 9)^(3/2)*x^3 - 9/128*(-4*x^2 + 9)^(3/2)*x + 81/256*\sqrt{-4*x^2 + 9}*x + 729/512*\arcsin(2/3*x)$

**Fricas [A]** time = 0.235527, size = 227, normalized size = 3.6

$$\frac{4608x^{11} - 68256x^9 + 277020x^7 - 295245x^5 - 314928x^3 + 2187(2x^6 - 81x^4 + 486x^2 + 9)(x^4 - 12x^2 + 27)\sqrt{-4x^2 + 9}}{768(2x^6 - 81x^4 + 486x^2 + 9)(x^4 - 12x^2 + 27)\sqrt{-4x^2 + 9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 + 9)*x^4,x, algorithm="fricas")`

[Out]  $-1/768*(4608*x^{11} - 68256*x^9 + 277020*x^7 - 295245*x^5 - 314928*x^3 + 2187*(2*x^6 - 81*x^4 + 486*x^2 + 9*(x^4 - 12*x^2 + 27))*\sqrt{-4*x^2 + 9} - 729)*\arctan(1/2*(\sqrt{-4*x^2 + 9} - 3)/x) - (256*x^{11} - 10512*x^9 + 67554*x^7 - 108621*x^5 - 65610*x^3 + 177147*x)*\sqrt{-4*x^2 + 9} + 531441*x)/(2*x^6 - 81*x^4 + 486*x^2 + 9*(x^4 - 12*x^2 + 27))*\sqrt{-4*x^2 + 9} - 729)$

**Sympy [A]** time = 15.3275, size = 167, normalized size = 2.65

$$\begin{cases} \frac{2ix^7}{3\sqrt{4x^2-9}} - \frac{15ix^5}{8\sqrt{4x^2-9}} - \frac{27ix^3}{64\sqrt{4x^2-9}} + \frac{729ix}{256\sqrt{4x^2-9}} - \frac{729i\operatorname{acosh}\left(\frac{2x}{3}\right)}{512} & \text{for } \frac{4|x^2|}{9} > 1 \\ -\frac{2x^7}{3\sqrt{-4x^2+9}} + \frac{15x^5}{8\sqrt{-4x^2+9}} + \frac{27x^3}{64\sqrt{-4x^2+9}} - \frac{729x}{256\sqrt{-4x^2+9}} + \frac{729\operatorname{asin}\left(\frac{2x}{3}\right)}{512} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(-4*x**2+9)**(1/2),x)`

[Out] `Piecewise((2*I*x**7/(3*sqrt(4*x**2 - 9)) - 15*I*x**5/(8*sqrt(4*x**2 - 9)) - 27*I*x**3/(64*sqrt(4*x**2 - 9)) + 729*I*x/(256*sqrt(4*x**2 - 9)) - 729*I*acosh(2*x/3)/512, 4*Abs(x**2)/9 > 1), (-2*x**7/(3*sqrt(-4*x**2 + 9)) + 15*x**5/(8*sqrt(-4*x**2 + 9)) + 27*x**3/(64*sqrt(-4*x**2 + 9)) - 729*x/(256*sqrt(-4*x**2 + 9)) + 729*asin(2*x/3)/512, True))`

**GIAC/XCAS [A]** time = 0.206875, size = 45, normalized size = 0.71

$$\frac{1}{768}(8(16x^2-9)x^2-243)\sqrt{-4x^2+9}x + \frac{729}{512}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-4*x^2 + 9)*x^4,x, algorithm="giac")
```

```
[Out] 1/768*(8*(16*x^2 - 9)*x^2 - 243)*sqrt(-4*x^2 + 9)*x + 729/512*arc  
sin(2/3*x)
```

$$3.454 \quad \int x^3 \sqrt{9 - 4x^2} dx$$

**Optimal.** Leaf size=31

$$\frac{1}{80} (9 - 4x^2)^{5/2} - \frac{3}{16} (9 - 4x^2)^{3/2}$$

[Out]  $(-3*(9 - 4*x^2)^(3/2))/16 + (9 - 4*x^2)^(5/2)/80$

**Rubi [A]** time = 0.0432047, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{1}{80} (9 - 4x^2)^{5/2} - \frac{3}{16} (9 - 4x^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x^3\*Sqrt[9 - 4\*x^2], x]

[Out]  $(-3*(9 - 4*x^2)^(3/2))/16 + (9 - 4*x^2)^(5/2)/80$

**Rubi in Sympy [A]** time = 5.54992, size = 24, normalized size = 0.77

$$\frac{(-4x^2 + 9)^{5/2}}{80} - \frac{3(-4x^2 + 9)^{3/2}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(-4\*x\*\*2+9)\*\*(1/2), x)

[Out]  $(-4*x**2 + 9)**(5/2)/80 - 3*(-4*x**2 + 9)**(3/2)/16$

**Mathematica [A]** time = 0.0124521, size = 22, normalized size = 0.71

$$-\frac{1}{40} (9 - 4x^2)^{3/2} (2x^2 + 3)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Sqrt[9 - 4\*x^2], x]

[Out]  $-((9 - 4*x^2)^(3/2)*(3 + 2*x^2))/40$

**Maple [A]** time = 0.005, size = 29, normalized size = 0.9

$$\frac{(2x - 3)(2x + 3)(2x^2 + 3)\sqrt{-4x^2 + 9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(-4\*x^2+9)^(1/2), x)

[Out]  $1/40*(2*x-3)*(2*x+3)*(2*x^2+3)*(-4*x^2+9)^(1/2)$

---

**Maxima [A]** time = 1.4936, size = 35, normalized size = 1.13

$$-\frac{1}{20}(-4x^2 + 9)^{\frac{3}{2}}x^2 - \frac{3}{40}(-4x^2 + 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 + 9)\*x^3,x, algorithm="maxima")

[Out] -1/20\*(-4\*x^2 + 9)^(3/2)\*x^2 - 3/40\*(-4\*x^2 + 9)^(3/2)

---

**Fricas [A]** time = 0.22821, size = 115, normalized size = 3.71

$$\frac{16x^{10} - 480x^8 + 2565x^6 - 3645x^4 + 15(4x^8 - 39x^6 + 81x^4)\sqrt{-4x^2 + 9}}{20(15x^4 - 135x^2 - (x^4 - 27x^2 + 81)\sqrt{-4x^2 + 9} + 243)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 + 9)\*x^3,x, algorithm="fricas")

[Out] 1/20\*(16\*x^10 - 480\*x^8 + 2565\*x^6 - 3645\*x^4 + 15\*(4\*x^8 - 39\*x^6 + 81\*x^4)\*sqrt(-4\*x^2 + 9))/(15\*x^4 - 135\*x^2 - (x^4 - 27\*x^2 + 81)\*sqrt(-4\*x^2 + 9) + 243)

---

**Sympy [A]** time = 1.79014, size = 44, normalized size = 1.42

$$\frac{x^4\sqrt{-4x^2 + 9}}{5} - \frac{3x^2\sqrt{-4x^2 + 9}}{20} - \frac{27\sqrt{-4x^2 + 9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(-4\*x\*\*2+9)\*\*(1/2),x)

[Out] x\*\*4\*sqrt(-4\*x\*\*2 + 9)/5 - 3\*x\*\*2\*sqrt(-4\*x\*\*2 + 9)/20 - 27\*sqrt(-4\*x\*\*2 + 9)/40

---

**GIAC/XCAS [A]** time = 0.201156, size = 43, normalized size = 1.39

$$\frac{1}{80}(4x^2 - 9)^2\sqrt{-4x^2 + 9} - \frac{3}{16}(-4x^2 + 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 + 9)\*x^3,x, algorithm="giac")

[Out] 1/80\*(4\*x^2 - 9)^2\*sqrt(-4\*x^2 + 9) - 3/16\*(-4\*x^2 + 9)^(3/2)



$$3.455 \quad \int x^2 \sqrt{9 - 4x^2} dx$$

**Optimal.** Leaf size=45

$$-\frac{9}{32} \sqrt{9 - 4x^2} x + \frac{1}{4} \sqrt{9 - 4x^2} x^3 + \frac{81}{64} \sin^{-1} \left( \frac{2x}{3} \right)$$

[Out]  $(-9*x*\text{Sqrt}[9 - 4*x^2])/32 + (x^3*\text{Sqrt}[9 - 4*x^2])/4 + (81*\text{ArcSin}[(2*x)/3])/64$

**Rubi [A]** time = 0.0396414, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{9}{32} \sqrt{9 - 4x^2} x + \frac{1}{4} \sqrt{9 - 4x^2} x^3 + \frac{81}{64} \sin^{-1} \left( \frac{2x}{3} \right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*\text{Sqrt}[9 - 4*x^2], x]$

[Out]  $(-9*x*\text{Sqrt}[9 - 4*x^2])/32 + (x^3*\text{Sqrt}[9 - 4*x^2])/4 + (81*\text{ArcSin}[(2*x)/3])/64$

**Rubi in Sympy [A]** time = 5.93338, size = 39, normalized size = 0.87

$$\frac{x^3 \sqrt{-4x^2 + 9}}{4} - \frac{9x \sqrt{-4x^2 + 9}}{32} + \frac{81 \operatorname{asin}\left(\frac{2x}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**2}*(-4*x^{**2}+9)^{**}(1/2), x)$

[Out]  $x^{**3}*\text{sqrt}(-4*x^{**2} + 9)/4 - 9*x*\text{sqrt}(-4*x^{**2} + 9)/32 + 81*\text{asin}(2*x/3)/64$

**Mathematica [A]** time = 0.0276222, size = 36, normalized size = 0.8

$$\sqrt{9 - 4x^2} \left( \frac{x^3}{4} - \frac{9x}{32} \right) + \frac{81}{64} \sin^{-1} \left( \frac{2x}{3} \right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^2*\text{Sqrt}[9 - 4*x^2], x]$

[Out]  $\text{Sqrt}[9 - 4*x^2]*((-9*x)/32 + x^3/4) + (81*\text{ArcSin}[(2*x)/3])/64$

**Maple [A]** time = 0.008, size = 32, normalized size = 0.7

$$-\frac{x}{16} (-4x^2 + 9)^{\frac{3}{2}} + \frac{9x}{32} \sqrt{-4x^2 + 9} + \frac{81}{64} \arcsin \left( \frac{2x}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^2*(-4*x^2+9)^{(1/2)}, x)$

[Out]  $-1/16 * x * (-4 * x^2 + 9)^{(3/2)} + 9/32 * x * (-4 * x^2 + 9)^{(1/2)} + 81/64 * \arcsin(2/3 * x)$

---

**Maxima [A]** time = 1.50466, size = 42, normalized size = 0.93

$$-\frac{1}{16} (-4x^2 + 9)^{\frac{3}{2}}x + \frac{9}{32} \sqrt{-4x^2 + 9}x + \frac{81}{64} \arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 + 9)*x^2,x, algorithm="maxima")`

[Out]  $-1/16 * (-4 * x^2 + 9)^{(3/2)} * x + 9/32 * \sqrt{-4 * x^2 + 9} * x + 81/64 * \arcsin(2/3 * x)$

---

**Fricas [A]** time = 0.231001, size = 178, normalized size = 3.96

$$\frac{192x^7 - 1512x^5 + 3402x^3 + 81\left(2x^4 - 36x^2 + 3(2x^2 - 9)\sqrt{-4x^2 + 9} + 81\right) \arctan\left(\frac{\sqrt{-4x^2 + 9} - 3}{2x}\right) - (16x^7 - 306x^5 + 972x^3 - 729x) \sqrt{-4x^2 + 9} - 2187x}{32\left(2x^4 - 36x^2 + 3(2x^2 - 9)\sqrt{-4x^2 + 9} + 81\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 + 9)*x^2,x, algorithm="fricas")`

[Out]  $-1/32 * (192 * x^7 - 1512 * x^5 + 3402 * x^3 + 81 * (2 * x^4 - 36 * x^2 + 3 * (2 * x^2 - 9) * \sqrt{-4 * x^2 + 9} + 81) * \arctan(1/2 * (\sqrt{-4 * x^2 + 9} - 3) / x) - (16 * x^7 - 306 * x^5 + 972 * x^3 - 729 * x) * \sqrt{-4 * x^2 + 9} - 2187 * x) / (2 * x^4 - 36 * x^2 + 3 * (2 * x^2 - 9) * \sqrt{-4 * x^2 + 9} + 81)$

---

**Sympy [A]** time = 9.01955, size = 124, normalized size = 2.76

$$\begin{cases} \frac{ix^5}{\sqrt{4x^2-9}} - \frac{27ix^3}{8\sqrt{4x^2-9}} + \frac{81ix}{32\sqrt{4x^2-9}} - \frac{81i \operatorname{acosh}\left(\frac{2x}{3}\right)}{64} & \text{for } \frac{4|x^2|}{9} > 1 \\ -\frac{x^5}{\sqrt{-4x^2+9}} + \frac{27x^3}{8\sqrt{-4x^2+9}} - \frac{81x}{32\sqrt{-4x^2+9}} + \frac{81 \operatorname{asin}\left(\frac{2x}{3}\right)}{64} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(-4*x**2+9)**(1/2),x)`

[Out] `Piecewise((I*x**5/sqrt(4*x**2 - 9) - 27*I*x**3/(8*sqrt(4*x**2 - 9)) + 81*I*x/(32*sqrt(4*x**2 - 9)) - 81*I*acosh(2*x/3)/64, 4*Abs(x**2)/9 > 1), (-x**5/sqrt(-4*x**2 + 9) + 27*x**3/(8*sqrt(-4*x**2 + 9)) - 81*x/(32*sqrt(-4*x**2 + 9)) + 81*asin(2*x/3)/64, True))`

---

**GIAC/XCAS [A]** time = 0.208278, size = 35, normalized size = 0.78

$$\frac{1}{32} (8x^2 - 9) \sqrt{-4x^2 + 9}x + \frac{81}{64} \arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 + 9)*x^2,x, algorithm="giac")`

[Out]  $1/32 * (8 * x^2 - 9) * \sqrt{-4 * x^2 + 9} * x + 81/64 * \arcsin(2/3 * x)$

$$3.456 \quad \int x\sqrt{9-4x^2} dx$$

**Optimal.** Leaf size=15

$$-\frac{1}{12} (9-4x^2)^{3/2}$$

[Out]  $-(9-4x^2)^{(3/2)}/12$

**Rubi [A]** time = 0.00931247, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{1}{12} (9-4x^2)^{3/2}$$

Antiderivative was successfully verified.

[In] `Int[x*Sqrt[9-4*x^2],x]`

[Out]  $-(9-4x^2)^{(3/2)}/12$

**Rubi in Sympy [A]** time = 1.9533, size = 12, normalized size = 0.8

$$-\frac{(-4x^2+9)^{\frac{3}{2}}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x*(-4*x**2+9)**(1/2),x)`

[Out]  $-(-4x^2+9)^{(3/2)}/12$

**Mathematica [A]** time = 0.00351085, size = 15, normalized size = 1.

$$-\frac{1}{12} (9-4x^2)^{3/2}$$

Antiderivative was successfully verified.

[In] `Integrate[x*Sqrt[9-4*x^2],x]`

[Out]  $-(9-4x^2)^{(3/2)}/12$

**Maple [A]** time = 0.004, size = 22, normalized size = 1.5

$$\frac{(2x-3)(2x+3)\sqrt{-4x^2+9}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(-4*x^2+9)^(1/2),x)`

[Out]  $1/12*(2*x-3)*(2*x+3)*(-4*x^2+9)^(1/2)$

---

**Maxima [A]** time = 1.3153, size = 15, normalized size = 1.

$$-\frac{1}{12}(-4x^2 + 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 + 9)\*x,x, algorithm="maxima")

[Out] -1/12\*(-4\*x^2 + 9)^(3/2)

---

**Fricas [A]** time = 0.22959, size = 88, normalized size = 5.87

$$\frac{8x^6 - 108x^4 + 243x^2 + 9(2x^4 - 9x^2)\sqrt{-4x^2 + 9}}{6(9x^2 - (x^2 - 9)\sqrt{-4x^2 + 9} - 27)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 + 9)\*x,x, algorithm="fricas")

[Out] 1/6\*(8\*x^6 - 108\*x^4 + 243\*x^2 + 9\*(2\*x^4 - 9\*x^2)\*sqrt(-4\*x^2 + 9))/(9\*x^2 - (x^2 - 9)\*sqrt(-4\*x^2 + 9) - 27)

---

**Sympy [A]** time = 0.467285, size = 27, normalized size = 1.8

$$\frac{x^2\sqrt{-4x^2 + 9}}{3} - \frac{3\sqrt{-4x^2 + 9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(-4\*x\*\*2+9)\*\*(1/2),x)

[Out] x\*\*2\*sqrt(-4\*x\*\*2 + 9)/3 - 3\*sqrt(-4\*x\*\*2 + 9)/4

---

**GIAC/XCAS [A]** time = 0.212605, size = 15, normalized size = 1.

$$-\frac{1}{12}(-4x^2 + 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 + 9)\*x,x, algorithm="giac")

[Out] -1/12\*(-4\*x^2 + 9)^(3/2)

$$3.457 \quad \int \sqrt{9 - 4x^2} dx$$

**Optimal.** Leaf size=27

$$\frac{1}{2}\sqrt{9 - 4x^2}x + \frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right)$$

[Out] (x\*Sqrt[9 - 4\*x^2])/2 + (9\*ArcSin[(2\*x)/3])/4

**Rubi [A]** time = 0.0143416, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{1}{2}\sqrt{9 - 4x^2}x + \frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[9 - 4\*x^2], x]

[Out] (x\*Sqrt[9 - 4\*x^2])/2 + (9\*ArcSin[(2\*x)/3])/4

**Rubi in Sympy [A]** time = 1.29574, size = 22, normalized size = 0.81

$$\frac{x\sqrt{-4x^2 + 9}}{2} + \frac{9 \operatorname{asin}\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-4\*x\*\*2+9)\*\*(1/2), x)

[Out] x\*sqrt(-4\*x\*\*2 + 9)/2 + 9\*asin(2\*x/3)/4

**Mathematica [A]** time = 0.0111783, size = 27, normalized size = 1.

$$\frac{1}{2}\sqrt{9 - 4x^2}x + \frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[9 - 4\*x^2], x]

[Out] (x\*Sqrt[9 - 4\*x^2])/2 + (9\*ArcSin[(2\*x)/3])/4

**Maple [A]** time = 0.004, size = 20, normalized size = 0.7

$$\frac{9}{4}\arcsin\left(\frac{2x}{3}\right) + \frac{x}{2}\sqrt{-4x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4\*x^2+9)^(1/2), x)

[Out] 9/4\*arcsin(2/3\*x)+1/2\*x\*(-4\*x^2+9)^(1/2)

---

**Maxima [A]** time = 1.52366, size = 26, normalized size = 0.96

$$\frac{1}{2} \sqrt{-4x^2 + 9}x + \frac{9}{4} \arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 + 9),x, algorithm="maxima")

[Out] 1/2\*sqrt(-4\*x^2 + 9)\*x + 9/4\*arcsin(2/3\*x)

---

**Fricas [A]** time = 0.22802, size = 119, normalized size = 4.41

$$\frac{12x^3 + 9\left(2x^2 + 3\sqrt{-4x^2 + 9} - 9\right) \arctan\left(\frac{\sqrt{-4x^2 + 9} - 3}{2x}\right) - (2x^3 - 9x)\sqrt{-4x^2 + 9} - 27x}{2\left(2x^2 + 3\sqrt{-4x^2 + 9} - 9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 + 9),x, algorithm="fricas")

[Out] -1/2\*(12\*x^3 + 9\*(2\*x^2 + 3\*sqrt(-4\*x^2 + 9) - 9)\*arctan(1/2\*(sqrt(-4\*x^2 + 9) - 3)/x) - (2\*x^3 - 9\*x)\*sqrt(-4\*x^2 + 9) - 27\*x)/(2\*x^2 + 3\*sqrt(-4\*x^2 + 9) - 9)

---

**Sympy [A]** time = 0.489743, size = 22, normalized size = 0.81

$$\frac{x\sqrt{-4x^2 + 9}}{2} + \frac{9 \operatorname{asin}\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4\*x\*\*2+9)\*\*(1/2),x)

[Out] x\*sqrt(-4\*x\*\*2 + 9)/2 + 9\*asin(2\*x/3)/4

---

**GIAC/XCAS [A]** time = 0.216991, size = 26, normalized size = 0.96

$$\frac{1}{2} \sqrt{-4x^2 + 9}x + \frac{9}{4} \arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 + 9),x, algorithm="giac")

[Out] 1/2\*sqrt(-4\*x^2 + 9)\*x + 9/4\*arcsin(2/3\*x)

$$3.458 \quad \int \frac{\sqrt{9-4x^2}}{x} dx$$

**Optimal.** Leaf size=30

$$\sqrt{9-4x^2} - 3 \tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

[Out] Sqrt[9 - 4\*x^2] - 3\*ArcTanh[Sqrt[9 - 4\*x^2]/3]

**Rubi [A]** time = 0.0505154, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\sqrt{9-4x^2} - 3 \tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[9 - 4\*x^2]/x, x]

[Out] Sqrt[9 - 4\*x^2] - 3\*ArcTanh[Sqrt[9 - 4\*x^2]/3]

**Rubi in Sympy [A]** time = 5.6659, size = 24, normalized size = 0.8

$$\sqrt{-4x^2+9} - 3 \operatorname{atanh}\left(\frac{\sqrt{-4x^2+9}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-4\*x\*\*2+9)\*\*(1/2)/x, x)

[Out] sqrt(-4\*x\*\*2 + 9) - 3\*atanh(sqrt(-4\*x\*\*2 + 9)/3)

**Mathematica [A]** time = 0.0128454, size = 32, normalized size = 1.07

$$\sqrt{9-4x^2} - 3 \log\left(\sqrt{9-4x^2} + 3\right) + 3 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[9 - 4\*x^2]/x, x]

[Out] Sqrt[9 - 4\*x^2] + 3\*Log[x] - 3\*Log[3 + Sqrt[9 - 4\*x^2]]

**Maple [A]** time = 0.006, size = 25, normalized size = 0.8

$$\sqrt{-4x^2+9} - 3 \operatorname{Artanh}\left(3 \frac{1}{\sqrt{-4x^2+9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4\*x^2+9)^(1/2)/x, x)

[Out] (-4\*x^2+9)^(1/2)-3\*arctanh(3/(-4\*x^2+9)^(1/2))

---

**Maxima [A]** time = 1.48048, size = 47, normalized size = 1.57

$$\sqrt{-4x^2 + 9} - 3 \log\left(\frac{6\sqrt{-4x^2 + 9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 + 9)/x,x, algorithm="maxima")

[Out] sqrt(-4\*x^2 + 9) - 3\*log(6\*sqrt(-4\*x^2 + 9)/abs(x) + 18/abs(x))

---

**Fricas [A]** time = 0.225252, size = 68, normalized size = 2.27

$$\frac{4x^2 - 3\left(\sqrt{-4x^2 + 9} - 3\right) \log\left(\frac{\sqrt{-4x^2 + 9} - 3}{x}\right)}{\sqrt{-4x^2 + 9} - 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 + 9)/x,x, algorithm="fricas")

[Out] -(4\*x^2 - 3\*(sqrt(-4\*x^2 + 9) - 3)\*log((sqrt(-4\*x^2 + 9) - 3)/x)) / (sqrt(-4\*x^2 + 9) - 3)

---

**Sympy [A]** time = 4.52126, size = 76, normalized size = 2.53

$$\begin{cases} i\sqrt{4x^2 - 9} - 3 \log(x) + \frac{3 \log(x^2)}{2} + 3i \operatorname{asin}\left(\frac{3}{2x}\right) & \text{for } \frac{4|x^2|}{9} > 1 \\ \sqrt{-4x^2 + 9} + \frac{3 \log(x^2)}{2} - 3 \log\left(\sqrt{-\frac{4x^2}{9} + 1} + 1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4\*x\*\*2+9)\*\*(1/2)/x,x)

[Out] Piecewise((I\*sqrt(4\*x\*\*2 - 9) - 3\*log(x) + 3\*log(x\*\*2)/2 + 3\*I\*asin(3/(2\*x)), 4\*Abs(x\*\*2)/9 > 1), (sqrt(-4\*x\*\*2 + 9) + 3\*log(x\*\*2)/2 - 3\*log(sqrt(-4\*x\*\*2/9 + 1) + 1), True))

---

**GIAC/XCAS [A]** time = 0.217346, size = 54, normalized size = 1.8

$$\sqrt{-4x^2 + 9} - \frac{3}{2} \ln\left(\sqrt{-4x^2 + 9} + 3\right) + \frac{3}{2} \ln\left(-\sqrt{-4x^2 + 9} + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 + 9)/x,x, algorithm="giac")

[Out] sqrt(-4\*x^2 + 9) - 3/2\*ln(sqrt(-4\*x^2 + 9) + 3) + 3/2\*ln(-sqrt(-4\*x^2 + 9) + 3)



$$3.459 \quad \int \frac{\sqrt{9-4x^2}}{x^2} dx$$

**Optimal.** Leaf size=25

$$-\frac{\sqrt{9-4x^2}}{x} - 2 \sin^{-1}\left(\frac{2x}{3}\right)$$

[Out] -(Sqrt[9 - 4\*x^2]/x) - 2\*ArcSin[(2\*x)/3]

**Rubi [A]** time = 0.0220104, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{\sqrt{9-4x^2}}{x} - 2 \sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[9 - 4\*x^2]/x^2, x]

[Out] -(Sqrt[9 - 4\*x^2]/x) - 2\*ArcSin[(2\*x)/3]

**Rubi in Sympy [A]** time = 3.28659, size = 20, normalized size = 0.8

$$-2 \operatorname{asin}\left(\frac{2x}{3}\right) - \frac{\sqrt{-4x^2+9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-4\*x\*\*2+9)\*\*(1/2)/x\*\*2, x)

[Out] -2\*asin(2\*x/3) - sqrt(-4\*x\*\*2 + 9)/x

**Mathematica [A]** time = 0.0153768, size = 25, normalized size = 1.

$$-\frac{\sqrt{9-4x^2}}{x} - 2 \sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[9 - 4\*x^2]/x^2, x]

[Out] -(Sqrt[9 - 4\*x^2]/x) - 2\*ArcSin[(2\*x)/3]

**Maple [A]** time = 0.006, size = 34, normalized size = 1.4

$$-\frac{1}{9x} (-4x^2 + 9)^{\frac{3}{2}} - \frac{4x}{9} \sqrt{-4x^2 + 9} - 2 \arcsin(2/3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4\*x^2+9)^(1/2)/x^2, x)

[Out]  $-1/9/x*(-4*x^2+9)^{(3/2)}-4/9*x*(-4*x^2+9)^{(1/2)}-2*\arcsin(2/3*x)$

**Maxima [A]** time = 1.50321, size = 28, normalized size = 1.12

$$-\frac{\sqrt{-4x^2+9}}{x} - 2 \arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 + 9)/x^2,x, algorithm="maxima")`

[Out]  $-\sqrt{-4*x^2 + 9}/x - 2*\arcsin(2/3*x)$

**Fricas [A]** time = 0.227527, size = 95, normalized size = 3.8

$$\frac{4x^2 + 4\left(\sqrt{-4x^2 + 9}x - 3x\right) \arctan\left(\frac{\sqrt{-4x^2 + 9} - 3}{2x}\right) + 3\sqrt{-4x^2 + 9} - 9}{\sqrt{-4x^2 + 9}x - 3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 + 9)/x^2,x, algorithm="fricas")`

[Out]  $(4*x^2 + 4*(\sqrt{-4*x^2 + 9}*x - 3*x)*\arctan(1/2*(\sqrt{-4*x^2 + 9} - 3)/x) + 3*\sqrt{-4*x^2 + 9} - 9)/(\sqrt{-4*x^2 + 9}*x - 3*x)$

**Sympy [A]** time = 0.669646, size = 20, normalized size = 0.8

$$-2 \operatorname{asin}\left(\frac{2x}{3}\right) - \frac{\sqrt{-4x^2 + 9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-4*x**2+9)**(1/2)/x**2,x)`

[Out]  $-2*\operatorname{asin}(2*x/3) - \sqrt{-4*x**2 + 9}/x$

**GIAC/XCAS [A]** time = 0.206805, size = 53, normalized size = 2.12

$$\frac{2x}{\sqrt{-4x^2+9}-3} - \frac{\sqrt{-4x^2+9}-3}{2x} - 2 \arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 + 9)/x^2,x, algorithm="giac")`

[Out]  $2*x/(\sqrt{-4*x^2 + 9} - 3) - 1/2*(\sqrt{-4*x^2 + 9} - 3)/x - 2*\arcsin(2/3*x)$

$$3.460 \quad \int \frac{\sqrt{9-4x^2}}{x^3} dx$$

**Optimal.** Leaf size=39

$$\frac{2}{3} \tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right) - \frac{\sqrt{9-4x^2}}{2x^2}$$

[Out] -Sqrt[9 - 4\*x^2]/(2\*x^2) + (2\*ArcTanh[Sqrt[9 - 4\*x^2]/3])/3

**Rubi [A]** time = 0.0518891, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{2}{3} \tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right) - \frac{\sqrt{9-4x^2}}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[9 - 4\*x^2]/x^3, x]

[Out] -Sqrt[9 - 4\*x^2]/(2\*x^2) + (2\*ArcTanh[Sqrt[9 - 4\*x^2]/3])/3

**Rubi in Sympy [A]** time = 5.75176, size = 31, normalized size = 0.79

$$\frac{2 \operatorname{atanh}\left(\frac{\sqrt{-4x^2+9}}{3}\right)}{3} - \frac{\sqrt{-4x^2+9}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-4\*x\*\*2+9)\*\*(1/2)/x\*\*3, x)

[Out] 2\*atanh(sqrt(-4\*x\*\*2 + 9)/3)/3 - sqrt(-4\*x\*\*2 + 9)/(2\*x\*\*2)

**Mathematica [A]** time = 0.0220078, size = 43, normalized size = 1.1

$$-\frac{\sqrt{9-4x^2}}{2x^2} + \frac{2}{3} \log\left(\sqrt{9-4x^2} + 3\right) - \frac{2 \log(x)}{3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[9 - 4\*x^2]/x^3, x]

[Out] -Sqrt[9 - 4\*x^2]/(2\*x^2) - (2\*Log[x])/3 + (2\*Log[3 + Sqrt[9 - 4\*x^2]])/3

**Maple [A]** time = 0.006, size = 41, normalized size = 1.1

$$-\frac{1}{18x^2} (-4x^2 + 9)^{\frac{3}{2}} - \frac{2}{9} \sqrt{-4x^2 + 9} + \frac{2}{3} \operatorname{Artanh}\left(3 \frac{1}{\sqrt{-4x^2 + 9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4\*x^2+9)^(1/2)/x^3, x)

[Out]  $-1/18/x^2 * (-4*x^2+9)^{(3/2)} - 2/9 * (-4*x^2+9)^{(1/2)} + 2/3 * \operatorname{arctanh}(3/(-4*x^2+9)^{(1/2)})$

**Maxima [A]** time = 1.49023, size = 69, normalized size = 1.77

$$-\frac{2}{9} \sqrt{-4x^2 + 9} - \frac{(-4x^2 + 9)^{\frac{3}{2}}}{18x^2} + \frac{2}{3} \log\left(\frac{6\sqrt{-4x^2 + 9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 + 9)/x^3,x, algorithm="maxima")`

[Out]  $-2/9*\operatorname{sqrt}(-4*x^2 + 9) - 1/18*(-4*x^2 + 9)^{(3/2)}/x^2 + 2/3*\log(6*\operatorname{sqrt}(-4*x^2 + 9)/\operatorname{abs}(x) + 18/\operatorname{abs}(x))$

**Fricas [A]** time = 0.232708, size = 131, normalized size = 3.36

$$\frac{36x^2 - 4\left(2x^4 + 3\sqrt{-4x^2 + 9}x^2 - 9x^2\right) \log\left(\frac{\sqrt{-4x^2 + 9} - 3}{x}\right) - 3(2x^2 - 9)\sqrt{-4x^2 + 9} - 81}{6\left(2x^4 + 3\sqrt{-4x^2 + 9}x^2 - 9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 + 9)/x^3,x, algorithm="fricas")`

[Out]  $1/6*(36*x^2 - 4*(2*x^4 + 3*\operatorname{sqrt}(-4*x^2 + 9)*x^2 - 9*x^2)*\log((\operatorname{sqrt}(-4*x^2 + 9) - 3)/x) - 3*(2*x^2 - 9)*\operatorname{sqrt}(-4*x^2 + 9) - 81)/(2*x^4 + 3*\operatorname{sqrt}(-4*x^2 + 9)*x^2 - 9*x^2)$

**Sympy [A]** time = 6.33424, size = 99, normalized size = 2.54

$$\begin{cases} \frac{2 \operatorname{acosh}\left(\frac{3}{2x}\right)}{3} + \frac{1}{x\sqrt{-1+\frac{9}{4x^2}}} - \frac{9}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for } \frac{9}{4} \left|\frac{1}{x^2}\right| > 1 \\ -\frac{2i \operatorname{asin}\left(\frac{3}{2x}\right)}{3} - \frac{i}{x\sqrt{1-\frac{9}{4x^2}}} + \frac{9i}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-4*x**2+9)**(1/2)/x**3,x)`

[Out] `Piecewise((2*acosh(3/(2*x))/3 + 1/(x*sqrt(-1 + 9/(4*x**2)))) - 9/(4*x**3*sqrt(-1 + 9/(4*x**2))), 9*Abs(x**(-2))/4 > 1), (-2*I*asin(3/(2*x))/3 - I/(x*sqrt(1 - 9/(4*x**2))) + 9*I/(4*x**3*sqrt(1 - 9/(4*x**2))), True))`

**GIAC/XCAS [A]** time = 0.208098, size = 61, normalized size = 1.56

$$-\frac{\sqrt{-4x^2 + 9}}{2x^2} + \frac{1}{3} \ln\left(\sqrt{-4x^2 + 9} + 3\right) - \frac{1}{3} \ln\left(-\sqrt{-4x^2 + 9} + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 + 9)/x^3,x, algorithm="giac")`

```
[Out] -1/2*sqrt(-4*x^2 + 9)/x^2 + 1/3*ln(sqrt(-4*x^2 + 9) + 3) - 1/3*ln  
(-sqrt(-4*x^2 + 9) + 3)
```

$$3.461 \quad \int \frac{\sqrt{9-4x^2}}{x^4} dx$$

**Optimal.** Leaf size=18

$$-\frac{(9-4x^2)^{3/2}}{27x^3}$$

[Out]  $-(9-4x^2)^{(3/2)}/(27x^3)$

**Rubi [A]** time = 0.0159054, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$-\frac{(9-4x^2)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[9 - 4\*x^2]/x^4, x]

[Out]  $-(9-4x^2)^{(3/2)}/(27x^3)$

**Rubi in Sympy [A]** time = 2.94244, size = 15, normalized size = 0.83

$$-\frac{(-4x^2+9)^{3/2}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-4\*x\*\*2+9)\*\*(1/2)/x\*\*4, x)

[Out]  $-(-4x^2+9)^{(3/2)}/(27x^3)$

**Mathematica [A]** time = 0.012615, size = 18, normalized size = 1.

$$-\frac{(9-4x^2)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[9 - 4\*x^2]/x^4, x]

[Out]  $-(9-4x^2)^{(3/2)}/(27x^3)$

**Maple [A]** time = 0.005, size = 25, normalized size = 1.4

$$\frac{(2x-3)(2x+3)\sqrt{-4x^2+9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4\*x^2+9)^(1/2)/x^4, x)

[Out]  $1/27/x^3*(2*x-3)*(2*x+3)*(-4*x^2+9)^(1/2)$

---

**Maxima [A]** time = 1.54915, size = 19, normalized size = 1.06

$$-\frac{(-4x^2 + 9)^{\frac{3}{2}}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 + 9)/x^4,x, algorithm="maxima")

[Out] -1/27\*(-4\*x^2 + 9)^(3/2)/x^3

---

**Fricas [A]** time = 0.229601, size = 101, normalized size = 5.61

$$\frac{16x^6 - 216x^4 + 729x^2 + 9(4x^4 - 21x^2 + 27)\sqrt{-4x^2 + 9} - 729}{27(9x^5 - 27x^3 - (x^5 - 9x^3)\sqrt{-4x^2 + 9})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 + 9)/x^4,x, algorithm="fricas")

[Out] 1/27\*(16\*x^6 - 216\*x^4 + 729\*x^2 + 9\*(4\*x^4 - 21\*x^2 + 27)\*sqrt(-4\*x^2 + 9) - 729)/(9\*x^5 - 27\*x^3 - (x^5 - 9\*x^3)\*sqrt(-4\*x^2 + 9))

---

**Sympy [A]** time = 3.73131, size = 78, normalized size = 4.33

$$\begin{cases} \frac{8\sqrt{-1+\frac{9}{4x^2}}}{27} - \frac{2\sqrt{-1+\frac{9}{4x^2}}}{3x^2} & \text{for } \frac{9|\frac{1}{x^2}|}{4} > 1 \\ \frac{8i\sqrt{1-\frac{9}{4x^2}}}{27} - \frac{2i\sqrt{1-\frac{9}{4x^2}}}{3x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4\*x\*\*2+9)\*\*(1/2)/x\*\*4,x)

[Out] Piecewise((8\*sqrt(-1 + 9/(4\*x\*\*2)))/27 - 2\*sqrt(-1 + 9/(4\*x\*\*2))/(3\*x\*\*2), 9\*Abs(x\*\*(-2))/4 > 1), (8\*I\*sqrt(1 - 9/(4\*x\*\*2)))/27 - 2\*I\*sqrt(1 - 9/(4\*x\*\*2))/(3\*x\*\*2), True))

---

**GIAC/XCAS [A]** time = 0.207816, size = 99, normalized size = 5.5

$$-\frac{2x^3\left(\frac{3(\sqrt{-4x^2+9}-3)^2}{x^2} - 4\right)}{27(\sqrt{-4x^2+9}-3)^3} + \frac{\sqrt{-4x^2+9}-3}{18x} - \frac{(\sqrt{-4x^2+9}-3)^3}{216x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 + 9)/x^4,x, algorithm="giac")

[Out] -2/27\*x^3\*(3\*(sqrt(-4\*x^2 + 9) - 3)^2/x^2 - 4)/(sqrt(-4\*x^2 + 9) - 3)^3 + 1/18\*(sqrt(-4\*x^2 + 9) - 3)/x - 1/216\*(sqrt(-4\*x^2 + 9) - 3)^3/x^3

$$3.462 \quad \int \frac{\sqrt{9-4x^2}}{x^5} dx$$

**Optimal.** Leaf size=57

$$\frac{\sqrt{9-4x^2}}{18x^2} + \frac{2}{27} \tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right) - \frac{\sqrt{9-4x^2}}{4x^4}$$

[Out] -Sqrt[9 - 4\*x^2]/(4\*x^4) + Sqrt[9 - 4\*x^2]/(18\*x^2) + (2\*ArcTanh[Sqrt[9 - 4\*x^2]/3])/27

**Rubi [A]** time = 0.0670115, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{\sqrt{9-4x^2}}{18x^2} + \frac{2}{27} \tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right) - \frac{\sqrt{9-4x^2}}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[9 - 4\*x^2]/x^5, x]

[Out] -Sqrt[9 - 4\*x^2]/(4\*x^4) + Sqrt[9 - 4\*x^2]/(18\*x^2) + (2\*ArcTanh[Sqrt[9 - 4\*x^2]/3])/27

**Rubi in Sympy [A]** time = 6.61916, size = 46, normalized size = 0.81

$$\frac{2 \operatorname{atanh}\left(\frac{\sqrt{-4x^2+9}}{3}\right)}{27} + \frac{\sqrt{-4x^2+9}}{18x^2} - \frac{\sqrt{-4x^2+9}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-4\*x\*\*2+9)\*\*(1/2)/x\*\*5, x)

[Out] 2\*atanh(sqrt(-4\*x\*\*2 + 9)/3)/27 + sqrt(-4\*x\*\*2 + 9)/(18\*x\*\*2) - sqrt(-4\*x\*\*2 + 9)/(4\*x\*\*4)

**Mathematica [A]** time = 0.0423702, size = 48, normalized size = 0.84

$$\frac{1}{108} \left( 8 \log(\sqrt{9-4x^2} + 3) + \frac{3\sqrt{9-4x^2}(2x^2-9)}{x^4} - 8 \log(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[9 - 4\*x^2]/x^5, x]

[Out] ((3\*Sqrt[9 - 4\*x^2]\*(-9 + 2\*x^2))/x^4 - 8\*Log[x] + 8\*Log[3 + Sqrt[9 - 4\*x^2]])/108

**Maple [A]** time = 0.008, size = 55, normalized size = 1.

$$-\frac{1}{36x^4}(-4x^2+9)^{\frac{3}{2}} - \frac{1}{162x^2}(-4x^2+9)^{\frac{3}{2}} - \frac{2}{81}\sqrt{-4x^2+9} + \frac{2}{27}\operatorname{Artanh}\left(3\frac{1}{\sqrt{-4x^2+9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((-4*x^2+9)^(1/2)/x^5,x)`

[Out]  $-1/36/x^4 * (-4*x^2+9)^(3/2) - 1/162/x^2 * (-4*x^2+9)^(3/2) - 2/81 * (-4*x^2+9)^(1/2) + 2/27 * \operatorname{arctanh}(3/(-4*x^2+9)^(1/2))$

**Maxima [A]** time = 1.51034, size = 88, normalized size = 1.54

$$-\frac{2}{81} \sqrt{-4x^2+9} - \frac{(-4x^2+9)^{\frac{3}{2}}}{162x^2} - \frac{(-4x^2+9)^{\frac{3}{2}}}{36x^4} + \frac{2}{27} \log\left(\frac{6\sqrt{-4x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 + 9)/x^5,x, algorithm="maxima")`

[Out]  $-2/81 * \operatorname{sqrt}(-4*x^2 + 9) - 1/162 * (-4*x^2 + 9)^(3/2)/x^2 - 1/36 * (-4*x^2 + 9)^(3/2)/x^4 + 2/27 * \log(6 * \operatorname{sqrt}(-4*x^2 + 9)/\operatorname{abs}(x) + 18/\operatorname{abs}(x))$

**Fricas [A]** time = 0.22692, size = 193, normalized size = 3.39

$$\frac{144x^6 - 1620x^4 + 5832x^2 + 8\left(2x^8 - 36x^6 + 81x^4 + 3(2x^6 - 9x^4)\sqrt{-4x^2+9}\right) \log\left(\frac{\sqrt{-4x^2+9}-3}{x}\right) - 3(4x^6 - 90x^4 + 486x^2 - 729)\sqrt{-4x^2+9}}{108\left(2x^8 - 36x^6 + 81x^4 + 3(2x^6 - 9x^4)\sqrt{-4x^2+9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 + 9)/x^5,x, algorithm="fricas")`

[Out]  $-1/108 * (144*x^6 - 1620*x^4 + 5832*x^2 + 8*(2*x^8 - 36*x^6 + 81*x^4 + 3*(2*x^6 - 9*x^4)*\operatorname{sqrt}(-4*x^2 + 9)) * \log((\operatorname{sqrt}(-4*x^2 + 9) - 3)/x) - 3*(4*x^6 - 90*x^4 + 486*x^2 - 729)*\operatorname{sqrt}(-4*x^2 + 9) - 6561)/(2*x^8 - 36*x^6 + 81*x^4 + 3*(2*x^6 - 9*x^4)*\operatorname{sqrt}(-4*x^2 + 9))$

**Sympy [A]** time = 12.6888, size = 141, normalized size = 2.47

$$\begin{cases} \frac{2 \operatorname{acosh}\left(\frac{3}{2x}\right)}{27} - \frac{1}{9x\sqrt{-1+\frac{9}{4x^2}}} + \frac{3}{4x^3\sqrt{-1+\frac{9}{4x^2}}} - \frac{9}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for } \frac{9\left|\frac{1}{x^2}\right|}{4} > 1 \\ -\frac{2i \operatorname{asin}\left(\frac{3}{2x}\right)}{27} + \frac{i}{9x\sqrt{1-\frac{9}{4x^2}}} - \frac{3i}{4x^3\sqrt{1-\frac{9}{4x^2}}} + \frac{9i}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-4*x**2+9)**(1/2)/x**5,x)`

[Out]  $\operatorname{Piecewise}((2*\operatorname{acosh}(3/(2*x)))/27 - 1/(9*x*\operatorname{sqrt}(-1 + 9/(4*x**2))) + 3/(4*x**3*\operatorname{sqrt}(-1 + 9/(4*x**2))) - 9/(8*x**5*\operatorname{sqrt}(-1 + 9/(4*x**2))), 9*\operatorname{Abs}(x**(-2))/4 > 1), (-2*I*\operatorname{asin}(3/(2*x)))/27 + I/(9*x*\operatorname{sqrt}(1 - 9/(4*x**2))) - 3*I/(4*x**3*\operatorname{sqrt}(1 - 9/(4*x**2))) + 9*I/(8*x**5*\operatorname{sqrt}(1 - 9/(4*x**2))), \operatorname{True}))$

**GIAC/XCAS [A]** time = 0.205775, size = 77, normalized size = 1.35

$$-\frac{(-4x^2+9)^{\frac{3}{2}}+9\sqrt{-4x^2+9}}{72x^4} + \frac{1}{27} \ln\left(\sqrt{-4x^2+9}+3\right) - \frac{1}{27} \ln\left(-\sqrt{-4x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-4*x^2 + 9)/x^5,x, algorithm="giac")
```

```
[Out] -1/72*((-4*x^2 + 9)^(3/2) + 9*sqrt(-4*x^2 + 9))/x^4 + 1/27*ln(sqrt(-4*x^2 + 9) + 3) - 1/27*ln(-sqrt(-4*x^2 + 9) + 3)
```

### 3.463 $\int x^5 \sqrt{-9 + 4x^2} dx$

**Optimal.** Leaf size=46

$$\frac{1}{448} (4x^2 - 9)^{7/2} + \frac{9}{160} (4x^2 - 9)^{5/2} + \frac{27}{64} (4x^2 - 9)^{3/2}$$

[Out]  $(27 * (-9 + 4 * x^2)^{(3/2)})/64 + (9 * (-9 + 4 * x^2)^{(5/2)})/160 + (-9 + 4 * x^2)^{(7/2)}/448$

**Rubi [A]** time = 0.0549916, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{1}{448} (4x^2 - 9)^{7/2} + \frac{9}{160} (4x^2 - 9)^{5/2} + \frac{27}{64} (4x^2 - 9)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x^5\*sqrt[-9 + 4\*x^2], x]

[Out]  $(27 * (-9 + 4 * x^2)^{(3/2)})/64 + (9 * (-9 + 4 * x^2)^{(5/2)})/160 + (-9 + 4 * x^2)^{(7/2)}/448$

**Rubi in Sympy [A]** time = 6.64265, size = 37, normalized size = 0.8

$$\frac{(4x^2 - 9)^{7/2}}{448} + \frac{9(4x^2 - 9)^{5/2}}{160} + \frac{27(4x^2 - 9)^{3/2}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(4\*x\*\*2-9)\*\*(1/2), x)

[Out]  $(4*x**2 - 9)**(7/2)/448 + 9*(4*x**2 - 9)**(5/2)/160 + 27*(4*x**2 - 9)**(3/2)/64$

**Mathematica [A]** time = 0.0153493, size = 27, normalized size = 0.59

$$\frac{1}{280} (4x^2 - 9)^{3/2} (10x^4 + 18x^2 + 27)$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*sqrt[-9 + 4\*x^2], x]

[Out]  $((-9 + 4 * x^2)^{(3/2)} * (27 + 18 * x^2 + 10 * x^4))/280$

**Maple [A]** time = 0.006, size = 34, normalized size = 0.7

$$\frac{(2x - 3)(2x + 3)(10x^4 + 18x^2 + 27)}{280} \sqrt{4x^2 - 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(4\*x^2-9)^(1/2), x)

[Out]  $1/280 * (2*x-3) * (2*x+3) * (10*x^4+18*x^2+27) * (4*x^2-9)^{(1/2)}$

**Maxima [A]** time = 1.49224, size = 54, normalized size = 1.17

$$\frac{1}{28} (4x^2 - 9)^{\frac{3}{2}} x^4 + \frac{9}{140} (4x^2 - 9)^{\frac{3}{2}} x^2 + \frac{27}{280} (4x^2 - 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 - 9)*x^5,x, algorithm="maxima")`

[Out]  $1/28 * (4*x^2 - 9)^{(3/2)} * x^4 + 9/140 * (4*x^2 - 9)^{(3/2)} * x^2 + 27/280 * (4*x^2 - 9)^{(3/2)}$

**Fricas [A]** time = 0.226623, size = 180, normalized size = 3.91

$$\frac{655360x^{14} - 3612672x^{12} + 5999616x^{10} - 4844448x^8 + 14451696x^6 - 28934010x^4 + 17360406x^2 - 2(163840x^{13} - 718848x^{11} + 794880x^9 - 655128x^7 + 3031182x^5 - 4133430x^3 + 1240029x) \sqrt{4x^2 - 9} - 1594323(8192x^7 - 32256x^5 + 36288x^3 - (4096x^6 - 11520x^4 + 7776x^2 - 729) \sqrt{4x^2 - 9}) - 10206x}{280(8192x^7 - 32256x^5 + 36288x^3 - (4096x^6 - 11520x^4 + 7776x^2 - 729) \sqrt{4x^2 - 9})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 - 9)*x^5,x, algorithm="fricas")`

[Out]  $-1/280 * (655360*x^{14} - 3612672*x^{12} + 5999616*x^{10} - 4844448*x^8 + 14451696*x^6 - 28934010*x^4 + 17360406*x^2 - 2 * (163840*x^{13} - 718848*x^{11} + 794880*x^9 - 655128*x^7 + 3031182*x^5 - 4133430*x^3 + 1240029*x) * \text{sqrt}(4*x^2 - 9) - 1594323) / (8192*x^7 - 32256*x^5 + 36288*x^3 - (4096*x^6 - 11520*x^4 + 7776*x^2 - 729) * \text{sqrt}(4*x^2 - 9) - 10206*x)$

**Sympy [A]** time = 5.95389, size = 61, normalized size = 1.33

$$\frac{x^6 \sqrt{4x^2 - 9}}{7} - \frac{9x^4 \sqrt{4x^2 - 9}}{140} - \frac{27x^2 \sqrt{4x^2 - 9}}{140} - \frac{243 \sqrt{4x^2 - 9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(4*x**2-9)**(1/2),x)`

[Out]  $x**6*\text{sqrt}(4*x**2 - 9)/7 - 9*x**4*\text{sqrt}(4*x**2 - 9)/140 - 27*x**2*\text{sqrt}(4*x**2 - 9)/140 - 243*\text{sqrt}(4*x**2 - 9)/280$

**GIAC/XCAS [A]** time = 0.203197, size = 46, normalized size = 1.

$$\frac{1}{448} (4x^2 - 9)^{\frac{7}{2}} + \frac{9}{160} (4x^2 - 9)^{\frac{5}{2}} + \frac{27}{64} (4x^2 - 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 - 9)*x^5,x, algorithm="giac")`

[Out]  $1/448 * (4*x^2 - 9)^{(7/2)} + 9/160 * (4*x^2 - 9)^{(5/2)} + 27/64 * (4*x^2 - 9)^{(3/2)}$

$$3.464 \quad \int x^4 \sqrt{-9 + 4x^2} dx$$

**Optimal.** Leaf size=72

$$-\frac{81}{256} \sqrt{4x^2 - 9} - \frac{729}{512} \tanh^{-1} \left( \frac{2x}{\sqrt{4x^2 - 9}} \right) + \frac{1}{6} \sqrt{4x^2 - 9} x^5 - \frac{3}{32} \sqrt{4x^2 - 9} x^3$$

[Out]  $(-81*x*\text{Sqrt}[-9 + 4*x^2])/256 - (3*x^3*\text{Sqrt}[-9 + 4*x^2])/32 + (x^5*\text{Sqrt}[-9 + 4*x^2])/6 - (729*\text{ArcTanh}[(2*x)/\text{Sqrt}[-9 + 4*x^2]])/512$

**Rubi [A]** time = 0.0653834, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{81}{256} \sqrt{4x^2 - 9} - \frac{729}{512} \tanh^{-1} \left( \frac{2x}{\sqrt{4x^2 - 9}} \right) + \frac{1}{6} \sqrt{4x^2 - 9} x^5 - \frac{3}{32} \sqrt{4x^2 - 9} x^3$$

Antiderivative was successfully verified.

[In] Int[x^4\*Sqrt[-9 + 4\*x^2], x]

[Out]  $(-81*x*\text{Sqrt}[-9 + 4*x^2])/256 - (3*x^3*\text{Sqrt}[-9 + 4*x^2])/32 + (x^5*\text{Sqrt}[-9 + 4*x^2])/6 - (729*\text{ArcTanh}[(2*x)/\text{Sqrt}[-9 + 4*x^2]])/512$

**Rubi in Sympy [A]** time = 8.20423, size = 65, normalized size = 0.9

$$\frac{x^5 \sqrt{4x^2 - 9}}{6} - \frac{3x^3 \sqrt{4x^2 - 9}}{32} - \frac{81x \sqrt{4x^2 - 9}}{256} - \frac{729 \operatorname{atanh} \left( \frac{2x}{\sqrt{4x^2 - 9}} \right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(4\*x\*\*2-9)\*\*(1/2), x)

[Out]  $x**5*\text{sqrt}(4*x**2 - 9)/6 - 3*x**3*\text{sqrt}(4*x**2 - 9)/32 - 81*x*\text{sqrt}(4*x**2 - 9)/256 - 729*\text{atanh}(2*x/\text{sqrt}(4*x**2 - 9))/512$

**Mathematica [A]** time = 0.0206437, size = 49, normalized size = 0.68

$$\frac{1}{768} x \sqrt{4x^2 - 9} (128x^4 - 72x^2 - 243) - \frac{729}{512} \log \left( \sqrt{4x^2 - 9} + 2x \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*Sqrt[-9 + 4\*x^2], x]

[Out]  $(x*\text{Sqrt}[-9 + 4*x^2]*(-243 - 72*x^2 + 128*x^4))/768 - (729*\text{Log}[2*x + \text{Sqrt}[-9 + 4*x^2]])/512$

**Maple [A]** time = 0.009, size = 61, normalized size = 0.9

$$\frac{x^3}{24} (4x^2 - 9)^{\frac{3}{2}} + \frac{9x}{128} (4x^2 - 9)^{\frac{3}{2}} + \frac{81x}{256} \sqrt{4x^2 - 9} - \frac{729\sqrt{4}}{1024} \ln \left( x\sqrt{4} + \sqrt{4x^2 - 9} \right)$$

Verification of antiderivative is not currently implemented for this CAS.



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(4*x^2 - 9)*x^4,x, algorithm="giac")
```

```
[Out] 1/768*(8*(16*x^2 - 9)*x^2 - 243)*sqrt(4*x^2 - 9)*x + 729/512*ln(abs(-2*x + sqrt(4*x^2 - 9)))
```

$$3.465 \quad \int x^3 \sqrt{-9 + 4x^2} dx$$

**Optimal.** Leaf size=31

$$\frac{1}{80} (4x^2 - 9)^{5/2} + \frac{3}{16} (4x^2 - 9)^{3/2}$$

[Out]  $(3 * (-9 + 4 * x^2)^{(3/2)})/16 + (-9 + 4 * x^2)^{(5/2)}/80$

**Rubi [A]** time = 0.0424921, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{1}{80} (4x^2 - 9)^{5/2} + \frac{3}{16} (4x^2 - 9)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x^3\*Sqrt[-9 + 4\*x^2],x]

[Out]  $(3 * (-9 + 4 * x^2)^{(3/2)})/16 + (-9 + 4 * x^2)^{(5/2)}/80$

**Rubi in Sympy [A]** time = 5.50386, size = 24, normalized size = 0.77

$$\frac{(4x^2 - 9)^{\frac{5}{2}}}{80} + \frac{3(4x^2 - 9)^{\frac{3}{2}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(4\*x\*\*2-9)\*\*(1/2),x)

[Out]  $(4 * x ** 2 - 9) ** (5/2) / 80 + 3 * (4 * x ** 2 - 9) ** (3/2) / 16$

**Mathematica [A]** time = 0.00876401, size = 22, normalized size = 0.71

$$\frac{1}{40} (2x^2 + 3) (4x^2 - 9)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Sqrt[-9 + 4\*x^2],x]

[Out]  $((3 + 2 * x^2) * (-9 + 4 * x^2)^{(3/2)})/40$

**Maple [A]** time = 0.007, size = 29, normalized size = 0.9

$$\frac{(2x - 3)(2x + 3)(2x^2 + 3)}{40} \sqrt{4x^2 - 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(4\*x^2-9)^(1/2),x)

[Out]  $1/40 * (2 * x - 3) * (2 * x + 3) * (2 * x^2 + 3) * (4 * x^2 - 9)^(1/2)$



---

**Maxima [A]** time = 1.48749, size = 35, normalized size = 1.13

$$\frac{1}{20} (4x^2 - 9)^{\frac{3}{2}} x^2 + \frac{3}{40} (4x^2 - 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 - 9)\*x^3,x, algorithm="maxima")

[Out] 1/20\*(4\*x^2 - 9)^(3/2)\*x^2 + 3/40\*(4\*x^2 - 9)^(3/2)

---

**Fricas [A]** time = 0.22268, size = 139, normalized size = 4.48

$$\frac{8192x^{10} - 38400x^8 + 30240x^6 + 77760x^4 - 109350x^2 - 2(2048x^9 - 7296x^7 + 648x^5 + 17010x^3 - 10935x)\sqrt{4x^2 - 9} + 19683}{40(512x^5 - 1440x^3 - (256x^4 - 432x^2 + 81)\sqrt{4x^2 - 9} + 810x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 - 9)\*x^3,x, algorithm="fricas")

[Out] -1/40\*(8192\*x^10 - 38400\*x^8 + 30240\*x^6 + 77760\*x^4 - 109350\*x^2 - 2\*(2048\*x^9 - 7296\*x^7 + 648\*x^5 + 17010\*x^3 - 10935\*x)\*sqrt(4\*x^2 - 9) + 19683)/(512\*x^5 - 1440\*x^3 - (256\*x^4 - 432\*x^2 + 81)\*sqrt(4\*x^2 - 9) + 810\*x)

---

**Sympy [A]** time = 1.77854, size = 44, normalized size = 1.42

$$\frac{x^4\sqrt{4x^2 - 9}}{5} - \frac{3x^2\sqrt{4x^2 - 9}}{20} - \frac{27\sqrt{4x^2 - 9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(4\*x\*\*2-9)\*\*(1/2),x)

[Out] x\*\*4\*sqrt(4\*x\*\*2 - 9)/5 - 3\*x\*\*2\*sqrt(4\*x\*\*2 - 9)/20 - 27\*sqrt(4\*x\*\*2 - 9)/40

---

**GIAC/XCAS [A]** time = 0.2021, size = 31, normalized size = 1.

$$\frac{1}{80} (4x^2 - 9)^{\frac{5}{2}} + \frac{3}{16} (4x^2 - 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 - 9)\*x^3,x, algorithm="giac")

[Out] 1/80\*(4\*x^2 - 9)^(5/2) + 3/16\*(4\*x^2 - 9)^(3/2)

### 3.466 $\int x^2 \sqrt{-9 + 4x^2} dx$

**Optimal.** Leaf size=54

$$-\frac{9}{32} \sqrt{4x^2 - 9} - \frac{81}{64} \tanh^{-1} \left( \frac{2x}{\sqrt{4x^2 - 9}} \right) + \frac{1}{4} \sqrt{4x^2 - 9} x^3$$

[Out]  $(-9*x*\text{Sqrt}[-9 + 4*x^2])/32 + (x^3*\text{Sqrt}[-9 + 4*x^2])/4 - (81*\text{ArcTanh}[(2*x)/\text{Sqrt}[-9 + 4*x^2]])/64$

**Rubi [A]** time = 0.0467754, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{9}{32} \sqrt{4x^2 - 9} - \frac{81}{64} \tanh^{-1} \left( \frac{2x}{\sqrt{4x^2 - 9}} \right) + \frac{1}{4} \sqrt{4x^2 - 9} x^3$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*\text{Sqrt}[-9 + 4*x^2], x]$

[Out]  $(-9*x*\text{Sqrt}[-9 + 4*x^2])/32 + (x^3*\text{Sqrt}[-9 + 4*x^2])/4 - (81*\text{ArcTanh}[(2*x)/\text{Sqrt}[-9 + 4*x^2]])/64$

**Rubi in Sympy [A]** time = 6.14923, size = 48, normalized size = 0.89

$$\frac{x^3 \sqrt{4x^2 - 9}}{4} - \frac{9x \sqrt{4x^2 - 9}}{32} - \frac{81 \operatorname{atanh}\left(\frac{2x}{\sqrt{4x^2 - 9}}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**2*(4*x**2-9)**(1/2), x)$

[Out]  $x**3*\text{sqrt}(4*x**2 - 9)/4 - 9*x*\text{sqrt}(4*x**2 - 9)/32 - 81*\text{atanh}(2*x/\text{sqrt}(4*x**2 - 9))/64$

**Mathematica [A]** time = 0.0230525, size = 46, normalized size = 0.85

$$\sqrt{4x^2 - 9} \left( \frac{x^3}{4} - \frac{9x}{32} \right) - \frac{81}{64} \log \left( \sqrt{4x^2 - 9} + 2x \right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^2*\text{Sqrt}[-9 + 4*x^2], x]$

[Out]  $\text{Sqrt}[-9 + 4*x^2]*((-9*x)/32 + x^3/4) - (81*\text{Log}[2*x + \text{Sqrt}[-9 + 4*x^2]])/64$

**Maple [A]** time = 0.007, size = 47, normalized size = 0.9

$$\frac{x}{16} (4x^2 - 9)^{\frac{3}{2}} + \frac{9x}{32} \sqrt{4x^2 - 9} - \frac{81\sqrt{4}}{128} \ln \left( x\sqrt{4} + \sqrt{4x^2 - 9} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(4*x^2-9)^(1/2),x)`

[Out]  $\frac{1}{16}x^2(4x^2-9)^{3/2} + \frac{9}{32}x^2(4x^2-9)^{1/2} - \frac{81}{128}\ln(x^2\sqrt{4x^2-9}) + \frac{81}{64}\ln(x^2+3)$

**Maxima [A]** time = 1.51012, size = 58, normalized size = 1.07

$$\frac{1}{16}(4x^2-9)^{3/2}x + \frac{9}{32}\sqrt{4x^2-9}x - \frac{81}{64}\log(8x+4\sqrt{4x^2-9})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 - 9)*x^2,x, algorithm="maxima")`

[Out]  $\frac{1}{16}(4x^2-9)^{3/2}x + \frac{9}{32}\sqrt{4x^2-9}x - \frac{81}{64}\log(8x+4\sqrt{4x^2-9})$

**Fricas [A]** time = 0.22284, size = 182, normalized size = 3.37

$$\frac{4096x^8 - 18432x^6 + 25920x^4 - 11664x^2 - 81(128x^4 - 288x^2 - 8(8x^3 - 9x)\sqrt{4x^2-9} + 81)\log(-2x + \sqrt{4x^2-9})}{64(128x^4 - 288x^2 - 8(8x^3 - 9x)\sqrt{4x^2-9} + 81)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 - 9)*x^2,x, algorithm="fricas")`

[Out]  $-\frac{1}{64}(4096x^8 - 18432x^6 + 25920x^4 - 11664x^2 - 81(128x^4 - 288x^2 - 8(8x^3 - 9x)\sqrt{4x^2-9} + 81)\log(-2x + \sqrt{4x^2-9}) - 2(1024x^7 - 3456x^5 + 3240x^3 - 729x)\sqrt{4x^2-9}) / (128x^4 - 288x^2 - 8(8x^3 - 9x)\sqrt{4x^2-9} + 81)$

**Sympy [A]** time = 9.21466, size = 124, normalized size = 2.3

$$\begin{cases} \frac{x^5}{\sqrt{4x^2-9}} - \frac{27x^3}{8\sqrt{4x^2-9}} + \frac{81x}{32\sqrt{4x^2-9}} - \frac{81\operatorname{acosh}(\frac{2x}{3})}{64} & \text{for } \frac{4|x^2|}{9} > 1 \\ -\frac{ix^5}{\sqrt{-4x^2+9}} + \frac{27ix^3}{8\sqrt{-4x^2+9}} - \frac{81ix}{32\sqrt{-4x^2+9}} + \frac{81i\operatorname{asin}(\frac{2x}{3})}{64} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(4*x**2-9)**(1/2),x)`

[Out]  $\operatorname{Piecewise}((x^5/\sqrt{4x^2-9} - 27x^3/(8\sqrt{4x^2-9}) + 81x/(32\sqrt{4x^2-9}) - 81\operatorname{acosh}(2x/3)/64, 4\operatorname{Abs}(x^2)/9 > 1), (-I*x^5/\sqrt{-4x^2+9} + 27*I*x^3/(8\sqrt{-4x^2+9}) - 81*I*x/(32\sqrt{-4x^2+9}) + 81*I*\operatorname{asin}(2x/3)/64, \operatorname{True}))$

**GIAC/XCAS [A]** time = 0.205321, size = 50, normalized size = 0.93

$$\frac{1}{32}(8x^2-9)\sqrt{4x^2-9}x + \frac{81}{64}\ln\left(\left|-2x + \sqrt{4x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 - 9)*x^2,x, algorithm="giac")`

```
[Out] 1/32*(8*x^2 - 9)*sqrt(4*x^2 - 9)*x + 81/64*ln(abs(-2*x + sqrt(4*x  
^2 - 9)))
```

$$3.467 \quad \int x\sqrt{-9 + 4x^2} dx$$

**Optimal.** Leaf size=15

$$\frac{1}{12} (4x^2 - 9)^{3/2}$$

[Out]  $(-9 + 4*x^2)^{(3/2)}/12$

**Rubi [A]** time = 0.00866834, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{1}{12} (4x^2 - 9)^{3/2}$$

Antiderivative was successfully verified.

[In] `Int[x*Sqrt[-9 + 4*x^2],x]`

[Out]  $(-9 + 4*x^2)^{(3/2)}/12$

**Rubi in Sympy [A]** time = 1.91575, size = 10, normalized size = 0.67

$$\frac{(4x^2 - 9)^{\frac{3}{2}}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x*(4*x**2-9)**(1/2),x)`

[Out]  $(4*x**2 - 9)**(3/2)/12$

**Mathematica [A]** time = 0.00302416, size = 15, normalized size = 1.

$$\frac{1}{12} (4x^2 - 9)^{3/2}$$

Antiderivative was successfully verified.

[In] `Integrate[x*Sqrt[-9 + 4*x^2],x]`

[Out]  $(-9 + 4*x^2)^{(3/2)}/12$

**Maple [A]** time = 0.005, size = 22, normalized size = 1.5

$$\frac{(2x - 3)(2x + 3)\sqrt{4x^2 - 9}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(4*x^2-9)^(1/2),x)`

[Out]  $1/12*(2*x-3)*(2*x+3)*(4*x^2-9)^(1/2)$

---

**Maxima [A]** time = 1.34624, size = 15, normalized size = 1.

$$\frac{1}{12} (4x^2 - 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 - 9)\*x,x, algorithm="maxima")

[Out] 1/12\*(4\*x^2 - 9)^(3/2)

---

**Fricas [A]** time = 0.22098, size = 99, normalized size = 6.6

$$\frac{256x^6 - 1296x^4 + 1944x^2 - 2(64x^5 - 252x^3 + 243x)\sqrt{4x^2 - 9} - 729}{12(32x^3 - (16x^2 - 9)\sqrt{4x^2 - 9} - 54x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 - 9)\*x,x, algorithm="fricas")

[Out] -1/12\*(256\*x^6 - 1296\*x^4 + 1944\*x^2 - 2\*(64\*x^5 - 252\*x^3 + 243\*x)\*sqrt(4\*x^2 - 9) - 729)/(32\*x^3 - (16\*x^2 - 9)\*sqrt(4\*x^2 - 9) - 54\*x)

---

**Sympy [A]** time = 0.469106, size = 27, normalized size = 1.8

$$\frac{x^2\sqrt{4x^2 - 9}}{3} - \frac{3\sqrt{4x^2 - 9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(4\*x\*\*2-9)\*\*(1/2),x)

[Out] x\*\*2\*sqrt(4\*x\*\*2 - 9)/3 - 3\*sqrt(4\*x\*\*2 - 9)/4

---

**GIAC/XCAS [A]** time = 0.201687, size = 15, normalized size = 1.

$$\frac{1}{12} (4x^2 - 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 - 9)\*x,x, algorithm="giac")

[Out] 1/12\*(4\*x^2 - 9)^(3/2)

$$3.468 \quad \int \sqrt{-9 + 4x^2} dx$$

**Optimal.** Leaf size=36

$$\frac{1}{2}x\sqrt{4x^2 - 9} - \frac{9}{4} \tanh^{-1}\left(\frac{2x}{\sqrt{4x^2 - 9}}\right)$$

[Out] (x\*Sqrt[-9 + 4\*x^2])/2 - (9\*ArcTanh[(2\*x)/Sqrt[-9 + 4\*x^2]])/4

**Rubi [A]** time = 0.0200079, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{1}{2}x\sqrt{4x^2 - 9} - \frac{9}{4} \tanh^{-1}\left(\frac{2x}{\sqrt{4x^2 - 9}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-9 + 4\*x^2], x]

[Out] (x\*Sqrt[-9 + 4\*x^2])/2 - (9\*ArcTanh[(2\*x)/Sqrt[-9 + 4\*x^2]])/4

**Rubi in Sympy [A]** time = 1.50947, size = 31, normalized size = 0.86

$$\frac{x\sqrt{4x^2 - 9}}{2} - \frac{9 \operatorname{atanh}\left(\frac{2x}{\sqrt{4x^2 - 9}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((4\*x\*\*2-9)\*\*(1/2), x)

[Out] x\*sqrt(4\*x\*\*2 - 9)/2 - 9\*atanh(2\*x/sqrt(4\*x\*\*2 - 9))/4

**Mathematica [A]** time = 0.0106599, size = 37, normalized size = 1.03

$$\frac{1}{2}x\sqrt{4x^2 - 9} - \frac{9}{4} \log\left(\sqrt{4x^2 - 9} + 2x\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-9 + 4\*x^2], x]

[Out] (x\*Sqrt[-9 + 4\*x^2])/2 - (9\*Log[2\*x + Sqrt[-9 + 4\*x^2]])/4

**Maple [A]** time = 0.003, size = 35, normalized size = 1.

$$\frac{x}{2}\sqrt{4x^2 - 9} - \frac{9\sqrt{4}}{8} \ln\left(x\sqrt{4} + \sqrt{4x^2 - 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^2-9)^(1/2), x)

[Out] 1/2\*x\*(4\*x^2-9)^(1/2)-9/8\*ln(x\*4^(1/2)+(4\*x^2-9)^(1/2))\*4^(1/2)

---

**Maxima [A]** time = 1.49419, size = 42, normalized size = 1.17

$$\frac{1}{2} \sqrt{4x^2 - 9}x - \frac{9}{4} \log(8x + 4\sqrt{4x^2 - 9})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 - 9), x, algorithm="maxima")

[Out] 1/2\*sqrt(4\*x^2 - 9)\*x - 9/4\*log(8\*x + 4\*sqrt(4\*x^2 - 9))

---

**Fricas [A]** time = 0.228234, size = 120, normalized size = 3.33

$$\frac{32x^4 - 72x^2 - 9(8x^2 - 4\sqrt{4x^2 - 9}x - 9) \log(-2x + \sqrt{4x^2 - 9}) - 2(8x^3 - 9x)\sqrt{4x^2 - 9}}{4(8x^2 - 4\sqrt{4x^2 - 9}x - 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 - 9), x, algorithm="fricas")

[Out] -1/4\*(32\*x^4 - 72\*x^2 - 9\*(8\*x^2 - 4\*sqrt(4\*x^2 - 9)\*x - 9)\*log(-2\*x + sqrt(4\*x^2 - 9)) - 2\*(8\*x^3 - 9\*x)\*sqrt(4\*x^2 - 9))/(8\*x^2 - 4\*sqrt(4\*x^2 - 9)\*x - 9)

---

**Sympy [A]** time = 0.491568, size = 22, normalized size = 0.61

$$\frac{x\sqrt{4x^2 - 9}}{2} - \frac{9 \operatorname{acosh}\left(\frac{2x}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x\*\*2-9)\*\*(1/2), x)

[Out] x\*sqrt(4\*x\*\*2 - 9)/2 - 9\*acosh(2\*x/3)/4

---

**GIAC/XCAS [A]** time = 0.205561, size = 41, normalized size = 1.14

$$\frac{1}{2} \sqrt{4x^2 - 9}x + \frac{9}{4} \ln\left(\left|-2x + \sqrt{4x^2 - 9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 - 9), x, algorithm="giac")

[Out] 1/2\*sqrt(4\*x^2 - 9)\*x + 9/4\*ln(abs(-2\*x + sqrt(4\*x^2 - 9)))



$$3.469 \quad \int \frac{\sqrt{-9+4x^2}}{x} dx$$

**Optimal.** Leaf size=30

$$\sqrt{4x^2 - 9} - 3 \tan^{-1} \left( \frac{1}{3} \sqrt{4x^2 - 9} \right)$$

[Out] Sqrt[-9 + 4\*x^2] - 3\*ArcTan[Sqrt[-9 + 4\*x^2]/3]

**Rubi [A]** time = 0.0488835, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\sqrt{4x^2 - 9} - 3 \tan^{-1} \left( \frac{1}{3} \sqrt{4x^2 - 9} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-9 + 4\*x^2]/x, x]

[Out] Sqrt[-9 + 4\*x^2] - 3\*ArcTan[Sqrt[-9 + 4\*x^2]/3]

**Rubi in Sympy [A]** time = 5.70377, size = 24, normalized size = 0.8

$$\sqrt{4x^2 - 9} - 3 \operatorname{atan} \left( \frac{\sqrt{4x^2 - 9}}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((4\*x\*\*2-9)\*\*(1/2)/x, x)

[Out] sqrt(4\*x\*\*2 - 9) - 3\*atan(sqrt(4\*x\*\*2 - 9)/3)

**Mathematica [A]** time = 0.0125494, size = 28, normalized size = 0.93

$$\sqrt{4x^2 - 9} + 3 \tan^{-1} \left( \frac{3}{\sqrt{4x^2 - 9}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-9 + 4\*x^2]/x, x]

[Out] Sqrt[-9 + 4\*x^2] + 3\*ArcTan[3/Sqrt[-9 + 4\*x^2]]

**Maple [A]** time = 0.007, size = 25, normalized size = 0.8

$$\sqrt{4x^2 - 9} + 3 \arctan \left( 3 \frac{1}{\sqrt{4x^2 - 9}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^2-9)^(1/2)/x, x)

[Out]  $(4x^2-9)^{1/2}+3\arctan(3/(4x^2-9)^{1/2})$

**Maxima [A]** time = 1.47554, size = 26, normalized size = 0.87

$$\sqrt{4x^2-9} + 3 \arcsin\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 - 9)/x,x, algorithm="maxima")`

[Out]  $\sqrt{4x^2-9} + 3\arcsin(3/2/\text{abs}(x))$

**Fricas [A]** time = 0.230551, size = 96, normalized size = 3.2

$$\frac{4x^2 + 6\left(2x - \sqrt{4x^2-9}\right) \arctan\left(-\frac{2}{3}x + \frac{1}{3}\sqrt{4x^2-9}\right) - 2\sqrt{4x^2-9}x - 9}{2x - \sqrt{4x^2-9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 - 9)/x,x, algorithm="fricas")`

[Out]  $-(4x^2 + 6(2x - \sqrt{4x^2-9})\arctan(-2/3x + 1/3\sqrt{4x^2-9}) - 2\sqrt{4x^2-9}x - 9)/(2x - \sqrt{4x^2-9})$

**Sympy [A]** time = 4.59978, size = 82, normalized size = 2.73

$$\begin{cases} \sqrt{4x^2-9} - 3i \log(x) + \frac{3i \log(x^2)}{2} + 3 \operatorname{asin}\left(\frac{3}{2x}\right) & \text{for } \frac{4|x^2|}{9} > 1 \\ i\sqrt{-4x^2+9} + \frac{3i \log(x^2)}{2} - 3i \log\left(\sqrt{-\frac{4x^2}{9}+1}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2-9)**(1/2)/x,x)`

[Out] `Piecewise((sqrt(4*x**2 - 9) - 3*I*log(x) + 3*I*log(x**2)/2 + 3*asin(3/(2*x)), 4*Abs(x**2)/9 > 1), (I*sqrt(-4*x**2 + 9) + 3*I*log(x**2)/2 - 3*I*log(sqrt(-4*x**2/9 + 1) + 1), True))`

**GIAC/XCAS [A]** time = 0.20434, size = 32, normalized size = 1.07

$$\sqrt{4x^2-9} - 3 \arctan\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 - 9)/x,x, algorithm="giac")`

[Out]  $\sqrt{4x^2-9} - 3\arctan(1/3\sqrt{4x^2-9})$

$$3.470 \quad \int \frac{\sqrt{-9+4x^2}}{x^2} dx$$

**Optimal.** Leaf size=34

$$2 \tanh^{-1} \left( \frac{2x}{\sqrt{4x^2-9}} \right) - \frac{\sqrt{4x^2-9}}{x}$$

[Out]  $-(\text{Sqrt}[-9 + 4*x^2]/x) + 2*\text{ArcTanh}[(2*x)/\text{Sqrt}[-9 + 4*x^2]]$

**Rubi [A]** time = 0.0277707, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$2 \tanh^{-1} \left( \frac{2x}{\sqrt{4x^2-9}} \right) - \frac{\sqrt{4x^2-9}}{x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[-9 + 4*x^2]/x^2, x]$

[Out]  $-(\text{Sqrt}[-9 + 4*x^2]/x) + 2*\text{ArcTanh}[(2*x)/\text{Sqrt}[-9 + 4*x^2]]$

**Rubi in Sympy [A]** time = 3.50471, size = 27, normalized size = 0.79

$$2 \operatorname{atanh} \left( \frac{2x}{\sqrt{4x^2-9}} \right) - \frac{\sqrt{4x^2-9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((4*x**2-9)**(1/2)/x**2, x)$

[Out]  $2*\operatorname{atanh}(2*x/\text{sqrt}(4*x**2 - 9)) - \text{sqrt}(4*x**2 - 9)/x$

**Mathematica [A]** time = 0.0135292, size = 35, normalized size = 1.03

$$2 \log \left( \sqrt{4x^2-9} + 2x \right) - \frac{\sqrt{4x^2-9}}{x}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Sqrt}[-9 + 4*x^2]/x^2, x]$

[Out]  $-(\text{Sqrt}[-9 + 4*x^2]/x) + 2*\text{Log}[2*x + \text{Sqrt}[-9 + 4*x^2]]$

**Maple [A]** time = 0.004, size = 48, normalized size = 1.4

$$\frac{1}{9x} (4x^2 - 9)^{\frac{3}{2}} - \frac{4x}{9} \sqrt{4x^2 - 9} + \ln \left( x\sqrt{4} + \sqrt{4x^2 - 9} \right) \sqrt{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((4*x^2-9)^(1/2)/x^2, x)$

[Out]  $1/9/x*(4*x^2-9)^{(3/2)}-4/9*x*(4*x^2-9)^{(1/2)}+\ln(x*4^{(1/2)}+(4*x^2-9)^{(1/2)})*4^{(1/2)}$

**Maxima [A]** time = 1.48656, size = 45, normalized size = 1.32

$$-\frac{\sqrt{4x^2-9}}{x} + 2 \log\left(8x + 4\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 - 9)/x^2,x, algorithm="maxima")`

[Out]  $-\text{sqrt}(4*x^2 - 9)/x + 2*\log(8*x + 4*\text{sqrt}(4*x^2 - 9))$

**Fricas [A]** time = 0.228035, size = 78, normalized size = 2.29

$$-\frac{2\left(2x^2 - \sqrt{4x^2 - 9}x\right) \log\left(-2x + \sqrt{4x^2 - 9}\right) + 9}{2x^2 - \sqrt{4x^2 - 9}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 - 9)/x^2,x, algorithm="fricas")`

[Out]  $-(2*(2*x^2 - \text{sqrt}(4*x^2 - 9)*x)*\log(-2*x + \text{sqrt}(4*x^2 - 9)) + 9)/(2*x^2 - \text{sqrt}(4*x^2 - 9)*x)$

**Sympy [A]** time = 0.681614, size = 19, normalized size = 0.56

$$2 \operatorname{acosh}\left(\frac{2x}{3}\right) - \frac{\sqrt{4x^2-9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2-9)**(1/2)/x**2,x)`

[Out]  $2*\operatorname{acosh}(2*x/3) - \text{sqrt}(4*x**2 - 9)/x$

**GIAC/XCAS [A]** time = 0.207545, size = 59, normalized size = 1.74

$$-\frac{36}{\left(2x - \sqrt{4x^2 - 9}\right)^2 + 9} - \ln\left(\left(2x - \sqrt{4x^2 - 9}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 - 9)/x^2,x, algorithm="giac")`

[Out]  $-36/((2*x - \text{sqrt}(4*x^2 - 9))^2 + 9) - \ln((2*x - \text{sqrt}(4*x^2 - 9))^2)$

$$3.471 \quad \int \frac{\sqrt{-9+4x^2}}{x^3} dx$$

**Optimal.** Leaf size=39

$$\frac{2}{3} \tan^{-1} \left( \frac{1}{3} \sqrt{4x^2 - 9} \right) - \frac{\sqrt{4x^2 - 9}}{2x^2}$$

[Out]  $-\text{Sqrt}[-9 + 4*x^2]/(2*x^2) + (2*\text{ArcTan}[\text{Sqrt}[-9 + 4*x^2]/3])/3$

**Rubi [A]** time = 0.0502188, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{2}{3} \tan^{-1} \left( \frac{1}{3} \sqrt{4x^2 - 9} \right) - \frac{\sqrt{4x^2 - 9}}{2x^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[-9 + 4*x^2]/x^3, x]$

[Out]  $-\text{Sqrt}[-9 + 4*x^2]/(2*x^2) + (2*\text{ArcTan}[\text{Sqrt}[-9 + 4*x^2]/3])/3$

**Rubi in Sympy [A]** time = 5.8086, size = 31, normalized size = 0.79

$$\frac{2 \operatorname{atan} \left( \frac{\sqrt{4x^2-9}}{3} \right)}{3} - \frac{\sqrt{4x^2-9}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((4*x**2-9)**(1/2)/x**3, x)$

[Out]  $2*\operatorname{atan}(\text{sqrt}(4*x**2 - 9)/3)/3 - \text{sqrt}(4*x**2 - 9)/(2*x**2)$

**Mathematica [A]** time = 0.0159016, size = 37, normalized size = 0.95

$$-\frac{\sqrt{4x^2-9}}{2x^2} - \frac{2}{3} \tan^{-1} \left( \frac{3}{\sqrt{4x^2-9}} \right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Sqrt}[-9 + 4*x^2]/x^3, x]$

[Out]  $-\text{Sqrt}[-9 + 4*x^2]/(2*x^2) - (2*\text{ArcTan}[3/\text{Sqrt}[-9 + 4*x^2]])/3$

**Maple [A]** time = 0.006, size = 41, normalized size = 1.1

$$\frac{1}{18x^2} (4x^2 - 9)^{\frac{3}{2}} - \frac{2}{9} \sqrt{4x^2 - 9} - \frac{2}{3} \arctan \left( 3 \frac{1}{\sqrt{4x^2 - 9}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((4*x^2-9)^(1/2)/x^3, x)$

[Out]  $1/18/x^2*(4*x^2-9)^{(3/2)}-2/9*(4*x^2-9)^{(1/2)}-2/3*\arctan(3/(4*x^2-9)^{(1/2)})$

**Maxima [A]** time = 1.48651, size = 47, normalized size = 1.21

$$-\frac{2}{9}\sqrt{4x^2-9} + \frac{(4x^2-9)^{\frac{3}{2}}}{18x^2} - \frac{2}{3}\arcsin\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 - 9)/x^3,x, algorithm="maxima")`

[Out]  $-2/9*\sqrt{4*x^2 - 9} + 1/18*(4*x^2 - 9)^{(3/2)}/x^2 - 2/3*\arcsin(3/2/abs(x))$

**Fricas [A]** time = 0.230625, size = 134, normalized size = 3.44

$$\frac{48x^3 + 8\left(8x^4 - 4\sqrt{4x^2 - 9}x^3 - 9x^2\right)\arctan\left(-\frac{2}{3}x + \frac{1}{3}\sqrt{4x^2 - 9}\right) - 3(8x^2 - 9)\sqrt{4x^2 - 9} - 108x}{6\left(8x^4 - 4\sqrt{4x^2 - 9}x^3 - 9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 - 9)/x^3,x, algorithm="fricas")`

[Out]  $1/6*(48*x^3 + 8*(8*x^4 - 4*\sqrt{4*x^2 - 9}*x^3 - 9*x^2)*\arctan(-2/3*x + 1/3*\sqrt{4*x^2 - 9}) - 3*(8*x^2 - 9)*\sqrt{4*x^2 - 9} - 108*x)/(8*x^4 - 4*\sqrt{4*x^2 - 9}*x^3 - 9*x^2)$

**Sympy [A]** time = 6.07295, size = 99, normalized size = 2.54

$$\begin{cases} \frac{2i \operatorname{acosh}\left(\frac{3}{2x}\right)}{3} + \frac{i}{x\sqrt{-1+\frac{9}{4x^2}}} - \frac{9i}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for } \frac{9\left|\frac{1}{x^2}\right|}{4} > 1 \\ -\frac{2 \operatorname{asin}\left(\frac{3}{2x}\right)}{3} - \frac{1}{x\sqrt{1-\frac{9}{4x^2}}} + \frac{9}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2-9)**(1/2)/x**3,x)`

[Out] `Piecewise(((2*I*acosh(3/(2*x)))/3 + I/(x*sqrt(-1 + 9/(4*x**2))) - 9*I/(4*x**3*sqrt(-1 + 9/(4*x**2))), 9*Abs(x**(-2))/4 > 1), (-2*asin(3/(2*x))/3 - 1/(x*sqrt(1 - 9/(4*x**2))) + 9/(4*x**3*sqrt(1 - 9/(4*x**2))), True))`

**GIAC/XCAS [A]** time = 0.204703, size = 39, normalized size = 1.

$$-\frac{\sqrt{4x^2-9}}{2x^2} + \frac{2}{3}\arctan\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 - 9)/x^3,x, algorithm="giac")`

[Out]  $-1/2*\sqrt{4*x^2 - 9}/x^2 + 2/3*\arctan(1/3*\sqrt{4*x^2 - 9})$

$$3.472 \quad \int \frac{\sqrt{-9+4x^2}}{x^4} dx$$

**Optimal.** Leaf size=18

$$\frac{(4x^2 - 9)^{3/2}}{27x^3}$$

[Out]  $(-9 + 4x^2)^{(3/2)}/(27x^3)$

**Rubi [A]** time = 0.015971, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{(4x^2 - 9)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-9 + 4\*x^2]/x^4, x]

[Out]  $(-9 + 4x^2)^{(3/2)}/(27x^3)$

**Rubi in Sympy [A]** time = 2.95962, size = 14, normalized size = 0.78

$$\frac{(4x^2 - 9)^{\frac{3}{2}}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((4\*x\*\*2-9)\*\*(1/2)/x\*\*4, x)

[Out]  $(4x^{**2} - 9)^{(3/2)}/(27x^{**3})$

**Mathematica [A]** time = 0.008341, size = 18, normalized size = 1.

$$\frac{(4x^2 - 9)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-9 + 4\*x^2]/x^4, x]

[Out]  $(-9 + 4x^2)^{(3/2)}/(27x^3)$

**Maple [A]** time = 0.005, size = 25, normalized size = 1.4

$$\frac{(2x - 3)(2x + 3)\sqrt{4x^2 - 9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4\*x^2-9)^(1/2)/x^4, x)

[Out]  $1/27/x^3*(2*x-3)*(2*x+3)*(4*x^2-9)^(1/2)$

---

**Maxima [A]** time = 1.47569, size = 19, normalized size = 1.06

$$\frac{(4x^2 - 9)^{\frac{3}{2}}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 - 9)/x^4,x, algorithm="maxima")

[Out] 1/27\*(4\*x^2 - 9)^(3/2)/x^3

---

**Fricas [A]** time = 0.222245, size = 92, normalized size = 5.11

$$\frac{32x^4 - 72x^2 - 2(8x^3 - 9x)\sqrt{4x^2 - 9} + 27}{32x^6 - 54x^4 - (16x^5 - 9x^3)\sqrt{4x^2 - 9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 - 9)/x^4,x, algorithm="fricas")

[Out] (32\*x^4 - 72\*x^2 - 2\*(8\*x^3 - 9\*x)\*sqrt(4\*x^2 - 9) + 27)/(32\*x^6 - 54\*x^4 - (16\*x^5 - 9\*x^3)\*sqrt(4\*x^2 - 9))

---

**Sympy [A]** time = 3.69334, size = 78, normalized size = 4.33

$$\begin{cases} \frac{8i\sqrt{-1+\frac{9}{4x^2}}}{27} - \frac{2i\sqrt{-1+\frac{9}{4x^2}}}{3x^2} & \text{for } \frac{9|\frac{1}{x^2}|}{4} > 1 \\ \frac{8\sqrt{1-\frac{9}{4x^2}}}{27} - \frac{2\sqrt{1-\frac{9}{4x^2}}}{3x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x\*\*2-9)\*\*(1/2)/x\*\*4,x)

[Out] Piecewise((8\*I\*sqrt(-1 + 9/(4\*x\*\*2)))/27 - 2\*I\*sqrt(-1 + 9/(4\*x\*\*2)))/(3\*x\*\*2), 9\*Abs(x\*\*(-2))/4 > 1), (8\*sqrt(1 - 9/(4\*x\*\*2)))/27 - 2\*sqrt(1 - 9/(4\*x\*\*2))/(3\*x\*\*2), True)

---

**GIAC/XCAS [A]** time = 0.205882, size = 57, normalized size = 3.17

$$\frac{16 \left( (2x - \sqrt{4x^2 - 9})^4 + 27 \right)}{\left( (2x - \sqrt{4x^2 - 9})^2 + 9 \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(4\*x^2 - 9)/x^4,x, algorithm="giac")

[Out] 16\*((2\*x - sqrt(4\*x^2 - 9))^4 + 27)/((2\*x - sqrt(4\*x^2 - 9))^2 + 9)^3



$$3.473 \quad \int \frac{\sqrt{-9+4x^2}}{x^5} dx$$

**Optimal.** Leaf size=57

$$\frac{\sqrt{4x^2-9}}{18x^2} + \frac{2}{27} \tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right) - \frac{\sqrt{4x^2-9}}{4x^4}$$

[Out] -Sqrt[-9 + 4\*x^2]/(4\*x^4) + Sqrt[-9 + 4\*x^2]/(18\*x^2) + (2\*ArcTan[Sqrt[-9 + 4\*x^2]/3])/27

**Rubi [A]** time = 0.0674758, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{\sqrt{4x^2-9}}{18x^2} + \frac{2}{27} \tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right) - \frac{\sqrt{4x^2-9}}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-9 + 4\*x^2]/x^5, x]

[Out] -Sqrt[-9 + 4\*x^2]/(4\*x^4) + Sqrt[-9 + 4\*x^2]/(18\*x^2) + (2\*ArcTan[Sqrt[-9 + 4\*x^2]/3])/27

**Rubi in Sympy [A]** time = 6.70903, size = 46, normalized size = 0.81

$$\frac{2 \operatorname{atan}\left(\frac{\sqrt{4x^2-9}}{3}\right)}{27} + \frac{\sqrt{4x^2-9}}{18x^2} - \frac{\sqrt{4x^2-9}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((4\*x\*\*2-9)\*\*(1/2)/x\*\*5, x)

[Out] 2\*atan(sqrt(4\*x\*\*2 - 9)/3)/27 + sqrt(4\*x\*\*2 - 9)/(18\*x\*\*2) - sqrt(4\*x\*\*2 - 9)/(4\*x\*\*4)

**Mathematica [A]** time = 0.0309164, size = 46, normalized size = 0.81

$$\left(\frac{1}{18x^2} - \frac{1}{4x^4}\right)\sqrt{4x^2-9} - \frac{2}{27} \tan^{-1}\left(\frac{3}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-9 + 4\*x^2]/x^5, x]

[Out] (-1/(4\*x^4) + 1/(18\*x^2))\*Sqrt[-9 + 4\*x^2] - (2\*ArcTan[3/Sqrt[-9 + 4\*x^2]])/27

**Maple [A]** time = 0.007, size = 55, normalized size = 1.

$$\frac{1}{36x^4} (4x^2-9)^{\frac{3}{2}} + \frac{1}{162x^2} (4x^2-9)^{\frac{3}{2}} - \frac{2}{81}\sqrt{4x^2-9} - \frac{2}{27} \arctan\left(3\frac{1}{\sqrt{4x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^2-9)^(1/2)/x^5,x)`

[Out]  $1/36/x^4*(4*x^2-9)^(3/2)+1/162/x^2*(4*x^2-9)^(3/2)-2/81*(4*x^2-9)^(1/2)-2/27*\arctan(3/(4*x^2-9)^(1/2))$

**Maxima [A]** time = 1.49376, size = 66, normalized size = 1.16

$$-\frac{2}{81}\sqrt{4x^2-9} + \frac{(4x^2-9)^{\frac{3}{2}}}{162x^2} + \frac{(4x^2-9)^{\frac{3}{2}}}{36x^4} - \frac{2}{27}\arcsin\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 - 9)/x^5,x, algorithm="maxima")`

[Out]  $-2/81*\sqrt{4*x^2 - 9} + 1/162*(4*x^2 - 9)^(3/2)/x^2 + 1/36*(4*x^2 - 9)^(3/2)/x^4 - 2/27*\arcsin(3/2/abs(x))$

**Fricas [A]** time = 0.225739, size = 196, normalized size = 3.44

$$\frac{1536x^7 - 12096x^5 + 27216x^3 - 16\left(128x^8 - 288x^6 + 81x^4 - 8(8x^7 - 9x^5)\sqrt{4x^2 - 9}\right)\arctan\left(-\frac{2}{3}x + \frac{1}{3}\sqrt{4x^2 - 9}\right) - 3}{108\left(128x^8 - 288x^6 + 81x^4 - 8(8x^7 - 9x^5)\sqrt{4x^2 - 9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(4*x^2 - 9)/x^5,x, algorithm="fricas")`

[Out]  $-1/108*(1536*x^7 - 12096*x^5 + 27216*x^3 - 16*(128*x^8 - 288*x^6 + 81*x^4 - 8*(8*x^7 - 9*x^5)*\sqrt{4*x^2 - 9}))*\arctan(-2/3*x + 1/3*\sqrt{4*x^2 - 9}) - 3*(256*x^6 - 1728*x^4 + 2754*x^2 - 729)*\sqrt{4*x^2 - 9} - 17496*x)/(128*x^8 - 288*x^6 + 81*x^4 - 8*(8*x^7 - 9*x^5)*\sqrt{4*x^2 - 9})$

**Sympy [A]** time = 12.5172, size = 141, normalized size = 2.47

$$\begin{cases} \frac{2i \operatorname{acosh}\left(\frac{3}{2x}\right)}{27} - \frac{i}{9x\sqrt{-1+\frac{9}{4x^2}}} + \frac{3i}{4x^3\sqrt{-1+\frac{9}{4x^2}}} - \frac{9i}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for } \frac{9}{4} \left|\frac{1}{x^2}\right| > 1 \\ -\frac{2 \operatorname{asin}\left(\frac{3}{2x}\right)}{27} + \frac{1}{9x\sqrt{1-\frac{9}{4x^2}}} - \frac{3}{4x^3\sqrt{1-\frac{9}{4x^2}}} + \frac{9}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2-9)**(1/2)/x**5,x)`

[Out] `Piecewise((2*I*acosh(3/(2*x))/27 - I/(9*x*sqrt(-1 + 9/(4*x**2))) + 3*I/(4*x**3*sqrt(-1 + 9/(4*x**2))) - 9*I/(8*x**5*sqrt(-1 + 9/(4*x**2))), 9*Abs(x**(-2))/4 > 1), (-2*asin(3/(2*x))/27 + 1/(9*x*sqrt(1 - 9/(4*x**2))) - 3/(4*x**3*sqrt(1 - 9/(4*x**2))) + 9/(8*x**5*sqrt(1 - 9/(4*x**2))), True))`

**GIAC/XCAS [A]** time = 0.203685, size = 55, normalized size = 0.96

$$\frac{(4x^2-9)^{\frac{3}{2}} - 9\sqrt{4x^2-9}}{72x^4} + \frac{2}{27}\arctan\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(4*x^2 - 9)/x^5,x, algorithm="giac")
```

```
[Out] 1/72*((4*x^2 - 9)^(3/2) - 9*sqrt(4*x^2 - 9))/x^4 + 2/27*arctan(1/3*sqrt(4*x^2 - 9))
```

$$3.474 \quad \int x^5 \sqrt{-9 - 4x^2} dx$$

**Optimal.** Leaf size=46

$$-\frac{1}{448} (-4x^2 - 9)^{7/2} - \frac{9}{160} (-4x^2 - 9)^{5/2} - \frac{27}{64} (-4x^2 - 9)^{3/2}$$

[Out]  $(-27*(-9 - 4*x^2)^(3/2))/64 - (9*(-9 - 4*x^2)^(5/2))/160 - (-9 - 4*x^2)^(7/2)/448$

**Rubi [A]** time = 0.0575953, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{1}{448} (-4x^2 - 9)^{7/2} - \frac{9}{160} (-4x^2 - 9)^{5/2} - \frac{27}{64} (-4x^2 - 9)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x^5\*Sqrt[-9 - 4\*x^2], x]

[Out]  $(-27*(-9 - 4*x^2)^(3/2))/64 - (9*(-9 - 4*x^2)^(5/2))/160 - (-9 - 4*x^2)^(7/2)/448$

**Rubi in Sympy [A]** time = 6.70208, size = 44, normalized size = 0.96

$$-\frac{(-4x^2 - 9)^{7/2}}{448} - \frac{9(-4x^2 - 9)^{5/2}}{160} - \frac{27(-4x^2 - 9)^{3/2}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(-4\*x\*\*2-9)\*\*(1/2), x)

[Out]  $-(-4*x**2 - 9)**(7/2)/448 - 9*(-4*x**2 - 9)**(5/2)/160 - 27*(-4*x**2 - 9)**(3/2)/64$

**Mathematica [A]** time = 0.0223057, size = 27, normalized size = 0.59

$$-\frac{1}{280} (-4x^2 - 9)^{3/2} (10x^4 - 18x^2 + 27)$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*Sqrt[-9 - 4\*x^2], x]

[Out]  $-((-9 - 4*x^2)^(3/2)*(27 - 18*x^2 + 10*x^4))/280$

**Maple [A]** time = 0.005, size = 24, normalized size = 0.5

$$-\frac{10x^4 - 18x^2 + 27}{280} (-4x^2 - 9)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(-4\*x^2-9)^(1/2), x)

[Out]  $-1/280 * (10 * x^4 - 18 * x^2 + 27) * (-4 * x^2 - 9)^{(3/2)}$

**Maxima [A]** time = 1.48544, size = 54, normalized size = 1.17

$$-\frac{1}{28} (-4x^2 - 9)^{\frac{3}{2}} x^4 + \frac{9}{140} (-4x^2 - 9)^{\frac{3}{2}} x^2 - \frac{27}{280} (-4x^2 - 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)*x^5,x, algorithm="maxima")`

[Out]  $-1/28 * (-4 * x^2 - 9)^{(3/2)} * x^4 + 9/140 * (-4 * x^2 - 9)^{(3/2)} * x^2 - 27/280 * (-4 * x^2 - 9)^{(3/2)}$

**Fricas [A]** time = 0.218158, size = 38, normalized size = 0.83

$$\frac{1}{280} (40x^6 + 18x^4 - 54x^2 + 243) \sqrt{-4x^2 - 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)*x^5,x, algorithm="fricas")`

[Out]  $1/280 * (40 * x^6 + 18 * x^4 - 54 * x^2 + 243) * \text{sqrt}(-4 * x^2 - 9)$

**Sympy [A]** time = 6.02091, size = 68, normalized size = 1.48

$$\frac{x^6 \sqrt{-4x^2 - 9}}{7} + \frac{9x^4 \sqrt{-4x^2 - 9}}{140} - \frac{27x^2 \sqrt{-4x^2 - 9}}{140} + \frac{243 \sqrt{-4x^2 - 9}}{280}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(-4*x**2-9)**(1/2),x)`

[Out]  $x**6*\text{sqrt}(-4*x**2 - 9)/7 + 9*x**4*\text{sqrt}(-4*x**2 - 9)/140 - 27*x**2*\text{sqrt}(-4*x**2 - 9)/140 + 243*\text{sqrt}(-4*x**2 - 9)/280$

**GIAC/XCAS [A]** time = 0.20336, size = 50, normalized size = 1.09

$$\frac{1}{448} (4x^2 + 9)^{\frac{7}{2}} i - \frac{9}{160} (4x^2 + 9)^{\frac{5}{2}} i + \frac{27}{64} (4x^2 + 9)^{\frac{3}{2}} i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)*x^5,x, algorithm="giac")`

[Out]  $1/448 * (4 * x^2 + 9)^{(7/2)} * i - 9/160 * (4 * x^2 + 9)^{(5/2)} * i + 27/64 * (4 * x^2 + 9)^{(3/2)} * i$

$$3.475 \quad \int x^4 \sqrt{-9 - 4x^2} dx$$

**Optimal.** Leaf size=72

$$-\frac{81}{256} \sqrt{-4x^2 - 9}x - \frac{729}{512} \tan^{-1} \left( \frac{2x}{\sqrt{-4x^2 - 9}} \right) + \frac{1}{6} \sqrt{-4x^2 - 9}x^5 + \frac{3}{32} \sqrt{-4x^2 - 9}x^3$$

[Out]  $(-81*x*\text{Sqrt}[-9 - 4*x^2])/256 + (3*x^3*\text{Sqrt}[-9 - 4*x^2])/32 + (x^5*\text{Sqrt}[-9 - 4*x^2])/6 - (729*\text{ArcTan}[(2*x)/\text{Sqrt}[-9 - 4*x^2]])/512$

**Rubi [A]** time = 0.0666989, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{81}{256} \sqrt{-4x^2 - 9}x - \frac{729}{512} \tan^{-1} \left( \frac{2x}{\sqrt{-4x^2 - 9}} \right) + \frac{1}{6} \sqrt{-4x^2 - 9}x^5 + \frac{3}{32} \sqrt{-4x^2 - 9}x^3$$

Antiderivative was successfully verified.

[In] Int[x^4\*Sqrt[-9 - 4\*x^2], x]

[Out]  $(-81*x*\text{Sqrt}[-9 - 4*x^2])/256 + (3*x^3*\text{Sqrt}[-9 - 4*x^2])/32 + (x^5*\text{Sqrt}[-9 - 4*x^2])/6 - (729*\text{ArcTan}[(2*x)/\text{Sqrt}[-9 - 4*x^2]])/512$

**Rubi in Sympy [A]** time = 7.78677, size = 71, normalized size = 0.99

$$\frac{x^5 \sqrt{-4x^2 - 9}}{6} + \frac{3x^3 \sqrt{-4x^2 - 9}}{32} - \frac{81x \sqrt{-4x^2 - 9}}{256} - \frac{729 \operatorname{atan} \left( \frac{2x}{\sqrt{-4x^2 - 9}} \right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(-4\*x\*\*2-9)\*\*(1/2), x)

[Out]  $x**5*\text{sqrt}(-4*x**2 - 9)/6 + 3*x**3*\text{sqrt}(-4*x**2 - 9)/32 - 81*x*\text{sqrt}(-4*x**2 - 9)/256 - 729*\text{atan}(2*x/\text{sqrt}(-4*x**2 - 9))/512$

**Mathematica [A]** time = 0.0302694, size = 48, normalized size = 0.67

$$\frac{1}{768} x \sqrt{-4x^2 - 9} (128x^4 + 72x^2 - 243) - \frac{729}{512} \tan^{-1} \left( \frac{2x}{\sqrt{-4x^2 - 9}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*Sqrt[-9 - 4\*x^2], x]

[Out]  $(x*\text{Sqrt}[-9 - 4*x^2]*(-243 + 72*x^2 + 128*x^4))/768 - (729*\text{ArcTan}[(2*x)/\text{Sqrt}[-9 - 4*x^2]])/512$

**Maple [A]** time = 0.012, size = 55, normalized size = 0.8

$$-\frac{x^3}{24} (-4x^2 - 9)^{\frac{3}{2}} + \frac{9x}{128} (-4x^2 - 9)^{\frac{3}{2}} + \frac{81x}{256} \sqrt{-4x^2 - 9} - \frac{729}{512} \arctan \left( 2 \frac{x}{\sqrt{-4x^2 - 9}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(-4*x^2-9)^(1/2),x)`

[Out]  $-1/24*x^3*(-4*x^2-9)^(3/2)+9/128*x*(-4*x^2-9)^(3/2)+81/256*x*(-4*x^2-9)^(1/2)-729/512*\arctan(2*x/(-4*x^2-9)^(1/2))$

**Maxima [A]** time = 1.50604, size = 61, normalized size = 0.85

$$-\frac{1}{24}(-4x^2-9)^{\frac{3}{2}}x^3 + \frac{9}{128}(-4x^2-9)^{\frac{3}{2}}x + \frac{81}{256}\sqrt{-4x^2-9} + \frac{729}{512}i \operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)*x^4,x, algorithm="maxima")`

[Out]  $-1/24*(-4*x^2 - 9)^(3/2)*x^3 + 9/128*(-4*x^2 - 9)^(3/2)*x + 81/256*\sqrt{-4*x^2 - 9}*x + 729/512*I*\operatorname{arcsinh}(2/3*x)$

**Fricas [A]** time = 0.235006, size = 97, normalized size = 1.35

$$\frac{1}{768}(128x^5 + 72x^3 - 243x)\sqrt{-4x^2 - 9} - \frac{729}{1024}i \log\left(-\frac{8x + 4i\sqrt{-4x^2 - 9}}{x}\right) + \frac{729}{1024}i \log\left(-\frac{8x - 4i\sqrt{-4x^2 - 9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)*x^4,x, algorithm="fricas")`

[Out]  $1/768*(128*x^5 + 72*x^3 - 243*x)*\sqrt{-4*x^2 - 9} - 729/1024*I*\log(-(8*x + 4*I*\sqrt{-4*x^2 - 9})/x) + 729/1024*I*\log(-(8*x - 4*I*\sqrt{-4*x^2 - 9})/x)$

**Sympy [A]** time = 15.3896, size = 83, normalized size = 1.15

$$\frac{2ix^7}{3\sqrt{4x^2+9}} + \frac{15ix^5}{8\sqrt{4x^2+9}} - \frac{27ix^3}{64\sqrt{4x^2+9}} - \frac{729ix}{256\sqrt{4x^2+9}} + \frac{729i \operatorname{asinh}\left(\frac{2x}{3}\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(-4*x**2-9)**(1/2),x)`

[Out]  $2*I*x**7/(3*\sqrt{4*x**2 + 9}) + 15*I*x**5/(8*\sqrt{4*x**2 + 9}) - 27*I*x**3/(64*\sqrt{4*x**2 + 9}) - 729*I*x/(256*\sqrt{4*x**2 + 9}) + 729*I*\operatorname{asinh}(2*x/3)/512$

**GIAC/XCAS [A]** time = 0.207161, size = 47, normalized size = 0.65

$$\frac{1}{768}(8(16x^2+9)x^2-243)\sqrt{-4x^2-9} + \frac{729}{512}i \operatorname{arcsin}\left(\frac{2}{3}ix\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)*x^4,x, algorithm="giac")`

```
[Out] 1/768*(8*(16*x^2 + 9)*x^2 - 243)*sqrt(-4*x^2 - 9)*x + 729/512*i*arcsin(2/3*i*x)
```



$$3.476 \quad \int x^3 \sqrt{-9 - 4x^2} dx$$

**Optimal.** Leaf size=31

$$\frac{1}{80} (-4x^2 - 9)^{5/2} + \frac{3}{16} (-4x^2 - 9)^{3/2}$$

[Out]  $(3 * (-9 - 4 * x^2)^{(3/2)})/16 + (-9 - 4 * x^2)^{(5/2)}/80$

**Rubi [A]** time = 0.0437391, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{1}{80} (-4x^2 - 9)^{5/2} + \frac{3}{16} (-4x^2 - 9)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x^3\*Sqrt[-9 - 4\*x^2],x]

[Out]  $(3 * (-9 - 4 * x^2)^{(3/2)})/16 + (-9 - 4 * x^2)^{(5/2)}/80$

**Rubi in Sympy [A]** time = 5.5061, size = 27, normalized size = 0.87

$$\frac{(-4x^2 - 9)^{\frac{5}{2}}}{80} + \frac{3(-4x^2 - 9)^{\frac{3}{2}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(-4\*x\*\*2-9)\*\*(1/2),x)

[Out]  $(-4 * x^2 - 9)^{(5/2)}/80 + 3 * (-4 * x^2 - 9)^{(3/2)}/16$

**Mathematica [A]** time = 0.0137714, size = 22, normalized size = 0.71

$$\frac{1}{40} (-4x^2 - 9)^{3/2} (3 - 2x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Sqrt[-9 - 4\*x^2],x]

[Out]  $((-9 - 4 * x^2)^{(3/2)} * (3 - 2 * x^2))/40$

**Maple [A]** time = 0.005, size = 19, normalized size = 0.6

$$-\frac{2x^2 - 3}{40} (-4x^2 - 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(-4\*x^2-9)^(1/2),x)

[Out]  $-1/40 * (2 * x^2 - 3) * (-4 * x^2 - 9)^{(3/2)}$

---

**Maxima [A]** time = 1.49885, size = 35, normalized size = 1.13

$$-\frac{1}{20}(-4x^2 - 9)^{\frac{3}{2}}x^2 + \frac{3}{40}(-4x^2 - 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 - 9)\*x^3,x, algorithm="maxima")

[Out] -1/20\*(-4\*x^2 - 9)^(3/2)\*x^2 + 3/40\*(-4\*x^2 - 9)^(3/2)

---

**Fricas [A]** time = 0.2342, size = 31, normalized size = 1.

$$\frac{1}{40}(8x^4 + 6x^2 - 27)\sqrt{-4x^2 - 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 - 9)\*x^3,x, algorithm="fricas")

[Out] 1/40\*(8\*x^4 + 6\*x^2 - 27)\*sqrt(-4\*x^2 - 9)

---

**Sympy [A]** time = 1.83754, size = 49, normalized size = 1.58

$$\frac{x^4\sqrt{-4x^2 - 9}}{5} + \frac{3x^2\sqrt{-4x^2 - 9}}{20} - \frac{27\sqrt{-4x^2 - 9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(-4\*x\*\*2-9)\*\*(1/2),x)

[Out] x\*\*4\*sqrt(-4\*x\*\*2 - 9)/5 + 3\*x\*\*2\*sqrt(-4\*x\*\*2 - 9)/20 - 27\*sqrt(-4\*x\*\*2 - 9)/40

---

**GIAC/XCAS [A]** time = 0.201951, size = 34, normalized size = 1.1

$$\frac{1}{80}(4x^2 + 9)^{\frac{5}{2}}i - \frac{3}{16}(4x^2 + 9)^{\frac{3}{2}}i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 - 9)\*x^3,x, algorithm="giac")

[Out] 1/80\*(4\*x^2 + 9)^(5/2)\*i - 3/16\*(4\*x^2 + 9)^(3/2)\*i

$$3.477 \quad \int x^2 \sqrt{-9 - 4x^2} dx$$

**Optimal.** Leaf size=54

$$\frac{9}{32} \sqrt{-4x^2 - 9}x + \frac{81}{64} \tan^{-1} \left( \frac{2x}{\sqrt{-4x^2 - 9}} \right) + \frac{1}{4} \sqrt{-4x^2 - 9}x^3$$

[Out] (9\*x\*Sqrt[-9 - 4\*x^2])/32 + (x^3\*Sqrt[-9 - 4\*x^2])/4 + (81\*ArcTan[(2\*x)/Sqrt[-9 - 4\*x^2]])/64

**Rubi [A]** time = 0.0469325, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{9}{32} \sqrt{-4x^2 - 9}x + \frac{81}{64} \tan^{-1} \left( \frac{2x}{\sqrt{-4x^2 - 9}} \right) + \frac{1}{4} \sqrt{-4x^2 - 9}x^3$$

Antiderivative was successfully verified.

[In] Int[x^2\*Sqrt[-9 - 4\*x^2], x]

[Out] (9\*x\*Sqrt[-9 - 4\*x^2])/32 + (x^3\*Sqrt[-9 - 4\*x^2])/4 + (81\*ArcTan[(2\*x)/Sqrt[-9 - 4\*x^2]])/64

**Rubi in Sympy [A]** time = 6.12681, size = 53, normalized size = 0.98

$$\frac{x^3 \sqrt{-4x^2 - 9}}{4} + \frac{9x \sqrt{-4x^2 - 9}}{32} + \frac{81 \operatorname{atan} \left( \frac{2x}{\sqrt{-4x^2 - 9}} \right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(-4\*x\*\*2-9)\*\*(1/2), x)

[Out] x\*\*3\*sqrt(-4\*x\*\*2 - 9)/4 + 9\*x\*sqrt(-4\*x\*\*2 - 9)/32 + 81\*atan(2\*x/sqrt(-4\*x\*\*2 - 9))/64

**Mathematica [A]** time = 0.0320073, size = 43, normalized size = 0.8

$$\frac{1}{64} \left( 2x \sqrt{-4x^2 - 9} (8x^2 + 9) + 81 \tan^{-1} \left( \frac{2x}{\sqrt{-4x^2 - 9}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Sqrt[-9 - 4\*x^2], x]

[Out] (2\*x\*Sqrt[-9 - 4\*x^2]\*(9 + 8\*x^2) + 81\*ArcTan[(2\*x)/Sqrt[-9 - 4\*x^2]])/64

**Maple [A]** time = 0.007, size = 41, normalized size = 0.8

$$-\frac{x}{16} (-4x^2 - 9)^{\frac{3}{2}} - \frac{9x}{32} \sqrt{-4x^2 - 9} + \frac{81}{64} \arctan \left( 2 \frac{x}{\sqrt{-4x^2 - 9}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(-4*x^2-9)^(1/2),x)`

[Out]  $-1/16*x*(-4*x^2-9)^(3/2)-9/32*x*(-4*x^2-9)^(1/2)+81/64*\arctan(2*x/(-4*x^2-9)^(1/2))$

**Maxima [A]** time = 1.50434, size = 42, normalized size = 0.78

$$-\frac{1}{16}(-4x^2-9)^{\frac{3}{2}}x - \frac{9}{32}\sqrt{-4x^2-9}x - \frac{81}{64}i \operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)*x^2,x, algorithm="maxima")`

[Out]  $-1/16*(-4*x^2 - 9)^(3/2)*x - 9/32*\sqrt{-4*x^2 - 9}*x - 81/64*I*\operatorname{arcsinh}(2/3*x)$

**Fricas [A]** time = 0.241599, size = 90, normalized size = 1.67

$$\frac{1}{32}(8x^3+9x)\sqrt{-4x^2-9} + \frac{81}{128}i \log\left(-\frac{8x+4i\sqrt{-4x^2-9}}{x}\right) - \frac{81}{128}i \log\left(-\frac{8x-4i\sqrt{-4x^2-9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)*x^2,x, algorithm="fricas")`

[Out]  $1/32*(8*x^3 + 9*x)*\sqrt{-4*x^2 - 9} + 81/128*I*\log(-(8*x + 4*I*\sqrt{-4*x^2 - 9})/x) - 81/128*I*\log(-(8*x - 4*I*\sqrt{-4*x^2 - 9})/x)$

**Sympy [A]** time = 9.06351, size = 61, normalized size = 1.13

$$\frac{ix^5}{\sqrt{4x^2+9}} + \frac{27ix^3}{8\sqrt{4x^2+9}} + \frac{81ix}{32\sqrt{4x^2+9}} - \frac{81i \operatorname{asinh}\left(\frac{2x}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(-4*x**2-9)**(1/2),x)`

[Out]  $I*x**5/\sqrt{4*x**2 + 9} + 27*I*x**3/(8*\sqrt{4*x**2 + 9}) + 81*I*x/(32*\sqrt{4*x**2 + 9}) - 81*I*\operatorname{asinh}(2*x/3)/64$

**GIAC/XCAS [A]** time = 0.2091, size = 38, normalized size = 0.7

$$\frac{1}{32}(8x^2+9)\sqrt{-4x^2-9}x - \frac{81}{64}i \operatorname{arcsin}\left(\frac{2}{3}ix\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)*x^2,x, algorithm="giac")`

[Out]  $1/32*(8*x^2 + 9)*\sqrt{-4*x^2 - 9}*x - 81/64*i*\operatorname{arcsin}(2/3*i*x)$

$$3.478 \quad \int x\sqrt{-9-4x^2} dx$$

**Optimal.** Leaf size=15

$$-\frac{1}{12}(-4x^2-9)^{3/2}$$

[Out]  $-( -9 - 4 * x^2 ) ^ { 3 / 2 } / 12$

**Rubi [A]** time = 0.00875601, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{1}{12}(-4x^2-9)^{3/2}$$

Antiderivative was successfully verified.

[In] `Int[x*Sqrt[-9 - 4*x^2],x]`

[Out]  $-( -9 - 4 * x^2 ) ^ { 3 / 2 } / 12$

**Rubi in Sympy [A]** time = 1.97019, size = 14, normalized size = 0.93

$$-\frac{(-4x^2-9)^{\frac{3}{2}}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x*(-4*x**2-9)**(1/2),x)`

[Out]  $-( -4 * x ** 2 - 9 ) ** ( 3 / 2 ) / 12$

**Mathematica [A]** time = 0.00321487, size = 15, normalized size = 1.

$$-\frac{1}{12}(-4x^2-9)^{3/2}$$

Antiderivative was successfully verified.

[In] `Integrate[x*Sqrt[-9 - 4*x^2],x]`

[Out]  $-( -9 - 4 * x^2 ) ^ { 3 / 2 } / 12$

**Maple [A]** time = 0.003, size = 12, normalized size = 0.8

$$-\frac{1}{12}(-4x^2-9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(-4*x^2-9)^(1/2),x)`

[Out]  $-1/12 * ( -4 * x^2 - 9 ) ^ { 3 / 2 }$

---

**Maxima [A]** time = 1.34706, size = 15, normalized size = 1.

$$-\frac{1}{12}(-4x^2 - 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)*x,x, algorithm="maxima")`

[Out] `-1/12*(-4*x^2 - 9)^(3/2)`

---

**Fricas [A]** time = 0.230497, size = 24, normalized size = 1.6

$$\frac{1}{12}(4x^2 + 9)\sqrt{-4x^2 - 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)*x,x, algorithm="fricas")`

[Out] `1/12*(4*x^2 + 9)*sqrt(-4*x^2 - 9)`

---

**Sympy [A]** time = 0.511571, size = 31, normalized size = 2.07

$$\frac{x^2\sqrt{-4x^2 - 9}}{3} + \frac{3\sqrt{-4x^2 - 9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-4*x**2-9)**(1/2),x)`

[Out] `x**2*sqrt(-4*x**2 - 9)/3 + 3*sqrt(-4*x**2 - 9)/4`

---

**GIAC/XCAS [A]** time = 0.20191, size = 16, normalized size = 1.07

$$\frac{1}{12}(4x^2 + 9)^{\frac{3}{2}}i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)*x,x, algorithm="giac")`

[Out] `1/12*(4*x^2 + 9)^(3/2)*i`

$$3.479 \quad \int \sqrt{-9 - 4x^2} dx$$

**Optimal.** Leaf size=36

$$\frac{1}{2}x\sqrt{-4x^2 - 9} - \frac{9}{4} \tan^{-1}\left(\frac{2x}{\sqrt{-4x^2 - 9}}\right)$$

[Out] (x\*Sqrt[-9 - 4\*x^2])/2 - (9\*ArcTan[(2\*x)/Sqrt[-9 - 4\*x^2]])/4

**Rubi [A]** time = 0.0196066, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{1}{2}x\sqrt{-4x^2 - 9} - \frac{9}{4} \tan^{-1}\left(\frac{2x}{\sqrt{-4x^2 - 9}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-9 - 4\*x^2], x]

[Out] (x\*Sqrt[-9 - 4\*x^2])/2 - (9\*ArcTan[(2\*x)/Sqrt[-9 - 4\*x^2]])/4

**Rubi in Sympy [A]** time = 1.4944, size = 34, normalized size = 0.94

$$\frac{x\sqrt{-4x^2 - 9}}{2} - \frac{9 \operatorname{atan}\left(\frac{2x}{\sqrt{-4x^2 - 9}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-4\*x\*\*2-9)\*\*(1/2), x)

[Out] x\*sqrt(-4\*x\*\*2 - 9)/2 - 9\*atan(2\*x/sqrt(-4\*x\*\*2 - 9))/4

**Mathematica [A]** time = 0.0145256, size = 36, normalized size = 1.

$$\frac{1}{4} \left( 2x\sqrt{-4x^2 - 9} - 9 \tan^{-1}\left(\frac{2x}{\sqrt{-4x^2 - 9}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-9 - 4\*x^2], x]

[Out] (2\*x\*Sqrt[-9 - 4\*x^2] - 9\*ArcTan[(2\*x)/Sqrt[-9 - 4\*x^2]])/4

**Maple [A]** time = 0.005, size = 29, normalized size = 0.8

$$-\frac{9}{4} \operatorname{arctan}\left(2 \frac{x}{\sqrt{-4x^2 - 9}}\right) + \frac{x}{2} \sqrt{-4x^2 - 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4\*x^2-9)^(1/2), x)

[Out]  $-9/4 \cdot \arctan(2x/(-4x^2-9)^{1/2}) + 1/2 \cdot x \cdot (-4x^2-9)^{1/2}$

**Maxima [A]** time = 1.49262, size = 26, normalized size = 0.72

$$\frac{1}{2} \sqrt{-4x^2 - 9x} + \frac{9}{4} i \operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9),x, algorithm="maxima")`

[Out]  $1/2 \cdot \sqrt{-4x^2 - 9} \cdot x + 9/4 \cdot I \cdot \operatorname{arcsinh}(2/3 \cdot x)$

**Fricas [A]** time = 0.230336, size = 80, normalized size = 2.22

$$\frac{1}{2} \sqrt{-4x^2 - 9x} - \frac{9}{8} i \log\left(-\frac{8x + 4i \sqrt{-4x^2 - 9}}{x}\right) + \frac{9}{8} i \log\left(-\frac{8x - 4i \sqrt{-4x^2 - 9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9),x, algorithm="fricas")`

[Out]  $1/2 \cdot \sqrt{-4x^2 - 9} \cdot x - 9/8 \cdot I \cdot \log(-(8x + 4 \cdot I \cdot \sqrt{-4x^2 - 9})/x) + 9/8 \cdot I \cdot \log(-(8x - 4 \cdot I \cdot \sqrt{-4x^2 - 9})/x)$

**Sympy [A]** time = 1.47484, size = 34, normalized size = 0.94

$$\frac{x \sqrt{-4x^2 - 9}}{2} - \frac{9 \operatorname{atan}\left(\frac{2x}{\sqrt{-4x^2 - 9}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-4*x**2-9)**(1/2),x)`

[Out]  $x \cdot \sqrt{-4x^2 - 9} / 2 - 9 \cdot \operatorname{atan}(2x / \sqrt{-4x^2 - 9}) / 4$

**GIAC/XCAS [A]** time = 0.202731, size = 28, normalized size = 0.78

$$\frac{9}{4} i \operatorname{arcsin}\left(\frac{2}{3}ix\right) + \frac{1}{2} \sqrt{-4x^2 - 9x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9),x, algorithm="giac")`

[Out]  $9/4 \cdot i \cdot \operatorname{arcsin}(2/3 \cdot i \cdot x) + 1/2 \cdot \sqrt{-4x^2 - 9} \cdot x$



$$3.480 \quad \int \frac{\sqrt{-9-4x^2}}{x} dx$$

**Optimal.** Leaf size=30

$$\sqrt{-4x^2 - 9} - 3 \tan^{-1} \left( \frac{1}{3} \sqrt{-4x^2 - 9} \right)$$

[Out] Sqrt[-9 - 4\*x^2] - 3\*ArcTan[Sqrt[-9 - 4\*x^2]/3]

**Rubi [A]** time = 0.0500217, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\sqrt{-4x^2 - 9} - 3 \tan^{-1} \left( \frac{1}{3} \sqrt{-4x^2 - 9} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-9 - 4\*x^2]/x, x]

[Out] Sqrt[-9 - 4\*x^2] - 3\*ArcTan[Sqrt[-9 - 4\*x^2]/3]

**Rubi in Sympy [A]** time = 5.5623, size = 27, normalized size = 0.9

$$\sqrt{-4x^2 - 9} - 3 \operatorname{atan} \left( \frac{\sqrt{-4x^2 - 9}}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*(-4\*x\*\*2-9)\*\*(1/2), x)

[Out] sqrt(-4\*x\*\*2 - 9) - 3\*atan(sqrt(-4\*x\*\*2 - 9)/3)

**Mathematica [A]** time = 0.0153361, size = 28, normalized size = 0.93

$$\sqrt{-4x^2 - 9} + 3 \tan^{-1} \left( \frac{3}{\sqrt{-4x^2 - 9}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-9 - 4\*x^2]/x, x]

[Out] Sqrt[-9 - 4\*x^2] + 3\*ArcTan[3/Sqrt[-9 - 4\*x^2]]

**Maple [A]** time = 0.007, size = 25, normalized size = 0.8

$$\sqrt{-4x^2 - 9} + 3 \arctan \left( 3 \frac{1}{\sqrt{-4x^2 - 9}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x\*(-4\*x^2-9)^(1/2), x)

[Out]  $(-4x^2-9)^{1/2}+3\arctan(3/(-4x^2-9)^{1/2})$

**Maxima [A]** time = 1.50193, size = 47, normalized size = 1.57

$$\sqrt{-4x^2-9} + 3i \log\left(\frac{6\sqrt{4x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)/x,x, algorithm="maxima")`

[Out] `sqrt(-4*x^2 - 9) + 3*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x))`

**Fricas [A]** time = 0.230703, size = 70, normalized size = 2.33

$$\sqrt{-4x^2-9} - \frac{3}{2}i \log\left(-\frac{6(i\sqrt{-4x^2-9}-3)}{x}\right) + \frac{3}{2}i \log\left(-\frac{6(-i\sqrt{-4x^2-9}-3)}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)/x,x, algorithm="fricas")`

[Out] `sqrt(-4*x^2 - 9) - 3/2*I*log(-6*(I*sqrt(-4*x^2 - 9) - 3)/x) + 3/2*I*log(-6*(-I*sqrt(-4*x^2 - 9) - 3)/x)`

**Sympy [A]** time = 4.31336, size = 44, normalized size = 1.47

$$\frac{2ix}{\sqrt{1+\frac{9}{4x^2}}} - 3i \operatorname{asinh}\left(\frac{3}{2x}\right) + \frac{9i}{2x\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x*(-4*x**2-9)**(1/2),x)`

[Out] `2*I*x/sqrt(1 + 9/(4*x**2)) - 3*I*asinh(3/(2*x)) + 9*I/(2*x*sqrt(1 + 9/(4*x**2)))`

**GIAC/XCAS [A]** time = 0.20272, size = 36, normalized size = 1.2

$$\sqrt{4x^2+9}i - 3 \arctan\left(\frac{1}{3}\sqrt{4x^2+9}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)/x,x, algorithm="giac")`

[Out] `sqrt(4*x^2 + 9)*i - 3*arctan(1/3*sqrt(4*x^2 + 9)*i)`

$$3.481 \quad \int \frac{\sqrt{-9-4x^2}}{x^2} dx$$

**Optimal.** Leaf size=34

$$-\frac{\sqrt{-4x^2-9}}{x} - 2 \tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

[Out] -(Sqrt[-9 - 4\*x^2]/x) - 2\*ArcTan[(2\*x)/Sqrt[-9 - 4\*x^2]]

**Rubi [A]** time = 0.0288548, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{\sqrt{-4x^2-9}}{x} - 2 \tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-9 - 4\*x^2]/x^2, x]

[Out] -(Sqrt[-9 - 4\*x^2]/x) - 2\*ArcTan[(2\*x)/Sqrt[-9 - 4\*x^2]]

**Rubi in Sympy [A]** time = 3.46473, size = 32, normalized size = 0.94

$$-2 \operatorname{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right) - \frac{\sqrt{-4x^2-9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-4\*x\*\*2-9)\*\*(1/2)/x\*\*2, x)

[Out] -2\*atan(2\*x/sqrt(-4\*x\*\*2 - 9)) - sqrt(-4\*x\*\*2 - 9)/x

**Mathematica [A]** time = 0.025055, size = 35, normalized size = 1.03

$$-\frac{\sqrt{-4x^2-9} + 2x \tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-9 - 4\*x^2]/x^2, x]

[Out] -((Sqrt[-9 - 4\*x^2] + 2\*x\*ArcTan[(2\*x)/Sqrt[-9 - 4\*x^2]])/x)

**Maple [A]** time = 0.005, size = 43, normalized size = 1.3

$$\frac{1}{9x} (-4x^2 - 9)^{\frac{3}{2}} + \frac{4x}{9} \sqrt{-4x^2 - 9} - 2 \operatorname{arctan}\left(2 \frac{x}{\sqrt{-4x^2 - 9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4\*x^2-9)^(1/2)/x^2, x)

[Out]  $1/9/x * (-4 * x^2 - 9)^{(3/2)} + 4/9 * x * (-4 * x^2 - 9)^{(1/2)} - 2 * \arctan(2 * x / (-4 * x^2 - 9)^{(1/2)})$

**Maxima [A]** time = 1.50435, size = 28, normalized size = 0.82

$$-\frac{\sqrt{-4x^2 - 9}}{x} + 2i \operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)/x^2,x, algorithm="maxima")`

[Out]  $-\sqrt{-4x^2 - 9}/x + 2 * I * \operatorname{arcsinh}(2/3 * x)$

**Fricas [A]** time = 0.230315, size = 86, normalized size = 2.53

$$\frac{-i x \log\left(-\frac{8x+4i\sqrt{-4x^2-9}}{x}\right) + i x \log\left(-\frac{8x-4i\sqrt{-4x^2-9}}{x}\right) - \sqrt{-4x^2 - 9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)/x^2,x, algorithm="fricas")`

[Out]  $(-I * x * \log(-(8 * x + 4 * I * \sqrt{-4 * x^2 - 9}))/x) + I * x * \log(-(8 * x - 4 * I * \sqrt{-4 * x^2 - 9}))/x - \sqrt{-4 * x^2 - 9}/x$

**Sympy [A]** time = 1.63291, size = 32, normalized size = 0.94

$$-2 \operatorname{atan}\left(\frac{2x}{\sqrt{-4x^2 - 9}}\right) - \frac{\sqrt{-4x^2 - 9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-4*x**2-9)**(1/2)/x**2,x)`

[Out]  $-2 * \operatorname{atan}(2 * x / \sqrt{-4 * x^2 - 9}) - \sqrt{-4 * x^2 - 9} / x$

**GIAC/XCAS [A]** time = 0.208263, size = 66, normalized size = 1.94

$$2i \operatorname{arcsin}\left(\frac{2}{3}ix\right) - \frac{\sqrt{4x^2 + 9i + 3i}}{2x} + \frac{2x}{\sqrt{4x^2 + 9i + 3i}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)/x^2,x, algorithm="giac")`

[Out]  $2 * i * \operatorname{arcsin}(2/3 * i * x) - 1/2 * (\sqrt{4 * x^2 + 9} * i + 3 * i) / x + 2 * x / (\sqrt{4 * x^2 + 9} * i + 3 * i)$

$$3.482 \quad \int \frac{\sqrt{-9-4x^2}}{x^3} dx$$

**Optimal.** Leaf size=39

$$-\frac{\sqrt{-4x^2-9}}{2x^2} - \frac{2}{3} \tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

[Out] -Sqrt[-9 - 4\*x^2]/(2\*x^2) - (2\*ArcTan[Sqrt[-9 - 4\*x^2]/3])/3

**Rubi [A]** time = 0.0510821, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{\sqrt{-4x^2-9}}{2x^2} - \frac{2}{3} \tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-9 - 4\*x^2]/x^3, x]

[Out] -Sqrt[-9 - 4\*x^2]/(2\*x^2) - (2\*ArcTan[Sqrt[-9 - 4\*x^2]/3])/3

**Rubi in Sympy [A]** time = 5.75434, size = 36, normalized size = 0.92

$$-\frac{2 \operatorname{atan}\left(\frac{\sqrt{-4x^2-9}}{3}\right)}{3} - \frac{\sqrt{-4x^2-9}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-4\*x\*\*2-9)\*\*(1/2)/x\*\*3, x)

[Out] -2\*atan(sqrt(-4\*x\*\*2 - 9)/3)/3 - sqrt(-4\*x\*\*2 - 9)/(2\*x\*\*2)

**Mathematica [A]** time = 0.0199935, size = 37, normalized size = 0.95

$$\frac{2}{3} \tan^{-1}\left(\frac{3}{\sqrt{-4x^2-9}}\right) - \frac{\sqrt{-4x^2-9}}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-9 - 4\*x^2]/x^3, x]

[Out] -Sqrt[-9 - 4\*x^2]/(2\*x^2) + (2\*ArcTan[3/Sqrt[-9 - 4\*x^2]])/3

**Maple [A]** time = 0.006, size = 41, normalized size = 1.1

$$\frac{1}{18x^2} (-4x^2-9)^{\frac{3}{2}} + \frac{2}{9}\sqrt{-4x^2-9} + \frac{2}{3} \arctan\left(3 \frac{1}{\sqrt{-4x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4\*x^2-9)^(1/2)/x^3, x)

[Out]  $1/18/x^2 * (-4*x^2-9)^{(3/2)} + 2/9 * (-4*x^2-9)^{(1/2)} + 2/3 * \arctan(3/(-4*x^2-9)^{(1/2)})$

**Maxima [A]** time = 1.49906, size = 69, normalized size = 1.77

$$\frac{2}{9} \sqrt{-4x^2 - 9} + \frac{(-4x^2 - 9)^{\frac{3}{2}}}{18x^2} + \frac{2}{3}i \log\left(\frac{6\sqrt{4x^2 + 9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)/x^3,x, algorithm="maxima")`

[Out]  $2/9 * \text{sqrt}(-4*x^2 - 9) + 1/18 * (-4*x^2 - 9)^{(3/2)}/x^2 + 2/3 * I * \log(6 * \text{sqrt}(4*x^2 + 9)/\text{abs}(x) + 18/\text{abs}(x))$

**Fricas [A]** time = 0.235661, size = 88, normalized size = 2.26

$$\frac{-2ix^2 \log\left(-\frac{4(i\sqrt{-4x^2-9}-3)}{3x}\right) + 2ix^2 \log\left(-\frac{4(-i\sqrt{-4x^2-9}-3)}{3x}\right) - 3\sqrt{-4x^2-9}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)/x^3,x, algorithm="fricas")`

[Out]  $1/6 * (-2 * I * x^2 * \log(-4/3 * (I * \text{sqrt}(-4 * x^2 - 9) - 3)/x) + 2 * I * x^2 * \log(-4/3 * (-I * \text{sqrt}(-4 * x^2 - 9) - 3)/x) - 3 * \text{sqrt}(-4 * x^2 - 9))/x^2$

**Sympy [A]** time = 6.0263, size = 27, normalized size = 0.69

$$-\frac{2i \operatorname{asinh}\left(\frac{3}{2x}\right)}{3} - \frac{i\sqrt{1 + \frac{9}{4x^2}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-4*x**2-9)**(1/2)/x**3,x)`

[Out]  $-2 * I * \operatorname{asinh}(3/(2*x))/3 - I * \text{sqrt}(1 + 9/(4*x**2))/x$

**GIAC/XCAS [A]** time = 0.20471, size = 42, normalized size = 1.08

$$-\frac{\sqrt{4x^2 + 9}i}{2x^2} - \frac{2}{3} \arctan\left(\frac{1}{3}\sqrt{4x^2 + 9}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)/x^3,x, algorithm="giac")`

[Out]  $-1/2 * \text{sqrt}(4 * x^2 + 9) * i/x^2 - 2/3 * \arctan(1/3 * \text{sqrt}(4 * x^2 + 9) * i)$

$$3.483 \quad \int \frac{\sqrt{-9-4x^2}}{x^4} dx$$

**Optimal.** Leaf size=18

$$\frac{(-4x^2 - 9)^{3/2}}{27x^3}$$

[Out]  $(-9 - 4*x^2)^{(3/2)}/(27*x^3)$

**Rubi [A]** time = 0.0160887, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{(-4x^2 - 9)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-9 - 4\*x^2]/x^4, x]

[Out]  $(-9 - 4*x^2)^{(3/2)}/(27*x^3)$

**Rubi in Sympy [A]** time = 2.94518, size = 15, normalized size = 0.83

$$\frac{(-4x^2 - 9)^{\frac{3}{2}}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-4\*x\*\*2-9)\*\*(1/2)/x\*\*4, x)

[Out]  $(-4*x**2 - 9)**(3/2)/(27*x**3)$

**Mathematica [A]** time = 0.0101905, size = 18, normalized size = 1.

$$\frac{(-4x^2 - 9)^{3/2}}{27x^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-9 - 4\*x^2]/x^4, x]

[Out]  $(-9 - 4*x^2)^{(3/2)}/(27*x^3)$

**Maple [A]** time = 0.006, size = 15, normalized size = 0.8

$$\frac{1}{27x^3} (-4x^2 - 9)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4\*x^2-9)^(1/2)/x^4, x)

[Out]  $1/27*(-4*x^2-9)^{(3/2)}/x^3$

---

**Maxima [A]** time = 1.49791, size = 19, normalized size = 1.06

$$\frac{(-4x^2 - 9)^{\frac{3}{2}}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 - 9)/x^4,x, algorithm="maxima")

[Out] 1/27\*(-4\*x^2 - 9)^(3/2)/x^3

---

**Fricas [A]** time = 0.225196, size = 19, normalized size = 1.06

$$\frac{(-4x^2 - 9)^{\frac{3}{2}}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 - 9)/x^4,x, algorithm="fricas")

[Out] 1/27\*(-4\*x^2 - 9)^(3/2)/x^3

---

**Sympy [A]** time = 3.64371, size = 37, normalized size = 2.06

$$-\frac{8i\sqrt{1 + \frac{9}{4x^2}}}{27} - \frac{2i\sqrt{1 + \frac{9}{4x^2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4\*x\*\*2-9)\*\*(1/2)/x\*\*4,x)

[Out] -8\*I\*sqrt(1 + 9/(4\*x\*\*2))/27 - 2\*I\*sqrt(1 + 9/(4\*x\*\*2))/(3\*x\*\*2)

---

**GIAC/XCAS [A]** time = 0.209531, size = 120, normalized size = 6.67

$$\frac{2x^3 \left( \frac{3(\sqrt{4x^2+9i+3i})^2}{x^2} - 4 \right)}{27(\sqrt{4x^2+9i+3i})^3} + \frac{(\sqrt{4x^2+9i+3i})^3}{216x^3} - \frac{\sqrt{4x^2+9i+3i}}{18x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-4\*x^2 - 9)/x^4,x, algorithm="giac")

[Out] 2/27\*x^3\*(3\*(sqrt(4\*x^2 + 9)\*i + 3\*i)^2/x^2 - 4)/(sqrt(4\*x^2 + 9)\*i + 3\*i)^3 + 1/216\*(sqrt(4\*x^2 + 9)\*i + 3\*i)^3/x^3 - 1/18\*(sqrt(4\*x^2 + 9)\*i + 3\*i)/x



$$3.484 \quad \int \frac{\sqrt{-9-4x^2}}{x^5} dx$$

**Optimal.** Leaf size=57

$$-\frac{\sqrt{-4x^2-9}}{18x^2} + \frac{2}{27} \tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right) - \frac{\sqrt{-4x^2-9}}{4x^4}$$

[Out] -Sqrt[-9 - 4\*x^2]/(4\*x^4) - Sqrt[-9 - 4\*x^2]/(18\*x^2) + (2\*ArcTan[Sqrt[-9 - 4\*x^2]/3])/27

**Rubi [A]** time = 0.0668579, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{\sqrt{-4x^2-9}}{18x^2} + \frac{2}{27} \tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right) - \frac{\sqrt{-4x^2-9}}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-9 - 4\*x^2]/x^5, x]

[Out] -Sqrt[-9 - 4\*x^2]/(4\*x^4) - Sqrt[-9 - 4\*x^2]/(18\*x^2) + (2\*ArcTan[Sqrt[-9 - 4\*x^2]/3])/27

**Rubi in Sympy [A]** time = 6.62009, size = 51, normalized size = 0.89

$$\frac{2 \operatorname{atan}\left(\frac{\sqrt{-4x^2-9}}{3}\right)}{27} - \frac{\sqrt{-4x^2-9}}{18x^2} - \frac{\sqrt{-4x^2-9}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-4\*x\*\*2-9)\*\*(1/2)/x\*\*5, x)

[Out] 2\*atan(sqrt(-4\*x\*\*2 - 9)/3)/27 - sqrt(-4\*x\*\*2 - 9)/(18\*x\*\*2) - sqrt(-4\*x\*\*2 - 9)/(4\*x\*\*4)

**Mathematica [A]** time = 0.034755, size = 44, normalized size = 0.77

$$-\frac{2}{27} \tan^{-1}\left(\frac{3}{\sqrt{-4x^2-9}}\right) - \frac{\sqrt{-4x^2-9}(2x^2+9)}{36x^4}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-9 - 4\*x^2]/x^5, x]

[Out] -(Sqrt[-9 - 4\*x^2]\*(9 + 2\*x^2))/(36\*x^4) - (2\*ArcTan[3/Sqrt[-9 - 4\*x^2]])/27

**Maple [A]** time = 0.007, size = 55, normalized size = 1.

$$\frac{1}{36x^4}(-4x^2-9)^{\frac{3}{2}} - \frac{1}{162x^2}(-4x^2-9)^{\frac{3}{2}} - \frac{2}{81}\sqrt{-4x^2-9} - \frac{2}{27} \arctan\left(3\frac{1}{\sqrt{-4x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-4*x^2-9)^(1/2)/x^5,x)`

[Out]  $\frac{1}{36}x^{-4}(-4x^2-9)^{3/2} - \frac{1}{162}x^{-2}(-4x^2-9)^{3/2} - \frac{2}{81}(-4x^2-9)^{1/2} - \frac{2}{27}\arctan\left(\frac{3}{(-4x^2-9)^{1/2}}\right)$

**Maxima [A]** time = 1.50731, size = 88, normalized size = 1.54

$$-\frac{2}{81}\sqrt{-4x^2-9} - \frac{(-4x^2-9)^{3/2}}{162x^2} + \frac{(-4x^2-9)^{3/2}}{36x^4} - \frac{2}{27}i \log\left(\frac{6\sqrt{4x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)/x^5,x, algorithm="maxima")`

[Out]  $-\frac{2}{81}\sqrt{-4x^2-9} - \frac{1}{162}(-4x^2-9)^{3/2}/x^2 + \frac{1}{36}(-4x^2-9)^{3/2}/x^4 - \frac{2}{27}I \log\left(\frac{6\sqrt{4x^2+9}}{\text{abs}(x)} + \frac{18}{\text{abs}(x)}\right)$

**Fricas [A]** time = 0.238835, size = 97, normalized size = 1.7

$$\frac{-4ix^4 \log\left(-\frac{4(i\sqrt{-4x^2-9+3})}{27x}\right) + 4ix^4 \log\left(-\frac{4(-i\sqrt{-4x^2-9+3})}{27x}\right) - 3(2x^2+9)\sqrt{-4x^2-9}}{108x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)/x^5,x, algorithm="fricas")`

[Out]  $\frac{1}{108}(-4I^2x^4 \log(-4/27(I\sqrt{-4x^2-9}+3)/x) + 4I^2x^4 \log(-4/27(-I\sqrt{-4x^2-9}+3)/x) - 3(2x^2+9)\sqrt{-4x^2-9})/x^4$

**Sympy [A]** time = 12.5321, size = 68, normalized size = 1.19

$$\frac{2i \operatorname{asinh}\left(\frac{3}{2x}\right)}{27} - \frac{i}{9x\sqrt{1+\frac{9}{4x^2}}} - \frac{3i}{4x^3\sqrt{1+\frac{9}{4x^2}}} - \frac{9i}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-4*x**2-9)**(1/2)/x**5,x)`

[Out]  $2I \operatorname{asinh}(3/(2x))/27 - I/(9x\sqrt{1+9/(4x^2)}) - 3I/(4x^3\sqrt{1+9/(4x^2)}) - 9I/(8x^5\sqrt{1+9/(4x^2)})$

**GIAC/XCAS [A]** time = 0.203618, size = 61, normalized size = 1.07

$$-\frac{(4x^2+9)^{3/2}i + 9\sqrt{4x^2+9}i}{72x^4} + \frac{2}{27}\arctan\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-4*x^2 - 9)/x^5,x, algorithm="giac")`

```
[Out] -1/72*((4*x^2 + 9)^(3/2)*i + 9*sqrt(4*x^2 + 9)*i)/x^4 + 2/27*arctan(1/3*sqrt(4*x^2 + 9)*i)
```

$$3.485 \quad \int \frac{x^5}{\sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=56

$$\frac{a^2\sqrt{a+bx^2}}{b^3} + \frac{(a+bx^2)^{5/2}}{5b^3} - \frac{2a(a+bx^2)^{3/2}}{3b^3}$$

[Out] (a^2\*Sqrt[a + b\*x^2])/b^3 - (2\*a\*(a + b\*x^2)^(3/2))/(3\*b^3) + (a + b\*x^2)^(5/2)/(5\*b^3)

**Rubi [A]** time = 0.0927026, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{a^2\sqrt{a+bx^2}}{b^3} + \frac{(a+bx^2)^{5/2}}{5b^3} - \frac{2a(a+bx^2)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5/Sqrt[a + b\*x^2], x]

[Out] (a^2\*Sqrt[a + b\*x^2])/b^3 - (2\*a\*(a + b\*x^2)^(3/2))/(3\*b^3) + (a + b\*x^2)^(5/2)/(5\*b^3)

**Rubi in Sympy [A]** time = 11.7623, size = 49, normalized size = 0.88

$$\frac{a^2\sqrt{a+bx^2}}{b^3} - \frac{2a(a+bx^2)^{\frac{3}{2}}}{3b^3} + \frac{(a+bx^2)^{\frac{5}{2}}}{5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5/(b\*x\*\*2+a)\*\*(1/2), x)

[Out] a\*\*2\*sqrt(a + b\*x\*\*2)/b\*\*3 - 2\*a\*(a + b\*x\*\*2)\*\*(3/2)/(3\*b\*\*3) + (a + b\*x\*\*2)\*\*(5/2)/(5\*b\*\*3)

**Mathematica [A]** time = 0.0268568, size = 39, normalized size = 0.7

$$\frac{\sqrt{a+bx^2}(8a^2-4abx^2+3b^2x^4)}{15b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/Sqrt[a + b\*x^2], x]

[Out] (Sqrt[a + b\*x^2]\*(8\*a^2 - 4\*a\*b\*x^2 + 3\*b^2\*x^4))/(15\*b^3)

**Maple [A]** time = 0.008, size = 36, normalized size = 0.6

$$\frac{3b^2x^4 - 4abx^2 + 8a^2}{15b^3} \sqrt{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b\*x^2+a)^(1/2), x)

[Out]  $1/15 * (b * x^2 + a)^{(1/2)} * (3 * b^2 * x^4 - 4 * a * b * x^2 + 8 * a^2) / b^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/sqrt(b*x^2 + a), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.234902, size = 47, normalized size = 0.84

$$\frac{(3b^2x^4 - 4abx^2 + 8a^2)\sqrt{bx^2 + a}}{15b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/sqrt(b*x^2 + a), x, algorithm="fricas")`

[Out]  $1/15 * (3 * b^2 * x^4 - 4 * a * b * x^2 + 8 * a^2) * \text{sqrt}(b * x^2 + a) / b^3$

**Sympy [A]** time = 2.67692, size = 68, normalized size = 1.21

$$\begin{cases} \frac{8a^2\sqrt{a+bx^2}}{15b^3} - \frac{4ax^2\sqrt{a+bx^2}}{15b^2} + \frac{x^4\sqrt{a+bx^2}}{5b} & \text{for } b \neq 0 \\ \frac{x^6}{6\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(b*x**2+a)**(1/2), x)`

[Out] `Piecewise((8*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*a*x**2*sqrt(a + b*x**2)/(15*b**2) + x**4*sqrt(a + b*x**2)/(5*b), Ne(b, 0)), (x**6/(6*sqrt(a)), True))`

**GIAC/XCAS [A]** time = 0.202612, size = 58, normalized size = 1.04

$$\frac{3(bx^2 + a)^{\frac{5}{2}} - 10(bx^2 + a)^{\frac{3}{2}}a + 15\sqrt{bx^2 + a}a^2}{15b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/sqrt(b*x^2 + a), x, algorithm="giac")`

[Out]  $1/15 * (3 * (b * x^2 + a)^{(5/2)} - 10 * (b * x^2 + a)^{(3/2)} * a + 15 * \text{sqrt}(b * x^2 + a) * a^2) / b^3$

$$3.486 \quad \int \frac{x^4}{\sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=73

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{8b^{5/2}} - \frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b}$$

[Out]  $(-3*a*x*\text{Sqrt}[a + b*x^2])/(8*b^2) + (x^3*\text{Sqrt}[a + b*x^2])/(4*b) + (3*a^2*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]])/(8*b^{(5/2)})$

**Rubi [A]** time = 0.0726057, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{3a^2 \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{8b^{5/2}} - \frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

[In] Int[x^4/Sqrt[a + b\*x^2], x]

[Out]  $(-3*a*x*\text{Sqrt}[a + b*x^2])/(8*b^2) + (x^3*\text{Sqrt}[a + b*x^2])/(4*b) + (3*a^2*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]])/(8*b^{(5/2)})$

**Rubi in Sympy [A]** time = 9.34801, size = 66, normalized size = 0.9

$$\frac{3a^2 \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{8b^{\frac{5}{2}}} - \frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(b\*x\*\*2+a)\*\*(1/2), x)

[Out]  $3*a^2*\operatorname{atanh}(\text{sqrt}(b)*x/\text{sqrt}(a + b*x^2))/(8*b^{(5/2)}) - 3*a*x*\text{sqrt}(a + b*x^2)/(8*b^2) + x^3*\text{sqrt}(a + b*x^2)/(4*b)$

**Mathematica [A]** time = 0.0546691, size = 67, normalized size = 0.92

$$\frac{3a^2 \log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right)}{8b^{5/2}} + \sqrt{a+bx^2} \left(\frac{x^3}{4b} - \frac{3ax}{8b^2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/Sqrt[a + b\*x^2], x]

[Out]  $\text{Sqrt}[a + b*x^2]*((-3*a*x)/(8*b^2) + x^3/(4*b)) + (3*a^2*\text{Log}[b*x + \text{Sqrt}[b]*\text{Sqrt}[a + b*x^2]])/(8*b^{(5/2)})$

**Maple [A]** time = 0.007, size = 59, normalized size = 0.8

$$\frac{x^3}{4b} \sqrt{bx^2 + a} - \frac{3ax}{8b^2} \sqrt{bx^2 + a} + \frac{3a^2}{8} \ln\left(x\sqrt{b} + \sqrt{bx^2 + a}\right) b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(b*x^2+a)^(1/2),x)`

[Out]  $\frac{1}{4}x^3(bx^2+a)^{1/2}/b - \frac{3}{8}ax(bx^2+a)^{1/2}/b^2 + \frac{3}{8}a^2/b^{5/2} \ln(xb^{1/2} + (bx^2+a)^{1/2})$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/sqrt(b*x^2 + a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.247362, size = 1, normalized size = 0.01

$$\left[ \frac{3a^2 \log\left(-2\sqrt{bx^2+ax} - (2bx^2+a)\sqrt{b}\right) + 2(2bx^3-3ax)\sqrt{bx^2+a}\sqrt{b}}{16b^{5/2}}, \frac{3a^2 \arctan\left(\frac{\sqrt{-bx}}{\sqrt{bx^2+a}}\right) + (2bx^3-3ax)\sqrt{bx^2+a}}{8\sqrt{-bb^2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/sqrt(b*x^2 + a),x, algorithm="fricas")`

[Out]  $\left[ \frac{1}{16}(3a^2 \log(-2\sqrt{bx^2+ax})b^2x - (2bx^2+a)\sqrt{bx^2+ax}) + 2(2bx^3-3ax)\sqrt{bx^2+ax}\sqrt{b} \right] / b^{5/2}, \frac{1}{8}(3a^2 \arctan(\sqrt{-b}x/\sqrt{bx^2+a}) + (2bx^3-3ax)\sqrt{bx^2+ax}) / (\sqrt{-b}b^2)$

**Sympy** [A] time = 12.8538, size = 95, normalized size = 1.3

$$-\frac{3a^{3/2}x}{8b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{\sqrt{ax}^3}{8b\sqrt{1+\frac{bx^2}{a}}} + \frac{3a^2 \operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{8b^{5/2}} + \frac{x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(b*x**2+a)**(1/2),x)`

[Out]  $-3a^{3/2}x/(8b^2\sqrt{1+bx^2/a}) - \sqrt{a}x^3/(8b\sqrt{1+bx^2/a}) + 3a^2 \operatorname{asinh}(\sqrt{b}x/\sqrt{a})/(8b^{5/2}) + x^5/(4\sqrt{a}\sqrt{1+bx^2/a})$

**GIAC/XCAS** [A] time = 0.20828, size = 73, normalized size = 1.

$$\frac{1}{8}\sqrt{bx^2+ax}\left(\frac{2x^2}{b} - \frac{3a}{b^2}\right) - \frac{3a^2 \ln\left(\left|-\sqrt{bx} + \sqrt{bx^2+a}\right|\right)}{8b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*x*(2*x^2/b - 3*a/b^2) - 3/8*a^2*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```



$$3.487 \quad \int \frac{x^3}{\sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=36

$$\frac{(a+bx^2)^{3/2}}{3b^2} - \frac{a\sqrt{a+bx^2}}{b^2}$$

[Out]  $-\left(\frac{a\sqrt{a+bx^2}}{b^2}\right) + \frac{(a+bx^2)^{3/2}}{3b^2}$

**Rubi [A]** time = 0.0669123, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{(a+bx^2)^{3/2}}{3b^2} - \frac{a\sqrt{a+bx^2}}{b^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[a + b\*x^2], x]

[Out]  $-\left(\frac{a\sqrt{a+bx^2}}{b^2}\right) + \frac{(a+bx^2)^{3/2}}{3b^2}$

**Rubi in Sympy [A]** time = 7.92724, size = 29, normalized size = 0.81

$$-\frac{a\sqrt{a+bx^2}}{b^2} + \frac{(a+bx^2)^{3/2}}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(b\*x\*\*2+a)\*\*(1/2), x)

[Out]  $-a\sqrt{a+bx^2}/b^2 + (a+bx^2)^{3/2}/(3b^2)$

**Mathematica [A]** time = 0.0199465, size = 27, normalized size = 0.75

$$\frac{(bx^2 - 2a)\sqrt{a+bx^2}}{3b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[a + b\*x^2], x]

[Out]  $\frac{(-2a + bx^2)\sqrt{a+bx^2}}{3b^2}$

**Maple [A]** time = 0.006, size = 25, normalized size = 0.7

$$-\frac{-bx^2 + 2a}{3b^2}\sqrt{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b\*x^2+a)^(1/2), x)

[Out]  $-1/3 * (b * x^2 + a)^{(1/2)} * (-b * x^2 + 2 * a) / b^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/sqrt(b*x^2 + a), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.226963, size = 31, normalized size = 0.86

$$\frac{\sqrt{bx^2 + a}(bx^2 - 2a)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/sqrt(b*x^2 + a), x, algorithm="fricas")`

[Out]  $1/3 * \sqrt{b * x^2 + a} * (b * x^2 - 2 * a) / b^2$

**Sympy [A]** time = 1.74897, size = 44, normalized size = 1.22

$$\begin{cases} -\frac{2a\sqrt{a+bx^2}}{3b^2} + \frac{x^2\sqrt{a+bx^2}}{3b} & \text{for } b \neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(b*x**2+a)**(1/2), x)`

[Out] `Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b), Ne(b, 0)), (x**4/(4*sqrt(a)), True))`

**GIAC/XCAS [A]** time = 0.200588, size = 36, normalized size = 1.

$$\frac{(bx^2 + a)^{\frac{3}{2}} - 3\sqrt{bx^2 + a}a}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/sqrt(b*x^2 + a), x, algorithm="giac")`

[Out]  $1/3 * ((b * x^2 + a)^{(3/2)} - 3 * \sqrt{b * x^2 + a} * a) / b^2$

$$3.488 \quad \int \frac{x^2}{\sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=49

$$\frac{x\sqrt{a+bx^2}}{2b} - \frac{a \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

[Out] (x\*sqrt[a + b\*x^2])/(2\*b) - (a\*ArcTanh[(sqrt[b]\*x)/sqrt[a + b\*x^2]])/(2\*b^(3/2))

**Rubi [A]** time = 0.0449893, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{x\sqrt{a+bx^2}}{2b} - \frac{a \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[a + b\*x^2], x]

[Out] (x\*sqrt[a + b\*x^2])/(2\*b) - (a\*ArcTanh[(sqrt[b]\*x)/sqrt[a + b\*x^2]])/(2\*b^(3/2))

**Rubi in Sympy [A]** time = 5.8361, size = 41, normalized size = 0.84

$$-\frac{a \operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}} + \frac{x\sqrt{a+bx^2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(b\*x\*\*2+a)\*\*(1/2), x)

[Out] -a\*atanh(sqrt(b)\*x/sqrt(a + b\*x\*\*2))/(2\*b\*\*(3/2)) + x\*sqrt(a + b\*x\*\*2)/(2\*b)

**Mathematica [A]** time = 0.0289751, size = 52, normalized size = 1.06

$$\frac{x\sqrt{a+bx^2}}{2b} - \frac{a \log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[a + b\*x^2], x]

[Out] (x\*sqrt[a + b\*x^2])/(2\*b) - (a\*Log[b\*x + sqrt[b]\*sqrt[a + b\*x^2]])/(2\*b^(3/2))

**Maple [A]** time = 0.008, size = 39, normalized size = 0.8

$$\frac{x}{2b} \sqrt{bx^2 + a} - \frac{a}{2} \ln\left(x\sqrt{b} + \sqrt{bx^2 + a}\right) b^{-3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*x^2+a)^(1/2),x)`

[Out] `1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/sqrt(b*x^2 + a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.235486, size = 1, normalized size = 0.02

$$\left[ \frac{2\sqrt{bx^2+a}\sqrt{bx} + a \log\left(2\sqrt{bx^2+abx} - (2bx^2+a)\sqrt{b}\right)}{4b^{\frac{3}{2}}}, \frac{\sqrt{bx^2+a}\sqrt{-bx} - a \arctan\left(\frac{\sqrt{-bx}}{\sqrt{bx^2+a}}\right)}{2\sqrt{-bb}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/sqrt(b*x^2 + a),x, algorithm="fricas")`

[Out] `[1/4*(2*sqrt(b*x^2 + a)*sqrt(b)*x + a*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/b^(3/2), 1/2*(sqrt(b*x^2 + a)*sqrt(-b)*x - a*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/(sqrt(-b)*b)]`

**Sympy [A]** time = 7.30377, size = 42, normalized size = 0.86

$$\frac{\sqrt{ax}\sqrt{1+\frac{bx^2}{a}}}{2b} - \frac{a \operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**2+a)**(1/2),x)`

[Out] `sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2))`

**GIAC/XCAS [A]** time = 0.209541, size = 54, normalized size = 1.1

$$\frac{\sqrt{bx^2+ax}}{2b} + \frac{a \ln\left(\left|-\sqrt{bx} + \sqrt{bx^2+a}\right|\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/sqrt(b*x^2 + a),x, algorithm="giac")`

[Out] `1/2*sqrt(b*x^2 + a)*x/b + 1/2*a*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)`

$$3.489 \quad \int \frac{x}{\sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=15

$$\frac{\sqrt{a+bx^2}}{b}$$

[Out] Sqrt[a + b\*x^2]/b

**Rubi [A]** time = 0.012049, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a + b\*x^2], x]

[Out] Sqrt[a + b\*x^2]/b

**Rubi in Sympy [A]** time = 2.18028, size = 10, normalized size = 0.67

$$\frac{\sqrt{a+bx^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(b\*x\*\*2+a)\*\*(1/2), x)

[Out] sqrt(a + b\*x\*\*2)/b

**Mathematica [A]** time = 0.00373068, size = 15, normalized size = 1.

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[a + b\*x^2], x]

[Out] Sqrt[a + b\*x^2]/b

**Maple [A]** time = 0.004, size = 14, normalized size = 0.9

$$\frac{1}{b} \sqrt{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b\*x^2+a)^(1/2), x)

[Out] (b\*x^2+a)^(1/2)/b

---

**Maxima [A]** time = 1.35436, size = 18, normalized size = 1.2

$$\frac{\sqrt{bx^2 + a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sqrt(b\*x^2 + a),x, algorithm="maxima")

[Out] sqrt(b\*x^2 + a)/b

---

**Fricas [A]** time = 0.228136, size = 18, normalized size = 1.2

$$\frac{\sqrt{bx^2 + a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sqrt(b\*x^2 + a),x, algorithm="fricas")

[Out] sqrt(b\*x^2 + a)/b

---

**Sympy [A]** time = 1.45599, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sqrt{a+bx^2}}{b} & \text{for } b \neq 0 \\ \frac{x^2}{2\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b\*x\*\*2+a)\*\*(1/2),x)

[Out] Piecewise((sqrt(a + b\*x\*\*2)/b, Ne(b, 0)), (x\*\*2/(2\*sqrt(a)), True))

---

**GIAC/XCAS [A]** time = 0.201714, size = 18, normalized size = 1.2

$$\frac{\sqrt{bx^2 + a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sqrt(b\*x^2 + a),x, algorithm="giac")

[Out] sqrt(b\*x^2 + a)/b

$$3.490 \quad \int \frac{1}{\sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

[Out] ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]]/Sqrt[b]

**Rubi [A]** time = 0.0183677, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b\*x^2], x]

[Out] ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]]/Sqrt[b]

**Rubi in Sympy [A]** time = 2.42764, size = 22, normalized size = 0.88

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+a)\*\*(1/2), x)

[Out] atanh(sqrt(b)\*x/sqrt(a + b\*x\*\*2))/sqrt(b)

**Mathematica [A]** time = 0.0105178, size = 25, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b\*x^2], x]

[Out] ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]]/Sqrt[b]

**Maple [A]** time = 0.004, size = 21, normalized size = 0.8

$$\frac{1}{\sqrt{b}} \ln\left(x\sqrt{b} + \sqrt{bx^2 + a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+a)^(1/2), x)

[Out]  $\ln(x \cdot b^{1/2} + (b \cdot x^2 + a)^{1/2}) / b^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(b*x^2 + a), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.246045, size = 1, normalized size = 0.04

$$\left[ \frac{\log\left(-2\sqrt{bx^2+ax} - (2bx^2+a)\sqrt{b}\right)}{2\sqrt{b}}, \frac{\arctan\left(\frac{\sqrt{-bx}}{\sqrt{bx^2+a}}\right)}{\sqrt{-b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(b*x^2 + a), x, algorithm="fricas")`

[Out]  $[1/2 \cdot \log(-2 \cdot \sqrt{bx^2 + ax} - (2bx^2 + a) \cdot \sqrt{b}) / \sqrt{b}, \arctan(\sqrt{-b} \cdot x / \sqrt{bx^2 + a}) / \sqrt{-b}]$

**Sympy [A]** time = 3.5889, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**2+a)**(1/2), x)`

[Out]  $\operatorname{asinh}(\sqrt{b} \cdot x / \sqrt{a}) / \sqrt{b}$

**GIAC/XCAS [A]** time = 0.20912, size = 31, normalized size = 1.24

$$-\frac{\ln\left(\left|-\sqrt{bx} + \sqrt{bx^2 + a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(b*x^2 + a), x, algorithm="giac")`

[Out]  $-\ln(\operatorname{abs}(-\sqrt{b} \cdot x + \sqrt{bx^2 + a})) / \sqrt{b}$



$$3.491 \quad \int \frac{1}{x\sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=25

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

[Out] -(ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]]/Sqrt[a])

**Rubi [A]** time = 0.0529997, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Sqrt[a + b\*x^2]), x]

[Out] -(ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]]/Sqrt[a])

**Rubi in Sympy [A]** time = 5.79395, size = 22, normalized size = 0.88

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(b\*x\*\*2+a)\*\*(1/2), x)

[Out] -atanh(sqrt(a + b\*x\*\*2)/sqrt(a))/sqrt(a)

**Mathematica [A]** time = 0.0283959, size = 31, normalized size = 1.24

$$\frac{\log(x) - \log\left(\sqrt{a}\sqrt{a+bx^2} + a\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Sqrt[a + b\*x^2]), x]

[Out] (Log[x] - Log[a + Sqrt[a]\*Sqrt[a + b\*x^2]])/Sqrt[a]

**Maple [A]** time = 0.006, size = 29, normalized size = 1.2

$$-1 \ln\left(\frac{1}{x}\left(2a + 2\sqrt{a}\sqrt{bx^2 + a}\right)\right) \frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b\*x^2+a)^(1/2), x)

[Out]  $-1/a^{(1/2)} * \ln((2*a+2*a^{(1/2)} * (b*x^2+a)^{(1/2)})/x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^2 + a)*x), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.232149, size = 1, normalized size = 0.04

$$\left[ \frac{\log\left(-\frac{(bx^2+2a)\sqrt{a}-2\sqrt{bx^2+aa}}{x^2}\right)}{2\sqrt{a}}, -\frac{\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{\sqrt{-a}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^2 + a)*x), x, algorithm="fricas")`

[Out]  $[1/2 * \log(-((b*x^2 + 2*a)*\sqrt{a} - 2*\sqrt{b*x^2 + a}*a)/x^2)/\sqrt{a}, -\arctan(\sqrt{-a}/\sqrt{b*x^2 + a})/\sqrt{-a}]$

**Sympy [A]** time = 3.66943, size = 19, normalized size = 0.76

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x**2+a)**(1/2), x)`

[Out]  $-\operatorname{asinh}(\sqrt{a}/(\sqrt{b}*x))/\sqrt{a}$

**GIAC/XCAS [A]** time = 0.203612, size = 30, normalized size = 1.2

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^2 + a)*x), x, algorithm="giac")`

[Out]  $\arctan(\sqrt{b*x^2 + a}/\sqrt{-a})/\sqrt{-a}$

$$3.492 \quad \int \frac{1}{x^2 \sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=19

$$-\frac{\sqrt{a+bx^2}}{ax}$$

[Out] -(Sqrt[a + b\*x^2]/(a\*x))

**Rubi [A]** time = 0.0227713, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*Sqrt[a + b\*x^2]), x]

[Out] -(Sqrt[a + b\*x^2]/(a\*x))

**Rubi in Sympy [A]** time = 3.21968, size = 14, normalized size = 0.74

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(b\*x\*\*2+a)\*\*(1/2), x)

[Out] -sqrt(a + b\*x\*\*2)/(a\*x)

**Mathematica [A]** time = 0.015603, size = 19, normalized size = 1.

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*Sqrt[a + b\*x^2]), x]

[Out] -(Sqrt[a + b\*x^2]/(a\*x))

**Maple [A]** time = 0.005, size = 18, normalized size = 1.

$$-\frac{1}{ax} \sqrt{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b\*x^2+a)^(1/2), x)

[Out] -(b\*x^2+a)^(1/2)/a/x

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^2 + a)*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.22212, size = 23, normalized size = 1.21

$$-\frac{\sqrt{bx^2 + a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^2 + a)*x^2),x, algorithm="fricas")`

[Out] `-sqrt(b*x^2 + a)/(a*x)`

---

**Sympy [A]** time = 1.86294, size = 19, normalized size = 1.

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2} + 1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(b*x**2+a)**(1/2),x)`

[Out] `-sqrt(b)*sqrt(a/(b*x**2) + 1)/a`

---

**GIAC/XCAS [A]** time = 0.207664, size = 41, normalized size = 2.16

$$\frac{2\sqrt{b}}{(\sqrt{bx} - \sqrt{bx^2 + a})^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^2 + a)*x^2),x, algorithm="giac")`

[Out] `2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)`

$$3.493 \quad \int \frac{1}{x^3 \sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=50

$$\frac{b \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{\sqrt{a+bx^2}}{2ax^2}$$

[Out]  $-\text{Sqrt}[a + b*x^2]/(2*a*x^2) + (b*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(2*a^{(3/2)})$

**Rubi [A]** time = 0.0801228, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{b \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^3*\text{Sqrt}[a + b*x^2]), x]$

[Out]  $-\text{Sqrt}[a + b*x^2]/(2*a*x^2) + (b*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(2*a^{(3/2)})$

**Rubi in Sympy [A]** time = 7.8013, size = 41, normalized size = 0.82

$$-\frac{\sqrt{a+bx^2}}{2ax^2} + \frac{b \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{**3}/(b*x^{**2}+a)^{(1/2)}, x)$

[Out]  $-\text{sqrt}(a + b*x^{**2})/(2*a*x^{**2}) + b*\text{atanh}(\text{sqrt}(a + b*x^{**2})/\text{sqrt}(a))/(2*a^{(3/2)})$

**Mathematica [A]** time = 0.0380223, size = 64, normalized size = 1.28

$$\frac{b \log\left(\sqrt{a}\sqrt{a+bx^2} + a\right)}{2a^{3/2}} - \frac{b \log(x)}{2a^{3/2}} - \frac{\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/(x^3*\text{Sqrt}[a + b*x^2]), x]$

[Out]  $-\text{Sqrt}[a + b*x^2]/(2*a*x^2) - (b*\text{Log}[x])/(2*a^{(3/2)}) + (b*\text{Log}[a + \text{Sqrt}[a]*\text{Sqrt}[a + b*x^2]])/(2*a^{(3/2)})$

**Maple [A]** time = 0.009, size = 48, normalized size = 1.

$$-\frac{1}{2ax^2}\sqrt{bx^2+a} + \frac{b}{2}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{bx^2+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(b*x^2+a)^(1/2),x)`

[Out]  $-1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^(3/2)*\ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^2 + a)*x^3),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.241474, size = 1, normalized size = 0.02

$$\left[ \frac{bx^2 \log\left(-\frac{(bx^2+2a)\sqrt{a+2\sqrt{bx^2+aa}}}{x^2}\right) - 2\sqrt{bx^2+a}\sqrt{a}}{4a^{\frac{3}{2}}x^2}, \frac{bx^2 \arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) - \sqrt{bx^2+a}\sqrt{-a}}{2\sqrt{-aa}x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^2 + a)*x^3),x, algorithm="fricas")`

[Out]  $[1/4*(b*x^2*\log(-((b*x^2 + 2*a)*\sqrt{a} + 2*\sqrt{b*x^2 + a}*a)/x^2) - 2*\sqrt{b*x^2 + a}*\sqrt{a})/(a^(3/2)*x^2), 1/2*(b*x^2*\arctan(\sqrt{-a}/\sqrt{b*x^2 + a}) - \sqrt{b*x^2 + a}*\sqrt{-a})/(a*x^2)]$

**Sympy [A]** time = 7.67278, size = 42, normalized size = 0.84

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2} + 1}}{2ax} + \frac{b \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(b*x**2+a)**(1/2),x)`

[Out]  $-\sqrt{b}*\sqrt{a/(b*x**2) + 1}/(2*a*x) + b*\operatorname{asinh}(\sqrt{a}/(\sqrt{b}*x))/(2*a**(3/2))$

**GIAC/XCAS [A]** time = 0.203229, size = 65, normalized size = 1.3

$$-\frac{1}{2}b\left(\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-aa}} + \frac{\sqrt{bx^2+a}}{abx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^2 + a)*x^3),x, algorithm="giac")`

[Out]  $-1/2*b*(\arctan(\sqrt{b*x^2 + a}/\sqrt{-a})/(\sqrt{-a}*a) + \sqrt{b*x^2 + a}/(a*b*x^2))$

$$3.494 \quad \int \frac{1}{x^4 \sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=44

$$\frac{2b\sqrt{a+bx^2}}{3a^2x} - \frac{\sqrt{a+bx^2}}{3ax^3}$$

[Out]  $-\text{Sqrt}[a + b*x^2]/(3*a*x^3) + (2*b*\text{Sqrt}[a + b*x^2])/(3*a^2*x)$

**Rubi [A]** time = 0.0452485, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{2b\sqrt{a+bx^2}}{3a^2x} - \frac{\sqrt{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^4*\text{Sqrt}[a + b*x^2]), x]$

[Out]  $-\text{Sqrt}[a + b*x^2]/(3*a*x^3) + (2*b*\text{Sqrt}[a + b*x^2])/(3*a^2*x)$

**Rubi in Sympy [A]** time = 5.30919, size = 36, normalized size = 0.82

$$-\frac{\sqrt{a+bx^2}}{3ax^3} + \frac{2b\sqrt{a+bx^2}}{3a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{**4}/(b*x^{**2}+a)^{(1/2)}, x)$

[Out]  $-\text{sqrt}(a + b*x^{**2})/(3*a*x^{**3}) + 2*b*\text{sqrt}(a + b*x^{**2})/(3*a^{**2}*x)$

**Mathematica [A]** time = 0.021124, size = 29, normalized size = 0.66

$$-\frac{(a - 2bx^2) \sqrt{a+bx^2}}{3a^2x^3}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/(x^4*\text{Sqrt}[a + b*x^2]), x]$

[Out]  $-((a - 2*b*x^2)*\text{Sqrt}[a + b*x^2])/(3*a^2*x^3)$

**Maple [A]** time = 0.005, size = 26, normalized size = 0.6

$$-\frac{-2bx^2 + a}{3a^2x^3} \sqrt{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/x^4/(b*x^2+a)^{(1/2)}, x)$

[Out]  $-1/3*(b*x^2+a)^{(1/2)}*(-2*b*x^2+a)/a^2/x^3$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^2 + a)*x^4),x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.223408, size = 36, normalized size = 0.82

$$\frac{(2bx^2 - a)\sqrt{bx^2 + a}}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^2 + a)*x^4),x, algorithm="fricas")`

[Out]  $1/3*(2*b*x^2 - a)*\sqrt{b*x^2 + a}/(a^2*x^3)$

---

**Sympy [A]** time = 2.5783, size = 46, normalized size = 1.05

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2} + 1}}{3ax^2} + \frac{2b^{3/2}\sqrt{\frac{a}{bx^2} + 1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(b*x**2+a)**(1/2),x)`

[Out]  $-\sqrt{b}*\sqrt{a/(b*x**2) + 1}/(3*a*x**2) + 2*b**(3/2)*\sqrt{a/(b*x**2) + 1}/(3*a**2)$

---

**GIAC/XCAS [A]** time = 0.210274, size = 74, normalized size = 1.68

$$\frac{4\left(3\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^2 - a\right)b^{3/2}}{3\left(\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^2 - a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^2 + a)*x^4),x, algorithm="giac")`

[Out]  $4/3*(3*(\sqrt{b}*x - \sqrt{b*x^2 + a})^2 - a)*b^{3/2}/((\sqrt{b}*x - \sqrt{b*x^2 + a})^2 - a)^3$



$$3.495 \quad \int \frac{1}{x^5 \sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=74

$$-\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}} + \frac{3b\sqrt{a+bx^2}}{8a^2x^2} - \frac{\sqrt{a+bx^2}}{4ax^4}$$

[Out]  $-\text{Sqrt}[a + b*x^2]/(4*a*x^4) + (3*b*\text{Sqrt}[a + b*x^2])/(8*a^2*x^2) - (3*b^2*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(8*a^{(5/2)})$

**Rubi [A]** time = 0.113404, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}} + \frac{3b\sqrt{a+bx^2}}{8a^2x^2} - \frac{\sqrt{a+bx^2}}{4ax^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^5*\text{Sqrt}[a + b*x^2]), x]$

[Out]  $-\text{Sqrt}[a + b*x^2]/(4*a*x^4) + (3*b*\text{Sqrt}[a + b*x^2])/(8*a^2*x^2) - (3*b^2*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(8*a^{(5/2)})$

**Rubi in Sympy [A]** time = 10.7609, size = 66, normalized size = 0.89

$$-\frac{\sqrt{a+bx^2}}{4ax^4} + \frac{3b\sqrt{a+bx^2}}{8a^2x^2} - \frac{3b^2 \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{**5}/(b*x^{**2}+a)^{(1/2}), x)$

[Out]  $-\text{sqrt}(a + b*x^{**2})/(4*a*x^{**4}) + 3*b*\text{sqrt}(a + b*x^{**2})/(8*a^{**2}*x^{**2}) - 3*b^{**2}*\text{atanh}(\text{sqrt}(a + b*x^{**2})/\text{sqrt}(a))/(8*a^{**}(5/2))$

**Mathematica [A]** time = 0.0647646, size = 78, normalized size = 1.05

$$\frac{-3b^2x^4 \log\left(\sqrt{a}\sqrt{a+bx^2} + a\right) + \sqrt{a}\sqrt{a+bx^2}(3bx^2 - 2a) + 3b^2x^4 \log(x)}{8a^{5/2}x^4}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/(x^5*\text{Sqrt}[a + b*x^2]), x]$

[Out]  $(\text{Sqrt}[a]*\text{Sqrt}[a + b*x^2]*(-2*a + 3*b*x^2) + 3*b^2*x^4*\text{Log}[x] - 3*b^2*x^4*\text{Log}[a + \text{Sqrt}[a]*\text{Sqrt}[a + b*x^2]])/(8*a^{(5/2)}*x^4)$

**Maple [A]** time = 0.01, size = 68, normalized size = 0.9

$$-\frac{1}{4ax^4}\sqrt{bx^2+a} + \frac{3b}{8a^2x^2}\sqrt{bx^2+a} - \frac{3b^2}{8}\ln\left(\frac{1}{x}(2a+2\sqrt{a}\sqrt{bx^2+a})\right)a^{-5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^5/(b*x^2+a)^(1/2),x)`

[Out]  $-1/4*(b*x^2+a)^(1/2)/a/x^4+3/8*b*(b*x^2+a)^(1/2)/a^2/x^2-3/8*b^2/a^(5/2)*\ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^2 + a)*x^5),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.241923, size = 1, normalized size = 0.01

$$\left[ \frac{3 b^2 x^4 \log\left(-\frac{(b x^2+2 a) \sqrt{a-2 \sqrt{b x^2+a a}}}{x^2}\right)+2\left(3 b x^2-2 a\right) \sqrt{b x^2+a} \sqrt{a}}{16 a^{\frac{5}{2}} x^4}, \frac{3 b^2 x^4 \arctan\left(\frac{\sqrt{-a}}{\sqrt{b x^2+a}}\right)-\left(3 b x^2-2 a\right) \sqrt{b x^2+a} \sqrt{-a}}{8 \sqrt{-a} a^2 x^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^2 + a)*x^5),x, algorithm="fricas")`

[Out]  $[1/16*(3*b^2*x^4*\log(-((b*x^2 + 2*a)*\sqrt{a} - 2*\sqrt{b*x^2 + a})*a)/x^2) + 2*(3*b*x^2 - 2*a)*\sqrt{b*x^2 + a}*\sqrt{a})/(a^(5/2)*x^4), -1/8*(3*b^2*x^4*\arctan(\sqrt{-a}/\sqrt{b*x^2 + a}) - (3*b*x^2 - 2*a)*\sqrt{b*x^2 + a}*\sqrt{-a})/(sqrt{-a}*a^2*x^4)]$

**Sympy [A]** time = 13.6364, size = 97, normalized size = 1.31

$$-\frac{1}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{\sqrt{b}}{8ax^3\sqrt{\frac{a}{bx^2}+1}}+\frac{3b^{\frac{3}{2}}}{8a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(b*x**2+a)**(1/2),x)`

[Out]  $-1/(4*\sqrt{b}*x**5*\sqrt{a/(b*x**2) + 1}) + \sqrt{b}/(8*a*x**3*\sqrt{a/(b*x**2) + 1}) + 3*b**(3/2)/(8*a**2*x*\sqrt{a/(b*x**2) + 1}) - 3*b**2*asinh(\sqrt{a}/(\sqrt{b}*x))/(8*a**(5/2))$

**GIAC/XCAS [A]** time = 0.20579, size = 89, normalized size = 1.2

$$\frac{1}{8}b^2\left(\frac{3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3(bx^2+a)^{\frac{3}{2}}-5\sqrt{bx^2+aa}}{a^2b^2x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^2 + a)*x^5),x, algorithm="giac")
```

```
[Out] 1/8*b^2*(3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(  
b*x^2 + a)^(3/2) - 5*sqrt(b*x^2 + a)*a)/(a^2*b^2*x^4))
```

$$3.496 \quad \int \frac{x^5}{(a+bx^2)^{3/2}} dx$$

**Optimal.** Leaf size=55

$$-\frac{a^2}{b^3\sqrt{a+bx^2}} - \frac{2a\sqrt{a+bx^2}}{b^3} + \frac{(a+bx^2)^{3/2}}{3b^3}$$

[Out]  $-(a^2/(b^3*\text{Sqrt}[a + b*x^2])) - (2*a*\text{Sqrt}[a + b*x^2])/b^3 + (a + b*x^2)^{(3/2)/(3*b^3)}$

**Rubi [A]** time = 0.0906742, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{a^2}{b^3\sqrt{a+bx^2}} - \frac{2a\sqrt{a+bx^2}}{b^3} + \frac{(a+bx^2)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a + b\*x^2)^(3/2), x]

[Out]  $-(a^2/(b^3*\text{Sqrt}[a + b*x^2])) - (2*a*\text{Sqrt}[a + b*x^2])/b^3 + (a + b*x^2)^{(3/2)/(3*b^3)}$

**Rubi in Sympy [A]** time = 11.6797, size = 48, normalized size = 0.87

$$-\frac{a^2}{b^3\sqrt{a+bx^2}} - \frac{2a\sqrt{a+bx^2}}{b^3} + \frac{(a+bx^2)^{\frac{3}{2}}}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5/(b\*x\*\*2+a)\*\*(3/2), x)

[Out]  $-a**2/(b**3*\text{sqrt}(a + b*x**2)) - 2*a*\text{sqrt}(a + b*x**2)/b**3 + (a + b*x**2)**(3/2)/(3*b**3)$

**Mathematica [A]** time = 0.0281339, size = 38, normalized size = 0.69

$$\frac{-8a^2 - 4abx^2 + b^2x^4}{3b^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a + b\*x^2)^(3/2), x]

[Out]  $(-8*a^2 - 4*a*b*x^2 + b^2*x^4)/(3*b^3*\text{Sqrt}[a + b*x^2])$

**Maple [A]** time = 0.006, size = 36, normalized size = 0.7

$$-\frac{-b^2x^4 + 4abx^2 + 8a^2}{3b^3} \frac{1}{\sqrt{bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(b*x^2+a)^(3/2),x)`

[Out]  $-1/3*(-b^2*x^4+4*a*b*x^2+8*a^2)/(b*x^2+a)^(1/2)/b^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2 + a)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.226775, size = 62, normalized size = 1.13

$$\frac{(b^2x^4 - 4abx^2 - 8a^2)\sqrt{bx^2 + a}}{3(b^4x^2 + ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2 + a)^(3/2),x, algorithm="fricas")`

[Out]  $1/3*(b^2*x^4 - 4*a*b*x^2 - 8*a^2)*\text{sqrt}(b*x^2 + a)/(b^4*x^2 + a*b^3)$

**Sympy [A]** time = 3.01689, size = 68, normalized size = 1.24

$$\begin{cases} -\frac{8a^2}{3b^3\sqrt{a+bx^2}} - \frac{4ax^2}{3b^2\sqrt{a+bx^2}} + \frac{x^4}{3b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(b*x**2+a)**(3/2),x)`

[Out] `Piecewise((-8*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*a*x**2/(3*b**2*sqrt(a + b*x**2)) + x**4/(3*b*sqrt(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(3/2)), True))`

**GIAC/XCAS [A]** time = 0.220025, size = 55, normalized size = 1.

$$\frac{(bx^2 + a)^{\frac{3}{2}} - 6\sqrt{bx^2 + a}a - \frac{3a^2}{\sqrt{bx^2 + a}}}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2 + a)^(3/2),x, algorithm="giac")`

[Out]  $1/3*((b*x^2 + a)^(3/2) - 6*\text{sqrt}(b*x^2 + a)*a - 3*a^2/\text{sqrt}(b*x^2 + a))/b^3$

$$3.497 \quad \int \frac{x^4}{(a+bx^2)^{3/2}} dx$$

**Optimal.** Leaf size=68

$$-\frac{3a \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{2b^{5/2}} + \frac{3x\sqrt{a+bx^2}}{2b^2} - \frac{x^3}{b\sqrt{a+bx^2}}$$

[Out]  $-(x^3/(b*\text{Sqrt}[a + b*x^2])) + (3*x*\text{Sqrt}[a + b*x^2])/(2*b^2) - (3*a*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]])/(2*b^{(5/2)})$

**Rubi [A]** time = 0.0717978, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{3a \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{2b^{5/2}} + \frac{3x\sqrt{a+bx^2}}{2b^2} - \frac{x^3}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a + b\*x^2)^(3/2), x]

[Out]  $-(x^3/(b*\text{Sqrt}[a + b*x^2])) + (3*x*\text{Sqrt}[a + b*x^2])/(2*b^2) - (3*a*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]])/(2*b^{(5/2)})$

**Rubi in Sympy [A]** time = 9.42943, size = 61, normalized size = 0.9

$$-\frac{3a \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{2b^{5/2}} - \frac{x^3}{b\sqrt{a+bx^2}} + \frac{3x\sqrt{a+bx^2}}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(b\*x\*\*2+a)\*\*(3/2), x)

[Out]  $-3*a*\operatorname{atanh}(\text{sqrt}(b)*x/\text{sqrt}(a + b*x**2))/(2*b^{(5/2)}) - x**3/(b*\text{sqrt}(a + b*x**2)) + 3*x*\text{sqrt}(a + b*x**2)/(2*b**2)$

**Mathematica [A]** time = 0.0936453, size = 61, normalized size = 0.9

$$\frac{3ax + bx^3}{2b^2\sqrt{a+bx^2}} - \frac{3a \log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a + b\*x^2)^(3/2), x]

[Out]  $(3*a*x + b*x^3)/(2*b^2*\text{Sqrt}[a + b*x^2]) - (3*a*\text{Log}[b*x + \text{Sqrt}[b]*\text{Sqrt}[a + b*x^2]])/(2*b^{(5/2)})$

**Maple [A]** time = 0.009, size = 57, normalized size = 0.8

$$\frac{x^3}{2b\sqrt{bx^2+a}} + \frac{3ax}{2b^2\sqrt{bx^2+a}} - \frac{3a}{2} \ln\left(x\sqrt{b} + \sqrt{bx^2+a}\right) b^{-5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(b*x^2+a)^(3/2),x)`

[Out]  $\frac{1}{2}x^3/b/(b*x^2+a)^{(1/2)}+3/2*a/b^2*x/(b*x^2+a)^{(1/2)}-3/2*a/b^{(5/2)}*\ln(x*b^{(1/2)}+(b*x^2+a)^{(1/2)})$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.246054, size = 1, normalized size = 0.01

$$\left[ \frac{2 (bx^3 + 3ax) \sqrt{bx^2 + a} \sqrt{b} + 3 (abx^2 + a^2) \log \left( 2 \sqrt{bx^2 + a} bx - (2bx^2 + a) \sqrt{b} \right)}{4 (b^3x^2 + ab^2) \sqrt{b}}, \frac{(bx^3 + 3ax) \sqrt{bx^2 + a} \sqrt{-b} - 3 (abx^2 + a^2) \log \left( 2 \sqrt{bx^2 + a} bx - (2bx^2 + a) \sqrt{-b} \right)}{2 (b^3x^2 + ab^2) \sqrt{-b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^(3/2),x, algorithm="fricas")`

[Out]  $\left[ \frac{1}{4} * (2 * (b * x^3 + 3 * a * x) * \text{sqrt}(b * x^2 + a) * \text{sqrt}(b) + 3 * (a * b * x^2 + a^2) * \log(2 * \text{sqrt}(b * x^2 + a) * b * x - (2 * b * x^2 + a) * \text{sqrt}(b))) / ((b^3 * x^2 + a * b^2) * \text{sqrt}(b)), \frac{1}{2} * ((b * x^3 + 3 * a * x) * \text{sqrt}(b * x^2 + a) * \text{sqrt}(-b) - 3 * (a * b * x^2 + a^2) * \arctan(\text{sqrt}(-b) * x / \text{sqrt}(b * x^2 + a))) / ((b^3 * x^2 + a * b^2) * \text{sqrt}(-b)) \right]$

**Sympy** [A] time = 10.6197, size = 71, normalized size = 1.04

$$\frac{3\sqrt{ax}}{2b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a \operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{ab}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(b*x**2+a)**(3/2),x)`

[Out]  $3*\text{sqrt}(a)*x/(2*b**2*\text{sqrt}(1+b*x**2/a)) - 3*a*\text{asinh}(\text{sqrt}(b)*x/\text{sqrt}(a))/(2*b**(5/2)) + x**3/(2*\text{sqrt}(a)*b*\text{sqrt}(1+b*x**2/a))$

**GIAC/XCAS** [A] time = 0.216809, size = 69, normalized size = 1.01

$$\frac{x\left(\frac{x^2}{b} + \frac{3a}{b^2}\right)}{2\sqrt{bx^2+a}} + \frac{3\ln\left(\left|-\sqrt{bx} + \sqrt{bx^2+a}\right|\right)}{2b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*x*(x^2/b + 3*a/b^2)/sqrt(b*x^2 + a) + 3/2*a*ln(abs(-sqrt(b)*x  
+ sqrt(b*x^2 + a)))/b^(5/2)
```



$$3.498 \quad \int \frac{x^3}{(a+bx^2)^{3/2}} dx$$

Optimal. Leaf size=32

$$\frac{a}{b^2\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}}{b^2}$$

[Out] a/(b^2\*Sqrt[a + b\*x^2]) + Sqrt[a + b\*x^2]/b^2

**Rubi [A]** time = 0.0642135, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{a}{b^2\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}}{b^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b\*x^2)^(3/2), x]

[Out] a/(b^2\*Sqrt[a + b\*x^2]) + Sqrt[a + b\*x^2]/b^2

**Rubi in Sympy [A]** time = 7.85634, size = 27, normalized size = 0.84

$$\frac{a}{b^2\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(b\*x\*\*2+a)\*\*(3/2), x)

[Out] a/(b\*\*2\*sqrt(a + b\*x\*\*2)) + sqrt(a + b\*x\*\*2)/b\*\*2

**Mathematica [A]** time = 0.0209656, size = 24, normalized size = 0.75

$$\frac{2a + bx^2}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b\*x^2)^(3/2), x]

[Out] (2\*a + b\*x^2)/(b^2\*Sqrt[a + b\*x^2])

**Maple [A]** time = 0.007, size = 23, normalized size = 0.7

$$\frac{bx^2 + 2a}{b^2} \frac{1}{\sqrt{bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b\*x^2+a)^(3/2), x)

[Out]  $(b^2 x^2 + 2 a) / (b^2 x^2 + a)^{1/2} / b^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 + a)^(3/2), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.227841, size = 46, normalized size = 1.44

$$\frac{(bx^2 + 2a)\sqrt{bx^2 + a}}{b^3x^2 + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 + a)^(3/2), x, algorithm="fricas")`

[Out]  $(b^2 x^2 + 2 a) \sqrt{b^2 x^2 + a} / (b^3 x^2 + a b^2)$

**Sympy [A]** time = 2.01207, size = 41, normalized size = 1.28

$$\begin{cases} \frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{3/2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(b*x**2+a)**(3/2), x)`

[Out] `Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(3/2)), True))`

**GIAC/XCAS [A]** time = 0.209427, size = 34, normalized size = 1.06

$$\frac{\sqrt{bx^2 + a} + \frac{a}{\sqrt{bx^2 + a}}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 + a)^(3/2), x, algorithm="giac")`

[Out]  $(\sqrt{b^2 x^2 + a} + a/\sqrt{b^2 x^2 + a})/b^2$

$$3.499 \quad \int \frac{x^2}{(a+bx^2)^{3/2}} dx$$

**Optimal.** Leaf size=43

$$\frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{b^{3/2}} - \frac{x}{b\sqrt{a+bx^2}}$$

[Out]  $-(x/(b*\text{Sqrt}[a + b*x^2])) + \text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]]/b^{(3/2)}$

**Rubi [A]** time = 0.0447909, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{b^{3/2}} - \frac{x}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2/(a + b*x^2)^{(3/2)}, x]$

[Out]  $-(x/(b*\text{Sqrt}[a + b*x^2])) + \text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]]/b^{(3/2)}$

**Rubi in Sympy [A]** time = 5.79736, size = 36, normalized size = 0.84

$$-\frac{x}{b\sqrt{a+bx^2}} + \frac{\text{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**2}/(b*x^{**2}+a)^{(3/2)}, x)$

[Out]  $-x/(b*\text{sqrt}(a + b*x^{**2})) + \text{atanh}(\text{sqrt}(b)*x/\text{sqrt}(a + b*x^{**2}))/b^{(3/2)}$

**Mathematica [A]** time = 0.0362189, size = 46, normalized size = 1.07

$$\frac{\log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right)}{b^{3/2}} - \frac{x}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^2/(a + b*x^2)^{(3/2)}, x]$

[Out]  $-(x/(b*\text{Sqrt}[a + b*x^2])) + \text{Log}[b*x + \text{Sqrt}[b]*\text{Sqrt}[a + b*x^2]]/b^{(3/2)}$

**Maple [A]** time = 0.007, size = 37, normalized size = 0.9

$$-\frac{x}{b\sqrt{bx^2+a}} + 1 \ln\left(x\sqrt{b} + \sqrt{bx^2+a}\right) b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*x^2+a)^(3/2),x)`

[Out] `-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.241935, size = 1, normalized size = 0.02

$$\left[ \begin{array}{l} \frac{2\sqrt{bx^2+a}\sqrt{bx} - (bx^2+a)\log\left(-2\sqrt{bx^2+abx} - (2bx^2+a)\sqrt{b}\right)}{2(b^2x^2+ab)\sqrt{b}}, \\ -\frac{\sqrt{bx^2+a}\sqrt{-bx} - (bx^2+a)\arctan\left(\frac{\sqrt{-bx}}{\sqrt{bx^2+a}}\right)}{(b^2x^2+ab)\sqrt{-b}} \end{array} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(3/2),x, algorithm="fricas")`

[Out] `[-1/2*(2*sqrt(b*x^2 + a)*sqrt(b)*x - (b*x^2 + a)*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/((b^2*x^2 + a*b)*sqrt(b)), - (sqrt(b*x^2 + a)*sqrt(-b)*x - (b*x^2 + a)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/((b^2*x^2 + a*b)*sqrt(-b))]`

**Sympy [A]** time = 5.57659, size = 37, normalized size = 0.86

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{ab}\sqrt{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**2+a)**(3/2),x)`

[Out] `asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))`

**GIAC/XCAS [A]** time = 0.216745, size = 53, normalized size = 1.23

$$-\frac{x}{\sqrt{bx^2+ab}} - \frac{\ln\left(\left|-\sqrt{bx} + \sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] -x/(sqrt(b*x^2 + a)*b) - ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

$$3.500 \quad \int \frac{x}{(a+bx^2)^{3/2}} dx$$

**Optimal.** Leaf size=16

$$-\frac{1}{b\sqrt{a+bx^2}}$$

[Out] -(1/(b\*Sqrt[a + b\*x^2]))

**Rubi [A]** time = 0.0111924, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{1}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*x^2)^(3/2), x]

[Out] -(1/(b\*Sqrt[a + b\*x^2]))

**Rubi in Sympy [A]** time = 2.17266, size = 14, normalized size = 0.88

$$-\frac{1}{b\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(b\*x\*\*2+a)\*\*(3/2), x)

[Out] -1/(b\*sqrt(a + b\*x\*\*2))

**Mathematica [A]** time = 0.00516101, size = 16, normalized size = 1.

$$-\frac{1}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*x^2)^(3/2), x]

[Out] -(1/(b\*Sqrt[a + b\*x^2]))

**Maple [A]** time = 0.003, size = 15, normalized size = 0.9

$$-\frac{1}{b} \frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b\*x^2+a)^(3/2), x)

[Out] -1/b/(b\*x^2+a)^(1/2)

---

**Maxima [A]** time = 1.32497, size = 19, normalized size = 1.19

$$-\frac{1}{\sqrt{bx^2 + ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b\*x^2 + a)^(3/2),x, algorithm="maxima")

[Out] -1/(sqrt(b\*x^2 + a)\*b)

---

**Fricas [A]** time = 0.226154, size = 32, normalized size = 2.

$$-\frac{\sqrt{bx^2 + a}}{b^2x^2 + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b\*x^2 + a)^(3/2),x, algorithm="fricas")

[Out] -sqrt(b\*x^2 + a)/(b^2\*x^2 + a\*b)

---

**Sympy [A]** time = 1.91106, size = 24, normalized size = 1.5

$$\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b\*x\*\*2+a)\*\*(3/2),x)

[Out] Piecewise((-1/(b\*sqrt(a + b\*x\*\*2)), Ne(b, 0)), (x\*\*2/(2\*a\*\*(3/2)), True))

---

**GIAC/XCAS [A]** time = 0.207064, size = 19, normalized size = 1.19

$$-\frac{1}{\sqrt{bx^2 + ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b\*x^2 + a)^(3/2),x, algorithm="giac")

[Out] -1/(sqrt(b\*x^2 + a)\*b)

$$3.501 \quad \int \frac{1}{(a+bx^2)^{3/2}} dx$$

**Optimal.** Leaf size=16

$$\frac{x}{a\sqrt{a+bx^2}}$$

[Out] x/(a\*Sqrt[a + b\*x^2])

**Rubi [A]** time = 0.00960781, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{x}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(-3/2), x]

[Out] x/(a\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 1.26499, size = 12, normalized size = 0.75

$$\frac{x}{a\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+a)\*\*(3/2), x)

[Out] x/(a\*sqrt(a + b\*x\*\*2))

**Mathematica [A]** time = 0.0121706, size = 16, normalized size = 1.

$$\frac{x}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(-3/2), x]

[Out] x/(a\*Sqrt[a + b\*x^2])

**Maple [A]** time = 0.005, size = 15, normalized size = 0.9

$$\frac{x}{a\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+a)^(3/2), x)

[Out] x/a/(b\*x^2+a)^(1/2)



---

**Maxima [A]** time = 1.32161, size = 19, normalized size = 1.19

$$\frac{x}{\sqrt{bx^2 + aa}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-3/2),x, algorithm="maxima")

[Out] x/(sqrt(b\*x^2 + a)\*a)

---

**Fricas [A]** time = 0.218333, size = 31, normalized size = 1.94

$$\frac{\sqrt{bx^2 + ax}}{abx^2 + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-3/2),x, algorithm="fricas")

[Out] sqrt(b\*x^2 + a)\*x/(a\*b\*x^2 + a^2)

---

**Sympy [A]** time = 1.78085, size = 17, normalized size = 1.06

$$\frac{x}{a^{\frac{3}{2}}\sqrt{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*2+a)\*\*(3/2),x)

[Out] x/(a\*\*(3/2)\*sqrt(1 + b\*x\*\*2/a))

---

**GIAC/XCAS [A]** time = 0.213166, size = 19, normalized size = 1.19

$$\frac{x}{\sqrt{bx^2 + aa}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-3/2),x, algorithm="giac")

[Out] x/(sqrt(b\*x^2 + a)\*a)

$$3.502 \quad \int \frac{1}{x(a+bx^2)^{3/2}} dx$$

**Optimal.** Leaf size=41

$$\frac{1}{a\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

[Out] 1/(a\*Sqrt[a + b\*x^2]) - ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]]/a^(3/2)

**Rubi [A]** time = 0.0773629, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{1}{a\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(a + b\*x^2)^(3/2)), x]

[Out] 1/(a\*Sqrt[a + b\*x^2]) - ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]]/a^(3/2)

**Rubi in Sympy [A]** time = 7.81388, size = 34, normalized size = 0.83

$$\frac{1}{a\sqrt{a+bx^2}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(b\*x\*\*2+a)\*\*(3/2), x)

[Out] 1/(a\*sqrt(a + b\*x\*\*2)) - atanh(sqrt(a + b\*x\*\*2)/sqrt(a))/a\*\*(3/2)

**Mathematica [A]** time = 0.0542374, size = 48, normalized size = 1.17

$$\frac{\frac{\sqrt{a}}{\sqrt{a+bx^2}} - \log\left(\sqrt{a}\sqrt{a+bx^2} + a\right) + \log(x)}{a^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(a + b\*x^2)^(3/2)), x]

[Out] (Sqrt[a]/Sqrt[a + b\*x^2] + Log[x] - Log[a + Sqrt[a]\*Sqrt[a + b\*x^2]])/a^(3/2)

**Maple [A]** time = 0.007, size = 43, normalized size = 1.1

$$\frac{1}{a\sqrt{bx^2+a}} - 1 \ln\left(\frac{1}{x}\left(2a + 2\sqrt{a}\sqrt{bx^2+a}\right)\right) a^{-3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(b*x^2+a)^(3/2),x)`

[Out]  $1/a/(b*x^2+a)^{(1/2)} - 1/a^{(3/2)} * \ln((2*a+2*a^{(1/2)}*(b*x^2+a)^{(1/2)})/x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/2)*x),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.23793, size = 1, normalized size = 0.02

$$\left[ \frac{(bx^2 + a) \log\left(-\frac{(bx^2+2a)\sqrt{a-2\sqrt{bx^2+aa}}}{x^2}\right) + 2\sqrt{bx^2+a}\sqrt{a}}{2(abx^2 + a^2)\sqrt{a}}, -\frac{(bx^2 + a) \arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) - \sqrt{bx^2+a}\sqrt{-a}}{(abx^2 + a^2)\sqrt{-a}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/2)*x),x, algorithm="fricas")`

[Out]  $[1/2*((b*x^2 + a)*\log(-((b*x^2 + 2*a)*\sqrt{a} - 2*\sqrt{b*x^2 + a})*a)/x^2) + 2*\sqrt{b*x^2 + a}*\sqrt{a})/((a*b*x^2 + a^2)*\sqrt{a}), -((b*x^2 + a)*\arctan(\sqrt{-a}/\sqrt{b*x^2 + a}) - \sqrt{b*x^2 + a}*\sqrt{-a})/((a*b*x^2 + a^2)*\sqrt{-a})]$

**Sympy [A]** time = 5.8383, size = 184, normalized size = 4.49

$$\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} + \frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} + \frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} - \frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x**2+a)**(3/2),x)`

[Out]  $2*a**3*\sqrt{1+b*x**2/a}/(2*a**(9/2)+2*a**(7/2)*b*x**2) + a**3*\log(b*x**2/a)/(2*a**(9/2)+2*a**(7/2)*b*x**2) - 2*a**3*\log(\sqrt{1+b*x**2/a}+1)/(2*a**(9/2)+2*a**(7/2)*b*x**2) + a**2*b*x**2*\log(b*x**2/a)/(2*a**(9/2)+2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*\log(\sqrt{1+b*x**2/a}+1)/(2*a**(9/2)+2*a**(7/2)*b*x**2)$

**GIAC/XCAS [A]** time = 0.210027, size = 53, normalized size = 1.29

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-aa}} + \frac{1}{\sqrt{bx^2+aa}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(3/2)*x),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + 1/(sqrt(b*x^2 + a)*a)
```

$$3.503 \quad \int \frac{1}{x^2(a+bx^2)^{3/2}} dx$$

**Optimal.** Leaf size=38

$$-\frac{2bx}{a^2\sqrt{a+bx^2}} - \frac{1}{ax\sqrt{a+bx^2}}$$

[Out]  $-(1/(a*x*\text{Sqrt}[a + b*x^2])) - (2*b*x)/(a^2*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.0344174, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{2bx}{a^2\sqrt{a+bx^2}} - \frac{1}{ax\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^2*(a + b*x^2)^(3/2)), x]$

[Out]  $-(1/(a*x*\text{Sqrt}[a + b*x^2])) - (2*b*x)/(a^2*\text{Sqrt}[a + b*x^2])$

**Rubi in Sympy [A]** time = 4.01587, size = 34, normalized size = 0.89

$$-\frac{1}{ax\sqrt{a+bx^2}} - \frac{2bx}{a^2\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{**2}/(b*x^{**2}+a)^{(3/2)}, x)$

[Out]  $-1/(a*x*\text{sqrt}(a + b*x^{**2})) - 2*b*x/(a^{**2}*\text{sqrt}(a + b*x^{**2}))$

**Mathematica [A]** time = 0.0247401, size = 27, normalized size = 0.71

$$-\frac{a+2bx^2}{a^2x\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/(x^2*(a + b*x^2)^(3/2)), x]$

[Out]  $-((a + 2*b*x^2)/(a^2*x*\text{Sqrt}[a + b*x^2]))$

**Maple [A]** time = 0.006, size = 26, normalized size = 0.7

$$-\frac{2bx^2+a}{a^2x} \frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/x^2/(b*x^2+a)^(3/2), x)$

[Out]  $-(2*b*x^2+a)/x/(b*x^2+a)^(1/2)/a^2$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/2)*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.227836, size = 47, normalized size = 1.24

$$-\frac{(2bx^2 + a)\sqrt{bx^2 + a}}{a^2bx^3 + a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/2)*x^2),x, algorithm="fricas")`

[Out]  $-(2*b*x^2 + a)*\text{sqrt}(b*x^2 + a)/(a^2*b*x^3 + a^3*x)$

---

**Sympy [A]** time = 2.70006, size = 46, normalized size = 1.21

$$-\frac{1}{a\sqrt{bx^2}\sqrt{\frac{a}{bx^2} + 1}} - \frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(b*x**2+a)**(3/2),x)`

[Out]  $-1/(a*\text{sqrt}(b)*x**2*\text{sqrt}(a/(b*x**2) + 1)) - 2*\text{sqrt}(b)/(a**2*\text{sqrt}(a/(b*x**2) + 1))$

---

**GIAC/XCAS [A]** time = 0.21463, size = 68, normalized size = 1.79

$$-\frac{bx}{\sqrt{bx^2 + aa^2}} + \frac{2\sqrt{b}}{\left(\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^2 - a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/2)*x^2),x, algorithm="giac")`

[Out]  $-b*x/(\text{sqrt}(b*x^2 + a)*a^2) + 2*\text{sqrt}(b)/(((\text{sqrt}(b)*x - \text{sqrt}(b*x^2 + a))^2 - a)*a)$

$$3.504 \quad \int \frac{1}{x^3(a+bx^2)^{3/2}} dx$$

**Optimal.** Leaf size=68

$$\frac{3b \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{3\sqrt{a+bx^2}}{2a^2x^2} + \frac{1}{ax^2\sqrt{a+bx^2}}$$

[Out]  $1/(a*x^2*\text{Sqrt}[a + b*x^2]) - (3*\text{Sqrt}[a + b*x^2])/(2*a^2*x^2) + (3*b*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(2*a^{(5/2)})$

**Rubi [A]** time = 0.108735, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{3b \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{3\sqrt{a+bx^2}}{2a^2x^2} + \frac{1}{ax^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(a + b\*x^2)^(3/2)), x]

[Out]  $1/(a*x^2*\text{Sqrt}[a + b*x^2]) - (3*\text{Sqrt}[a + b*x^2])/(2*a^2*x^2) + (3*b*\text{ArcTanh}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[a]])/(2*a^{(5/2)})$

**Rubi in Sympy [A]** time = 10.664, size = 63, normalized size = 0.93

$$\frac{1}{ax^2\sqrt{a+bx^2}} - \frac{3\sqrt{a+bx^2}}{2a^2x^2} + \frac{3b \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(b\*x\*\*2+a)\*\*(3/2), x)

[Out]  $1/(a*x**2*\text{sqrt}(a + b*x**2)) - 3*\text{sqrt}(a + b*x**2)/(2*a**2*x**2) + 3*b*\operatorname{atanh}(\text{sqrt}(a + b*x**2)/\text{sqrt}(a))/(2*a**(5/2))$

**Mathematica [A]** time = 0.151314, size = 67, normalized size = 0.99

$$\frac{-\frac{\sqrt{a}(a+3bx^2)}{x^2\sqrt{a+bx^2}} + 3b \log\left(\sqrt{a}\sqrt{a+bx^2} + a\right) - 3b \log(x)}{2a^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a + b\*x^2)^(3/2)), x]

[Out]  $(-((\text{Sqrt}[a]*(a + 3*b*x^2))/(x^2*\text{Sqrt}[a + b*x^2])) - 3*b*\text{Log}[x] + 3*b*\text{Log}[a + \text{Sqrt}[a]*\text{Sqrt}[a + b*x^2]])/(2*a^{(5/2)})$

**Maple [A]** time = 0.007, size = 63, normalized size = 0.9

$$-\frac{1}{2ax^2} \frac{1}{\sqrt{bx^2+a}} - \frac{3b}{2a^2} \frac{1}{\sqrt{bx^2+a}} + \frac{3b}{2} \ln\left(\frac{1}{x} \left(2a + 2\sqrt{a}\sqrt{bx^2+a}\right)\right) a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(b*x^2+a)^(3/2), x)`

[Out]  $-1/2/a/x^2/(b*x^2+a)^{(1/2)} - 3/2*b/a^2/(b*x^2+a)^{(1/2)} + 3/2*b/a^{(5/2)} * \ln((2*a+2*a^{(1/2)}*(b*x^2+a)^{(1/2)})/x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/2)*x^3), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.23984, size = 1, normalized size = 0.01

$$\left[ \begin{array}{l} \frac{2(3bx^2 + a)\sqrt{bx^2 + a}\sqrt{a} - 3(b^2x^4 + abx^2) \log\left(-\frac{(bx^2+2a)\sqrt{a+2\sqrt{bx^2+aa}}}{x^2}\right)}{4(a^2bx^4 + a^3x^2)\sqrt{a}}, \\ \frac{(3bx^2 + a)\sqrt{bx^2 + a}\sqrt{-a} - 3(b^2x^4 + abx^2) \arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{2(a^2bx^4 + a^3x^2)\sqrt{-a}} \end{array} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/2)*x^3), x, algorithm="fricas")`

[Out]  $[-1/4*(2*(3*b*x^2 + a)*\sqrt{b*x^2 + a}*\sqrt{a} - 3*(b^2*x^4 + a*b*x^2)*\log(-((b*x^2 + 2*a)*\sqrt{a} + 2*\sqrt{b*x^2 + a}*a)/x^2))/((a^2*b*x^4 + a^3*x^2)*\sqrt{a}), -1/2*((3*b*x^2 + a)*\sqrt{b*x^2 + a})*\sqrt{-a} - 3*(b^2*x^4 + a*b*x^2)*\arctan(\sqrt{-a}/\sqrt{b*x^2 + a})]/((a^2*b*x^4 + a^3*x^2)*\sqrt{-a})]$

**Sympy [A]** time = 11.295, size = 73, normalized size = 1.07

$$-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2} + 1}} - \frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2} + 1}} + \frac{3b \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(b*x**2+a)**(3/2), x)`

[Out]  $-1/(2*a*\sqrt{b}*x**3*\sqrt{a/(b*x**2) + 1}) - 3*\sqrt{b}/(2*a**2*x*\sqrt{a/(b*x**2) + 1}) + 3*b*\operatorname{asinh}(\sqrt{a}/(\sqrt{b}*x))/(2*a**(5/2))$

**GIAC/XCAS [A]** time = 0.215673, size = 89, normalized size = 1.31

$$-\frac{1}{2}b\left(\frac{3 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{3bx^2 + a}{((bx^2 + a)^{\frac{3}{2}} - \sqrt{bx^2 + aa})a^2}\right)$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(3/2)*x^3),x, algorithm="giac")
```

```
[Out] -1/2*b*(3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*b*x^2 + a)/(((b*x^2 + a)^(3/2) - sqrt(b*x^2 + a)*a)*a^2))
```

$$3.505 \quad \int \frac{1}{x^4(a+bx^2)^{3/2}} dx$$

**Optimal.** Leaf size=66

$$\frac{8b^2x}{3a^3\sqrt{a+bx^2}} + \frac{4b}{3a^2x\sqrt{a+bx^2}} - \frac{1}{3ax^3\sqrt{a+bx^2}}$$

[Out]  $-1/(3*a*x^3*\text{Sqrt}[a + b*x^2]) + (4*b)/(3*a^2*x*\text{Sqrt}[a + b*x^2]) + (8*b^2*x)/(3*a^3*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.0592301, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{8b^2x}{3a^3\sqrt{a+bx^2}} + \frac{4b}{3a^2x\sqrt{a+bx^2}} - \frac{1}{3ax^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a + b\*x^2)^(3/2)), x]

[Out]  $-1/(3*a*x^3*\text{Sqrt}[a + b*x^2]) + (4*b)/(3*a^2*x*\text{Sqrt}[a + b*x^2]) + (8*b^2*x)/(3*a^3*\text{Sqrt}[a + b*x^2])$

**Rubi in Sympy [A]** time = 6.98637, size = 60, normalized size = 0.91

$$-\frac{1}{3ax^3\sqrt{a+bx^2}} + \frac{4b}{3a^2x\sqrt{a+bx^2}} + \frac{8b^2x}{3a^3\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(b\*x\*\*2+a)\*\*(3/2), x)

[Out]  $-1/(3*a*x**3*\text{sqrt}(a + b*x**2)) + 4*b/(3*a**2*x*\text{sqrt}(a + b*x**2)) + 8*b**2*x/(3*a**3*\text{sqrt}(a + b*x**2))$

**Mathematica [A]** time = 0.0315122, size = 42, normalized size = 0.64

$$\frac{-a^2 + 4abx^2 + 8b^2x^4}{3a^3x^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a + b\*x^2)^(3/2)), x]

[Out]  $(-a^2 + 4*a*b*x^2 + 8*b^2*x^4)/(3*a^3*x^3*\text{Sqrt}[a + b*x^2])$

**Maple [A]** time = 0.007, size = 37, normalized size = 0.6

$$-\frac{-8b^2x^4 - 4abx^2 + a^2}{3a^3x^3} \frac{1}{\sqrt{bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(b*x^2+a)^(3/2), x)`

[Out]  $-1/3 * (-8*b^2*x^4 - 4*a*b*x^2 + a^2) / x^3 / (b*x^2 + a)^{(1/2)} / a^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/2)*x^4), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.229318, size = 68, normalized size = 1.03

$$\frac{(8b^2x^4 + 4abx^2 - a^2)\sqrt{bx^2 + a}}{3(a^3bx^5 + a^4x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/2)*x^4), x, algorithm="fricas")`

[Out]  $1/3 * (8*b^2*x^4 + 4*a*b*x^2 - a^2) * \text{sqrt}(b*x^2 + a) / (a^3*b*x^5 + a^4*x^3)$

**Sympy [A]** time = 4.23255, size = 233, normalized size = 3.53

$$\begin{aligned} & -\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2} + 1}}{3a^5b^4x^2 + 6a^4b^5x^4 + 3a^3b^6x^6} + \frac{3a^2b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2} + 1}}{3a^5b^4x^2 + 6a^4b^5x^4 + 3a^3b^6x^6} \\ & + \frac{12ab^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2} + 1}}{3a^5b^4x^2 + 6a^4b^5x^4 + 3a^3b^6x^6} + \frac{8b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2} + 1}}{3a^5b^4x^2 + 6a^4b^5x^4 + 3a^3b^6x^6} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(b*x**2+a)**(3/2), x)`

[Out]  $-a^{**3}b^{**9/2} * \text{sqrt}(a/(b*x^{**2}) + 1) / (3*a^{**5}b^{**4}x^{**2} + 6*a^{**4}b^{**5}x^{**4} + 3*a^{**3}b^{**6}x^{**6}) + 3*a^{**2}b^{**11/2} * x^{**2} * \text{sqrt}(a/(b*x^{**2}) + 1) / (3*a^{**5}b^{**4}x^{**2} + 6*a^{**4}b^{**5}x^{**4} + 3*a^{**3}b^{**6}x^{**6}) + 12*a*b^{**13/2} * x^{**4} * \text{sqrt}(a/(b*x^{**2}) + 1) / (3*a^{**5}b^{**4}x^{**2} + 6*a^{**4}b^{**5}x^{**4} + 3*a^{**3}b^{**6}x^{**6}) + 8*b^{**15/2} * x^{**6} * \text{sqrt}(a/(b*x^{**2}) + 1) / (3*a^{**5}b^{**4}x^{**2} + 6*a^{**4}b^{**5}x^{**4} + 3*a^{**3}b^{**6}x^{**6})$

**GIAC/XCAS [A]** time = 0.223298, size = 143, normalized size = 2.17

$$\frac{b^2x}{\sqrt{bx^2 + aa^3}} - \frac{2 \left( 3 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^4 b^{\frac{3}{2}} - 12 \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 ab^{\frac{3}{2}} + 5 a^2 b^{\frac{3}{2}} \right)}{3 \left( \left( \sqrt{bx} - \sqrt{bx^2 + a} \right)^2 - a \right)^3 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(3/2)*x^4),x, algorithm="giac")
```

```
[Out] b^2*x/(sqrt(b*x^2 + a)*a^3) - 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a)
)^4*b^(3/2) - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 5*a^
2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3*a^2)
```

$$3.506 \quad \int \frac{x^6}{(a+bx^2)^{5/2}} dx$$

Optimal. Leaf size=91

$$-\frac{5a \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{2b^{7/2}} + \frac{5x\sqrt{a+bx^2}}{2b^3} - \frac{5x^3}{3b^2\sqrt{a+bx^2}} - \frac{x^5}{3b(a+bx^2)^{3/2}}$$

[Out]  $-x^5/(3*b*(a + b*x^2)^(3/2)) - (5*x^3)/(3*b^2*\text{Sqrt}[a + b*x^2]) + (5*x*\text{Sqrt}[a + b*x^2])/(2*b^3) - (5*a*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]])/(2*b^(7/2))$

Rubi [A] time = 0.099484, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{5a \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{2b^{7/2}} + \frac{5x\sqrt{a+bx^2}}{2b^3} - \frac{5x^3}{3b^2\sqrt{a+bx^2}} - \frac{x^5}{3b(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a + b\*x^2)^(5/2), x]

[Out]  $-x^5/(3*b*(a + b*x^2)^(3/2)) - (5*x^3)/(3*b^2*\text{Sqrt}[a + b*x^2]) + (5*x*\text{Sqrt}[a + b*x^2])/(2*b^3) - (5*a*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]])/(2*b^(7/2))$

Rubi in Sympy [A] time = 13.6176, size = 83, normalized size = 0.91

$$-\frac{5a \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{2b^{7/2}} - \frac{x^5}{3b(a+bx^2)^{3/2}} - \frac{5x^3}{3b^2\sqrt{a+bx^2}} + \frac{5x\sqrt{a+bx^2}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6/(b\*x\*\*2+a)\*\*(5/2), x)

[Out]  $-5*a*\operatorname{atanh}(\text{sqrt}(b)*x/\text{sqrt}(a + b*x**2))/(2*b**(7/2)) - x**5/(3*b*(a + b*x**2)**(3/2)) - 5*x**3/(3*b**2*\text{sqrt}(a + b*x**2)) + 5*x*\text{sqrt}(a + b*x**2)/(2*b**3)$

Mathematica [A] time = 0.148745, size = 73, normalized size = 0.8

$$\frac{15a^2x + 20abx^3 + 3b^2x^5}{6b^3(a+bx^2)^{3/2}} - \frac{5a \log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a + b\*x^2)^(5/2), x]

[Out]  $(15*a^2*x + 20*a*b*x^3 + 3*b^2*x^5)/(6*b^3*(a + b*x^2)^(3/2)) - (5*a*\text{Log}[b*x + \text{Sqrt}[b]*\text{Sqrt}[a + b*x^2]])/(2*b^(7/2))$

**Maple [A]** time = 0.013, size = 75, normalized size = 0.8

$$\frac{x^5}{2b} (bx^2 + a)^{-\frac{3}{2}} + \frac{5ax^3}{6b^2} (bx^2 + a)^{-\frac{3}{2}} + \frac{5ax}{2b^3} \frac{1}{\sqrt{bx^2 + a}} - \frac{5a}{2} \ln(x\sqrt{b} + \sqrt{bx^2 + a}) b^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(b\*x^2+a)^(5/2), x)

[Out] 1/2\*x^5/b/(b\*x^2+a)^(3/2)+5/6\*a/b^2\*x^3/(b\*x^2+a)^(3/2)+5/2\*a/b^3\*x/(b\*x^2+a)^(1/2)-5/2\*a/b^(7/2)\*ln(x\*b^(1/2)+(b\*x^2+a)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b\*x^2 + a)^(5/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.255383, size = 1, normalized size = 0.01

$$\left[ \frac{2(3b^2x^5 + 20abx^3 + 15a^2x)\sqrt{bx^2 + a}\sqrt{b} + 15(ab^2x^4 + 2a^2bx^2 + a^3)\log(2\sqrt{bx^2 + abx} - (2bx^2 + a)\sqrt{b})}{12(b^5x^4 + 2ab^4x^2 + a^2b^3)\sqrt{b}}, \frac{(3b^2x^5 + 20abx^3 + 15a^2x)\sqrt{bx^2 + a}\sqrt{b}}{12(b^5x^4 + 2ab^4x^2 + a^2b^3)\sqrt{b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b\*x^2 + a)^(5/2), x, algorithm="fricas")

[Out] [1/12\*(2\*(3\*b^2\*x^5 + 20\*a\*b\*x^3 + 15\*a^2\*x)\*sqrt(b\*x^2 + a)\*sqrt(b) + 15\*(a\*b^2\*x^4 + 2\*a^2\*b\*x^2 + a^3)\*log(2\*sqrt(b\*x^2 + a)\*b\*x - (2\*b\*x^2 + a)\*sqrt(b)))/((b^5\*x^4 + 2\*a\*b^4\*x^2 + a^2\*b^3)\*sqrt(b)), 1/6\*((3\*b^2\*x^5 + 20\*a\*b\*x^3 + 15\*a^2\*x)\*sqrt(b\*x^2 + a)\*sqrt(-b) - 15\*(a\*b^2\*x^4 + 2\*a^2\*b\*x^2 + a^3)\*arctan(sqrt(-b)\*x/sqrt(b\*x^2 + a)))/((b^5\*x^4 + 2\*a\*b^4\*x^2 + a^2\*b^3)\*sqrt(-b))]

**Sympy [A]** time = 16.6398, size = 367, normalized size = 4.03

$$\begin{aligned} & -\frac{15a^{\frac{81}{2}}b^{22}\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} - \frac{15a^{\frac{79}{2}}b^{23}x^2\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} \\ & + \frac{15a^{40}b^{\frac{45}{2}}x}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} + \frac{20a^{39}b^{\frac{47}{2}}x^3}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} \\ & + \frac{3a^{38}b^{\frac{49}{2}}x^5}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*6/(b\*x\*\*2+a)\*\*(5/2), x)

```
[Out] -15*a**(81/2)*b**22*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(
6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*
x**2*sqrt(1 + b*x**2/a)) - 15*a**(79/2)*b**23*x**2*sqrt(1 + b*x**
2/a)*asinh(sqrt(b)*x/sqrt(a))/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x
**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 15*a**4
0*b**(45/2)*x/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(7
7/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 20*a**39*b**(47/2)*x**3
/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2
)*x**2*sqrt(1 + b*x**2/a)) + 3*a**38*b**(49/2)*x**5/(6*a**(79/2)*
b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1
+ b*x**2/a))
```

**GIAC/XCAS [A]** time = 0.235957, size = 88, normalized size = 0.97

$$\frac{\left(x^2\left(\frac{3x^2}{b} + \frac{20a}{b^2}\right) + \frac{15a^2}{b^3}\right)x}{6(bx^2 + a)^{\frac{3}{2}}} + \frac{5 \operatorname{aln}\left(\left|-\sqrt{bx} + \sqrt{bx^2 + a}\right|\right)}{2b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(x^2*(3*x^2/b + 20*a/b^2) + 15*a^2/b^3)*x/(b*x^2 + a)^(3/2) +
5/2*a*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

$$3.507 \quad \int \frac{x^5}{(a+bx^2)^{5/2}} dx$$

**Optimal.** Leaf size=54

$$-\frac{a^2}{3b^3(a+bx^2)^{3/2}} + \frac{2a}{b^3\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}}{b^3}$$

[Out]  $-a^2/(3*b^3*(a + b*x^2)^(3/2)) + (2*a)/(b^3*sqrt[a + b*x^2]) + sqrt[a + b*x^2]/b^3$

**Rubi [A]** time = 0.0915382, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{a^2}{3b^3(a+bx^2)^{3/2}} + \frac{2a}{b^3\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}}{b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a + b\*x^2)^(5/2), x]

[Out]  $-a^2/(3*b^3*(a + b*x^2)^(3/2)) + (2*a)/(b^3*sqrt[a + b*x^2]) + sqrt[a + b*x^2]/b^3$

**Rubi in Sympy [A]** time = 11.5525, size = 48, normalized size = 0.89

$$-\frac{a^2}{3b^3(a+bx^2)^{3/2}} + \frac{2a}{b^3\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5/(b\*x\*\*2+a)\*\*(5/2), x)

[Out]  $-a**2/(3*b**3*(a + b*x**2)**(3/2)) + 2*a/(b**3*sqrt(a + b*x**2)) + sqrt(a + b*x**2)/b**3$

**Mathematica [A]** time = 0.0313702, size = 39, normalized size = 0.72

$$\frac{8a^2 + 12abx^2 + 3b^2x^4}{3b^3(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a + b\*x^2)^(5/2), x]

[Out]  $(8*a^2 + 12*a*b*x^2 + 3*b^2*x^4)/(3*b^3*(a + b*x^2)^(3/2))$

**Maple [A]** time = 0.006, size = 36, normalized size = 0.7

$$\frac{3b^2x^4 + 12abx^2 + 8a^2}{3b^3} (bx^2 + a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(x^5/(b*x^2+a)^(5/2),x)`

[Out]  $1/3*(3*b^2*x^4+12*a*b*x^2+8*a^2)/(b*x^2+a)^(3/2)/b^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2 + a)^(5/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.23619, size = 78, normalized size = 1.44

$$\frac{(3b^2x^4 + 12abx^2 + 8a^2)\sqrt{bx^2 + a}}{3(b^5x^4 + 2ab^4x^2 + a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2 + a)^(5/2),x, algorithm="fricas")`

[Out]  $1/3*(3*b^2*x^4 + 12*a*b*x^2 + 8*a^2)*\text{sqrt}(b*x^2 + a)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)$

**Sympy [A]** time = 3.66451, size = 138, normalized size = 2.56

$$\begin{cases} \frac{8a^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} + \frac{12abx^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} + \frac{3b^2x^4}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(b*x**2+a)**(5/2),x)`

[Out] `Piecewise((8*a**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)) + 12*a*b*x**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)) + 3*b**2*x**4/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(5/2)), True))`

**GIAC/XCAS [A]** time = 0.212483, size = 58, normalized size = 1.07

$$\frac{3\sqrt{bx^2 + a} + \frac{6(bx^2+a)a-a^2}{(bx^2+a)^{\frac{3}{2}}}}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2 + a)^(5/2),x, algorithm="giac")`

[Out]  $1/3*(3*\text{sqrt}(b*x^2 + a) + (6*(b*x^2 + a)*a - a^2)/(b*x^2 + a)^(3/2))/b^3$

$$3.508 \quad \int \frac{x^4}{(a+bx^2)^{5/2}} dx$$

Optimal. Leaf size=64

$$\frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{b^{5/2}} - \frac{x}{b^2\sqrt{a+bx^2}} - \frac{x^3}{3b(a+bx^2)^{3/2}}$$

[Out]  $-x^3/(3*b*(a + b*x^2)^(3/2)) - x/(b^2*sqrt[a + b*x^2]) + ArcTanh[(sqrt[b]*x)/sqrt[a + b*x^2]]/b^(5/2)$

Rubi [A] time = 0.0694782, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{b^{5/2}} - \frac{x}{b^2\sqrt{a+bx^2}} - \frac{x^3}{3b(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a + b\*x^2)^(5/2), x]

[Out]  $-x^3/(3*b*(a + b*x^2)^(3/2)) - x/(b^2*sqrt[a + b*x^2]) + ArcTanh[(sqrt[b]*x)/sqrt[a + b*x^2]]/b^(5/2)$

Rubi in Sympy [A] time = 9.33941, size = 54, normalized size = 0.84

$$-\frac{x^3}{3b(a+bx^2)^{3/2}} - \frac{x}{b^2\sqrt{a+bx^2}} + \frac{\operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(b\*x\*\*2+a)\*\*(5/2), x)

[Out]  $-x**3/(3*b*(a + b*x**2)**(3/2)) - x/(b**2*sqrt(a + b*x**2)) + \operatorname{atanh}(sqrt(b)*x/sqrt(a + b*x**2))/b**(5/2)$

Mathematica [A] time = 0.120747, size = 58, normalized size = 0.91

$$\frac{\log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right)}{b^{5/2}} - \frac{x(3a+4bx^2)}{3b^2(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a + b\*x^2)^(5/2), x]

[Out]  $-(x*(3*a + 4*b*x^2))/(3*b^2*(a + b*x^2)^(3/2)) + \operatorname{Log}[b*x + \operatorname{Sqrt}[b]*\operatorname{Sqrt}[a + b*x^2]]/b^(5/2)$

Maple [A] time = 0.008, size = 54, normalized size = 0.8

$$-\frac{x^3}{3b}(bx^2+a)^{-3/2} - \frac{x}{b^2} \frac{1}{\sqrt{bx^2+a}} + 1 \ln\left(x\sqrt{b} + \sqrt{bx^2+a}\right) b^{-5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(b*x^2+a)^(5/2),x)`

[Out]  $-1/3*x^3/b/(b*x^2+a)^{(3/2)}-x/b^2/(b*x^2+a)^{(1/2)}+1/b^{(5/2)}*\ln(x*b^{(1/2)}+(b*x^2+a)^{(1/2)})$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^(5/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.247527, size = 1, normalized size = 0.02

$$\left[ \begin{array}{l} \frac{2(4bx^3 + 3ax)\sqrt{bx^2 + a}\sqrt{b} - 3(b^2x^4 + 2abx^2 + a^2)\log(-2\sqrt{bx^2 + a}bx - (2bx^2 + a)\sqrt{b})}{6(b^4x^4 + 2ab^3x^2 + a^2b^2)\sqrt{b}}, \\ -\frac{(4bx^3 + 3ax)\sqrt{bx^2 + a}\sqrt{-b} - 3(b^2x^4 + 2abx^2 + a^2)\arctan\left(\frac{\sqrt{-bx}}{\sqrt{bx^2 + a}}\right)}{3(b^4x^4 + 2ab^3x^2 + a^2b^2)\sqrt{-b}} \end{array} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^(5/2),x, algorithm="fricas")`

[Out]  $[-1/6*(2*(4*b*x^3 + 3*a*x)*\sqrt{b*x^2 + a}*\sqrt{b} - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*\log(-2*\sqrt{b*x^2 + a}*b*x - (2*b*x^2 + a)*\sqrt{b}))/((b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*\sqrt{b}), -1/3*((4*b*x^3 + 3*a*x)*\sqrt{b*x^2 + a}*\sqrt{-b} - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*\arctan(\sqrt{-b}*x/\sqrt{b*x^2 + a}))/((b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*\sqrt{-b})]$

**Sympy** [A] time = 10.0111, size = 303, normalized size = 4.73

$$\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1 + \frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1 + \frac{bx^2}{a}} + 3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1 + \frac{bx^2}{a}}} + \frac{3a^{\frac{37}{2}}b^{12}x^2\sqrt{1 + \frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1 + \frac{bx^2}{a}} + 3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1 + \frac{bx^2}{a}}} - \frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1 + \frac{bx^2}{a}} + 3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1 + \frac{bx^2}{a}}} - \frac{4a^{18}b^{\frac{25}{2}}x^3}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1 + \frac{bx^2}{a}} + 3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(b*x**2+a)**(5/2),x)`

[Out]  $3*a^{(39/2)}*b^{11}*\sqrt{1 + b*x^{**2}/a}*\operatorname{asinh}(\sqrt{b}*x/\sqrt{a})/(3*a^{(39/2)}*b^{(27/2)}*\sqrt{1 + b*x^{**2}/a} + 3*a^{(37/2)}*b^{(29/2)}*x^{**2}*\sqrt{1 + b*x^{**2}/a}) + 3*a^{(37/2)}*b^{12}*x^{**2}*\sqrt{1 + b*x^{**2}/a}*\operatorname{asinh}(\sqrt{b}*x/\sqrt{a})/(3*a^{(39/2)}*b^{(27/2)}*\sqrt{1 + b*x^{**2}/a} + 3*a^{(37/2)}*b^{(29/2)}*x^{**2}*\sqrt{1 + b*x^{**2}/a}) - 3*a^{19}*b^{(23/2)}*x/(3*a^{(39/2)}*b^{(27/2)}*\sqrt{1 + b*x^{**2}/a} + 3*a^{(37/2)}*b^{(29/2)}*x^2*\sqrt{1 + b*x^{**2}/a}) - 4*a^{18}*b^{(25/2)}*x^3/(3*a^{(39/2)}*b^{(27/2)}*\sqrt{1 + b*x^{**2}/a} + 3*a^{(37/2)}*b^{(29/2)}*x^2*\sqrt{1 + b*x^{**2}/a})$

```
*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 4*a**18*b**(25/2)*x**3/(3*a
**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**
2*sqrt(1 + b*x**2/a))
```

**GIAC/XCAS [A]** time = 0.221788, size = 69, normalized size = 1.08

$$\frac{x\left(\frac{4x^2}{b} + \frac{3a}{b^2}\right)}{3(bx^2 + a)^{\frac{3}{2}}} - \frac{\ln\left(\left|-\sqrt{b}x + \sqrt{bx^2 + a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*(4*x^2/b + 3*a/b^2)/(b*x^2 + a)^(3/2) - ln(abs(-sqrt(b)*x
+ sqrt(b*x^2 + a)))/b^(5/2)
```

$$3.509 \quad \int \frac{x^3}{(a+bx^2)^{5/2}} dx$$

Optimal. Leaf size=36

$$\frac{a}{3b^2(a+bx^2)^{3/2}} - \frac{1}{b^2\sqrt{a+bx^2}}$$

[Out]  $a/(3*b^2*(a + b*x^2)^(3/2)) - 1/(b^2*Sqrt[a + b*x^2])$

Rubi [A] time = 0.0653482, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{a}{3b^2(a+bx^2)^{3/2}} - \frac{1}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b\*x^2)^(5/2), x]

[Out]  $a/(3*b^2*(a + b*x^2)^(3/2)) - 1/(b^2*Sqrt[a + b*x^2])$

Rubi in Sympy [A] time = 7.92136, size = 31, normalized size = 0.86

$$\frac{a}{3b^2(a+bx^2)^{3/2}} - \frac{1}{b^2\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(b\*x\*\*2+a)\*\*(5/2), x)

[Out]  $a/(3*b**2*(a + b*x**2)**(3/2)) - 1/(b**2*sqrt(a + b*x**2))$

Mathematica [A] time = 0.0236391, size = 28, normalized size = 0.78

$$-\frac{2a + 3bx^2}{3b^2(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b\*x^2)^(5/2), x]

[Out]  $-(2*a + 3*b*x^2)/(3*b^2*(a + b*x^2)^(3/2))$

Maple [A] time = 0.008, size = 25, normalized size = 0.7

$$-\frac{3bx^2 + 2a}{3b^2} (bx^2 + a)^{-3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b\*x^2+a)^(5/2), x)

[Out]  $-1/3*(3*b*x^2+2*a)/(b*x^2+a)^(3/2)/b^2$

---

**Maxima [A]** time = 1.32183, size = 45, normalized size = 1.25

$$-\frac{x^2}{(bx^2 + a)^{\frac{3}{2}}b} - \frac{2a}{3(bx^2 + a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b\*x^2 + a)^(5/2), x, algorithm="maxima")

[Out] -x^2/((b\*x^2 + a)^(3/2)\*b) - 2/3\*a/((b\*x^2 + a)^(3/2)\*b^2)

---

**Fricas [A]** time = 0.231581, size = 63, normalized size = 1.75

$$-\frac{(3bx^2 + 2a)\sqrt{bx^2 + a}}{3(b^4x^4 + 2ab^3x^2 + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b\*x^2 + a)^(5/2), x, algorithm="fricas")

[Out] -1/3\*(3\*b\*x^2 + 2\*a)\*sqrt(b\*x^2 + a)/(b^4\*x^4 + 2\*a\*b^3\*x^2 + a^2\*b^2)

---

**Sympy [A]** time = 3.42026, size = 92, normalized size = 2.56

$$\begin{cases} -\frac{2a}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} - \frac{3bx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(b\*x\*\*2+a)\*\*(5/2), x)

[Out] Piecewise((-2\*a/(3\*a\*b\*\*2\*sqrt(a + b\*x\*\*2)) + 3\*b\*\*3\*x\*\*2\*sqrt(a + b\*x\*\*2)) - 3\*b\*x\*\*2/(3\*a\*b\*\*2\*sqrt(a + b\*x\*\*2)) + 3\*b\*\*3\*x\*\*2\*sqrt(a + b\*x\*\*2)), Ne(b, 0)), (x\*\*4/(4\*a\*\*(5/2)), True))

---

**GIAC/XCAS [A]** time = 0.214547, size = 32, normalized size = 0.89

$$-\frac{3bx^2 + 2a}{3(bx^2 + a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b\*x^2 + a)^(5/2), x, algorithm="giac")

[Out] -1/3\*(3\*b\*x^2 + 2\*a)/((b\*x^2 + a)^(3/2)\*b^2)

$$3.510 \quad \int \frac{x^2}{(a+bx^2)^{5/2}} dx$$

**Optimal.** Leaf size=21

$$\frac{x^3}{3a(a+bx^2)^{3/2}}$$

[Out]  $x^3/(3*a*(a+b*x^2)^(3/2))$

**Rubi [A]** time = 0.0241773, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{x^3}{3a(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2/(a+b*x^2)^(5/2), x]$

[Out]  $x^3/(3*a*(a+b*x^2)^(3/2))$

**Rubi in Sympy [A]** time = 3.49363, size = 15, normalized size = 0.71

$$\frac{x^3}{3a(a+bx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**2}/(b*x^{**2}+a)^{(5/2)}, x)$

[Out]  $x^{**3}/(3*a*(a+b*x^{**2})^{(3/2)})$

**Mathematica [A]** time = 0.0218737, size = 21, normalized size = 1.

$$\frac{x^3}{3a(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^2/(a+b*x^2)^(5/2), x]$

[Out]  $x^3/(3*a*(a+b*x^2)^(3/2))$

**Maple [A]** time = 0.007, size = 18, normalized size = 0.9

$$\frac{x^3}{3a}(bx^2+a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^2/(b*x^2+a)^(5/2), x)$

[Out]  $1/3 * x^3/a/(b * x^2+a)^{(3/2)}$

**Maxima [A]** time = 1.36187, size = 46, normalized size = 2.19

$$-\frac{x}{3(bx^2+a)^{\frac{3}{2}}b} + \frac{x}{3\sqrt{bx^2+aab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(5/2),x, algorithm="maxima")`

[Out]  $-1/3 * x/((b * x^2 + a)^{(3/2)} * b) + 1/3 * x/(\text{sqrt}(b * x^2 + a) * a * b)$

**Fricas [A]** time = 0.230388, size = 50, normalized size = 2.38

$$\frac{\sqrt{bx^2+ax^3}}{3(ab^2x^4+2a^2bx^2+a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(5/2),x, algorithm="fricas")`

[Out]  $1/3 * \text{sqrt}(b * x^2 + a) * x^3/(a * b^2 * x^4 + 2 * a^2 * b * x^2 + a^3)$

**Sympy [A]** time = 2.42274, size = 44, normalized size = 2.1

$$\frac{x^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**2+a)**(5/2),x)`

[Out]  $x^{**3}/(3 * a^{** (5/2)} * \text{sqrt}(1 + b * x^{**2}/a) + 3 * a^{** (3/2)} * b * x^{**2} * \text{sqrt}(1 + b * x^{**2}/a))$

**GIAC/XCAS [A]** time = 0.213209, size = 23, normalized size = 1.1

$$\frac{x^3}{3(bx^2+a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(5/2),x, algorithm="giac")`

[Out]  $1/3 * x^3/((b * x^2 + a)^{(3/2)} * a)$



$$3.511 \quad \int \frac{x}{(a+bx^2)^{5/2}} dx$$

**Optimal.** Leaf size=18

$$-\frac{1}{3b(a+bx^2)^{3/2}}$$

[Out] -1/(3\*b\*(a + b\*x^2)^(3/2))

**Rubi [A]** time = 0.0112618, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{1}{3b(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*x^2)^(5/2), x]

[Out] -1/(3\*b\*(a + b\*x^2)^(3/2))

**Rubi in Sympy [A]** time = 2.15649, size = 15, normalized size = 0.83

$$-\frac{1}{3b(a+bx^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(b\*x\*\*2+a)\*\*(5/2), x)

[Out] -1/(3\*b\*(a + b\*x\*\*2)\*\*(3/2))

**Mathematica [A]** time = 0.00619615, size = 18, normalized size = 1.

$$-\frac{1}{3b(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*x^2)^(5/2), x]

[Out] -1/(3\*b\*(a + b\*x^2)^(3/2))

**Maple [A]** time = 0.005, size = 15, normalized size = 0.8

$$-\frac{1}{3b}(bx^2+a)^{-3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b\*x^2+a)^(5/2), x)

[Out] -1/3/b/(b\*x^2+a)^(3/2)

---

**Maxima [A]** time = 1.32354, size = 19, normalized size = 1.06

$$-\frac{1}{3(bx^2 + a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^(5/2),x, algorithm="maxima")`

[Out] `-1/3/((b*x^2 + a)^(3/2)*b)`

---

**Fricas [A]** time = 0.233697, size = 47, normalized size = 2.61

$$-\frac{\sqrt{bx^2 + a}}{3(b^3x^4 + 2ab^2x^2 + a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^(5/2),x, algorithm="fricas")`

[Out] `-1/3*sqrt(b*x^2 + a)/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)`

---

**Sympy [A]** time = 3.25096, size = 46, normalized size = 2.56

$$\begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x**2+a)**(5/2),x)`

[Out] `Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(5/2)), True))`

---

**GIAC/XCAS [A]** time = 0.218557, size = 19, normalized size = 1.06

$$-\frac{1}{3(bx^2 + a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^(5/2),x, algorithm="giac")`

[Out] `-1/3/((b*x^2 + a)^(3/2)*b)`

$$3.512 \quad \int \frac{1}{(a+bx^2)^{5/2}} dx$$

**Optimal.** Leaf size=39

$$\frac{2x}{3a^2\sqrt{a+bx^2}} + \frac{x}{3a(a+bx^2)^{3/2}}$$

[Out]  $x/(3*a*(a + b*x^2)^{(3/2)}) + (2*x)/(3*a^2*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.021003, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{2x}{3a^2\sqrt{a+bx^2}} + \frac{x}{3a(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(-5/2), x]

[Out]  $x/(3*a*(a + b*x^2)^{(3/2)}) + (2*x)/(3*a^2*\text{Sqrt}[a + b*x^2])$

**Rubi in Sympy [A]** time = 2.0296, size = 32, normalized size = 0.82

$$\frac{x}{3a(a+bx^2)^{3/2}} + \frac{2x}{3a^2\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+a)\*\*(5/2), x)

[Out]  $x/(3*a*(a + b*x**2)**(3/2)) + 2*x/(3*a**2*\text{sqrt}(a + b*x**2))$

**Mathematica [A]** time = 0.0200485, size = 29, normalized size = 0.74

$$\frac{x(3a+2bx^2)}{3a^2(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(-5/2), x]

[Out]  $(x*(3*a + 2*b*x^2))/(3*a^2*(a + b*x^2)^{(3/2)})$

**Maple [A]** time = 0.005, size = 26, normalized size = 0.7

$$\frac{x(2bx^2+3a)}{3a^2}(bx^2+a)^{-3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+a)^(5/2), x)

[Out]  $1/3*x*(2*b*x^2+3*a)/(b*x^2+a)^{(3/2)}/a^2$

---

**Maxima [A]** time = 1.3479, size = 42, normalized size = 1.08

$$\frac{2x}{3\sqrt{bx^2 + aa^2}} + \frac{x}{3(bx^2 + a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-5/2),x, algorithm="maxima")

[Out] 2/3\*x/(sqrt(b\*x^2 + a)\*a^2) + 1/3\*x/((b\*x^2 + a)^(3/2)\*a)

---

**Fricas [A]** time = 0.238363, size = 63, normalized size = 1.62

$$\frac{(2bx^3 + 3ax)\sqrt{bx^2 + a}}{3(a^2b^2x^4 + 2a^3bx^2 + a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-5/2),x, algorithm="fricas")

[Out] 1/3\*(2\*b\*x^3 + 3\*a\*x)\*sqrt(b\*x^2 + a)/(a^2\*b^2\*x^4 + 2\*a^3\*b\*x^2 + a^4)

---

**Sympy [A]** time = 2.72637, size = 95, normalized size = 2.44

$$\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1 + \frac{bx^2}{a}} + 3a^{\frac{5}{2}}bx^2\sqrt{1 + \frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1 + \frac{bx^2}{a}} + 3a^{\frac{5}{2}}bx^2\sqrt{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*2+a)\*\*(5/2),x)

[Out] 3\*a\*x/(3\*a\*\*(7/2)\*sqrt(1 + b\*x\*\*2/a) + 3\*a\*\*(5/2)\*b\*x\*\*2\*sqrt(1 + b\*x\*\*2/a)) + 2\*b\*x\*\*3/(3\*a\*\*(7/2)\*sqrt(1 + b\*x\*\*2/a) + 3\*a\*\*(5/2)\*b\*x\*\*2\*sqrt(1 + b\*x\*\*2/a))

---

**GIAC/XCAS [A]** time = 0.217708, size = 36, normalized size = 0.92

$$\frac{x\left(\frac{2bx^2}{a^2} + \frac{3}{a}\right)}{3(bx^2 + a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-5/2),x, algorithm="giac")

[Out] 1/3\*x\*(2\*b\*x^2/a^2 + 3/a)/(b\*x^2 + a)^(3/2)

$$3.513 \quad \int \frac{1}{x(a+bx^2)^{5/2}} dx$$

**Optimal.** Leaf size=59

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{1}{a^2\sqrt{a+bx^2}} + \frac{1}{3a(a+bx^2)^{3/2}}$$

[Out] 1/(3\*a\*(a + b\*x^2)^(3/2)) + 1/(a^2\*Sqrt[a + b\*x^2]) - ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]]/a^(5/2)

**Rubi [A]** time = 0.104154, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{1}{a^2\sqrt{a+bx^2}} + \frac{1}{3a(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(a + b\*x^2)^(5/2)), x]

[Out] 1/(3\*a\*(a + b\*x^2)^(3/2)) + 1/(a^2\*Sqrt[a + b\*x^2]) - ArcTanh[Sqrt[a + b\*x^2]/Sqrt[a]]/a^(5/2)

**Rubi in Sympy [A]** time = 10.5706, size = 51, normalized size = 0.86

$$\frac{1}{3a(a+bx^2)^{\frac{3}{2}}} + \frac{1}{a^2\sqrt{a+bx^2}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(b\*x\*\*2+a)\*\*(5/2), x)

[Out] 1/(3\*a\*(a + b\*x\*\*2)\*\*(3/2)) + 1/(a\*\*2\*sqrt(a + b\*x\*\*2)) - atanh(sqrt(a + b\*x\*\*2)/sqrt(a))/a\*\*(5/2)

**Mathematica [A]** time = 0.22258, size = 63, normalized size = 1.07

$$\frac{\sqrt{a}(4a+3bx^2)}{(a+bx^2)^{3/2}} - 3 \log\left(\sqrt{a}\sqrt{a+bx^2} + a\right) + 3 \log(x)$$

$$3a^{5/2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(a + b\*x^2)^(5/2)), x]

[Out] ((Sqrt[a]\*(4\*a + 3\*b\*x^2))/(a + b\*x^2)^(3/2) + 3\*Log[x] - 3\*Log[a + Sqrt[a]\*Sqrt[a + b\*x^2]])/(3\*a^(5/2))

**Maple [A]** time = 0.008, size = 57, normalized size = 1.

$$\frac{1}{3a}(bx^2 + a)^{-\frac{3}{2}} + \frac{1}{a^2\sqrt{bx^2 + a}} - 1 \ln\left(\frac{1}{x}\left(2a + 2\sqrt{a}\sqrt{bx^2 + a}\right)\right) a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(b*x^2+a)^(5/2),x)`

[Out]  $1/3/a/(b*x^2+a)^{(3/2)}+1/a^2/(b*x^2+a)^{(1/2)}-1/a^{(5/2)}*\ln((2*a+2*a^{(1/2)}*(b*x^2+a)^{(1/2)})/x)$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/2)*x),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.252786, size = 1, normalized size = 0.02

$$\left[ \frac{2(3bx^2 + 4a)\sqrt{bx^2 + a}\sqrt{a} + 3(b^2x^4 + 2abx^2 + a^2)\log\left(-\frac{(bx^2+2a)\sqrt{a-2}\sqrt{bx^2+aa}}{x^2}\right)}{6(a^2b^2x^4 + 2a^3bx^2 + a^4)\sqrt{a}}, \frac{(3bx^2 + 4a)\sqrt{bx^2 + a}\sqrt{-a} - 3(b^2x^4 + a^2)\sqrt{-a}}{3(a^2b^2x^4 + 2a^3bx^2 + a^4)\sqrt{-a}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/2)*x),x, algorithm="fricas")`

[Out]  $[1/6*(2*(3*b*x^2 + 4*a)*\sqrt{b*x^2 + a}*\sqrt{a} + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*\log(-((b*x^2 + 2*a)*\sqrt{a} - 2*\sqrt{b*x^2 + a})*a/x^2))/((a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*\sqrt{a}), 1/3*((3*b*x^2 + 4*a)*\sqrt{b*x^2 + a}*\sqrt{-a} - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*\arctan(\sqrt{-a}/\sqrt{b*x^2 + a}))/((a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*\sqrt{-a})]$

**Sympy** [A] time = 9.83845, size = 740, normalized size = 12.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x**2+a)**(5/2),x)`

[Out]  $8*a^{**7}*\sqrt{1 + b*x**2/a}/(6*a^{**}(19/2) + 18*a^{**}(17/2)*b*x**2 + 18*a^{**}(15/2)*b**2*x**4 + 6*a^{**}(13/2)*b**3*x**6) + 3*a^{**7}*\log(b*x**2/a)/(6*a^{**}(19/2) + 18*a^{**}(17/2)*b*x**2 + 18*a^{**}(15/2)*b**2*x**4 + 6*a^{**}(13/2)*b**3*x**6) - 6*a^{**7}*\log(\sqrt{1 + b*x**2/a} + 1)/(6*a^{**}(19/2) + 18*a^{**}(17/2)*b*x**2 + 18*a^{**}(15/2)*b**2*x**4 + 6*a^{**}(13/2)*b**3*x**6) + 14*a^{**6}*b*x**2*\sqrt{1 + b*x**2/a}/(6*a^{**}(19/2) + 18*a^{**}(17/2)*b*x**2 + 18*a^{**}(15/2)*b**2*x**4 + 6*a^{**}(13/2)*b**3*x**6) + 9*a^{**6}*b*x**2*\log(b*x**2/a)/(6*a^{**}(19/2) + 18*a^{**}(17/2)*b*x**2 + 18*a^{**}(15/2)*b**2*x**4 + 6*a^{**}(13/2)*b**3*x**6) - 18*a^{**6}*b*x**2*\log(\sqrt{1 + b*x**2/a} + 1)/(6*a^{**}(19/2) + 18*a^{**}(17/2)*b*x**2 + 18*a^{**}(15/2)*b**2*x**4 + 6*a^{**}(13/2)*b**3*x**6) + 6*a^{**5}*b**2*x**4*\sqrt{1 + b*x**2/a}/(6*a^{**}(19/2) + 18*a^{**}(17/2)*b*x**2 + 18*a^{**}(15/2)*b**2*x**4 + 6*a^{**}(13/2)*b**3*x**6) + 9*a^{**5}*b**2*x**4*\log(b*x**2/a)/(6*a^{**}(19/2) + 18*a^{**}(17/2)*b*x**2 + 18*a^{**}(15/2)*b**2*x**4 + 6*a^{**}(13/2)*b**3*x**6) - 18*a^{**5}*b**2*x**4*\log(\sqrt{1 + b*x**2/a} + 1)/(6*a^{**}(19/2) + 18*a^{**}(17/2)*b*x**2 + 18*a^{**}(15/2)*b**2*x**4 + 6*a^{**}(13/2)*b**3*x**6)$

$$\frac{15}{2} b^2 x^4 + 6 a^{13/2} b^3 x^6 + 3 a^4 b^3 x^6 \log\left(\frac{b x^2/a}{6 a^{19/2} + 18 a^{17/2} b x^2 + 18 a^{15/2} b^2 x^4 + 6 a^{13/2} b^3 x^6} - 6 a^4 b^3 x^6 \log(\sqrt{1 + b x^2/a} + 1)\right) / (6 a^{19/2} + 18 a^{17/2} b x^2 + 18 a^{15/2} b^2 x^4 + 6 a^{13/2} b^3 x^6)$$

**GIAC/XCAS [A]** time = 0.213141, size = 68, normalized size = 1.15

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-aa^2}} + \frac{3bx^2 + 4a}{3(bx^2 + a)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/2)\*x),x, algorithm="giac")

[Out] arctan(sqrt(b\*x^2 + a)/sqrt(-a))/(sqrt(-a)\*a^2) + 1/3\*(3\*b\*x^2 + 4\*a)/((b\*x^2 + a)^(3/2)\*a^2)

$$3.514 \quad \int \frac{1}{x^2(a+bx^2)^{5/2}} dx$$

**Optimal.** Leaf size=60

$$-\frac{8bx}{3a^3\sqrt{a+bx^2}} - \frac{4bx}{3a^2(a+bx^2)^{3/2}} - \frac{1}{ax(a+bx^2)^{3/2}}$$

[Out]  $-(1/(a*x*(a+b*x^2)^(3/2))) - (4*b*x)/(3*a^2*(a+b*x^2)^(3/2)) - (8*b*x)/(3*a^3*\text{Sqrt}[a+b*x^2])$

**Rubi [A]** time = 0.0480806, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{8bx}{3a^3\sqrt{a+bx^2}} - \frac{4bx}{3a^2(a+bx^2)^{3/2}} - \frac{1}{ax(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a+b\*x^2)^(5/2)),x]

[Out]  $-(1/(a*x*(a+b*x^2)^(3/2))) - (4*b*x)/(3*a^2*(a+b*x^2)^(3/2)) - (8*b*x)/(3*a^3*\text{Sqrt}[a+b*x^2])$

**Rubi in Sympy [A]** time = 5.40753, size = 56, normalized size = 0.93

$$-\frac{1}{ax(a+bx^2)^{\frac{3}{2}}} - \frac{4bx}{3a^2(a+bx^2)^{\frac{3}{2}}} - \frac{8bx}{3a^3\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(b\*x\*\*2+a)\*\*(5/2),x)

[Out]  $-1/(a*x*(a+b*x**2)**(3/2)) - 4*b*x/(3*a**2*(a+b*x**2)**(3/2)) - 8*b*x/(3*a**3*\text{sqrt}(a+b*x**2))$

**Mathematica [A]** time = 0.033547, size = 42, normalized size = 0.7

$$-\frac{3a^2 + 12abx^2 + 8b^2x^4}{3a^3x(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a+b\*x^2)^(5/2)),x]

[Out]  $-(3*a^2 + 12*a*b*x^2 + 8*b^2*x^4)/(3*a^3*x*(a+b*x^2)^(3/2))$

**Maple [A]** time = 0.008, size = 39, normalized size = 0.7

$$-\frac{8b^2x^4 + 12abx^2 + 3a^2}{3a^3x} (bx^2 + a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(1/x^2/(b*x^2+a)^(5/2),x)`

[Out]  $-1/3*(8*b^2*x^4+12*a*b*x^2+3*a^2)/x/(b*x^2+a)^(3/2)/a^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/2)*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.240366, size = 80, normalized size = 1.33

$$-\frac{(8b^2x^4 + 12abx^2 + 3a^2)\sqrt{bx^2 + a}}{3(a^3b^2x^5 + 2a^4bx^3 + a^5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/2)*x^2),x, algorithm="fricas")`

[Out]  $-1/3*(8*b^2*x^4 + 12*a*b*x^2 + 3*a^2)*\text{sqrt}(b*x^2 + a)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)$

**Sympy [A]** time = 4.82485, size = 165, normalized size = 2.75

$$-\frac{3a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2} + 1}}{3a^5b^4 + 6a^4b^5x^2 + 3a^3b^6x^4} - \frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2} + 1}}{3a^5b^4 + 6a^4b^5x^2 + 3a^3b^6x^4} - \frac{8b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2} + 1}}{3a^5b^4 + 6a^4b^5x^2 + 3a^3b^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(b*x**2+a)**(5/2),x)`

[Out]  $-3*a**2*b**(9/2)*\text{sqrt}(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*\text{sqrt}(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*\text{sqrt}(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4)$

**GIAC/XCAS [A]** time = 0.213645, size = 86, normalized size = 1.43

$$-\frac{x\left(\frac{5b^2x^2}{a^3} + \frac{6b}{a^2}\right)}{3(bx^2 + a)^{\frac{3}{2}}} + \frac{2\sqrt{b}}{\left(\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^2 - a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/2)*x^2),x, algorithm="giac")`

[Out]  $-1/3*x*(5*b^2*x^2/a^3 + 6*b/a^2)/(b*x^2 + a)^(3/2) + 2*\text{sqrt}(b)/((\text{sqrt}(b)*x - \text{sqrt}(b*x^2 + a))^2 - a)*a^2)$

$$3.515 \quad \int \frac{1}{x^3(a+bx^2)^{5/2}} dx$$

**Optimal.** Leaf size=92

$$\frac{5b \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}} - \frac{5\sqrt{a+bx^2}}{2a^3x^2} + \frac{5}{3a^2x^2\sqrt{a+bx^2}} + \frac{1}{3ax^2(a+bx^2)^{3/2}}$$

[Out]  $1/(3*a*x^2*(a+b*x^2)^(3/2)) + 5/(3*a^2*x^2*Sqrt[a+b*x^2]) - (5*Sqrt[a+b*x^2])/(2*a^3*x^2) + (5*b*ArcTanh[Sqrt[a+b*x^2]/Sqrt[a]])/(2*a^(7/2))$

**Rubi [A]** time = 0.14283, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{5b \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}} - \frac{5\sqrt{a+bx^2}}{2a^3x^2} + \frac{5}{3a^2x^2\sqrt{a+bx^2}} + \frac{1}{3ax^2(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(a+b\*x^2)^(5/2)),x]

[Out]  $1/(3*a*x^2*(a+b*x^2)^(3/2)) + 5/(3*a^2*x^2*Sqrt[a+b*x^2]) - (5*Sqrt[a+b*x^2])/(2*a^3*x^2) + (5*b*ArcTanh[Sqrt[a+b*x^2]/Sqrt[a]])/(2*a^(7/2))$

**Rubi in Sympy [A]** time = 14.0995, size = 85, normalized size = 0.92

$$\frac{1}{3ax^2(a+bx^2)^{3/2}} + \frac{5}{3a^2x^2\sqrt{a+bx^2}} - \frac{5\sqrt{a+bx^2}}{2a^3x^2} + \frac{5b \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(b\*x\*\*2+a)\*\*(5/2),x)

[Out]  $1/(3*a*x**2*(a+b*x**2)**(3/2)) + 5/(3*a**2*x**2*sqrt(a+b*x**2)) - 5*sqrt(a+b*x**2)/(2*a**3*x**2) + 5*b*atanh(sqrt(a+b*x**2)/sqrt(a))/(2*a**(7/2))$

**Mathematica [A]** time = 0.183753, size = 80, normalized size = 0.87

$$\frac{-\frac{\sqrt{a}(3a^2+20abx^2+15b^2x^4)}{x^2(a+bx^2)^{3/2}} + 15b \log\left(\sqrt{a}\sqrt{a+bx^2} + a\right) - 15b \log(x)}{6a^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a+b\*x^2)^(5/2)),x]

[Out]  $(-((Sqrt[a]*(3*a^2+20*a*b*x^2+15*b^2*x^4))/(x^2*(a+b*x^2)^(3/2))) - 15*b*Log[x] + 15*b*Log[a+Sqrt[a]*Sqrt[a+b*x^2]])/(6*a^(7/2))$

**Maple [A]** time = 0.008, size = 78, normalized size = 0.9

$$-\frac{1}{2ax^2} (bx^2 + a)^{-\frac{3}{2}} - \frac{5b}{6a^2} (bx^2 + a)^{-\frac{3}{2}} - \frac{5b}{2a^3} \frac{1}{\sqrt{bx^2 + a}} + \frac{5b}{2} \ln\left(\frac{1}{x} (2a + 2\sqrt{a}\sqrt{bx^2 + a})\right) a^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(b\*x^2+a)^(5/2), x)

[Out] -1/2/a/x^2/(b\*x^2+a)^(3/2)-5/6\*b/a^2/(b\*x^2+a)^(3/2)-5/2\*b/a^3/(b\*x^2+a)^(1/2)+5/2\*b/a^(7/2)\*ln((2\*a+2\*a^(1/2)\*(b\*x^2+a)^(1/2))/x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/2)\*x^3), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.255272, size = 1, normalized size = 0.01

$$\left[ \frac{2(15b^2x^4 + 20abx^2 + 3a^2)\sqrt{bx^2 + a}\sqrt{a} - 15(b^3x^6 + 2ab^2x^4 + a^2bx^2) \log\left(-\frac{(bx^2+2a)\sqrt{a+2\sqrt{bx^2+aa}}}{x^2}\right)}{12(a^3b^2x^6 + 2a^4bx^4 + a^5x^2)\sqrt{a}}, \right. \\ \left. \frac{(15b^2x^4 + 20abx^2 + 3a^2)\sqrt{bx^2 + a}\sqrt{-a} - 15(b^3x^6 + 2ab^2x^4 + a^2bx^2) \arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{6(a^3b^2x^6 + 2a^4bx^4 + a^5x^2)\sqrt{-a}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/2)\*x^3), x, algorithm="fricas")

[Out] [-1/12\*(2\*(15\*b^2\*x^4 + 20\*a\*b\*x^2 + 3\*a^2)\*sqrt(b\*x^2 + a)\*sqrt(a) - 15\*(b^3\*x^6 + 2\*a\*b^2\*x^4 + a^2\*b\*x^2)\*log(-((b\*x^2 + 2\*a)\*sqrt(a) + 2\*sqrt(b\*x^2 + a)\*a)/x^2))/((a^3\*b^2\*x^6 + 2\*a^4\*b\*x^4 + a^5\*x^2)\*sqrt(a)), -1/6\*((15\*b^2\*x^4 + 20\*a\*b\*x^2 + 3\*a^2)\*sqrt(b\*x^2 + a)\*sqrt(-a) - 15\*(b^3\*x^6 + 2\*a\*b^2\*x^4 + a^2\*b\*x^2)\*arctan(sqrt(-a)/sqrt(b\*x^2 + a)))/((a^3\*b^2\*x^6 + 2\*a^4\*b\*x^4 + a^5\*x^2)\*sqrt(-a))]

**Sympy [A]** time = 18.0549, size = 864, normalized size = 9.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(b\*x\*\*2+a)\*\*(5/2), x)

[Out] -6\*a\*\*17\*sqrt(1 + b\*x\*\*2/a)/(12\*a\*\*(39/2)\*x\*\*2 + 36\*a\*\*(37/2)\*b\*x\*\*4 + 36\*a\*\*(35/2)\*b\*\*2\*x\*\*6 + 12\*a\*\*(33/2)\*b\*\*3\*x\*\*8) - 46\*a\*\*16\*b\*x\*\*2\*sqrt(1 + b\*x\*\*2/a)/(12\*a\*\*(39/2)\*x\*\*2 + 36\*a\*\*(37/2)\*b\*x\*\*4 + 36\*a\*\*(35/2)\*b\*\*2\*x\*\*6 + 12\*a\*\*(33/2)\*b\*\*3\*x\*\*8)

```

*4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 15*a**16*
b*x**2*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 3
6*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 30*a**16*b*x**2
*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*
x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70*a**1
5*b**2*x**4*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*
b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a
**15*b**2*x**4*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x
**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a**15
*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a
*(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8)
- 30*a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*
a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**
8) - 45*a**14*b**3*x**6*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**
(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8)
+ 90*a**14*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x
**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*
b**3*x**8) - 15*a**13*b**4*x**8*log(b*x**2/a)/(12*a**(39/2)*x**2
+ 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**
3*x**8) + 30*a**13*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(12*a**
(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a
*(33/2)*b**3*x**8)

```

---

**GIAC/XCAS [A]** time = 0.230744, size = 100, normalized size = 1.09

$$-\frac{1}{6}b\left(\frac{15\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3} + \frac{2(6bx^2+7a)}{(bx^2+a)^{\frac{3}{2}}a^3} + \frac{3\sqrt{bx^2+a}}{a^3bx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/2)\*x^3),x, algorithm="giac")

[Out] -1/6\*b\*(15\*arctan(sqrt(b\*x^2 + a)/sqrt(-a))/(sqrt(-a)\*a^3) + 2\*(6\*b\*x^2 + 7\*a)/((b\*x^2 + a)^(3/2)\*a^3) + 3\*sqrt(b\*x^2 + a)/(a^3\*b\*x^2))

$$3.516 \quad \int \frac{1}{x^4(a+bx^2)^{5/2}} dx$$

**Optimal.** Leaf size=86

$$\frac{16b^2x}{3a^4\sqrt{a+bx^2}} + \frac{8b^2x}{3a^3(a+bx^2)^{3/2}} + \frac{2b}{a^2x(a+bx^2)^{3/2}} - \frac{1}{3ax^3(a+bx^2)^{3/2}}$$

[Out]  $-1/(3*a*x^3*(a+b*x^2)^(3/2)) + (2*b)/(a^2*x*(a+b*x^2)^(3/2)) + (8*b^2*x)/(3*a^3*(a+b*x^2)^(3/2)) + (16*b^2*x)/(3*a^4*\text{Sqrt}[a+b*x^2])$

**Rubi [A]** time = 0.0782973, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{16b^2x}{3a^4\sqrt{a+bx^2}} + \frac{8b^2x}{3a^3(a+bx^2)^{3/2}} + \frac{2b}{a^2x(a+bx^2)^{3/2}} - \frac{1}{3ax^3(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a+b\*x^2)^(5/2)),x]

[Out]  $-1/(3*a*x^3*(a+b*x^2)^(3/2)) + (2*b)/(a^2*x*(a+b*x^2)^(3/2)) + (8*b^2*x)/(3*a^3*(a+b*x^2)^(3/2)) + (16*b^2*x)/(3*a^4*\text{Sqrt}[a+b*x^2])$

**Rubi in Sympy [A]** time = 8.95041, size = 80, normalized size = 0.93

$$-\frac{1}{3ax^3(a+bx^2)^{\frac{3}{2}}} + \frac{2b}{a^2x(a+bx^2)^{\frac{3}{2}}} + \frac{8b^2x}{3a^3(a+bx^2)^{\frac{3}{2}}} + \frac{16b^2x}{3a^4\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(b\*x\*\*2+a)\*\*(5/2),x)

[Out]  $-1/(3*a*x**3*(a+b*x**2)**(3/2)) + 2*b/(a**2*x*(a+b*x**2)**(3/2)) + 8*b**2*x/(3*a**3*(a+b*x**2)**(3/2)) + 16*b**2*x/(3*a**4*\text{sqrt}(a+b*x**2))$

**Mathematica [A]** time = 0.0402852, size = 53, normalized size = 0.62

$$\frac{-a^3 + 6a^2bx^2 + 24ab^2x^4 + 16b^3x^6}{3a^4x^3(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a+b\*x^2)^(5/2)),x]

[Out]  $(-a^3 + 6*a^2*b*x^2 + 24*a*b^2*x^4 + 16*b^3*x^6)/(3*a^4*x^3*(a+b*x^2)^(3/2))$

**Maple [A]** time = 0.007, size = 48, normalized size = 0.6

$$-\frac{-16b^3x^6 - 24ab^2x^4 - 6a^2bx^2 + a^3}{3x^3a^4} (bx^2 + a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(b*x^2+a)^(5/2),x)`

[Out]  $-1/3 * (-16 * b^3 * x^6 - 24 * a * b^2 * x^4 - 6 * a^2 * b * x^2 + a^3) / x^3 / (b * x^2 + a)^{(3/2)} / a^4$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/2)*x^4),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.249279, size = 97, normalized size = 1.13

$$\frac{(16b^3x^6 + 24ab^2x^4 + 6a^2bx^2 - a^3)\sqrt{bx^2 + a}}{3(a^4b^2x^7 + 2a^5bx^5 + a^6x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/2)*x^4),x, algorithm="fricas")`

[Out]  $1/3 * (16 * b^3 * x^6 + 24 * a * b^2 * x^4 + 6 * a^2 * b * x^2 - a^3) * \text{sqrt}(b * x^2 + a) / (a^4 * b^2 * x^7 + 2 * a^5 * b * x^5 + a^6 * x^3)$

**Sympy [A]** time = 7.003, size = 354, normalized size = 4.12

$$\begin{aligned} & -\frac{a^4 b^{\frac{19}{2}} \sqrt{\frac{a}{bx^2} + 1}}{3a^7 b^9 x^2 + 9a^6 b^{10} x^4 + 9a^5 b^{11} x^6 + 3a^4 b^{12} x^8} + \frac{5a^3 b^{\frac{21}{2}} x^2 \sqrt{\frac{a}{bx^2} + 1}}{3a^7 b^9 x^2 + 9a^6 b^{10} x^4 + 9a^5 b^{11} x^6 + 3a^4 b^{12} x^8} \\ & + \frac{30a^2 b^{\frac{23}{2}} x^4 \sqrt{\frac{a}{bx^2} + 1}}{3a^7 b^9 x^2 + 9a^6 b^{10} x^4 + 9a^5 b^{11} x^6 + 3a^4 b^{12} x^8} + \frac{40ab^{\frac{25}{2}} x^6 \sqrt{\frac{a}{bx^2} + 1}}{3a^7 b^9 x^2 + 9a^6 b^{10} x^4 + 9a^5 b^{11} x^6 + 3a^4 b^{12} x^8} \\ & + \frac{16b^{\frac{27}{2}} x^8 \sqrt{\frac{a}{bx^2} + 1}}{3a^7 b^9 x^2 + 9a^6 b^{10} x^4 + 9a^5 b^{11} x^6 + 3a^4 b^{12} x^8} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(b*x**2+a)**(5/2),x)`

[Out]  $-a^{**4} * b^{** (19/2)} * \text{sqrt}(a / (b * x^{**2}) + 1) / (3 * a^{**7} * b^{**9} * x^{**2} + 9 * a^{**6} * b^{**10} * x^{**4} + 9 * a^{**5} * b^{**11} * x^{**6} + 3 * a^{**4} * b^{**12} * x^{**8}) + 5 * a^{**3} * b^{** (21/2)} * x^{**2} * \text{sqrt}(a / (b * x^{**2}) + 1) / (3 * a^{**7} * b^{**9} * x^{**2} + 9 * a^{**6} * b^{**10} * x^{**4} + 9 * a^{**5} * b^{**11} * x^{**6} + 3 * a^{**4} * b^{**12} * x^{**8}) + 30 * a^{**2} * b^{** (23/2)} * x^{**4} * \text{sqrt}(a / (b * x^{**2}) + 1) / (3 * a^{**7} * b^{**9} * x^{**2} + 9 * a^{**6} * b^{**10} * x^{**4} + 9 * a^{**5} * b^{**11} * x^{**6} + 3 * a^{**4} * b^{**12} * x^{**8}) + 40 * a * b^{** (25/2)} * x^{**6} * \text{sqrt}(a / (b * x^{**2}) + 1) / (3 * a^{**7} * b^{**9} * x^{**2} + 9 * a^{**6} * b^{**10} * x^{**4} + 9 * a^{**5} * b^{**11} * x^{**6} + 3 * a^{**4} * b^{**12} * x^{**8}) + 16 * b^{** (27/2)} * x^{**8} * \text{sqrt}(a / (b * x^{**2}) + 1) / (3 * a^{**7} * b^{**9} * x^{**2} + 9 * a^{**6} * b^{**10} * x^{**4} + 9 * a^{**5} * b^{**11} * x^{**6} + 3 * a^{**4} * b^{**12} * x^{**8})$

**GIAC/XCAS [A]** time = 0.228498, size = 163, normalized size = 1.9

$$\frac{x\left(\frac{8b^3x^2}{a^4} + \frac{9b^2}{a^3}\right)}{3(bx^2 + a)^{\frac{3}{2}}} - \frac{4\left(3\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^4 b^{\frac{3}{2}} - 9\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^2 ab^{\frac{3}{2}} + 4a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^2 - a\right)^3 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/2)\*x^4),x, algorithm="giac")

[Out] 1/3\*x\*(8\*b^3\*x^2/a^4 + 9\*b^2/a^3)/(b\*x^2 + a)^(3/2) - 4/3\*(3\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^4\*b^(3/2) - 9\*(sqrt(b)\*x - sqrt(b\*x^2 + a))^2\*a\*b^(3/2) + 4\*a^2\*b^(3/2))/(((sqrt(b)\*x - sqrt(b\*x^2 + a))^2 - a)^3\*a^3)

$$3.517 \quad \int \frac{x^{10}}{(a+bx^2)^{9/2}} dx$$

**Optimal.** Leaf size=131

$$-\frac{9a \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{2b^{11/2}} + \frac{9x\sqrt{a+bx^2}}{2b^5} - \frac{3x^3}{b^4\sqrt{a+bx^2}} - \frac{3x^5}{5b^3(a+bx^2)^{3/2}} - \frac{9x^7}{35b^2(a+bx^2)^{5/2}} - \frac{x^9}{7b(a+bx^2)^{7/2}}$$

[Out]  $-x^9/(7*b*(a + b*x^2)^(7/2)) - (9*x^7)/(35*b^2*(a + b*x^2)^(5/2)) - (3*x^5)/(5*b^3*(a + b*x^2)^(3/2)) - (3*x^3)/(b^4*\text{Sqrt}[a + b*x^2]) + (9*x*\text{Sqrt}[a + b*x^2])/(2*b^5) - (9*a*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]])/(2*b^(11/2))$

**Rubi [A]** time = 0.172094, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{9a \tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{2b^{11/2}} + \frac{9x\sqrt{a+bx^2}}{2b^5} - \frac{3x^3}{b^4\sqrt{a+bx^2}} - \frac{3x^5}{5b^3(a+bx^2)^{3/2}} - \frac{9x^7}{35b^2(a+bx^2)^{5/2}} - \frac{x^9}{7b(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[x^10/(a + b\*x^2)^(9/2), x]

[Out]  $-x^9/(7*b*(a + b*x^2)^(7/2)) - (9*x^7)/(35*b^2*(a + b*x^2)^(5/2)) - (3*x^5)/(5*b^3*(a + b*x^2)^(3/2)) - (3*x^3)/(b^4*\text{Sqrt}[a + b*x^2]) + (9*x*\text{Sqrt}[a + b*x^2])/(2*b^5) - (9*a*\text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]])/(2*b^(11/2))$

**Rubi in Sympy [A]** time = 22.8746, size = 122, normalized size = 0.93

$$-\frac{9a \operatorname{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{2b^{\frac{11}{2}}} - \frac{x^9}{7b(a+bx^2)^{\frac{7}{2}}} - \frac{9x^7}{35b^2(a+bx^2)^{\frac{5}{2}}} - \frac{3x^5}{5b^3(a+bx^2)^{\frac{3}{2}}} - \frac{3x^3}{b^4\sqrt{a+bx^2}} + \frac{9x\sqrt{a+bx^2}}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*10/(b\*x\*\*2+a)\*\*(9/2), x)

[Out]  $-9*a*\operatorname{atanh}(\text{sqrt}(b)*x/\text{sqrt}(a + b*x**2))/(2*b**(11/2)) - x**9/(7*b*(a + b*x**2)**(7/2)) - 9*x**7/(35*b**2*(a + b*x**2)**(5/2)) - 3*x**5/(5*b**3*(a + b*x**2)**(3/2)) - 3*x**3/(b**4*\text{sqrt}(a + b*x**2)) + 9*x*\text{sqrt}(a + b*x**2)/(2*b**5)$

**Mathematica [A]** time = 0.200454, size = 95, normalized size = 0.73

$$\frac{315a^4x + 1050a^3bx^3 + 1218a^2b^2x^5 + 528ab^3x^7 + 35b^4x^9}{70b^5(a+bx^2)^{7/2}} - \frac{9a \log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right)}{2b^{11/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^10/(a + b\*x^2)^(9/2), x]

[Out]  $(315*a^4*x + 1050*a^3*b*x^3 + 1218*a^2*b^2*x^5 + 528*a*b^3*x^7 + 35*b^4*x^9)/(70*b^5*(a + b*x^2)^(7/2)) - (9*a*\text{Log}[b*x + \text{Sqrt}[b]*\text{Sqrt}[a + b*x^2]])/(2*b^(11/2))$



**Maple [A]** time = 0.048, size = 111, normalized size = 0.9

$$\frac{x^9}{2b} (bx^2 + a)^{-\frac{7}{2}} + \frac{9ax^7}{14b^2} (bx^2 + a)^{-\frac{7}{2}} + \frac{9ax^5}{10b^3} (bx^2 + a)^{-\frac{5}{2}} \\ + \frac{3ax^3}{2b^4} (bx^2 + a)^{-\frac{3}{2}} + \frac{9ax}{2b^5} \frac{1}{\sqrt{bx^2 + a}} - \frac{9a}{2} \ln(x\sqrt{b} + \sqrt{bx^2 + a}) b^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^10/(b\*x^2+a)^(9/2), x)

[Out] 1/2\*x^9/b/(b\*x^2+a)^(7/2)+9/14\*a/b^2\*x^7/(b\*x^2+a)^(7/2)+9/10\*a/b^3\*x^5/(b\*x^2+a)^(5/2)+3/2\*a/b^4\*x^3/(b\*x^2+a)^(3/2)+9/2\*a/b^5\*x/(b\*x^2+a)^(1/2)-9/2\*a/b^(11/2)\*ln(x\*b^(1/2)+(b\*x^2+a)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10/(b\*x^2 + a)^(9/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.324428, size = 1, normalized size = 0.01

$$\left[ \frac{2(35b^4x^9 + 528ab^3x^7 + 1218a^2b^2x^5 + 1050a^3bx^3 + 315a^4x)\sqrt{bx^2+a}\sqrt{b} + 315(ab^4x^8 + 4a^2b^3x^6 + 6a^3b^2x^4 + 4a^4bx^2 + a^5)}{140(b^9x^8 + 4ab^8x^6 + 6a^2b^7x^4 + 4a^3b^6x^2 + a^4b^5)\sqrt{b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10/(b\*x^2 + a)^(9/2), x, algorithm="fricas")

[Out] [1/140\*(2\*(35\*b^4\*x^9 + 528\*a\*b^3\*x^7 + 1218\*a^2\*b^2\*x^5 + 1050\*a^3\*b\*x^3 + 315\*a^4\*x)\*sqrt(b\*x^2 + a)\*sqrt(b) + 315\*(a\*b^4\*x^8 + 4\*a^2\*b^3\*x^6 + 6\*a^3\*b^2\*x^4 + 4\*a^4\*b\*x^2 + a^5)\*log(2\*sqrt(b\*x^2 + a)\*b\*x - (2\*b\*x^2 + a)\*sqrt(b)))/((b^9\*x^8 + 4\*a\*b^8\*x^6 + 6\*a^2\*b^7\*x^4 + 4\*a^3\*b^6\*x^2 + a^4\*b^5)\*sqrt(b)), 1/70\*((35\*b^4\*x^9 + 528\*a\*b^3\*x^7 + 1218\*a^2\*b^2\*x^5 + 1050\*a^3\*b\*x^3 + 315\*a^4\*x)\*sqrt(b\*x^2 + a)\*sqrt(-b) - 315\*(a\*b^4\*x^8 + 4\*a^2\*b^3\*x^6 + 6\*a^3\*b^2\*x^4 + 4\*a^4\*b\*x^2 + a^5)\*arctan(sqrt(-b)\*x/sqrt(b\*x^2 + a)))/((b^9\*x^8 + 4\*a\*b^8\*x^6 + 6\*a^2\*b^7\*x^4 + 4\*a^3\*b^6\*x^2 + a^4\*b^5)\*sqrt(-b))]

**Sympy [A]** time = 45.5557, size = 3181, normalized size = 24.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*10/(b\*x\*\*2+a)\*\*(9/2), x)

[Out] -315\*a\*\*(311/2)\*b\*\*66\*sqrt(1 + b\*x\*\*2/a)\*asinh(sqrt(b)\*x/sqrt(a))/(70\*a\*\*(309/2)\*b\*\*(143/2)\*sqrt(1 + b\*x\*\*2/a) + 420\*a\*\*(307/2)\*b\*



) + 70\*a\*\*(297/2)\*b\*\*(155/2)\*x\*\*12\*sqrt(1 + b\*x\*\*2/a) + 2907\*a\*\*150\*b\*\*(143/2)\*x\*\*11/(70\*a\*\*(309/2)\*b\*\*(143/2)\*sqrt(1 + b\*x\*\*2/a) + 420\*a\*\*(307/2)\*b\*\*(145/2)\*x\*\*2\*sqrt(1 + b\*x\*\*2/a) + 1050\*a\*\*(305/2)\*b\*\*(147/2)\*x\*\*4\*sqrt(1 + b\*x\*\*2/a) + 1400\*a\*\*(303/2)\*b\*\*(149/2)\*x\*\*6\*sqrt(1 + b\*x\*\*2/a) + 1050\*a\*\*(301/2)\*b\*\*(151/2)\*x\*\*8\*sqrt(1 + b\*x\*\*2/a) + 420\*a\*\*(299/2)\*b\*\*(153/2)\*x\*\*10\*sqrt(1 + b\*x\*\*2/a) + 70\*a\*\*(297/2)\*b\*\*(155/2)\*x\*\*12\*sqrt(1 + b\*x\*\*2/a) + 633\*a\*\*149\*b\*\*(145/2)\*x\*\*13/(70\*a\*\*(309/2)\*b\*\*(143/2)\*sqrt(1 + b\*x\*\*2/a) + 420\*a\*\*(307/2)\*b\*\*(145/2)\*x\*\*2\*sqrt(1 + b\*x\*\*2/a) + 1050\*a\*\*(305/2)\*b\*\*(147/2)\*x\*\*4\*sqrt(1 + b\*x\*\*2/a) + 1400\*a\*\*(303/2)\*b\*\*(149/2)\*x\*\*6\*sqrt(1 + b\*x\*\*2/a) + 1050\*a\*\*(301/2)\*b\*\*(151/2)\*x\*\*8\*sqrt(1 + b\*x\*\*2/a) + 420\*a\*\*(299/2)\*b\*\*(153/2)\*x\*\*10\*sqrt(1 + b\*x\*\*2/a) + 70\*a\*\*(297/2)\*b\*\*(155/2)\*x\*\*12\*sqrt(1 + b\*x\*\*2/a) + 35\*a\*\*148\*b\*\*(147/2)\*x\*\*15/(70\*a\*\*(309/2)\*b\*\*(143/2)\*sqrt(1 + b\*x\*\*2/a) + 420\*a\*\*(307/2)\*b\*\*(145/2)\*x\*\*2\*sqrt(1 + b\*x\*\*2/a) + 1050\*a\*\*(305/2)\*b\*\*(147/2)\*x\*\*4\*sqrt(1 + b\*x\*\*2/a) + 1400\*a\*\*(303/2)\*b\*\*(149/2)\*x\*\*6\*sqrt(1 + b\*x\*\*2/a) + 1050\*a\*\*(301/2)\*b\*\*(151/2)\*x\*\*8\*sqrt(1 + b\*x\*\*2/a) + 420\*a\*\*(299/2)\*b\*\*(153/2)\*x\*\*10\*sqrt(1 + b\*x\*\*2/a) + 70\*a\*\*(297/2)\*b\*\*(155/2)\*x\*\*12\*sqrt(1 + b\*x\*\*2/a)

**GIAC/XCAS [A]** time = 0.227116, size = 123, normalized size = 0.94

$$\frac{\left(\left(x^2\left(\frac{35x^2}{b} + \frac{528a}{b^2}\right) + \frac{1218a^2}{b^3}\right)x^2 + \frac{1050a^3}{b^4}\right)x^2 + \frac{315a^4}{b^5}x}{70(bx^2 + a)^{\frac{7}{2}}} + \frac{9a \ln\left(\left|-\sqrt{bx} + \sqrt{bx^2 + a}\right|\right)}{2b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10/(b\*x^2 + a)^(9/2),x, algorithm="giac")

[Out] 1/70\*(((x^2\*(35\*x^2/b + 528\*a/b^2) + 1218\*a^2/b^3)\*x^2 + 1050\*a^3/b^4)\*x^2 + 315\*a^4/b^5)\*x/(b\*x^2 + a)^(7/2) + 9/2\*a\*ln(abs(-sqrt(b)\*x + sqrt(b\*x^2 + a)))/b^(11/2)

$$3.518 \quad \int \frac{x^9}{(a+bx^2)^{9/2}} dx$$

**Optimal.** Leaf size=94

$$-\frac{a^4}{7b^5(a+bx^2)^{7/2}} + \frac{4a^3}{5b^5(a+bx^2)^{5/2}} - \frac{2a^2}{b^5(a+bx^2)^{3/2}} + \frac{4a}{b^5\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}}{b^5}$$

[Out]  $-a^4/(7*b^5*(a + b*x^2)^(7/2)) + (4*a^3)/(5*b^5*(a + b*x^2)^(5/2)) - (2*a^2)/(b^5*(a + b*x^2)^(3/2)) + (4*a)/(b^5*sqrt[a + b*x^2]) + sqrt[a + b*x^2]/b^5$

**Rubi [A]** time = 0.14519, antiderivative size = 94, normalized size of antiderivative = 1., number of rules used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{a^4}{7b^5(a+bx^2)^{7/2}} + \frac{4a^3}{5b^5(a+bx^2)^{5/2}} - \frac{2a^2}{b^5(a+bx^2)^{3/2}} + \frac{4a}{b^5\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}}{b^5}$$

Antiderivative was successfully verified.

[In] Int[x^9/(a + b\*x^2)^(9/2), x]

[Out]  $-a^4/(7*b^5*(a + b*x^2)^(7/2)) + (4*a^3)/(5*b^5*(a + b*x^2)^(5/2)) - (2*a^2)/(b^5*(a + b*x^2)^(3/2)) + (4*a)/(b^5*sqrt[a + b*x^2]) + sqrt[a + b*x^2]/b^5$

**Rubi in Sympy [A]** time = 19.1032, size = 87, normalized size = 0.93

$$-\frac{a^4}{7b^5(a+bx^2)^{7/2}} + \frac{4a^3}{5b^5(a+bx^2)^{5/2}} - \frac{2a^2}{b^5(a+bx^2)^{3/2}} + \frac{4a}{b^5\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*9/(b\*x\*\*2+a)\*\*(9/2), x)

[Out]  $-a**4/(7*b**5*(a + b*x**2)**(7/2)) + 4*a**3/(5*b**5*(a + b*x**2)**(5/2)) - 2*a**2/(b**5*(a + b*x**2)**(3/2)) + 4*a/(b**5*sqrt(a + b*x**2)) + sqrt(a + b*x**2)/b**5$

**Mathematica [A]** time = 0.040633, size = 61, normalized size = 0.65

$$\frac{128a^4 + 448a^3bx^2 + 560a^2b^2x^4 + 280ab^3x^6 + 35b^4x^8}{35b^5(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^9/(a + b\*x^2)^(9/2), x]

[Out]  $(128*a^4 + 448*a^3*b*x^2 + 560*a^2*b^2*x^4 + 280*a*b^3*x^6 + 35*b^4*x^8)/(35*b^5*(a + b*x^2)^(7/2))$

**Maple [A]** time = 0.008, size = 58, normalized size = 0.6

$$\frac{35x^8b^4 + 280ax^6b^3 + 560a^2x^4b^2 + 448a^3x^2b + 128a^4}{35b^5} (bx^2 + a)^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^9/(b*x^2+a)^(9/2),x)`

[Out]  $1/35*(35*b^4*x^8+280*a*b^3*x^6+560*a^2*b^2*x^4+448*a^3*b*x^2+128*a^4)/(b*x^2+a)^(7/2)/b^5$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9/(b*x^2 + a)^(9/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.25747, size = 138, normalized size = 1.47

$$\frac{(35b^4x^8 + 280ab^3x^6 + 560a^2b^2x^4 + 448a^3bx^2 + 128a^4)\sqrt{bx^2 + a}}{35(b^9x^8 + 4ab^8x^6 + 6a^2b^7x^4 + 4a^3b^6x^2 + a^4b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^9/(b*x^2 + a)^(9/2),x, algorithm="fricas")`

[Out]  $1/35*(35*b^4*x^8 + 280*a*b^3*x^6 + 560*a^2*b^2*x^4 + 448*a^3*b*x^2 + 128*a^4)*\sqrt{b*x^2 + a}/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)$

**Sympy [A]** time = 20.2494, size = 454, normalized size = 4.83

$$\left\{ \frac{128a^4}{35a^3b^5\sqrt{a+bx^2}+105a^2b^6x^2\sqrt{a+bx^2}+105ab^7x^4\sqrt{a+bx^2}+35b^8x^6\sqrt{a+bx^2}} + \frac{448a^3bx^2}{35a^3b^5\sqrt{a+bx^2}+105a^2b^6x^2\sqrt{a+bx^2}+105ab^7x^4\sqrt{a+bx^2}+35b^8x^6\sqrt{a+bx^2}} + \frac{x^{10}}{10a^2} \right\}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**9/(b*x**2+a)**(9/2),x)`

[Out] `Piecewise(((128*a**4/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**6*sqrt(a + b*x**2)) + 448*a**3*b*x**2/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**6*sqrt(a + b*x**2)) + 560*a**2*b**2*x**4/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**6*sqrt(a + b*x**2)) + 280*a*b**3*x**6/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**6*sqrt(a + b*x**2)) + 35*b**4*x**8/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**6*sqrt(a + b*x**2))), Ne(b, 0)), (x**10/(10*a**(9/2)), True))`

**GIAC/XCAS [A]** time = 0.215655, size = 96, normalized size = 1.02

$$\frac{35 \sqrt{bx^2 + a} + \frac{140 (bx^2 + a)^3 a - 70 (bx^2 + a)^2 a^2 + 28 (bx^2 + a) a^3 - 5 a^4}{(bx^2 + a)^{\frac{7}{2}}}}{35 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(b\*x^2 + a)^(9/2),x, algorithm="giac")

[Out] 1/35\*(35\*sqrt(b\*x^2 + a) + (140\*(b\*x^2 + a)^3\*a - 70\*(b\*x^2 + a)^2\*a^2 + 28\*(b\*x^2 + a)\*a^3 - 5\*a^4)/(b\*x^2 + a)^(7/2))/b^5

$$3.519 \quad \int \frac{x^8}{(a+bx^2)^{9/2}} dx$$

**Optimal.** Leaf size=106

$$\frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{b^{9/2}} - \frac{x}{b^4\sqrt{a+bx^2}} - \frac{x^3}{3b^3(a+bx^2)^{3/2}} - \frac{x^5}{5b^2(a+bx^2)^{5/2}} - \frac{x^7}{7b(a+bx^2)^{7/2}}$$

[Out]  $-x^7/(7*b*(a + b*x^2)^{(7/2)}) - x^5/(5*b^2*(a + b*x^2)^{(5/2)}) - x^3/(3*b^3*(a + b*x^2)^{(3/2)}) - x/(b^4*\text{Sqrt}[a + b*x^2]) + \text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]]/b^{(9/2)}$

**Rubi [A]** time = 0.133731, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{b^{9/2}} - \frac{x}{b^4\sqrt{a+bx^2}} - \frac{x^3}{3b^3(a+bx^2)^{3/2}} - \frac{x^5}{5b^2(a+bx^2)^{5/2}} - \frac{x^7}{7b(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[x^8/(a + b\*x^2)^(9/2), x]

[Out]  $-x^7/(7*b*(a + b*x^2)^{(7/2)}) - x^5/(5*b^2*(a + b*x^2)^{(5/2)}) - x^3/(3*b^3*(a + b*x^2)^{(3/2)}) - x/(b^4*\text{Sqrt}[a + b*x^2]) + \text{ArcTanh}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a + b*x^2]]/b^{(9/2)}$

**Rubi in Sympy [A]** time = 17.6918, size = 92, normalized size = 0.87

$$-\frac{x^7}{7b(a+bx^2)^{\frac{7}{2}}} - \frac{x^5}{5b^2(a+bx^2)^{\frac{5}{2}}} - \frac{x^3}{3b^3(a+bx^2)^{\frac{3}{2}}} - \frac{x}{b^4\sqrt{a+bx^2}} + \frac{\text{atanh}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*8/(b\*x\*\*2+a)\*\*(9/2), x)

[Out]  $-x^{**7}/(7*b*(a + b*x^{**2})^{**}(7/2)) - x^{**5}/(5*b^{**2}*(a + b*x^{**2})^{**}(5/2)) - x^{**3}/(3*b^{**3}*(a + b*x^{**2})^{**}(3/2)) - x/(b^{**4}*\text{sqrt}(a + b*x^{**2})) + \text{atanh}(\text{sqrt}(b)*x/\text{sqrt}(a + b*x^{**2}))/b^{**}(9/2)$

**Mathematica [A]** time = 0.132754, size = 80, normalized size = 0.75

$$\frac{\log\left(\sqrt{b}\sqrt{a+bx^2} + bx\right)}{b^{9/2}} - \frac{x(105a^3 + 350a^2bx^2 + 406ab^2x^4 + 176b^3x^6)}{105b^4(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(a + b\*x^2)^(9/2), x]

[Out]  $-(x*(105*a^3 + 350*a^2*b*x^2 + 406*a*b^2*x^4 + 176*b^3*x^6))/(105*b^4*(a + b*x^2)^{(7/2)}) + \text{Log}[b*x + \text{Sqrt}[b]*\text{Sqrt}[a + b*x^2]]/b^{(9/2)}$

**Maple [A]** time = 0.024, size = 88, normalized size = 0.8

$$-\frac{x^7}{7b}(bx^2+a)^{-\frac{7}{2}} - \frac{x^5}{5b^2}(bx^2+a)^{-\frac{5}{2}} - \frac{x^3}{3b^3}(bx^2+a)^{-\frac{3}{2}} - \frac{x}{b^4\sqrt{bx^2+a}} + 1 \ln\left(x\sqrt{b} + \sqrt{bx^2+a}\right) b^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(b\*x^2+a)^(9/2), x)

[Out]  $-\frac{1}{7}x^7/b/(b*x^2+a)^{(7/2)} - \frac{1}{5}x^5/b^2/(b*x^2+a)^{(5/2)} - \frac{1}{3}x^3/b^3/(b*x^2+a)^{(3/2)} - x/b^4/(b*x^2+a)^{(1/2)} + 1/b^{(9/2)} \ln(x*b^{(1/2)} + (b*x^2+a)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b\*x^2 + a)^(9/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.271229, size = 1, normalized size = 0.01

$$\frac{2(176b^3x^7 + 406ab^2x^5 + 350a^2bx^3 + 105a^3x)\sqrt{bx^2+a}\sqrt{b} - 105(b^4x^8 + 4ab^3x^6 + 6a^2b^2x^4 + 4a^3bx^2 + a^4)\log(-2\sqrt{bx^2+a}\sqrt{b})}{210(b^8x^8 + 4ab^7x^6 + 6a^2b^6x^4 + 4a^3b^5x^2 + a^4b^4)\sqrt{b}} \\ + \frac{(176b^3x^7 + 406ab^2x^5 + 350a^2bx^3 + 105a^3x)\sqrt{bx^2+a}\sqrt{-b} - 105(b^4x^8 + 4ab^3x^6 + 6a^2b^2x^4 + 4a^3bx^2 + a^4)\arctan\left(\frac{\sqrt{-b}}{\sqrt{bx^2+a}}\right)}{105(b^8x^8 + 4ab^7x^6 + 6a^2b^6x^4 + 4a^3b^5x^2 + a^4b^4)\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b\*x^2 + a)^(9/2), x, algorithm="fricas")

[Out]  $[-1/210*(2*(176*b^3*x^7 + 406*a*b^2*x^5 + 350*a^2*b*x^3 + 105*a^3*x)*\sqrt{b*x^2+a}*\sqrt{b} - 105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*\log(-2*\sqrt{b*x^2+a}*\sqrt{b}))/((b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)*\sqrt{b}), -1/105*((176*b^3*x^7 + 406*a*b^2*x^5 + 350*a^2*b*x^3 + 105*a^3*x)*\sqrt{b*x^2+a}*\sqrt{-b} - 105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*\arctan(\sqrt{-b}*x/\sqrt{b*x^2+a}))/((b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)*\sqrt{-b})]$

**Sympy [A]** time = 31.4447, size = 2980, normalized size = 28.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*8/(b\*x\*\*2+a)\*\*(9/2), x)

[Out]  $105*a**(205/2)*b**45*\sqrt{1+b*x**2/a}*\operatorname{asinh}(\sqrt{b}*x/\sqrt{a})/(105*a**(205/2)*b**(99/2)*\sqrt{1+b*x**2/a}) + 630*a**(203/2)*b**$



$$\begin{aligned}
& (101/2)*x^{**2}*sqrt(1 + b*x^{**2}/a) + 1575*a^{**}(201/2)*b^{**}(103/2)*x^{**4} \\
& *sqrt(1 + b*x^{**2}/a) + 2100*a^{**}(199/2)*b^{**}(105/2)*x^{**6}*sqrt(1 + b* \\
& x^{**2}/a) + 1575*a^{**}(197/2)*b^{**}(107/2)*x^{**8}*sqrt(1 + b*x^{**2}/a) + 63 \\
& 0*a^{**}(195/2)*b^{**}(109/2)*x^{**10}*sqrt(1 + b*x^{**2}/a) + 105*a^{**}(193/2) \\
& *b^{**}(111/2)*x^{**12}*sqrt(1 + b*x^{**2}/a) + 630*a^{**}(203/2)*b^{**46}*x^{**2} \\
& *sqrt(1 + b*x^{**2}/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a^{**}(205/2)*b^{**}( \\
& 99/2)*sqrt(1 + b*x^{**2}/a) + 630*a^{**}(203/2)*b^{**}(101/2)*x^{**2}*sqrt(1 \\
& + b*x^{**2}/a) + 1575*a^{**}(201/2)*b^{**}(103/2)*x^{**4}*sqrt(1 + b*x^{**2}/a) \\
& + 2100*a^{**}(199/2)*b^{**}(105/2)*x^{**6}*sqrt(1 + b*x^{**2}/a) + 1575*a^{**}(1 \\
& 97/2)*b^{**}(107/2)*x^{**8}*sqrt(1 + b*x^{**2}/a) + 630*a^{**}(195/2)*b^{**}(109 \\
& /2)*x^{**10}*sqrt(1 + b*x^{**2}/a) + 105*a^{**}(193/2)*b^{**}(111/2)*x^{**12}*sq \\
& rt(1 + b*x^{**2}/a) + 1575*a^{**}(201/2)*b^{**47}*x^{**4}*sqrt(1 + b*x^{**2}/a) \\
& *asinh(sqrt(b)*x/sqrt(a))/(105*a^{**}(205/2)*b^{**}(99/2)*sqrt(1 + b*x* \\
& **2/a) + 630*a^{**}(203/2)*b^{**}(101/2)*x^{**2}*sqrt(1 + b*x^{**2}/a) + 1575* \\
& a^{**}(201/2)*b^{**}(103/2)*x^{**4}*sqrt(1 + b*x^{**2}/a) + 2100*a^{**}(199/2)*b \\
& ** (105/2)*x^{**6}*sqrt(1 + b*x^{**2}/a) + 1575*a^{**}(197/2)*b^{**}(107/2)*x* \\
& **8*sqrt(1 + b*x^{**2}/a) + 630*a^{**}(195/2)*b^{**}(109/2)*x^{**10}*sqrt(1 + \\
& b*x^{**2}/a) + 105*a^{**}(193/2)*b^{**}(111/2)*x^{**12}*sqrt(1 + b*x^{**2}/a) + \\
& 2100*a^{**}(199/2)*b^{**48}*x^{**6}*sqrt(1 + b*x^{**2}/a)*asinh(sqrt(b)*x/sq \\
& rt(a))/(105*a^{**}(205/2)*b^{**}(99/2)*sqrt(1 + b*x^{**2}/a) + 630*a^{**}(203 \\
& /2)*b^{**}(101/2)*x^{**2}*sqrt(1 + b*x^{**2}/a) + 1575*a^{**}(201/2)*b^{**}(103/ \\
& 2)*x^{**4}*sqrt(1 + b*x^{**2}/a) + 2100*a^{**}(199/2)*b^{**}(105/2)*x^{**6}*sqrt \\
& (1 + b*x^{**2}/a) + 1575*a^{**}(197/2)*b^{**}(107/2)*x^{**8}*sqrt(1 + b*x^{**2}/ \\
& a) + 630*a^{**}(195/2)*b^{**}(109/2)*x^{**10}*sqrt(1 + b*x^{**2}/a) + 105*a^{**} \\
& (193/2)*b^{**}(111/2)*x^{**12}*sqrt(1 + b*x^{**2}/a) + 1575*a^{**}(197/2)*b* \\
& **49*x^{**8}*sqrt(1 + b*x^{**2}/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a^{**}(205 \\
& /2)*b^{**}(99/2)*sqrt(1 + b*x^{**2}/a) + 630*a^{**}(203/2)*b^{**}(101/2)*x^{**2} \\
& *sqrt(1 + b*x^{**2}/a) + 1575*a^{**}(201/2)*b^{**}(103/2)*x^{**4}*sqrt(1 + b* \\
& x^{**2}/a) + 2100*a^{**}(199/2)*b^{**}(105/2)*x^{**6}*sqrt(1 + b*x^{**2}/a) + 15 \\
& 75*a^{**}(197/2)*b^{**}(107/2)*x^{**8}*sqrt(1 + b*x^{**2}/a) + 630*a^{**}(195/2) \\
& *b^{**}(109/2)*x^{**10}*sqrt(1 + b*x^{**2}/a) + 105*a^{**}(193/2)*b^{**}(111/2)* \\
& x^{**12}*sqrt(1 + b*x^{**2}/a) + 630*a^{**}(195/2)*b^{**50}*x^{**10}*sqrt(1 + b \\
& *x^{**2}/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a^{**}(205/2)*b^{**}(99/2)*sqrt( \\
& 1 + b*x^{**2}/a) + 630*a^{**}(203/2)*b^{**}(101/2)*x^{**2}*sqrt(1 + b*x^{**2}/a) \\
& + 1575*a^{**}(201/2)*b^{**}(103/2)*x^{**4}*sqrt(1 + b*x^{**2}/a) + 2100*a^{**}( \\
& 199/2)*b^{**}(105/2)*x^{**6}*sqrt(1 + b*x^{**2}/a) + 1575*a^{**}(197/2)*b^{**}(1 \\
& 07/2)*x^{**8}*sqrt(1 + b*x^{**2}/a) + 630*a^{**}(195/2)*b^{**}(109/2)*x^{**10}*s \\
& qrt(1 + b*x^{**2}/a) + 105*a^{**}(193/2)*b^{**}(111/2)*x^{**12}*sqrt(1 + b*x* \\
& **2/a) + 105*a^{**}(193/2)*b^{**51}*x^{**12}*sqrt(1 + b*x^{**2}/a)*asinh(sqrt \\
& (b)*x/sqrt(a))/(105*a^{**}(205/2)*b^{**}(99/2)*sqrt(1 + b*x^{**2}/a) + 630 \\
& *a^{**}(203/2)*b^{**}(101/2)*x^{**2}*sqrt(1 + b*x^{**2}/a) + 1575*a^{**}(201/2)* \\
& b^{**}(103/2)*x^{**4}*sqrt(1 + b*x^{**2}/a) + 2100*a^{**}(199/2)*b^{**}(105/2)*x \\
& **6*sqrt(1 + b*x^{**2}/a) + 1575*a^{**}(197/2)*b^{**}(107/2)*x^{**8}*sqrt(1 + \\
& b*x^{**2}/a) + 630*a^{**}(195/2)*b^{**}(109/2)*x^{**10}*sqrt(1 + b*x^{**2}/a) + \\
& 105*a^{**}(193/2)*b^{**}(111/2)*x^{**12}*sqrt(1 + b*x^{**2}/a) - 105*a^{**}102 \\
& *b^{**}(91/2)*x/(105*a^{**}(205/2)*b^{**}(99/2)*sqrt(1 + b*x^{**2}/a) + 630*a \\
& ** (203/2)*b^{**}(101/2)*x^{**2}*sqrt(1 + b*x^{**2}/a) + 1575*a^{**}(201/2)*b* \\
& ** (103/2)*x^{**4}*sqrt(1 + b*x^{**2}/a) + 2100*a^{**}(199/2)*b^{**}(105/2)*x* \\
& **6*sqrt(1 + b*x^{**2}/a) + 1575*a^{**}(197/2)*b^{**}(107/2)*x^{**8}*sqrt(1 + b \\
& *x^{**2}/a) + 630*a^{**}(195/2)*b^{**}(109/2)*x^{**10}*sqrt(1 + b*x^{**2}/a) + 1 \\
& 05*a^{**}(193/2)*b^{**}(111/2)*x^{**12}*sqrt(1 + b*x^{**2}/a) - 665*a^{**}101*b \\
& ** (93/2)*x^{**3}/(105*a^{**}(205/2)*b^{**}(99/2)*sqrt(1 + b*x^{**2}/a) + 630* \\
& a^{**}(203/2)*b^{**}(101/2)*x^{**2}*sqrt(1 + b*x^{**2}/a) + 1575*a^{**}(201/2)*b \\
& ** (103/2)*x^{**4}*sqrt(1 + b*x^{**2}/a) + 2100*a^{**}(199/2)*b^{**}(105/2)*x* \\
& **6*sqrt(1 + b*x^{**2}/a) + 1575*a^{**}(197/2)*b^{**}(107/2)*x^{**8}*sqrt(1 + \\
& b*x^{**2}/a) + 630*a^{**}(195/2)*b^{**}(109/2)*x^{**10}*sqrt(1 + b*x^{**2}/a) + \\
& 105*a^{**}(193/2)*b^{**}(111/2)*x^{**12}*sqrt(1 + b*x^{**2}/a) - 1771*a^{**}100 \\
& *b^{**}(95/2)*x^{**5}/(105*a^{**}(205/2)*b^{**}(99/2)*sqrt(1 + b*x^{**2}/a) + 63 \\
& 0*a^{**}(203/2)*b^{**}(101/2)*x^{**2}*sqrt(1 + b*x^{**2}/a) + 1575*a^{**}(201/2) \\
& *b^{**}(103/2)*x^{**4}*sqrt(1 + b*x^{**2}/a) + 2100*a^{**}(199/2)*b^{**}(105/2)* \\
& x^{**6}*sqrt(1 + b*x^{**2}/a) + 1575*a^{**}(197/2)*b^{**}(107/2)*x^{**8}*sqrt(1 \\
& + b*x^{**2}/a) + 630*a^{**}(195/2)*b^{**}(109/2)*x^{**10}*sqrt(1 + b*x^{**2}/a) \\
& + 105*a^{**}(193/2)*b^{**}(111/2)*x^{**12}*sqrt(1 + b*x^{**2}/a) - 2549*a^{**}9 \\
& 9*b^{**}(97/2)*x^{**7}/(105*a^{**}(205/2)*b^{**}(99/2)*sqrt(1 + b*x^{**2}/a) + 6 \\
& 30*a^{**}(203/2)*b^{**}(101/2)*x^{**2}*sqrt(1 + b*x^{**2}/a) + 1575*a^{**}(201/2) \\
& )*b^{**}(103/2)*x^{**4}*sqrt(1 + b*x^{**2}/a) + 2100*a^{**}(199/2)*b^{**}(105/2) \\
& *x^{**6}*sqrt(1 + b*x^{**2}/a) + 1575*a^{**}(197/2)*b^{**}(107/2)*x^{**8}*sqrt(1 \\
& + b*x^{**2}/a) + 630*a^{**}(195/2)*b^{**}(109/2)*x^{**10}*sqrt(1 + b*x^{**2}/a) \\
& + 105*a^{**}(193/2)*b^{**}(111/2)*x^{**12}*sqrt(1 + b*x^{**2}/a) - 2096*a^{**} \\
& 98*b^{**}(99/2)*x^{**9}/(105*a^{**}(205/2)*b^{**}(99/2)*sqrt(1 + b*x^{**2}/a) + \\
& 630*a^{**}(203/2)*b^{**}(101/2)*x^{**2}*sqrt(1 + b*x^{**2}/a) + 1575*a^{**}(201/ \\
& 2)*b^{**}(103/2)*x^{**4}*sqrt(1 + b*x^{**2}/a) + 2100*a^{**}(199/2)*b^{**}(105/2) \\
& )*x^{**6}*sqrt(1 + b*x^{**2}/a) + 1575*a^{**}(197/2)*b^{**}(107/2)*x^{**8}*sqrt( \\
& 1 + b*x^{**2}/a) + 630*a^{**}(195/2)*b^{**}(109/2)*x^{**10}*sqrt(1 + b*x^{**2}/a)
\end{aligned}$$

) + 105\*a\*\*(193/2)\*b\*\*(111/2)\*x\*\*12\*sqrt(1 + b\*x\*\*2/a) - 934\*a\*\*97\*b\*\*(101/2)\*x\*\*11/(105\*a\*\*(205/2)\*b\*\*(99/2)\*sqrt(1 + b\*x\*\*2/a) + 630\*a\*\*(203/2)\*b\*\*(101/2)\*x\*\*2\*sqrt(1 + b\*x\*\*2/a) + 1575\*a\*\*(201/2)\*b\*\*(103/2)\*x\*\*4\*sqrt(1 + b\*x\*\*2/a) + 2100\*a\*\*(199/2)\*b\*\*(105/2)\*x\*\*6\*sqrt(1 + b\*x\*\*2/a) + 1575\*a\*\*(197/2)\*b\*\*(107/2)\*x\*\*8\*sqrt(1 + b\*x\*\*2/a) + 630\*a\*\*(195/2)\*b\*\*(109/2)\*x\*\*10\*sqrt(1 + b\*x\*\*2/a) + 105\*a\*\*(193/2)\*b\*\*(111/2)\*x\*\*12\*sqrt(1 + b\*x\*\*2/a) - 176\*a\*\*96\*b\*\*(103/2)\*x\*\*13/(105\*a\*\*(205/2)\*b\*\*(99/2)\*sqrt(1 + b\*x\*\*2/a) + 630\*a\*\*(203/2)\*b\*\*(101/2)\*x\*\*2\*sqrt(1 + b\*x\*\*2/a) + 1575\*a\*\*(201/2)\*b\*\*(103/2)\*x\*\*4\*sqrt(1 + b\*x\*\*2/a) + 2100\*a\*\*(199/2)\*b\*\*(105/2)\*x\*\*6\*sqrt(1 + b\*x\*\*2/a) + 1575\*a\*\*(197/2)\*b\*\*(107/2)\*x\*\*8\*sqrt(1 + b\*x\*\*2/a) + 630\*a\*\*(195/2)\*b\*\*(109/2)\*x\*\*10\*sqrt(1 + b\*x\*\*2/a) + 105\*a\*\*(193/2)\*b\*\*(111/2)\*x\*\*12\*sqrt(1 + b\*x\*\*2/a)

**GIAC/XCAS [A]** time = 0.224491, size = 105, normalized size = 0.99

$$\frac{\left(2 \left(x^2 \left(\frac{88x^2}{b} + \frac{203a}{b^2}\right) + \frac{175a^2}{b^3}\right)x^2 + \frac{105a^3}{b^4}\right)x}{105(bx^2 + a)^{\frac{7}{2}}} - \frac{\ln\left(\left|-\sqrt{bx} + \sqrt{bx^2 + a}\right|\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(b\*x^2 + a)^(9/2),x, algorithm="giac")

[Out] -1/105\*(2\*(x^2\*(88\*x^2/b + 203\*a/b^2) + 175\*a^2/b^3)\*x^2 + 105\*a^3/b^4)\*x/(b\*x^2 + a)^(7/2) - ln(abs(-sqrt(b)\*x + sqrt(b\*x^2 + a)))/b^(9/2)

$$3.520 \quad \int \frac{x^7}{(a+bx^2)^{9/2}} dx$$

**Optimal.** Leaf size=75

$$\frac{a^3}{7b^4(a+bx^2)^{7/2}} - \frac{3a^2}{5b^4(a+bx^2)^{5/2}} + \frac{a}{b^4(a+bx^2)^{3/2}} - \frac{1}{b^4\sqrt{a+bx^2}}$$

[Out]  $a^3/(7*b^4*(a + b*x^2)^(7/2)) - (3*a^2)/(5*b^4*(a + b*x^2)^(5/2)) + a/(b^4*(a + b*x^2)^(3/2)) - 1/(b^4*sqrt[a + b*x^2])$

**Rubi [A]** time = 0.121862, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{a^3}{7b^4(a+bx^2)^{7/2}} - \frac{3a^2}{5b^4(a+bx^2)^{5/2}} + \frac{a}{b^4(a+bx^2)^{3/2}} - \frac{1}{b^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^7/(a + b\*x^2)^(9/2), x]

[Out]  $a^3/(7*b^4*(a + b*x^2)^(7/2)) - (3*a^2)/(5*b^4*(a + b*x^2)^(5/2)) + a/(b^4*(a + b*x^2)^(3/2)) - 1/(b^4*sqrt[a + b*x^2])$

**Rubi in Sympy [A]** time = 15.7117, size = 68, normalized size = 0.91

$$\frac{a^3}{7b^4(a+bx^2)^{7/2}} - \frac{3a^2}{5b^4(a+bx^2)^{5/2}} + \frac{a}{b^4(a+bx^2)^{3/2}} - \frac{1}{b^4\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*7/(b\*x\*\*2+a)\*\*(9/2), x)

[Out]  $a**3/(7*b**4*(a + b*x**2)**(7/2)) - 3*a**2/(5*b**4*(a + b*x**2)**(5/2)) + a/(b**4*(a + b*x**2)**(3/2)) - 1/(b**4*sqrt(a + b*x**2))$

**Mathematica [A]** time = 0.0408353, size = 50, normalized size = 0.67

$$-\frac{16a^3 + 56a^2bx^2 + 70ab^2x^4 + 35b^3x^6}{35b^4(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a + b\*x^2)^(9/2), x]

[Out]  $-(16*a^3 + 56*a^2*b*x^2 + 70*a*b^2*x^4 + 35*b^3*x^6)/(35*b^4*(a + b*x^2)^(7/2))$

**Maple [A]** time = 0.008, size = 47, normalized size = 0.6

$$-\frac{35b^3x^6 + 70ab^2x^4 + 56a^2bx^2 + 16a^3}{35b^4} (bx^2 + a)^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7/(b*x^2+a)^(9/2),x)`

[Out]  $-1/35*(35*b^3*x^6+70*a*b^2*x^4+56*a^2*b*x^2+16*a^3)/(b*x^2+a)^(7/2)/b^4$

**Maxima [A]** time = 1.35235, size = 99, normalized size = 1.32

$$-\frac{x^6}{(bx^2+a)^{\frac{7}{2}}b} - \frac{2ax^4}{(bx^2+a)^{\frac{7}{2}}b^2} - \frac{8a^2x^2}{5(bx^2+a)^{\frac{7}{2}}b^3} - \frac{16a^3}{35(bx^2+a)^{\frac{7}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^2 + a)^(9/2),x, algorithm="maxima")`

[Out]  $-x^6/((b*x^2 + a)^(7/2)*b) - 2*a*x^4/((b*x^2 + a)^(7/2)*b^2) - 8/5*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) - 16/35*a^3/((b*x^2 + a)^(7/2)*b^4)$

**Fricas [A]** time = 0.254301, size = 123, normalized size = 1.64

$$-\frac{(35b^3x^6 + 70ab^2x^4 + 56a^2bx^2 + 16a^3)\sqrt{bx^2 + a}}{35(b^8x^8 + 4ab^7x^6 + 6a^2b^6x^4 + 4a^3b^5x^2 + a^4b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^2 + a)^(9/2),x, algorithm="fricas")`

[Out]  $-1/35*(35*b^3*x^6 + 70*a*b^2*x^4 + 56*a^2*b*x^2 + 16*a^3)*\text{sqrt}(b*x^2 + a)/(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)$

**Sympy [A]** time = 19.9163, size = 364, normalized size = 4.85

$$\left\{ \begin{array}{l} -\frac{16a^3}{35a^3b^4\sqrt{a+bx^2}+105a^2b^5x^2\sqrt{a+bx^2}+105ab^6x^4\sqrt{a+bx^2}+35b^7x^6\sqrt{a+bx^2}} - \frac{56a^2bx^2}{35a^3b^4\sqrt{a+bx^2}+105a^2b^5x^2\sqrt{a+bx^2}+105ab^6x^4\sqrt{a+bx^2}+35b^7x^6\sqrt{a+bx^2}} - \frac{x^8}{8a^2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7/(b*x**2+a)**(9/2),x)`

[Out] `Piecewise((-16*a**3/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a + b*x**2)) - 56*a**2*b*x**2/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a + b*x**2)) - 70*a*b**2*x**4/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a + b*x**2)) - 35*b**3*x**6/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**8/(8*a**(9/2)), True))`

**GIAC/XCAS [A]** time = 0.21424, size = 74, normalized size = 0.99

$$-\frac{35(bx^2+a)^3 - 35(bx^2+a)^2a + 21(bx^2+a)a^2 - 5a^3}{35(bx^2+a)^{\frac{7}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^2 + a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*(35*(b*x^2 + a)^3 - 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2  
- 5*a^3)/((b*x^2 + a)^(7/2)*b^4)
```

$$3.521 \quad \int \frac{x^6}{(a+bx^2)^{9/2}} dx$$

Optimal. Leaf size=21

$$\frac{x^7}{7a(a+bx^2)^{7/2}}$$

[Out]  $x^7/(7*a*(a+b*x^2)^(7/2))$

**Rubi [A]** time = 0.024336, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{x^7}{7a(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^6/(a+b*x^2)^(9/2), x]$

[Out]  $x^7/(7*a*(a+b*x^2)^(7/2))$

**Rubi in Sympy [A]** time = 3.23318, size = 15, normalized size = 0.71

$$\frac{x^7}{7a(a+bx^2)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**6}/(b*x^{**2}+a)^{(9/2)}, x)$

[Out]  $x^{**7}/(7*a*(a+b*x^{**2})^{(7/2)})$

**Mathematica [A]** time = 0.030527, size = 21, normalized size = 1.

$$\frac{x^7}{7a(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^6/(a+b*x^2)^(9/2), x]$

[Out]  $x^7/(7*a*(a+b*x^2)^(7/2))$

**Maple [A]** time = 0.007, size = 18, normalized size = 0.9

$$\frac{x^7}{7a}(bx^2+a)^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^6/(b*x^2+a)^(9/2), x)$

[Out]  $1/7 * x^7 / a / (b * x^2 + a)^{(7/2)}$

**Maxima [A]** time = 1.35908, size = 139, normalized size = 6.62

$$-\frac{x^5}{2(bx^2 + a)^{\frac{7}{2}}b} - \frac{5ax^3}{8(bx^2 + a)^{\frac{7}{2}}b^2} + \frac{x}{14(bx^2 + a)^{\frac{3}{2}}b^3} + \frac{x}{7\sqrt{bx^2 + a}ab^3} + \frac{3ax}{56(bx^2 + a)^{\frac{5}{2}}b^3} - \frac{15a^2x}{56(bx^2 + a)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^2 + a)^(9/2), x, algorithm="maxima")`

[Out]  $-1/2 * x^5 / ((b * x^2 + a)^{(7/2)} * b) - 5/8 * a * x^3 / ((b * x^2 + a)^{(7/2)} * b^2) + 1/14 * x / ((b * x^2 + a)^{(3/2)} * b^3) + 1/7 * x / (\text{sqrt}(b * x^2 + a) * a * b^3) + 3/56 * a * x / ((b * x^2 + a)^{(5/2)} * b^3) - 15/56 * a^2 * x / ((b * x^2 + a)^{(7/2)} * b^3)$

**Fricas [A]** time = 0.248511, size = 80, normalized size = 3.81

$$\frac{\sqrt{bx^2 + ax^7}}{7(ab^4x^8 + 4a^2b^3x^6 + 6a^3b^2x^4 + 4a^4bx^2 + a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^2 + a)^(9/2), x, algorithm="fricas")`

[Out]  $1/7 * \text{sqrt}(b * x^2 + a) * x^7 / (a * b^4 * x^8 + 4 * a^2 * b^3 * x^6 + 6 * a^3 * b^2 * x^4 + 4 * a^4 * b * x^2 + a^5)$

**Sympy [A]** time = 9.70337, size = 95, normalized size = 4.52

$$\frac{x^7}{7a^{\frac{9}{2}}\sqrt{1 + \frac{bx^2}{a}} + 21a^{\frac{7}{2}}bx^2\sqrt{1 + \frac{bx^2}{a}} + 21a^{\frac{5}{2}}b^2x^4\sqrt{1 + \frac{bx^2}{a}} + 7a^{\frac{3}{2}}b^3x^6\sqrt{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(b*x**2+a)**(9/2), x)`

[Out]  $x^{**7} / (7 * a^{** (9/2)} * \text{sqrt}(1 + b * x^{**2} / a) + 21 * a^{** (7/2)} * b * x^{**2} * \text{sqrt}(1 + b * x^{**2} / a) + 21 * a^{** (5/2)} * b^2 * x^{**4} * \text{sqrt}(1 + b * x^{**2} / a) + 7 * a^{** (3/2)} * b^3 * x^{**6} * \text{sqrt}(1 + b * x^{**2} / a))$

**GIAC/XCAS [A]** time = 0.229299, size = 23, normalized size = 1.1

$$\frac{x^7}{7(bx^2 + a)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^2 + a)^(9/2), x, algorithm="giac")`

[Out]  $1/7 * x^7 / ((b * x^2 + a)^{(7/2)} * a)$

$$3.522 \quad \int \frac{x^5}{(a+bx^2)^{9/2}} dx$$

**Optimal.** Leaf size=59

$$-\frac{a^2}{7b^3(a+bx^2)^{7/2}} + \frac{2a}{5b^3(a+bx^2)^{5/2}} - \frac{1}{3b^3(a+bx^2)^{3/2}}$$

[Out]  $-a^2/(7*b^3*(a + b*x^2)^(7/2)) + (2*a)/(5*b^3*(a + b*x^2)^(5/2)) - 1/(3*b^3*(a + b*x^2)^(3/2))$

**Rubi [A]** time = 0.093691, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{a^2}{7b^3(a+bx^2)^{7/2}} + \frac{2a}{5b^3(a+bx^2)^{5/2}} - \frac{1}{3b^3(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a + b\*x^2)^(9/2), x]

[Out]  $-a^2/(7*b^3*(a + b*x^2)^(7/2)) + (2*a)/(5*b^3*(a + b*x^2)^(5/2)) - 1/(3*b^3*(a + b*x^2)^(3/2))$

**Rubi in Sympy [A]** time = 11.6875, size = 53, normalized size = 0.9

$$-\frac{a^2}{7b^3(a+bx^2)^{7/2}} + \frac{2a}{5b^3(a+bx^2)^{5/2}} - \frac{1}{3b^3(a+bx^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5/(b\*x\*\*2+a)\*\*(9/2), x)

[Out]  $-a**2/(7*b**3*(a + b*x**2)**(7/2)) + 2*a/(5*b**3*(a + b*x**2)**(5/2)) - 1/(3*b**3*(a + b*x**2)**(3/2))$

**Mathematica [A]** time = 0.0340043, size = 39, normalized size = 0.66

$$-\frac{8a^2 + 28abx^2 + 35b^2x^4}{105b^3(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a + b\*x^2)^(9/2), x]

[Out]  $-(8*a^2 + 28*a*b*x^2 + 35*b^2*x^4)/(105*b^3*(a + b*x^2)^(7/2))$

**Maple [A]** time = 0.007, size = 36, normalized size = 0.6

$$-\frac{35b^2x^4 + 28abx^2 + 8a^2}{105b^3}(bx^2 + a)^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(x^5/(b*x^2+a)^(9/2),x)`

[Out]  $-1/105*(35*b^2*x^4+28*a*b*x^2+8*a^2)/(b*x^2+a)^(7/2)/b^3$

**Maxima [A]** time = 1.35407, size = 72, normalized size = 1.22

$$-\frac{x^4}{3(bx^2+a)^{\frac{7}{2}}b} - \frac{4ax^2}{15(bx^2+a)^{\frac{7}{2}}b^2} - \frac{8a^2}{105(bx^2+a)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2+a)^(9/2),x, algorithm="maxima")`

[Out]  $-1/3*x^4/((b*x^2+a)^(7/2)*b) - 4/15*a*x^2/((b*x^2+a)^(7/2)*b^2) - 8/105*a^2/((b*x^2+a)^(7/2)*b^3)$

**Fricas [A]** time = 0.2535, size = 108, normalized size = 1.83

$$-\frac{(35b^2x^4 + 28abx^2 + 8a^2)\sqrt{bx^2+a}}{105(b^7x^8 + 4ab^6x^6 + 6a^2b^5x^4 + 4a^3b^4x^2 + a^4b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2+a)^(9/2),x, algorithm="fricas")`

[Out]  $-1/105*(35*b^2*x^4 + 28*a*b*x^2 + 8*a^2)*\text{sqrt}(b*x^2+a)/(b^7*x^8 + 4*a*b^6*x^6 + 6*a^2*b^5*x^4 + 4*a^3*b^4*x^2 + a^4*b^3)$

**Sympy [A]** time = 19.6059, size = 272, normalized size = 4.61

$$\left\{ \begin{array}{l} \frac{8a^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}} - \frac{28abx^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}} \\ \frac{x^6}{6a^2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(b*x**2+a)**(9/2),x)`

[Out] `Piecewise((-8*a**2/(105*a**3*b**3*sqrt(a+b*x**2))+315*a**2*b**4*x**2*sqrt(a+b*x**2)+315*a*b**5*x**4*sqrt(a+b*x**2)+105*b**6*x**6*sqrt(a+b*x**2))-28*a*b*x**2/(105*a**3*b**3*sqrt(a+b*x**2))+315*a**2*b**4*x**2*sqrt(a+b*x**2)+315*a*b**5*x**4*sqrt(a+b*x**2)+105*b**6*x**6*sqrt(a+b*x**2))-35*b**2*x**4/(105*a**3*b**3*sqrt(a+b*x**2))+315*a**2*b**4*x**2*sqrt(a+b*x**2)+315*a*b**5*x**4*sqrt(a+b*x**2)+105*b**6*x**6*sqrt(a+b*x**2)), Ne(b, 0)), (x**6/(6*a**(9/2)), True))`

**GIAC/XCAS [A]** time = 0.216118, size = 55, normalized size = 0.93

$$\frac{35(bx^2+a)^2-42(bx^2+a)a+15a^2}{105(bx^2+a)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2 + a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(35*(b*x^2 + a)^2 - 42*(b*x^2 + a)*a + 15*a^2)/((b*x^2 + a)^(7/2)*b^3)
```

$$3.523 \quad \int \frac{x^4}{(a+bx^2)^{9/2}} dx$$

Optimal. Leaf size=44

$$\frac{2bx^7}{35a^2(a+bx^2)^{7/2}} + \frac{x^5}{5a(a+bx^2)^{7/2}}$$

[Out]  $x^5/(5*a*(a+b*x^2)^(7/2)) + (2*b*x^7)/(35*a^2*(a+b*x^2)^(7/2))$

**Rubi [A]** time = 0.0493929, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{2bx^7}{35a^2(a+bx^2)^{7/2}} + \frac{x^5}{5a(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a+b\*x^2)^(9/2),x]

[Out]  $x^5/(5*a*(a+b*x^2)^(7/2)) + (2*b*x^7)/(35*a^2*(a+b*x^2)^(7/2))$

**Rubi in Sympy [A]** time = 5.53708, size = 37, normalized size = 0.84

$$\frac{x^5}{5a(a+bx^2)^{7/2}} + \frac{2bx^7}{35a^2(a+bx^2)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(b\*x\*\*2+a)\*\*(9/2),x)

[Out]  $x**5/(5*a*(a+b*x**2)**(7/2)) + 2*b*x**7/(35*a**2*(a+b*x**2)**(7/2))$

**Mathematica [A]** time = 0.0332654, size = 31, normalized size = 0.7

$$\frac{x^5(7a+2bx^2)}{35a^2(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a+b\*x^2)^(9/2),x]

[Out]  $(x^5*(7*a+2*b*x^2))/(35*a^2*(a+b*x^2)^(7/2))$

**Maple [A]** time = 0.008, size = 28, normalized size = 0.6

$$\frac{x^5(2bx^2+7a)}{35a^2}(bx^2+a)^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(b*x^2+a)^(9/2),x)`

[Out]  $1/35*x^5*(2*b*x^2+7*a)/(b*x^2+a)^(7/2)/a^2$

**Maxima [A]** time = 1.34811, size = 115, normalized size = 2.61

$$-\frac{x^3}{4(bx^2+a)^{\frac{7}{2}}b} + \frac{3x}{140(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2x}{35\sqrt{bx^2+aa^2}b^2} + \frac{x}{35(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3ax}{28(bx^2+a)^{\frac{7}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^(9/2),x, algorithm="maxima")`

[Out]  $-1/4*x^3/((b*x^2 + a)^(7/2)*b) + 3/140*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*a*x/((b*x^2 + a)^(7/2)*b^2)$

**Fricas [A]** time = 0.252244, size = 96, normalized size = 2.18

$$\frac{(2bx^7 + 7ax^5)\sqrt{bx^2 + a}}{35(a^2b^4x^8 + 4a^3b^3x^6 + 6a^4b^2x^4 + 4a^5bx^2 + a^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^(9/2),x, algorithm="fricas")`

[Out]  $1/35*(2*b*x^7 + 7*a*x^5)*sqrt(b*x^2 + a)/(a^2*b^4*x^8 + 4*a^3*b^3*x^6 + 6*a^4*b^2*x^4 + 4*a^5*b*x^2 + a^6)$

**Sympy [A]** time = 10.0758, size = 199, normalized size = 4.52

$$\frac{7ax^5}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}} + 105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}} + 105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}} + 35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^7}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}} + 105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}} + 105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}} + 35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(b*x**2+a)**(9/2),x)`

[Out]  $7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))$

**GIAC/XCAS [A]** time = 0.221414, size = 39, normalized size = 0.89

$$\frac{x^5\left(\frac{2bx^2}{a^2} + \frac{7}{a}\right)}{35(bx^2+a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2 + a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*x^5*(2*b*x^2/a^2 + 7/a)/(b*x^2 + a)^(7/2)
```

$$3.524 \quad \int \frac{x^3}{(a+bx^2)^{9/2}} dx$$

**Optimal.** Leaf size=38

$$\frac{a}{7b^2(a+bx^2)^{7/2}} - \frac{1}{5b^2(a+bx^2)^{5/2}}$$

[Out]  $a/(7*b^2*(a + b*x^2)^(7/2)) - 1/(5*b^2*(a + b*x^2)^(5/2))$

**Rubi [A]** time = 0.0649565, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{a}{7b^2(a+bx^2)^{7/2}} - \frac{1}{5b^2(a+bx^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b\*x^2)^(9/2), x]

[Out]  $a/(7*b^2*(a + b*x^2)^(7/2)) - 1/(5*b^2*(a + b*x^2)^(5/2))$

**Rubi in Sympy [A]** time = 8.10256, size = 32, normalized size = 0.84

$$\frac{a}{7b^2(a+bx^2)^{7/2}} - \frac{1}{5b^2(a+bx^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(b\*x\*\*2+a)\*\*(9/2), x)

[Out]  $a/(7*b**2*(a + b*x**2)**(7/2)) - 1/(5*b**2*(a + b*x**2)**(5/2))$

**Mathematica [A]** time = 0.0274773, size = 28, normalized size = 0.74

$$-\frac{2a + 7bx^2}{35b^2(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b\*x^2)^(9/2), x]

[Out]  $-(2*a + 7*b*x^2)/(35*b^2*(a + b*x^2)^(7/2))$

**Maple [A]** time = 0.008, size = 25, normalized size = 0.7

$$-\frac{7bx^2 + 2a}{35b^2} (bx^2 + a)^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b\*x^2+a)^(9/2), x)

[Out]  $-1/35*(7*b*x^2+2*a)/(b*x^2+a)^(7/2)/b^2$

---

**Maxima [A]** time = 1.35186, size = 45, normalized size = 1.18

$$-\frac{x^2}{5(bx^2 + a)^{\frac{7}{2}}b} - \frac{2a}{35(bx^2 + a)^{\frac{7}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b\*x^2 + a)^(9/2), x, algorithm="maxima")

[Out] -1/5\*x^2/((b\*x^2 + a)^(7/2)\*b) - 2/35\*a/((b\*x^2 + a)^(7/2)\*b^2)

---

**Fricas [A]** time = 0.25093, size = 93, normalized size = 2.45

$$-\frac{(7bx^2 + 2a)\sqrt{bx^2 + a}}{35(b^6x^8 + 4ab^5x^6 + 6a^2b^4x^4 + 4a^3b^3x^2 + a^4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b\*x^2 + a)^(9/2), x, algorithm="fricas")

[Out] -1/35\*(7\*b\*x^2 + 2\*a)\*sqrt(b\*x^2 + a)/(b^6\*x^8 + 4\*a\*b^5\*x^6 + 6\*a^2\*b^4\*x^4 + 4\*a^3\*b^3\*x^2 + a^4\*b^2)

---

**Sympy [A]** time = 19.2669, size = 180, normalized size = 4.74

$$\left\{ \begin{array}{l} -\frac{2a}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+bx^2}+105ab^4x^4\sqrt{a+bx^2}+35b^5x^6\sqrt{a+bx^2}} - \frac{7bx^2}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+bx^2}+105ab^4x^4\sqrt{a+bx^2}+35b^5x^6\sqrt{a+bx^2}} \\ \frac{x^4}{4a^{\frac{9}{2}}} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(b\*x\*\*2+a)\*\*(9/2), x)

[Out] Piecewise((-2\*a/(35\*a\*\*3\*b\*\*2\*sqrt(a + b\*x\*\*2) + 105\*a\*\*2\*b\*\*3\*x\*\*2\*sqrt(a + b\*x\*\*2) + 105\*a\*b\*\*4\*x\*\*4\*sqrt(a + b\*x\*\*2) + 35\*b\*\*5\*x\*\*6\*sqrt(a + b\*x\*\*2)) - 7\*b\*x\*\*2/(35\*a\*\*3\*b\*\*2\*sqrt(a + b\*x\*\*2) + 105\*a\*\*2\*b\*\*3\*x\*\*2\*sqrt(a + b\*x\*\*2) + 105\*a\*b\*\*4\*x\*\*4\*sqrt(a + b\*x\*\*2) + 35\*b\*\*5\*x\*\*6\*sqrt(a + b\*x\*\*2)), Ne(b, 0)), (x\*\*4/(4\*a\*\*2), True))

---

**GIAC/XCAS [A]** time = 0.219402, size = 32, normalized size = 0.84

$$-\frac{7bx^2 + 2a}{35(bx^2 + a)^{\frac{7}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b\*x^2 + a)^(9/2), x, algorithm="giac")

[Out] -1/35\*(7\*b\*x^2 + 2\*a)/((b\*x^2 + a)^(7/2)\*b^2)

$$3.525 \quad \int \frac{x^2}{(a+bx^2)^{9/2}} dx$$

Optimal. Leaf size=68

$$\frac{8b^2x^7}{105a^3(a+bx^2)^{7/2}} + \frac{4bx^5}{15a^2(a+bx^2)^{7/2}} + \frac{x^3}{3a(a+bx^2)^{7/2}}$$

[Out]  $x^3/(3*a*(a+b*x^2)^{(7/2)}) + (4*b*x^5)/(15*a^2*(a+b*x^2)^{(7/2)}) + (8*b^2*x^7)/(105*a^3*(a+b*x^2)^{(7/2)})$

Rubi [A] time = 0.0748306, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{8b^2x^7}{105a^3(a+bx^2)^{7/2}} + \frac{4bx^5}{15a^2(a+bx^2)^{7/2}} + \frac{x^3}{3a(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a+b\*x^2)^(9/2),x]

[Out]  $x^3/(3*a*(a+b*x^2)^{(7/2)}) + (4*b*x^5)/(15*a^2*(a+b*x^2)^{(7/2)}) + (8*b^2*x^7)/(105*a^3*(a+b*x^2)^{(7/2)})$

Rubi in Sympy [A] time = 8.83357, size = 61, normalized size = 0.9

$$\frac{x^3}{3a(a+bx^2)^{7/2}} + \frac{4bx^5}{15a^2(a+bx^2)^{7/2}} + \frac{8b^2x^7}{105a^3(a+bx^2)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(b\*x\*\*2+a)\*\*(9/2),x)

[Out]  $x**3/(3*a*(a+b*x**2)**(7/2)) + 4*b*x**5/(15*a**2*(a+b*x**2)**(7/2)) + 8*b**2*x**7/(105*a**3*(a+b*x**2)**(7/2))$

Mathematica [A] time = 0.0336033, size = 42, normalized size = 0.62

$$\frac{x^3(35a^2+28abx^2+8b^2x^4)}{105a^3(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a+b\*x^2)^(9/2),x]

[Out]  $(x^3*(35*a^2+28*a*b*x^2+8*b^2*x^4))/(105*a^3*(a+b*x^2)^{(7/2)})$

Maple [A] time = 0.007, size = 39, normalized size = 0.6

$$\frac{x^3(8b^2x^4+28abx^2+35a^2)}{105a^3}(bx^2+a)^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(x^2/(b*x^2+a)^(9/2),x)`

[Out]  $1/105*x^3*(8*b^2*x^4+28*a*b*x^2+35*a^2)/(b*x^2+a)^(7/2)/a^3$

**Maxima [A]** time = 1.34966, size = 95, normalized size = 1.4

$$-\frac{x}{7(bx^2+a)^{\frac{7}{2}}b} + \frac{8x}{105\sqrt{bx^2+aa^3}b} + \frac{4x}{105(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{x}{35(bx^2+a)^{\frac{5}{2}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")`

[Out]  $-1/7*x/((b*x^2+a)^(7/2)*b) + 8/105*x/(\sqrt{b*x^2+a}*a^3*b) + 4/105*x/((b*x^2+a)^(3/2)*a^2*b) + 1/35*x/((b*x^2+a)^(5/2)*a*b)$

**Fricas [A]** time = 0.255757, size = 111, normalized size = 1.63

$$\frac{(8b^2x^7 + 28abx^5 + 35a^2x^3)\sqrt{bx^2+a}}{105(a^3b^4x^8 + 4a^4b^3x^6 + 6a^5b^2x^4 + 4a^6bx^2 + a^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")`

[Out]  $1/105*(8*b^2*x^7 + 28*a*b*x^5 + 35*a^2*x^3)*\sqrt{b*x^2+a}/(a^3*b^4*x^8 + 4*a^4*b^3*x^6 + 6*a^5*b^2*x^4 + 4*a^6*b*x^2 + a^7)$

**Sympy [A]** time = 10.9184, size = 517, normalized size = 7.6

$$\frac{35a^5x^3}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}} + 420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}} + 630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}} + 420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}} + 105a^{\frac{11}{2}}b^4x^8\sqrt{1+\frac{bx^2}{a}}} + \frac{63a^4bx^5}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}} + 420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}} + 630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}} + 420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}} + 105a^{\frac{11}{2}}b^4x^8\sqrt{1+\frac{bx^2}{a}}} + \frac{36a^3b^2x^7}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}} + 420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}} + 630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}} + 420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}} + 105a^{\frac{11}{2}}b^4x^8\sqrt{1+\frac{bx^2}{a}}} + \frac{8a^2b^3x^9}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}} + 420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}} + 630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}} + 420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}} + 105a^{\frac{11}{2}}b^4x^8\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**2+a)**(9/2),x)`

[Out]  $35*a**5*x**3/(105*a**(19/2)*\sqrt{1+b*x**2/a} + 420*a**(17/2)*b*x**2*\sqrt{1+b*x**2/a} + 630*a**(15/2)*b**2*x**4*\sqrt{1+b*x**2/a} + 420*a**(13/2)*b**3*x**6*\sqrt{1+b*x**2/a} + 105*a**(11/2)*b**4*x**8*\sqrt{1+b*x**2/a}) + 63*a**4*b*x**5/(105*a**(19/2)*\sqrt{1+b*x**2/a} + 420*a**(17/2)*b*x**2*\sqrt{1+b*x**2/a} + 630*a**(15/2)*b**2*x**4*\sqrt{1+b*x**2/a} + 420*a**(13/2)*b**3*x**6*\sqrt{1+b*x**2/a} + 105*a**(11/2)*b**4*x**8*\sqrt{1+b*x**2/a}) + 36*a**3*b**2*x**7/(105*a**(19/2)*\sqrt{1+b*x**2/a} + 420*a**(17/2)*b*x**2*\sqrt{1+b*x**2/a} + 630*a**(15/2)*b**2*x**4*\sqrt{1+b*x**2/a} + 420*a**(13/2)*b**3*x**6*\sqrt{1+b*x**2/a} + 105*a**(11/2)*b**4*x**8*\sqrt{1+b*x**2/a}) + 8*a**2*b**3*x**9/(105*a**(19/2)*\sqrt{1+b*x**2/a} + 420*a**(17/2)*b*x**2*\sqrt{1+b*x**2/a} + 630*a**(15/2)*b**2*x**4*\sqrt{1+b*x**2/a} + 420*a**(13/2)*b**3*x**6*\sqrt{1+b*x**2/a} + 105*a**(11/2)*b**4*x**8*\sqrt{1+b*x**2/a})$

$11/2) * b^{**4} * x^{**8} * \text{sqrt}(1 + b * x^{**2} / a)) + 8 * a^{**2} * b^{**3} * x^{**9} / (105 * a^{**2} * (1$   
 $9/2) * \text{sqrt}(1 + b * x^{**2} / a) + 420 * a^{**2} * (17/2) * b * x^{**2} * \text{sqrt}(1 + b * x^{**2} / a)$   
 $+ 630 * a^{**2} * (15/2) * b^{**2} * x^{**4} * \text{sqrt}(1 + b * x^{**2} / a) + 420 * a^{**2} * (13/2) * b^{**3}$   
 $3 * x^{**6} * \text{sqrt}(1 + b * x^{**2} / a) + 105 * a^{**2} * (11/2) * b^{**4} * x^{**8} * \text{sqrt}(1 + b * x^{**2} / a))$

**GIAC/XCAS [A]** time = 0.218991, size = 58, normalized size = 0.85

$$\frac{\left(4x^2\left(\frac{2b^2x^2}{a^3} + \frac{7b}{a^2}\right) + \frac{35}{a}\right)x^3}{105(bx^2 + a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b\*x^2 + a)^(9/2),x, algorithm="giac")

[Out] 1/105\*(4\*x^2\*(2\*b^2\*x^2/a^3 + 7\*b/a^2) + 35/a)\*x^3/(b\*x^2 + a)^(7/2)

$$3.526 \quad \int \frac{x}{(a+bx^2)^{9/2}} dx$$

**Optimal.** Leaf size=18

$$-\frac{1}{7b(a+bx^2)^{7/2}}$$

[Out]  $-1/(7*b*(a + b*x^2)^(7/2))$

**Rubi [A]** time = 0.0109965, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{1}{7b(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x/(a + b*x^2)^(9/2), x]$

[Out]  $-1/(7*b*(a + b*x^2)^(7/2))$

**Rubi in Sympy [A]** time = 2.16294, size = 15, normalized size = 0.83

$$-\frac{1}{7b(a+bx^2)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x/(b*x^2+a)^(9/2), x)$

[Out]  $-1/(7*b*(a + b*x^2)^(7/2))$

**Mathematica [A]** time = 0.00838707, size = 18, normalized size = 1.

$$-\frac{1}{7b(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x/(a + b*x^2)^(9/2), x]$

[Out]  $-1/(7*b*(a + b*x^2)^(7/2))$

**Maple [A]** time = 0.005, size = 15, normalized size = 0.8

$$-\frac{1}{7b}(bx^2+a)^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x/(b*x^2+a)^(9/2), x)$

[Out]  $-1/7/b/(b*x^2+a)^(7/2)$

---

**Maxima [A]** time = 1.34539, size = 19, normalized size = 1.06

$$-\frac{1}{7(bx^2 + a)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^(9/2),x, algorithm="maxima")`

[Out] `-1/7/((b*x^2 + a)^(7/2)*b)`

---

**Fricas [A]** time = 0.252929, size = 77, normalized size = 4.28

$$-\frac{\sqrt{bx^2 + a}}{7(b^5x^8 + 4ab^4x^6 + 6a^2b^3x^4 + 4a^3b^2x^2 + a^4b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^(9/2),x, algorithm="fricas")`

[Out] `-1/7*sqrt(b*x^2 + a)/(b^5*x^8 + 4*a*b^4*x^6 + 6*a^2*b^3*x^4 + 4*a^3*b^2*x^2 + a^4*b)`

---

**Sympy [A]** time = 18.901, size = 90, normalized size = 5.

$$\begin{cases} -\frac{1}{7a^3b\sqrt{a+bx^2}+21a^2b^2x^2\sqrt{a+bx^2}+21ab^3x^4\sqrt{a+bx^2}+7b^4x^6\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^9}{2a^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x**2+a)**(9/2),x)`

[Out] `Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqrt(a + b*x**2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(9/2)), True))`

---

**GIAC/XCAS [A]** time = 0.211185, size = 19, normalized size = 1.06

$$-\frac{1}{7(bx^2 + a)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^(9/2),x, algorithm="giac")`

[Out] `-1/7/((b*x^2 + a)^(7/2)*b)`

$$3.527 \quad \int \frac{1}{(a+bx^2)^{9/2}} dx$$

**Optimal.** Leaf size=77

$$\frac{16x}{35a^4\sqrt{a+bx^2}} + \frac{8x}{35a^3(a+bx^2)^{3/2}} + \frac{6x}{35a^2(a+bx^2)^{5/2}} + \frac{x}{7a(a+bx^2)^{7/2}}$$

[Out]  $x/(7*a*(a+b*x^2)^(7/2)) + (6*x)/(35*a^2*(a+b*x^2)^(5/2)) + (8*x)/(35*a^3*(a+b*x^2)^(3/2)) + (16*x)/(35*a^4*\text{Sqrt}[a+b*x^2])$

**Rubi [A]** time = 0.0494499, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{16x}{35a^4\sqrt{a+bx^2}} + \frac{8x}{35a^3(a+bx^2)^{3/2}} + \frac{6x}{35a^2(a+bx^2)^{5/2}} + \frac{x}{7a(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(-9/2), x]

[Out]  $x/(7*a*(a+b*x^2)^(7/2)) + (6*x)/(35*a^2*(a+b*x^2)^(5/2)) + (8*x)/(35*a^3*(a+b*x^2)^(3/2)) + (16*x)/(35*a^4*\text{Sqrt}[a+b*x^2])$

**Rubi in Sympy [A]** time = 5.0471, size = 70, normalized size = 0.91

$$\frac{x}{7a(a+bx^2)^{7/2}} + \frac{6x}{35a^2(a+bx^2)^{5/2}} + \frac{8x}{35a^3(a+bx^2)^{3/2}} + \frac{16x}{35a^4\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+a)\*\*(9/2), x)

[Out]  $x/(7*a*(a+b*x**2)**(7/2)) + 6*x/(35*a**2*(a+b*x**2)**(5/2)) + 8*x/(35*a**3*(a+b*x**2)**(3/2)) + 16*x/(35*a**4*\text{sqrt}(a+b*x**2))$

**Mathematica [A]** time = 0.0295028, size = 51, normalized size = 0.66

$$\frac{x(35a^3 + 70a^2bx^2 + 56ab^2x^4 + 16b^3x^6)}{35a^4(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(-9/2), x]

[Out]  $(x*(35*a^3 + 70*a^2*b*x^2 + 56*a*b^2*x^4 + 16*b^3*x^6))/(35*a^4*(a+b*x^2)^(7/2))$

**Maple [A]** time = 0.005, size = 48, normalized size = 0.6

$$\frac{x(16b^3x^6 + 56ab^2x^4 + 70a^2bx^2 + 35a^3)}{35a^4} (bx^2 + a)^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^2+a)^(9/2),x)`

[Out]  $1/35*x*(16*b^3*x^6+56*a*b^2*x^4+70*a^2*b*x^2+35*a^3)/(b*x^2+a)^(7/2)/a^4$

**Maxima [A]** time = 1.3403, size = 82, normalized size = 1.06

$$\frac{16x}{35\sqrt{bx^2+aa^4}} + \frac{8x}{35(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6x}{35(bx^2+a)^{\frac{5}{2}}a^2} + \frac{x}{7(bx^2+a)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(-9/2),x, algorithm="maxima")`

[Out]  $16/35*x/(\sqrt{b*x^2 + a}*a^4) + 8/35*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*x/((b*x^2 + a)^(7/2)*a)$

**Fricas [A]** time = 0.259011, size = 123, normalized size = 1.6

$$\frac{(16b^3x^7 + 56ab^2x^5 + 70a^2bx^3 + 35a^3x)\sqrt{bx^2 + a}}{35(a^4b^4x^8 + 4a^5b^3x^6 + 6a^6b^2x^4 + 4a^7bx^2 + a^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(-9/2),x, algorithm="fricas")`

[Out]  $1/35*(16*b^3*x^7 + 56*a*b^2*x^5 + 70*a^2*b*x^3 + 35*a^3*x)*\sqrt{b*x^2 + a}/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)$

**Sympy [A]** time = 12.0295, size = 1265, normalized size = 16.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**2+a)**(9/2),x)`

[Out]  $35*a^{14}*x/(35*a^{(37/2)}*\sqrt{1 + b*x^2/a} + 210*a^{(35/2)}*b*x^{2*\sqrt{1 + b*x^2/a}} + 525*a^{(33/2)}*b^{*2*x^{*4}}*\sqrt{1 + b*x^2/a} + 700*a^{(31/2)}*b^{*3*x^{*6}}*\sqrt{1 + b*x^2/a} + 525*a^{(29/2)}*b^{*4*x^{*8}}*\sqrt{1 + b*x^2/a} + 210*a^{(27/2)}*b^{*5*x^{*10}}*\sqrt{1 + b*x^2/a} + 35*a^{(25/2)}*b^{*6*x^{*12}}*\sqrt{1 + b*x^2/a}) + 175*a^{13}*b*x^3/(35*a^{(37/2)}*\sqrt{1 + b*x^2/a} + 210*a^{(35/2)}*b*x^{2*\sqrt{1 + b*x^2/a}} + 525*a^{(33/2)}*b^{*2*x^{*4}}*\sqrt{1 + b*x^2/a} + 700*a^{(31/2)}*b^{*3*x^{*6}}*\sqrt{1 + b*x^2/a} + 525*a^{(29/2)}*b^{*4*x^{*8}}*\sqrt{1 + b*x^2/a} + 210*a^{(27/2)}*b^{*5*x^{*10}}*\sqrt{1 + b*x^2/a} + 35*a^{(25/2)}*b^{*6*x^{*12}}*\sqrt{1 + b*x^2/a}) + 371*a^{12}*b^{*2*x^{*5}}/(35*a^{(37/2)}*\sqrt{1 + b*x^2/a} + 210*a^{(35/2)}*b*x^{2*\sqrt{1 + b*x^2/a}} + 525*a^{(33/2)}*b^{*2*x^{*4}}*\sqrt{1 + b*x^2/a} + 700*a^{(31/2)}*b^{*3*x^{*6}}*\sqrt{1 + b*x^2/a} + 525*a^{(29/2)}*b^{*4*x^{*8}}*\sqrt{1 + b*x^2/a} + 210*a^{(27/2)}*b^{*5*x^{*10}}*\sqrt{1 + b*x^2/a} + 35*a^{(25/2)}*b^{*6*x^{*12}}*\sqrt{1 + b*x^2/a}) + 429*a^{11}*b^{*3*x^{*7}}/(35*a^{(37/2)}*\sqrt{1 + b*x^2/a} + 210*a^{(35/2)}*b*x^{2*\sqrt{1 + b*x^2/a}} + 525*a^{(33/2)}*b^{*2*x^{*4}}*\sqrt{1 + b*x^2/a} + 700*a^{(31/2)}*b^{*3*x^{*6}}*\sqrt{1 + b*x^2/a} + 525*a^{(29/2)}*b^{*4*x^{*8}}*\sqrt{1 + b*x^2/a} + 210*a^{(27/2)}*b^{*5*x^{*10}}*\sqrt{1 + b*x^2/a} + 35*a^{(25/2)}*b^{*6*x^{*12}}*\sqrt{1 + b*x^2/a}) + 286*a^{10}*b^{*4*x^{*9}}/(35*a^{(37/2)}*\sqrt{1 + b*x^2/a} + 210*a^{(35/2)}*b*x^{2*\sqrt{1 + b*x^2/a}} + 525*a^{(33/2)}*b^{*2*x^{*4}}*\sqrt{1 + b*x^2/a} + 700*a^{(31/2)}*b^{*3*x^{*6}}*\sqrt{1 + b*x^2/a} + 525*a^{(29/2)}*b^{*4*x^{*8}}*\sqrt{1 + b*x^2/a} + 210*a^{(27/2)}*b^{*5*x^{*10}}*\sqrt{1 + b*x^2/a} + 35*a^{(25/2)}*b^{*6*x^{*12}}*\sqrt{1 + b*x^2/a})$

$1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*$   
 $a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*$   
 $sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a)$   
 $+ 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**$   
 $11/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1$   
 $+ b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a*$   
 $** (31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*s$   
 $qrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) +$   
 $35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x**13$   
 $/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 +$   
 $b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**$   
 $(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sq$   
 $r$   
 $t(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 3$   
 $5*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a))$

**GIAC/XCAS [A]** time = 0.215918, size = 74, normalized size = 0.96

$$\frac{\left(2\left(4x^2\left(\frac{2b^3x^2}{a^4} + \frac{7b^2}{a^3}\right) + \frac{35b}{a^2}\right)x^2 + \frac{35}{a}\right)x}{35(bx^2 + a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-9/2),x, algorithm="giac")

[Out] 1/35\*(2\*(4\*x^2\*(2\*b^3\*x^2/a^4 + 7\*b^2/a^3) + 35\*b/a^2)\*x^2 + 35/a)\*x/(b\*x^2 + a)^(7/2)

$$3.528 \quad \int \frac{1}{x(a+bx^2)^{9/2}} dx$$

**Optimal.** Leaf size=95

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}} + \frac{1}{a^4\sqrt{a+bx^2}} + \frac{1}{3a^3(a+bx^2)^{3/2}} + \frac{1}{5a^2(a+bx^2)^{5/2}} + \frac{1}{7a(a+bx^2)^{7/2}}$$

[Out]  $1/(7*a*(a+b*x^2)^(7/2)) + 1/(5*a^2*(a+b*x^2)^(5/2)) + 1/(3*a^3*(a+b*x^2)^(3/2)) + 1/(a^4*sqrt[a+b*x^2]) - \text{ArcTanh}[sqrt[a+b*x^2]/sqrt[a]]/a^(9/2)$

**Rubi [A]** time = 0.166498, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}} + \frac{1}{a^4\sqrt{a+bx^2}} + \frac{1}{3a^3(a+bx^2)^{3/2}} + \frac{1}{5a^2(a+bx^2)^{5/2}} + \frac{1}{7a(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(a+b\*x^2)^(9/2)),x]

[Out]  $1/(7*a*(a+b*x^2)^(7/2)) + 1/(5*a^2*(a+b*x^2)^(5/2)) + 1/(3*a^3*(a+b*x^2)^(3/2)) + 1/(a^4*sqrt[a+b*x^2]) - \text{ArcTanh}[sqrt[a+b*x^2]/sqrt[a]]/a^(9/2)$

**Rubi in Sympy [A]** time = 17.449, size = 85, normalized size = 0.89

$$\frac{1}{7a(a+bx^2)^{7/2}} + \frac{1}{5a^2(a+bx^2)^{5/2}} + \frac{1}{3a^3(a+bx^2)^{3/2}} + \frac{1}{a^4\sqrt{a+bx^2}} - \frac{\text{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(b\*x\*\*2+a)\*\*(9/2),x)

[Out]  $1/(7*a*(a+b*x**2)**(7/2)) + 1/(5*a**2*(a+b*x**2)**(5/2)) + 1/(3*a**3*(a+b*x**2)**(3/2)) + 1/(a**4*sqrt(a+b*x**2)) - \text{atanh}(sqrt(a+b*x**2)/sqrt(a))/a**(9/2)$

**Mathematica [A]** time = 0.260244, size = 86, normalized size = 0.91

$$-\frac{\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{a^{9/2}} + \frac{\log(x)}{a^{9/2}} + \frac{176a^3+406a^2bx^2+350ab^2x^4+105b^3x^6}{105a^4(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(a+b\*x^2)^(9/2)),x]

[Out]  $(176*a^3+406*a^2*b*x^2+350*a*b^2*x^4+105*b^3*x^6)/(105*a^4*(a+b*x^2)^(7/2)) + \text{Log}[x]/a^(9/2) - \text{Log}[a+sqrt[a]*sqrt[a+b*x^2]]/a^(9/2)$



**Maple [A]** time = 0.009, size = 85, normalized size = 0.9

$$\frac{1}{7a} (bx^2 + a)^{-\frac{7}{2}} + \frac{1}{5a^2} (bx^2 + a)^{-\frac{5}{2}} + \frac{1}{3a^3} (bx^2 + a)^{-\frac{3}{2}} + \frac{1}{a^4} \frac{1}{\sqrt{bx^2 + a}} - 1 \ln \left( \frac{1}{x} \left( 2a + 2\sqrt{a}\sqrt{bx^2 + a} \right) \right) a^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b\*x^2+a)^(9/2), x)

[Out] 1/7/a/(b\*x^2+a)^(7/2)+1/5/a^2/(b\*x^2+a)^(5/2)+1/3/a^3/(b\*x^2+a)^(3/2)+1/a^4/(b\*x^2+a)^(1/2)-1/a^(9/2)\*ln((2\*a+2\*a^(1/2)\*(b\*x^2+a)^(1/2))/x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(9/2)\*x), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.275331, size = 1, normalized size = 0.01

$$\left[ \frac{2(105b^3x^6 + 350ab^2x^4 + 406a^2bx^2 + 176a^3)\sqrt{bx^2 + a}\sqrt{a} + 105(b^4x^8 + 4ab^3x^6 + 6a^2b^2x^4 + 4a^3bx^2 + a^4)\log\left(-\frac{bx^2+2a}{\sqrt{bx^2+a}}\right)}{210(a^4b^4x^8 + 4a^5b^3x^6 + 6a^6b^2x^4 + 4a^7bx^2 + a^8)\sqrt{a}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(9/2)\*x), x, algorithm="fricas")

[Out] [1/210\*(2\*(105\*b^3\*x^6 + 350\*a\*b^2\*x^4 + 406\*a^2\*b\*x^2 + 176\*a^3)\*sqrt(b\*x^2 + a)\*sqrt(a) + 105\*(b^4\*x^8 + 4\*a\*b^3\*x^6 + 6\*a^2\*b^2\*x^4 + 4\*a^3\*b\*x^2 + a^4)\*log(-((b\*x^2 + 2\*a)\*sqrt(a) - 2\*sqrt(b\*x^2 + a)\*a)/x^2))/((a^4\*b^4\*x^8 + 4\*a^5\*b^3\*x^6 + 6\*a^6\*b^2\*x^4 + 4\*a^7\*b\*x^2 + a^8)\*sqrt(a)), 1/105\*((105\*b^3\*x^6 + 350\*a\*b^2\*x^4 + 406\*a^2\*b\*x^2 + 176\*a^3)\*sqrt(b\*x^2 + a)\*sqrt(-a) - 105\*(b^4\*x^8 + 4\*a\*b^3\*x^6 + 6\*a^2\*b^2\*x^4 + 4\*a^3\*b\*x^2 + a^4)\*arctan(sqrt(-a)/sqrt(b\*x^2 + a)))/((a^4\*b^4\*x^8 + 4\*a^5\*b^3\*x^6 + 6\*a^6\*b^2\*x^4 + 4\*a^7\*b\*x^2 + a^8)\*sqrt(-a))]

**Sympy [A]** time = 30.8073, size = 5250, normalized size = 55.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b\*x\*\*2+a)\*\*(9/2), x)

[Out] 352\*a\*\*32\*sqrt(1 + b\*x\*\*2/a)/(210\*a\*\*(73/2) + 2100\*a\*\*(71/2)\*b\*x\*\*2 + 9450\*a\*\*(69/2)\*b\*\*2\*x\*\*4 + 25200\*a\*\*(67/2)\*b\*\*3\*x\*\*6 + 44100\*a\*\*(65/2)\*b\*\*4\*x\*\*8 + 52920\*a\*\*(63/2)\*b\*\*5\*x\*\*10 + 44100\*a\*\*(61/2)\*b\*\*6\*x\*\*12 + 25200\*a\*\*(59/2)\*b\*\*7\*x\*\*14 + 9450\*a\*\*(57/2)\*b\*\*8\*x\*\*16 + 2100\*a\*\*(55/2)\*b\*\*9\*x\*\*18 + 210\*a\*\*(53/2)\*b\*\*10\*x\*\*20) +





$$\begin{aligned}
& 63/2) * b^{5} x^{10} + 44100 * a^{(61/2)} * b^{6} x^{12} + 25200 * a^{(59/2)} * b^{7} x^{14} + 9450 * a^{(57/2)} * b^{8} x^{16} + 2100 * a^{(55/2)} * b^{9} x^{18} \\
& + 210 * a^{(53/2)} * b^{10} x^{20}) + 105 * a^{22} * b^{10} x^{20} * \log(b x^2 / a) / (210 * a^{(73/2)} + 2100 * a^{(71/2)} * b x^2 + 9450 * a^{(69/2)} * b^2 x^4 \\
& + 25200 * a^{(67/2)} * b^3 x^6 + 44100 * a^{(65/2)} * b^4 x^8 + 52920 * a^{(63/2)} * b^5 x^{10} + 44100 * a^{(61/2)} * b^6 x^{12} + 25200 * a^{(59/2)} * b^7 x^{14} \\
& + 9450 * a^{(57/2)} * b^8 x^{16} + 2100 * a^{(55/2)} * b^9 x^{18} + 210 * a^{(53/2)} * b^{10} x^{20}) - 210 * a^{22} * b^{10} x^{20} * \log(\sqrt{1 + b x^2 / a} + 1) / (210 * a^{(73/2)} + 2100 * a^{(71/2)} * b x^2 + 9450 * a^{(69/2)} * b^2 x^4 \\
& + 25200 * a^{(67/2)} * b^3 x^6 + 44100 * a^{(65/2)} * b^4 x^8 + 52920 * a^{(63/2)} * b^5 x^{10} + 44100 * a^{(61/2)} * b^6 x^{12} + 25200 * a^{(59/2)} * b^7 x^{14} + 9450 * a^{(57/2)} * b^8 x^{16} \\
& + 2100 * a^{(55/2)} * b^9 x^{18} + 210 * a^{(53/2)} * b^{10} x^{20})
\end{aligned}$$

**GIAC/XCAS [A]** time = 0.213111, size = 109, normalized size = 1.15

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4} + \frac{105(bx^2+a)^3 + 35(bx^2+a)^2a + 21(bx^2+a)a^2 + 15a^3}{105(bx^2+a)^{7/2}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(9/2)\*x),x, algorithm="giac")

[Out] arctan(sqrt(b\*x^2 + a)/sqrt(-a))/(sqrt(-a)\*a^4) + 1/105\*(105\*(b\*x^2 + a)^3 + 35\*(b\*x^2 + a)^2\*a + 21\*(b\*x^2 + a)\*a^2 + 15\*a^3)/((b\*x^2 + a)^(7/2)\*a^4)

$$3.529 \quad \int \frac{1}{x^2(a+bx^2)^{9/2}} dx$$

**Optimal.** Leaf size=100

$$-\frac{128bx}{35a^5\sqrt{a+bx^2}} - \frac{64bx}{35a^4(a+bx^2)^{3/2}} - \frac{48bx}{35a^3(a+bx^2)^{5/2}} - \frac{8bx}{7a^2(a+bx^2)^{7/2}} - \frac{1}{ax(a+bx^2)^{7/2}}$$

[Out]  $-(1/(a*x*(a+b*x^2)^(7/2))) - (8*b*x)/(7*a^2*(a+b*x^2)^(7/2)) - (48*b*x)/(35*a^3*(a+b*x^2)^(5/2)) - (64*b*x)/(35*a^4*(a+b*x^2)^(3/2)) - (128*b*x)/(35*a^5*\text{Sqrt}[a+b*x^2])$

**Rubi [A]** time = 0.0814021, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{128bx}{35a^5\sqrt{a+bx^2}} - \frac{64bx}{35a^4(a+bx^2)^{3/2}} - \frac{48bx}{35a^3(a+bx^2)^{5/2}} - \frac{8bx}{7a^2(a+bx^2)^{7/2}} - \frac{1}{ax(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a+b\*x^2)^(9/2)),x]

[Out]  $-(1/(a*x*(a+b*x^2)^(7/2))) - (8*b*x)/(7*a^2*(a+b*x^2)^(7/2)) - (48*b*x)/(35*a^3*(a+b*x^2)^(5/2)) - (64*b*x)/(35*a^4*(a+b*x^2)^(3/2)) - (128*b*x)/(35*a^5*\text{Sqrt}[a+b*x^2])$

**Rubi in Sympy [A]** time = 9.28247, size = 97, normalized size = 0.97

$$-\frac{1}{ax(a+bx^2)^{7/2}} - \frac{8bx}{7a^2(a+bx^2)^{7/2}} - \frac{48bx}{35a^3(a+bx^2)^{5/2}} - \frac{64bx}{35a^4(a+bx^2)^{3/2}} - \frac{128bx}{35a^5\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(b\*x\*\*2+a)\*\*(9/2),x)

[Out]  $-1/(a*x*(a+b*x**2)**(7/2)) - 8*b*x/(7*a**2*(a+b*x**2)**(7/2)) - 48*b*x/(35*a**3*(a+b*x**2)**(5/2)) - 64*b*x/(35*a**4*(a+b*x**2)**(3/2)) - 128*b*x/(35*a**5*\text{sqrt}(a+b*x**2))$

**Mathematica [A]** time = 0.0480096, size = 64, normalized size = 0.64

$$-\frac{35a^4 + 280a^3bx^2 + 560a^2b^2x^4 + 448ab^3x^6 + 128b^4x^8}{35a^5x(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a+b\*x^2)^(9/2)),x]

[Out]  $-(35*a^4 + 280*a^3*b*x^2 + 560*a^2*b^2*x^4 + 448*a*b^3*x^6 + 128*b^4*x^8)/(35*a^5*x*(a+b*x^2)^(7/2))$

**Maple [A]** time = 0.008, size = 61, normalized size = 0.6

$$-\frac{128b^4x^8 + 448b^3x^6a + 560b^2x^4a^2 + 280bx^2a^3 + 35a^4}{35a^5x} (bx^2 + a)^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(b*x^2+a)^(9/2), x)`

[Out]  $-1/35 * (128 * b^4 * x^8 + 448 * a * b^3 * x^6 + 560 * a^2 * b^2 * x^4 + 280 * a^3 * b * x^2 + 35 * a^4) / x / (b * x^2 + a)^{7/2} / a^5$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(9/2)*x^2), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.292337, size = 139, normalized size = 1.39

$$\frac{(128 b^4 x^8 + 448 a b^3 x^6 + 560 a^2 b^2 x^4 + 280 a^3 b x^2 + 35 a^4) \sqrt{b x^2 + a}}{35 (a^5 b^4 x^9 + 4 a^6 b^3 x^7 + 6 a^7 b^2 x^5 + 4 a^8 b x^3 + a^9 x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(9/2)*x^2), x, algorithm="fricas")`

[Out]  $-1/35 * (128 * b^4 * x^8 + 448 * a * b^3 * x^6 + 560 * a^2 * b^2 * x^4 + 280 * a^3 * b * x^2 + 35 * a^4) * \text{sqrt}(b * x^2 + a) / (a^5 * b^4 * x^9 + 4 * a^6 * b^3 * x^7 + 6 * a^7 * b^2 * x^5 + 4 * a^8 * b * x^3 + a^9 * x)$

**Sympy [A]** time = 19.8757, size = 400, normalized size = 4.

$$\begin{aligned} & \frac{35 a^4 b^{\frac{33}{2}} \sqrt{\frac{a}{b x^2} + 1}}{35 a^9 b^{16} + 140 a^8 b^{17} x^2 + 210 a^7 b^{18} x^4 + 140 a^6 b^{19} x^6 + 35 a^5 b^{20} x^8} \\ & - \frac{280 a^3 b^{\frac{35}{2}} x^2 \sqrt{\frac{a}{b x^2} + 1}}{35 a^9 b^{16} + 140 a^8 b^{17} x^2 + 210 a^7 b^{18} x^4 + 140 a^6 b^{19} x^6 + 35 a^5 b^{20} x^8} \\ & - \frac{560 a^2 b^{\frac{37}{2}} x^4 \sqrt{\frac{a}{b x^2} + 1}}{35 a^9 b^{16} + 140 a^8 b^{17} x^2 + 210 a^7 b^{18} x^4 + 140 a^6 b^{19} x^6 + 35 a^5 b^{20} x^8} \\ & - \frac{448 a b^{\frac{39}{2}} x^6 \sqrt{\frac{a}{b x^2} + 1}}{35 a^9 b^{16} + 140 a^8 b^{17} x^2 + 210 a^7 b^{18} x^4 + 140 a^6 b^{19} x^6 + 35 a^5 b^{20} x^8} \\ & - \frac{128 b^{\frac{41}{2}} x^8 \sqrt{\frac{a}{b x^2} + 1}}{35 a^9 b^{16} + 140 a^8 b^{17} x^2 + 210 a^7 b^{18} x^4 + 140 a^6 b^{19} x^6 + 35 a^5 b^{20} x^8} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(b*x**2+a)**(9/2), x)`

[Out]  $-35 * a^{**4} * b^{** (33/2)} * \text{sqrt}(a / (b * x^{**2}) + 1) / (35 * a^{**9} * b^{**16} + 140 * a^{**8} * b^{**17} * x^{**2} + 210 * a^{**7} * b^{**18} * x^{**4} + 140 * a^{**6} * b^{**19} * x^{**6} + 35 * a^{**5} * b^{**20} * x^{**8}) - 280 * a^{**3} * b^{** (35/2)} * x^{**2} * \text{sqrt}(a / (b * x^{**2}) + 1) / (35 * a^{**9} * b^{**16} + 140 * a^{**8} * b^{**17} * x^{**2} + 210 * a^{**7} * b^{**18} * x^{**4} + 140 * a^{**6} * b^{**19} * x^{**6} + 35 * a^{**5} * b^{**20} * x^{**8}) - 560 * a^{**2} * b^{** (37/2)} * x^{**4} * \text{sqrt}(a / (b * x^{**2}) + 1) / (35 * a^{**9} * b^{**16} + 140 * a^{**8} * b^{**17} * x^{**2} + 210 * a^{**7} * b^{**18} * x^{**4} + 140 * a^{**6} * b^{**19} * x^{**6} + 35 * a^{**5} * b^{**20} * x^{**8}) - 448 * a * b^{** ($

$$\frac{39}{2} x^6 \sqrt{\frac{a}{bx^2} + 1} / (35 a^9 b^{16} + 140 a^8 b^{17} x^2 + 210 a^7 b^{18} x^4 + 140 a^6 b^{19} x^6 + 35 a^5 b^{20} x^8) - 128 b (41/2) x^8 \sqrt{\frac{a}{bx^2} + 1} / (35 a^9 b^{16} + 140 a^8 b^{17} x^2 + 210 a^7 b^{18} x^4 + 140 a^6 b^{19} x^6 + 35 a^5 b^{20} x^8)$$

**GIAC/XCAS [A]** time = 0.215283, size = 122, normalized size = 1.22

$$-\frac{\left(x^2 \left(\frac{93 b^4 x^2}{a^5} + \frac{308 b^3}{a^4}\right) + \frac{350 b^2}{a^3}\right) x^2 + \frac{140 b}{a^2} x}{35 (bx^2 + a)^{\frac{7}{2}}} + \frac{2 \sqrt{b}}{\left(\left(\sqrt{bx} - \sqrt{bx^2 + a}\right)^2 - a\right) a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(9/2)\*x^2),x, algorithm="giac")

[Out] -1/35\*((x^2\*(93\*b^4\*x^2/a^5 + 308\*b^3/a^4) + 350\*b^2/a^3)\*x^2 + 140\*b/a^2)\*x/(b\*x^2 + a)^(7/2) + 2\*sqrt(b)/(((sqrt(b)\*x - sqrt(b\*x^2 + a))^2 - a)\*a^4)

$$3.530 \quad \int \frac{1}{x^3(a+bx^2)^{9/2}} dx$$

**Optimal.** Leaf size=132

$$\frac{9b \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}} - \frac{9\sqrt{a+bx^2}}{2a^5x^2} + \frac{3}{a^4x^2\sqrt{a+bx^2}} + \frac{3}{5a^3x^2(a+bx^2)^{3/2}} + \frac{9}{35a^2x^2(a+bx^2)^{5/2}} + \frac{1}{7ax^2(a+bx^2)^{7/2}}$$

[Out] 1/(7\*a\*x^2\*(a+b\*x^2)^(7/2)) + 9/(35\*a^2\*x^2\*(a+b\*x^2)^(5/2)) + 3/(5\*a^3\*x^2\*(a+b\*x^2)^(3/2)) + 3/(a^4\*x^2\*Sqrt[a+b\*x^2]) - (9\*Sqrt[a+b\*x^2])/(2\*a^5\*x^2) + (9\*b\*ArcTanh[Sqrt[a+b\*x^2]/Sqrt[a]])/(2\*a^(11/2))

**Rubi [A]** time = 0.214556, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{9b \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}} - \frac{9\sqrt{a+bx^2}}{2a^5x^2} + \frac{3}{a^4x^2\sqrt{a+bx^2}} + \frac{3}{5a^3x^2(a+bx^2)^{3/2}} + \frac{9}{35a^2x^2(a+bx^2)^{5/2}} + \frac{1}{7ax^2(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(a+b\*x^2)^(9/2)),x]

[Out] 1/(7\*a\*x^2\*(a+b\*x^2)^(7/2)) + 9/(35\*a^2\*x^2\*(a+b\*x^2)^(5/2)) + 3/(5\*a^3\*x^2\*(a+b\*x^2)^(3/2)) + 3/(a^4\*x^2\*Sqrt[a+b\*x^2]) - (9\*Sqrt[a+b\*x^2])/(2\*a^5\*x^2) + (9\*b\*ArcTanh[Sqrt[a+b\*x^2]/Sqrt[a]])/(2\*a^(11/2))

**Rubi in Sympy [A]** time = 22.1782, size = 124, normalized size = 0.94

$$\frac{1}{7ax^2(a+bx^2)^{7/2}} + \frac{9}{35a^2x^2(a+bx^2)^{5/2}} + \frac{3}{5a^3x^2(a+bx^2)^{3/2}} + \frac{3}{a^4x^2\sqrt{a+bx^2}} - \frac{9\sqrt{a+bx^2}}{2a^5x^2} + \frac{9b \operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(b\*x\*\*2+a)\*\*(9/2),x)

[Out] 1/(7\*a\*x\*\*2\*(a+b\*x\*\*2)\*\*(7/2)) + 9/(35\*a\*\*2\*x\*\*2\*(a+b\*x\*\*2)\*\*(5/2)) + 3/(5\*a\*\*3\*x\*\*2\*(a+b\*x\*\*2)\*\*(3/2)) + 3/(a\*\*4\*x\*\*2\*sqrt(a+b\*x\*\*2)) - 9\*sqrt(a+b\*x\*\*2)/(2\*a\*\*5\*x\*\*2) + 9\*b\*atanh(sqrt(a+b\*x\*\*2)/sqrt(a))/(2\*a\*\*(11/2))

**Mathematica [A]** time = 0.223041, size = 102, normalized size = 0.77

$$\frac{-\sqrt{a}(35a^4+528a^3bx^2+1218a^2b^2x^4+1050ab^3x^6+315b^4x^8)}{x^2(a+bx^2)^{7/2}} + 315b \log\left(\sqrt{a}\sqrt{a+bx^2}+a\right) - 315b \log(x)}{70a^{11/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a+b\*x^2)^(9/2)),x]



[Out]  $(-\left(\sqrt{a} \left(35 a^4 + 528 a^3 b x^2 + 1218 a^2 b^2 x^4 + 1050 a b^3 x^6 + 315 b^4 x^8\right)\right) / \left(x^2 (a + b x^2)^{7/2}\right) - 315 b \operatorname{Log}[x] + 315 b \operatorname{Log}[a + \sqrt{a} \sqrt{a + b x^2}]) / (70 a^{11/2})$

**Maple [A]** time = 0.009, size = 108, normalized size = 0.8

$$-\frac{1}{2 a x^2} (b x^2 + a)^{-\frac{7}{2}} - \frac{9 b}{14 a^2} (b x^2 + a)^{-\frac{7}{2}} - \frac{9 b}{10 a^3} (b x^2 + a)^{-\frac{5}{2}} - \frac{3 b}{2 a^4} (b x^2 + a)^{-\frac{3}{2}} - \frac{9 b}{2 a^5} \frac{1}{\sqrt{b x^2 + a}} + \frac{9 b}{2} \ln\left(\frac{1}{x} \left(2 a + 2 \sqrt{a} \sqrt{b x^2 + a}\right)\right) a^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(b*x^2+a)^(9/2), x)`

[Out]  $-1/2/a/x^2/(b*x^2+a)^{7/2} - 9/14*b/a^2/(b*x^2+a)^{7/2} - 9/10*b/a^3/(b*x^2+a)^{5/2} - 3/2*b/a^4/(b*x^2+a)^{3/2} - 9/2*b/a^5/(b*x^2+a)^{1/2} + 9/2*b/a^{11/2} * \ln((2*a+2*a^{1/2}*(b*x^2+a)^{1/2})/x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(9/2)*x^3), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.291056, size = 1, normalized size = 0.01

$$\left[ \frac{2 (315 b^4 x^8 + 1050 a b^3 x^6 + 1218 a^2 b^2 x^4 + 528 a^3 b x^2 + 35 a^4) \sqrt{b x^2 + a} \sqrt{a} - 315 (b^5 x^{10} + 4 a b^4 x^8 + 6 a^2 b^3 x^6 + 4 a^3 b^2 x^4 + 4 a^4 b x^2 + a^5) \sqrt{a}}{140 (a^5 b^4 x^{10} + 4 a^6 b^3 x^8 + 6 a^7 b^2 x^6 + 4 a^8 b x^4 + a^9 x^2) \sqrt{a}} \right]$$

$$\frac{(315 b^4 x^8 + 1050 a b^3 x^6 + 1218 a^2 b^2 x^4 + 528 a^3 b x^2 + 35 a^4) \sqrt{b x^2 + a} \sqrt{-a} - 315 (b^5 x^{10} + 4 a b^4 x^8 + 6 a^2 b^3 x^6 + 4 a^3 b^2 x^4 + 4 a^4 b x^2 + a^5) \sqrt{-a}}{70 (a^5 b^4 x^{10} + 4 a^6 b^3 x^8 + 6 a^7 b^2 x^6 + 4 a^8 b x^4 + a^9 x^2) \sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(9/2)*x^3), x, algorithm="fricas")`

[Out]  $[-1/140 * (2 * (315 * b^4 * x^8 + 1050 * a * b^3 * x^6 + 1218 * a^2 * b^2 * x^4 + 528 * a^3 * b * x^2 + 35 * a^4) * \operatorname{sqrt}(b * x^2 + a) * \operatorname{sqrt}(a) - 315 * (b^5 * x^{10} + 4 * a * b^4 * x^8 + 6 * a^2 * b^3 * x^6 + 4 * a^3 * b^2 * x^4 + a^4 * b * x^2) * \log(-((b * x^2 + 2 * a) * \operatorname{sqrt}(a) + 2 * \operatorname{sqrt}(b * x^2 + a) * a) / x^2)) / ((a^5 * b^4 * x^{10} + 4 * a^6 * b^3 * x^8 + 6 * a^7 * b^2 * x^6 + 4 * a^8 * b * x^4 + a^9 * x^2) * \operatorname{sqrt}(a)), -1/70 * ((315 * b^4 * x^8 + 1050 * a * b^3 * x^6 + 1218 * a^2 * b^2 * x^4 + 528 * a^3 * b * x^2 + 35 * a^4) * \operatorname{sqrt}(b * x^2 + a) * \operatorname{sqrt}(-a) - 315 * (b^5 * x^{10} + 4 * a * b^4 * x^8 + 6 * a^2 * b^3 * x^6 + 4 * a^3 * b^2 * x^4 + a^4 * b * x^2) * \arctan(\operatorname{sqrt}(-a) / \operatorname{sqrt}(b * x^2 + a))) / ((a^5 * b^4 * x^{10} + 4 * a^6 * b^3 * x^8 + 6 * a^7 * b^2 * x^6 + 4 * a^8 * b * x^4 + a^9 * x^2) * \operatorname{sqrt}(-a))]$

**Sympy [A]** time = 50.5972, size = 5540, normalized size = 41.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(b\*x\*\*2+a)\*\*(9/2),x)

[Out] 
$$\begin{aligned} & -70*a^{49}\sqrt{1 + b*x^2/a}/(140*a^{(107/2)}*x^2 + 1400*a^{(105/2)}*b*x^4 + 6300*a^{(103/2)}*b^2*x^6 + 16800*a^{(101/2)}*b^3*x^8 + 29400*a^{(99/2)}*b^4*x^{10} + 35280*a^{(97/2)}*b^5*x^{12} + 29400*a^{(95/2)}*b^6*x^{14} + 16800*a^{(93/2)}*b^7*x^{16} + 6300*a^{(91/2)}*b^8*x^{18} + 1400*a^{(89/2)}*b^9*x^{20} + 140*a^{(87/2)}*b^{10}*x^{22}) \\ & - 1476*a^{48}*b*x^2*\sqrt{1 + b*x^2/a}/(140*a^{(107/2)}*x^2 + 1400*a^{(105/2)}*b*x^4 + 6300*a^{(103/2)}*b^2*x^6 + 16800*a^{(101/2)}*b^3*x^8 + 29400*a^{(99/2)}*b^4*x^{10} + 35280*a^{(97/2)}*b^5*x^{12} + 29400*a^{(95/2)}*b^6*x^{14} + 16800*a^{(93/2)}*b^7*x^{16} + 6300*a^{(91/2)}*b^8*x^{18} + 1400*a^{(89/2)}*b^9*x^{20} + 140*a^{(87/2)}*b^{10}*x^{22}) \\ & - 315*a^{48}*b*x^2*\log(b*x^2/a)/(140*a^{(107/2)}*x^2 + 1400*a^{(105/2)}*b*x^4 + 6300*a^{(103/2)}*b^2*x^6 + 16800*a^{(101/2)}*b^3*x^8 + 29400*a^{(99/2)}*b^4*x^{10} + 35280*a^{(97/2)}*b^5*x^{12} + 29400*a^{(95/2)}*b^6*x^{14} + 16800*a^{(93/2)}*b^7*x^{16} + 6300*a^{(91/2)}*b^8*x^{18} + 1400*a^{(89/2)}*b^9*x^{20} + 140*a^{(87/2)}*b^{10}*x^{22}) \\ & + 630*a^{48}*b*x^2*\log(\sqrt{1 + b*x^2/a} + 1)/(140*a^{(107/2)}*x^2 + 1400*a^{(105/2)}*b*x^4 + 6300*a^{(103/2)}*b^2*x^6 + 16800*a^{(101/2)}*b^3*x^8 + 29400*a^{(99/2)}*b^4*x^{10} + 35280*a^{(97/2)}*b^5*x^{12} + 29400*a^{(95/2)}*b^6*x^{14} + 16800*a^{(93/2)}*b^7*x^{16} + 6300*a^{(91/2)}*b^8*x^{18} + 1400*a^{(89/2)}*b^9*x^{20} + 140*a^{(87/2)}*b^{10}*x^{22}) \\ & - 9822*a^{47}*b^2*x^4*\sqrt{1 + b*x^2/a}/(140*a^{(107/2)}*x^2 + 1400*a^{(105/2)}*b*x^4 + 6300*a^{(103/2)}*b^2*x^6 + 16800*a^{(101/2)}*b^3*x^8 + 29400*a^{(99/2)}*b^4*x^{10} + 35280*a^{(97/2)}*b^5*x^{12} + 29400*a^{(95/2)}*b^6*x^{14} + 16800*a^{(93/2)}*b^7*x^{16} + 6300*a^{(91/2)}*b^8*x^{18} + 1400*a^{(89/2)}*b^9*x^{20} + 140*a^{(87/2)}*b^{10}*x^{22}) \\ & - 3150*a^{47}*b^2*x^4*\log(b*x^2/a)/(140*a^{(107/2)}*x^2 + 1400*a^{(105/2)}*b*x^4 + 6300*a^{(103/2)}*b^2*x^6 + 16800*a^{(101/2)}*b^3*x^8 + 29400*a^{(99/2)}*b^4*x^{10} + 35280*a^{(97/2)}*b^5*x^{12} + 29400*a^{(95/2)}*b^6*x^{14} + 16800*a^{(93/2)}*b^7*x^{16} + 6300*a^{(91/2)}*b^8*x^{18} + 1400*a^{(89/2)}*b^9*x^{20} + 140*a^{(87/2)}*b^{10}*x^{22}) \\ & + 6300*a^{47}*b^2*x^4*\log(\sqrt{1 + b*x^2/a} + 1)/(140*a^{(107/2)}*x^2 + 1400*a^{(105/2)}*b*x^4 + 6300*a^{(103/2)}*b^2*x^6 + 16800*a^{(101/2)}*b^3*x^8 + 29400*a^{(99/2)}*b^4*x^{10} + 35280*a^{(97/2)}*b^5*x^{12} + 29400*a^{(95/2)}*b^6*x^{14} + 16800*a^{(93/2)}*b^7*x^{16} + 6300*a^{(91/2)}*b^8*x^{18} + 1400*a^{(89/2)}*b^9*x^{20} + 140*a^{(87/2)}*b^{10}*x^{22}) \\ & - 14175*a^{46}*b^3*x^6*\sqrt{1 + b*x^2/a}/(140*a^{(107/2)}*x^2 + 1400*a^{(105/2)}*b*x^4 + 6300*a^{(103/2)}*b^2*x^6 + 16800*a^{(101/2)}*b^3*x^8 + 29400*a^{(99/2)}*b^4*x^{10} + 35280*a^{(97/2)}*b^5*x^{12} + 29400*a^{(95/2)}*b^6*x^{14} + 16800*a^{(93/2)}*b^7*x^{16} + 6300*a^{(91/2)}*b^8*x^{18} + 1400*a^{(89/2)}*b^9*x^{20} + 140*a^{(87/2)}*b^{10}*x^{22}) \\ & + 1400*a^{(105/2)}*b*x^4 + 6300*a^{(103/2)}*b^2*x^6 + 16800*a^{(101/2)}*b^3*x^8 + 29400*a^{(99/2)}*b^4*x^{10} + 35280*a^{(97/2)}*b^5*x^{12} + 29400*a^{(95/2)}*b^6*x^{14} + 16800*a^{(93/2)}*b^7*x^{16} + 6300*a^{(91/2)}*b^8*x^{18} + 1400*a^{(89/2)}*b^9*x^{20} + 140*a^{(87/2)}*b^{10}*x^{22}) \\ & - 71940*a^{45}*b^4*x^8*\sqrt{1 + b*x^2/a}/(140*a^{(107/2)}*x^2 + 1400*a^{(105/2)}*b*x^4 + 6300*a^{(103/2)}*b^2*x^6 + 16800*a^{(101/2)}*b^3*x^8 + 29400*a^{(99/2)}*b^4*x^{10} + 35280*a^{(97/2)}*b^5*x^{12} + 29400*a^{(95/2)}*b^6*x^{14} + 16800*a^{(93/2)}*b^7*x^{16} + 6300*a^{(91/2)}*b^8*x^{18} + 1400*a^{(89/2)}*b^9*x^{20} + 140*a^{(87/2)}*b^{10}*x^{22}) \\ & - 37800*a^{45}*b^4*x^8*\log(b*x^2/a)/(140*a^{(107/2)}*x^2 + 1400*a^{(105/2)}*b*x^4 + 6300*a^{(103/2)}*b^2*x^6 + 16800*a^{(101/2)}*b^3*x^8 + 29400*a^{(99/2)}*b^4*x^{10} + 35280*a^{(97/2)}*b^5*x^{12} + 29400*a^{(95/2)}*b^6*x^{14} + 16800*a^{(93/2)}*b^7*x^{16} + 6300*a^{(91/2)}*b^8*x^{18} + 1400*a^{(89/2)}*b^9*x^{20} + 140*a^{(87/2)}*b^{10}*x^{22}) \\ & + 75600*a^{45}*b^4*x^8*\log(\sqrt{1 + b*x^2/a} + 1)/(140*a^{(107/2)}*x^2 + 1400*a^{(105/2)}*b*x^4 + 6300*a^{(103/2)}*b^2*x^6 + 16800*a^{(101/2)}*b^3*x^8 + 29400*a^{(99/2)}*b^4*x^{10} + 35280*a^{(97/2)}*b^5*x^{12} + 29400*a^{(95/2)}*b^6*x^{14} + 16800*a^{(93/2)}*b^7*x^{16} + 6300*a^{(91/2)}*b^8*x^{18} + 1400*a^{(89/2)}*b^9*x^{20} + 140*a^{(87/2)}*b^{10}*x^{22}) \end{aligned}$$



$$\begin{aligned}
& 6 + 6300*a^{(91/2)}*b^{*8}*x^{*18} + 1400*a^{(89/2)}*b^{*9}*x^{*20} + 140*a^{(87/2)}*b^{*10}*x^{*22} + 28350*a^{*40}*b^{*9}*x^{*18}*\log(\sqrt{1 + b*x^{*2}/a} + 1)/(140*a^{(107/2)}*x^{*2} + 1400*a^{(105/2)}*b*x^{*4} + 6300*a^{(103/2)}*b^{*2}*x^{*6} + 16800*a^{(101/2)}*b^{*3}*x^{*8} + 29400*a^{(99/2)}*b^{*4}*x^{*10} + 35280*a^{(97/2)}*b^{*5}*x^{*12} + 29400*a^{(95/2)}*b^{*6}*x^{*14} + 16800*a^{(93/2)}*b^{*7}*x^{*16} + 6300*a^{(91/2)}*b^{*8}*x^{*18} + 1400*a^{(89/2)}*b^{*9}*x^{*20} + 140*a^{(87/2)}*b^{*10}*x^{*22} - 630*a^{*39}*b^{*10}*x^{*20}*\sqrt{1 + b*x^{*2}/a}/(140*a^{(107/2)}*x^{*2} + 1400*a^{(105/2)}*b*x^{*4} + 6300*a^{(103/2)}*b^{*2}*x^{*6} + 16800*a^{(101/2)}*b^{*3}*x^{*8} + 29400*a^{(99/2)}*b^{*4}*x^{*10} + 35280*a^{(97/2)}*b^{*5}*x^{*12} + 29400*a^{(95/2)}*b^{*6}*x^{*14} + 16800*a^{(93/2)}*b^{*7}*x^{*16} + 6300*a^{(91/2)}*b^{*8}*x^{*18} + 1400*a^{(89/2)}*b^{*9}*x^{*20} + 140*a^{(87/2)}*b^{*10}*x^{*22} - 3150*a^{*39}*b^{*10}*x^{*20}*\log(b*x^{*2}/a)/(140*a^{(107/2)}*x^{*2} + 1400*a^{(105/2)}*b*x^{*4} + 6300*a^{(103/2)}*b^{*2}*x^{*6} + 16800*a^{(101/2)}*b^{*3}*x^{*8} + 29400*a^{(99/2)}*b^{*4}*x^{*10} + 35280*a^{(97/2)}*b^{*5}*x^{*12} + 29400*a^{(95/2)}*b^{*6}*x^{*14} + 16800*a^{(93/2)}*b^{*7}*x^{*16} + 6300*a^{(91/2)}*b^{*8}*x^{*18} + 1400*a^{(89/2)}*b^{*9}*x^{*20} + 140*a^{(87/2)}*b^{*10}*x^{*22} + 6300*a^{*39}*b^{*10}*x^{*20}*\log(\sqrt{1 + b*x^{*2}/a} + 1)/(140*a^{(107/2)}*x^{*2} + 1400*a^{(105/2)}*b*x^{*4} + 6300*a^{(103/2)}*b^{*2}*x^{*6} + 16800*a^{(101/2)}*b^{*3}*x^{*8} + 29400*a^{(99/2)}*b^{*4}*x^{*10} + 35280*a^{(97/2)}*b^{*5}*x^{*12} + 29400*a^{(95/2)}*b^{*6}*x^{*14} + 16800*a^{(93/2)}*b^{*7}*x^{*16} + 6300*a^{(91/2)}*b^{*8}*x^{*18} + 1400*a^{(89/2)}*b^{*9}*x^{*20} + 140*a^{(87/2)}*b^{*10}*x^{*22} - 315*a^{*38}*b^{*11}*x^{*22}*\log(b*x^{*2}/a)/(140*a^{(107/2)}*x^{*2} + 1400*a^{(105/2)}*b*x^{*4} + 6300*a^{(103/2)}*b^{*2}*x^{*6} + 16800*a^{(101/2)}*b^{*3}*x^{*8} + 29400*a^{(99/2)}*b^{*4}*x^{*10} + 35280*a^{(97/2)}*b^{*5}*x^{*12} + 29400*a^{(95/2)}*b^{*6}*x^{*14} + 16800*a^{(93/2)}*b^{*7}*x^{*16} + 6300*a^{(91/2)}*b^{*8}*x^{*18} + 1400*a^{(89/2)}*b^{*9}*x^{*20} + 140*a^{(87/2)}*b^{*10}*x^{*22} + 630*a^{*38}*b^{*11}*x^{*22}*\log(\sqrt{1 + b*x^{*2}/a} + 1)/(140*a^{(107/2)}*x^{*2} + 1400*a^{(105/2)}*b*x^{*4} + 6300*a^{(103/2)}*b^{*2}*x^{*6} + 16800*a^{(101/2)}*b^{*3}*x^{*8} + 29400*a^{(99/2)}*b^{*4}*x^{*10} + 35280*a^{(97/2)}*b^{*5}*x^{*12} + 29400*a^{(95/2)}*b^{*6}*x^{*14} + 16800*a^{(93/2)}*b^{*7}*x^{*16} + 6300*a^{(91/2)}*b^{*8}*x^{*18} + 1400*a^{(89/2)}*b^{*9}*x^{*20} + 140*a^{(87/2)}*b^{*10}*x^{*22}
\end{aligned}$$

**GIAC/XCAS [A]** time = 0.214913, size = 142, normalized size = 1.08

$$-\frac{1}{70}b\left(\frac{315\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-aa^5}} + \frac{2\left(140(bx^2+a)^3 + 35(bx^2+a)^2a + 14(bx^2+a)a^2 + 5a^3\right)}{(bx^2+a)^{\frac{7}{2}}a^5} + \frac{35\sqrt{bx^2+a}}{a^5bx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(9/2)\*x^3),x, algorithm="giac")

[Out] -1/70\*b\*(315\*arctan(sqrt(b\*x^2 + a)/sqrt(-a))/(sqrt(-a)\*a^5) + 2\*(140\*(b\*x^2 + a)^3 + 35\*(b\*x^2 + a)^2\*a + 14\*(b\*x^2 + a)\*a^2 + 5\*a^3)/((b\*x^2 + a)^(7/2)\*a^5) + 35\*sqrt(b\*x^2 + a)/(a^5\*b\*x^2))

$$3.531 \quad \int \frac{1}{x^4(a+bx^2)^{9/2}} dx$$

**Optimal.** Leaf size=132

$$\frac{256b^2x}{21a^6\sqrt{a+bx^2}} + \frac{128b^2x}{21a^5(a+bx^2)^{3/2}} + \frac{32b^2x}{7a^4(a+bx^2)^{5/2}} \\ + \frac{80b^2x}{21a^3(a+bx^2)^{7/2}} + \frac{10b}{3a^2x(a+bx^2)^{7/2}} - \frac{1}{3ax^3(a+bx^2)^{7/2}}$$

[Out]  $-1/(3*a*x^3*(a+b*x^2)^(7/2)) + (10*b)/(3*a^2*x*(a+b*x^2)^(7/2)) + (80*b^2*x)/(21*a^3*(a+b*x^2)^(7/2)) + (32*b^2*x)/(7*a^4*(a+b*x^2)^(5/2)) + (128*b^2*x)/(21*a^5*(a+b*x^2)^(3/2)) + (256*b^2*x)/(21*a^6*\text{Sqrt}[a+b*x^2])$

**Rubi [A]** time = 0.120486, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{256b^2x}{21a^6\sqrt{a+bx^2}} + \frac{128b^2x}{21a^5(a+bx^2)^{3/2}} + \frac{32b^2x}{7a^4(a+bx^2)^{5/2}} \\ + \frac{80b^2x}{21a^3(a+bx^2)^{7/2}} + \frac{10b}{3a^2x(a+bx^2)^{7/2}} - \frac{1}{3ax^3(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a+b\*x^2)^(9/2)),x]

[Out]  $-1/(3*a*x^3*(a+b*x^2)^(7/2)) + (10*b)/(3*a^2*x*(a+b*x^2)^(7/2)) + (80*b^2*x)/(21*a^3*(a+b*x^2)^(7/2)) + (32*b^2*x)/(7*a^4*(a+b*x^2)^(5/2)) + (128*b^2*x)/(21*a^5*(a+b*x^2)^(3/2)) + (256*b^2*x)/(21*a^6*\text{Sqrt}[a+b*x^2])$

**Rubi in Sympy [A]** time = 14.2896, size = 126, normalized size = 0.95

$$-\frac{1}{3ax^3(a+bx^2)^{7/2}} + \frac{10b}{3a^2x(a+bx^2)^{7/2}} + \frac{80b^2x}{21a^3(a+bx^2)^{7/2}} + \frac{32b^2x}{7a^4(a+bx^2)^{5/2}} + \frac{128b^2x}{21a^5(a+bx^2)^{3/2}} + \frac{256b^2x}{21a^6\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(b\*x\*\*2+a)\*\*(9/2),x)

[Out]  $-1/(3*a*x**3*(a+b*x**2)**(7/2)) + 10*b/(3*a**2*x*(a+b*x**2)**(7/2)) + 80*b**2*x/(21*a**3*(a+b*x**2)**(7/2)) + 32*b**2*x/(7*a**4*(a+b*x**2)**(5/2)) + 128*b**2*x/(21*a**5*(a+b*x**2)**(3/2)) + 256*b**2*x/(21*a**6*\text{sqrt}(a+b*x**2))$

**Mathematica [A]** time = 0.0521774, size = 75, normalized size = 0.57

$$\frac{-7a^5 + 70a^4bx^2 + 560a^3b^2x^4 + 1120a^2b^3x^6 + 896ab^4x^8 + 256b^5x^{10}}{21a^6x^3(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a+b\*x^2)^(9/2)),x]

[Out]  $(-7*a^5 + 70*a^4*b*x^2 + 560*a^3*b^2*x^4 + 1120*a^2*b^3*x^6 + 896*a*b^4*x^8 + 256*b^5*x^{10})/(21*a^6*x^3*(a+b*x^2)^(7/2))$

---

**Maple [A]** time = 0.009, size = 72, normalized size = 0.6

$$-\frac{-256 b^5 x^{10} - 896 a b^4 x^8 - 1120 a^2 b^3 x^6 - 560 a^3 b^2 x^4 - 70 a^4 b x^2 + 7 a^5}{21 x^3 a^6} (b x^2 + a)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b\*x^2+a)^(9/2), x)

[Out] -1/21\*(-256\*b^5\*x^10-896\*a\*b^4\*x^8-1120\*a^2\*b^3\*x^6-560\*a^3\*b^2\*x^4-70\*a^4\*b\*x^2+7\*a^5)/x^3/(b\*x^2+a)^(7/2)/a^6

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(9/2)\*x^4), x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.337246, size = 157, normalized size = 1.19

$$\frac{(256 b^5 x^{10} + 896 a b^4 x^8 + 1120 a^2 b^3 x^6 + 560 a^3 b^2 x^4 + 70 a^4 b x^2 - 7 a^5) \sqrt{b x^2 + a}}{21 (a^6 b^4 x^{11} + 4 a^7 b^3 x^9 + 6 a^8 b^2 x^7 + 4 a^9 b x^5 + a^{10} x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(9/2)\*x^4), x, algorithm="fricas")

[Out] 1/21\*(256\*b^5\*x^10 + 896\*a\*b^4\*x^8 + 1120\*a^2\*b^3\*x^6 + 560\*a^3\*b^2\*x^4 + 70\*a^4\*b\*x^2 - 7\*a^5)\*sqrt(b\*x^2 + a)/(a^6\*b^4\*x^11 + 4\*a^7\*b^3\*x^9 + 6\*a^8\*b^2\*x^7 + 4\*a^9\*b\*x^5 + a^10\*x^3)

---

**Sympy [A]** time = 28.7195, size = 668, normalized size = 5.06

$$\begin{aligned} & -\frac{7a^6 b^{\frac{51}{2}} \sqrt{\frac{a}{bx^2} + 1}}{21a^{11} b^{25} x^2 + 105a^{10} b^{26} x^4 + 210a^9 b^{27} x^6 + 210a^8 b^{28} x^8 + 105a^7 b^{29} x^{10} + 21a^6 b^{30} x^{12}} \\ & + \frac{63a^5 b^{\frac{53}{2}} x^2 \sqrt{\frac{a}{bx^2} + 1}}{21a^{11} b^{25} x^2 + 105a^{10} b^{26} x^4 + 210a^9 b^{27} x^6 + 210a^8 b^{28} x^8 + 105a^7 b^{29} x^{10} + 21a^6 b^{30} x^{12}} \\ & + \frac{630a^4 b^{\frac{55}{2}} x^4 \sqrt{\frac{a}{bx^2} + 1}}{21a^{11} b^{25} x^2 + 105a^{10} b^{26} x^4 + 210a^9 b^{27} x^6 + 210a^8 b^{28} x^8 + 105a^7 b^{29} x^{10} + 21a^6 b^{30} x^{12}} \\ & + \frac{1680a^3 b^{\frac{57}{2}} x^6 \sqrt{\frac{a}{bx^2} + 1}}{21a^{11} b^{25} x^2 + 105a^{10} b^{26} x^4 + 210a^9 b^{27} x^6 + 210a^8 b^{28} x^8 + 105a^7 b^{29} x^{10} + 21a^6 b^{30} x^{12}} \\ & + \frac{2016a^2 b^{\frac{59}{2}} x^8 \sqrt{\frac{a}{bx^2} + 1}}{21a^{11} b^{25} x^2 + 105a^{10} b^{26} x^4 + 210a^9 b^{27} x^6 + 210a^8 b^{28} x^8 + 105a^7 b^{29} x^{10} + 21a^6 b^{30} x^{12}} \\ & + \frac{1152ab^{\frac{61}{2}} x^{10} \sqrt{\frac{a}{bx^2} + 1}}{21a^{11} b^{25} x^2 + 105a^{10} b^{26} x^4 + 210a^9 b^{27} x^6 + 210a^8 b^{28} x^8 + 105a^7 b^{29} x^{10} + 21a^6 b^{30} x^{12}} \\ & + \frac{256b^{\frac{63}{2}} x^{12} \sqrt{\frac{a}{bx^2} + 1}}{21a^{11} b^{25} x^2 + 105a^{10} b^{26} x^4 + 210a^9 b^{27} x^6 + 210a^8 b^{28} x^8 + 105a^7 b^{29} x^{10} + 21a^6 b^{30} x^{12}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(b\*x\*\*2+a)\*\*(9/2),x)

[Out] 
$$\frac{-7a^6b^{51/2}\sqrt{a/(bx^2)+1}/(21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12})+63a^5b^{53/2}x^2\sqrt{a/(bx^2)+1}/(21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12})+630a^4b^{55/2}x^4\sqrt{a/(bx^2)+1}/(21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12})+1680a^3b^{57/2}x^6\sqrt{a/(bx^2)+1}/(21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12})+2016a^2b^{59/2}x^8\sqrt{a/(bx^2)+1}/(21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12})+1152ab^{61/2}x^{10}\sqrt{a/(bx^2)+1}/(21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12})+256b^{63/2}x^{12}\sqrt{a/(bx^2)+1}/(21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12})$$

---

**GIAC/XCAS [A]** time = 0.220819, size = 198, normalized size = 1.5

$$\frac{\left(x^2\left(\frac{158b^5x^2}{a^6}+\frac{511b^4}{a^5}\right)+\frac{560b^3}{a^4}\right)x^2+\frac{210b^2}{a^3}x}{21(bx^2+a)^{7/2}}-\frac{4\left(6\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^4b^{3/2}-15\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^2ab^{3/2}+7a^2b^{3/2}\right)}{3\left(\left(\sqrt{bx}-\sqrt{bx^2+a}\right)^2-a\right)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(9/2)\*x^4),x, algorithm="giac")

[Out] 
$$\frac{1}{21}\left(\frac{x^2(158b^5x^2/a^6+511b^4/a^5)+560b^3/a^4}{(bx^2+a)^{7/2}}-4/3\left(6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4b^{3/2}-15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2ab^{3/2}+7a^2b^{3/2}\right)/\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3a^5\right)$$

$$3.532 \quad \int \frac{x^5}{\sqrt{9+4x^2}} dx$$

**Optimal.** Leaf size=46

$$\frac{1}{320} (4x^2 + 9)^{5/2} - \frac{3}{32} (4x^2 + 9)^{3/2} + \frac{81}{64} \sqrt{4x^2 + 9}$$

[Out] (81\*Sqrt[9 + 4\*x^2])/64 - (3\*(9 + 4\*x^2)^(3/2))/32 + (9 + 4\*x^2)^(5/2)/320

**Rubi [A]** time = 0.0556297, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{1}{320} (4x^2 + 9)^{5/2} - \frac{3}{32} (4x^2 + 9)^{3/2} + \frac{81}{64} \sqrt{4x^2 + 9}$$

Antiderivative was successfully verified.

[In] Int[x^5/Sqrt[9 + 4\*x^2], x]

[Out] (81\*Sqrt[9 + 4\*x^2])/64 - (3\*(9 + 4\*x^2)^(3/2))/32 + (9 + 4\*x^2)^(5/2)/320

**Rubi in Sympy [A]** time = 6.7358, size = 37, normalized size = 0.8

$$\frac{(4x^2 + 9)^{5/2}}{320} - \frac{3(4x^2 + 9)^{3/2}}{32} + \frac{81\sqrt{4x^2 + 9}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5/(4\*x\*\*2+9)\*\*(1/2), x)

[Out] (4\*x\*\*2 + 9)\*\*(5/2)/320 - 3\*(4\*x\*\*2 + 9)\*\*(3/2)/32 + 81\*sqrt(4\*x\*\*2 + 9)/64

**Mathematica [A]** time = 0.014406, size = 27, normalized size = 0.59

$$\frac{1}{40} \sqrt{4x^2 + 9} (2x^4 - 6x^2 + 27)$$

Antiderivative was successfully verified.

[In] Integrate[x^5/Sqrt[9 + 4\*x^2], x]

[Out] (Sqrt[9 + 4\*x^2]\*(27 - 6\*x^2 + 2\*x^4))/40

**Maple [A]** time = 0.004, size = 24, normalized size = 0.5

$$\frac{2x^4 - 6x^2 + 27}{40} \sqrt{4x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(4\*x^2+9)^(1/2), x)



[Out]  $1/40 * (4 * x^2 + 9)^{(1/2)} * (2 * x^4 - 6 * x^2 + 27)$

**Maxima [A]** time = 1.49696, size = 54, normalized size = 1.17

$$\frac{1}{20} \sqrt{4x^2 + 9} x^4 - \frac{3}{20} \sqrt{4x^2 + 9} x^2 + \frac{27}{40} \sqrt{4x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/sqrt(4*x^2 + 9),x, algorithm="maxima")`

[Out]  $1/20 * \text{sqrt}(4 * x^2 + 9) * x^4 - 3/20 * \text{sqrt}(4 * x^2 + 9) * x^2 + 27/40 * \text{sqrt}(4 * x^2 + 9)$

**Fricas [A]** time = 0.223038, size = 139, normalized size = 3.02

$$\frac{2048x^{10} + 1920x^8 + 11880x^6 + 85050x^4 + 109350x^2 - 2(512x^9 - 96x^7 + 3402x^5 + 17010x^3 + 10935x)\sqrt{4x^2 + 9} + 19683}{40(512x^5 + 1440x^3 - (256x^4 + 432x^2 + 81)\sqrt{4x^2 + 9} + 810x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/sqrt(4*x^2 + 9),x, algorithm="fricas")`

[Out]  $-1/40 * (2048 * x^{10} + 1920 * x^8 + 11880 * x^6 + 85050 * x^4 + 109350 * x^2 - 2 * (512 * x^9 - 96 * x^7 + 3402 * x^5 + 17010 * x^3 + 10935 * x) * \text{sqrt}(4 * x^2 + 9) + 19683) / (512 * x^5 + 1440 * x^3 - (256 * x^4 + 432 * x^2 + 81) * \text{sqrt}(4 * x^2 + 9) + 810 * x)$

**Sympy [A]** time = 3.42433, size = 44, normalized size = 0.96

$$\frac{x^4 \sqrt{4x^2 + 9}}{20} - \frac{3x^2 \sqrt{4x^2 + 9}}{20} + \frac{27 \sqrt{4x^2 + 9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(4*x**2+9)**(1/2),x)`

[Out]  $x^{**4} * \text{sqrt}(4 * x^{**2} + 9) / 20 - 3 * x^{**2} * \text{sqrt}(4 * x^{**2} + 9) / 20 + 27 * \text{sqrt}(4 * x^{**2} + 9) / 40$

**GIAC/XCAS [A]** time = 0.209852, size = 46, normalized size = 1.

$$\frac{1}{320} (4x^2 + 9)^{5/2} - \frac{3}{32} (4x^2 + 9)^{3/2} + \frac{81}{64} \sqrt{4x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/sqrt(4*x^2 + 9),x, algorithm="giac")`

[Out]  $1/320 * (4 * x^2 + 9)^{(5/2)} - 3/32 * (4 * x^2 + 9)^{(3/2)} + 81/64 * \text{sqrt}(4 * x^2 + 9)$

$$3.533 \quad \int \frac{x^4}{\sqrt{9+4x^2}} dx$$

**Optimal.** Leaf size=45

$$-\frac{27}{128}\sqrt{4x^2+9}x + \frac{1}{16}\sqrt{4x^2+9}x^3 + \frac{243}{256}\sinh^{-1}\left(\frac{2x}{3}\right)$$

[Out]  $(-27*x*\text{Sqrt}[9 + 4*x^2])/128 + (x^3*\text{Sqrt}[9 + 4*x^2])/16 + (243*\text{ArcSinh}[(2*x)/3])/256$

**Rubi [A]** time = 0.0391307, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{27}{128}\sqrt{4x^2+9}x + \frac{1}{16}\sqrt{4x^2+9}x^3 + \frac{243}{256}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

[In] Int[x^4/Sqrt[9 + 4\*x^2], x]

[Out]  $(-27*x*\text{Sqrt}[9 + 4*x^2])/128 + (x^3*\text{Sqrt}[9 + 4*x^2])/16 + (243*\text{ArcSinh}[(2*x)/3])/256$

**Rubi in Sympy [A]** time = 5.33303, size = 39, normalized size = 0.87

$$\frac{x^3\sqrt{4x^2+9}}{16} - \frac{27x\sqrt{4x^2+9}}{128} + \frac{243 \operatorname{asinh}\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(4\*x\*\*2+9)\*\*(1/2), x)

[Out]  $x**3*\text{sqrt}(4*x**2 + 9)/16 - 27*x*\text{sqrt}(4*x**2 + 9)/128 + 243*\text{asinh}(2*x/3)/256$

**Mathematica [A]** time = 0.0295799, size = 36, normalized size = 0.8

$$\sqrt{4x^2+9}\left(\frac{x^3}{16} - \frac{27x}{128}\right) + \frac{243}{256}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/Sqrt[9 + 4\*x^2], x]

[Out]  $\text{Sqrt}[9 + 4*x^2]*((-27*x)/128 + x^3/16) + (243*\text{ArcSinh}[(2*x)/3])/256$

**Maple [A]** time = 0.007, size = 34, normalized size = 0.8

$$\frac{243}{256}\operatorname{Arcsinh}\left(\frac{2x}{3}\right) - \frac{27x}{128}\sqrt{4x^2+9} + \frac{x^3}{16}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(4*x^2+9)^(1/2),x)`

[Out]  $243/256 \cdot \operatorname{arcsinh}(2/3 \cdot x) - 27/128 \cdot x \cdot (4 \cdot x^2 + 9)^{1/2} + 1/16 \cdot x^3 \cdot (4 \cdot x^2 + 9)^{1/2}$

**Maxima [A]** time = 1.482, size = 45, normalized size = 1.

$$\frac{1}{16} \sqrt{4x^2 + 9} x^3 - \frac{27}{128} \sqrt{4x^2 + 9} x + \frac{243}{256} \operatorname{arsinh}\left(\frac{2}{3} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/sqrt(4*x^2 + 9),x, algorithm="maxima")`

[Out]  $1/16 \cdot \sqrt{4 \cdot x^2 + 9} \cdot x^3 - 27/128 \cdot \sqrt{4 \cdot x^2 + 9} \cdot x + 243/256 \cdot \operatorname{arcsinh}(2/3 \cdot x)$

**Fricas [A]** time = 0.2331, size = 176, normalized size = 3.91

$$\frac{4096x^8 - 36288x^4 - 34992x^2 + 243 \left(128x^4 + 288x^2 - 8(8x^3 + 9x)\sqrt{4x^2 + 9} + 81\right) \log\left(-2x + \sqrt{4x^2 + 9}\right) - 2(1024x^7 - 1152x^5 - 7128x^3 - 2187x) \sqrt{4x^2 + 9}}{256 \left(128x^4 + 288x^2 - 8(8x^3 + 9x)\sqrt{4x^2 + 9} + 81\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/sqrt(4*x^2 + 9),x, algorithm="fricas")`

[Out]  $-1/256 \cdot (4096 \cdot x^8 - 36288 \cdot x^4 - 34992 \cdot x^2 + 243 \cdot (128 \cdot x^4 + 288 \cdot x^2 - 8 \cdot (8 \cdot x^3 + 9 \cdot x) \cdot \sqrt{4 \cdot x^2 + 9} + 81) \cdot \log(-2 \cdot x + \sqrt{4 \cdot x^2 + 9})) - 2 \cdot (1024 \cdot x^7 - 1152 \cdot x^5 - 7128 \cdot x^3 - 2187 \cdot x) \cdot \sqrt{4 \cdot x^2 + 9} / (128 \cdot x^4 + 288 \cdot x^2 - 8 \cdot (8 \cdot x^3 + 9 \cdot x) \cdot \sqrt{4 \cdot x^2 + 9} + 81)$

**Sympy [A]** time = 1.97666, size = 39, normalized size = 0.87

$$\frac{x^3 \sqrt{4x^2 + 9}}{16} - \frac{27x \sqrt{4x^2 + 9}}{128} + \frac{243 \operatorname{asinh}\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(4*x**2+9)**(1/2),x)`

[Out]  $x^{**3} \cdot \sqrt{4 \cdot x^{**2} + 9} / 16 - 27 \cdot x \cdot \sqrt{4 \cdot x^{**2} + 9} / 128 + 243 \cdot \operatorname{asinh}(2 \cdot x / 3) / 256$

**GIAC/XCAS [A]** time = 0.212123, size = 49, normalized size = 1.09

$$\frac{1}{128} (8x^2 - 27) \sqrt{4x^2 + 9} x - \frac{243}{256} \ln\left(-2x + \sqrt{4x^2 + 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/sqrt(4*x^2 + 9),x, algorithm="giac")`

[Out]  $1/128 \cdot (8 \cdot x^2 - 27) \cdot \sqrt{4 \cdot x^2 + 9} \cdot x - 243/256 \cdot \ln(-2 \cdot x + \sqrt{4 \cdot x^2 + 9})$

$$3.534 \quad \int \frac{x^3}{\sqrt{9+4x^2}} dx$$

**Optimal.** Leaf size=31

$$\frac{1}{48} (4x^2 + 9)^{3/2} - \frac{9}{16} \sqrt{4x^2 + 9}$$

[Out]  $(-9*\text{Sqrt}[9 + 4*x^2])/16 + (9 + 4*x^2)^(3/2)/48$

**Rubi [A]** time = 0.0415693, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{1}{48} (4x^2 + 9)^{3/2} - \frac{9}{16} \sqrt{4x^2 + 9}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3/\text{Sqrt}[9 + 4*x^2], x]$

[Out]  $(-9*\text{Sqrt}[9 + 4*x^2])/16 + (9 + 4*x^2)^(3/2)/48$

**Rubi in Sympy [A]** time = 5.62222, size = 24, normalized size = 0.77

$$\frac{(4x^2 + 9)^{3/2}}{48} - \frac{9\sqrt{4x^2 + 9}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**3}/(4*x^{**2}+9)^{(1/2)}, x)$

[Out]  $(4*x^{**2} + 9)^{(3/2)}/48 - 9*\text{sqrt}(4*x^{**2} + 9)/16$

**Mathematica [A]** time = 0.00943886, size = 22, normalized size = 0.71

$$\frac{1}{24} (2x^2 - 9) \sqrt{4x^2 + 9}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^3/\text{Sqrt}[9 + 4*x^2], x]$

[Out]  $((-9 + 2*x^2)*\text{Sqrt}[9 + 4*x^2])/24$

**Maple [A]** time = 0.005, size = 19, normalized size = 0.6

$$\frac{2x^2 - 9}{24} \sqrt{4x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^3/(4*x^2+9)^(1/2), x)$

[Out]  $1/24*(4*x^2+9)^(1/2)*(2*x^2-9)$

---

**Maxima [A]** time = 1.50334, size = 35, normalized size = 1.13

$$\frac{1}{12} \sqrt{4x^2 + 9} x^2 - \frac{3}{8} \sqrt{4x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/sqrt(4\*x^2 + 9),x, algorithm="maxima")

[Out] 1/12\*sqrt(4\*x^2 + 9)\*x^2 - 3/8\*sqrt(4\*x^2 + 9)

---

**Fricas [A]** time = 0.23286, size = 99, normalized size = 3.19

$$-\frac{128x^6 - 216x^4 - 1458x^2 - 2(32x^5 - 90x^3 - 243x)\sqrt{4x^2 + 9} - 729}{24(32x^3 - (16x^2 + 9)\sqrt{4x^2 + 9} + 54x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/sqrt(4\*x^2 + 9),x, algorithm="fricas")

[Out] -1/24\*(128\*x^6 - 216\*x^4 - 1458\*x^2 - 2\*(32\*x^5 - 90\*x^3 - 243\*x)\*sqrt(4\*x^2 + 9) - 729)/(32\*x^3 - (16\*x^2 + 9)\*sqrt(4\*x^2 + 9) + 54\*x)

---

**Sympy [A]** time = 0.96614, size = 27, normalized size = 0.87

$$\frac{x^2\sqrt{4x^2 + 9}}{12} - \frac{3\sqrt{4x^2 + 9}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(4\*x\*\*2+9)\*\*(1/2),x)

[Out] x\*\*2\*sqrt(4\*x\*\*2 + 9)/12 - 3\*sqrt(4\*x\*\*2 + 9)/8

---

**GIAC/XCAS [A]** time = 0.208748, size = 31, normalized size = 1.

$$\frac{1}{48} (4x^2 + 9)^{\frac{3}{2}} - \frac{9}{16} \sqrt{4x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/sqrt(4\*x^2 + 9),x, algorithm="giac")

[Out] 1/48\*(4\*x^2 + 9)^(3/2) - 9/16\*sqrt(4\*x^2 + 9)

$$3.535 \quad \int \frac{x^2}{\sqrt{9+4x^2}} dx$$

**Optimal.** Leaf size=27

$$\frac{1}{8}x\sqrt{4x^2+9} - \frac{9}{16}\sinh^{-1}\left(\frac{2x}{3}\right)$$

[Out] (x\*Sqrt[9 + 4\*x^2])/8 - (9\*ArcSinh[(2\*x)/3])/16

**Rubi [A]** time = 0.0226055, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{1}{8}x\sqrt{4x^2+9} - \frac{9}{16}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[9 + 4\*x^2], x]

[Out] (x\*Sqrt[9 + 4\*x^2])/8 - (9\*ArcSinh[(2\*x)/3])/16

**Rubi in Sympy [A]** time = 3.63142, size = 22, normalized size = 0.81

$$\frac{x\sqrt{4x^2+9}}{8} - \frac{9\operatorname{asinh}\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(4\*x\*\*2+9)\*\*(1/2), x)

[Out] x\*sqrt(4\*x\*\*2 + 9)/8 - 9\*asinh(2\*x/3)/16

**Mathematica [A]** time = 0.0142908, size = 27, normalized size = 1.

$$\frac{1}{8}x\sqrt{4x^2+9} - \frac{9}{16}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[9 + 4\*x^2], x]

[Out] (x\*Sqrt[9 + 4\*x^2])/8 - (9\*ArcSinh[(2\*x)/3])/16

**Maple [A]** time = 0.006, size = 20, normalized size = 0.7

$$-\frac{9}{16}\operatorname{Arcsinh}\left(\frac{2x}{3}\right) + \frac{x}{8}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(4\*x^2+9)^(1/2), x)

[Out] -9/16\*arcsinh(2/3\*x)+1/8\*x\*(4\*x^2+9)^(1/2)

---

**Maxima [A]** time = 1.50378, size = 26, normalized size = 0.96

$$\frac{1}{8} \sqrt{4x^2 + 9x} - \frac{9}{16} \operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/sqrt(4\*x^2 + 9),x, algorithm="maxima")

[Out] 1/8\*sqrt(4\*x^2 + 9)\*x - 9/16\*arcsinh(2/3\*x)

---

**Fricas [A]** time = 0.233359, size = 120, normalized size = 4.44

$$\frac{32x^4 + 72x^2 - 9\left(8x^2 - 4\sqrt{4x^2 + 9}x + 9\right) \log\left(-2x + \sqrt{4x^2 + 9}\right) - 2(8x^3 + 9x)\sqrt{4x^2 + 9}}{16\left(8x^2 - 4\sqrt{4x^2 + 9}x + 9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/sqrt(4\*x^2 + 9),x, algorithm="fricas")

[Out] -1/16\*(32\*x^4 + 72\*x^2 - 9\*(8\*x^2 - 4\*sqrt(4\*x^2 + 9)\*x + 9)\*log(-2\*x + sqrt(4\*x^2 + 9)) - 2\*(8\*x^3 + 9\*x)\*sqrt(4\*x^2 + 9))/(8\*x^2 - 4\*sqrt(4\*x^2 + 9)\*x + 9)

---

**Sympy [A]** time = 0.533, size = 22, normalized size = 0.81

$$\frac{x\sqrt{4x^2 + 9}}{8} - \frac{9 \operatorname{asinh}\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(4\*x\*\*2+9)\*\*(1/2),x)

[Out] x\*sqrt(4\*x\*\*2 + 9)/8 - 9\*asinh(2\*x/3)/16

---

**GIAC/XCAS [A]** time = 0.213792, size = 39, normalized size = 1.44

$$\frac{1}{8} \sqrt{4x^2 + 9x} + \frac{9}{16} \ln\left(-2x + \sqrt{4x^2 + 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/sqrt(4\*x^2 + 9),x, algorithm="giac")

[Out] 1/8\*sqrt(4\*x^2 + 9)\*x + 9/16\*ln(-2\*x + sqrt(4\*x^2 + 9))

$$3.536 \quad \int \frac{x}{\sqrt{9+4x^2}} dx$$

**Optimal.** Leaf size=15

$$\frac{1}{4}\sqrt{4x^2+9}$$

[Out] Sqrt[9 + 4\*x^2]/4

**Rubi [A]** time = 0.00887441, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{1}{4}\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[9 + 4\*x^2], x]

[Out] Sqrt[9 + 4\*x^2]/4

**Rubi in Sympy [A]** time = 1.93707, size = 10, normalized size = 0.67

$$\frac{\sqrt{4x^2+9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(4\*x\*\*2+9)\*\*(1/2), x)

[Out] sqrt(4\*x\*\*2 + 9)/4

**Mathematica [A]** time = 0.00244051, size = 15, normalized size = 1.

$$\frac{1}{4}\sqrt{4x^2+9}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[9 + 4\*x^2], x]

[Out] Sqrt[9 + 4\*x^2]/4

**Maple [A]** time = 0.004, size = 12, normalized size = 0.8

$$\frac{1}{4}\sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(4\*x^2+9)^(1/2), x)

[Out] 1/4\*(4\*x^2+9)^(1/2)



---

**Maxima [A]** time = 1.34624, size = 15, normalized size = 1.

$$\frac{1}{4} \sqrt{4x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sqrt(4\*x^2 + 9),x, algorithm="maxima")

[Out] 1/4\*sqrt(4\*x^2 + 9)

---

**Fricas [A]** time = 0.23241, size = 51, normalized size = 3.4

$$\frac{4x^2 - 2\sqrt{4x^2 + 9}x + 9}{4(2x - \sqrt{4x^2 + 9})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sqrt(4\*x^2 + 9),x, algorithm="fricas")

[Out] -1/4\*(4\*x^2 - 2\*sqrt(4\*x^2 + 9)\*x + 9)/(2\*x - sqrt(4\*x^2 + 9))

---

**Sympy [A]** time = 0.307005, size = 10, normalized size = 0.67

$$\frac{\sqrt{4x^2 + 9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(4\*x\*\*2+9)\*\*(1/2),x)

[Out] sqrt(4\*x\*\*2 + 9)/4

---

**GIAC/XCAS [A]** time = 0.216557, size = 15, normalized size = 1.

$$\frac{1}{4} \sqrt{4x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sqrt(4\*x^2 + 9),x, algorithm="giac")

[Out] 1/4\*sqrt(4\*x^2 + 9)

$$3.537 \quad \int \frac{1}{\sqrt{9+4x^2}} dx$$

**Optimal.** Leaf size=10

$$\frac{1}{2} \sinh^{-1} \left( \frac{2x}{3} \right)$$

[Out] ArcSinh[(2\*x)/3]/2

**Rubi [A]** time = 0.00681308, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{1}{2} \sinh^{-1} \left( \frac{2x}{3} \right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[9 + 4\*x^2], x]

[Out] ArcSinh[(2\*x)/3]/2

**Rubi in Sympy [A]** time = 1.07034, size = 7, normalized size = 0.7

$$\frac{\operatorname{asinh} \left( \frac{2x}{3} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(4\*x\*\*2+9)\*\*(1/2), x)

[Out] asinh(2\*x/3)/2

**Mathematica [A]** time = 0.00567938, size = 10, normalized size = 1.

$$\frac{1}{2} \sinh^{-1} \left( \frac{2x}{3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[9 + 4\*x^2], x]

[Out] ArcSinh[(2\*x)/3]/2

**Maple [A]** time = 0.004, size = 7, normalized size = 0.7

$$\frac{1}{2} \operatorname{Arcsinh} \left( \frac{2x}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4\*x^2+9)^(1/2), x)

[Out] 1/2\*arcsinh(2/3\*x)

---

**Maxima [A]** time = 1.50096, size = 8, normalized size = 0.8

$$\frac{1}{2} \operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(4*x^2 + 9),x, algorithm="maxima")`

[Out] `1/2*arcsinh(2/3*x)`

---

**Fricas [A]** time = 0.230023, size = 22, normalized size = 2.2

$$-\frac{1}{2} \log\left(-2x + \sqrt{4x^2 + 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(4*x^2 + 9),x, algorithm="fricas")`

[Out] `-1/2*log(-2*x + sqrt(4*x^2 + 9))`

---

**Sympy [A]** time = 0.322207, size = 7, normalized size = 0.7

$$\frac{\operatorname{asinh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(4*x**2+9)**(1/2),x)`

[Out] `asinh(2*x/3)/2`

---

**GIAC/XCAS [A]** time = 0.213354, size = 22, normalized size = 2.2

$$-\frac{1}{2} \ln\left(-2x + \sqrt{4x^2 + 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(4*x^2 + 9),x, algorithm="giac")`

[Out] `-1/2*ln(-2*x + sqrt(4*x^2 + 9))`

$$3.538 \quad \int \frac{1}{x\sqrt{9+4x^2}} dx$$

**Optimal.** Leaf size=20

$$-\frac{1}{3} \tanh^{-1} \left( \frac{1}{3} \sqrt{4x^2 + 9} \right)$$

[Out] -ArcTanh[Sqrt[9 + 4\*x^2]/3]/3

**Rubi [A]** time = 0.0334648, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{1}{3} \tanh^{-1} \left( \frac{1}{3} \sqrt{4x^2 + 9} \right)$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Sqrt[9 + 4\*x^2]), x]

[Out] -ArcTanh[Sqrt[9 + 4\*x^2]/3]/3

**Rubi in Sympy [A]** time = 4.53177, size = 15, normalized size = 0.75

$$-\frac{\operatorname{atanh} \left( \frac{\sqrt{4x^2+9}}{3} \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(4\*x\*\*2+9)\*\*(1/2), x)

[Out] -atanh(sqrt(4\*x\*\*2 + 9)/3)/3

**Mathematica [A]** time = 0.00908784, size = 25, normalized size = 1.25

$$\frac{\log(x)}{3} - \frac{1}{3} \log \left( \sqrt{4x^2 + 9} + 3 \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Sqrt[9 + 4\*x^2]), x]

[Out] Log[x]/3 - Log[3 + Sqrt[9 + 4\*x^2]]/3

**Maple [A]** time = 0.005, size = 15, normalized size = 0.8

$$-\frac{1}{3} \operatorname{Artanh} \left( 3 \frac{1}{\sqrt{4x^2 + 9}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(4\*x^2+9)^(1/2), x)

[Out] -1/3\*arctanh(3/(4\*x^2+9)^(1/2))

---

**Maxima [A]** time = 1.48464, size = 12, normalized size = 0.6

$$-\frac{1}{3} \operatorname{arsinh}\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(4*x^2 + 9)*x),x, algorithm="maxima")`

[Out] `-1/3*arcsinh(3/2/abs(x))`

---

**Fricas [A]** time = 0.231263, size = 47, normalized size = 2.35

$$-\frac{1}{3} \log\left(-2x + \sqrt{4x^2 + 9} + 3\right) + \frac{1}{3} \log\left(-2x + \sqrt{4x^2 + 9} - 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(4*x^2 + 9)*x),x, algorithm="fricas")`

[Out] `-1/3*log(-2*x + sqrt(4*x^2 + 9) + 3) + 1/3*log(-2*x + sqrt(4*x^2 + 9) - 3)`

---

**Sympy [A]** time = 3.56007, size = 8, normalized size = 0.4

$$\frac{\operatorname{asinh}\left(\frac{3}{2x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(4*x**2+9)**(1/2),x)`

[Out] `-asinh(3/(2*x))/3`

---

**GIAC/XCAS [A]** time = 0.210693, size = 39, normalized size = 1.95

$$-\frac{1}{6} \ln\left(\sqrt{4x^2 + 9} + 3\right) + \frac{1}{6} \ln\left(\sqrt{4x^2 + 9} - 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(4*x^2 + 9)*x),x, algorithm="giac")`

[Out] `-1/6*ln(sqrt(4*x^2 + 9) + 3) + 1/6*ln(sqrt(4*x^2 + 9) - 3)`

$$3.539 \quad \int \frac{1}{x^2 \sqrt{9+4x^2}} dx$$

**Optimal.** Leaf size=18

$$-\frac{\sqrt{4x^2+9}}{9x}$$

[Out] -Sqrt[9 + 4\*x^2]/(9\*x)

**Rubi [A]** time = 0.0166369, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$-\frac{\sqrt{4x^2+9}}{9x}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*Sqrt[9 + 4\*x^2]), x]

[Out] -Sqrt[9 + 4\*x^2]/(9\*x)

**Rubi in Sympy [A]** time = 3.03171, size = 14, normalized size = 0.78

$$-\frac{\sqrt{4x^2+9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(4\*x\*\*2+9)\*\*(1/2), x)

[Out] -sqrt(4\*x\*\*2 + 9)/(9\*x)

**Mathematica [A]** time = 0.0093787, size = 18, normalized size = 1.

$$-\frac{\sqrt{4x^2+9}}{9x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*Sqrt[9 + 4\*x^2]), x]

[Out] -Sqrt[9 + 4\*x^2]/(9\*x)

**Maple [A]** time = 0.004, size = 15, normalized size = 0.8

$$-\frac{1}{9x} \sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(4\*x^2+9)^(1/2), x)

[Out] -1/9\*(4\*x^2+9)^(1/2)/x

---

**Maxima [A]** time = 1.49391, size = 19, normalized size = 1.06

$$-\frac{\sqrt{4x^2 + 9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(4\*x^2 + 9)\*x^2),x, algorithm="maxima")

[Out] -1/9\*sqrt(4\*x^2 + 9)/x

---

**Fricas [A]** time = 0.227803, size = 27, normalized size = 1.5

$$\frac{1}{2x^2 - \sqrt{4x^2 + 9}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(4\*x^2 + 9)\*x^2),x, algorithm="fricas")

[Out] 1/(2\*x^2 - sqrt(4\*x^2 + 9)\*x)

---

**Sympy [A]** time = 2.49722, size = 15, normalized size = 0.83

$$-\frac{2\sqrt{1 + \frac{9}{4x^2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(4\*x\*\*2+9)\*\*(1/2),x)

[Out] -2\*sqrt(1 + 9/(4\*x\*\*2))/9

---

**GIAC/XCAS [A]** time = 0.211642, size = 31, normalized size = 1.72

$$\frac{4}{(2x - \sqrt{4x^2 + 9})^2 - 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(4\*x^2 + 9)\*x^2),x, algorithm="giac")

[Out] 4/((2\*x - sqrt(4\*x^2 + 9))^2 - 9)

$$3.540 \quad \int \frac{1}{x^3 \sqrt{9+4x^2}} dx$$

**Optimal.** Leaf size=39

$$\frac{2}{27} \tanh^{-1} \left( \frac{1}{3} \sqrt{4x^2 + 9} \right) - \frac{\sqrt{4x^2 + 9}}{18x^2}$$

[Out] -Sqrt[9 + 4\*x^2]/(18\*x^2) + (2\*ArcTanh[Sqrt[9 + 4\*x^2]/3])/27

**Rubi [A]** time = 0.0508805, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{2}{27} \tanh^{-1} \left( \frac{1}{3} \sqrt{4x^2 + 9} \right) - \frac{\sqrt{4x^2 + 9}}{18x^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*Sqrt[9 + 4\*x^2]),x]

[Out] -Sqrt[9 + 4\*x^2]/(18\*x^2) + (2\*ArcTanh[Sqrt[9 + 4\*x^2]/3])/27

**Rubi in Sympy [A]** time = 5.47355, size = 31, normalized size = 0.79

$$\frac{2 \operatorname{atanh} \left( \frac{\sqrt{4x^2+9}}{3} \right)}{27} - \frac{\sqrt{4x^2 + 9}}{18x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(4\*x\*\*2+9)\*\*(1/2),x)

[Out] 2\*atanh(sqrt(4\*x\*\*2 + 9)/3)/27 - sqrt(4\*x\*\*2 + 9)/(18\*x\*\*2)

**Mathematica [A]** time = 0.0259705, size = 43, normalized size = 1.1

$$-\frac{\sqrt{4x^2 + 9}}{18x^2} + \frac{2}{27} \log \left( \sqrt{4x^2 + 9} + 3 \right) - \frac{2 \log(x)}{27}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*Sqrt[9 + 4\*x^2]),x]

[Out] -Sqrt[9 + 4\*x^2]/(18\*x^2) - (2\*Log[x])/27 + (2\*Log[3 + Sqrt[9 + 4\*x^2]])/27

**Maple [A]** time = 0.006, size = 30, normalized size = 0.8

$$-\frac{1}{18x^2} \sqrt{4x^2 + 9} + \frac{2}{27} \operatorname{Artanh} \left( 3 \frac{1}{\sqrt{4x^2 + 9}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(4\*x^2+9)^(1/2),x)



[Out]  $-1/18*(4*x^2+9)^{(1/2)}/x^2+2/27*\operatorname{arctanh}(3/(4*x^2+9)^{(1/2)})$

**Maxima [A]** time = 1.49394, size = 32, normalized size = 0.82

$$-\frac{\sqrt{4x^2+9}}{18x^2} + \frac{2}{27} \operatorname{arsinh}\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(4*x^2 + 9)*x^3),x, algorithm="maxima")`

[Out]  $-1/18*\sqrt{4*x^2 + 9}/x^2 + 2/27*\operatorname{arcsinh}(3/2/\operatorname{abs}(x))$

**Fricas [A]** time = 0.2346, size = 189, normalized size = 4.85

$$\frac{48x^3 + 4\left(8x^4 - 4\sqrt{4x^2 + 9}x^3 + 9x^2\right) \log\left(-2x + \sqrt{4x^2 + 9} + 3\right) - 4\left(8x^4 - 4\sqrt{4x^2 + 9}x^3 + 9x^2\right) \log\left(-2x + \sqrt{4x^2 + 9} - 3\right) + 108x}{54\left(8x^4 - 4\sqrt{4x^2 + 9}x^3 + 9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(4*x^2 + 9)*x^3),x, algorithm="fricas")`

[Out]  $1/54*(48*x^3 + 4*(8*x^4 - 4*\sqrt{4*x^2 + 9}*x^3 + 9*x^2)*\log(-2*x + \sqrt{4*x^2 + 9} + 3) - 4*(8*x^4 - 4*\sqrt{4*x^2 + 9}*x^3 + 9*x^2)*\log(-2*x + \sqrt{4*x^2 + 9} - 3) - 3*(8*x^4 + 9*x^2)*\sqrt{4*x^2 + 9} + 108*x)/(8*x^4 - 4*\sqrt{4*x^2 + 9}*x^3 + 9*x^2)$

**Sympy [A]** time = 7.92405, size = 44, normalized size = 1.13

$$\frac{2 \operatorname{asinh}\left(\frac{3}{2x}\right)}{27} - \frac{1}{9x\sqrt{1 + \frac{9}{4x^2}}} - \frac{1}{4x^3\sqrt{1 + \frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(4*x**2+9)**(1/2),x)`

[Out]  $2*\operatorname{asinh}(3/(2*x))/27 - 1/(9*x*\sqrt{1 + 9/(4*x**2)}) - 1/(4*x**3*\sqrt{1 + 9/(4*x**2)})$

**GIAC/XCAS [A]** time = 0.211226, size = 58, normalized size = 1.49

$$-\frac{\sqrt{4x^2+9}}{18x^2} + \frac{1}{27} \ln\left(\sqrt{4x^2+9} + 3\right) - \frac{1}{27} \ln\left(\sqrt{4x^2+9} - 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(4*x^2 + 9)*x^3),x, algorithm="giac")`

[Out]  $-1/18*\sqrt{4*x^2 + 9}/x^2 + 1/27*\ln(\sqrt{4*x^2 + 9} + 3) - 1/27*\ln(\sqrt{4*x^2 + 9} - 3)$

$$3.541 \quad \int \frac{1}{x^4 \sqrt{9+4x^2}} dx$$

**Optimal.** Leaf size=37

$$\frac{8\sqrt{4x^2+9}}{243x} - \frac{\sqrt{4x^2+9}}{27x^3}$$

[Out]  $-\text{Sqrt}[9 + 4*x^2]/(27*x^3) + (8*\text{Sqrt}[9 + 4*x^2])/(243*x)$

**Rubi [A]** time = 0.0317295, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{8\sqrt{4x^2+9}}{243x} - \frac{\sqrt{4x^2+9}}{27x^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^4*\text{Sqrt}[9 + 4*x^2]), x]$

[Out]  $-\text{Sqrt}[9 + 4*x^2]/(27*x^3) + (8*\text{Sqrt}[9 + 4*x^2])/(243*x)$

**Rubi in Sympy [A]** time = 4.63728, size = 29, normalized size = 0.78

$$\frac{8\sqrt{4x^2+9}}{243x} - \frac{\sqrt{4x^2+9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{**4}/(4*x^{**2}+9)^{(1/2)}, x)$

[Out]  $8*\text{sqrt}(4*x^{**2} + 9)/(243*x) - \text{sqrt}(4*x^{**2} + 9)/(27*x^{**3})$

**Mathematica [A]** time = 0.0123385, size = 27, normalized size = 0.73

$$\left( \frac{8}{243x} - \frac{1}{27x^3} \right) \sqrt{4x^2+9}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/(x^4*\text{Sqrt}[9 + 4*x^2]), x]$

[Out]  $(-1/(27*x^3) + 8/(243*x))*\text{Sqrt}[9 + 4*x^2]$

**Maple [A]** time = 0.004, size = 22, normalized size = 0.6

$$\frac{8x^2-9}{243x^3} \sqrt{4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/x^4/(4*x^2+9)^{(1/2)}, x)$

[Out]  $1/243*(4*x^2+9)^{(1/2)}*(8*x^2-9)/x^3$

---

**Maxima [A]** time = 1.50699, size = 39, normalized size = 1.05

$$\frac{8\sqrt{4x^2+9}}{243x} - \frac{\sqrt{4x^2+9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(4\*x^2 + 9)\*x^4),x, algorithm="maxima")

[Out] 8/243\*sqrt(4\*x^2 + 9)/x - 1/27\*sqrt(4\*x^2 + 9)/x^3

---

**Fricas [A]** time = 0.223352, size = 74, normalized size = 2.

$$\frac{4x^2 - 2\sqrt{4x^2+9}x + 3}{32x^6 + 54x^4 - (16x^5 + 9x^3)\sqrt{4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(4\*x^2 + 9)\*x^4),x, algorithm="fricas")

[Out] (4\*x^2 - 2\*sqrt(4\*x^2 + 9)\*x + 3)/(32\*x^6 + 54\*x^4 - (16\*x^5 + 9\*x^3)\*sqrt(4\*x^2 + 9))

---

**Sympy [A]** time = 5.75664, size = 32, normalized size = 0.86

$$\frac{16\sqrt{1+\frac{9}{4x^2}}}{243} - \frac{2\sqrt{1+\frac{9}{4x^2}}}{27x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(4\*x\*\*2+9)\*\*(1/2),x)

[Out] 16\*sqrt(1 + 9/(4\*x\*\*2))/243 - 2\*sqrt(1 + 9/(4\*x\*\*2))/(27\*x\*\*2)

---

**GIAC/XCAS [A]** time = 0.215509, size = 57, normalized size = 1.54

$$\frac{32\left(\left(2x - \sqrt{4x^2+9}\right)^2 - 3\right)}{\left(\left(2x - \sqrt{4x^2+9}\right)^2 - 9\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(4\*x^2 + 9)\*x^4),x, algorithm="giac")

[Out] 32\*((2\*x - sqrt(4\*x^2 + 9))^2 - 3)/((2\*x - sqrt(4\*x^2 + 9))^2 - 9)^3

$$3.542 \quad \int \frac{1}{x^5 \sqrt{9+4x^2}} dx$$

**Optimal.** Leaf size=57

$$\frac{\sqrt{4x^2+9}}{54x^2} - \frac{2}{81} \tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right) - \frac{\sqrt{4x^2+9}}{36x^4}$$

[Out] -Sqrt[9 + 4\*x^2]/(36\*x^4) + Sqrt[9 + 4\*x^2]/(54\*x^2) - (2\*ArcTanh[Sqrt[9 + 4\*x^2]/3])/81

**Rubi [A]** time = 0.0699204, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{\sqrt{4x^2+9}}{54x^2} - \frac{2}{81} \tanh^{-1}\left(\frac{1}{3}\sqrt{4x^2+9}\right) - \frac{\sqrt{4x^2+9}}{36x^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5\*Sqrt[9 + 4\*x^2]), x]

[Out] -Sqrt[9 + 4\*x^2]/(36\*x^4) + Sqrt[9 + 4\*x^2]/(54\*x^2) - (2\*ArcTanh[Sqrt[9 + 4\*x^2]/3])/81

**Rubi in Sympy [A]** time = 6.56026, size = 46, normalized size = 0.81

$$-\frac{2 \operatorname{atanh}\left(\frac{\sqrt{4x^2+9}}{3}\right)}{81} + \frac{\sqrt{4x^2+9}}{54x^2} - \frac{\sqrt{4x^2+9}}{36x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*5/(4\*x\*\*2+9)\*\*(1/2), x)

[Out] -2\*atanh(sqrt(4\*x\*\*2 + 9)/3)/81 + sqrt(4\*x\*\*2 + 9)/(54\*x\*\*2) - sqrt(4\*x\*\*2 + 9)/(36\*x\*\*4)

**Mathematica [A]** time = 0.0395131, size = 48, normalized size = 0.84

$$\frac{1}{324} \left( -8 \log\left(\sqrt{4x^2+9}+3\right) + \frac{3\sqrt{4x^2+9}(2x^2-3)}{x^4} + 8 \log(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5\*Sqrt[9 + 4\*x^2]), x]

[Out] ((3\*(-3 + 2\*x^2)\*Sqrt[9 + 4\*x^2])/x^4 + 8\*Log[x] - 8\*Log[3 + Sqrt[9 + 4\*x^2]])/324

**Maple [A]** time = 0.007, size = 44, normalized size = 0.8

$$-\frac{1}{36x^4}\sqrt{4x^2+9} + \frac{1}{54x^2}\sqrt{4x^2+9} - \frac{2}{81} \operatorname{Artanh}\left(3\frac{1}{\sqrt{4x^2+9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^5/(4*x^2+9)^(1/2),x)`

[Out]  $-1/36*(4*x^2+9)^(1/2)/x^4+1/54*(4*x^2+9)^(1/2)/x^2-2/81*\operatorname{arctanh}(3/(4*x^2+9)^(1/2))$

**Maxima [A]** time = 1.50026, size = 51, normalized size = 0.89

$$\frac{\sqrt{4x^2+9}}{54x^2} - \frac{\sqrt{4x^2+9}}{36x^4} - \frac{2}{81} \operatorname{arsinh}\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(4*x^2 + 9)*x^5),x, algorithm="maxima")`

[Out]  $1/54*\sqrt{4*x^2 + 9}/x^2 - 1/36*\sqrt{4*x^2 + 9}/x^4 - 2/81*\operatorname{arcsinh}(3/2/\operatorname{abs}(x))$

**Fricas [A]** time = 0.23085, size = 269, normalized size = 4.72

$$\frac{1536x^7 + 2880x^5 - 3888x^3 + 8(128x^8 + 288x^6 + 81x^4 - 8(8x^7 + 9x^5)\sqrt{4x^2+9}) \log(-2x + \sqrt{4x^2+9} + 3) - 8(128x^8 + 288x^6 + 81x^4 - 8(8x^7 + 9x^5)\sqrt{4x^2+9}) \log(-2x + \sqrt{4x^2+9} - 3)}{324(128x^8 + 288x^6 + 81x^4 - 8(8x^7 + 9x^5)\sqrt{4x^2+9})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(4*x^2 + 9)*x^5),x, algorithm="fricas")`

[Out]  $-1/324*(1536*x^7 + 2880*x^5 - 3888*x^3 + 8*(128*x^8 + 288*x^6 + 81*x^4 - 8*(8*x^7 + 9*x^5)*\sqrt{4*x^2 + 9})*\log(-2*x + \sqrt{4*x^2 + 9} + 3) - 8*(128*x^8 + 288*x^6 + 81*x^4 - 8*(8*x^7 + 9*x^5)*\sqrt{4*x^2 + 9})*\log(-2*x + \sqrt{4*x^2 + 9} - 3) - 3*(256*x^6 + 192*x^4 - 702*x^2 - 243)*\sqrt{4*x^2 + 9} - 5832*x)/(128*x^8 + 288*x^6 + 81*x^4 - 8*(8*x^7 + 9*x^5)*\sqrt{4*x^2 + 9})$

**Sympy [A]** time = 16.8939, size = 63, normalized size = 1.11

$$-\frac{2 \operatorname{asinh}\left(\frac{3}{2x}\right)}{81} + \frac{1}{27x\sqrt{1 + \frac{9}{4x^2}}} + \frac{1}{36x^3\sqrt{1 + \frac{9}{4x^2}}} - \frac{1}{8x^5\sqrt{1 + \frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(4*x**2+9)**(1/2),x)`

[Out]  $-2*\operatorname{asinh}(3/(2*x))/81 + 1/(27*x*\sqrt{1 + 9/(4*x**2)}) + 1/(36*x**3*\sqrt{1 + 9/(4*x**2)}) - 1/(8*x**5*\sqrt{1 + 9/(4*x**2)})$

**GIAC/XCAS [A]** time = 0.215255, size = 74, normalized size = 1.3

$$\frac{(4x^2+9)^{\frac{3}{2}} - 15\sqrt{4x^2+9}}{216x^4} - \frac{1}{81} \ln(\sqrt{4x^2+9} + 3) + \frac{1}{81} \ln(\sqrt{4x^2+9} - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(4*x^2 + 9)*x^5),x, algorithm="giac")`

```
[Out] 1/216*((4*x^2 + 9)^(3/2) - 15*sqrt(4*x^2 + 9))/x^4 - 1/81*ln(sqrt(4*x^2 + 9) + 3) + 1/81*ln(sqrt(4*x^2 + 9) - 3)
```

$$3.543 \quad \int \frac{x^5}{\sqrt{9-4x^2}} dx$$

**Optimal.** Leaf size=46

$$-\frac{1}{320} (9-4x^2)^{5/2} + \frac{3}{32} (9-4x^2)^{3/2} - \frac{81}{64} \sqrt{9-4x^2}$$

[Out]  $(-81*\text{Sqrt}[9 - 4*x^2])/64 + (3*(9 - 4*x^2)^(3/2))/32 - (9 - 4*x^2)^(5/2)/320$

**Rubi [A]** time = 0.0570818, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{1}{320} (9-4x^2)^{5/2} + \frac{3}{32} (9-4x^2)^{3/2} - \frac{81}{64} \sqrt{9-4x^2}$$

Antiderivative was successfully verified.

[In] Int[x^5/Sqrt[9 - 4\*x^2], x]

[Out]  $(-81*\text{Sqrt}[9 - 4*x^2])/64 + (3*(9 - 4*x^2)^(3/2))/32 - (9 - 4*x^2)^(5/2)/320$

**Rubi in Sympy [A]** time = 6.79475, size = 37, normalized size = 0.8

$$-\frac{(-4x^2+9)^{5/2}}{320} + \frac{3(-4x^2+9)^{3/2}}{32} - \frac{81\sqrt{-4x^2+9}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5/(-4\*x\*\*2+9)\*\*(1/2), x)

[Out]  $-(-4*x**2 + 9)**(5/2)/320 + 3*(-4*x**2 + 9)**(3/2)/32 - 81*\text{sqrt}(-4*x**2 + 9)/64$

**Mathematica [A]** time = 0.0145931, size = 27, normalized size = 0.59

$$-\frac{1}{40} \sqrt{9-4x^2} (2x^4 + 6x^2 + 27)$$

Antiderivative was successfully verified.

[In] Integrate[x^5/Sqrt[9 - 4\*x^2], x]

[Out]  $-(\text{Sqrt}[9 - 4*x^2]*(27 + 6*x^2 + 2*x^4))/40$

**Maple [A]** time = 0.005, size = 34, normalized size = 0.7

$$\frac{(2x-3)(2x+3)(2x^4+6x^2+27)}{40} \frac{1}{\sqrt{-4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(-4\*x^2+9)^(1/2), x)

[Out]  $1/40 * (2*x-3) * (2*x+3) * (2*x^4+6*x^2+27)/(-4*x^2+9)^{(1/2)}$

**Maxima [A]** time = 1.50049, size = 54, normalized size = 1.17

$$-\frac{1}{20} \sqrt{-4x^2 + 9}x^4 - \frac{3}{20} \sqrt{-4x^2 + 9}x^2 - \frac{27}{40} \sqrt{-4x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/sqrt(-4*x^2 + 9),x, algorithm="maxima")`

[Out]  $-1/20 * \text{sqrt}(-4*x^2 + 9) * x^4 - 3/20 * \text{sqrt}(-4*x^2 + 9) * x^2 - 27/40 * \text{sqrt}(-4*x^2 + 9)$

**Fricas [A]** time = 0.223721, size = 99, normalized size = 2.15

$$\frac{4x^{10} - 105x^8 + 270x^6 + 15(x^8 - 6x^6)\sqrt{-4x^2 + 9}}{20(15x^4 - 135x^2 - (x^4 - 27x^2 + 81)\sqrt{-4x^2 + 9} + 243)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/sqrt(-4*x^2 + 9),x, algorithm="fricas")`

[Out]  $-1/20 * (4*x^{10} - 105*x^8 + 270*x^6 + 15*(x^8 - 6*x^6)*\text{sqrt}(-4*x^2 + 9)) / (15*x^4 - 135*x^2 - (x^4 - 27*x^2 + 81)*\text{sqrt}(-4*x^2 + 9) + 243)$

**Sympy [A]** time = 3.44526, size = 46, normalized size = 1.

$$-\frac{x^4\sqrt{-4x^2 + 9}}{20} - \frac{3x^2\sqrt{-4x^2 + 9}}{20} - \frac{27\sqrt{-4x^2 + 9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(-4*x**2+9)**(1/2),x)`

[Out]  $-x**4*\text{sqrt}(-4*x**2 + 9)/20 - 3*x**2*\text{sqrt}(-4*x**2 + 9)/20 - 27*\text{sqrt}(-4*x**2 + 9)/40$

**GIAC/XCAS [A]** time = 0.208853, size = 58, normalized size = 1.26

$$-\frac{1}{320} (4x^2 - 9)^2 \sqrt{-4x^2 + 9} + \frac{3}{32} (-4x^2 + 9)^{\frac{3}{2}} - \frac{81}{64} \sqrt{-4x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/sqrt(-4*x^2 + 9),x, algorithm="giac")`

[Out]  $-1/320 * (4*x^2 - 9)^2 * \text{sqrt}(-4*x^2 + 9) + 3/32 * (-4*x^2 + 9)^{(3/2)} - 81/64 * \text{sqrt}(-4*x^2 + 9)$



$$3.544 \quad \int \frac{x^4}{\sqrt{9-4x^2}} dx$$

**Optimal.** Leaf size=45

$$-\frac{27}{128}\sqrt{9-4x^2}x - \frac{1}{16}\sqrt{9-4x^2}x^3 + \frac{243}{256}\sin^{-1}\left(\frac{2x}{3}\right)$$

[Out]  $(-27*x*\text{Sqrt}[9 - 4*x^2])/128 - (x^3*\text{Sqrt}[9 - 4*x^2])/16 + (243*\text{ArcSin}[(2*x)/3])/256$

**Rubi [A]** time = 0.040779, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{27}{128}\sqrt{9-4x^2}x - \frac{1}{16}\sqrt{9-4x^2}x^3 + \frac{243}{256}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^4/\text{Sqrt}[9 - 4*x^2], x]$

[Out]  $(-27*x*\text{Sqrt}[9 - 4*x^2])/128 - (x^3*\text{Sqrt}[9 - 4*x^2])/16 + (243*\text{ArcSin}[(2*x)/3])/256$

**Rubi in Sympy [A]** time = 5.48068, size = 39, normalized size = 0.87

$$-\frac{x^3\sqrt{-4x^2+9}}{16} - \frac{27x\sqrt{-4x^2+9}}{128} + \frac{243\text{asin}\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**4}/(-4*x^{**2}+9)^{(1/2)}, x)$

[Out]  $-x^{**3}*\text{sqrt}(-4*x^{**2} + 9)/16 - 27*x*\text{sqrt}(-4*x^{**2} + 9)/128 + 243*\text{asin}(2*x/3)/256$

**Mathematica [A]** time = 0.0308707, size = 36, normalized size = 0.8

$$\sqrt{9-4x^2}\left(-\frac{x^3}{16} - \frac{27x}{128}\right) + \frac{243}{256}\sin^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^4/\text{Sqrt}[9 - 4*x^2], x]$

[Out]  $\text{Sqrt}[9 - 4*x^2]*((-27*x)/128 - x^3/16) + (243*\text{ArcSin}[(2*x)/3])/256$

**Maple [A]** time = 0.01, size = 34, normalized size = 0.8

$$\frac{243}{256}\arcsin\left(\frac{2x}{3}\right) - \frac{27x}{128}\sqrt{-4x^2+9} - \frac{x^3}{16}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(-4*x^2+9)^(1/2),x)`

[Out]  $243/256 \cdot \arcsin(2/3 \cdot x) - 27/128 \cdot x \cdot (-4 \cdot x^2 + 9)^{1/2} - 1/16 \cdot x^3 \cdot (-4 \cdot x^2 + 9)^{1/2}$

**Maxima [A]** time = 1.53899, size = 45, normalized size = 1.

$$-\frac{1}{16} \sqrt{-4x^2 + 9}x^3 - \frac{27}{128} \sqrt{-4x^2 + 9}x + \frac{243}{256} \arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/sqrt(-4*x^2 + 9),x, algorithm="maxima")`

[Out]  $-1/16 \cdot \sqrt{-4 \cdot x^2 + 9} \cdot x^3 - 27/128 \cdot \sqrt{-4 \cdot x^2 + 9} \cdot x + 243/256 \cdot \arcsin(2/3 \cdot x)$

**Fricas [A]** time = 0.230018, size = 178, normalized size = 3.96

$$\frac{192x^7 - 648x^5 - 2430x^3 - 243 \left( 2x^4 - 36x^2 + 3(2x^2 - 9)\sqrt{-4x^2 + 9} + 81 \right) \arctan\left(\frac{\sqrt{-4x^2 + 9} - 3}{2x}\right) - (16x^7 - 234x^5 - 324x^3 + 2187x) \sqrt{-4x^2 + 9} + 65}{128 \left( 2x^4 - 36x^2 + 3(2x^2 - 9)\sqrt{-4x^2 + 9} + 81 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/sqrt(-4*x^2 + 9),x, algorithm="fricas")`

[Out]  $1/128 \cdot (192 \cdot x^7 - 648 \cdot x^5 - 2430 \cdot x^3 - 243 \cdot (2 \cdot x^4 - 36 \cdot x^2 + 3 \cdot (2 \cdot x^2 - 9) \cdot \sqrt{-4 \cdot x^2 + 9} + 81) \cdot \arctan(1/2 \cdot (\sqrt{-4 \cdot x^2 + 9} - 3) / x) - (16 \cdot x^7 - 234 \cdot x^5 - 324 \cdot x^3 + 2187 \cdot x) \cdot \sqrt{-4 \cdot x^2 + 9} + 65) / (2 \cdot x^4 - 36 \cdot x^2 + 3 \cdot (2 \cdot x^2 - 9) \cdot \sqrt{-4 \cdot x^2 + 9} + 81)$

**Sympy [A]** time = 1.99993, size = 39, normalized size = 0.87

$$-\frac{x^3 \sqrt{-4x^2 + 9}}{16} - \frac{27x \sqrt{-4x^2 + 9}}{128} + \frac{243 \operatorname{asin}\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(-4*x**2+9)**(1/2),x)`

[Out]  $-x^{**3} \cdot \sqrt{-4 \cdot x^{**2} + 9} / 16 - 27 \cdot x \cdot \sqrt{-4 \cdot x^{**2} + 9} / 128 + 243 \cdot \operatorname{asin}(2 \cdot x / 3) / 256$

**GIAC/XCAS [A]** time = 0.21582, size = 35, normalized size = 0.78

$$-\frac{1}{128} (8x^2 + 27) \sqrt{-4x^2 + 9}x + \frac{243}{256} \arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/sqrt(-4*x^2 + 9),x, algorithm="giac")`

[Out]  $-1/128 \cdot (8 \cdot x^2 + 27) \cdot \sqrt{-4 \cdot x^2 + 9} \cdot x + 243/256 \cdot \arcsin(2/3 \cdot x)$

$$3.545 \quad \int \frac{x^3}{\sqrt{9-4x^2}} dx$$

**Optimal.** Leaf size=31

$$\frac{1}{48} (9 - 4x^2)^{3/2} - \frac{9}{16} \sqrt{9 - 4x^2}$$

[Out]  $(-9*\text{Sqrt}[9 - 4*x^2])/16 + (9 - 4*x^2)^(3/2)/48$

**Rubi [A]** time = 0.0438246, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{1}{48} (9 - 4x^2)^{3/2} - \frac{9}{16} \sqrt{9 - 4x^2}$$

Antiderivative was successfully verified.

[In] `Int[x^3/Sqrt[9 - 4*x^2], x]`

[Out]  $(-9*\text{Sqrt}[9 - 4*x^2])/16 + (9 - 4*x^2)^(3/2)/48$

**Rubi in Sympy [A]** time = 5.63449, size = 24, normalized size = 0.77

$$\frac{(-4x^2 + 9)^{3/2}}{48} - \frac{9\sqrt{-4x^2 + 9}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3/(-4*x**2+9)**(1/2), x)`

[Out]  $(-4*x**2 + 9)**(3/2)/48 - 9*\text{sqrt}(-4*x**2 + 9)/16$

**Mathematica [A]** time = 0.00855155, size = 22, normalized size = 0.71

$$-\frac{1}{24} \sqrt{9 - 4x^2} (2x^2 + 9)$$

Antiderivative was successfully verified.

[In] `Integrate[x^3/Sqrt[9 - 4*x^2], x]`

[Out]  $-(\text{Sqrt}[9 - 4*x^2]*(9 + 2*x^2))/24$

**Maple [A]** time = 0.006, size = 29, normalized size = 0.9

$$\frac{(2x - 3)(2x + 3)(2x^2 + 9)}{24} \frac{1}{\sqrt{-4x^2 + 9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(-4*x^2+9)^(1/2), x)`

[Out]  $1/24*(2*x-3)*(2*x+3)*(2*x^2+9)/(-4*x^2+9)^(1/2)$

---

**Maxima [A]** time = 1.48752, size = 35, normalized size = 1.13

$$-\frac{1}{12}\sqrt{-4x^2+9}x^2 - \frac{3}{8}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/sqrt(-4\*x^2 + 9),x, algorithm="maxima")

[Out] -1/12\*sqrt(-4\*x^2 + 9)\*x^2 - 3/8\*sqrt(-4\*x^2 + 9)

---

**Fricas [A]** time = 0.224675, size = 70, normalized size = 2.26

$$-\frac{4x^6 + 9\sqrt{-4x^2+9}x^4 - 27x^4}{12(9x^2 - (x^2 - 9)\sqrt{-4x^2+9} - 27)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/sqrt(-4\*x^2 + 9),x, algorithm="fricas")

[Out] -1/12\*(4\*x^6 + 9\*sqrt(-4\*x^2 + 9)\*x^4 - 27\*x^4)/(9\*x^2 - (x^2 - 9)\*sqrt(-4\*x^2 + 9) - 27)

---

**Sympy [A]** time = 0.979596, size = 29, normalized size = 0.94

$$-\frac{x^2\sqrt{-4x^2+9}}{12} - \frac{3\sqrt{-4x^2+9}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(-4\*x\*\*2+9)\*\*(1/2),x)

[Out] -x\*\*2\*sqrt(-4\*x\*\*2 + 9)/12 - 3\*sqrt(-4\*x\*\*2 + 9)/8

---

**GIAC/XCAS [A]** time = 0.208117, size = 31, normalized size = 1.

$$\frac{1}{48}(-4x^2+9)^{\frac{3}{2}} - \frac{9}{16}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/sqrt(-4\*x^2 + 9),x, algorithm="giac")

[Out] 1/48\*(-4\*x^2 + 9)^(3/2) - 9/16\*sqrt(-4\*x^2 + 9)

$$3.546 \quad \int \frac{x^2}{\sqrt{9-4x^2}} dx$$

**Optimal.** Leaf size=27

$$\frac{9}{16} \sin^{-1}\left(\frac{2x}{3}\right) - \frac{1}{8}x\sqrt{9-4x^2}$$

[Out]  $-(x*\text{Sqrt}[9 - 4*x^2])/8 + (9*\text{ArcSin}[(2*x)/3])/16$

**Rubi [A]** time = 0.0231098, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{9}{16} \sin^{-1}\left(\frac{2x}{3}\right) - \frac{1}{8}x\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2/\text{Sqrt}[9 - 4*x^2], x]$

[Out]  $-(x*\text{Sqrt}[9 - 4*x^2])/8 + (9*\text{ArcSin}[(2*x)/3])/16$

**Rubi in Sympy [A]** time = 3.70202, size = 22, normalized size = 0.81

$$-\frac{x\sqrt{-4x^2+9}}{8} + \frac{9 \operatorname{asin}\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**2/(-4*x**2+9)**(1/2), x)$

[Out]  $-x*\text{sqrt}(-4*x**2 + 9)/8 + 9*\text{asin}(2*x/3)/16$

**Mathematica [A]** time = 0.0149899, size = 27, normalized size = 1.

$$\frac{9}{16} \sin^{-1}\left(\frac{2x}{3}\right) - \frac{1}{8}x\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^2/\text{Sqrt}[9 - 4*x^2], x]$

[Out]  $-(x*\text{Sqrt}[9 - 4*x^2])/8 + (9*\text{ArcSin}[(2*x)/3])/16$

**Maple [A]** time = 0.007, size = 20, normalized size = 0.7

$$\frac{9}{16} \arcsin\left(\frac{2x}{3}\right) - \frac{x}{8}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^2/(-4*x^2+9)^(1/2), x)$

[Out]  $9/16*\arcsin(2/3*x)-1/8*x*(-4*x^2+9)^(1/2)$

---

**Maxima [A]** time = 1.48908, size = 26, normalized size = 0.96

$$-\frac{1}{8}\sqrt{-4x^2+9}x + \frac{9}{16}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/sqrt(-4\*x^2 + 9),x, algorithm="maxima")

[Out] -1/8\*sqrt(-4\*x^2 + 9)\*x + 9/16\*arcsin(2/3\*x)

---

**Fricas [A]** time = 0.239089, size = 119, normalized size = 4.41

$$\frac{12x^3 - 9\left(2x^2 + 3\sqrt{-4x^2+9} - 9\right)\arctan\left(\frac{\sqrt{-4x^2+9}-3}{2x}\right) - (2x^3 - 9x)\sqrt{-4x^2+9} - 27x}{8\left(2x^2 + 3\sqrt{-4x^2+9} - 9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/sqrt(-4\*x^2 + 9),x, algorithm="fricas")

[Out] 1/8\*(12\*x^3 - 9\*(2\*x^2 + 3\*sqrt(-4\*x^2 + 9) - 9)\*arctan(1/2\*(sqrt(-4\*x^2 + 9) - 3)/x) - (2\*x^3 - 9\*x)\*sqrt(-4\*x^2 + 9) - 27\*x)/(2\*x^2 + 3\*sqrt(-4\*x^2 + 9) - 9)

---

**Sympy [A]** time = 0.536504, size = 22, normalized size = 0.81

$$-\frac{x\sqrt{-4x^2+9}}{8} + \frac{9\operatorname{asin}\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(-4\*x\*\*2+9)\*\*(1/2),x)

[Out] -x\*sqrt(-4\*x\*\*2 + 9)/8 + 9\*asin(2\*x/3)/16

---

**GIAC/XCAS [A]** time = 0.213276, size = 26, normalized size = 0.96

$$-\frac{1}{8}\sqrt{-4x^2+9}x + \frac{9}{16}\arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/sqrt(-4\*x^2 + 9),x, algorithm="giac")

[Out] -1/8\*sqrt(-4\*x^2 + 9)\*x + 9/16\*arcsin(2/3\*x)

$$3.547 \quad \int \frac{x}{\sqrt{9-4x^2}} dx$$

**Optimal.** Leaf size=15

$$-\frac{1}{4}\sqrt{9-4x^2}$$

[Out] -Sqrt[9 - 4\*x^2]/4

**Rubi [A]** time = 0.00878129, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{1}{4}\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[9 - 4\*x^2], x]

[Out] -Sqrt[9 - 4\*x^2]/4

**Rubi in Sympy [A]** time = 2.00313, size = 12, normalized size = 0.8

$$-\frac{\sqrt{-4x^2+9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(-4\*x\*\*2+9)\*\*(1/2), x)

[Out] -sqrt(-4\*x\*\*2 + 9)/4

**Mathematica [A]** time = 0.00295312, size = 15, normalized size = 1.

$$-\frac{1}{4}\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[9 - 4\*x^2], x]

[Out] -Sqrt[9 - 4\*x^2]/4

**Maple [A]** time = 0.004, size = 22, normalized size = 1.5

$$\frac{(2x-3)(2x+3)}{4} \frac{1}{\sqrt{-4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-4\*x^2+9)^(1/2), x)

[Out] 1/4\*(2\*x-3)\*(2\*x+3)/(-4\*x^2+9)^(1/2)

---

**Maxima [A]** time = 1.35076, size = 15, normalized size = 1.

$$-\frac{1}{4}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sqrt(-4\*x^2 + 9),x, algorithm="maxima")

[Out] -1/4\*sqrt(-4\*x^2 + 9)

---

**Fricas [A]** time = 0.236573, size = 23, normalized size = 1.53

$$\frac{x^2}{\sqrt{-4x^2+9}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sqrt(-4\*x^2 + 9),x, algorithm="fricas")

[Out] x^2/(sqrt(-4\*x^2 + 9) - 3)

---

**Sympy [A]** time = 0.323522, size = 12, normalized size = 0.8

$$-\frac{\sqrt{-4x^2+9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-4\*x\*\*2+9)\*\*(1/2),x)

[Out] -sqrt(-4\*x\*\*2 + 9)/4

---

**GIAC/XCAS [A]** time = 0.207186, size = 15, normalized size = 1.

$$-\frac{1}{4}\sqrt{-4x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sqrt(-4\*x^2 + 9),x, algorithm="giac")

[Out] -1/4\*sqrt(-4\*x^2 + 9)



$$3.548 \quad \int \frac{1}{\sqrt{9-4x^2}} dx$$

**Optimal.** Leaf size=10

$$\frac{1}{2} \sin^{-1} \left( \frac{2x}{3} \right)$$

[Out] ArcSin[(2\*x)/3]/2

**Rubi [A]** time = 0.00719738, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{1}{2} \sin^{-1} \left( \frac{2x}{3} \right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[9 - 4\*x^2], x]

[Out] ArcSin[(2\*x)/3]/2

**Rubi in Sympy [A]** time = 1.12555, size = 7, normalized size = 0.7

$$\frac{\text{asin} \left( \frac{2x}{3} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-4\*x\*\*2+9)\*\*(1/2), x)

[Out] asin(2\*x/3)/2

**Mathematica [A]** time = 0.00621087, size = 10, normalized size = 1.

$$\frac{1}{2} \sin^{-1} \left( \frac{2x}{3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[9 - 4\*x^2], x]

[Out] ArcSin[(2\*x)/3]/2

**Maple [A]** time = 0.003, size = 7, normalized size = 0.7

$$\frac{1}{2} \arcsin \left( \frac{2x}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-4\*x^2+9)^(1/2), x)

[Out] 1/2\*arcsin(2/3\*x)

---

**Maxima [A]** time = 1.48279, size = 8, normalized size = 0.8

$$\frac{1}{2} \arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-4*x^2 + 9),x, algorithm="maxima")`

[Out] `1/2*arcsin(2/3*x)`

---

**Fricas [A]** time = 0.238376, size = 26, normalized size = 2.6

$$-\arctan\left(\frac{\sqrt{-4x^2 + 9} - 3}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-4*x^2 + 9),x, algorithm="fricas")`

[Out] `-arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x)`

---

**Sympy [A]** time = 0.31483, size = 7, normalized size = 0.7

$$\frac{\operatorname{asin}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-4*x**2+9)**(1/2),x)`

[Out] `asin(2*x/3)/2`

---

**GIAC/XCAS [A]** time = 0.216396, size = 8, normalized size = 0.8

$$\frac{1}{2} \arcsin\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-4*x^2 + 9),x, algorithm="giac")`

[Out] `1/2*arcsin(2/3*x)`

$$3.549 \quad \int \frac{1}{x\sqrt{9-4x^2}} dx$$

**Optimal.** Leaf size=20

$$-\frac{1}{3} \tanh^{-1} \left( \frac{1}{3} \sqrt{9-4x^2} \right)$$

[Out] -ArcTanh[Sqrt[9 - 4\*x^2]/3]/3

**Rubi [A]** time = 0.0356243, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{1}{3} \tanh^{-1} \left( \frac{1}{3} \sqrt{9-4x^2} \right)$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Sqrt[9 - 4\*x^2]), x]

[Out] -ArcTanh[Sqrt[9 - 4\*x^2]/3]/3

**Rubi in Sympy [A]** time = 4.59675, size = 15, normalized size = 0.75

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{-4x^2+9}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(-4\*x\*\*2+9)\*\*(1/2), x)

[Out] -atanh(sqrt(-4\*x\*\*2 + 9)/3)/3

**Mathematica [A]** time = 0.00978476, size = 25, normalized size = 1.25

$$\frac{\log(x)}{3} - \frac{1}{3} \log(\sqrt{9-4x^2} + 3)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Sqrt[9 - 4\*x^2]), x]

[Out] Log[x]/3 - Log[3 + Sqrt[9 - 4\*x^2]]/3

**Maple [A]** time = 0.006, size = 15, normalized size = 0.8

$$-\frac{1}{3} \operatorname{Artanh} \left( 3 \frac{1}{\sqrt{-4x^2+9}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-4\*x^2+9)^(1/2), x)

[Out] -1/3\*arctanh(3/(-4\*x^2+9)^(1/2))

---

**Maxima [A]** time = 1.48371, size = 34, normalized size = 1.7

$$-\frac{1}{3} \log\left(\frac{6\sqrt{-4x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-4\*x^2 + 9)\*x), x, algorithm="maxima")

[Out] -1/3\*log(6\*sqrt(-4\*x^2 + 9)/abs(x) + 18/abs(x))

---

**Fricas [A]** time = 0.238921, size = 24, normalized size = 1.2

$$\frac{1}{3} \log\left(\frac{\sqrt{-4x^2+9}-3}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-4\*x^2 + 9)\*x), x, algorithm="fricas")

[Out] 1/3\*log((sqrt(-4\*x^2 + 9) - 3)/x)

---

**Sympy [A]** time = 3.77272, size = 27, normalized size = 1.35

$$\begin{cases} -\frac{\operatorname{acosh}\left(\frac{3}{2x}\right)}{3} & \text{for } \frac{9\left|\frac{1}{x^2}\right|}{4} > 1 \\ \frac{i \operatorname{asin}\left(\frac{3}{2x}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-4\*x\*\*2+9)\*\*(1/2), x)

[Out] Piecewise((-acosh(3/(2\*x))/3, 9\*Abs(x\*\*(-2))/4 > 1), (I\*asin(3/(2\*x))/3, True))

---

**GIAC/XCAS [A]** time = 0.21457, size = 42, normalized size = 2.1

$$-\frac{1}{6} \ln\left(\sqrt{-4x^2+9}+3\right) + \frac{1}{6} \ln\left(-\sqrt{-4x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-4\*x^2 + 9)\*x), x, algorithm="giac")

[Out] -1/6\*ln(sqrt(-4\*x^2 + 9) + 3) + 1/6\*ln(-sqrt(-4\*x^2 + 9) + 3)

$$3.550 \quad \int \frac{1}{x^2 \sqrt{9-4x^2}} dx$$

**Optimal.** Leaf size=18

$$-\frac{\sqrt{9-4x^2}}{9x}$$

[Out] -Sqrt[9 - 4\*x^2]/(9\*x)

**Rubi [A]** time = 0.0174813, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$-\frac{\sqrt{9-4x^2}}{9x}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*Sqrt[9 - 4\*x^2]), x]

[Out] -Sqrt[9 - 4\*x^2]/(9\*x)

**Rubi in Sympy [A]** time = 2.99591, size = 14, normalized size = 0.78

$$-\frac{\sqrt{-4x^2+9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(-4\*x\*\*2+9)\*\*(1/2), x)

[Out] -sqrt(-4\*x\*\*2 + 9)/(9\*x)

**Mathematica [A]** time = 0.0106142, size = 18, normalized size = 1.

$$-\frac{\sqrt{9-4x^2}}{9x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*Sqrt[9 - 4\*x^2]), x]

[Out] -Sqrt[9 - 4\*x^2]/(9\*x)

**Maple [A]** time = 0.005, size = 25, normalized size = 1.4

$$\frac{(2x-3)(2x+3)}{9x} \frac{1}{\sqrt{-4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(-4\*x^2+9)^(1/2), x)

[Out] 1/9/x\*(2\*x-3)\*(2\*x+3)/(-4\*x^2+9)^(1/2)

---

**Maxima [A]** time = 1.47881, size = 19, normalized size = 1.06

$$-\frac{\sqrt{-4x^2 + 9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-4\*x^2 + 9)\*x^2),x, algorithm="maxima")

[Out] -1/9\*sqrt(-4\*x^2 + 9)/x

---

**Fricas [A]** time = 0.234267, size = 50, normalized size = 2.78

$$\frac{4x^2 + 3\sqrt{-4x^2 + 9} - 9}{9(\sqrt{-4x^2 + 9}x - 3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-4\*x^2 + 9)\*x^2),x, algorithm="fricas")

[Out] 1/9\*(4\*x^2 + 3\*sqrt(-4\*x^2 + 9) - 9)/(sqrt(-4\*x^2 + 9)\*x - 3\*x)

---

**Sympy [A]** time = 2.61198, size = 37, normalized size = 2.06

$$\begin{cases} -\frac{i\sqrt{4x^2-9}}{9x} & \text{for } \frac{4|x^2|}{9} > 1 \\ -\frac{\sqrt{-4x^2+9}}{9x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(-4\*x\*\*2+9)\*\*(1/2),x)

[Out] Piecewise((-I\*sqrt(4\*x\*\*2 - 9)/(9\*x), 4\*Abs(x\*\*2)/9 > 1), (-sqrt(-4\*x\*\*2 + 9)/(9\*x), True))

---

**GIAC/XCAS [A]** time = 0.218706, size = 45, normalized size = 2.5

$$\frac{2x}{9(\sqrt{-4x^2 + 9} - 3)} - \frac{\sqrt{-4x^2 + 9} - 3}{18x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-4\*x^2 + 9)\*x^2),x, algorithm="giac")

[Out] 2/9\*x/(sqrt(-4\*x^2 + 9) - 3) - 1/18\*(sqrt(-4\*x^2 + 9) - 3)/x

$$3.551 \quad \int \frac{1}{x^3 \sqrt{9-4x^2}} dx$$

**Optimal.** Leaf size=39

$$-\frac{\sqrt{9-4x^2}}{18x^2} - \frac{2}{27} \tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

[Out] -Sqrt[9 - 4\*x^2]/(18\*x^2) - (2\*ArcTanh[Sqrt[9 - 4\*x^2]/3])/27

**Rubi [A]** time = 0.0526097, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{\sqrt{9-4x^2}}{18x^2} - \frac{2}{27} \tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*Sqrt[9 - 4\*x^2]),x]

[Out] -Sqrt[9 - 4\*x^2]/(18\*x^2) - (2\*ArcTanh[Sqrt[9 - 4\*x^2]/3])/27

**Rubi in Sympy [A]** time = 5.47995, size = 32, normalized size = 0.82

$$-\frac{2 \operatorname{atanh}\left(\frac{\sqrt{-4x^2+9}}{3}\right)}{27} - \frac{\sqrt{-4x^2+9}}{18x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(-4\*x\*\*2+9)\*\*(1/2),x)

[Out] -2\*atanh(sqrt(-4\*x\*\*2 + 9)/3)/27 - sqrt(-4\*x\*\*2 + 9)/(18\*x\*\*2)

**Mathematica [A]** time = 0.0269669, size = 43, normalized size = 1.1

$$-\frac{\sqrt{9-4x^2}}{18x^2} - \frac{2}{27} \log\left(\sqrt{9-4x^2}+3\right) + \frac{2 \log(x)}{27}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*Sqrt[9 - 4\*x^2]),x]

[Out] -Sqrt[9 - 4\*x^2]/(18\*x^2) + (2\*Log[x])/27 - (2\*Log[3 + Sqrt[9 - 4\*x^2]])/27

**Maple [A]** time = 0.007, size = 30, normalized size = 0.8

$$-\frac{1}{18x^2} \sqrt{-4x^2+9} - \frac{2}{27} \operatorname{Artanh}\left(3 \frac{1}{\sqrt{-4x^2+9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(-4\*x^2+9)^(1/2),x)

[Out]  $-1/18 * (-4 * x^2 + 9)^{(1/2)} / x^2 - 2/27 * \operatorname{arctanh}(3 / (-4 * x^2 + 9)^{(1/2)})$

**Maxima [A]** time = 1.49114, size = 54, normalized size = 1.38

$$-\frac{\sqrt{-4x^2+9}}{18x^2} - \frac{2}{27} \log\left(\frac{6\sqrt{-4x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-4*x^2 + 9)*x^3),x, algorithm="maxima")`

[Out]  $-1/18 * \operatorname{sqrt}(-4 * x^2 + 9) / x^2 - 2/27 * \log(6 * \operatorname{sqrt}(-4 * x^2 + 9) / \operatorname{abs}(x) + 18 / \operatorname{abs}(x))$

**Fricas [A]** time = 0.236477, size = 131, normalized size = 3.36

$$\frac{36x^2 + 4\left(2x^4 + 3\sqrt{-4x^2+9}x^2 - 9x^2\right) \log\left(\frac{\sqrt{-4x^2+9}-3}{x}\right) - 3(2x^2 - 9)\sqrt{-4x^2+9} - 81}{54\left(2x^4 + 3\sqrt{-4x^2+9}x^2 - 9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-4*x^2 + 9)*x^3),x, algorithm="fricas")`

[Out]  $1/54 * (36 * x^2 + 4 * (2 * x^4 + 3 * \operatorname{sqrt}(-4 * x^2 + 9) * x^2 - 9 * x^2) * \log((\operatorname{sqrt}(-4 * x^2 + 9) - 3) / x) - 3 * (2 * x^2 - 9) * \operatorname{sqrt}(-4 * x^2 + 9) - 81) / (2 * x^4 + 3 * \operatorname{sqrt}(-4 * x^2 + 9) * x^2 - 9 * x^2)$

**Sympy [A]** time = 8.13158, size = 100, normalized size = 2.56

$$\begin{cases} -\frac{2 \operatorname{acosh}\left(\frac{3}{2x}\right)}{27} + \frac{1}{9x\sqrt{-1+\frac{9}{4x^2}}} - \frac{1}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for } \frac{9\left|\frac{1}{x^2}\right|}{4} > 1 \\ \frac{2i \operatorname{asin}\left(\frac{3}{2x}\right)}{27} - \frac{i}{9x\sqrt{1-\frac{9}{4x^2}}} + \frac{i}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(-4*x**2+9)**(1/2),x)`

[Out] `Piecewise((-2*acosh(3/(2*x))/27 + 1/(9*x*sqrt(-1 + 9/(4*x**2)))) - 1/(4*x**3*sqrt(-1 + 9/(4*x**2))), 9*Abs(x**(-2))/4 > 1), (2*I*asin(3/(2*x))/27 - I/(9*x*sqrt(1 - 9/(4*x**2))) + I/(4*x**3*sqrt(1 - 9/(4*x**2))), True))`

**GIAC/XCAS [A]** time = 0.235299, size = 61, normalized size = 1.56

$$-\frac{\sqrt{-4x^2+9}}{18x^2} - \frac{1}{27} \ln\left(\sqrt{-4x^2+9} + 3\right) + \frac{1}{27} \ln\left(-\sqrt{-4x^2+9} + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-4*x^2 + 9)*x^3),x, algorithm="giac")`

[Out]  $-1/18 * \operatorname{sqrt}(-4 * x^2 + 9) / x^2 - 1/27 * \ln(\operatorname{sqrt}(-4 * x^2 + 9) + 3) + 1/27 * \ln(-\operatorname{sqrt}(-4 * x^2 + 9) + 3)$



$$3.552 \quad \int \frac{1}{x^4 \sqrt{9-4x^2}} dx$$

**Optimal.** Leaf size=37

$$-\frac{8\sqrt{9-4x^2}}{243x} - \frac{\sqrt{9-4x^2}}{27x^3}$$

[Out]  $-\text{Sqrt}[9 - 4*x^2]/(27*x^3) - (8*\text{Sqrt}[9 - 4*x^2])/(243*x)$

**Rubi [A]** time = 0.033899, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{8\sqrt{9-4x^2}}{243x} - \frac{\sqrt{9-4x^2}}{27x^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^4*\text{Sqrt}[9 - 4*x^2]), x]$

[Out]  $-\text{Sqrt}[9 - 4*x^2]/(27*x^3) - (8*\text{Sqrt}[9 - 4*x^2])/(243*x)$

**Rubi in Sympy [A]** time = 4.5108, size = 31, normalized size = 0.84

$$-\frac{8\sqrt{-4x^2+9}}{243x} - \frac{\sqrt{-4x^2+9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{**4}/(-4*x^{**2}+9)^{(1/2)}, x)$

[Out]  $-8*\text{sqrt}(-4*x^{**2} + 9)/(243*x) - \text{sqrt}(-4*x^{**2} + 9)/(27*x^{**3})$

**Mathematica [A]** time = 0.0132803, size = 27, normalized size = 0.73

$$\left(-\frac{1}{27x^3} - \frac{8}{243x}\right)\sqrt{9-4x^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/(x^4*\text{Sqrt}[9 - 4*x^2]), x]$

[Out]  $(-1/(27*x^3) - 8/(243*x))*\text{Sqrt}[9 - 4*x^2]$

**Maple [A]** time = 0.005, size = 32, normalized size = 0.9

$$\frac{(2x-3)(2x+3)(8x^2+9)}{243x^3} \frac{1}{\sqrt{-4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/x^4/(-4*x^2+9)^{(1/2)}, x)$

[Out]  $1/243*(2*x-3)*(2*x+3)*(8*x^2+9)/x^3/(-4*x^2+9)^{(1/2)}$

---

**Maxima [A]** time = 1.4848, size = 39, normalized size = 1.05

$$-\frac{8\sqrt{-4x^2+9}}{243x} - \frac{\sqrt{-4x^2+9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-4\*x^2 + 9)\*x^4),x, algorithm="maxima")

[Out] -8/243\*sqrt(-4\*x^2 + 9)/x - 1/27\*sqrt(-4\*x^2 + 9)/x^3

---

**Fricas [A]** time = 0.23215, size = 101, normalized size = 2.73

$$-\frac{32x^6 - 324x^4 + 243x^2 + 9(8x^4 - 15x^2 - 27)\sqrt{-4x^2 + 9} + 729}{243(9x^5 - 27x^3 - (x^5 - 9x^3)\sqrt{-4x^2 + 9})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-4\*x^2 + 9)\*x^4),x, algorithm="fricas")

[Out] -1/243\*(32\*x^6 - 324\*x^4 + 243\*x^2 + 9\*(8\*x^4 - 15\*x^2 - 27)\*sqrt(-4\*x^2 + 9) + 729)/(9\*x^5 - 27\*x^3 - (x^5 - 9\*x^3)\*sqrt(-4\*x^2 + 9))

---

**Sympy [A]** time = 5.83143, size = 82, normalized size = 2.22

$$\begin{cases} -\frac{16\sqrt{-1+\frac{9}{4x^2}}}{243} - \frac{2\sqrt{-1+\frac{9}{4x^2}}}{27x^2} & \text{for } \frac{9|\frac{1}{x^2}|}{4} > 1 \\ -\frac{16i\sqrt{1-\frac{9}{4x^2}}}{243} - \frac{2i\sqrt{1-\frac{9}{4x^2}}}{27x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(-4\*x\*\*2+9)\*\*(1/2),x)

[Out] Piecewise((-16\*sqrt(-1 + 9/(4\*x\*\*2)))/243 - 2\*sqrt(-1 + 9/(4\*x\*\*2))/(27\*x\*\*2), 9\*Abs(x\*\*(-2))/4 > 1), (-16\*I\*sqrt(1 - 9/(4\*x\*\*2)))/243 - 2\*I\*sqrt(1 - 9/(4\*x\*\*2))/(27\*x\*\*2), True))

---

**GIAC/XCAS [A]** time = 0.230884, size = 99, normalized size = 2.68

$$\frac{2x^3\left(\frac{9(\sqrt{-4x^2+9}-3)^2}{x^2} + 4\right)}{243(\sqrt{-4x^2+9}-3)^3} - \frac{\sqrt{-4x^2+9}-3}{54x} - \frac{(\sqrt{-4x^2+9}-3)^3}{1944x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-4\*x^2 + 9)\*x^4),x, algorithm="giac")

[Out] 2/243\*x^3\*(9\*(sqrt(-4\*x^2 + 9) - 3)^2/x^2 + 4)/(sqrt(-4\*x^2 + 9) - 3)^3 - 1/54\*(sqrt(-4\*x^2 + 9) - 3)/x - 1/1944\*(sqrt(-4\*x^2 + 9) - 3)^3/x^3

$$3.553 \quad \int \frac{1}{x^5 \sqrt{9-4x^2}} dx$$

**Optimal.** Leaf size=57

$$-\frac{\sqrt{9-4x^2}}{54x^2} - \frac{2}{81} \tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right) - \frac{\sqrt{9-4x^2}}{36x^4}$$

[Out] -Sqrt[9 - 4\*x^2]/(36\*x^4) - Sqrt[9 - 4\*x^2]/(54\*x^2) - (2\*ArcTanh[Sqrt[9 - 4\*x^2]/3])/81

**Rubi [A]** time = 0.0721632, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{\sqrt{9-4x^2}}{54x^2} - \frac{2}{81} \tanh^{-1}\left(\frac{1}{3}\sqrt{9-4x^2}\right) - \frac{\sqrt{9-4x^2}}{36x^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5\*Sqrt[9 - 4\*x^2]), x]

[Out] -Sqrt[9 - 4\*x^2]/(36\*x^4) - Sqrt[9 - 4\*x^2]/(54\*x^2) - (2\*ArcTanh[Sqrt[9 - 4\*x^2]/3])/81

**Rubi in Sympy [A]** time = 6.62152, size = 48, normalized size = 0.84

$$-\frac{2 \operatorname{atanh}\left(\frac{\sqrt{-4x^2+9}}{3}\right)}{81} - \frac{\sqrt{-4x^2+9}}{54x^2} - \frac{\sqrt{-4x^2+9}}{36x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*5/(-4\*x\*\*2+9)\*\*(1/2), x)

[Out] -2\*atanh(sqrt(-4\*x\*\*2 + 9)/3)/81 - sqrt(-4\*x\*\*2 + 9)/(54\*x\*\*2) - sqrt(-4\*x\*\*2 + 9)/(36\*x\*\*4)

**Mathematica [A]** time = 0.0420362, size = 48, normalized size = 0.84

$$\frac{1}{324} \left( -8 \log\left(\sqrt{9-4x^2}+3\right) - \frac{3\sqrt{9-4x^2}(2x^2+3)}{x^4} + 8 \log(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5\*Sqrt[9 - 4\*x^2]), x]

[Out] ((-3\*Sqrt[9 - 4\*x^2]\*(3 + 2\*x^2))/x^4 + 8\*Log[x] - 8\*Log[3 + Sqrt[9 - 4\*x^2]])/324

**Maple [A]** time = 0.007, size = 44, normalized size = 0.8

$$-\frac{1}{36x^4} \sqrt{-4x^2+9} - \frac{1}{54x^2} \sqrt{-4x^2+9} - \frac{2}{81} \operatorname{Artanh}\left(3 \frac{1}{\sqrt{-4x^2+9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^5/(-4*x^2+9)^(1/2),x)`

[Out]  $-1/36*(-4*x^2+9)^(1/2)/x^4 - 1/54*(-4*x^2+9)^(1/2)/x^2 - 2/81*\operatorname{arctanh}(3/(-4*x^2+9)^(1/2))$

**Maxima [A]** time = 1.50629, size = 73, normalized size = 1.28

$$-\frac{\sqrt{-4x^2+9}}{54x^2} - \frac{\sqrt{-4x^2+9}}{36x^4} - \frac{2}{81} \log\left(\frac{6\sqrt{-4x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-4*x^2+9)*x^5),x, algorithm="maxima")`

[Out]  $-1/54*\sqrt{-4*x^2+9}/x^2 - 1/36*\sqrt{-4*x^2+9}/x^4 - 2/81*\log(6*\sqrt{-4*x^2+9}/\operatorname{abs}(x) + 18/\operatorname{abs}(x))$

**Fricas [A]** time = 0.234977, size = 186, normalized size = 3.26

$$\frac{144x^6 - 756x^4 + 8(2x^8 - 36x^6 + 81x^4 + 3(2x^6 - 9x^4)\sqrt{-4x^2+9})\log\left(\frac{\sqrt{-4x^2+9}-3}{x}\right) - 3(4x^6 - 66x^4 + 54x^2 + 243)\sqrt{-4x^2+9}}{324(2x^8 - 36x^6 + 81x^4 + 3(2x^6 - 9x^4)\sqrt{-4x^2+9})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-4*x^2+9)*x^5),x, algorithm="fricas")`

[Out]  $1/324*(144*x^6 - 756*x^4 + 8*(2*x^8 - 36*x^6 + 81*x^4 + 3*(2*x^6 - 9*x^4)*\sqrt{-4*x^2+9})*\log((\sqrt{-4*x^2+9}-3)/x) - 3*(4*x^6 - 66*x^4 + 54*x^2 + 243)*\sqrt{-4*x^2+9} + 2187)/(2*x^8 - 36*x^6 + 81*x^4 + 3*(2*x^6 - 9*x^4)*\sqrt{-4*x^2+9})$

**Sympy [A]** time = 17.1808, size = 138, normalized size = 2.42

$$\begin{cases} -\frac{2 \operatorname{acosh}\left(\frac{3}{2x}\right)}{81} + \frac{1}{27x\sqrt{-1+\frac{9}{4x^2}}} - \frac{1}{36x^3\sqrt{-1+\frac{9}{4x^2}}} - \frac{1}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for } \frac{9}{4} \left|\frac{1}{x^2}\right| > 1 \\ \frac{2i \operatorname{asin}\left(\frac{3}{2x}\right)}{81} - \frac{i}{27x\sqrt{1-\frac{9}{4x^2}}} + \frac{i}{36x^3\sqrt{1-\frac{9}{4x^2}}} + \frac{i}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(-4*x**2+9)**(1/2),x)`

[Out]  $\operatorname{Piecewise}((-2*\operatorname{acosh}(3/(2*x))/81 + 1/(27*x*\sqrt{-1+9/(4*x**2)})) - 1/(36*x**3*\sqrt{-1+9/(4*x**2)}) - 1/(8*x**5*\sqrt{-1+9/(4*x**2)}), 9*\operatorname{Abs}(x**(-2))/4 > 1), (2*I*\operatorname{asin}(3/(2*x))/81 - I/(27*x*\sqrt{1-9/(4*x**2)}) + I/(36*x**3*\sqrt{1-9/(4*x**2)}) + I/(8*x**5*\sqrt{1-9/(4*x**2)}), \operatorname{True}))$

**GIAC/XCAS [A]** time = 0.215896, size = 77, normalized size = 1.35

$$\frac{(-4x^2+9)^{\frac{3}{2}} - 15\sqrt{-4x^2+9}}{216x^4} - \frac{1}{81} \ln\left(\sqrt{-4x^2+9}+3\right) + \frac{1}{81} \ln\left(-\sqrt{-4x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-4*x^2 + 9)*x^5),x, algorithm="giac")
```

```
[Out] 1/216*((-4*x^2 + 9)^(3/2) - 15*sqrt(-4*x^2 + 9))/x^4 - 1/81*ln(sqrt(-4*x^2 + 9) + 3) + 1/81*ln(-sqrt(-4*x^2 + 9) + 3)
```

$$3.554 \quad \int \frac{x^5}{\sqrt{-9+4x^2}} dx$$

**Optimal.** Leaf size=46

$$\frac{1}{320} (4x^2 - 9)^{5/2} + \frac{3}{32} (4x^2 - 9)^{3/2} + \frac{81}{64} \sqrt{4x^2 - 9}$$

[Out] (81\*Sqrt[-9 + 4\*x^2])/64 + (3\*(-9 + 4\*x^2)^(3/2))/32 + (-9 + 4\*x^2)^(5/2)/320

**Rubi [A]** time = 0.0569192, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{1}{320} (4x^2 - 9)^{5/2} + \frac{3}{32} (4x^2 - 9)^{3/2} + \frac{81}{64} \sqrt{4x^2 - 9}$$

Antiderivative was successfully verified.

[In] Int[x^5/Sqrt[-9 + 4\*x^2], x]

[Out] (81\*Sqrt[-9 + 4\*x^2])/64 + (3\*(-9 + 4\*x^2)^(3/2))/32 + (-9 + 4\*x^2)^(5/2)/320

**Rubi in Sympy [A]** time = 6.79158, size = 37, normalized size = 0.8

$$\frac{(4x^2 - 9)^{5/2}}{320} + \frac{3(4x^2 - 9)^{3/2}}{32} + \frac{81\sqrt{4x^2 - 9}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5/(4\*x\*\*2-9)\*\*(1/2), x)

[Out] (4\*x\*\*2 - 9)\*\*(5/2)/320 + 3\*(4\*x\*\*2 - 9)\*\*(3/2)/32 + 81\*sqrt(4\*x\*\*2 - 9)/64

**Mathematica [A]** time = 0.0145695, size = 27, normalized size = 0.59

$$\frac{1}{40} \sqrt{4x^2 - 9} (2x^4 + 6x^2 + 27)$$

Antiderivative was successfully verified.

[In] Integrate[x^5/Sqrt[-9 + 4\*x^2], x]

[Out] (Sqrt[-9 + 4\*x^2]\*(27 + 6\*x^2 + 2\*x^4))/40

**Maple [A]** time = 0.004, size = 34, normalized size = 0.7

$$\frac{(2x - 3)(2x + 3)(2x^4 + 6x^2 + 27)}{40} \frac{1}{\sqrt{4x^2 - 9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(4\*x^2-9)^(1/2), x)

[Out]  $1/40 * (2*x-3) * (2*x+3) * (2*x^4+6*x^2+27)/(4*x^2-9)^{(1/2)}$

**Maxima [A]** time = 1.49926, size = 54, normalized size = 1.17

$$\frac{1}{20} \sqrt{4x^2 - 9}x^4 + \frac{3}{20} \sqrt{4x^2 - 9}x^2 + \frac{27}{40} \sqrt{4x^2 - 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/sqrt(4*x^2 - 9),x, algorithm="maxima")`

[Out]  $1/20 * \text{sqrt}(4*x^2 - 9)*x^4 + 3/20 * \text{sqrt}(4*x^2 - 9)*x^2 + 27/40 * \text{sqrt}(4*x^2 - 9)$

**Fricas [A]** time = 0.237268, size = 139, normalized size = 3.02

$$\frac{2048x^{10} - 1920x^8 + 11880x^6 - 85050x^4 + 109350x^2 - 2(512x^9 + 96x^7 + 3402x^5 - 17010x^3 + 10935x)\sqrt{4x^2 - 9} - 19683}{40(512x^5 - 1440x^3 - (256x^4 - 432x^2 + 81)\sqrt{4x^2 - 9} + 810x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/sqrt(4*x^2 - 9),x, algorithm="fricas")`

[Out]  $-1/40 * (2048*x^{10} - 1920*x^8 + 11880*x^6 - 85050*x^4 + 109350*x^2 - 2*(512*x^9 + 96*x^7 + 3402*x^5 - 17010*x^3 + 10935*x)*\text{sqrt}(4*x^2 - 9) - 19683)/(512*x^5 - 1440*x^3 - (256*x^4 - 432*x^2 + 81)*\text{sqrt}(4*x^2 - 9) + 810*x)$

**Sympy [A]** time = 3.46276, size = 44, normalized size = 0.96

$$\frac{x^4\sqrt{4x^2 - 9}}{20} + \frac{3x^2\sqrt{4x^2 - 9}}{20} + \frac{27\sqrt{4x^2 - 9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(4*x**2-9)**(1/2),x)`

[Out]  $x**4*\text{sqrt}(4*x**2 - 9)/20 + 3*x**2*\text{sqrt}(4*x**2 - 9)/20 + 27*\text{sqrt}(4*x**2 - 9)/40$

**GIAC/XCAS [A]** time = 0.213607, size = 46, normalized size = 1.

$$\frac{1}{320} (4x^2 - 9)^{\frac{5}{2}} + \frac{3}{32} (4x^2 - 9)^{\frac{3}{2}} + \frac{81}{64} \sqrt{4x^2 - 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/sqrt(4*x^2 - 9),x, algorithm="giac")`

[Out]  $1/320 * (4*x^2 - 9)^{(5/2)} + 3/32 * (4*x^2 - 9)^{(3/2)} + 81/64 * \text{sqrt}(4*x^2 - 9)$

$$3.555 \quad \int \frac{x^4}{\sqrt{-9+4x^2}} dx$$

**Optimal.** Leaf size=54

$$\frac{27}{128} \sqrt{4x^2 - 9}x + \frac{243}{256} \tanh^{-1} \left( \frac{2x}{\sqrt{4x^2 - 9}} \right) + \frac{1}{16} \sqrt{4x^2 - 9}x^3$$

[Out] (27\*x\*Sqrt[-9 + 4\*x^2])/128 + (x^3\*Sqrt[-9 + 4\*x^2])/16 + (243\*ArcTanh[(2\*x)/Sqrt[-9 + 4\*x^2]])/256

**Rubi [A]** time = 0.0478432, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{27}{128} \sqrt{4x^2 - 9}x + \frac{243}{256} \tanh^{-1} \left( \frac{2x}{\sqrt{4x^2 - 9}} \right) + \frac{1}{16} \sqrt{4x^2 - 9}x^3$$

Antiderivative was successfully verified.

[In] Int[x^4/Sqrt[-9 + 4\*x^2], x]

[Out] (27\*x\*Sqrt[-9 + 4\*x^2])/128 + (x^3\*Sqrt[-9 + 4\*x^2])/16 + (243\*ArcTanh[(2\*x)/Sqrt[-9 + 4\*x^2]])/256

**Rubi in Sympy [A]** time = 5.66536, size = 48, normalized size = 0.89

$$\frac{x^3 \sqrt{4x^2 - 9}}{16} + \frac{27x \sqrt{4x^2 - 9}}{128} + \frac{243 \operatorname{atanh} \left( \frac{2x}{\sqrt{4x^2 - 9}} \right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(4\*x\*\*2-9)\*\*(1/2), x)

[Out] x\*\*3\*sqrt(4\*x\*\*2 - 9)/16 + 27\*x\*sqrt(4\*x\*\*2 - 9)/128 + 243\*atanh(2\*x/sqrt(4\*x\*\*2 - 9))/256

**Mathematica [A]** time = 0.02782, size = 46, normalized size = 0.85

$$\frac{243}{256} \log \left( \sqrt{4x^2 - 9} + 2x \right) + \sqrt{4x^2 - 9} \left( \frac{x^3}{16} + \frac{27x}{128} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/Sqrt[-9 + 4\*x^2], x]

[Out] Sqrt[-9 + 4\*x^2]\*((27\*x)/128 + x^3/16) + (243\*Log[2\*x + Sqrt[-9 + 4\*x^2]])/256

**Maple [A]** time = 0.009, size = 49, normalized size = 0.9

$$\frac{x^3}{16} \sqrt{4x^2 - 9} + \frac{27x}{128} \sqrt{4x^2 - 9} + \frac{243 \sqrt{4}}{512} \ln \left( x\sqrt{4} + \sqrt{4x^2 - 9} \right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(x^4/(4*x^2-9)^(1/2),x)`

[Out]  $\frac{1}{16}x^3(4x^2-9)^{1/2} + \frac{27}{128}x(4x^2-9)^{1/2} + \frac{243}{512}\ln(x^4(1/2)+(4x^2-9)^{1/2})^{4^{1/2}}$

**Maxima [A]** time = 1.50061, size = 61, normalized size = 1.13

$$\frac{1}{16}\sqrt{4x^2-9}x^3 + \frac{27}{128}\sqrt{4x^2-9}x + \frac{243}{256}\log\left(8x + 4\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/sqrt(4*x^2 - 9),x, algorithm="maxima")`

[Out]  $\frac{1}{16}\sqrt{4x^2-9}x^3 + \frac{27}{128}\sqrt{4x^2-9}x + \frac{243}{256}\log(8x + 4\sqrt{4x^2-9})$

**Fricas [A]** time = 0.222266, size = 176, normalized size = 3.26

$$\frac{4096x^8 - 36288x^4 + 34992x^2 + 243\left(128x^4 - 288x^2 - 8(8x^3 - 9x)\sqrt{4x^2-9} + 81\right)\log\left(-2x + \sqrt{4x^2-9}\right) - 2(1024x^7 - 1152x^5 + 7128x^3 + 2187x)\sqrt{4x^2-9}}{256\left(128x^4 - 288x^2 - 8(8x^3 - 9x)\sqrt{4x^2-9} + 81\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/sqrt(4*x^2 - 9),x, algorithm="fricas")`

[Out]  $-\frac{1}{256}(4096x^8 - 36288x^4 + 34992x^2 + 243(128x^4 - 288x^2 - 8(8x^3 - 9x)\sqrt{4x^2-9} + 81)\log(-2x + \sqrt{4x^2-9}) - 2(1024x^7 - 1152x^5 + 7128x^3 + 2187x)\sqrt{4x^2-9}) / (128x^4 - 288x^2 - 8(8x^3 - 9x)\sqrt{4x^2-9} + 81)$

**Sympy [A]** time = 1.93608, size = 39, normalized size = 0.72

$$\frac{x^3\sqrt{4x^2-9}}{16} + \frac{27x\sqrt{4x^2-9}}{128} + \frac{243\operatorname{acosh}\left(\frac{2x}{3}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(4*x**2-9)**(1/2),x)`

[Out]  $x^3\sqrt{4x^2-9}/16 + 27x\sqrt{4x^2-9}/128 + 243\operatorname{acosh}(2x/3)/256$

**GIAC/XCAS [A]** time = 0.214604, size = 50, normalized size = 0.93

$$\frac{1}{128}(8x^2 + 27)\sqrt{4x^2-9}x - \frac{243}{256}\ln\left(\left|-2x + \sqrt{4x^2-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/sqrt(4*x^2 - 9),x, algorithm="giac")`

[Out]  $\frac{1}{128}(8x^2 + 27)\sqrt{4x^2-9}x - \frac{243}{256}\ln(\operatorname{abs}(-2x + \sqrt{4x^2-9}))$

$$3.556 \quad \int \frac{x^3}{\sqrt{-9+4x^2}} dx$$

**Optimal.** Leaf size=31

$$\frac{1}{48} (4x^2 - 9)^{3/2} + \frac{9}{16} \sqrt{4x^2 - 9}$$

[Out] (9\*Sqrt[-9 + 4\*x^2])/16 + (-9 + 4\*x^2)^(3/2)/48

**Rubi [A]** time = 0.0430051, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{1}{48} (4x^2 - 9)^{3/2} + \frac{9}{16} \sqrt{4x^2 - 9}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[-9 + 4\*x^2], x]

[Out] (9\*Sqrt[-9 + 4\*x^2])/16 + (-9 + 4\*x^2)^(3/2)/48

**Rubi in Sympy [A]** time = 5.60549, size = 24, normalized size = 0.77

$$\frac{(4x^2 - 9)^{3/2}}{48} + \frac{9\sqrt{4x^2 - 9}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(4\*x\*\*2-9)\*\*(1/2), x)

[Out] (4\*x\*\*2 - 9)\*\*(3/2)/48 + 9\*sqrt(4\*x\*\*2 - 9)/16

**Mathematica [A]** time = 0.00827988, size = 22, normalized size = 0.71

$$\frac{1}{24} (2x^2 + 9) \sqrt{4x^2 - 9}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[-9 + 4\*x^2], x]

[Out] ((9 + 2\*x^2)\*Sqrt[-9 + 4\*x^2])/24

**Maple [A]** time = 0.004, size = 29, normalized size = 0.9

$$\frac{(2x - 3)(2x + 3)(2x^2 + 9)}{24} \frac{1}{\sqrt{4x^2 - 9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(4\*x^2-9)^(1/2), x)

[Out] 1/24\*(2\*x-3)\*(2\*x+3)\*(2\*x^2+9)/(4\*x^2-9)^(1/2)

---

**Maxima [A]** time = 1.49032, size = 35, normalized size = 1.13

$$\frac{1}{12} \sqrt{4x^2 - 9} x^2 + \frac{3}{8} \sqrt{4x^2 - 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/sqrt(4\*x^2 - 9),x, algorithm="maxima")

[Out] 1/12\*sqrt(4\*x^2 - 9)\*x^2 + 3/8\*sqrt(4\*x^2 - 9)

---

**Fricas [A]** time = 0.216032, size = 99, normalized size = 3.19

$$-\frac{128x^6 + 216x^4 - 1458x^2 - 2(32x^5 + 90x^3 - 243x)\sqrt{4x^2 - 9} + 729}{24(32x^3 - (16x^2 - 9)\sqrt{4x^2 - 9} - 54x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/sqrt(4\*x^2 - 9),x, algorithm="fricas")

[Out] -1/24\*(128\*x^6 + 216\*x^4 - 1458\*x^2 - 2\*(32\*x^5 + 90\*x^3 - 243\*x)\*sqrt(4\*x^2 - 9) + 729)/(32\*x^3 - (16\*x^2 - 9)\*sqrt(4\*x^2 - 9) - 54\*x)

---

**Sympy [A]** time = 0.960798, size = 27, normalized size = 0.87

$$\frac{x^2\sqrt{4x^2 - 9}}{12} + \frac{3\sqrt{4x^2 - 9}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(4\*x\*\*2-9)\*\*(1/2),x)

[Out] x\*\*2\*sqrt(4\*x\*\*2 - 9)/12 + 3\*sqrt(4\*x\*\*2 - 9)/8

---

**GIAC/XCAS [A]** time = 0.209018, size = 31, normalized size = 1.

$$\frac{1}{48} (4x^2 - 9)^{\frac{3}{2}} + \frac{9}{16} \sqrt{4x^2 - 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/sqrt(4\*x^2 - 9),x, algorithm="giac")

[Out] 1/48\*(4\*x^2 - 9)^(3/2) + 9/16\*sqrt(4\*x^2 - 9)

$$3.557 \quad \int \frac{x^2}{\sqrt{-9+4x^2}} dx$$

**Optimal.** Leaf size=36

$$\frac{1}{8}\sqrt{4x^2-9}x + \frac{9}{16} \tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

[Out] (x\*Sqrt[-9 + 4\*x^2])/8 + (9\*ArcTanh[(2\*x)/Sqrt[-9 + 4\*x^2]])/16

**Rubi [A]** time = 0.0291869, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{1}{8}\sqrt{4x^2-9}x + \frac{9}{16} \tanh^{-1}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[-9 + 4\*x^2], x]

[Out] (x\*Sqrt[-9 + 4\*x^2])/8 + (9\*ArcTanh[(2\*x)/Sqrt[-9 + 4\*x^2]])/16

**Rubi in Sympy [A]** time = 3.79431, size = 31, normalized size = 0.86

$$\frac{x\sqrt{4x^2-9}}{8} + \frac{9 \operatorname{atanh}\left(\frac{2x}{\sqrt{4x^2-9}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(4\*x\*\*2-9)\*\*(1/2), x)

[Out] x\*sqrt(4\*x\*\*2 - 9)/8 + 9\*atanh(2\*x/sqrt(4\*x\*\*2 - 9))/16

**Mathematica [A]** time = 0.0151419, size = 37, normalized size = 1.03

$$\frac{1}{8}\sqrt{4x^2-9}x + \frac{9}{16} \log\left(\sqrt{4x^2-9} + 2x\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[-9 + 4\*x^2], x]

[Out] (x\*Sqrt[-9 + 4\*x^2])/8 + (9\*Log[2\*x + Sqrt[-9 + 4\*x^2]])/16

**Maple [A]** time = 0.008, size = 35, normalized size = 1.

$$\frac{x}{8}\sqrt{4x^2-9} + \frac{9\sqrt{4}}{32} \ln\left(x\sqrt{4} + \sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(4\*x^2-9)^(1/2), x)

[Out] 1/8\*x\*(4\*x^2-9)^(1/2)+9/32\*ln(x\*4^(1/2)+(4\*x^2-9)^(1/2))\*4^(1/2)

---

**Maxima [A]** time = 1.48117, size = 42, normalized size = 1.17

$$\frac{1}{8} \sqrt{4x^2 - 9} + \frac{9}{16} \log(8x + 4\sqrt{4x^2 - 9})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/sqrt(4\*x^2 - 9), x, algorithm="maxima")

[Out] 1/8\*sqrt(4\*x^2 - 9)\*x + 9/16\*log(8\*x + 4\*sqrt(4\*x^2 - 9))

---

**Fricas [A]** time = 0.217976, size = 120, normalized size = 3.33

$$\frac{32x^4 - 72x^2 + 9(8x^2 - 4\sqrt{4x^2 - 9}x - 9) \log(-2x + \sqrt{4x^2 - 9}) - 2(8x^3 - 9x)\sqrt{4x^2 - 9}}{16(8x^2 - 4\sqrt{4x^2 - 9}x - 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/sqrt(4\*x^2 - 9), x, algorithm="fricas")

[Out] -1/16\*(32\*x^4 - 72\*x^2 + 9\*(8\*x^2 - 4\*sqrt(4\*x^2 - 9)\*x - 9)\*log(-2\*x + sqrt(4\*x^2 - 9)) - 2\*(8\*x^3 - 9\*x)\*sqrt(4\*x^2 - 9))/(8\*x^2 - 4\*sqrt(4\*x^2 - 9)\*x - 9)

---

**Sympy [A]** time = 0.543233, size = 22, normalized size = 0.61

$$\frac{x\sqrt{4x^2 - 9}}{8} + \frac{9 \operatorname{acosh}\left(\frac{2x}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(4\*x\*\*2-9)\*\*(1/2), x)

[Out] x\*sqrt(4\*x\*\*2 - 9)/8 + 9\*acosh(2\*x/3)/16

---

**GIAC/XCAS [A]** time = 0.21347, size = 41, normalized size = 1.14

$$\frac{1}{8} \sqrt{4x^2 - 9} - \frac{9}{16} \ln\left(\left|-2x + \sqrt{4x^2 - 9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/sqrt(4\*x^2 - 9), x, algorithm="giac")

[Out] 1/8\*sqrt(4\*x^2 - 9)\*x - 9/16\*ln(abs(-2\*x + sqrt(4\*x^2 - 9)))

$$3.558 \quad \int \frac{x}{\sqrt{-9+4x^2}} dx$$

**Optimal.** Leaf size=15

$$\frac{1}{4}\sqrt{4x^2-9}$$

[Out] Sqrt[-9 + 4\*x^2]/4

**Rubi [A]** time = 0.0087077, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{1}{4}\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[-9 + 4\*x^2], x]

[Out] Sqrt[-9 + 4\*x^2]/4

**Rubi in Sympy [A]** time = 1.96512, size = 10, normalized size = 0.67

$$\frac{\sqrt{4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(4\*x\*\*2-9)\*\*(1/2), x)

[Out] sqrt(4\*x\*\*2 - 9)/4

**Mathematica [A]** time = 0.00256402, size = 15, normalized size = 1.

$$\frac{1}{4}\sqrt{4x^2-9}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[-9 + 4\*x^2], x]

[Out] Sqrt[-9 + 4\*x^2]/4

**Maple [A]** time = 0.004, size = 22, normalized size = 1.5

$$\frac{(2x-3)(2x+3)}{4} \frac{1}{\sqrt{4x^2-9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(4\*x^2-9)^(1/2), x)

[Out] 1/4\*(2\*x-3)\*(2\*x+3)/(4\*x^2-9)^(1/2)

---

**Maxima [A]** time = 1.34712, size = 15, normalized size = 1.

$$\frac{1}{4} \sqrt{4x^2 - 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sqrt(4\*x^2 - 9),x, algorithm="maxima")

[Out] 1/4\*sqrt(4\*x^2 - 9)

---

**Fricas [A]** time = 0.214052, size = 51, normalized size = 3.4

$$\frac{4x^2 - 2\sqrt{4x^2 - 9}x - 9}{4(2x - \sqrt{4x^2 - 9})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sqrt(4\*x^2 - 9),x, algorithm="fricas")

[Out] -1/4\*(4\*x^2 - 2\*sqrt(4\*x^2 - 9)\*x - 9)/(2\*x - sqrt(4\*x^2 - 9))

---

**Sympy [A]** time = 0.310601, size = 10, normalized size = 0.67

$$\frac{\sqrt{4x^2 - 9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(4\*x\*\*2-9)\*\*(1/2),x)

[Out] sqrt(4\*x\*\*2 - 9)/4

---

**GIAC/XCAS [A]** time = 0.215375, size = 15, normalized size = 1.

$$\frac{1}{4} \sqrt{4x^2 - 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sqrt(4\*x^2 - 9),x, algorithm="giac")

[Out] 1/4\*sqrt(4\*x^2 - 9)

$$3.559 \quad \int \frac{1}{\sqrt{-9+4x^2}} dx$$

**Optimal.** Leaf size=19

$$\frac{1}{2} \tanh^{-1} \left( \frac{2x}{\sqrt{4x^2 - 9}} \right)$$

[Out] ArcTanh[(2\*x)/Sqrt[-9 + 4\*x^2]]/2

**Rubi [A]** time = 0.0123318, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{1}{2} \tanh^{-1} \left( \frac{2x}{\sqrt{4x^2 - 9}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-9 + 4\*x^2], x]

[Out] ArcTanh[(2\*x)/Sqrt[-9 + 4\*x^2]]/2

**Rubi in Sympy [A]** time = 1.21891, size = 15, normalized size = 0.79

$$\frac{\operatorname{atanh} \left( \frac{2x}{\sqrt{4x^2-9}} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(4\*x\*\*2-9)\*\*(1/2), x)

[Out] atanh(2\*x/sqrt(4\*x\*\*2 - 9))/2

**Mathematica [B]** time = 0.00505957, size = 43, normalized size = 2.26

$$\frac{1}{4} \log \left( \frac{2x}{\sqrt{4x^2 - 9}} + 1 \right) - \frac{1}{4} \log \left( 1 - \frac{2x}{\sqrt{4x^2 - 9}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-9 + 4\*x^2], x]

[Out] -Log[1 - (2\*x)/Sqrt[-9 + 4\*x^2]]/4 + Log[1 + (2\*x)/Sqrt[-9 + 4\*x^2]]/4

**Maple [A]** time = 0.003, size = 22, normalized size = 1.2

$$\frac{\sqrt{4}}{4} \ln \left( x\sqrt{4} + \sqrt{4x^2 - 9} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4\*x^2-9)^(1/2), x)



[Out]  $\frac{1}{4} \ln(x \sqrt{4} + (\sqrt{4x^2 - 9}) \sqrt{4}) \sqrt{4}$

**Maxima [A]** time = 1.49784, size = 24, normalized size = 1.26

$$\frac{1}{2} \log(8x + 4\sqrt{4x^2 - 9})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(4*x^2 - 9),x, algorithm="maxima")`

[Out]  $\frac{1}{2} \log(8x + 4\sqrt{4x^2 - 9})$

**Fricas [A]** time = 0.211873, size = 22, normalized size = 1.16

$$-\frac{1}{2} \log(-2x + \sqrt{4x^2 - 9})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(4*x^2 - 9),x, algorithm="fricas")`

[Out]  $-\frac{1}{2} \log(-2x + \sqrt{4x^2 - 9})$

**Sympy [A]** time = 0.31438, size = 7, normalized size = 0.37

$$\frac{\operatorname{acosh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(4*x**2-9)**(1/2),x)`

[Out]  $\operatorname{acosh}(2x/3)/2$

**GIAC/XCAS [A]** time = 0.215207, size = 23, normalized size = 1.21

$$-\frac{1}{2} \ln\left(\left|-2x + \sqrt{4x^2 - 9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(4*x^2 - 9),x, algorithm="giac")`

[Out]  $-\frac{1}{2} \ln(\operatorname{abs}(-2x + \sqrt{4x^2 - 9}))$

$$3.560 \quad \int \frac{1}{x\sqrt{-9+4x^2}} dx$$

**Optimal.** Leaf size=20

$$\frac{1}{3} \tan^{-1} \left( \frac{1}{3} \sqrt{4x^2 - 9} \right)$$

[Out] ArcTan[Sqrt[-9 + 4\*x^2]/3]/3

**Rubi [A]** time = 0.0339633, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{1}{3} \tan^{-1} \left( \frac{1}{3} \sqrt{4x^2 - 9} \right)$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Sqrt[-9 + 4\*x^2]), x]

[Out] ArcTan[Sqrt[-9 + 4\*x^2]/3]/3

**Rubi in Sympy [A]** time = 4.48021, size = 14, normalized size = 0.7

$$\frac{\operatorname{atan} \left( \frac{\sqrt{4x^2-9}}{3} \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(4\*x\*\*2-9)\*\*(1/2), x)

[Out] atan(sqrt(4\*x\*\*2 - 9)/3)/3

**Mathematica [A]** time = 0.00948078, size = 18, normalized size = 0.9

$$-\frac{1}{3} \tan^{-1} \left( \frac{3}{\sqrt{4x^2 - 9}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Sqrt[-9 + 4\*x^2]), x]

[Out] -ArcTan[3/Sqrt[-9 + 4\*x^2]]/3

**Maple [A]** time = 0.007, size = 15, normalized size = 0.8

$$-\frac{1}{3} \operatorname{arctan} \left( 3 \frac{1}{\sqrt{4x^2 - 9}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(4\*x^2-9)^(1/2), x)

[Out]  $-1/3 \cdot \arctan(3/(4 \cdot x^2 - 9)^{1/2})$

**Maxima [A]** time = 1.48231, size = 12, normalized size = 0.6

$$-\frac{1}{3} \arcsin\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(4*x^2 - 9)*x), x, algorithm="maxima")`

[Out]  $-1/3 \cdot \arcsin(3/2/\text{abs}(x))$

**Fricas [A]** time = 0.229463, size = 24, normalized size = 1.2

$$\frac{2}{3} \arctan\left(-\frac{2}{3}x + \frac{1}{3}\sqrt{4x^2 - 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(4*x^2 - 9)*x), x, algorithm="fricas")`

[Out]  $2/3 \cdot \arctan(-2/3 \cdot x + 1/3 \cdot \text{sqrt}(4 \cdot x^2 - 9))$

**Sympy [A]** time = 3.67349, size = 27, normalized size = 1.35

$$\begin{cases} \frac{i \operatorname{acosh}\left(\frac{3}{2x}\right)}{3} & \text{for } \frac{9}{4} \left|\frac{1}{x^2}\right| > 1 \\ -\frac{\operatorname{asin}\left(\frac{3}{2x}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(4*x**2-9)**(1/2), x)`

[Out] `Piecewise((I*acosh(3/(2*x)))/3, 9*Abs(x**(-2))/4 > 1), (-asin(3/(2*x)))/3, True))`

**GIAC/XCAS [A]** time = 0.207089, size = 19, normalized size = 0.95

$$\frac{1}{3} \arctan\left(\frac{1}{3}\sqrt{4x^2 - 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(4*x^2 - 9)*x), x, algorithm="giac")`

[Out]  $1/3 \cdot \arctan(1/3 \cdot \text{sqrt}(4 \cdot x^2 - 9))$

$$3.561 \quad \int \frac{1}{x^2 \sqrt{-9+4x^2}} dx$$

**Optimal.** Leaf size=18

$$\frac{\sqrt{4x^2 - 9}}{9x}$$

[Out] Sqrt[-9 + 4\*x^2]/(9\*x)

**Rubi [A]** time = 0.0160932, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{\sqrt{4x^2 - 9}}{9x}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*Sqrt[-9 + 4\*x^2]),x]

[Out] Sqrt[-9 + 4\*x^2]/(9\*x)

**Rubi in Sympy [A]** time = 3.04114, size = 12, normalized size = 0.67

$$\frac{\sqrt{4x^2 - 9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(4\*x\*\*2-9)\*\*(1/2),x)

[Out] sqrt(4\*x\*\*2 - 9)/(9\*x)

**Mathematica [A]** time = 0.00913327, size = 18, normalized size = 1.

$$\frac{\sqrt{4x^2 - 9}}{9x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*Sqrt[-9 + 4\*x^2]),x]

[Out] Sqrt[-9 + 4\*x^2]/(9\*x)

**Maple [A]** time = 0.005, size = 25, normalized size = 1.4

$$\frac{(2x - 3)(2x + 3)}{9x} \frac{1}{\sqrt{4x^2 - 9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(4\*x^2-9)^(1/2),x)

[Out] 1/9/x\*(2\*x-3)\*(2\*x+3)/(4\*x^2-9)^(1/2)

---

**Maxima [A]** time = 1.4843, size = 19, normalized size = 1.06

$$\frac{\sqrt{4x^2 - 9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(4\*x^2 - 9)\*x^2),x, algorithm="maxima")

[Out] 1/9\*sqrt(4\*x^2 - 9)/x

---

**Fricas [A]** time = 0.218556, size = 27, normalized size = 1.5

$$\frac{1}{2x^2 - \sqrt{4x^2 - 9}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(4\*x^2 - 9)\*x^2),x, algorithm="fricas")

[Out] 1/(2\*x^2 - sqrt(4\*x^2 - 9)\*x)

---

**Sympy [A]** time = 2.59728, size = 39, normalized size = 2.17

$$\begin{cases} \frac{2i\sqrt{-1+\frac{9}{4x^2}}}{9} & \text{for } \frac{9\left|\frac{1}{x^2}\right|}{4} > 1 \\ \frac{2\sqrt{1-\frac{9}{4x^2}}}{9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(4\*x\*\*2-9)\*\*(1/2),x)

[Out] Piecewise((2\*I\*sqrt(-1 + 9/(4\*x\*\*2)))/9, 9\*Abs(x\*\*(-2))/4 > 1), (2\*sqrt(1 - 9/(4\*x\*\*2)))/9, True))

---

**GIAC/XCAS [A]** time = 0.219679, size = 31, normalized size = 1.72

$$\frac{4}{(2x - \sqrt{4x^2 - 9})^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(4\*x^2 - 9)\*x^2),x, algorithm="giac")

[Out] 4/((2\*x - sqrt(4\*x^2 - 9))^2 + 9)

$$3.562 \quad \int \frac{1}{x^3 \sqrt{-9+4x^2}} dx$$

**Optimal.** Leaf size=39

$$\frac{\sqrt{4x^2-9}}{18x^2} + \frac{2}{27} \tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

[Out] Sqrt[-9 + 4\*x^2]/(18\*x^2) + (2\*ArcTan[Sqrt[-9 + 4\*x^2]/3])/27

**Rubi [A]** time = 0.0507426, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{\sqrt{4x^2-9}}{18x^2} + \frac{2}{27} \tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*Sqrt[-9 + 4\*x^2]),x]

[Out] Sqrt[-9 + 4\*x^2]/(18\*x^2) + (2\*ArcTan[Sqrt[-9 + 4\*x^2]/3])/27

**Rubi in Sympy [A]** time = 5.46903, size = 31, normalized size = 0.79

$$\frac{2 \operatorname{atan}\left(\frac{\sqrt{4x^2-9}}{3}\right)}{27} + \frac{\sqrt{4x^2-9}}{18x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(4\*x\*\*2-9)\*\*(1/2),x)

[Out] 2\*atan(sqrt(4\*x\*\*2 - 9)/3)/27 + sqrt(4\*x\*\*2 - 9)/(18\*x\*\*2)

**Mathematica [A]** time = 0.0227073, size = 37, normalized size = 0.95

$$\frac{\sqrt{4x^2-9}}{18x^2} - \frac{2}{27} \tan^{-1}\left(\frac{3}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*Sqrt[-9 + 4\*x^2]),x]

[Out] Sqrt[-9 + 4\*x^2]/(18\*x^2) - (2\*ArcTan[3/Sqrt[-9 + 4\*x^2]])/27

**Maple [A]** time = 0.005, size = 30, normalized size = 0.8

$$\frac{1}{18x^2} \sqrt{4x^2-9} - \frac{2}{27} \arctan\left(3 \frac{1}{\sqrt{4x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(4\*x^2-9)^(1/2),x)

[Out]  $1/18*(4*x^2-9)^{(1/2)}/x^2-2/27*\arctan(3/(4*x^2-9)^{(1/2)})$

**Maxima [A]** time = 1.59341, size = 32, normalized size = 0.82

$$\frac{\sqrt{4x^2-9}}{18x^2} - \frac{2}{27} \arcsin\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(4*x^2 - 9)*x^3),x, algorithm="maxima")`

[Out]  $1/18*\sqrt{4*x^2 - 9}/x^2 - 2/27*\arcsin(3/2/abs(x))$

**Fricas [A]** time = 0.219368, size = 134, normalized size = 3.44

$$\frac{48x^3 - 8(8x^4 - 4\sqrt{4x^2-9}x^3 - 9x^2) \arctan\left(-\frac{2}{3}x + \frac{1}{3}\sqrt{4x^2-9}\right) - 3(8x^2-9)\sqrt{4x^2-9} - 108x}{54(8x^4 - 4\sqrt{4x^2-9}x^3 - 9x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(4*x^2 - 9)*x^3),x, algorithm="fricas")`

[Out]  $-1/54*(48*x^3 - 8*(8*x^4 - 4*\sqrt{4*x^2 - 9}*x^3 - 9*x^2)*\arctan(-2/3*x + 1/3*\sqrt{4*x^2 - 9}) - 3*(8*x^2 - 9)*\sqrt{4*x^2 - 9} - 108*x)/(8*x^4 - 4*\sqrt{4*x^2 - 9}*x^3 - 9*x^2)$

**Sympy [A]** time = 8.16821, size = 100, normalized size = 2.56

$$\begin{cases} \frac{2i \operatorname{acosh}\left(\frac{3}{2x}\right)}{27} - \frac{i}{9x\sqrt{-1+\frac{9}{4x^2}}} + \frac{i}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for } \frac{9}{4} \left|\frac{1}{x^2}\right| > 1 \\ -\frac{2 \operatorname{asin}\left(\frac{3}{2x}\right)}{27} + \frac{1}{9x\sqrt{1-\frac{9}{4x^2}}} - \frac{1}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(4*x**2-9)**(1/2),x)`

[Out] `Piecewise((2*I*acosh(3/(2*x))/27 - I/(9*x*sqrt(-1 + 9/(4*x**2)))) + I/(4*x**3*sqrt(-1 + 9/(4*x**2))), 9*Abs(x**(-2))/4 > 1), (-2*asin(3/(2*x))/27 + 1/(9*x*sqrt(1 - 9/(4*x**2))) - 1/(4*x**3*sqrt(1 - 9/(4*x**2))), True))`

**GIAC/XCAS [A]** time = 0.216261, size = 39, normalized size = 1.

$$\frac{\sqrt{4x^2-9}}{18x^2} + \frac{2}{27} \arctan\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(4*x^2 - 9)*x^3),x, algorithm="giac")`

[Out]  $1/18*\sqrt{4*x^2 - 9}/x^2 + 2/27*\arctan(1/3*\sqrt{4*x^2 - 9})$

$$3.563 \quad \int \frac{1}{x^4 \sqrt{-9+4x^2}} dx$$

**Optimal.** Leaf size=37

$$\frac{8\sqrt{4x^2-9}}{243x} + \frac{\sqrt{4x^2-9}}{27x^3}$$

[Out] Sqrt[-9 + 4\*x^2]/(27\*x^3) + (8\*Sqrt[-9 + 4\*x^2])/(243\*x)

**Rubi [A]** time = 0.0316089, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{8\sqrt{4x^2-9}}{243x} + \frac{\sqrt{4x^2-9}}{27x^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*Sqrt[-9 + 4\*x^2]), x]

[Out] Sqrt[-9 + 4\*x^2]/(27\*x^3) + (8\*Sqrt[-9 + 4\*x^2])/(243\*x)

**Rubi in Sympy [A]** time = 4.50453, size = 29, normalized size = 0.78

$$\frac{8\sqrt{4x^2-9}}{243x} + \frac{\sqrt{4x^2-9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(4\*x\*\*2-9)\*\*(1/2), x)

[Out] 8\*sqrt(4\*x\*\*2 - 9)/(243\*x) + sqrt(4\*x\*\*2 - 9)/(27\*x\*\*3)

**Mathematica [A]** time = 0.0118691, size = 27, normalized size = 0.73

$$\left( \frac{1}{27x^3} + \frac{8}{243x} \right) \sqrt{4x^2-9}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*Sqrt[-9 + 4\*x^2]), x]

[Out] (1/(27\*x^3) + 8/(243\*x))\*Sqrt[-9 + 4\*x^2]

**Maple [A]** time = 0.005, size = 32, normalized size = 0.9

$$\frac{(2x-3)(2x+3)(8x^2+9)}{243x^3} \frac{1}{\sqrt{4x^2-9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(4\*x^2-9)^(1/2), x)

[Out] 1/243\*(2\*x-3)\*(2\*x+3)\*(8\*x^2+9)/x^3/(4\*x^2-9)^(1/2)



---

**Maxima [A]** time = 1.50597, size = 39, normalized size = 1.05

$$\frac{8\sqrt{4x^2-9}}{243x} + \frac{\sqrt{4x^2-9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(4\*x^2 - 9)\*x^4),x, algorithm="maxima")

[Out] 8/243\*sqrt(4\*x^2 - 9)/x + 1/27\*sqrt(4\*x^2 - 9)/x^3

---

**Fricas [A]** time = 0.222752, size = 74, normalized size = 2.

$$\frac{4x^2 - 2\sqrt{4x^2-9}x - 3}{32x^6 - 54x^4 - (16x^5 - 9x^3)\sqrt{4x^2-9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(4\*x^2 - 9)\*x^4),x, algorithm="fricas")

[Out] (4\*x^2 - 2\*sqrt(4\*x^2 - 9)\*x - 3)/(32\*x^6 - 54\*x^4 - (16\*x^5 - 9\*x^3)\*sqrt(4\*x^2 - 9))

---

**Sympy [A]** time = 5.81184, size = 70, normalized size = 1.89

$$\begin{cases} \frac{8\sqrt{4x^2-9}}{243x} + \frac{\sqrt{4x^2-9}}{27x^3} & \text{for } \frac{4|x^2|}{9} > 1 \\ \frac{8i\sqrt{-4x^2+9}}{243x} + \frac{i\sqrt{-4x^2+9}}{27x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(4\*x\*\*2-9)\*\*(1/2),x)

[Out] Piecewise((8\*sqrt(4\*x\*\*2 - 9)/(243\*x) + sqrt(4\*x\*\*2 - 9)/(27\*x\*\*3), 4\*Abs(x\*\*2)/9 > 1), (8\*I\*sqrt(-4\*x\*\*2 + 9)/(243\*x) + I\*sqrt(-4\*x\*\*2 + 9)/(27\*x\*\*3), True))

---

**GIAC/XCAS [A]** time = 0.220725, size = 57, normalized size = 1.54

$$\frac{32 \left( (2x - \sqrt{4x^2-9})^2 + 3 \right)}{\left( (2x - \sqrt{4x^2-9})^2 + 9 \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(4\*x^2 - 9)\*x^4),x, algorithm="giac")

[Out] 32\*((2\*x - sqrt(4\*x^2 - 9))^2 + 3)/((2\*x - sqrt(4\*x^2 - 9))^2 + 9)^3

$$3.564 \quad \int \frac{1}{x^5 \sqrt{-9+4x^2}} dx$$

**Optimal.** Leaf size=57

$$\frac{\sqrt{4x^2-9}}{54x^2} + \frac{2}{81} \tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right) + \frac{\sqrt{4x^2-9}}{36x^4}$$

[Out] Sqrt[-9 + 4\*x^2]/(36\*x^4) + Sqrt[-9 + 4\*x^2]/(54\*x^2) + (2\*ArcTan[Sqrt[-9 + 4\*x^2]/3])/81

**Rubi [A]** time = 0.0690367, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{\sqrt{4x^2-9}}{54x^2} + \frac{2}{81} \tan^{-1}\left(\frac{1}{3}\sqrt{4x^2-9}\right) + \frac{\sqrt{4x^2-9}}{36x^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5\*Sqrt[-9 + 4\*x^2]), x]

[Out] Sqrt[-9 + 4\*x^2]/(36\*x^4) + Sqrt[-9 + 4\*x^2]/(54\*x^2) + (2\*ArcTan[Sqrt[-9 + 4\*x^2]/3])/81

**Rubi in Sympy [A]** time = 6.53081, size = 46, normalized size = 0.81

$$\frac{2 \operatorname{atan}\left(\frac{\sqrt{4x^2-9}}{3}\right)}{81} + \frac{\sqrt{4x^2-9}}{54x^2} + \frac{\sqrt{4x^2-9}}{36x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*5/(4\*x\*\*2-9)\*\*(1/2), x)

[Out] 2\*atan(sqrt(4\*x\*\*2 - 9)/3)/81 + sqrt(4\*x\*\*2 - 9)/(54\*x\*\*2) + sqrt(4\*x\*\*2 - 9)/(36\*x\*\*4)

**Mathematica [A]** time = 0.0307532, size = 46, normalized size = 0.81

$$\left(\frac{1}{36x^4} + \frac{1}{54x^2}\right)\sqrt{4x^2-9} - \frac{2}{81} \tan^{-1}\left(\frac{3}{\sqrt{4x^2-9}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5\*Sqrt[-9 + 4\*x^2]), x]

[Out] (1/(36\*x^4) + 1/(54\*x^2))\*Sqrt[-9 + 4\*x^2] - (2\*ArcTan[3/Sqrt[-9 + 4\*x^2]])/81

**Maple [A]** time = 0.008, size = 44, normalized size = 0.8

$$\frac{1}{36x^4}\sqrt{4x^2-9} + \frac{1}{54x^2}\sqrt{4x^2-9} - \frac{2}{81} \arctan\left(3\frac{1}{\sqrt{4x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^5/(4*x^2-9)^(1/2),x)`

[Out]  $\frac{1}{36} \cdot (4x^2 - 9)^{1/2} / x^4 + \frac{1}{54} \cdot (4x^2 - 9)^{1/2} / x^2 - \frac{2}{81} \cdot \arctan\left(\frac{3}{\sqrt{4x^2 - 9}}\right)$

**Maxima [A]** time = 1.49224, size = 51, normalized size = 0.89

$$\frac{\sqrt{4x^2 - 9}}{54x^2} + \frac{\sqrt{4x^2 - 9}}{36x^4} - \frac{2}{81} \arcsin\left(\frac{3}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(4*x^2 - 9)*x^5),x, algorithm="maxima")`

[Out]  $\frac{1}{54} \cdot \sqrt{4x^2 - 9} / x^2 + \frac{1}{36} \cdot \sqrt{4x^2 - 9} / x^4 - \frac{2}{81} \cdot \arcsin\left(\frac{3}{2/\text{abs}(x)}\right)$

**Fricas [A]** time = 0.223273, size = 196, normalized size = 3.44

$$\frac{1536x^7 - 2880x^5 - 3888x^3 - 16 \left(128x^8 - 288x^6 + 81x^4 - 8(8x^7 - 9x^5)\sqrt{4x^2 - 9}\right) \arctan\left(-\frac{2}{3}x + \frac{1}{3}\sqrt{4x^2 - 9}\right) - 3 \left(128x^8 - 288x^6 + 81x^4 - 8(8x^7 - 9x^5)\sqrt{4x^2 - 9}\right)}{324 \left(128x^8 - 288x^6 + 81x^4 - 8(8x^7 - 9x^5)\sqrt{4x^2 - 9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(4*x^2 - 9)*x^5),x, algorithm="fricas")`

[Out]  $-\frac{1}{324} \cdot (1536x^7 - 2880x^5 - 3888x^3 - 16 \cdot (128x^8 - 288x^6 + 81x^4 - 8 \cdot (8x^7 - 9x^5) \cdot \sqrt{4x^2 - 9})) \cdot \arctan\left(-\frac{2}{3}x + \frac{1}{3} \cdot \sqrt{4x^2 - 9}\right) - \frac{3 \cdot (128x^8 - 288x^6 + 81x^4 - 8 \cdot (8x^7 - 9x^5) \cdot \sqrt{4x^2 - 9})}{324 \cdot (128x^8 - 288x^6 + 81x^4 - 8 \cdot (8x^7 - 9x^5) \cdot \sqrt{4x^2 - 9})}$

**Sympy [A]** time = 16.8471, size = 138, normalized size = 2.42

$$\begin{cases} \frac{2i \operatorname{acosh}\left(\frac{3}{2x}\right)}{81} - \frac{i}{27x\sqrt{-1+\frac{9}{4x^2}}} + \frac{i}{36x^3\sqrt{-1+\frac{9}{4x^2}}} + \frac{i}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for } \left|\frac{1}{x^2}\right| > 1 \\ -\frac{2 \operatorname{asin}\left(\frac{3}{2x}\right)}{81} + \frac{1}{27x\sqrt{1-\frac{9}{4x^2}}} - \frac{1}{36x^3\sqrt{1-\frac{9}{4x^2}}} - \frac{1}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(4*x**2-9)**(1/2),x)`

[Out]  $\text{Piecewise}\left(\left(\frac{2 \cdot I \cdot \operatorname{acosh}\left(\frac{3}{2x}\right)}{81} - \frac{I}{27x \sqrt{-1 + \frac{9}{4x^2}}}\right) - \frac{I}{36x^3 \sqrt{-1 + \frac{9}{4x^2}}}\right) + \frac{I}{8x^5 \sqrt{-1 + \frac{9}{4x^2}}}, \left(\frac{-2 \cdot \operatorname{asin}\left(\frac{3}{2x}\right)}{81} + \frac{1}{27x \sqrt{1 - \frac{9}{4x^2}}}\right) - \frac{1}{36x^3 \sqrt{1 - \frac{9}{4x^2}}}\right) - \frac{1}{8x^5 \sqrt{1 - \frac{9}{4x^2}}}, \text{True})$

**GIAC/XCAS [A]** time = 0.217551, size = 55, normalized size = 0.96

$$\frac{(4x^2 - 9)^{\frac{3}{2}} + 15\sqrt{4x^2 - 9}}{216x^4} + \frac{2}{81} \arctan\left(\frac{1}{3}\sqrt{4x^2 - 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(4*x^2 - 9)*x^5),x, algorithm="giac")
```

```
[Out] 1/216*((4*x^2 - 9)^(3/2) + 15*sqrt(4*x^2 - 9))/x^4 + 2/81*arctan(1/3*sqrt(4*x^2 - 9))
```

$$3.565 \quad \int \frac{x^5}{\sqrt{-9-4x^2}} dx$$

**Optimal.** Leaf size=46

$$-\frac{1}{320}(-4x^2-9)^{5/2} - \frac{3}{32}(-4x^2-9)^{3/2} - \frac{81}{64}\sqrt{-4x^2-9}$$

[Out]  $(-81*\text{Sqrt}[-9 - 4*x^2])/64 - (3*(-9 - 4*x^2)^(3/2))/32 - (-9 - 4*x^2)^(5/2)/320$

**Rubi [A]** time = 0.0589636, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{1}{320}(-4x^2-9)^{5/2} - \frac{3}{32}(-4x^2-9)^{3/2} - \frac{81}{64}\sqrt{-4x^2-9}$$

Antiderivative was successfully verified.

[In] Int[x^5/Sqrt[-9 - 4\*x^2], x]

[Out]  $(-81*\text{Sqrt}[-9 - 4*x^2])/64 - (3*(-9 - 4*x^2)^(3/2))/32 - (-9 - 4*x^2)^(5/2)/320$

**Rubi in Sympy [A]** time = 6.76829, size = 44, normalized size = 0.96

$$-\frac{(-4x^2-9)^{5/2}}{320} - \frac{3(-4x^2-9)^{3/2}}{32} - \frac{81\sqrt{-4x^2-9}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5/(-4\*x\*\*2-9)\*\*(1/2), x)

[Out]  $-(-4*x**2 - 9)**(5/2)/320 - 3*(-4*x**2 - 9)**(3/2)/32 - 81*\text{sqrt}(-4*x**2 - 9)/64$

**Mathematica [A]** time = 0.0143208, size = 27, normalized size = 0.59

$$-\frac{1}{40}\sqrt{-4x^2-9}(2x^4-6x^2+27)$$

Antiderivative was successfully verified.

[In] Integrate[x^5/Sqrt[-9 - 4\*x^2], x]

[Out]  $-(\text{Sqrt}[-9 - 4*x^2]*(27 - 6*x^2 + 2*x^4))/40$

**Maple [A]** time = 0.004, size = 24, normalized size = 0.5

$$-\frac{2x^4-6x^2+27}{40}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(-4\*x^2-9)^(1/2), x)

[Out]  $-1/40 * (2 * x^4 - 6 * x^2 + 27) * (-4 * x^2 - 9)^{(1/2)}$

**Maxima [A]** time = 1.4862, size = 54, normalized size = 1.17

$$-\frac{1}{20} \sqrt{-4x^2 - 9}x^4 + \frac{3}{20} \sqrt{-4x^2 - 9}x^2 - \frac{27}{40} \sqrt{-4x^2 - 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/sqrt(-4*x^2 - 9),x, algorithm="maxima")`

[Out]  $-1/20 * \text{sqrt}(-4 * x^2 - 9) * x^4 + 3/20 * \text{sqrt}(-4 * x^2 - 9) * x^2 - 27/40 * \text{sqrt}(-4 * x^2 - 9)$

**Fricas [A]** time = 0.21638, size = 31, normalized size = 0.67

$$-\frac{1}{40} (2x^4 - 6x^2 + 27) \sqrt{-4x^2 - 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/sqrt(-4*x^2 - 9),x, algorithm="fricas")`

[Out]  $-1/40 * (2 * x^4 - 6 * x^2 + 27) * \text{sqrt}(-4 * x^2 - 9)$

**Sympy [A]** time = 3.54761, size = 49, normalized size = 1.07

$$-\frac{x^4 \sqrt{-4x^2 - 9}}{20} + \frac{3x^2 \sqrt{-4x^2 - 9}}{20} - \frac{27 \sqrt{-4x^2 - 9}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(-4*x**2-9)**(1/2),x)`

[Out]  $-x^{**4} * \text{sqrt}(-4 * x^{**2} - 9) / 20 + 3 * x^{**2} * \text{sqrt}(-4 * x^{**2} - 9) / 20 - 27 * \text{sqrt}(-4 * x^{**2} - 9) / 40$

**GIAC/XCAS [A]** time = 0.219703, size = 50, normalized size = 1.09

$$-\frac{1}{320} (4x^2 + 9)^{\frac{5}{2}}i + \frac{3}{32} (4x^2 + 9)^{\frac{3}{2}}i - \frac{81}{64} \sqrt{4x^2 + 9}i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/sqrt(-4*x^2 - 9),x, algorithm="giac")`

[Out]  $-1/320 * (4 * x^2 + 9)^{(5/2)} * i + 3/32 * (4 * x^2 + 9)^{(3/2)} * i - 81/64 * \text{sqrt}(4 * x^2 + 9) * i$

$$3.566 \quad \int \frac{x^4}{\sqrt{-9-4x^2}} dx$$

**Optimal.** Leaf size=54

$$\frac{27}{128} \sqrt{-4x^2-9}x + \frac{243}{256} \tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right) - \frac{1}{16} \sqrt{-4x^2-9}x^3$$

[Out] (27\*x\*Sqrt[-9 - 4\*x^2])/128 - (x^3\*Sqrt[-9 - 4\*x^2])/16 + (243\*ArcTan[(2\*x)/Sqrt[-9 - 4\*x^2]])/256

**Rubi [A]** time = 0.0482272, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{27}{128} \sqrt{-4x^2-9}x + \frac{243}{256} \tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right) - \frac{1}{16} \sqrt{-4x^2-9}x^3$$

Antiderivative was successfully verified.

[In] Int[x^4/Sqrt[-9 - 4\*x^2], x]

[Out] (27\*x\*Sqrt[-9 - 4\*x^2])/128 - (x^3\*Sqrt[-9 - 4\*x^2])/16 + (243\*ArcTan[(2\*x)/Sqrt[-9 - 4\*x^2]])/256

**Rubi in Sympy [A]** time = 5.58185, size = 53, normalized size = 0.98

$$-\frac{x^3 \sqrt{-4x^2-9}}{16} + \frac{27x \sqrt{-4x^2-9}}{128} + \frac{243 \operatorname{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(-4\*x\*\*2-9)\*\*(1/2), x)

[Out] -x\*\*3\*sqrt(-4\*x\*\*2 - 9)/16 + 27\*x\*sqrt(-4\*x\*\*2 - 9)/128 + 243\*atan(2\*x/sqrt(-4\*x\*\*2 - 9))/256

**Mathematica [A]** time = 0.0382876, size = 43, normalized size = 0.8

$$\frac{1}{256} \left( 2x \sqrt{-4x^2-9} (27-8x^2) + 243 \tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/Sqrt[-9 - 4\*x^2], x]

[Out] (2\*x\*(27 - 8\*x^2)\*Sqrt[-9 - 4\*x^2] + 243\*ArcTan[(2\*x)/Sqrt[-9 - 4\*x^2]])/256

**Maple [A]** time = 0.009, size = 43, normalized size = 0.8

$$\frac{243}{256} \arctan\left(2 \frac{x}{\sqrt{-4x^2-9}}\right) + \frac{27x}{128} \sqrt{-4x^2-9} - \frac{x^3}{16} \sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(-4*x^2-9)^(1/2),x)`

[Out]  $243/256 \cdot \arctan(2x/(-4x^2-9)^{1/2}) + 27/128 \cdot x \cdot (-4x^2-9)^{1/2} - 1/16 \cdot x^3 \cdot (-4x^2-9)^{1/2}$

**Maxima [A]** time = 1.50375, size = 45, normalized size = 0.83

$$-\frac{1}{16} \sqrt{-4x^2-9}x^3 + \frac{27}{128} \sqrt{-4x^2-9}x - \frac{243}{256}i \operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/sqrt(-4*x^2 - 9),x, algorithm="maxima")`

[Out]  $-1/16 \cdot \sqrt{-4x^2-9} \cdot x^3 + 27/128 \cdot \sqrt{-4x^2-9} \cdot x - 243/256 \cdot I \cdot \operatorname{arcsinh}(2/3 \cdot x)$

**Fricas [A]** time = 0.225959, size = 90, normalized size = 1.67

$$-\frac{1}{128} (8x^3 - 27x) \sqrt{-4x^2-9} + \frac{243}{512}i \log\left(-\frac{8x + 4i\sqrt{-4x^2-9}}{x}\right) - \frac{243}{512}i \log\left(-\frac{8x - 4i\sqrt{-4x^2-9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/sqrt(-4*x^2 - 9),x, algorithm="fricas")`

[Out]  $-1/128 \cdot (8x^3 - 27x) \cdot \sqrt{-4x^2-9} + 243/512 \cdot I \cdot \log(- (8x + 4 \cdot I \cdot \sqrt{-4x^2-9})/x) - 243/512 \cdot I \cdot \log(- (8x - 4 \cdot I \cdot \sqrt{-4x^2-9})/x)$

**Sympy [A]** time = 3.00486, size = 53, normalized size = 0.98

$$-\frac{x^3 \sqrt{-4x^2-9}}{16} + \frac{27x \sqrt{-4x^2-9}}{128} + \frac{243 \operatorname{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(-4*x**2-9)**(1/2),x)`

[Out]  $-x^3 \cdot \sqrt{-4x^2-9}/16 + 27x \cdot \sqrt{-4x^2-9}/128 + 243 \cdot \operatorname{atan}(2x/\sqrt{-4x^2-9})/256$

**GIAC/XCAS [A]** time = 0.226112, size = 38, normalized size = 0.7

$$-\frac{1}{128} (8x^2 - 27) \sqrt{-4x^2-9}x - \frac{243}{256}i \operatorname{arcsin}\left(\frac{2}{3}ix\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/sqrt(-4*x^2 - 9),x, algorithm="giac")`

[Out]  $-1/128 \cdot (8x^2 - 27) \cdot \sqrt{-4x^2-9} \cdot x - 243/256 \cdot i \cdot \operatorname{arcsin}(2/3 \cdot i \cdot x)$



$$3.567 \quad \int \frac{x^3}{\sqrt{-9-4x^2}} dx$$

**Optimal.** Leaf size=31

$$\frac{1}{48} (-4x^2 - 9)^{3/2} + \frac{9}{16} \sqrt{-4x^2 - 9}$$

[Out] (9\*Sqrt[-9 - 4\*x^2])/16 + (-9 - 4\*x^2)^(3/2)/48

**Rubi [A]** time = 0.0437958, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{1}{48} (-4x^2 - 9)^{3/2} + \frac{9}{16} \sqrt{-4x^2 - 9}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[-9 - 4\*x^2], x]

[Out] (9\*Sqrt[-9 - 4\*x^2])/16 + (-9 - 4\*x^2)^(3/2)/48

**Rubi in Sympy [A]** time = 5.65234, size = 27, normalized size = 0.87

$$\frac{(-4x^2 - 9)^{3/2}}{48} + \frac{9\sqrt{-4x^2 - 9}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(-4\*x\*\*2-9)\*\*(1/2), x)

[Out] (-4\*x\*\*2 - 9)\*\*(3/2)/48 + 9\*sqrt(-4\*x\*\*2 - 9)/16

**Mathematica [A]** time = 0.0109383, size = 22, normalized size = 0.71

$$\frac{1}{24} \sqrt{-4x^2 - 9} (9 - 2x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[-9 - 4\*x^2], x]

[Out] (Sqrt[-9 - 4\*x^2]\*(9 - 2\*x^2))/24

**Maple [A]** time = 0.004, size = 19, normalized size = 0.6

$$-\frac{2x^2 - 9}{24} \sqrt{-4x^2 - 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(-4\*x^2-9)^(1/2), x)

[Out] -1/24\*(2\*x^2-9)\*(-4\*x^2-9)^(1/2)

---

**Maxima [A]** time = 1.4853, size = 35, normalized size = 1.13

$$-\frac{1}{12}\sqrt{-4x^2-9}x^2 + \frac{3}{8}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/sqrt(-4\*x^2 - 9),x, algorithm="maxima")

[Out] -1/12\*sqrt(-4\*x^2 - 9)\*x^2 + 3/8\*sqrt(-4\*x^2 - 9)

---

**Fricas [A]** time = 0.210213, size = 24, normalized size = 0.77

$$-\frac{1}{24}(2x^2-9)\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/sqrt(-4\*x^2 - 9),x, algorithm="fricas")

[Out] -1/24\*(2\*x^2 - 9)\*sqrt(-4\*x^2 - 9)

---

**Sympy [A]** time = 1.02515, size = 31, normalized size = 1.

$$-\frac{x^2\sqrt{-4x^2-9}}{12} + \frac{3\sqrt{-4x^2-9}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(-4\*x\*\*2-9)\*\*(1/2),x)

[Out] -x\*\*2\*sqrt(-4\*x\*\*2 - 9)/12 + 3\*sqrt(-4\*x\*\*2 - 9)/8

---

**GIAC/XCAS [A]** time = 0.218521, size = 34, normalized size = 1.1

$$-\frac{1}{48}(4x^2+9)^{\frac{3}{2}}i + \frac{9}{16}\sqrt{4x^2+9}i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/sqrt(-4\*x^2 - 9),x, algorithm="giac")

[Out] -1/48\*(4\*x^2 + 9)^(3/2)\*i + 9/16\*sqrt(4\*x^2 + 9)\*i

$$3.568 \quad \int \frac{x^2}{\sqrt{-9-4x^2}} dx$$

**Optimal.** Leaf size=36

$$-\frac{1}{8}\sqrt{-4x^2-9}x - \frac{9}{16}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

[Out]  $-(x*\text{Sqrt}[-9 - 4*x^2])/8 - (9*\text{ArcTan}[(2*x)/\text{Sqrt}[-9 - 4*x^2]])/16$

**Rubi [A]** time = 0.0300093, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{1}{8}\sqrt{-4x^2-9}x - \frac{9}{16}\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2/\text{Sqrt}[-9 - 4*x^2], x]$

[Out]  $-(x*\text{Sqrt}[-9 - 4*x^2])/8 - (9*\text{ArcTan}[(2*x)/\text{Sqrt}[-9 - 4*x^2]])/16$

**Rubi in Sympy [A]** time = 3.87726, size = 36, normalized size = 1.

$$-\frac{x\sqrt{-4x^2-9}}{8} - \frac{9\text{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**2}/(-4*x^{**2}-9)^{(1/2)}, x)$

[Out]  $-x*\text{sqrt}(-4*x^{**2} - 9)/8 - 9*\text{atan}(2*x/\text{sqrt}(-4*x^{**2} - 9))/16$

**Mathematica [A]** time = 0.0187488, size = 36, normalized size = 1.

$$\frac{1}{16}\left(-2\sqrt{-4x^2-9}x - 9\tan^{-1}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)\right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^2/\text{Sqrt}[-9 - 4*x^2], x]$

[Out]  $(-2*x*\text{Sqrt}[-9 - 4*x^2] - 9*\text{ArcTan}[(2*x)/\text{Sqrt}[-9 - 4*x^2]])/16$

**Maple [A]** time = 0.009, size = 29, normalized size = 0.8

$$-\frac{9}{16}\arctan\left(2\frac{x}{\sqrt{-4x^2-9}}\right) - \frac{x}{8}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^2/(-4*x^2-9)^{(1/2)}, x)$

[Out]  $-9/16 \cdot \arctan(2x/(-4x^2-9)^{1/2}) - 1/8 \cdot x \cdot (-4x^2-9)^{1/2}$

**Maxima [A]** time = 1.48551, size = 26, normalized size = 0.72

$$-\frac{1}{8} \sqrt{-4x^2 - 9x} + \frac{9}{16} i \operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/sqrt(-4*x^2 - 9),x, algorithm="maxima")`

[Out]  $-1/8 \cdot \sqrt{-4x^2 - 9} \cdot x + 9/16 \cdot I \cdot \operatorname{arcsinh}(2/3 \cdot x)$

**Fricas [A]** time = 0.221632, size = 80, normalized size = 2.22

$$-\frac{1}{8} \sqrt{-4x^2 - 9x} - \frac{9}{32} i \log\left(-\frac{8x + 4i \sqrt{-4x^2 - 9}}{x}\right) + \frac{9}{32} i \log\left(-\frac{8x - 4i \sqrt{-4x^2 - 9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/sqrt(-4*x^2 - 9),x, algorithm="fricas")`

[Out]  $-1/8 \cdot \sqrt{-4x^2 - 9} \cdot x - 9/32 \cdot I \cdot \log(-(8x + 4 \cdot I \cdot \sqrt{-4x^2 - 9})/x) + 9/32 \cdot I \cdot \log(-(8x - 4 \cdot I \cdot \sqrt{-4x^2 - 9})/x)$

**Sympy [A]** time = 1.5029, size = 36, normalized size = 1.

$$-\frac{x \sqrt{-4x^2 - 9}}{8} - \frac{9 \operatorname{atan}\left(\frac{2x}{\sqrt{-4x^2 - 9}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(-4*x**2-9)**(1/2),x)`

[Out]  $-x \cdot \sqrt{-4x^2 - 9}/8 - 9 \cdot \operatorname{atan}(2x/\sqrt{-4x^2 - 9})/16$

**GIAC/XCAS [A]** time = 0.228331, size = 28, normalized size = 0.78

$$\frac{9}{16} i \operatorname{arcsin}\left(\frac{2}{3}ix\right) - \frac{1}{8} \sqrt{-4x^2 - 9x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/sqrt(-4*x^2 - 9),x, algorithm="giac")`

[Out]  $9/16 \cdot i \cdot \operatorname{arcsin}(2/3 \cdot i \cdot x) - 1/8 \cdot \sqrt{-4x^2 - 9} \cdot x$

$$3.569 \quad \int \frac{x}{\sqrt{-9-4x^2}} dx$$

**Optimal.** Leaf size=15

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

[Out] -Sqrt[-9 - 4\*x^2]/4

**Rubi [A]** time = 0.00876209, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[-9 - 4\*x^2],x]

[Out] -Sqrt[-9 - 4\*x^2]/4

**Rubi in Sympy [A]** time = 1.98666, size = 14, normalized size = 0.93

$$-\frac{\sqrt{-4x^2-9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(-4\*x\*\*2-9)\*\*(1/2),x)

[Out] -sqrt(-4\*x\*\*2 - 9)/4

**Mathematica [A]** time = 0.00248883, size = 15, normalized size = 1.

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[-9 - 4\*x^2],x]

[Out] -Sqrt[-9 - 4\*x^2]/4

**Maple [A]** time = 0.004, size = 12, normalized size = 0.8

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-4\*x^2-9)^(1/2),x)

[Out] -1/4\*(-4\*x^2-9)^(1/2)

---

**Maxima [A]** time = 1.34056, size = 15, normalized size = 1.

$$-\frac{1}{4}\sqrt{-4x^2 - 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/sqrt(-4*x^2 - 9),x, algorithm="maxima")`

[Out] `-1/4*sqrt(-4*x^2 - 9)`

---

**Fricas [A]** time = 0.227362, size = 15, normalized size = 1.

$$-\frac{1}{4}\sqrt{-4x^2 - 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/sqrt(-4*x^2 - 9),x, algorithm="fricas")`

[Out] `-1/4*sqrt(-4*x^2 - 9)`

---

**Sympy [A]** time = 0.361372, size = 14, normalized size = 0.93

$$-\frac{\sqrt{-4x^2 - 9}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-4*x**2-9)**(1/2),x)`

[Out] `-sqrt(-4*x**2 - 9)/4`

---

**GIAC/XCAS [A]** time = 0.220944, size = 16, normalized size = 1.07

$$-\frac{1}{4}\sqrt{4x^2 + 9}i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/sqrt(-4*x^2 - 9),x, algorithm="giac")`

[Out] `-1/4*sqrt(4*x^2 + 9)*i`

$$3.570 \quad \int \frac{1}{\sqrt{-9-4x^2}} dx$$

**Optimal.** Leaf size=19

$$\frac{1}{2} \tan^{-1} \left( \frac{2x}{\sqrt{-4x^2-9}} \right)$$

[Out] ArcTan[(2\*x)/Sqrt[-9 - 4\*x^2]]/2

**Rubi [A]** time = 0.012007, antiderivative size = 19, normalized size of antiderivative = 1., number of rules used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{1}{2} \tan^{-1} \left( \frac{2x}{\sqrt{-4x^2-9}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-9 - 4\*x^2], x]

[Out] ArcTan[(2\*x)/Sqrt[-9 - 4\*x^2]]/2

**Rubi in Sympy [A]** time = 1.2204, size = 17, normalized size = 0.89

$$\frac{\operatorname{atan} \left( \frac{2x}{\sqrt{-4x^2-9}} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-4\*x\*\*2-9)\*\*(1/2), x)

[Out] atan(2\*x/sqrt(-4\*x\*\*2 - 9))/2

**Mathematica [A]** time = 0.00581921, size = 19, normalized size = 1.

$$\frac{1}{2} \tan^{-1} \left( \frac{2x}{\sqrt{-4x^2-9}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-9 - 4\*x^2], x]

[Out] ArcTan[(2\*x)/Sqrt[-9 - 4\*x^2]]/2

**Maple [A]** time = 0.003, size = 16, normalized size = 0.8

$$\frac{1}{2} \arctan \left( 2 \frac{x}{\sqrt{-4x^2-9}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-4\*x^2-9)^(1/2), x)

[Out]  $1/2 \cdot \arctan(2 \cdot x / (-4 \cdot x^2 - 9)^{(1/2)})$

**Maxima [A]** time = 1.49873, size = 8, normalized size = 0.42

$$-\frac{1}{2}i \operatorname{arsinh}\left(\frac{2}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-4*x^2 - 9),x, algorithm="maxima")`

[Out]  $-1/2 \cdot I \cdot \operatorname{arcsinh}(2/3 \cdot x)$

**Fricas [A]** time = 0.254671, size = 63, normalized size = 3.32

$$\frac{1}{4}i \log\left(-\frac{8x + 4i\sqrt{-4x^2 - 9}}{x}\right) - \frac{1}{4}i \log\left(-\frac{8x - 4i\sqrt{-4x^2 - 9}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-4*x^2 - 9),x, algorithm="fricas")`

[Out]  $1/4 \cdot I \cdot \log(-(8 \cdot x + 4 \cdot I \cdot \sqrt{-4 \cdot x^2 - 9})/x) - 1/4 \cdot I \cdot \log(-(8 \cdot x - 4 \cdot I \cdot \sqrt{-4 \cdot x^2 - 9})/x)$

**Sympy [A]** time = 1.29645, size = 17, normalized size = 0.89

$$\frac{\operatorname{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-4*x**2-9)**(1/2),x)`

[Out]  $\operatorname{atan}(2 \cdot x / \sqrt{-4 \cdot x^2 - 9}) / 2$

**GIAC/XCAS [A]** time = 0.223282, size = 11, normalized size = 0.58

$$-\frac{1}{2}i \operatorname{arcsin}\left(\frac{2}{3}ix\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-4*x^2 - 9),x, algorithm="giac")`

[Out]  $-1/2 \cdot i \cdot \operatorname{arcsin}(2/3 \cdot i \cdot x)$



$$3.571 \quad \int \frac{1}{x\sqrt{-9-4x^2}} dx$$

**Optimal.** Leaf size=20

$$\frac{1}{3} \tan^{-1} \left( \frac{1}{3} \sqrt{-4x^2 - 9} \right)$$

[Out] ArcTan[Sqrt[-9 - 4\*x^2]/3]/3

**Rubi [A]** time = 0.0347329, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{1}{3} \tan^{-1} \left( \frac{1}{3} \sqrt{-4x^2 - 9} \right)$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Sqrt[-9 - 4\*x^2]), x]

[Out] ArcTan[Sqrt[-9 - 4\*x^2]/3]/3

**Rubi in Sympy [A]** time = 4.5437, size = 15, normalized size = 0.75

$$\frac{\operatorname{atan} \left( \frac{\sqrt{-4x^2-9}}{3} \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(-4\*x\*\*2-9)\*\*(1/2), x)

[Out] atan(sqrt(-4\*x\*\*2 - 9)/3)/3

**Mathematica [A]** time = 0.0104522, size = 27, normalized size = 1.35

$$\frac{1}{3} \tan^{-1} \left( \frac{3\sqrt{-4x^2 - 9}}{4x^2 + 9} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Sqrt[-9 - 4\*x^2]), x]

[Out] ArcTan[(3\*Sqrt[-9 - 4\*x^2])/(9 + 4\*x^2)]/3

**Maple [A]** time = 0.005, size = 15, normalized size = 0.8

$$-\frac{1}{3} \arctan \left( 3 \frac{1}{\sqrt{-4x^2 - 9}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-4\*x^2-9)^(1/2), x)

[Out]  $-1/3 \cdot \arctan(3/(-4x^2-9)^{(1/2)})$

**Maxima [A]** time = 1.49123, size = 34, normalized size = 1.7

$$-\frac{1}{3}i \log\left(\frac{6\sqrt{4x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-4*x^2 - 9)*x), x, algorithm="maxima")`

[Out]  $-1/3 \cdot I \cdot \log(6 \cdot \text{sqrt}(4 \cdot x^2 + 9)/\text{abs}(x) + 18/\text{abs}(x))$

**Fricas [A]** time = 0.234414, size = 58, normalized size = 2.9

$$-\frac{1}{6}i \log\left(-\frac{2(i\sqrt{-4x^2-9}+3)}{3x}\right) + \frac{1}{6}i \log\left(-\frac{2(-i\sqrt{-4x^2-9}+3)}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-4*x^2 - 9)*x), x, algorithm="fricas")`

[Out]  $-1/6 \cdot I \cdot \log(-2/3 \cdot (I \cdot \text{sqrt}(-4 \cdot x^2 - 9) + 3)/x) + 1/6 \cdot I \cdot \log(-2/3 \cdot (-I \cdot \text{sqrt}(-4 \cdot x^2 - 9) + 3)/x)$

**Sympy [A]** time = 3.62174, size = 8, normalized size = 0.4

$$\frac{i \operatorname{asinh}\left(\frac{3}{2x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-4*x**2-9)**(1/2), x)`

[Out]  $I \cdot \operatorname{asinh}(3/(2 \cdot x))/3$

**GIAC/XCAS [A]** time = 0.217058, size = 20, normalized size = 1.

$$\frac{1}{3} \arctan\left(\frac{1}{3} \sqrt{4x^2+9}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-4*x^2 - 9)*x), x, algorithm="giac")`

[Out]  $1/3 \cdot \arctan(1/3 \cdot \text{sqrt}(4 \cdot x^2 + 9) \cdot i)$

$$3.572 \quad \int \frac{1}{x^2 \sqrt{-9-4x^2}} dx$$

**Optimal.** Leaf size=18

$$\frac{\sqrt{-4x^2 - 9}}{9x}$$

[Out] Sqrt[-9 - 4\*x^2]/(9\*x)

**Rubi [A]** time = 0.0173373, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{\sqrt{-4x^2 - 9}}{9x}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*Sqrt[-9 - 4\*x^2]), x]

[Out] Sqrt[-9 - 4\*x^2]/(9\*x)

**Rubi in Sympy [A]** time = 3.06647, size = 14, normalized size = 0.78

$$\frac{\sqrt{-4x^2 - 9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(-4\*x\*\*2-9)\*\*(1/2), x)

[Out] sqrt(-4\*x\*\*2 - 9)/(9\*x)

**Mathematica [A]** time = 0.00968013, size = 18, normalized size = 1.

$$\frac{\sqrt{-4x^2 - 9}}{9x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*Sqrt[-9 - 4\*x^2]), x]

[Out] Sqrt[-9 - 4\*x^2]/(9\*x)

**Maple [A]** time = 0.004, size = 15, normalized size = 0.8

$$\frac{1}{9x} \sqrt{-4x^2 - 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(-4\*x^2-9)^(1/2), x)

[Out] 1/9/x\*(-4\*x^2-9)^(1/2)

---

**Maxima [A]** time = 1.49007, size = 19, normalized size = 1.06

$$\frac{\sqrt{-4x^2 - 9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-4\*x^2 - 9)\*x^2),x, algorithm="maxima")

[Out] 1/9\*sqrt(-4\*x^2 - 9)/x

---

**Fricas [A]** time = 0.224122, size = 19, normalized size = 1.06

$$\frac{\sqrt{-4x^2 - 9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-4\*x^2 - 9)\*x^2),x, algorithm="fricas")

[Out] 1/9\*sqrt(-4\*x^2 - 9)/x

---

**Sympy [A]** time = 2.57854, size = 15, normalized size = 0.83

$$\frac{2i\sqrt{1 + \frac{9}{4x^2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(-4\*x\*\*2-9)\*\*(1/2),x)

[Out] 2\*I\*sqrt(1 + 9/(4\*x\*\*2))/9

---

**GIAC/XCAS [A]** time = 0.229299, size = 55, normalized size = 3.06

$$\frac{\sqrt{4x^2 + 9}i + 3i}{18x} - \frac{2x}{9(\sqrt{4x^2 + 9}i + 3i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-4\*x^2 - 9)\*x^2),x, algorithm="giac")

[Out] 1/18\*(sqrt(4\*x^2 + 9)\*i + 3\*i)/x - 2/9\*x/(sqrt(4\*x^2 + 9)\*i + 3\*i)

$$3.573 \quad \int \frac{1}{x^3 \sqrt{-9-4x^2}} dx$$

**Optimal.** Leaf size=39

$$\frac{\sqrt{-4x^2-9}}{18x^2} - \frac{2}{27} \tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

[Out] Sqrt[-9 - 4\*x^2]/(18\*x^2) - (2\*ArcTan[Sqrt[-9 - 4\*x^2]/3])/27

**Rubi [A]** time = 0.0525262, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{\sqrt{-4x^2-9}}{18x^2} - \frac{2}{27} \tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*Sqrt[-9 - 4\*x^2]), x]

[Out] Sqrt[-9 - 4\*x^2]/(18\*x^2) - (2\*ArcTan[Sqrt[-9 - 4\*x^2]/3])/27

**Rubi in Sympy [A]** time = 5.41894, size = 34, normalized size = 0.87

$$-\frac{2 \operatorname{atan}\left(\frac{\sqrt{-4x^2-9}}{3}\right)}{27} + \frac{\sqrt{-4x^2-9}}{18x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(-4\*x\*\*2-9)\*\*(1/2), x)

[Out] -2\*atan(sqrt(-4\*x\*\*2 - 9)/3)/27 + sqrt(-4\*x\*\*2 - 9)/(18\*x\*\*2)

**Mathematica [A]** time = 0.0303392, size = 37, normalized size = 0.95

$$\frac{1}{54} \left( \frac{3\sqrt{-4x^2-9}}{x^2} + 4 \tan^{-1}\left(\frac{3}{\sqrt{-4x^2-9}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*Sqrt[-9 - 4\*x^2]), x]

[Out] ((3\*Sqrt[-9 - 4\*x^2])/x^2 + 4\*ArcTan[3/Sqrt[-9 - 4\*x^2]])/54

**Maple [A]** time = 0.007, size = 30, normalized size = 0.8

$$\frac{1}{18x^2} \sqrt{-4x^2-9} + \frac{2}{27} \arctan\left(3 \frac{1}{\sqrt{-4x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(-4\*x^2-9)^(1/2), x)

[Out]  $1/18 * (-4 * x^2 - 9)^{(1/2)} / x^2 + 2/27 * \arctan(3 / (-4 * x^2 - 9)^{(1/2)})$

**Maxima [A]** time = 1.49288, size = 54, normalized size = 1.38

$$\frac{\sqrt{-4x^2 - 9}}{18x^2} + \frac{2}{27}i \log\left(\frac{6\sqrt{4x^2 + 9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-4*x^2 - 9)*x^3),x, algorithm="maxima")`

[Out]  $1/18 * \sqrt{-4 * x^2 - 9} / x^2 + 2/27 * I * \log(6 * \sqrt{4 * x^2 + 9} / \text{abs}(x) + 18 / \text{abs}(x))$

**Fricas [A]** time = 0.241184, size = 88, normalized size = 2.26

$$\frac{-2ix^2 \log\left(-\frac{4(i\sqrt{-4x^2-9}-3)}{27x}\right) + 2ix^2 \log\left(-\frac{4(-i\sqrt{-4x^2-9}-3)}{27x}\right) + 3\sqrt{-4x^2-9}}{54x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-4*x^2 - 9)*x^3),x, algorithm="fricas")`

[Out]  $1/54 * (-2 * I * x^2 * \log(-4/27 * (I * \sqrt{-4 * x^2 - 9} - 3) / x) + 2 * I * x^2 * \log(-4/27 * (-I * \sqrt{-4 * x^2 - 9} - 3) / x) + 3 * \sqrt{-4 * x^2 - 9}) / x^2$

**Sympy [A]** time = 8.03448, size = 46, normalized size = 1.18

$$-\frac{2i \operatorname{asinh}\left(\frac{3}{2x}\right)}{27} + \frac{i}{9x\sqrt{1 + \frac{9}{4x^2}}} + \frac{i}{4x^3\sqrt{1 + \frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(-4*x**2-9)**(1/2),x)`

[Out]  $-2 * I * \operatorname{asinh}(3 / (2 * x)) / 27 + I / (9 * x * \sqrt{1 + 9 / (4 * x^2)}) + I / (4 * x^3 * \sqrt{1 + 9 / (4 * x^2)})$

**GIAC/XCAS [A]** time = 0.221213, size = 42, normalized size = 1.08

$$\frac{\sqrt{4x^2 + 9}i}{18x^2} - \frac{2}{27} \arctan\left(\frac{1}{3} \sqrt{4x^2 + 9}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-4*x^2 - 9)*x^3),x, algorithm="giac")`

[Out]  $1/18 * \sqrt{4 * x^2 + 9} * i / x^2 - 2/27 * \arctan(1/3 * \sqrt{4 * x^2 + 9} * i)$

$$3.574 \quad \int \frac{1}{x^4 \sqrt{-9-4x^2}} dx$$

**Optimal.** Leaf size=37

$$\frac{\sqrt{-4x^2-9}}{27x^3} - \frac{8\sqrt{-4x^2-9}}{243x}$$

[Out] Sqrt[-9 - 4\*x^2]/(27\*x^3) - (8\*Sqrt[-9 - 4\*x^2])/(243\*x)

**Rubi [A]** time = 0.0327919, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{\sqrt{-4x^2-9}}{27x^3} - \frac{8\sqrt{-4x^2-9}}{243x}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*Sqrt[-9 - 4\*x^2]),x]

[Out] Sqrt[-9 - 4\*x^2]/(27\*x^3) - (8\*Sqrt[-9 - 4\*x^2])/(243\*x)

**Rubi in Sympy [A]** time = 4.53566, size = 32, normalized size = 0.86

$$-\frac{8\sqrt{-4x^2-9}}{243x} + \frac{\sqrt{-4x^2-9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(-4\*x\*\*2-9)\*\*(1/2),x)

[Out] -8\*sqrt(-4\*x\*\*2 - 9)/(243\*x) + sqrt(-4\*x\*\*2 - 9)/(27\*x\*\*3)

**Mathematica [A]** time = 0.0149915, size = 27, normalized size = 0.73

$$\left(\frac{1}{27x^3} - \frac{8}{243x}\right)\sqrt{-4x^2-9}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*Sqrt[-9 - 4\*x^2]),x]

[Out] (1/(27\*x^3) - 8/(243\*x))\*Sqrt[-9 - 4\*x^2]

**Maple [A]** time = 0.005, size = 22, normalized size = 0.6

$$-\frac{8x^2-9}{243x^3}\sqrt{-4x^2-9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(-4\*x^2-9)^(1/2),x)

[Out] -1/243\*(8\*x^2-9)/x^3\*(-4\*x^2-9)^(1/2)

---

**Maxima [A]** time = 1.49112, size = 39, normalized size = 1.05

$$-\frac{8\sqrt{-4x^2-9}}{243x} + \frac{\sqrt{-4x^2-9}}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-4\*x^2 - 9)\*x^4),x, algorithm="maxima")

[Out] -8/243\*sqrt(-4\*x^2 - 9)/x + 1/27\*sqrt(-4\*x^2 - 9)/x^3

---

**Fricas [A]** time = 0.226117, size = 28, normalized size = 0.76

$$-\frac{(8x^2-9)\sqrt{-4x^2-9}}{243x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-4\*x^2 - 9)\*x^4),x, algorithm="fricas")

[Out] -1/243\*(8\*x^2 - 9)\*sqrt(-4\*x^2 - 9)/x^3

---

**Sympy [A]** time = 5.8704, size = 36, normalized size = 0.97

$$-\frac{16i\sqrt{1+\frac{9}{4x^2}}}{243} + \frac{2i\sqrt{1+\frac{9}{4x^2}}}{27x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(-4\*x\*\*2-9)\*\*(1/2),x)

[Out] -16\*I\*sqrt(1 + 9/(4\*x\*\*2))/243 + 2\*I\*sqrt(1 + 9/(4\*x\*\*2))/(27\*x\*\*2)

---

**GIAC/XCAS [A]** time = 0.229191, size = 120, normalized size = 3.24

$$\frac{2x^3\left(\frac{9(\sqrt{4x^2+9i+3i})^2}{x^2}+4\right)}{243(\sqrt{4x^2+9i+3i})^3} - \frac{(\sqrt{4x^2+9i+3i})^3}{1944x^3} - \frac{\sqrt{4x^2+9i+3i}}{54x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-4\*x^2 - 9)\*x^4),x, algorithm="giac")

[Out] 2/243\*x^3\*(9\*(sqrt(4\*x^2 + 9)\*i + 3\*i)^2/x^2 + 4)/(sqrt(4\*x^2 + 9)\*i + 3\*i)^3 - 1/1944\*(sqrt(4\*x^2 + 9)\*i + 3\*i)^3/x^3 - 1/54\*(sqrt(4\*x^2 + 9)\*i + 3\*i)/x



$$3.575 \quad \int \frac{1}{x^5 \sqrt{-9-4x^2}} dx$$

**Optimal.** Leaf size=57

$$-\frac{\sqrt{-4x^2-9}}{54x^2} + \frac{2}{81} \tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right) + \frac{\sqrt{-4x^2-9}}{36x^4}$$

[Out] Sqrt[-9 - 4\*x^2]/(36\*x^4) - Sqrt[-9 - 4\*x^2]/(54\*x^2) + (2\*ArcTan[Sqrt[-9 - 4\*x^2]/3])/81

**Rubi [A]** time = 0.0711409, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{\sqrt{-4x^2-9}}{54x^2} + \frac{2}{81} \tan^{-1}\left(\frac{1}{3}\sqrt{-4x^2-9}\right) + \frac{\sqrt{-4x^2-9}}{36x^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5\*Sqrt[-9 - 4\*x^2]), x]

[Out] Sqrt[-9 - 4\*x^2]/(36\*x^4) - Sqrt[-9 - 4\*x^2]/(54\*x^2) + (2\*ArcTan[Sqrt[-9 - 4\*x^2]/3])/81

**Rubi in Sympy [A]** time = 6.44451, size = 51, normalized size = 0.89

$$\frac{2 \operatorname{atan}\left(\frac{\sqrt{-4x^2-9}}{3}\right)}{81} - \frac{\sqrt{-4x^2-9}}{54x^2} + \frac{\sqrt{-4x^2-9}}{36x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*5/(-4\*x\*\*2-9)\*\*(1/2), x)

[Out] 2\*atan(sqrt(-4\*x\*\*2 - 9)/3)/81 - sqrt(-4\*x\*\*2 - 9)/(54\*x\*\*2) + sqrt(-4\*x\*\*2 - 9)/(36\*x\*\*4)

**Mathematica [A]** time = 0.0384991, size = 44, normalized size = 0.77

$$\frac{1}{324} \left( \frac{3\sqrt{-4x^2-9}(3-2x^2)}{x^4} - 8 \tan^{-1}\left(\frac{3}{\sqrt{-4x^2-9}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5\*Sqrt[-9 - 4\*x^2]), x]

[Out] ((3\*Sqrt[-9 - 4\*x^2]\*(3 - 2\*x^2))/x^4 - 8\*ArcTan[3/Sqrt[-9 - 4\*x^2]])/324

**Maple [A]** time = 0.007, size = 44, normalized size = 0.8

$$\frac{1}{36x^4} \sqrt{-4x^2-9} - \frac{1}{54x^2} \sqrt{-4x^2-9} - \frac{2}{81} \arctan\left(3 \frac{1}{\sqrt{-4x^2-9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^5/(-4*x^2-9)^(1/2),x)`

[Out]  $\frac{1}{36}(-4x^2-9)^{1/2}/x^4 - \frac{1}{54}(-4x^2-9)^{1/2}/x^2 - \frac{2}{81}\arctan\left(\frac{3}{(-4x^2-9)^{1/2}}\right)$

**Maxima [A]** time = 1.50385, size = 73, normalized size = 1.28

$$-\frac{\sqrt{-4x^2-9}}{54x^2} + \frac{\sqrt{-4x^2-9}}{36x^4} - \frac{2}{81}i \log\left(\frac{6\sqrt{4x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-4*x^2 - 9)*x^5),x, algorithm="maxima")`

[Out]  $-\frac{1}{54}\sqrt{-4x^2-9}/x^2 + \frac{1}{36}\sqrt{-4x^2-9}/x^4 - \frac{2}{81}I \log\left(\frac{6\sqrt{4x^2+9}}{\text{abs}(x)} + \frac{18}{\text{abs}(x)}\right)$

**Fricas [A]** time = 0.255535, size = 97, normalized size = 1.7

$$\frac{-4ix^4 \log\left(-\frac{4(i\sqrt{-4x^2-9}+3)}{81x}\right) + 4ix^4 \log\left(-\frac{4(-i\sqrt{-4x^2-9}+3)}{81x}\right) - 3(2x^2-3)\sqrt{-4x^2-9}}{324x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-4*x^2 - 9)*x^5),x, algorithm="fricas")`

[Out]  $\frac{1}{324}(-4I^2x^4 \log(-4/81(I\sqrt{-4x^2-9}+3)/x) + 4I^2x^4 \log(-4/81(-I\sqrt{-4x^2-9}+3)/x) - 3(2x^2-3)\sqrt{-4x^2-9})/x^4$

**Sympy [A]** time = 17.0118, size = 65, normalized size = 1.14

$$\frac{2i \operatorname{asinh}\left(\frac{3}{2x}\right)}{81} - \frac{i}{27x\sqrt{1+\frac{9}{4x^2}}} - \frac{i}{36x^3\sqrt{1+\frac{9}{4x^2}}} + \frac{i}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(-4*x**2-9)**(1/2),x)`

[Out]  $2I \operatorname{asinh}(3/(2x))/81 - I/(27x\sqrt{1+9/(4x^2)}) - I/(36x^3\sqrt{1+9/(4x^2)}) + I/(8x^5\sqrt{1+9/(4x^2)})$

**GIAC/XCAS [A]** time = 0.218736, size = 61, normalized size = 1.07

$$-\frac{(4x^2+9)^{\frac{3}{2}}i - 15\sqrt{4x^2+9}i}{216x^4} + \frac{2}{81} \arctan\left(\frac{1}{3}\sqrt{4x^2+9}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-4*x^2 - 9)*x^5),x, algorithm="giac")`

[Out]  $-\frac{1}{216}((4x^2+9)^{3/2}i - 15\sqrt{4x^2+9}i)/x^4 + \frac{2}{81}\arctan(1/3\sqrt{4x^2+9}i)$

$$3.576 \quad \int \frac{1}{\sqrt{9+bx^2}} dx$$

**Optimal.** Leaf size=17

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

[Out] ArcSinh[(Sqrt[b]\*x)/3]/Sqrt[b]

**Rubi [A]** time = 0.0111053, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[9 + b\*x^2], x]

[Out] ArcSinh[(Sqrt[b]\*x)/3]/Sqrt[b]

**Rubi in Sympy [A]** time = 1.86675, size = 14, normalized size = 0.82

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+9)\*\*(1/2), x)

[Out] asinh(sqrt(b)\*x/3)/sqrt(b)

**Mathematica [A]** time = 0.0124646, size = 17, normalized size = 1.

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[9 + b\*x^2], x]

[Out] ArcSinh[(Sqrt[b]\*x)/3]/Sqrt[b]

**Maple [A]** time = 0.004, size = 21, normalized size = 1.2

$$\frac{1}{\sqrt{b}} \ln\left(x\sqrt{b} + \sqrt{bx^2 + 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+9)^(1/2), x)

[Out]  $\ln(x \cdot b^{1/2} + (b \cdot x^2 + 9)^{1/2}) / b^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(b*x^2 + 9), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.24976, size = 1, normalized size = 0.06

$$\left[ \frac{\log\left(\frac{3bx - \sqrt{bx^2+9}(bx-3\sqrt{b}) - (bx^2+9)\sqrt{b}}{\sqrt{bx^2+9}-3}\right)}{\sqrt{b}}, \frac{2 \arctan\left(\frac{\sqrt{bx^2+9}\sqrt{-b}-3\sqrt{-b}}{bx}\right)}{\sqrt{-b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(b*x^2 + 9), x, algorithm="fricas")`

[Out]  $[\log((3 \cdot b \cdot x - \sqrt{b \cdot x^2 + 9}) \cdot (b \cdot x - 3 \cdot \sqrt{b}) - (b \cdot x^2 + 9) \cdot \sqrt{b}) / (\sqrt{b \cdot x^2 + 9} - 3) / \sqrt{b}, 2 \cdot \arctan((\sqrt{b \cdot x^2 + 9}) \cdot \sqrt{-b} - 3 \cdot \sqrt{-b}) / (b \cdot x) / \sqrt{-b}]$

**Sympy [A]** time = 3.35143, size = 14, normalized size = 0.82

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**2+9)**(1/2), x)`

[Out] `asinh(sqrt(b)*x/3)/sqrt(b)`

**GIAC/XCAS [A]** time = 0.222086, size = 30, normalized size = 1.76

$$-\frac{\ln\left(-\sqrt{b}x + \sqrt{bx^2 + 9}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(b*x^2 + 9), x, algorithm="giac")`

[Out] `-ln(-sqrt(b)*x + sqrt(b*x^2 + 9))/sqrt(b)`

$$3.577 \quad \int \frac{1}{\sqrt{9-bx^2}} dx$$

**Optimal.** Leaf size=17

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

[Out] ArcSin[(Sqrt[b]\*x)/3]/Sqrt[b]

**Rubi [A]** time = 0.0113191, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[9 - b\*x^2], x]

[Out] ArcSin[(Sqrt[b]\*x)/3]/Sqrt[b]

**Rubi in Sympy [A]** time = 1.97757, size = 14, normalized size = 0.82

$$\frac{\text{asin}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-b\*x\*\*2+9)\*\*(1/2), x)

[Out] asin(sqrt(b)\*x/3)/sqrt(b)

**Mathematica [A]** time = 0.0127152, size = 17, normalized size = 1.

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[9 - b\*x^2], x]

[Out] ArcSin[(Sqrt[b]\*x)/3]/Sqrt[b]

**Maple [A]** time = 0.004, size = 21, normalized size = 1.2

$$1 \arctan\left(x\sqrt{b}\frac{1}{\sqrt{-bx^2+9}}\right) \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^2+9)^(1/2), x)

[Out]  $1/b^{(1/2)} * \arctan(b^{(1/2)} * x / (-b * x^2 + 9)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-b*x^2 + 9), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.244151, size = 1, normalized size = 0.06

$$\left[ \frac{\log\left(\frac{\sqrt{-bx^2+9}bx-3bx+(bx^2+3\sqrt{-bx^2+9})\sqrt{-b}}{\sqrt{-bx^2+9}-3}\right)}{\sqrt{-b}}, -\frac{2\arctan\left(\frac{\sqrt{-bx^2+9}-3}{\sqrt{bx}}\right)}{\sqrt{b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-b*x^2 + 9), x, algorithm="fricas")`

[Out]  $[\log((\sqrt{-bx^2 + 9}) * b * x - 3 * b * x + (b * x^2 + 3 * \sqrt{-bx^2 + 9}) * \sqrt{-b}) / (\sqrt{-bx^2 + 9} - 3) / \sqrt{-b}, -2 * \arctan((\sqrt{-bx^2 + 9} - 3) / (\sqrt{b} * x)) / \sqrt{b}]$

**Sympy [A]** time = 3.52015, size = 39, normalized size = 2.29

$$\begin{cases} -\frac{i \operatorname{acosh}\left(\frac{\sqrt{bx}}{3}\right)}{\sqrt{b}} & \text{for } \frac{|bx^2|}{9} > 1 \\ \frac{\operatorname{asin}\left(\frac{\sqrt{bx}}{3}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**2+9)**(1/2), x)`

[Out] `Piecewise((-I*acosh(sqrt(b)*x/3)/sqrt(b), Abs(b*x**2)/9 > 1), (asin(sqrt(b)*x/3)/sqrt(b), True))`

**GIAC/XCAS [A]** time = 0.217633, size = 36, normalized size = 2.12

$$\frac{\ln\left(-\sqrt{-bx} + \sqrt{-bx^2 + 9}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-b*x^2 + 9), x, algorithm="giac")`

[Out]  $-\ln(-\sqrt{-b} * x + \sqrt{-b * x^2 + 9}) / \sqrt{-b}$

$$3.578 \quad \int \frac{1}{\sqrt{-9+bx^2}} dx$$

**Optimal.** Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-9}}\right)}{\sqrt{b}}$$

[Out] ArcTanh[(Sqrt[b]\*x)/Sqrt[-9 + b\*x^2]]/Sqrt[b]

**Rubi [A]** time = 0.0182614, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-9}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-9 + b\*x^2], x]

[Out] ArcTanh[(Sqrt[b]\*x)/Sqrt[-9 + b\*x^2]]/Sqrt[b]

**Rubi in Sympy [A]** time = 2.33443, size = 22, normalized size = 0.88

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-9}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2-9)\*\*(1/2), x)

[Out] atanh(sqrt(b)\*x/sqrt(b\*x\*\*2 - 9))/sqrt(b)

**Mathematica [A]** time = 0.012144, size = 25, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-9}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-9 + b\*x^2], x]

[Out] ArcTanh[(Sqrt[b]\*x)/Sqrt[-9 + b\*x^2]]/Sqrt[b]

**Maple [A]** time = 0.004, size = 21, normalized size = 0.8

$$1 \ln\left(x\sqrt{b} + \sqrt{bx^2 - 9}\right) \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2-9)^(1/2), x)

[Out]  $\ln(x \cdot b^{1/2} + (b \cdot x^2 - 9)^{1/2}) / b^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(b*x^2 - 9), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.253168, size = 1, normalized size = 0.04

$$\left[ \frac{\log\left(2\sqrt{bx^2 - 9}bx + (2bx^2 - 9)\sqrt{b}\right)}{2\sqrt{b}}, \frac{\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2 - 9}}\right)}{\sqrt{-b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(b*x^2 - 9), x, algorithm="fricas")`

[Out]  $[1/2 \cdot \log(2 \cdot \sqrt{bx^2 - 9} \cdot bx + (2 \cdot bx^2 - 9) \cdot \sqrt{b}) / \sqrt{b}, \arctan(\sqrt{-b} \cdot x / \sqrt{bx^2 - 9}) / \sqrt{-b}]$

**Sympy [A]** time = 3.56038, size = 39, normalized size = 1.56

$$\begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{bx}}{3}\right)}{\sqrt{b}} & \text{for } \frac{|bx^2|}{9} > 1 \\ -\frac{i \operatorname{asin}\left(\frac{\sqrt{bx}}{3}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**2-9)**(1/2), x)`

[Out] `Piecewise((acosh(sqrt(b)*x/3)/sqrt(b), Abs(b*x**2)/9 > 1), (-I*asin(sqrt(b)*x/3)/sqrt(b), True))`

**GIAC/XCAS [A]** time = 0.217286, size = 31, normalized size = 1.24

$$-\frac{\ln\left(-\sqrt{bx} + \sqrt{bx^2 - 9}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(b*x^2 - 9), x, algorithm="giac")`

[Out]  $-\ln(\operatorname{abs}(-\sqrt{b} \cdot x + \sqrt{bx^2 - 9})) / \sqrt{b}$



$$3.579 \quad \int \frac{1}{\sqrt{-9-bx^2}} dx$$

**Optimal.** Leaf size=26

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2-9}}\right)}{\sqrt{b}}$$

[Out] ArcTan[(Sqrt[b]\*x)/Sqrt[-9 - b\*x^2]]/Sqrt[b]

**Rubi [A]** time = 0.0172256, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2-9}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-9 - b\*x^2], x]

[Out] ArcTan[(Sqrt[b]\*x)/Sqrt[-9 - b\*x^2]]/Sqrt[b]

**Rubi in Sympy [A]** time = 2.07762, size = 24, normalized size = 0.92

$$\frac{\text{atan}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2-9}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-b\*x\*\*2-9)\*\*(1/2), x)

[Out] atan(sqrt(b)\*x/sqrt(-b\*x\*\*2 - 9))/sqrt(b)

**Mathematica [A]** time = 0.0116653, size = 26, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2-9}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-9 - b\*x^2], x]

[Out] ArcTan[(Sqrt[b]\*x)/Sqrt[-9 - b\*x^2]]/Sqrt[b]

**Maple [A]** time = 0.004, size = 21, normalized size = 0.8

$$1 \arctan\left(x\sqrt{b}\frac{1}{\sqrt{-bx^2-9}}\right) \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^2-9)^(1/2), x)

[Out]  $\arctan(x \cdot b^{1/2} / (-b \cdot x^2 - 9)^{1/2}) / b^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-b*x^2 - 9), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.252234, size = 1, normalized size = 0.04

$$\left[ \frac{\log\left(-2\sqrt{-bx^2 - 9}bx - (2bx^2 + 9)\sqrt{-b}\right)}{2\sqrt{-b}}, \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{-bx^2 - 9}}\right)}{\sqrt{b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-b*x^2 - 9), x, algorithm="fricas")`

[Out]  $[1/2 \cdot \log(-2 \cdot \sqrt{-bx^2 - 9} \cdot bx - (2 \cdot bx^2 + 9) \cdot \sqrt{-b}) / \sqrt{-b}, \arctan(\sqrt{b} \cdot x / \sqrt{-bx^2 - 9}) / \sqrt{b}]$

**Sympy [A]** time = 3.36872, size = 17, normalized size = 0.65

$$-\frac{i \operatorname{asinh}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**2-9)**(1/2), x)`

[Out]  $-I \cdot \operatorname{asinh}(\sqrt{b} \cdot x / 3) / \sqrt{b}$

**GIAC/XCAS [A]** time = 0.216764, size = 38, normalized size = 1.46

$$-\frac{\ln\left(\left|-\sqrt{-b}x + \sqrt{-bx^2 - 9}\right|\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-b*x^2 - 9), x, algorithm="giac")`

[Out]  $-\ln(\operatorname{abs}(-\sqrt{-b} \cdot x + \sqrt{-bx^2 - 9})) / \sqrt{-b}$

$$3.580 \quad \int \frac{1}{\sqrt{\pi+bx^2}} dx$$

**Optimal.** Leaf size=19

$$\frac{\sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

[Out] ArcSinh[(Sqrt[b]\*x)/Sqrt[Pi]]/Sqrt[b]

**Rubi [A]** time = 0.0187497, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{\sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[Pi + b\*x^2], x]

[Out] ArcSinh[(Sqrt[b]\*x)/Sqrt[Pi]]/Sqrt[b]

**Rubi in Sympy [A]** time = 2.03723, size = 17, normalized size = 0.89

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+pi)\*\*(1/2), x)

[Out] asinh(sqrt(b)\*x/sqrt(pi))/sqrt(b)

**Mathematica [A]** time = 0.0138223, size = 19, normalized size = 1.

$$\frac{\sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[Pi + b\*x^2], x]

[Out] ArcSinh[(Sqrt[b]\*x)/Sqrt[Pi]]/Sqrt[b]

**Maple [A]** time = 0.007, size = 21, normalized size = 1.1

$$1 \ln\left(x\sqrt{b} + \sqrt{bx^2 + \pi}\right) \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+Pi)^(1/2), x)

[Out]  $\ln(x \cdot b^{1/2} + (b \cdot x^2 + \pi)^{1/2}) / b^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(pi + b*x^2), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.231859, size = 1, normalized size = 0.05

$$\left[ \frac{\log\left(-2\sqrt{\pi + bx^2}bx - (\pi + 2bx^2)\sqrt{b}\right)}{2\sqrt{b}}, \frac{\arctan\left(\frac{\sqrt{-b}x}{\sqrt{\pi + bx^2}}\right)}{\sqrt{-b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(pi + b*x^2), x, algorithm="fricas")`

[Out]  $[1/2 \cdot \log(-2 \cdot \sqrt{\pi + b \cdot x^2} \cdot b \cdot x - (\pi + 2 \cdot b \cdot x^2) \cdot \sqrt{b}) / \sqrt{b}, \arctan(\sqrt{-b} \cdot x / \sqrt{\pi + b \cdot x^2}) / \sqrt{-b}]$

**Sympy [A]** time = 3.42307, size = 17, normalized size = 0.89

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**2+pi)**(1/2), x)`

[Out]  $\operatorname{asinh}(\sqrt{b} \cdot x / \sqrt{\pi}) / \sqrt{b}$

**GIAC/XCAS [A]** time = 0.225098, size = 30, normalized size = 1.58

$$\frac{\ln\left(-\sqrt{b}x + \sqrt{\pi + bx^2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(pi + b*x^2), x, algorithm="giac")`

[Out]  $-\ln(-\sqrt{b} \cdot x + \sqrt{\pi + b \cdot x^2}) / \sqrt{b}$

$$3.581 \quad \int \frac{1}{\sqrt{\pi - bx^2}} dx$$

**Optimal.** Leaf size=19

$$\frac{\sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

[Out] ArcSin[(Sqrt[b]\*x)/Sqrt[Pi]]/Sqrt[b]

**Rubi [A]** time = 0.0138777, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{\sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[Pi - b\*x^2], x]

[Out] ArcSin[(Sqrt[b]\*x)/Sqrt[Pi]]/Sqrt[b]

**Rubi in Sympy [A]** time = 2.23522, size = 17, normalized size = 0.89

$$\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-b\*x\*\*2+pi)\*\*(1/2), x)

[Out] asin(sqrt(b)\*x/sqrt(pi))/sqrt(b)

**Mathematica [A]** time = 0.0132435, size = 19, normalized size = 1.

$$\frac{\sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[Pi - b\*x^2], x]

[Out] ArcSin[(Sqrt[b]\*x)/Sqrt[Pi]]/Sqrt[b]

**Maple [A]** time = 0.008, size = 21, normalized size = 1.1

$$1 \arctan\left(x\sqrt{b}\frac{1}{\sqrt{-bx^2 + \pi}}\right) \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^2+Pi)^(1/2), x)

[Out]  $1/b^{(1/2)} * \arctan(b^{(1/2)} * x / (-b * x^2 + \pi)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(pi - b*x^2), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.225467, size = 1, normalized size = 0.05

$$\left[ \frac{\log\left(2\sqrt{\pi - bx^2}bx - (\pi - 2bx^2)\sqrt{-b}\right)}{2\sqrt{-b}}, \frac{\arctan\left(\frac{\sqrt{bx}}{\sqrt{\pi - bx^2}}\right)}{\sqrt{b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(pi - b*x^2), x, algorithm="fricas")`

[Out]  $[1/2 * \log(2 * \sqrt{\pi - b * x^2} * b * x - (\pi - 2 * b * x^2) * \sqrt{-b}) / \sqrt{-b}, \arctan(\sqrt{b} * x / \sqrt{\pi - b * x^2}) / \sqrt{b}]$

**Sympy [A]** time = 3.60544, size = 46, normalized size = 2.42

$$\begin{cases} -\frac{i \operatorname{acosh}\left(\frac{\sqrt{bx}}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{for } \frac{|bx^2|}{\pi} > 1 \\ \frac{\operatorname{asin}\left(\frac{\sqrt{bx}}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**2+pi)**(1/2), x)`

[Out] `Piecewise((-I*acosh(sqrt(b)*x/sqrt(pi))/sqrt(b), Abs(b*x**2)/pi > 1), (asin(sqrt(b)*x/sqrt(pi))/sqrt(b), True))`

**GIAC/XCAS [A]** time = 0.223298, size = 38, normalized size = 2.

$$-\frac{\ln\left(\left|-\sqrt{-bx} + \sqrt{\pi - bx^2}\right|\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(pi - b*x^2), x, algorithm="giac")`

[Out]  $-\ln(\operatorname{abs}(-\sqrt{-b} * x + \sqrt{\pi - b * x^2})) / \sqrt{-b}$

$$3.582 \quad \int \frac{1}{\sqrt{-\pi+bx^2}} dx$$

**Optimal.** Leaf size=27

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-\pi}}\right)}{\sqrt{b}}$$

[Out] ArcTanh[(Sqrt[b]\*x)/Sqrt[-Pi + b\*x^2]]/Sqrt[b]

**Rubi [A]** time = 0.0193695, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-Pi + b\*x^2], x]

[Out] ArcTanh[(Sqrt[b]\*x)/Sqrt[-Pi + b\*x^2]]/Sqrt[b]

**Rubi in Sympy [A]** time = 2.64392, size = 22, normalized size = 0.81

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-\pi}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2-pi)\*\*(1/2), x)

[Out] atanh(sqrt(b)\*x/sqrt(b\*x\*\*2 - pi))/sqrt(b)

**Mathematica [A]** time = 0.011545, size = 27, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-Pi + b\*x^2], x]

[Out] ArcTanh[(Sqrt[b]\*x)/Sqrt[-Pi + b\*x^2]]/Sqrt[b]

**Maple [A]** time = 0.005, size = 23, normalized size = 0.9

$$1 \ln\left(x\sqrt{b} + \sqrt{bx^2 - \pi}\right) \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2-Pi)^(1/2), x)

[Out]  $\ln(x \cdot b^{1/2} + (b \cdot x^2 - \pi)^{1/2}) / b^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-pi + b*x^2), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.227114, size = 1, normalized size = 0.04

$$\left[ \frac{\log\left(2\sqrt{-\pi + bx^2}bx - (\pi - 2bx^2)\sqrt{b}\right)}{2\sqrt{b}}, \frac{\arctan\left(\frac{\sqrt{-bx}}{\sqrt{-\pi + bx^2}}\right)}{\sqrt{-b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-pi + b*x^2), x, algorithm="fricas")`

[Out]  $[1/2 \cdot \log(2 \cdot \sqrt{-\pi + b \cdot x^2} \cdot b \cdot x - (\pi - 2 \cdot b \cdot x^2) \cdot \sqrt{b}) / \sqrt{b}, \arctan(\sqrt{-b} \cdot x / \sqrt{-\pi + b \cdot x^2}) / \sqrt{-b}]$

**Sympy [A]** time = 3.65351, size = 46, normalized size = 1.7

$$\begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{bx}}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{for } \frac{|bx^2|}{\pi} > 1 \\ -\frac{i \operatorname{asin}\left(\frac{\sqrt{bx}}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**2-pi)**(1/2), x)`

[Out]  $\text{Piecewise}\left(\frac{\operatorname{acosh}(\sqrt{b} \cdot x / \sqrt{\pi})}{\sqrt{b}}, \operatorname{Abs}(b \cdot x^2) / \pi > 1\right), \left(-i \operatorname{asin}(\sqrt{b} \cdot x / \sqrt{\pi}) / \sqrt{b}, \text{True}\right)$

**GIAC/XCAS [A]** time = 0.228399, size = 34, normalized size = 1.26

$$\frac{\ln\left(\left|-\sqrt{bx} + \sqrt{-\pi + bx^2}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-pi + b*x^2), x, algorithm="giac")`

[Out]  $-\ln(\operatorname{abs}(-\sqrt{b} \cdot x + \sqrt{-\pi + b \cdot x^2})) / \sqrt{b}$



$$3.583 \quad \int \frac{1}{\sqrt{-\pi - bx^2}} dx$$

**Optimal.** Leaf size=28

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2-\pi}}\right)}{\sqrt{b}}$$

[Out] ArcTan[(Sqrt[b]\*x)/Sqrt[-Pi - b\*x^2]]/Sqrt[b]

**Rubi [A]** time = 0.0192495, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2-\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-Pi - b\*x^2], x]

[Out] ArcTan[(Sqrt[b]\*x)/Sqrt[-Pi - b\*x^2]]/Sqrt[b]

**Rubi in Sympy [A]** time = 2.34624, size = 24, normalized size = 0.86

$$\frac{\text{atan}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2-\pi}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-b\*x\*\*2-pi)\*\*(1/2), x)

[Out] atan(sqrt(b)\*x/sqrt(-b\*x\*\*2 - pi))/sqrt(b)

**Mathematica [A]** time = 0.0110877, size = 28, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2-\pi}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-Pi - b\*x^2], x]

[Out] ArcTan[(Sqrt[b]\*x)/Sqrt[-Pi - b\*x^2]]/Sqrt[b]

**Maple [A]** time = 0.006, size = 23, normalized size = 0.8

$$1 \arctan\left(x\sqrt{b}\frac{1}{\sqrt{-bx^2-\pi}}\right) \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^2-Pi)^(1/2), x)

[Out]  $\arctan(x \cdot b^{1/2} / (-b \cdot x^2 - \pi)^{1/2}) / b^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-pi - b*x^2), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.252196, size = 1, normalized size = 0.04

$$\left[ \frac{\log\left(-2\sqrt{-\pi - bx^2}bx - (\pi + 2bx^2)\sqrt{-b}\right)}{2\sqrt{-b}}, \frac{\arctan\left(\frac{\sqrt{bx}}{\sqrt{-\pi - bx^2}}\right)}{\sqrt{b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-pi - b*x^2), x, algorithm="fricas")`

[Out]  $[1/2 \cdot \log(-2 \cdot \sqrt{-\pi - b \cdot x^2} \cdot b \cdot x - (\pi + 2 \cdot b \cdot x^2) \cdot \sqrt{-b}) / \sqrt{-b}, \arctan(\sqrt{b} \cdot x / \sqrt{-\pi - b \cdot x^2}) / \sqrt{b}]$

**Sympy [A]** time = 3.47881, size = 20, normalized size = 0.71

$$-\frac{i \operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**2-pi)**(1/2), x)`

[Out]  $-I \cdot \operatorname{asinh}(\sqrt{b} \cdot x / \sqrt{\pi}) / \sqrt{b}$

**GIAC/XCAS [A]** time = 0.214004, size = 41, normalized size = 1.46

$$-\frac{\ln\left(\left|-\sqrt{-bx} + \sqrt{-\pi - bx^2}\right|\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-pi - b*x^2), x, algorithm="giac")`

[Out]  $-\ln(\operatorname{abs}(-\sqrt{-b} \cdot x + \sqrt{-\pi - b \cdot x^2})) / \sqrt{-b}$

$$3.584 \quad \int \frac{1}{\sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

[Out] ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]]/Sqrt[b]

**Rubi [A]** time = 0.0183529, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b\*x^2], x]

[Out] ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]]/Sqrt[b]

**Rubi in Sympy [A]** time = 2.41956, size = 22, normalized size = 0.88

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+a)\*\*(1/2), x)

[Out] atanh(sqrt(b)\*x/sqrt(a + b\*x\*\*2))/sqrt(b)

**Mathematica [A]** time = 0.0104122, size = 25, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b\*x^2], x]

[Out] ArcTanh[(Sqrt[b]\*x)/Sqrt[a + b\*x^2]]/Sqrt[b]

**Maple [A]** time = 0., size = 21, normalized size = 0.8

$$\frac{1}{\sqrt{b}} \ln\left(x\sqrt{b} + \sqrt{bx^2 + a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+a)^(1/2), x)

[Out]  $\ln(x \cdot b^{1/2} + (b \cdot x^2 + a)^{1/2}) / b^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(b*x^2 + a), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.230075, size = 1, normalized size = 0.04

$$\left[ \frac{\log\left(-2\sqrt{bx^2+ax} - (2bx^2+a)\sqrt{b}\right)}{2\sqrt{b}}, \frac{\arctan\left(\frac{\sqrt{-bx}}{\sqrt{bx^2+a}}\right)}{\sqrt{-b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(b*x^2 + a), x, algorithm="fricas")`

[Out]  $[1/2 \cdot \log(-2 \cdot \sqrt{bx^2 + a} \cdot b \cdot x - (2 \cdot b \cdot x^2 + a) \cdot \sqrt{b}) / \sqrt{b}, \arctan(\sqrt{-b} \cdot x / \sqrt{bx^2 + a}) / \sqrt{-b}]$

**Sympy [A]** time = 3.58701, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**2+a)**(1/2), x)`

[Out]  $\operatorname{asinh}(\sqrt{b} \cdot x / \sqrt{a}) / \sqrt{b}$

**GIAC/XCAS [A]** time = 0.216712, size = 31, normalized size = 1.24

$$-\frac{\ln\left(\left|-\sqrt{bx} + \sqrt{bx^2 + a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(b*x^2 + a), x, algorithm="giac")`

[Out]  $-\ln(\operatorname{abs}(-\sqrt{b} \cdot x + \sqrt{bx^2 + a})) / \sqrt{b}$

$$3.585 \quad \int \frac{1}{\sqrt{a-bx^2}} dx$$

**Optimal.** Leaf size=26

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}$$

[Out] ArcTan[(Sqrt[b]\*x)/Sqrt[a - b\*x^2]]/Sqrt[b]

**Rubi [A]** time = 0.0180275, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a - b\*x^2], x]

[Out] ArcTan[(Sqrt[b]\*x)/Sqrt[a - b\*x^2]]/Sqrt[b]

**Rubi in Sympy [A]** time = 2.32923, size = 22, normalized size = 0.85

$$\frac{\text{atan}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-b\*x\*\*2+a)\*\*(1/2), x)

[Out] atan(sqrt(b)\*x/sqrt(a - b\*x\*\*2))/sqrt(b)

**Mathematica [A]** time = 0.0104618, size = 26, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a - b\*x^2], x]

[Out] ArcTan[(Sqrt[b]\*x)/Sqrt[a - b\*x^2]]/Sqrt[b]

**Maple [A]** time = 0.004, size = 21, normalized size = 0.8

$$1 \arctan\left(x\sqrt{b}\frac{1}{\sqrt{-bx^2+a}}\right) \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^2+a)^(1/2), x)

[Out]  $\arctan(x \cdot b^{1/2} / (-b \cdot x^2 + a)^{1/2}) / b^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-b*x^2 + a), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.233812, size = 1, normalized size = 0.04

$$\left[ \frac{\log\left(2\sqrt{-bx^2+abx+(2bx^2-a)\sqrt{-b}}\right)}{2\sqrt{-b}}, \frac{\arctan\left(\frac{\sqrt{bx}}{\sqrt{-bx^2+a}}\right)}{\sqrt{b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-b*x^2 + a), x, algorithm="fricas")`

[Out]  $[1/2 \cdot \log(2 \cdot \sqrt{-bx^2 + a} \cdot bx + (2 \cdot bx^2 - a) \cdot \sqrt{-b}) / \sqrt{-b}, \arctan(\sqrt{b} \cdot x / \sqrt{-bx^2 + a}) / \sqrt{b}]$

**Sympy [A]** time = 3.68542, size = 46, normalized size = 1.77

$$\begin{cases} -\frac{i \operatorname{acosh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{\sqrt{b}} & \text{for } \left|\frac{bx^2}{a}\right| > 1 \\ \frac{\operatorname{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**2+a)**(1/2), x)`

[Out] `Piecewise((-I*acosh(sqrt(b)*x/sqrt(a))/sqrt(b), Abs(b*x**2/a) > 1), (asin(sqrt(b)*x/sqrt(a))/sqrt(b), True))`

**GIAC/XCAS [A]** time = 0.217663, size = 38, normalized size = 1.46

$$-\frac{\ln\left(\left|-\sqrt{-bx} + \sqrt{-bx^2 + a}\right|\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-b*x^2 + a), x, algorithm="giac")`

[Out]  $-\ln(\operatorname{abs}(-\sqrt{-b} \cdot x + \sqrt{-bx^2 + a})) / \sqrt{-b}$

$$3.586 \quad \int \frac{1}{\sqrt{-a+bx^2}} dx$$

**Optimal.** Leaf size=27

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-a}}\right)}{\sqrt{b}}$$

[Out] ArcTanh[(Sqrt[b]\*x)/Sqrt[-a + b\*x^2]]/Sqrt[b]

**Rubi [A]** time = 0.0192454, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-a}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-a + b\*x^2], x]

[Out] ArcTanh[(Sqrt[b]\*x)/Sqrt[-a + b\*x^2]]/Sqrt[b]

**Rubi in Sympy [A]** time = 2.62496, size = 22, normalized size = 0.81

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{-a+bx^2}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2-a)\*\*(1/2), x)

[Out] atanh(sqrt(b)\*x/sqrt(-a + b\*x\*\*2))/sqrt(b)

**Mathematica [A]** time = 0.0119709, size = 27, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-a}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-a + b\*x^2], x]

[Out] ArcTanh[(Sqrt[b]\*x)/Sqrt[-a + b\*x^2]]/Sqrt[b]

**Maple [A]** time = 0.003, size = 23, normalized size = 0.9

$$\frac{1}{\sqrt{b}} \ln\left(x\sqrt{b} + \sqrt{bx^2 - a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2-a)^(1/2), x)

[Out]  $\ln(x \cdot b^{1/2} + (b \cdot x^2 - a)^{1/2}) / b^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(b*x^2 - a), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.227841, size = 1, normalized size = 0.04

$$\left[ \frac{\log\left(2\sqrt{bx^2 - ax} + (2bx^2 - a)\sqrt{b}\right)}{2\sqrt{b}}, \frac{\arctan\left(\frac{\sqrt{-bx}}{\sqrt{bx^2 - a}}\right)}{\sqrt{-b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(b*x^2 - a), x, algorithm="fricas")`

[Out]  $[1/2 \cdot \log(2 \cdot \sqrt{bx^2 - a} \cdot b \cdot x + (2 \cdot b \cdot x^2 - a) \cdot \sqrt{b}) / \sqrt{b}, \arctan(\sqrt{-b} \cdot x / \sqrt{bx^2 - a}) / \sqrt{-b}]$

**Sympy [A]** time = 3.78788, size = 46, normalized size = 1.7

$$\begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{\sqrt{b}} & \text{for } \left|\frac{bx^2}{a}\right| > 1 \\ -\frac{i \operatorname{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**2-a)**(1/2), x)`

[Out] `Piecewise((acosh(sqrt(b)*x/sqrt(a))/sqrt(b), Abs(b*x**2/a) > 1), (-I*asin(sqrt(b)*x/sqrt(a))/sqrt(b), True))`

**GIAC/XCAS [A]** time = 0.220033, size = 34, normalized size = 1.26

$$-\frac{\ln\left(\left|-\sqrt{bx} + \sqrt{bx^2 - a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(b*x^2 - a), x, algorithm="giac")`

[Out]  $-\ln(\operatorname{abs}(-\sqrt{b} \cdot x + \sqrt{bx^2 - a})) / \sqrt{b}$



$$3.587 \quad \int \frac{1}{\sqrt{-a-bx^2}} dx$$

**Optimal.** Leaf size=28

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}$$

[Out] ArcTan[(Sqrt[b]\*x)/Sqrt[-a - b\*x^2]]/Sqrt[b]

**Rubi [A]** time = 0.0189193, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-a - b\*x^2], x]

[Out] ArcTan[(Sqrt[b]\*x)/Sqrt[-a - b\*x^2]]/Sqrt[b]

**Rubi in Sympy [A]** time = 2.27965, size = 24, normalized size = 0.86

$$\frac{\text{atan}\left(\frac{\sqrt{b}x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-b\*x\*\*2-a)\*\*(1/2), x)

[Out] atan(sqrt(b)\*x/sqrt(-a - b\*x\*\*2))/sqrt(b)

**Mathematica [A]** time = 0.0112304, size = 28, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-a - b\*x^2], x]

[Out] ArcTan[(Sqrt[b]\*x)/Sqrt[-a - b\*x^2]]/Sqrt[b]

**Maple [A]** time = 0.004, size = 23, normalized size = 0.8

$$1 \arctan\left(x\sqrt{b}\frac{1}{\sqrt{-bx^2 - a}}\right) \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^2-a)^(1/2), x)

[Out]  $\arctan(x \cdot b^{1/2} / (-b \cdot x^2 - a)^{1/2}) / b^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-b*x^2 - a), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.227189, size = 1, normalized size = 0.04

$$\left[ \frac{\log\left(-2\sqrt{-bx^2 - abx} - (2bx^2 + a)\sqrt{-b}\right)}{2\sqrt{-b}}, \frac{\arctan\left(\frac{\sqrt{bx}}{\sqrt{-bx^2 - a}}\right)}{\sqrt{b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-b*x^2 - a), x, algorithm="fricas")`

[Out]  $[1/2 \cdot \log(-2 \cdot \sqrt{-bx^2 - a} \cdot bx - (2 \cdot bx^2 + a) \cdot \sqrt{-b}) / \sqrt{-b}, \arctan(\sqrt{b} \cdot x / \sqrt{-bx^2 - a}) / \sqrt{b}]$

**Sympy [A]** time = 3.59286, size = 20, normalized size = 0.71

$$-\frac{i \operatorname{asinh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**2-a)**(1/2), x)`

[Out]  $-I \cdot \operatorname{asinh}(\sqrt{b} \cdot x / \sqrt{a}) / \sqrt{b}$

**GIAC/XCAS [A]** time = 0.215438, size = 41, normalized size = 1.46

$$-\frac{\ln\left(\left|-\sqrt{-bx} + \sqrt{-bx^2 - a}\right|\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(-b*x^2 - a), x, algorithm="giac")`

[Out]  $-\ln(\operatorname{abs}(-\sqrt{-b} \cdot x + \sqrt{-bx^2 - a})) / \sqrt{-b}$

$$3.588 \quad \int \frac{1}{\sqrt{a^2-x^2}} dx$$

**Optimal.** Leaf size=16

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

[Out] ArcTan[x/Sqrt[a^2 - x^2]]

**Rubi [A]** time = 0.0103035, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a^2 - x^2], x]

[Out] ArcTan[x/Sqrt[a^2 - x^2]]

**Rubi in Sympy [A]** time = 2.22693, size = 12, normalized size = 0.75

$$\text{atan}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a\*\*2-x\*\*2)\*\*(1/2), x)

[Out] atan(x/sqrt(a\*\*2 - x\*\*2))

**Mathematica [A]** time = 0.00619231, size = 16, normalized size = 1.

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a^2 - x^2], x]

[Out] ArcTan[x/Sqrt[a^2 - x^2]]

**Maple [A]** time = 0.005, size = 15, normalized size = 0.9

$$\arctan\left(x\frac{1}{\sqrt{a^2-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2-x^2)^(1/2), x)

[Out] arctan(x/(a^2-x^2)^(1/2))

---

**Maxima [A]** time = 1.50435, size = 11, normalized size = 0.69

$$\arcsin\left(\frac{x}{\sqrt{a^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(a^2 - x^2), x, algorithm="maxima")`

[Out] `arcsin(x/sqrt(a^2))`

---

**Fricas [A]** time = 0.221823, size = 31, normalized size = 1.94

$$-2 \arctan\left(-\frac{a - \sqrt{a^2 - x^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(a^2 - x^2), x, algorithm="fricas")`

[Out] `-2*arctan(-(a - sqrt(a^2 - x^2))/x)`

---

**Sympy [A]** time = 3.51868, size = 19, normalized size = 1.19

$$\begin{cases} -i \operatorname{acosh}\left(\frac{x}{a}\right) & \text{for } \left|\frac{x^2}{a^2}\right| > 1 \\ \operatorname{asin}\left(\frac{x}{a}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a**2-x**2)**(1/2), x)`

[Out] `Piecewise((-I*acosh(x/a), Abs(x**2/a**2) > 1), (asin(x/a), True))`

---

**GIAC/XCAS [A]** time = 0.222653, size = 12, normalized size = 0.75

$$\arcsin\left(\frac{x}{a}\right) \operatorname{sign}(a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/sqrt(a^2 - x^2), x, algorithm="giac")`

[Out] `arcsin(x/a)*sign(a)`

### 3.589 $\int (cx)^{7/2} \sqrt{a+bx^2} dx$

**Optimal.** Leaf size=184

$$\frac{10a^{11/4}c^{7/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}} - \frac{20a^2c^3\sqrt{cx}\sqrt{a+bx^2}}{231b^2} + \frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c} + \frac{4ac(cx)^{5/2}\sqrt{a+bx^2}}{77b}$$

[Out]  $(-20*a^2*c^3*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(231*b^2) + (4*a*c*(c*x)^(5/2)*\text{Sqrt}[a + b*x^2])/(77*b) + (2*(c*x)^(9/2)*\text{Sqrt}[a + b*x^2])/(11*c) + (10*a^(11/4)*c^(7/2)*( \text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^(1/4)*\text{Sqrt}[c*x])/(a^(1/4)*\text{Sqrt}[c])], 1/2])/(231*b^(9/4)*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.325662, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{10a^{11/4}c^{7/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}} - \frac{20a^2c^3\sqrt{cx}\sqrt{a+bx^2}}{231b^2} + \frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c} + \frac{4ac(cx)^{5/2}\sqrt{a+bx^2}}{77b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*x)^(7/2)*\text{Sqrt}[a + b*x^2], x]$

[Out]  $(-20*a^2*c^3*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(231*b^2) + (4*a*c*(c*x)^(5/2)*\text{Sqrt}[a + b*x^2])/(77*b) + (2*(c*x)^(9/2)*\text{Sqrt}[a + b*x^2])/(11*c) + (10*a^(11/4)*c^(7/2)*( \text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^(1/4)*\text{Sqrt}[c*x])/(a^(1/4)*\text{Sqrt}[c])], 1/2])/(231*b^(9/4)*\text{Sqrt}[a + b*x^2])$

**Rubi in Sympy [A]** time = 30.2627, size = 170, normalized size = 0.92

$$\frac{10a^{11/4}c^{7/2} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) F\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}} - \frac{20a^2c^3\sqrt{cx}\sqrt{a+bx^2}}{231b^2} + \frac{4ac(cx)^{5/2}\sqrt{a+bx^2}}{77b} + \frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c*x)**(7/2)*(b*x**2+a)**(1/2), x)$

[Out]  $10*a**(11/4)*c**(7/2)*\text{sqrt}((a + b*x**2)/(\text{sqrt}(a) + \text{sqrt}(b)*x)**2)*(\text{sqrt}(a) + \text{sqrt}(b)*x)*\text{elliptic\_f}(2*\text{atan}(b**(1/4)*\text{sqrt}(c*x)/(a**(1/4)*\text{sqrt}(c))), 1/2)/(231*b**(9/4)*\text{sqrt}(a + b*x**2)) - 20*a**2*c**3*\text{sqrt}(c*x)*\text{sqrt}(a + b*x**2)/(231*b**2) + 4*a*c*(c*x)**(5/2)*\text{sqrt}(a + b*x**2)/(77*b) + 2*(c*x)**(9/2)*\text{sqrt}(a + b*x**2)/(11*c)$

**Mathematica [C]** time = 0.260193, size = 155, normalized size = 0.84

$$\frac{2c^3\sqrt{cx}\left(10ia^3\sqrt{x}\sqrt{\frac{a}{bx^2}}+1F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\right)-1\right)+\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(-10a^3-4a^2bx^2+27ab^2x^4+21b^3x^6\right)}{231b^2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(7/2)\*Sqrt[a + b\*x^2],x]

[Out] (2\*c^3\*Sqrt[c\*x]\*(Sqrt[(I\*Sqrt[a])/Sqrt[b]]\*(-10\*a^3 - 4\*a^2\*b\*x^2 + 27\*a\*b^2\*x^4 + 21\*b^3\*x^6) + (10\*I)\*a^3\*Sqrt[1 + a/(b\*x^2)]\*Sqrt[x]\*EllipticF[I\*ArcSinh[Sqrt[(I\*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1]))/(231\*Sqrt[(I\*Sqrt[a])/Sqrt[b]]\*b^2\*Sqrt[a + b\*x^2])

**Maple [A]** time = 0.089, size = 152, normalized size = 0.8

$$\frac{2c^3}{231b^3x}\sqrt{cx}\left(21x^7b^4+5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{bx}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}\sqrt{2a^3+27x^5ab^3}-\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(7/2)\*(b\*x^2+a)^(1/2),x)

[Out] 2/231\*c^3/x\*(c\*x)^(1/2)/(b\*x^2+a)^(1/2)\*(21\*x^7\*b^4+5\*((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*((-b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*(-x\*b/(-a\*b)^(1/2))^(1/2)\*EllipticF(((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2),1/2\*2^(1/2))\*(-a\*b)^(1/2)\*2^(1/2)\*a^3+27\*x^5\*a\*b^3-3-4\*x^3\*a^2\*b^2-10\*x\*a^3\*b)/b^3

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + a}(cx)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b\*x^2 + a)\*(c\*x)^(7/2),x, algorithm="maxima")

[Out] integrate(sqrt(b\*x^2 + a)\*(c\*x)^(7/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^2 + a}\sqrt{cx}^3x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b\*x^2 + a)\*(c\*x)^(7/2),x, algorithm="fricas")

[Out] integral(sqrt(b\*x^2 + a)\*sqrt(c\*x)\*c^3\*x^3, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(7/2)*(b*x**2+a)**(1/2),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + a} (cx)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*(c*x)^(7/2),x, algorithm="giac")`

[Out] `integrate(sqrt(b*x^2 + a)*(c*x)^(7/2), x)`

### 3.590 $\int (cx)^{5/2} \sqrt{a + bx^2} dx$

**Optimal.** Leaf size=301

$$\frac{2a^{9/4}c^{5/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}} + \frac{4a^{9/4}c^{5/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}} - \frac{4a^2c^2\sqrt{cx}\sqrt{a+bx^2}}{15b^{3/2}(\sqrt{a} + \sqrt{bx})} + \frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c} + \frac{4ac(cx)^{3/2}\sqrt{a+bx^2}}{45b}$$

[Out]  $(4*a*c*(c*x)^{(3/2)}*\text{Sqrt}[a + b*x^2])/(45*b) + (2*(c*x)^{(7/2)}*\text{Sqrt}[a + b*x^2])/(9*c) - (4*a^2*c^2*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(15*b^{(3/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) + (4*a^{(9/4)}*c^{(5/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(15*b^{(7/4)}*\text{Sqrt}[a + b*x^2]) - (2*a^{(9/4)}*c^{(5/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(15*b^{(7/4)}*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.600756, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$\frac{2a^{9/4}c^{5/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}} + \frac{4a^{9/4}c^{5/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}} - \frac{4a^2c^2\sqrt{cx}\sqrt{a+bx^2}}{15b^{3/2}(\sqrt{a} + \sqrt{bx})} + \frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c} + \frac{4ac(cx)^{3/2}\sqrt{a+bx^2}}{45b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*x)^{(5/2)}*\text{Sqrt}[a + b*x^2], x]$

[Out]  $(4*a*c*(c*x)^{(3/2)}*\text{Sqrt}[a + b*x^2])/(45*b) + (2*(c*x)^{(7/2)}*\text{Sqrt}[a + b*x^2])/(9*c) - (4*a^2*c^2*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(15*b^{(3/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) + (4*a^{(9/4)}*c^{(5/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(15*b^{(7/4)}*\text{Sqrt}[a + b*x^2]) - (2*a^{(9/4)}*c^{(5/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(15*b^{(7/4)}*\text{Sqrt}[a + b*x^2])$



**Rubi in Sympy [A]** time = 57.923, size = 277, normalized size = 0.92

$$\frac{4a^{\frac{9}{4}}c^{\frac{5}{2}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{7}{4}}\sqrt{a+bx^2}} - \frac{2a^{\frac{9}{4}}c^{\frac{5}{2}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{7}{4}}\sqrt{a+bx^2}} - \frac{4a^2c^2\sqrt{cx}\sqrt{a+bx^2}}{15b^{\frac{3}{2}}(\sqrt{a}+\sqrt{bx})} + \frac{4ac(cx)^{\frac{3}{2}}\sqrt{a+bx^2}}{45b} + \frac{2(cx)^{\frac{7}{2}}\sqrt{a+bx^2}}{9c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x)**(5/2)*(b*x**2+a)**(1/2),x)`

[Out]  $4*a^{9/4}*c^{5/2}*\sqrt{(a+b*x^2)/(\sqrt{a}+\sqrt{b}*x)^2}*(\sqrt{a}+\sqrt{b}*x)*\operatorname{elliptic\_e}(2*\operatorname{atan}(b^{1/4}*\sqrt{c*x}/(a^{1/4}*\sqrt{c})),1/2)/(15*b^{7/4}*\sqrt{a+b*x^2}) - 2*a^{9/4}*c^{5/2}*\sqrt{(a+b*x^2)/(\sqrt{a}+\sqrt{b}*x)^2}*(\sqrt{a}+\sqrt{b}*x)*\operatorname{elliptic\_f}(2*\operatorname{atan}(b^{1/4}*\sqrt{c*x}/(a^{1/4}*\sqrt{c})),1/2)/(15*b^{7/4}*\sqrt{a+b*x^2}) - 4*a^{9/4}*c^{5/2}*\sqrt{c*x}*\sqrt{a+b*x^2}/(15*b^{3/2}*(\sqrt{a}+\sqrt{b}*x)) + 4*a*c*(c*x)^{3/2}/(45*b) + 2*(c*x)^{7/2}*\sqrt{a+b*x^2}/(9*c)$

**Mathematica [C]** time = 0.342254, size = 191, normalized size = 0.63

$$\frac{2c^2\sqrt{cx}\left(6a^{5/2}\sqrt{\frac{bx^2}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle|-1\right)-6a^{5/2}\sqrt{\frac{bx^2}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle|-1\right)+\sqrt{bx}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}(2a^2+7abx)\right)}{45b^{3/2}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c*x)^(5/2)*Sqrt[a+b*x^2],x]`

[Out]  $(2*c^2*\sqrt{c*x}*(\sqrt{b}*x*\sqrt{(I*\sqrt{b}*x)/\sqrt{a}})^2*a^2+7*a*b*x^2+5*b^2*x^4)-6*a^{5/2}*\sqrt{1+(b*x^2)/a}*\operatorname{EllipticE}[I*\operatorname{ArcSinh}[\sqrt{(I*\sqrt{b}*x)/\sqrt{a}}],-1]+6*a^{5/2}*\sqrt{1+(b*x^2)/a}*\operatorname{EllipticF}[I*\operatorname{ArcSinh}[\sqrt{(I*\sqrt{b}*x)/\sqrt{a}}],-1])/(45*b^{3/2}*\sqrt{(I*\sqrt{b}*x)/\sqrt{a}}*\sqrt{a+b*x^2})$

**Maple [A]** time = 0.032, size = 221, normalized size = 0.7

$$-\frac{2c^2}{45b^2x}\sqrt{cx}\left(-5b^3x^6+6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2,\sqrt{2}\right)\sqrt{2}a^3-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(5/2)*(b*x^2+a)^(1/2),x)`

[Out]  $-2/45*c^2/x*(c*x)^{1/2}/(b*x^2+a)^{1/2}/b^2*(-5*b^3*x^6+6*((b*x+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}*((-b*x+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}*(-x*b/(-a*b)^{1/2})^{1/2}*\operatorname{EllipticE}(((b*x+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2},1/2,2^{1/2})*2^{1/2}*a^3-3*((b*x+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}*((-b*x+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}*(-x*b/(-a*b)^{1/2})^{1/2}*\operatorname{EllipticF}(((b*x+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2},1/2,2^{1/2}))$

$)^{1/2}, 1/2 \cdot 2^{1/2}) \cdot 2^{1/2} \cdot a^3 - 7 \cdot a \cdot b^2 \cdot x^4 - 2 \cdot a^2 \cdot b \cdot x^2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + a} (cx)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b\*x^2 + a)\*(c\*x)^(5/2),x, algorithm="maxima")

[Out] integrate(sqrt(b\*x^2 + a)\*(c\*x)^(5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^2 + a}\sqrt{cx}c^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b\*x^2 + a)\*(c\*x)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b\*x^2 + a)\*sqrt(c\*x)\*c^2\*x^2, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(5/2)\*(b\*x\*\*2+a)\*\*(1/2),x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + a} (cx)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b\*x^2 + a)\*(c\*x)^(5/2),x, algorithm="giac")

[Out] integrate(sqrt(b\*x^2 + a)\*(c\*x)^(5/2), x)

### 3.591 $\int (cx)^{3/2} \sqrt{a + bx^2} dx$

**Optimal.** Leaf size=153

$$-\frac{2a^{7/4}c^{3/2}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{21b^{5/4}\sqrt{a+bx^2}} + \frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c} + \frac{4ac\sqrt{cx}\sqrt{a+bx^2}}{21b}$$

[Out]  $(4*a*c*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(21*b) + (2*(c*x)^{(5/2)}*\text{Sqrt}[a + b*x^2])/(7*c) - (2*a^{(7/4)}*c^{(3/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(21*b^{(5/4)}*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.249118, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$-\frac{2a^{7/4}c^{3/2}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{21b^{5/4}\sqrt{a+bx^2}} + \frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c} + \frac{4ac\sqrt{cx}\sqrt{a+bx^2}}{21b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*x)^{(3/2)}*\text{Sqrt}[a + b*x^2], x]$

[Out]  $(4*a*c*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(21*b) + (2*(c*x)^{(5/2)}*\text{Sqrt}[a + b*x^2])/(7*c) - (2*a^{(7/4)}*c^{(3/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(21*b^{(5/4)}*\text{Sqrt}[a + b*x^2])$

**Rubi in Sympy [A]** time = 22.866, size = 139, normalized size = 0.91

$$-\frac{2a^{7/4}c^{3/2}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a} + \sqrt{bx})F\left(2\text{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{21b^{5/4}\sqrt{a+bx^2}} + \frac{4ac\sqrt{cx}\sqrt{a+bx^2}}{21b} + \frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c*x)**(3/2)*(b*x**2+a)**(1/2), x)$

[Out]  $-2*a^{(7/4)}*c^{(3/2)}*\text{sqrt}((a + b*x**2)/(\text{sqrt}(a) + \text{sqrt}(b)*x)**2)*(\text{sqrt}(a) + \text{sqrt}(b)*x)*\text{elliptic\_f}(2*\text{atan}(b^{(1/4)}*\text{sqrt}(c*x)/(a^{(1/4)}*\text{sqrt}(c))), 1/2)/(21*b^{(5/4)}*\text{sqrt}(a + b*x**2)) + 4*a*c*\text{sqrt}(c*x)*\text{sqrt}(a + b*x**2)/(21*b) + 2*(c*x)**(5/2)*\text{sqrt}(a + b*x**2)/(7*c)$

**Mathematica [C]** time = 0.148747, size = 142, normalized size = 0.93

$$\frac{2c\sqrt{cx}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}(2a^2 + 5abx^2 + 3b^2x^4) - 2ia^2\sqrt{x}\sqrt{\frac{a}{bx^2}} + 1F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\right) - 1\right)}{21b\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(c*x)^{(3/2)}*\text{Sqrt}[a + b*x^2], x]$

[Out]  $(2*c*\text{Sqrt}[c*x]*(\text{Sqrt}[(I*\text{Sqrt}[a])/ \text{Sqrt}[b]])*(2*a^2 + 5*a*b*x^2 + 3*b^2*x^4) - (2*I)*a^2*\text{Sqrt}[1 + a/(b*x^2)]*\text{Sqrt}[x]*\text{EllipticF}[I*\text{ArcSinh}[\text{Sqrt}[(I*\text{Sqrt}[a])/ \text{Sqrt}[b]]/\text{Sqrt}[x]], -1)]/(21*\text{Sqrt}[(I*\text{Sqrt}[a])/ \text{Sqrt}[b]]*b*\text{Sqrt}[a + b*x^2])$

**Maple [A]** time = 0.032, size = 138, normalized size = 0.9

$$-\frac{2c}{21b^2x}\sqrt{cx}\left(\sqrt{-ab}\sqrt{1\left(bx+\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{1\left(-bx+\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-bx}\frac{1}{\sqrt{-ab}}\text{EllipticF}\left(\sqrt{1\left(bx+\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}},\frac{\sqrt{2}}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(3/2)*(b*x^2+a)^(1/2),x)`

[Out]  $-2/21*c/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*((-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*\text{EllipticF}(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^2-3*b^3*x^5-5*a*b^2*x^3-2*a^2*b*x)/b^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + a}(cx)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*(c*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*x^2 + a)*(c*x)^(3/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^2 + a}\sqrt{cxcx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*(c*x)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*x^2 + a)*sqrt(c*x)*c*x, x)`

**Sympy [A]** time = 21.2517, size = 46, normalized size = 0.3

$$\frac{\sqrt{ac}^{\frac{3}{2}}x^{\frac{5}{2}}\left(\frac{5}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{5}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(3/2)*(b*x**2+a)**(1/2),x)`

[Out] `sqrt(a)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4, ), b*x**2*exp_polar(I*pi)/a)/(2*gamma(9/4))`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + a} (cx)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*(c*x)^(3/2),x, algorithm="giac")`

[Out] `integrate(sqrt(b*x^2 + a)*(c*x)^(3/2), x)`

### 3.592 $\int \sqrt{cx} \sqrt{a + bx^2} dx$

**Optimal.** Leaf size=269

$$\frac{2a^{5/4}\sqrt{c}\left(\sqrt{a} + \sqrt{bx}\right) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^2}} - \frac{4a^{5/4}\sqrt{c}\left(\sqrt{a} + \sqrt{bx}\right) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^2}} + \frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c} + \frac{4a\sqrt{cx}\sqrt{a+bx^2}}{5\sqrt{b}\left(\sqrt{a} + \sqrt{bx}\right)}$$

[Out]  $(2*(c*x)^{(3/2)}*\text{Sqrt}[a + b*x^2])/(5*c) + (4*a*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(5*\text{Sqrt}[b]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) - (4*a^{(5/4)}*\text{Sqrt}[c]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(5*b^{(3/4)}*\text{Sqrt}[a + b*x^2]) + (2*a^{(5/4)}*\text{Sqrt}[c]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(5*b^{(3/4)}*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.505198, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{2a^{5/4}\sqrt{c}\left(\sqrt{a} + \sqrt{bx}\right) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^2}} - \frac{4a^{5/4}\sqrt{c}\left(\sqrt{a} + \sqrt{bx}\right) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^2}} + \frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c} + \frac{4a\sqrt{cx}\sqrt{a+bx^2}}{5\sqrt{b}\left(\sqrt{a} + \sqrt{bx}\right)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c\*x]\*Sqrt[a + b\*x^2], x]

[Out]  $(2*(c*x)^{(3/2)}*\text{Sqrt}[a + b*x^2])/(5*c) + (4*a*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(5*\text{Sqrt}[b]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) - (4*a^{(5/4)}*\text{Sqrt}[c]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(5*b^{(3/4)}*\text{Sqrt}[a + b*x^2]) + (2*a^{(5/4)}*\text{Sqrt}[c]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(5*b^{(3/4)}*\text{Sqrt}[a + b*x^2])$

**Rubi in Sympy [A]** time = 47.4581, size = 246, normalized size = 0.91

$$\frac{4a^{5/4}\sqrt{c} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) E\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^2}} + \frac{2a^{5/4}\sqrt{c} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) F\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^2}} + \frac{4a\sqrt{cx}\sqrt{a+bx^2}}{5\sqrt{b}\left(\sqrt{a} + \sqrt{bx}\right)} + \frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(1/2)\*(b\*x\*\*2+a)\*\*(1/2), x)

[Out]  $-4a^{5/4}\sqrt{c}\sqrt{(a+bx^2)/(\sqrt{a}+\sqrt{b}x)^2}(\sqrt{a}+\sqrt{b}x)\operatorname{elliptic}_e(2\operatorname{atan}(b^{1/4}\sqrt{cx}/(a^{1/4}\sqrt{c})), 1/2)/(5b^{3/4}\sqrt{a+bx^2})+2a^{5/4}\sqrt{c}\sqrt{(a+bx^2)/(\sqrt{a}+\sqrt{b}x)^2}(\sqrt{a}+\sqrt{b}x)\operatorname{elliptic}_f(2\operatorname{atan}(b^{1/4}\sqrt{cx}/(a^{1/4}\sqrt{c})), 1/2)/(5b^{3/4}\sqrt{a+bx^2})+4a\sqrt{c}\sqrt{a+bx^2}/(5\sqrt{b}(\sqrt{a}+\sqrt{b}x))+2(cx)^{3/2}\sqrt{a+bx^2}/(5c)$

**Mathematica [C]** time = 0.209423, size = 174, normalized size = 0.65

$$\frac{2\sqrt{cx}\left(-2a^{3/2}\sqrt{\frac{bx^2}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\right)-1\right)+2a^{3/2}\sqrt{\frac{bx^2}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\right)-1+\sqrt{bx}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}(a+bx^2)}{5\sqrt{b}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c\*x]\*Sqrt[a + b\*x^2], x]

[Out]  $(2\sqrt{c}x)(\sqrt{b}x\sqrt{(I\sqrt{b}x)/\sqrt{a}})(a+bx^2)+2a^{3/2}\sqrt{1+(bx^2)/a}\operatorname{EllipticE}[I\operatorname{ArcSinh}[\sqrt{(I\sqrt{b}x)/\sqrt{a}}], -1]-2a^{3/2}\sqrt{1+(bx^2)/a}\operatorname{EllipticF}[I\operatorname{ArcSinh}[\sqrt{(I\sqrt{b}x)/\sqrt{a}}], -1)]/(5\sqrt{b}\sqrt{(I\sqrt{b}x)/\sqrt{a}})\sqrt{a+bx^2}$

**Maple [A]** time = 0.03, size = 205, normalized size = 0.8

$$\frac{2}{5bx}\sqrt{cx}\left(2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}, 1/2\sqrt{2}\right)\sqrt{2a^2}-\sqrt{1+(bx+\sqrt{-ab})}\frac{1}{\sqrt{-ab}}\sqrt{1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(1/2)\*(b\*x^2+a)^(1/2), x)

[Out]  $2/5(c*x)^{1/2}/(b*x^2+a)^{1/2}/b(2*((b*x+(-a*b))^{1/2})/(-a*b)^{(1/2)})^{1/2}*((-b*x+(-a*b))^{1/2})/(-a*b)^{(1/2)})^{1/2}*(-x*b/(-a*b))^{1/2})^{1/2}\operatorname{EllipticE}(((b*x+(-a*b))^{1/2})/(-a*b)^{(1/2)})^{1/2}, 1/2*2^{1/2})^{1/2}*a^2-(((b*x+(-a*b))^{1/2})/(-a*b)^{(1/2)})^{1/2}*(-b*x+(-a*b))^{1/2}/(-a*b)^{(1/2)})^{1/2}*(-x*b/(-a*b))^{1/2})^{1/2}\operatorname{EllipticF}(((b*x+(-a*b))^{1/2})/(-a*b)^{(1/2)})^{1/2}, 1/2*2^{1/2})^{1/2}*a^2+b^2*x^4+a*b*x^2)/x$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2+a}\sqrt{cx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b\*x^2 + a)\*sqrt(c\*x), x, algorithm="maxima")

[Out] integrate(sqrt(b\*x^2 + a)\*sqrt(c\*x), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{bx^2+a}\sqrt{cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*sqrt(c*x),x, algorithm="fricas")`

[Out] `integral(sqrt(b*x^2 + a)*sqrt(c*x), x)`

**Sympy [A]** time = 3.3459, size = 46, normalized size = 0.17

$$\frac{\sqrt{a}\sqrt{cx}^{\frac{3}{2}} \left(\frac{3}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{3}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2 \left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(1/2)*(b*x**2+a)**(1/2),x)`

[Out] `sqrt(a)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4, ), b*x**2*exp_polar(I*pi)/a)/(2*gamma(7/4))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + a}\sqrt{cx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*sqrt(c*x),x, algorithm="giac")`

[Out] `integrate(sqrt(b*x^2 + a)*sqrt(c*x), x)`



$$3.593 \quad \int \frac{\sqrt{a+bx^2}}{\sqrt{cx}} dx$$

**Optimal.** Leaf size=126

$$\frac{2a^{3/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}} + \frac{2\sqrt{cx}\sqrt{a+bx^2}}{3c}$$

[Out] (2\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(3\*c) + (2\*a^(3/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(3\*b^(1/4)\*Sqrt[c]\*Sqrt[a + b\*x^2])

**Rubi [A]** time = 0.200497, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{2a^{3/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}} + \frac{2\sqrt{cx}\sqrt{a+bx^2}}{3c}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x^2]/Sqrt[c\*x], x]

[Out] (2\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(3\*c) + (2\*a^(3/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(3\*b^(1/4)\*Sqrt[c]\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 17.3573, size = 114, normalized size = 0.9

$$\frac{2a^{3/4} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) F\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}} + \frac{2\sqrt{cx}\sqrt{a+bx^2}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/2)/(c\*x)\*\*(1/2), x)

[Out] 2\*a\*\*(3/4)\*sqrt((a + b\*x\*\*2)/(sqrt(a) + sqrt(b)\*x)\*\*2)\*(sqrt(a) + sqrt(b)\*x)\*elliptic\_f(2\*atan(b\*\*(1/4)\*sqrt(c\*x)/(a\*\*(1/4)\*sqrt(c))), 1/2)/(3\*b\*\*(1/4)\*sqrt(c)\*sqrt(a + b\*x\*\*2)) + 2\*sqrt(c\*x)\*sqrt(a + b\*x\*\*2)/(3\*c)

**Mathematica [C]** time = 0.239399, size = 103, normalized size = 0.82

$$\frac{2x \left( \frac{2ia\sqrt{x}\sqrt{\frac{a}{bx^2}+1} F\left(i \sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right) \middle| -1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}} + a + bx^2 \right)}{3\sqrt{cx}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x^2]/Sqrt[c\*x], x]

[Out]  $(2*x*(a + b*x^2 + ((2*I)*a*Sqrt[1 + a/(b*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1])/Sqrt[(I*Sqrt[a])/Sqrt[b]]))/(3*Sqrt[c*x]*Sqrt[a + b*x^2])$

**Maple [A]** time = 0.03, size = 119, normalized size = 0.9

$$\frac{2}{3b} \left( \sqrt{-ab} \sqrt{1 \left( bx + \sqrt{-ab} \right) \frac{1}{\sqrt{-ab}}} \sqrt{1 \left( -bx + \sqrt{-ab} \right) \frac{1}{\sqrt{-ab}}} \sqrt{-bx \frac{1}{\sqrt{-ab}}} \text{EllipticF} \left( \sqrt{1 \left( bx + \sqrt{-ab} \right) \frac{1}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2} \right) \sqrt{2a + b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(1/2)/(c\*x)^(1/2), x)

[Out]  $2/3/(b*x^2+a)^{(1/2)}*((-a*b)^{(1/2)}*((b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*((-b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}*EllipticF((b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)})^2)^{(1/2)*a+b^2*x^3+a*b*x)/(c*x)^{(1/2)}/b$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b\*x^2 + a)/sqrt(c\*x), x, algorithm="maxima")

[Out] integrate(sqrt(b\*x^2 + a)/sqrt(c\*x), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\sqrt{bx^2 + a}}{\sqrt{cx}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b\*x^2 + a)/sqrt(c\*x), x, algorithm="fricas")

[Out] integral(sqrt(b\*x^2 + a)/sqrt(c\*x), x)

**Sympy [A]** time = 2.88542, size = 46, normalized size = 0.37

$$\frac{\sqrt{a}\sqrt{x} \left( \frac{1}{4} \right) {}_2F_1 \left( -\frac{1}{2}, \frac{1}{4} \middle| \frac{bx^2 e^{i\pi}}{a} \right)}{2\sqrt{c} \left( \frac{5}{4} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/2)/(c\*x)\*\*(1/2), x)

[Out]  $\sqrt{a} \sqrt{x} \gamma(1/4) \operatorname{hyper}((-1/2, 1/4), (5/4, ), b x^2 \exp_{\text{polar}}(I \pi)/a) / (2 \sqrt{c} \gamma(5/4))$

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/sqrt(c*x),x, algorithm="giac")`

[Out] `integrate(sqrt(b*x^2 + a)/sqrt(c*x), x)`

$$3.594 \quad \int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}} dx$$

**Optimal.** Leaf size=263

$$\frac{2\sqrt[4]{a}\sqrt[4]{b}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{c^{3/2}\sqrt{a+bx^2}} - \frac{4\sqrt[4]{a}\sqrt[4]{b}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{c^{3/2}\sqrt{a+bx^2}} + \frac{4\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{c^2(\sqrt{a} + \sqrt{bx})} - \frac{2\sqrt{a+bx^2}}{c\sqrt{cx}}$$

[Out]  $(-2*\text{Sqrt}[a + b*x^2])/(c*\text{Sqrt}[c*x]) + (4*\text{Sqrt}[b]*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(c^2*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) - (4*a^{(1/4)}*b^{(1/4)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(c^{(3/2)}*\text{Sqrt}[a + b*x^2]) + (2*a^{(1/4)}*b^{(1/4)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(c^{(3/2)}*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.508974, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{2\sqrt[4]{a}\sqrt[4]{b}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{c^{3/2}\sqrt{a+bx^2}} - \frac{4\sqrt[4]{a}\sqrt[4]{b}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{c^{3/2}\sqrt{a+bx^2}} + \frac{4\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{c^2(\sqrt{a} + \sqrt{bx})} - \frac{2\sqrt{a+bx^2}}{c\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x^2]/(c\*x)^(3/2), x]

[Out]  $(-2*\text{Sqrt}[a + b*x^2])/(c*\text{Sqrt}[c*x]) + (4*\text{Sqrt}[b]*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(c^2*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) - (4*a^{(1/4)}*b^{(1/4)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(c^{(3/2)}*\text{Sqrt}[a + b*x^2]) + (2*a^{(1/4)}*b^{(1/4)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(c^{(3/2)}*\text{Sqrt}[a + b*x^2])$

**Rubi in Sympy [A]** time = 48.0827, size = 241, normalized size = 0.92

$$\frac{4\sqrt[4]{a}\sqrt[4]{b} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) E\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{c^{\frac{3}{2}}\sqrt{a+bx^2}} + \frac{2\sqrt[4]{a}\sqrt[4]{b} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) F\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{c^{\frac{3}{2}}\sqrt{a+bx^2}} + \frac{4\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{c^2(\sqrt{a} + \sqrt{bx})} - \frac{2\sqrt{a+bx^2}}{c\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/2)/(c\*x)\*\*(3/2), x)

```
[Out] -4*a**(1/4)*b**(1/4)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*
(sqrt(a) + sqrt(b)*x)*elliptic_e(2*atan(b**(1/4)*sqrt(c*x)/(a**(1
/4)*sqrt(c))), 1/2)/(c**(3/2)*sqrt(a + b*x**2)) + 2*a**(1/4)*b**(
1/4)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(
b)*x)*elliptic_f(2*atan(b**(1/4)*sqrt(c*x)/(a**(1/4)*sqrt(c))), 1
/2)/(c**(3/2)*sqrt(a + b*x**2)) + 4*sqrt(b)*sqrt(c*x)*sqrt(a + b*
x**2)/(c**2*(sqrt(a) + sqrt(b)*x)) - 2*sqrt(a + b*x**2)/(c*sqrt(c
*x))
```

**Mathematica [C]** time = 0.207506, size = 174, normalized size = 0.66

$$\frac{x \left( -2\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}} (a + bx^2) - 4\sqrt{a}\sqrt{bx}\sqrt{\frac{bx^2}{a}} + 1F \left( i \sinh^{-1} \left( \sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}} \right) \middle| -1 \right) + 4\sqrt{a}\sqrt{bx}\sqrt{\frac{bx^2}{a}} + 1E \left( i \sinh^{-1} \left( \sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}} \right) \middle| -1 \right) \right)}{(cx)^{3/2} \sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}} \sqrt{a + bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(c*x)^(3/2),x]
```

```
[Out] (x*(-2*Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*(a + b*x^2) + 4*Sqrt[a]*Sqrt[b
]*x*Sqrt[1 + (b*x^2)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sq
rt[a]]], -1] - 4*Sqrt[a]*Sqrt[b]*x*Sqrt[1 + (b*x^2)/a]*EllipticF[
I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]], -1]))/(Sqrt[(I*Sqrt[b]*x)
/Sqrt[a]]*(c*x)^(3/2)*Sqrt[a + b*x^2])
```

**Maple [A]** time = 0.042, size = 194, normalized size = 0.7

$$2 \frac{1}{\sqrt{bx^2 + ac}\sqrt{cx}} \left( 2 \sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{-bx + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-\frac{bx}{\sqrt{-ab}}} \text{EllipticE} \left( \sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \sqrt{2} \right) \sqrt{2a} - \sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{-bx + \sqrt{-ab}}{\sqrt{-ab}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(c*x)^(3/2),x)
```

```
[Out] 2*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2))*2^(1/2)*a-((b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2), 1/2*2^(1/2))*2^(1/2)*a-b*x^2-a)/(b*x^2+a)^(1/2)/
c/(c*x)^(1/2)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{(cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)/(c*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(c*x)^(3/2), x)
```

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2 + a}}{\sqrt{cxcx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b\*x^2 + a)/(c\*x)^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(b\*x^2 + a)/(sqrt(c\*x)\*c\*x), x)

**Sympy** [A] time = 5.13953, size = 49, normalized size = 0.19

$$\frac{\sqrt{a} \left(-\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2c^{\frac{3}{2}} \sqrt{x} \left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/2)/(c\*x)\*\*(3/2), x)

[Out] sqrt(a)\*gamma(-1/4)\*hyper((-1/2, -1/4), (3/4, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*c\*\*(3/2)\*sqrt(x)\*gamma(3/4))

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{(cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b\*x^2 + a)/(c\*x)^(3/2), x, algorithm="giac")

[Out] integrate(sqrt(b\*x^2 + a)/(c\*x)^(3/2), x)

$$3.595 \quad \int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}} dx$$

**Optimal.** Leaf size=126

$$\frac{2b^{3/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{3\sqrt[4]{ac^5/2}\sqrt{a+bx^2}} - \frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}}$$

[Out]  $(-2*\text{Sqrt}[a + b*x^2])/(3*c*(c*x)^{(3/2)}) + (2*b^{(3/4)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(3*a^{(1/4)}*c^{(5/2)}*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.199549, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{2b^{3/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{3\sqrt[4]{ac^5/2}\sqrt{a+bx^2}} - \frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[a + b*x^2]/(c*x)^{(5/2)}, x]$

[Out]  $(-2*\text{Sqrt}[a + b*x^2])/(3*c*(c*x)^{(3/2)}) + (2*b^{(3/4)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(3*a^{(1/4)}*c^{(5/2)}*\text{Sqrt}[a + b*x^2])$

**Rubi in Sympy [A]** time = 18.0486, size = 114, normalized size = 0.9

$$-\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}} + \frac{2b^{3/4} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) F\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{3\sqrt[4]{ac^5/2}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x**2+a)**(1/2)/(c*x)**(5/2), x)$

[Out]  $-2*\text{sqrt}(a + b*x**2)/(3*c*(c*x)**(3/2)) + 2*b**(3/4)*\text{sqrt}((a + b*x**2)/(\text{sqrt}(a) + \text{sqrt}(b)*x)**2)*(\text{sqrt}(a) + \text{sqrt}(b)*x)*\text{elliptic\_f}(2*\text{atan}(b**(1/4)*\text{sqrt}(c*x)/(a**(1/4)*\text{sqrt}(c))), 1/2)/(3*a**(1/4)*c**(5/2)*\text{sqrt}(a + b*x**2))$

**Mathematica [C]** time = 0.280169, size = 106, normalized size = 0.84

$$2x \left( \frac{2ibx^{5/2} \sqrt{\frac{a}{bx^2}+1} F\left(i \sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right) \middle| -1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}} - a - bx^2 \right) \frac{1}{3(cx)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x^2]/(c\*x)^(5/2), x]

[Out] (2\*x\*(-a - b\*x^2 + ((2\*I)\*b\*Sqrt[1 + a/(b\*x^2)]\*x^(5/2)\*EllipticF[I\*ArcSinh[Sqrt[(I\*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1])/Sqrt[(I\*Sqrt[a])/Sqrt[b]]))/(3\*(c\*x)^(5/2)\*Sqrt[a + b\*x^2])

**Maple [A]** time = 0.034, size = 120, normalized size = 1.

$$\frac{2}{3xc^2} \left( \sqrt{1(bx + \sqrt{-ab})} \frac{1}{\sqrt{-ab}} \sqrt{1(-bx + \sqrt{-ab})} \frac{1}{\sqrt{-ab}} \sqrt{-bx} \frac{1}{\sqrt{-ab}} \text{EllipticF} \left( \sqrt{1(bx + \sqrt{-ab})} \frac{1}{\sqrt{-ab}}, \frac{\sqrt{2}}{2} \right) \sqrt{-ab} \sqrt{2x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(1/2)/(c\*x)^(5/2), x)

[Out] 2/3/(b\*x^2+a)^(1/2)/x\*(((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2))\*((-b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*(-x\*b/(-a\*b)^(1/2))^(1/2)\*EllipticF(((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2), 1/2\*2^(1/2))\*(-a\*b)^(1/2)\*2^(1/2)\*x-b\*x^2-a)/c^2/(c\*x)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{(cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b\*x^2 + a)/(c\*x)^(5/2), x, algorithm="maxima")

[Out] integrate(sqrt(b\*x^2 + a)/(c\*x)^(5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\sqrt{bx^2 + a}}{\sqrt{cx}c^2x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(b\*x^2 + a)/(c\*x)^(5/2), x, algorithm="fricas")

[Out] integral(sqrt(b\*x^2 + a)/(sqrt(c\*x)\*c^2\*x^2), x)

**Sympy [A]** time = 24.1588, size = 49, normalized size = 0.39

$$\frac{\sqrt{a} \left(-\frac{3}{4}\right) {}_2F_1 \left( \left. \begin{matrix} -\frac{3}{4}, -\frac{1}{2} \\ \frac{1}{4} \end{matrix} \right| \frac{bx^2 e^{i\pi}}{a} \right)}{2c^{\frac{5}{2}} x^{\frac{3}{2}} \left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/2)/(c\*x)\*\*(5/2), x)



```
[Out] sqrt(a)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4, ), b*x**2*exp_polar(
I*pi)/a)/(2*c**(5/2)*x**(3/2)*gamma(1/4))
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{(cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(c*x)^(5/2), x)
```

$$3.596 \quad \int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}} dx$$

**Optimal.** Leaf size=303

$$\frac{2b^{5/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5a^{3/4}c^{7/2}\sqrt{a+bx^2}} - \frac{4b^{5/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5a^{3/4}c^{7/2}\sqrt{a+bx^2}} + \frac{4b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5ac^4(\sqrt{a} + \sqrt{bx})} - \frac{4b\sqrt{a+bx^2}}{5ac^3\sqrt{cx}} - \frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}$$

[Out]  $(-2*\text{Sqrt}[a + b*x^2])/(5*c*(c*x)^(5/2)) - (4*b*\text{Sqrt}[a + b*x^2])/(5*a*c^3*\text{Sqrt}[c*x]) + (4*b^(3/2)*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(5*a*c^4*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) - (4*b^(5/4)*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^(1/4)*\text{Sqrt}[c*x])/(a^(1/4)*\text{Sqrt}[c])], 1/2])/(5*a^(3/4)*c^(7/2)*\text{Sqrt}[a + b*x^2]) + (2*b^(5/4)*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^(1/4)*\text{Sqrt}[c*x])/(a^(1/4)*\text{Sqrt}[c])], 1/2])/(5*a^(3/4)*c^(7/2)*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.599759, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$\frac{2b^{5/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5a^{3/4}c^{7/2}\sqrt{a+bx^2}} - \frac{4b^{5/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5a^{3/4}c^{7/2}\sqrt{a+bx^2}} + \frac{4b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5ac^4(\sqrt{a} + \sqrt{bx})} - \frac{4b\sqrt{a+bx^2}}{5ac^3\sqrt{cx}} - \frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x^2]/(c\*x)^(7/2), x]

[Out]  $(-2*\text{Sqrt}[a + b*x^2])/(5*c*(c*x)^(5/2)) - (4*b*\text{Sqrt}[a + b*x^2])/(5*a*c^3*\text{Sqrt}[c*x]) + (4*b^(3/2)*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(5*a*c^4*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) - (4*b^(5/4)*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^(1/4)*\text{Sqrt}[c*x])/(a^(1/4)*\text{Sqrt}[c])], 1/2])/(5*a^(3/4)*c^(7/2)*\text{Sqrt}[a + b*x^2]) + (2*b^(5/4)*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^(1/4)*\text{Sqrt}[c*x])/(a^(1/4)*\text{Sqrt}[c])], 1/2])/(5*a^(3/4)*c^(7/2)*\text{Sqrt}[a + b*x^2])$

**Rubi in Sympy [A]** time = 59.8426, size = 277, normalized size = 0.91

$$-\frac{2\sqrt{a+bx^2}}{5c(cx)^{\frac{5}{2}}} + \frac{4b^{\frac{3}{2}}\sqrt{cx}\sqrt{a+bx^2}}{5ac^4(\sqrt{a}+\sqrt{bx})} - \frac{4b\sqrt{a+bx^2}}{5ac^3\sqrt{cx}}$$

$$-\frac{4b^{\frac{5}{4}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{5a^{\frac{3}{4}}c^{\frac{7}{2}}\sqrt{a+bx^2}}$$

$$+\frac{2b^{\frac{5}{4}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{5a^{\frac{3}{4}}c^{\frac{7}{2}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(1/2)/(c*x)**(7/2),x)`

[Out]  $-2\sqrt{a+bx^2}/(5c(c*x)^{(5/2)}) + 4b^{(3/2)}\sqrt{c*x}\sqrt{a+bx^2}/(5a*c^{(4/2)}(\sqrt{a}+\sqrt{b*x})) - 4b\sqrt{a+bx^2}/(5a*c^{(3/2)}\sqrt{c*x}) - 4b^{(5/4)}\sqrt{a+bx^2}/(\sqrt{a}+\sqrt{b*x})E(2\operatorname{atan}(b^{(1/4)}\sqrt{c*x}/(a^{(1/4)}\sqrt{c})),1/2)/(5a^{(3/4)}c^{(7/2)}\sqrt{a+bx^2}) + 2b^{(5/4)}\sqrt{a+bx^2}/(\sqrt{a}+\sqrt{b*x})F(2\operatorname{atan}(b^{(1/4)}\sqrt{c*x}/(a^{(1/4)}\sqrt{c})),1/2)/(5a^{(3/4)}c^{(7/2)}\sqrt{a+bx^2})$

**Mathematica [C]** time = 0.289352, size = 196, normalized size = 0.65

$$\frac{x\left(-2\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}(a^2+3abx^2+2b^2x^4) - 4\sqrt{ab}^{3/2}x^3\sqrt{\frac{bx^2}{a}} + 1F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle|-1\right) + 4\sqrt{ab}^{3/2}x^3\sqrt{\frac{bx^2}{a}} + 1E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle|-1\right)\right)}{5a(cx)^{7/2}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[a + b*x^2]/(c*x)^(7/2),x]`

[Out]  $(x*(-2\sqrt{(I\sqrt{b}x)/\sqrt{a}})(a^2+3a*b*x^2+2*b^2*x^4) + 4\sqrt{a}*b^{(3/2)}*x^3*\sqrt{1+(b*x^2)/a}*EllipticE[I*ArcSinh[Sqrt[(I\sqrt{b}x)/\sqrt{a}]]],-1) - 4\sqrt{a}*b^{(3/2)}*x^3*\sqrt{1+(b*x^2)/a}*EllipticF[I*ArcSinh[Sqrt[(I\sqrt{b}x)/\sqrt{a}]]],-1))/(5*a*\sqrt{(I\sqrt{b}x)/\sqrt{a}}*(c*x)^(7/2)*\sqrt{a+b*x^2})$

**Maple [A]** time = 0.041, size = 219, normalized size = 0.7

$$\frac{2}{5x^2c^3a}\left(2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{2x^2ab} - \sqrt{1\left(\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}\right)\frac{1}{\sqrt{-ab}}}\sqrt{1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/2)/(c*x)^(7/2),x)`

[Out]  $2/5/x^2*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a*b - ((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a*b - 2*b^2*x^4 - 3*a*b*x^2 - a^2)/(b*x^2+a)^(1/2)/c^3/(c*x)^(1/2)/a$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{(cx)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/(c*x)^(7/2), x, algorithm="maxima")`

[Out] `integrate(sqrt(b*x^2 + a)/(c*x)^(7/2), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2 + a}}{\sqrt{cx}c^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/(c*x)^(7/2), x, algorithm="fricas")`

[Out] `integral(sqrt(b*x^2 + a)/(sqrt(c*x)*c^3*x^3), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/2)/(c*x)**(7/2), x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{(cx)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)/(c*x)^(7/2), x, algorithm="giac")`

[Out] `integrate(sqrt(b*x^2 + a)/(c*x)^(7/2), x)`

$$3.597 \quad \int (cx)^{7/2} (a + bx^2)^{3/2} dx$$

**Optimal.** Leaf size=212

$$\frac{4a^{15/4}c^{7/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}} - \frac{8a^3c^3\sqrt{cx}\sqrt{a+bx^2}}{231b^2} + \frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b} + \frac{2(cx)^{9/2}(a+bx^2)^{3/2}}{15c} + \frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c}$$

[Out]  $(-8*a^3*c^3*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(231*b^2) + (8*a^2*c*(c*x)^{5/2}*\text{Sqrt}[a + b*x^2])/(385*b) + (4*a*(c*x)^{9/2}*\text{Sqrt}[a + b*x^2])/(55*c) + (2*(c*x)^{9/2}*(a + b*x^2)^{3/2})/(15*c) + (4*a^{15/4}*c^{7/2}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{1/4}*\text{Sqrt}[c*x])/(a^{1/4}*\text{Sqrt}[c])], 1/2])/(231*b^{9/4}*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.366531, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{4a^{15/4}c^{7/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}} - \frac{8a^3c^3\sqrt{cx}\sqrt{a+bx^2}}{231b^2} + \frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b} + \frac{2(cx)^{9/2}(a+bx^2)^{3/2}}{15c} + \frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*x)^{7/2}*(a + b*x^2)^{3/2}, x]$

[Out]  $(-8*a^3*c^3*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(231*b^2) + (8*a^2*c*(c*x)^{5/2}*\text{Sqrt}[a + b*x^2])/(385*b) + (4*a*(c*x)^{9/2}*\text{Sqrt}[a + b*x^2])/(55*c) + (2*(c*x)^{9/2}*(a + b*x^2)^{3/2})/(15*c) + (4*a^{15/4}*c^{7/2}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{1/4}*\text{Sqrt}[c*x])/(a^{1/4}*\text{Sqrt}[c])], 1/2])/(231*b^{9/4}*\text{Sqrt}[a + b*x^2])$

**Rubi in Sympy [A]** time = 37.1248, size = 196, normalized size = 0.92

$$\frac{4a^{15/4}c^{7/2}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a} + \sqrt{bx}) F\left(2 \text{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}} - \frac{8a^3c^3\sqrt{cx}\sqrt{a+bx^2}}{231b^2} + \frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b} + \frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c} + \frac{2(cx)^{9/2}(a+bx^2)^{3/2}}{15c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c*x)**(7/2)*(b*x**2+a)**(3/2), x)$

[Out]  $4*a^{15/4}*c^{7/2}*\text{sqrt}((a + b*x**2)/(\text{sqrt}(a) + \text{sqrt}(b)*x)**2)*(\text{sqrt}(a) + \text{sqrt}(b)*x)*\text{elliptic\_f}(2*\text{atan}(b^{1/4}*\text{sqrt}(c*x)/(a^{1/4}*\text{sqrt}(c))), 1/2)/(231*b^{9/4}*\text{sqrt}(a + b*x**2)) - 8*a^3*c^3*\text{sqrt}(c*x)*\text{sqrt}(a + b*x**2)/(231*b^2) + 8*a^2*c*(c*x)**(5/2)*\text{sqrt}(a + b*x**2)/(385*b) + 4*a*(c*x)**(9/2)*\text{sqrt}(a + b*x**2)/(55*c) + 2*(c*x)**(9/2)*(a + b*x**2)**(3/2)/(15*c)$

**Mathematica [C]** time = 0.203836, size = 166, normalized size = 0.78

$$\frac{2c^3\sqrt{cx}\left(20ia^4\sqrt{x}\sqrt{\frac{a}{bx^2}}+1F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)-1\right)+\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\left(-20a^4-8a^3bx^2+131a^2b^2x^4+196ab^3x^6+77b^4x^8\right)\right)}{1155b^2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(7/2)\*(a + b\*x^2)^(3/2), x]

[Out] (2\*c^3\*Sqrt[c\*x]\*(Sqrt[(I\*Sqrt[a])/Sqrt[b]]\*(-20\*a^4 - 8\*a^3\*b\*x^2 + 131\*a^2\*b^2\*x^4 + 196\*a\*b^3\*x^6 + 77\*b^4\*x^8) + (20\*I)\*a^4\*Sqrt[1 + a/(b\*x^2)]\*Sqrt[x]\*EllipticF[I\*ArcSinh[Sqrt[(I\*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1]))/(1155\*Sqrt[(I\*Sqrt[a])/Sqrt[b]]\*b^2\*Sqrt[a + b\*x^2])

**Maple [A]** time = 0.034, size = 163, normalized size = 0.8

$$\frac{2c^3}{1155b^3x}\sqrt{cx}\left(77b^5x^9+196ab^4x^7+10\sqrt{-ab}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(7/2)\*(b\*x^2+a)^(3/2), x)

[Out] 2/1155\*c^3/x\*(c\*x)^(1/2)/(b\*x^2+a)^(1/2)\*(77\*b^5\*x^9+196\*a\*b^4\*x^7+10\*(-a\*b)^(1/2)\*((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*((-b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*(-x\*b/(-a\*b)^(1/2))^(1/2)\*EllipticF(((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2),1/2\*2^(1/2))\*2^(1/2)\*a^4+131\*a^2\*b^3\*x^5-8\*a^3\*b^2\*x^3-20\*a^4\*b\*x)/b^3

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{3}{2}}(cx)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)\*(c\*x)^(7/2), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(3/2)\*(c\*x)^(7/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bc^3x^5+ac^3x^3\right)\sqrt{bx^2+a}\sqrt{cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)\*(c\*x)^(7/2), x, algorithm="fricas")

[Out] integral((b\*c^3\*x^5 + a\*c^3\*x^3)\*sqrt(b\*x^2 + a)\*sqrt(c\*x), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(7/2)*(b*x**2+a)**(3/2),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{3}{2}} (cx)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)*(c*x)^(7/2),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(3/2)*(c*x)^(7/2), x)`

$$3.598 \quad \int (cx)^{5/2} (a + bx^2)^{3/2} dx$$

**Optimal.** Leaf size=329

$$\frac{4a^{13/4}c^{5/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^2}} + \frac{8a^{13/4}c^{5/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^2}} - \frac{8a^3c^2\sqrt{cx}\sqrt{a+bx^2}}{65b^{3/2}(\sqrt{a} + \sqrt{bx})} + \frac{8a^2c(cx)^{3/2}\sqrt{a+bx^2}}{195b} + \frac{2(cx)^{7/2}(a+bx^2)^{3/2}}{13c} + \frac{4a(cx)^{7/2}\sqrt{a+bx^2}}{39c}$$

[Out]  $(8*a^2*c*(c*x)^{(3/2)}*\text{Sqrt}[a + b*x^2])/(195*b) + (4*a*(c*x)^{(7/2)}*\text{Sqrt}[a + b*x^2])/(39*c) - (8*a^3*c^2*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(65*b^{(3/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) + (2*(c*x)^{(7/2)}*(a + b*x^2)^{(3/2)})/(13*c) + (8*a^{(13/4)}*c^{(5/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(65*b^{(7/4)}*\text{Sqrt}[a + b*x^2]) - (4*a^{(13/4)}*c^{(5/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(65*b^{(7/4)}*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.670314, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$\frac{4a^{13/4}c^{5/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^2}} + \frac{8a^{13/4}c^{5/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^2}} - \frac{8a^3c^2\sqrt{cx}\sqrt{a+bx^2}}{65b^{3/2}(\sqrt{a} + \sqrt{bx})} + \frac{8a^2c(cx)^{3/2}\sqrt{a+bx^2}}{195b} + \frac{2(cx)^{7/2}(a+bx^2)^{3/2}}{13c} + \frac{4a(cx)^{7/2}\sqrt{a+bx^2}}{39c}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(5/2)\*(a + b\*x^2)^(3/2), x]

[Out]  $(8*a^2*c*(c*x)^{(3/2)}*\text{Sqrt}[a + b*x^2])/(195*b) + (4*a*(c*x)^{(7/2)}*\text{Sqrt}[a + b*x^2])/(39*c) - (8*a^3*c^2*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(65*b^{(3/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) + (2*(c*x)^{(7/2)}*(a + b*x^2)^{(3/2)})/(13*c) + (8*a^{(13/4)}*c^{(5/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(65*b^{(7/4)}*\text{Sqrt}[a + b*x^2]) - (4*a^{(13/4)}*c^{(5/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(65*b^{(7/4)}*\text{Sqrt}[a + b*x^2])$



**Rubi in Sympy [A]** time = 68.9929, size = 303, normalized size = 0.92

$$\frac{8a^{\frac{13}{4}}c^{\frac{5}{2}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{65b^{\frac{7}{4}}\sqrt{a+bx^2}} - \frac{4a^{\frac{13}{4}}c^{\frac{5}{2}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{65b^{\frac{7}{4}}\sqrt{a+bx^2}} - \frac{8a^3c^2\sqrt{cx}\sqrt{a+bx^2}}{65b^{\frac{3}{2}}(\sqrt{a}+\sqrt{bx})} + \frac{8a^2c(cx)^{\frac{3}{2}}\sqrt{a+bx^2}}{195b} + \frac{4a(cx)^{\frac{7}{2}}\sqrt{a+bx^2}}{39c} + \frac{2(cx)^{\frac{7}{2}}(a+bx^2)^{\frac{3}{2}}}{13c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x)**(5/2)*(b*x**2+a)**(3/2),x)`

[Out] `8*a**(13/4)*c**(5/2)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*elliptic_e(2*atan(b**(1/4)*sqrt(c*x)/(a**(1/4)*sqrt(c))), 1/2)/(65*b**(7/4)*sqrt(a + b*x**2)) - 4*a**(13/4)*c**(5/2)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*elliptic_f(2*atan(b**(1/4)*sqrt(c*x)/(a**(1/4)*sqrt(c))), 1/2)/(65*b**(7/4)*sqrt(a + b*x**2)) - 8*a**3*c**2*sqrt(c*x)*sqrt(a + b*x**2)/(65*b**(3/2)*(sqrt(a) + sqrt(b)*x)) + 8*a**2*c*(c*x)**(3/2)*sqrt(a + b*x**2)/(195*b) + 4*a*(c*x)**(7/2)*sqrt(a + b*x**2)/(39*c) + 2*(c*x)**(7/2)*(a + b*x**2)**(3/2)/(13*c)`

**Mathematica [C]** time = 0.310453, size = 202, normalized size = 0.61

$$\frac{2c^2\sqrt{cx}\left(12a^{7/2}\sqrt{\frac{bx^2}{a}}+1F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle|-1\right)-12a^{7/2}\sqrt{\frac{bx^2}{a}}+1E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle|-1\right)+\sqrt{bx}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}(4a^3+29)\right)}{195b^{3/2}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c*x)^(5/2)*(a + b*x^2)^(3/2),x]`

[Out] `(2*c^2*Sqrt[c*x]*(Sqrt[b]*x*Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*(4*a^3 + 29*a^2*b*x^2 + 40*a*b^2*x^4 + 15*b^3*x^6) - 12*a^(7/2)*Sqrt[1 + (b*x^2)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]], -1] + 12*a^(7/2)*Sqrt[1 + (b*x^2)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]], -1))/(195*b^(3/2)*Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*Sqrt[a + b*x^2])`

**Maple [A]** time = 0.033, size = 232, normalized size = 0.7

$$-\frac{2c^2}{195b^2x}\sqrt{cx}\left(-15x^8b^4-40x^6ab^3+12\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx}{\sqrt{-ab}}}\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{1}{2}\sqrt{2}\right)\sqrt{2a^4}-\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(5/2)*(b*x^2+a)^(3/2),x)`

[Out] `-2/195*c^2/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b^2*(-15*x^8*b^4-40*x^6*a*b^3+12*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^4-6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)`

$$\frac{(-x^2 b / (-a^2 b))^{1/2} \operatorname{EllipticF}\left(\frac{(b^2 x + (-a^2 b))^{1/2}}{(-a^2 b)^{1/2}}, \frac{1}{2} \sqrt{2} \sqrt{a^4 - 29 x^4 a^2 b^2 - 4 x^2 a^3 b}\right)}{(-a^2 b)^{1/2}}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{3}{2}} (cx)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)\*(c\*x)^(5/2),x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(3/2)\*(c\*x)^(5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(bc^2x^4 + ac^2x^2\right)\sqrt{bx^2 + a}\sqrt{cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)\*(c\*x)^(5/2),x, algorithm="fricas")

[Out] integral((b\*c^2\*x^4 + a\*c^2\*x^2)\*sqrt(b\*x^2 + a)\*sqrt(c\*x), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(5/2)\*(b\*x\*\*2+a)\*\*(3/2),x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{3}{2}} (cx)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)\*(c\*x)^(5/2),x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(3/2)\*(c\*x)^(5/2), x)

### 3.599 $\int (cx)^{3/2} (a + bx^2)^{3/2} dx$

**Optimal.** Leaf size=181

$$\frac{4a^{11/4}c^{3/2}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{77b^{5/4}\sqrt{a+bx^2}} + \frac{8a^2c\sqrt{cx}\sqrt{a+bx^2}}{77b} + \frac{12a(cx)^{5/2}\sqrt{a+bx^2}}{77c} + \frac{2(cx)^{5/2}(a+bx^2)^{3/2}}{11c}$$

[Out]  $(8*a^2*c*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(77*b) + (12*a*(c*x)^{(5/2)}*\text{Sqrt}[a + b*x^2])/(77*c) + (2*(c*x)^{(5/2)}*(a + b*x^2)^{(3/2)})/(11*c) - (4*a^{(11/4)}*c^{(3/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(77*b^{(5/4)}*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.305132, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{4a^{11/4}c^{3/2}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{77b^{5/4}\sqrt{a+bx^2}} + \frac{8a^2c\sqrt{cx}\sqrt{a+bx^2}}{77b} + \frac{12a(cx)^{5/2}\sqrt{a+bx^2}}{77c} + \frac{2(cx)^{5/2}(a+bx^2)^{3/2}}{11c}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*x)^{(3/2)}*(a + b*x^2)^{(3/2)}, x]$

[Out]  $(8*a^2*c*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(77*b) + (12*a*(c*x)^{(5/2)}*\text{Sqrt}[a + b*x^2])/(77*c) + (2*(c*x)^{(5/2)}*(a + b*x^2)^{(3/2)})/(11*c) - (4*a^{(11/4)}*c^{(3/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(77*b^{(5/4)}*\text{Sqrt}[a + b*x^2])$

**Rubi in Sympy [A]** time = 28.5825, size = 165, normalized size = 0.91

$$\frac{4a^{\frac{11}{4}}c^{\frac{3}{2}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a} + \sqrt{bx})F\left(2\text{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{77b^{\frac{5}{4}}\sqrt{a+bx^2}} + \frac{8a^2c\sqrt{cx}\sqrt{a+bx^2}}{77b} + \frac{12a(cx)^{\frac{5}{2}}\sqrt{a+bx^2}}{77c} + \frac{2(cx)^{\frac{5}{2}}(a+bx^2)^{\frac{3}{2}}}{11c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c*x)**(3/2)*(b*x**2+a)**(3/2), x)$

[Out]  $-4*a^{(11/4)}*c^{(3/2)}*\text{sqrt}((a + b*x**2)/(\text{sqrt}(a) + \text{sqrt}(b)*x)**2)*(\text{sqrt}(a) + \text{sqrt}(b)*x)*\text{elliptic\_f}(2*\text{atan}(b^{(1/4)}*\text{sqrt}(c*x)/(a^{(1/4)}*\text{sqrt}(c))), 1/2)/(77*b^{(5/4)}*\text{sqrt}(a + b*x**2)) + 8*a^{(11/4)}*c^{(3/2)}*\text{sqrt}(c*x)*\text{sqrt}(a + b*x**2)/(77*b) + 12*a*(c*x)**(5/2)*\text{sqrt}(a + b*x**2)/(77*c) + 2*(c*x)**(5/2)*(a + b*x**2)**(3/2)/(11*c)$

**Mathematica [C]** time = 0.165292, size = 153, normalized size = 0.85

$$\frac{2c\sqrt{cx} \left( \sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} (4a^3 + 17a^2bx^2 + 20ab^2x^4 + 7b^3x^6) - 4ia^3\sqrt{x}\sqrt{\frac{a}{bx^2}} + 1F \left( i \sinh^{-1} \left( \frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}} \right) \middle| -1 \right) \right)}{77b\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(3/2)\*(a + b\*x^2)^(3/2), x]

[Out] (2\*c\*Sqrt[c\*x]\*(Sqrt[(I\*Sqrt[a])/Sqrt[b]]\*(4\*a^3 + 17\*a^2\*b\*x^2 + 20\*a\*b^2\*x^4 + 7\*b^3\*x^6) - (4\*I)\*a^3\*Sqrt[1 + a/(b\*x^2)]\*Sqrt[x])\*EllipticF[I\*ArcSinh[Sqrt[(I\*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1])/ (77\*Sqrt[(I\*Sqrt[a])/Sqrt[b]]\*b\*Sqrt[a + b\*x^2])

**Maple [A]** time = 0.017, size = 150, normalized size = 0.8

$$-\frac{2c}{77b^2x}\sqrt{cx} \left( -7x^7b^4 + 2\sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx + \sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{bx}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2\sqrt{2}\right)\sqrt{-ab}\sqrt{2a^3} - 20x^5ab^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(3/2)\*(b\*x^2+a)^(3/2), x)

[Out] -2/77\*c/x\*(c\*x)^(1/2)/(b\*x^2+a)^(1/2)\*(-7\*x^7\*b^4+2\*((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*((-b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*(-x\*b/(-a\*b)^(1/2))^(1/2)\*EllipticF(((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2), 1/2\*2^(1/2))\*(-a\*b)^(1/2)\*2^(1/2)\*a^3-20\*x^5\*a\*b^3-17\*x^3\*a^2\*b^2-4\*x\*a^3\*b)/b^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{3}{2}} (cx)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)\*(c\*x)^(3/2), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(3/2)\*(c\*x)^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bc^3 + acx\right)\sqrt{bx^2 + a}\sqrt{cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)\*(c\*x)^(3/2), x, algorithm="fricas")

[Out] integral((b\*c\*x^3 + a\*c\*x)\*sqrt(b\*x^2 + a)\*sqrt(c\*x), x)

**Sympy [A]** time = 67.9741, size = 46, normalized size = 0.25

$$\frac{a^{\frac{3}{2}} c^{\frac{3}{2}} x^{\frac{5}{2}} \left(\frac{5}{4}\right) {}_2F_1\left(-\frac{3}{2}, \frac{5}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2 \left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(3/2)\*(b\*x\*\*2+a)\*\*(3/2),x)

[Out] a\*\*(3/2)\*c\*\*(3/2)\*x\*\*(5/2)\*gamma(5/4)\*hyper((-3/2, 5/4), (9/4, ), b\*x\*\*2\*exp\_polar(1\*pi)/a)/(2\*gamma(9/4))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{3}{2}} (cx)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)\*(c\*x)^(3/2),x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(3/2)\*(c\*x)^(3/2), x)

$$3.600 \quad \int \sqrt{cx} (a + bx^2)^{3/2} dx$$

**Optimal.** Leaf size=297

$$\frac{4a^{9/4}\sqrt{c}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^2}} - \frac{8a^{9/4}\sqrt{c}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^2}} + \frac{8a^2\sqrt{cx}\sqrt{a+bx^2}}{15\sqrt{b}(\sqrt{a} + \sqrt{bx})} + \frac{4a(cx)^{3/2}\sqrt{a+bx^2}}{15c} + \frac{2(cx)^{3/2}(a+bx^2)^{3/2}}{9c}$$

[Out]  $(4*a*(c*x)^{(3/2)}*\text{Sqrt}[a + b*x^2])/(15*c) + (8*a^2*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(15*\text{Sqrt}[b]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) + (2*(c*x)^{(3/2)}*(a + b*x^2)^{(3/2)})/(9*c) - (8*a^{(9/4)}*\text{Sqrt}[c]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(15*b^{(3/4)}*\text{Sqrt}[a + b*x^2]) + (4*a^{(9/4)}*\text{Sqrt}[c]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(15*b^{(3/4)}*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.579623, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{4a^{9/4}\sqrt{c}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^2}} - \frac{8a^{9/4}\sqrt{c}(\sqrt{a} + \sqrt{bx})\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^2}} + \frac{8a^2\sqrt{cx}\sqrt{a+bx^2}}{15\sqrt{b}(\sqrt{a} + \sqrt{bx})} + \frac{4a(cx)^{3/2}\sqrt{a+bx^2}}{15c} + \frac{2(cx)^{3/2}(a+bx^2)^{3/2}}{9c}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c\*x]\*(a + b\*x^2)^(3/2), x]

[Out]  $(4*a*(c*x)^{(3/2)}*\text{Sqrt}[a + b*x^2])/(15*c) + (8*a^2*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(15*\text{Sqrt}[b]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) + (2*(c*x)^{(3/2)}*(a + b*x^2)^{(3/2)})/(9*c) - (8*a^{(9/4)}*\text{Sqrt}[c]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(15*b^{(3/4)}*\text{Sqrt}[a + b*x^2]) + (4*a^{(9/4)}*\text{Sqrt}[c]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(15*b^{(3/4)}*\text{Sqrt}[a + b*x^2])$

**Rubi in Sympy [A]** time = 55.5755, size = 272, normalized size = 0.92

$$\frac{8a^{\frac{9}{4}}\sqrt{c}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{3}{4}}\sqrt{a+bx^2}} + \frac{4a^{\frac{9}{4}}\sqrt{c}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{3}{4}}\sqrt{a+bx^2}} + \frac{8a^2\sqrt{cx}\sqrt{a+bx^2}}{15\sqrt{b}(\sqrt{a}+\sqrt{bx})} + \frac{4a(cx)^{\frac{3}{2}}\sqrt{a+bx^2}}{15c} + \frac{2(cx)^{\frac{3}{2}}(a+bx^2)^{\frac{3}{2}}}{9c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x)**(1/2)*(b*x**2+a)**(3/2),x)`

[Out] `-8*a**(9/4)*sqrt(c)*sqrt((a+b*x**2)/(sqrt(a)+sqrt(b)*x)**2)*(sqrt(a)+sqrt(b)*x)*elliptic_e(2*atan(b**(1/4)*sqrt(c*x)/(a**(1/4)*sqrt(c))),1/2)/(15*b**(3/4)*sqrt(a+b*x**2))+4*a**(9/4)*sqrt(c)*sqrt((a+b*x**2)/(sqrt(a)+sqrt(b)*x)**2)*(sqrt(a)+sqrt(b)*x)*elliptic_f(2*atan(b**(1/4)*sqrt(c*x)/(a**(1/4)*sqrt(c))),1/2)/(15*b**(3/4)*sqrt(a+b*x**2))+8*a**2*sqrt(c*x)*sqrt(a+b*x**2)/(15*sqrt(b)*(sqrt(a)+sqrt(b)*x))+4*a*(c*x)**(3/2)*sqrt(a+b*x**2)/(15*c)+2*(c*x)**(3/2)*(a+b*x**2)**(3/2)/(9*c)`

**Mathematica [C]** time = 0.266469, size = 188, normalized size = 0.63

$$\frac{2\sqrt{cx}\left(-12a^{5/2}\sqrt{\frac{bx^2}{a}}+1F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle|-1\right)+12a^{5/2}\sqrt{\frac{bx^2}{a}}+1E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle|-1\right)+\sqrt{bx}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\left(11a^2+16\right)\right)}{45\sqrt{b}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[c*x]*(a+b*x^2)^(3/2),x]`

[Out] `(2*Sqrt[c*x]*(Sqrt[b]*x*Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*(11*a^2+16*a*b*x^2+5*b^2*x^4)+12*a^(5/2)*Sqrt[1+(b*x^2)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]],-1]-12*a^(5/2)*Sqrt[1+(b*x^2)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]],-1])/ (45*Sqrt[b]*Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*Sqrt[a+b*x^2])`

**Maple [A]** time = 0.017, size = 218, normalized size = 0.7

$$\frac{2}{45bx}\sqrt{cx}\left(5b^3x^6+12\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx}{\sqrt{-ab}}}\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{1}{2}\sqrt{2}\right)\sqrt{2a^3-6}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx}{\sqrt{-ab}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(1/2)*(b*x^2+a)^(3/2),x)`

[Out] `2/45*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b*(5*b^3*x^6+12*((b*x+(-a*b))^(1/2)/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3-6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a^3+16*a*b^2*x^4+11*a^2*b*x^2)/x`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{3}{2}} \sqrt{cx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)\*sqrt(c\*x),x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(3/2)\*sqrt(c\*x), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^{\frac{3}{2}} \sqrt{cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)\*sqrt(c\*x),x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(3/2)\*sqrt(c\*x), x)

---

**Sympy [A]** time = 21.4824, size = 46, normalized size = 0.15

$$\frac{a^{\frac{3}{2}} \sqrt{cx}^{\frac{3}{2}} \left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2 \left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(1/2)\*(b\*x\*\*2+a)\*\*(3/2),x)

[Out] a\*\*(3/2)\*sqrt(c)\*x\*\*(3/2)\*gamma(3/4)\*hyper((-3/2, 3/4), (7/4, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*gamma(7/4))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{3}{2}} \sqrt{cx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)\*sqrt(c\*x),x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(3/2)\*sqrt(c\*x), x)



$$3.601 \quad \int \frac{(a+bx^2)^{3/2}}{\sqrt{cx}} dx$$

**Optimal.** Leaf size=152

$$\frac{4a^{7/4} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}} + \frac{4a\sqrt{cx}\sqrt{a+bx^2}}{7c} + \frac{2\sqrt{cx}(a+bx^2)^{3/2}}{7c}$$

[Out] (4\*a\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(7\*c) + (2\*Sqrt[c\*x]\*(a + b\*x^2)^(3/2))/(7\*c) + (4\*a^(7/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(7\*b^(1/4)\*Sqrt[c]\*Sqrt[a + b\*x^2])

**Rubi [A]** time = 0.244705, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{4a^{7/4} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}} + \frac{4a\sqrt{cx}\sqrt{a+bx^2}}{7c} + \frac{2\sqrt{cx}(a+bx^2)^{3/2}}{7c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(3/2)/Sqrt[c\*x], x]

[Out] (4\*a\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(7\*c) + (2\*Sqrt[c\*x]\*(a + b\*x^2)^(3/2))/(7\*c) + (4\*a^(7/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(7\*b^(1/4)\*Sqrt[c]\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 22.3054, size = 138, normalized size = 0.91

$$\frac{4a^{7/4} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) F\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}} + \frac{4a\sqrt{cx}\sqrt{a+bx^2}}{7c} + \frac{2\sqrt{cx}(a+bx^2)^{3/2}}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(3/2)/(c\*x)\*\*(1/2), x)

[Out] 4\*a\*\*(7/4)\*sqrt((a + b\*x\*\*2)/(sqrt(a) + sqrt(b)\*x)\*\*2)\*(sqrt(a) + sqrt(b)\*x)\*elliptic\_f(2\*atan(b\*\*(1/4)\*sqrt(c\*x)/(a\*\*(1/4)\*sqrt(c))), 1/2)/(7\*b\*\*(1/4)\*sqrt(c)\*sqrt(a + b\*x\*\*2)) + 4\*a\*sqrt(c\*x)\*sqrt(a + b\*x\*\*2)/(7\*c) + 2\*sqrt(c\*x)\*(a + b\*x\*\*2)\*\*(3/2)/(7\*c)

**Mathematica [C]** time = 0.117673, size = 141, normalized size = 0.93

$$\frac{\sqrt{x}\sqrt{a+bx^2}\left(\frac{6a\sqrt{x}}{7} + \frac{2}{7}bx^{5/2}\right)}{\sqrt{cx}} + \frac{8ia^2x^{3/2}\sqrt{\frac{a}{bx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\right)-1}{7\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{cx}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(3/2)/Sqrt[c\*x], x]

[Out] (Sqrt[x]\*Sqrt[a + b\*x^2]\*((6\*a\*Sqrt[x])/7 + (2\*b\*x^(5/2))/7))/Sqrt[c\*x] + (((8\*I)/7)\*a^2\*Sqrt[1 + a/(b\*x^2)]\*x^(3/2)\*EllipticF[I\*ArcSinh[Sqrt[(I\*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1])/(Sqrt[(I\*Sqrt[a])/Sqrt[b]]\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])

**Maple [A]** time = 0.018, size = 134, normalized size = 0.9

$$\frac{2}{7b} \left( 2\sqrt{-ab} \sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{-bx + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-\frac{bx}{\sqrt{-ab}}} \operatorname{EllipticF} \left( \sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \sqrt{2} \right) \sqrt{2a^2 + b^3x^5 + 4ab^2x^3 + 3a^2bx} \right) \frac{1}{\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(3/2)/(c\*x)^(1/2), x)

[Out] 2/7/(b\*x^2+a)^(1/2)\*(2\*(-a\*b)^(1/2)\*((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*((-b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*(-x\*b/(-a\*b)^(1/2))^(1/2)\*EllipticF(((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2), 1/2\*2^(1/2))\*2^(1/2)\*a^2+b^3\*x^5+4\*a\*b^2\*x^3+3\*a^2\*b\*x)/b/(c\*x)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{3}{2}}}{\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/sqrt(c\*x), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(3/2)/sqrt(c\*x), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{(bx^2 + a)^{\frac{3}{2}}}{\sqrt{cx}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/sqrt(c\*x), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(3/2)/sqrt(c\*x), x)

**Sympy [A]** time = 12.1133, size = 46, normalized size = 0.3

$$\frac{a^{\frac{3}{2}} \sqrt{x} \left( \frac{1}{4} \right) {}_2F_1 \left( -\frac{3}{2}, \frac{1}{4} \middle| \frac{bx^2 e^{i\pi}}{a} \right)}{2\sqrt{c} \left( \frac{5}{4} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(3/2)/(c\*x)\*\*(1/2),x)

[Out] a\*\*(3/2)\*sqrt(x)\*gamma(1/4)\*hyper((-3/2, 1/4), (5/4,)), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*sqrt(c)\*gamma(5/4))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{3}{2}}}{\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/sqrt(c\*x),x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(3/2)/sqrt(c\*x), x)

$$3.602 \quad \int \frac{(a+bx^2)^{3/2}}{(cx)^{3/2}} dx$$

**Optimal.** Leaf size=296

$$\frac{12a^{5/4}\sqrt[4]{b}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5c^{3/2}\sqrt{a+bx^2}} - \frac{24a^{5/4}\sqrt[4]{b}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5c^{3/2}\sqrt{a+bx^2}} + \frac{12b(cx)^{3/2}\sqrt{a+bx^2}}{5c^3} + \frac{24a\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{5c^2(\sqrt{a} + \sqrt{bx})} - \frac{2(a+bx^2)^{3/2}}{c\sqrt{cx}}$$

[Out] (12\*b\*(c\*x)^(3/2)\*Sqrt[a + b\*x^2])/(5\*c^3) + (24\*a\*Sqrt[b]\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(5\*c^2\*(Sqrt[a] + Sqrt[b]\*x)) - (2\*(a + b\*x^2)^(3/2))/(c\*Sqrt[c\*x]) - (24\*a^(5/4)\*b^(1/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticE[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(5\*c^(3/2)\*Sqrt[a + b\*x^2]) + (12\*a^(5/4)\*b^(1/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(5\*c^(3/2)\*Sqrt[a + b\*x^2])

**Rubi [A]** time = 0.575039, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$\frac{12a^{5/4}\sqrt[4]{b}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5c^{3/2}\sqrt{a+bx^2}} - \frac{24a^{5/4}\sqrt[4]{b}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5c^{3/2}\sqrt{a+bx^2}} + \frac{12b(cx)^{3/2}\sqrt{a+bx^2}}{5c^3} + \frac{24a\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{5c^2(\sqrt{a} + \sqrt{bx})} - \frac{2(a+bx^2)^{3/2}}{c\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(3/2)/(c\*x)^(3/2), x]

[Out] (12\*b\*(c\*x)^(3/2)\*Sqrt[a + b\*x^2])/(5\*c^3) + (24\*a\*Sqrt[b]\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(5\*c^2\*(Sqrt[a] + Sqrt[b]\*x)) - (2\*(a + b\*x^2)^(3/2))/(c\*Sqrt[c\*x]) - (24\*a^(5/4)\*b^(1/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticE[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(5\*c^(3/2)\*Sqrt[a + b\*x^2]) + (12\*a^(5/4)\*b^(1/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(5\*c^(3/2)\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 56.639, size = 274, normalized size = 0.93

$$\frac{24a^{\frac{5}{4}}\sqrt[4]{b}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{5c^{\frac{3}{2}}\sqrt{a+bx^2}} + \frac{12a^{\frac{5}{4}}\sqrt[4]{b}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{5c^{\frac{3}{2}}\sqrt{a+bx^2}} + \frac{24a\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{5c^2(\sqrt{a}+\sqrt{bx})} + \frac{12b(cx)^{\frac{3}{2}}\sqrt{a+bx^2}}{5c^3} - \frac{2(a+bx^2)^{\frac{3}{2}}}{c\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(3/2)/(c*x)**(3/2), x)`

[Out] `-24*a**(5/4)*b**(1/4)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*sqrt(a) + sqrt(b)*x*elliptic_e(2*atan(b**(1/4)*sqrt(c*x)/(a**(1/4)*sqrt(c))), 1/2)/(5*c**(3/2)*sqrt(a + b*x**2)) + 12*a**(5/4)*b**(1/4)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*elliptic_f(2*atan(b**(1/4)*sqrt(c*x)/(a**(1/4)*sqrt(c))), 1/2)/(5*c**(3/2)*sqrt(a + b*x**2)) + 24*a*sqrt(b)*sqrt(c*x)*sqrt(a + b*x**2)/(5*c**2*(sqrt(a) + sqrt(b)*x)) + 12*b*(c*x)**(3/2)*sqrt(a + b*x**2)/(5*c**3) - 2*(a + b*x**2)**(3/2)/(c*sqrt(c*x))`

**Mathematica [C]** time = 0.367438, size = 190, normalized size = 0.64

$$\frac{x\left(-24a^{3/2}\sqrt{bx}\sqrt{\frac{bx^2}{a}} + 1F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{a}}\right)\middle|-1\right) + 24a^{3/2}\sqrt{bx}\sqrt{\frac{bx^2}{a}} + 1E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{a}}\right)\middle|-1\right) + 2\sqrt{\frac{i\sqrt{bx}}{a}}(-5a^2 - 5cx)\right)}{5(cx)^{3/2}\sqrt{\frac{i\sqrt{bx}}{a}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(3/2)/(c*x)^(3/2), x]`

[Out] `(x*(2*Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*(-5*a^2 - 4*a*b*x^2 + b^2*x^4) + 24*a^(3/2)*Sqrt[b]*x*Sqrt[1 + (b*x^2)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]], -1] - 24*a^(3/2)*Sqrt[b]*x*Sqrt[1 + (b*x^2)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]], -1]))/(5*Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*(c*x)^(3/2)*Sqrt[a + b*x^2])`

**Maple [A]** time = 0.023, size = 208, normalized size = 0.7

$$\frac{2}{5c}\left(12\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}, 1/2\sqrt{2}\right)\sqrt{2a^2} - 6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{bx}{\sqrt{-ab}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(3/2)/(c*x)^(3/2), x)`

[Out] `2/5*(12*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2))*2^(1/2)*a^2-6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2))*2^(1/2)*a^2+b^2*x^4-4*a*b*x^2-5*a^2)/(b*x^2+a)^(1/2)/c/(c*x)^(1/2)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{3}{2}}}{(cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(3/2), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(3/2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{3}{2}}}{\sqrt{cx}cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(3/2), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(3/2)/(sqrt(c\*x)\*c\*x), x)

---

**Sympy [A]** time = 12.5809, size = 49, normalized size = 0.17

$$\frac{a^{\frac{3}{2}} \left(-\frac{1}{4}\right) {}_2F_1\left(-\frac{3}{2}, -\frac{1}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2c^{\frac{3}{2}} \sqrt{x} \left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(3/2)/(c\*x)\*\*(3/2), x)

[Out] a\*\*(3/2)\*gamma(-1/4)\*hyper((-3/2, -1/4), (3/4, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*c\*\*(3/2)\*sqrt(x)\*gamma(3/4))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{3}{2}}}{(cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(3/2), x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(3/2), x)

$$3.603 \quad \int \frac{(a+bx^2)^{3/2}}{(cx)^{5/2}} dx$$

**Optimal.** Leaf size=152

$$\frac{4a^{3/4}b^{3/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{3c^{5/2}\sqrt{a+bx^2}} + \frac{4b\sqrt{cx}\sqrt{a+bx^2}}{3c^3} - \frac{2(a+bx^2)^{3/2}}{3c(cx)^{3/2}}$$

[Out] (4\*b\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(3\*c^3) - (2\*(a + b\*x^2)^(3/2))/(3\*c\*(c\*x)^(3/2)) + (4\*a^(3/4)\*b^(3/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(3\*c^(5/2)\*Sqrt[a + b\*x^2])

**Rubi [A]** time = 0.249651, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{4a^{3/4}b^{3/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{3c^{5/2}\sqrt{a+bx^2}} + \frac{4b\sqrt{cx}\sqrt{a+bx^2}}{3c^3} - \frac{2(a+bx^2)^{3/2}}{3c(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(3/2)/(c\*x)^(5/2), x]

[Out] (4\*b\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(3\*c^3) - (2\*(a + b\*x^2)^(3/2))/(3\*c\*(c\*x)^(3/2)) + (4\*a^(3/4)\*b^(3/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(3\*c^(5/2)\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 23.3014, size = 139, normalized size = 0.91

$$\frac{4a^{3/4}b^{3/4} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) F\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{3c^{5/2}\sqrt{a+bx^2}} + \frac{4b\sqrt{cx}\sqrt{a+bx^2}}{3c^3} - \frac{2(a+bx^2)^{3/2}}{3c(cx)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(3/2)/(c\*x)\*\*(5/2), x)

[Out] 4\*a\*\*(3/4)\*b\*\*(3/4)\*sqrt((a + b\*x\*\*2)/(sqrt(a) + sqrt(b)\*x)\*\*2)\*(sqrt(a) + sqrt(b)\*x)\*elliptic\_f(2\*atan(b\*\*(1/4)\*sqrt(c\*x)/(a\*\*(1/4)\*sqrt(c))), 1/2)/(3\*c\*\*(5/2)\*sqrt(a + b\*x\*\*2)) + 4\*b\*sqrt(c\*x)\*sqrt(a + b\*x\*\*2)/(3\*c\*\*3) - 2\*(a + b\*x\*\*2)\*\*(3/2)/(3\*c\*(c\*x)\*\*(3/2))

**Mathematica [C]** time = 0.152072, size = 130, normalized size = 0.86

$$\frac{x \left( 8iabx^{5/2} \sqrt{\frac{a}{bx^2} + 1} F\left(i \sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right) \middle| -1 \right) - 2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} (a^2 - b^2x^4) \right)}{3\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}(cx)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(3/2)/(c\*x)^(5/2), x]

[Out] (x\*(-2\*Sqrt[(I\*Sqrt[a])/Sqrt[b]]\*(a^2 - b^2\*x^4) + (8\*I)\*a\*b\*Sqrt[1 + a/(b\*x^2)]\*x^(5/2)\*EllipticF[I\*ArcSinh[Sqrt[(I\*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1]))/(3\*Sqrt[(I\*Sqrt[a])/Sqrt[b]]\*(c\*x)^(5/2)\*Sqrt[a + b\*x^2])

**Maple [A]** time = 0.02, size = 125, normalized size = 0.8

$$\frac{2}{3xc^2} \left( 2\sqrt{-ab} \sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{-bx + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-\frac{bx}{\sqrt{-ab}}} \operatorname{EllipticF} \left( \sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \sqrt{2} \right) \sqrt{2xa + b^2x^4 - a^2} \right) \frac{1}{\sqrt{cx}} \frac{1}{\sqrt{bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(3/2)/(c\*x)^(5/2), x)

[Out] 2/3/(b\*x^2+a)^(1/2)/x\*(2\*(-a\*b)^(1/2)\*((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*((-b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*(-x\*b/(-a\*b)^(1/2))^(1/2)\*EllipticF((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2), 1/2\*2^(1/2))\*2^(1/2)\*x\*a+b^2\*x^4-a^2)/c^2/(c\*x)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(5/2), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{(bx^2 + a)^{\frac{3}{2}}}{\sqrt{cxc^2x^2}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(5/2), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(3/2)/(sqrt(c\*x)\*c^2\*x^2), x)

**Sympy [A]** time = 53.0876, size = 49, normalized size = 0.32

$$\frac{a^{\frac{3}{2}} \left(-\frac{3}{4}\right) {}_2F_1 \left( \begin{matrix} -\frac{3}{2}, -\frac{3}{4} \\ \frac{1}{4} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a} \right)}{2c^{\frac{5}{2}} x^{\frac{3}{2}} \left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(3/2)/(c\*x)\*\*(5/2), x)



[Out]  $a^{3/2} \gamma(-3/4) \text{hyper}((-3/2, -3/4), (1/4, ), b x^2 \exp_{\text{polar}}(i \pi)/a) / (2 c^{5/2} x^{3/2} \gamma(1/4))$

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{3/2}}{(cx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/(c*x)^(5/2), x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(3/2)/(c*x)^(5/2), x)`

$$3.604 \quad \int \frac{(a+bx^2)^{3/2}}{(cx)^{7/2}} dx$$

**Optimal.** Leaf size=297

$$\frac{12\sqrt[4]{ab}^{5/4} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5c^{7/2}\sqrt{a+bx^2}} - \frac{24\sqrt[4]{ab}^{5/4} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5c^{7/2}\sqrt{a+bx^2}} + \frac{24b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5c^4(\sqrt{a} + \sqrt{bx})} - \frac{12b\sqrt{a+bx^2}}{5c^3\sqrt{cx}} - \frac{2(a+bx^2)^{3/2}}{5c(cx)^{5/2}}$$

[Out]  $(-12*b*\text{Sqrt}[a + b*x^2])/(5*c^3*\text{Sqrt}[c*x]) + (24*b^{(3/2)}*\text{Sqrt}[c*x] * \text{Sqrt}[a + b*x^2])/(5*c^4*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) - (2*(a + b*x^2)^{(3/2)})/(5*c*(c*x)^{(5/2)}) - (24*a^{(1/4)}*b^{(5/4)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(5*c^{(7/2)}*\text{Sqrt}[a + b*x^2]) + (12*a^{(1/4)}*b^{(5/4)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(5*c^{(7/2)}*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.580355, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{12\sqrt[4]{ab}^{5/4} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5c^{7/2}\sqrt{a+bx^2}} - \frac{24\sqrt[4]{ab}^{5/4} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5c^{7/2}\sqrt{a+bx^2}} + \frac{24b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5c^4(\sqrt{a} + \sqrt{bx})} - \frac{12b\sqrt{a+bx^2}}{5c^3\sqrt{cx}} - \frac{2(a+bx^2)^{3/2}}{5c(cx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(3/2)/(c\*x)^(7/2), x]

[Out]  $(-12*b*\text{Sqrt}[a + b*x^2])/(5*c^3*\text{Sqrt}[c*x]) + (24*b^{(3/2)}*\text{Sqrt}[c*x] * \text{Sqrt}[a + b*x^2])/(5*c^4*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) - (2*(a + b*x^2)^{(3/2)})/(5*c*(c*x)^{(5/2)}) - (24*a^{(1/4)}*b^{(5/4)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(5*c^{(7/2)}*\text{Sqrt}[a + b*x^2]) + (12*a^{(1/4)}*b^{(5/4)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(5*c^{(7/2)}*\text{Sqrt}[a + b*x^2])$

**Rubi in Sympy [A]** time = 57.7429, size = 274, normalized size = 0.92

$$\frac{24\sqrt[4]{ab}^{\frac{5}{4}} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) E\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{5c^{\frac{7}{2}}\sqrt{a+bx^2}} + \frac{12\sqrt[4]{ab}^{\frac{5}{4}} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) F\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{5c^{\frac{7}{2}}\sqrt{a+bx^2}} + \frac{24b^{\frac{3}{2}}\sqrt{cx}\sqrt{a+bx^2}}{5c^4(\sqrt{a} + \sqrt{bx})} - \frac{12b\sqrt{a+bx^2}}{5c^3\sqrt{cx}} - \frac{2(a+bx^2)^{\frac{3}{2}}}{5c(cx)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**(3/2)/(c*x)**(7/2), x)
```

```
[Out] -24*a**(1/4)*b**(5/4)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)
*(sqrt(a) + sqrt(b)*x)*elliptic_e(2*atan(b**(1/4)*sqrt(c*x)/(a**(1/4)*sqrt(c))), 1/2)/(5*c**(7/2)*sqrt(a + b*x**2)) + 12*a**(1/4)*
b**(5/4)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + s
qrt(b)*x)*elliptic_f(2*atan(b**(1/4)*sqrt(c*x)/(a**(1/4)*sqrt(c)
), 1/2)/(5*c**(7/2)*sqrt(a + b*x**2)) + 24*b**(3/2)*sqrt(c*x)*sq
rt(a + b*x**2)/(5*c**4*(sqrt(a) + sqrt(b)*x)) - 12*b*sqrt(a + b*x*
*2)/(5*c**3*sqrt(c*x)) - 2*(a + b*x**2)**(3/2)/(5*c*(c*x)**(5/2))
```

**Mathematica [C]** time = 0.327321, size = 193, normalized size = 0.65

$$\frac{x\left(-2\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\left(a^2 + 8abx^2 + 7b^2x^4\right) - 24\sqrt{ab}^{3/2}x^3\sqrt{\frac{bx^2}{a}} + 1F\left(i \sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle|-1\right) + 24\sqrt{ab}^{3/2}x^3\sqrt{\frac{bx^2}{a}} + 1E\left(i \sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle|-1\right)\right)}{5(cx)^{7/2}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(c*x)^(7/2), x]
```

```
[Out] (x*(-2*Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*(a^2 + 8*a*b*x^2 + 7*b^2*x^4)
+ 24*Sqrt[a]*b^(3/2)*x^3*Sqrt[1 + (b*x^2)/a]*EllipticE[I*ArcSinh[
Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]], -1] - 24*Sqrt[a]*b^(3/2)*x^3*Sqrt[1
+ (b*x^2)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]], -
1]))/(5*Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*(c*x)^(7/2)*Sqrt[a + b*x^2])
```

**Maple [A]** time = 0.023, size = 216, normalized size = 0.7

$$\frac{2}{5x^2c^3}\left(12\sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx + \sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticE}\left(\sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2\sqrt{2}\right)\sqrt{2x^2ab} - 6\sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx + \sqrt{-ab}}{\sqrt{-ab}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(c*x)^(7/2), x)
```

```
[Out] 2/5/x^2*(12*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE((
(b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2))*2^(1/2)*x^2*a
*b-6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF((b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2))*2^(1/2)*x^2*a*b-7*b^
2*x^4-8*a*b*x^2-a^2)/(b*x^2+a)^(1/2)/c^3/(c*x)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{3}{2}}}{(cx)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(7/2), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(7/2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{3}{2}}}{\sqrt{cx}c^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(7/2), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(3/2)/(sqrt(c\*x)\*c^3\*x^3), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(3/2)/(c\*x)\*\*(7/2), x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{3}{2}}}{(cx)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(7/2), x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(7/2), x)

$$3.605 \quad \int \frac{(a+bx^2)^{3/2}}{(cx)^{9/2}} dx$$

**Optimal.** Leaf size=152

$$\frac{4b^{7/4} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{7\sqrt[4]{ac}^{9/2} \sqrt{a+bx^2}} - \frac{4b\sqrt{a+bx^2}}{7c^3(cx)^{3/2}} - \frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}}$$

[Out]  $(-4*b*\text{Sqrt}[a + b*x^2])/(7*c^3*(c*x)^(3/2)) - (2*(a + b*x^2)^(3/2))/(7*c*(c*x)^(7/2)) + (4*b^(7/4)*(Sqrt[a] + Sqrt[b]*x)*\text{Sqrt}[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^(1/4)*\text{Sqrt}[c*x])/(a^(1/4)*\text{Sqrt}[c])], 1/2])/(7*a^(1/4)*c^(9/2)*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.256562, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{4b^{7/4} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{7\sqrt[4]{ac}^{9/2} \sqrt{a+bx^2}} - \frac{4b\sqrt{a+bx^2}}{7c^3(cx)^{3/2}} - \frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^(3/2)/(c*x)^(9/2), x]$

[Out]  $(-4*b*\text{Sqrt}[a + b*x^2])/(7*c^3*(c*x)^(3/2)) - (2*(a + b*x^2)^(3/2))/(7*c*(c*x)^(7/2)) + (4*b^(7/4)*(Sqrt[a] + Sqrt[b]*x)*\text{Sqrt}[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^(1/4)*\text{Sqrt}[c*x])/(a^(1/4)*\text{Sqrt}[c])], 1/2])/(7*a^(1/4)*c^(9/2)*\text{Sqrt}[a + b*x^2])$

**Rubi in Sympy [A]** time = 23.6911, size = 139, normalized size = 0.91

$$-\frac{4b\sqrt{a+bx^2}}{7c^3(cx)^{3/2}} - \frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}} + \frac{4b^{7/4} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) F\left(2 \text{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{7\sqrt[4]{ac}^{9/2} \sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x**2+a)**(3/2)/(c*x)**(9/2), x)$

[Out]  $-4*b*\text{sqrt}(a + b*x**2)/(7*c**3*(c*x)**(3/2)) - 2*(a + b*x**2)**(3/2)/(7*c*(c*x)**(7/2)) + 4*b**(7/4)*\text{sqrt}((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*\text{elliptic\_f}(2*\text{atan}(b**(1/4)*\text{sqrt}(c*x)/(a**(1/4)*\text{sqrt}(c))), 1/2)/(7*a**(1/4)*c**(9/2)*\text{sqrt}(a + b*x**2))$

**Mathematica [C]** time = 0.242044, size = 121, normalized size = 0.8

$$\frac{x^{9/2} \left( -\frac{2(a+bx^2)(a+3bx^2)}{x^{7/2}} + \frac{8ib^2x \sqrt{\frac{a}{bx^2}+1} F\left(i \sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right) \middle| -1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}} \right)}{7(cx)^{9/2} \sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(3/2)/(c\*x)^(9/2), x]

[Out] (x^(9/2)\*((-2\*(a + b\*x^2)\*(a + 3\*b\*x^2))/x^(7/2) + ((8\*I)\*b^2\*Sqrt[1 + a/(b\*x^2)]\*x\*EllipticF[I\*ArcSinh[Sqrt[(I\*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1])/Sqrt[(I\*Sqrt[a])/Sqrt[b]]))/(7\*(c\*x)^(9/2)\*Sqrt[a + b\*x^2])

**Maple [A]** time = 0.04, size = 135, normalized size = 0.9

$$\frac{2}{7x^3c^4} \left( 2 \sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{-bx + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-\frac{bx}{\sqrt{-ab}}} \text{EllipticF} \left( \sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \sqrt{2} \right) \sqrt{-ab} \sqrt{2} x^3 b - 3 b^2 x^4 - 4 abx^2 - a^2 \right) \frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(3/2)/(c\*x)^(9/2), x)

[Out] 2/7/(b\*x^2+a)^(1/2)/x^3\*(2\*((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*((-b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*(-x\*b/(-a\*b)^(1/2))^(1/2)\*EllipticF(((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2), 1/2\*2^(1/2))\*(-a\*b)^(1/2)\*2^(1/2)\*x^3\*b-3\*b^2\*x^4-4\*a\*b\*x^2-a^2)/c^4/(c\*x)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{3}{2}}}{(cx)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(9/2), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(9/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(bx^2 + a)^{\frac{3}{2}}}{\sqrt{cx}c^4x^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(9/2), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(3/2)/(sqrt(c\*x)\*c^4\*x^4), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(3/2)/(c\*x)\*\*(9/2), x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{3}{2}}}{(cx)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/2)/(c*x)^(9/2), x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(3/2)/(c*x)^(9/2), x)`

$$3.606 \quad \int \frac{(a+bx^2)^{3/2}}{(cx)^{11/2}} dx$$

**Optimal.** Leaf size=331

$$\frac{4b^{9/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{15a^{3/4}c^{11/2}\sqrt{a+bx^2}} - \frac{8b^{9/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{15a^{3/4}c^{11/2}\sqrt{a+bx^2}} + \frac{8b^{5/2}\sqrt{cx}\sqrt{a+bx^2}}{15ac^6(\sqrt{a} + \sqrt{bx})} - \frac{8b^2\sqrt{a+bx^2}}{15ac^5\sqrt{cx}} - \frac{4b\sqrt{a+bx^2}}{15c^3(cx)^{5/2}} - \frac{2(a+bx^2)^{3/2}}{9c(cx)^{9/2}}$$

[Out]  $(-4*b*\text{Sqrt}[a + b*x^2])/(15*c^3*(c*x)^(5/2)) - (8*b^2*\text{Sqrt}[a + b*x^2])/(15*a*c^5*\text{Sqrt}[c*x]) + (8*b^(5/2)*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(15*a*c^6*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) - (2*(a + b*x^2)^(3/2))/(9*c*(c*x)^(9/2)) - (8*b^(9/4)*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^(1/4)*\text{Sqrt}[c*x])/(a^(1/4)*\text{Sqrt}[c])], 1/2])/(15*a^(3/4)*c^(11/2)*\text{Sqrt}[a + b*x^2]) + (4*b^(9/4)*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^(1/4)*\text{Sqrt}[c*x])/(a^(1/4)*\text{Sqrt}[c])], 1/2])/(15*a^(3/4)*c^(11/2)*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.686009, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$\frac{4b^{9/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{15a^{3/4}c^{11/2}\sqrt{a+bx^2}} - \frac{8b^{9/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{15a^{3/4}c^{11/2}\sqrt{a+bx^2}} + \frac{8b^{5/2}\sqrt{cx}\sqrt{a+bx^2}}{15ac^6(\sqrt{a} + \sqrt{bx})} - \frac{8b^2\sqrt{a+bx^2}}{15ac^5\sqrt{cx}} - \frac{4b\sqrt{a+bx^2}}{15c^3(cx)^{5/2}} - \frac{2(a+bx^2)^{3/2}}{9c(cx)^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(3/2)/(c\*x)^(11/2), x]

[Out]  $(-4*b*\text{Sqrt}[a + b*x^2])/(15*c^3*(c*x)^(5/2)) - (8*b^2*\text{Sqrt}[a + b*x^2])/(15*a*c^5*\text{Sqrt}[c*x]) + (8*b^(5/2)*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(15*a*c^6*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) - (2*(a + b*x^2)^(3/2))/(9*c*(c*x)^(9/2)) - (8*b^(9/4)*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^(1/4)*\text{Sqrt}[c*x])/(a^(1/4)*\text{Sqrt}[c])], 1/2])/(15*a^(3/4)*c^(11/2)*\text{Sqrt}[a + b*x^2]) + (4*b^(9/4)*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^(1/4)*\text{Sqrt}[c*x])/(a^(1/4)*\text{Sqrt}[c])], 1/2])/(15*a^(3/4)*c^(11/2)*\text{Sqrt}[a + b*x^2])$



**Rubi in Sympy [A]** time = 70.9209, size = 304, normalized size = 0.92

$$\frac{4b\sqrt{a+bx^2}}{15c^3(cx)^{\frac{5}{2}}} - \frac{2(a+bx^2)^{\frac{3}{2}}}{9c(cx)^{\frac{9}{2}}} + \frac{8b^{\frac{5}{2}}\sqrt{cx}\sqrt{a+bx^2}}{15ac^6(\sqrt{a}+\sqrt{bx})} - \frac{8b^2\sqrt{a+bx^2}}{15ac^5\sqrt{cx}}$$

$$- \frac{8b^{\frac{9}{4}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{15a^{\frac{3}{4}}c^{\frac{11}{2}}\sqrt{a+bx^2}}$$

$$+ \frac{4b^{\frac{9}{4}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{15a^{\frac{3}{4}}c^{\frac{11}{2}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(3/2)/(c*x)**(11/2), x)`

[Out]  $-4*b*\sqrt{a+b*x^2}/(15*c^3*(c*x)^{(5/2)}) - 2*(a+b*x^2)^{(3/2)}/(9*c*(c*x)^{(9/2)}) + 8*b^{(5/2)}*\sqrt{c*x}*\sqrt{a+b*x^2}/(15*a*c^6*(\sqrt{a}+\sqrt{b*x})) - 8*b^{(5/2)}*\sqrt{a+b*x^2}/(15*a*c^5*\sqrt{c*x}) - 8*b^{(9/4)}*\sqrt{(a+b*x^2)/(\sqrt{a}+\sqrt{b*x})}*\operatorname{elliptic}_e(2*\operatorname{atan}(b^{(1/4)}*\sqrt{c*x}/(a^{(1/4)}*\sqrt{c})), 1/2)/(15*a^{(3/4)}*c^{(11/2)}*\sqrt{a+b*x^2}) + 4*b^{(9/4)}*\sqrt{(a+b*x^2)/(\sqrt{a}+\sqrt{b*x})}*\operatorname{elliptic}_f(2*\operatorname{atan}(b^{(1/4)}*\sqrt{c*x}/(a^{(1/4)}*\sqrt{c})), 1/2)/(15*a^{(3/4)}*c^{(11/2)}*\sqrt{a+b*x^2})$

**Mathematica [C]** time = 0.424198, size = 213, normalized size = 0.64

$$\frac{2\sqrt{cx}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}(5a^3+16a^2bx^2+23ab^2x^4+12b^3x^6)+12\sqrt{ab^{5/2}}x^5\sqrt{\frac{bx^2}{a}}+1F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle|-1\right)-12\sqrt{ab^{5/2}}x^5\sqrt{\frac{bx^2}{a}}\right)}{45ac^6x^5\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(3/2)/(c*x)^(11/2), x]`

[Out]  $(-2*\operatorname{Sqrt}[c*x]*(\operatorname{Sqrt}[(I*\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]*(5*a^3+16*a^2*b*x^2+23*a*b^2*x^4+12*b^3*x^6)-12*\operatorname{Sqrt}[a]*b^{(5/2)}*x^5*\operatorname{Sqrt}[1+(b*x^2)/a]*\operatorname{EllipticE}[I*\operatorname{ArcSinh}[\operatorname{Sqrt}[(I*\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]], -1]+12*\operatorname{Sqrt}[a]*b^{(5/2)}*x^5*\operatorname{Sqrt}[1+(b*x^2)/a]*\operatorname{EllipticF}[I*\operatorname{ArcSinh}[\operatorname{Sqrt}[(I*\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]], -1))/(45*a*c^6*x^5*\operatorname{Sqrt}[(I*\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]*\operatorname{Sqrt}[a+b*x^2])$

**Maple [A]** time = 0.046, size = 234, normalized size = 0.7

$$\frac{2}{45ax^4c^5}\left(12\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}, 1/2\sqrt{2}\right)\sqrt{2}x^4ab^2-6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(3/2)/(c*x)^(11/2), x)`

[Out]  $2/45/x^4*(12*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*\operatorname{EllipticE}((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2))*2^(1/2)*x^4*a*b^2-6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*\operatorname{EllipticF}((b*x$

$$\frac{+(-a*b)^{(1/2))/(-a*b)^{(1/2))^{(1/2)}, 1/2*2^{(1/2))} * 2^{(1/2)} * x^4 * a * b^2 - 12*b^3*x^6 - 23*a*b^2*x^4 - 16*a^2*b*x^2 - 5*a^3)/(b*x^2+a)^{(1/2)}/a/c^5/(c*x)^{(1/2)}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(11/2), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(11/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{3}{2}}}{\sqrt{cx}c^5x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(11/2), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(3/2)/(sqrt(c\*x)\*c^5\*x^5), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(3/2)/(c\*x)\*\*(11/2), x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(11/2), x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(3/2)/(c\*x)^(11/2), x)

### 3.607 $\int (cx)^{5/2} \sqrt{3a - 2ax^2} dx$

**Optimal.** Leaf size=128

$$-\frac{3\sqrt[4]{6}ac^2\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6x}}}{\sqrt{6}}\right)\middle|2\right)}{5\sqrt{x}\sqrt{3a-2ax^2}} + \frac{2\sqrt{3a-2ax^2}(cx)^{7/2}}{9c} - \frac{2}{15}c\sqrt{3a-2ax^2}(cx)^{3/2}$$

[Out]  $(-2*c*(c*x)^{(3/2)}*\text{Sqrt}[3*a - 2*a*x^2])/15 + (2*(c*x)^{(7/2)}*\text{Sqrt}[3*a - 2*a*x^2])/(9*c) - (3*6^{(1/4)}*a*c^2*\text{Sqrt}[c*x]*\text{Sqrt}[3 - 2*x^2]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[3 - \text{Sqrt}[6]*x]/\text{Sqrt}[6]], 2])/(5*\text{Sqrt}[x]*\text{Sqrt}[3*a - 2*a*x^2])$

**Rubi [A]** time = 0.194163, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$-\frac{3\sqrt[4]{6}ac^2\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6x}}}{\sqrt{6}}\right)\middle|2\right)}{5\sqrt{x}\sqrt{3a-2ax^2}} + \frac{2\sqrt{3a-2ax^2}(cx)^{7/2}}{9c} - \frac{2}{15}c\sqrt{3a-2ax^2}(cx)^{3/2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*x)^{(5/2)}*\text{Sqrt}[3*a - 2*a*x^2], x]$

[Out]  $(-2*c*(c*x)^{(3/2)}*\text{Sqrt}[3*a - 2*a*x^2])/15 + (2*(c*x)^{(7/2)}*\text{Sqrt}[3*a - 2*a*x^2])/(9*c) - (3*6^{(1/4)}*a*c^2*\text{Sqrt}[c*x]*\text{Sqrt}[3 - 2*x^2]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[3 - \text{Sqrt}[6]*x]/\text{Sqrt}[6]], 2])/(5*\text{Sqrt}[x]*\text{Sqrt}[3*a - 2*a*x^2])$

**Rubi in Sympy [A]** time = 54.6855, size = 199, normalized size = 1.55

$$\frac{3\sqrt[4]{2} \cdot 3^{\frac{3}{4}} ac^{\frac{5}{2}} \sqrt{-\frac{2x^2}{3} + 1} E\left(\text{asin}\left(\frac{\sqrt[4]{2} \cdot 3^{\frac{3}{4}} \sqrt{cx}}{3\sqrt{c}}\right)\middle|-1\right)}{5\sqrt{-2ax^2 + 3a}} - \frac{3\sqrt[4]{2} \cdot 3^{\frac{3}{4}} ac^{\frac{5}{2}} \sqrt{-\frac{2x^2}{3} + 1} F\left(\text{asin}\left(\frac{\sqrt[4]{2} \cdot 3^{\frac{3}{4}} \sqrt{cx}}{3\sqrt{c}}\right)\middle|-1\right)}{5\sqrt{-2ax^2 + 3a}} - \frac{2c(cx)^{\frac{3}{2}} \sqrt{-2ax^2 + 3a}}{15} + \frac{2(cx)^{\frac{7}{2}} \sqrt{-2ax^2 + 3a}}{9c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c*x)**(5/2)*(-2*a*x**2+3*a)**(1/2), x)$

[Out]  $3*2**(1/4)*3**(3/4)*a*c**(5/2)*\text{sqrt}(-2*x**2/3 + 1)*\text{elliptic\_e}(\text{asin}(2**(1/4)*3**(3/4)*\text{sqrt}(c*x)/(3*\text{sqrt}(c))), -1)/(5*\text{sqrt}(-2*a*x**2 + 3*a)) - 3*2**(1/4)*3**(3/4)*a*c**(5/2)*\text{sqrt}(-2*x**2/3 + 1)*\text{elliptic\_f}(\text{asin}(2**(1/4)*3**(3/4)*\text{sqrt}(c*x)/(3*\text{sqrt}(c))), -1)/(5*\text{sqrt}(-2*a*x**2 + 3*a)) - 2*c*(c*x)**(3/2)*\text{sqrt}(-2*a*x**2 + 3*a)/15 + 2*(c*x)**(7/2)*\text{sqrt}(-2*a*x**2 + 3*a)/(9*c)$

**Mathematica [A]** time = 0.187564, size = 112, normalized size = 0.88

$$\frac{\sqrt{a(3-2x^2)}(cx)^{5/2} \left( 2\sqrt{3-2x^2}(5x^2-3)x^{3/2} - 27\sqrt[4]{6}F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\sqrt{x}\right)\middle|-1\right) + 27\sqrt[4]{6}E\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\sqrt{x}\right)\middle|-1\right) \right)}{45x^{5/2}\sqrt{3-2x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(5/2)\*Sqrt[3\*a - 2\*a\*x^2],x]

[Out] ((c\*x)^(5/2)\*Sqrt[a\*(3 - 2\*x^2)]\*(2\*x^(3/2)\*Sqrt[3 - 2\*x^2]\*(-3 + 5\*x^2) + 27\*6^(1/4)\*EllipticE[ArcSin[(2/3)^(1/4)\*Sqrt[x]], -1] - 27\*6^(1/4)\*EllipticF[ArcSin[(2/3)^(1/4)\*Sqrt[x]], -1]))/(45\*x^(5/2)\*Sqrt[3 - 2\*x^2])

**Maple [B]** time = 0.059, size = 237, normalized size = 1.9

$$\frac{c^2}{180x(2x^2-3)}\sqrt{cx}\sqrt{-a(2x^2-3)}\left(80x^6+18\sqrt{(-2x+\sqrt{3}\sqrt{2})}\sqrt{3}\sqrt{2}\sqrt{3}\sqrt{-x\sqrt{3}\sqrt{2}}\text{EllipticE}\left(\frac{1}{6}\sqrt{3}\sqrt{2}\sqrt{(2x+\sqrt{3}\sqrt{2})}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(5/2)\*(-2\*a\*x^2+3\*a)^(1/2),x)

[Out] 1/180\*c^2/x\*(c\*x)^(1/2)\*(-a\*(2\*x^2-3))^(1/2)\*(80\*x^6+18\*((-2\*x+3^(1/2)\*2^(1/2))^3^(1/2)\*2^(1/2))^(1/2)\*3^(1/2)\*(-x\*3^(1/2)\*2^(1/2))^3^(1/2)\*2^(1/2)\*EllipticE(1/6\*3^(1/2)\*2^(1/2)\*((2\*x+3^(1/2)\*2^(1/2))^3^(1/2)\*2^(1/2))^(1/2),1/2\*2^(1/2))\*((2\*x+3^(1/2)\*2^(1/2))^3^(1/2)\*2^(1/2))^(1/2)\*2^(1/2)-9\*((-2\*x+3^(1/2)\*2^(1/2))^3^(1/2)\*2^(1/2))^(1/2)\*3^(1/2)\*(-x\*3^(1/2)\*2^(1/2))^3^(1/2)\*2^(1/2)\*EllipticF(1/6\*3^(1/2)\*2^(1/2)\*((2\*x+3^(1/2)\*2^(1/2))^3^(1/2)\*2^(1/2))^(1/2),1/2\*2^(1/2))\*((2\*x+3^(1/2)\*2^(1/2))^3^(1/2)\*2^(1/2))^(1/2)\*2^(1/2)-168\*x^4+72\*x^2)/(2\*x^2-3)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-2ax^2 + 3a}(cx)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-2\*a\*x^2 + 3\*a)\*(c\*x)^(5/2),x, algorithm="maxima")

[Out] integrate(sqrt(-2\*a\*x^2 + 3\*a)\*(c\*x)^(5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-2ax^2 + 3a}\sqrt{cx}c^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-2\*a\*x^2 + 3\*a)\*(c\*x)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(-2\*a\*x^2 + 3\*a)\*sqrt(c\*x)\*c^2\*x^2, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(5/2)\*(-2\*a\*x\*\*2+3\*a)\*\*(1/2),x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-2ax^2 + 3a} (cx)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-2*a*x^2 + 3*a)*(c*x)^(5/2),x, algorithm="giac")`

[Out] `integrate(sqrt(-2*a*x^2 + 3*a)*(c*x)^(5/2), x)`

### 3.608 $\int (cx)^{3/2} \sqrt{3a - 2ax^2} dx$

**Optimal.** Leaf size=117

$$\frac{6^{3/4} ac^{3/2} \sqrt{3 - 2x^2} F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}} \sqrt{cx}}{\sqrt{c}}\right) \middle| -1\right)}{7\sqrt{a(3 - 2x^2)}} + \frac{2\sqrt{3a - 2ax^2}(cx)^{5/2}}{7c} - \frac{2}{7}c\sqrt{3a - 2ax^2}\sqrt{cx}$$

[Out]  $(-2*c*\text{Sqrt}[c*x]*\text{Sqrt}[3*a - 2*a*x^2])/7 + (2*(c*x)^{(5/2)}*\text{Sqrt}[3*a - 2*a*x^2])/(7*c) + (6^{(3/4)}*a*c^{(3/2)}*\text{Sqrt}[3 - 2*x^2]*\text{EllipticF}[\text{ArcSin}[\frac{(2/3)^{(1/4)}*\text{Sqrt}[c*x]}{\text{Sqrt}[c]}], -1])/(7*\text{Sqrt}[a*(3 - 2*x^2)])$

**Rubi [A]** time = 0.195791, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$\frac{6^{3/4} ac^{3/2} \sqrt{3 - 2x^2} F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}} \sqrt{cx}}{\sqrt{c}}\right) \middle| -1\right)}{7\sqrt{a(3 - 2x^2)}} + \frac{2\sqrt{3a - 2ax^2}(cx)^{5/2}}{7c} - \frac{2}{7}c\sqrt{3a - 2ax^2}\sqrt{cx}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*x)^{(3/2)}*\text{Sqrt}[3*a - 2*a*x^2], x]$

[Out]  $(-2*c*\text{Sqrt}[c*x]*\text{Sqrt}[3*a - 2*a*x^2])/7 + (2*(c*x)^{(5/2)}*\text{Sqrt}[3*a - 2*a*x^2])/(7*c) + (6^{(3/4)}*a*c^{(3/2)}*\text{Sqrt}[3 - 2*x^2]*\text{EllipticF}[\text{ArcSin}[\frac{(2/3)^{(1/4)}*\text{Sqrt}[c*x]}{\text{Sqrt}[c]}], -1])/(7*\text{Sqrt}[a*(3 - 2*x^2)])$

**Rubi in Sympy [A]** time = 15.3583, size = 124, normalized size = 1.06

$$\frac{3 \cdot 2^{\frac{3}{4}} \sqrt[4]{3} ac^{\frac{3}{2}} \sqrt{-\frac{2x^2}{3}} + 1F\left(\text{asin}\left(\frac{\sqrt[4]{2 \cdot 3^{\frac{3}{4}} \sqrt{cx}}}{3\sqrt{c}}\right) \middle| -1\right)}{7\sqrt{-2ax^2 + 3a}} - \frac{2c\sqrt{cx}\sqrt{-2ax^2 + 3a}}{7} + \frac{2(cx)^{\frac{5}{2}}\sqrt{-2ax^2 + 3a}}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c*x)**(3/2)*(-2*a*x**2+3*a)**(1/2), x)$

[Out]  $3*2^{(3/4)}*3^{(1/4)}*a*c^{(3/2)}*\text{sqrt}(-2*x**2/3 + 1)*\text{elliptic\_f}(\text{asin}(2^{(1/4)}*3^{(3/4)}*\text{sqrt}(c*x)/(3*\text{sqrt}(c))), -1)/(7*\text{sqrt}(-2*a*x**2 + 3*a)) - 2*c*\text{sqrt}(c*x)*\text{sqrt}(-2*a*x**2 + 3*a)/7 + 2*(c*x)**(5/2)*\text{sqrt}(-2*a*x**2 + 3*a)/(7*c)$

**Mathematica [A]** time = 0.176619, size = 86, normalized size = 0.74

$$\frac{\sqrt{a(3 - 2x^2)}(cx)^{3/2} \left( 2\sqrt{2 - \frac{3}{x^2}}(x^2 - 1)x^{3/2} + 6^{3/4} F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}}{\sqrt{x}}\right) \middle| -1\right) \right)}{7\sqrt{2 - \frac{3}{x^2}}x^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(3/2)\*Sqrt[3\*a - 2\*a\*x^2], x]

[Out] ((c\*x)^(3/2)\*Sqrt[a\*(3 - 2\*x^2)]\*(2\*Sqrt[2 - 3/x^2]\*x^(3/2)\*(-1 + x^2) + 6^(3/4)\*EllipticF[ArcSin[(3/2)^(1/4)/Sqrt[x]], -1]))/(7\*Sqrt[2 - 3/x^2]\*x^(5/2))

**Maple [A]** time = 0.041, size = 133, normalized size = 1.1

$$-\frac{c}{14x(2x^2-3)}\sqrt{cx}\sqrt{-a(2x^2-3)}\left(-8x^5+\sqrt{(-2x+\sqrt{3}\sqrt{2})\sqrt{3}\sqrt{2}}\sqrt{-x\sqrt{3}\sqrt{2}}\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{2}}{6}\sqrt{(2x+\sqrt{3}\sqrt{2})\sqrt{3}\sqrt{2}},\frac{\sqrt{3}\sqrt{2}}{6}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(3/2)\*(-2\*a\*x^2+3\*a)^(1/2), x)

[Out] -1/14\*c\*(c\*x)^(1/2)\*(-a\*(2\*x^2-3))^(1/2)\*(-8\*x^5+((-2\*x+3^(1/2)\*2^(1/2))^3^(1/2)\*2^(1/2))^(1/2)\*(-x\*3^(1/2)\*2^(1/2))^(1/2)\*EllipticF(1/6\*3^(1/2)\*2^(1/2)\*((2\*x+3^(1/2)\*2^(1/2))^3^(1/2)\*2^(1/2))^(1/2), 1/2\*2^(1/2))\*((2\*x+3^(1/2)\*2^(1/2))^3^(1/2)\*2^(1/2))^(1/2)+20\*x^3-12\*x)/x/(2\*x^2-3)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-2ax^2 + 3a}(cx)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-2\*a\*x^2 + 3\*a)\*(c\*x)^(3/2), x, algorithm="maxima")

[Out] integrate(sqrt(-2\*a\*x^2 + 3\*a)\*(c\*x)^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-2ax^2 + 3a}\sqrt{cxcx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-2\*a\*x^2 + 3\*a)\*(c\*x)^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(-2\*a\*x^2 + 3\*a)\*sqrt(c\*x)\*c\*x, x)

**Sympy [A]** time = 21.2369, size = 53, normalized size = 0.45

$$\frac{\sqrt{3}\sqrt{ac}^{\frac{3}{2}}x^{\frac{5}{2}}\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{4} \\ \frac{9}{4} \end{matrix} \middle| \frac{2x^2e^{2i\pi}}{3}\right)}{2\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(3/2)\*(-2\*a\*x\*\*2+3\*a)\*\*(1/2), x)

[Out]  $\sqrt{3} \sqrt{a} c^{3/2} x^{5/2} \gamma(5/4) \text{hyper}((-1/2, 5/4), (9/4, ), 2x^2 \exp(\text{polar}(2i\pi)/3)) / (2\gamma(9/4))$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-2ax^2 + 3a} (cx)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2),x, algorithm="giac")`

[Out] `integrate(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2), x)`



### 3.609 $\int \sqrt{cx} \sqrt{3a - 2ax^2} dx$

**Optimal.** Leaf size=99

$$\frac{2\sqrt{3a - 2ax^2}(cx)^{3/2}}{5c} - \frac{6\sqrt[4]{6a}\sqrt{3 - 2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6x}}}{\sqrt{6}}\right)\middle|2\right)}{5\sqrt{x}\sqrt{3a - 2ax^2}}$$

[Out] (2\*(c\*x)^(3/2)\*Sqrt[3\*a - 2\*a\*x^2])/(5\*c) - (6\*6^(1/4)\*a\*Sqrt[c\*x]\*Sqrt[3 - 2\*x^2]\*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]\*x]/Sqrt[6]]], 2))/(5\*Sqrt[x]\*Sqrt[3\*a - 2\*a\*x^2])

**Rubi [A]** time = 0.136814, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$\frac{2\sqrt{3a - 2ax^2}(cx)^{3/2}}{5c} - \frac{6\sqrt[4]{6a}\sqrt{3 - 2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6x}}}{\sqrt{6}}\right)\middle|2\right)}{5\sqrt{x}\sqrt{3a - 2ax^2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c\*x]\*Sqrt[3\*a - 2\*a\*x^2], x]

[Out] (2\*(c\*x)^(3/2)\*Sqrt[3\*a - 2\*a\*x^2])/(5\*c) - (6\*6^(1/4)\*a\*Sqrt[c\*x]\*Sqrt[3 - 2\*x^2]\*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]\*x]/Sqrt[6]]], 2))/(5\*Sqrt[x]\*Sqrt[3\*a - 2\*a\*x^2])

**Rubi in Sympy [A]** time = 50.228, size = 173, normalized size = 1.75

$$\frac{6\sqrt{2} \cdot 3^{\frac{3}{4}} a \sqrt{c} \sqrt{-\frac{2x^2}{3}} + 1E\left(\operatorname{asin}\left(\frac{\sqrt[4]{2} \cdot 3^{\frac{3}{4}} \sqrt{cx}}{3\sqrt{c}}\right)\middle|-1\right)}{5\sqrt{-2ax^2 + 3a}} - \frac{6\sqrt{2} \cdot 3^{\frac{3}{4}} a \sqrt{c} \sqrt{-\frac{2x^2}{3}} + 1F\left(\operatorname{asin}\left(\frac{\sqrt[4]{2} \cdot 3^{\frac{3}{4}} \sqrt{cx}}{3\sqrt{c}}\right)\middle|-1\right)}{5\sqrt{-2ax^2 + 3a}} + \frac{2(cx)^{\frac{3}{2}} \sqrt{-2ax^2 + 3a}}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(1/2)\*(-2\*a\*x\*\*2+3\*a)\*\*(1/2), x)

[Out] 6\*2\*\*(1/4)\*3\*\*(3/4)\*a\*sqrt(c)\*sqrt(-2\*x\*\*2/3 + 1)\*elliptic\_e(asin(2\*\*(1/4)\*3\*\*(3/4)\*sqrt(c\*x)/(3\*sqrt(c))), -1)/(5\*sqrt(-2\*a\*x\*\*2 + 3\*a)) - 6\*2\*\*(1/4)\*3\*\*(3/4)\*a\*sqrt(c)\*sqrt(-2\*x\*\*2/3 + 1)\*elliptic\_f(asin(2\*\*(1/4)\*3\*\*(3/4)\*sqrt(c\*x)/(3\*sqrt(c))), -1)/(5\*sqrt(-2\*a\*x\*\*2 + 3\*a)) + 2\*(c\*x)\*\*(3/2)\*sqrt(-2\*a\*x\*\*2 + 3\*a)/(5\*c)

**Mathematica [A]** time = 0.0959984, size = 106, normalized size = 1.07

$$\frac{2}{5}x\sqrt{a(3 - 2x^2)}\sqrt{cx} + \frac{6\sqrt[4]{6}\sqrt{a(3 - 2x^2)}\sqrt{cx}\left(E\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\sqrt{x}\right)\middle|-1\right) - F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\sqrt{x}\right)\middle|-1\right)\right)}{5\sqrt{3 - 2x^2}\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c\*x]\*Sqrt[3\*a - 2\*a\*x^2], x]

[Out]  $(2*x*\text{Sqrt}[c*x]*\text{Sqrt}[a*(3-2*x^2)])/5 + (6*6^{1/4}*\text{Sqrt}[c*x]*\text{Sqrt}[a*(3-2*x^2)]*(\text{EllipticE}[\text{ArcSin}[(2/3)^{1/4}*\text{Sqrt}[x]], -1] - \text{EllipticF}[\text{ArcSin}[(2/3)^{1/4}*\text{Sqrt}[x]], -1]))/(5*\text{Sqrt}[x]*\text{Sqrt}[3-2*x^2])$

**Maple [B]** time = 0.036, size = 229, normalized size = 2.3

$$\frac{1}{10x(2x^2-3)}\sqrt{cx}\sqrt{-a(2x^2-3)}\left(2\sqrt{(-2x+\sqrt{3}\sqrt{2})}\sqrt{3}\sqrt{2}\sqrt{3}\sqrt{-x\sqrt{3}\sqrt{2}}\text{EllipticE}\left(\frac{1}{6}\sqrt{3}\sqrt{2}\sqrt{(2x+\sqrt{3}\sqrt{2})}\sqrt{3}\sqrt{2}, 1/2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2), x)`

[Out]  $1/10*(c*x)^{1/2}*(-a*(2*x^2-3))^{1/2}*(2*((-2*x+3^{1/2})^{1/2})^{1/2})^{1/2}*(3^{1/2})^{1/2}*(-x*3^{1/2})^{1/2}*(2^{1/2})^{1/2}*\text{EllipticE}(1/6*3^{1/2})^{1/2}*(2*x+3^{1/2})^{1/2}*(2^{1/2})^{1/2}*(3^{1/2})^{1/2}*(2^{1/2})^{1/2})^{1/2}*(1/2*2^{1/2})^{1/2}*((2*x+3^{1/2})^{1/2})^{1/2}*(3^{1/2})^{1/2}*(2^{1/2})^{1/2})^{1/2}-( (-2*x+3^{1/2})^{1/2}*(2^{1/2})^{1/2}*(3^{1/2})^{1/2}*(2^{1/2})^{1/2})^{1/2}*(3^{1/2})^{1/2}*(-x*3^{1/2})^{1/2}*(2^{1/2})^{1/2})*\text{EllipticF}(1/6*3^{1/2})^{1/2}*(2*x+3^{1/2})^{1/2}*(2^{1/2})^{1/2}*(3^{1/2})^{1/2}*(2^{1/2})^{1/2})^{1/2}*(1/2*2^{1/2})^{1/2}*(2*x+3^{1/2})^{1/2}*(2^{1/2})^{1/2}*(3^{1/2})^{1/2}*(2^{1/2})^{1/2})^{1/2}+8*x^4-12*x^2)/x/(2*x^2-3)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-2ax^2 + 3a}\sqrt{cx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x), x, algorithm="maxima")`

[Out] `integrate(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-2ax^2 + 3a}\sqrt{cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x), x, algorithm="fricas")`

[Out] `integral(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x), x)`

**Sympy [A]** time = 3.20509, size = 53, normalized size = 0.54

$$\frac{\sqrt{3}\sqrt{a}\sqrt{cx}^{\frac{3}{2}}\left(\frac{3}{4}\right) {}_2F_1\left(\frac{-\frac{1}{2}, \frac{3}{4}}{\frac{7}{4}} \middle| \frac{2x^2 e^{2i\pi}}{3}\right)}{2\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(1/2)*(-2*a*x**2+3*a)**(1/2), x)`

[Out]  $\sqrt{3} \sqrt{a} \sqrt{c} x^{3/2} \gamma(3/4) \operatorname{hyper}\left(-1/2, 3/4, (7/4, ), 2x^2 \exp_{\text{polar}}(2i\pi)/3\right) / (2\gamma(7/4))$

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-2ax^2 + 3a} \sqrt{cx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x), x, algorithm="giac")`

[Out] `integrate(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x), x)`

$$3.610 \quad \int \frac{\sqrt{3a-2ax^2}}{\sqrt{cx}} dx$$

**Optimal.** Leaf size=94

$$\frac{2\sqrt{3a-2ax^2}\sqrt{cx}}{3c} + \frac{2 \cdot 2^{3/4} a \sqrt{3-2x^2} F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right) \middle| -1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{a(3-2x^2)}}$$

[Out] (2\*Sqrt[c\*x]\*Sqrt[3\*a - 2\*a\*x^2])/(3\*c) + (2\*2^(3/4)\*a\*Sqrt[3 - 2\*x^2]\*EllipticF[ArcSin[((2/3)^(1/4)\*Sqrt[c\*x])/Sqrt[c]], -1])/(3^(1/4)\*Sqrt[c]\*Sqrt[a\*(3 - 2\*x^2)])

**Rubi [A]** time = 0.13336, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{2\sqrt{3a-2ax^2}\sqrt{cx}}{3c} + \frac{2 \cdot 2^{3/4} a \sqrt{3-2x^2} F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right) \middle| -1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{a(3-2x^2)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3\*a - 2\*a\*x^2]/Sqrt[c\*x], x]

[Out] (2\*Sqrt[c\*x]\*Sqrt[3\*a - 2\*a\*x^2])/(3\*c) + (2\*2^(3/4)\*a\*Sqrt[3 - 2\*x^2]\*EllipticF[ArcSin[((2/3)^(1/4)\*Sqrt[c\*x])/Sqrt[c]], -1])/(3^(1/4)\*Sqrt[c]\*Sqrt[a\*(3 - 2\*x^2)])

**Rubi in Sympy [A]** time = 12.0209, size = 97, normalized size = 1.03

$$\frac{2 \cdot 2^{\frac{3}{4}} \sqrt[4]{3a} \sqrt{-\frac{2x^2}{3}} + 1 F\left(\operatorname{asin}\left(\frac{\sqrt[4]{2 \cdot 3^{\frac{3}{4}} \sqrt{cx}}}{3\sqrt{c}}\right) \middle| -1\right)}{\sqrt{c}\sqrt{-2ax^2 + 3a}} + \frac{2\sqrt{cx}\sqrt{-2ax^2 + 3a}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-2\*a\*x\*\*2+3\*a)\*\*(1/2)/(c\*x)\*\*(1/2), x)

[Out] 2\*2\*\*(3/4)\*3\*\*(1/4)\*a\*sqrt(-2\*x\*\*2/3 + 1)\*elliptic\_f(asin(2\*\*(1/4)\*3\*\*(3/4)\*sqrt(c\*x)/(3\*sqrt(c))), -1)/(sqrt(c)\*sqrt(-2\*a\*x\*\*2 + 3\*a)) + 2\*sqrt(c\*x)\*sqrt(-2\*a\*x\*\*2 + 3\*a)/(3\*c)

**Mathematica [A]** time = 0.0676898, size = 80, normalized size = 0.85

$$\frac{2\sqrt{a(3-2x^2)}\left(\sqrt{2-\frac{3}{x^2}x^{3/2}} + 6^{3/4}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}}{\sqrt{x}}\right) \middle| -1\right)\right)}{3\sqrt{2-\frac{3}{x^2}}\sqrt{x}\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3\*a - 2\*a\*x^2]/Sqrt[c\*x], x]

[Out] (2\*Sqrt[a\*(3 - 2\*x^2)]\*(Sqrt[2 - 3/x^2]\*x^(3/2) + 6^(3/4)\*EllipticF[ArcSin[(3/2)^(1/4)/Sqrt[x]], -1]))/(3\*Sqrt[2 - 3/x^2]\*Sqrt[x]\*Sqrt[c\*x])

**Maple [A]** time = 0.039, size = 124, normalized size = 1.3

$$-\frac{1}{6x^2-9}\sqrt{-a(2x^2-3)}\left(\sqrt{(-2x+\sqrt{3}\sqrt{2})}\sqrt{3}\sqrt{2}\sqrt{-x\sqrt{3}\sqrt{2}}\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{2}}{6}\sqrt{(2x+\sqrt{3}\sqrt{2})}\sqrt{3}\sqrt{2},\frac{\sqrt{2}}{2}\right)\sqrt{(2x+\sqrt{3}\sqrt{2})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2\*a\*x^2+3\*a)^(1/2)/(c\*x)^(1/2), x)

[Out] -1/3\*(-a\*(2\*x^2-3))^(1/2)\*((( -2\*x+3^(1/2)\*2^(1/2))^3^(1/2)\*2^(1/2))^(1/2)\*(-x\*3^(1/2)\*2^(1/2))^(1/2)\*EllipticF(1/6\*3^(1/2)\*2^(1/2)\*((2\*x+3^(1/2)\*2^(1/2))^3^(1/2)\*2^(1/2))^(1/2), 1/2\*2^(1/2))\*((2\*x+3^(1/2)\*2^(1/2))^3^(1/2)\*2^(1/2))^(1/2)-4\*x^3+6\*x)/(c\*x)^(1/2)/(2\*x^2-3)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-2ax^2 + 3a}}{\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-2\*a\*x^2 + 3\*a)/sqrt(c\*x), x, algorithm="maxima")

[Out] integrate(sqrt(-2\*a\*x^2 + 3\*a)/sqrt(c\*x), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-2ax^2 + 3a}}{\sqrt{cx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-2\*a\*x^2 + 3\*a)/sqrt(c\*x), x, algorithm="fricas")

[Out] integral(sqrt(-2\*a\*x^2 + 3\*a)/sqrt(c\*x), x)

**Sympy [A]** time = 2.74152, size = 53, normalized size = 0.56

$$\frac{\sqrt{3}\sqrt{a}\sqrt{x}\left(\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{1}{4} \middle| \frac{2x^2 e^{2i\pi}}{3}\right)}{2\sqrt{c}\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*a\*x\*\*2+3\*a)\*\*(1/2)/(c\*x)\*\*(1/2), x)

```
[Out] sqrt(3)*sqrt(a)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4, ), 2*x
**2*exp_polar(2*I*pi)/3)/(2*sqrt(c)*gamma(5/4))
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-2ax^2 + 3a}}{\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-2*a*x^2 + 3*a)/sqrt(c*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-2*a*x^2 + 3*a)/sqrt(c*x), x)
```

$$3.611 \quad \int \frac{\sqrt{3a-2ax^2}}{(cx)^{3/2}} dx$$

**Optimal.** Leaf size=98

$$\frac{4\sqrt[4]{6a}\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6x}}}{\sqrt{6}}\right)\middle|2\right)}{c^2\sqrt{x}\sqrt{3a-2ax^2}} - \frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}$$

[Out]  $(-2*\text{Sqrt}[3*a - 2*a*x^2])/(c*\text{Sqrt}[c*x]) + (4*6^{(1/4)}*a*\text{Sqrt}[c*x]*\text{Sqrt}[3 - 2*x^2]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[3 - \text{Sqrt}[6]*x]/\text{Sqrt}[6]], 2])/(c^2*\text{Sqrt}[x]*\text{Sqrt}[3*a - 2*a*x^2])$

**Rubi [A]** time = 0.141477, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$\frac{4\sqrt[4]{6a}\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6x}}}{\sqrt{6}}\right)\middle|2\right)}{c^2\sqrt{x}\sqrt{3a-2ax^2}} - \frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[3*a - 2*a*x^2]/(c*x)^{(3/2)}, x]$

[Out]  $(-2*\text{Sqrt}[3*a - 2*a*x^2])/(c*\text{Sqrt}[c*x]) + (4*6^{(1/4)}*a*\text{Sqrt}[c*x]*\text{Sqrt}[3 - 2*x^2]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[3 - \text{Sqrt}[6]*x]/\text{Sqrt}[6]], 2])/(c^2*\text{Sqrt}[x]*\text{Sqrt}[3*a - 2*a*x^2])$

**Rubi in Sympy [A]** time = 50.7487, size = 168, normalized size = 1.71

$$\frac{4\sqrt[4]{2} \cdot 3^{3/4} a \sqrt{-\frac{2x^2}{3}} + 1E\left(\text{asin}\left(\frac{\sqrt[4]{2} \cdot 3^{3/4} \sqrt{cx}}{3\sqrt{c}}\right)\middle|-1\right)}{c^{3/2} \sqrt{-2ax^2 + 3a}} + \frac{4\sqrt[4]{2} \cdot 3^{3/4} a \sqrt{-\frac{2x^2}{3}} + 1F\left(\text{asin}\left(\frac{\sqrt[4]{2} \cdot 3^{3/4} \sqrt{cx}}{3\sqrt{c}}\right)\middle|-1\right)}{c^{3/2} \sqrt{-2ax^2 + 3a}} - \frac{2\sqrt{-2ax^2 + 3a}}{c\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-2*a*x**2+3*a)**(1/2)/(c*x)**(3/2), x)$

[Out]  $-4*2^{(1/4)}*3^{(3/4)}*a*\text{sqrt}(-2*x**2/3 + 1)*\text{elliptic\_e}(\text{asin}(2^{(1/4)}*3^{(3/4)}*\text{sqrt}(c*x)/(3*\text{sqrt}(c))), -1)/(c^{(3/2)}*\text{sqrt}(-2*a*x**2 + 3*a)) + 4*2^{(1/4)}*3^{(3/4)}*a*\text{sqrt}(-2*x**2/3 + 1)*\text{elliptic\_f}(\text{asin}(2^{(1/4)}*3^{(3/4)}*\text{sqrt}(c*x)/(3*\text{sqrt}(c))), -1)/(c^{(3/2)}*\text{sqrt}(-2*a*x**2 + 3*a)) - 2*\text{sqrt}(-2*a*x**2 + 3*a)/(c*\text{sqrt}(c*x))$

**Mathematica [A]** time = 0.13006, size = 83, normalized size = 0.85

$$\frac{2x\sqrt{a(3-2x^2)}\left(\frac{2\sqrt[4]{6}\sqrt{x}\left(E\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\sqrt{x}\right)\middle|-1\right)-F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\sqrt{x}\right)\middle|-1\right)\right)}{\sqrt{3-2x^2}}-1\right)}{(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3\*a - 2\*a\*x^2]/(c\*x)^(3/2), x]

[Out] (2\*x\*Sqrt[a\*(3 - 2\*x^2)]\*(-1 - (2\*6^(1/4)\*Sqrt[x]\*(EllipticE[ArcSin[(2/3)^(1/4)\*Sqrt[x]], -1] - EllipticF[ArcSin[(2/3)^(1/4)\*Sqrt[x]], -1]))/Sqrt[3 - 2\*x^2]))/(c\*x)^(3/2)

**Maple [B]** time = 0.049, size = 225, normalized size = 2.3

$$-\frac{1}{3c(2x^2-3)}\sqrt{-a(2x^2-3)}\left(2\sqrt{(-2x+\sqrt{3}\sqrt{2})}\sqrt{3}\sqrt{2}\sqrt{3}\sqrt{-x\sqrt{3}\sqrt{2}}\text{EllipticE}\left(\frac{1}{6}\sqrt{3}\sqrt{2},\sqrt{(2x+\sqrt{3}\sqrt{2})}\sqrt{3}\sqrt{2},\frac{1}{2}\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2\*a\*x^2+3\*a)^(1/2)/(c\*x)^(3/2), x)

[Out] -1/3\*(-a\*(2\*x^2-3))^(1/2)\*(2\*((-2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2)\*3^(1/2)\*(-x\*3^(1/2)\*2^(1/2))^(1/2)\*EllipticE(1/6\*3^(1/2)\*2^(1/2)\*((2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2), 1/2\*2^(1/2))\*((2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2)\*2^(1/2)-((-2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2)\*3^(1/2)\*(-x\*3^(1/2)\*2^(1/2))^(1/2)\*EllipticF(1/6\*3^(1/2)\*2^(1/2)\*((2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2), 1/2\*2^(1/2))\*((2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2)\*2^(1/2)+12\*x^2-18)/c/(c\*x)^(1/2)/(2\*x^2-3)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-2ax^2 + 3a}}{(cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-2\*a\*x^2 + 3\*a)/(c\*x)^(3/2), x, algorithm="maxima")

[Out] integrate(sqrt(-2\*a\*x^2 + 3\*a)/(c\*x)^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-2ax^2 + 3a}}{\sqrt{cxcx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-2\*a\*x^2 + 3\*a)/(c\*x)^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(-2\*a\*x^2 + 3\*a)/(sqrt(c\*x)\*c\*x), x)

**Sympy [A]** time = 5.07206, size = 56, normalized size = 0.57

$$\frac{\sqrt{3}\sqrt{a}\left(-\frac{1}{4}\right) {}_2F_1\left(\left(-\frac{1}{2}, -\frac{1}{4}\right)\left|\frac{2x^2 e^{2i\pi}}{3}\right.\right)}{2c^{\frac{3}{2}}\sqrt{x}\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((-2\*a\*x\*\*2+3\*a)\*\*(1/2)/(c\*x)\*\*(3/2),x)

[Out] sqrt(3)\*sqrt(a)\*gamma(-1/4)\*hyper((-1/2, -1/4), (3/4, ), 2\*x\*\*2\*exp\_polar(2\*I\*pi)/3)/(2\*c\*\*(3/2)\*sqrt(x)\*gamma(3/4))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-2ax^2 + 3a}}{(cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-2\*a\*x^2 + 3\*a)/(c\*x)^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(-2\*a\*x^2 + 3\*a)/(c\*x)^(3/2), x)

$$3.612 \quad \int \frac{\sqrt{3a-2ax^2}}{(cx)^{5/2}} dx$$

Optimal. Leaf size=96

$$\frac{4 \cdot 2^{3/4} a \sqrt{3-2x^2} F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}} \sqrt{cx}}{\sqrt{c}}\right) \middle| -1\right)}{3 \sqrt[4]{3} c^{5/2} \sqrt{a(3-2x^2)}} - \frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}}$$

[Out]  $(-2 \cdot \text{Sqrt}[3 \cdot a - 2 \cdot a \cdot x^2]) / (3 \cdot c \cdot (c \cdot x)^{(3/2)}) - (4 \cdot 2^{(3/4)} \cdot a \cdot \text{Sqrt}[3 - 2 \cdot x^2] \cdot \text{EllipticF}[\text{ArcSin}[(2/3)^{(1/4)} \cdot \text{Sqrt}[c \cdot x]] / \text{Sqrt}[c]], -1)) / (3 \cdot 3^{(1/4)} \cdot c^{(5/2)} \cdot \text{Sqrt}[a \cdot (3 - 2 \cdot x^2)])$

Rubi [A] time = 0.132355, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{4 \cdot 2^{3/4} a \sqrt{3-2x^2} F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}} \sqrt{cx}}{\sqrt{c}}\right) \middle| -1\right)}{3 \sqrt[4]{3} c^{5/2} \sqrt{a(3-2x^2)}} - \frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3\*a - 2\*a\*x^2]/(c\*x)^(5/2), x]

[Out]  $(-2 \cdot \text{Sqrt}[3 \cdot a - 2 \cdot a \cdot x^2]) / (3 \cdot c \cdot (c \cdot x)^{(3/2)}) - (4 \cdot 2^{(3/4)} \cdot a \cdot \text{Sqrt}[3 - 2 \cdot x^2] \cdot \text{EllipticF}[\text{ArcSin}[(2/3)^{(1/4)} \cdot \text{Sqrt}[c \cdot x]] / \text{Sqrt}[c]], -1)) / (3 \cdot 3^{(1/4)} \cdot c^{(5/2)} \cdot \text{Sqrt}[a \cdot (3 - 2 \cdot x^2)])$

Rubi in Sympy [A] time = 12.1028, size = 100, normalized size = 1.04

$$\frac{4 \cdot 2^{3/4} \sqrt[4]{3a} \sqrt{-\frac{2x^2}{3}} + 1 F\left(\text{asin}\left(\frac{\sqrt[4]{2 \cdot 3^{3/4}} \sqrt{cx}}{3 \sqrt{c}}\right) \middle| -1\right)}{3c^{5/2} \sqrt{-2ax^2 + 3a}} - \frac{2\sqrt{-2ax^2 + 3a}}{3c(cx)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-2\*a\*x\*\*2+3\*a)\*\*(1/2)/(c\*x)\*\*(5/2), x)

[Out]  $-4 \cdot 2^{(3/4)} \cdot 3^{(1/4)} \cdot a \cdot \text{sqrt}(-2 \cdot x^2/3 + 1) \cdot \text{elliptic\_f}(\text{asin}(2^{(1/4)} \cdot 3^{(3/4)} \cdot \text{sqrt}(c \cdot x) / (3 \cdot \text{sqrt}(c))), -1) / (3 \cdot c^{(5/2)} \cdot \text{sqrt}(-2 \cdot a \cdot x^2 + 3 \cdot a)) - 2 \cdot \text{sqrt}(-2 \cdot a \cdot x^2 + 3 \cdot a) / (3 \cdot c \cdot (c \cdot x)^{(3/2)})$

Mathematica [A] time = 0.077536, size = 78, normalized size = 0.81

$$\frac{2x\sqrt{a(3-2x^2)} \left( 3\sqrt{2-\frac{3}{x^2}} + 2 \cdot 6^{3/4} \sqrt{x} F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}}{\sqrt{x}}\right) \middle| -1\right) \right)}{9\sqrt{2-\frac{3}{x^2}}(cx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3\*a - 2\*a\*x^2]/(c\*x)^(5/2), x]

[Out]  $(-2*x*\text{Sqrt}[a*(3 - 2*x^2)]*(3*\text{Sqrt}[2 - 3/x^2] + 2*6^{3/4}*\text{Sqrt}[x]*\text{EllipticF}[\text{ArcSin}[(3/2)^{1/4}/\text{Sqrt}[x]], -1]))/(9*\text{Sqrt}[2 - 3/x^2]*(c*x)^{5/2})$

**Maple [A]** time = 0.041, size = 129, normalized size = 1.3

$$\frac{2}{9xc^2(2x^2-3)}\sqrt{-a(2x^2-3)}\left(\sqrt{(-2x+\sqrt{3}\sqrt{2})}\sqrt{3}\sqrt{2}\sqrt{-x\sqrt{3}\sqrt{2}}\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{2}}{6}\sqrt{(2x+\sqrt{3}\sqrt{2})}\sqrt{3}\sqrt{2}, \frac{\sqrt{2}}{2}\right)\sqrt{(2x+\sqrt{3}\sqrt{2})}\sqrt{3}\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2\*a\*x^2+3\*a)^(1/2)/(c\*x)^(5/2), x)

[Out]  $2/9*(-a*(2*x^2-3))^{1/2}*((( -2*x+3^{1/2})^{1/2})^{1/2})^{1/2})^{1/2}*(-x*3^{1/2})^{1/2})^{1/2}*\text{EllipticF}(1/6*3^{1/2})^{1/2})^{1/2}*((2*x+3^{1/2})^{1/2})^{1/2})^{1/2}, 1/2*2^{1/2})^{1/2}*((2*x+3^{1/2})^{1/2})^{1/2})^{1/2})^{1/2}*x-6*x^2+9)/x/c^2/(c*x)^{1/2}/(2*x^2-3)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-2ax^2 + 3a}}{(cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-2\*a\*x^2 + 3\*a)/(c\*x)^(5/2), x, algorithm="maxima")

[Out] integrate(sqrt(-2\*a\*x^2 + 3\*a)/(c\*x)^(5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-2ax^2 + 3a}}{\sqrt{cxc^2x^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-2\*a\*x^2 + 3\*a)/(c\*x)^(5/2), x, algorithm="fricas")

[Out] integral(sqrt(-2\*a\*x^2 + 3\*a)/(sqrt(c\*x)\*c^2\*x^2), x)

**Sympy [A]** time = 24.0874, size = 49, normalized size = 0.51

$$\frac{\sqrt{2}i\sqrt{a}\left(-\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{2}, \frac{1}{4}\left|\frac{3}{2x^2}\right.\right)}{2c^{\frac{5}{2}}\sqrt{x}\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2\*a\*x\*\*2+3\*a)\*\*(1/2)/(c\*x)\*\*(5/2), x)

[Out]  $\sqrt{2} \cdot I \cdot \sqrt{a} \cdot \text{gamma}(-1/4) \cdot \text{hyper}((-1/2, 1/4), (5/4, ), 3/(2 \cdot x^{2})) / (2 \cdot c^{5/2} \cdot \sqrt{x} \cdot \text{gamma}(3/4))$

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-2ax^2 + 3a}}{(cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(5/2), x, algorithm="giac")`

[Out] `integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(5/2), x)`

$$3.613 \quad \int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=156

$$\frac{5a^{7/4}c^{7/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{21b^{9/4}\sqrt{a+bx^2}} - \frac{10ac^3\sqrt{cx}\sqrt{a+bx^2}}{21b^2} + \frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b}$$

[Out]  $(-10*a*c^3*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(21*b^2) + (2*c*(c*x)^{(5/2)}*\text{Sqrt}[a + b*x^2])/(7*b) + (5*a^{(7/4)}*c^{(7/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(21*b^{(9/4)}*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.260321, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{5a^{7/4}c^{7/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{21b^{9/4}\sqrt{a+bx^2}} - \frac{10ac^3\sqrt{cx}\sqrt{a+bx^2}}{21b^2} + \frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*x)^{(7/2)}/\text{Sqrt}[a + b*x^2], x]$

[Out]  $(-10*a*c^3*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(21*b^2) + (2*c*(c*x)^{(5/2)}*\text{Sqrt}[a + b*x^2])/(7*b) + (5*a^{(7/4)}*c^{(7/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(21*b^{(9/4)}*\text{Sqrt}[a + b*x^2])$

**Rubi in Sympy [A]** time = 24.0598, size = 144, normalized size = 0.92

$$\frac{5a^{7/4}c^{7/2} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) F\left(2 \text{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{21b^{9/4}\sqrt{a+bx^2}} - \frac{10ac^3\sqrt{cx}\sqrt{a+bx^2}}{21b^2} + \frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c*x)^{(7/2)}/(b*x^2+a)^{(1/2)}, x)$

[Out]  $5*a^{(7/4)}*c^{(7/2)}*\text{sqrt}((a + b*x^2)/(\text{sqrt}(a) + \text{sqrt}(b)*x)^2)*(\text{sqrt}(a) + \text{sqrt}(b)*x)*\text{elliptic\_f}(2*\text{atan}(b^{(1/4)}*\text{sqrt}(c*x)/(a^{(1/4)}*\text{sqrt}(c))), 1/2)/(21*b^{(9/4)}*\text{sqrt}(a + b*x^2)) - 10*a*c^3*\text{sqrt}(c*x)*\text{sqrt}(a + b*x^2)/(21*b^2) + 2*c*(c*x)^{(5/2)}*\text{sqrt}(a + b*x^2)/(7*b)$

**Mathematica [C]** time = 0.171411, size = 144, normalized size = 0.92

$$\frac{2c^3\sqrt{cx} \left( \sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} (-5a^2 - 2abx^2 + 3b^2x^4) + 5ia^2\sqrt{x}\sqrt{\frac{a}{bx^2}} + 1F\left(i \sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right) \middle| -1 \right) \right)}{21b^2 \sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} \sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(7/2)/Sqrt[a + b\*x^2], x]

[Out] (2\*c^3\*Sqrt[c\*x]\*(Sqrt[(I\*Sqrt[a])/Sqrt[b]]\*(-5\*a^2 - 2\*a\*b\*x^2 + 3\*b^2\*x^4) + (5\*I)\*a^2\*Sqrt[1 + a/(b\*x^2)]\*Sqrt[x]\*EllipticF[I\*ArcSinh[Sqrt[(I\*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1]))/(21\*Sqrt[(I\*Sqrt[a])/Sqrt[b]]\*b^2\*Sqrt[a + b\*x^2])

**Maple [A]** time = 0.034, size = 141, normalized size = 0.9

$$\frac{c^3}{21 b^3 x} \sqrt{c x} \left( 5 \sqrt{-a b} \sqrt{\frac{b x + \sqrt{-a b}}{\sqrt{-a b}}} \sqrt{\frac{-b x + \sqrt{-a b}}{\sqrt{-a b}}} \sqrt{-\frac{b x}{\sqrt{-a b}}} \operatorname{EllipticF} \left( \sqrt{\frac{b x + \sqrt{-a b}}{\sqrt{-a b}}}, 1/2 \sqrt{2} \right) \sqrt{2 a^2 + 6 b^3 x^5 - 4 a b^2 x^3 - 10 a^2 b x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(7/2)/(b\*x^2+a)^(1/2), x)

[Out] 1/21\*c^3/x\*(c\*x)^(1/2)/(b\*x^2+a)^(1/2)\*(5\*(-a\*b)^(1/2)\*((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*((-b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*(-x\*b/(-a\*b)^(1/2))^(1/2)\*EllipticF((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2), 1/2\*2^(1/2))\*2^(1/2)\*a^2+6\*b^3\*x^5-4\*a\*b^2\*x^3-10\*a^2\*b\*x)/b^3

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(c x)^{\frac{7}{2}}}{\sqrt{b x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(7/2)/sqrt(b\*x^2 + a), x, algorithm="maxima")

[Out] integrate((c\*x)^(7/2)/sqrt(b\*x^2 + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{\sqrt{c x} c^3 x^3}{\sqrt{b x^2 + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(7/2)/sqrt(b\*x^2 + a), x, algorithm="fricas")

[Out] integral(sqrt(c\*x)\*c^3\*x^3/sqrt(b\*x^2 + a), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(7/2)/(b\*x\*\*2+a)\*\*(1/2), x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{7}{2}}}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(7/2)/sqrt(b\*x^2 + a),x, algorithm="giac")

[Out] integrate((c\*x)^(7/2)/sqrt(b\*x^2 + a), x)

$$3.614 \quad \int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=273

$$\begin{aligned} & \frac{3a^{5/4}c^{5/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}} \\ & + \frac{6a^{5/4}c^{5/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}} \\ & - \frac{6ac^2\sqrt{cx}\sqrt{a+bx^2}}{5b^{3/2}(\sqrt{a} + \sqrt{bx})} + \frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b} \end{aligned}$$

[Out] (2\*c\*(c\*x)^(3/2)\*Sqrt[a + b\*x^2])/(5\*b) - (6\*a\*c^2\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(5\*b^(3/2)\*(Sqrt[a] + Sqrt[b]\*x)) + (6\*a^(5/4)\*c^(5/2)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticE[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(5\*b^(7/4)\*Sqrt[a + b\*x^2]) - (3\*a^(5/4)\*c^(5/2)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(5\*b^(7/4)\*Sqrt[a + b\*x^2])

**Rubi [A]** time = 0.514288, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\begin{aligned} & \frac{3a^{5/4}c^{5/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}} \\ & + \frac{6a^{5/4}c^{5/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}} \\ & - \frac{6ac^2\sqrt{cx}\sqrt{a+bx^2}}{5b^{3/2}(\sqrt{a} + \sqrt{bx})} + \frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(5/2)/Sqrt[a + b\*x^2], x]

[Out] (2\*c\*(c\*x)^(3/2)\*Sqrt[a + b\*x^2])/(5\*b) - (6\*a\*c^2\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(5\*b^(3/2)\*(Sqrt[a] + Sqrt[b]\*x)) + (6\*a^(5/4)\*c^(5/2)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticE[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(5\*b^(7/4)\*Sqrt[a + b\*x^2]) - (3\*a^(5/4)\*c^(5/2)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(5\*b^(7/4)\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 48.8616, size = 252, normalized size = 0.92

$$\begin{aligned} & \frac{6a^{\frac{5}{4}}c^{\frac{5}{2}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a} + \sqrt{bx}) E\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5b^{\frac{7}{4}}\sqrt{a+bx^2}} \\ & - \frac{3a^{\frac{5}{4}}c^{\frac{5}{2}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a} + \sqrt{bx}) F\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5b^{\frac{7}{4}}\sqrt{a+bx^2}} - \frac{6ac^2\sqrt{cx}\sqrt{a+bx^2}}{5b^{\frac{3}{2}}(\sqrt{a} + \sqrt{bx})} + \frac{2c(cx)^{\frac{3}{2}}\sqrt{a+bx^2}}{5b} \end{aligned}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x)**(5/2)/(b*x**2+a)**(1/2),x)`

[Out]  $6*a^{5/4}*c^{5/2}*sqrt((a+b*x^2)/(sqrt(a)+sqrt(b)*x)^2)*(sqrt(a)+sqrt(b)*x)*elliptic_e(2*atan(b^{1/4}*sqrt(c*x)/(a^{1/4}*sqrt(c))),1/2)/(5*b^{7/4}*sqrt(a+b*x^2))-3*a^{5/4}*c^{5/2}*sqrt((a+b*x^2)/(sqrt(a)+sqrt(b)*x)^2)*(sqrt(a)+sqrt(b)*x)*elliptic_f(2*atan(b^{1/4}*sqrt(c*x)/(a^{1/4}*sqrt(c))),1/2)/(5*b^{7/4}*sqrt(a+b*x^2))-6*a*c^2*sqrt(c*x)*sqrt(a+b*x^2)/(5*b^{3/2}*(sqrt(a)+sqrt(b)*x))+2*c*(c*x)^{3/2}*sqrt(a+b*x^2)/(5*b)$

**Mathematica [C]** time = 0.229875, size = 177, normalized size = 0.65

$$\frac{2c^2\sqrt{cx}\left(3a^{3/2}\sqrt{\frac{bx^2}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)-1\right)-3a^{3/2}\sqrt{\frac{bx^2}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)-1\right)+\sqrt{bx}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}(a+bx^2)\right)}{5b^{3/2}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c*x)^(5/2)/Sqrt[a+b*x^2],x]`

[Out]  $(2*c^2*sqrt(c*x)*(sqrt(b)*x*sqrt((I*sqrt(b)*x)/sqrt(a))*(a+b*x^2)-3*a^{3/2}*sqrt(1+(b*x^2)/a)*EllipticE[I*ArcSinh[Sqrt[(I*sqrt(b)*x)/sqrt(a)]],-1]+3*a^{3/2}*sqrt(1+(b*x^2)/a)*EllipticF[I*ArcSinh[Sqrt[(I*sqrt(b)*x)/sqrt(a)]],-1]))/(5*b^{3/2}*sqrt((I*sqrt(b)*x)/sqrt(a))*sqrt(a+b*x^2))$

**Maple [A]** time = 0.034, size = 210, normalized size = 0.8

$$-\frac{c^2}{5b^2x}\sqrt{cx}\left(6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx}{\sqrt{-ab}}}\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2,\sqrt{2}\right)\sqrt{2a^2}-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(5/2)/(b*x^2+a)^(1/2),x)`

[Out]  $-1/5*c^2/x*(c*x)^{1/2}/(b*x^2+a)^{1/2}/b^2*(6*((b*x+(-a*b))^{1/2})/((-a*b))^{1/2})^{1/2}*((-b*x+(-a*b))^{1/2})/((-a*b))^{1/2})^{1/2}*(-x*b/((-a*b))^{1/2})^{1/2}*EllipticE(((b*x+(-a*b))^{1/2})/((-a*b))^{1/2})^{1/2},1/2*2^{1/2})^{1/2}*a^2-3*((b*x+(-a*b))^{1/2})/((-a*b))^{1/2})^{1/2}*((-b*x+(-a*b))^{1/2})/((-a*b))^{1/2})^{1/2}*(-x*b/((-a*b))^{1/2})^{1/2}*EllipticF(((b*x+(-a*b))^{1/2})/((-a*b))^{1/2})^{1/2},1/2*2^{1/2})^{1/2}*a^2-2*b^2*x^4-2*a*b*x^2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{\sqrt{bx^2+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(5/2)/sqrt(b*x^2+a),x,algorithm="maxima")`

[Out] `integrate((c*x)^(5/2)/sqrt(b*x^2+a),x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}c^2x^2}{\sqrt{bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(5/2)/sqrt(b*x^2 + a),x, algorithm="fricas")`

[Out] `integral(sqrt(c*x)*c^2*x^2/sqrt(b*x^2 + a), x)`

---

**Sympy [A]** time = 140.009, size = 44, normalized size = 0.16

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\left(\frac{7}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{7}{4} \middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(5/2)/(b*x**2+a)**(1/2),x)`

[Out] `c**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(11/4))`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{\sqrt{bx^2+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(5/2)/sqrt(b*x^2 + a),x, algorithm="giac")`

[Out] `integrate((c*x)^(5/2)/sqrt(b*x^2 + a), x)`

$$3.615 \quad \int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=127

$$\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b} - \frac{a^{3/4}c^{3/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}$$

[Out] (2\*c\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(3\*b) - (a^(3/4)\*c^(3/2)\*(Sqrt[a + Sqrt[b]\*x]\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(3\*b^(5/4)\*Sqrt[a + b\*x^2])

**Rubi [A]** time = 0.201396, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b} - \frac{a^{3/4}c^{3/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(3/2)/Sqrt[a + b\*x^2], x]

[Out] (2\*c\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(3\*b) - (a^(3/4)\*c^(3/2)\*(Sqrt[a + Sqrt[b]\*x]\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(3\*b^(5/4)\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 17.58, size = 114, normalized size = 0.9

$$-\frac{a^{\frac{3}{4}}c^{\frac{3}{2}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a} + \sqrt{bx})F\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{3b^{\frac{5}{4}}\sqrt{a+bx^2}} + \frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(3/2)/(b\*x\*\*2+a)\*\*(1/2), x)

[Out] -a\*\*(3/4)\*c\*\*(3/2)\*sqrt((a + b\*x\*\*2)/(sqrt(a) + sqrt(b)\*x)\*\*2)\*(sqrt(a) + sqrt(b)\*x)\*elliptic\_f(2\*atan(b\*\*(1/4)\*sqrt(c\*x)/(a\*\*(1/4)\*sqrt(c))), 1/2)/(3\*b\*\*(5/4)\*sqrt(a + b\*x\*\*2)) + 2\*c\*sqrt(c\*x)\*sqrt(a + b\*x\*\*2)/(3\*b)

**Mathematica [C]** time = 0.210603, size = 106, normalized size = 0.83

$$\frac{2c\sqrt{cx} \left( -\frac{ia\sqrt{x}\sqrt{\frac{a}{bx^2}+1}F\left(i \sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right) \middle| -1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}} + a + bx^2 \right)}{3b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(3/2)/Sqrt[a + b\*x^2], x]

[Out] (2\*c\*Sqrt[c\*x]\*(a + b\*x^2 - (I\*a\*Sqrt[1 + a/(b\*x^2)]\*Sqrt[x]\*EllipticF[I\*ArcSinh[Sqrt[(I\*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1])/Sqrt[(I\*Sqrt[a])/Sqrt[b]]))/(3\*b\*Sqrt[a + b\*x^2])

**Maple [A]** time = 0.015, size = 125, normalized size = 1.

$$-\frac{c}{3b^2x}\sqrt{cx}\left(\sqrt{-ab}\sqrt{1\left(bx+\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{1\left(-bx+\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}}\sqrt{-bx}\frac{1}{\sqrt{-ab}}\text{EllipticF}\left(\sqrt{1\left(bx+\sqrt{-ab}\right)}\frac{1}{\sqrt{-ab}},\frac{\sqrt{2}}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(3/2)/(b\*x^2+a)^(1/2), x)

[Out] -1/3\*c/x\*(c\*x)^(1/2)/(b\*x^2+a)^(1/2)\*((-a\*b)^(1/2)\*((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*((-b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*(-x\*b/(-a\*b)^(1/2))^(1/2)\*EllipticF(((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2), 1/2\*2^(1/2))\*2^(1/2)\*a-2\*b^2\*x^3-2\*a\*b\*x)/b^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{\sqrt{bx^2+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(3/2)/sqrt(b\*x^2 + a), x, algorithm="maxima")

[Out] integrate((c\*x)^(3/2)/sqrt(b\*x^2 + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}cx}{\sqrt{bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(3/2)/sqrt(b\*x^2 + a), x, algorithm="fricas")

[Out] integral(sqrt(c\*x)\*c\*x/sqrt(b\*x^2 + a), x)

**Sympy [A]** time = 10.994, size = 44, normalized size = 0.35

$$\frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\left(\frac{5}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{5}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt{a}\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(3/2)/(b\*x\*\*2+a)\*\*(1/2), x)

[Out]  $c^{3/2} x^{5/2} \gamma(5/4) \text{hyper}((1/2, 5/4), (9/4, ), b x^2 \exp_{\text{polar}}(i\pi)/a) / (2 \sqrt{a} \gamma(9/4))$

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{3/2}}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(3/2)/sqrt(b*x^2 + a),x, algorithm="giac")`

[Out] `integrate((c*x)^(3/2)/sqrt(b*x^2 + a), x)`

$$3.616 \quad \int \frac{\sqrt{cx}}{\sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=236

$$\frac{\sqrt[4]{a}\sqrt{c}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^2}} - \frac{2\sqrt[4]{a}\sqrt{c}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^2}} + \frac{2\sqrt{cx}\sqrt{a+bx^2}}{\sqrt{b}(\sqrt{a} + \sqrt{bx})}$$

[Out] (2\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(Sqrt[b]\*(Sqrt[a] + Sqrt[b]\*x)) - (2\*a^(1/4)\*Sqrt[c]\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticE[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(b^(3/4)\*Sqrt[a + b\*x^2]) + (a^(1/4)\*Sqrt[c]\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(b^(3/4)\*Sqrt[a + b\*x^2])

**Rubi [A]** time = 0.434289, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{\sqrt[4]{a}\sqrt{c}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^2}} - \frac{2\sqrt[4]{a}\sqrt{c}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^2}} + \frac{2\sqrt{cx}\sqrt{a+bx^2}}{\sqrt{b}(\sqrt{a} + \sqrt{bx})}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c\*x]/Sqrt[a + b\*x^2], x]

[Out] (2\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(Sqrt[b]\*(Sqrt[a] + Sqrt[b]\*x)) - (2\*a^(1/4)\*Sqrt[c]\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticE[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(b^(3/4)\*Sqrt[a + b\*x^2]) + (a^(1/4)\*Sqrt[c]\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(b^(3/4)\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 39.2971, size = 216, normalized size = 0.92

$$\frac{2\sqrt[4]{a}\sqrt{c} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) E\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^2}} + \frac{\sqrt[4]{a}\sqrt{c} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) F\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^2}} + \frac{2\sqrt{cx}\sqrt{a+bx^2}}{\sqrt{b}(\sqrt{a} + \sqrt{bx})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(1/2)/(b\*x\*\*2+a)\*\*(1/2), x)

[Out]  $-2*a^{1/4}*sqrt(c)*sqrt((a + b*x^2)/(sqrt(a) + sqrt(b)*x)^2)*(sqrt(a) + sqrt(b)*x)*elliptic_e(2*atan(b^{1/4}*sqrt(c*x)/(a^{1/4}*sqrt(c))), 1/2)/(b^{3/4}*sqrt(a + b*x^2)) + a^{1/4}*sqrt(c)*sqrt((a + b*x^2)/(sqrt(a) + sqrt(b)*x)^2)*(sqrt(a) + sqrt(b)*x)*elliptic_f(2*atan(b^{1/4}*sqrt(c*x)/(a^{1/4}*sqrt(c))), 1/2)/(b^{3/4}*sqrt(a + b*x^2)) + 2*sqrt(c*x)*sqrt(a + b*x^2)/(sqrt(b)*(sqrt(a) + sqrt(b)*x))$

**Mathematica [C]** time = 0.0941198, size = 111, normalized size = 0.47

$$\frac{2ix\sqrt{cx}\sqrt{\frac{bx^2}{a}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle| -1\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle| -1\right)\right)}{\left(\frac{i\sqrt{bx}}{\sqrt{a}}\right)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c\*x]/Sqrt[a + b\*x^2], x]

[Out]  $((2*I)*x*Sqrt[c*x]*Sqrt[1 + (b*x^2)/a]*(EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]], -1] - EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]], -1])/((I*Sqrt[b]*x)/Sqrt[a])^{3/2}*Sqrt[a + b*x^2])$

**Maple [A]** time = 0.015, size = 132, normalized size = 0.6

$$\frac{a\sqrt{2}}{bx}\sqrt{cx}\sqrt{1\left(bx + \sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}\sqrt{1\left(-bx + \sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\left(2\text{EllipticE}\left(\sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2\sqrt{2}\right) - \text{EllipticF}\left(\sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2\sqrt{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(1/2)/(b\*x^2+a)^(1/2), x)

[Out]  $(c*x)^{1/2}/(b*x^2+a)^{1/2}*a/b*((b*x+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}*2^{1/2}*((-b*x+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}*(-x*b/(-a*b)^{1/2})^{1/2}*(2*EllipticE(((b*x+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}, 1/2*2^{1/2}) - EllipticF(((b*x+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}, 1/2*2^{1/2}))/x$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x)/sqrt(b\*x^2 + a), x, algorithm="maxima")

[Out] integrate(sqrt(c\*x)/sqrt(b\*x^2 + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}}{\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x)/sqrt(b\*x^2 + a),x, algorithm="fricas")

[Out] integral(sqrt(c\*x)/sqrt(b\*x^2 + a), x)

**Sympy [A]** time = 2.60509, size = 44, normalized size = 0.19

$$\frac{\sqrt{c}x^{\frac{3}{2}} \left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt{a} \left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(1/2)/(b\*x\*\*2+a)\*\*(1/2),x)

[Out] sqrt(c)\*x\*\*(3/2)\*gamma(3/4)\*hyper((1/2, 3/4), (7/4, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*sqrt(a)\*gamma(7/4))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x)/sqrt(b\*x^2 + a),x, algorithm="giac")

[Out] integrate(sqrt(c\*x)/sqrt(b\*x^2 + a), x)



$$3.617 \quad \int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=97

$$\frac{(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

[Out] ((Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/ (a^(1/4)\*b^(1/4)\*Sqrt[c]\*Sqrt[a + b\*x^2])

**Rubi [A]** time = 0.156603, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[c\*x]\*Sqrt[a + b\*x^2]), x]

[Out] ((Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/ (a^(1/4)\*b^(1/4)\*Sqrt[c]\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 12.7659, size = 88, normalized size = 0.91

$$\frac{\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) F\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(1/2)/(b\*x\*\*2+a)\*\*(1/2), x)

[Out] sqrt((a + b\*x\*\*2)/(sqrt(a) + sqrt(b)\*x)\*\*2)\*(sqrt(a) + sqrt(b)\*x)\*elliptic\_f(2\*atan(b\*\*(1/4)\*sqrt(c\*x)/(a\*\*(1/4)\*sqrt(c))), 1/2)/(a\*\*(1/4)\*b\*\*(1/4)\*sqrt(c)\*sqrt(a + b\*x\*\*2))

**Mathematica [C]** time = 0.049535, size = 90, normalized size = 0.93

$$\frac{2ix^{3/2} \sqrt{\frac{a}{bx^2}} + 1 F\left(i \sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right) \middle| -1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{cx}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[c\*x]\*Sqrt[a + b\*x^2]), x]

[Out] ((2\*I)\*Sqrt[1 + a/(b\*x^2)]\*x^(3/2)\*EllipticF[I\*ArcSinh[Sqrt[(I\*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1])/ (Sqrt[(I\*Sqrt[a])/Sqrt[b]]\*Sqrt[c]\*

$x] * \text{Sqrt}[a + b * x^2])$

**Maple [A]** time = 0.022, size = 104, normalized size = 1.1

$$\frac{\sqrt{2}}{b} \sqrt{-ab} \sqrt{1 \left( bx + \sqrt{-ab} \right) \frac{1}{\sqrt{-ab}}} \sqrt{1 \left( -bx + \sqrt{-ab} \right) \frac{1}{\sqrt{-ab}}} \sqrt{-bx \frac{1}{\sqrt{-ab}}} \text{EllipticF} \left( \sqrt{1 \left( bx + \sqrt{-ab} \right) \frac{1}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2} \right) \frac{1}{\sqrt{cx}} \frac{1}{\sqrt{bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(1/2)/(b*x^2+a)^(1/2),x)`

[Out]  $1/(b*x^2+a)^{(1/2)} * (-a*b)^{(1/2)} * ((b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)} * 2^{(1/2)} * ((-b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)} * (-x*b/(-a*b)^{(1/2)})^{(1/2)} * \text{EllipticF}(((b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}, 1/2 * 2^{(1/2)})/b/(c*x)^{(1/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2 + a}\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^2 + a)*sqrt(c*x)),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(b*x^2 + a)*sqrt(c*x)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{\sqrt{bx^2 + a}\sqrt{cx}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^2 + a)*sqrt(c*x)),x, algorithm="fricas")`

[Out] `integral(1/(sqrt(b*x^2 + a)*sqrt(c*x)), x)`

**Sympy [A]** time = 3.45906, size = 44, normalized size = 0.45

$$\frac{\sqrt{x} \left( \frac{1}{4} \right) {}_2F_1 \left( \frac{1}{4}, \frac{1}{2} \middle| \frac{bx^2 e^{i\pi}}{a} \right)}{2\sqrt{a}\sqrt{c} \left( \frac{5}{4} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(1/2)/(b*x**2+a)**(1/2),x)`

[Out] `sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4, ), b*x**2*exp_polar(I*pi/a)/(2*sqrt(a)*sqrt(c))*gamma(5/4))`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2 + a}\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^2 + a)*sqrt(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(c*x)), x)
```

$$3.618 \quad \int \frac{1}{(cx)^{3/2} \sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=268

$$\frac{\sqrt[4]{b} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{a^{3/4} c^{3/2} \sqrt{a+bx^2}} - \frac{2\sqrt[4]{b} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{a^{3/4} c^{3/2} \sqrt{a+bx^2}} + \frac{2\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{ac^2 (\sqrt{a} + \sqrt{bx})} - \frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}$$

[Out]  $(-2*\text{Sqrt}[a + b*x^2])/(a*c*\text{Sqrt}[c*x]) + (2*\text{Sqrt}[b]*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(a*c^2*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) - (2*b^{1/4}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{1/4}*\text{Sqrt}[c*x])/(a^{1/4}*\text{Sqrt}[c])], 1/2])/(a^{3/4}*c^{3/2}*\text{Sqrt}[a + b*x^2]) + (b^{1/4}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{1/4}*\text{Sqrt}[c*x])/(a^{1/4}*\text{Sqrt}[c])], 1/2])/(a^{3/4}*c^{3/2}*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.513263, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{\sqrt[4]{b} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{a^{3/4} c^{3/2} \sqrt{a+bx^2}} - \frac{2\sqrt[4]{b} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{a^{3/4} c^{3/2} \sqrt{a+bx^2}} + \frac{2\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{ac^2 (\sqrt{a} + \sqrt{bx})} - \frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(3/2)\*Sqrt[a + b\*x^2]), x]

[Out]  $(-2*\text{Sqrt}[a + b*x^2])/(a*c*\text{Sqrt}[c*x]) + (2*\text{Sqrt}[b]*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(a*c^2*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) - (2*b^{1/4}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{1/4}*\text{Sqrt}[c*x])/(a^{1/4}*\text{Sqrt}[c])], 1/2])/(a^{3/4}*c^{3/2}*\text{Sqrt}[a + b*x^2]) + (b^{1/4}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{1/4}*\text{Sqrt}[c*x])/(a^{1/4}*\text{Sqrt}[c])], 1/2])/(a^{3/4}*c^{3/2}*\text{Sqrt}[a + b*x^2])$

**Rubi in Sympy [A]** time = 49.8933, size = 243, normalized size = 0.91

$$\frac{2\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{ac^2 (\sqrt{a} + \sqrt{bx})} - \frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}} - \frac{2\sqrt[4]{b} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) E\left(2 \text{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{a^{3/4} c^{3/2} \sqrt{a+bx^2}} + \frac{\sqrt[4]{b} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) F\left(2 \text{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{a^{3/4} c^{3/2} \sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(3/2)/(b\*x\*\*2+a)\*\*(1/2), x)

[Out]  $2\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}/(a^{3/2}(\sqrt{a}+\sqrt{b}x)) - 2\sqrt{a+bx^2}/(a^{3/2}\sqrt{cx}) - 2b^{1/4}\sqrt{(a+bx^2)/(\sqrt{a}+\sqrt{b}x)^2}(\sqrt{a}+\sqrt{b}x)\text{elliptic}_e(2\text{atan}(b^{1/4}\sqrt{cx}/(a^{1/4}\sqrt{c})), 1/2)/(a^{3/4}c^{3/2}\sqrt{a+bx^2}) + b^{1/4}\sqrt{(a+bx^2)/(\sqrt{a}+\sqrt{b}x)^2}(\sqrt{a}+\sqrt{b}x)\text{elliptic}_f(2\text{atan}(b^{1/4}\sqrt{cx}/(a^{1/4}\sqrt{c})), 1/2)/(a^{3/4}c^{3/2}\sqrt{a+bx^2})$

**Mathematica [C]** time = 0.19961, size = 176, normalized size = 0.66

$$\frac{2x\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}(a+bx^2) + \sqrt{a}\sqrt{bx}\sqrt{\frac{bx^2}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle| -1\right) - \sqrt{a}\sqrt{bx}\sqrt{\frac{bx^2}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle| -1\right)\right)}{a(cx)^{3/2}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(3/2)\*Sqrt[a + b\*x^2]), x]

[Out]  $(-2x*(\text{Sqrt}[(I*\text{Sqrt}[b]*x)/\text{Sqrt}[a]]*(a+b*x^2) - \text{Sqrt}[a]*\text{Sqrt}[b]*x*\text{Sqrt}[1+(b*x^2)/a]*\text{EllipticE}[I*\text{ArcSinh}[\text{Sqrt}[(I*\text{Sqrt}[b]*x)/\text{Sqrt}[a]]], -1] + \text{Sqrt}[a]*\text{Sqrt}[b]*x*\text{Sqrt}[1+(b*x^2)/a]*\text{EllipticF}[I*\text{ArcSinh}[\text{Sqrt}[(I*\text{Sqrt}[b]*x)/\text{Sqrt}[a]]], -1]))/(a*\text{Sqrt}[(I*\text{Sqrt}[b]*x)/\text{Sqrt}[a]]*(c*x)^(3/2)*\text{Sqrt}[a+b*x^2])$

**Maple [A]** time = 0.023, size = 196, normalized size = 0.7

$$\frac{1}{ac}\left(2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{bx}{\sqrt{-ab}}}\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}, 1/2\sqrt{2}\right)\sqrt{2a} - \sqrt{1+(bx+\sqrt{-ab})}\frac{1}{\sqrt{-ab}}\sqrt{1(-bx+\sqrt{-ab})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x)^(3/2)/(b\*x^2+a)^(1/2), x)

[Out]  $(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*\text{EllipticE}((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2))*2^(1/2)*a - ((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*\text{EllipticF}((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2))*2^(1/2)*a - 2*b*x^2 - 2*a)/(b*x^2+a)^(1/2)/c/(c*x)^(1/2)/a$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}(cx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(b\*x^2 + a)\*(c\*x)^(3/2)), x, algorithm="maxima")

[Out] integrate(1/(sqrt(b\*x^2 + a)\*(c\*x)^(3/2)), x)

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{bx^2 + a}\sqrt{cxcx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^2 + a)*(c*x)^(3/2)),x, algorithm="fricas")`

[Out] `integral(1/(sqrt(b*x^2 + a)*sqrt(c*x)*c*x), x)`

**Sympy** [A] time = 7.78478, size = 48, normalized size = 0.18

$$\frac{\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{4} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt{ac}^{\frac{3}{2}}\sqrt{x}\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(3/2)/(b*x**2+a)**(1/2),x)`

[Out] `gamma(-1/4)*hyper((-1/4, 1/2), (3/4, ), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*c**(3/2)*sqrt(x)*gamma(3/4))`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2 + a}(cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^2 + a)*(c*x)^(3/2)),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(b*x^2 + a)*(c*x)^(3/2)), x)`

$$3.619 \quad \int \frac{1}{(cx)^{5/2} \sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=129

$$\frac{b^{3/4} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{3a^{5/4}c^{5/2}\sqrt{a+bx^2}} - \frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}$$

[Out]  $(-2*\text{Sqrt}[a + b*x^2])/(3*a*c*(c*x)^{(3/2)}) - (b^{(3/4)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(3*a^{(5/4)}*c^{(5/2)}*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.207491, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{b^{3/4} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{3a^{5/4}c^{5/2}\sqrt{a+bx^2}} - \frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(5/2)\*Sqrt[a + b\*x^2]),x]

[Out]  $(-2*\text{Sqrt}[a + b*x^2])/(3*a*c*(c*x)^{(3/2)}) - (b^{(3/4)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(3*a^{(5/4)}*c^{(5/2)}*\text{Sqrt}[a + b*x^2])$

**Rubi in Sympy [A]** time = 18.0708, size = 116, normalized size = 0.9

$$\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}} - \frac{b^{3/4} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) F\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{3a^{5/4}c^{5/2}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(5/2)/(b\*x\*\*2+a)\*\*(1/2),x)

[Out]  $-2*\text{sqrt}(a + b*x**2)/(3*a*c*(c*x)**(3/2)) - b**(3/4)*\text{sqrt}((a + b*x**2)/(\text{sqrt}(a) + \text{sqrt}(b)*x)**2)*(\text{sqrt}(a) + \text{sqrt}(b)*x)*\text{elliptic\_f}(2*\text{atan}(b**(1/4)*\text{sqrt}(c*x)/(a**(1/4)*\text{sqrt}(c))), 1/2)/(3*a**(5/4)*c**(5/2)*\text{sqrt}(a + b*x**2))$

**Mathematica [C]** time = 0.273826, size = 109, normalized size = 0.84

$$\frac{2x \left( \frac{ibx^{5/2} \sqrt{\frac{a}{bx^2} + 1} F\left(i \sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right) \middle| -1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}} - a - bx^2 \right)}{3a(cx)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(5/2)\*Sqrt[a + b\*x^2]),x]

[Out] (2\*x\*(-a - b\*x^2 - (I\*b\*Sqrt[1 + a/(b\*x^2)]\*x^(5/2)\*EllipticF[I\*ArcSinh[Sqrt[(I\*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1])/Sqrt[(I\*Sqrt[a])/Sqrt[b]]))/(3\*a\*(c\*x)^(5/2)\*Sqrt[a + b\*x^2])

**Maple [A]** time = 0.023, size = 123, normalized size = 1.

$$-\frac{1}{3axc^2} \left( \sqrt{1 \left( bx + \sqrt{-ab} \right) \frac{1}{\sqrt{-ab}}} \sqrt{1 \left( -bx + \sqrt{-ab} \right) \frac{1}{\sqrt{-ab}}} \sqrt{-bx \frac{1}{\sqrt{-ab}}} \text{EllipticF} \left( \sqrt{1 \left( bx + \sqrt{-ab} \right) \frac{1}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2} \right) \sqrt{-ab} \sqrt{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x)^(5/2)/(b\*x^2+a)^(1/2),x)

[Out] -1/3/(b\*x^2+a)^(1/2)/x\*(((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*((-b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*(-x\*b/(-a\*b)^(1/2))^(1/2)\*EllipticF((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2),1/2\*2^(1/2))\*(-a\*b)^(1/2)\*2^(1/2)\*x+2\*b\*x^2+2\*a)/a/c^2/(c\*x)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2 + a}(cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(b\*x^2 + a)\*(c\*x)^(5/2)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b\*x^2 + a)\*(c\*x)^(5/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{\sqrt{bx^2 + a} \sqrt{cxc^2x^2}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(b\*x^2 + a)\*(c\*x)^(5/2)),x, algorithm="fricas")

[Out] integral(1/(sqrt(b\*x^2 + a)\*sqrt(c\*x)\*c^2\*x^2), x)

**Sympy [A]** time = 53.1272, size = 48, normalized size = 0.37

$$\frac{\left(-\frac{3}{4}\right) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt{ac}^{\frac{5}{2}} x^{\frac{3}{2}} \left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(5/2)/(b\*x\*\*2+a)\*\*(1/2),x)



[Out]  $\text{gamma}(-3/4) \cdot \text{hyper}((-3/4, 1/2), (1/4, ), b \cdot x^{22} \cdot \text{exp\_polar}(I \cdot \text{pi})/a) / (2 \cdot \text{sqrt}(a) \cdot c^{5/2} \cdot x^{3/2} \cdot \text{gamma}(1/4))$

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2 + a} (cx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(b*x^2 + a)*(c*x)^(5/2)),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(b*x^2 + a)*(c*x)^(5/2)), x)`

$$3.620 \quad \int \frac{1}{(cx)^{7/2} \sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=306

$$\begin{aligned} & \frac{3b^{5/4} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5a^{7/4}c^{7/2}\sqrt{a+bx^2}} \\ & + \frac{6b^{5/4} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5a^{7/4}c^{7/2}\sqrt{a+bx^2}} \\ & - \frac{6b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5a^2c^4(\sqrt{a} + \sqrt{bx})} + \frac{6b\sqrt{a+bx^2}}{5a^2c^3\sqrt{cx}} - \frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}} \end{aligned}$$

[Out]  $(-2*\text{Sqrt}[a + b*x^2])/(5*a*c*(c*x)^{(5/2)}) + (6*b*\text{Sqrt}[a + b*x^2])/(5*a^2*c^3*\text{Sqrt}[c*x]) - (6*b^{(3/2)}*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(5*a^2*c^4*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) + (6*b^{(5/4)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(5*a^{(7/4)}*c^{(7/2)}*\text{Sqrt}[a + b*x^2]) - (3*b^{(5/4)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(5*a^{(7/4)}*c^{(7/2)}*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.594835, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\begin{aligned} & \frac{3b^{5/4} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5a^{7/4}c^{7/2}\sqrt{a+bx^2}} \\ & + \frac{6b^{5/4} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5a^{7/4}c^{7/2}\sqrt{a+bx^2}} \\ & - \frac{6b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5a^2c^4(\sqrt{a} + \sqrt{bx})} + \frac{6b\sqrt{a+bx^2}}{5a^2c^3\sqrt{cx}} - \frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(7/2)\*Sqrt[a + b\*x^2]),x]

[Out]  $(-2*\text{Sqrt}[a + b*x^2])/(5*a*c*(c*x)^{(5/2)}) + (6*b*\text{Sqrt}[a + b*x^2])/(5*a^2*c^3*\text{Sqrt}[c*x]) - (6*b^{(3/2)}*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(5*a^2*c^4*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) + (6*b^{(5/4)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(5*a^{(7/4)}*c^{(7/2)}*\text{Sqrt}[a + b*x^2]) - (3*b^{(5/4)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(5*a^{(7/4)}*c^{(7/2)}*\text{Sqrt}[a + b*x^2])$

**Rubi in Sympy [A]** time = 60.6029, size = 282, normalized size = 0.92

$$\begin{aligned} & -\frac{2\sqrt{a+bx^2}}{5ac(cx)^{\frac{5}{2}}} - \frac{6b^{\frac{3}{2}}\sqrt{cx}\sqrt{a+bx^2}}{5a^2c^4(\sqrt{a}+\sqrt{bx})} + \frac{6b\sqrt{a+bx^2}}{5a^2c^3\sqrt{cx}} \\ & + \frac{6b^{\frac{5}{4}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{5a^{\frac{7}{4}}c^{\frac{7}{2}}\sqrt{a+bx^2}} \\ & - \frac{3b^{\frac{5}{4}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{5a^{\frac{7}{4}}c^{\frac{7}{2}}\sqrt{a+bx^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(c*x)**(7/2)/(b*x**2+a)**(1/2),x)`

[Out] `-2*sqrt(a + b*x**2)/(5*a*c*(c*x)**(5/2)) - 6*b**(3/2)*sqrt(c*x)*sqrt(a + b*x**2)/(5*a**2*c**4*(sqrt(a) + sqrt(b)*x)) + 6*b*sqrt(a + b*x**2)/(5*a**2*c**3*sqrt(c*x)) + 6*b**(5/4)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*elliptic_e(2*atan(b**(1/4)*sqrt(c*x)/(a**(1/4)*sqrt(c))), 1/2)/(5*a**(7/4)*c**(7/2)*sqrt(a + b*x**2)) - 3*b**(5/4)*sqrt((a + b*x**2)/(sqrt(a) + sqrt(b)*x)**2)*(sqrt(a) + sqrt(b)*x)*elliptic_f(2*atan(b**(1/4)*sqrt(c*x)/(a**(1/4)*sqrt(c))), 1/2)/(5*a**(7/4)*c**(7/2)*sqrt(a + b*x**2))`

**Mathematica [C]** time = 0.26398, size = 198, normalized size = 0.65

$$\frac{x\left(2\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\left(-a^2+2abx^2+3b^2x^4\right)+6\sqrt{ab}^{3/2}x^3\sqrt{\frac{bx^2}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle|-1\right)-6\sqrt{ab}^{3/2}x^3\sqrt{\frac{bx^2}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle|-1\right)\right)}{5a^2(cx)^{7/2}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/((c*x)^(7/2)*Sqrt[a + b*x^2]),x]`

[Out] `(x*(2*Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*(-a^2+2*a*b*x^2+3*b^2*x^4)-6*Sqrt[a]*b^(3/2)*x^3*Sqrt[1+(b*x^2)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]],-1]+6*Sqrt[a]*b^(3/2)*x^3*Sqrt[1+(b*x^2)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]],-1])/(5*a^2*Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*(c*x)^(7/2)*Sqrt[a + b*x^2])`

**Maple [A]** time = 0.025, size = 219, normalized size = 0.7

$$-\frac{1}{5x^2c^3a^2}\left(6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2,\sqrt{2}\right)\sqrt{2}x^2ab-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(7/2)/(b*x^2+a)^(1/2),x)`

[Out] `-1/5/x^2*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^2*a*b-3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2))`

$$\frac{a^2 b^2 x^4 - 4 a^2 b x^2 + 2 a^2}{(b x^2 + a)^{1/2} c^3 (c x)^{1/2} a^2}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2 + a} (cx)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(b\*x^2 + a)\*(c\*x)^(7/2)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b\*x^2 + a)\*(c\*x)^(7/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{bx^2 + a}\sqrt{cx^3x^3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(b\*x^2 + a)\*(c\*x)^(7/2)),x, algorithm="fricas")

[Out] integral(1/(sqrt(b\*x^2 + a)\*sqrt(c\*x)\*c^3\*x^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(7/2)/(b\*x\*\*2+a)\*\*(1/2),x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2 + a} (cx)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(b\*x^2 + a)\*(c\*x)^(7/2)),x, algorithm="giac")

[Out] integrate(1/(sqrt(b\*x^2 + a)\*(c\*x)^(7/2)), x)

$$3.621 \quad \int \frac{(cx)^{7/2}}{(a+bx^2)^{3/2}} dx$$

**Optimal.** Leaf size=153

$$\frac{5a^{3/4}c^{7/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{6b^{9/4}\sqrt{a+bx^2}} + \frac{5c^3\sqrt{cx}\sqrt{a+bx^2}}{3b^2} - \frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}$$

[Out]  $-\left(\frac{c(c*x)^{(5/2)}}{b*\text{Sqrt}[a + b*x^2]}\right) + \left(\frac{5*c^3*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2]}{3*b^2} - \frac{5*a^{(3/4)}*c^{(7/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2]}{(6*b^{(9/4)}*\text{Sqrt}[a + b*x^2])}\right)$

**Rubi [A]** time = 0.251851, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{5a^{3/4}c^{7/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{6b^{9/4}\sqrt{a+bx^2}} + \frac{5c^3\sqrt{cx}\sqrt{a+bx^2}}{3b^2} - \frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(7/2)/(a + b\*x^2)^(3/2), x]

[Out]  $-\left(\frac{c(c*x)^{(5/2)}}{b*\text{Sqrt}[a + b*x^2]}\right) + \left(\frac{5*c^3*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2]}{3*b^2} - \frac{5*a^{(3/4)}*c^{(7/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2]}{(6*b^{(9/4)}*\text{Sqrt}[a + b*x^2])}\right)$

**Rubi in Sympy [A]** time = 24.5488, size = 139, normalized size = 0.91

$$\frac{5a^{\frac{3}{4}}c^{\frac{7}{2}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a} + \sqrt{bx}) F\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{6b^{\frac{9}{4}}\sqrt{a+bx^2}} - \frac{c(cx)^{\frac{5}{2}}}{b\sqrt{a+bx^2}} + \frac{5c^3\sqrt{cx}\sqrt{a+bx^2}}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(7/2)/(b\*x\*\*2+a)\*\*(3/2), x)

[Out]  $-5*a^{(3/4)}*c^{(7/2)}*\text{sqrt}((a + b*x**2)/(\text{sqrt}(a) + \text{sqrt}(b)*x)**2)*(\text{sqrt}(a) + \text{sqrt}(b)*x)*\text{elliptic\_f}(2*\text{atan}(b^{(1/4)}*\text{sqrt}(c*x)/(a^{(1/4)}*\text{sqrt}(c))), 1/2)/(6*b^{(9/4)}*\text{sqrt}(a + b*x**2)) - c*(c*x)**(5/2)/(b*\text{sqrt}(a + b*x**2)) + 5*c**3*\text{sqrt}(c*x)*\text{sqrt}(a + b*x**2)/(3*b**2)$

**Mathematica [C]** time = 0.127496, size = 131, normalized size = 0.86

$$\frac{c^3\sqrt{cx}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}(5a + 2bx^2) - 5ia\sqrt{x}\sqrt{\frac{a}{bx^2}} + 1F\left(i \sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right) \middle| -1\right)\right)}{3b^2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(7/2)/(a + b\*x^2)^(3/2), x]

[Out] (c^3\*Sqrt[c\*x]\*(Sqrt[(I\*Sqrt[a])/Sqrt[b]]\*(5\*a + 2\*b\*x^2) - (5\*I)\*a\*Sqrt[1 + a/(b\*x^2)]\*Sqrt[x]\*EllipticF[I\*ArcSinh[Sqrt[(I\*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1]))/(3\*Sqrt[(I\*Sqrt[a])/Sqrt[b]]\*b^2\*Sqrt[a + b\*x^2])

**Maple [A]** time = 0.048, size = 128, normalized size = 0.8

$$-\frac{c^3}{6b^3x}\sqrt{cx}\left(5\sqrt{-ab}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}, 1/2\sqrt{2}\right)\sqrt{2a-4b^2x^3-10abx}\right)\frac{1}{\sqrt{bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(7/2)/(b\*x^2+a)^(3/2), x)

[Out] -1/6\*c^3/x\*(c\*x)^(1/2)\*(5\*(-a\*b)^(1/2)\*((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*((-b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*(-x\*b/(-a\*b)^(1/2))^(1/2)\*EllipticF(((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2), 1/2\*2^(1/2))\*2^(1/2)\*a-4\*b^2\*x^3-10\*a\*b\*x)/(b\*x^2+a)^(1/2)/b^3

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{7}{2}}}{(bx^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(7/2)/(b\*x^2 + a)^(3/2), x, algorithm="maxima")

[Out] integrate((c\*x)^(7/2)/(b\*x^2 + a)^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{cx}c^3x^3}{(bx^2 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(7/2)/(b\*x^2 + a)^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(c\*x)\*c^3\*x^3/(b\*x^2 + a)^(3/2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(7/2)/(b\*x\*\*2+a)\*\*(3/2), x)

[Out] Timed out

---

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{7}{2}}}{(bx^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(7/2)/(b\*x^2 + a)^(3/2), x, algorithm="giac")

[Out] integrate((c\*x)^(7/2)/(b\*x^2 + a)^(3/2), x)

$$3.622 \quad \int \frac{(cx)^{5/2}}{(a+bx^2)^{3/2}} dx$$

**Optimal.** Leaf size=266

$$\frac{3\sqrt[4]{ac}^{5/2} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^2}} - \frac{3\sqrt[4]{ac}^{5/2} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{b^{7/4}\sqrt{a+bx^2}} + \frac{3c^2\sqrt{cx}\sqrt{a+bx^2}}{b^{3/2}(\sqrt{a} + \sqrt{bx})} - \frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}$$

[Out]  $-\left(\frac{c(c*x)^{(3/2)}}{(b*\text{Sqrt}[a + b*x^2])}\right) + (3*c^2*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(b^{(3/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) - (3*a^{(1/4)}*c^{(5/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(b^{(7/4)}*\text{Sqrt}[a + b*x^2]) + (3*a^{(1/4)}*c^{(5/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(2*b^{(7/4)}*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.510318, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{3\sqrt[4]{ac}^{5/2} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^2}} - \frac{3\sqrt[4]{ac}^{5/2} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{b^{7/4}\sqrt{a+bx^2}} + \frac{3c^2\sqrt{cx}\sqrt{a+bx^2}}{b^{3/2}(\sqrt{a} + \sqrt{bx})} - \frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(5/2)/(a + b\*x^2)^(3/2), x]

[Out]  $-\left(\frac{c(c*x)^{(3/2)}}{(b*\text{Sqrt}[a + b*x^2])}\right) + (3*c^2*\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(b^{(3/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) - (3*a^{(1/4)}*c^{(5/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(b^{(7/4)}*\text{Sqrt}[a + b*x^2]) + (3*a^{(1/4)}*c^{(5/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(2*b^{(7/4)}*\text{Sqrt}[a + b*x^2])$

**Rubi in Sympy [A]** time = 49.2374, size = 243, normalized size = 0.91

$$\frac{3\sqrt[4]{ac}^{5/2} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) E\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{b^{7/4}\sqrt{a+bx^2}} + \frac{3\sqrt[4]{ac}^{5/2} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) F\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^2}} - \frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}} + \frac{3c^2\sqrt{cx}\sqrt{a+bx^2}}{b^{3/2}(\sqrt{a} + \sqrt{bx})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(5/2)/(b\*x\*\*2+a)\*\*(3/2), x)



[Out]  $-3a^{1/4}c^{5/2}\sqrt{(a+bx^2)/(\sqrt{a}+\sqrt{b}x)^2} \cdot (\sqrt{a}+\sqrt{b}x)\operatorname{elliptic}_e(2\operatorname{atan}(b^{1/4}\sqrt{cx}/(a^{1/4}\sqrt{c})), 1/2)/(b^{7/4}\sqrt{a+bx^2}) + 3a^{1/4}c^{5/2}\sqrt{(a+bx^2)/(\sqrt{a}+\sqrt{b}x)^2} \cdot (\sqrt{a}+\sqrt{b}x)\operatorname{elliptic}_f(2\operatorname{atan}(b^{1/4}\sqrt{cx}/(a^{1/4}\sqrt{c})), 1/2)/(2b^{7/4}\sqrt{a+bx^2}) - c^{3/2}\sqrt{cx}/(b\sqrt{a+bx^2}) + 3c^2\sqrt{cx}\sqrt{a+bx^2}/(b^{3/2}(\sqrt{a}+\sqrt{b}x))$

**Mathematica [C]** time = 0.149333, size = 168, normalized size = 0.63

$$\frac{c^2\sqrt{cx}\left(3\sqrt{a}\sqrt{\frac{bx^2}{a}}+1F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle| -1\right)-3\sqrt{a}\sqrt{\frac{bx^2}{a}}+1E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle| -1\right)+\sqrt{bx}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)}{b^{3/2}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(5/2)/(a+b\*x^2)^(3/2),x]

[Out]  $-\left((c^2\sqrt{cx}\left(\sqrt{b}x\sqrt{\frac{(I\sqrt{b}x)}{\sqrt{a}}}\right)-3\sqrt{a}\sqrt{1+(b^2x^2)/a}\operatorname{EllipticE}\left[\operatorname{ArcSinh}\left[\sqrt{\frac{(I\sqrt{b}x)}{\sqrt{a}}}\right], -1\right]+3\sqrt{a}\sqrt{1+(b^2x^2)/a}\operatorname{EllipticF}\left[\operatorname{ArcSinh}\left[\sqrt{\frac{(I\sqrt{b}x)}{\sqrt{a}}}\right], -1\right]\right)/\left(b^{3/2}\sqrt{a+bx^2}\right)\right)$

**Maple [A]** time = 0.045, size = 197, normalized size = 0.7

$$\frac{c^2}{2b^2x}\sqrt{cx}\left(6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}, 1/2\sqrt{2}\right)\sqrt{2a}-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(5/2)/(b\*x^2+a)^(3/2),x)

[Out]  $1/2c^2/x(c*x)^{1/2}\left(6\left(\frac{(b^2x+(-a^2b)^{1/2})}{(-a^2b)^{1/2}}\right)^{1/2}\left(\frac{(-b^2x+(-a^2b)^{1/2})}{(-a^2b)^{1/2}}\right)^{1/2}\left(-x\sqrt{b}/(-a^2b)^{1/2}\right)^{1/2}\operatorname{EllipticE}\left(\frac{(b^2x+(-a^2b)^{1/2})}{(-a^2b)^{1/2}}\right)^{1/2}, 1/2\sqrt{2}\right)^2-3\left(\frac{(b^2x+(-a^2b)^{1/2})}{(-a^2b)^{1/2}}\right)^{1/2}\left(\frac{(-b^2x+(-a^2b)^{1/2})}{(-a^2b)^{1/2}}\right)^{1/2}\left(-x\sqrt{b}/(-a^2b)^{1/2}\right)^{1/2}\operatorname{EllipticF}\left(\frac{(b^2x+(-a^2b)^{1/2})}{(-a^2b)^{1/2}}\right)^{1/2}, 1/2\sqrt{2}\right)^2-2^2a-2^2b^2x^2/(b^2x^2+a)^{1/2}/b^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{5/2}}{(bx^2+a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(5/2)/(b\*x^2+a)^(3/2),x, algorithm="maxima")

[Out] integrate((c\*x)^(5/2)/(b\*x^2+a)^(3/2), x)

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\sqrt{cx} c^2 x^2}{(bx^2 + a)^{\frac{3}{2}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(5/2)/(b*x^2 + a)^(3/2), x, algorithm="fricas")`

[Out] `integral(sqrt(c*x)*c^2*x^2/(b*x^2 + a)^(3/2), x)`

**Sympy** [A] time = 170.819, size = 44, normalized size = 0.17

$$\frac{c^{\frac{5}{2}} x^{\frac{7}{2}} \left(\frac{7}{4}\right) {}_2F_1\left(\frac{3}{2}, \frac{7}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}} \left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(5/2)/(b*x**2+a)**(3/2), x)`

[Out] `c**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/2, 7/4), (11/4, ), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(11/4))`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{(bx^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(5/2)/(b*x^2 + a)^(3/2), x, algorithm="giac")`

[Out] `integrate((c*x)^(5/2)/(b*x^2 + a)^(3/2), x)`

$$3.623 \quad \int \frac{(cx)^{3/2}}{(a+bx^2)^{3/2}} dx$$

**Optimal.** Leaf size=125

$$\frac{c^{3/2} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{2\sqrt[4]{ab^{5/4}}\sqrt{a+bx^2}} - \frac{c\sqrt{cx}}{b\sqrt{a+bx^2}}$$

[Out] -((c\*Sqrt[c\*x])/(b\*Sqrt[a + b\*x^2])) + (c^(3/2)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(2\*a^(1/4)\*b^(5/4)\*Sqrt[a + b\*x^2])

**Rubi [A]** time = 0.20044, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{c^{3/2} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{2\sqrt[4]{ab^{5/4}}\sqrt{a+bx^2}} - \frac{c\sqrt{cx}}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(3/2)/(a + b\*x^2)^(3/2), x]

[Out] -((c\*Sqrt[c\*x])/(b\*Sqrt[a + b\*x^2])) + (c^(3/2)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(2\*a^(1/4)\*b^(5/4)\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 18.3454, size = 110, normalized size = 0.88

$$-\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}} + \frac{c^{\frac{3}{2}} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) F\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{2\sqrt[4]{ab^{\frac{5}{4}}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(3/2)/(b\*x\*\*2+a)\*\*(3/2), x)

[Out] -c\*sqrt(c\*x)/(b\*sqrt(a + b\*x\*\*2)) + c\*\*(3/2)\*sqrt((a + b\*x\*\*2)/(sqrt(a) + sqrt(b)\*x)\*\*2)\*(sqrt(a) + sqrt(b)\*x)\*elliptic\_f(2\*atan(b\*\*(1/4)\*sqrt(c\*x)/(a\*\*(1/4)\*sqrt(c))), 1/2)/(2\*a\*\*(1/4)\*b\*\*(5/4)\*sqrt(a + b\*x\*\*2))

**Mathematica [C]** time = 0.0963709, size = 115, normalized size = 0.92

$$-\frac{c\sqrt{cx} \left( \sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} - i\sqrt{x}\sqrt{\frac{a}{bx^2}} + 1F\left(i \sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right) \middle| -1 \right) \right)}{b\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(3/2)/(a + b\*x^2)^(3/2), x]

[Out]  $-\left(\frac{c \sqrt{cx} \left(\sqrt{-ab} \sqrt{1 \left(bx + \sqrt{-ab}\right) \frac{1}{\sqrt{-ab}}} \sqrt{1 \left(-bx + \sqrt{-ab}\right) \frac{1}{\sqrt{-ab}}} \sqrt{-bx} \frac{1}{\sqrt{-ab}} \operatorname{EllipticF}\left(\sqrt{1 \left(bx + \sqrt{-ab}\right) \frac{1}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right)\right)}{\left(\sqrt{1 \left(bx + \sqrt{-ab}\right) \frac{1}{\sqrt{-ab}}}\right)^{3/2} b \sqrt{a + bx^2}}\right) - \frac{1}{\sqrt{bx^2 + a}} \sqrt{cx} \operatorname{EllipticF}\left(\operatorname{ArcSinh}\left(\frac{\sqrt{bx^2 + a}}{\sqrt{bx^2 + a}}\right), \frac{\sqrt{2}}{2}\right)$

**Maple [A]** time = 0.017, size = 115, normalized size = 0.9

$$\frac{c}{2b^2x} \sqrt{cx} \left( \sqrt{-ab} \sqrt{1 \left(bx + \sqrt{-ab}\right) \frac{1}{\sqrt{-ab}}} \sqrt{1 \left(-bx + \sqrt{-ab}\right) \frac{1}{\sqrt{-ab}}} \sqrt{-bx} \frac{1}{\sqrt{-ab}} \operatorname{EllipticF}\left(\sqrt{1 \left(bx + \sqrt{-ab}\right) \frac{1}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(3/2)/(b*x^2+a)^(3/2), x)`

[Out]  $\frac{1}{2} \frac{c}{x} (cx)^{1/2} \left( (-ab)^{1/2} \left( \frac{bx + (-ab)^{1/2}}{(-ab)^{1/2}} \right)^{1/2} \left( \frac{-bx + (-ab)^{1/2}}{(-ab)^{1/2}} \right)^{1/2} (-x) \frac{1}{(-ab)^{1/2}} \right)^{1/2} \operatorname{EllipticF}\left(\frac{bx + (-ab)^{1/2}}{(-ab)^{1/2}}, \frac{1}{2}\right) - 2 \frac{1}{b} \frac{1}{(bx^2 + a)^{1/2}}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{(bx^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(3/2)/(b*x^2 + a)^(3/2), x, algorithm="maxima")`

[Out] `integrate((c*x)^(3/2)/(b*x^2 + a)^(3/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{cx} cx}{(bx^2 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(3/2)/(b*x^2 + a)^(3/2), x, algorithm="fricas")`

[Out] `integral(sqrt(c*x)*c*x/(b*x^2 + a)^(3/2), x)`

**Sympy [A]** time = 13.0531, size = 44, normalized size = 0.35

$$\frac{c^{\frac{3}{2}} x^{\frac{5}{2}} \left(\frac{5}{4}\right) {}_2F_1\left(\frac{5}{4}, \frac{3}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}} \left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(3/2)/(b*x**2+a)**(3/2), x)`

[Out]  $c^{3/2} x^{5/2} \gamma(5/4) \operatorname{hyper}\left(\left(\frac{5}{4}, \frac{3}{2}\right), \left(\frac{9}{4},\right), \frac{bx^2 \exp(i\pi)}{a}\right) / (2a^{3/2} \gamma(9/4))$

---

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{(bx^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(3/2)/(b\*x^2 + a)^(3/2),x, algorithm="giac")

[Out] integrate((c\*x)^(3/2)/(b\*x^2 + a)^(3/2), x)

$$3.624 \quad \int \frac{\sqrt{cx}}{(a+bx^2)^{3/2}} dx$$

**Optimal.** Leaf size=266

$$\frac{\sqrt{c}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a} + \sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^2}} + \frac{\sqrt{c}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a} + \sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{a+bx^2}} + \frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}} - \frac{\sqrt{cx}\sqrt{a+bx^2}}{a\sqrt{b}(\sqrt{a} + \sqrt{bx})}$$

[Out]  $(c*x)^{(3/2)}/(a*c*\text{Sqrt}[a + b*x^2]) - (\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(a*\text{Sqrt}[b]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) + (\text{Sqrt}[c]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{1/4}*\text{Sqrt}[c*x])/(a^{1/4}*\text{Sqrt}[c])], 1/2])/(a^{3/4}*b^{3/4}*\text{Sqrt}[a + b*x^2]) - (\text{Sqrt}[c]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{1/4}*\text{Sqrt}[c*x])/(a^{1/4}*\text{Sqrt}[c])], 1/2])/(2*a^{3/4}*b^{3/4}*\text{Sqrt}[a + b*x^2])$

**Rubi [A]** time = 0.500484, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{\sqrt{c}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a} + \sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^2}} + \frac{\sqrt{c}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a} + \sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{a+bx^2}} + \frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}} - \frac{\sqrt{cx}\sqrt{a+bx^2}}{a\sqrt{b}(\sqrt{a} + \sqrt{bx})}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c\*x]/(a + b\*x^2)^(3/2), x]

[Out]  $(c*x)^{(3/2)}/(a*c*\text{Sqrt}[a + b*x^2]) - (\text{Sqrt}[c*x]*\text{Sqrt}[a + b*x^2])/(a*\text{Sqrt}[b]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)) + (\text{Sqrt}[c]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{1/4}*\text{Sqrt}[c*x])/(a^{1/4}*\text{Sqrt}[c])], 1/2])/(a^{3/4}*b^{3/4}*\text{Sqrt}[a + b*x^2]) - (\text{Sqrt}[c]*(\text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a + b*x^2)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{1/4}*\text{Sqrt}[c*x])/(a^{1/4}*\text{Sqrt}[c])], 1/2])/(2*a^{3/4}*b^{3/4}*\text{Sqrt}[a + b*x^2])$

**Rubi in Sympy [A]** time = 47.9834, size = 236, normalized size = 0.89

$$\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}} - \frac{\sqrt{cx}\sqrt{a+bx^2}}{a\sqrt{b}(\sqrt{a} + \sqrt{bx})} + \frac{\sqrt{c} \sqrt{\frac{a+bx^2}{(\sqrt{a} + \sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) E\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{a+bx^2}} - \frac{\sqrt{c} \sqrt{\frac{a+bx^2}{(\sqrt{a} + \sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) F\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(1/2)/(b\*x\*\*2+a)\*\*(3/2), x)

[Out]  $(c*x)^{(3/2)/(a*c*\sqrt{a+b*x^2})} - \sqrt{c*x}*\sqrt{a+b*x^2}/(a*\sqrt{b}*(\sqrt{a}+\sqrt{b}*x)) + \sqrt{c}*\sqrt{(a+b*x^2)/(\sqrt{a}+\sqrt{b}*x)^2}*(\sqrt{a}+\sqrt{b}*x)*\text{elliptic}_e(2*\text{atan}(b^{1/4}*\sqrt{c*x}/(a^{1/4}*\sqrt{c})), 1/2)/(a^{3/4}*b^{3/4}*\sqrt{a+b*x^2}) - \sqrt{c}*\sqrt{(a+b*x^2)/(\sqrt{a}+\sqrt{b}*x)^2}*(\sqrt{a}+\sqrt{b}*x)*\text{elliptic}_f(2*\text{atan}(b^{1/4}*\sqrt{c*x}/(a^{1/4}*\sqrt{c})), 1/2)/(2*a^{3/4}*b^{3/4}*\sqrt{a+b*x^2})$

**Mathematica [C]** time = 0.137594, size = 166, normalized size = 0.62

$$\frac{\sqrt{cx} \left( \sqrt{a} \sqrt{\frac{bx^2}{a}} + 1F \left( i \sinh^{-1} \left( \sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}} \right) \middle| -1 \right) - \sqrt{a} \sqrt{\frac{bx^2}{a}} + 1E \left( i \sinh^{-1} \left( \sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}} \right) \middle| -1 \right) + \sqrt{bx} \sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}} \right)}{a\sqrt{b} \sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}} \sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c\*x]/(a + b\*x^2)^(3/2), x]

[Out]  $(\text{Sqrt}[c*x]*(\text{Sqrt}[b]*x*\text{Sqrt}[(I*\text{Sqrt}[b]*x)/\text{Sqrt}[a]] - \text{Sqrt}[a]*\text{Sqrt}[1 + (b*x^2)/a]*\text{EllipticE}[I*\text{ArcSinh}[\text{Sqrt}[(I*\text{Sqrt}[b]*x)/\text{Sqrt}[a]]], -1] + \text{Sqrt}[a]*\text{Sqrt}[1 + (b*x^2)/a]*\text{EllipticF}[I*\text{ArcSinh}[\text{Sqrt}[(I*\text{Sqrt}[b]*x)/\text{Sqrt}[a]]], -1]))/(a*\text{Sqrt}[b]*\text{Sqrt}[(I*\text{Sqrt}[b]*x)/\text{Sqrt}[a]]*\text{Sqrt}[a + b*x^2])$

**Maple [A]** time = 0.018, size = 197, normalized size = 0.7

$$-\frac{1}{2abx} \sqrt{cx} \left( 2 \sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{-bx + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-\frac{bx}{\sqrt{-ab}}} \text{EllipticE} \left( \sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \sqrt{2} \right) \sqrt{2a} - \sqrt{1 \left( bx + \sqrt{-ab} \right) \frac{1}{\sqrt{-ab}}} \sqrt{1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(1/2)/(b\*x^2+a)^(3/2), x)

[Out]  $-1/2*(c*x)^{(1/2)}*(2*((b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*((-b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}*\text{EllipticE}(((b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)})^2*(1/2)*a - ((b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*((-b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}*\text{EllipticF}(((b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)})^2*(1/2)*a - 2*b*x^2)/(b*x^2+a)^{(1/2)}/b/x/a$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{(bx^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x)/(b\*x^2 + a)^(3/2), x, algorithm="maxima")

[Out] integrate(sqrt(c\*x)/(b\*x^2 + a)^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\sqrt{cx}}{(bx^2 + a)^{\frac{3}{2}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x)/(b*x^2 + a)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(c*x)/(b*x^2 + a)^(3/2), x)`

**Sympy [A]** time = 5.24471, size = 44, normalized size = 0.17

$$\frac{\sqrt{cx}^{\frac{3}{2}} \left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{3}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}} \left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(1/2)/(b*x**2+a)**(3/2),x)`

[Out] `sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(7/4))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{(bx^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x)/(b*x^2 + a)^(3/2),x, algorithm="giac")`

[Out] `integrate(sqrt(c*x)/(b*x^2 + a)^(3/2), x)`



$$3.625 \quad \int \frac{1}{\sqrt{cx}(a+bx^2)^{3/2}} dx$$

**Optimal.** Leaf size=126

$$\frac{(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}} + \frac{\sqrt{cx}}{ac\sqrt{a+bx^2}}$$

[Out] Sqrt[c\*x]/(a\*c\*Sqrt[a + b\*x^2]) + ((Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(2\*a^(5/4)\*b^(1/4)\*Sqrt[c]\*Sqrt[a + b\*x^2])

**Rubi [A]** time = 0.202381, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}} + \frac{\sqrt{cx}}{ac\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[c\*x]\*(a + b\*x^2)^(3/2)), x]

[Out] Sqrt[c\*x]/(a\*c\*Sqrt[a + b\*x^2]) + ((Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(2\*a^(5/4)\*b^(1/4)\*Sqrt[c]\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 17.8272, size = 110, normalized size = 0.87

$$\frac{\sqrt{cx}}{ac\sqrt{a+bx^2}} + \frac{\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) F\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(1/2)/(b\*x\*\*2+a)\*\*(3/2), x)

[Out] sqrt(c\*x)/(a\*c\*sqrt(a + b\*x\*\*2)) + sqrt((a + b\*x\*\*2)/(sqrt(a) + sqrt(b)\*x)\*\*2)\*(sqrt(a) + sqrt(b)\*x)\*elliptic\_f(2\*atan(b\*\*(1/4)\*sqrt(c\*x)/(a\*\*(1/4)\*sqrt(c))), 1/2)/(2\*a\*\*(5/4)\*b\*\*(1/4)\*sqrt(c)\*sqrt(a + b\*x\*\*2))

**Mathematica [C]** time = 0.0884251, size = 117, normalized size = 0.93

$$\frac{x}{a\sqrt{cx}\sqrt{a+bx^2}} + \frac{ix^{3/2}\sqrt{\frac{a}{bx^2}} + 1F\left(i \sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right) \middle| -1\right)}{a\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{cx}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[c\*x]\*(a + b\*x^2)^(3/2)), x]

[Out]  $x/(a*\sqrt{c*x}*\sqrt{a+b*x^2}) + (I*\sqrt{1+a/(b*x^2)})*x^{3/2}*EllipticF[I*ArcSinh[\sqrt{(I*\sqrt{a})/\sqrt{b}}]/\sqrt{x}], -1)/(a*\sqrt{(I*\sqrt{a})/\sqrt{b}}*\sqrt{c*x}*\sqrt{a+b*x^2})$

**Maple [A]** time = 0.025, size = 114, normalized size = 0.9

$$\frac{1}{2ab} \left( \sqrt{-ab} \sqrt{1 \left( bx + \sqrt{-ab} \right) \frac{1}{\sqrt{-ab}}} \sqrt{1 \left( -bx + \sqrt{-ab} \right) \frac{1}{\sqrt{-ab}}} \sqrt{-bx \frac{1}{\sqrt{-ab}}} EllipticF \left( \sqrt{1 \left( bx + \sqrt{-ab} \right) \frac{1}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2} \right) \sqrt{2} + 2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(1/2)/(b*x^2+a)^(3/2), x)`

[Out]  $1/2*((-a*b)^{1/2}*((b*x+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}*((-b*x+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}*(-x*b/(-a*b)^{1/2})^{1/2}*EllipticF((b*x+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}, 1/2*2^{1/2})^{1/2}+2*b*x)/(b*x^2+a)^{1/2}/b/a/(c*x)^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{2}} \sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/2)*sqrt(c*x)), x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(3/2)*sqrt(c*x)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{(bx^2 + a)^{\frac{3}{2}} \sqrt{cx}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/2)*sqrt(c*x)), x, algorithm="fricas")`

[Out] `integral(1/((b*x^2 + a)^(3/2)*sqrt(c*x)), x)`

**Sympy [A]** time = 8.88972, size = 44, normalized size = 0.35

$$\frac{\sqrt{x} \left( \frac{1}{4} \right) {}_2F_1 \left( \frac{1}{4}, \frac{3}{2} \middle| \frac{bx^2 e^{i\pi}}{a} \right)}{2a^{\frac{3}{2}} \sqrt{c} \left( \frac{5}{4} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(1/2)/(b*x**2+a)**(3/2), x)`

[Out] `sqrt(x)*gamma(1/4)*hyper((1/4, 3/2), (5/4, ), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*sqrt(c)*gamma(5/4))`

---

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{2}} \sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/2)*sqrt(c*x)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(3/2)*sqrt(c*x)), x)`

$$3.626 \quad \int \frac{1}{(cx)^{3/2}(a+bx^2)^{3/2}} dx$$

**Optimal.** Leaf size=296

$$\frac{3\sqrt[4]{b}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{2a^{7/4}c^{3/2}\sqrt{a+bx^2}} - \frac{3\sqrt[4]{b}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{a^{7/4}c^{3/2}\sqrt{a+bx^2}} + \frac{3\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{a^2c^2(\sqrt{a} + \sqrt{bx})} - \frac{3\sqrt{a+bx^2}}{a^2c\sqrt{cx}} + \frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}}$$

[Out] 1/(a\*c\*Sqrt[c\*x]\*Sqrt[a + b\*x^2]) - (3\*Sqrt[a + b\*x^2])/(a^2\*c\*Sqrt[c\*x]) + (3\*Sqrt[b]\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(a^2\*c^2\*(Sqrt[a] + Sqrt[b]\*x)) - (3\*b^(1/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticE[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(a^(7/4)\*c^(3/2)\*Sqrt[a + b\*x^2]) + (3\*b^(1/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(2\*a^(7/4)\*c^(3/2)\*Sqrt[a + b\*x^2])

**Rubi [A]** time = 0.595667, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$\frac{3\sqrt[4]{b}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{2a^{7/4}c^{3/2}\sqrt{a+bx^2}} - \frac{3\sqrt[4]{b}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{a^{7/4}c^{3/2}\sqrt{a+bx^2}} + \frac{3\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{a^2c^2(\sqrt{a} + \sqrt{bx})} - \frac{3\sqrt{a+bx^2}}{a^2c\sqrt{cx}} + \frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(3/2)\*(a + b\*x^2)^(3/2)),x]

[Out] 1/(a\*c\*Sqrt[c\*x]\*Sqrt[a + b\*x^2]) - (3\*Sqrt[a + b\*x^2])/(a^2\*c\*Sqrt[c\*x]) + (3\*Sqrt[b]\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(a^2\*c^2\*(Sqrt[a] + Sqrt[b]\*x)) - (3\*b^(1/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticE[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(a^(7/4)\*c^(3/2)\*Sqrt[a + b\*x^2]) + (3\*b^(1/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(2\*a^(7/4)\*c^(3/2)\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 59.1573, size = 272, normalized size = 0.92

$$\frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}} + \frac{3\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{a^2c^2(\sqrt{a}+\sqrt{bx})} - \frac{3\sqrt{a+bx^2}}{a^2c\sqrt{cx}}$$

$$- \frac{3\sqrt[4]{b}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{a^{\frac{7}{4}}c^{\frac{3}{2}}\sqrt{a+bx^2}}$$

$$+ \frac{3\sqrt[4]{b}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{2a^{\frac{7}{4}}c^{\frac{3}{2}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(c*x)**(3/2)/(b*x**2+a)**(3/2),x)`

[Out] `1/(a*c*sqrt(c*x)*sqrt(a+b*x**2))+3*sqrt(b)*sqrt(c*x)*sqrt(a+b*x**2)/(a**2*c**2*(sqrt(a)+sqrt(b)*x))-3*sqrt(a+b*x**2)/(a**2*c*sqrt(c*x))-3*b**(1/4)*sqrt((a+b*x**2)/(sqrt(a)+sqrt(b)*x)**2)*(sqrt(a)+sqrt(b)*x)*elliptic_e(2*atan(b**(1/4)*sqrt(c*x)/(a**(1/4)*sqrt(c))),1/2)/(a**(7/4)*c**(3/2)*sqrt(a+b*x**2))+3*b**(1/4)*sqrt((a+b*x**2)/(sqrt(a)+sqrt(b)*x)**2)*(sqrt(a)+sqrt(b)*x)*elliptic_f(2*atan(b**(1/4)*sqrt(c*x)/(a**(1/4)*sqrt(c))),1/2)/(2*a**(7/4)*c**(3/2)*sqrt(a+b*x**2))`

**Mathematica [C]** time = 0.220314, size = 180, normalized size = 0.61

$$\frac{x\left(-\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}(2a+3bx^2)-3\sqrt{a}\sqrt{bx}\sqrt{\frac{bx^2}{a}}+1F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle|-1\right)+3\sqrt{a}\sqrt{bx}\sqrt{\frac{bx^2}{a}}+1E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle|-1\right)\right)}{a^2(cx)^{3/2}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/((c*x)^(3/2)*(a+b*x^2)^(3/2)),x]`

[Out] `(x*(-(Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*(2*a+3*b*x^2))+3*Sqrt[a]*Sqrt[b]*x*Sqrt[1+(b*x^2)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]],-1]-3*Sqrt[a]*Sqrt[b]*x*Sqrt[1+(b*x^2)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]],-1]))/(a^2*Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*(c*x)^(3/2)*Sqrt[a+b*x^2])`

**Maple [A]** time = 0.027, size = 197, normalized size = 0.7

$$\frac{1}{2a^2c}\left(6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{2a}-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{bx}{\sqrt{-ab}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(3/2)/(b*x^2+a)^(3/2),x)`

[Out] `1/2*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a-3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*a-6*b*x^2-4*a)/(b*x^2+a)^(3/2)/c/(c*x)^(3/2)/a^2`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{2}} (cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(3/2)),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(3/2)), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bcx^3 + acx)\sqrt{bx^2 + a}\sqrt{cx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(3/2)),x, algorithm="fricas")`

[Out] `integral(1/((b*c*x^3 + a*c*x)*sqrt(b*x^2 + a)*sqrt(c*x)), x)`

---

**Sympy [A]** time = 28.1362, size = 48, normalized size = 0.16

$$\frac{\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{3}{2} \\ \frac{3}{4} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}c^{\frac{3}{2}}\sqrt{x}\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(3/2)/(b*x**2+a)**(3/2),x)`

[Out] `gamma(-1/4)*hyper((-1/4, 3/2), (3/4, ), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*c**(3/2)*sqrt(x)*gamma(3/4)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{2}} (cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(3/2)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(3/2)), x)`

$$3.627 \quad \int \frac{1}{(cx)^{5/2}(a+bx^2)^{3/2}} dx$$

**Optimal.** Leaf size=154

$$\frac{5b^{3/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{6a^{9/4}c^{5/2}\sqrt{a+bx^2}} - \frac{5\sqrt{a+bx^2}}{3a^2c(cx)^{3/2}} + \frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}$$

[Out] 1/(a\*c\*(c\*x)^(3/2)\*Sqrt[a + b\*x^2]) - (5\*Sqrt[a + b\*x^2])/(3\*a^2\*c\*(c\*x)^(3/2)) - (5\*b^(3/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(6\*a^(9/4)\*c^(5/2)\*Sqrt[a + b\*x^2])

**Rubi [A]** time = 0.258346, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{5b^{3/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{6a^{9/4}c^{5/2}\sqrt{a+bx^2}} - \frac{5\sqrt{a+bx^2}}{3a^2c(cx)^{3/2}} + \frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(5/2)\*(a + b\*x^2)^(3/2)), x]

[Out] 1/(a\*c\*(c\*x)^(3/2)\*Sqrt[a + b\*x^2]) - (5\*Sqrt[a + b\*x^2])/(3\*a^2\*c\*(c\*x)^(3/2)) - (5\*b^(3/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(6\*a^(9/4)\*c^(5/2)\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 23.6931, size = 139, normalized size = 0.9

$$\frac{1}{ac(cx)^{\frac{3}{2}}\sqrt{a+bx^2}} - \frac{5\sqrt{a+bx^2}}{3a^2c(cx)^{\frac{3}{2}}} - \frac{5b^{\frac{3}{4}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a} + \sqrt{bx}) F\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{6a^{\frac{9}{4}}c^{\frac{5}{2}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(5/2)/(b\*x\*\*2+a)\*\*(3/2), x)

[Out] 1/(a\*c\*(c\*x)\*\*(3/2)\*sqrt(a + b\*x\*\*2)) - 5\*sqrt(a + b\*x\*\*2)/(3\*a\*\*2\*c\*(c\*x)\*\*(3/2)) - 5\*b\*\*(3/4)\*sqrt((a + b\*x\*\*2)/(sqrt(a) + sqrt(b)\*x)\*\*2)\*(sqrt(a) + sqrt(b)\*x)\*elliptic\_f(2\*atan(b\*\*(1/4)\*sqrt(c\*x)/(a\*\*(1/4)\*sqrt(c))), 1/2)/(6\*a\*\*(9/4)\*c\*\*(5/2)\*sqrt(a + b\*x\*\*2))

**Mathematica [C]** time = 0.157612, size = 130, normalized size = 0.84

$$\frac{x \left( -\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} (2a + 5bx^2) - 5ibx^{5/2} \sqrt{\frac{a}{bx^2}} + 1 F\left( i \sinh^{-1}\left( \frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}} \right) \middle| -1 \right) \right)}{3a^2 \sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} (cx)^{5/2} \sqrt{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(5/2)\*(a + b\*x^2)^(3/2)),x]

[Out] (x\*(-(Sqrt[(I\*Sqrt[a])/Sqrt[b]]\*(2\*a + 5\*b\*x^2)) - (5\*I)\*b\*Sqrt[1 + a/(b\*x^2)]\*x^(5/2)\*EllipticF[I\*ArcSinh[Sqrt[(I\*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1]))/(3\*a^2\*Sqrt[(I\*Sqrt[a])/Sqrt[b]]\*(c\*x)^(5/2)\*Sqrt[a + b\*x^2])

**Maple [A]** time = 0.027, size = 124, normalized size = 0.8

$$-\frac{1}{6a^2xc^2} \left( 5 \sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{-bx + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{bx}{\sqrt{-ab}}} \operatorname{EllipticF} \left( \sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \sqrt{2} \right) \sqrt{-ab} \sqrt{2x + 10bx^2 + 4a} \right) \frac{1}{\sqrt{cx}} \frac{1}{\sqrt{bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x)^(5/2)/(b\*x^2+a)^(3/2),x)

[Out] -1/6/x\*(5\*((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*((-b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*(-x\*b/(-a\*b)^(1/2))^(1/2)\*EllipticF((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2),1/2\*2^(1/2))\*(-a\*b)^(1/2)\*2^(1/2)\*x+10\*b\*x^2+4\*a)/(b\*x^2+a)^(1/2)/a^2/c^2/(c\*x)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{2}} (cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(3/2)\*(c\*x)^(5/2)),x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(3/2)\*(c\*x)^(5/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{1}{(bc^2x^4 + ac^2x^2)\sqrt{bx^2 + a}\sqrt{cx}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(3/2)\*(c\*x)^(5/2)),x, algorithm="fricas")

[Out] integral(1/((b\*c^2\*x^4 + a\*c^2\*x^2)\*sqrt(b\*x^2 + a)\*sqrt(c\*x)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(5/2)/(b\*x\*\*2+a)\*\*(3/2),x)



[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{2}} (cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(5/2)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(5/2)), x)`

$$3.628 \quad \int \frac{1}{(cx)^{7/2}(a+bx^2)^{3/2}} dx$$

**Optimal.** Leaf size=331

$$\frac{21b^{5/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{10a^{11/4}c^{7/2}\sqrt{a+bx^2}} + \frac{21b^{5/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5a^{11/4}c^{7/2}\sqrt{a+bx^2}} - \frac{21b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5a^3c^4(\sqrt{a} + \sqrt{bx})} + \frac{21b\sqrt{a+bx^2}}{5a^3c^3\sqrt{cx}} - \frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{5/2}} + \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}$$

[Out] 1/(a\*c\*(c\*x)^(5/2)\*Sqrt[a + b\*x^2]) - (7\*Sqrt[a + b\*x^2])/(5\*a^2\*c\*(c\*x)^(5/2)) + (21\*b\*Sqrt[a + b\*x^2])/(5\*a^3\*c^3\*Sqrt[c\*x]) - (21\*b^(3/2)\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(5\*a^3\*c^4\*(Sqrt[a] + Sqrt[b]\*x)) + (21\*b^(5/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticE[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(5\*a^(11/4)\*c^(7/2)\*Sqrt[a + b\*x^2]) - (21\*b^(5/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(10\*a^(11/4)\*c^(7/2)\*Sqrt[a + b\*x^2])

**Rubi [A]** time = 0.679896, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$\frac{21b^{5/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{10a^{11/4}c^{7/2}\sqrt{a+bx^2}} + \frac{21b^{5/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{5a^{11/4}c^{7/2}\sqrt{a+bx^2}} - \frac{21b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5a^3c^4(\sqrt{a} + \sqrt{bx})} + \frac{21b\sqrt{a+bx^2}}{5a^3c^3\sqrt{cx}} - \frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{5/2}} + \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(7/2)\*(a + b\*x^2)^(3/2)),x]

[Out] 1/(a\*c\*(c\*x)^(5/2)\*Sqrt[a + b\*x^2]) - (7\*Sqrt[a + b\*x^2])/(5\*a^2\*c\*(c\*x)^(5/2)) + (21\*b\*Sqrt[a + b\*x^2])/(5\*a^3\*c^3\*Sqrt[c\*x]) - (21\*b^(3/2)\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(5\*a^3\*c^4\*(Sqrt[a] + Sqrt[b]\*x)) + (21\*b^(5/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticE[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(5\*a^(11/4)\*c^(7/2)\*Sqrt[a + b\*x^2]) - (21\*b^(5/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(10\*a^(11/4)\*c^(7/2)\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 71.0126, size = 306, normalized size = 0.92

$$\frac{1}{ac(cx)^{\frac{5}{2}}\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{\frac{5}{2}}} - \frac{21b^{\frac{3}{2}}\sqrt{cx}\sqrt{a+bx^2}}{5a^3c^4(\sqrt{a}+\sqrt{bx})} + \frac{21b\sqrt{a+bx^2}}{5a^3c^3\sqrt{cx}}$$

$$+ \frac{21b^{\frac{5}{4}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{5a^{\frac{11}{4}}c^{\frac{7}{2}}\sqrt{a+bx^2}}$$

$$- \frac{21b^{\frac{5}{4}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{10a^{\frac{11}{4}}c^{\frac{7}{2}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(c*x)**(7/2)/(b*x**2+a)**(3/2),x)`

[Out]  $1/(a*c*(c*x)^{(5/2)*\sqrt{a+b*x^2}}) - 7*\sqrt{a+b*x^2}/(5*a^{**2}*c*(c*x)^{(5/2)}) - 21*b^{(3/2)*\sqrt{c*x}}*\sqrt{a+b*x^2}/(5*a^{**3}*c^{**4}*(\sqrt{a}+\sqrt{b}*x)) + 21*b*\sqrt{a+b*x^2}/(5*a^{**3}*c^{**3}*\sqrt{c*x}) + 21*b^{(5/4)*\sqrt{(a+b*x^2)/(\sqrt{a}+\sqrt{b}*x)^{**2}}*(\sqrt{a}+\sqrt{b}*x)*\operatorname{elliptic}_e(2*\operatorname{atan}(b^{(1/4)*\sqrt{c*x}}/(a^{(1/4)*\sqrt{c}})), 1/2)/(5*a^{(11/4)*c^{(7/2)*\sqrt{a+b*x^2}}}) - 21*b^{(5/4)*\sqrt{(a+b*x^2)/(\sqrt{a}+\sqrt{b}*x)^{**2}}*(\sqrt{a}+\sqrt{b}*x)*\operatorname{elliptic}_f(2*\operatorname{atan}(b^{(1/4)*\sqrt{c*x}}/(a^{(1/4)*\sqrt{c}})), 1/2)/(10*a^{(11/4)*c^{(7/2)*\sqrt{a+b*x^2}}})$

**Mathematica [C]** time = 0.236381, size = 197, normalized size = 0.6

$$x \left( \sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}} (-2a^2 + 14abx^2 + 21b^2x^4) + 21\sqrt{ab}^{3/2}x^3\sqrt{\frac{bx^2}{a}} + 1F \left( i \sinh^{-1} \left( \sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}} \right) \middle| -1 \right) - 21\sqrt{ab}^{3/2}x^3\sqrt{\frac{bx^2}{a}} + 1E \left( i \sinh^{-1} \left( \sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}} \right) \right) \right) / (5a^3(cx)^{7/2}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\sqrt{a+bx^2})$$

Antiderivative was successfully verified.

[In] `Integrate[1/((c*x)^(7/2)*(a+b*x^2)^(3/2)),x]`

[Out]  $(x*(\operatorname{Sqrt}[(I*\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]*(-2*a^2+14*a*b*x^2+21*b^2*x^4)) - 21*\operatorname{Sqrt}[a]*b^{(3/2)*x^3*\operatorname{Sqrt}[1+(b*x^2)/a]}*\operatorname{EllipticE}[I*\operatorname{ArcSinh}[\operatorname{Sqrt}[(I*\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]], -1) + 21*\operatorname{Sqrt}[a]*b^{(3/2)*x^3*\operatorname{Sqrt}[1+(b*x^2)/a]}*\operatorname{EllipticF}[I*\operatorname{ArcSinh}[\operatorname{Sqrt}[(I*\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]], -1)]/(5*a^3*\operatorname{Sqrt}[(I*\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]*(c*x)^{(7/2)*\operatorname{Sqrt}[a+b*x^2]})$

**Maple [A]** time = 0.027, size = 219, normalized size = 0.7

$$\frac{1}{10x^2c^3a^3} \left( 21\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}, 1/2\sqrt{2}\right)\sqrt{2}x^2ab - 42\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}, 1/2\sqrt{2}\right)\sqrt{2}x^2ab \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(7/2)/(b*x^2+a)^(3/2),x)`

[Out]  $1/10*(21*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*\operatorname{EllipticF}((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2))*2^(1/2)*x^2*a*b - 42*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*\operatorname{EllipticE}((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2))$

$$b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)})*2^{(1/2)}*x^2*a*b+42*b^2*x^4+28*a*b*x^2-4*a^2)/x^2/(b*x^2+a)^{(1/2)}/c^3/(c*x)^{(1/2)}/a^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{2}} (cx)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(3/2)\*(c\*x)^(7/2)),x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(3/2)\*(c\*x)^(7/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bc^3x^5 + ac^3x^3)\sqrt{bx^2 + a}\sqrt{cx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(3/2)\*(c\*x)^(7/2)),x, algorithm="fricas")

[Out] integral(1/((b\*c^3\*x^5 + a\*c^3\*x^3)\*sqrt(b\*x^2 + a)\*sqrt(c\*x)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(7/2)/(b\*x\*\*2+a)\*\*(3/2),x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{2}} (cx)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(3/2)\*(c\*x)^(7/2)),x, algorithm="giac")

[Out] integrate(1/((b\*x^2 + a)^(3/2)\*(c\*x)^(7/2)), x)

$$3.629 \quad \int \frac{(cx)^{7/2}}{(a+bx^2)^{5/2}} dx$$

**Optimal.** Leaf size=155

$$\frac{5c^{7/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{12\sqrt[4]{ab}^{9/4}\sqrt{a+bx^2}} - \frac{5c^3\sqrt{cx}}{6b^2\sqrt{a+bx^2}} - \frac{c(cx)^{5/2}}{3b(a+bx^2)^{3/2}}$$

[Out]  $-(c*(c*x)^{(5/2)})/(3*b*(a+b*x^2)^{(3/2)}) - (5*c^3*\text{Sqrt}[c*x])/(6*b^2*\text{Sqrt}[a+b*x^2]) + (5*c^{(7/2)}*(\text{Sqrt}[a]+\text{Sqrt}[b]*x)*\text{Sqrt}[(a+b*x^2)/(\text{Sqrt}[a]+\text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(12*a^{(1/4)}*b^{(9/4)}*\text{Sqrt}[a+b*x^2])$

**Rubi [A]** time = 0.253884, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{5c^{7/2}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{12\sqrt[4]{ab}^{9/4}\sqrt{a+bx^2}} - \frac{5c^3\sqrt{cx}}{6b^2\sqrt{a+bx^2}} - \frac{c(cx)^{5/2}}{3b(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(7/2)/(a+b\*x^2)^(5/2),x]

[Out]  $-(c*(c*x)^{(5/2)})/(3*b*(a+b*x^2)^{(3/2)}) - (5*c^3*\text{Sqrt}[c*x])/(6*b^2*\text{Sqrt}[a+b*x^2]) + (5*c^{(7/2)}*(\text{Sqrt}[a]+\text{Sqrt}[b]*x)*\text{Sqrt}[(a+b*x^2)/(\text{Sqrt}[a]+\text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(12*a^{(1/4)}*b^{(9/4)}*\text{Sqrt}[a+b*x^2])$

**Rubi in Sympy [A]** time = 25.2179, size = 141, normalized size = 0.91

$$-\frac{c(cx)^{5/2}}{3b(a+bx^2)^{3/2}} - \frac{5c^3\sqrt{cx}}{6b^2\sqrt{a+bx^2}} + \frac{5c^{7/2}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})F\left(2\text{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{12\sqrt[4]{ab}^{9/4}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(7/2)/(b\*x\*\*2+a)\*\*(5/2),x)

[Out]  $-c*(c*x)^{(5/2)}/(3*b*(a+b*x^2)^{(3/2)}) - 5*c^3*\text{sqrt}(c*x)/(6*b^2*\text{sqrt}(a+b*x^2)) + 5*c^{(7/2)}*\text{sqrt}((a+b*x^2)/(\text{sqrt}(a)+\text{sqrt}(b)*x)^2)*(\text{sqrt}(a)+\text{sqrt}(b)*x)*\text{elliptic\_f}(2*\text{atan}(b^{(1/4)}*\text{sqrt}(c*x)/(a^{(1/4)}*\text{sqrt}(c))), 1/2)/(12*a^{(1/4)}*b^{(9/4)}*\text{sqrt}(a+b*x^2))$

**Mathematica [C]** time = 0.47241, size = 117, normalized size = 0.75

$$\frac{c^3\sqrt{cx} \left( \frac{5i\sqrt{x}\sqrt{\frac{a}{bx^2}+1}(a+bx^2)F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right) \middle| -1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}} - 5a - 7bx^2 \right)}{6b^2(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(7/2)/(a + b\*x^2)^(5/2), x]

[Out] (c^3\*Sqrt[c\*x]\*(-5\*a - 7\*b\*x^2 + ((5\*I)\*Sqrt[1 + a/(b\*x^2)]\*Sqrt[x]\*(a + b\*x^2)\*EllipticF[I\*ArcSinh[Sqrt[(I\*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1])/Sqrt[(I\*Sqrt[a])/Sqrt[b]]))/(6\*b^2\*(a + b\*x^2)^(3/2))

**Maple [A]** time = 0.051, size = 219, normalized size = 1.4

$$\frac{c^3}{12b^3x} \left( 5 \sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{-bx + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{bx}{\sqrt{-ab}}} \operatorname{EllipticF} \left( \sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \sqrt{2} \right) \sqrt{-ab} \sqrt{2x^2b + 5} \sqrt{-ab} \sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{-bx + \sqrt{-ab}}{\sqrt{-ab}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(7/2)/(b\*x^2+a)^(5/2), x)

[Out] 1/12\*(5\*((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*((-b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*(-x\*b/(-a\*b)^(1/2))^(1/2)\*EllipticF(((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2), 1/2\*2^(1/2))\*(-a\*b)^(1/2)\*2^(1/2)\*x^2\*b+5\*(-a\*b)^(1/2)\*((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*((-b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*(-x\*b/(-a\*b)^(1/2))^(1/2)\*EllipticF(((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2), 1/2\*2^(1/2))\*2^(1/2)\*a-14\*b^2\*x^3-10\*a\*b\*x)\*c^3/x\*(c\*x)^(1/2)/b^3/(b\*x^2+a)^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{7}{2}}}{(bx^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(7/2)/(b\*x^2 + a)^(5/2), x, algorithm="maxima")

[Out] integrate((c\*x)^(7/2)/(b\*x^2 + a)^(5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{\sqrt{cx} c^3 x^3}{(b^2 x^4 + 2 a b x^2 + a^2) \sqrt{b x^2 + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(7/2)/(b\*x^2 + a)^(5/2), x, algorithm="fricas")

[Out] integral(sqrt(c\*x)\*c^3\*x^3/((b^2\*x^4 + 2\*a\*b\*x^2 + a^2)\*sqrt(b\*x^2 + a)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{7}{2}}}{(bx^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(7/2)/(b*x^2 + a)^(5/2), x)
```

$$3.630 \quad \int \frac{(cx)^{5/2}}{(a+bx^2)^{5/2}} dx$$

**Optimal.** Leaf size=304

$$\frac{c^{5/2} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{4a^{3/4}b^{7/4}\sqrt{a+bx^2}} + \frac{c^{5/2} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{2a^{3/4}b^{7/4}\sqrt{a+bx^2}} - \frac{c^2\sqrt{cx}\sqrt{a+bx^2}}{2ab^{3/2}(\sqrt{a} + \sqrt{bx})} + \frac{c(cx)^{3/2}}{2ab\sqrt{a+bx^2}} - \frac{c(cx)^{3/2}}{3b(a+bx^2)^{3/2}}$$

[Out]  $-(c*(c*x)^{(3/2)})/(3*b*(a+b*x^2)^{(3/2)}) + (c*(c*x)^{(3/2)})/(2*a*b*\text{Sqrt}[a+b*x^2]) - (c^2*\text{Sqrt}[c*x]*\text{Sqrt}[a+b*x^2])/(2*a*b^{(3/2)}*(\text{Sqrt}[a]+\text{Sqrt}[b]*x)) + (c^{(5/2)}*(\text{Sqrt}[a]+\text{Sqrt}[b]*x)*\text{Sqrt}[(a+b*x^2)/(\text{Sqrt}[a]+\text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(2*a^{(3/4)}*b^{(7/4)}*\text{Sqrt}[a+b*x^2]) - (c^{(5/2)}*(\text{Sqrt}[a]+\text{Sqrt}[b]*x)*\text{Sqrt}[(a+b*x^2)/(\text{Sqrt}[a]+\text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(4*a^{(3/4)}*b^{(7/4)}*\text{Sqrt}[a+b*x^2])$

**Rubi [A]** time = 0.593803, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$\frac{c^{5/2} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{4a^{3/4}b^{7/4}\sqrt{a+bx^2}} + \frac{c^{5/2} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{2a^{3/4}b^{7/4}\sqrt{a+bx^2}} - \frac{c^2\sqrt{cx}\sqrt{a+bx^2}}{2ab^{3/2}(\sqrt{a} + \sqrt{bx})} + \frac{c(cx)^{3/2}}{2ab\sqrt{a+bx^2}} - \frac{c(cx)^{3/2}}{3b(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(5/2)/(a+b\*x^2)^(5/2), x]

[Out]  $-(c*(c*x)^{(3/2)})/(3*b*(a+b*x^2)^{(3/2)}) + (c*(c*x)^{(3/2)})/(2*a*b*\text{Sqrt}[a+b*x^2]) - (c^2*\text{Sqrt}[c*x]*\text{Sqrt}[a+b*x^2])/(2*a*b^{(3/2)}*(\text{Sqrt}[a]+\text{Sqrt}[b]*x)) + (c^{(5/2)}*(\text{Sqrt}[a]+\text{Sqrt}[b]*x)*\text{Sqrt}[(a+b*x^2)/(\text{Sqrt}[a]+\text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(2*a^{(3/4)}*b^{(7/4)}*\text{Sqrt}[a+b*x^2]) - (c^{(5/2)}*(\text{Sqrt}[a]+\text{Sqrt}[b]*x)*\text{Sqrt}[(a+b*x^2)/(\text{Sqrt}[a]+\text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(4*a^{(3/4)}*b^{(7/4)}*\text{Sqrt}[a+b*x^2])$



**Rubi in Sympy [A]** time = 59.2727, size = 269, normalized size = 0.88

$$\begin{aligned} & -\frac{c(cx)^{\frac{3}{2}}}{3b(a+bx^2)^{\frac{3}{2}}} + \frac{c(cx)^{\frac{3}{2}}}{2ab\sqrt{a+bx^2}} - \frac{c^2\sqrt{cx}\sqrt{a+bx^2}}{2ab^{\frac{3}{2}}(\sqrt{a}+\sqrt{bx})} \\ & + \frac{c^{\frac{5}{2}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{2a^{\frac{3}{4}}b^{\frac{7}{4}}\sqrt{a+bx^2}} \\ & - \frac{c^{\frac{5}{2}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{4a^{\frac{3}{4}}b^{\frac{7}{4}}\sqrt{a+bx^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x)**(5/2)/(b*x**2+a)**(5/2),x)`

[Out]  $-c*(c*x)^{(3/2)}/(3*b*(a+b*x^2)^{(3/2)}) + c*(c*x)^{(3/2)}/(2*a*b*\sqrt{a+b*x^2}) - c**2*\sqrt{c*x}*\sqrt{a+b*x^2}/(2*a*b^{(3/2)}*(\sqrt{a}+\sqrt{b}*x)) + c**(5/2)*\sqrt{c*x}*(\sqrt{a}+\sqrt{b}*x)*\operatorname{elliptic}_e(2*\operatorname{atan}(b^{(1/4)}*\sqrt{c*x}/(a^{(1/4)}*\sqrt{c})),1/2)/(2*a^{(3/4)}*b^{(7/4)}*\sqrt{a+b*x^2}) - c**(5/2)*\sqrt{c*x}*(\sqrt{a}+\sqrt{b}*x)*\operatorname{elliptic}_f(2*\operatorname{atan}(b^{(1/4)}*\sqrt{c*x}/(a^{(1/4)}*\sqrt{c})),1/2)/(4*a^{(3/4)}*b^{(7/4)}*\sqrt{a+b*x^2})$

**Mathematica [C]** time = 0.312484, size = 195, normalized size = 0.64

$$\frac{c^2\sqrt{cx}\left(\sqrt{bx}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}(a+3bx^2)+3\sqrt{a}(a+bx^2)\sqrt{\frac{bx^2}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle|-1\right)-3\sqrt{a}(a+bx^2)\sqrt{\frac{bx^2}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\right)\right)}{6ab^{3/2}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c*x)^(5/2)/(a+b*x^2)^(5/2),x]`

[Out]  $(c^2*\sqrt{c*x}*(\sqrt{b}*x*\sqrt{(I*\sqrt{b}*x)/\sqrt{a}})^*(a+3*b*x^2) - 3*\sqrt{a}*(a+b*x^2)*\sqrt{1+(b*x^2)/a}*\operatorname{EllipticE}[I*\operatorname{ArcSinh}[\sqrt{(I*\sqrt{b}*x)/\sqrt{a}}],-1] + 3*\sqrt{a}*(a+b*x^2)*\sqrt{1+(b*x^2)/a}*\operatorname{EllipticF}[I*\operatorname{ArcSinh}[\sqrt{(I*\sqrt{b}*x)/\sqrt{a}}],-1])/(6*a*b^{(3/2)}*\sqrt{(I*\sqrt{b}*x)/\sqrt{a}})^*(a+b*x^2)^{(3/2)}$

**Maple [A]** time = 0.049, size = 385, normalized size = 1.3

$$\frac{c^2}{12ab^2x}\left(3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2,\sqrt{2}\right)\sqrt{2x^2ab}-6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(5/2)/(b*x^2+a)^(5/2),x)`

[Out]  $1/12*(3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*\operatorname{EllipticF}((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))/(-a*b)^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a*b-6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*\operatorname{EllipticE}((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))/(-a*b)^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a*b+3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)$

$$2))^{(1/2)} * (-x*b/(-a*b)^{(1/2)})^{(1/2)} * \text{EllipticF}(((b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)}) * 2^{(1/2)} * a^2 - 6 * ((b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)} * ((-b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)} * (-x*b/(-a*b)^{(1/2)})^{(1/2)} * \text{EllipticE}(((b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)}) * 2^{(1/2)} * a^2 + 6 * b^2 * x^4 + 2 * a * b * x^2) * c^{1/2} / x * (c*x)^{(1/2)} / b^2 / a / (b*x^2+a)^{(3/2)}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{(bx^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(5/2)/(b\*x^2 + a)^(5/2), x, algorithm="maxima")

[Out] integrate((c\*x)^(5/2)/(b\*x^2 + a)^(5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c}xc^2x^2}{(b^2x^4 + 2abx^2 + a^2)\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(5/2)/(b\*x^2 + a)^(5/2), x, algorithm="fricas")

[Out] integral(sqrt(c\*x)\*c^2\*x^2/((b^2\*x^4 + 2\*a\*b\*x^2 + a^2)\*sqrt(b\*x^2 + a)), x)

**Sympy [A]** time = 170.984, size = 44, normalized size = 0.14

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\left(\frac{7}{4}\right) {}_2F_1\left(\frac{7}{4}, \frac{5}{2} \middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(5/2)/(b\*x\*\*2+a)\*\*(5/2), x)

[Out] c\*\*(5/2)\*x\*\*(7/2)\*gamma(7/4)\*hyper((7/4, 5/2), (11/4, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*a\*\*(5/2)\*gamma(11/4))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{(bx^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(5/2)/(b\*x^2 + a)^(5/2), x, algorithm="giac")

[Out] integrate((c\*x)^(5/2)/(b\*x^2 + a)^(5/2), x)

$$3.631 \quad \int \frac{(cx)^{3/2}}{(a+bx^2)^{5/2}} dx$$

**Optimal.** Leaf size=156

$$\frac{c^{3/2} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{12a^{5/4}b^{5/4}\sqrt{a+bx^2}} + \frac{c\sqrt{cx}}{6ab\sqrt{a+bx^2}} - \frac{c\sqrt{cx}}{3b(a+bx^2)^{3/2}}$$

[Out]  $-(c*\text{Sqrt}[c*x])/(3*b*(a+b*x^2)^(3/2)) + (c*\text{Sqrt}[c*x])/(6*a*b*\text{Sqrt}[a+b*x^2]) + (c^(3/2)*( \text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a+b*x^2)/( \text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^(1/4)*\text{Sqrt}[c*x])/(a^(1/4)*\text{Sqrt}[c])], 1/2])/(12*a^(5/4)*b^(5/4)*\text{Sqrt}[a+b*x^2])$

**Rubi [A]** time = 0.25216, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{c^{3/2} (\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{12a^{5/4}b^{5/4}\sqrt{a+bx^2}} + \frac{c\sqrt{cx}}{6ab\sqrt{a+bx^2}} - \frac{c\sqrt{cx}}{3b(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(3/2)/(a+b\*x^2)^(5/2),x]

[Out]  $-(c*\text{Sqrt}[c*x])/(3*b*(a+b*x^2)^(3/2)) + (c*\text{Sqrt}[c*x])/(6*a*b*\text{Sqrt}[a+b*x^2]) + (c^(3/2)*( \text{Sqrt}[a] + \text{Sqrt}[b]*x)*\text{Sqrt}[(a+b*x^2)/( \text{Sqrt}[a] + \text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^(1/4)*\text{Sqrt}[c*x])/(a^(1/4)*\text{Sqrt}[c])], 1/2])/(12*a^(5/4)*b^(5/4)*\text{Sqrt}[a+b*x^2])$

**Rubi in Sympy [A]** time = 24.4707, size = 136, normalized size = 0.87

$$-\frac{c\sqrt{cx}}{3b(a+bx^2)^{3/2}} + \frac{c\sqrt{cx}}{6ab\sqrt{a+bx^2}} + \frac{c^{3/2} \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} (\sqrt{a} + \sqrt{bx}) F\left(2 \text{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{12a^{5/4}b^{5/4}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(3/2)/(b\*x\*\*2+a)\*\*(5/2),x)

[Out]  $-c*\text{sqrt}(c*x)/(3*b*(a+b*x**2)**(3/2)) + c*\text{sqrt}(c*x)/(6*a*b*\text{sqrt}(a+b*x**2)) + c**(3/2)*\text{sqrt}((a+b*x**2)/( \text{sqrt}(a) + \text{sqrt}(b)*x)**2)*(\text{sqrt}(a) + \text{sqrt}(b)*x)*\text{elliptic}_f(2*\text{atan}(b**(1/4)*\text{sqrt}(c*x)/(a**(1/4)*\text{sqrt}(c))), 1/2)/(12*a**(5/4)*b**(5/4)*\text{sqrt}(a+b*x**2))$

**Mathematica [C]** time = 0.189829, size = 137, normalized size = 0.88

$$\frac{c\sqrt{cx} \left( \sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} (bx^2 - a) + i\sqrt{x} \sqrt{\frac{a}{bx^2} + 1} (a + bx^2) F\left(i \sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right) - 1 \right) \right)}{6ab\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}} (a + bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(3/2)/(a+b\*x^2)^(5/2),x]

[Out]  $(c \sqrt{c x} (\sqrt{(I \sqrt{a})/\sqrt{b}} (-a + b x^2) + I \sqrt{1 + a/(b x^2)}) \sqrt{x} (a + b x^2) \text{EllipticF}[I \text{ArcSinh}[\sqrt{(I \sqrt{a})/\sqrt{b}}/\sqrt{x}], -1]))/(6 a \sqrt{(I \sqrt{a})/\sqrt{b}} b (a + b x^2)^{3/2})$

**Maple [A]** time = 0.019, size = 218, normalized size = 1.4

$$\frac{c}{12 a b^2 x} \left( \sqrt{1 (b x + \sqrt{-a b})} \frac{1}{\sqrt{-a b}} \sqrt{1 (-b x + \sqrt{-a b})} \frac{1}{\sqrt{-a b}} \sqrt{-b x} \frac{1}{\sqrt{-a b}} \text{EllipticF} \left( \sqrt{1 (b x + \sqrt{-a b})} \frac{1}{\sqrt{-a b}}, \frac{\sqrt{2}}{2} \right) \sqrt{-a b} \sqrt{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(3/2)/(b*x^2+a)^(5/2),x)`

[Out]  $1/12 * (((b*x + (-a*b)^{1/2})/(-a*b)^{1/2})^{1/2} * ((-b*x + (-a*b)^{1/2})/(-a*b)^{1/2})^{1/2} * (-x*b/(-a*b)^{1/2})^{1/2} * \text{EllipticF}(((b*x + (-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}, 1/2 * 2^{1/2})) * (-a*b)^{1/2} * 2^{1/2} * x^2 * b + (-a*b)^{1/2} * ((b*x + (-a*b)^{1/2})/(-a*b)^{1/2})^{1/2} * ((-b*x + (-a*b)^{1/2})/(-a*b)^{1/2})^{1/2} * (-x*b/(-a*b)^{1/2})^{1/2} * \text{EllipticF}(((b*x + (-a*b)^{1/2})/(-a*b)^{1/2})^{1/2}, 1/2 * 2^{1/2})) * 2^{1/2} * a + 2 * b^2 * x^3 - 2 * a * b * x) * c/x * (c*x)^{1/2}/a/b^2/(b*x^2+a)^{3/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{(bx^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(3/2)/(b*x^2 + a)^(5/2),x, algorithm="maxima")`

[Out] `integrate((c*x)^(3/2)/(b*x^2 + a)^(5/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\sqrt{c x} c x}{(b^2 x^4 + 2 a b x^2 + a^2) \sqrt{b x^2 + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(3/2)/(b*x^2 + a)^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(c*x)*c*x/((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)), x)`

**Sympy [A]** time = 55.0959, size = 44, normalized size = 0.28

$$\frac{c^{\frac{3}{2}} x^{\frac{5}{2}} \left( \frac{5}{4} \right) {}_2F_1 \left( \frac{5}{4}, \frac{5}{2} \middle| \frac{b x^2 e^{i \pi}}{a} \right)}{2 a^{\frac{5}{2}} \left( \frac{9}{4} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(3/2)/(b\*x\*\*2+a)\*\*(5/2),x)

[Out] c\*\*(3/2)\*x\*\*(5/2)\*gamma(5/4)\*hyper((5/4, 5/2), (9/4, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*a\*\*(5/2)\*gamma(9/4))

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{(bx^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(3/2)/(b\*x^2 + a)^(5/2),x, algorithm="giac")

[Out] integrate((c\*x)^(3/2)/(b\*x^2 + a)^(5/2), x)

$$3.632 \quad \int \frac{\sqrt{cx}}{(a+bx^2)^{5/2}} dx$$

**Optimal.** Leaf size=302

$$\frac{\sqrt{c}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a} + \sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{4a^{7/4}b^{3/4}\sqrt{a+bx^2}} + \frac{\sqrt{c}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a} + \sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{2a^{7/4}b^{3/4}\sqrt{a+bx^2}} + \frac{(cx)^{3/2}}{2a^2c\sqrt{a+bx^2}} - \frac{\sqrt{cx}\sqrt{a+bx^2}}{2a^2\sqrt{b}(\sqrt{a} + \sqrt{bx})} + \frac{(cx)^{3/2}}{3ac(a+bx^2)^{3/2}}$$

[Out] (c\*x)^(3/2)/(3\*a\*c\*(a + b\*x^2)^(3/2)) + (c\*x)^(3/2)/(2\*a^2\*c\*Sqrt[a + b\*x^2]) - (Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(2\*a^2\*Sqrt[b]\*(Sqrt[a] + Sqrt[b]\*x)) + (Sqrt[c]\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticE[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(2\*a^(7/4)\*b^(3/4)\*Sqrt[a + b\*x^2]) - (Sqrt[c]\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(4\*a^(7/4)\*b^(3/4)\*Sqrt[a + b\*x^2])

**Rubi [A]** time = 0.582577, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{\sqrt{c}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a} + \sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{4a^{7/4}b^{3/4}\sqrt{a+bx^2}} + \frac{\sqrt{c}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a} + \sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{2a^{7/4}b^{3/4}\sqrt{a+bx^2}} + \frac{(cx)^{3/2}}{2a^2c\sqrt{a+bx^2}} - \frac{\sqrt{cx}\sqrt{a+bx^2}}{2a^2\sqrt{b}(\sqrt{a} + \sqrt{bx})} + \frac{(cx)^{3/2}}{3ac(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c\*x]/(a + b\*x^2)^(5/2), x]

[Out] (c\*x)^(3/2)/(3\*a\*c\*(a + b\*x^2)^(3/2)) + (c\*x)^(3/2)/(2\*a^2\*c\*Sqrt[a + b\*x^2]) - (Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(2\*a^2\*Sqrt[b]\*(Sqrt[a] + Sqrt[b]\*x)) + (Sqrt[c]\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticE[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(2\*a^(7/4)\*b^(3/4)\*Sqrt[a + b\*x^2]) - (Sqrt[c]\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(4\*a^(7/4)\*b^(3/4)\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 57.1746, size = 267, normalized size = 0.88

$$\frac{(cx)^{\frac{3}{2}}}{3ac(a+bx^2)^{\frac{3}{2}}} + \frac{(cx)^{\frac{3}{2}}}{2a^2c\sqrt{a+bx^2}} - \frac{\sqrt{cx}\sqrt{a+bx^2}}{2a^2\sqrt{b}(\sqrt{a}+\sqrt{bx})}$$

$$+ \frac{\sqrt{c}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\right)\left|\frac{1}{2}\right.}{2a^{\frac{7}{4}}b^{\frac{3}{4}}\sqrt{a+bx^2}}$$

$$- \frac{\sqrt{c}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\right)\left|\frac{1}{2}\right.}{4a^{\frac{7}{4}}b^{\frac{3}{4}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x)**(1/2)/(b*x**2+a)**(5/2), x)`

[Out]  $(c*x)^{(3/2)/(3*a*c*(a+b*x^2)^{(3/2)})} + (c*x)^{(3/2)/(2*a^{**2}*c*\sqrt{a+b*x^2})} - \sqrt{c*x}*\sqrt{a+b*x^2}/(2*a^{**2}*\sqrt{b}*(\sqrt{a}+\sqrt{b*x})) + \sqrt{c}*\sqrt{(a+b*x^2)/(\sqrt{a}+\sqrt{b*x})}*(\sqrt{a}+\sqrt{b*x})*\operatorname{elliptic}_e(2*\operatorname{atan}(b^{**1/4}*\sqrt{c*x}/(a^{**1/4}*\sqrt{c})), 1/2)/(2*a^{**7/4}*b^{**3/4}*\sqrt{a+b*x^2}) - \sqrt{c}*\sqrt{(a+b*x^2)/(\sqrt{a}+\sqrt{b*x})}*(\sqrt{a}+\sqrt{b*x})^{**2}*(\sqrt{a}+\sqrt{b*x})*\operatorname{elliptic}_f(2*\operatorname{atan}(b^{**1/4}*\sqrt{c*x}/(a^{**1/4}*\sqrt{c})), 1/2)/(4*a^{**7/4}*b^{**3/4}*\sqrt{a+b*x^2})$

**Mathematica [C]** time = 0.431896, size = 194, normalized size = 0.64

$$\frac{ix\sqrt{cx}\left(\sqrt{bx}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}(5a+3bx^2)+3\sqrt{a}(a+bx^2)\sqrt{\frac{bx^2}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\right)-1\right)-3\sqrt{a}(a+bx^2)\sqrt{\frac{bx^2}{a}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\right)}{6a^{5/2}\left(\frac{i\sqrt{bx}}{\sqrt{a}}\right)^{3/2}(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[c*x]/(a+b*x^2)^(5/2), x]`

[Out]  $((I/6)*x*\sqrt{c*x}*(\sqrt{b}*x*\sqrt{(I*\sqrt{b}*x)/\sqrt{a}})^{(5*a+3*b*x^2)} - 3*\sqrt{a}*(a+b*x^2)*\sqrt{1+(b*x^2)/a}*\operatorname{EllipticE}[I*\operatorname{ArcSinh}[\sqrt{(I*\sqrt{b}*x)/\sqrt{a}}], -1] + 3*\sqrt{a}*(a+b*x^2)*\sqrt{1+(b*x^2)/a}*\operatorname{EllipticF}[I*\operatorname{ArcSinh}[\sqrt{(I*\sqrt{b}*x)/\sqrt{a}}], -1]))/(a^{(5/2)}*((I*\sqrt{b}*x)/\sqrt{a})^{(3/2)}*(a+b*x^2)^{(3/2)})$

**Maple [A]** time = 0.02, size = 382, normalized size = 1.3

$$-\frac{1}{12a^2bx}\left(6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}, 1/2\sqrt{2}\right)\sqrt{2x^2ab}-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(1/2)/(b*x^2+a)^(5/2), x)`

[Out]  $-1/12*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*\operatorname{EllipticE}(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2))*2^(1/2)*x^2*a*b-3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*\operatorname{EllipticF}(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2), 1/2*2^(1/2))*2^(1/2)*x^2*a*b+6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)$

$$\begin{aligned} & /2))^{1/2} * (-x*b/(-a*b)^{1/2})^{1/2} * \text{EllipticE}(((b*x+(-a*b)^{1/2}) \\ & )/(-a*b)^{1/2})^{1/2}, 1/2*2^{1/2})^{1/2} * a^2 - 3*((b*x+(-a*b)^{1/2}) \\ & )/(-a*b)^{1/2})^{1/2} * ((-b*x+(-a*b)^{1/2})/(-a*b)^{1/2})^{1/2} * \\ & (-x*b/(-a*b)^{1/2})^{1/2} * \text{EllipticF}(((b*x+(-a*b)^{1/2})/(-a*b)^{1/2}) \\ & )^{1/2}, 1/2*2^{1/2})^{1/2} * a^2 - 6*b^2*x^4 - 10*a*b*x^2) * (c*x)^{1/2} \\ & /b/a^2/x/(b*x^2+a)^{3/2} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{(bx^2 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x)/(b\*x^2 + a)^(5/2), x, algorithm="maxima")

[Out] integrate(sqrt(c\*x)/(b\*x^2 + a)^(5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}}{(b^2x^4 + 2abx^2 + a^2)\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x)/(b\*x^2 + a)^(5/2), x, algorithm="fricas")

[Out] integral(sqrt(c\*x)/((b^2\*x^4 + 2\*a\*b\*x^2 + a^2)\*sqrt(b\*x^2 + a)), x)

**Sympy [A]** time = 25.4123, size = 44, normalized size = 0.15

$$\frac{\sqrt{cx}^{3/2} \left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{5}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{5/2} \left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(1/2)/(b\*x\*\*2+a)\*\*(5/2), x)

[Out] sqrt(c)\*x\*\*(3/2)\*gamma(3/4)\*hyper((3/4, 5/2), (7/4, ), b\*x\*\*2\*exp\_polar(I\*pi/a)/(2\*a\*\*(5/2)\*gamma(7/4))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{(bx^2 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x)/(b\*x^2 + a)^(5/2), x, algorithm="giac")

[Out] integrate(sqrt(c\*x)/(b\*x^2 + a)^(5/2), x)



$$3.633 \quad \int \frac{1}{\sqrt{cx}(a+bx^2)^{5/2}} dx$$

**Optimal.** Leaf size=157

$$\frac{5(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{12a^{9/4}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}} + \frac{5\sqrt{cx}}{6a^2c\sqrt{a+bx^2}} + \frac{\sqrt{cx}}{3ac(a+bx^2)^{3/2}}$$

[Out] Sqrt[c\*x]/(3\*a\*c\*(a + b\*x^2)^(3/2)) + (5\*Sqrt[c\*x])/(6\*a^2\*c\*Sqrt[a + b\*x^2]) + (5\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(12\*a^(9/4)\*b^(1/4)\*Sqrt[c]\*Sqrt[a + b\*x^2])

**Rubi [A]** time = 0.257674, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{5(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{12a^{9/4}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}} + \frac{5\sqrt{cx}}{6a^2c\sqrt{a+bx^2}} + \frac{\sqrt{cx}}{3ac(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[c\*x]\*(a + b\*x^2)^(5/2)), x]

[Out] Sqrt[c\*x]/(3\*a\*c\*(a + b\*x^2)^(3/2)) + (5\*Sqrt[c\*x])/(6\*a^2\*c\*Sqrt[a + b\*x^2]) + (5\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(12\*a^(9/4)\*b^(1/4)\*Sqrt[c]\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 23.5794, size = 139, normalized size = 0.89

$$\frac{\sqrt{cx}}{3ac(a+bx^2)^{3/2}} + \frac{5\sqrt{cx}}{6a^2c\sqrt{a+bx^2}} + \frac{5\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a} + \sqrt{bx}) F\left(2 \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{12a^{9/4}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(1/2)/(b\*x\*\*2+a)\*\*(5/2), x)

[Out] sqrt(c\*x)/(3\*a\*c\*(a + b\*x\*\*2)\*\*(3/2)) + 5\*sqrt(c\*x)/(6\*a\*\*2\*c\*sqr t(a + b\*x\*\*2)) + 5\*sqrt((a + b\*x\*\*2)/(sqrt(a) + sqrt(b)\*x)\*\*2)\*(s qrt(a) + sqrt(b)\*x)\*elliptic\_f(2\*atan(b\*\*(1/4)\*sqrt(c\*x)/(a\*\*(1/4) )\*sqrt(c))), 1/2)/(12\*a\*\*(9/4)\*b\*\*(1/4)\*sqrt(c)\*sqrt(a + b\*x\*\*2))

**Mathematica [C]** time = 0.333311, size = 115, normalized size = 0.73

$$x \left( \frac{\left( 5i\sqrt{x}\sqrt{\frac{a}{bx^2}+1}(a+bx^2) F\left(i \sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right) \middle| -1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}} + 7a + 5bx^2 \right)}{6a^2\sqrt{cx}(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[c\*x]\*(a + b\*x^2)^(5/2)),x]

[Out] (x\*(7\*a + 5\*b\*x^2 + ((5\*I)\*Sqrt[1 + a/(b\*x^2)]\*Sqrt[x]\*(a + b\*x^2)\*EllipticF[I\*ArcSinh[Sqrt[(I\*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1])/Sqrt[(I\*Sqrt[a])/Sqrt[b]])/(6\*a^2\*Sqrt[c\*x]\*(a + b\*x^2)^(3/2))

**Maple [A]** time = 0.028, size = 216, normalized size = 1.4

$$\frac{1}{12 a^2 b} \left( 5 \sqrt{\frac{b x + \sqrt{-a b}}{\sqrt{-a b}}} \sqrt{\frac{-b x + \sqrt{-a b}}{\sqrt{-a b}}} \sqrt{-\frac{b x}{\sqrt{-a b}}} \operatorname{EllipticF} \left( \sqrt{\frac{b x + \sqrt{-a b}}{\sqrt{-a b}}}, 1/2 \sqrt{2} \right) \sqrt{-a b} \sqrt{2 x^2 b + 5} \sqrt{-a b} \sqrt{\frac{b x + \sqrt{-a b}}{\sqrt{-a b}}} \sqrt{-\frac{b x}{\sqrt{-a b}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x)^(1/2)/(b\*x^2+a)^(5/2),x)

[Out] 1/12\*(5\*((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*((-b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*(-x\*b/(-a\*b)^(1/2))^(1/2)\*EllipticF(((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2),1/2\*2^(1/2))\*(-a\*b)^(1/2)\*2^(1/2)\*x^2\*b+5\*(-a\*b)^(1/2)\*((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*((-b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*(-x\*b/(-a\*b)^(1/2))^(1/2)\*EllipticF(((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2),1/2\*2^(1/2))\*2^(1/2)\*a+10\*b^2\*x^3+14\*a\*b\*x)/(c\*x)^(1/2)/a^2/b/(b\*x^2+a)^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b x^2 + a)^{\frac{5}{2}} \sqrt{c x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/2)\*sqrt(c\*x)),x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(5/2)\*sqrt(c\*x)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{1}{(b^2 x^4 + 2 a b x^2 + a^2) \sqrt{b x^2 + a} \sqrt{c x}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/2)\*sqrt(c\*x)),x, algorithm="fricas")

[Out] integral(1/((b^2\*x^4 + 2\*a\*b\*x^2 + a^2)\*sqrt(b\*x^2 + a)\*sqrt(c\*x)), x)

**Sympy [A]** time = 54.9977, size = 44, normalized size = 0.28

$$\frac{\sqrt{x} \left( \frac{1}{4} \right) {}_2F_1 \left( \frac{1}{4}, \frac{5}{2} \middle| \frac{b x^2 e^{i \pi}}{a} \right)}{2 a^{\frac{5}{2}} \sqrt{c} \left( \frac{5}{4} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(1/2)/(b*x**2+a)**(5/2),x)`

[Out] `sqrt(x)*gamma(1/4)*hyper((1/4, 5/2), (5/4, ), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*sqrt(c)*gamma(5/4))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{2}} \sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/2)*sqrt(c*x)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(5/2)*sqrt(c*x)), x)`

$$3.634 \quad \int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/2}} dx$$

**Optimal.** Leaf size=333

$$\frac{7\sqrt[4]{b}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{4a^{11/4}c^{3/2}\sqrt{a+bx^2}} - \frac{7\sqrt[4]{b}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{2a^{11/4}c^{3/2}\sqrt{a+bx^2}} + \frac{7\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{2a^3c^2(\sqrt{a} + \sqrt{bx})} - \frac{7\sqrt{a+bx^2}}{2a^3c\sqrt{cx}} + \frac{7}{6a^2c\sqrt{cx}\sqrt{a+bx^2}} + \frac{1}{3ac\sqrt{cx}(a+bx^2)^{3/2}}$$

[Out] 1/(3\*a\*c\*Sqrt[c\*x]\*(a + b\*x^2)^(3/2)) + 7/(6\*a^2\*c\*Sqrt[c\*x]\*Sqrt[a + b\*x^2]) - (7\*Sqrt[a + b\*x^2])/(2\*a^3\*c\*Sqrt[c\*x]) + (7\*Sqrt[b]\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(2\*a^3\*c^2\*(Sqrt[a] + Sqrt[b]\*x)) - (7\*b^(1/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticE[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(2\*a^(11/4)\*c^(3/2)\*Sqrt[a + b\*x^2]) + (7\*b^(1/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(4\*a^(11/4)\*c^(3/2)\*Sqrt[a + b\*x^2])

**Rubi [A]** time = 0.670973, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$\frac{7\sqrt[4]{b}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{4a^{11/4}c^{3/2}\sqrt{a+bx^2}} - \frac{7\sqrt[4]{b}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{2a^{11/4}c^{3/2}\sqrt{a+bx^2}} + \frac{7\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{2a^3c^2(\sqrt{a} + \sqrt{bx})} - \frac{7\sqrt{a+bx^2}}{2a^3c\sqrt{cx}} + \frac{7}{6a^2c\sqrt{cx}\sqrt{a+bx^2}} + \frac{1}{3ac\sqrt{cx}(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(3/2)\*(a + b\*x^2)^(5/2)),x]

[Out] 1/(3\*a\*c\*Sqrt[c\*x]\*(a + b\*x^2)^(3/2)) + 7/(6\*a^2\*c\*Sqrt[c\*x]\*Sqrt[a + b\*x^2]) - (7\*Sqrt[a + b\*x^2])/(2\*a^3\*c\*Sqrt[c\*x]) + (7\*Sqrt[b]\*Sqrt[c\*x]\*Sqrt[a + b\*x^2])/(2\*a^3\*c^2\*(Sqrt[a] + Sqrt[b]\*x)) - (7\*b^(1/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticE[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(2\*a^(11/4)\*c^(3/2)\*Sqrt[a + b\*x^2]) + (7\*b^(1/4)\*(Sqrt[a] + Sqrt[b]\*x)\*Sqrt[(a + b\*x^2)/(Sqrt[a] + Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(4\*a^(11/4)\*c^(3/2)\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 69.1994, size = 304, normalized size = 0.91

$$\frac{1}{3ac\sqrt{cx}(a+bx^2)^{\frac{3}{2}}} + \frac{7}{6a^2c\sqrt{cx}\sqrt{a+bx^2}} + \frac{7\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{2a^3c^2(\sqrt{a}+\sqrt{bx})}$$

$$- \frac{7\sqrt{a+bx^2}}{2a^3c\sqrt{cx}} - \frac{7\sqrt[4]{b}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{2a^{\frac{11}{4}}c^{\frac{3}{2}}\sqrt{a+bx^2}}$$

$$+ \frac{7\sqrt[4]{b}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{4a^{\frac{11}{4}}c^{\frac{3}{2}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(c*x)**(3/2)/(b*x**2+a)**(5/2),x)`

[Out] `1/(3*a*c*sqrt(c*x)*(a+b*x**2)**(3/2))+7/(6*a**2*c*sqrt(c*x)*sqrt(a+b*x**2))+7*sqrt(b)*sqrt(c*x)*sqrt(a+b*x**2)/(2*a**3*c**2*(sqrt(a)+sqrt(b)*x))-7*sqrt(a+b*x**2)/(2*a**3*c*sqrt(c*x))-7*b**(1/4)*sqrt((a+b*x**2)/(sqrt(a)+sqrt(b)*x)**2)*(sqrt(a)+sqrt(b)*x)*elliptic_e(2*atan(b**(1/4)*sqrt(c*x)/(a**(1/4)*sqrt(c))),1/2)/(2*a**(11/4)*c**(3/2)*sqrt(a+b*x**2))+7*b**(1/4)*sqrt((a+b*x**2)/(sqrt(a)+sqrt(b)*x)**2)*(sqrt(a)+sqrt(b)*x)*elliptic_f(2*atan(b**(1/4)*sqrt(c*x)/(a**(1/4)*sqrt(c))),1/2)/(4*a**(11/4)*c**(3/2)*sqrt(a+b*x**2))`

**Mathematica [C]** time = 0.422165, size = 208, normalized size = 0.62

$$x\left(-\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}(12a^2+35abx^2+21b^2x^4)-21\sqrt{a}\sqrt{bx}(a+bx^2)\sqrt{\frac{bx^2}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right)\middle|-1\right)+21\sqrt{a}\sqrt{bx}(a+bx^2)\right)$$

$$6a^3(cx)^{3/2}\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}(a+bx^2)^{3/2}$$

Antiderivative was successfully verified.

[In] `Integrate[1/((c*x)^(3/2)*(a+b*x^2)^(5/2)),x]`

[Out] `(x*(-(Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*(12*a^2+35*a*b*x^2+21*b^2*x^4))+21*Sqrt[a]*Sqrt[b]*x*(a+b*x^2)*Sqrt[1+(b*x^2)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]],-1]-21*Sqrt[a]*Sqrt[b]*x*(a+b*x^2)*Sqrt[1+(b*x^2)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]],-1]))/(6*a^3*Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*(c*x)^(3/2)*(a+b*x^2)^(3/2))`

**Maple [A]** time = 0.03, size = 384, normalized size = 1.2

$$\frac{1}{12ca^3}\left(42\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{2x^2ab}-21\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(3/2)/(b*x^2+a)^(5/2),x)`

[Out] `1/12*(42*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*a*b-21*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))`

$$b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)})*2^{(1/2)}*x^2*a*b+42*((b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*((-b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}*EllipticE(((b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)})*2^{(1/2)}*a^2-21*((b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*((-b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}*(-x*b/(-a*b)^{(1/2)})^{(1/2)}*EllipticF(((b*x+(-a*b)^{(1/2)})/(-a*b)^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)})*2^{(1/2)}*a^2-42*b^2*x^4-70*a*b*x^2-24*a^2)/a^3/c/(c*x)^{(1/2)}/(b*x^2+a)^{(3/2)}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{2}} (cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/2)\*(c\*x)^(3/2)),x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(5/2)\*(c\*x)^(3/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(b^2cx^5 + 2abcx^3 + a^2cx)\sqrt{bx^2 + a}\sqrt{cx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/2)\*(c\*x)^(3/2)),x, algorithm="fricas")

[Out] integral(1/((b^2\*c\*x^5 + 2\*a\*b\*c\*x^3 + a^2\*c\*x)\*sqrt(b\*x^2 + a)\*sqrt(c\*x)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(3/2)/(b\*x\*\*2+a)\*\*(5/2),x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{2}} (cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/2)\*(c\*x)^(3/2)),x, algorithm="giac")

[Out] integrate(1/((b\*x^2 + a)^(5/2)\*(c\*x)^(3/2)), x)

$$3.635 \quad \int \frac{1}{(cx)^{5/2}(a+bx^2)^{5/2}} dx$$

**Optimal.** Leaf size=185

$$\frac{5b^{3/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{4a^{13/4}c^{5/2}\sqrt{a+bx^2}} - \frac{5\sqrt{a+bx^2}}{2a^3c(cx)^{3/2}} + \frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}} + \frac{1}{3ac(cx)^{3/2}(a+bx^2)^{3/2}}$$

[Out] 1/(3\*a\*c\*(c\*x)^(3/2)\*(a+b\*x^2)^(3/2)) + 3/(2\*a^2\*c\*(c\*x)^(3/2)\*Sqrt[a+b\*x^2]) - (5\*Sqrt[a+b\*x^2])/(2\*a^3\*c\*(c\*x)^(3/2)) - (5\*b^(3/4)\*(Sqrt[a]+Sqrt[b]\*x)\*Sqrt[(a+b\*x^2)/(Sqrt[a]+Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(4\*a^(13/4)\*c^(5/2)\*Sqrt[a+b\*x^2])

**Rubi [A]** time = 0.312186, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{5b^{3/4}(\sqrt{a} + \sqrt{bx}) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right) \middle| \frac{1}{2}\right)}{4a^{13/4}c^{5/2}\sqrt{a+bx^2}} - \frac{5\sqrt{a+bx^2}}{2a^3c(cx)^{3/2}} + \frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}} + \frac{1}{3ac(cx)^{3/2}(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(5/2)\*(a+b\*x^2)^(5/2)),x]

[Out] 1/(3\*a\*c\*(c\*x)^(3/2)\*(a+b\*x^2)^(3/2)) + 3/(2\*a^2\*c\*(c\*x)^(3/2)\*Sqrt[a+b\*x^2]) - (5\*Sqrt[a+b\*x^2])/(2\*a^3\*c\*(c\*x)^(3/2)) - (5\*b^(3/4)\*(Sqrt[a]+Sqrt[b]\*x)\*Sqrt[(a+b\*x^2)/(Sqrt[a]+Sqrt[b]\*x)^2]\*EllipticF[2\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(a^(1/4)\*Sqrt[c])], 1/2])/(4\*a^(13/4)\*c^(5/2)\*Sqrt[a+b\*x^2])

**Rubi in Sympy [A]** time = 30.1621, size = 167, normalized size = 0.9

$$\frac{1}{3ac(cx)^{\frac{3}{2}}(a+bx^2)^{\frac{3}{2}}} + \frac{3}{2a^2c(cx)^{\frac{3}{2}}\sqrt{a+bx^2}} - \frac{5\sqrt{a+bx^2}}{2a^3c(cx)^{\frac{3}{2}}} - \frac{5b^{\frac{3}{4}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{4a^{\frac{13}{4}}c^{\frac{5}{2}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(5/2)/(b\*x\*\*2+a)\*\*(5/2),x)

[Out] 1/(3\*a\*c\*(c\*x)\*\*(3/2)\*(a+b\*x\*\*2)\*\*(3/2)) + 3/(2\*a\*\*2\*c\*(c\*x)\*\*(3/2)\*sqrt(a+b\*x\*\*2)) - 5\*sqrt(a+b\*x\*\*2)/(2\*a\*\*3\*c\*(c\*x)\*\*(3/2)) - 5\*b\*\*(3/4)\*sqrt((a+b\*x\*\*2)/(sqrt(a)+sqrt(b)\*x)\*\*2)\*(sqrt(a)+sqrt(b)\*x)\*elliptic\_f(2\*atan(b\*\*(1/4)\*sqrt(c\*x)/(a\*\*(1/4)\*sqrt(c))), 1/2)/(4\*a\*\*(13/4)\*c\*\*(5/2)\*sqrt(a+b\*x\*\*2))

**Mathematica [C]** time = 0.458683, size = 127, normalized size = 0.69

$$\frac{x \left( -4a^2 - 21abx^2 - \frac{15ibx^{5/2} \sqrt{\frac{a}{bx^2} + 1} (a + bx^2) F \left( i \sinh^{-1} \left( \frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}} \right) \middle| -1 \right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}} - 15b^2x^4 \right)}{6a^3(cx)^{5/2} (a + bx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(5/2)\*(a + b\*x^2)^(5/2)), x]

[Out] (x\*(-4\*a^2 - 21\*a\*b\*x^2 - 15\*b^2\*x^4 - ((15\*I)\*b\*Sqrt[1 + a/(b\*x^2)]\*x^(5/2)\*(a + b\*x^2)\*EllipticF[I\*ArcSinh[Sqrt[(I\*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1])/Sqrt[(I\*Sqrt[a])/Sqrt[b]]))/(6\*a^3\*(c\*x)^(5/2)\*(a + b\*x^2)^(3/2))

**Maple [A]** time = 0.028, size = 227, normalized size = 1.2

$$-\frac{1}{12xc^2a^3} \left( 15 \sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{-bx + \sqrt{-ab}}{\sqrt{-ab}}} \sqrt{-\frac{bx}{\sqrt{-ab}}} \text{EllipticF} \left( \sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}}, 1/2 \sqrt{2} \right) \sqrt{-ab} \sqrt{2} x^3 b + 15 \sqrt{-ab} \sqrt{\frac{bx + \sqrt{-ab}}{\sqrt{-ab}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x)^(5/2)/(b\*x^2+a)^(5/2), x)

[Out] -1/12\*(15\*((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*((-b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*(-x\*b/(-a\*b)^(1/2))^(1/2)\*EllipticF(((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2), 1/2\*2^(1/2))\*(-a\*b)^(1/2)\*2^(1/2)\*x^3\*b+15\*(-a\*b)^(1/2)\*((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*((-b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2)\*(-x\*b/(-a\*b)^(1/2))^(1/2)\*EllipticF(((b\*x+(-a\*b)^(1/2))/(-a\*b)^(1/2))^(1/2), 1/2\*2^(1/2))\*2^(1/2)\*x\*a+30\*b^2\*x^4+42\*a\*b\*x^2+8\*a^2)/x/c^2/(c\*x)^(1/2)/a^3/(b\*x^2+a)^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{2}} (cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/2)\*(c\*x)^(5/2)), x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(5/2)\*(c\*x)^(5/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{(b^2c^2x^6 + 2abc^2x^4 + a^2c^2x^2)\sqrt{bx^2 + a}\sqrt{cx}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/2)\*(c\*x)^(5/2)), x, algorithm="fricas")



[Out] `integral(1/((b^2*c^2*x^6 + 2*a*b*c^2*x^4 + a^2*c^2*x^2)*sqrt(b*x^2 + a)*sqrt(c*x)), x)`

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(5/2)/(b*x**2+a)**(5/2), x)`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{2}} (cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(5/2)), x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(5/2)), x)`

$$3.636 \quad \int \frac{1}{(cx)^{7/2}(a+bx^2)^{5/2}} dx$$

**Optimal.** Leaf size=362

$$\begin{aligned} & \frac{77b^{5/4} \left( \sqrt{a} + \sqrt{bx} \right) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F \left( 2 \tan^{-1} \left( \frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}} \right) \middle| \frac{1}{2} \right)}{20a^{15/4}c^{7/2}\sqrt{a+bx^2}} \\ & + \frac{77b^{5/4} \left( \sqrt{a} + \sqrt{bx} \right) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E \left( 2 \tan^{-1} \left( \frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}} \right) \middle| \frac{1}{2} \right)}{10a^{15/4}c^{7/2}\sqrt{a+bx^2}} - \frac{77b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{10a^4c^4 \left( \sqrt{a} + \sqrt{bx} \right)} \\ & + \frac{77b\sqrt{a+bx^2}}{10a^4c^3\sqrt{cx}} - \frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{5/2}} + \frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}} + \frac{1}{3ac(cx)^{5/2}(a+bx^2)^{3/2}} \end{aligned}$$

[Out]  $1/(3*a*c*(c*x)^{(5/2)}*(a+b*x^2)^{(3/2)}) + 11/(6*a^2*c*(c*x)^{(5/2)}*\text{Sqrt}[a+b*x^2]) - (77*\text{Sqrt}[a+b*x^2])/(30*a^3*c*(c*x)^{(5/2)}) + (77*b*\text{Sqrt}[a+b*x^2])/(10*a^4*c^3*\text{Sqrt}[c*x]) - (77*b^{(3/2)}*\text{Sqrt}[c*x]*\text{Sqrt}[a+b*x^2])/(10*a^4*c^4*(\text{Sqrt}[a]+\text{Sqrt}[b]*x)) + (77*b^{(5/4)}*(\text{Sqrt}[a]+\text{Sqrt}[b]*x)*\text{Sqrt}[(a+b*x^2)/(\text{Sqrt}[a]+\text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(10*a^{(15/4)}*c^{(7/2)}*\text{Sqrt}[a+b*x^2]) - (77*b^{(5/4)}*(\text{Sqrt}[a]+\text{Sqrt}[b]*x)*\text{Sqrt}[(a+b*x^2)/(\text{Sqrt}[a]+\text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(20*a^{(15/4)}*c^{(7/2)}*\text{Sqrt}[a+b*x^2])$

**Rubi [A]** time = 0.769094, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$\begin{aligned} & \frac{77b^{5/4} \left( \sqrt{a} + \sqrt{bx} \right) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} F \left( 2 \tan^{-1} \left( \frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}} \right) \middle| \frac{1}{2} \right)}{20a^{15/4}c^{7/2}\sqrt{a+bx^2}} \\ & + \frac{77b^{5/4} \left( \sqrt{a} + \sqrt{bx} \right) \sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}} E \left( 2 \tan^{-1} \left( \frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}} \right) \middle| \frac{1}{2} \right)}{10a^{15/4}c^{7/2}\sqrt{a+bx^2}} - \frac{77b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{10a^4c^4 \left( \sqrt{a} + \sqrt{bx} \right)} \\ & + \frac{77b\sqrt{a+bx^2}}{10a^4c^3\sqrt{cx}} - \frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{5/2}} + \frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}} + \frac{1}{3ac(cx)^{5/2}(a+bx^2)^{3/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(7/2)\*(a+b\*x^2)^(5/2)),x]

[Out]  $1/(3*a*c*(c*x)^{(5/2)}*(a+b*x^2)^{(3/2)}) + 11/(6*a^2*c*(c*x)^{(5/2)}*\text{Sqrt}[a+b*x^2]) - (77*\text{Sqrt}[a+b*x^2])/(30*a^3*c*(c*x)^{(5/2)}) + (77*b*\text{Sqrt}[a+b*x^2])/(10*a^4*c^3*\text{Sqrt}[c*x]) - (77*b^{(3/2)}*\text{Sqrt}[c*x]*\text{Sqrt}[a+b*x^2])/(10*a^4*c^4*(\text{Sqrt}[a]+\text{Sqrt}[b]*x)) + (77*b^{(5/4)}*(\text{Sqrt}[a]+\text{Sqrt}[b]*x)*\text{Sqrt}[(a+b*x^2)/(\text{Sqrt}[a]+\text{Sqrt}[b]*x)^2]*\text{EllipticE}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(10*a^{(15/4)}*c^{(7/2)}*\text{Sqrt}[a+b*x^2]) - (77*b^{(5/4)}*(\text{Sqrt}[a]+\text{Sqrt}[b]*x)*\text{Sqrt}[(a+b*x^2)/(\text{Sqrt}[a]+\text{Sqrt}[b]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(a^{(1/4)}*\text{Sqrt}[c])], 1/2])/(20*a^{(15/4)}*c^{(7/2)}*\text{Sqrt}[a+b*x^2])$

**Rubi in Sympy [A]** time = 82.228, size = 333, normalized size = 0.92

$$\frac{1}{3ac(cx)^{\frac{5}{2}}(a+bx^2)^{\frac{3}{2}}} + \frac{11}{6a^2c(cx)^{\frac{5}{2}}\sqrt{a+bx^2}} - \frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{\frac{5}{2}}} - \frac{77b^{\frac{3}{2}}\sqrt{cx}\sqrt{a+bx^2}}{10a^4c^4(\sqrt{a}+\sqrt{bx})}$$

$$+ \frac{77b\sqrt{a+bx^2}}{10a^4c^3\sqrt{cx}} + \frac{77b^{\frac{5}{4}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{10a^{\frac{15}{4}}c^{\frac{7}{2}}\sqrt{a+bx^2}}$$

$$- \frac{77b^{\frac{5}{4}}\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{bx})^2}}(\sqrt{a}+\sqrt{bx})F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\middle|\frac{1}{2}\right)}{20a^{\frac{15}{4}}c^{\frac{7}{2}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(c*x)**(7/2)/(b*x**2+a)**(5/2),x)`

[Out]  $1/(3*a*c*(c*x)^{(5/2)}*(a+b*x^2)^{(3/2)}) + 11/(6*a**2*c*(c*x)^{(5/2)}*\sqrt{a+b*x^2}) - 77*\sqrt{a+b*x^2}/(30*a**3*c*(c*x)^{(5/2)}) - 77*b^{(3/2)}*\sqrt{c*x}*\sqrt{a+b*x^2}/(10*a**4*c**4*(\sqrt{a}+\sqrt{b*x})) + 77*b*\sqrt{a+b*x^2}/(10*a**4*c**3*\sqrt{c*x}) + 77*b^{(5/4)}*\sqrt{(a+b*x^2)/(\sqrt{a}+\sqrt{b*x})^2}/(10*a^{(15/4)}*c^{(7/2)}*\sqrt{a+bx^2}) * \operatorname{elliptic}_e(2*\operatorname{atan}(b^{(1/4)}*\sqrt{c*x}/(a^{(1/4)}*\sqrt{c})), 1/2)/(10*a^{(15/4)}*c^{(7/2)}*\sqrt{a+bx^2}) - 77*b^{(5/4)}*\sqrt{(a+b*x^2)/(\sqrt{a}+\sqrt{b*x})^2}/(20*a^{(15/4)}*c^{(7/2)}*\sqrt{a+bx^2}) * \operatorname{elliptic}_f(2*\operatorname{atan}(b^{(1/4)}*\sqrt{c*x}/(a^{(1/4)}*\sqrt{c})), 1/2)/(20*a^{(15/4)}*c^{(7/2)}*\sqrt{a+bx^2})$

**Mathematica [C]** time = 0.405609, size = 222, normalized size = 0.61

$$x \left( \sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}} (-12a^3 + 132a^2bx^2 + 385ab^2x^4 + 231b^3x^6) + 231\sqrt{ab}^{3/2}x^3(a+bx^2) \sqrt{\frac{bx^2}{a}} + 1F\left(i \sinh^{-1}\left(\sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}}\right) \middle| -1 \right) - 231 \right)$$

$$30a^4(cx)^{7/2} \sqrt{\frac{i\sqrt{bx}}{\sqrt{a}}} (a+bx^2)^{3/2}$$

Antiderivative was successfully verified.

[In] `Integrate[1/((c*x)^(7/2)*(a+b*x^2)^(5/2)),x]`

[Out]  $(x*(\operatorname{Sqrt}[(I*\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]*(-12*a^3 + 132*a^2*b*x^2 + 385*a*b^2*x^4 + 231*b^3*x^6) - 231*\operatorname{Sqrt}[a]*b^{(3/2)}*x^3*(a+b*x^2)*\operatorname{Sqrt}[1 + (b*x^2)/a]*\operatorname{EllipticE}[I*\operatorname{ArcSinh}[\operatorname{Sqrt}[(I*\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]], -1] + 231*\operatorname{Sqrt}[a]*b^{(3/2)}*x^3*(a+b*x^2)*\operatorname{Sqrt}[1 + (b*x^2)/a]*\operatorname{EllipticF}[I*\operatorname{ArcSinh}[\operatorname{Sqrt}[(I*\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]], -1))/(30*a^4*\operatorname{Sqrt}[(I*\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]*(c*x)^{(7/2)}*(a+b*x^2)^{(3/2)})$

**Maple [A]** time = 0.03, size = 410, normalized size = 1.1

$$-\frac{1}{60x^2a^4c^3} \left( 462 \sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{bx}{\sqrt{-ab}}} \operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}, 1/2, \sqrt{2}\right) \sqrt{2}x^4ab^2 - 231 \sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}} \sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(7/2)/(b*x^2+a)^(5/2),x)`

[Out]  $-1/60*(462*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*\operatorname{EllipticE}(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2), 1/2, 2^(1/2))*2^(1/2)*x^4*a*b^2 - 231*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*\operatorname{EllipticF}(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2), 1/2, 2^(1/2))*2^(1/2)*x^4*a*b^2 - 231*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*\operatorname{EllipticE}(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2), 1/2, 2^(1/2))*2^(1/2)*x^4*a*b^2 - 231*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)*\operatorname{EllipticF}(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2), 1/2, 2^(1/2))*2^(1/2)*x^4*a*b^2)$

$$\begin{aligned} & 2)) / (-a*b)^{(1/2)})^{(1/2)} * (-x*b / (-a*b)^{(1/2)})^{(1/2)} * \text{EllipticF}((b*x \\ & + (-a*b)^{(1/2)}) / (-a*b)^{(1/2)})^{(1/2)}, 1/2 * 2^{(1/2)}) * 2^{(1/2)} * x^4 * a * b^2 \\ & + 462 * ((b*x + (-a*b)^{(1/2)}) / (-a*b)^{(1/2)})^{(1/2)} * ((-b*x + (-a*b)^{(1/2)}) \\ & / (-a*b)^{(1/2)})^{(1/2)} * (-x*b / (-a*b)^{(1/2)})^{(1/2)} * \text{EllipticE}((b*x + (- \\ & a*b)^{(1/2)}) / (-a*b)^{(1/2)})^{(1/2)}, 1/2 * 2^{(1/2)}) * 2^{(1/2)} * x^2 * a^2 * b - 23 \\ & 1 * ((b*x + (-a*b)^{(1/2)}) / (-a*b)^{(1/2)})^{(1/2)} * ((-b*x + (-a*b)^{(1/2)}) / (- \\ & a*b)^{(1/2)})^{(1/2)} * (-x*b / (-a*b)^{(1/2)})^{(1/2)} * \text{EllipticF}((b*x + (-a*b) \\ & )^{(1/2)}) / (-a*b)^{(1/2)})^{(1/2)}, 1/2 * 2^{(1/2)}) * 2^{(1/2)} * x^2 * a^2 * b - 462 * b \\ & ^3 * x^6 - 770 * a * b^2 * x^4 - 264 * a^2 * b * x^2 + 24 * a^3) / x^2 / a^4 / c^3 / (c*x)^{(1/2)} \\ & ) / (b*x^2 + a)^{(3/2)} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{2}} (cx)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/2)\*(c\*x)^(7/2)),x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(5/2)\*(c\*x)^(7/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(b^2c^3x^7 + 2abc^3x^5 + a^2c^3x^3)\sqrt{bx^2 + a}\sqrt{cx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/2)\*(c\*x)^(7/2)),x, algorithm="fricas")

[Out] integral(1/((b^2\*c^3\*x^7 + 2\*a\*b\*c^3\*x^5 + a^2\*c^3\*x^3)\*sqrt(b\*x^2 + a)\*sqrt(c\*x)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(7/2)/(b\*x\*\*2+a)\*\*(5/2),x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{2}} (cx)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/2)\*(c\*x)^(7/2)),x, algorithm="giac")

[Out] integrate(1/((b\*x^2 + a)^(5/2)\*(c\*x)^(7/2)), x)

$$3.637 \quad \int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}} dx$$

**Optimal.** Leaf size=107

$$\frac{9\sqrt[4]{3}c^2\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{5\cdot 2^{3/4}\sqrt{x}\sqrt{3a-2ax^2}} - \frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}$$

[Out]  $-(c*(c*x)^{(3/2)*\text{Sqrt}[3*a - 2*a*x^2]})/(5*a) - (9*3^{(1/4)}*c^2*\text{Sqrt}[c*x]*\text{Sqrt}[3 - 2*x^2]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[3 - \text{Sqrt}[6]*x]/\text{Sqrt}[6]], 2])/(5*2^{(3/4)}*\text{Sqrt}[x]*\text{Sqrt}[3*a - 2*a*x^2])$

**Rubi [A]** time = 0.148801, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$\frac{9\sqrt[4]{3}c^2\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{5\cdot 2^{3/4}\sqrt{x}\sqrt{3a-2ax^2}} - \frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*x)^{(5/2)}/\text{Sqrt}[3*a - 2*a*x^2], x]$

[Out]  $-(c*(c*x)^{(3/2)*\text{Sqrt}[3*a - 2*a*x^2]})/(5*a) - (9*3^{(1/4)}*c^2*\text{Sqrt}[c*x]*\text{Sqrt}[3 - 2*x^2]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[3 - \text{Sqrt}[6]*x]/\text{Sqrt}[6]], 2])/(5*2^{(3/4)}*\text{Sqrt}[x]*\text{Sqrt}[3*a - 2*a*x^2])$

**Rubi in Sympy [A]** time = 48.4722, size = 170, normalized size = 1.59

$$\frac{9\sqrt[4]{2} \cdot 3^{\frac{3}{4}}c^{\frac{5}{2}}\sqrt{-\frac{2x^2}{3}+1}E\left(\text{asin}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}\sqrt{cx}}{3\sqrt{c}}\right)\middle|-1\right)}{10\sqrt{-2ax^2+3a}} - \frac{9\sqrt[4]{2} \cdot 3^{\frac{3}{4}}c^{\frac{5}{2}}\sqrt{-\frac{2x^2}{3}+1}F\left(\text{asin}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}\sqrt{cx}}{3\sqrt{c}}\right)\middle|-1\right)}{10\sqrt{-2ax^2+3a}} - \frac{c(cx)^{\frac{3}{2}}\sqrt{-2ax^2+3a}}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c*x)**(5/2)/(-2*a*x**2+3*a)**(1/2), x)$

[Out]  $9*2^{(1/4)}*3^{(3/4)}*c^{(5/2)}*\text{sqrt}(-2*x**2/3 + 1)*\text{elliptic\_e}(\text{asin}(2^{(1/4)}*3^{(3/4)}*\text{sqrt}(c*x)/(3*\text{sqrt}(c))), -1)/(10*\text{sqrt}(-2*a*x**2 + 3*a)) - 9*2^{(1/4)}*3^{(3/4)}*c^{(5/2)}*\text{sqrt}(-2*x**2/3 + 1)*\text{elliptic\_f}(\text{asin}(2^{(1/4)}*3^{(3/4)}*\text{sqrt}(c*x)/(3*\text{sqrt}(c))), -1)/(10*\text{sqrt}(-2*a*x**2 + 3*a)) - c*(c*x)**(3/2)*\text{sqrt}(-2*a*x**2 + 3*a)/(5*a)$

**Mathematica [A]** time = 0.135091, size = 112, normalized size = 1.05

$$\frac{(cx)^{5/2}\left(-9\sqrt[4]{6}\sqrt{3-2x^2}F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\sqrt{x}\right)\middle|-1\right)+9\sqrt[4]{6}\sqrt{3-2x^2}E\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\sqrt{x}\right)\middle|-1\right)+2(2x^2-3)x^{3/2}\right)}{10x^{5/2}\sqrt{a(3-2x^2)}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(c*x)^{(5/2)}/\text{Sqrt}[3*a - 2*a*x^2], x]$

```
[Out] ((c*x)^(5/2)*(2*x^(3/2)*(-3 + 2*x^2) + 9*6^(1/4)*Sqrt[3 - 2*x^2]*
EllipticE[ArcSin[(2/3)^(1/4)*Sqrt[x]], -1] - 9*6^(1/4)*Sqrt[3 - 2
*x^2]*EllipticF[ArcSin[(2/3)^(1/4)*Sqrt[x]], -1]))/(10*x^(5/2)*Sqr
rt[a*(3 - 2*x^2)])
```

**Maple [B]** time = 0.041, size = 235, normalized size = 2.2

$$\frac{c^2}{40ax(2x^2-3)}\sqrt{cx}\sqrt{-a(2x^2-3)}\left(6\sqrt{\left(-2x+\sqrt{3}\sqrt{2}\right)\sqrt{3}\sqrt{2}\sqrt{3}}\sqrt{-x\sqrt{3}\sqrt{2}}\text{EllipticE}\left(\frac{1}{6}\sqrt{3}\sqrt{2}\sqrt{\left(2x+\sqrt{3}\sqrt{2}\right)\sqrt{3}\sqrt{2}},\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x)
```

```
[Out] 1/40*c^2/x*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)/a*(6*((-2*x+3^(1/2))*2
^(1/2))*3^(1/2)*2^(1/2))^(1/2)*3^(1/2)*(-x*3^(1/2)*2^(1/2))^(1/2)
*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+3^(1/2))*2^(1/2))*3^(1/2)*2^(
1/2))^(1/2),1/2*2^(1/2))*((2*x+3^(1/2))*2^(1/2))*3^(1/2)*2^(1/2))^(
1/2)*2^(1/2)-3*((-2*x+3^(1/2))*2^(1/2))*3^(1/2)*2^(1/2))^(1/2)*3^
(1/2)*(-x*3^(1/2)*2^(1/2))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((
2*x+3^(1/2))*2^(1/2))*3^(1/2)*2^(1/2))^(1/2),1/2*2^(1/2))*((2*x+3^
(1/2))*2^(1/2))*3^(1/2)*2^(1/2))^(1/2)*2^(1/2)-16*x^4+24*x^2)/(2*x
^2-3)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{\sqrt{-2ax^2+3a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/sqrt(-2*a*x^2 + 3*a),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(5/2)/sqrt(-2*a*x^2 + 3*a), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}c^2x^2}{\sqrt{-2ax^2+3a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/sqrt(-2*a*x^2 + 3*a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x)*c^2*x^2/sqrt(-2*a*x^2 + 3*a), x)
```

**Sympy [A]** time = 139.309, size = 51, normalized size = 0.48

$$\frac{\sqrt{3}c^{\frac{5}{2}}x^{\frac{7}{2}}\left(\frac{7}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{7}{4} \middle| \frac{2x^2 e^{2i\pi}}{3}\right)}{6\sqrt{a}\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(5/2)/(-2\*a\*x\*\*2+3\*a)\*\*(1/2),x)

[Out] sqrt(3)\*c\*\*(5/2)\*x\*\*(7/2)\*gamma(7/4)\*hyper((1/2, 7/4), (11/4, ), 2\*x\*\*2\*exp\_polar(2\*I\*pi)/3)/(6\*sqrt(a)\*gamma(11/4))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{\sqrt{-2ax^2 + 3a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(5/2)/sqrt(-2\*a\*x^2 + 3\*a),x, algorithm="giac")

[Out] integrate((c\*x)^(5/2)/sqrt(-2\*a\*x^2 + 3\*a), x)

$$3.638 \quad \int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}} dx$$

**Optimal.** Leaf size=88

$$\frac{c^{3/2}\sqrt{3-2x^2}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle| -1\right)}{\sqrt[4]{6}\sqrt{a(3-2x^2)}} - \frac{c\sqrt{3a-2ax^2}\sqrt{cx}}{3a}$$

[Out]  $-(c*\text{Sqrt}[c*x]*\text{Sqrt}[3*a - 2*a*x^2])/(3*a) + (c^{(3/2)}*\text{Sqrt}[3 - 2*x^2]*\text{EllipticF}[\text{ArcSin}[(2/3)^{(1/4)}*\text{Sqrt}[c*x]]/\text{Sqrt}[c], -1])/(6^{(1/4)}*\text{Sqrt}[a*(3 - 2*x^2)])$

**Rubi [A]** time = 0.143669, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{c^{3/2}\sqrt{3-2x^2}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle| -1\right)}{\sqrt[4]{6}\sqrt{a(3-2x^2)}} - \frac{c\sqrt{3a-2ax^2}\sqrt{cx}}{3a}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*x)^{(3/2)}/\text{Sqrt}[3*a - 2*a*x^2], x]$

[Out]  $-(c*\text{Sqrt}[c*x]*\text{Sqrt}[3*a - 2*a*x^2])/(3*a) + (c^{(3/2)}*\text{Sqrt}[3 - 2*x^2]*\text{EllipticF}[\text{ArcSin}[(2/3)^{(1/4)}*\text{Sqrt}[c*x]]/\text{Sqrt}[c], -1])/(6^{(1/4)}*\text{Sqrt}[a*(3 - 2*x^2)])$

**Rubi in Sympy [A]** time = 11.406, size = 95, normalized size = 1.08

$$\frac{2^{3/4}\sqrt[3]{3}c^{3/4}\sqrt{-\frac{2x^2}{3}} + 1F\left(\text{asin}\left(\frac{\sqrt[4]{2}\cdot 3^{3/4}\sqrt{cx}}{3\sqrt{c}}\right)\middle| -1\right)}{2\sqrt{-2ax^2 + 3a}} - \frac{c\sqrt{cx}\sqrt{-2ax^2 + 3a}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c*x)**(3/2)/(-2*a*x**2+3*a)**(1/2), x)$

[Out]  $2^{(3/4)}*3^{(1/4)}*c^{(3/2)}*\text{sqrt}(-2*x**2/3 + 1)*\text{elliptic\_f}(\text{asin}(2^{(1/4)}*3^{(3/4)}*\text{sqrt}(c*x)/(3*\text{sqrt}(c))), -1)/(2*\text{sqrt}(-2*a*x**2 + 3*a)) - c*\text{sqrt}(c*x)*\text{sqrt}(-2*a*x**2 + 3*a)/(3*a)$

**Mathematica [A]** time = 0.0956983, size = 89, normalized size = 1.01

$$\frac{(2x^2 - 3)(cx)^{3/2}\left(2\sqrt{2 - \frac{3}{x^2}x^{3/2}} - 6^{3/4}F\left(\sin^{-1}\left(\frac{\sqrt[4]{3}}{\sqrt{x}}\right)\middle| -1\right)\right)}{6\sqrt{2 - \frac{3}{x^2}x^{5/2}}\sqrt{a(3-2x^2)}}$$

Antiderivative was successfully verified.



[In] Integrate[(c\*x)^(3/2)/Sqrt[3\*a - 2\*a\*x^2], x]

[Out] ((c\*x)^(3/2)\*(-3 + 2\*x^2)\*(2\*Sqrt[2 - 3/x^2]\*x^(3/2) - 6^(3/4)\*EllipticF[ArcSin[(3/2)^(1/4)/Sqrt[x]], -1]))/(6\*Sqrt[2 - 3/x^2]\*x^(5/2)\*Sqrt[a\*(3 - 2\*x^2)])

**Maple [A]** time = 0.038, size = 131, normalized size = 1.5

$$-\frac{c}{12ax(2x^2-3)}\sqrt{cx}\sqrt{-a(2x^2-3)}\left(\sqrt{(-2x+\sqrt{3}\sqrt{2})}\sqrt{3}\sqrt{2}\sqrt{-x\sqrt{3}\sqrt{2}}\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{2}}{6}\sqrt{(2x+\sqrt{3}\sqrt{2})}\sqrt{3}\sqrt{2},\frac{\sqrt{2}}{2}\right)\sqrt{\dots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(3/2)/(-2\*a\*x^2+3\*a)^(1/2), x)

[Out] -1/12\*c\*(c\*x)^(1/2)\*(-a\*(2\*x^2-3))^(1/2)\*((( -2\*x+3^(1/2)\*2^(1/2) ) \* 3^(1/2)\*2^(1/2))^(1/2)\*(-x\*3^(1/2)\*2^(1/2))^(1/2)\*EllipticF(1/6\* 3^(1/2)\*2^(1/2)\*((2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2), 1/2 \* 2^(1/2))\*((2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2)+8\*x^3-12\*x)/x/a/(2\*x^2-3)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{\sqrt{-2ax^2+3a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(3/2)/sqrt(-2\*a\*x^2 + 3\*a), x, algorithm="maxima")

[Out] integrate((c\*x)^(3/2)/sqrt(-2\*a\*x^2 + 3\*a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c}cx}{\sqrt{-2ax^2+3a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(3/2)/sqrt(-2\*a\*x^2 + 3\*a), x, algorithm="fricas")

[Out] integral(sqrt(c\*x)\*c\*x/sqrt(-2\*a\*x^2 + 3\*a), x)

**Sympy [A]** time = 10.9506, size = 51, normalized size = 0.58

$$\frac{\sqrt{3}c^{\frac{3}{2}}x^{\frac{5}{2}}\left(\frac{5}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{5}{4} \middle| \frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(3/2)/(-2\*a\*x\*\*2+3\*a)\*\*(1/2), x)

[Out]  $\sqrt{3} c^{3/2} x^{5/2} \gamma(5/4) \text{hyper}((1/2, 5/4), (9/4, ), 2 x^2 \exp_{\text{polar}}(2 i \pi)/3) / (6 \sqrt{a} \gamma(9/4))$

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{3/2}}{\sqrt{-2ax^2 + 3a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(3/2)/sqrt(-2*a*x^2 + 3*a), x, algorithm="giac")`

[Out] `integrate((c*x)^(3/2)/sqrt(-2*a*x^2 + 3*a), x)`

$$3.639 \quad \int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}} dx$$

**Optimal.** Leaf size=67

$$\frac{\sqrt[4]{6}\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6x}}}{\sqrt{6}}\right)\middle|2\right)}{\sqrt{x}\sqrt{3a-2ax^2}}$$

[Out] -((6^(1/4)\*Sqrt[c\*x]\*Sqrt[3-2\*x^2]\*EllipticE[ArcSin[Sqrt[3-Sqrt[6]\*x]/Sqrt[6]], 2])/(Sqrt[x]\*Sqrt[3\*a-2\*a\*x^2]))

**Rubi [A]** time = 0.101799, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{\sqrt[4]{6}\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6x}}}{\sqrt{6}}\right)\middle|2\right)}{\sqrt{x}\sqrt{3a-2ax^2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c\*x]/Sqrt[3\*a-2\*a\*x^2], x]

[Out] -((6^(1/4)\*Sqrt[c\*x]\*Sqrt[3-2\*x^2]\*EllipticE[ArcSin[Sqrt[3-Sqrt[6]\*x]/Sqrt[6]], 2])/(Sqrt[x]\*Sqrt[3\*a-2\*a\*x^2]))

**Rubi in Sympy [A]** time = 44.3939, size = 138, normalized size = 2.06

$$\frac{\sqrt[4]{2} \cdot 3^{\frac{3}{4}} \sqrt{c} \sqrt{-\frac{2x^2}{3} + 1} E\left(\operatorname{asin}\left(\frac{\sqrt[4]{2} \cdot 3^{\frac{3}{4}} \sqrt{cx}}{3\sqrt{c}}\right)\middle|-1\right)}{\sqrt{-2ax^2 + 3a}} - \frac{\sqrt[4]{2} \cdot 3^{\frac{3}{4}} \sqrt{c} \sqrt{-\frac{2x^2}{3} + 1} F\left(\operatorname{asin}\left(\frac{\sqrt[4]{2} \cdot 3^{\frac{3}{4}} \sqrt{cx}}{3\sqrt{c}}\right)\middle|-1\right)}{\sqrt{-2ax^2 + 3a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(1/2)/(-2\*a\*x\*\*2+3\*a)\*\*(1/2), x)

[Out] 2\*\*(1/4)\*3\*\*(3/4)\*sqrt(c)\*sqrt(-2\*x\*\*2/3+1)\*elliptic\_e(asin(2\*\*(1/4)\*3\*\*(3/4)\*sqrt(c\*x)/(3\*sqrt(c))), -1)/sqrt(-2\*a\*x\*\*2+3\*a) - 2\*\*(1/4)\*3\*\*(3/4)\*sqrt(c)\*sqrt(-2\*x\*\*2/3+1)\*elliptic\_f(asin(2\*\*(1/4)\*3\*\*(3/4)\*sqrt(c\*x)/(3\*sqrt(c))), -1)/sqrt(-2\*a\*x\*\*2+3\*a)

**Mathematica [A]** time = 0.0965715, size = 77, normalized size = 1.15

$$\frac{\sqrt[4]{6}\sqrt{3-2x^2}\sqrt{cx}\left(E\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\sqrt{x}\right)\middle|-1\right)-F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\sqrt{x}\right)\middle|-1\right)\right)}{\sqrt{x}\sqrt{a(3-2x^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c\*x]/Sqrt[3\*a-2\*a\*x^2], x]

[Out] (6^(1/4)\*Sqrt[c\*x]\*Sqrt[3-2\*x^2]\*(EllipticE[ArcSin[(2/3)^(1/4)\*Sqrt[x]], -1] - EllipticF[ArcSin[(2/3)^(1/4)\*Sqrt[x]], -1]))/(Sqrt[x]\*Sqrt[a\*(3-2\*x^2)])

---

**Maple [B]** time = 0.02, size = 165, normalized size = 2.5

$$\frac{\sqrt{3}\sqrt{2}}{12ax(2x^2-3)}\sqrt{cx}\sqrt{-a(2x^2-3)}\sqrt{(2x+\sqrt{3}\sqrt{2})\sqrt{3}\sqrt{2}}\sqrt{(-2x+\sqrt{3}\sqrt{2})\sqrt{3}\sqrt{2}}\sqrt{-x\sqrt{3}\sqrt{2}}\left(2\operatorname{EllipticE}\left(\frac{1}{6}\sqrt{3}\sqrt{2}\sqrt{(2x+\sqrt{3}\sqrt{2})\sqrt{3}\sqrt{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x)`

[Out] `1/12*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*2^(1/2)*((2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2)*((-2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2)*3^(1/2)*(-x*3^(1/2)*2^(1/2))^(1/2)*(2*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2),1/2*2^(1/2))-EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2),1/2*2^(1/2)))/x/a/(2*x^2-3)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\sqrt{-2ax^2+3a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x)/sqrt(-2*a*x^2+3*a),x,algorithm="maxima")`

[Out] `integrate(sqrt(c*x)/sqrt(-2*a*x^2+3*a),x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{cx}}{\sqrt{-2ax^2+3a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x)/sqrt(-2*a*x^2+3*a),x,algorithm="fricas")`

[Out] `integral(sqrt(c*x)/sqrt(-2*a*x^2+3*a),x)`

---

**Sympy [A]** time = 2.54772, size = 51, normalized size = 0.76

$$\frac{\sqrt{3}\sqrt{c}x^{\frac{3}{2}}\left(\frac{3}{4}\right)_2F_1\left(\frac{1}{2},\frac{3}{4}\left|\frac{2x^2e^{2i\pi}}{3}\right.\right)}{6\sqrt{a}\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(1/2)/(-2*a*x**2+3*a)**(1/2),x)`

[Out] `sqrt(3)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((1/2,3/4),(7/4),2*x**2*exp_polar(2*I*pi)/3)/(6*sqrt(a)*gamma(7/4))`

---

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\sqrt{-2ax^2 + 3a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x)/sqrt(-2*a*x^2 + 3*a), x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/sqrt(-2*a*x^2 + 3*a), x)
```

$$3.640 \quad \int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}} dx$$

**Optimal.** Leaf size=63

$$\frac{2^{3/4}\sqrt{3-2x^2}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle| -1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{a(3-2x^2)}}$$

[Out] (2^(3/4)\*Sqrt[3 - 2\*x^2]\*EllipticF[ArcSin[((2/3)^(1/4)\*Sqrt[c\*x])/Sqrt[c]], -1])/(3^(1/4)\*Sqrt[c]\*Sqrt[a\*(3 - 2\*x^2)])

**Rubi [A]** time = 0.0920981, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{2^{3/4}\sqrt{3-2x^2}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle| -1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{a(3-2x^2)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[c\*x]\*Sqrt[3\*a - 2\*a\*x^2]),x]

[Out] (2^(3/4)\*Sqrt[3 - 2\*x^2]\*EllipticF[ArcSin[((2/3)^(1/4)\*Sqrt[c\*x])/Sqrt[c]], -1])/(3^(1/4)\*Sqrt[c]\*Sqrt[a\*(3 - 2\*x^2)])

**Rubi in Sympy [A]** time = 8.28505, size = 68, normalized size = 1.08

$$\frac{2^{\frac{3}{4}}\sqrt[4]{3}\sqrt{-\frac{2x^2}{3}+1}F\left(\operatorname{asin}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}\sqrt{cx}}{3\sqrt{c}}\right)\middle| -1\right)}{\sqrt{c}\sqrt{-2ax^2+3a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(1/2)/(-2\*a\*x\*\*2+3\*a)\*\*(1/2),x)

[Out] 2\*\*(3/4)\*3\*\*(1/4)\*sqrt(-2\*x\*\*2/3 + 1)\*elliptic\_f(asin(2\*\*(1/4)\*3\*\*(3/4)\*sqrt(c\*x)/(3\*sqrt(c))), -1)/(sqrt(c)\*sqrt(-2\*a\*x\*\*2 + 3\*a))

**Mathematica [A]** time = 0.0946721, size = 64, normalized size = 1.02

$$\frac{2^{3/4}\sqrt{2-\frac{3}{x^2}x^{3/2}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}}{\sqrt{x}}\right)\middle| -1\right)}{\sqrt[4]{3}\sqrt{a(3-2x^2)}\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[c\*x]\*Sqrt[3\*a - 2\*a\*x^2]),x]

[Out]  $-\left(\frac{2^{3/4} \sqrt{2 - 3/x^2} x^{3/2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{3}{2}\right]^{1/4}/\sqrt{x}\right], -1\right)}{3^{1/4} \sqrt{c x} \sqrt{a(3 - 2x^2)}}\right)$

**Maple [B]** time = 0.027, size = 117, normalized size = 1.9

$$-\frac{1}{6a(2x^2-3)}\sqrt{-a(2x^2-3)}\sqrt{(2x+\sqrt{3}\sqrt{2})}\sqrt{3}\sqrt{2}\sqrt{(-2x+\sqrt{3}\sqrt{2})}\sqrt{3}\sqrt{2}\sqrt{-x\sqrt{3}\sqrt{2}}\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{2}}{6}\sqrt{(2x+\sqrt{3}\sqrt{2})}\sqrt{-x\sqrt{3}\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2), x)`

[Out]  $-1/6*(-a*(2*x^2-3))^{1/2}*((2*x+3^{1/2})^{2^{1/2}})^{3^{1/2}}*2^{1/2})^{1/2}*(-2*x+3^{1/2})^{2^{1/2}})^{3^{1/2}}*2^{1/2})^{1/2}*(-x^{3^{1/2}})^{2^{1/2}})^{1/2}*\text{EllipticF}(1/6*3^{1/2}*2^{1/2}*((2*x+3^{1/2})^{2^{1/2}})^{3^{1/2}}*2^{1/2})^{1/2}, 1/2*2^{1/2})/(c*x)^{1/2}/a/(2*x^2-3)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2ax^2 + 3a}\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)), x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-2ax^2 + 3a}\sqrt{cx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)), x, algorithm="fricas")`

[Out] `integral(1/(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)), x)`

**Sympy [A]** time = 3.41868, size = 51, normalized size = 0.81

$$\frac{\sqrt{3}\sqrt{x} \left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{2x^2 e^{2i\pi}}{3}\right)}{6\sqrt{a}\sqrt{c} \left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(1/2)/(-2*a*x**2+3*a)**(1/2), x)`

[Out] `sqrt(3)*sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4, ), 2*x**2*exp_polar(2*I*pi)/3)/(6*sqrt(a)*sqrt(c)*gamma(5/4))`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2ax^2 + 3a}\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-2\*a\*x^2 + 3\*a)\*sqrt(c\*x)),x, algorithm="giac")

[Out] integrate(1/(sqrt(-2\*a\*x^2 + 3\*a)\*sqrt(c\*x)), x)



$$3.641 \quad \int \frac{1}{(cx)^{3/2} \sqrt{3a-2ax^2}} dx$$

**Optimal.** Leaf size=107

$$\frac{2\sqrt[4]{2}\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6x}}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}c^2\sqrt{x}\sqrt{3a-2ax^2}} - \frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}$$

[Out]  $(-2*\text{Sqrt}[3*a - 2*a*x^2])/(3*a*c*\text{Sqrt}[c*x]) + (2*2^{1/4}*\text{Sqrt}[c*x] * \text{Sqrt}[3 - 2*x^2]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[3 - \text{Sqrt}[6]*x]/\text{Sqrt}[6]], 2 ])/(3^{3/4}*c^2*\text{Sqrt}[x]*\text{Sqrt}[3*a - 2*a*x^2])$

**Rubi [A]** time = 0.143461, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$\frac{2\sqrt[4]{2}\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6x}}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}c^2\sqrt{x}\sqrt{3a-2ax^2}} - \frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/((c*x)^{(3/2)}*\text{Sqrt}[3*a - 2*a*x^2]), x]$

[Out]  $(-2*\text{Sqrt}[3*a - 2*a*x^2])/(3*a*c*\text{Sqrt}[c*x]) + (2*2^{1/4}*\text{Sqrt}[c*x] * \text{Sqrt}[3 - 2*x^2]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[3 - \text{Sqrt}[6]*x]/\text{Sqrt}[6]], 2 ])/(3^{3/4}*c^2*\text{Sqrt}[x]*\text{Sqrt}[3*a - 2*a*x^2])$

**Rubi in Sympy [A]** time = 48.6144, size = 172, normalized size = 1.61

$$\frac{2\sqrt[4]{2} \cdot 3^{3/4} \sqrt{-\frac{2x^2}{3} + 1} E\left(\text{asin}\left(\frac{\sqrt[4]{2} \cdot 3^{3/4} \sqrt{cx}}{3\sqrt{c}}\right)\middle|-1\right)}{3c^{3/2} \sqrt{-2ax^2 + 3a}} + \frac{2\sqrt[4]{2} \cdot 3^{3/4} \sqrt{-\frac{2x^2}{3} + 1} F\left(\text{asin}\left(\frac{\sqrt[4]{2} \cdot 3^{3/4} \sqrt{cx}}{3\sqrt{c}}\right)\middle|-1\right)}{3c^{3/2} \sqrt{-2ax^2 + 3a}} - \frac{2\sqrt{-2ax^2 + 3a}}{3ac\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/(c*x)^{(3/2)}/(-2*a*x^2+3*a)^{(1/2}), x)$

[Out]  $-2*2^{1/4}*3^{3/4}*\text{sqrt}(-2*x^2/3 + 1)*\text{elliptic\_e}(\text{asin}(2^{1/4} * 3^{3/4}*\text{sqrt}(c*x)/(3*\text{sqrt}(c))), -1)/(3*c^{3/2}*\text{sqrt}(-2*a*x^2 + 3*a)) + 2*2^{1/4}*3^{3/4}*\text{sqrt}(-2*x^2/3 + 1)*\text{elliptic\_f}(\text{asin}(2^{1/4} * 3^{3/4}*\text{sqrt}(c*x)/(3*\text{sqrt}(c))), -1)/(3*c^{3/2}*\text{sqrt}(-2*a*x^2 + 3*a)) - 2*\text{sqrt}(-2*a*x^2 + 3*a)/(3*a*c*\text{sqrt}(c*x))$

**Mathematica [A]** time = 0.108577, size = 110, normalized size = 1.03

$$\frac{x \left( 4x^2 + 2\sqrt[4]{6}\sqrt{3-2x^2}\sqrt{x}F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\sqrt{x}\right)\middle|-1\right) - 2\sqrt[4]{6}\sqrt{3-2x^2}\sqrt{x}E\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\sqrt{x}\right)\middle|-1\right) - 6 \right)}{3\sqrt{a(3-2x^2)}(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/((c*x)^{(3/2)}*\text{Sqrt}[3*a - 2*a*x^2]), x]$

[Out]  $(x^*(-6 + 4*x^2 - 2*6^{(1/4)}*\text{Sqrt}[x]*\text{Sqrt}[3 - 2*x^2]*\text{EllipticE}[\text{ArcSin}[(2/3)^{(1/4)}*\text{Sqrt}[x]], -1] + 2*6^{(1/4)}*\text{Sqrt}[x]*\text{Sqrt}[3 - 2*x^2]*\text{EllipticF}[\text{ArcSin}[(2/3)^{(1/4)}*\text{Sqrt}[x]], -1]))/(3*(c*x)^{(3/2)}*\text{Sqrt}[a*(3 - 2*x^2)])$

**Maple [B]** time = 0.029, size = 228, normalized size = 2.1

$$-\frac{1}{18ac(2x^2-3)}\sqrt{-a(2x^2-3)}\left(2\sqrt{(-2x+\sqrt{3}\sqrt{2})}\sqrt{3}\sqrt{2}\sqrt{3}\sqrt{-x\sqrt{3}\sqrt{2}}\text{EllipticE}\left(\frac{1}{6}\sqrt{3}\sqrt{2}\sqrt{(2x+\sqrt{3}\sqrt{2})}\sqrt{3}\sqrt{2}, \frac{1}{2}\sqrt{3}\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2), x)`

[Out]  $-1/18*(-a*(2*x^2-3))^{(1/2)}*(2*((-2*x+3^{(1/2)}*2^{(1/2)})*3^{(1/2)}*2^{(1/2)})^{(1/2)}*3^{(1/2)}*(-x*3^{(1/2)}*2^{(1/2)})^{(1/2)}*\text{EllipticE}(1/6*3^{(1/2)}*2^{(1/2)}*((2*x+3^{(1/2)}*2^{(1/2)})*3^{(1/2)}*2^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)}*((2*x+3^{(1/2)}*2^{(1/2)})*3^{(1/2)}*2^{(1/2)})^{(1/2)}*2^{(1/2)}-((-2*x+3^{(1/2)}*2^{(1/2)})*3^{(1/2)}*2^{(1/2)})^{(1/2)}*3^{(1/2)}*(-x*3^{(1/2)}*2^{(1/2)})^{(1/2)}*\text{EllipticF}(1/6*3^{(1/2)}*2^{(1/2)}*((2*x+3^{(1/2)}*2^{(1/2)})*3^{(1/2)}*2^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)}*((2*x+3^{(1/2)}*2^{(1/2)})*3^{(1/2)}*2^{(1/2)})^{(1/2)}*2^{(1/2)}+24*x^2-36)/c/(c*x)^{(1/2)}/a/(2*x^2-3)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2ax^2+3a}(cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2)), x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-2ax^2+3a}\sqrt{cxcx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2)), x, algorithm="fricas")`

[Out] `integral(1/(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*c*x), x)`

**Sympy [A]** time = 7.69047, size = 54, normalized size = 0.5

$$\frac{\sqrt{3}\left(-\frac{1}{4}\right) {}_2F_1\left(\left.-\frac{1}{4}, \frac{1}{2}\right|\frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{ac}^{\frac{3}{2}}\sqrt{x}\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(3/2)/(-2*a*x**2+3*a)**(1/2), x)`

```
[Out] sqrt(3)*gamma(-1/4)*hyper((-1/4, 1/2), (3/4, ), 2*x**2*exp_polar(2
*I*pi)/3)/(6*sqrt(a)*c**(3/2)*sqrt(x)*gamma(3/4))
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2ax^2 + 3a}(cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2)), x)
```

$$3.642 \quad \int \frac{1}{(cx)^{5/2} \sqrt{3a-2ax^2}} dx$$

**Optimal.** Leaf size=98

$$\frac{2 \cdot 2^{3/4} \sqrt{3-2x^2} F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}} \sqrt{cx}}{\sqrt{c}}\right) \middle| -1\right)}{9 \sqrt[4]{3} c^{5/2} \sqrt{a(3-2x^2)}} - \frac{2 \sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}$$

[Out]  $(-2 \cdot \text{Sqrt}[3 \cdot a - 2 \cdot a \cdot x^2]) / (9 \cdot a \cdot c \cdot (c \cdot x)^{(3/2)}) + (2 \cdot 2^{(3/4)} \cdot \text{Sqrt}[3 - 2 \cdot x^2] \cdot \text{EllipticF}[\text{ArcSin}[(2/3)^{(1/4)} \cdot \text{Sqrt}[c \cdot x]) / \text{Sqrt}[c]], -1]) / (9 \cdot 3^{(1/4)} \cdot c^{(5/2)} \cdot \text{Sqrt}[a \cdot (3 - 2 \cdot x^2)])$

**Rubi [A]** time = 0.134777, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{2 \cdot 2^{3/4} \sqrt{3-2x^2} F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}} \sqrt{cx}}{\sqrt{c}}\right) \middle| -1\right)}{9 \sqrt[4]{3} c^{5/2} \sqrt{a(3-2x^2)}} - \frac{2 \sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/((c \cdot x)^{(5/2)} \cdot \text{Sqrt}[3 \cdot a - 2 \cdot a \cdot x^2]), x]$

[Out]  $(-2 \cdot \text{Sqrt}[3 \cdot a - 2 \cdot a \cdot x^2]) / (9 \cdot a \cdot c \cdot (c \cdot x)^{(3/2)}) + (2 \cdot 2^{(3/4)} \cdot \text{Sqrt}[3 - 2 \cdot x^2] \cdot \text{EllipticF}[\text{ArcSin}[(2/3)^{(1/4)} \cdot \text{Sqrt}[c \cdot x]) / \text{Sqrt}[c]], -1]) / (9 \cdot 3^{(1/4)} \cdot c^{(5/2)} \cdot \text{Sqrt}[a \cdot (3 - 2 \cdot x^2)])$

**Rubi in Sympy [A]** time = 11.7122, size = 99, normalized size = 1.01

$$\frac{2 \cdot 2^{3/4} \sqrt[4]{3} \sqrt{-\frac{2x^2}{3}} + 1 F\left(\text{asin}\left(\frac{\sqrt[4]{2 \cdot 3^{3/4}} \sqrt{cx}}{3 \sqrt{c}}\right) \middle| -1\right)}{9c^{5/2} \sqrt{-2ax^2 + 3a}} - \frac{2 \sqrt{-2ax^2 + 3a}}{9ac(cx)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/(c \cdot x)^{(5/2)} / (-2 \cdot a \cdot x^2 + 3 \cdot a)^{(1/2)}, x)$

[Out]  $2 \cdot 2^{(3/4)} \cdot 3^{(1/4)} \cdot \text{sqrt}(-2 \cdot x^2 / 3 + 1) \cdot \text{elliptic\_f}(\text{asin}(2^{(1/4)} \cdot 3^{(3/4)} \cdot \text{sqrt}(c \cdot x) / (3 \cdot \text{sqrt}(c))), -1) / (9 \cdot c^{(5/2)} \cdot \text{sqrt}(-2 \cdot a \cdot x^2 + 3 \cdot a)) - 2 \cdot \text{sqrt}(-2 \cdot a \cdot x^2 + 3 \cdot a) / (9 \cdot a \cdot c \cdot (c \cdot x)^{(3/2)})$

**Mathematica [A]** time = 0.0991967, size = 84, normalized size = 0.86

$$\frac{2x(2x^2 - 3) \left( 6^{3/4} \sqrt{x} F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}}{\sqrt{x}}\right) \middle| -1\right) - 3 \sqrt{2 - \frac{3}{x^2}} \right)}{27 \sqrt{2 - \frac{3}{x^2}} \sqrt{a(3-2x^2)} (cx)^{5/2}}$$

Antiderivative was successfully verified.



```
[Out] sqrt(3)*gamma(-3/4)*hyper((-3/4, 1/2), (1/4, ), 2*x**2*exp_polar(2
*I*pi)/3)/(6*sqrt(a)*c**(5/2)*x**(3/2)*gamma(1/4))
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2ax^2 + 3a}(cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(5/2)), x)
```

$$3.643 \quad \int \frac{(cx)^{5/2}}{(3a-2ax^2)^{3/2}} dx$$

**Optimal.** Leaf size=110

$$\frac{3\sqrt[4]{3}c^2\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{2^{2^{3/4}}a\sqrt{x}\sqrt{3a-2ax^2}} + \frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}}$$

[Out] (c\*(c\*x)^(3/2))/(2\*a\*Sqrt[3\*a - 2\*a\*x^2]) + (3\*3^(1/4)\*c^2\*Sqrt[c\*x]\*Sqrt[3 - 2\*x^2]\*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]\*x]/Sqrt[6]], 2])/(2\*2^(3/4)\*a\*Sqrt[x]\*Sqrt[3\*a - 2\*a\*x^2])

**Rubi [A]** time = 0.146379, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$\frac{3\sqrt[4]{3}c^2\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{2^{2^{3/4}}a\sqrt{x}\sqrt{3a-2ax^2}} + \frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(5/2)/(3\*a - 2\*a\*x^2)^(3/2), x]

[Out] (c\*(c\*x)^(3/2))/(2\*a\*Sqrt[3\*a - 2\*a\*x^2]) + (3\*3^(1/4)\*c^2\*Sqrt[c\*x]\*Sqrt[3 - 2\*x^2]\*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]\*x]/Sqrt[6]], 2])/(2\*2^(3/4)\*a\*Sqrt[x]\*Sqrt[3\*a - 2\*a\*x^2])

**Rubi in Sympy [A]** time = 52.0957, size = 173, normalized size = 1.57

$$\frac{3\sqrt[4]{2} \cdot 3^{\frac{3}{4}} c^{\frac{5}{2}} \sqrt{-\frac{2x^2}{3} + 1} E\left(\operatorname{asin}\left(\frac{\sqrt[4]{2} \cdot 3^{\frac{3}{4}} \sqrt{cx}}{3\sqrt{c}}\right)\middle|-1\right)}{4a\sqrt{-2ax^2 + 3a}} + \frac{3\sqrt[4]{2} \cdot 3^{\frac{3}{4}} c^{\frac{5}{2}} \sqrt{-\frac{2x^2}{3} + 1} F\left(\operatorname{asin}\left(\frac{\sqrt[4]{2} \cdot 3^{\frac{3}{4}} \sqrt{cx}}{3\sqrt{c}}\right)\middle|-1\right)}{4a\sqrt{-2ax^2 + 3a}} + \frac{c(cx)^{\frac{3}{2}}}{2a\sqrt{-2ax^2 + 3a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(5/2)/(-2\*a\*x\*\*2+3\*a)\*\*(3/2), x)

[Out] -3\*2\*\*(1/4)\*3\*\*(3/4)\*c\*\*(5/2)\*sqrt(-2\*x\*\*2/3 + 1)\*elliptic\_e(asin(2\*\*(1/4)\*3\*\*(3/4)\*sqrt(c\*x)/(3\*sqrt(c))), -1)/(4\*a\*sqrt(-2\*a\*x\*\*2 + 3\*a)) + 3\*2\*\*(1/4)\*3\*\*(3/4)\*c\*\*(5/2)\*sqrt(-2\*x\*\*2/3 + 1)\*elliptic\_f(asin(2\*\*(1/4)\*3\*\*(3/4)\*sqrt(c\*x)/(3\*sqrt(c))), -1)/(4\*a\*sqrt(-2\*a\*x\*\*2 + 3\*a)) + c\*(c\*x)\*\*(3/2)/(2\*a\*sqrt(-2\*a\*x\*\*2 + 3\*a))

**Mathematica [A]** time = 0.104991, size = 108, normalized size = 0.98

$$\frac{(cx)^{5/2} \left( 2x^{3/2} + 3\sqrt[4]{6}\sqrt{3-2x^2} F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}\sqrt{x}\right)\middle|-1\right) - 3\sqrt[4]{6}\sqrt{3-2x^2} E\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}\sqrt{x}\right)\middle|-1\right) \right)}{4ax^{5/2}\sqrt{a(3-2x^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(5/2)/(3\*a - 2\*a\*x^2)^(3/2), x]

[Out]  $((c*x)^{(5/2)}*(2*x^{(3/2)} - 3*6^{(1/4)}*\text{Sqrt}[3 - 2*x^2]*\text{EllipticE}[\text{ArcSin}[(2/3)^{(1/4)}*\text{Sqrt}[x]], -1] + 3*6^{(1/4)}*\text{Sqrt}[3 - 2*x^2]*\text{EllipticF}[\text{ArcSin}[(2/3)^{(1/4)}*\text{Sqrt}[x]], -1]))/(4*a*x^{(5/2)}*\text{Sqrt}[a*(3 - 2*x^2)])$

**Maple [B]** time = 0.054, size = 229, normalized size = 2.1

$$\frac{c^2}{16 a^2 x (2 x^2 - 3)} \sqrt{c x} \sqrt{-a (2 x^2 - 3)} \left( \sqrt{(-2 x + \sqrt{3} \sqrt{2})} \sqrt{3} \sqrt{2} \sqrt{3} \sqrt{-x \sqrt{3} \sqrt{2}} \text{EllipticF} \left( \frac{\sqrt{3} \sqrt{2}}{6} \sqrt{(2 x + \sqrt{3} \sqrt{2})} \sqrt{3} \sqrt{2}, \frac{\sqrt{2}}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x)`

[Out]  $1/16*c^2*(c*x)^{(1/2)}*(-a*(2*x^2-3))^{(1/2)}*((( -2*x+3^{(1/2)}*2^{(1/2)})^{(1/2)}*3^{(1/2)}*2^{(1/2)})^{(1/2)}*3^{(1/2)}*2^{(1/2)})^{(1/2)}*(-x*3^{(1/2)}*2^{(1/2)})^{(1/2)}*\text{EllipticF}(1/6*3^{(1/2)}*2^{(1/2)}*((2*x+3^{(1/2)}*2^{(1/2)})^{(1/2)}*3^{(1/2)}*2^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)}*((2*x+3^{(1/2)}*2^{(1/2)})^{(1/2)}*3^{(1/2)}*2^{(1/2)})^{(1/2)})^{(1/2)}*2^{(1/2)}-2*(( -2*x+3^{(1/2)}*2^{(1/2)})^{(1/2)}*3^{(1/2)}*2^{(1/2)})^{(1/2)}*3^{(1/2)}*(-x*3^{(1/2)}*2^{(1/2)})^{(1/2)}*\text{EllipticE}(1/6*3^{(1/2)}*2^{(1/2)}*((2*x+3^{(1/2)}*2^{(1/2)})^{(1/2)}*3^{(1/2)}*2^{(1/2)})^{(1/2)}, 1/2*2^{(1/2)}*((2*x+3^{(1/2)}*2^{(1/2)})^{(1/2)}*3^{(1/2)}*2^{(1/2)})^{(1/2)})^{(1/2)}*2^{(1/2)}-8*x^2)/x/a^2/(2*x^2-3)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{(-2ax^2 + 3a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(5/2)/(-2*a*x^2 + 3*a)^(3/2),x, algorithm="maxima")`

[Out] `integrate((c*x)^(5/2)/(-2*a*x^2 + 3*a)^(3/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{\sqrt{c x} c^2 x^2}{(2 a x^2 - 3 a) \sqrt{-2 a x^2 + 3 a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(5/2)/(-2*a*x^2 + 3*a)^(3/2),x, algorithm="fricas")`

[Out] `integral(-sqrt(c*x)*c^2*x^2/((2*a*x^2 - 3*a)*sqrt(-2*a*x^2 + 3*a)), x)`

**Sympy [A]** time = 170.871, size = 51, normalized size = 0.46

$$\frac{\sqrt{3} c^{\frac{5}{2}} x^{\frac{7}{2}} \left( \frac{7}{4} \right) {}_2F_1 \left( \frac{3}{2}, \frac{7}{4} \middle| \frac{2 x^2 e^{2 i \pi}}{3} \right)}{18 a^{\frac{3}{2}} \left( \frac{11}{4} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((c\*x)\*\*(5/2)/(-2\*a\*x\*\*2+3\*a)\*\*(3/2),x)

[Out] sqrt(3)\*c\*\*(5/2)\*x\*\*(7/2)\*gamma(7/4)\*hyper((3/2, 7/4), (11/4, ), 2\*x\*\*2\*exp\_polar(2\*I\*pi)/3)/(18\*a\*\*(3/2)\*gamma(11/4))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{(-2ax^2 + 3a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(5/2)/(-2\*a\*x^2 + 3\*a)^(3/2),x, algorithm="giac")

[Out] integrate((c\*x)^(5/2)/(-2\*a\*x^2 + 3\*a)^(3/2), x)

$$3.644 \quad \int \frac{(cx)^{3/2}}{(3a-2ax^2)^{3/2}} dx$$

**Optimal.** Leaf size=94

$$\frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}} - \frac{c^{3/2}\sqrt{3-2x^2} F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right) \middle| -1\right)}{2\sqrt[4]{6a}\sqrt{a(3-2x^2)}}$$

[Out] (c\*Sqrt[c\*x])/(2\*a\*Sqrt[3\*a - 2\*a\*x^2]) - (c^(3/2)\*Sqrt[3 - 2\*x^2])\*EllipticF[ArcSin[((2/3)^(1/4)\*Sqrt[c\*x])/Sqrt[c]], -1]/(2\*6^(1/4)\*a\*Sqrt[a\*(3 - 2\*x^2)])

**Rubi [A]** time = 0.139715, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}} - \frac{c^{3/2}\sqrt{3-2x^2} F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right) \middle| -1\right)}{2\sqrt[4]{6a}\sqrt{a(3-2x^2)}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(3/2)/(3\*a - 2\*a\*x^2)^(3/2), x]

[Out] (c\*Sqrt[c\*x])/(2\*a\*Sqrt[3\*a - 2\*a\*x^2]) - (c^(3/2)\*Sqrt[3 - 2\*x^2])\*EllipticF[ArcSin[((2/3)^(1/4)\*Sqrt[c\*x])/Sqrt[c]], -1]/(2\*6^(1/4)\*a\*Sqrt[a\*(3 - 2\*x^2)])

**Rubi in Sympy [A]** time = 12.7823, size = 97, normalized size = 1.03

$$-\frac{2^{\frac{3}{4}}\sqrt[4]{3}c^{\frac{3}{2}}\sqrt{-\frac{2x^2}{3}} + 1F\left(\operatorname{asin}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}\sqrt{cx}}{3\sqrt{c}}\right) \middle| -1\right)}{4a\sqrt{-2ax^2+3a}} + \frac{c\sqrt{cx}}{2a\sqrt{-2ax^2+3a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(3/2)/(-2\*a\*x\*\*2+3\*a)\*\*(3/2), x)

[Out] -2\*\*(3/4)\*3\*\*(1/4)\*c\*\*(3/2)\*sqrt(-2\*x\*\*2/3 + 1)\*elliptic\_f(asin(2\*\*(1/4)\*3\*\*(3/4)\*sqrt(c\*x)/(3\*sqrt(c))), -1)/(4\*a\*sqrt(-2\*a\*x\*\*2 + 3\*a)) + c\*sqrt(c\*x)/(2\*a\*sqrt(-2\*a\*x\*\*2 + 3\*a))

**Mathematica [A]** time = 0.119088, size = 91, normalized size = 0.97

$$\frac{(cx)^{3/2} \left( 6^{3/4} (2x^2 - 3) F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}}{\sqrt{x}}\right) \middle| -1\right) + 6\sqrt{2 - \frac{3}{x^2}}x^{3/2} \right)}{12a\sqrt{2 - \frac{3}{x^2}}x^{5/2}\sqrt{a(3-2x^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(3/2)/(3\*a - 2\*a\*x^2)^(3/2), x]

[Out] ((c\*x)^(3/2)\*(6\*Sqrt[2 - 3/x^2]\*x^(3/2) + 6^(3/4)\*(-3 + 2\*x^2)\*EllipticF[ArcSin[(3/2)^(1/4)/Sqrt[x]], -1]))/(12\*a\*Sqrt[2 - 3/x^2]\*x^(5/2)\*Sqrt[a\*(3 - 2\*x^2)])

**Maple [A]** time = 0.046, size = 126, normalized size = 1.3

$$\frac{c}{24 a^2 x (2 x^2 - 3)} \sqrt{c x} \sqrt{-a (2 x^2 - 3)} \left( \sqrt{(-2 x + \sqrt{3} \sqrt{2})} \sqrt{3} \sqrt{2} \sqrt{-x \sqrt{3} \sqrt{2}} \text{EllipticF} \left( \frac{\sqrt{3} \sqrt{2}}{6} \sqrt{(2 x + \sqrt{3} \sqrt{2})} \sqrt{3} \sqrt{2}, \frac{\sqrt{2}}{2} \right) \sqrt{(-2 x + \sqrt{3} \sqrt{2})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(3/2)/(-2\*a\*x^2+3\*a)^(3/2), x)

[Out] 1/24\*c\*(c\*x)^(1/2)\*(-a\*(2\*x^2-3))^(1/2)\*((( -2\*x+3^(1/2)\*2^(1/2) ) \* 3^(1/2)\*2^(1/2))^(1/2)\*(-x\*3^(1/2)\*2^(1/2))^(1/2)\*EllipticF(1/6\*3^(1/2)\*2^(1/2)\*((2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2), 1/2\*2^(1/2))\*((2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2)-12\*x)/x/a^2/(2\*x^2-3)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{(-2ax^2 + 3a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(3/2)/(-2\*a\*x^2 + 3\*a)^(3/2), x, algorithm="maxima")

[Out] integrate((c\*x)^(3/2)/(-2\*a\*x^2 + 3\*a)^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{\sqrt{c x} c x}{(2 a x^2 - 3 a) \sqrt{-2 a x^2 + 3 a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(3/2)/(-2\*a\*x^2 + 3\*a)^(3/2), x, algorithm="fricas")

[Out] integral(-sqrt(c\*x)\*c\*x/((2\*a\*x^2 - 3\*a)\*sqrt(-2\*a\*x^2 + 3\*a)), x)

**Sympy [A]** time = 12.8282, size = 51, normalized size = 0.54

$$\frac{\sqrt{3} c^{\frac{3}{2}} x^{\frac{5}{2}} \left( \frac{5}{4} \right) {}_2F_1 \left( \frac{5}{4}, \frac{3}{2} \middle| \frac{2 x^2 e^{2 i \pi}}{3} \right)}{18 a^{\frac{3}{2}} \left( \frac{9}{4} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(3/2)/(-2\*a\*x\*\*2+3\*a)\*\*(3/2),x)

[Out] sqrt(3)\*c\*\*(3/2)\*x\*\*(5/2)\*gamma(5/4)\*hyper((5/4, 3/2), (9/4, ), 2\*x\*\*2\*exp\_polar(2\*I\*pi)/3)/(18\*a\*\*(3/2)\*gamma(9/4))

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{(-2ax^2 + 3a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(3/2)/(-2\*a\*x^2 + 3\*a)^(3/2),x, algorithm="giac")

[Out] integrate((c\*x)^(3/2)/(-2\*a\*x^2 + 3\*a)^(3/2), x)

$$3.645 \quad \int \frac{\sqrt{cx}}{(3a-2ax^2)^{3/2}} dx$$

**Optimal.** Leaf size=101

$$\frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}} + \frac{\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6x}}}{\sqrt{6}}\right)\middle|2\right)}{6^{3/4}a\sqrt{x}\sqrt{3a-2ax^2}}$$

[Out] (c\*x)^(3/2)/(3\*a\*c\*Sqrt[3\*a - 2\*a\*x^2]) + (Sqrt[c\*x]\*Sqrt[3 - 2\*x^2]\*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]\*x]/Sqrt[6]], 2])/(6^(3/4)\*a\*Sqrt[x]\*Sqrt[3\*a - 2\*a\*x^2])

**Rubi [A]** time = 0.141017, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$\frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}} + \frac{\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6x}}}{\sqrt{6}}\right)\middle|2\right)}{6^{3/4}a\sqrt{x}\sqrt{3a-2ax^2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c\*x]/(3\*a - 2\*a\*x^2)^(3/2), x]

[Out] (c\*x)^(3/2)/(3\*a\*c\*Sqrt[3\*a - 2\*a\*x^2]) + (Sqrt[c\*x]\*Sqrt[3 - 2\*x^2]\*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]\*x]/Sqrt[6]], 2])/(6^(3/4)\*a\*Sqrt[x]\*Sqrt[3\*a - 2\*a\*x^2])

**Rubi in Sympy [A]** time = 51.4448, size = 170, normalized size = 1.68

$$\frac{\sqrt[4]{2} \cdot 3^{3/4} \sqrt{c} \sqrt{-\frac{2x^2}{3} + 1} E\left(\operatorname{asin}\left(\frac{\sqrt[4]{2} \cdot 3^{3/4} \sqrt{cx}}{3\sqrt{c}}\right)\middle|-1\right)}{6a\sqrt{-2ax^2 + 3a}} + \frac{\sqrt[4]{2} \cdot 3^{3/4} \sqrt{c} \sqrt{-\frac{2x^2}{3} + 1} F\left(\operatorname{asin}\left(\frac{\sqrt[4]{2} \cdot 3^{3/4} \sqrt{cx}}{3\sqrt{c}}\right)\middle|-1\right)}{6a\sqrt{-2ax^2 + 3a}} + \frac{(cx)^{3/2}}{3ac\sqrt{-2ax^2 + 3a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(1/2)/(-2\*a\*x\*\*2+3\*a)\*\*(3/2), x)

[Out] -2\*\*(1/4)\*3\*\*(3/4)\*sqrt(c)\*sqrt(-2\*x\*\*2/3 + 1)\*elliptic\_e(asin(2\*\*(1/4)\*3\*\*(3/4)\*sqrt(c\*x)/(3\*sqrt(c))), -1)/(6\*a\*sqrt(-2\*a\*x\*\*2 + 3\*a)) + 2\*\*(1/4)\*3\*\*(3/4)\*sqrt(c)\*sqrt(-2\*x\*\*2/3 + 1)\*elliptic\_f(asin(2\*\*(1/4)\*3\*\*(3/4)\*sqrt(c\*x)/(3\*sqrt(c))), -1)/(6\*a\*sqrt(-2\*a\*x\*\*2 + 3\*a)) + (c\*x)\*\*(3/2)/(3\*a\*c\*sqrt(-2\*a\*x\*\*2 + 3\*a))

**Mathematica [A]** time = 0.0797538, size = 107, normalized size = 1.06

$$\frac{\sqrt{cx} \left( 2x^{3/2} + \sqrt[4]{6}\sqrt{3-2x^2} F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\sqrt{x}\right)\middle|-1\right) - \sqrt[4]{6}\sqrt{3-2x^2} E\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\sqrt{x}\right)\middle|-1\right) \right)}{6a\sqrt{x}\sqrt{a(3-2x^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c\*x]/(3\*a - 2\*a\*x^2)^(3/2), x]

[Out]  $(\text{Sqrt}[c*x] * (2*x^{3/2}) - 6^{1/4} * \text{Sqrt}[3 - 2*x^2] * \text{EllipticE}[\text{ArcSin}[(2/3)^{1/4} * \text{Sqrt}[x]], -1] + 6^{1/4} * \text{Sqrt}[3 - 2*x^2] * \text{EllipticF}[\text{ArcSin}[(2/3)^{1/4} * \text{Sqrt}[x]], -1])) / (6*a*\text{Sqrt}[x]*\text{Sqrt}[a*(3 - 2*x^2)])$

**Maple [B]** time = 0.023, size = 227, normalized size = 2.3

$$-\frac{1}{72 a^2 x (2 x^2 - 3)} \sqrt{c x} \sqrt{-a (2 x^2 - 3)} \left( 2 \sqrt{(-2 x + \sqrt{3} \sqrt{2})} \sqrt{3} \sqrt{2} \sqrt{3} \sqrt{-x \sqrt{3} \sqrt{2}} \text{EllipticE} \left( \frac{1}{6} \sqrt{3} \sqrt{2} \sqrt{(2 x + \sqrt{3} \sqrt{2})} \sqrt{3} \sqrt{2}, \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((c*x)^{(1/2)} / (-2*a*x^2 + 3*a)^{(3/2)}, x)$

[Out]  $-1/72*(c*x)^{(1/2)}*(-a*(2*x^2-3))^{(1/2)}*(2*((-2*x+3^{1/2})^{2^{1/2}})^{3^{1/2}})^{(1/2)}*(3^{1/2})^{(1/2)}*(-x*3^{1/2})^{2^{1/2}})^{(1/2)}*\text{EllipticE}(1/6*3^{1/2})^{2^{1/2}}*((2*x+3^{1/2})^{2^{1/2}})^{3^{1/2}})^{(1/2)}*(1/2*2^{1/2})^{(1/2)}*((2*x+3^{1/2})^{2^{1/2}})^{3^{1/2}})^{(1/2)}*2^{1/2} - ((-2*x+3^{1/2})^{2^{1/2}})^{3^{1/2}})^{(1/2)}*(3^{1/2})^{(1/2)}*(-x*3^{1/2})^{2^{1/2}})^{(1/2)}*\text{EllipticF}(1/6*3^{1/2})^{2^{1/2}}*((2*x+3^{1/2})^{2^{1/2}})^{3^{1/2}})^{(1/2)}, 1/2*2^{1/2})^{(1/2)}*((2*x+3^{1/2})^{2^{1/2}})^{3^{1/2}})^{(1/2)}*2^{1/2} + 24*x^2/a^2/x/(2*x^2-3)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{(-2ax^2 + 3a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\text{sqrt}(c*x) / (-2*a*x^2 + 3*a)^{(3/2)}, x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}(\text{sqrt}(c*x) / (-2*a*x^2 + 3*a)^{(3/2)}, x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{\sqrt{cx}}{(2ax^2 - 3a)\sqrt{-2ax^2 + 3a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\text{sqrt}(c*x) / (-2*a*x^2 + 3*a)^{(3/2)}, x, \text{algorithm}="fricas")$

[Out]  $\text{integral}(-\text{sqrt}(c*x) / ((2*a*x^2 - 3*a)*\text{sqrt}(-2*a*x^2 + 3*a)), x)$

**Sympy [A]** time = 5.1292, size = 51, normalized size = 0.5

$$\frac{\sqrt{3}\sqrt{cx}^{\frac{3}{2}} \left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{3}{2} \middle| \frac{2x^2 e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}} \left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*x)^{(1/2)} / (-2*a*x^2 + 3*a)^{(3/2)}, x)$

```
[Out] sqrt(3)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/2), (7/4, ), 2*x
**2*exp_polar(2*I*pi)/3)/(18*a**(3/2)*gamma(7/4))
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{(-2ax^2 + 3a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x)/(-2*a*x^2 + 3*a)^(3/2), x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(-2*a*x^2 + 3*a)^(3/2), x)
```

$$3.646 \quad \int \frac{1}{\sqrt{cx}(3a-2ax^2)^{3/2}} dx$$

**Optimal.** Leaf size=96

$$\frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}} + \frac{\sqrt{3-2x^2} F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right) \middle| -1\right)}{3\sqrt[4]{6a}\sqrt{c}\sqrt{a(3-2x^2)}}$$

[Out] Sqrt[c\*x]/(3\*a\*c\*Sqrt[3\*a - 2\*a\*x^2]) + (Sqrt[3 - 2\*x^2]\*Elliptic F[ArcSin[((2/3)^(1/4)\*Sqrt[c\*x])/Sqrt[c]], -1])/(3\*6^(1/4)\*a\*Sqrt[c]\*Sqrt[a\*(3 - 2\*x^2)])

**Rubi [A]** time = 0.133548, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}} + \frac{\sqrt{3-2x^2} F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right) \middle| -1\right)}{3\sqrt[4]{6a}\sqrt{c}\sqrt{a(3-2x^2)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[c\*x]\*(3\*a - 2\*a\*x^2)^(3/2)), x]

[Out] Sqrt[c\*x]/(3\*a\*c\*Sqrt[3\*a - 2\*a\*x^2]) + (Sqrt[3 - 2\*x^2]\*Elliptic F[ArcSin[((2/3)^(1/4)\*Sqrt[c\*x])/Sqrt[c]], -1])/(3\*6^(1/4)\*a\*Sqrt[c]\*Sqrt[a\*(3 - 2\*x^2)])

**Rubi in Sympy [A]** time = 12.5306, size = 97, normalized size = 1.01

$$\frac{\sqrt{cx}}{3ac\sqrt{-2ax^2+3a}} + \frac{2^{\frac{3}{4}}\sqrt[4]{3}\sqrt{-\frac{2x^2}{3}} + 1 F\left(\operatorname{asin}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}\sqrt{cx}}{3\sqrt{c}}\right) \middle| -1\right)}{6a\sqrt{c}\sqrt{-2ax^2+3a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(1/2)/(-2\*a\*x\*\*2+3\*a)\*\*(3/2), x)

[Out] sqrt(c\*x)/(3\*a\*c\*sqrt(-2\*a\*x\*\*2 + 3\*a)) + 2\*\*(3/4)\*3\*\*(1/4)\*sqrt(-2\*x\*\*2/3 + 1)\*elliptic\_f(asin(2\*\*(1/4)\*3\*\*(3/4)\*sqrt(c\*x)/(3\*sqrt(c))), -1)/(6\*a\*sqrt(c)\*sqrt(-2\*a\*x\*\*2 + 3\*a))

**Mathematica [A]** time = 0.101973, size = 92, normalized size = 0.96

$$\frac{6\sqrt{2-\frac{3}{x^2}x^{3/2}} - 6^{3/4}(2x^2-3) F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}}{\sqrt{x}}\right) \middle| -1\right)}{18a\sqrt{2-\frac{3}{x^2}}\sqrt{x}\sqrt{a(3-2x^2)}\sqrt{cx}}$$

Antiderivative was successfully verified.



[In] Integrate[1/(Sqrt[c\*x]\*(3\*a - 2\*a\*x^2)^(3/2)),x]

[Out] (6\*Sqrt[2 - 3/x^2]\*x^(3/2) - 6^(3/4)\*(-3 + 2\*x^2)\*EllipticF[ArcSin[(3/2)^(1/4)/Sqrt[x]], -1])/(18\*a\*Sqrt[2 - 3/x^2]\*Sqrt[x]\*Sqrt[c\*x]\*Sqrt[a\*(3 - 2\*x^2)])

**Maple [A]** time = 0.034, size = 122, normalized size = 1.3

$$-\frac{1}{36 a^2 (2 x^2 - 3)} \sqrt{-a (2 x^2 - 3)} \left( \sqrt{(-2 x + \sqrt{3} \sqrt{2})} \sqrt{3} \sqrt{2} \sqrt{-x \sqrt{3} \sqrt{2}} \text{EllipticF} \left( \frac{\sqrt{3} \sqrt{2}}{6} \sqrt{(2 x + \sqrt{3} \sqrt{2})} \sqrt{3} \sqrt{2}, \frac{\sqrt{2}}{2} \right) \sqrt{(2 x + \sqrt{3} \sqrt{2})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x)^(1/2)/(-2\*a\*x^2+3\*a)^(3/2),x)

[Out] -1/36\*(-a\*(2\*x^2-3))^(1/2)\*((( -2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2)\*(-x\*3^(1/2)\*2^(1/2))^(1/2)\*EllipticF(1/6\*3^(1/2)\*2^(1/2)\*((2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2),1/2\*2^(1/2))\*((2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2)+12\*x)/a^2/(c\*x)^(1/2)/(2\*x^2-3)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-2 a x^2 + 3 a)^{\frac{3}{2}} \sqrt{c x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-2\*a\*x^2 + 3\*a)^(3/2)\*sqrt(c\*x)),x, algorithm="maxima")

[Out] integrate(1/((-2\*a\*x^2 + 3\*a)^(3/2)\*sqrt(c\*x)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{1}{(2 a x^2 - 3 a) \sqrt{-2 a x^2 + 3 a} \sqrt{c x}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-2\*a\*x^2 + 3\*a)^(3/2)\*sqrt(c\*x)),x, algorithm="fricas")

[Out] integral(-1/((2\*a\*x^2 - 3\*a)\*sqrt(-2\*a\*x^2 + 3\*a)\*sqrt(c\*x)), x)

**Sympy [A]** time = 8.6981, size = 51, normalized size = 0.53

$$\frac{\sqrt{3} \sqrt{x} \left( \frac{1}{4} \right) {}_2F_1 \left( \frac{1}{4}, \frac{3}{2} \middle| \frac{2 x^2 e^{2 i \pi}}{3} \right)}{18 a^{\frac{3}{2}} \sqrt{c} \left( \frac{5}{4} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(1/2)/(-2\*a\*x\*\*2+3\*a)\*\*(3/2),x)

[Out]  $\sqrt{3} \sqrt{x} \gamma(1/4) \text{hyper}((1/4, 3/2), (5/4, ), 2*x**2* \text{exp\_polar}(2*I*pi)/3)/(18*a**(3/2)*\sqrt{c}*\gamma(5/4))$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-2ax^2 + 3a)^{\frac{3}{2}} \sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-2*a*x^2 + 3*a)^(3/2)*sqrt(c*x)),x, algorithm="giac")`

[Out] `integrate(1/((-2*a*x^2 + 3*a)^(3/2)*sqrt(c*x)), x)`

$$3.647 \quad \int \frac{1}{(cx)^{3/2}(3a-2ax^2)^{3/2}} dx$$

**Optimal.** Leaf size=140

$$-\frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}} + \frac{\sqrt[4]{2}\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}ac^2\sqrt{x}\sqrt{3a-2ax^2}} + \frac{1}{3ac\sqrt{3a-2ax^2}\sqrt{cx}}$$

[Out] 1/(3\*a\*c\*Sqrt[c\*x]\*Sqrt[3\*a - 2\*a\*x^2]) - Sqrt[3\*a - 2\*a\*x^2]/(3\*a^2\*c\*Sqrt[c\*x]) + (2^(1/4)\*Sqrt[c\*x]\*Sqrt[3 - 2\*x^2]\*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]\*x]/Sqrt[6]], 2])/(3^(3/4)\*a\*c^2\*Sqrt[x]\*Sqrt[3\*a - 2\*a\*x^2])

**Rubi [A]** time = 0.188963, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$-\frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}} + \frac{\sqrt[4]{2}\sqrt{3-2x^2}\sqrt{cx}E\left(\sin^{-1}\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}ac^2\sqrt{x}\sqrt{3a-2ax^2}} + \frac{1}{3ac\sqrt{3a-2ax^2}\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(3/2)\*(3\*a - 2\*a\*x^2)^(3/2)), x]

[Out] 1/(3\*a\*c\*Sqrt[c\*x]\*Sqrt[3\*a - 2\*a\*x^2]) - Sqrt[3\*a - 2\*a\*x^2]/(3\*a^2\*c\*Sqrt[c\*x]) + (2^(1/4)\*Sqrt[c\*x]\*Sqrt[3 - 2\*x^2]\*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]\*x]/Sqrt[6]], 2])/(3^(3/4)\*a\*c^2\*Sqrt[x]\*Sqrt[3\*a - 2\*a\*x^2])

**Rubi in Sympy [A]** time = 56.3224, size = 199, normalized size = 1.42

$$\frac{1}{3ac\sqrt{cx}\sqrt{-2ax^2+3a}} - \frac{\sqrt[4]{2} \cdot 3^{\frac{3}{4}} \sqrt{-\frac{2x^2}{3}} + 1E\left(\operatorname{asin}\left(\frac{\sqrt[4]{2} \cdot 3^{\frac{3}{4}} \sqrt{cx}}{3\sqrt{c}}\right)\middle|-1\right)}{3ac^{\frac{3}{2}}\sqrt{-2ax^2+3a}} + \frac{\sqrt[4]{2} \cdot 3^{\frac{3}{4}} \sqrt{-\frac{2x^2}{3}} + 1F\left(\operatorname{asin}\left(\frac{\sqrt[4]{2} \cdot 3^{\frac{3}{4}} \sqrt{cx}}{3\sqrt{c}}\right)\middle|-1\right)}{3ac^{\frac{3}{2}}\sqrt{-2ax^2+3a}} - \frac{\sqrt{-2ax^2+3a}}{3a^2c\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(3/2)/(-2\*a\*x\*\*2+3\*a)\*\*(3/2), x)

[Out] 1/(3\*a\*c\*sqrt(c\*x)\*sqrt(-2\*a\*x\*\*2 + 3\*a)) - 2\*\*(1/4)\*3\*\*(3/4)\*sqrt(-2\*x\*\*2/3 + 1)\*elliptic\_e(asin(2\*\*(1/4)\*3\*\*(3/4)\*sqrt(c\*x)/(3\*sqrt(c))), -1)/(3\*a\*c\*\*(3/2)\*sqrt(-2\*a\*x\*\*2 + 3\*a)) + 2\*\*(1/4)\*3\*\*(3/4)\*sqrt(-2\*x\*\*2/3 + 1)\*elliptic\_f(asin(2\*\*(1/4)\*3\*\*(3/4)\*sqrt(c\*x)/(3\*sqrt(c))), -1)/(3\*a\*c\*\*(3/2)\*sqrt(-2\*a\*x\*\*2 + 3\*a)) - sqrt(-2\*a\*x\*\*2 + 3\*a)/(3\*a\*\*2\*c\*sqrt(c\*x))

**Mathematica [A]** time = 0.104092, size = 112, normalized size = 0.8

$$\frac{x \left( 2x^2 + \sqrt[4]{6}\sqrt{3-2x^2}\sqrt{x}F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\sqrt{x}\right)\middle|-1\right) - \sqrt[4]{6}\sqrt{3-2x^2}\sqrt{x}E\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}\sqrt{x}\right)\middle|-1\right) - 2 \right)}{3a\sqrt{a(3-2x^2)}(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(3/2)\*(3\*a - 2\*a\*x^2)^(3/2)),x]

[Out] (x\*(-2 + 2\*x^2 - 6^(1/4)\*Sqrt[x]\*Sqrt[3 - 2\*x^2]\*EllipticE[ArcSin[(2/3)^(1/4)\*Sqrt[x]], -1] + 6^(1/4)\*Sqrt[x]\*Sqrt[3 - 2\*x^2]\*EllipticF[ArcSin[(2/3)^(1/4)\*Sqrt[x]], -1])/(3\*a\*(c\*x)^(3/2)\*Sqrt[a\*(3 - 2\*x^2)])

**Maple [B]** time = 0.031, size = 228, normalized size = 1.6

$$-\frac{1}{36 a^2 c (2 x^2 - 3)} \sqrt{-a (2 x^2 - 3)} \left( 2 \sqrt{(-2 x + \sqrt{3} \sqrt{2})} \sqrt{3} \sqrt{2} \sqrt{3} \sqrt{-x \sqrt{3} \sqrt{2}} \text{EllipticE} \left( \frac{1}{6} \sqrt{3} \sqrt{2} \sqrt{(2 x + \sqrt{3} \sqrt{2})} \sqrt{3} \sqrt{2}, \frac{1}{2} \right) \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x)^(3/2)/(-2\*a\*x^2+3\*a)^(3/2),x)

[Out] -1/36\*(-a\*(2\*x^2-3))^(1/2)\*(2\*((-2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2)\*3^(1/2)\*(-x\*3^(1/2)\*2^(1/2))^(1/2)\*EllipticE(1/6\*3^(1/2)\*2^(1/2)\*((2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2),1/2\*2^(1/2))\*((2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2)\*2^(1/2)-((-2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2)\*3^(1/2)\*(-x\*3^(1/2)\*2^(1/2))^(1/2)\*EllipticF(1/6\*3^(1/2)\*2^(1/2)\*((2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2),1/2\*2^(1/2))\*((2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2)+24\*x^2-24)/a^2/c/(c\*x)^(1/2)/(2\*x^2-3)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-2ax^2 + 3a)^{\frac{3}{2}} (cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-2\*a\*x^2 + 3\*a)^(3/2)\*(c\*x)^(3/2)),x, algorithm="maxima")

[Out] integrate(1/((-2\*a\*x^2 + 3\*a)^(3/2)\*(c\*x)^(3/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{1}{(2acx^3 - 3acx)\sqrt{-2ax^2 + 3a}\sqrt{cx}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-2\*a\*x^2 + 3\*a)^(3/2)\*(c\*x)^(3/2)),x, algorithm="fricas")

[Out] integral(-1/((2\*a\*c\*x^3 - 3\*a\*c\*x)\*sqrt(-2\*a\*x^2 + 3\*a)\*sqrt(c\*x)), x)

**Sympy [A]** time = 28.2296, size = 54, normalized size = 0.39

$$\frac{\sqrt{3} \left(-\frac{1}{4}\right) {}_2F_1 \left( -\frac{1}{4}, \frac{3}{2} \middle| \frac{2x^2 e^{2i\pi}}{3} \right)}{18a^{\frac{3}{2}} c^{\frac{3}{2}} \sqrt{x} \left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(3/2)/(-2\*a\*x\*\*2+3\*a)\*\*(3/2),x)

[Out] sqrt(3)\*gamma(-1/4)\*hyper((-1/4, 3/2), (3/4, ), 2\*x\*\*2\*exp\_polar(2\*I\*pi)/3)/(18\*a\*\*(3/2)\*c\*\*(3/2)\*sqrt(x)\*gamma(3/4))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-2ax^2 + 3a)^{\frac{3}{2}} (cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-2\*a\*x^2 + 3\*a)^(3/2)\*(c\*x)^(3/2)),x, algorithm="giac")

[Out] integrate(1/((-2\*a\*x^2 + 3\*a)^(3/2)\*(c\*x)^(3/2)), x)

$$3.648 \quad \int \frac{1}{(cx)^{5/2}(3a-2ax^2)^{3/2}} dx$$

**Optimal.** Leaf size=132

$$\frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}} + \frac{5 \cdot 2^{3/4} \sqrt{3-2x^2} F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}} \sqrt{cx}}{\sqrt{c}}\right) \middle| -1\right)}{27\sqrt[4]{3}ac^{5/2}\sqrt{a(3-2x^2)}} + \frac{1}{3ac\sqrt{3a-2ax^2}(cx)^{3/2}}$$

[Out] 1/(3\*a\*c\*(c\*x)^(3/2)\*Sqrt[3\*a - 2\*a\*x^2]) - (5\*Sqrt[3\*a - 2\*a\*x^2])/(27\*a^2\*c\*(c\*x)^(3/2)) + (5\*2^(3/4)\*Sqrt[3 - 2\*x^2]\*EllipticF[ArcSin[((2/3)^(1/4)\*Sqrt[c\*x])/Sqrt[c]], -1])/(27\*3^(1/4)\*a\*c^(5/2)\*Sqrt[a\*(3 - 2\*x^2)])

**Rubi [A]** time = 0.179135, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$\frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}} + \frac{5 \cdot 2^{3/4} \sqrt{3-2x^2} F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{2}{3}} \sqrt{cx}}{\sqrt{c}}\right) \middle| -1\right)}{27\sqrt[4]{3}ac^{5/2}\sqrt{a(3-2x^2)}} + \frac{1}{3ac\sqrt{3a-2ax^2}(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(5/2)\*(3\*a - 2\*a\*x^2)^(3/2)), x]

[Out] 1/(3\*a\*c\*(c\*x)^(3/2)\*Sqrt[3\*a - 2\*a\*x^2]) - (5\*Sqrt[3\*a - 2\*a\*x^2])/(27\*a^2\*c\*(c\*x)^(3/2)) + (5\*2^(3/4)\*Sqrt[3 - 2\*x^2]\*EllipticF[ArcSin[((2/3)^(1/4)\*Sqrt[c\*x])/Sqrt[c]], -1])/(27\*3^(1/4)\*a\*c^(5/2)\*Sqrt[a\*(3 - 2\*x^2)])

**Rubi in Sympy [A]** time = 16.7497, size = 129, normalized size = 0.98

$$\frac{1}{3ac(cx)^{\frac{3}{2}}\sqrt{-2ax^2+3a}} + \frac{5 \cdot 2^{\frac{3}{4}} \sqrt[3]{3} \sqrt{-\frac{2x^2}{3}} + 1 F\left(\operatorname{asin}\left(\frac{\sqrt[4]{2} \cdot 3^{\frac{3}{4}} \sqrt{cx}}{3\sqrt{c}}\right) \middle| -1\right)}{27ac^{\frac{5}{2}}\sqrt{-2ax^2+3a}} - \frac{5\sqrt{-2ax^2+3a}}{27a^2c(cx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(5/2)/(-2\*a\*x\*\*2+3\*a)\*\*(3/2), x)

[Out] 1/(3\*a\*c\*(c\*x)\*\*(3/2)\*sqrt(-2\*a\*x\*\*2 + 3\*a)) + 5\*2\*\*(3/4)\*3\*\*(1/4)\*sqrt(-2\*x\*\*2/3 + 1)\*elliptic\_f(asin(2\*\*(1/4)\*3\*\*(3/4)\*sqrt(c\*x)/(3\*sqrt(c))), -1)/(27\*a\*c\*\*(5/2)\*sqrt(-2\*a\*x\*\*2 + 3\*a)) - 5\*sqrt(-2\*a\*x\*\*2 + 3\*a)/(27\*a\*\*2\*c\*(c\*x)\*\*(3/2))

**Mathematica [A]** time = 0.144872, size = 95, normalized size = 0.72

$$\frac{x \left( 6\sqrt{2 - \frac{3}{x^2}} (5x^2 - 3) - 5 \cdot 6^{3/4} \sqrt{x} (2x^2 - 3) F\left(\sin^{-1}\left(\frac{\sqrt[4]{\frac{3}{2}}}{\sqrt{x}}\right) \middle| -1\right) \right)}{81a\sqrt{2 - \frac{3}{x^2}}\sqrt{a(3-2x^2)}(cx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(5/2)\*(3\*a - 2\*a\*x^2)^(3/2)), x]

[Out] (x\*(6\*Sqrt[2 - 3/x^2]\*(-3 + 5\*x^2) - 5\*6^(3/4)\*Sqrt[x]\*(-3 + 2\*x^2)\*EllipticF[ArcSin[(3/2)^(1/4)/Sqrt[x]], -1]))/(81\*a\*Sqrt[2 - 3/x^2]\*(c\*x)^(5/2)\*Sqrt[a\*(3 - 2\*x^2)])

**Maple [A]** time = 0.029, size = 133, normalized size = 1.

$$-\frac{1}{162 a^2 x c^2 (2 x^2 - 3)} \sqrt{-a (2 x^2 - 3)} \left( 5 \sqrt{(-2 x + \sqrt{3} \sqrt{2})} \sqrt{3} \sqrt{2} \sqrt{-x \sqrt{3} \sqrt{2}} \text{EllipticF} \left( \frac{1}{6} \sqrt{3} \sqrt{2} \sqrt{(2 x + \sqrt{3} \sqrt{2})} \sqrt{3} \sqrt{2}, \frac{1}{2} \right) \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x)^(5/2)/(-2\*a\*x^2+3\*a)^(3/2), x)

[Out] -1/162\*(-a\*(2\*x^2-3))^(1/2)\*(5\*((-2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2)\*(-x\*3^(1/2)\*2^(1/2))^(1/2)\*EllipticF(1/6\*3^(1/2)\*2^(1/2)\*((2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2), 1/2\*2^(1/2))\*((2\*x+3^(1/2)\*2^(1/2))\*3^(1/2)\*2^(1/2))^(1/2)\*x+60\*x^2-36)/x/a^2/c^2/(c\*x)^(1/2)/(2\*x^2-3)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-2ax^2 + 3a)^{\frac{3}{2}}(cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-2\*a\*x^2 + 3\*a)^(3/2)\*(c\*x)^(5/2)), x, algorithm="maxima")

[Out] integrate(1/((-2\*a\*x^2 + 3\*a)^(3/2)\*(c\*x)^(5/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{1}{(2ac^2x^4 - 3ac^2x^2)\sqrt{-2ax^2 + 3a}\sqrt{cx}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-2\*a\*x^2 + 3\*a)^(3/2)\*(c\*x)^(5/2)), x, algorithm="fricas")

[Out] integral(-1/((2\*a\*c^2\*x^4 - 3\*a\*c^2\*x^2)\*sqrt(-2\*a\*x^2 + 3\*a)\*sqrt(c\*x)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(5/2)/(-2\*a\*x\*\*2+3\*a)\*\*(3/2), x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-2ax^2 + 3a)^{\frac{3}{2}} (cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-2*a*x^2 + 3*a)^(3/2)*(c*x)^(5/2)),x, algorithm="giac")`

[Out] `integrate(1/((-2*a*x^2 + 3*a)^(3/2)*(c*x)^(5/2)), x)`



$$3.649 \quad \int \frac{1}{\sqrt{x}\sqrt{1-a^2x^2}} dx$$

**Optimal.** Leaf size=21

$$\frac{2F(\sin^{-1}(\sqrt{a}\sqrt{x})|-1)}{\sqrt{a}}$$

[Out] (2\*EllipticF[ArcSin[Sqrt[a]\*Sqrt[x]], -1])/Sqrt[a]

**Rubi [A]** time = 0.0329877, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{2F(\sin^{-1}(\sqrt{a}\sqrt{x})|-1)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]\*Sqrt[1 - a^2\*x^2]), x]

[Out] (2\*EllipticF[ArcSin[Sqrt[a]\*Sqrt[x]], -1])/Sqrt[a]

**Rubi in Sympy [A]** time = 5.3998, size = 20, normalized size = 0.95

$$\frac{2F(\operatorname{asin}(\sqrt{a}\sqrt{x})|-1)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*(1/2)/(-a\*\*2\*x\*\*2+1)\*\*(1/2), x)

[Out] 2\*elliptic\_f(asin(sqrt(a)\*sqrt(x)), -1)/sqrt(a)

**Mathematica [C]** time = 0.125551, size = 65, normalized size = 3.1

$$\frac{2i\sqrt{-\frac{1}{a}}ax\sqrt{1-\frac{1}{a^2x^2}}F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{1}{a}}}{\sqrt{x}}\right)\right|-1)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]\*Sqrt[1 - a^2\*x^2]), x]

[Out] ((-2\*I)\*Sqrt[-a^(-1)]\*a\*Sqrt[1 - 1/(a^2\*x^2)]\*x\*EllipticF[I\*ArcSinh[Sqrt[-a^(-1)]]/Sqrt[x]], -1))/Sqrt[1 - a^2\*x^2]

**Maple [B]** time = 0.073, size = 66, normalized size = 3.1

$$-\frac{1}{a(a^2x^2-1)}\sqrt{-a^2x^2+1}\sqrt{ax+1}\sqrt{-2ax+2}\sqrt{-ax}\operatorname{EllipticF}\left(\sqrt{ax+1}, \frac{\sqrt{2}}{2}\right)\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^(1/2)/(-a^2*x^2+1)^(1/2),x)`

[Out]  $-1/x^{1/2} * (-a^2*x^2+1)^{1/2} * (a*x+1)^{1/2} * (-2*a*x+2)^{1/2} * (-a*x)^{1/2} * \text{EllipticF}((a*x+1)^{1/2}, 1/2*2^{1/2})/a/(a^2*x^2-1)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2x^2 + 1}\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-a^2*x^2 + 1)*sqrt(x)),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(-a^2*x^2 + 1)*sqrt(x)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-a^2x^2 + 1}\sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-a^2*x^2 + 1)*sqrt(x)),x, algorithm="fricas")`

[Out] `integral(1/(sqrt(-a^2*x^2 + 1)*sqrt(x)), x)`

**Sympy [A]** time = 2.30182, size = 36, normalized size = 1.71

$$\frac{\sqrt{x} \left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{5}{4}, a^2 x^2 e^{2i\pi}\right)}{2 \left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**(1/2)/(-a**2*x**2+1)**(1/2),x)`

[Out] `sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4, ), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(5/4))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2x^2 + 1}\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-a^2*x^2 + 1)*sqrt(x)),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(-a^2*x^2 + 1)*sqrt(x)), x)`

$$3.650 \quad \int \frac{1}{\sqrt{x}\sqrt{1+ax^2}} dx$$

**Optimal.** Leaf size=67

$$\frac{(\sqrt{ax} + 1) \sqrt{\frac{ax^2+1}{(\sqrt{ax}+1)^2}} F\left(2 \tan^{-1}(\sqrt[4]{a}\sqrt{x}) \mid \frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{ax^2 + 1}}$$

[Out]  $((1 + \text{Sqrt}[a]*x)*\text{Sqrt}[(1 + a*x^2)/(1 + \text{Sqrt}[a]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[a^{(1/4)}*\text{Sqrt}[x]], 1/2])/(a^{(1/4)}*\text{Sqrt}[1 + a*x^2])$

**Rubi [A]** time = 0.091526, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{(\sqrt{ax} + 1) \sqrt{\frac{ax^2+1}{(\sqrt{ax}+1)^2}} F\left(2 \tan^{-1}(\sqrt[4]{a}\sqrt{x}) \mid \frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{ax^2 + 1}}$$

Antiderivative was successfully verified.

[In] `Int[1/(Sqrt[x]*Sqrt[1 + a*x^2]), x]`

[Out]  $((1 + \text{Sqrt}[a]*x)*\text{Sqrt}[(1 + a*x^2)/(1 + \text{Sqrt}[a]*x)^2]*\text{EllipticF}[2*\text{ArcTan}[a^{(1/4)}*\text{Sqrt}[x]], 1/2])/(a^{(1/4)}*\text{Sqrt}[1 + a*x^2])$

**Rubi in Sympy [A]** time = 5.40887, size = 60, normalized size = 0.9

$$\frac{\sqrt{\frac{ax^2+1}{(\sqrt{ax}+1)^2}} (\sqrt{ax} + 1) F\left(2 \operatorname{atan}\left(\sqrt[4]{a}\sqrt{x}\right) \mid \frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{ax^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**(1/2)/(a*x**2+1)**(1/2), x)`

[Out]  $\text{sqrt}((a*x**2 + 1)/(\text{sqrt}(a)*x + 1)**2)*(\text{sqrt}(a)*x + 1)*\text{elliptic}_f(2*\text{atan}(a**(1/4)*\text{sqrt}(x)), 1/2)/(a**(1/4)*\text{sqrt}(a*x**2 + 1))$

**Mathematica [C]** time = 0.099452, size = 68, normalized size = 1.01

$$\frac{2ix \sqrt{\frac{a+\frac{1}{x^2}}{a}} F\left(i \sinh^{-1}\left(\frac{\sqrt{\frac{i}{a}}}{\sqrt{x}}\right) \mid -1\right)}{\sqrt{\frac{i}{a}}\sqrt{ax^2 + 1}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(Sqrt[x]*Sqrt[1 + a*x^2]), x]`

[Out]  $((2*I)*\text{Sqrt}[(a + x^{(-2)})/a]*x*\text{EllipticF}[I*\text{ArcSinh}[\text{Sqrt}[I/\text{Sqrt}[a]]/\text{Sqrt}[x]], -1])/(\text{Sqrt}[I/\text{Sqrt}[a]]*\text{Sqrt}[1 + a*x^2])$

**Maple [A]** time = 0.061, size = 73, normalized size = 1.1

$$-\sqrt{2}\sqrt{-x\sqrt{-a}+1}\sqrt{x\sqrt{-a}+1}\sqrt{x\sqrt{-a}}\text{EllipticF}\left(\sqrt{-x\sqrt{-a}+1}, \frac{\sqrt{2}}{2}\right) \frac{1}{\sqrt{x}} \frac{1}{\sqrt{ax^2+1}} \frac{1}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(1/2)/(a\*x^2+1)^(1/2), x)

[Out] -1/x^(1/2)/(a\*x^2+1)^(1/2)\*(-x\*(-a)^(1/2)+1)^(1/2)\*2^(1/2)\*(x\*(-a)^(1/2)+1)^(1/2)\*(x\*(-a)^(1/2))^(1/2)\*EllipticF((-x\*(-a)^(1/2)+1)^(1/2), 1/2\*2^(1/2))/(-a)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^2+1}\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(a\*x^2 + 1)\*sqrt(x)), x, algorithm="maxima")

[Out] integrate(1/(sqrt(a\*x^2 + 1)\*sqrt(x)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{ax^2+1}\sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(a\*x^2 + 1)\*sqrt(x)), x, algorithm="fricas")

[Out] integral(1/(sqrt(a\*x^2 + 1)\*sqrt(x)), x)

**Sympy [A]** time = 2.13592, size = 32, normalized size = 0.48

$$\frac{\sqrt{x} \left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{5}{4}, ax^2 e^{i\pi}\right)}{2 \left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(1/2)/(a\*x\*\*2+1)\*\*(1/2), x)

[Out] sqrt(x)\*gamma(1/4)\*hyper((1/4, 1/2), (5/4, ), a\*x\*\*2\*exp\_polar(I\*pi))/(2\*gamma(5/4))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^2+1}\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(a*x^2 + 1)*sqrt(x)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x^2 + 1)*sqrt(x)), x)
```

$$3.651 \quad \int x^m (a + bx^2)^{3/2} dx$$

**Optimal.** Leaf size=50

$$\frac{x^{m+1} (a + bx^2)^{5/2} {}_2F_1\left(1, \frac{m+6}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{a(m+1)}$$

[Out] (x^(1 + m)\*(a + b\*x^2)^(5/2)\*Hypergeometric2F1[1, (6 + m)/2, (3 + m)/2, -(b\*x^2)/a])/(a\*(1 + m))

**Rubi [A]** time = 0.0700529, antiderivative size = 64, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{ax^{m+1}\sqrt{a+bx^2} {}_2F_1\left(-\frac{3}{2}, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{(m+1)\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

[In] Int[x^m\*(a + b\*x^2)^(3/2), x]

[Out] (a\*x^(1 + m)\*Sqrt[a + b\*x^2]\*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2, -(b\*x^2)/a])/((1 + m)\*Sqrt[1 + (b\*x^2)/a])

**Rubi in Sympy [A]** time = 7.94928, size = 54, normalized size = 1.08

$$\frac{ax^{m+1}\sqrt{a+bx^2} {}_2F_1\left(-\frac{3}{2}, \frac{m}{2} + \frac{1}{2}; \frac{m}{2} + \frac{3}{2}; -\frac{bx^2}{a}\right)}{\sqrt{1 + \frac{bx^2}{a}}(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*m\*(b\*x\*\*2+a)\*\*(3/2), x)

[Out] a\*x\*\*(m + 1)\*sqrt(a + b\*x\*\*2)\*hyper((-3/2, m/2 + 1/2), (m/2 + 3/2, ), -b\*x\*\*2/a)/(sqrt(1 + b\*x\*\*2/a)\*(m + 1))

**Mathematica [B]** time = 0.117617, size = 109, normalized size = 2.18

$$\frac{x^{m+1}\sqrt{a+bx^2}\left(b(m+1)x^2 {}_2F_1\left(-\frac{1}{2}, \frac{m+3}{2}; \frac{m+5}{2}; -\frac{bx^2}{a}\right) + a(m+3) {}_2F_1\left(-\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)\right)}{(m+1)(m+3)\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*(a + b\*x^2)^(3/2), x]

[Out] (x^(1 + m)\*Sqrt[a + b\*x^2]\*(a\*(3 + m)\*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, -(b\*x^2)/a] + b\*(1 + m)\*x^2\*Hypergeometric2F1[-1/2, (3 + m)/2, (5 + m)/2, -(b\*x^2)/a]))/((1 + m)\*(3 + m)\*Sqrt[1 + (b\*x^2)/a])

**Maple [F]** time = 0.028, size = 0, normalized size = 0.

$$\int x^m (bx^2 + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*(b\*x^2+a)^(3/2),x)

[Out] int(x^m\*(b\*x^2+a)^(3/2),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{3}{2}} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)\*x^m,x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(3/2)\*x^m, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^{\frac{3}{2}} x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)\*x^m,x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(3/2)\*x^m, x)

---

**Sympy [A]** time = 22.4915, size = 54, normalized size = 1.08

$$\frac{a^{\frac{3}{2}} x x^m \left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(-\frac{3}{2}, \frac{m}{2} + \frac{1}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{2\left(\frac{m}{2} + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*(b\*x\*\*2+a)\*\*(3/2),x)

[Out] a\*\*(3/2)\*x\*x\*\*m\*gamma(m/2 + 1/2)\*hyper((-3/2, m/2 + 1/2), (m/2 + 3/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*gamma(m/2 + 3/2))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{3}{2}} x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/2)\*x^m,x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(3/2)\*x^m, x)

### 3.652 $\int x^m \sqrt{a + bx^2} dx$

**Optimal.** Leaf size=50

$$\frac{x^{m+1} (a + bx^2)^{3/2} {}_2F_1\left(1, \frac{m+4}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{a(m+1)}$$

[Out]  $(x^{(1+m)}*(a+b*x^2)^{(3/2)}*Hypergeometric2F1[1, (4+m)/2, (3+m)/2, -(b*x^2)/a])/ (a*(1+m))$

**Rubi [A]** time = 0.0641649, antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{x^{m+1} \sqrt{a + bx^2} {}_2F_1\left(-\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{(m+1) \sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Sqrt[a + b\*x^2], x]

[Out]  $(x^{(1+m)}*Sqrt[a+b*x^2]*Hypergeometric2F1[-1/2, (1+m)/2, (3+m)/2, -(b*x^2)/a])/((1+m)*Sqrt[1+(b*x^2)/a])$

**Rubi in Sympy [A]** time = 7.85796, size = 53, normalized size = 1.06

$$\frac{x^{m+1} \sqrt{a + bx^2} {}_2F_1\left(-\frac{1}{2}, \frac{m}{2} + \frac{1}{2}; \frac{m}{2} + \frac{3}{2}; -\frac{bx^2}{a}\right)}{\sqrt{1 + \frac{bx^2}{a}} (m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*m\*(b\*x\*\*2+a)\*\*(1/2), x)

[Out]  $x^{(m+1)}*sqrt(a+b*x**2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2, ), -b*x**2/a)/(sqrt(1 + b*x**2/a)*(m+1))$

**Mathematica [A]** time = 0.0276401, size = 63, normalized size = 1.26

$$\frac{x^{m+1} \sqrt{a + bx^2} {}_2F_1\left(-\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{(m+1) \sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*Sqrt[a + b\*x^2], x]

[Out]  $(x^{(1+m)}*Sqrt[a+b*x^2]*Hypergeometric2F1[-1/2, (1+m)/2, (3+m)/2, -(b*x^2)/a])/((1+m)*Sqrt[1+(b*x^2)/a])$

**Maple [F]** time = 0.026, size = 0, normalized size = 0.

$$\int x^m \sqrt{bx^2 + a} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*(b*x^2+a)^(1/2),x)`

[Out] `int(x^m*(b*x^2+a)^(1/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + ax^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*x^m,x, algorithm="maxima")`

[Out] `integrate(sqrt(b*x^2 + a)*x^m, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^2 + ax^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*x^m,x, algorithm="fricas")`

[Out] `integral(sqrt(b*x^2 + a)*x^m, x)`

**Sympy [A]** time = 3.6877, size = 54, normalized size = 1.08

$$\frac{\sqrt{ax}x^m \left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(-\frac{1}{2}, \frac{m}{2} + \frac{1}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\left(\frac{m}{2} + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*(b*x**2+a)**(1/2),x)`

[Out] `sqrt(a)*x*x**m*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2, ), b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + ax^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(b*x^2 + a)*x^m,x, algorithm="giac")`

[Out] `integrate(sqrt(b*x^2 + a)*x^m, x)`

$$3.653 \quad \int \frac{x^m}{\sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=50

$$\frac{x^{m+1}\sqrt{a+bx^2} {}_2F_1\left(1, \frac{m+2}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{a(m+1)}$$

[Out] (x^(1 + m)\*Sqrt[a + b\*x^2]\*Hypergeometric2F1[1, (2 + m)/2, (3 + m)/2, -((b\*x^2)/a)]/(a\*(1 + m))

**Rubi [A]** time = 0.0644855, antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}} {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^m/Sqrt[a + b\*x^2], x]

[Out] (x^(1 + m)\*Sqrt[1 + (b\*x^2)/a]\*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -((b\*x^2)/a)]/((1 + m)\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 8.33755, size = 53, normalized size = 1.06

$$\frac{x^{m+1}\sqrt{a+bx^2} {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + \frac{1}{2}; \frac{m}{2} + \frac{3}{2}; -\frac{bx^2}{a}\right)}{a\sqrt{1 + \frac{bx^2}{a}}(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*m/(b\*x\*\*2+a)\*\*(1/2), x)

[Out] x\*\*(m + 1)\*sqrt(a + b\*x\*\*2)\*hyper((1/2, m/2 + 1/2), (m/2 + 3/2, ), -b\*x\*\*2/a)/(a\*sqrt(1 + b\*x\*\*2/a)\*(m + 1))

**Mathematica [A]** time = 0.0454513, size = 63, normalized size = 1.26

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}} {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^m/Sqrt[a + b\*x^2], x]

[Out] (x^(1 + m)\*Sqrt[1 + (b\*x^2)/a]\*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -((b\*x^2)/a)]/((1 + m)\*Sqrt[a + b\*x^2])

**Maple [F]** time = 0.028, size = 0, normalized size = 0.

$$\int x^m \frac{1}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m/(b*x^2+a)^(1/2),x)`

[Out] `int(x^m/(b*x^2+a)^(1/2),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/sqrt(b*x^2 + a),x, algorithm="maxima")`

[Out] `integrate(x^m/sqrt(b*x^2 + a), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/sqrt(b*x^2 + a),x, algorithm="fricas")`

[Out] `integral(x^m/sqrt(b*x^2 + a), x)`

**Sympy** [A] time = 2.93202, size = 53, normalized size = 1.06

$$\frac{xx^m \left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + \frac{1}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt{a} \left(\frac{m}{2} + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m/(b*x**2+a)**(1/2),x)`

[Out] `x*x**m*gamma(m/2 + 1/2)*hyper((1/2, m/2 + 1/2), (m/2 + 3/2, ), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 3/2))`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/sqrt(b*x^2 + a),x, algorithm="giac")`

[Out] `integrate(x^m/sqrt(b*x^2 + a), x)`

$$3.654 \quad \int \frac{x^m}{(a+bx^2)^{3/2}} dx$$

**Optimal.** Leaf size=48

$$\frac{x^{m+1} {}_2F_1\left(1, \frac{m}{2}, \frac{m+3}{2}, -\frac{bx^2}{a}\right)}{a(m+1)\sqrt{a+bx^2}}$$

[Out] (x^(1 + m)\*Hypergeometric2F1[1, m/2, (3 + m)/2, -((b\*x^2)/a)]/(a\*(1 + m)\*Sqrt[a + b\*x^2])

**Rubi [A]** time = 0.0696581, antiderivative size = 66, normalized size of antiderivative = 1.38, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{x^{m+1} \sqrt{\frac{bx^2}{a} + 1} {}_2F_1\left(\frac{3}{2}, \frac{m+1}{2}, \frac{m+3}{2}, -\frac{bx^2}{a}\right)}{a(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^m/(a + b\*x^2)^(3/2), x]

[Out] (x^(1 + m)\*Sqrt[1 + (b\*x^2)/a]\*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, -((b\*x^2)/a)]/(a\*(1 + m)\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 8.34955, size = 54, normalized size = 1.12

$$\frac{x^{m+1} \sqrt{a+bx^2} {}_2F_1\left(\frac{3}{2}, \frac{m}{2} + \frac{1}{2}, \frac{m}{2} + \frac{3}{2}, -\frac{bx^2}{a}\right)}{a^2 \sqrt{1 + \frac{bx^2}{a}} (m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*m/(b\*x\*\*2+a)\*\*(3/2), x)

[Out] x\*\*(m + 1)\*sqrt(a + b\*x\*\*2)\*hyper((3/2, m/2 + 1/2), (m/2 + 3/2, ), -b\*x\*\*2/a)/(a\*\*2\*sqrt(1 + b\*x\*\*2/a)\*(m + 1))

**Mathematica [A]** time = 0.0495535, size = 66, normalized size = 1.38

$$\frac{x^{m+1} \sqrt{\frac{bx^2}{a} + 1} {}_2F_1\left(\frac{3}{2}, \frac{m+1}{2}, \frac{m+3}{2}, -\frac{bx^2}{a}\right)}{a(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^m/(a + b\*x^2)^(3/2), x]

[Out] (x^(1 + m)\*Sqrt[1 + (b\*x^2)/a]\*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, -((b\*x^2)/a)]/(a\*(1 + m)\*Sqrt[a + b\*x^2])

**Maple [F]** time = 0.028, size = 0, normalized size = 0.

$$\int x^m (bx^2 + a)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m/(b*x^2+a)^(3/2),x)`

[Out] `int(x^m/(b*x^2+a)^(3/2),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/(b*x^2 + a)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x^m/(b*x^2 + a)^(3/2), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{(bx^2 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/(b*x^2 + a)^(3/2),x, algorithm="fricas")`

[Out] `integral(x^m/(b*x^2 + a)^(3/2), x)`

**Sympy** [A] time = 5.65628, size = 53, normalized size = 1.1

$$\frac{xx^m \left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(\frac{3}{2}, \frac{m}{2} + \frac{1}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}} \left(\frac{m}{2} + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m/(b*x**2+a)**(3/2),x)`

[Out] `x*x**m*gamma(m/2 + 1/2)*hyper((3/2, m/2 + 1/2), (m/2 + 3/2, ), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m/2 + 3/2))`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/(b*x^2 + a)^(3/2),x, algorithm="giac")`

[Out] `integrate(x^m/(b*x^2 + a)^(3/2), x)`

$$3.655 \quad \int \frac{x^m}{(a+bx^2)^{5/2}} dx$$

**Optimal.** Leaf size=50

$$\frac{x^{m+1} {}_2F_1\left(1, \frac{m-2}{2}, \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{a(m+1)(a+bx^2)^{3/2}}$$

[Out] (x^(1 + m)\*Hypergeometric2F1[1, (-2 + m)/2, (3 + m)/2, -((b\*x^2)/a)]/(a\*(1 + m)\*(a + b\*x^2)^(3/2))

**Rubi [A]** time = 0.0676396, antiderivative size = 66, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{x^{m+1} \sqrt{\frac{bx^2}{a} + 1} {}_2F_1\left(\frac{5}{2}, \frac{m+1}{2}, \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{a^2(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^m/(a + b\*x^2)^(5/2), x]

[Out] (x^(1 + m)\*Sqrt[1 + (b\*x^2)/a]\*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2, -((b\*x^2)/a)]/(a^2\*(1 + m)\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 8.40576, size = 54, normalized size = 1.08

$$\frac{x^{m+1} \sqrt{a+bx^2} {}_2F_1\left(\frac{5}{2}, \frac{m}{2} + \frac{1}{2} \middle| \frac{m}{2} + \frac{3}{2}; -\frac{bx^2}{a}\right)}{a^3 \sqrt{1 + \frac{bx^2}{a}} (m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*m/(b\*x\*\*2+a)\*\*(5/2), x)

[Out] x\*\*(m + 1)\*sqrt(a + b\*x\*\*2)\*hyper((5/2, m/2 + 1/2), (m/2 + 3/2, ), -b\*x\*\*2/a)/(a\*\*3\*sqrt(1 + b\*x\*\*2/a)\*(m + 1))

**Mathematica [A]** time = 0.0491804, size = 66, normalized size = 1.32

$$\frac{x^{m+1} \sqrt{\frac{bx^2}{a} + 1} {}_2F_1\left(\frac{5}{2}, \frac{m+1}{2}, \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{a^2(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^m/(a + b\*x^2)^(5/2), x]

[Out] (x^(1 + m)\*Sqrt[1 + (b\*x^2)/a]\*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2, -((b\*x^2)/a)]/(a^2\*(1 + m)\*Sqrt[a + b\*x^2])

**Maple [F]** time = 0.027, size = 0, normalized size = 0.

$$\int x^m (bx^2 + a)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m/(b*x^2+a)^(5/2),x)`

[Out] `int(x^m/(b*x^2+a)^(5/2),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/(b*x^2 + a)^(5/2),x, algorithm="maxima")`

[Out] `integrate(x^m/(b*x^2 + a)^(5/2), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{(b^2x^4 + 2abx^2 + a^2)\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/(b*x^2 + a)^(5/2),x, algorithm="fricas")`

[Out] `integral(x^m/((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)), x)`

**Sympy** [A] time = 26.206, size = 53, normalized size = 1.06

$$\frac{xx^m \left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(\frac{5}{2}, \frac{m}{2} + \frac{1}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}} \left(\frac{m}{2} + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m/(b*x**2+a)**(5/2),x)`

[Out] `x*x**m*gamma(m/2 + 1/2)*hyper((5/2, m/2 + 1/2), (m/2 + 3/2, ), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*gamma(m/2 + 3/2))`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{(bx^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/(b*x^2 + a)^(5/2),x, algorithm="giac")`

[Out] `integrate(x^m/(b*x^2 + a)^(5/2), x)`

$$3.656 \quad \int \frac{x^{2+m}}{\sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=50

$$\frac{x^{m+3}\sqrt{a+bx^2} {}_2F_1\left(1, \frac{m+4}{2}, \frac{m+5}{2}; -\frac{bx^2}{a}\right)}{a(m+3)}$$

[Out] (x^(3 + m)\*Sqrt[a + b\*x^2]\*Hypergeometric2F1[1, (4 + m)/2, (5 + m)/2, -((b\*x^2)/a)]/(a\*(3 + m))

**Rubi [A]** time = 0.0694405, antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{x^{m+3}\sqrt{\frac{bx^2}{a}+1} {}_2F_1\left(\frac{1}{2}, \frac{m+3}{2}, \frac{m+5}{2}; -\frac{bx^2}{a}\right)}{(m+3)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^(2 + m)/Sqrt[a + b\*x^2], x]

[Out] (x^(3 + m)\*Sqrt[1 + (b\*x^2)/a]\*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, -((b\*x^2)/a)]/((3 + m)\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 8.58471, size = 53, normalized size = 1.06

$$\frac{x^{m+3}\sqrt{a+bx^2} {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + \frac{5}{2}; -\frac{bx^2}{a}\right)}{a\sqrt{1 + \frac{bx^2}{a}}(m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(2+m)/(b\*x\*\*2+a)\*\*(1/2), x)

[Out] x\*\*(m + 3)\*sqrt(a + b\*x\*\*2)\*hyper((1/2, m/2 + 3/2), (m/2 + 5/2, ), -b\*x\*\*2/a)/(a\*sqrt(1 + b\*x\*\*2/a)\*(m + 3))

**Mathematica [A]** time = 0.0734742, size = 96, normalized size = 1.92

$$\frac{x^{m+1}\sqrt{a+bx^2}\left({}_2F_1\left(-\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}; -\frac{bx^2}{a}\right) - {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}; -\frac{bx^2}{a}\right)\right)}{b(m+1)\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(2 + m)/Sqrt[a + b\*x^2], x]

[Out] (x^(1 + m)\*Sqrt[a + b\*x^2]\*(Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, -((b\*x^2)/a)] - Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -((b\*x^2)/a)])/(b\*(1 + m)\*Sqrt[1 + (b\*x^2)/a])

**Maple [F]** time = 0.03, size = 0, normalized size = 0.

$$\int x^{2+m} \frac{1}{\sqrt{bx^2+a}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(2+m)/(b*x^2+a)^(1/2),x)`

[Out] `int(x^(2+m)/(b*x^2+a)^(1/2),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m+2}}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(m + 2)/sqrt(b*x^2 + a),x, algorithm="maxima")`

[Out] `integrate(x^(m + 2)/sqrt(b*x^2 + a), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{m+2}}{\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(m + 2)/sqrt(b*x^2 + a),x, algorithm="fricas")`

[Out] `integral(x^(m + 2)/sqrt(b*x^2 + a), x)`

**Sympy** [A] time = 18.0904, size = 54, normalized size = 1.08

$$\frac{x^3 x^m \left(\frac{m}{2} + \frac{3}{2}\right) {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + \frac{3}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt{a} \left(\frac{m}{2} + \frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(2+m)/(b*x**2+a)**(1/2),x)`

[Out] `x**3*x**m*gamma(m/2 + 3/2)*hyper((1/2, m/2 + 3/2), (m/2 + 5/2, ), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 5/2))`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m+2}}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(m + 2)/sqrt(b*x^2 + a),x, algorithm="giac")`

[Out] `integrate(x^(m + 2)/sqrt(b*x^2 + a), x)`

$$3.657 \quad \int \frac{x^{1+m}}{\sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=50

$$\frac{x^{m+2}\sqrt{a+bx^2} {}_2F_1\left(1, \frac{m+3}{2}, \frac{m+4}{2}; -\frac{bx^2}{a}\right)}{a(m+2)}$$

[Out] (x^(2 + m)\*Sqrt[a + b\*x^2]\*Hypergeometric2F1[1, (3 + m)/2, (4 + m)/2, -((b\*x^2)/a)]/(a\*(2 + m))

**Rubi [A]** time = 0.0687624, antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{x^{m+2}\sqrt{\frac{bx^2}{a} + 1} {}_2F_1\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}; -\frac{bx^2}{a}\right)}{(m+2)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^(1 + m)/Sqrt[a + b\*x^2], x]

[Out] (x^(2 + m)\*Sqrt[1 + (b\*x^2)/a]\*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -((b\*x^2)/a)]/((2 + m)\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 8.46599, size = 49, normalized size = 0.98

$$\frac{x^{m+2}\sqrt{a+bx^2} {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + 1; \frac{m}{2} + 2; -\frac{bx^2}{a}\right)}{a\sqrt{1 + \frac{bx^2}{a}}(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(1+m)/(b\*x\*\*2+a)\*\*(1/2), x)

[Out] x\*\*(m + 2)\*sqrt(a + b\*x\*\*2)\*hyper((1/2, m/2 + 1), (m/2 + 2, ), -b\*x\*\*2/a)/(a\*sqrt(1 + b\*x\*\*2/a)\*(m + 2))

**Mathematica [A]** time = 0.0488652, size = 63, normalized size = 1.26

$$\frac{x^{m+2}\sqrt{\frac{bx^2}{a} + 1} {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + 1; \frac{m}{2} + 2; -\frac{bx^2}{a}\right)}{(m+2)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(1 + m)/Sqrt[a + b\*x^2], x]

[Out] (x^(2 + m)\*Sqrt[1 + (b\*x^2)/a]\*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, -((b\*x^2)/a)]/((2 + m)\*Sqrt[a + b\*x^2])

**Maple [F]** time = 0.029, size = 0, normalized size = 0.

$$\int x^{1+m} \frac{1}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1+m)/(b*x^2+a)^(1/2),x)`

[Out] `int(x^(1+m)/(b*x^2+a)^(1/2),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m+1}}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(m + 1)/sqrt(b*x^2 + a),x, algorithm="maxima")`

[Out] `integrate(x^(m + 1)/sqrt(b*x^2 + a), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{m+1}}{\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(m + 1)/sqrt(b*x^2 + a),x, algorithm="fricas")`

[Out] `integral(x^(m + 1)/sqrt(b*x^2 + a), x)`

**Sympy** [A] time = 8.28546, size = 48, normalized size = 0.96

$$\frac{x^2 x^m \left(\frac{m}{2} + 1\right) {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + 1 \mid \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt{a} \left(\frac{m}{2} + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1+m)/(b*x**2+a)**(1/2),x)`

[Out] `x**2*x**m*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2, ), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 2))`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m+1}}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(m + 1)/sqrt(b*x^2 + a),x, algorithm="giac")`

[Out] `integrate(x^(m + 1)/sqrt(b*x^2 + a), x)`

$$3.658 \quad \int \frac{x^m}{\sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=50

$$\frac{x^{m+1}\sqrt{a+bx^2} {}_2F_1\left(1, \frac{m+2}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{a(m+1)}$$

[Out] (x^(1 + m)\*Sqrt[a + b\*x^2]\*Hypergeometric2F1[1, (2 + m)/2, (3 + m)/2, -(b\*x^2)/a])/(a\*(1 + m))

**Rubi [A]** time = 0.0636763, antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a} + 1} {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^m/Sqrt[a + b\*x^2], x]

[Out] (x^(1 + m)\*Sqrt[1 + (b\*x^2)/a]\*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(b\*x^2)/a])/((1 + m)\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 8.3375, size = 53, normalized size = 1.06

$$\frac{x^{m+1}\sqrt{a+bx^2} {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + \frac{1}{2}; \frac{m}{2} + \frac{3}{2}; -\frac{bx^2}{a}\right)}{a\sqrt{1 + \frac{bx^2}{a}}(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*m/(b\*x\*\*2+a)\*\*(1/2), x)

[Out] x\*\*(m + 1)\*sqrt(a + b\*x\*\*2)\*hyper((1/2, m/2 + 1/2), (m/2 + 3/2, ), -b\*x\*\*2/a)/(a\*sqrt(1 + b\*x\*\*2/a)\*(m + 1))

**Mathematica [A]** time = 0.0154907, size = 63, normalized size = 1.26

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a} + 1} {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{(m+1)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^m/Sqrt[a + b\*x^2], x]

[Out] (x^(1 + m)\*Sqrt[1 + (b\*x^2)/a]\*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(b\*x^2)/a])/((1 + m)\*Sqrt[a + b\*x^2])

**Maple [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \frac{1}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m/(b*x^2+a)^(1/2),x)`

[Out] `int(x^m/(b*x^2+a)^(1/2),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/sqrt(b*x^2 + a),x, algorithm="maxima")`

[Out] `integrate(x^m/sqrt(b*x^2 + a), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/sqrt(b*x^2 + a),x, algorithm="fricas")`

[Out] `integral(x^m/sqrt(b*x^2 + a), x)`

**Sympy** [A] time = 2.9253, size = 53, normalized size = 1.06

$$\frac{xx^m \left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + \frac{1}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt{a} \left(\frac{m}{2} + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m/(b*x**2+a)**(1/2),x)`

[Out] `x*x**m*gamma(m/2 + 1/2)*hyper((1/2, m/2 + 1/2), (m/2 + 3/2, ), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 3/2))`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m/sqrt(b*x^2 + a),x, algorithm="giac")`

[Out] `integrate(x^m/sqrt(b*x^2 + a), x)`

$$3.659 \quad \int \frac{x^{-1+m}}{\sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=46

$$\frac{x^m \sqrt{a+bx^2} {}_2F_1\left(1, \frac{m+1}{2}; \frac{m+2}{2}; -\frac{bx^2}{a}\right)}{am}$$

[Out] (x^m\*Sqrt[a + b\*x^2]\*Hypergeometric2F1[1, (1 + m)/2, (2 + m)/2, -((b\*x^2)/a)])/(a\*m)

**Rubi [A]** time = 0.0659517, antiderivative size = 57, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{x^m \sqrt{\frac{bx^2}{a} + 1} {}_2F_1\left(\frac{1}{2}, \frac{m}{2}; \frac{m+2}{2}; -\frac{bx^2}{a}\right)}{m\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + m)/Sqrt[a + b\*x^2], x]

[Out] (x^m\*Sqrt[1 + (b\*x^2)/a]\*Hypergeometric2F1[1/2, m/2, (2 + m)/2, -((b\*x^2)/a)])/(m\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 8.13028, size = 44, normalized size = 0.96

$$\frac{x^m \sqrt{a+bx^2} {}_2F_1\left(\frac{1}{2}, \frac{m}{2} \middle| \frac{m}{2} + 1; -\frac{bx^2}{a}\right)}{am\sqrt{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(-1+m)/(b\*x\*\*2+a)\*\*(1/2), x)

[Out] x\*\*m\*sqrt(a + b\*x\*\*2)\*hyper((1/2, m/2), (m/2 + 1, ), -b\*x\*\*2/a)/(a\*m\*sqrt(1 + b\*x\*\*2/a))

**Mathematica [B]** time = 0.103636, size = 105, normalized size = 2.28

$$\frac{x^m \sqrt{a+bx^2} \left( a(m+2) {}_2F_1\left(-\frac{1}{2}, \frac{m}{2}; \frac{m}{2} + 1; -\frac{bx^2}{a}\right) - bmx^2 {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + 1; \frac{m}{2} + 2; -\frac{bx^2}{a}\right) \right)}{a^2 m(m+2) \sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + m)/Sqrt[a + b\*x^2], x]

[Out] (x^m\*Sqrt[a + b\*x^2]\*(a\*(2 + m)\*Hypergeometric2F1[-1/2, m/2, 1 + m/2, -((b\*x^2)/a)] - b\*m\*x^2\*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, -((b\*x^2)/a)]))/(a^2\*m\*(2 + m)\*Sqrt[1 + (b\*x^2)/a])

**Maple [F]** time = 0.032, size = 0, normalized size = 0.

$$\int x^{-1+m} \frac{1}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-1+m)/(b*x^2+a)^(1/2),x)`

[Out] `int(x^(-1+m)/(b*x^2+a)^(1/2),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m-1}}{\sqrt{bx^2+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(m-1)/sqrt(b*x^2+a),x, algorithm="maxima")`

[Out] `integrate(x^(m-1)/sqrt(b*x^2+a),x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{m-1}}{\sqrt{bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(m-1)/sqrt(b*x^2+a),x, algorithm="fricas")`

[Out] `integral(x^(m-1)/sqrt(b*x^2+a),x)`

**Sympy** [A] time = 21.6267, size = 41, normalized size = 0.89

$$\frac{x^m \left(\frac{m}{2}\right) {}_2F_1\left(\frac{1}{2}, \frac{m}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt{a} \left(\frac{m}{2} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1+m)/(b*x**2+a)**(1/2),x)`

[Out] `x**m*gamma(m/2)*hyper((1/2, m/2), (m/2 + 1, ), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 1))`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m-1}}{\sqrt{bx^2+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(m-1)/sqrt(b*x^2+a),x, algorithm="giac")`

[Out] `integrate(x^(m-1)/sqrt(b*x^2+a),x)`

$$3.660 \quad \int \frac{x^{-2+m}}{\sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=51

$$-\frac{x^{m-1}\sqrt{a+bx^2} {}_2F_1\left(1, \frac{m}{2}; \frac{m+1}{2}; -\frac{bx^2}{a}\right)}{a(1-m)}$$

[Out]  $-\left(\left(x^{(-1+m)}\sqrt{a+bx^2}\right)\text{Hypergeometric2F1}\left[1, m/2, (1+m)/2, -\left(\frac{bx^2}{a}\right)\right]\right)/\left(a(1-m)\right)$

**Rubi [A]** time = 0.0715114, antiderivative size = 66, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$-\frac{x^{m-1}\sqrt{\frac{bx^2}{a}+1} {}_2F_1\left(\frac{1}{2}, \frac{m-1}{2}; \frac{m+1}{2}; -\frac{bx^2}{a}\right)}{(1-m)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^(-2 + m)/Sqrt[a + b\*x^2], x]

[Out]  $-\left(\left(x^{(-1+m)}\sqrt{1+\frac{bx^2}{a}}\right)\text{Hypergeometric2F1}\left[1/2, (-1+m)/2, (1+m)/2, -\left(\frac{bx^2}{a}\right)\right]\right)/\left((1-m)\sqrt{a+bx^2}\right)$

**Rubi in Sympy [A]** time = 8.68175, size = 54, normalized size = 1.06

$$-\frac{x^{m-1}\sqrt{a+bx^2} {}_2F_1\left(\frac{1}{2}, \frac{m}{2} - \frac{1}{2}; \frac{m}{2} + \frac{1}{2}; -\frac{bx^2}{a}\right)}{a\sqrt{1+\frac{bx^2}{a}}(-m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(-2+m)/(b\*x\*\*2+a)\*\*(1/2), x)

[Out]  $-x^{(m-1)}\sqrt{a+bx^2}\text{hyper}\left(\left(1/2, m/2 - 1/2\right), \left(m/2 + 1/2, \right), -bx^2/a\right)/\left(a\sqrt{1+bx^2/a}\right)(-m+1)$

**Mathematica [B]** time = 0.130774, size = 110, normalized size = 2.16

$$\frac{x^{m-1}\sqrt{a+bx^2}\left(a(m+1) {}_2F_1\left(-\frac{1}{2}, \frac{m-1}{2}; \frac{m+1}{2}; -\frac{bx^2}{a}\right) - b(m-1)x^2 {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)\right)}{a^2(m^2-1)\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-2 + m)/Sqrt[a + b\*x^2], x]

[Out]  $\left(x^{(-1+m)}\sqrt{a+bx^2}\right)\left(a(1+m)\text{Hypergeometric2F1}\left[-1/2, (-1+m)/2, (1+m)/2, -\left(\frac{bx^2}{a}\right)\right] - b(-1+m)x^2\text{Hypergeometric2F1}\left[1/2, (1+m)/2, (3+m)/2, -\left(\frac{bx^2}{a}\right)\right]\right)/\left(a^2(-1+m^2)\sqrt{1+\frac{bx^2}{a}}\right)$



**Maple [F]** time = 0.03, size = 0, normalized size = 0.

$$\int x^{-2+m} \frac{1}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-2+m)/(b\*x^2+a)^(1/2), x)

[Out] int(x^(-2+m)/(b\*x^2+a)^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m-2}}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(m - 2)/sqrt(b\*x^2 + a), x, algorithm="maxima")

[Out] integrate(x^(m - 2)/sqrt(b\*x^2 + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{m-2}}{\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(m - 2)/sqrt(b\*x^2 + a), x, algorithm="fricas")

[Out] integral(x^(m - 2)/sqrt(b\*x^2 + a), x)

**Sympy [A]** time = 157.304, size = 53, normalized size = 1.04

$$\frac{x^m \left(\frac{m}{2} - \frac{1}{2}\right) {}_2F_1\left(\frac{1}{2}, \frac{m}{2} - \frac{1}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt{ax} \left(\frac{m}{2} + \frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(-2+m)/(b\*x\*\*2+a)\*\*(1/2), x)

[Out] x\*\*m\*gamma(m/2 - 1/2)\*hyper((1/2, m/2 - 1/2), (m/2 + 1/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*sqrt(a)\*x\*gamma(m/2 + 1/2))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{m-2}}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(m - 2)/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)/sqrt(b*x^2 + a), x)
```

$$3.661 \quad \int \frac{x^{1+m}(a(2+m)+b(3+m)x^2)}{\sqrt{a+bx^2}} dx$$

**Optimal.** Leaf size=17

$$x^{m+2}\sqrt{a+bx^2}$$

[Out]  $x^{(2+m)}\sqrt{a+bx^2}$

**Rubi [A]** time = 0.0213, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$

$$x^{m+2}\sqrt{a+bx^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^{(1+m)}(a(2+m)+b(3+m)x^2))/\sqrt{a+bx^2}, x]$

[Out]  $x^{(2+m)}\sqrt{a+bx^2}$

**Rubi in Sympy [A]** time = 8.33823, size = 14, normalized size = 0.82

$$x^{m+2}\sqrt{a+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{(1+m)}(a(2+m)+b(3+m)x^2)/(b*x^2+a)^{(1/2)}, x)$

[Out]  $x^{(m+2)}\sqrt{a+bx^2}$

**Mathematica [C]** time = 0.152603, size = 97, normalized size = 5.71

$$\frac{x^{m+2}\sqrt{a+bx^2} \left( (m+3) {}_2F_1\left(-\frac{1}{2}, \frac{m}{2}+1; \frac{m}{2}+2; -\frac{bx^2}{a}\right) - {}_2F_1\left(\frac{1}{2}, \frac{m}{2}+1; \frac{m}{2}+2; -\frac{bx^2}{a}\right) \right)}{(m+2)\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x^{(1+m)}(a(2+m)+b(3+m)x^2))/\sqrt{a+bx^2}, x]$

[Out]  $(x^{(2+m)}\sqrt{a+bx^2}) \left( (3+m) \text{Hypergeometric2F1}\left[-\frac{1}{2}, 1+\frac{m}{2}, 2+\frac{m}{2}, -\frac{(bx^2)}{a}\right] - \text{Hypergeometric2F1}\left[\frac{1}{2}, 1+\frac{m}{2}, 2+\frac{m}{2}, -\frac{(bx^2)}{a}\right] \right) / ((2+m)\sqrt{1+\frac{(bx^2)}{a}})$

**Maple [A]** time = 0.015, size = 16, normalized size = 0.9

$$x^{2+m}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^{(1+m)}(a(2+m)+b(3+m)x^2)/(b*x^2+a)^{(1/2)}, x)$

[Out]  $x^{(2+m)} \cdot (b \cdot x^2 + a)^{(1/2)}$

**Maxima [A]** time = 1.49765, size = 22, normalized size = 1.29

$$\sqrt{bx^2 + ax^2} x^m$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*(m+3)*x^2 + a*(m+2))*x^(m+1)/sqrt(b*x^2 + a),x, algorithm="maxima")`

[Out] `sqrt(b*x^2 + a)*x^2*x^m`

**Fricas [A]** time = 0.237351, size = 22, normalized size = 1.29

$$\sqrt{bx^2 + ax} x^{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*(m+3)*x^2 + a*(m+2))*x^(m+1)/sqrt(b*x^2 + a),x, algorithm="fricas")`

[Out] `sqrt(b*x^2 + a)*x*x^(m+1)`

**Sympy [A]** time = 68.0022, size = 202, normalized size = 11.88

$$\frac{\sqrt{am} x^m \left(\frac{m}{2} + 1\right) {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + 1 \mid \frac{bx^2 e^{i\pi}}{a}\right)}{2\left(\frac{m}{2} + 2\right)} + \frac{\sqrt{ax^2} x^m \left(\frac{m}{2} + 1\right) {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + 1 \mid \frac{bx^2 e^{i\pi}}{a}\right)}{\left(\frac{m}{2} + 2\right)} \\ + \frac{bm x^4 x^m \left(\frac{m}{2} + 2\right) {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + 2 \mid \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt{a}\left(\frac{m}{2} + 3\right)} + \frac{3bx^4 x^m \left(\frac{m}{2} + 2\right) {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + 2 \mid \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt{a}\left(\frac{m}{2} + 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x** (1+m) * (a * (2+m)+b * (3+m) * x** 2)/(b * x** 2+a)** (1/2), x)`

[Out] `sqrt(a)*m*x**2*x**m*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2, ), b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 2)) + sqrt(a)*x**2*x**m*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2, ), b*x**2*exp_polar(I*pi)/a)/gamma(m/2 + 2) + b*m*x**4*x**m*gamma(m/2 + 2)*hyper((1/2, m/2 + 2), (m/2 + 3, ), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 3)) + 3*b*x**4*x**m*gamma(m/2 + 2)*hyper((1/2, m/2 + 2), (m/2 + 3, ), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 3))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b(m+3)x^2 + a(m+2))x^{m+1}}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*(m+3)*x^2 + a*(m+2))*x^(m+1)/sqrt(b*x^2 + a),x, algorithm="giac")`

```
[Out] integrate((b*(m + 3)*x^2 + a*(m + 2))*x^(m + 1)/sqrt(b*x^2 + a),  
x)
```

$$3.662 \quad \int \left( \frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}} + \frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}} \right) dx$$

**Optimal.** Leaf size=17

$$x^{m+2}\sqrt{a+bx^2}$$

[Out]  $x^{(2+m)}\sqrt{a+bx^2}$

**Rubi [C]** time = 0.188891, antiderivative size = 127, normalized size of antiderivative = 7.47, number of steps used = 5, number of rules used = 2, integrand size = 43,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$

$$\frac{ax^{m+2}\sqrt{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, -\frac{bx^2}{a}\right)}{\sqrt{a+bx^2}} + \frac{b(m+3)x^{m+4}\sqrt{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{2}, \frac{m+4}{2}, \frac{m+6}{2}, -\frac{bx^2}{a}\right)}{(m+4)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*(2+m)*x^{(1+m)})/\sqrt{a+bx^2} + (b*(3+m)*x^{(3+m)})/\sqrt{a+bx^2}]$

[Out]  $(a*x^{(2+m)}*\sqrt{1+(b*x^2)/a}*\text{Hypergeometric2F1}[1/2, (2+m)/2, (4+m)/2, -((b*x^2)/a)]/\sqrt{a+bx^2} + (b*(3+m)*x^{(4+m)}*\sqrt{1+(b*x^2)/a}*\text{Hypergeometric2F1}[1/2, (4+m)/2, (6+m)/2, -((b*x^2)/a)]/((4+m)*\sqrt{a+bx^2})$

**Rubi in Sympy [A]** time = 19.3719, size = 100, normalized size = 5.88

$$\frac{x^{m+2}\sqrt{a+bx^2} {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + 1, \frac{m}{2} + 2, -\frac{bx^2}{a}\right)}{\sqrt{1 + \frac{bx^2}{a}}} + \frac{bx^{m+4}\sqrt{a+bx^2} (m+3) {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + 2, \frac{m}{2} + 3, -\frac{bx^2}{a}\right)}{a\sqrt{1 + \frac{bx^2}{a}} (m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(a*(2+m)*x^{(1+m)}/(b*x^{(2+a)})^{(1/2)}+b*(3+m)*x^{(3+m)}/(b*x^{(2+a)})^{(1/2)})$

[Out]  $x^{(m+2)}*\sqrt{a+bx^2}*\text{hyper}((1/2, m/2 + 1), (m/2 + 2, ), -b*x^{(2+a)}/\sqrt{1+b*x^{(2/a)}}) + b*x^{(m+4)}*\sqrt{a+bx^2}*(m+3)*\text{hyper}((1/2, m/2 + 2), (m/2 + 3, ), -b*x^{(2/a)}/(a*\sqrt{1+b*x^{(2/a)}}))$

**Mathematica [C]** time = 0.0467591, size = 97, normalized size = 5.71

$$\frac{x^{m+2}\sqrt{a+bx^2} \left( (m+3) {}_2F_1\left(-\frac{1}{2}, \frac{m}{2} + 1; \frac{m}{2} + 2; -\frac{bx^2}{a}\right) - {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + 1; \frac{m}{2} + 2; -\frac{bx^2}{a}\right) \right)}{(m+2)\sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a*(2+m)*x^{(1+m)})/\sqrt{a+bx^2} + (b*(3+m)*x^{(3+m)})/\sqrt{a+bx^2}]$

[Out]  $(x^{(2+m)}*\sqrt{a+bx^2}*((3+m)*\text{Hypergeometric2F1}[-1/2, 1+m/2, 2+m/2, -((b*x^2)/a)] - \text{Hypergeometric2F1}[1/2, 1+m/2, 2+m/2, -((b*x^2)/a)])/((2+m)*\sqrt{1+(b*x^2)/a})$

**Maple [F]** time = 0.041, size = 0, normalized size = 0.

$$\int a(2+m)x^{1+m}\frac{1}{\sqrt{bx^2+a}} + b(3+m)x^{3+m}\frac{1}{\sqrt{bx^2+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a\*(2+m)\*x^(1+m)/(b\*x^2+a)^(1/2)+b\*(3+m)\*x^(3+m)/(b\*x^2+a)^(1/2),x)

[Out] int(a\*(2+m)\*x^(1+m)/(b\*x^2+a)^(1/2)+b\*(3+m)\*x^(3+m)/(b\*x^2+a)^(1/2),x)

**Maxima [A]** time = 1.49329, size = 22, normalized size = 1.29

$$\sqrt{bx^2+ax^2}x^m$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b\*(m+3)\*x^(m+3)/sqrt(b\*x^2+a)+a\*(m+2)\*x^(m+1)/sqrt(b\*x^2+a)

[Out] sqrt(b\*x^2+a)\*x^2\*x^m

**Fricas [A]** time = 0.27008, size = 24, normalized size = 1.41

$$\frac{\sqrt{bx^2+ax^{m+3}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b\*(m+3)\*x^(m+3)/sqrt(b\*x^2+a)+a\*(m+2)\*x^(m+1)/sqrt(b\*x^2+a)

[Out] sqrt(b\*x^2+a)\*x^(m+3)/x

**Sympy [A]** time = 49.1256, size = 105, normalized size = 6.18

$$\frac{\sqrt{ax^2}x^m(m+2)\left(\frac{m}{2}+1\right) {}_2F_1\left(\frac{1}{2}, \frac{m}{2}+1 \mid \frac{bx^2e^{i\pi}}{a}\right)}{2\left(\frac{m}{2}+2\right)} + \frac{bx^4x^m(m+3)\left(\frac{m}{2}+2\right) {}_2F_1\left(\frac{1}{2}, \frac{m}{2}+2 \mid \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\left(\frac{m}{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a\*(2+m)\*x\*\*(1+m)/(b\*x\*\*2+a)\*\*(1/2)+b\*(3+m)\*x\*\*(3+m)/(b\*x\*\*2+a)\*\*(1/2)

[Out] sqrt(a)\*x\*\*2\*x\*\*m\*(m+2)\*gamma(m/2+1)\*hyper((1/2, m/2+1), (m/2+2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*gamma(m/2+2))+b\*x\*\*4\*x\*\*m\*(m+3)\*gamma(m/2+2)\*hyper((1/2, m/2+2), (m/2+3, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*sqrt(a)\*gamma(m/2+3))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b(m+3)x^{m+3}}{\sqrt{bx^2+a}} + \frac{a(m+2)x^{m+1}}{\sqrt{bx^2+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*(m + 3)*x^(m + 3)/sqrt(b*x^2 + a) + a*(m + 2)*x^(m + 1)/sqrt(b*x^2 + a)
```

```
[Out] integrate(b*(m + 3)*x^(m + 3)/sqrt(b*x^2 + a) + a*(m + 2)*x^(m + 1)/sqrt(b*x^2 + a), x)
```



$$3.663 \quad \int \frac{x^{-1+m}(am+b(-1+m)x^2)}{(a+bx^2)^{3/2}} dx$$

**Optimal.** Leaf size=15

$$\frac{x^m}{\sqrt{a+bx^2}}$$

[Out] x^m/Sqrt[a + b\*x^2]

**Rubi [A]** time = 0.0200751, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$

$$\frac{x^m}{\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + m)\*(a\*m + b\*(-1 + m)\*x^2))/(a + b\*x^2)^(3/2), x]

[Out] x^m/Sqrt[a + b\*x^2]

**Rubi in Sympy [A]** time = 7.75963, size = 12, normalized size = 0.8

$$\frac{x^m}{\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(-1+m)\*(a\*m+b\*(-1+m)\*x\*\*2)/(b\*x\*\*2+a)\*\*(3/2), x)

[Out] x\*\*m/sqrt(a + b\*x\*\*2)

**Mathematica [C]** time = 0.182035, size = 131, normalized size = 8.73

$$\frac{x^m \sqrt{a+bx^2} \left( a(m+2) {}_2F_1 \left( -\frac{1}{2}, \frac{m}{2}, \frac{m}{2} + 1; -\frac{bx^2}{a} \right) - bx^2 \left( m {}_2F_1 \left( \frac{1}{2}, \frac{m}{2} + 1; \frac{m}{2} + 2; -\frac{bx^2}{a} \right) + {}_2F_1 \left( \frac{3}{2}, \frac{m}{2} + 1; \frac{m}{2} + 2; -\frac{bx^2}{a} \right) \right) \right)}{a^2(m+2)\sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1 + m)\*(a\*m + b\*(-1 + m)\*x^2))/(a + b\*x^2)^(3/2), x]

[Out] (x^m\*Sqrt[a + b\*x^2]\*(a\*(2 + m)\*Hypergeometric2F1[-1/2, m/2, 1 + m/2, -(b\*x^2)/a]) - b\*x^2\*(m\*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, -(b\*x^2)/a]) + Hypergeometric2F1[3/2, 1 + m/2, 2 + m/2, -(b\*x^2)/a]))/(a^2\*(2 + m)\*Sqrt[1 + (b\*x^2)/a])

**Maple [A]** time = 0.01, size = 14, normalized size = 0.9

$$x^m \frac{1}{\sqrt{bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-1+m)*(a*m+b*(-1+m)*x^2)/(b*x^2+a)^(3/2),x)`

[Out] `x^m/(b*x^2+a)^(1/2)`

**Maxima [A]** time = 1.53085, size = 18, normalized size = 1.2

$$\frac{x^m}{\sqrt{bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*(m-1)*x^2+a*m)*x^(m-1)/(b*x^2+a)^(3/2),x,algorithm="maxima")`

[Out] `x^m/sqrt(b*x^2+a)`

**Fricas [A]** time = 0.227398, size = 22, normalized size = 1.47

$$\frac{xx^{m-1}}{\sqrt{bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*(m-1)*x^2+a*m)*x^(m-1)/(b*x^2+a)^(3/2),x,algorithm="fricas")`

[Out] `x*x^(m-1)/sqrt(b*x^2+a)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1+m)*(a*m+b*(-1+m)*x**2)/(b*x**2+a)**(3/2),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b(m-1)x^2 + am)x^{m-1}}{(bx^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*(m-1)*x^2+a*m)*x^(m-1)/(b*x^2+a)^(3/2),x,algorithm="giac")`

[Out] `integrate((b*(m-1)*x^2+a*m)*x^(m-1)/(b*x^2+a)^(3/2),x)`

$$3.664 \quad \int \left( -\frac{bx^{1+m}}{(a+bx^2)^{3/2}} + \frac{mx^{-1+m}}{\sqrt{a+bx^2}} \right) dx$$

**Optimal.** Leaf size=15

$$\frac{x^m}{\sqrt{a+bx^2}}$$

[Out] x^m/Sqrt[a + b\*x^2]

**Rubi [C]** time = 0.180465, antiderivative size = 123, normalized size of antiderivative = 8.2, number of steps used = 5, number of rules used = 2, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{x^m \sqrt{\frac{bx^2}{a} + 1} {}_2F_1\left(\frac{1}{2}, \frac{m}{2}; \frac{m+2}{2}; -\frac{bx^2}{a}\right)}{\sqrt{a+bx^2}} - \frac{bx^{m+2} \sqrt{\frac{bx^2}{a} + 1} {}_2F_1\left(\frac{3}{2}, \frac{m+2}{2}; \frac{m+4}{2}; -\frac{bx^2}{a}\right)}{a(m+2)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[-((b\*x^(1+m))/(a+b\*x^2)^(3/2)) + (m\*x^(-1+m))/Sqrt[a+b\*x^2], x]

[Out] (x^m\*Sqrt[1 + (b\*x^2)/a]\*Hypergeometric2F1[1/2, m/2, (2 + m)/2, -(b\*x^2)/a])/Sqrt[a + b\*x^2] - (b\*x^(2 + m)\*Sqrt[1 + (b\*x^2)/a]\*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, -(b\*x^2)/a])/(a\*(2 + m)\*Sqrt[a + b\*x^2])

**Rubi in Sympy [A]** time = 17.8506, size = 97, normalized size = 6.47

$$\frac{x^m \sqrt{a+bx^2} {}_2F_1\left(\frac{1}{2}, \frac{m}{2} \middle| -\frac{bx^2}{a}\right)}{a\sqrt{1+\frac{bx^2}{a}}} - \frac{bx^{m+2} \sqrt{a+bx^2} {}_2F_1\left(\frac{3}{2}, \frac{m}{2} + 1 \middle| -\frac{bx^2}{a}\right)}{a^2\sqrt{1+\frac{bx^2}{a}}(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(-b\*x\*\*(1+m)/(b\*x\*\*2+a)\*\*(3/2)+m\*x\*\*(-1+m)/(b\*x\*\*2+a)\*\*(1/2), x)

[Out] x\*\*m\*sqrt(a + b\*x\*\*2)\*hyper((1/2, m/2), (m/2 + 1, ), -b\*x\*\*2/a)/(a\*sqrt(1 + b\*x\*\*2/a)) - b\*x\*\*(m + 2)\*sqrt(a + b\*x\*\*2)\*hyper((3/2, m/2 + 1), (m/2 + 2, ), -b\*x\*\*2/a)/(a\*\*2\*sqrt(1 + b\*x\*\*2/a)\*(m + 2))

**Mathematica [C]** time = 0.0546339, size = 131, normalized size = 8.73

$$\frac{x^m \sqrt{a+bx^2} \left( a(m+2) {}_2F_1\left(-\frac{1}{2}, \frac{m}{2}; \frac{m}{2} + 1; -\frac{bx^2}{a}\right) - bx^2 \left( m {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + 1; \frac{m}{2} + 2; -\frac{bx^2}{a}\right) + {}_2F_1\left(\frac{3}{2}, \frac{m}{2} + 1; \frac{m}{2} + 2; -\frac{bx^2}{a}\right) \right) \right)}{a^2(m+2)\sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[-((b\*x^(1+m))/(a+b\*x^2)^(3/2)) + (m\*x^(-1+m))/Sqrt[a+b\*x^2], x]

[Out] (x^m\*Sqrt[a + b\*x^2]\*(a\*(2 + m)\*Hypergeometric2F1[-1/2, m/2, 1 + m/2, -(b\*x^2)/a]) - b\*x^2\*(m\*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, -(b\*x^2)/a]) + Hypergeometric2F1[3/2, 1 + m/2, 2 + m/2, -(b\*x^2)/a]))/(a^2\*(2 + m)\*Sqrt[1 + (b\*x^2)/a])

---

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int -bx^{1+m} (bx^2 + a)^{-\frac{3}{2}} + mx^{-1+m} \frac{1}{\sqrt{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-b\*x^(1+m)/(b\*x^2+a)^(3/2)+m\*x^(-1+m)/(b\*x^2+a)^(1/2), x)

[Out] int(-b\*x^(1+m)/(b\*x^2+a)^(3/2)+m\*x^(-1+m)/(b\*x^2+a)^(1/2), x)

---

**Maxima [A]** time = 1.49654, size = 18, normalized size = 1.2

$$\frac{x^m}{\sqrt{bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(m\*x^(m - 1)/sqrt(b\*x^2 + a) - b\*x^(m + 1)/(b\*x^2 + a)^(3/2), x, algorithm

[Out] x^m/sqrt(b\*x^2 + a)

---

**Fricas [A]** time = 0.226878, size = 35, normalized size = 2.33

$$\frac{\sqrt{bx^2 + a}x^{m+1}}{bx^3 + ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(m\*x^(m - 1)/sqrt(b\*x^2 + a) - b\*x^(m + 1)/(b\*x^2 + a)^(3/2), x, algorithm

[Out] sqrt(b\*x^2 + a)\*x^(m + 1)/(b\*x^3 + a\*x)

---

**Sympy [A]** time = 53.0037, size = 94, normalized size = 6.27

$$\frac{mx^m \left(\frac{m}{2}\right) {}_2F_1\left(\frac{1}{2}, \frac{m}{2} \left| \frac{bx^2 e^{i\pi}}{a} \right. \right)}{2\sqrt{a} \left(\frac{m}{2} + 1\right)} - \frac{bx^2 x^m \left(\frac{m}{2} + 1\right) {}_2F_1\left(\frac{3}{2}, \frac{m}{2} + 1 \left| \frac{bx^2 e^{i\pi}}{a} \right. \right)}{2a^{\frac{3}{2}} \left(\frac{m}{2} + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-b\*x\*\*(1+m)/(b\*x\*\*2+a)\*\*(3/2)+m\*x\*\*(-1+m)/(b\*x\*\*2+a)\*\*(1/2), x)

[Out] m\*x\*\*m\*gamma(m/2)\*hyper((1/2, m/2), (m/2 + 1, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*sqrt(a)\*gamma(m/2 + 1)) - b\*x\*\*2\*x\*\*m\*gamma(m/2 + 1)\*hyper((3/2, m/2 + 1), (m/2 + 2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*a\*\* (3/2)\*gamma(m/2 + 2))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{mx^{m-1}}{\sqrt{bx^2 + a}} - \frac{bx^{m+1}}{(bx^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(m*x^(m - 1)/sqrt(b*x^2 + a) - b*x^(m + 1)/(b*x^2 + a)^(3/2), x, algorithm="default")
```

```
[Out] integrate(m*x^(m - 1)/sqrt(b*x^2 + a) - b*x^(m + 1)/(b*x^2 + a)^(3/2), x)
```

### 3.665 $\int x^7 \sqrt[3]{a + bx^2} dx$

**Optimal.** Leaf size=80

$$-\frac{3a^3 (a + bx^2)^{4/3}}{8b^4} + \frac{9a^2 (a + bx^2)^{7/3}}{14b^4} + \frac{3 (a + bx^2)^{13/3}}{26b^4} - \frac{9a (a + bx^2)^{10/3}}{20b^4}$$

[Out]  $(-3*a^3*(a + b*x^2)^(4/3))/(8*b^4) + (9*a^2*(a + b*x^2)^(7/3))/(14*b^4) - (9*a*(a + b*x^2)^(10/3))/(20*b^4) + (3*(a + b*x^2)^(13/3))/(26*b^4)$

**Rubi [A]** time = 0.124718, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{3a^3 (a + bx^2)^{4/3}}{8b^4} + \frac{9a^2 (a + bx^2)^{7/3}}{14b^4} + \frac{3 (a + bx^2)^{13/3}}{26b^4} - \frac{9a (a + bx^2)^{10/3}}{20b^4}$$

Antiderivative was successfully verified.

[In] Int[x^7\*(a + b\*x^2)^(1/3), x]

[Out]  $(-3*a^3*(a + b*x^2)^(4/3))/(8*b^4) + (9*a^2*(a + b*x^2)^(7/3))/(14*b^4) - (9*a*(a + b*x^2)^(10/3))/(20*b^4) + (3*(a + b*x^2)^(13/3))/(26*b^4)$

**Rubi in Sympy [A]** time = 15.3545, size = 75, normalized size = 0.94

$$-\frac{3a^3 (a + bx^2)^{4/3}}{8b^4} + \frac{9a^2 (a + bx^2)^{7/3}}{14b^4} - \frac{9a (a + bx^2)^{10/3}}{20b^4} + \frac{3 (a + bx^2)^{13/3}}{26b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*7\*(b\*x\*\*2+a)\*\*(1/3), x)

[Out]  $-3*a**3*(a + b*x**2)**(4/3)/(8*b**4) + 9*a**2*(a + b*x**2)**(7/3)/(14*b**4) - 9*a*(a + b*x**2)**(10/3)/(20*b**4) + 3*(a + b*x**2)**(13/3)/(26*b**4)$

**Mathematica [A]** time = 0.0279051, size = 61, normalized size = 0.76

$$\frac{3\sqrt[3]{a + bx^2} (-81a^4 + 27a^3bx^2 - 18a^2b^2x^4 + 14ab^3x^6 + 140b^4x^8)}{3640b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^7\*(a + b\*x^2)^(1/3), x]

[Out]  $(3*(a + b*x^2)^(1/3)*(-81*a^4 + 27*a^3*b*x^2 - 18*a^2*b^2*x^4 + 14*a*b^3*x^6 + 140*b^4*x^8))/(3640*b^4)$

**Maple [A]** time = 0.008, size = 47, normalized size = 0.6

$$-\frac{-420b^3x^6 + 378ab^2x^4 - 324a^2bx^2 + 243a^3}{3640b^4} (bx^2 + a)^{4/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(b*x^2+a)^(1/3),x)`

[Out] 
$$-3/3640*(b*x^2+a)^{(4/3)}*(-140*b^3*x^6+126*a*b^2*x^4-108*a^2*b*x^2+81*a^3)/b^4$$

**Maxima [A]** time = 1.34985, size = 86, normalized size = 1.08

$$\frac{3(bx^2+a)^{\frac{13}{3}}}{26b^4} - \frac{9(bx^2+a)^{\frac{10}{3}}a}{20b^4} + \frac{9(bx^2+a)^{\frac{7}{3}}a^2}{14b^4} - \frac{3(bx^2+a)^{\frac{4}{3}}a^3}{8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*x^7,x, algorithm="maxima")`

[Out] 
$$3/26*(b*x^2 + a)^{(13/3)}/b^4 - 9/20*(b*x^2 + a)^{(10/3)}*a/b^4 + 9/14*(b*x^2 + a)^{(7/3)}*a^2/b^4 - 3/8*(b*x^2 + a)^{(4/3)}*a^3/b^4$$

**Fricas [A]** time = 0.204991, size = 77, normalized size = 0.96

$$\frac{3(140b^4x^8 + 14ab^3x^6 - 18a^2b^2x^4 + 27a^3bx^2 - 81a^4)(bx^2 + a)^{\frac{1}{3}}}{3640b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*x^7,x, algorithm="fricas")`

[Out] 
$$3/3640*(140*b^4*x^8 + 14*a*b^3*x^6 - 18*a^2*b^2*x^4 + 27*a^3*b*x^2 - 81*a^4)*(b*x^2 + a)^{(1/3)}/b^4$$

**Sympy [A]** time = 9.7601, size = 1795, normalized size = 22.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(b*x**2+a)**(1/3),x)`

[Out] 
$$\begin{aligned} & -243*a**(73/3)*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) \\ & + 243*a**(73/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) - 1377*a** \\ & *(70/3)*b*x**2*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) \\ & - 3213*a**(67/3)*b**2*x**4*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) \\ & + 3645*a**(67/3)*b**2*x**4/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) \\ & - 3927*a**(64/3)*b**3*x**6*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) \end{aligned}$$

```

2800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x
**10 + 3640*a**14*b**10*x**12) + 4860*a**(64/3)*b**3*x**6/(3640*a
**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800
*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10
+ 3640*a**14*b**10*x**12) - 2163*a**(61/3)*b**4*x**8*(1 + b*x**2
/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18
*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 2184
0*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 3645*a**(61/3)*b**
4*x**8/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**
6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a
**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 1827*a**(58/3)*b**5*x
**10*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**
2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b
**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 145
8*a**(58/3)*b**5*x**10/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 +
54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8
*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 6573*a
**(55/3)*b**6*x**12*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 2184
0*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6
+ 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**
10*x**12) + 243*a**(55/3)*b**6*x**12/(3640*a**20*b**4 + 21840*a
**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 5
4600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*
x**12) + 8787*a**(52/3)*b**7*x**14*(1 + b*x**2/a)**(1/3)/(3640*a
**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*
a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10
+ 3640*a**14*b**10*x**12) + 6498*a**(49/3)*b**8*x**16*(1 + b*x**2
/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18
*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 2184
0*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 2562*a**(46/3)*b**
9*x**18*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5
*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**
16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) +
420*a**(43/3)*b**10*x**20*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4
+ 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b
**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a
**14*b**10*x**12)

```

**GIAC/XCAS [A]** time = 0.217679, size = 77, normalized size = 0.96

$$\frac{3 \left( 140 (bx^2 + a)^{\frac{13}{3}} - 546 (bx^2 + a)^{\frac{10}{3}} a + 780 (bx^2 + a)^{\frac{7}{3}} a^2 - 455 (bx^2 + a)^{\frac{4}{3}} a^3 \right)}{3640 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)\*x^7,x, algorithm="giac")

[Out] 3/3640\*(140\*(b\*x^2 + a)^(13/3) - 546\*(b\*x^2 + a)^(10/3)\*a + 780\*(b\*x^2 + a)^(7/3)\*a^2 - 455\*(b\*x^2 + a)^(4/3)\*a^3)/b^4



$$3.666 \quad \int x^5 \sqrt[3]{a + bx^2} dx$$

**Optimal.** Leaf size=59

$$\frac{3a^2 (a + bx^2)^{4/3}}{8b^3} + \frac{3 (a + bx^2)^{10/3}}{20b^3} - \frac{3a (a + bx^2)^{7/3}}{7b^3}$$

[Out]  $(3*a^2*(a + b*x^2)^(4/3))/(8*b^3) - (3*a*(a + b*x^2)^(7/3))/(7*b^3) + (3*(a + b*x^2)^(10/3))/(20*b^3)$

**Rubi [A]** time = 0.0959325, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{3a^2 (a + bx^2)^{4/3}}{8b^3} + \frac{3 (a + bx^2)^{10/3}}{20b^3} - \frac{3a (a + bx^2)^{7/3}}{7b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(a + b\*x^2)^(1/3), x]

[Out]  $(3*a^2*(a + b*x^2)^(4/3))/(8*b^3) - (3*a*(a + b*x^2)^(7/3))/(7*b^3) + (3*(a + b*x^2)^(10/3))/(20*b^3)$

**Rubi in Sympy [A]** time = 11.5431, size = 54, normalized size = 0.92

$$\frac{3a^2 (a + bx^2)^{4/3}}{8b^3} - \frac{3a (a + bx^2)^{7/3}}{7b^3} + \frac{3 (a + bx^2)^{10/3}}{20b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(b\*x\*\*2+a)\*\*(1/3), x)

[Out]  $3*a**2*(a + b*x**2)**(4/3)/(8*b**3) - 3*a*(a + b*x**2)**(7/3)/(7*b**3) + 3*(a + b*x**2)**(10/3)/(20*b**3)$

**Mathematica [A]** time = 0.0236736, size = 50, normalized size = 0.85

$$\frac{\sqrt[3]{a + bx^2} (9a^3 - 3a^2bx^2 + 2ab^2x^4 + 14b^3x^6)}{280b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*x^2)^(1/3), x]

[Out]  $(3*(a + b*x^2)^(1/3)*(9*a^3 - 3*a^2*b*x^2 + 2*a*b^2*x^4 + 14*b^3*x^6))/(280*b^3)$

**Maple [A]** time = 0.008, size = 36, normalized size = 0.6

$$\frac{42b^2x^4 - 36abx^2 + 27a^2}{280b^3} (bx^2 + a)^{4/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(b*x^2+a)^(1/3),x)`

[Out]  $3/280*(b*x^2+a)^{4/3}*(14*b^2*x^4-12*a*b*x^2+9*a^2)/b^3$

**Maxima [A]** time = 1.35652, size = 63, normalized size = 1.07

$$\frac{3(bx^2+a)^{\frac{10}{3}}}{20b^3} - \frac{3(bx^2+a)^{\frac{7}{3}}a}{7b^3} + \frac{3(bx^2+a)^{\frac{4}{3}}a^2}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*x^5,x, algorithm="maxima")`

[Out]  $3/20*(b*x^2 + a)^{10/3}/b^3 - 3/7*(b*x^2 + a)^{7/3}*a/b^3 + 3/8*(b*x^2 + a)^{4/3}*a^2/b^3$

**Fricas [A]** time = 0.203324, size = 62, normalized size = 1.05

$$\frac{3(14b^3x^6 + 2ab^2x^4 - 3a^2bx^2 + 9a^3)(bx^2 + a)^{\frac{1}{3}}}{280b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*x^5,x, algorithm="fricas")`

[Out]  $3/280*(14*b^3*x^6 + 2*a*b^2*x^4 - 3*a^2*b*x^2 + 9*a^3)*(b*x^2 + a)^{1/3}/b^3$

**Sympy [A]** time = 6.42102, size = 700, normalized size = 11.86

$$\frac{27a^{\frac{34}{3}} \sqrt[3]{1 + \frac{bx^2}{a}}}{280a^8b^3 + 840a^7b^4x^2 + 840a^6b^5x^4 + 280a^5b^6x^6} - \frac{280a^8b^3 + 840a^7b^4x^2 + 840a^6b^5x^4 + 280a^5b^6x^6}{27a^{\frac{34}{3}}}$$

$$+ \frac{72a^{\frac{31}{3}} bx^2 \sqrt[3]{1 + \frac{bx^2}{a}}}{280a^8b^3 + 840a^7b^4x^2 + 840a^6b^5x^4 + 280a^5b^6x^6} - \frac{81a^{\frac{31}{3}} bx^2}{280a^8b^3 + 840a^7b^4x^2 + 840a^6b^5x^4 + 280a^5b^6x^6}$$

$$+ \frac{60a^{\frac{28}{3}} b^2x^4 \sqrt[3]{1 + \frac{bx^2}{a}}}{280a^8b^3 + 840a^7b^4x^2 + 840a^6b^5x^4 + 280a^5b^6x^6} - \frac{81a^{\frac{28}{3}} b^2x^4}{280a^8b^3 + 840a^7b^4x^2 + 840a^6b^5x^4 + 280a^5b^6x^6}$$

$$+ \frac{60a^{\frac{25}{3}} b^3x^6 \sqrt[3]{1 + \frac{bx^2}{a}}}{280a^8b^3 + 840a^7b^4x^2 + 840a^6b^5x^4 + 280a^5b^6x^6} - \frac{27a^{\frac{25}{3}} b^3x^6}{280a^8b^3 + 840a^7b^4x^2 + 840a^6b^5x^4 + 280a^5b^6x^6}$$

$$+ \frac{135a^{\frac{22}{3}} b^4x^8 \sqrt[3]{1 + \frac{bx^2}{a}}}{280a^8b^3 + 840a^7b^4x^2 + 840a^6b^5x^4 + 280a^5b^6x^6} - \frac{132a^{\frac{19}{3}} b^5x^{10} \sqrt[3]{1 + \frac{bx^2}{a}}}{280a^8b^3 + 840a^7b^4x^2 + 840a^6b^5x^4 + 280a^5b^6x^6}$$

$$+ \frac{42a^{\frac{16}{3}} b^6x^{12} \sqrt[3]{1 + \frac{bx^2}{a}}}{280a^8b^3 + 840a^7b^4x^2 + 840a^6b^5x^4 + 280a^5b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(b\*x\*\*2+a)\*\*(1/3),x)

[Out]  $27*a^{34/3}*(1 + b*x^2/a)^{1/3}/(280*a^8*b^3 + 840*a^7*b^4*x^2 + 840*a^6*b^5*x^4 + 280*a^5*b^6*x^6) - 27*a^{34/3}/(280*a^8*b^3 + 840*a^7*b^4*x^2 + 840*a^6*b^5*x^4 + 280*a^5*b^6*x^6) + 72*a^{31/3}*b*x^2*(1 + b*x^2/a)^{1/3}/(280*a^8*b^3 + 840*a^7*b^4*x^2 + 840*a^6*b^5*x^4 + 280*a^5*b^6*x^6) - 81*a^{31/3}*b*x^2/(280*a^8*b^3 + 840*a^7*b^4*x^2 + 840*a^6*b^5*x^4 + 280*a^5*b^6*x^6) + 60*a^{28/3}*b^2*x^4*(1 + b*x^2/a)^{1/3}/(280*a^8*b^3 + 840*a^7*b^4*x^2 + 840*a^6*b^5*x^4 + 280*a^5*b^6*x^6) - 81*a^{28/3}*b^2*x^4/(280*a^8*b^3 + 840*a^7*b^4*x^2 + 840*a^6*b^5*x^4 + 280*a^5*b^6*x^6) + 60*a^{25/3}*b^3*x^6*(1 + b*x^2/a)^{1/3}/(280*a^8*b^3 + 840*a^7*b^4*x^2 + 840*a^6*b^5*x^4 + 280*a^5*b^6*x^6) - 27*a^{25/3}*b^3*x^6/(280*a^8*b^3 + 840*a^7*b^4*x^2 + 840*a^6*b^5*x^4 + 280*a^5*b^6*x^6) + 135*a^{22/3}*b^4*x^8*(1 + b*x^2/a)^{1/3}/(280*a^8*b^3 + 840*a^7*b^4*x^2 + 840*a^6*b^5*x^4 + 280*a^5*b^6*x^6) + 132*a^{19/3}*b^5*x^{10}*(1 + b*x^2/a)^{1/3}/(280*a^8*b^3 + 840*a^7*b^4*x^2 + 840*a^6*b^5*x^4 + 280*a^5*b^6*x^6) + 42*a^{16/3}*b^6*x^{12}*(1 + b*x^2/a)^{1/3}/(280*a^8*b^3 + 840*a^7*b^4*x^2 + 840*a^6*b^5*x^4 + 280*a^5*b^6*x^6)$

**GIAC/XCAS [A]** time = 0.21708, size = 58, normalized size = 0.98

$$\frac{3 \left( 14 (bx^2 + a)^{\frac{10}{3}} - 40 (bx^2 + a)^{\frac{7}{3}} a + 35 (bx^2 + a)^{\frac{4}{3}} a^2 \right)}{280 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(1/3)*x^5,x, algorithm="giac")
```

```
[Out] 3/280*(14*(b*x^2 + a)^(10/3) - 40*(b*x^2 + a)^(7/3)*a + 35*(b*x^2 + a)^(4/3)*a^2)/b^3
```

$$3.667 \quad \int x^3 \sqrt[3]{a + bx^2} dx$$

**Optimal.** Leaf size=38

$$\frac{3(a + bx^2)^{7/3}}{14b^2} - \frac{3a(a + bx^2)^{4/3}}{8b^2}$$

[Out]  $(-3*a*(a + b*x^2)^(4/3))/(8*b^2) + (3*(a + b*x^2)^(7/3))/(14*b^2)$

**Rubi [A]** time = 0.067535, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{3(a + bx^2)^{7/3}}{14b^2} - \frac{3a(a + bx^2)^{4/3}}{8b^2}$$

Antiderivative was successfully verified.

[In] `Int[x^3*(a + b*x^2)^(1/3), x]`

[Out]  $(-3*a*(a + b*x^2)^(4/3))/(8*b^2) + (3*(a + b*x^2)^(7/3))/(14*b^2)$

**Rubi in Sympy [A]** time = 7.81525, size = 34, normalized size = 0.89

$$-\frac{3a(a + bx^2)^{4/3}}{8b^2} + \frac{3(a + bx^2)^{7/3}}{14b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3*(b*x**2+a)**(1/3), x)`

[Out]  $-3*a*(a + b*x**2)**(4/3)/(8*b**2) + 3*(a + b*x**2)**(7/3)/(14*b**2)$

**Mathematica [A]** time = 0.0191875, size = 38, normalized size = 1.

$$\frac{3\sqrt[3]{a + bx^2}(-3a^2 + abx^2 + 4b^2x^4)}{56b^2}$$

Antiderivative was successfully verified.

[In] `Integrate[x^3*(a + b*x^2)^(1/3), x]`

[Out]  $(3*(a + b*x^2)^(1/3)*(-3*a^2 + a*b*x^2 + 4*b^2*x^4))/(56*b^2)$

**Maple [A]** time = 0.007, size = 25, normalized size = 0.7

$$-\frac{-12bx^2 + 9a}{56b^2} (bx^2 + a)^{4/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(b*x^2+a)^(1/3), x)`

[Out]  $-3/56 * (b * x^2 + a)^{4/3} * (-4 * b * x^2 + 3 * a) / b^2$

**Maxima [A]** time = 1.39595, size = 41, normalized size = 1.08

$$\frac{3 (bx^2 + a)^{\frac{7}{3}}}{14 b^2} - \frac{3 (bx^2 + a)^{\frac{4}{3}} a}{8 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*x^3,x, algorithm="maxima")`

[Out]  $3/14 * (b * x^2 + a)^{7/3} / b^2 - 3/8 * (b * x^2 + a)^{4/3} * a / b^2$

**Fricas [A]** time = 0.203093, size = 46, normalized size = 1.21

$$\frac{3 (4 b^2 x^4 + a b x^2 - 3 a^2) (b x^2 + a)^{\frac{1}{3}}}{56 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*x^3,x, algorithm="fricas")`

[Out]  $3/56 * (4 * b^2 * x^4 + a * b * x^2 - 3 * a^2) * (b * x^2 + a)^{1/3} / b^2$

**Sympy [A]** time = 4.05133, size = 223, normalized size = 5.87

$$\begin{aligned} & -\frac{9 a^{\frac{13}{3}} \sqrt[3]{1 + \frac{b x^2}{a}}}{56 a^2 b^2 + 56 a b^3 x^2} + \frac{9 a^{\frac{13}{3}}}{56 a^2 b^2 + 56 a b^3 x^2} - \frac{6 a^{\frac{10}{3}} b x^2 \sqrt[3]{1 + \frac{b x^2}{a}}}{56 a^2 b^2 + 56 a b^3 x^2} \\ & + \frac{9 a^{\frac{10}{3}} b x^2}{56 a^2 b^2 + 56 a b^3 x^2} + \frac{15 a^{\frac{7}{3}} b^2 x^4 \sqrt[3]{1 + \frac{b x^2}{a}}}{56 a^2 b^2 + 56 a b^3 x^2} + \frac{12 a^{\frac{4}{3}} b^3 x^6 \sqrt[3]{1 + \frac{b x^2}{a}}}{56 a^2 b^2 + 56 a b^3 x^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**2+a)**(1/3),x)`

[Out]  $-9 * a^{13/3} * (1 + b * x^2 / a)^{1/3} / (56 * a^2 * b^2 + 56 * a * b^3 * x^2) + 9 * a^{13/3} / (56 * a^2 * b^2 + 56 * a * b^3 * x^2) - 6 * a^{10/3} * b * x^2 * (1 + b * x^2 / a)^{1/3} / (56 * a^2 * b^2 + 56 * a * b^3 * x^2) + 9 * a^{10/3} * b * x^2 / (56 * a^2 * b^2 + 56 * a * b^3 * x^2) + 15 * a^{7/3} * b^2 * x^4 * (1 + b * x^2 / a)^{1/3} / (56 * a^2 * b^2 + 56 * a * b^3 * x^2) + 12 * a^{4/3} * b^3 * x^6 * (1 + b * x^2 / a)^{1/3} / (56 * a^2 * b^2 + 56 * a * b^3 * x^2)$

**GIAC/XCAS [A]** time = 0.218817, size = 39, normalized size = 1.03

$$\frac{3 \left( 4 (bx^2 + a)^{\frac{7}{3}} - 7 (bx^2 + a)^{\frac{4}{3}} a \right)}{56 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*x^3,x, algorithm="giac")`

[Out]  $3/56 * (4 * (b * x^2 + a)^{7/3} - 7 * (b * x^2 + a)^{4/3} * a) / b^2$

$$3.668 \quad \int x \sqrt[3]{a + bx^2} dx$$

**Optimal.** Leaf size=18

$$\frac{3(a + bx^2)^{4/3}}{8b}$$

[Out]  $(3*(a + b*x^2)^(4/3))/(8*b)$

**Rubi [A]** time = 0.0115885, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{3(a + bx^2)^{4/3}}{8b}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*x^2)^(1/3), x]

[Out]  $(3*(a + b*x^2)^(4/3))/(8*b)$

**Rubi in Sympy [A]** time = 2.15678, size = 14, normalized size = 0.78

$$\frac{3(a + bx^2)^{\frac{4}{3}}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(b\*x\*\*2+a)\*\*(1/3), x)

[Out]  $3*(a + b*x**2)**(4/3)/(8*b)$

**Mathematica [A]** time = 0.00760632, size = 18, normalized size = 1.

$$\frac{3(a + bx^2)^{4/3}}{8b}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*x^2)^(1/3), x]

[Out]  $(3*(a + b*x^2)^(4/3))/(8*b)$

**Maple [A]** time = 0.004, size = 15, normalized size = 0.8

$$\frac{3}{8b} (bx^2 + a)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(b\*x^2+a)^(1/3), x)

[Out]  $3/8*(b*x^2+a)^(4/3)/b$

---

**Maxima [A]** time = 1.33752, size = 19, normalized size = 1.06

$$\frac{3 (bx^2 + a)^{\frac{4}{3}}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*x,x, algorithm="maxima")`

[Out] `3/8*(b*x^2 + a)^(4/3)/b`

---

**Fricas [A]** time = 0.203082, size = 19, normalized size = 1.06

$$\frac{3 (bx^2 + a)^{\frac{4}{3}}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*x,x, algorithm="fricas")`

[Out] `3/8*(b*x^2 + a)^(4/3)/b`

---

**Sympy [A]** time = 0.497763, size = 42, normalized size = 2.33

$$\begin{cases} \frac{3a\sqrt[3]{a+bx^2}}{8b} + \frac{3x^2\sqrt[3]{a+bx^2}}{8} & \text{for } b \neq 0 \\ \frac{\sqrt[3]{ax^2}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x**2+a)**(1/3),x)`

[Out] `Piecewise((3*a*(a + b*x**2)**(1/3)/(8*b) + 3*x**2*(a + b*x**2)**(1/3)/8, Ne(b, 0)), (a**(1/3)*x**2/2, True))`

---

**GIAC/XCAS [A]** time = 0.221284, size = 19, normalized size = 1.06

$$\frac{3 (bx^2 + a)^{\frac{4}{3}}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*x,x, algorithm="giac")`

[Out] `3/8*(b*x^2 + a)^(4/3)/b`



$$3.669 \quad \int \frac{\sqrt[3]{a+bx^2}}{x} dx$$

**Optimal.** Leaf size=101

$$\frac{3}{2}\sqrt[3]{a+bx^2} + \frac{3}{4}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right) - \frac{1}{2}\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right) - \frac{1}{2}\sqrt[3]{a}\log(x)$$

[Out] (3\*(a + b\*x^2)^(1/3))/2 - (Sqrt[3]\*a^(1/3)\*ArcTan[(a^(1/3) + 2\*(a + b\*x^2)^(1/3))/(Sqrt[3]\*a^(1/3))])/2 - (a^(1/3)\*Log[x])/2 + (3\*a^(1/3)\*Log[a^(1/3) - (a + b\*x^2)^(1/3)])/4

**Rubi [A]** time = 0.189221, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{3}{2}\sqrt[3]{a+bx^2} + \frac{3}{4}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right) - \frac{1}{2}\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right) - \frac{1}{2}\sqrt[3]{a}\log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/3)/x, x]

[Out] (3\*(a + b\*x^2)^(1/3))/2 - (Sqrt[3]\*a^(1/3)\*ArcTan[(a^(1/3) + 2\*(a + b\*x^2)^(1/3))/(Sqrt[3]\*a^(1/3))])/2 - (a^(1/3)\*Log[x])/2 + (3\*a^(1/3)\*Log[a^(1/3) - (a + b\*x^2)^(1/3)])/4

**Rubi in Sympy [A]** time = 10.4473, size = 94, normalized size = 0.93

$$-\frac{\sqrt[3]{a}\log(x^2)}{4} + \frac{3\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4} - \frac{\sqrt{3}\sqrt[3]{a}\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} + \frac{2\sqrt[3]{a+bx^2}}{3}\right)}{\sqrt[3]{a}}\right)}{2} + \frac{3\sqrt[3]{a+bx^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/3)/x, x)

[Out] -a\*\*(1/3)\*log(x\*\*2)/4 + 3\*a\*\*(1/3)\*log(a\*\*(1/3) - (a + b\*x\*\*2)\*\*(1/3))/4 - sqrt(3)\*a\*\*(1/3)\*atan(sqrt(3)\*(a\*\*(1/3)/3 + 2\*(a + b\*x\*\*2)\*\*(1/3)/3)/a\*\*(1/3))/2 + 3\*(a + b\*x\*\*2)\*\*(1/3)/2

**Mathematica [C]** time = 0.0455867, size = 61, normalized size = 0.6

$$\frac{6(a+bx^2) - 3a\left(\frac{a}{bx^2} + 1\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{a}{bx^2}\right)}{4(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/3)/x, x]

[Out] (6\*(a + b\*x^2) - 3\*a\*(1 + a/(b\*x^2))^(2/3)\*Hypergeometric2F1[2/3, 2/3, 5/3, -(a/(b\*x^2))])/(4\*(a + b\*x^2)^(2/3))

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x} \sqrt[3]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/3)/x,x)`

[Out] `int((b*x^2+a)^(1/3)/x,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/x,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.214155, size = 131, normalized size = 1.3

$$-\frac{1}{2} \sqrt{3} a^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(2(bx^2 + a)^{\frac{1}{3}} + a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right) - \frac{1}{4} a^{\frac{1}{3}} \log\left(\left(bx^2 + a\right)^{\frac{2}{3}} + \left(bx^2 + a\right)^{\frac{1}{3}} a^{\frac{1}{3}} + a^{\frac{2}{3}}\right) + \frac{1}{2} a^{\frac{1}{3}} \log\left(\left(bx^2 + a\right)^{\frac{1}{3}} - a^{\frac{1}{3}}\right) + \frac{3}{2} (bx^2 + a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/x,x, algorithm="fricas")`

[Out] `-1/2*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)) - 1/4*a^(1/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 1/2*a^(1/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/2*(b*x^2 + a)^(1/3)`

**Sympy [A]** time = 3.84707, size = 46, normalized size = 0.46

$$-\frac{\sqrt[3]{bx^2} \left(-\frac{1}{3}\right) {}_2F_1\left(-\frac{1}{3}, -\frac{1}{3} \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/3)/x,x)`

[Out] `-b**(1/3)*x**(2/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3, ), a*exp_polar(I*pi)/(b*x**2))/(2*gamma(2/3))`

**GIAC/XCAS [A]** time = 0.574102, size = 132, normalized size = 1.31

$$-\frac{1}{2}\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)-\frac{1}{4}a^{\frac{1}{3}}\ln\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) \\ +\frac{1}{2}a^{\frac{1}{3}}\ln\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+\frac{3}{2}\left(bx^2+a\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)/x,x, algorithm="giac")

[Out] -1/2\*sqrt(3)\*a^(1/3)\*arctan(1/3\*sqrt(3)\*(2\*(b\*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)) - 1/4\*a^(1/3)\*ln((b\*x^2 + a)^(2/3) + (b\*x^2 + a)^(1/3)\*a^(1/3) + a^(2/3)) + 1/2\*a^(1/3)\*ln(abs((b\*x^2 + a)^(1/3) - a^(1/3))) + 3/2\*(b\*x^2 + a)^(1/3)

$$3.670 \quad \int \frac{\sqrt[3]{a+bx^2}}{x^3} dx$$

**Optimal.** Leaf size=107

$$\frac{b \log(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{4a^{2/3}} - \frac{b \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}a^{2/3}} - \frac{b \log(x)}{6a^{2/3}} - \frac{\sqrt[3]{a+bx^2}}{2x^2}$$

[Out]  $-(a + b*x^2)^{(1/3)}/(2*x^2) - (b*ArcTan[(a^{(1/3)} + 2*(a + b*x^2)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(2*Sqrt[3]*a^{(2/3)}) - (b*Log[x])/(6*a^{(2/3)}) + (b*Log[a^{(1/3)} - (a + b*x^2)^{(1/3)}])/(4*a^{(2/3)})$

**Rubi [A]** time = 0.172615, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{b \log(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{4a^{2/3}} - \frac{b \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}a^{2/3}} - \frac{b \log(x)}{6a^{2/3}} - \frac{\sqrt[3]{a+bx^2}}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/3)/x^3, x]

[Out]  $-(a + b*x^2)^{(1/3)}/(2*x^2) - (b*ArcTan[(a^{(1/3)} + 2*(a + b*x^2)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(2*Sqrt[3]*a^{(2/3)}) - (b*Log[x])/(6*a^{(2/3)}) + (b*Log[a^{(1/3)} - (a + b*x^2)^{(1/3)}])/(4*a^{(2/3)})$

**Rubi in Sympy [A]** time = 10.8191, size = 99, normalized size = 0.93

$$-\frac{\sqrt[3]{a+bx^2}}{2x^2} - \frac{b \log(x^2)}{12a^{2/3}} + \frac{b \log(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{4a^{2/3}} - \frac{\sqrt{3}b \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} + 2\frac{\sqrt[3]{a+bx^2}}{3}\right)}{\sqrt[3]{a}}\right)}{6a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/3)/x\*\*3, x)

[Out]  $-(a + b*x^2)^{(1/3)}/(2*x^2) - b*\log(x^2)/(12*a^{(2/3)}) + b*\log(a^{(1/3)} - (a + b*x^2)^{(1/3)})/(4*a^{(2/3)}) - \sqrt{3}*b*\operatorname{atan}(\sqrt{3}*(a^{(1/3)}/3 + 2*(a + b*x^2)^{(1/3)}/3)/a^{(1/3)})/(6*a^{(2/3)})$

**Mathematica [C]** time = 0.0446799, size = 67, normalized size = 0.63

$$\frac{-bx^2 \left(\frac{a}{bx^2} + 1\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{a}{bx^2}\right) - 2(a + bx^2)}{4x^2 (a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/3)/x^3, x]

[Out]  $(-2*(a + b*x^2) - b*(1 + a/(b*x^2))^{2/3}*x^2*\operatorname{Hypergeometric2F1}[2/3, 2/3, 5/3, -(a/(b*x^2))])/(4*x^2*(a + b*x^2)^{2/3})$

---

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} \sqrt[3]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(1/3)/x^3, x)

[Out] int((b\*x^2+a)^(1/3)/x^3, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)/x^3, x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.216654, size = 193, normalized size = 1.8

$$\frac{\sqrt{3} \left( \sqrt{3} b x^2 \log \left( a^2 + (b x^2 + a)^{\frac{1}{3}} (a^2)^{\frac{1}{3}} a + (b x^2 + a)^{\frac{2}{3}} (a^2)^{\frac{2}{3}} \right) - 2 \sqrt{3} b x^2 \log \left( -a + (b x^2 + a)^{\frac{1}{3}} (a^2)^{\frac{1}{3}} \right) + 6 b x^2 \arctan \left( \frac{\sqrt{3} a + \sqrt{3} b x^2}{a} \right) \right)}{36 (a^2)^{\frac{1}{3}} x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)/x^3, x, algorithm="fricas")

[Out] -1/36 \* sqrt(3) \* (sqrt(3) \* b \* x^2 \* log(a^2 + (b \* x^2 + a)^(1/3) \* (a^2)^(1/3) \* a + (b \* x^2 + a)^(2/3) \* (a^2)^(2/3)) - 2 \* sqrt(3) \* b \* x^2 \* log(-a + (b \* x^2 + a)^(1/3) \* (a^2)^(1/3)) + 6 \* b \* x^2 \* arctan(1/3 \* (sqrt(3) \* a + 2 \* sqrt(3) \* (b \* x^2 + a)^(1/3) \* (a^2)^(1/3)) / a) + 6 \* sqrt(3) \* (b \* x^2 + a)^(1/3) \* (a^2)^(1/3)) / ((a^2)^(1/3) \* x^2)

---

**Sympy [A]** time = 4.40067, size = 42, normalized size = 0.39

$$\frac{\sqrt[3]{b} \left( \frac{2}{3} \right) {}_2F_1 \left( \left. \begin{matrix} -\frac{1}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix} \right| \frac{a e^{i\pi}}{b x^2} \right)}{2 x^{\frac{4}{3}} \left( \frac{5}{3} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/3)/x\*\*3, x)

[Out] -b\*\*(1/3) \* gamma(2/3) \* hyper((-1/3, 2/3), (5/3, ), a \* exp\_polar(I \* pi) / (b \* x\*\*2)) / (2 \* x\*\*(4/3) \* gamma(5/3))

---

**GIAC/XCAS [A]** time = 0.581174, size = 143, normalized size = 1.34

$$-\frac{1}{12}b \left( \frac{2\sqrt{3} \arctan\left(\frac{\sqrt{3}(2(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}})}{3a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}} + \frac{\ln\left((bx^2+a)^{\frac{2}{3}} + (bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}} - \frac{2\ln\left(\left|(bx^2+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}} + \frac{6(bx^2+a)^{\frac{1}{3}}}{bx^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)/x^3,x, algorithm="giac")

[Out] -1/12\*b\*(2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(b\*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(2/3) + ln((b\*x^2 + a)^(2/3) + (b\*x^2 + a)^(1/3)\*a^(1/3) + a^(2/3))/a^(2/3) - 2\*ln(abs((b\*x^2 + a)^(1/3) - a^(1/3)))/a^(2/3) + 6\*(b\*x^2 + a)^(1/3)/(b\*x^2)

$$3.671 \quad \int \frac{\sqrt[3]{a+bx^2}}{x^5} dx$$

**Optimal.** Leaf size=135

$$-\frac{b^2 \log(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{12a^{5/3}} + \frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}a^{5/3}} + \frac{b^2 \log(x)}{18a^{5/3}} - \frac{b\sqrt[3]{a+bx^2}}{12ax^2} - \frac{\sqrt[3]{a+bx^2}}{4x^4}$$

[Out]  $-(a + b*x^2)^{(1/3)}/(4*x^4) - (b*(a + b*x^2)^{(1/3)})/(12*a*x^2) + (b^2*ArcTan[(a^{(1/3)} + 2*(a + b*x^2)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(6*Sqrt[3]*a^{(5/3)}) + (b^2*Log[x])/(18*a^{(5/3)}) - (b^2*Log[a^{(1/3)} - (a + b*x^2)^{(1/3)}])/(12*a^{(5/3)})$

**Rubi [A]** time = 0.231337, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$-\frac{b^2 \log(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{12a^{5/3}} + \frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}a^{5/3}} + \frac{b^2 \log(x)}{18a^{5/3}} - \frac{b\sqrt[3]{a+bx^2}}{12ax^2} - \frac{\sqrt[3]{a+bx^2}}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/3)/x^5, x]

[Out]  $-(a + b*x^2)^{(1/3)}/(4*x^4) - (b*(a + b*x^2)^{(1/3)})/(12*a*x^2) + (b^2*ArcTan[(a^{(1/3)} + 2*(a + b*x^2)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(6*Sqrt[3]*a^{(5/3)}) + (b^2*Log[x])/(18*a^{(5/3)}) - (b^2*Log[a^{(1/3)} - (a + b*x^2)^{(1/3)}])/(12*a^{(5/3)})$

**Rubi in Sympy [A]** time = 15.649, size = 122, normalized size = 0.9

$$-\frac{\sqrt[3]{a+bx^2}}{4x^4} - \frac{b\sqrt[3]{a+bx^2}}{12ax^2} + \frac{b^2 \log(x^2)}{36a^{5/3}} - \frac{b^2 \log(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{12a^{5/3}} + \frac{\sqrt{3}b^2 \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} + \frac{\sqrt[3]{a+bx^2}}{3}\right)}{\sqrt[3]{a}}\right)}{18a^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/3)/x\*\*5, x)

[Out]  $-(a + b*x^2)^{(1/3)}/(4*x^4) - b*(a + b*x^2)^{(1/3)}/(12*a*x^2) + b^2*2*log(x^2)/(36*a^{(5/3)}) - b^2*2*log(a^{(1/3)} - (a + b*x^2)^{(1/3)})/(12*a^{(5/3)}) + sqrt(3)*b^2*atan(sqrt(3)*(a^{(1/3)}/3 + 2*(a + b*x^2)^{(1/3)}/3)/a^{(1/3)})/(18*a^{(5/3)})$

**Mathematica [C]** time = 0.0526654, size = 82, normalized size = 0.61

$$\frac{-3a^2 + b^2x^4 \left(\frac{a}{bx^2} + 1\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{a}{bx^2}\right) - 4abx^2 - b^2x^4}{12ax^4(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/3)/x^5, x]

[Out]  $(-3*a^2 - 4*a*b*x^2 - b^2*x^4 + b^2*(1 + a/(b*x^2))^{2/3}) * x^4 * \text{Hypergeometric2F1}[2/3, 2/3, 5/3, -(a/(b*x^2))]/(12*a*x^4*(a + b*x^2)^{2/3})$

**Maple [F]** time = 0.045, size = 0, normalized size = 0.

$$\int \frac{1}{x^5} \sqrt[3]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/3)/x^5,x)`

[Out] `int((b*x^2+a)^(1/3)/x^5,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/x^5,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.220502, size = 232, normalized size = 1.72

$$\frac{\sqrt{3} \left( \sqrt{3} b^2 x^4 \log \left( a^2 - (b x^2 + a)^{\frac{1}{3}} (-a^2)^{\frac{1}{3}} a + (b x^2 + a)^{\frac{2}{3}} (-a^2)^{\frac{2}{3}} \right) - 2 \sqrt{3} b^2 x^4 \log \left( a + (b x^2 + a)^{\frac{1}{3}} (-a^2)^{\frac{1}{3}} \right) - 6 b^2 x^4 \arctan \left( \frac{a + (b x^2 + a)^{\frac{1}{3}} (-a^2)^{\frac{1}{3}}}{108 (-a^2)^{\frac{1}{3}} a x^4} \right) \right)}{108 (-a^2)^{\frac{1}{3}} a x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/x^5,x, algorithm="fricas")`

[Out]  $-1/108 * \sqrt{3} * (\sqrt{3} * b^2 * x^4 * \log(a^2 - (b*x^2 + a)^{1/3} * (-a^2)^{1/3} * a + (b*x^2 + a)^{2/3} * (-a^2)^{2/3}) - 2 * \sqrt{3} * b^2 * x^4 * \log(a + (b*x^2 + a)^{1/3} * (-a^2)^{1/3}) - 6 * b^2 * x^4 * \arctan(-1/3 * (\sqrt{3} * a - 2 * \sqrt{3} * (b*x^2 + a)^{1/3} * (-a^2)^{1/3}) / a) + 3 * \sqrt{3} * (b*x^2 + 3*a) * (b*x^2 + a)^{1/3} * (-a^2)^{1/3}) / ((-a^2)^{1/3} * a * x^4)$

**Sympy [A]** time = 5.38724, size = 42, normalized size = 0.31

$$\frac{\sqrt[3]{b} \left( \frac{5}{3} \right) {}_2F_1 \left( \begin{matrix} -\frac{1}{3}, \frac{5}{3} \\ \frac{8}{3} \end{matrix} \middle| \frac{a e^{i\pi}}{b x^2} \right)}{2 x^{\frac{10}{3}} \left( \frac{8}{3} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/3)/x**5,x)`

[Out]  $-b^{1/3} * \gamma(5/3) * \text{hyper}((-1/3, 5/3), (8/3, ), a * \exp\_polar(i * \pi) / (b * x^{10/3})) / (2 * x^{10/3} * \gamma(8/3))$



GIAC/XCAS [A] time = 0.601307, size = 167, normalized size = 1.24

$$\frac{1}{36} b^2 \left( \frac{2 \sqrt{3} \arctan\left(\frac{\sqrt{3} \left(2 (bx^2+a)^{\frac{1}{3}} + a^{\frac{1}{3}}\right)}{3 a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}} + \frac{\ln\left(\left(bx^2+a\right)^{\frac{2}{3}} + \left(bx^2+a\right)^{\frac{1}{3}} a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}} - \frac{2 \ln\left(\left| \left(bx^2+a\right)^{\frac{1}{3}} - a^{\frac{1}{3}} \right|\right)}{a^{\frac{5}{3}}} - \frac{3 \left(\left(bx^2+a\right)^{\frac{4}{3}} + 2 \left(bx^2+a\right)^{\frac{1}{3}} a\right)}{a^{\frac{5}{3}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)/x^5,x, algorithm="giac")

[Out] 1/36\*b^2\*(2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(b\*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(5/3) + ln((b\*x^2 + a)^(2/3) + (b\*x^2 + a)^(1/3)\*a^(1/3) + a^(2/3))/a^(5/3) - 2\*ln(abs((b\*x^2 + a)^(1/3) - a^(1/3)))/a^(5/3) - 3\*((b\*x^2 + a)^(4/3) + 2\*(b\*x^2 + a)^(1/3)\*a)/(a\*b^2\*x^4)

### 3.672 $\int x^4 \sqrt[3]{a + bx^2} dx$

**Optimal.** Leaf size=314

$$\frac{54a^2x\sqrt[3]{a+bx^2}}{935b^2} + 54 \cdot 3^{3/4} \sqrt{2-\sqrt{3}} a^3 \left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right) \sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}} F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle| -7+4\sqrt{3}\right)$$


---


$$935b^3x \sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}$$

$$+ \frac{3}{17}x^5\sqrt[3]{a+bx^2} + \frac{6ax^3\sqrt[3]{a+bx^2}}{187b}$$

[Out]  $(-54*a^2*x*(a+b*x^2)^{(1/3)})/(935*b^2) + (6*a*x^3*(a+b*x^2)^{(1/3)})/(187*b) + (3*x^5*(a+b*x^2)^{(1/3)})/17 - (54*3^{(3/4)}*Sqrt[2 - Sqrt[3]]*a^3*(a^{(1/3)} - (a+b*x^2)^{(1/3)})*Sqrt[(a^{(2/3)} + a^{(1/3)}*(a+b*x^2)^{(1/3)} + (a+b*x^2)^{(2/3)})/((1 - Sqrt[3])*a^{(1/3)} - (a+b*x^2)^{(1/3)})^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^{(1/3)} - (a+b*x^2)^{(1/3)})/((1 - Sqrt[3])*a^{(1/3)} - (a+b*x^2)^{(1/3)})], -7 + 4*Sqrt[3]])/(935*b^3*x*Sqrt[-((a^{(1/3)}*(a^{(1/3)} - (a+b*x^2)^{(1/3)})))/((1 - Sqrt[3])*a^{(1/3)} - (a+b*x^2)^{(1/3)})^2])]$

**Rubi [A]** time = 0.631319, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{54a^2x\sqrt[3]{a+bx^2}}{935b^2} + 54 \cdot 3^{3/4} \sqrt{2-\sqrt{3}} a^3 \left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right) \sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}} F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle| -7+4\sqrt{3}\right)$$


---


$$935b^3x \sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}$$

$$+ \frac{3}{17}x^5\sqrt[3]{a+bx^2} + \frac{6ax^3\sqrt[3]{a+bx^2}}{187b}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a + b\*x^2)^(1/3), x]

[Out]  $(-54*a^2*x*(a+b*x^2)^{(1/3)})/(935*b^2) + (6*a*x^3*(a+b*x^2)^{(1/3)})/(187*b) + (3*x^5*(a+b*x^2)^{(1/3)})/17 - (54*3^{(3/4)}*Sqrt[2 - Sqrt[3]]*a^3*(a^{(1/3)} - (a+b*x^2)^{(1/3)})*Sqrt[(a^{(2/3)} + a^{(1/3)}*(a+b*x^2)^{(1/3)} + (a+b*x^2)^{(2/3)})/((1 - Sqrt[3])*a^{(1/3)} - (a+b*x^2)^{(1/3)})^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^{(1/3)} - (a+b*x^2)^{(1/3)})/((1 - Sqrt[3])*a^{(1/3)} - (a+b*x^2)^{(1/3)})], -7 + 4*Sqrt[3]])/(935*b^3*x*Sqrt[-((a^{(1/3)}*(a^{(1/3)} - (a+b*x^2)^{(1/3)})))/((1 - Sqrt[3])*a^{(1/3)} - (a+b*x^2)^{(1/3)})^2])]$

**Rubi in Sympy [A]** time = 21.0271, size = 264, normalized size = 0.84

$$\frac{54 \cdot 3^{\frac{3}{4}} a^3 \sqrt{\frac{a^{\frac{2}{3}} + \sqrt[3]{a} \sqrt{a + bx^2} + (a + bx^2)^{\frac{2}{3}}}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}} \sqrt{-\sqrt{3} + 2} (\sqrt[3]{a} - \sqrt[3]{a + bx^2}) F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}(1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}{-\sqrt[3]{a}(-1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}\right)\right) \Big|_{-7 + 4\sqrt{3}}}{935b^3 x \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{a + bx^2})}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}} - \frac{54a^2 x \sqrt[3]{a + bx^2}}{935b^2} + \frac{6ax^3 \sqrt[3]{a + bx^2}}{187b} + \frac{3x^5 \sqrt[3]{a + bx^2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**4*(b*x**2+a)**(1/3),x)`

[Out]  $-54 \cdot 3^{3/4} \cdot a^{3/4} \cdot \sqrt{(a^{2/3} + a^{1/3} \cdot (a + b \cdot x^2)^{1/3}) + (a + b \cdot x^2)^{2/3}} / (a^{1/3} \cdot (-1 + \sqrt{3}) + (a + b \cdot x^2)^{1/3}) \cdot \sqrt{-\sqrt{3} + 2} \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3}) \cdot \operatorname{elliptic\_f}(\operatorname{asin}((a^{1/3} \cdot (1 + \sqrt{3}) - (a + b \cdot x^2)^{1/3}) / (-a^{1/3} \cdot (-1 + \sqrt{3}) - (a + b \cdot x^2)^{1/3})), -7 + 4 \cdot \sqrt{3}) / (935 \cdot b^3 \cdot x \cdot \sqrt{-a^{1/3} \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3}) / (a^{1/3} \cdot (-1 + \sqrt{3}) + (a + b \cdot x^2)^{1/3})}) - 54 \cdot a^2 \cdot x \cdot (a + b \cdot x^2)^{1/3} / (935 \cdot b^2) + 6 \cdot a \cdot x^3 \cdot (a + b \cdot x^2)^{1/3} / (187 \cdot b) + 3 \cdot x^5 \cdot (a + b \cdot x^2)^{1/3} / 17$

**Mathematica [C]** time = 0.0606291, size = 90, normalized size = 0.29

$$\frac{3 \left( 18a^3 x \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{1}{2}, \frac{2}{3}; \frac{3}{2}; -\frac{bx^2}{a} \right) - 18a^3 x - 8a^2 bx^3 + 65ab^2 x^5 + 55b^3 x^7 \right)}{935b^2 (a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[x^4*(a + b*x^2)^(1/3),x]`

[Out]  $(3 \cdot (-18 \cdot a^3 \cdot x - 8 \cdot a^2 \cdot b \cdot x^3 + 65 \cdot a \cdot b^2 \cdot x^5 + 55 \cdot b^3 \cdot x^7 + 18 \cdot a^3 \cdot x \cdot (1 + (b \cdot x^2)/a)^{2/3}) \cdot \operatorname{Hypergeometric2F1}[1/2, 2/3, 3/2, -(b \cdot x^2)/a]) / (935 \cdot b^2 \cdot (a + b \cdot x^2)^{2/3})$

**Maple [F]** time = 0.04, size = 0, normalized size = 0.

$$\int x^4 \sqrt[3]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(b*x^2+a)^(1/3),x)`

[Out] `int(x^4*(b*x^2+a)^(1/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{3}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)\*x^4,x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(1/3)\*x^4, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^{\frac{1}{3}}x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)\*x^4,x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(1/3)\*x^4, x)

**Sympy [A]** time = 2.74801, size = 29, normalized size = 0.09

$$\frac{\sqrt[3]{ax^5} {}_2F_1\left(-\frac{1}{3}, \frac{5}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(b\*x\*\*2+a)\*\*(1/3), x)

[Out] a\*\*(1/3)\*x\*\*5\*hyper((-1/3, 5/2), (7/2,), b\*x\*\*2\*exp\_polar(I\*pi)/a)/5

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{3}}x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)\*x^4,x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(1/3)\*x^4, x)

### 3.673 $\int x^2 \sqrt[3]{a + bx^2} dx$

**Optimal.** Leaf size=290

$$6 \cdot 3^{3/4} \sqrt{2 - \sqrt{3}} a^2 \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \middle| -7 + 4\sqrt{3} \right)$$


---


$$55b^2 x \sqrt{-\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}$$

$$+ \frac{6ax \sqrt[3]{a + bx^2}}{55b} + \frac{3}{11} x^3 \sqrt[3]{a + bx^2}$$

[Out] (6\*a\*x\*(a + b\*x^2)^(1/3))/(55\*b) + (3\*x^3\*(a + b\*x^2)^(1/3))/11 + (6\*3^(3/4)\*Sqrt[2 - Sqrt[3]]\*a^2\*(a^(1/3) - (a + b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a + b\*x^2)^(1/3) + (a + b\*x^2)^(2/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]\*EllipticF[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))], -7 + 4\*Sqrt[3]])/(55\*b^2\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a + b\*x^2)^(1/3)))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]))]

**Rubi [A]** time = 0.41542, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$6 \cdot 3^{3/4} \sqrt{2 - \sqrt{3}} a^2 \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \middle| -7 + 4\sqrt{3} \right)$$


---


$$55b^2 x \sqrt{-\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}$$

$$+ \frac{6ax \sqrt[3]{a + bx^2}}{55b} + \frac{3}{11} x^3 \sqrt[3]{a + bx^2}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*x^2)^(1/3), x]

[Out] (6\*a\*x\*(a + b\*x^2)^(1/3))/(55\*b) + (3\*x^3\*(a + b\*x^2)^(1/3))/11 + (6\*3^(3/4)\*Sqrt[2 - Sqrt[3]]\*a^2\*(a^(1/3) - (a + b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a + b\*x^2)^(1/3) + (a + b\*x^2)^(2/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]\*EllipticF[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))], -7 + 4\*Sqrt[3]])/(55\*b^2\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a + b\*x^2)^(1/3)))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]))]

**Rubi in Sympy [A]** time = 15.7747, size = 240, normalized size = 0.83

$$6 \cdot 3^{3/4} a^2 \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( \sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2} \right)^2}} \sqrt{-\sqrt{3} + 2} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) F \left( \operatorname{asin} \left( \frac{\sqrt[3]{a}(1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}{-\sqrt[3]{a}(-1 + \sqrt{3}) - \sqrt[3]{a + bx^2}} \right) \middle| -7 + 4\sqrt{3} \right)$$


---


$$55b^2 x \sqrt{-\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( \sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2} \right)^2}}$$

$$+ \frac{6ax \sqrt[3]{a + bx^2}}{55b} + \frac{3x^3 \sqrt[3]{a + bx^2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2*(b*x**2+a)**(1/3),x)`

[Out]  $6 \cdot 3^{3/4} \cdot a^{2/3} \sqrt{(a^{2/3} + a^{1/3})(a + b^2 x^2)^{1/3} + (a + b^2 x^2)^{2/3}} / (a^{1/3}(-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3}) + (a + b^2 x^2)^{2/3} \sqrt{-\sqrt{3} + 2} (a^{1/3} - (a + b^2 x^2)^{1/3}) \operatorname{elliptic}_f(\operatorname{asin}((a^{1/3}(1 + \sqrt{3}) - (a + b^2 x^2)^{1/3}) / (-a^{1/3}(-1 + \sqrt{3}) - (a + b^2 x^2)^{1/3})), -7 + 4\sqrt{3}) / (55 b^2 x \sqrt{-a^{1/3}(a^{1/3} - (a + b^2 x^2)^{1/3}) / (a^{1/3}(-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3})} + 6 a x (a + b^2 x^2)^{1/3} / (55 b) + 3 x^3 (a + b^2 x^2)^{1/3} / 11$

**Mathematica [C]** time = 0.0553379, size = 78, normalized size = 0.27

$$\frac{3x \left( -2a^2 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{1}{2}, \frac{2}{3}; \frac{3}{2}; -\frac{bx^2}{a} \right) + 2a^2 + 7abx^2 + 5b^2x^4 \right)}{55b(a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[x^2*(a + b*x^2)^(1/3),x]`

[Out]  $(3x^3(2a^2 + 7abx^2 + 5b^2x^4 - 2a^2(1 + (bx^2)/a)^{2/3}) \operatorname{Hypergeometric2F1}[1/2, 2/3, 3/2, -(bx^2)/a]) / (55b^2(a + b^2x^2)^{2/3})$

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int x^2 \sqrt[3]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^2+a)^(1/3),x)`

[Out] `int(x^2*(b*x^2+a)^(1/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{1/3} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*x^2,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(1/3)*x^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( (bx^2 + a)^{1/3} x^2, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*x^2,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(1/3)*x^2, x)`

---

**Sympy [A]** time = 2.37917, size = 29, normalized size = 0.1

$$\frac{\sqrt[3]{ax^3} {}_2F_1\left(-\frac{1}{3}, \frac{3}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**2+a)**(1/3), x)`

[Out] `a**(1/3)*x**3*hyper((-1/3, 3/2), (5/2, ), b*x**2*exp_polar(I*pi)/a)/3`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{3}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*x^2, x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(1/3)*x^2, x)`

3.674  $\int \sqrt[3]{a + bx^2} dx$ **Optimal.** Leaf size=266

$$\frac{3}{5}x\sqrt[3]{a + bx^2} - \frac{2 \cdot 3^{3/4} \sqrt{2 - \sqrt{3}} a \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \middle| -7 + 4\sqrt{3} \right)}{5bx \sqrt{-\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}$$

[Out]  $(3*x*(a + b*x^2)^{(1/3)})/5 - (2*3^{(3/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*a*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(5*b*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2])$

**Rubi [A]** time = 0.30191, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{3}{5}x\sqrt[3]{a + bx^2} - \frac{2 \cdot 3^{3/4} \sqrt{2 - \sqrt{3}} a \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \middle| -7 + 4\sqrt{3} \right)}{5bx \sqrt{-\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/3), x]

[Out]  $(3*x*(a + b*x^2)^{(1/3)})/5 - (2*3^{(3/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*a*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(5*b*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2])$

**Rubi in Sympy [A]** time = 8.11757, size = 216, normalized size = 0.81

$$2 \cdot 3^{3/4} a \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( \sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2} \right)^2}} \sqrt{-\sqrt{3} + 2} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) F \left( \text{asin} \left( \frac{\sqrt[3]{a}(1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}{-\sqrt[3]{a}(-1 + \sqrt{3}) - \sqrt[3]{a + bx^2}} \right) \middle| -7 + 4\sqrt{3} \right) - \frac{3x\sqrt[3]{a + bx^2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/3), x)



[Out]  $-2 \cdot 3^{3/4} \cdot a \cdot \sqrt{(a^{2/3} + a^{1/3} \cdot (a + b \cdot x^2)^{1/3}) + (a + b \cdot x^2)^{2/3}} / (a^{1/3} \cdot (-1 + \sqrt{3})) + (a + b \cdot x^2)^{1/3} \cdot \sqrt{-\sqrt{3} + 2} \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3}) \cdot \text{elliptic\_f}(\text{asin}((a^{1/3} \cdot (1 + \sqrt{3})) - (a + b \cdot x^2)^{1/3}) / (-a^{1/3} \cdot (-1 + \sqrt{3}) - (a + b \cdot x^2)^{1/3})), -7 + 4 \cdot \sqrt{3}) / (5 \cdot b \cdot x \cdot \sqrt{-a^{1/3} \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3}) / (a^{1/3} \cdot (-1 + \sqrt{3})) + (a + b \cdot x^2)^{1/3}}) + 3 \cdot x \cdot (a + b \cdot x^2)^{1/3} / 5$

**Mathematica [C]** time = 0.0344254, size = 63, normalized size = 0.24

$$\frac{2ax \left(\frac{bx^2}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{3}{2}; -\frac{bx^2}{a}\right) + 3x(a + bx^2)}{5(a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/3), x]

[Out]  $(3 \cdot x \cdot (a + b \cdot x^2) + 2 \cdot a \cdot x \cdot (1 + (b \cdot x^2)/a)^{2/3} \cdot \text{Hypergeometric2F1}[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, -((b \cdot x^2)/a)]) / (5 \cdot (a + b \cdot x^2)^{2/3})$

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int \sqrt[3]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(1/3), x)

[Out] int((b\*x^2+a)^(1/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(1/3), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^{1/3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(1/3), x)

**Sympy [A]** time = 2.20161, size = 26, normalized size = 0.1

$$\sqrt[3]{ax^2} F_1 \left( -\frac{1}{3}, \frac{1}{2} \middle| \frac{bx^2 e^{i\pi}}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/3), x)

[Out] a\*\*(1/3)\*x\*hyper((-1/3, 1/2), (3/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3), x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(1/3), x)

$$3.675 \quad \int \frac{\sqrt[3]{a+bx^2}}{x^2} dx$$

**Optimal.** Leaf size=260

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle| -7+4\sqrt{3}\right)}{\sqrt[3]{3}x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}-\frac{\sqrt[3]{a+bx^2}}{x}}$$

[Out]  $-\left((a+b*x^2)^{(1/3)}/x\right)-\left(2*\text{Sqrt}[2-\text{Sqrt}[3]]*(a^{(1/3)}-(a+b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)}+a^{(1/3)}*(a+b*x^2)^{(1/3)}+(a+b*x^2)^{(2/3)})/((1-\text{Sqrt}[3])*a^{(1/3)}-(a+b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}(((1+\text{Sqrt}[3])*a^{(1/3)}-(a+b*x^2)^{(1/3)})/((1-\text{Sqrt}[3])*a^{(1/3)}-(a+b*x^2)^{(1/3)})), -7+4*\text{Sqrt}[3]]/(3^{(1/4)}*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)}-(a+b*x^2)^{(1/3)})/((1-\text{Sqrt}[3])*a^{(1/3)}-(a+b*x^2)^{(1/3)})^2)])]$

**Rubi [A]** time = 0.312366, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle| -7+4\sqrt{3}\right)}{\sqrt[3]{3}x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}-\frac{\sqrt[3]{a+bx^2}}{x}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a+b*x^2)^{(1/3)}/x^2, x]$

[Out]  $-\left((a+b*x^2)^{(1/3)}/x\right)-\left(2*\text{Sqrt}[2-\text{Sqrt}[3]]*(a^{(1/3)}-(a+b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)}+a^{(1/3)}*(a+b*x^2)^{(1/3)}+(a+b*x^2)^{(2/3)})/((1-\text{Sqrt}[3])*a^{(1/3)}-(a+b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}(((1+\text{Sqrt}[3])*a^{(1/3)}-(a+b*x^2)^{(1/3)})/((1-\text{Sqrt}[3])*a^{(1/3)}-(a+b*x^2)^{(1/3)})), -7+4*\text{Sqrt}[3]]/(3^{(1/4)}*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)}-(a+b*x^2)^{(1/3)})/((1-\text{Sqrt}[3])*a^{(1/3)}-(a+b*x^2)^{(1/3)})^2)])]$

**Rubi in Sympy [A]** time = 9.82465, size = 211, normalized size = 0.81

$$\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)F\left(\text{asin}\left(\frac{\sqrt[3]{a}(1+\sqrt{3})-\sqrt[3]{a+bx^2}}{-\sqrt[3]{a}(-1+\sqrt{3})-\sqrt[3]{a+bx^2}}\right)\middle| -7+4\sqrt{3}\right)}{3x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)^2}}-\frac{\sqrt[3]{a+bx^2}}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(1/3)/x**2,x)`

[Out] 
$$-2 \cdot 3^{3/4} \cdot \sqrt{(a^{2/3} + a^{1/3} \cdot (a + b \cdot x^{**2})^{1/3} + (a + b \cdot x^{**2})^{2/3}) / (a^{1/3} \cdot (-1 + \sqrt{3}) + (a + b \cdot x^{**2})^{1/3})} \cdot \sqrt{2} \cdot \sqrt{-\sqrt{3} + 2} \cdot (a^{1/3} - (a + b \cdot x^{**2})^{1/3}) \cdot \text{elliptic\_f}(\text{asin}((a^{1/3} \cdot (1 + \sqrt{3}) - (a + b \cdot x^{**2})^{1/3}) / (-a^{1/3} \cdot (-1 + \sqrt{3}) - (a + b \cdot x^{**2})^{1/3})), -7 + 4 \cdot \sqrt{3}) / (3 \cdot x \cdot \sqrt{-a^{1/3} \cdot (a^{1/3} - (a + b \cdot x^{**2})^{1/3}) / (a^{1/3} \cdot (-1 + \sqrt{3}) + (a + b \cdot x^{**2})^{1/3})} - (a + b \cdot x^{**2})^{1/3} / x$$

**Mathematica [C]** time = 0.0387698, size = 68, normalized size = 0.26

$$\frac{2bx \left(\frac{a+bx^2}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{3}{2}; -\frac{bx^2}{a}\right)}{3(a+bx^2)^{2/3}} - \frac{\sqrt[3]{a+bx^2}}{x}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(1/3)/x^2,x]`

[Out] 
$$-((a + b \cdot x^2)^{1/3} / x) + (2 \cdot b \cdot x \cdot ((a + b \cdot x^2) / a)^{2/3} \cdot \text{Hypergeometric2F1}[1/2, 2/3, 3/2, -((b \cdot x^2) / a)]) / (3 \cdot (a + b \cdot x^2)^{2/3})$$

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \sqrt[3]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/3)/x^2,x)`

[Out] `int((b*x^2+a)^(1/3)/x^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{1/3}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/x^2,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(1/3)/x^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{1/3}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/x^2,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(1/3)/x^2, x)`

---

**Sympy [A]** time = 2.39789, size = 29, normalized size = 0.11

$$\frac{\sqrt[3]{a} {}_2F_1\left(-\frac{1}{2}, -\frac{1}{3} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/3)/x**2, x)`

[Out] `-a**(1/3)*hyper((-1/2, -1/3), (1/2,), b*x**2*exp_polar(I*pi)/a)/x`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{3}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/x^2, x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(1/3)/x^2, x)`



Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(1/3)/x**4,x)`

[Out]  $-(a + b x^2)^{1/3}/(3 x^3) + 2 \cdot 3^{3/4} b \sqrt{(a^{2/3} + a^{1/3})(a + b x^2)^{1/3} + (a + b x^2)^{2/3}}/(a^{1/3}(-1 + \sqrt{3})) + (a + b x^2)^{1/3})^2 \sqrt{-\sqrt{3} + 2} (a^{1/3} - (a + b x^2)^{1/3}) \operatorname{elliptic}_f(\operatorname{asin}((a^{1/3}(1 + \sqrt{3}) - (a + b x^2)^{1/3})/(-a^{1/3}(-1 + \sqrt{3}) - (a + b x^2)^{1/3}))), -7 + 4 \sqrt{3})/(27 a x \sqrt{-a^{1/3}(a^{1/3} - (a + b x^2)^{1/3})/(a^{1/3}(-1 + \sqrt{3})) + (a + b x^2)^{1/3})} - 2 b (a + b x^2)^{1/3}/(9 a x)$

**Mathematica [C]** time = 0.0449163, size = 88, normalized size = 0.3

$$\left(-\frac{2b}{9ax} - \frac{1}{3x^3}\right) \sqrt[3]{a + bx^2} - \frac{2b^2 x \left(\frac{a+bx^2}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, -\frac{bx^2}{a}\right)}{27a(a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(1/3)/x^4,x]`

[Out]  $(-1/(3 x^3) - (2 b)/(9 a x)) (a + b x^2)^{1/3} - (2 b^2 x ((a + b x^2)/a)^{2/3} \operatorname{Hypergeometric2F1}[1/2, 2/3, 3/2, -((b x^2)/a)])/(27 a (a + b x^2)^{2/3})$

**Maple [F]** time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} \sqrt[3]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/3)/x^4,x)`

[Out] `int((b*x^2+a)^(1/3)/x^4,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{1/3}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/x^4,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(1/3)/x^4, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(bx^2 + a)^{1/3}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)/x^4,x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(1/3)/x^4, x)

**Sympy [A]** time = 2.8454, size = 34, normalized size = 0.12

$$\frac{\sqrt[3]{a} {}_2F_1\left(-\frac{3}{2}, -\frac{1}{3} \middle| -\frac{1}{2}, \frac{bx^2 e^{i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/3)/x\*\*4,x)

[Out] -a\*\*(1/3)\*hyper((-3/2, -1/3), (-1/2,), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(3\*x\*\*3)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{3}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)/x^4,x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(1/3)/x^4, x)



$$3.677 \quad \int x^7 (a + bx^2)^{2/3} dx$$

**Optimal.** Leaf size=80

$$-\frac{3a^3 (a + bx^2)^{5/3}}{10b^4} + \frac{9a^2 (a + bx^2)^{8/3}}{16b^4} + \frac{3 (a + bx^2)^{14/3}}{28b^4} - \frac{9a (a + bx^2)^{11/3}}{22b^4}$$

[Out]  $(-3*a^3*(a + b*x^2)^(5/3))/(10*b^4) + (9*a^2*(a + b*x^2)^(8/3))/(16*b^4) - (9*a*(a + b*x^2)^(11/3))/(22*b^4) + (3*(a + b*x^2)^(14/3))/(28*b^4)$

**Rubi [A]** time = 0.123728, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{3a^3 (a + bx^2)^{5/3}}{10b^4} + \frac{9a^2 (a + bx^2)^{8/3}}{16b^4} + \frac{3 (a + bx^2)^{14/3}}{28b^4} - \frac{9a (a + bx^2)^{11/3}}{22b^4}$$

Antiderivative was successfully verified.

[In] Int[x^7\*(a + b\*x^2)^(2/3), x]

[Out]  $(-3*a^3*(a + b*x^2)^(5/3))/(10*b^4) + (9*a^2*(a + b*x^2)^(8/3))/(16*b^4) - (9*a*(a + b*x^2)^(11/3))/(22*b^4) + (3*(a + b*x^2)^(14/3))/(28*b^4)$

**Rubi in Sympy [A]** time = 15.4796, size = 75, normalized size = 0.94

$$-\frac{3a^3 (a + bx^2)^{\frac{5}{3}}}{10b^4} + \frac{9a^2 (a + bx^2)^{\frac{8}{3}}}{16b^4} - \frac{9a (a + bx^2)^{\frac{11}{3}}}{22b^4} + \frac{3 (a + bx^2)^{\frac{14}{3}}}{28b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*7\*(b\*x\*\*2+a)\*\*(2/3), x)

[Out]  $-3*a**3*(a + b*x**2)**(5/3)/(10*b**4) + 9*a**2*(a + b*x**2)**(8/3)/(16*b**4) - 9*a*(a + b*x**2)**(11/3)/(22*b**4) + 3*(a + b*x**2)**(14/3)/(28*b**4)$

**Mathematica [A]** time = 0.0298714, size = 61, normalized size = 0.76

$$\frac{3 (a + bx^2)^{2/3} (-81a^4 + 54a^3bx^2 - 45a^2b^2x^4 + 40ab^3x^6 + 220b^4x^8)}{6160b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^7\*(a + b\*x^2)^(2/3), x]

[Out]  $(3*(a + b*x^2)^(2/3)*(-81*a^4 + 54*a^3*b*x^2 - 45*a^2*b^2*x^4 + 40*a*b^3*x^6 + 220*b^4*x^8))/(6160*b^4)$

**Maple [A]** time = 0.008, size = 47, normalized size = 0.6

$$-\frac{-660b^3x^6 + 540ab^2x^4 - 405a^2bx^2 + 243a^3}{6160b^4} (bx^2 + a)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(b*x^2+a)^(2/3),x)`

[Out]  $-3/6160*(b*x^2+a)^{5/3}*(-220*b^3*x^6+180*a*b^2*x^4-135*a^2*b*x^2+81*a^3)/b^4$

**Maxima [A]** time = 1.34686, size = 86, normalized size = 1.08

$$\frac{3(bx^2+a)^{\frac{14}{3}}}{28b^4} - \frac{9(bx^2+a)^{\frac{11}{3}}a}{22b^4} + \frac{9(bx^2+a)^{\frac{8}{3}}a^2}{16b^4} - \frac{3(bx^2+a)^{\frac{5}{3}}a^3}{10b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3)*x^7,x, algorithm="maxima")`

[Out]  $3/28*(b*x^2 + a)^{14/3}/b^4 - 9/22*(b*x^2 + a)^{11/3}*a/b^4 + 9/16*(b*x^2 + a)^{8/3}*a^2/b^4 - 3/10*(b*x^2 + a)^{5/3}*a^3/b^4$

**Fricas [A]** time = 0.211101, size = 77, normalized size = 0.96

$$\frac{3(220b^4x^8 + 40ab^3x^6 - 45a^2b^2x^4 + 54a^3bx^2 - 81a^4)(bx^2 + a)^{\frac{2}{3}}}{6160b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3)*x^7,x, algorithm="fricas")`

[Out]  $3/6160*(220*b^4*x^8 + 40*a*b^3*x^6 - 45*a^2*b^2*x^4 + 54*a^3*b*x^2 - 81*a^4)*(b*x^2 + a)^{2/3}/b^4$

**Sympy [A]** time = 11.0237, size = 1795, normalized size = 22.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(b*x**2+a)**(2/3),x)`

[Out]  $-243*a**(74/3)*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 243*a**(74/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) - 1296*a**(71/3)*b*x**2*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 1458*a**(71/3)*b*x**2/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) - 2808*a**(68/3)*b**2*x**4*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 3645*a**(68/3)*b**2*x**4/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) - 3120*a**(65/3)*b**3*x**6*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**8$

```

*4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15
*b**9*x**10 + 6160*a**14*b**10*x**12) + 4860*a**((65/3)*b**3*x**6/
(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4
+ 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**
*9*x**10 + 6160*a**14*b**10*x**12) - 1050*a**((62/3)*b**4*x**8*(1
+ b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 924
00*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**
*8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 3645*a**((
62/3)*b**4*x**8/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*
a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8
+ 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 4032*a**((59/
3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**1
9*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92
400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x
**12) + 1458*a**((59/3)*b**5*x**10/(6160*a**20*b**4 + 36960*a**19*
b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 9240
0*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**
12) + 11004*a**((56/3)*b**6*x**12*(1 + b*x**2/a)**(2/3)/(6160*a**2
0*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a
**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 +
6160*a**14*b**10*x**12) + 243*a**((56/3)*b**6*x**12/(6160*a**20*b
**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**1
7*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 61
60*a**14*b**10*x**12) + 14352*a**((53/3)*b**7*x**14*(1 + b*x**2/a)
** (2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**
*6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*
a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 10485*a**((50/3)*b**8
*x**16*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*
x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**
16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) +
4080*a**((47/3)*b**9*x**18*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4
+ 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b
**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*
a**14*b**10*x**12) + 660*a**((44/3)*b**10*x**20*(1 + b*x**2/a)**(2
/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x
**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**1
5*b**9*x**10 + 6160*a**14*b**10*x**12)

```

---

**GIAC/XCAS [A]** time = 0.217977, size = 77, normalized size = 0.96

$$\frac{3 \left( 220 (bx^2 + a)^{\frac{14}{3}} - 840 (bx^2 + a)^{\frac{11}{3}} a + 1155 (bx^2 + a)^{\frac{8}{3}} a^2 - 616 (bx^2 + a)^{\frac{5}{3}} a^3 \right)}{6160 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(2/3)\*x^7,x, algorithm="giac")

[Out] 3/6160\*(220\*(b\*x^2 + a)^(14/3) - 840\*(b\*x^2 + a)^(11/3)\*a + 1155\*(b\*x^2 + a)^(8/3)\*a^2 - 616\*(b\*x^2 + a)^(5/3)\*a^3)/b^4

$$3.678 \quad \int x^5 (a + bx^2)^{2/3} dx$$

**Optimal.** Leaf size=59

$$\frac{3a^2 (a + bx^2)^{5/3}}{10b^3} + \frac{3 (a + bx^2)^{11/3}}{22b^3} - \frac{3a (a + bx^2)^{8/3}}{8b^3}$$

[Out]  $(3*a^2*(a + b*x^2)^(5/3))/(10*b^3) - (3*a*(a + b*x^2)^(8/3))/(8*b^3) + (3*(a + b*x^2)^(11/3))/(22*b^3)$

**Rubi [A]** time = 0.0957364, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{3a^2 (a + bx^2)^{5/3}}{10b^3} + \frac{3 (a + bx^2)^{11/3}}{22b^3} - \frac{3a (a + bx^2)^{8/3}}{8b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(a + b\*x^2)^(2/3), x]

[Out]  $(3*a^2*(a + b*x^2)^(5/3))/(10*b^3) - (3*a*(a + b*x^2)^(8/3))/(8*b^3) + (3*(a + b*x^2)^(11/3))/(22*b^3)$

**Rubi in Sympy [A]** time = 11.6083, size = 54, normalized size = 0.92

$$\frac{3a^2 (a + bx^2)^{5/3}}{10b^3} - \frac{3a (a + bx^2)^{8/3}}{8b^3} + \frac{3 (a + bx^2)^{11/3}}{22b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(b\*x\*\*2+a)\*\*(2/3), x)

[Out]  $3*a**2*(a + b*x**2)**(5/3)/(10*b**3) - 3*a*(a + b*x**2)**(8/3)/(8*b**3) + 3*(a + b*x**2)**(11/3)/(22*b**3)$

**Mathematica [A]** time = 0.0236759, size = 50, normalized size = 0.85

$$\frac{3 (a + bx^2)^{2/3} (9a^3 - 6a^2bx^2 + 5ab^2x^4 + 20b^3x^6)}{440b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*x^2)^(2/3), x]

[Out]  $(3*(a + b*x^2)^(2/3)*(9*a^3 - 6*a^2*b*x^2 + 5*a*b^2*x^4 + 20*b^3*x^6))/(440*b^3)$

**Maple [A]** time = 0.007, size = 36, normalized size = 0.6

$$\frac{60 b^2 x^4 - 45 a b x^2 + 27 a^2}{440 b^3} (b x^2 + a)^{5/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(b*x^2+a)^(2/3),x)`

[Out]  $3/440*(b*x^2+a)^(5/3)*(20*b^2*x^4-15*a*b*x^2+9*a^2)/b^3$

**Maxima [A]** time = 1.3386, size = 63, normalized size = 1.07

$$\frac{3(bx^2+a)^{\frac{11}{3}}}{22b^3} - \frac{3(bx^2+a)^{\frac{8}{3}}a}{8b^3} + \frac{3(bx^2+a)^{\frac{5}{3}}a^2}{10b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^(2/3)*x^5,x, algorithm="maxima")`

[Out]  $3/22*(b*x^2+a)^(11/3)/b^3 - 3/8*(b*x^2+a)^(8/3)*a/b^3 + 3/10*(b*x^2+a)^(5/3)*a^2/b^3$

**Fricas [A]** time = 0.210268, size = 62, normalized size = 1.05

$$\frac{3(20b^3x^6 + 5ab^2x^4 - 6a^2bx^2 + 9a^3)(bx^2 + a)^{\frac{2}{3}}}{440b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^(2/3)*x^5,x, algorithm="fricas")`

[Out]  $3/440*(20*b^3*x^6 + 5*a*b^2*x^4 - 6*a^2*b*x^2 + 9*a^3)*(b*x^2 + a)^(2/3)/b^3$

**Sympy [A]** time = 7.0286, size = 700, normalized size = 11.86

$$\begin{aligned} & \frac{27a^{\frac{35}{3}} \left(1 + \frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3 + 1320a^7b^4x^2 + 1320a^6b^5x^4 + 440a^5b^6x^6} \\ & - \frac{27a^{\frac{35}{3}}}{440a^8b^3 + 1320a^7b^4x^2 + 1320a^6b^5x^4 + 440a^5b^6x^6} \\ & + \frac{63a^{\frac{32}{3}}bx^2 \left(1 + \frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3 + 1320a^7b^4x^2 + 1320a^6b^5x^4 + 440a^5b^6x^6} \\ & - \frac{81a^{\frac{32}{3}}bx^2}{440a^8b^3 + 1320a^7b^4x^2 + 1320a^6b^5x^4 + 440a^5b^6x^6} \\ & + \frac{42a^{\frac{29}{3}}b^2x^4 \left(1 + \frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3 + 1320a^7b^4x^2 + 1320a^6b^5x^4 + 440a^5b^6x^6} \\ & - \frac{81a^{\frac{29}{3}}b^2x^4}{440a^8b^3 + 1320a^7b^4x^2 + 1320a^6b^5x^4 + 440a^5b^6x^6} \\ & + \frac{78a^{\frac{26}{3}}b^3x^6 \left(1 + \frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3 + 1320a^7b^4x^2 + 1320a^6b^5x^4 + 440a^5b^6x^6} \\ & - \frac{27a^{\frac{26}{3}}b^3x^6}{440a^8b^3 + 1320a^7b^4x^2 + 1320a^6b^5x^4 + 440a^5b^6x^6} \\ & + \frac{207a^{\frac{23}{3}}b^4x^8 \left(1 + \frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3 + 1320a^7b^4x^2 + 1320a^6b^5x^4 + 440a^5b^6x^6} \\ & - \frac{207a^{\frac{23}{3}}b^4x^8}{440a^8b^3 + 1320a^7b^4x^2 + 1320a^6b^5x^4 + 440a^5b^6x^6} \\ & + \frac{195a^{\frac{20}{3}}b^5x^{10} \left(1 + \frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3 + 1320a^7b^4x^2 + 1320a^6b^5x^4 + 440a^5b^6x^6} \\ & - \frac{195a^{\frac{20}{3}}b^5x^{10}}{440a^8b^3 + 1320a^7b^4x^2 + 1320a^6b^5x^4 + 440a^5b^6x^6} \\ & + \frac{60a^{\frac{17}{3}}b^6x^{12} \left(1 + \frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3 + 1320a^7b^4x^2 + 1320a^6b^5x^4 + 440a^5b^6x^6} \\ & - \frac{60a^{\frac{17}{3}}b^6x^{12}}{440a^8b^3 + 1320a^7b^4x^2 + 1320a^6b^5x^4 + 440a^5b^6x^6} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(b\*x\*\*2+a)\*\*(2/3),x)

[Out] 
$$\frac{27 a^{35/3} (1 + b x^2/a)^{2/3} (440 a^8 b^3 + 1320 a^7 b^4 x^2 + 1320 a^6 b^5 x^4 + 440 a^5 b^6 x^6) - 27 a^{35/3} (440 a^8 b^3 + 1320 a^7 b^4 x^2 + 1320 a^6 b^5 x^4 + 440 a^5 b^6 x^6) + 63 a^{32/3} b^2 (1 + b x^2/a)^{2/3} (440 a^8 b^3 + 1320 a^7 b^4 x^2 + 1320 a^6 b^5 x^4 + 440 a^5 b^6 x^6) - 81 a^{32/3} b^2 (440 a^8 b^3 + 1320 a^7 b^4 x^2 + 1320 a^6 b^5 x^4 + 440 a^5 b^6 x^6) + 42 a^{29/3} b^2 x^4 (1 + b x^2/a)^{2/3} (440 a^8 b^3 + 1320 a^7 b^4 x^2 + 1320 a^6 b^5 x^4 + 440 a^5 b^6 x^6) - 81 a^{29/3} b^2 x^4 (440 a^8 b^3 + 1320 a^7 b^4 x^2 + 1320 a^6 b^5 x^4 + 440 a^5 b^6 x^6) + 78 a^{26/3} b^3 x^6 (1 + b x^2/a)^{2/3} (440 a^8 b^3 + 1320 a^7 b^4 x^2 + 1320 a^6 b^5 x^4 + 440 a^5 b^6 x^6) - 27 a^{26/3} b^3 x^6 (440 a^8 b^3 + 1320 a^7 b^4 x^2 + 1320 a^6 b^5 x^4 + 440 a^5 b^6 x^6) + 207 a^{23/3} b^4 x^8 (1 + b x^2/a)^{2/3} (440 a^8 b^3 + 1320 a^7 b^4 x^2 + 1320 a^6 b^5 x^4 + 440 a^5 b^6 x^6) + 195 a^{20/3} b^5 x^{10} (1 + b x^2/a)^{2/3} (440 a^8 b^3 + 1320 a^7 b^4 x^2 + 1320 a^6 b^5 x^4 + 440 a^5 b^6 x^6) + 60 a^{17/3} b^6 x^{12} (1 + b x^2/a)^{2/3} (440 a^8 b^3 + 1320 a^7 b^4 x^2 + 1320 a^6 b^5 x^4 + 440 a^5 b^6 x^6)}$$

**GIAC/XCAS [A]** time = 0.214432, size = 58, normalized size = 0.98

$$\frac{3 \left( 20 (bx^2 + a)^{\frac{11}{3}} - 55 (bx^2 + a)^{\frac{8}{3}} a + 44 (bx^2 + a)^{\frac{5}{3}} a^2 \right)}{440 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(2/3)\*x^5,x, algorithm="giac")

[Out] 
$$\frac{3}{440} (20 (b x^2 + a)^{11/3} - 55 (b x^2 + a)^{8/3} a + 44 (b x^2 + a)^{5/3} a^2) / b^3$$

$$3.679 \quad \int x^3 (a + bx^2)^{2/3} dx$$

**Optimal.** Leaf size=38

$$\frac{3(a+bx^2)^{8/3}}{16b^2} - \frac{3a(a+bx^2)^{5/3}}{10b^2}$$

[Out] (-3\*a\*(a + b\*x^2)^(5/3))/(10\*b^2) + (3\*(a + b\*x^2)^(8/3))/(16\*b^2)

**Rubi [A]** time = 0.0674809, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{3(a+bx^2)^{8/3}}{16b^2} - \frac{3a(a+bx^2)^{5/3}}{10b^2}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*x^2)^(2/3), x]

[Out] (-3\*a\*(a + b\*x^2)^(5/3))/(10\*b^2) + (3\*(a + b\*x^2)^(8/3))/(16\*b^2)

**Rubi in Sympy [A]** time = 7.8195, size = 34, normalized size = 0.89

$$-\frac{3a(a+bx^2)^{5/3}}{10b^2} + \frac{3(a+bx^2)^{8/3}}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(b\*x\*\*2+a)\*\*(2/3), x)

[Out] -3\*a\*(a + b\*x\*\*2)\*\*(5/3)/(10\*b\*\*2) + 3\*(a + b\*x\*\*2)\*\*(8/3)/(16\*b\*\*2)

**Mathematica [A]** time = 0.0205877, size = 39, normalized size = 1.03

$$\frac{3(a+bx^2)^{2/3}(-3a^2+2abx^2+5b^2x^4)}{80b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*x^2)^(2/3), x]

[Out] (3\*(a + b\*x^2)^(2/3)\*(-3\*a^2 + 2\*a\*b\*x^2 + 5\*b^2\*x^4))/(80\*b^2)

**Maple [A]** time = 0.007, size = 25, normalized size = 0.7

$$-\frac{-15bx^2+9a}{80b^2}(bx^2+a)^{5/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(b\*x^2+a)^(2/3), x)

[Out]  $-3/80 * (b * x^2 + a)^{5/3} * (-5 * b * x^2 + 3 * a) / b^2$

**Maxima [A]** time = 1.33989, size = 41, normalized size = 1.08

$$\frac{3 (bx^2 + a)^{\frac{8}{3}}}{16 b^2} - \frac{3 (bx^2 + a)^{\frac{5}{3}} a}{10 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3)*x^3,x, algorithm="maxima")`

[Out]  $3/16 * (b * x^2 + a)^{8/3} / b^2 - 3/10 * (b * x^2 + a)^{5/3} * a / b^2$

**Fricas [A]** time = 0.208908, size = 47, normalized size = 1.24

$$\frac{3 (5 b^2 x^4 + 2 a b x^2 - 3 a^2) (b x^2 + a)^{\frac{2}{3}}}{80 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3)*x^3,x, algorithm="fricas")`

[Out]  $3/80 * (5 * b^2 * x^4 + 2 * a * b * x^2 - 3 * a^2) * (b * x^2 + a)^{2/3} / b^2$

**Sympy [A]** time = 2.22428, size = 66, normalized size = 1.74

$$\begin{cases} -\frac{9a^2(a+bx^2)^{\frac{2}{3}}}{80b^2} + \frac{3ax^2(a+bx^2)^{\frac{2}{3}}}{40b} + \frac{3x^4(a+bx^2)^{\frac{2}{3}}}{16} & \text{for } b \neq 0 \\ \frac{a^{\frac{2}{3}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**2+a)**(2/3),x)`

[Out] `Piecewise((-9*a**2*(a + b*x**2)**(2/3)/(80*b**2) + 3*a*x**2*(a + b*x**2)**(2/3)/(40*b) + 3*x**4*(a + b*x**2)**(2/3)/16, Ne(b, 0)), (a**(2/3)*x**4/4, True))`

**GIAC/XCAS [A]** time = 0.214446, size = 39, normalized size = 1.03

$$\frac{3 \left( 5 (bx^2 + a)^{\frac{8}{3}} - 8 (bx^2 + a)^{\frac{5}{3}} a \right)}{80 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3)*x^3,x, algorithm="giac")`

[Out]  $3/80 * (5 * (b * x^2 + a)^{8/3} - 8 * (b * x^2 + a)^{5/3} * a) / b^2$



$$3.680 \quad \int x (a + bx^2)^{2/3} dx$$

**Optimal.** Leaf size=18

$$\frac{3 (a + bx^2)^{5/3}}{10b}$$

[Out] (3\*(a + b\*x^2)^(5/3))/(10\*b)

**Rubi [A]** time = 0.0113831, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{3 (a + bx^2)^{5/3}}{10b}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*x^2)^(2/3), x]

[Out] (3\*(a + b\*x^2)^(5/3))/(10\*b)

**Rubi in Sympy [A]** time = 2.14743, size = 14, normalized size = 0.78

$$\frac{3 (a + bx^2)^{\frac{5}{3}}}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(b\*x\*\*2+a)\*\*(2/3), x)

[Out] 3\*(a + b\*x\*\*2)\*\*(5/3)/(10\*b)

**Mathematica [A]** time = 0.00827124, size = 18, normalized size = 1.

$$\frac{3 (a + bx^2)^{5/3}}{10b}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*x^2)^(2/3), x]

[Out] (3\*(a + b\*x^2)^(5/3))/(10\*b)

**Maple [A]** time = 0.005, size = 15, normalized size = 0.8

$$\frac{3}{10b} (bx^2 + a)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(b\*x^2+a)^(2/3), x)

[Out] 3/10\*(b\*x^2+a)^(5/3)/b

---

**Maxima [A]** time = 1.32962, size = 19, normalized size = 1.06

$$\frac{3 (bx^2 + a)^{\frac{5}{3}}}{10 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3)*x,x, algorithm="maxima")`

[Out] `3/10*(b*x^2 + a)^(5/3)/b`

---

**Fricas [A]** time = 0.208338, size = 19, normalized size = 1.06

$$\frac{3 (bx^2 + a)^{\frac{5}{3}}}{10 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3)*x,x, algorithm="fricas")`

[Out] `3/10*(b*x^2 + a)^(5/3)/b`

---

**Sympy [A]** time = 1.06609, size = 42, normalized size = 2.33

$$\begin{cases} \frac{3a(a+bx^2)^{\frac{2}{3}}}{10b} + \frac{3x^2(a+bx^2)^{\frac{2}{3}}}{10} & \text{for } b \neq 0 \\ \frac{a^{\frac{2}{3}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x**2+a)**(2/3),x)`

[Out] `Piecewise((3*a*(a + b*x**2)**(2/3)/(10*b) + 3*x**2*(a + b*x**2)**(2/3)/10, Ne(b, 0)), (a**(2/3)*x**2/2, True))`

---

**GIAC/XCAS [A]** time = 0.21362, size = 19, normalized size = 1.06

$$\frac{3 (bx^2 + a)^{\frac{5}{3}}}{10 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3)*x,x, algorithm="giac")`

[Out] `3/10*(b*x^2 + a)^(5/3)/b`

$$3.681 \quad \int \frac{(a+bx^2)^{2/3}}{x} dx$$

**Optimal.** Leaf size=101

$$\frac{3}{4}a^{2/3} \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right) + \frac{1}{2}\sqrt{3}a^{2/3} \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right) - \frac{1}{2}a^{2/3} \log(x) + \frac{3}{4}(a+bx^2)^{2/3}$$

[Out] (3\*(a + b\*x^2)^(2/3))/4 + (Sqrt[3]\*a^(2/3)\*ArcTan[(a^(1/3) + 2\*(a + b\*x^2)^(1/3))/(Sqrt[3]\*a^(1/3))])/2 - (a^(2/3)\*Log[x])/2 + (3\*a^(2/3)\*Log[a^(1/3) - (a + b\*x^2)^(1/3)])/4

**Rubi [A]** time = 0.176451, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{3}{4}a^{2/3} \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right) + \frac{1}{2}\sqrt{3}a^{2/3} \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right) - \frac{1}{2}a^{2/3} \log(x) + \frac{3}{4}(a+bx^2)^{2/3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(2/3)/x, x]

[Out] (3\*(a + b\*x^2)^(2/3))/4 + (Sqrt[3]\*a^(2/3)\*ArcTan[(a^(1/3) + 2\*(a + b\*x^2)^(1/3))/(Sqrt[3]\*a^(1/3))])/2 - (a^(2/3)\*Log[x])/2 + (3\*a^(2/3)\*Log[a^(1/3) - (a + b\*x^2)^(1/3)])/4

**Rubi in Sympy [A]** time = 10.2652, size = 94, normalized size = 0.93

$$-\frac{a^{2/3} \log(x^2)}{4} + \frac{3a^{2/3} \log(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{4} + \frac{\sqrt{3}a^{2/3} \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} + 2\frac{\sqrt[3]{a+bx^2}}{3}\right)}{\sqrt[3]{a}}\right)}{2} + \frac{3(a+bx^2)^{2/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(2/3)/x, x)

[Out] -a\*\*(2/3)\*log(x\*\*2)/4 + 3\*a\*\*(2/3)\*log(a\*\*(1/3) - (a + b\*x\*\*2)\*\*(1/3))/4 + sqrt(3)\*a\*\*(2/3)\*atan(sqrt(3)\*(a\*\*(1/3)/3 + 2\*(a + b\*x\*\*2)\*\*(1/3)/3)/a\*\*(1/3))/2 + 3\*(a + b\*x\*\*2)\*\*(2/3)/4

**Mathematica [C]** time = 0.0471863, size = 61, normalized size = 0.6

$$\frac{3(a+bx^2) - 6a\sqrt[3]{\frac{a}{bx^2}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{a}{bx^2}\right)}{4\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(2/3)/x, x]

[Out] (3\*(a + b\*x^2) - 6\*a\*(1 + a/(b\*x^2))^(1/3)\*Hypergeometric2F1[1/3, 1/3, 4/3, -a/(b\*x^2)])/(4\*(a + b\*x^2)^(1/3))

**Maple [F]** time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{x} (bx^2 + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(2/3)/x,x)`

[Out] `int((b*x^2+a)^(2/3)/x,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3)/x,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.220509, size = 162, normalized size = 1.6

$$\begin{aligned} & \frac{1}{2} \sqrt{3} (a^2)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(2(bx^2 + a)^{\frac{1}{3}}a + (a^2)^{\frac{2}{3}}\right)}{3(a^2)^{\frac{2}{3}}}\right) \\ & - \frac{1}{4} (a^2)^{\frac{1}{3}} \log\left((bx^2 + a)^{\frac{2}{3}}a + (a^2)^{\frac{1}{3}}a + (bx^2 + a)^{\frac{1}{3}}(a^2)^{\frac{2}{3}}\right) \\ & + \frac{1}{2} (a^2)^{\frac{1}{3}} \log\left((bx^2 + a)^{\frac{1}{3}}a - (a^2)^{\frac{2}{3}}\right) + \frac{3}{4} (bx^2 + a)^{\frac{2}{3}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3)/x,x, algorithm="fricas")`

[Out] `1/2*sqrt(3)*(a^2)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3)*a + (a^2)^(2/3))/(a^2)^(2/3)) - 1/4*(a^2)^(1/3)*log((b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(2/3)) + 1/2*(a^2)^(1/3)*log((b*x^2 + a)^(1/3)*a - (a^2)^(2/3)) + 3/4*(b*x^2 + a)^(2/3)`

**Sympy [A]** time = 3.91863, size = 46, normalized size = 0.46

$$-\frac{b^{\frac{2}{3}}x^{\frac{4}{3}}\left(-\frac{2}{3}\right) {}_2F_1\left(-\frac{2}{3}, -\frac{2}{3} \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(2/3)/x,x)`

[Out] `-b**(2/3)*x**(4/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3, ), a*exp_polar(I*pi)/(b*x**2))/(2*gamma(1/3))`

**GIAC/XCAS [A]** time = 0.595562, size = 132, normalized size = 1.31

$$\frac{1}{2} \sqrt{3} a^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2(bx^2 + a)^{\frac{1}{3}} + a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right) - \frac{1}{4} a^{\frac{2}{3}} \ln\left(\left(bx^2 + a\right)^{\frac{2}{3}} + \left(bx^2 + a\right)^{\frac{1}{3}} a^{\frac{1}{3}} + a^{\frac{2}{3}}\right) + \frac{1}{2} a^{\frac{2}{3}} \ln\left(\left|\left(bx^2 + a\right)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right) + \frac{3}{4} (bx^2 + a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(2/3)/x,x, algorithm="giac")

[Out] 1/2\*sqrt(3)\*a^(2/3)\*arctan(1/3\*sqrt(3)\*(2\*(b\*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)) - 1/4\*a^(2/3)\*ln((b\*x^2 + a)^(2/3) + (b\*x^2 + a)^(1/3)\*a^(1/3) + a^(2/3)) + 1/2\*a^(2/3)\*ln(abs((b\*x^2 + a)^(1/3) - a^(1/3))) + 3/4\*(b\*x^2 + a)^(2/3)

$$3.682 \quad \int \frac{(a+bx^2)^{2/3}}{x^3} dx$$

**Optimal.** Leaf size=104

$$-\frac{(a+bx^2)^{2/3}}{2x^2} + \frac{b \log(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{2\sqrt[3]{a}} + \frac{b \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}} - \frac{b \log(x)}{3\sqrt[3]{a}}$$

[Out]  $-(a + b*x^2)^{(2/3)}/(2*x^2) + (b*ArcTan[(a^{(1/3)} + 2*(a + b*x^2)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(1/3)}) - (b*Log[x])/(3*a^{(1/3)}) + (b*Log[a^{(1/3)} - (a + b*x^2)^{(1/3)}])/(2*a^{(1/3)})$

**Rubi [A]** time = 0.163719, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\frac{(a+bx^2)^{2/3}}{2x^2} + \frac{b \log(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{2\sqrt[3]{a}} + \frac{b \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}} - \frac{b \log(x)}{3\sqrt[3]{a}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(2/3)/x^3, x]

[Out]  $-(a + b*x^2)^{(2/3)}/(2*x^2) + (b*ArcTan[(a^{(1/3)} + 2*(a + b*x^2)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(1/3)}) - (b*Log[x])/(3*a^{(1/3)}) + (b*Log[a^{(1/3)} - (a + b*x^2)^{(1/3)}])/(2*a^{(1/3)})$

**Rubi in Sympy [A]** time = 10.6862, size = 99, normalized size = 0.95

$$-\frac{(a+bx^2)^{2/3}}{2x^2} - \frac{b \log(x^2)}{6\sqrt[3]{a}} + \frac{b \log(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{2\sqrt[3]{a}} + \frac{\sqrt{3}b \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} + 2\frac{\sqrt[3]{a+bx^2}}{3}\right)}{\sqrt[3]{a}}\right)}{3\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(2/3)/x\*\*3, x)

[Out]  $-(a + b*x^2)^{(2/3)}/(2*x^2) - b*\log(x^2)/(6*a^{(1/3)}) + b*\log(a^{(1/3)} - (a + b*x^2)^{(1/3)})/(2*a^{(1/3)}) + \sqrt{3}*b*\operatorname{atan}(\sqrt{3}*(a^{(1/3)}/3 + 2*(a + b*x^2)^{(1/3)}/3)/a^{(1/3)})/(3*a^{(1/3)})$

**Mathematica [C]** time = 0.0420195, size = 67, normalized size = 0.64

$$\frac{-2bx^2 \sqrt[3]{\frac{a}{bx^2}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{a}{bx^2}\right) - a - bx^2}{2x^2 \sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(2/3)/x^3, x]

[Out]  $(-a - b*x^2 - 2*b*(1 + a/(b*x^2))^{(1/3)}*x^2*\operatorname{Hypergeometric2F1}[1/3, 1/3, 4/3, -(a/(b*x^2))])/(2*x^2*(a + b*x^2)^{(1/3)})$

---

**Maple [F]** time = 0.045, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} (bx^2 + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(2/3)/x^3,x)`

[Out] `int((b*x^2+a)^(2/3)/x^3,x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3)/x^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.220306, size = 173, normalized size = 1.66

$$\frac{\sqrt{3} \left( \sqrt{3} b x^2 \log \left( (b x^2 + a)^{\frac{2}{3}} a^{\frac{1}{3}} + (b x^2 + a)^{\frac{1}{3}} a^{\frac{2}{3}} + a \right) - 2 \sqrt{3} b x^2 \log \left( (b x^2 + a)^{\frac{1}{3}} a^{\frac{2}{3}} - a \right) - 6 b x^2 \arctan \left( \frac{2 \sqrt{3} (b x^2 + a)^{\frac{1}{3}} a^{\frac{2}{3}} + \sqrt{3} a}{3 a} \right) \right)}{18 a^{\frac{1}{3}} x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3)/x^3,x, algorithm="fricas")`

[Out] `-1/18*sqrt(3)*(sqrt(3)*b*x^2*log((b*x^2 + a)^(2/3)*a^(1/3) + (b*x^2 + a)^(1/3)*a^(2/3) + a) - 2*sqrt(3)*b*x^2*log((b*x^2 + a)^(1/3)*a^(2/3) - a) - 6*b*x^2*arctan(1/3*(2*sqrt(3)*(b*x^2 + a)^(1/3)*a^(2/3) + sqrt(3)*a)/a) + 3*sqrt(3)*(b*x^2 + a)^(2/3)*a^(1/3))/(a^(1/3)*x^2)`

---

**Sympy [A]** time = 4.56695, size = 42, normalized size = 0.4

$$\frac{b^{\frac{2}{3}} \left( \frac{1}{3} \right) {}_2F_1 \left( \begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{a e^{i\pi}}{b x^2} \right)}{2 x^{\frac{2}{3}} \left( \frac{4}{3} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(2/3)/x**3,x)`

[Out] `-b**(2/3)*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), a*exp_polar(I*pi)/(b*x**2))/(2*x**(2/3)*gamma(4/3))`

---

**GIAC/XCAS [A]** time = 0.585341, size = 144, normalized size = 1.38

$$\frac{1}{6} \left( \frac{2\sqrt{3} \arctan\left(\frac{\sqrt{3}(2(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}})}{3a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}} - \frac{\ln\left((bx^2+a)^{\frac{2}{3}} + (bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}} + \frac{2\ln\left(\left|(bx^2+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{a^{\frac{1}{3}}} - \frac{3(bx^2+a)^{\frac{2}{3}}}{bx^2} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(2/3)/x^3,x, algorithm="giac")

[Out] 1/6\*(2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(b\*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) - ln((b\*x^2 + a)^(2/3) + (b\*x^2 + a)^(1/3)\*a^(1/3) + a^(2/3))/a^(1/3) + 2\*ln(abs((b\*x^2 + a)^(1/3) - a^(1/3)))/a^(1/3) - 3\*(b\*x^2 + a)^(2/3)/(b\*x^2))\*b



$$3.683 \quad \int \frac{(a+bx^2)^{2/3}}{x^5} dx$$

**Optimal.** Leaf size=135

$$\frac{b^2 \log(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{12a^{4/3}} - \frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}a^{4/3}} + \frac{b^2 \log(x)}{18a^{4/3}} - \frac{b(a+bx^2)^{2/3}}{6ax^2} - \frac{(a+bx^2)^{2/3}}{4x^4}$$

[Out]  $-(a + b*x^2)^{(2/3)}/(4*x^4) - (b*(a + b*x^2)^{(2/3)})/(6*a*x^2) - (b^2*ArcTan[(a^{(1/3)} + 2*(a + b*x^2)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(6*Sqrt[3]*a^{(4/3)}) + (b^2*Log[x])/(18*a^{(4/3)}) - (b^2*Log[a^{(1/3)} - (a + b*x^2)^{(1/3)}])/(12*a^{(4/3)})$

**Rubi [A]** time = 0.219052, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$\frac{b^2 \log(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{12a^{4/3}} - \frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}a^{4/3}} + \frac{b^2 \log(x)}{18a^{4/3}} - \frac{b(a+bx^2)^{2/3}}{6ax^2} - \frac{(a+bx^2)^{2/3}}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(2/3)/x^5, x]

[Out]  $-(a + b*x^2)^{(2/3)}/(4*x^4) - (b*(a + b*x^2)^{(2/3)})/(6*a*x^2) - (b^2*ArcTan[(a^{(1/3)} + 2*(a + b*x^2)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(6*Sqrt[3]*a^{(4/3)}) + (b^2*Log[x])/(18*a^{(4/3)}) - (b^2*Log[a^{(1/3)} - (a + b*x^2)^{(1/3)}])/(12*a^{(4/3)})$

**Rubi in Sympy [A]** time = 15.4821, size = 122, normalized size = 0.9

$$\frac{(a+bx^2)^{\frac{2}{3}}}{4x^4} - \frac{b(a+bx^2)^{\frac{2}{3}}}{6ax^2} + \frac{b^2 \log(x^2)}{36a^{\frac{4}{3}}} - \frac{b^2 \log(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{12a^{\frac{4}{3}}} - \frac{\sqrt{3}b^2 \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} + 2\frac{\sqrt[3]{a+bx^2}}{3}\right)}{\sqrt[3]{a}}\right)}{18a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(2/3)/x\*\*5, x)

[Out]  $-(a + b*x^2)^{(2/3)}/(4*x^4) - b*(a + b*x^2)^{(2/3)}/(6*a*x^2) + b^2*2*log(x^2)/(36*a^{(4/3)}) - b^2*2*log(a^{(1/3)} - (a + b*x^2)^{(1/3)})/(12*a^{(4/3)}) - sqrt(3)*b^2*atan(sqrt(3)*(a^{(1/3)}/3 + 2*(a + b*x^2)^{(1/3)}/3)/a^{(1/3)})/(18*a^{(4/3)})$

**Mathematica [C]** time = 0.0455761, size = 83, normalized size = 0.61

$$\frac{-3a^2 + 2b^2x^4\sqrt{\frac{a}{bx^2}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{a}{bx^2}\right) - 5abx^2 - 2b^2x^4}{12ax^4\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(2/3)/x^5, x]

[Out]  $(-3*a^2 - 5*a*b*x^2 - 2*b^2*x^4 + 2*b^2*(1 + a/(b*x^2))^{1/3}) * x^4$   
 $* \text{Hypergeometric2F1}[1/3, 1/3, 4/3, -(a/(b*x^2))]/(12*a*x^4*(a + b$   
 $* x^2)^{1/3})$

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{x^5} (bx^2 + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(2/3)/x^5,x)`

[Out] `int((b*x^2+a)^(2/3)/x^5,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3)/x^5,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.220498, size = 219, normalized size = 1.62

$$\frac{\sqrt{3} \left( \sqrt{3} b^2 x^4 \log \left( (bx^2 + a)^{\frac{2}{3}} (-a)^{\frac{1}{3}} - (bx^2 + a)^{\frac{1}{3}} (-a)^{\frac{2}{3}} - a \right) - 2 \sqrt{3} b^2 x^4 \log \left( (bx^2 + a)^{\frac{1}{3}} (-a)^{\frac{2}{3}} - a \right) - 6 b^2 x^4 \arctan \left( \frac{2\sqrt{3}}{\dots} \right) \right)}{108 (-a)^{\frac{1}{3}} a x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3)/x^5,x, algorithm="fricas")`

[Out]  $-1/108 * \text{sqrt}(3) * (\text{sqrt}(3) * b^2 * x^4 * \log((b*x^2 + a)^{2/3} * (-a)^{1/3})$   
 $- (b*x^2 + a)^{1/3} * (-a)^{2/3} - a) - 2 * \text{sqrt}(3) * b^2 * x^4 * \log((b*x^2$   
 $+ a)^{1/3} * (-a)^{2/3} - a) - 6 * b^2 * x^4 * \arctan(1/3 * (2 * \text{sqrt}(3) * (b$   
 $* x^2 + a)^{1/3} * (-a)^{2/3} + \text{sqrt}(3) * a) / a) + 3 * \text{sqrt}(3) * (2 * b * x^2 +$   
 $3 * a) * (b * x^2 + a)^{2/3} * (-a)^{1/3}) / ((-a)^{1/3} * a * x^4)$

**Sympy [A]** time = 5.76814, size = 42, normalized size = 0.31

$$\frac{b^{\frac{2}{3}} \left( \frac{4}{3} \right) {}_2F_1 \left( \begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{ae^{i\pi}}{bx^2} \right)}{2x^{\frac{8}{3}} \left( \frac{7}{3} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(2/3)/x**5,x)`

[Out]  $-b^{2/3} * \text{gamma}(4/3) * \text{hyper}((-2/3, 4/3), (7/3, ), a * \text{exp\_polar}(I * \text{pi})$   
 $/(b * x^{**2})) / (2 * x^{**8/3}) * \text{gamma}(7/3)$

---

**GIAC/XCAS [A]** time = 0.611818, size = 170, normalized size = 1.26

$$-\frac{1}{36}b^2 \left( \frac{2\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}} - \frac{\ln\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}} + \frac{2\ln\left(\left|(bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}} + \frac{3\left(2(bx^2+a)^{\frac{5}{3}}+(bx^2+a)^{\frac{2}{3}}a\right)}{a^{\frac{4}{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(2/3)/x^5,x, algorithm="giac")

[Out] -1/36\*b^2\*(2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(b\*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(4/3) - ln((b\*x^2 + a)^(2/3) + (b\*x^2 + a)^(1/3)\*a^(1/3) + a^(2/3))/a^(4/3) + 2\*ln(abs((b\*x^2 + a)^(1/3) - a^(1/3)))/a^(4/3) + 3\*(2\*(b\*x^2 + a)^(5/3) + (b\*x^2 + a)^(2/3)\*a)/(a\*b^2\*x^4)

$$3.684 \quad \int x^4 (a + bx^2)^{2/3} dx$$

**Optimal.** Leaf size=601

$$\frac{108\sqrt{23}^{3/4}a^{10/3} \left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)^2}} F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}\right) \mid -7 + 4\sqrt{3}\right)}{1729b^3x \sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)^2}}}$$

$$+ \frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3} \left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)^2}} E\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}\right) \mid -7 + 4\sqrt{3}\right)}{1729b^3x \sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)^2}}}$$

$$- \frac{324a^3x}{1729b^2 \left((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)} - \frac{108a^2x(a+bx^2)^{2/3}}{1729b^2} + \frac{3}{19}x^5(a+bx^2)^{2/3} + \frac{12ax^3(a+bx^2)^{2/3}}{247b}$$

[Out]  $(-108*a^2*x*(a + b*x^2)^(2/3))/(1729*b^2) + (12*a*x^3*(a + b*x^2)^(2/3))/(247*b) + (3*x^5*(a + b*x^2)^(2/3))/19 - (324*a^3*x)/(1729*b^2*((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3))) + (162*3^(1/4)*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^(10/3)*(a^(1/3) - (a + b*x^2)^(1/3))*\text{Sqrt}[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3))]^2*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3)]/((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*\text{Sqrt}[3])]/(1729*b^3*x*\text{Sqrt}[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3)))^2]) - (108*\text{Sqrt}[2]*3^(3/4)*a^(10/3)*(a^(1/3) - (a + b*x^2)^(1/3))*\text{Sqrt}[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3))]^2*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3)]/((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*\text{Sqrt}[3])]/(1729*b^3*x*\text{Sqrt}[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3)))^2])$

**Rubi [A]** time = 1.03129, antiderivative size = 601, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{108\sqrt{23}^{3/4}a^{10/3} \left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)^2}} F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}\right) \mid -7 + 4\sqrt{3}\right)}{1729b^3x \sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)^2}}}$$

$$+ \frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3} \left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)^2}} E\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}\right) \mid -7 + 4\sqrt{3}\right)}{1729b^3x \sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)^2}}}$$

$$- \frac{324a^3x}{1729b^2 \left((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)} - \frac{108a^2x(a+bx^2)^{2/3}}{1729b^2} + \frac{3}{19}x^5(a+bx^2)^{2/3} + \frac{12ax^3(a+bx^2)^{2/3}}{247b}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a + b\*x^2)^(2/3), x]

[Out]  $(-108*a^2*x*(a + b*x^2)^{(2/3)})/(1729*b^2) + (12*a*x^3*(a + b*x^2)^{(2/3)})/(247*b) + (3*x^5*(a + b*x^2)^{(2/3)})/19 - (324*a^3*x)/(1729*b^2*((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})) + (162*3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^{(10/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})]/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticE}[\text{ArcSin}(((1 + \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3])]/(1729*b^3*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]) - (108*\text{Sqrt}[2]*3^{(3/4)}*a^{(10/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})]/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}(((1 + \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})), -7 + 4*\text{Sqrt}[3])]/(1729*b^3*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2])]$

**Rubi in Sympy [A]** time = 46.8648, size = 502, normalized size = 0.84

$$\frac{162\sqrt[4]{3}a^{\frac{10}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a+bx^2+(a+bx^2)^{\frac{2}{3}}}}{(\sqrt[3]{a(-1+\sqrt{3})}+\sqrt[3]{a+bx^2})^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)E\left(\text{asin}\left(\frac{\sqrt[3]{a(1+\sqrt{3})}-\sqrt[3]{a+bx^2}}{-\sqrt[3]{a(-1+\sqrt{3})}-\sqrt[3]{a+bx^2}}\right)\right)\left|-7+4\sqrt{3}\right.}{1729b^3x\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{a+bx^2})}{(\sqrt[3]{a(-1+\sqrt{3})}+\sqrt[3]{a+bx^2})^2}}}$$

$$\frac{108\sqrt{2}\cdot 3^{\frac{3}{4}}a^{\frac{10}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a+bx^2+(a+bx^2)^{\frac{2}{3}}}}{(\sqrt[3]{a(-1+\sqrt{3})}+\sqrt[3]{a+bx^2})^2}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)F\left(\text{asin}\left(\frac{\sqrt[3]{a(1+\sqrt{3})}-\sqrt[3]{a+bx^2}}{-\sqrt[3]{a(-1+\sqrt{3})}-\sqrt[3]{a+bx^2}}\right)\right)\left|-7+4\sqrt{3}\right.}{1729b^3x\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{a+bx^2})}{(\sqrt[3]{a(-1+\sqrt{3})}+\sqrt[3]{a+bx^2})^2}}}$$

$$+\frac{324a^3x}{1729b^2\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)}-\frac{108a^2x(a+bx^2)^{\frac{2}{3}}}{1729b^2}+\frac{12ax^3(a+bx^2)^{\frac{2}{3}}}{247b}+\frac{3x^5(a+bx^2)^{\frac{2}{3}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**4*(b*x**2+a)**(2/3),x)`

[Out]  $162*3^{(1/4)}*a^{(10/3)}*\text{sqrt}((a^{(2/3)} + a^{(1/3)}*(a + b*x**2))^{(1/3)} + (a + b*x**2)^{(2/3)})/(a^{(1/3)}*(-1 + \text{sqrt}(3)) + (a + b*x**2)^{(1/3)})^{(1/3)}*2*\text{sqrt}(\text{sqrt}(3) + 2)*(a^{(1/3)} - (a + b*x**2)^{(1/3)})*\text{elliptic}_e(\text{asin}((a^{(1/3)}*(1 + \text{sqrt}(3)) - (a + b*x**2)^{(1/3)})/(-a^{(1/3)}*(-1 + \text{sqrt}(3)) - (a + b*x**2)^{(1/3)})), -7 + 4*\text{sqrt}(3))/((1729*b**3*x*\text{sqrt}(-a^{(1/3)}*(a^{(1/3)} - (a + b*x**2)^{(1/3)}))/((a^{(1/3)}*(-1 + \text{sqrt}(3)) + (a + b*x**2)^{(1/3)})^{(1/3)} - (a + b*x**2)^{(1/3)})) - 108*\text{sqrt}(2)*3^{(3/4)}*a^{(10/3)}*\text{sqrt}((a^{(2/3)} + a^{(1/3)}*(a + b*x**2)^{(1/3)} + (a + b*x**2)^{(2/3)}))/((a^{(1/3)}*(-1 + \text{sqrt}(3)) + (a + b*x**2)^{(1/3)})^{(1/3)} - (a + b*x**2)^{(1/3)})))*\text{elliptic}_f(\text{asin}((a^{(1/3)}*(1 + \text{sqrt}(3)) - (a + b*x**2)^{(1/3)})/(-a^{(1/3)}*(-1 + \text{sqrt}(3)) - (a + b*x**2)^{(1/3)})), -7 + 4*\text{sqrt}(3))/((1729*b**3*x*\text{sqrt}(-a^{(1/3)}*(a^{(1/3)}*(-1 + \text{sqrt}(3)) - (a + b*x**2)^{(1/3)}))/((a^{(1/3)}*(-1 + \text{sqrt}(3)) + (a + b*x**2)^{(1/3)})^{(1/3)} - (a + b*x**2)^{(1/3)})) + 324*a**3*x/(1729*b**2*(a^{(1/3)}*(-1 + \text{sqrt}(3)) + (a + b*x**2)^{(1/3)})) - 108*a**2*x*(a + b*x**2)^{(2/3)}/(1729*b**2) + 12*a*x**3*(a + b*x**2)^{(2/3)}/(247*b) + 3*x**5*(a + b*x**2)^{(2/3)}/19$

**Mathematica [C]** time = 0.0583495, size = 90, normalized size = 0.15

$$\frac{3\left(36a^3x\sqrt{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) - 36a^3x - 8a^2bx^3 + 119ab^2x^5 + 91b^3x^7\right)}{1729b^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a + b\*x^2)^(2/3), x]

[Out] (3\*(-36\*a^3\*x - 8\*a^2\*b\*x^3 + 119\*a\*b^2\*x^5 + 91\*b^3\*x^7 + 36\*a^3\*x\*(1 + (b\*x^2)/a)^(1/3)\*Hypergeometric2F1[1/3, 1/2, 3/2, -((b\*x^2)/a)]))/(1729\*b^2\*(a + b\*x^2)^(1/3))

**Maple** [F] time = 0.039, size = 0, normalized size = 0.

$$\int x^4 (bx^2 + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(b\*x^2+a)^(2/3), x)

[Out] int(x^4\*(b\*x^2+a)^(2/3), x)

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{2}{3}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(2/3)\*x^4, x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(2/3)\*x^4, x)

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^{\frac{2}{3}} x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(2/3)\*x^4, x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(2/3)\*x^4, x)

**Sympy** [A] time = 3.28974, size = 29, normalized size = 0.05

$$\frac{a^{\frac{2}{3}} x^5 {}_2F_1\left(-\frac{2}{3}, \frac{5}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(b\*x\*\*2+a)\*\*(2/3), x)

[Out] a\*\*(2/3)\*x\*\*5\*hyper((-2/3, 5/2), (7/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/5

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{2}{3}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3)*x^4,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(2/3)*x^4, x)`

### 3.685 $\int x^2 (a + bx^2)^{2/3} dx$

**Optimal.** Leaf size=577

$$\frac{12\sqrt{2}3^{3/4}a^{7/3} \left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)^2}} F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}\right) \mid -7 + 4\sqrt{3}\right)}{91b^2x \sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)^2}}}$$

$$\frac{18\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3} \left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)^2}} E\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}\right) \mid -7 + 4\sqrt{3}\right)}{91b^2x \sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)^2}}}$$

$$+ \frac{36a^2x}{91b \left(\left(1 - \sqrt{3}\right) \sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)} + \frac{12ax (a + bx^2)^{2/3}}{91b} + \frac{3}{13}x^3 (a + bx^2)^{2/3}$$

[Out]  $(12*a*x*(a + b*x^2)^(2/3))/(91*b) + (3*x^3*(a + b*x^2)^(2/3))/13 + (36*a^2*x)/(91*b*((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3))) - (18*3^(1/4)*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^(7/3)*(a^(1/3) - (a + b*x^2)^(1/3))*\text{Sqrt}[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))]/((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3)]/((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3)), -7 + 4*\text{Sqrt}[3]]/(91*b^2*x*\text{Sqrt}[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) + (12*\text{Sqrt}[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a + b*x^2)^(1/3))*\text{Sqrt}[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))]/((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3)]/((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3)), -7 + 4*\text{Sqrt}[3]]/(91*b^2*x*\text{Sqrt}[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])]$

**Rubi [A]** time = 0.884357, antiderivative size = 577, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{12\sqrt{2}3^{3/4}a^{7/3} \left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)^2}} F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}\right) \mid -7 + 4\sqrt{3}\right)}{91b^2x \sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)^2}}}$$

$$\frac{18\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3} \left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)^2}} E\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}\right) \mid -7 + 4\sqrt{3}\right)}{91b^2x \sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)^2}}}$$

$$+ \frac{36a^2x}{91b \left(\left(1 - \sqrt{3}\right) \sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)} + \frac{12ax (a + bx^2)^{2/3}}{91b} + \frac{3}{13}x^3 (a + bx^2)^{2/3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*(a + b*x^2)^(2/3), x]$

[Out]  $(12*a*x*(a + b*x^2)^(2/3))/(91*b) + (3*x^3*(a + b*x^2)^(2/3))/13 + (36*a^2*x)/(91*b*((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3))) -$



$$\frac{(18 \cdot 3^{1/4} \cdot \sqrt{2 + \sqrt{3}}) \cdot a^{7/3} \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3}) \cdot \sqrt{(a^{2/3} + a^{1/3} \cdot (a + b \cdot x^2)^{1/3} + (a + b \cdot x^2)^{2/3})}}{((1 - \sqrt{3}) \cdot a^{1/3} - (a + b \cdot x^2)^{1/3})^2} \cdot \text{EllipticE}\left[\text{ArcSin}\left[\frac{((1 + \sqrt{3}) \cdot a^{1/3} - (a + b \cdot x^2)^{1/3})}{((1 - \sqrt{3}) \cdot a^{1/3} - (a + b \cdot x^2)^{1/3})}\right], -7 + 4 \cdot \sqrt{3}\right] / (91 \cdot b^2 \cdot x \cdot \sqrt{-(a^{1/3} \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3})) / ((1 - \sqrt{3}) \cdot a^{1/3} - (a + b \cdot x^2)^{1/3})^2}) + (12 \cdot \sqrt{2} \cdot 3^{3/4} \cdot a^{7/3} \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3}) \cdot \sqrt{(a^{2/3} + a^{1/3} \cdot (a + b \cdot x^2)^{1/3} + (a + b \cdot x^2)^{2/3})}}{((1 - \sqrt{3}) \cdot a^{1/3} - (a + b \cdot x^2)^{1/3})^2} \cdot \text{EllipticF}\left[\text{ArcSin}\left[\frac{((1 + \sqrt{3}) \cdot a^{1/3} - (a + b \cdot x^2)^{1/3})}{((1 - \sqrt{3}) \cdot a^{1/3} - (a + b \cdot x^2)^{1/3})}\right], -7 + 4 \cdot \sqrt{3}\right] / (91 \cdot b^2 \cdot x \cdot \sqrt{-(a^{1/3} \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3})) / ((1 - \sqrt{3}) \cdot a^{1/3} - (a + b \cdot x^2)^{1/3})^2})$$

**Rubi in Sympy [A]** time = 37.8257, size = 476, normalized size = 0.82

$$\frac{18 \sqrt[3]{3} a^{\frac{7}{3}} \sqrt{\frac{a^{\frac{2}{3}} + \sqrt[3]{a} \sqrt{a + bx^2} + (a + bx^2)^{\frac{2}{3}}}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}} \sqrt{\sqrt{3} + 2} \left(\sqrt[3]{a} - \sqrt[3]{a + bx^2}\right) E\left(\text{asin}\left(\frac{\sqrt[3]{a}(1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}{-\sqrt[3]{a}(-1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}\right)\right) \Big|_{-7 + 4\sqrt{3}}}{91 b^2 x \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{a + bx^2})}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}} + \frac{12 \sqrt{2} \cdot 3^{\frac{3}{4}} a^{\frac{7}{3}} \sqrt{\frac{a^{\frac{2}{3}} + \sqrt[3]{a} \sqrt{a + bx^2} + (a + bx^2)^{\frac{2}{3}}}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}} \left(\sqrt[3]{a} - \sqrt[3]{a + bx^2}\right) F\left(\text{asin}\left(\frac{\sqrt[3]{a}(1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}{-\sqrt[3]{a}(-1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}\right)\right) \Big|_{-7 + 4\sqrt{3}}}{91 b^2 x \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{a + bx^2})}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}} - \frac{36 a^2 x}{91 b (\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})} + \frac{12 a x (a + bx^2)^{\frac{2}{3}}}{91 b} + \frac{3 x^3 (a + bx^2)^{\frac{2}{3}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2*(b*x**2+a)**(2/3),x)`

[Out]  $-18 \cdot 3^{1/4} \cdot a^{7/3} \cdot \sqrt{(a^{2/3} + a^{1/3} \cdot (a + b \cdot x^2)^{1/3} + (a + b \cdot x^2)^{2/3})} / (a^{1/3} \cdot (-1 + \sqrt{3}) + (a + b \cdot x^2)^{1/3})^2 \cdot \sqrt{(\sqrt{3} + 2) \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3})} \cdot \text{elliptic\_e}(\text{asin}((a^{1/3} \cdot (1 + \sqrt{3}) - (a + b \cdot x^2)^{1/3}) / (-a^{1/3} \cdot (-1 + \sqrt{3}) - (a + b \cdot x^2)^{1/3})), -7 + 4 \cdot \sqrt{3}) / (91 \cdot b^2 \cdot x \cdot \sqrt{-(a^{1/3} \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3})) / (a^{1/3} \cdot (-1 + \sqrt{3}) - (a + b \cdot x^2)^{1/3})^2}) + 12 \cdot \sqrt{2} \cdot 3^{3/4} \cdot a^{7/3} \cdot \sqrt{(a^{2/3} + a^{1/3} \cdot (a + b \cdot x^2)^{1/3} + (a + b \cdot x^2)^{2/3})} / (a^{1/3} \cdot (-1 + \sqrt{3}) + (a + b \cdot x^2)^{1/3})^2 \cdot \text{elliptic\_f}(\text{asin}((a^{1/3} \cdot (1 + \sqrt{3}) - (a + b \cdot x^2)^{1/3}) / (-a^{1/3} \cdot (-1 + \sqrt{3}) - (a + b \cdot x^2)^{1/3})), -7 + 4 \cdot \sqrt{3}) / (91 \cdot b^2 \cdot x \cdot \sqrt{-(a^{1/3} \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3})) / (a^{1/3} \cdot (-1 + \sqrt{3}) - (a + b \cdot x^2)^{1/3})^2}) - 36 \cdot a^2 \cdot x / (91 \cdot b \cdot (a^{1/3} \cdot (-1 + \sqrt{3}) + (a + b \cdot x^2)^{1/3})) + 12 \cdot a \cdot x \cdot (a + b \cdot x^2)^{2/3} / (91 \cdot b) + 3 \cdot x^3 \cdot (a + b \cdot x^2)^{2/3} / 13$

**Mathematica [C]** time = 0.0533776, size = 79, normalized size = 0.14

$$\frac{3 \left( -4a^2 x \sqrt{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) + 4a^2 x + 11abx^3 + 7b^2 x^5 \right)}{91 b \sqrt[3]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*x^2)^(2/3), x]

[Out]  $(3*(4*a^2*x + 11*a*b*x^3 + 7*b^2*x^5 - 4*a^2*x*(1 + (b*x^2)/a))^{1/3} * \text{Hypergeometric2F1}[1/3, 1/2, 3/2, -((b*x^2)/a)]) / (91*b*(a + b*x^2)^{1/3})$

**Maple** [F] time = 0.035, size = 0, normalized size = 0.

$$\int x^2 (bx^2 + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(b\*x^2+a)^(2/3), x)

[Out] int(x^2\*(b\*x^2+a)^(2/3), x)

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{2}{3}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(2/3)\*x^2, x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(2/3)\*x^2, x)

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^{\frac{2}{3}} x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(2/3)\*x^2, x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(2/3)\*x^2, x)

**Sympy** [A] time = 2.69102, size = 29, normalized size = 0.05

$$\frac{a^{\frac{2}{3}} x^3 {}_2F_1\left(-\frac{2}{3}, \frac{3}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(b\*x\*\*2+a)\*\*(2/3), x)

[Out]  $a^{2/3} x^3 \text{hyper}((-2/3, 3/2), (5/2, ), b*x**2*\text{exp\_polar}(I*\text{pi})/a) / 3$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{2}{3}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(2/3)*x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(2/3)*x^2, x)
```

### 3.686 $\int (a + bx^2)^{2/3} dx$

**Optimal.** Leaf size=550

$$\frac{4\sqrt{23}^{3/4} a^{4/3} \left(\sqrt[3]{a} - \sqrt[3]{a + bx^2}\right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left((1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2}\right)^2}} F\left(\sin^{-1}\left(\frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}\right) \mid -7 + 4\sqrt{3}\right)}{7bx \sqrt{\frac{\sqrt[3]{a} \left(\sqrt[3]{a} - \sqrt[3]{a + bx^2}\right)}{\left((1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2}\right)^2}}}$$

$$+ \frac{6\sqrt{3}\sqrt{2 + \sqrt{3}} a^{4/3} \left(\sqrt[3]{a} - \sqrt[3]{a + bx^2}\right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left((1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2}\right)^2}} E\left(\sin^{-1}\left(\frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}\right) \mid -7 + 4\sqrt{3}\right)}{7bx \sqrt{\frac{\sqrt[3]{a} \left(\sqrt[3]{a} - \sqrt[3]{a + bx^2}\right)}{\left((1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2}\right)^2}}}$$

$$- \frac{12ax}{7\left((1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2}\right)} + \frac{3}{7}x(a + bx^2)^{2/3}$$

[Out] (3\*x\*(a + b\*x^2)^(2/3))/7 - (12\*a\*x)/(7\*((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))) + (6\*3^(1/4)\*Sqrt[2 + Sqrt[3]]\*a^(4/3)\*(a^(1/3) - (a + b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a + b\*x^2)^(1/3) + (a + b\*x^2)^(2/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))]^2\*EllipticE[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))/( (1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))], -7 + 4\*Sqrt[3]])/(7\*b\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a + b\*x^2)^(1/3)))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3)))^2]) - (4\*Sqrt[2]\*3^(3/4)\*a^(4/3)\*(a^(1/3) - (a + b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a + b\*x^2)^(1/3) + (a + b\*x^2)^(2/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))]^2\*EllipticF[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))/( (1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))], -7 + 4\*Sqrt[3]])/(7\*b\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a + b\*x^2)^(1/3)))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3)))^2])

**Rubi [A]** time = 0.732176, antiderivative size = 550, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$

$$\frac{4\sqrt{23}^{3/4} a^{4/3} \left(\sqrt[3]{a} - \sqrt[3]{a + bx^2}\right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left((1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2}\right)^2}} F\left(\sin^{-1}\left(\frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}\right) \mid -7 + 4\sqrt{3}\right)}{7bx \sqrt{\frac{\sqrt[3]{a} \left(\sqrt[3]{a} - \sqrt[3]{a + bx^2}\right)}{\left((1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2}\right)^2}}}$$

$$+ \frac{6\sqrt{3}\sqrt{2 + \sqrt{3}} a^{4/3} \left(\sqrt[3]{a} - \sqrt[3]{a + bx^2}\right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left((1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2}\right)^2}} E\left(\sin^{-1}\left(\frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}\right) \mid -7 + 4\sqrt{3}\right)}{7bx \sqrt{\frac{\sqrt[3]{a} \left(\sqrt[3]{a} - \sqrt[3]{a + bx^2}\right)}{\left((1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2}\right)^2}}}$$

$$- \frac{12ax}{7\left((1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2}\right)} + \frac{3}{7}x(a + bx^2)^{2/3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(2/3), x]

[Out] (3\*x\*(a + b\*x^2)^(2/3))/7 - (12\*a\*x)/(7\*((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))) + (6\*3^(1/4)\*Sqrt[2 + Sqrt[3]]\*a^(4/3)\*(a^(1/3) - (a + b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a + b\*x^2)^(1/3) + (a + b\*x^2)^(2/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))]^2\*EllipticE[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))/( (1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))], -7 + 4\*Sqrt[3]])/(7\*b\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a + b\*x^2)^(1/3)))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3)))^2]) - (4\*Sqrt[2]\*3^(3/4)\*a^(4/3)\*(a^(1/3) - (a + b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a + b\*x^2)^(1/3) + (a + b\*x^2)^(2/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))]^2\*EllipticF[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))/( (1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))], -7 + 4\*Sqrt[3]])/(7\*b\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a + b\*x^2)^(1/3)))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3)))^2])

$$\begin{aligned}
& + (a + b^2 x^2)^{2/3} / ((1 - \sqrt{3}) a^{1/3} - (a + b^2 x^2)^{1/3}) \\
& \cdot \text{EllipticE}[\text{ArcSin}(((1 + \sqrt{3}) a^{1/3} - (a + b^2 x^2)^{1/3}) / \\
& ((1 - \sqrt{3}) a^{1/3} - (a + b^2 x^2)^{1/3})), -7 + 4\sqrt{3}] / (7 \\
& b^2 x \sqrt{-(a^{1/3} (a^{1/3} - (a + b^2 x^2)^{1/3})) / ((1 - \sqrt{3}) \\
& ) a^{1/3} - (a + b^2 x^2)^{1/3})^2}) - (4 \sqrt{2} \cdot 3^{3/4} a^{4/3} \\
& (a^{1/3} - (a + b^2 x^2)^{1/3}) \sqrt{(a^{2/3} + a^{1/3} (a + b^2 x^2)^{1/3} + \\
& (a + b^2 x^2)^{2/3})} / ((1 - \sqrt{3}) a^{1/3} - (a + b^2 x^2)^{1/3})^2) \\
& \cdot \text{EllipticF}[\text{ArcSin}(((1 + \sqrt{3}) a^{1/3} - (a + b^2 x^2)^{1/3}) / \\
& ((1 - \sqrt{3}) a^{1/3} - (a + b^2 x^2)^{1/3})), -7 + 4\sqrt{3}] \\
& ] / (7 b^2 x \sqrt{-(a^{1/3} (a^{1/3} - (a + b^2 x^2)^{1/3})) / ((1 - \sqrt{3}) \\
& ) a^{1/3} - (a + b^2 x^2)^{1/3})^2})
\end{aligned}$$

**Rubi in Sympy [A]** time = 26.8005, size = 450, normalized size = 0.82

$$\begin{aligned}
& \frac{6\sqrt{3}a^{\frac{4}{3}} \sqrt{\frac{a^{\frac{2}{3}} + \sqrt[3]{a}\sqrt{a+bx^2} + (a+bx^2)^{\frac{2}{3}}}{(\sqrt[3]{a}(-1+\sqrt{3}) + \sqrt[3]{a+bx^2})^2}} \sqrt{\sqrt{3}+2} (\sqrt[3]{a} - \sqrt[3]{a+bx^2}) E\left(\text{asin}\left(\frac{\sqrt[3]{a}(1+\sqrt{3}) - \sqrt[3]{a+bx^2}}{-\sqrt[3]{a}(-1+\sqrt{3}) - \sqrt[3]{a+bx^2}}\right)\right) \Big|_{-7+4\sqrt{3}}}{7bx \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{(\sqrt[3]{a}(-1+\sqrt{3}) + \sqrt[3]{a+bx^2})^2}}} \\
& - \frac{4\sqrt{2} \cdot 3^{\frac{3}{4}} a^{\frac{4}{3}} \sqrt{\frac{a^{\frac{2}{3}} + \sqrt[3]{a}\sqrt{a+bx^2} + (a+bx^2)^{\frac{2}{3}}}{(\sqrt[3]{a}(-1+\sqrt{3}) + \sqrt[3]{a+bx^2})^2}} (\sqrt[3]{a} - \sqrt[3]{a+bx^2}) F\left(\text{asin}\left(\frac{\sqrt[3]{a}(1+\sqrt{3}) - \sqrt[3]{a+bx^2}}{-\sqrt[3]{a}(-1+\sqrt{3}) - \sqrt[3]{a+bx^2}}\right)\right) \Big|_{-7+4\sqrt{3}}}{7bx \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{(\sqrt[3]{a}(-1+\sqrt{3}) + \sqrt[3]{a+bx^2})^2}}} \\
& + \frac{12ax}{7(\sqrt[3]{a}(-1+\sqrt{3}) + \sqrt[3]{a+bx^2})} + \frac{3x(a+bx^2)^{\frac{2}{3}}}{7}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(2/3),x)`

[Out]  $6 \cdot 3^{1/4} \cdot a^{4/3} \cdot \sqrt{(a^{2/3} + a^{1/3} (a + b^2 x^2)^{1/3}) / (a^{1/3} (-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3})} \cdot \sqrt{(a^{1/3} (a^{1/3} - (a + b^2 x^2)^{1/3})) / (a^{1/3} (-1 + \sqrt{3}) - (a + b^2 x^2)^{1/3})} \cdot \text{elliptic\_e}(\text{asin}((a^{1/3} (1 + \sqrt{3}) - (a + b^2 x^2)^{1/3}) / (-a^{1/3} (-1 + \sqrt{3}) - (a + b^2 x^2)^{1/3})), -7 + 4\sqrt{3}) / (7 b^2 x \sqrt{-(a^{1/3} (a^{1/3} - (a + b^2 x^2)^{1/3})) / (a^{1/3} (-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3})^2}) - 4 \sqrt{2} \cdot 3^{3/4} a^{4/3} \sqrt{(a^{2/3} + a^{1/3} (a + b^2 x^2)^{1/3} + (a + b^2 x^2)^{2/3})} / (a^{1/3} (-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3})^2) \cdot \text{elliptic\_f}(\text{asin}((a^{1/3} (1 + \sqrt{3}) - (a + b^2 x^2)^{1/3}) / (-a^{1/3} (-1 + \sqrt{3}) - (a + b^2 x^2)^{1/3})), -7 + 4\sqrt{3}) / (7 b^2 x \sqrt{-(a^{1/3} (a^{1/3} - (a + b^2 x^2)^{1/3})) / (a^{1/3} (-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3})^2}) + 12 a x / (7 (a^{1/3} (-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3})) + 3 x (a + b^2 x^2)^{2/3} / 7$

**Mathematica [C]** time = 0.0405825, size = 63, normalized size = 0.11

$$\frac{4ax \sqrt{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) + 3x(a+bx^2)}{7\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(2/3),x]`

[Out]  $(3*x*(a + b*x^2) + 4*a*x*(1 + (b*x^2)/a)^{(1/3)}*Hypergeometric2F1[1/3, 1/2, 3/2, -((b*x^2)/a)])/(7*(a + b*x^2)^{(1/3)})$

**Maple [F]** time = 0.042, size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(2/3), x)`

[Out] `int((b*x^2+a)^(2/3), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3), x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(2/3), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3), x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(2/3), x)`

**Sympy [A]** time = 2.27398, size = 26, normalized size = 0.05

$$a^{\frac{2}{3}} x {}_2F_1\left(-\frac{2}{3}, \frac{1}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(2/3), x)`

[Out] `a**(2/3)*x*hyper((-2/3, 1/2), (3/2, ), b*x**2*exp_polar(I*pi)/a)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(2/3), x)
```

$$3.687 \quad \int \frac{(a+bx^2)^{2/3}}{x^2} dx$$

**Optimal.** Leaf size=538

$$\frac{4\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|_{-7+4\sqrt{3}}\right)}{\sqrt[4]{3}x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$+ \frac{2\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|_{-7+4\sqrt{3}}\right)}{x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$- \frac{4bx}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}} - \frac{(a+bx^2)^{2/3}}{x}$$

[Out]  $-\left((a+b*x^2)^{(2/3)}/x\right) - (4*b*x)/((1-\text{Sqrt}[3])*a^{(1/3)} - (a+b*x^2)^{(1/3)}) + (2*3^{(1/4)}*\text{Sqrt}[2+\text{Sqrt}[3]]*a^{(1/3)}*(a^{(1/3)} - (a+b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)}+a^{(1/3)}*(a+b*x^2)^{(1/3)}+(a+b*x^2)^{(2/3)})]/((1-\text{Sqrt}[3])*a^{(1/3)} - (a+b*x^2)^{(1/3)})^2)*\text{EllipticE}[\text{ArcSin}[(\left(1+\text{Sqrt}[3]\right)*a^{(1/3)} - (a+b*x^2)^{(1/3)})/(\left(1-\text{Sqrt}[3]\right)*a^{(1/3)} - (a+b*x^2)^{(1/3)})], -7+4*\text{Sqrt}[3]]/(x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a+b*x^2)^{(1/3)}))/((1-\text{Sqrt}[3])*a^{(1/3)} - (a+b*x^2)^{(1/3)})^2)]) - (4*\text{Sqrt}[2]*a^{(1/3)}*(a^{(1/3)} - (a+b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)}+a^{(1/3)}*(a+b*x^2)^{(1/3)}+(a+b*x^2)^{(2/3)})]/((1-\text{Sqrt}[3])*a^{(1/3)} - (a+b*x^2)^{(1/3)})^2)*\text{EllipticF}[\text{ArcSin}[(\left(1+\text{Sqrt}[3]\right)*a^{(1/3)} - (a+b*x^2)^{(1/3)})/(\left(1-\text{Sqrt}[3]\right)*a^{(1/3)} - (a+b*x^2)^{(1/3)})], -7+4*\text{Sqrt}[3]]/(3^{(1/4)}*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a+b*x^2)^{(1/3)}))/((1-\text{Sqrt}[3])*a^{(1/3)} - (a+b*x^2)^{(1/3)})^2)])$

**Rubi [A]** time = 0.724622, antiderivative size = 538, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{4\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|_{-7+4\sqrt{3}}\right)}{\sqrt[4]{3}x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$+ \frac{2\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|_{-7+4\sqrt{3}}\right)}{x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$- \frac{4bx}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}} - \frac{(a+bx^2)^{2/3}}{x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a+b*x^2)^{(2/3)}/x^2, x]$

[Out]  $-\left((a+b*x^2)^{(2/3)}/x\right) - (4*b*x)/((1-\text{Sqrt}[3])*a^{(1/3)} - (a+b*x^2)^{(1/3)}) + (2*3^{(1/4)}*\text{Sqrt}[2+\text{Sqrt}[3]]*a^{(1/3)}*(a^{(1/3)} - (a$



$$+ b^*x^2)^{(1/3)} * \text{Sqrt}[(a^{(2/3)} + a^{(1/3)} * (a + b^*x^2)^{(1/3)} + (a + b^*x^2)^{(2/3)}) / ((1 - \text{Sqrt}[3]) * a^{(1/3)} - (a + b^*x^2)^{(1/3)})^2] * \text{EllipticE}[\text{ArcSin}[\frac{(1 + \text{Sqrt}[3]) * a^{(1/3)} - (a + b^*x^2)^{(1/3)}}{(1 - \text{Sqrt}[3]) * a^{(1/3)} - (a + b^*x^2)^{(1/3)}}], -7 + 4 * \text{Sqrt}[3]] / (x * \text{Sqrt}[-((a^{(1/3)} * (a^{(1/3)} - (a + b^*x^2)^{(1/3)})) / ((1 - \text{Sqrt}[3]) * a^{(1/3)} - (a + b^*x^2)^{(1/3)})^2)] - (4 * \text{Sqrt}[2] * a^{(1/3)} * (a^{(1/3)} - (a + b^*x^2)^{(1/3)}) * \text{Sqrt}[(a^{(2/3)} + a^{(1/3)} * (a + b^*x^2)^{(1/3)} + (a + b^*x^2)^{(2/3)}) / ((1 - \text{Sqrt}[3]) * a^{(1/3)} - (a + b^*x^2)^{(1/3)})^2] * \text{EllipticF}[\text{ArcSin}[\frac{(1 + \text{Sqrt}[3]) * a^{(1/3)} - (a + b^*x^2)^{(1/3)}}{(1 - \text{Sqrt}[3]) * a^{(1/3)} - (a + b^*x^2)^{(1/3)}}], -7 + 4 * \text{Sqrt}[3]]) / (3^{(1/4)} * x * \text{Sqrt}[-((a^{(1/3)} * (a^{(1/3)} - (a + b^*x^2)^{(1/3)})) / ((1 - \text{Sqrt}[3]) * a^{(1/3)} - (a + b^*x^2)^{(1/3)})^2)])$$

**Rubi in Sympy [A]** time = 28.162, size = 439, normalized size = 0.82

$$\frac{2\sqrt[3]{3}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt{a+bx^2+(a+bx^2)^{\frac{2}{3}}}}{(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2})^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)E\left(\text{asin}\left(\frac{\sqrt[3]{a}(1+\sqrt{3})-\sqrt[3]{a+bx^2}}{-\sqrt[3]{a}(-1+\sqrt{3})-\sqrt[3]{a+bx^2}}\right)\right)\Big|_{-7+4\sqrt{3}}}{x\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{a+bx^2})}{(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2})^2}}}$$

$$\frac{4\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt{a+bx^2+(a+bx^2)^{\frac{2}{3}}}}{(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2})^2}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)F\left(\text{asin}\left(\frac{\sqrt[3]{a}(1+\sqrt{3})-\sqrt[3]{a+bx^2}}{-\sqrt[3]{a}(-1+\sqrt{3})-\sqrt[3]{a+bx^2}}\right)\right)\Big|_{-7+4\sqrt{3}}}{3x\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{a+bx^2})}{(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2})^2}}}$$

$$+\frac{4bx}{\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}}-\frac{(a+bx^2)^{\frac{2}{3}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(2/3)/x**2,x)`

[Out]  $2 * 3^{(1/4)} * a^{(1/3)} * \text{sqrt}((a^{(2/3)} + a^{(1/3)} * (a + b^*x^2)^{(1/3)} + (a + b^*x^2)^{(2/3)}) / (a^{(1/3)} * (-1 + \text{sqrt}(3)) + (a + b^*x^2)^{(1/3)})^{(2/3)} * \text{sqrt}(\text{sqrt}(3) + 2) * (a^{(1/3)} - (a + b^*x^2)^{(1/3)}) * \text{elliptic\_e}(\text{asin}((a^{(1/3)} * (1 + \text{sqrt}(3)) - (a + b^*x^2)^{(1/3)}) / (-a^{(1/3)} * (-1 + \text{sqrt}(3)) - (a + b^*x^2)^{(1/3)})), -7 + 4 * \text{sqrt}(3)) / (x * \text{sqrt}(-a^{(1/3)} * (a^{(1/3)} - (a + b^*x^2)^{(1/3)}) / (a^{(1/3)} * (-1 + \text{sqrt}(3)) + (a + b^*x^2)^{(1/3)})^{(2/3)} - 4 * \text{sqrt}(2) * 3^{(3/4)} * a^{(1/3)} * \text{sqrt}((a^{(2/3)} + a^{(1/3)} * (a + b^*x^2)^{(1/3)} + (a + b^*x^2)^{(2/3)}) / (a^{(1/3)} * (-1 + \text{sqrt}(3)) + (a + b^*x^2)^{(1/3)})^{(2/3)} * \text{elliptic\_f}(\text{asin}((a^{(1/3)} * (1 + \text{sqrt}(3)) - (a + b^*x^2)^{(1/3)}) / (-a^{(1/3)} * (-1 + \text{sqrt}(3)) - (a + b^*x^2)^{(1/3)})), -7 + 4 * \text{sqrt}(3)) / (3 * x * \text{sqrt}(-a^{(1/3)} * (a^{(1/3)} - (a + b^*x^2)^{(1/3)}) / (a^{(1/3)} * (-1 + \text{sqrt}(3)) + (a + b^*x^2)^{(1/3)})^{(2/3)})) + 4 * b * x / (a^{(1/3)} * (-1 + \text{sqrt}(3)) + (a + b^*x^2)^{(1/3)}) - (a + b^*x^2)^{(2/3)} / x$

**Mathematica [C]** time = 0.0415476, size = 68, normalized size = 0.13

$$\frac{4bx\sqrt[3]{\frac{a+bx^2}{a}}{}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right)}{3\sqrt[3]{a+bx^2}} - \frac{(a+bx^2)^{2/3}}{x}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(2/3)/x^2,x]`

[Out]  $-\frac{(a + b x^2)^{2/3}}{x} + \frac{4 b x (a + b x^2/a)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{3}{2}, -\frac{(b x^2)}{a}\right]}{3 (a + b x^2)^{1/3}}$

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (bx^2 + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(2/3)/x^2,x)`

[Out] `int((b*x^2+a)^(2/3)/x^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{2}{3}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3)/x^2,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(2/3)/x^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(bx^2 + a)^{\frac{2}{3}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3)/x^2,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(2/3)/x^2, x)`

**Sympy [A]** time = 2.58172, size = 29, normalized size = 0.05

$$\frac{a^{\frac{2}{3}} {}_2F_1\left(-\frac{2}{3}, -\frac{1}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(2/3)/x**2,x)`

[Out] `-a**(2/3)*hyper((-2/3, -1/2), (1/2,), b*x**2*exp_polar(I*pi)/a)/x`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{2}{3}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(2/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(2/3)/x^2, x)
```

$$3.688 \quad \int \frac{(a+bx^2)^{2/3}}{x^4} dx$$

**Optimal.** Leaf size=575

$$\frac{4\sqrt{2}b \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1+\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{9\sqrt[3]{3}a^{2/3}x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$+ \frac{2\sqrt{2+\sqrt{3}}b \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1+\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{3 \cdot 3^{3/4} a^{2/3} x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$- \frac{4b^2x}{9a \left( (1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)} - \frac{4b(a+bx^2)^{2/3}}{9ax} - \frac{(a+bx^2)^{2/3}}{3x^3}$$

[Out]  $-(a + b*x^2)^{(2/3)}/(3*x^3) - (4*b*(a + b*x^2)^{(2/3)})/(9*a*x) - (4*b^2*x)/(9*a*((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})) + (2*\text{Sqrt}[2 + \text{Sqrt}[3]]*b*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(3*3^{(3/4)}*a^{(2/3)}*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]) - (4*\text{Sqrt}[2]*b*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(9*3^{(1/4)}*a^{(2/3)}*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2])]$

**Rubi [A]** time = 0.875573, antiderivative size = 575, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{4\sqrt{2}b \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1+\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{9\sqrt[3]{3}a^{2/3}x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$+ \frac{2\sqrt{2+\sqrt{3}}b \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1+\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{3 \cdot 3^{3/4} a^{2/3} x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$- \frac{4b^2x}{9a \left( (1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)} - \frac{4b(a+bx^2)^{2/3}}{9ax} - \frac{(a+bx^2)^{2/3}}{3x^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^{(2/3)}/x^4, x]$

[Out]  $-(a + b*x^2)^{(2/3)}/(3*x^3) - (4*b*(a + b*x^2)^{(2/3)})/(9*a*x) - (4*b^2*x)/(9*a*((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})) + (2*\text{Sqrt}[2 + \text{Sqrt}[3]]*b*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(3*3^{(3/4)}*a^{(2/3)}*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]) - (4*\text{Sqrt}[2]*b*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(9*3^{(1/4)}*a^{(2/3)}*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2])]$

```

rt[2 + Sqrt[3]]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a
^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1
/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3
))], -7 + 4*Sqrt[3]]/(3*3^(3/4)*a^(2/3)*x*Sqrt[-((a^(1/3)*(a^(1/
3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/
3))^2]) - (4*Sqrt[2]*b*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/
3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))], -7 + 4*Sqrt[3]]/(9*3^(1/4)*a^(2/3)*x*Sqrt[-((a^(1/3)*
(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^
2)^(1/3))^2)])

```

**Rubi in Sympy [A]** time = 36.8668, size = 471, normalized size = 0.82

$$\begin{aligned}
& -\frac{(a+bx^2)^{\frac{2}{3}}}{3x^3} + \frac{4b^2x}{9a(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2})} - \frac{4b(a+bx^2)^{\frac{2}{3}}}{9ax} \\
& + \frac{2\sqrt[3]{3}b \sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a+bx^2+(a+bx^2)^{\frac{2}{3}}}}{(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2})^2}} \sqrt{\sqrt{3}+2} (\sqrt[3]{a}-\sqrt[3]{a+bx^2}) E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}(1+\sqrt{3})-\sqrt[3]{a+bx^2}}{-\sqrt[3]{a}(-1+\sqrt{3})-\sqrt[3]{a+bx^2}}\right)\right) \Big|_{-7+4\sqrt{3}}}{9a^{\frac{2}{3}}x \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{a+bx^2})}{(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2})^2}}} \\
& + \frac{4\sqrt{2} \cdot 3^{\frac{3}{4}}b \sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a+bx^2+(a+bx^2)^{\frac{2}{3}}}}{(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2})^2}} (\sqrt[3]{a}-\sqrt[3]{a+bx^2}) F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}(1+\sqrt{3})-\sqrt[3]{a+bx^2}}{-\sqrt[3]{a}(-1+\sqrt{3})-\sqrt[3]{a+bx^2}}\right)\right) \Big|_{-7+4\sqrt{3}}}{27a^{\frac{2}{3}}x \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{a+bx^2})}{(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2})^2}}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(2/3)/x**4,x)`

```

[Out] -(a + b*x**2)**(2/3)/(3*x**3) + 4*b**2*x/(9*a*(a**(1/3)*(-1 + sq
t(3)) + (a + b*x**2)**(1/3))) - 4*b*(a + b*x**2)**(2/3)/(9*a*x) +
2*3**(1/4)*b*sqrt((a**(2/3) + a**(1/3)*(a + b*x**2)**(1/3) + (a
+ b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a + b*x**2)**(1/3)
**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a + b*x**2)**(1/3))*elliptic_
e(asin((a**(1/3)*(1 + sqrt(3)) - (a + b*x**2)**(1/3))/(-a**(1/3)*
(-1 + sqrt(3)) - (a + b*x**2)**(1/3))), -7 + 4*sqrt(3))/(9*a**(2/
3)*x*sqrt(-a**(1/3)*(a**(1/3) - (a + b*x**2)**(1/3))/(a**(1/3)*(-
1 + sqrt(3)) + (a + b*x**2)**(1/3))**2) - 4*sqrt(2)*3**(3/4)*b*s
qrt((a**(2/3) + a**(1/3)*(a + b*x**2)**(1/3) + (a + b*x**2)**(2/3
)))/(a**(1/3)*(-1 + sqrt(3)) + (a + b*x**2)**(1/3))**2*(a**(1/3)
- (a + b*x**2)**(1/3))*elliptic_f(asin((a**(1/3)*(1 + sqrt(3)) -
(a + b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)) - (a + b*x**2)**(1
/3))), -7 + 4*sqrt(3))/(27*a**(2/3)*x*sqrt(-a**(1/3)*(a**(1/3) -
(a + b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a + b*x**2)**(1/
3))**2))

```

**Mathematica [C]** time = 0.0493884, size = 88, normalized size = 0.15

$$\frac{4b^2x\sqrt[3]{\frac{a+bx^2}{a}}{}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right)}{27a\sqrt[3]{a+bx^2}} + \left(-\frac{4b}{9ax} - \frac{1}{3x^3}\right)(a+bx^2)^{2/3}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(2/3)/x^4,x]`

[Out]  $(-1/(3*x^3) - (4*b)/(9*a*x)) * (a + b*x^2)^{(2/3)} + (4*b^2*x * ((a + b*x^2)/a)^{(1/3)} * \text{Hypergeometric2F1}[1/3, 1/2, 3/2, -((b*x^2)/a)]) / (27*a*(a + b*x^2)^{(1/3)})$

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} (bx^2 + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(2/3)/x^4, x)`

[Out] `int((b*x^2+a)^(2/3)/x^4, x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{2}{3}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3)/x^4, x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(2/3)/x^4, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{2}{3}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(2/3)/x^4, x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(2/3)/x^4, x)`

**Sympy [A]** time = 3.08705, size = 34, normalized size = 0.06

$$-\frac{a^{\frac{2}{3}} {}_2F_1\left(-\frac{3}{2}, -\frac{2}{3} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(2/3)/x**4, x)`

[Out] `-a**(2/3)*hyper((-3/2, -2/3), (-1/2, ), b*x**2*exp_polar(I*pi)/a)/(3*x**3)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{2}{3}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(2/3)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(2/3)/x^4, x)
```

$$3.689 \quad \int x^7 (a + bx^2)^{4/3} dx$$

**Optimal.** Leaf size=80

$$-\frac{3a^3 (a + bx^2)^{7/3}}{14b^4} + \frac{9a^2 (a + bx^2)^{10/3}}{20b^4} + \frac{3 (a + bx^2)^{16/3}}{32b^4} - \frac{9a (a + bx^2)^{13/3}}{26b^4}$$

[Out]  $(-3*a^3*(a + b*x^2)^(7/3))/(14*b^4) + (9*a^2*(a + b*x^2)^(10/3))/(20*b^4) - (9*a*(a + b*x^2)^(13/3))/(26*b^4) + (3*(a + b*x^2)^(16/3))/(32*b^4)$

**Rubi [A]** time = 0.131948, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{3a^3 (a + bx^2)^{7/3}}{14b^4} + \frac{9a^2 (a + bx^2)^{10/3}}{20b^4} + \frac{3 (a + bx^2)^{16/3}}{32b^4} - \frac{9a (a + bx^2)^{13/3}}{26b^4}$$

Antiderivative was successfully verified.

[In] Int[x^7\*(a + b\*x^2)^(4/3), x]

[Out]  $(-3*a^3*(a + b*x^2)^(7/3))/(14*b^4) + (9*a^2*(a + b*x^2)^(10/3))/(20*b^4) - (9*a*(a + b*x^2)^(13/3))/(26*b^4) + (3*(a + b*x^2)^(16/3))/(32*b^4)$

**Rubi in Sympy [A]** time = 15.5822, size = 75, normalized size = 0.94

$$-\frac{3a^3 (a + bx^2)^{7/3}}{14b^4} + \frac{9a^2 (a + bx^2)^{10/3}}{20b^4} - \frac{9a (a + bx^2)^{13/3}}{26b^4} + \frac{3 (a + bx^2)^{16/3}}{32b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*7\*(b\*x\*\*2+a)\*\*(4/3), x)

[Out]  $-3*a^3*(a + b*x^2)**(7/3)/(14*b^4) + 9*a^2*(a + b*x^2)**(10/3)/(20*b^4) - 9*a*(a + b*x^2)**(13/3)/(26*b^4) + 3*(a + b*x^2)**(16/3)/(32*b^4)$

**Mathematica [A]** time = 0.0454171, size = 50, normalized size = 0.62

$$\frac{3 (a + bx^2)^{7/3} (-81a^3 + 189a^2bx^2 - 315ab^2x^4 + 455b^3x^6)}{14560b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^7\*(a + b\*x^2)^(4/3), x]

[Out]  $(3*(a + b*x^2)^(7/3)*(-81*a^3 + 189*a^2*b*x^2 - 315*a*b^2*x^4 + 455*b^3*x^6))/(14560*b^4)$

**Maple [A]** time = 0.009, size = 47, normalized size = 0.6

$$-\frac{-1365b^3x^6 + 945ab^2x^4 - 567a^2bx^2 + 243a^3}{14560b^4} (bx^2 + a)^{7/3}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(b*x^2+a)^(4/3),x)`

[Out] 
$$-3/14560*(b*x^2+a)^(7/3)*(-455*b^3*x^6+315*a*b^2*x^4-189*a^2*b*x^2+81*a^3)/b^4$$

**Maxima [A]** time = 1.35407, size = 86, normalized size = 1.08

$$\frac{3(bx^2+a)^{\frac{16}{3}}}{32b^4} - \frac{9(bx^2+a)^{\frac{13}{3}}a}{26b^4} + \frac{9(bx^2+a)^{\frac{10}{3}}a^2}{20b^4} - \frac{3(bx^2+a)^{\frac{7}{3}}a^3}{14b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^(4/3)*x^7,x, algorithm="maxima")`

[Out] 
$$3/32*(b*x^2+a)^(16/3)/b^4 - 9/26*(b*x^2+a)^(13/3)*a/b^4 + 9/20*(b*x^2+a)^(10/3)*a^2/b^4 - 3/14*(b*x^2+a)^(7/3)*a^3/b^4$$

**Fricas [A]** time = 0.206399, size = 92, normalized size = 1.15

$$\frac{3(455b^5x^{10} + 595ab^4x^8 + 14a^2b^3x^6 - 18a^3b^2x^4 + 27a^4bx^2 - 81a^5)(bx^2+a)^{\frac{1}{3}}}{14560b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^(4/3)*x^7,x, algorithm="fricas")`

[Out] 
$$3/14560*(455*b^5*x^{10} + 595*a*b^4*x^8 + 14*a^2*b^3*x^6 - 18*a^3*b^2*x^4 + 27*a^4*b*x^2 - 81*a^5)*(b*x^2+a)^(1/3)/b^4$$

**Sympy [A]** time = 20.8648, size = 136, normalized size = 1.7

$$\begin{cases} \frac{-243a^5\sqrt[3]{a+bx^2}}{14560b^4} + \frac{81a^4x^2\sqrt[3]{a+bx^2}}{14560b^3} - \frac{27a^3x^4\sqrt[3]{a+bx^2}}{7280b^2} + \frac{3a^2x^6\sqrt[3]{a+bx^2}}{1040b} + \frac{51ax^8\sqrt[3]{a+bx^2}}{416} + \frac{3bx^{10}\sqrt[3]{a+bx^2}}{32} & \text{for } b \neq 0 \\ \frac{a^{\frac{4}{3}}x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(b*x**2+a)**(4/3),x)`

[Out] `Piecewise((-243*a**5*(a+b*x**2)**(1/3)/(14560*b**4) + 81*a**4*x**2*(a+b*x**2)**(1/3)/(14560*b**3) - 27*a**3*x**4*(a+b*x**2)**(1/3)/(7280*b**2) + 3*a**2*x**6*(a+b*x**2)**(1/3)/(1040*b) + 51*a*x**8*(a+b*x**2)**(1/3)/416 + 3*b*x**10*(a+b*x**2)**(1/3)/32, Ne(b, 0)), (a**(4/3)*x**8/8, True))`

**GIAC/XCAS [A]** time = 0.220508, size = 181, normalized size = 2.26

$$3 \left( \frac{4 \left( 140(bx^2+a)^{\frac{13}{3}} - 546(bx^2+a)^{\frac{10}{3}}a + 780(bx^2+a)^{\frac{7}{3}}a^2 - 455(bx^2+a)^{\frac{4}{3}}a^3 \right) a}{b^3} + \frac{455(bx^2+a)^{\frac{16}{3}} - 2240(bx^2+a)^{\frac{13}{3}}a + 4368(bx^2+a)^{\frac{10}{3}}a^2 - 4160(bx^2+a)^{\frac{7}{3}}a^3 + 14560b}{14560b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(4/3)*x^7,x, algorithm="giac")
```

```
[Out] 3/14560*(4*(140*(b*x^2 + a)^(13/3) - 546*(b*x^2 + a)^(10/3)*a + 780*(b*x^2 + a)^(7/3)*a^2 - 455*(b*x^2 + a)^(4/3)*a^3)*a/b^3 + (455*(b*x^2 + a)^(16/3) - 2240*(b*x^2 + a)^(13/3)*a + 4368*(b*x^2 + a)^(10/3)*a^2 - 4160*(b*x^2 + a)^(7/3)*a^3 + 1820*(b*x^2 + a)^(4/3)*a^4)/b^3)/b
```

$$3.690 \quad \int x^5 (a + bx^2)^{4/3} dx$$

**Optimal.** Leaf size=59

$$\frac{3a^2 (a + bx^2)^{7/3}}{14b^3} + \frac{3 (a + bx^2)^{13/3}}{26b^3} - \frac{3a (a + bx^2)^{10/3}}{10b^3}$$

[Out]  $(3*a^2*(a + b*x^2)^(7/3))/(14*b^3) - (3*a*(a + b*x^2)^(10/3))/(10*b^3) + (3*(a + b*x^2)^(13/3))/(26*b^3)$

**Rubi [A]** time = 0.102202, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{3a^2 (a + bx^2)^{7/3}}{14b^3} + \frac{3 (a + bx^2)^{13/3}}{26b^3} - \frac{3a (a + bx^2)^{10/3}}{10b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(a + b\*x^2)^(4/3), x]

[Out]  $(3*a^2*(a + b*x^2)^(7/3))/(14*b^3) - (3*a*(a + b*x^2)^(10/3))/(10*b^3) + (3*(a + b*x^2)^(13/3))/(26*b^3)$

**Rubi in Sympy [A]** time = 11.6929, size = 54, normalized size = 0.92

$$\frac{3a^2 (a + bx^2)^{7/3}}{14b^3} - \frac{3a (a + bx^2)^{10/3}}{10b^3} + \frac{3 (a + bx^2)^{13/3}}{26b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(b\*x\*\*2+a)\*\*(4/3), x)

[Out]  $3*a**2*(a + b*x**2)**(7/3)/(14*b**3) - 3*a*(a + b*x**2)**(10/3)/(10*b**3) + 3*(a + b*x**2)**(13/3)/(26*b**3)$

**Mathematica [A]** time = 0.0342535, size = 39, normalized size = 0.66

$$\frac{3 (a + bx^2)^{7/3} (9a^2 - 21abx^2 + 35b^2x^4)}{910b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*x^2)^(4/3), x]

[Out]  $(3*(a + b*x^2)^(7/3)*(9*a^2 - 21*a*b*x^2 + 35*b^2*x^4))/(910*b^3)$

**Maple [A]** time = 0.007, size = 36, normalized size = 0.6

$$\frac{105 b^2 x^4 - 63 a b x^2 + 27 a^2}{910 b^3} (b x^2 + a)^{7/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(b*x^2+a)^(4/3),x)`

[Out]  $3/910*(b*x^2+a)^{(7/3)}*(35*b^2*x^4-21*a*b*x^2+9*a^2)/b^3$

**Maxima [A]** time = 1.34555, size = 63, normalized size = 1.07

$$\frac{3(bx^2+a)^{\frac{13}{3}}}{26b^3} - \frac{3(bx^2+a)^{\frac{10}{3}}a}{10b^3} + \frac{3(bx^2+a)^{\frac{7}{3}}a^2}{14b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^(4/3)*x^5,x, algorithm="maxima")`

[Out]  $3/26*(b*x^2+a)^{(13/3)}/b^3 - 3/10*(b*x^2+a)^{(10/3)}*a/b^3 + 3/14*(b*x^2+a)^{(7/3)}*a^2/b^3$

**Fricas [A]** time = 0.208169, size = 77, normalized size = 1.31

$$\frac{3(35b^4x^8 + 49ab^3x^6 + 2a^2b^2x^4 - 3a^3bx^2 + 9a^4)(bx^2+a)^{\frac{1}{3}}}{910b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^(4/3)*x^5,x, algorithm="fricas")`

[Out]  $3/910*(35*b^4*x^8 + 49*a*b^3*x^6 + 2*a^2*b^2*x^4 - 3*a^3*b*x^2 + 9*a^4)*(b*x^2+a)^{(1/3)}/b^3$

**Sympy [A]** time = 12.5168, size = 112, normalized size = 1.9

$$\begin{cases} \frac{27a^4\sqrt[3]{a+bx^2}}{910b^3} - \frac{9a^3x^2\sqrt[3]{a+bx^2}}{910b^2} + \frac{3a^2x^4\sqrt[3]{a+bx^2}}{455b} + \frac{21ax^6\sqrt[3]{a+bx^2}}{130} + \frac{3bx^8\sqrt[3]{a+bx^2}}{26} & \text{for } b \neq 0 \\ \frac{a^{\frac{4}{3}}x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(b*x**2+a)**(4/3),x)`

[Out] `Piecewise(((27*a**4*(a+b*x**2)**(1/3))/(910*b**3) - 9*a**3*x**2*(a+b*x**2)**(1/3)/(910*b**2) + 3*a**2*x**4*(a+b*x**2)**(1/3)/(455*b) + 21*a*x**6*(a+b*x**2)**(1/3)/130 + 3*b*x**8*(a+b*x**2)**(1/3)/26, Ne(b, 0)), (a**(4/3)*x**6/6, True))`

**GIAC/XCAS [A]** time = 0.214999, size = 143, normalized size = 2.42

$$\frac{3 \left( \frac{13 \left( 14(bx^2+a)^{\frac{10}{3}} - 40(bx^2+a)^{\frac{7}{3}}a + 35(bx^2+a)^{\frac{4}{3}}a^2 \right) a}{b^2} + \frac{140(bx^2+a)^{\frac{13}{3}} - 546(bx^2+a)^{\frac{10}{3}}a + 780(bx^2+a)^{\frac{7}{3}}a^2 - 455(bx^2+a)^{\frac{4}{3}}a^3}{b^2} \right)}{3640b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^(4/3)*x^5,x, algorithm="giac")`

```
[Out] 3/3640*(13*(14*(b*x^2 + a)^(10/3) - 40*(b*x^2 + a)^(7/3)*a + 35*(
b*x^2 + a)^(4/3)*a^2)*a/b^2 + (140*(b*x^2 + a)^(13/3) - 546*(b*x^
2 + a)^(10/3)*a + 780*(b*x^2 + a)^(7/3)*a^2 - 455*(b*x^2 + a)^(4/
3)*a^3)/b^2)/b
```

$$3.691 \quad \int x^3 (a + bx^2)^{4/3} dx$$

**Optimal.** Leaf size=38

$$\frac{3(a + bx^2)^{10/3}}{20b^2} - \frac{3a(a + bx^2)^{7/3}}{14b^2}$$

[Out]  $(-3*a*(a + b*x^2)^{(7/3)})/(14*b^2) + (3*(a + b*x^2)^{(10/3)})/(20*b^2)$

**Rubi [A]** time = 0.0702891, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{3(a + bx^2)^{10/3}}{20b^2} - \frac{3a(a + bx^2)^{7/3}}{14b^2}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*x^2)^(4/3), x]

[Out]  $(-3*a*(a + b*x^2)^{(7/3)})/(14*b^2) + (3*(a + b*x^2)^{(10/3)})/(20*b^2)$

**Rubi in Sympy [A]** time = 7.84642, size = 34, normalized size = 0.89

$$-\frac{3a(a + bx^2)^{7/3}}{14b^2} + \frac{3(a + bx^2)^{10/3}}{20b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(b\*x\*\*2+a)\*\*(4/3), x)

[Out]  $-3*a*(a + b*x**2)**(7/3)/(14*b**2) + 3*(a + b*x**2)**(10/3)/(20*b**2)$

**Mathematica [A]** time = 0.0350308, size = 28, normalized size = 0.74

$$\frac{3(a + bx^2)^{7/3}(7bx^2 - 3a)}{140b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*x^2)^(4/3), x]

[Out]  $(3*(a + b*x^2)^{(7/3)}*(-3*a + 7*b*x^2))/(140*b^2)$

**Maple [A]** time = 0.008, size = 25, normalized size = 0.7

$$-\frac{-21bx^2 + 9a}{140b^2} (bx^2 + a)^{7/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(b\*x^2+a)^(4/3), x)

[Out]  $-3/140 * (b * x^2 + a)^{7/3} * (-7 * b * x^2 + 3 * a) / b^2$

**Maxima [A]** time = 1.35679, size = 41, normalized size = 1.08

$$\frac{3 (bx^2 + a)^{\frac{10}{3}}}{20 b^2} - \frac{3 (bx^2 + a)^{\frac{7}{3}} a}{14 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)*x^3,x, algorithm="maxima")`

[Out]  $3/20 * (b * x^2 + a)^{10/3} / b^2 - 3/14 * (b * x^2 + a)^{7/3} * a / b^2$

**Fricas [A]** time = 0.206714, size = 61, normalized size = 1.61

$$\frac{3 (7 b^3 x^6 + 11 a b^2 x^4 + a^2 b x^2 - 3 a^3) (b x^2 + a)^{\frac{1}{3}}}{140 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)*x^3,x, algorithm="fricas")`

[Out]  $3/140 * (7 * b^3 * x^6 + 11 * a * b^2 * x^4 + a^2 * b * x^2 - 3 * a^3) * (b * x^2 + a)^{1/3} / b^2$

**Sympy [A]** time = 7.49349, size = 88, normalized size = 2.32

$$\begin{cases} \frac{-9a^3 \sqrt[3]{a+bx^2}}{140b^2} + \frac{3a^2x^2 \sqrt[3]{a+bx^2}}{140b} + \frac{33ax^4 \sqrt[3]{a+bx^2}}{140} + \frac{3bx^6 \sqrt[3]{a+bx^2}}{20} & \text{for } b \neq 0 \\ \frac{a^{\frac{4}{3}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**2+a)**(4/3),x)`

[Out] `Piecewise((-9*a**3*(a + b*x**2)**(1/3)/(140*b**2) + 3*a**2*x**2*(a + b*x**2)**(1/3)/(140*b) + 33*a*x**4*(a + b*x**2)**(1/3)/140 + 3*b*x**6*(a + b*x**2)**(1/3)/20, Ne(b, 0)), (a**(4/3)*x**4/4, True))`

**GIAC/XCAS [A]** time = 0.216017, size = 105, normalized size = 2.76

$$\frac{3 \left( \frac{5 \left( 4 (bx^2+a)^{\frac{7}{3}} - 7 (bx^2+a)^{\frac{4}{3}} a \right) a}{b} + \frac{14 (bx^2+a)^{\frac{10}{3}} - 40 (bx^2+a)^{\frac{7}{3}} a + 35 (bx^2+a)^{\frac{4}{3}} a^2}{b} \right)}{280 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)*x^3,x, algorithm="giac")`

[Out]  $3/280 * (5 * (4 * (b * x^2 + a)^{7/3} - 7 * (b * x^2 + a)^{4/3} * a) * a / b + (14 * (b * x^2 + a)^{10/3} - 40 * (b * x^2 + a)^{7/3} * a + 35 * (b * x^2 + a)^{4/3} * a^2) / b) / b$

$$3.692 \quad \int x (a + bx^2)^{4/3} dx$$

**Optimal.** Leaf size=18

$$\frac{3 (a + bx^2)^{7/3}}{14b}$$

[Out] (3\*(a + b\*x^2)^(7/3))/(14\*b)

**Rubi [A]** time = 0.011641, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{3 (a + bx^2)^{7/3}}{14b}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*x^2)^(4/3), x]

[Out] (3\*(a + b\*x^2)^(7/3))/(14\*b)

**Rubi in Sympy [A]** time = 2.15158, size = 14, normalized size = 0.78

$$\frac{3 (a + bx^2)^{7/3}}{14b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(b\*x\*\*2+a)\*\*(4/3), x)

[Out] 3\*(a + b\*x\*\*2)\*\*(7/3)/(14\*b)

**Mathematica [A]** time = 0.00991019, size = 18, normalized size = 1.

$$\frac{3 (a + bx^2)^{7/3}}{14b}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*x^2)^(4/3), x]

[Out] (3\*(a + b\*x^2)^(7/3))/(14\*b)

**Maple [A]** time = 0.003, size = 15, normalized size = 0.8

$$\frac{3}{14b} (bx^2 + a)^{7/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(b\*x^2+a)^(4/3), x)

[Out] 3/14\*(b\*x^2+a)^(7/3)/b



---

**Maxima [A]** time = 1.33952, size = 19, normalized size = 1.06

$$\frac{3 (bx^2 + a)^{\frac{7}{3}}}{14b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)*x,x, algorithm="maxima")`

[Out] `3/14*(b*x^2 + a)^(7/3)/b`

---

**Fricas [A]** time = 0.208104, size = 43, normalized size = 2.39

$$\frac{3 (b^2x^4 + 2abx^2 + a^2)(bx^2 + a)^{\frac{1}{3}}}{14b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)*x,x, algorithm="fricas")`

[Out] `3/14*(b^2*x^4 + 2*a*b*x^2 + a^2)*(b*x^2 + a)^(1/3)/b`

---

**Sympy [A]** time = 3.88725, size = 65, normalized size = 3.61

$$\begin{cases} \frac{3a^2\sqrt[3]{a+bx^2}}{14b} + \frac{3ax^2\sqrt[3]{a+bx^2}}{7} + \frac{3bx^4\sqrt[3]{a+bx^2}}{14} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{4}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x**2+a)**(4/3),x)`

[Out] `Piecewise((3*a**2*(a + b*x**2)**(1/3)/(14*b) + 3*a*x**2*(a + b*x**2)**(1/3)/7 + 3*b*x**4*(a + b*x**2)**(1/3)/14, Ne(b, 0)), (a**(4/3)*x**2/2, True))`

---

**GIAC/XCAS [A]** time = 0.213338, size = 19, normalized size = 1.06

$$\frac{3 (bx^2 + a)^{\frac{7}{3}}}{14b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)*x,x, algorithm="giac")`

[Out] `3/14*(b*x^2 + a)^(7/3)/b`

$$3.693 \quad \int \frac{(a+bx^2)^{4/3}}{x} dx$$

**Optimal.** Leaf size=117

$$\frac{3}{4}a^{4/3} \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right) - \frac{1}{2}\sqrt{3}a^{4/3} \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right) - \frac{1}{2}a^{4/3} \log(x) + \frac{3}{2}a\sqrt[3]{a+bx^2} + \frac{3}{8}(a+bx^2)^{4/3}$$

[Out] (3\*a\*(a + b\*x^2)^(1/3))/2 + (3\*(a + b\*x^2)^(4/3))/8 - (Sqrt[3]\*a^(4/3)\*ArcTan[(a^(1/3) + 2\*(a + b\*x^2)^(1/3))/(Sqrt[3]\*a^(1/3))])/2 - (a^(4/3)\*Log[x])/2 + (3\*a^(4/3)\*Log[a^(1/3) - (a + b\*x^2)^(1/3)])/4

**Rubi [A]** time = 0.224914, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{3}{4}a^{4/3} \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right) - \frac{1}{2}\sqrt{3}a^{4/3} \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right) - \frac{1}{2}a^{4/3} \log(x) + \frac{3}{2}a\sqrt[3]{a+bx^2} + \frac{3}{8}(a+bx^2)^{4/3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(4/3)/x, x]

[Out] (3\*a\*(a + b\*x^2)^(1/3))/2 + (3\*(a + b\*x^2)^(4/3))/8 - (Sqrt[3]\*a^(4/3)\*ArcTan[(a^(1/3) + 2\*(a + b\*x^2)^(1/3))/(Sqrt[3]\*a^(1/3))])/2 - (a^(4/3)\*Log[x])/2 + (3\*a^(4/3)\*Log[a^(1/3) - (a + b\*x^2)^(1/3)])/4

**Rubi in Sympy [A]** time = 13.4781, size = 109, normalized size = 0.93

$$-\frac{a^{4/3} \log(x^2)}{4} + \frac{3a^{4/3} \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4} - \frac{\sqrt{3}a^{4/3} \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} + \frac{\sqrt[3]{a+bx^2}}{3}\right)}{\sqrt[3]{a}}\right)}{2} + \frac{3a\sqrt[3]{a+bx^2}}{2} + \frac{3(a+bx^2)^{4/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(4/3)/x, x)

[Out] -a\*\*(4/3)\*log(x\*\*2)/4 + 3\*a\*\*(4/3)\*log(a\*\*(1/3) - (a + b\*x\*\*2)\*\*(1/3))/4 - sqrt(3)\*a\*\*(4/3)\*atan(sqrt(3)\*(a\*\*(1/3)/3 + 2\*(a + b\*x\*\*2)\*\*(1/3)/3)/a\*\*(1/3))/2 + 3\*a\*(a + b\*x\*\*2)\*\*(1/3)/2 + 3\*(a + b\*x\*\*2)\*\*(4/3)/8

**Mathematica [C]** time = 0.0513, size = 76, normalized size = 0.65

$$\frac{3(5a^2 + 6abx^2 + b^2x^4) - 6a^2\left(\frac{a}{bx^2} + 1\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{a}{bx^2}\right)}{8(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(4/3)/x, x]

[Out] (3\*(5\*a^2 + 6\*a\*b\*x^2 + b^2\*x^4) - 6\*a^2\*(1 + a/(b\*x^2))^(2/3)\*Hypergeometric2F1[2/3, 2/3, 5/3, -(a/(b\*x^2))])/(8\*(a + b\*x^2)^(2/3))

))

---

**Maple [F]** time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{x} (bx^2 + a)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(4/3)/x,x)`

[Out] `int((b*x^2+a)^(4/3)/x,x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)/x,x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.215985, size = 143, normalized size = 1.22

$$-\frac{1}{2} \sqrt{3} a^{\frac{4}{3}} \arctan\left(\frac{\sqrt{3}(2(bx^2 + a)^{\frac{1}{3}} + a^{\frac{1}{3}})}{3a^{\frac{1}{3}}}\right) - \frac{1}{4} a^{\frac{4}{3}} \log\left((bx^2 + a)^{\frac{2}{3}} + (bx^2 + a)^{\frac{1}{3}} a^{\frac{1}{3}} + a^{\frac{2}{3}}\right) + \frac{1}{2} a^{\frac{4}{3}} \log\left((bx^2 + a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right) + \frac{3}{8} (bx^2 + 5a)(bx^2 + a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)/x,x, algorithm="fricas")`

[Out] `-1/2*sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)) - 1/4*a^(4/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 1/2*a^(4/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/8*(b*x^2 + 5*a)*(b*x^2 + a)^(1/3)`

---

**Sympy [A]** time = 5.39158, size = 49, normalized size = 0.42

$$-\frac{b^{\frac{4}{3}} x^{\frac{8}{3}} \left(-\frac{4}{3}\right) {}_2F_1\left(-\frac{4}{3}, -\frac{4}{3} \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(4/3)/x,x)`

[Out] `-b**(4/3)*x**(8/3)*gamma(-4/3)*hyper((-4/3, -4/3), (-1/3,), a*exp_polar(I*pi)/(b*x**2))/(2*gamma(-1/3))`

---

**GIAC/XCAS [A]** time = 0.596838, size = 149, normalized size = 1.27

$$-\frac{1}{2} \sqrt{3} a^{\frac{4}{3}} \arctan\left(\frac{\sqrt{3}\left(2(bx^2 + a)^{\frac{1}{3}} + a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right) - \frac{1}{4} a^{\frac{4}{3}} \ln\left(\left(bx^2 + a\right)^{\frac{2}{3}} + \left(bx^2 + a\right)^{\frac{1}{3}} a^{\frac{1}{3}} + a^{\frac{2}{3}}\right) + \frac{1}{2} a^{\frac{4}{3}} \ln\left(\left|(bx^2 + a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right) + \frac{3}{8} (bx^2 + a)^{\frac{4}{3}} + \frac{3}{2} (bx^2 + a)^{\frac{1}{3}} a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/x,x, algorithm="giac")

[Out] -1/2\*sqrt(3)\*a^(4/3)\*arctan(1/3\*sqrt(3)\*(2\*(b\*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)) - 1/4\*a^(4/3)\*ln((b\*x^2 + a)^(2/3) + (b\*x^2 + a)^(1/3)\*a^(1/3) + a^(2/3)) + 1/2\*a^(4/3)\*ln(abs((b\*x^2 + a)^(1/3) - a^(1/3))) + 3/8\*(b\*x^2 + a)^(4/3) + 3/2\*(b\*x^2 + a)^(1/3)\*a

$$3.694 \quad \int \frac{(a+bx^2)^{4/3}}{x^3} dx$$

**Optimal.** Leaf size=116

$$-\frac{(a+bx^2)^{4/3}}{2x^2} + 2b\sqrt[3]{a+bx^2} + \sqrt[3]{ab} \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right) - \frac{2\sqrt[3]{ab} \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}} - \frac{2}{3}\sqrt[3]{ab} \log(x)$$

[Out] 2\*b\*(a + b\*x^2)^(1/3) - (a + b\*x^2)^(4/3)/(2\*x^2) - (2\*a^(1/3)\*b\*ArcTan[(a^(1/3) + 2\*(a + b\*x^2)^(1/3))/(Sqrt[3]\*a^(1/3))]/Sqrt[3] - (2\*a^(1/3)\*b\*Log[x])/3 + a^(1/3)\*b\*Log[a^(1/3) - (a + b\*x^2)^(1/3)]

**Rubi [A]** time = 0.205856, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$-\frac{(a+bx^2)^{4/3}}{2x^2} + 2b\sqrt[3]{a+bx^2} + \sqrt[3]{ab} \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right) - \frac{2\sqrt[3]{ab} \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}} - \frac{2}{3}\sqrt[3]{ab} \log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(4/3)/x^3, x]

[Out] 2\*b\*(a + b\*x^2)^(1/3) - (a + b\*x^2)^(4/3)/(2\*x^2) - (2\*a^(1/3)\*b\*ArcTan[(a^(1/3) + 2\*(a + b\*x^2)^(1/3))/(Sqrt[3]\*a^(1/3))]/Sqrt[3] - (2\*a^(1/3)\*b\*Log[x])/3 + a^(1/3)\*b\*Log[a^(1/3) - (a + b\*x^2)^(1/3)]

**Rubi in Sympy [A]** time = 13.5848, size = 112, normalized size = 0.97

$$-\frac{\sqrt[3]{ab} \log(x^2)}{3} + \sqrt[3]{ab} \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right) - \frac{2\sqrt{3}\sqrt[3]{ab} \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} + \frac{2\sqrt[3]{a+bx^2}}{3}\right)}{\sqrt[3]{a}}\right)}{3} + 2b\sqrt[3]{a+bx^2} - \frac{(a+bx^2)^{4/3}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(4/3)/x\*\*3, x)

[Out] -a\*\*(1/3)\*b\*log(x\*\*2)/3 + a\*\*(1/3)\*b\*log(a\*\*(1/3) - (a + b\*x\*\*2)\*\*(1/3)) - 2\*sqrt(3)\*a\*\*(1/3)\*b\*atan(sqrt(3)\*(a\*\*(1/3)/3 + 2\*(a + b\*x\*\*2)\*\*(1/3)/3)/a\*\*(1/3))/3 + 2\*b\*(a + b\*x\*\*2)\*\*(1/3) - (a + b\*x\*\*2)\*\*(4/3)/(2\*x\*\*2)

**Mathematica [C]** time = 0.0653677, size = 73, normalized size = 0.63

$$\frac{-\frac{a^2}{2x^2} - ab\left(\frac{a}{bx^2} + 1\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{bx^2}\right) + ab + \frac{3b^2x^2}{2}}{(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(4/3)/x^3, x]

[Out]  $(a*b - a^2/(2*x^2) + (3*b^2*x^2)/2 - a*b*(1 + a/(b*x^2))^{2/3}) * \text{Hypergeometric2F1}[2/3, 2/3, 5/3, -(a/(b*x^2))]/(a + b*x^2)^{2/3}$

**Maple [F]** time = 0.052, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} (bx^2 + a)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(4/3)/x^3,x)`

[Out] `int((b*x^2+a)^(4/3)/x^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)/x^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.217566, size = 185, normalized size = 1.59

$$\frac{\sqrt{3} \left( 2 \sqrt{3} a^{\frac{1}{3}} b x^2 \log \left( (b x^2 + a)^{\frac{2}{3}} + (b x^2 + a)^{\frac{1}{3}} a^{\frac{1}{3}} + a^{\frac{2}{3}} \right) - 4 \sqrt{3} a^{\frac{1}{3}} b x^2 \log \left( (b x^2 + a)^{\frac{1}{3}} - a^{\frac{1}{3}} \right) + 12 a^{\frac{1}{3}} b x^2 \arctan \left( \frac{2 \sqrt{3} (b x^2 + a)^{\frac{1}{3}}}{3 a^{\frac{1}{3}}} \right) \right)}{18 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)/x^3,x, algorithm="fricas")`

[Out]  $-1/18 * \sqrt{3} * (2 * \sqrt{3} * a^{1/3} * b * x^2 * \log((b * x^2 + a)^{2/3} + (b * x^2 + a)^{1/3} * a^{1/3} + a^{2/3}) - 4 * \sqrt{3} * a^{1/3} * b * x^2 * \log((b * x^2 + a)^{1/3} - a^{1/3}) + 12 * a^{1/3} * b * x^2 * \arctan(1/3 * (2 * \sqrt{3} * (b * x^2 + a)^{1/3} + \sqrt{3} * a^{1/3}) / a^{1/3}) - 3 * \sqrt{3} * (3 * b * x^2 - a) * (b * x^2 + a)^{1/3}) / x^2$

**Sympy [A]** time = 6.15417, size = 46, normalized size = 0.4

$$\frac{b^{\frac{4}{3}} x^{\frac{2}{3}} \left(-\frac{1}{3}\right) {}_2F_1\left(-\frac{4}{3}, -\frac{1}{3} \middle| \frac{a e^{i\pi}}{b x^2}\right)}{2 \left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(4/3)/x**3,x)`

[Out]  $-b^{4/3} * x^{2/3} * \text{gamma}(-1/3) * \text{hyper}((-4/3, -1/3), (2/3, ), a * \text{exp\_polar}(I * \text{pi}) / (b * x^2)) / (2 * \text{gamma}(2/3))$

**GIAC/XCAS [A]** time = 0.585777, size = 161, normalized size = 1.39

$$-\frac{1}{6} \left( 4 \sqrt{3} a^{\frac{1}{3}} \arctan \left( \frac{\sqrt{3} \left( 2 (bx^2 + a)^{\frac{1}{3}} + a^{\frac{1}{3}} \right)}{3 a^{\frac{1}{3}}} \right) + 2 a^{\frac{1}{3}} \ln \left( (bx^2 + a)^{\frac{2}{3}} + (bx^2 + a)^{\frac{1}{3}} a^{\frac{1}{3}} + a^{\frac{2}{3}} \right) - 4 a^{\frac{1}{3}} \ln \left( \left| (bx^2 + a)^{\frac{1}{3}} - a^{\frac{1}{3}} \right| \right) \right) - 9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/x^3,x, algorithm="giac")

[Out] -1/6\*(4\*sqrt(3)\*a^(1/3)\*arctan(1/3\*sqrt(3)\*(2\*(b\*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)) + 2\*a^(1/3)\*ln((b\*x^2 + a)^(2/3) + (b\*x^2 + a)^(1/3)\*a^(1/3) + a^(2/3)) - 4\*a^(1/3)\*ln(abs((b\*x^2 + a)^(1/3) - a^(1/3)))) - 9\*(b\*x^2 + a)^(1/3) + 3\*(b\*x^2 + a)^(1/3)\*a/(b\*x^2))\*  
b

$$3.695 \quad \int \frac{(a+bx^2)^{4/3}}{x^5} dx$$

**Optimal.** Leaf size=132

$$\frac{b^2 \log(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{6a^{2/3}} - \frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}} - \frac{b^2 \log(x)}{9a^{2/3}} - \frac{b\sqrt[3]{a+bx^2}}{3x^2} - \frac{(a+bx^2)^{4/3}}{4x^4}$$

[Out]  $-(b*(a + b*x^2)^{(1/3)})/(3*x^2) - (a + b*x^2)^{(4/3)}/(4*x^4) - (b^2 * \text{ArcTan}[(a^{(1/3)} + 2*(a + b*x^2)^{(1/3)})/(\text{Sqrt}[3]*a^{(1/3)})])/(3*\text{Sqrt}[3]*a^{(2/3)}) - (b^2*\text{Log}[x])/(9*a^{(2/3)}) + (b^2*\text{Log}[a^{(1/3)} - (a + b*x^2)^{(1/3)}])/(6*a^{(2/3)})$

**Rubi [A]** time = 0.222703, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{b^2 \log(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{6a^{2/3}} - \frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}} - \frac{b^2 \log(x)}{9a^{2/3}} - \frac{b\sqrt[3]{a+bx^2}}{3x^2} - \frac{(a+bx^2)^{4/3}}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(4/3)/x^5, x]

[Out]  $-(b*(a + b*x^2)^{(1/3)})/(3*x^2) - (a + b*x^2)^{(4/3)}/(4*x^4) - (b^2 * \text{ArcTan}[(a^{(1/3)} + 2*(a + b*x^2)^{(1/3)})/(\text{Sqrt}[3]*a^{(1/3)})])/(3*\text{Sqrt}[3]*a^{(2/3)}) - (b^2*\text{Log}[x])/(9*a^{(2/3)}) + (b^2*\text{Log}[a^{(1/3)} - (a + b*x^2)^{(1/3)}])/(6*a^{(2/3)})$

**Rubi in Sympy [A]** time = 15.0453, size = 121, normalized size = 0.92

$$-\frac{b\sqrt[3]{a+bx^2}}{3x^2} - \frac{(a+bx^2)^{4/3}}{4x^4} - \frac{b^2 \log(x^2)}{18a^{2/3}} + \frac{b^2 \log(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{6a^{2/3}} - \frac{\sqrt{3}b^2 \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} + \frac{2\sqrt[3]{a+bx^2}}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(4/3)/x\*\*5, x)

[Out]  $-b*(a + b*x^2)^{(1/3)}/(3*x^2) - (a + b*x^2)^{(4/3)}/(4*x^4) - b^2*\log(x^2)/(18*a^{(2/3)}) + b^2*\log(a^{(1/3)} - (a + b*x^2)^{(1/3)})/(6*a^{(2/3)}) - \text{sqrt}(3)*b^2*\operatorname{atan}(\text{sqrt}(3)*(a^{(1/3)}/3 + 2*(a + b*x^2)^{(1/3)}/3)/a^{(1/3)})/(9*a^{(2/3)})$

**Mathematica [C]** time = 0.0502357, size = 80, normalized size = 0.61

$$\frac{-3a^2 - 2b^2x^4 \left(\frac{a}{bx^2} + 1\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{a}{bx^2}\right) - 10abx^2 - 7b^2x^4}{12x^4(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(4/3)/x^5, x]





---

**GIAC/XCAS [A]** time = 0.593371, size = 167, normalized size = 1.27

$$-\frac{1}{36}b^2 \left( \frac{4\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}} + \frac{2 \ln\left((bx^2+a)^{\frac{2}{3}} + (bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}} - \frac{4 \ln\left(\left|(bx^2+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}} + \frac{3\left(7(bx^2+a)^{\frac{4}{3}} - 4(bx^2+a)^{\frac{1}{3}}a\right)}{b^2 a^{\frac{2}{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/x^5,x, algorithm="giac")

[Out] -1/36\*b^2\*(4\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(b\*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(2/3) + 2\*ln((b\*x^2 + a)^(2/3) + (b\*x^2 + a)^(1/3)\*a^(1/3) + a^(2/3))/a^(2/3) - 4\*ln(abs((b\*x^2 + a)^(1/3) - a^(1/3)))/a^(2/3) + 3\*(7\*(b\*x^2 + a)^(4/3) - 4\*(b\*x^2 + a)^(1/3)\*a)/(b^2\*x^4)

$$3.696 \quad \int x^4 (a + bx^2)^{4/3} dx$$

**Optimal.** Leaf size=335

$$\frac{\frac{432a^3x\sqrt[3]{a+bx^2}}{21505b^2} + \frac{48a^2x^3\sqrt[3]{a+bx^2}}{4301b}}{432 \cdot 3^{3/4} \sqrt{2-\sqrt{3}} a^4 (\sqrt[3]{a}-\sqrt[3]{a+bx^2}) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2})^2}} F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right) \mid -7+4\sqrt{3}\right)} - \frac{21505b^3x \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{a+bx^2})}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2})^2}}}{21505b^3x} + \frac{3}{23}x^5(a+bx^2)^{4/3} + \frac{24}{391}ax^5\sqrt[3]{a+bx^2}$$

[Out]  $(-432 \cdot a^3 \cdot x \cdot (a + b \cdot x^2)^{(1/3)}) / (21505 \cdot b^2) + (48 \cdot a^2 \cdot x^3 \cdot (a + b \cdot x^2)^{(1/3)}) / (4301 \cdot b) + (24 \cdot a \cdot x^5 \cdot (a + b \cdot x^2)^{(1/3)}) / 391 + (3 \cdot x^5 \cdot (a + b \cdot x^2)^{(4/3)}) / 23 - (432 \cdot 3^{(3/4)} \cdot \text{Sqrt}[2 - \text{Sqrt}[3]] \cdot a^4 \cdot (a^{(1/3)} - (a + b \cdot x^2)^{(1/3)}) \cdot \text{Sqrt}[(a^{(2/3)} + a^{(1/3)} \cdot (a + b \cdot x^2)^{(1/3)} + (a + b \cdot x^2)^{(2/3)}) / ((1 - \text{Sqrt}[3]) \cdot a^{(1/3)} - (a + b \cdot x^2)^{(1/3)})^2] \cdot \text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3]) \cdot a^{(1/3)} - (a + b \cdot x^2)^{(1/3)}) / ((1 - \text{Sqrt}[3]) \cdot a^{(1/3)} - (a + b \cdot x^2)^{(1/3)})], -7 + 4 \cdot \text{Sqrt}[3]]) / (21505 \cdot b^3 \cdot x \cdot \text{Sqrt}[-((a^{(1/3)} \cdot (a^{(1/3)} - (a + b \cdot x^2)^{(1/3)})) / ((1 - \text{Sqrt}[3]) \cdot a^{(1/3)} - (a + b \cdot x^2)^{(1/3)})^2])]$

**Rubi [A]** time = 0.594663, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{\frac{432a^3x\sqrt[3]{a+bx^2}}{21505b^2} + \frac{48a^2x^3\sqrt[3]{a+bx^2}}{4301b}}{432 \cdot 3^{3/4} \sqrt{2-\sqrt{3}} a^4 (\sqrt[3]{a}-\sqrt[3]{a+bx^2}) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2})^2}} F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right) \mid -7+4\sqrt{3}\right)} - \frac{21505b^3x \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{a+bx^2})}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2})^2}}}{21505b^3x} + \frac{3}{23}x^5(a+bx^2)^{4/3} + \frac{24}{391}ax^5\sqrt[3]{a+bx^2}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a + b\*x^2)^(4/3), x]

[Out]  $(-432 \cdot a^3 \cdot x \cdot (a + b \cdot x^2)^{(1/3)}) / (21505 \cdot b^2) + (48 \cdot a^2 \cdot x^3 \cdot (a + b \cdot x^2)^{(1/3)}) / (4301 \cdot b) + (24 \cdot a \cdot x^5 \cdot (a + b \cdot x^2)^{(1/3)}) / 391 + (3 \cdot x^5 \cdot (a + b \cdot x^2)^{(4/3)}) / 23 - (432 \cdot 3^{(3/4)} \cdot \text{Sqrt}[2 - \text{Sqrt}[3]] \cdot a^4 \cdot (a^{(1/3)} - (a + b \cdot x^2)^{(1/3)}) \cdot \text{Sqrt}[(a^{(2/3)} + a^{(1/3)} \cdot (a + b \cdot x^2)^{(1/3)} + (a + b \cdot x^2)^{(2/3)}) / ((1 - \text{Sqrt}[3]) \cdot a^{(1/3)} - (a + b \cdot x^2)^{(1/3)})^2] \cdot \text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3]) \cdot a^{(1/3)} - (a + b \cdot x^2)^{(1/3)}) / ((1 - \text{Sqrt}[3]) \cdot a^{(1/3)} - (a + b \cdot x^2)^{(1/3)})], -7 + 4 \cdot \text{Sqrt}[3]]) / (21505 \cdot b^3 \cdot x \cdot \text{Sqrt}[-((a^{(1/3)} \cdot (a^{(1/3)} - (a + b \cdot x^2)^{(1/3)})) / ((1 - \text{Sqrt}[3]) \cdot a^{(1/3)} - (a + b \cdot x^2)^{(1/3)})^2])]$

**Rubi in Sympy [A]** time = 26.8838, size = 284, normalized size = 0.85

$$\frac{432 \cdot 3^{\frac{3}{4}} a^4 \sqrt{\frac{a^{\frac{2}{3}} + \sqrt[3]{a} \sqrt[3]{a + bx^2 + (a + bx^2)^{\frac{2}{3}}}}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}} \sqrt{-\sqrt{3} + 2} (\sqrt[3]{a} - \sqrt[3]{a + bx^2}) F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}(1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}{-\sqrt[3]{a}(-1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}\right)\right) \Big|_{-7 + 4\sqrt{3}}}{21505 b^3 x \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{a + bx^2})}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}} - \frac{432 a^3 x \sqrt[3]{a + bx^2}}{21505 b^2} + \frac{48 a^2 x^3 \sqrt[3]{a + bx^2}}{4301 b} + \frac{24 a x^5 \sqrt[3]{a + bx^2}}{391} + \frac{3 x^5 (a + bx^2)^{\frac{4}{3}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**4*(b*x**2+a)**(4/3),x)`

[Out] `-432*3**(3/4)*a**4*sqrt((a**(2/3) + a**(1/3)*(a + b*x**2)**(1/3) + (a + b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a + b*x**2)**(1/3))**2)*sqrt(-sqrt(3) + 2)*(a**(1/3) - (a + b*x**2)**(1/3))*elliptic_f(asin((a**(1/3)*(1 + sqrt(3)) - (a + b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)) - (a + b*x**2)**(1/3))), -7 + 4*sqrt(3))/(21505*b**3*x*sqrt(-a**(1/3)*(a**(1/3) - (a + b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a + b*x**2)**(1/3))**2)) - 432*a**3*x*(a + b*x**2)**(1/3)/(21505*b**2) + 48*a**2*x**3*(a + b*x**2)**(1/3)/(4301*b) + 24*a*x**5*(a + b*x**2)**(1/3)/391 + 3*x**5*(a + b*x**2)**(4/3)/23`

**Mathematica [C]** time = 0.0698203, size = 100, normalized size = 0.3

$$\frac{3x \left( 144a^4 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{1}{2}, \frac{2}{3}; \frac{3}{2}; -\frac{bx^2}{a} \right) - 144a^4 - 64a^3bx^2 + 1455a^2b^2x^4 + 2310ab^3x^6 + 935b^4x^8 \right)}{21505b^2(a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[x^4*(a + b*x^2)^(4/3),x]`

[Out] `(3*x*(-144*a^4 - 64*a^3*b*x^2 + 1455*a^2*b^2*x^4 + 2310*a*b^3*x^6 + 935*b^4*x^8 + 144*a^4*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -(b*x^2)/a]))/(21505*b^2*(a + b*x^2)^(2/3))`

**Maple [F]** time = 0.037, size = 0, normalized size = 0.

$$\int x^4 (bx^2 + a)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(b*x^2+a)^(4/3),x)`

[Out] `int(x^4*(b*x^2+a)^(4/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{4}{3}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)\*x^4,x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(4/3)\*x^4, x)

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^6 + ax^4)(bx^2 + a)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)\*x^4,x, algorithm="fricas")

[Out] integral((b\*x^6 + a\*x^4)\*(b\*x^2 + a)^(1/3), x)

**Sympy** [A] time = 5.99838, size = 29, normalized size = 0.09

$$\frac{a^{\frac{4}{3}}x^5 {}_2F_1\left(-\frac{4}{3}, \frac{5}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(b\*x\*\*2+a)\*\*(4/3), x)

[Out] a\*\*(4/3)\*x\*\*5\*hyper((-4/3, 5/2), (7/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/5

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{4}{3}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)\*x^4,x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(4/3)\*x^4, x)

$$3.697 \quad \int x^2 (a + bx^2)^{4/3} dx$$

**Optimal.** Leaf size=311

$$\frac{48a^2x\sqrt[3]{a+bx^2}}{935b} + \frac{48 \cdot 3^{3/4} \sqrt{2-\sqrt{3}} a^3 \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} F\left( \sin^{-1}\left( \frac{(1+\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7+4\sqrt{3} \right)}{935b^2x \sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{\left( (1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$+ \frac{24}{187} ax^3 \sqrt[3]{a+bx^2} + \frac{3}{17} x^3 (a+bx^2)^{4/3}$$

[Out] (48\*a^2\*x\*(a + b\*x^2)^(1/3))/(935\*b) + (24\*a\*x^3\*(a + b\*x^2)^(1/3))/187 + (3\*x^3\*(a + b\*x^2)^(4/3))/17 + (48\*3^(3/4)\*Sqrt[2 - Sqrt[3]]\*a^3\*(a^(1/3) - (a + b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a + b\*x^2)^(1/3) + (a + b\*x^2)^(2/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]\*EllipticF[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))], -7 + 4\*Sqrt[3]])/(935\*b^2\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a + b\*x^2)^(1/3)))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]))]

**Rubi [A]** time = 0.493214, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{48a^2x\sqrt[3]{a+bx^2}}{935b} + \frac{48 \cdot 3^{3/4} \sqrt{2-\sqrt{3}} a^3 \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} F\left( \sin^{-1}\left( \frac{(1+\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7+4\sqrt{3} \right)}{935b^2x \sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{\left( (1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$+ \frac{24}{187} ax^3 \sqrt[3]{a+bx^2} + \frac{3}{17} x^3 (a+bx^2)^{4/3}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*x^2)^(4/3), x]

[Out] (48\*a^2\*x\*(a + b\*x^2)^(1/3))/(935\*b) + (24\*a\*x^3\*(a + b\*x^2)^(1/3))/187 + (3\*x^3\*(a + b\*x^2)^(4/3))/17 + (48\*3^(3/4)\*Sqrt[2 - Sqrt[3]]\*a^3\*(a^(1/3) - (a + b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a + b\*x^2)^(1/3) + (a + b\*x^2)^(2/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]\*EllipticF[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))], -7 + 4\*Sqrt[3]])/(935\*b^2\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a + b\*x^2)^(1/3)))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]))]

**Rubi in Sympy [A]** time = 21.0105, size = 260, normalized size = 0.84

$$\frac{48 \cdot 3^{\frac{3}{4}} a^3 \sqrt{\frac{a^{\frac{2}{3}} + \sqrt[3]{a} \sqrt{a + bx^2 + (a+bx^2)^{\frac{2}{3}}}}{(\sqrt[3]{a}(-1+\sqrt{3}) + \sqrt[3]{a+bx^2})^2}} \sqrt{-\sqrt{3} + 2} (\sqrt[3]{a} - \sqrt[3]{a+bx^2}) F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}(1+\sqrt{3}) - \sqrt[3]{a+bx^2}}{-\sqrt[3]{a}(-1+\sqrt{3}) - \sqrt[3]{a+bx^2}}\right)\right) \Big|_{-7+4\sqrt{3}}}{935b^2x \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{(\sqrt[3]{a}(-1+\sqrt{3}) + \sqrt[3]{a+bx^2})^2}} + \frac{48a^2x\sqrt[3]{a+bx^2}}{935b} + \frac{24ax^3\sqrt[3]{a+bx^2}}{187} + \frac{3x^3(a+bx^2)^{\frac{4}{3}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2*(b*x**2+a)**(4/3),x)`

[Out] `48*3**(3/4)*a**3*sqrt((a**(2/3) + a**(1/3)*(a + b*x**2)**(1/3) + (a + b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a + b*x**2)**(1/3))**2)*sqrt(-sqrt(3) + 2)*(a**(1/3) - (a + b*x**2)**(1/3))*elliptic_f(asin((a**(1/3)*(1 + sqrt(3)) - (a + b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)) - (a + b*x**2)**(1/3))), -7 + 4*sqrt(3))/(935*b**2*x*sqrt(-a**(1/3)*(a**(1/3) - (a + b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a + b*x**2)**(1/3))**2)) + 48*a**2*x*(a + b*x**2)**(1/3)/(935*b) + 24*a*x**3*(a + b*x**2)**(1/3)/187 + 3*x**3*(a + b*x**2)**(4/3)/17`

**Mathematica [C]** time = 0.0705652, size = 90, normalized size = 0.29

$$\frac{3 \left( -16a^3x \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{1}{2}, \frac{2}{3}, \frac{3}{2}; -\frac{bx^2}{a} \right) + 16a^3x + 111a^2bx^3 + 150ab^2x^5 + 55b^3x^7 \right)}{935b(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[x^2*(a + b*x^2)^(4/3),x]`

[Out] `(3*(16*a^3*x + 111*a^2*b*x^3 + 150*a*b^2*x^5 + 55*b^3*x^7 - 16*a^3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -(b*x^2)/a]))/(935*b*(a + b*x^2)^(2/3))`

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int x^2 (bx^2 + a)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^2+a)^(4/3),x)`

[Out] `int(x^2*(b*x^2+a)^(4/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{4}{3}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)\*x^2,x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(4/3)\*x^2, x)

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^4 + ax^2)(bx^2 + a)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)\*x^2,x, algorithm="fricas")

[Out] integral((b\*x^4 + a\*x^2)\*(b\*x^2 + a)^(1/3), x)

**Sympy** [A] time = 4.4478, size = 29, normalized size = 0.09

$$\frac{a^{\frac{4}{3}}x^3 {}_2F_1\left(-\frac{4}{3}, \frac{3}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(b\*x\*\*2+a)\*\*(4/3), x)

[Out] a\*\*(4/3)\*x\*\*3\*hyper((-4/3, 3/2), (5/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/3

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{4}{3}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)\*x^2,x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(4/3)\*x^2, x)



### 3.698 $\int (a + bx^2)^{4/3} dx$

**Optimal.** Leaf size=285

$$\frac{16 \cdot 3^{3/4} \sqrt{2 - \sqrt{3}} a^2 \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \middle| -7 + 4\sqrt{3} \right)}{55bx \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}}$$

$$+ \frac{24}{55} ax \sqrt[3]{a + bx^2} + \frac{3}{11} x (a + bx^2)^{4/3}$$

[Out] (24\*a\*x\*(a + b\*x^2)^(1/3))/55 + (3\*x\*(a + b\*x^2)^(4/3))/11 - (16\*3^(3/4)\*Sqrt[2 - Sqrt[3]]\*a^2\*(a^(1/3) - (a + b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a + b\*x^2)^(1/3) + (a + b\*x^2)^(2/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]\*EllipticF[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))], -7 + 4\*Sqrt[3]])/(55\*b\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a + b\*x^2)^(1/3)))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]))

**Rubi [A]** time = 0.379512, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{16 \cdot 3^{3/4} \sqrt{2 - \sqrt{3}} a^2 \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \middle| -7 + 4\sqrt{3} \right)}{55bx \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}}$$

$$+ \frac{24}{55} ax \sqrt[3]{a + bx^2} + \frac{3}{11} x (a + bx^2)^{4/3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(4/3), x]

[Out] (24\*a\*x\*(a + b\*x^2)^(1/3))/55 + (3\*x\*(a + b\*x^2)^(4/3))/11 - (16\*3^(3/4)\*Sqrt[2 - Sqrt[3]]\*a^2\*(a^(1/3) - (a + b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a + b\*x^2)^(1/3) + (a + b\*x^2)^(2/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]\*EllipticF[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))], -7 + 4\*Sqrt[3]])/(55\*b\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a + b\*x^2)^(1/3)))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]))

**Rubi in Sympy [A]** time = 10.9512, size = 235, normalized size = 0.82

$$\frac{16 \cdot 3^{3/4} a^2 \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( \sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2} \right)^2}} \sqrt{-\sqrt{3} + 2} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) F \left( \operatorname{asin} \left( \frac{\sqrt[3]{a}(1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}{-\sqrt[3]{a}(-1 + \sqrt{3}) - \sqrt[3]{a + bx^2}} \right) \middle| -7 + 4\sqrt{3} \right)}{55bx \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( \sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2} \right)^2}}}$$

$$+ \frac{24ax \sqrt[3]{a + bx^2}}{55} + \frac{3x (a + bx^2)^{4/3}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(4/3),x)`

[Out]  $-16 \cdot 3^{3/4} \cdot a^{2/3} \sqrt{(a^{2/3} + a^{1/3}(a + b^2 x^2)^{1/3} + (a + b^2 x^2)^{2/3}) / (a^{1/3}(-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3})} + (a + b^2 x^2)^{2/3} \sqrt{-\sqrt{3} + 2} \cdot (a^{1/3} - (a + b^2 x^2)^{1/3}) \cdot \text{elliptic\_f}(\text{asin}((a^{1/3}(1 + \sqrt{3}) - (a + b^2 x^2)^{1/3}) / (-a^{1/3}(-1 + \sqrt{3}) - (a + b^2 x^2)^{1/3})), -7 + 4\sqrt{3}) / (55 \cdot b^2 x^2 \sqrt{-a^{1/3}(a^{1/3} - (a + b^2 x^2)^{1/3}) / (a^{1/3}(-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3})} + 24 \cdot a \cdot x \cdot (a + b^2 x^2)^{1/3}) / 55 + 3 \cdot x \cdot (a + b^2 x^2)^{4/3} / 11$

**Mathematica [C]** time = 0.0432367, size = 76, normalized size = 0.27

$$\frac{16a^2x \left(\frac{bx^2}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{3}{2}; -\frac{bx^2}{a}\right) + 39a^2x + 54abx^3 + 15b^2x^5}{55(a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(4/3),x]`

[Out]  $(39 \cdot a^2 \cdot x + 54 \cdot a \cdot b \cdot x^3 + 15 \cdot b^2 \cdot x^5 + 16 \cdot a^2 \cdot x \cdot (1 + (b \cdot x^2)/a)^{2/3} \cdot \text{Hypergeometric2F1}[1/2, 2/3, 3/2, -((b \cdot x^2)/a)]) / (55 \cdot (a + b \cdot x^2)^{2/3})$

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(4/3),x)`

[Out] `int((b*x^2+a)^(4/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(4/3), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^{\frac{4}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(4/3), x)`

**Sympy [A]** time = 3.59642, size = 26, normalized size = 0.09

$$a^{\frac{4}{3}} x {}_2F_1\left(-\frac{4}{3}, \frac{1}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(4/3), x)`

[Out] `a**(4/3)*x*hyper((-4/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3), x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(4/3), x)`

$$3.699 \quad \int \frac{(a+bx^2)^{4/3}}{x^2} dx$$

**Optimal.** Leaf size=280

$$\frac{16\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|_{-7+4\sqrt{3}}\right)}{5\sqrt[4]{3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{(a+bx^2)^{4/3}}{x}+\frac{8}{5}bx\sqrt[3]{a+bx^2}$$

[Out] (8\*b\*x\*(a+b\*x^2)^(1/3))/5 - (a+b\*x^2)^(4/3)/x - (16\*Sqrt[2 - Sqrt[3]]\*a\*(a^(1/3) - (a+b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a+b\*x^2)^(1/3) + (a+b\*x^2)^(2/3))/((1 - Sqrt[3])\*a^(1/3) - (a+b\*x^2)^(1/3))]^2]\*EllipticF[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a+b\*x^2)^(1/3))/((1 - Sqrt[3])\*a^(1/3) - (a+b\*x^2)^(1/3))], -7 + 4\*Sqrt[3]])/(5\*3^(1/4)\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a+b\*x^2)^(1/3)))/(1 - Sqrt[3])\*a^(1/3) - (a+b\*x^2)^(1/3))^2]))

**Rubi [A]** time = 0.371953, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{16\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|_{-7+4\sqrt{3}}\right)}{5\sqrt[4]{3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{(a+bx^2)^{4/3}}{x}+\frac{8}{5}bx\sqrt[3]{a+bx^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(4/3)/x^2, x]

[Out] (8\*b\*x\*(a+b\*x^2)^(1/3))/5 - (a+b\*x^2)^(4/3)/x - (16\*Sqrt[2 - Sqrt[3]]\*a\*(a^(1/3) - (a+b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a+b\*x^2)^(1/3) + (a+b\*x^2)^(2/3))/((1 - Sqrt[3])\*a^(1/3) - (a+b\*x^2)^(1/3))]^2]\*EllipticF[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a+b\*x^2)^(1/3))/((1 - Sqrt[3])\*a^(1/3) - (a+b\*x^2)^(1/3))], -7 + 4\*Sqrt[3]])/(5\*3^(1/4)\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a+b\*x^2)^(1/3)))/(1 - Sqrt[3])\*a^(1/3) - (a+b\*x^2)^(1/3))^2]))

**Rubi in Sympy [A]** time = 12.6351, size = 228, normalized size = 0.81

$$\frac{16\cdot 3^{\frac{3}{4}}a\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}(1+\sqrt{3})-\sqrt[3]{a+bx^2}}{-\sqrt[3]{a}(-1+\sqrt{3})-\sqrt[3]{a+bx^2}}\right)\middle|_{-7+4\sqrt{3}}\right)}{15x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)^2}}}$$

$$+\frac{8bx\sqrt[3]{a+bx^2}}{5}-\frac{(a+bx^2)^{\frac{4}{3}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(4/3)/x**2,x)`

[Out]  $-16 \cdot 3^{3/4} \cdot a \cdot \sqrt{(a^{2/3} + a^{1/3} \cdot (a + b \cdot x^{**2})^{1/3} + (a + b \cdot x^{**2})^{2/3}) / (a^{1/3} \cdot (-1 + \sqrt{3}) + (a + b \cdot x^{**2})^{1/3})} \cdot \sqrt{-\sqrt{3} + 2} \cdot (a^{1/3} - (a + b \cdot x^{**2})^{1/3}) \cdot \text{elliptic\_f}(\text{asin}((a^{1/3} \cdot (1 + \sqrt{3}) - (a + b \cdot x^{**2})^{1/3}) / (-a^{1/3} \cdot (-1 + \sqrt{3}) - (a + b \cdot x^{**2})^{1/3})), -7 + 4 \cdot \sqrt{3}) / (15 \cdot x \cdot \sqrt{-a^{1/3} \cdot (a^{1/3} - (a + b \cdot x^{**2})^{1/3}) / (a^{1/3} \cdot (-1 + \sqrt{3}) + (a + b \cdot x^{**2})^{1/3})}) + 8 \cdot b \cdot x \cdot (a + b \cdot x^{**2})^{1/3} / 5 - (a + b \cdot x^{**2})^{4/3} / x$

**Mathematica [C]** time = 0.0860569, size = 78, normalized size = 0.28

$$\frac{16abx \left(\frac{a+bx^2}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{3}{2}; -\frac{bx^2}{a}\right)}{15(a+bx^2)^{2/3}} + \sqrt[3]{a+bx^2} \left(\frac{3bx}{5} - \frac{a}{x}\right)$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(4/3)/x^2, x]`

[Out]  $(-(a/x) + (3 \cdot b \cdot x)/5) \cdot (a + b \cdot x^2)^{1/3} + (16 \cdot a \cdot b \cdot x \cdot ((a + b \cdot x^2)/a)^{2/3} \cdot \text{Hypergeometric2F1}[1/2, 2/3, 3/2, -(b \cdot x^2)/a]) / (15 \cdot (a + b \cdot x^2)^{2/3})$

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (bx^2 + a)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(4/3)/x^2,x)`

[Out] `int((b*x^2+a)^(4/3)/x^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{4}{3}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)/x^2,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(4/3)/x^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{4}{3}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/x^2,x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(4/3)/x^2, x)

**Sympy [A]** time = 4.18818, size = 29, normalized size = 0.1

$$-\frac{a^{\frac{4}{3}} {}_2F_1\left(-\frac{4}{3}, -\frac{1}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(4/3)/x\*\*2,x)

[Out] -a\*\*(4/3)\*hyper((-4/3, -1/2), (1/2,), b\*x\*\*2\*exp\_polar(I\*pi)/a)/x

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{4}{3}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/x^2,x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(4/3)/x^2, x)

$$3.700 \quad \int \frac{(a+bx^2)^{4/3}}{x^4} dx$$

**Optimal.** Leaf size=284

$$\frac{16\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle| -7+4\sqrt{3}\right)}{9\sqrt[4]{3}x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{8b\sqrt[3]{a+bx^2}}{9x}-\frac{(a+bx^2)^{4/3}}{3x^3}$$

[Out]  $(-8*b*(a+b*x^2)^(1/3))/(9*x) - (a+b*x^2)^(4/3)/(3*x^3) - (16*\text{Sqrt}[2-\text{Sqrt}[3]]*b*(a^(1/3)-(a+b*x^2)^(1/3))*\text{Sqrt}[(a^(2/3)+a^(1/3)*(a+b*x^2)^(1/3)+(a+b*x^2)^(2/3)]/((1-\text{Sqrt}[3])*a^(1/3)-(a+b*x^2)^(1/3))^2]*\text{EllipticF}[\text{ArcSin}(((1+\text{Sqrt}[3])*a^(1/3)-(a+b*x^2)^(1/3))/((1-\text{Sqrt}[3])*a^(1/3)-(a+b*x^2)^(1/3)))],-7+4*\text{Sqrt}[3])]/(9*3^(1/4)*x*\text{Sqrt}[-((a^(1/3)*(a^(1/3)-(a+b*x^2)^(1/3)))/((1-\text{Sqrt}[3])*a^(1/3)-(a+b*x^2)^(1/3))^2])$

**Rubi [A]** time = 0.392893, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{16\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle| -7+4\sqrt{3}\right)}{9\sqrt[4]{3}x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{8b\sqrt[3]{a+bx^2}}{9x}-\frac{(a+bx^2)^{4/3}}{3x^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a+b*x^2)^(4/3)/x^4,x]$

[Out]  $(-8*b*(a+b*x^2)^(1/3))/(9*x) - (a+b*x^2)^(4/3)/(3*x^3) - (16*\text{Sqrt}[2-\text{Sqrt}[3]]*b*(a^(1/3)-(a+b*x^2)^(1/3))*\text{Sqrt}[(a^(2/3)+a^(1/3)*(a+b*x^2)^(1/3)+(a+b*x^2)^(2/3)]/((1-\text{Sqrt}[3])*a^(1/3)-(a+b*x^2)^(1/3))^2]*\text{EllipticF}[\text{ArcSin}(((1+\text{Sqrt}[3])*a^(1/3)-(a+b*x^2)^(1/3))/((1-\text{Sqrt}[3])*a^(1/3)-(a+b*x^2)^(1/3)))],-7+4*\text{Sqrt}[3])]/(9*3^(1/4)*x*\text{Sqrt}[-((a^(1/3)*(a^(1/3)-(a+b*x^2)^(1/3)))/((1-\text{Sqrt}[3])*a^(1/3)-(a+b*x^2)^(1/3))^2])$

**Rubi in Sympy [A]** time = 14.2426, size = 233, normalized size = 0.82

$$\frac{16\cdot 3^{\frac{3}{4}}b\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)F\left(\text{asin}\left(\frac{\sqrt[3]{a}(1+\sqrt{3})-\sqrt[3]{a+bx^2}}{-\sqrt[3]{a}(-1+\sqrt{3})-\sqrt[3]{a+bx^2}}\right)\middle| -7+4\sqrt{3}\right)}{27x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{8b\sqrt[3]{a+bx^2}}{9x}-\frac{(a+bx^2)^{\frac{4}{3}}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(4/3)/x**4,x)`

[Out]  $-16 \cdot 3^{3/4} \cdot b \cdot \sqrt{(a^{2/3} + a^{1/3} \cdot (a + b \cdot x^2)^{1/3}) + (a + b \cdot x^2)^{2/3}} / (a^{1/3} \cdot (-1 + \sqrt{3})) + (a + b \cdot x^2)^{1/3} \cdot \sqrt{-\sqrt{3} + 2} \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3}) \cdot \text{elliptic\_f}(\text{asin}((a^{1/3} \cdot (1 + \sqrt{3})) - (a + b \cdot x^2)^{1/3}) / (-a^{1/3} \cdot (-1 + \sqrt{3}) - (a + b \cdot x^2)^{1/3})), -7 + 4 \cdot \sqrt{3}) / (27 \cdot x \cdot \sqrt{-a^{1/3} \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3}) / (a^{1/3} \cdot (-1 + \sqrt{3})) + (a + b \cdot x^2)^{1/3}}) - 8 \cdot b \cdot (a + b \cdot x^2)^{1/3} / (9 \cdot x) - (a + b \cdot x^2)^{4/3} / (3 \cdot x^3)$

**Mathematica [C]** time = 0.045003, size = 80, normalized size = 0.28

$$\frac{-9a^2 + 16b^2x^4 \left(\frac{bx^2}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{3}{2}; -\frac{bx^2}{a}\right) - 42abx^2 - 33b^2x^4}{27x^3 (a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(4/3)/x^4,x]`

[Out]  $(-9 \cdot a^2 - 42 \cdot a \cdot b \cdot x^2 - 33 \cdot b^2 \cdot x^4 + 16 \cdot b^2 \cdot x^4 \cdot (1 + (b \cdot x^2)/a)^{2/3}) \cdot \text{Hypergeometric2F1}[1/2, 2/3, 3/2, -(b \cdot x^2)/a] / (27 \cdot x^3 \cdot (a + b \cdot x^2)^{2/3})$

**Maple [F]** time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} (bx^2 + a)^{4/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(4/3)/x^4,x)`

[Out] `int((b*x^2+a)^(4/3)/x^4,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{4/3}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)/x^4,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(4/3)/x^4, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{4/3}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((b\*x^2 + a)^(4/3)/x^4,x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(4/3)/x^4, x)

**Sympy [A]** time = 4.29696, size = 34, normalized size = 0.12

$$-\frac{a^{\frac{4}{3}} {}_2F_1\left(-\frac{3}{2}, -\frac{4}{3} \middle| -\frac{1}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(4/3)/x\*\*4,x)

[Out] -a\*\*(4/3)\*hyper((-3/2, -4/3), (-1/2,), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(3\*x\*\*3)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{4}{3}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/x^4,x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(4/3)/x^4, x)

$$3.701 \quad \int x (-1 + x^2)^{7/3} dx$$

**Optimal.** Leaf size=13

$$\frac{3}{20} (x^2 - 1)^{10/3}$$

[Out] (3\*(-1 + x^2)^(10/3))/20

**Rubi [A]** time = 0.00776087, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{3}{20} (x^2 - 1)^{10/3}$$

Antiderivative was successfully verified.

[In] Int[x\*(-1 + x^2)^(7/3), x]

[Out] (3\*(-1 + x^2)^(10/3))/20

**Rubi in Sympy [A]** time = 1.63431, size = 10, normalized size = 0.77

$$\frac{3(x^2 - 1)^{\frac{10}{3}}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(x\*\*2-1)\*\*(7/3), x)

[Out] 3\*(x\*\*2 - 1)\*\*(10/3)/20

**Mathematica [A]** time = 0.00778583, size = 13, normalized size = 1.

$$\frac{3}{20} (x^2 - 1)^{10/3}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(-1 + x^2)^(7/3), x]

[Out] (3\*(-1 + x^2)^(10/3))/20

**Maple [A]** time = 0.003, size = 16, normalized size = 1.2

$$\frac{(-3 + 3x)(1 + x)}{20} (x^2 - 1)^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(x^2-1)^(7/3), x)

[Out] 3/20\*(-1+x)\*(1+x)\*(x^2-1)^(7/3)

---

**Maxima [A]** time = 1.33677, size = 12, normalized size = 0.92

$$\frac{3}{20} (x^2 - 1)^{\frac{10}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2 - 1)^(7/3)*x,x, algorithm="maxima")`

[Out] `3/20*(x^2 - 1)^(10/3)`

---

**Fricas [A]** time = 0.206275, size = 32, normalized size = 2.46

$$\frac{3}{20} (x^6 - 3x^4 + 3x^2 - 1)(x^2 - 1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2 - 1)^(7/3)*x,x, algorithm="fricas")`

[Out] `3/20*(x^6 - 3*x^4 + 3*x^2 - 1)*(x^2 - 1)^(1/3)`

---

**Sympy [A]** time = 10.4779, size = 56, normalized size = 4.31

$$\frac{3x^6\sqrt[3]{x^2-1}}{20} - \frac{9x^4\sqrt[3]{x^2-1}}{20} + \frac{9x^2\sqrt[3]{x^2-1}}{20} - \frac{3\sqrt[3]{x^2-1}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x**2-1)**(7/3),x)`

[Out] `3*x**6*(x**2 - 1)**(1/3)/20 - 9*x**4*(x**2 - 1)**(1/3)/20 + 9*x**2*(x**2 - 1)**(1/3)/20 - 3*(x**2 - 1)**(1/3)/20`

---

**GIAC/XCAS [A]** time = 0.21132, size = 12, normalized size = 0.92

$$\frac{3}{20} (x^2 - 1)^{\frac{10}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2 - 1)^(7/3)*x,x, algorithm="giac")`

[Out] `3/20*(x^2 - 1)^(10/3)`

$$3.702 \quad \int \frac{x^7}{\sqrt[3]{a + bx^2}} dx$$

**Optimal.** Leaf size=80

$$-\frac{3a^3 (a + bx^2)^{2/3}}{4b^4} + \frac{9a^2 (a + bx^2)^{5/3}}{10b^4} + \frac{3 (a + bx^2)^{11/3}}{22b^4} - \frac{9a (a + bx^2)^{8/3}}{16b^4}$$

[Out]  $(-3*a^3*(a + b*x^2)^(2/3))/(4*b^4) + (9*a^2*(a + b*x^2)^(5/3))/(10*b^4) - (9*a*(a + b*x^2)^(8/3))/(16*b^4) + (3*(a + b*x^2)^(11/3))/(22*b^4)$

**Rubi [A]** time = 0.126352, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{3a^3 (a + bx^2)^{2/3}}{4b^4} + \frac{9a^2 (a + bx^2)^{5/3}}{10b^4} + \frac{3 (a + bx^2)^{11/3}}{22b^4} - \frac{9a (a + bx^2)^{8/3}}{16b^4}$$

Antiderivative was successfully verified.

[In] Int[x^7/(a + b\*x^2)^(1/3), x]

[Out]  $(-3*a^3*(a + b*x^2)^(2/3))/(4*b^4) + (9*a^2*(a + b*x^2)^(5/3))/(10*b^4) - (9*a*(a + b*x^2)^(8/3))/(16*b^4) + (3*(a + b*x^2)^(11/3))/(22*b^4)$

**Rubi in Sympy [A]** time = 15.4855, size = 75, normalized size = 0.94

$$-\frac{3a^3 (a + bx^2)^{2/3}}{4b^4} + \frac{9a^2 (a + bx^2)^{5/3}}{10b^4} - \frac{9a (a + bx^2)^{8/3}}{16b^4} + \frac{3 (a + bx^2)^{11/3}}{22b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*7/(b\*x\*\*2+a)\*\*(1/3), x)

[Out]  $-3*a**3*(a + b*x**2)**(2/3)/(4*b**4) + 9*a**2*(a + b*x**2)**(5/3)/(10*b**4) - 9*a*(a + b*x**2)**(8/3)/(16*b**4) + 3*(a + b*x**2)**(11/3)/(22*b**4)$

**Mathematica [A]** time = 0.0317634, size = 50, normalized size = 0.62

$$\frac{3 (a + bx^2)^{2/3} (-81a^3 + 54a^2bx^2 - 45ab^2x^4 + 40b^3x^6)}{880b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a + b\*x^2)^(1/3), x]

[Out]  $(3*(a + b*x^2)^(2/3)*(-81*a^3 + 54*a^2*b*x^2 - 45*a*b^2*x^4 + 40*b^3*x^6))/(880*b^4)$

**Maple [A]** time = 0.008, size = 47, normalized size = 0.6

$$-\frac{-120b^3x^6 + 135ab^2x^4 - 162a^2bx^2 + 243a^3}{880b^4} (bx^2 + a)^{2/3}$$



```

00*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**1
2) + 4860*a**(62/3)*b**3*x**6/(880*a**20*b**4 + 5280*a**19*b**5*x
**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16
*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) - 171
0*a**(59/3)*b**4*x**8*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 528
0*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6
+ 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**1
0*x**12) + 3645*a**(59/3)*b**4*x**8/(880*a**20*b**4 + 5280*a**19*
b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200
*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12)
+ 72*a**(56/3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4
+ 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7
*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14
*b**10*x**12) + 1458*a**(56/3)*b**5*x**10/(880*a**20*b**4 + 5280*
a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 +
13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*
x**12) + 1104*a**(53/3)*b**6*x**12*(1 + b*x**2/a)**(2/3)/(880*a**
20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a*
**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 8
80*a**14*b**10*x**12) + 243*a**(53/3)*b**6*x**12/(880*a**20*b**4
+ 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7
*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14
*b**10*x**12) + 1152*a**(50/3)*b**7*x**14*(1 + b*x**2/a)**(2/3)/(
880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 1
7600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x*
**10 + 880*a**14*b**10*x**12) + 585*a**(47/3)*b**8*x**16*(1 + b*x*
**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18
*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280
*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 120*a**(44/3)*b**9*x
**18*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2
+ 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b*
**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12)

```

**GIAC/XCAS [A]** time = 0.213983, size = 77, normalized size = 0.96

$$\frac{3 \left( 40 (bx^2 + a)^{\frac{11}{3}} - 165 (bx^2 + a)^{\frac{8}{3}} a + 264 (bx^2 + a)^{\frac{5}{3}} a^2 - 220 (bx^2 + a)^{\frac{2}{3}} a^3 \right)}{880 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b\*x^2 + a)^(1/3),x, algorithm="giac")

[Out] 3/880\*(40\*(b\*x^2 + a)^(11/3) - 165\*(b\*x^2 + a)^(8/3)\*a + 264\*(b\*x^2 + a)^(5/3)\*a^2 - 220\*(b\*x^2 + a)^(2/3)\*a^3)/b^4

$$3.703 \quad \int \frac{x^5}{\sqrt[3]{a + bx^2}} dx$$

**Optimal.** Leaf size=59

$$\frac{3a^2 (a + bx^2)^{2/3}}{4b^3} + \frac{3 (a + bx^2)^{8/3}}{16b^3} - \frac{3a (a + bx^2)^{5/3}}{5b^3}$$

[Out]  $(3*a^2*(a + b*x^2)^(2/3))/(4*b^3) - (3*a*(a + b*x^2)^(5/3))/(5*b^3) + (3*(a + b*x^2)^(8/3))/(16*b^3)$

**Rubi [A]** time = 0.0978418, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{3a^2 (a + bx^2)^{2/3}}{4b^3} + \frac{3 (a + bx^2)^{8/3}}{16b^3} - \frac{3a (a + bx^2)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a + b\*x^2)^(1/3), x]

[Out]  $(3*a^2*(a + b*x^2)^(2/3))/(4*b^3) - (3*a*(a + b*x^2)^(5/3))/(5*b^3) + (3*(a + b*x^2)^(8/3))/(16*b^3)$

**Rubi in Sympy [A]** time = 11.5459, size = 54, normalized size = 0.92

$$\frac{3a^2 (a + bx^2)^{2/3}}{4b^3} - \frac{3a (a + bx^2)^{5/3}}{5b^3} + \frac{3 (a + bx^2)^{8/3}}{16b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5/(b\*x\*\*2+a)\*\*(1/3), x)

[Out]  $3*a^2*(a + b*x^2)**(2/3)/(4*b^3) - 3*a*(a + b*x^2)**(5/3)/(5*b^3) + 3*(a + b*x^2)**(8/3)/(16*b^3)$

**Mathematica [A]** time = 0.0258725, size = 39, normalized size = 0.66

$$\frac{3 (a + bx^2)^{2/3} (9a^2 - 6abx^2 + 5b^2x^4)}{80b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a + b\*x^2)^(1/3), x]

[Out]  $(3*(a + b*x^2)^(2/3)*(9*a^2 - 6*a*b*x^2 + 5*b^2*x^4))/(80*b^3)$

**Maple [A]** time = 0.006, size = 36, normalized size = 0.6

$$\frac{15b^2x^4 - 18abx^2 + 27a^2}{80b^3} (bx^2 + a)^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(b*x^2+a)^(1/3),x)`

[Out]  $3/80*(b*x^2+a)^(2/3)*(5*b^2*x^4-6*a*b*x^2+9*a^2)/b^3$

**Maxima [A]** time = 1.34512, size = 63, normalized size = 1.07

$$\frac{3(bx^2+a)^{\frac{8}{3}}}{16b^3} - \frac{3(bx^2+a)^{\frac{5}{3}}a}{5b^3} + \frac{3(bx^2+a)^{\frac{2}{3}}a^2}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2+a)^(1/3),x, algorithm="maxima")`

[Out]  $3/16*(b*x^2+a)^(8/3)/b^3 - 3/5*(b*x^2+a)^(5/3)*a/b^3 + 3/4*(b*x^2+a)^(2/3)*a^2/b^3$

**Fricas [A]** time = 0.212686, size = 47, normalized size = 0.8

$$\frac{3(5b^2x^4 - 6abx^2 + 9a^2)(bx^2 + a)^{\frac{2}{3}}}{80b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2+a)^(1/3),x, algorithm="fricas")`

[Out]  $3/80*(5*b^2*x^4 - 6*a*b*x^2 + 9*a^2)*(b*x^2 + a)^(2/3)/b^3$

**Sympy [A]** time = 5.83966, size = 631, normalized size = 10.69

$$\begin{aligned} & \frac{27a^{\frac{32}{3}} \left(1 + \frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3 + 240a^7b^4x^2 + 240a^6b^5x^4 + 80a^5b^6x^6} - \frac{27a^{\frac{32}{3}}}{80a^8b^3 + 240a^7b^4x^2 + 240a^6b^5x^4 + 80a^5b^6x^6} \\ & + \frac{63a^{\frac{29}{3}}bx^2 \left(1 + \frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3 + 240a^7b^4x^2 + 240a^6b^5x^4 + 80a^5b^6x^6} - \frac{81a^{\frac{29}{3}}bx^2}{80a^8b^3 + 240a^7b^4x^2 + 240a^6b^5x^4 + 80a^5b^6x^6} \\ & + \frac{42a^{\frac{26}{3}}b^2x^4 \left(1 + \frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3 + 240a^7b^4x^2 + 240a^6b^5x^4 + 80a^5b^6x^6} - \frac{81a^{\frac{26}{3}}b^2x^4}{80a^8b^3 + 240a^7b^4x^2 + 240a^6b^5x^4 + 80a^5b^6x^6} \\ & + \frac{18a^{\frac{23}{3}}b^3x^6 \left(1 + \frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3 + 240a^7b^4x^2 + 240a^6b^5x^4 + 80a^5b^6x^6} - \frac{27a^{\frac{23}{3}}b^3x^6}{80a^8b^3 + 240a^7b^4x^2 + 240a^6b^5x^4 + 80a^5b^6x^6} \\ & + \frac{27a^{\frac{20}{3}}b^4x^8 \left(1 + \frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3 + 240a^7b^4x^2 + 240a^6b^5x^4 + 80a^5b^6x^6} + \frac{15a^{\frac{17}{3}}b^5x^{10} \left(1 + \frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3 + 240a^7b^4x^2 + 240a^6b^5x^4 + 80a^5b^6x^6} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(b*x**2+a)**(1/3),x)`

[Out]  $27*a**(32/3)*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) - 27*a**(32/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 63*a**(29/3)*b*x**2*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) - 81*a**(29/3)*b*x**2/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 42*a**(26/3)*b**2*x**4*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) - 81*a**(26/3)*b**2*x**4/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 15*a**(17/3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6)$



$$\begin{aligned}
& 3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) \\
& + 18*a**(23/3)*b**3*x**6*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) - 27*a** \\
& *(23/3)*b**3*x**6/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 27*a**(20/3)*b**4*x**8*(1 + b*x** \\
& *2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 15*a**(17/3)*b**5*x**10*(1 + b*x**2/a) \\
& *(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6)
\end{aligned}$$

**GIAC/XCAS [A]** time = 0.215365, size = 58, normalized size = 0.98

$$\frac{3 \left( 5 (bx^2 + a)^{\frac{8}{3}} - 16 (bx^2 + a)^{\frac{5}{3}} a + 20 (bx^2 + a)^{\frac{2}{3}} a^2 \right)}{80 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b\*x^2 + a)^(1/3),x, algorithm="giac")

[Out] 3/80\*(5\*(b\*x^2 + a)^(8/3) - 16\*(b\*x^2 + a)^(5/3)\*a + 20\*(b\*x^2 + a)^(2/3)\*a^2)/b^3

$$3.704 \quad \int \frac{x^3}{\sqrt[3]{a + bx^2}} dx$$

**Optimal.** Leaf size=38

$$\frac{3(a + bx^2)^{5/3}}{10b^2} - \frac{3a(a + bx^2)^{2/3}}{4b^2}$$

[Out]  $(-3*a*(a + b*x^2)^(2/3))/(4*b^2) + (3*(a + b*x^2)^(5/3))/(10*b^2)$

**Rubi [A]** time = 0.068741, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{3(a + bx^2)^{5/3}}{10b^2} - \frac{3a(a + bx^2)^{2/3}}{4b^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b\*x^2)^(1/3), x]

[Out]  $(-3*a*(a + b*x^2)^(2/3))/(4*b^2) + (3*(a + b*x^2)^(5/3))/(10*b^2)$

**Rubi in Sympy [A]** time = 7.89859, size = 34, normalized size = 0.89

$$-\frac{3a(a + bx^2)^{2/3}}{4b^2} + \frac{3(a + bx^2)^{5/3}}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(b\*x\*\*2+a)\*\*(1/3), x)

[Out]  $-3*a*(a + b*x**2)**(2/3)/(4*b**2) + 3*(a + b*x**2)**(5/3)/(10*b**2)$

**Mathematica [A]** time = 0.0215342, size = 28, normalized size = 0.74

$$\frac{3(a + bx^2)^{2/3}(2bx^2 - 3a)}{20b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b\*x^2)^(1/3), x]

[Out]  $(3*(a + b*x^2)^(2/3)*(-3*a + 2*b*x^2))/(20*b^2)$

**Maple [A]** time = 0.007, size = 25, normalized size = 0.7

$$-\frac{-6bx^2 + 9a}{20b^2} (bx^2 + a)^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b\*x^2+a)^(1/3), x)

[Out]  $-3/20 * (b * x^2 + a)^{(2/3)} * (-2 * b * x^2 + 3 * a) / b^2$

**Maxima [A]** time = 1.34245, size = 41, normalized size = 1.08

$$\frac{3 (bx^2 + a)^{\frac{5}{3}}}{10 b^2} - \frac{3 (bx^2 + a)^{\frac{2}{3}} a}{4 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 + a)^(1/3),x, algorithm="maxima")`

[Out]  $3/10 * (b * x^2 + a)^{(5/3)} / b^2 - 3/4 * (b * x^2 + a)^{(2/3)} * a / b^2$

**Fricas [A]** time = 0.214361, size = 32, normalized size = 0.84

$$\frac{3 (2bx^2 - 3a) (bx^2 + a)^{\frac{2}{3}}}{20 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 + a)^(1/3),x, algorithm="fricas")`

[Out]  $3/20 * (2 * b * x^2 - 3 * a) * (b * x^2 + a)^{(2/3)} / b^2$

**Sympy [A]** time = 3.82915, size = 178, normalized size = 4.68

$$\frac{9a^{\frac{11}{3}} \left(1 + \frac{bx^2}{a}\right)^{\frac{2}{3}}}{20a^2b^2 + 20ab^3x^2} + \frac{9a^{\frac{11}{3}}}{20a^2b^2 + 20ab^3x^2} - \frac{3a^{\frac{8}{3}}bx^2 \left(1 + \frac{bx^2}{a}\right)^{\frac{2}{3}}}{20a^2b^2 + 20ab^3x^2} + \frac{9a^{\frac{8}{3}}bx^2}{20a^2b^2 + 20ab^3x^2} + \frac{6a^{\frac{5}{3}}b^2x^4 \left(1 + \frac{bx^2}{a}\right)^{\frac{2}{3}}}{20a^2b^2 + 20ab^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(b*x**2+a)**(1/3),x)`

[Out]  $-9 * a^{11/3} * (1 + b * x^2 / a)^{2/3} / (20 * a^2 * b^2 + 20 * a * b^3 * x^2) + 9 * a^{11/3} / (20 * a^2 * b^2 + 20 * a * b^3 * x^2) - 3 * a^{8/3} * (8/3) * b * x^2 * (1 + b * x^2 / a)^{2/3} / (20 * a^2 * b^2 + 20 * a * b^3 * x^2) + 9 * a^{8/3} * (8/3) * b * x^2 / (20 * a^2 * b^2 + 20 * a * b^3 * x^2) + 6 * a^{5/3} * b^2 * x^4 * (1 + b * x^2 / a)^{2/3} / (20 * a^2 * b^2 + 20 * a * b^3 * x^2)$

**GIAC/XCAS [A]** time = 0.213093, size = 39, normalized size = 1.03

$$\frac{3 \left(2 (bx^2 + a)^{\frac{5}{3}} - 5 (bx^2 + a)^{\frac{2}{3}} a\right)}{20 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 + a)^(1/3),x, algorithm="giac")`

[Out]  $3/20 * (2 * (b * x^2 + a)^{(5/3)} - 5 * (b * x^2 + a)^{(2/3)} * a) / b^2$

$$3.705 \quad \int \frac{x}{\sqrt[3]{a + bx^2}} dx$$

**Optimal.** Leaf size=18

$$\frac{3(a + bx^2)^{2/3}}{4b}$$

[Out] (3\*(a + b\*x^2)^(2/3))/(4\*b)

**Rubi [A]** time = 0.0115472, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{3(a + bx^2)^{2/3}}{4b}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*x^2)^(1/3), x]

[Out] (3\*(a + b\*x^2)^(2/3))/(4\*b)

**Rubi in Sympy [A]** time = 2.14296, size = 14, normalized size = 0.78

$$\frac{3(a + bx^2)^{\frac{2}{3}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(b\*x\*\*2+a)\*\*(1/3), x)

[Out] 3\*(a + b\*x\*\*2)\*\*(2/3)/(4\*b)

**Mathematica [A]** time = 0.00494022, size = 18, normalized size = 1.

$$\frac{3(a + bx^2)^{2/3}}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*x^2)^(1/3), x]

[Out] (3\*(a + b\*x^2)^(2/3))/(4\*b)

**Maple [A]** time = 0.004, size = 15, normalized size = 0.8

$$\frac{3}{4b} (bx^2 + a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b\*x^2+a)^(1/3), x)

[Out]  $3/4 * (b * x^2 + a)^{(2/3)} / b$

**Maxima [A]** time = 1.34506, size = 19, normalized size = 1.06

$$\frac{3 (bx^2 + a)^{\frac{2}{3}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^(1/3),x, algorithm="maxima")`

[Out]  $3/4 * (b * x^2 + a)^{(2/3)} / b$

**Fricas [A]** time = 0.2124, size = 19, normalized size = 1.06

$$\frac{3 (bx^2 + a)^{\frac{2}{3}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^(1/3),x, algorithm="fricas")`

[Out]  $3/4 * (b * x^2 + a)^{(2/3)} / b$

**Sympy [A]** time = 1.54547, size = 24, normalized size = 1.33

$$\begin{cases} \frac{3(a+bx^2)^{\frac{2}{3}}}{4b} & \text{for } b \neq 0 \\ \frac{x^2}{2\sqrt[3]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x**2+a)**(1/3),x)`

[Out] `Piecewise((3*(a + b*x**2)**(2/3)/(4*b), Ne(b, 0)), (x**2/(2*a**(1/3)), True))`

**GIAC/XCAS [A]** time = 0.210255, size = 19, normalized size = 1.06

$$\frac{3 (bx^2 + a)^{\frac{2}{3}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^(1/3),x, algorithm="giac")`

[Out]  $3/4 * (b * x^2 + a)^{(2/3)} / b$

$$3.706 \quad \int \frac{1}{x \sqrt[3]{a + bx^2}} dx$$

**Optimal.** Leaf size=86

$$\frac{3 \log(\sqrt[3]{a} - \sqrt[3]{a + bx^2})}{4\sqrt[3]{a}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{a + bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{a}} - \frac{\log(x)}{2\sqrt[3]{a}}$$

[Out] (Sqrt[3]\*ArcTan[(a^(1/3) + 2\*(a + b\*x^2)^(1/3))/(Sqrt[3]\*a^(1/3))])/(2\*a^(1/3)) - Log[x]/(2\*a^(1/3)) + (3\*Log[a^(1/3) - (a + b\*x^2)^(1/3)])/(4\*a^(1/3))

**Rubi [A]** time = 0.144326, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{3 \log(\sqrt[3]{a} - \sqrt[3]{a + bx^2})}{4\sqrt[3]{a}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{a + bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{a}} - \frac{\log(x)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(a + b\*x^2)^(1/3)), x]

[Out] (Sqrt[3]\*ArcTan[(a^(1/3) + 2\*(a + b\*x^2)^(1/3))/(Sqrt[3]\*a^(1/3))])/(2\*a^(1/3)) - Log[x]/(2\*a^(1/3)) + (3\*Log[a^(1/3) - (a + b\*x^2)^(1/3)])/(4\*a^(1/3))

**Rubi in Sympy [A]** time = 7.83303, size = 80, normalized size = 0.93

$$-\frac{\log(x^2)}{4\sqrt[3]{a}} + \frac{3 \log(\sqrt[3]{a} - \sqrt[3]{a + bx^2})}{4\sqrt[3]{a}} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} + \frac{2\sqrt[3]{a + bx^2}}{3}\right)}{\sqrt[3]{a}}\right)}{2\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(b\*x\*\*2+a)\*\*(1/3), x)

[Out] -log(x\*\*2)/(4\*a\*\*(1/3)) + 3\*log(a\*\*(1/3) - (a + b\*x\*\*2)\*\*(1/3))/(4\*a\*\*(1/3)) + sqrt(3)\*atan(sqrt(3)\*(a\*\*(1/3)/3 + 2\*(a + b\*x\*\*2)\*\*(1/3)/3)/a\*\*(1/3))/(2\*a\*\*(1/3))

**Mathematica [C]** time = 0.0380073, size = 48, normalized size = 0.56

$$-\frac{3\sqrt[3]{\frac{a}{bx^2}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{a}{bx^2}\right)}{2\sqrt[3]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(a + b\*x^2)^(1/3)), x]

[Out] (-3\*(1 + a/(b\*x^2))^(1/3)\*Hypergeometric2F1[1/3, 1/3, 4/3, -(a/(b\*x^2))])/(2\*(a + b\*x^2)^(1/3))

---

**Maple [F]** time = 0.027, size = 0, normalized size = 0.

$$\int \frac{1}{x \sqrt[3]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b\*x^2+a)^(1/3), x)

[Out] int(1/x/(b\*x^2+a)^(1/3), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/3)\*x), x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.223546, size = 117, normalized size = 1.36

$$\frac{2\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2(bx^2+a)^{\frac{1}{3}}a^{\frac{2}{3}}+a\right)}{3a}\right) - \log\left(\left(bx^2+a\right)^{\frac{2}{3}}a^{\frac{1}{3}} + \left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+a\right) + 2\log\left(\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}-a\right)}{4a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/3)\*x), x, algorithm="fricas")

[Out] 1/4\*(2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(b\*x^2 + a)^(1/3)\*a^(2/3) + a)/a) - log((b\*x^2 + a)^(2/3)\*a^(1/3) + (b\*x^2 + a)^(1/3)\*a^(2/3) + a) + 2\*log((b\*x^2 + a)^(1/3)\*a^(2/3) - a))/a^(1/3)

---

**Sympy [A]** time = 3.64169, size = 41, normalized size = 0.48

$$\frac{\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{3} \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2\sqrt[3]{bx^{\frac{2}{3}}}\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b\*x\*\*2+a)\*\*(1/3), x)

[Out] -gamma(1/3)\*hyper((1/3, 1/3), (4/3, ), a\*exp\_polar(I\*pi)/(b\*x\*\*2))/(2\*b\*\*(1/3)\*x\*\*(2/3)\*gamma(4/3))

---

**GIAC/XCAS [A]** time = 0.605376, size = 117, normalized size = 1.36

$$\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{2a^{\frac{1}{3}}} - \frac{\ln\left(\left(bx^2+a\right)^{\frac{2}{3}} + \left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{4a^{\frac{1}{3}}} + \frac{\ln\left(\left|\left(bx^2+a\right)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{2a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/3)\*x),x, algorithm="giac")

[Out] 1/2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(b\*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) - 1/4\*ln((b\*x^2 + a)^(2/3) + (b\*x^2 + a)^(1/3)\*a^(1/3) + a^(2/3))/a^(1/3) + 1/2\*ln(abs((b\*x^2 + a)^(1/3) - a^(1/3)))/a^(1/3)



$$3.707 \quad \int \frac{1}{x^3 \sqrt[3]{a + bx^2}} dx$$

**Optimal.** Leaf size=110

$$-\frac{b \log(\sqrt[3]{a} - \sqrt[3]{a + bx^2})}{4a^{4/3}} - \frac{b \tan^{-1}\left(\frac{2\sqrt[3]{a + bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}a^{4/3}} + \frac{b \log(x)}{6a^{4/3}} - \frac{(a + bx^2)^{2/3}}{2ax^2}$$

[Out]  $-(a + b*x^2)^{(2/3)}/(2*a*x^2) - (b*ArcTan[(a^{(1/3)} + 2*(a + b*x^2)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(2*Sqrt[3]*a^{(4/3)}) + (b*Log[x])/(6*a^{(4/3)}) - (b*Log[a^{(1/3)} - (a + b*x^2)^{(1/3)}])/(4*a^{(4/3)})$

**Rubi [A]** time = 0.1727, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\frac{b \log(\sqrt[3]{a} - \sqrt[3]{a + bx^2})}{4a^{4/3}} - \frac{b \tan^{-1}\left(\frac{2\sqrt[3]{a + bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}a^{4/3}} + \frac{b \log(x)}{6a^{4/3}} - \frac{(a + bx^2)^{2/3}}{2ax^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(a + b\*x^2)^(1/3)), x]

[Out]  $-(a + b*x^2)^{(2/3)}/(2*a*x^2) - (b*ArcTan[(a^{(1/3)} + 2*(a + b*x^2)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(2*Sqrt[3]*a^{(4/3)}) + (b*Log[x])/(6*a^{(4/3)}) - (b*Log[a^{(1/3)} - (a + b*x^2)^{(1/3)}])/(4*a^{(4/3)})$

**Rubi in Sympy [A]** time = 10.9878, size = 100, normalized size = 0.91

$$-\frac{(a + bx^2)^{2/3}}{2ax^2} + \frac{b \log(x^2)}{12a^{4/3}} - \frac{b \log(\sqrt[3]{a} - \sqrt[3]{a + bx^2})}{4a^{4/3}} - \frac{\sqrt{3}b \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} + 2\frac{\sqrt[3]{a + bx^2}}{3}\right)}{\sqrt[3]{a}}\right)}{6a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(b\*x\*\*2+a)\*\*(1/3), x)

[Out]  $-(a + b*x**2)**(2/3)/(2*a*x**2) + b*\log(x**2)/(12*a**(4/3)) - b*\log(a**(1/3) - (a + b*x**2)**(1/3))/(4*a**(4/3)) - \sqrt{3}*b*\operatorname{atan}(\sqrt{3}*(a**(1/3)/3 + 2*(a + b*x**2)**(1/3)/3)/a**(1/3))/(6*a**(4/3))$

**Mathematica [C]** time = 0.0479722, size = 69, normalized size = 0.63

$$\frac{bx^2 \sqrt[3]{\frac{a}{bx^2}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{a}{bx^2}\right) - a - bx^2}{2ax^2 \sqrt[3]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a + b\*x^2)^(1/3)), x]

[Out]  $(-a - b*x^2 + b*(1 + a/(b*x^2))^{(1/3)}*x^2*\operatorname{Hypergeometric2F1}[1/3, 1/3, 4/3, -(a/(b*x^2))])/(2*a*x^2*(a + b*x^2)^{(1/3)})$

---

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \sqrt[3]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(b*x^2+a)^(1/3), x)`

[Out] `int(1/x^3/(b*x^2+a)^(1/3), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/3)*x^3), x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.22369, size = 197, normalized size = 1.79

$$\frac{\sqrt{3} \left( \sqrt{3} b x^2 \log \left( (b x^2 + a)^{\frac{2}{3}} (-a)^{\frac{1}{3}} - (b x^2 + a)^{\frac{1}{3}} (-a)^{\frac{2}{3}} - a \right) - 2 \sqrt{3} b x^2 \log \left( (b x^2 + a)^{\frac{1}{3}} (-a)^{\frac{2}{3}} - a \right) - 6 b x^2 \arctan \left( \frac{2 \sqrt{3} (b x^2 + a)^{\frac{1}{3}} (-a)^{\frac{1}{3}}}{36 (-a)^{\frac{1}{3}} a x^2} \right) \right)}{36 (-a)^{\frac{1}{3}} a x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/3)*x^3), x, algorithm="fricas")`

[Out] `-1/36*sqrt(3)*(sqrt(3)*b*x^2*log((b*x^2 + a)^(2/3)*(-a)^(1/3) - (b*x^2 + a)^(1/3)*(-a)^(2/3) - a) - 2*sqrt(3)*b*x^2*log((b*x^2 + a)^(1/3)*(-a)^(2/3) - a) - 6*b*x^2*arctan(1/3*(2*sqrt(3)*(b*x^2 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 6*sqrt(3)*(b*x^2 + a)^(2/3)*(-a)^(1/3)/((-a)^(1/3)*a*x^2)`

---

**Sympy [A]** time = 4.37401, size = 41, normalized size = 0.37

$$\frac{\left(\frac{4}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{4}{3} \middle| \frac{a e^{i\pi}}{b x^2}\right)}{2 \sqrt[3]{b x^{\frac{8}{3}} \left(\frac{7}{3}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(b*x**2+a)**(1/3), x)`

[Out] `-gamma(4/3)*hyper((1/3, 4/3), (7/3, ), a*exp_polar(I*pi)/(b*x**2))/(2*b**(1/3)*x**(8/3)*gamma(7/3))`

---

**GIAC/XCAS [A]** time = 0.591303, size = 149, normalized size = 1.35

$$-\frac{1}{12}b \left( \frac{2\sqrt{3} \arctan\left(\frac{\sqrt{3}(2(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}})}{3a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}} - \frac{\ln\left((bx^2+a)^{\frac{2}{3}} + (bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}} + \frac{2\ln\left(\left|(bx^2+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}} + \frac{6(bx^2+a)^{\frac{2}{3}}}{abx^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/3)\*x^3),x, algorithm="giac")

[Out] -1/12\*b\*(2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(b\*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(4/3) - ln((b\*x^2 + a)^(2/3) + (b\*x^2 + a)^(1/3)\*a^(1/3) + a^(2/3))/a^(4/3) + 2\*ln(abs((b\*x^2 + a)^(1/3) - a^(1/3)))/a^(4/3) + 6\*(b\*x^2 + a)^(2/3)/(a\*b\*x^2)

$$3.708 \quad \int \frac{1}{x^5 \sqrt[3]{a + bx^2}} dx$$

**Optimal.** Leaf size=138

$$\frac{b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a + bx^2}\right)}{6a^{7/3}} + \frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a + bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}} - \frac{b^2 \log(x)}{9a^{7/3}} + \frac{b(a + bx^2)^{2/3}}{3a^2x^2} - \frac{(a + bx^2)^{2/3}}{4ax^4}$$

[Out]  $-(a + b*x^2)^{(2/3)}/(4*a*x^4) + (b*(a + b*x^2)^{(2/3)})/(3*a^2*x^2) + (b^2*ArcTan[(a^{(1/3)} + 2*(a + b*x^2)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(3*Sqrt[3]*a^{(7/3)}) - (b^2*Log[x])/(9*a^{(7/3)}) + (b^2*Log[a^{(1/3)}] - (a + b*x^2)^{(1/3)})/(6*a^{(7/3)})$

**Rubi [A]** time = 0.228244, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a + bx^2}\right)}{6a^{7/3}} + \frac{b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a + bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}} - \frac{b^2 \log(x)}{9a^{7/3}} + \frac{b(a + bx^2)^{2/3}}{3a^2x^2} - \frac{(a + bx^2)^{2/3}}{4ax^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5\*(a + b\*x^2)^(1/3)), x]

[Out]  $-(a + b*x^2)^{(2/3)}/(4*a*x^4) + (b*(a + b*x^2)^{(2/3)})/(3*a^2*x^2) + (b^2*ArcTan[(a^{(1/3)} + 2*(a + b*x^2)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(3*Sqrt[3]*a^{(7/3)}) - (b^2*Log[x])/(9*a^{(7/3)}) + (b^2*Log[a^{(1/3)}] - (a + b*x^2)^{(1/3)})/(6*a^{(7/3)})$

**Rubi in Sympy [A]** time = 15.961, size = 126, normalized size = 0.91

$$-\frac{(a + bx^2)^{\frac{2}{3}}}{4ax^4} + \frac{b(a + bx^2)^{\frac{2}{3}}}{3a^2x^2} - \frac{b^2 \log(x^2)}{18a^{\frac{7}{3}}} + \frac{b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a + bx^2}\right)}{6a^{\frac{7}{3}}} + \frac{\sqrt{3}b^2 \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} + 2\frac{\sqrt[3]{a + bx^2}}{3}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*5/(b\*x\*\*2+a)\*\*(1/3), x)

[Out]  $-(a + b*x^2)^{(2/3)}/(4*a*x^4) + b*(a + b*x^2)^{(2/3)}/(3*a^2*x^2) - b^2*log(x^2)/(18*a^{(7/3)}) + b^2*log(a^{(1/3)} - (a + b*x^2)^{(1/3)})/(6*a^{(7/3)}) + sqrt(3)*b^2*atan(sqrt(3)*(a^{(1/3)}/3 + 2*(a + b*x^2)^{(1/3)}/3)/a^{(1/3)})/(9*a^{(7/3)})$

**Mathematica [C]** time = 0.0539015, size = 82, normalized size = 0.59

$$\frac{-3a^2 - 4b^2x^4\sqrt[3]{\frac{a}{bx^2}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{a}{bx^2}\right) + abx^2 + 4b^2x^4}{12a^2x^4\sqrt[3]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5\*(a + b\*x^2)^(1/3)), x]

[Out]  $(-3*a^2 + a*b*x^2 + 4*b^2*x^4 - 4*b^2*(1 + a/(b*x^2))^{1/3}*x^4*Hypergeometric2F1[1/3, 1/3, 4/3, -(a/(b*x^2))])/(12*a^2*x^4*(a + b*x^2)^{1/3})$

---

**Maple [F]** time = 0.042, size = 0, normalized size = 0.

$$\int \frac{1}{x^5 \sqrt[3]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^5/(b*x^2+a)^(1/3), x)`

[Out] `int(1/x^5/(b*x^2+a)^(1/3), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/3)*x^5), x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.222643, size = 196, normalized size = 1.42

$$\frac{\sqrt{3} \left( 2 \sqrt{3} b^2 x^4 \log \left( (bx^2 + a)^{\frac{2}{3}} a^{\frac{1}{3}} + (bx^2 + a)^{\frac{1}{3}} a^{\frac{2}{3}} + a \right) - 4 \sqrt{3} b^2 x^4 \log \left( (bx^2 + a)^{\frac{1}{3}} a^{\frac{2}{3}} - a \right) - 12 b^2 x^4 \arctan \left( \frac{2 \sqrt{3} (bx^2 + a)^{\frac{1}{3}}}{3 a} \right) \right)}{108 a^{\frac{7}{3}} x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/3)*x^5), x, algorithm="fricas")`

[Out]  $-1/108*\sqrt{3}*(2*\sqrt{3}*b^2*x^4*\log((b*x^2 + a)^{2/3}*a^{1/3} + (b*x^2 + a)^{1/3}*a^{2/3} + a) - 4*\sqrt{3}*b^2*x^4*\log((b*x^2 + a)^{1/3}*a^{2/3} - a) - 12*b^2*x^4*\arctan(1/3*(2*\sqrt{3}*(b*x^2 + a)^{1/3}*a^{2/3} + \sqrt{3}*a)/a) - 3*\sqrt{3}*(4*b*x^2 - 3*a)*(b*x^2 + a)^{2/3}*a^{1/3})/(a^{7/3}*x^4)$

---

**Sympy [A]** time = 5.55842, size = 41, normalized size = 0.3

$$\frac{\left(\frac{7}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{7}{3} \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2\sqrt[3]{bx^{\frac{14}{3}}}\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(b*x**2+a)**(1/3), x)`

[Out]  $-\text{gamma}(7/3)*\text{hyper}((1/3, 7/3), (10/3, ), a*\text{exp\_polar}(I*\text{pi})/(b*x**2))/(2*b**(1/3)*x**(14/3)*\text{gamma}(10/3))$

---

**GIAC/XCAS [A]** time = 0.598851, size = 171, normalized size = 1.24

$$\frac{1}{36} b^2 \left( \frac{4 \sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{7}{3}}} - \frac{2 \ln\left(\left(bx^2+a\right)^{\frac{2}{3}} + \left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{a^{\frac{7}{3}}} + \frac{4 \ln\left(\left| \left(bx^2+a\right)^{\frac{1}{3}} - a^{\frac{1}{3}} \right| \right)}{a^{\frac{7}{3}}} + \frac{3\left(4(bx^2+a)^{\frac{5}{3}} - 7(bx^2+a)^{\frac{2}{3}}a\right)}{a^2 b^2 x^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/3)\*x^5),x, algorithm="giac")

[Out] 1/36\*b^2\*(4\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(b\*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(7/3) - 2\*ln((b\*x^2 + a)^(2/3) + (b\*x^2 + a)^(1/3)\*a^(1/3) + a^(2/3))/a^(7/3) + 4\*ln(abs((b\*x^2 + a)^(1/3) - a^(1/3)))/a^(7/3) + 3\*(4\*(b\*x^2 + a)^(5/3) - 7\*(b\*x^2 + a)^(2/3)\*a)/(a^2\*b^2\*x^4)

$$3.709 \quad \int \frac{x^4}{\sqrt[3]{a + bx^2}} dx$$

**Optimal.** Leaf size=580

$$\begin{aligned} & 27\sqrt{2}3^{3/4}a^{7/3} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \mid -7 + 4\sqrt{3} \right) \\ & \frac{91b^3x \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} - \sqrt[3]{a + bx^2})}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}}{81\sqrt[4]{3} \sqrt{2 + \sqrt{3}} a^{7/3} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \mid -7 + 4\sqrt{3} \right)} \\ & + \frac{182b^3x \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} - \sqrt[3]{a + bx^2})}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}}{81a^2x} - \frac{27ax (a + bx^2)^{2/3}}{91b^2} + \frac{3x^3 (a + bx^2)^{2/3}}{13b} \end{aligned}$$

[Out]  $(-27*a*x*(a + b*x^2)^{(2/3)})/(91*b^2) + (3*x^3*(a + b*x^2)^{(2/3)})/(13*b) - (81*a^2*x)/(91*b^2*((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})) + (81*3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^{(7/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3]]/(182*b^3*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]) - (27*\text{Sqrt}[2]*3^{(3/4)}*a^{(7/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3]]/(91*b^3*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2])$

**Rubi [A]** time = 0.875128, antiderivative size = 580, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\begin{aligned} & 27\sqrt{2}3^{3/4}a^{7/3} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \mid -7 + 4\sqrt{3} \right) \\ & \frac{91b^3x \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} - \sqrt[3]{a + bx^2})}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}}{81\sqrt[4]{3} \sqrt{2 + \sqrt{3}} a^{7/3} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \mid -7 + 4\sqrt{3} \right)} \\ & + \frac{182b^3x \sqrt{\frac{\sqrt[3]{a} (\sqrt[3]{a} - \sqrt[3]{a + bx^2})}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}}{81a^2x} - \frac{27ax (a + bx^2)^{2/3}}{91b^2} + \frac{3x^3 (a + bx^2)^{2/3}}{13b} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^4/(a + b*x^2)^{(1/3)}, x]$

[Out]  $(-27*a*x*(a + b*x^2)^{(2/3)})/(91*b^2) + (3*x^3*(a + b*x^2)^{(2/3)})/(13*b) - (81*a^2*x)/(91*b^2*((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)}))$

$$\begin{aligned} & (1/3))) + (81 \cdot 3^{1/4} \cdot \text{Sqrt}[2 + \text{Sqrt}[3]] \cdot a^{7/3} \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3}) \cdot \text{Sqrt}[(a^{2/3} + a^{1/3} \cdot (a + b \cdot x^2)^{1/3} + (a + b \cdot x^2)^{2/3}) / ((1 - \text{Sqrt}[3]) \cdot a^{1/3} - (a + b \cdot x^2)^{1/3})^2] \cdot \text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3]) \cdot a^{1/3} - (a + b \cdot x^2)^{1/3}) / ((1 - \text{Sqrt}[3]) \cdot a^{1/3} - (a + b \cdot x^2)^{1/3})], -7 + 4 \cdot \text{Sqrt}[3]]) / (182 \cdot b^3 \cdot x \cdot \text{Sqrt}[-((a^{1/3} \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3})) / ((1 - \text{Sqrt}[3]) \cdot a^{1/3} - (a + b \cdot x^2)^{1/3}))^2]) - (27 \cdot \text{Sqrt}[2] \cdot 3^{3/4} \cdot a^{7/3} \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3}) \cdot \text{Sqrt}[(a^{2/3} + a^{1/3} \cdot (a + b \cdot x^2)^{1/3} + (a + b \cdot x^2)^{2/3}) / ((1 - \text{Sqrt}[3]) \cdot a^{1/3} - (a + b \cdot x^2)^{1/3})^2] \cdot \text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3]) \cdot a^{1/3} - (a + b \cdot x^2)^{1/3}) / ((1 - \text{Sqrt}[3]) \cdot a^{1/3} - (a + b \cdot x^2)^{1/3})], -7 + 4 \cdot \text{Sqrt}[3]]) / (91 \cdot b^3 \cdot x \cdot \text{Sqrt}[-((a^{1/3} \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3})) / ((1 - \text{Sqrt}[3]) \cdot a^{1/3} - (a + b \cdot x^2)^{1/3}))^2]) \end{aligned}$$

**Rubi in Sympy [A]** time = 38.0188, size = 481, normalized size = 0.83

$$\begin{aligned} & 81 \sqrt[3]{3} a^{\frac{7}{3}} \sqrt{\frac{a^{\frac{2}{3}} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{\frac{2}{3}}}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}} \sqrt{\sqrt{3} + 2} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) E \left( \text{asin} \left( \frac{\sqrt[3]{a}(1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}{-\sqrt[3]{a}(-1 + \sqrt{3}) - \sqrt[3]{a + bx^2}} \right) \middle| -7 + 4\sqrt{3} \right) \\ & \frac{182 b^3 x \sqrt{-\frac{\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{a + bx^2})}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}}}{27 \sqrt{2} \cdot 3^{\frac{3}{4}} a^{\frac{7}{3}} \sqrt{\frac{a^{\frac{2}{3}} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{\frac{2}{3}}}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}}} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) F \left( \text{asin} \left( \frac{\sqrt[3]{a}(1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}{-\sqrt[3]{a}(-1 + \sqrt{3}) - \sqrt[3]{a + bx^2}} \right) \middle| -7 + 4\sqrt{3} \right) \\ & \frac{91 b^3 x \sqrt{-\frac{\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{a + bx^2})}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}}}{+ \frac{81 a^2 x}{91 b^2 (\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})} - \frac{27 a x (a + bx^2)^{\frac{2}{3}}}{91 b^2} + \frac{3 x^3 (a + bx^2)^{\frac{2}{3}}}{13 b}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**4/(b*x**2+a)**(1/3),x)`

[Out]  $81 \cdot 3^{1/4} \cdot a^{7/3} \cdot \text{sqrt}((a^{2/3} + a^{1/3} \cdot (a + b \cdot x^2)^{1/3}) + (a + b \cdot x^2)^{2/3}) / (a^{1/3} \cdot (-1 + \text{sqrt}(3)) + (a + b \cdot x^2)^{1/3}) \cdot \text{sqrt}(\text{sqrt}(3) + 2) \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3}) \cdot \text{elliptic}_e(\text{asin}((a^{1/3} \cdot (1 + \text{sqrt}(3)) - (a + b \cdot x^2)^{1/3}) / (-a^{1/3} \cdot (-1 + \text{sqrt}(3)) - (a + b \cdot x^2)^{1/3})), -7 + 4 \cdot \text{sqrt}(3)) / (182 \cdot b^3 \cdot x \cdot \text{sqrt}(-a^{1/3} \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3})) / (a^{1/3} \cdot (-1 + \text{sqrt}(3)) + (a + b \cdot x^2)^{1/3})) - 27 \cdot \text{sqrt}(2) \cdot 3^{3/4} \cdot a^{7/3} \cdot \text{sqrt}((a^{2/3} + a^{1/3} \cdot (a + b \cdot x^2)^{1/3} + (a + b \cdot x^2)^{2/3}) / (a^{1/3} \cdot (-1 + \text{sqrt}(3)) + (a + b \cdot x^2)^{1/3})) \cdot \text{elliptic}_f(\text{asin}((a^{1/3} \cdot (1 + \text{sqrt}(3)) - (a + b \cdot x^2)^{1/3}) / (-a^{1/3} \cdot (-1 + \text{sqrt}(3)) - (a + b \cdot x^2)^{1/3})), -7 + 4 \cdot \text{sqrt}(3)) / (91 \cdot b^3 \cdot x \cdot \text{sqrt}(-a^{1/3} \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3})) / (a^{1/3} \cdot (-1 + \text{sqrt}(3)) + (a + b \cdot x^2)^{1/3})) + 81 \cdot a^2 \cdot x / (91 \cdot b^2 \cdot (a^{1/3} \cdot (-1 + \text{sqrt}(3)) + (a + b \cdot x^2)^{1/3})) - 27 \cdot a \cdot x \cdot (a + b \cdot x^2)^{2/3} / (91 \cdot b^2) + 3 \cdot x^3 \cdot (a + b \cdot x^2)^{2/3} / (13 \cdot b)$

**Mathematica [C]** time = 0.0587118, size = 79, normalized size = 0.14

$$\frac{3 \left( 9 a^2 x \sqrt{\frac{b x^2}{a}} + {}_2F_1 \left( \frac{1}{3}, \frac{1}{2}; \frac{3}{2}; -\frac{b x^2}{a} \right) - 9 a^2 x - 2 a b x^3 + 7 b^2 x^5 \right)}{91 b^2 \sqrt[3]{a + b x^2}}$$

Antiderivative was successfully verified.



[In] Integrate[x^4/(a + b\*x^2)^(1/3), x]

[Out] (3\*(-9\*a^2\*x - 2\*a\*b\*x^3 + 7\*b^2\*x^5 + 9\*a^2\*x\*(1 + (b\*x^2)/a)^(1/3))\*Hypergeometric2F1[1/3, 1/2, 3/2, -((b\*x^2)/a)])/(91\*b^2\*(a + b\*x^2)^(1/3))

**Maple** [F] time = 0.036, size = 0, normalized size = 0.

$$\int x^4 \frac{1}{\sqrt[3]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b\*x^2+a)^(1/3), x)

[Out] int(x^4/(b\*x^2+a)^(1/3), x)

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2 + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b\*x^2 + a)^(1/3), x, algorithm="maxima")

[Out] integrate(x^4/(b\*x^2 + a)^(1/3), x)

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{(bx^2 + a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b\*x^2 + a)^(1/3), x, algorithm="fricas")

[Out] integral(x^4/(b\*x^2 + a)^(1/3), x)

**Sympy** [A] time = 2.38402, size = 27, normalized size = 0.05

$$\frac{x^5 {}_2F_1\left(\frac{1}{3}, \frac{5}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4/(b\*x\*\*2+a)\*\*(1/3), x)

[Out] x\*\*5\*hyper((1/3, 5/2), (7/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(5\*a\*\*(1/3))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2 + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^(1/3), x, algorithm="giac")`

[Out] `integrate(x^4/(b*x^2 + a)^(1/3), x)`

$$3.710 \quad \int \frac{x^2}{\sqrt[3]{a + bx^2}} dx$$

**Optimal.** Leaf size=556

$$\frac{3\sqrt{2}3^{3/4}a^{4/3} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3})\sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1 + \sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \mid -7 + 4\sqrt{3} \right)}{7b^2x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3})\sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}}$$

$$\frac{9\sqrt[4]{3}\sqrt{2 + \sqrt{3}}a^{4/3} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3})\sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1 + \sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \mid -7 + 4\sqrt{3} \right)}{14b^2x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3})\sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}}$$

$$+ \frac{9ax}{7b \left( (1 - \sqrt{3})\sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)} + \frac{3x(a + bx^2)^{2/3}}{7b}$$

[Out]  $(3*x*(a + b*x^2)^(2/3))/(7*b) + (9*a*x)/(7*b*((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3))) - (9*3^(1/4)*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^(4/3)*(a^(1/3) - (a + b*x^2)^(1/3))*\text{Sqrt}[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3))]^2)*\text{EllipticE}[\text{ArcSin}[\frac{(1 + \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3)}{(1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3)}], -7 + 4*\text{Sqrt}[3]])/(14*b^2*x*\text{Sqrt}[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) + (3*\text{Sqrt}[2]*3^(3/4)*a^(4/3)*(a^(1/3) - (a + b*x^2)^(1/3))*\text{Sqrt}[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3))]^2)*\text{EllipticF}[\text{ArcSin}[\frac{(1 + \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3)}{(1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3)}], -7 + 4*\text{Sqrt}[3]])/(7*b^2*x*\text{Sqrt}[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])$

**Rubi [A]** time = 0.754629, antiderivative size = 556, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{3\sqrt{2}3^{3/4}a^{4/3} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3})\sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1 + \sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \mid -7 + 4\sqrt{3} \right)}{7b^2x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3})\sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}}$$

$$\frac{9\sqrt[4]{3}\sqrt{2 + \sqrt{3}}a^{4/3} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3})\sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1 + \sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3})\sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \mid -7 + 4\sqrt{3} \right)}{14b^2x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3})\sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}}$$

$$+ \frac{9ax}{7b \left( (1 - \sqrt{3})\sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)} + \frac{3x(a + bx^2)^{2/3}}{7b}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*x^2)^(1/3), x]

[Out]  $(3*x*(a + b*x^2)^(2/3))/(7*b) + (9*a*x)/(7*b*((1 - \text{Sqrt}[3])*a^(1/3) - (a + b*x^2)^(1/3))) - (9*3^(1/4)*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^(4/3)*($

$$a^{1/3} - (a + b^2 x^2)^{1/3}) \sqrt{(a^{2/3} + a^{1/3} (a + b^2 x^2)^{1/3} + (a + b^2 x^2)^{2/3}) / ((1 - \sqrt{3}) a^{1/3} - (a + b^2 x^2)^{1/3})^2} \operatorname{EllipticE}[\operatorname{ArcSin}(((1 + \sqrt{3}) a^{1/3} - (a + b^2 x^2)^{1/3}) / ((1 - \sqrt{3}) a^{1/3} - (a + b^2 x^2)^{1/3}))], -7 + 4\sqrt{3}] / (14 b^2 x \sqrt{-(a^{1/3} (a^{1/3} - (a + b^2 x^2)^{1/3})) / ((1 - \sqrt{3}) a^{1/3} - (a + b^2 x^2)^{1/3})^2}) + (3 \sqrt{2} \cdot 3^{3/4} a^{3/4} \sqrt{(a^{2/3} + \sqrt{a} \sqrt{a + b^2 x^2} + (a + b^2 x^2)^{2/3}) / ((\sqrt{a}(-1 + \sqrt{3}) + \sqrt{a + b^2 x^2})^2)} \sqrt{\sqrt{3} + 2} (\sqrt{a} - \sqrt{a + b^2 x^2}) E\left(\operatorname{asin}\left(\frac{\sqrt{a}(1 + \sqrt{3}) - \sqrt{a + b^2 x^2}}{-\sqrt{a}(-1 + \sqrt{3}) - \sqrt{a + b^2 x^2}}\right)\right) \Big|_{-7 + 4\sqrt{3}}) + \frac{14 b^2 x \sqrt{\frac{\sqrt{a}(\sqrt{a} - \sqrt{a + b^2 x^2})}{(\sqrt{a}(-1 + \sqrt{3}) + \sqrt{a + b^2 x^2})^2}}}{3 \sqrt{2} \cdot 3^{3/4} a^{3/4} \sqrt{(a^{2/3} + \sqrt{a} \sqrt{a + b^2 x^2} + (a + b^2 x^2)^{2/3}) / ((\sqrt{a}(-1 + \sqrt{3}) + \sqrt{a + b^2 x^2})^2)} (\sqrt{a} - \sqrt{a + b^2 x^2}) F\left(\operatorname{asin}\left(\frac{\sqrt{a}(1 + \sqrt{3}) - \sqrt{a + b^2 x^2}}{-\sqrt{a}(-1 + \sqrt{3}) - \sqrt{a + b^2 x^2}}\right)\right) \Big|_{-7 + 4\sqrt{3}}) + \frac{7 b^2 x \sqrt{\frac{\sqrt{a}(\sqrt{a} - \sqrt{a + b^2 x^2})}{(\sqrt{a}(-1 + \sqrt{3}) + \sqrt{a + b^2 x^2})^2}}}{\frac{9 a x}{7 b (\sqrt{a}(-1 + \sqrt{3}) + \sqrt{a + b^2 x^2})} + \frac{3 x (a + b^2 x^2)^{2/3}}{7 b}}$$

**Rubi in Sympy [A]** time = 29.9619, size = 456, normalized size = 0.82

$$9 \sqrt[4]{3} a^{\frac{4}{3}} \sqrt{\frac{a^{\frac{2}{3}} + \sqrt{a} \sqrt{a + b^2 x^2} + (a + b^2 x^2)^{\frac{2}{3}}}{(\sqrt{a}(-1 + \sqrt{3}) + \sqrt{a + b^2 x^2})^2}} \sqrt{\sqrt{3} + 2} (\sqrt{a} - \sqrt{a + b^2 x^2}) E\left(\operatorname{asin}\left(\frac{\sqrt{a}(1 + \sqrt{3}) - \sqrt{a + b^2 x^2}}{-\sqrt{a}(-1 + \sqrt{3}) - \sqrt{a + b^2 x^2}}\right)\right) \Big|_{-7 + 4\sqrt{3}}) + \frac{14 b^2 x \sqrt{\frac{\sqrt{a}(\sqrt{a} - \sqrt{a + b^2 x^2})}{(\sqrt{a}(-1 + \sqrt{3}) + \sqrt{a + b^2 x^2})^2}}}{3 \sqrt{2} \cdot 3^{\frac{3}{4}} a^{\frac{4}{3}} \sqrt{\frac{a^{\frac{2}{3}} + \sqrt{a} \sqrt{a + b^2 x^2} + (a + b^2 x^2)^{\frac{2}{3}}}{(\sqrt{a}(-1 + \sqrt{3}) + \sqrt{a + b^2 x^2})^2}} (\sqrt{a} - \sqrt{a + b^2 x^2}) F\left(\operatorname{asin}\left(\frac{\sqrt{a}(1 + \sqrt{3}) - \sqrt{a + b^2 x^2}}{-\sqrt{a}(-1 + \sqrt{3}) - \sqrt{a + b^2 x^2}}\right)\right) \Big|_{-7 + 4\sqrt{3}}) + \frac{7 b^2 x \sqrt{\frac{\sqrt{a}(\sqrt{a} - \sqrt{a + b^2 x^2})}{(\sqrt{a}(-1 + \sqrt{3}) + \sqrt{a + b^2 x^2})^2}}}{\frac{9 a x}{7 b (\sqrt{a}(-1 + \sqrt{3}) + \sqrt{a + b^2 x^2})} + \frac{3 x (a + b^2 x^2)^{\frac{2}{3}}}{7 b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2/(b*x**2+a)**(1/3),x)`

[Out]  $-9 \cdot 3^{1/4} \cdot a^{4/3} \sqrt{(a^{2/3} + a^{1/3} (a + b^2 x^2)^{1/3} + (a + b^2 x^2)^{2/3}) / (a^{1/3} (-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3})^2} \sqrt{(\sqrt{3} + 2) (a^{1/3} - (a + b^2 x^2)^{1/3})} \operatorname{elliptic\_e}(\operatorname{asin}((a^{1/3} (1 + \sqrt{3}) - (a + b^2 x^2)^{1/3}) / (-a^{1/3} (-1 + \sqrt{3}) - (a + b^2 x^2)^{1/3})), -7 + 4\sqrt{3}) / (14 b^2 x \sqrt{-(a^{1/3} (a^{1/3} - (a + b^2 x^2)^{1/3})) / (a^{1/3} (-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3})^2}) + 3 \sqrt{2} \cdot 3^{3/4} a^{3/4} \sqrt{(a^{2/3} + \sqrt{a} \sqrt{a + b^2 x^2} + (a + b^2 x^2)^{2/3}) / (a^{1/3} (-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3})^2} (\sqrt{a} - \sqrt{a + b^2 x^2}) \operatorname{elliptic\_f}(\operatorname{asin}((a^{1/3} (1 + \sqrt{3}) - (a + b^2 x^2)^{1/3}) / (-a^{1/3} (-1 + \sqrt{3}) - (a + b^2 x^2)^{1/3})), -7 + 4\sqrt{3}) / (7 b^2 x \sqrt{-(a^{1/3} (a^{1/3} - (a + b^2 x^2)^{1/3})) / (a^{1/3} (-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3})^2}) - 9 a x / (7 b (a^{1/3} (-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3})) + 3 x (a + b^2 x^2)^{2/3} / (7 b)$

**Mathematica [C]** time = 0.0484029, size = 62, normalized size = 0.11

$$\frac{3x \left( -a \sqrt[3]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) + a + bx^2 \right)}{7b \sqrt[3]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] `Integrate[x^2/(a + b*x^2)^(1/3),x]`

[Out]  $(3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^{1/3}) * \text{Hypergeometric2F1}[1/3, 1/2, 3/2, -((b*x^2)/a)]) / (7*b*(a + b*x^2)^{1/3})$

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt[3]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*x^2+a)^(1/3), x)`

[Out] `int(x^2/(b*x^2+a)^(1/3), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2 + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(1/3), x, algorithm="maxima")`

[Out] `integrate(x^2/(b*x^2 + a)^(1/3), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{(bx^2 + a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(1/3), x, algorithm="fricas")`

[Out] `integral(x^2/(b*x^2 + a)^(1/3), x)`

**Sympy [A]** time = 2.22013, size = 27, normalized size = 0.05

$$\frac{x^3 {}_2F_1\left(\frac{1}{3}, \frac{3}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{3\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**2+a)**(1/3), x)`

[Out] `x**3*hyper((1/3, 3/2), (5/2, ), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/3))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2 + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2 + a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(1/3), x)
```

$$3.711 \quad \int \frac{1}{\sqrt[3]{a+bx^2}} dx$$

**Optimal.** Leaf size=529

$$\frac{\sqrt{23}^{3/4} \sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{bx \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$+ \frac{3\sqrt[3]{3} \sqrt{2 + \sqrt{3}} \sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{2bx \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$- \frac{3x}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2}}$$

[Out]  $(-3*x)/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)}) + (3*3^{(1/4)}*S$   
 $\text{qrt}[2 + \text{Sqrt}[3]]*a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3]$   
 $)*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3])$   
 $)*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3]]/(2*b*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a$   
 $+ b*x^2)^{(1/3)))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2])$   
 $)- (\text{Sqrt}[2]*3^{(3/4)}*a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3]$   
 $)*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3])$   
 $)*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3]]/(b*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a$   
 $+ b*x^2)^{(1/3)))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2])$   
 $)]$

**Rubi [A]** time = 0.618768, antiderivative size = 529, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$\frac{\sqrt{23}^{3/4} \sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{bx \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$+ \frac{3\sqrt[3]{3} \sqrt{2 + \sqrt{3}} \sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{2bx \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$- \frac{3x}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^{-1/3}, x]$

[Out]  $(-3*x)/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)}) + (3*3^{(1/4)}*S$   
 $\text{qrt}[2 + \text{Sqrt}[3]]*a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3]$   
 $)*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3])$   
 $)*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3]]/(2*b*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a$   
 $+ b*x^2)^{(1/3)))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2])$   
 $)- (\text{Sqrt}[2]*3^{(3/4)}*a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3]$   
 $)*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3])$   
 $)*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3]]/(b*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a$   
 $+ b*x^2)^{(1/3)))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2])$   
 $)]$





---

**Maple [F]** time = 0.037, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+a)^(1/3), x)

[Out] int(1/(b\*x^2+a)^(1/3), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-1/3), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(-1/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-1/3), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(-1/3), x)

---

**Sympy [A]** time = 2.14711, size = 24, normalized size = 0.05

$$\frac{x {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*2+a)\*\*(1/3), x)

[Out] x\*hyper((1/3, 1/2), (3/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/a\*\*(1/3)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(-1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-1/3), x)
```

$$3.712 \quad \int \frac{1}{x^2 \sqrt[3]{a + bx^2}} dx$$

**Optimal.** Leaf size=546

$$\frac{\sqrt{2} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \mid -7 + 4\sqrt{3} \right)}{\sqrt[3]{3} a^{2/3} x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}}$$

$$+ \frac{\sqrt[3]{3} \sqrt{2 + \sqrt{3}} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \mid -7 + 4\sqrt{3} \right)}{2 a^{2/3} x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}}$$

$$- \frac{bx}{a \left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)} - \frac{(a + bx^2)^{2/3}}{ax}$$

[Out]  $-\left((a + b*x^2)^{(2/3)}/(a*x)\right) - (b*x)/(a*((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})) + (3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3]]/(2*a^{(2/3)}*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2)]) - (\text{Sqrt}[2]*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3]]/(3^{(1/4)}*a^{(2/3)}*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2)])$

**Rubi [A]** time = 0.732836, antiderivative size = 546, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{\sqrt{2} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \mid -7 + 4\sqrt{3} \right)}{\sqrt[3]{3} a^{2/3} x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}}$$

$$+ \frac{\sqrt[3]{3} \sqrt{2 + \sqrt{3}} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \mid -7 + 4\sqrt{3} \right)}{2 a^{2/3} x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}}$$

$$- \frac{bx}{a \left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)} - \frac{(a + bx^2)^{2/3}}{ax}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a + b\*x^2)^(1/3)), x]

[Out]  $-\left((a + b*x^2)^{(2/3)}/(a*x)\right) - (b*x)/(a*((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})) + (3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3]]/(2*a^{(2/3)}*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2)]) - (\text{Sqrt}[2]*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3]]/(3^{(1/4)}*a^{(2/3)}*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2)])$

$$\begin{aligned} & x^2)^{(1/3)}) * \text{Sqrt}[(a^{(2/3)} + a^{(1/3)} * (a + b * x^2)^{(1/3)} + (a + b * x^2)^{(2/3)}) / ((1 - \text{Sqrt}[3]) * a^{(1/3)} - (a + b * x^2)^{(1/3)})^2] * \text{EllipticE}[\text{ArcSin}[\frac{(1 + \text{Sqrt}[3]) * a^{(1/3)} - (a + b * x^2)^{(1/3)}}{(1 - \text{Sqrt}[3]) * a^{(1/3)} - (a + b * x^2)^{(1/3)}}], -7 + 4 * \text{Sqrt}[3]]] / (2 * a^{(2/3)} * x * \text{Sqrt}[-((a^{(1/3)} * (a^{(1/3)} - (a + b * x^2)^{(1/3)})) / ((1 - \text{Sqrt}[3]) * a^{(1/3)} - (a + b * x^2)^{(1/3)})^2)]) - (\text{Sqrt}[2] * (a^{(1/3)} - (a + b * x^2)^{(1/3)}) * \text{Sqrt}[(a^{(2/3)} + a^{(1/3)} * (a + b * x^2)^{(1/3)} + (a + b * x^2)^{(2/3)}) / ((1 - \text{Sqrt}[3]) * a^{(1/3)} - (a + b * x^2)^{(1/3)})^2] * \text{EllipticF}[\text{ArcSin}[\frac{(1 + \text{Sqrt}[3]) * a^{(1/3)} - (a + b * x^2)^{(1/3)}}{(1 - \text{Sqrt}[3]) * a^{(1/3)} - (a + b * x^2)^{(1/3)}}], -7 + 4 * \text{Sqrt}[3]]] / (3^{(1/4)} * a^{(2/3)} * x * \text{Sqrt}[-((a^{(1/3)} * (a^{(1/3)} - (a + b * x^2)^{(1/3)})) / ((1 - \text{Sqrt}[3]) * a^{(1/3)} - (a + b * x^2)^{(1/3)})^2)]) \end{aligned}$$

**Rubi in Sympy [A]** time = 29.2212, size = 439, normalized size = 0.8

$$\begin{aligned} & \frac{bx}{a \left( \sqrt[3]{a} (-1 + \sqrt{3}) + \sqrt[3]{a + bx^2} \right)} - \frac{(a + bx^2)^{\frac{2}{3}}}{ax} \\ & + \frac{\sqrt[3]{3} \sqrt{\frac{a^{\frac{2}{3}} + \sqrt[3]{a} \sqrt[3]{a + bx^2 + (a + bx^2)^{\frac{2}{3}}}}{\left( \sqrt[3]{a} (-1 + \sqrt{3}) + \sqrt[3]{a + bx^2} \right)^2}} \sqrt{\sqrt{3} + 2} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) E \left( \text{asin} \left( \frac{\sqrt[3]{a} (1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}{-\sqrt[3]{a} (-1 + \sqrt{3}) - \sqrt[3]{a + bx^2}} \right) \right) \Big|_{-7 + 4\sqrt{3}}}{2a^{\frac{2}{3}} x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( \sqrt[3]{a} (-1 + \sqrt{3}) + \sqrt[3]{a + bx^2} \right)^2}}} \\ & + \frac{\sqrt{2} \cdot 3^{\frac{3}{4}} \sqrt{\frac{a^{\frac{2}{3}} + \sqrt[3]{a} \sqrt[3]{a + bx^2 + (a + bx^2)^{\frac{2}{3}}}}{\left( \sqrt[3]{a} (-1 + \sqrt{3}) + \sqrt[3]{a + bx^2} \right)^2}} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) F \left( \text{asin} \left( \frac{\sqrt[3]{a} (1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}{-\sqrt[3]{a} (-1 + \sqrt{3}) - \sqrt[3]{a + bx^2}} \right) \right) \Big|_{-7 + 4\sqrt{3}}}{3a^{\frac{2}{3}} x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( \sqrt[3]{a} (-1 + \sqrt{3}) + \sqrt[3]{a + bx^2} \right)^2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**2/(b*x**2+a)**(1/3),x)`

[Out]  $b * x / (a * (a^{(1/3)} * (-1 + \text{sqrt}(3)) + (a + b * x^2)^{(1/3)})) - (a + b * x^2)^{(2/3)} / (a * x) + 3^{(1/4)} * \text{sqrt}((a^{(2/3)} + a^{(1/3)} * (a + b * x^2)^{(1/3)} + (a + b * x^2)^{(2/3)}) / (a^{(1/3)} * (-1 + \text{sqrt}(3)) + (a + b * x^2)^{(1/3)})^2) * \text{sqrt}(\text{sqrt}(3) + 2) * (a^{(1/3)} - (a + b * x^2)^{(1/3)}) * \text{elliptic\_e}(\text{asin}((a^{(1/3)} * (1 + \text{sqrt}(3)) - (a + b * x^2)^{(1/3)}) / (-a^{(1/3)} * (-1 + \text{sqrt}(3)) - (a + b * x^2)^{(1/3)})), -7 + 4 * \text{sqrt}(3)) / (2 * a^{(2/3)} * x * \text{sqrt}(-a^{(1/3)} * (a^{(1/3)} - (a + b * x^2)^{(1/3)})) / (a^{(1/3)} * (-1 + \text{sqrt}(3)) + (a + b * x^2)^{(1/3)})^2) - \text{sqrt}(2) * 3^{(3/4)} * \text{sqrt}((a^{(2/3)} + a^{(1/3)} * (a + b * x^2)^{(1/3)} + (a + b * x^2)^{(2/3)}) / (a^{(1/3)} * (-1 + \text{sqrt}(3)) + (a + b * x^2)^{(1/3)})^2) * (a^{(1/3)} - (a + b * x^2)^{(1/3)}) * \text{elliptic\_f}(\text{asin}((a^{(1/3)} * (1 + \text{sqrt}(3)) - (a + b * x^2)^{(1/3)}) / (-a^{(1/3)} * (-1 + \text{sqrt}(3)) - (a + b * x^2)^{(1/3)})), -7 + 4 * \text{sqrt}(3)) / (3 * a^{(2/3)} * x * \text{sqrt}(-a^{(1/3)} * (a^{(1/3)} - (a + b * x^2)^{(1/3)})) / (a^{(1/3)} * (-1 + \text{sqrt}(3)) + (a + b * x^2)^{(1/3)})^2)$

**Mathematica [C]** time = 0.0461355, size = 69, normalized size = 0.13

$$\frac{bx^2 \sqrt[3]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) - 3(a + bx^2)}{3ax \sqrt[3]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^2*(a + b*x^2)^(1/3)),x]`

[Out]  $(-3*(a + b*x^2) + b*x^2*(1 + (b*x^2)/a)^{(1/3)}*\text{Hypergeometric2F1}[1/3, 1/2, 3/2, -((b*x^2)/a)])/(3*a*x*(a + b*x^2)^{(1/3)})$

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt[3]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(b*x^2+a)^(1/3), x)`

[Out] `int(1/x^2/(b*x^2+a)^(1/3), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{3}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/3)*x^2), x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(1/3)*x^2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{1}{3}} x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/3)*x^2), x, algorithm="fricas")`

[Out] `integral(1/((b*x^2 + a)^(1/3)*x^2), x)`

**Sympy [A]** time = 2.38922, size = 27, normalized size = 0.05

$$\frac{{}_2F_1\left(-\frac{1}{2}, \frac{1}{3} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{\sqrt[3]{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(b*x**2+a)**(1/3), x)`

[Out] `-hyper((-1/2, 1/3), (1/2, ), b*x**2*exp_polar(I*pi)/a)/(a**(1/3)*x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{3}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(1/3)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/3)*x^2), x)
```

$$3.713 \quad \int \frac{1}{x^4 \sqrt[3]{a + bx^2}} dx$$

**Optimal.** Leaf size=578

$$\frac{5\sqrt{2}b \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \mid -7 + 4\sqrt{3} \right)}{9\sqrt[3]{3}a^{5/3}x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}}$$

$$\frac{5\sqrt{2 + \sqrt{3}}b \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \mid -7 + 4\sqrt{3} \right)}{6 \cdot 3^{3/4} a^{5/3} x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}}$$

$$+ \frac{5b^2x}{9a^2 \left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)} + \frac{5b(a + bx^2)^{2/3}}{9a^2x} - \frac{(a + bx^2)^{2/3}}{3ax^3}$$

[Out]  $-(a + b*x^2)^{(2/3)}/(3*a*x^3) + (5*b*(a + b*x^2)^{(2/3)})/(9*a^2*x) + (5*b^2*x)/(9*a^2*((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})) - (5*\text{Sqrt}[2 + \text{Sqrt}[3]]*b*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(6*3^{(3/4)}*a^{(5/3)}*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2])) + (5*\text{Sqrt}[2]*b*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(9*3^{(1/4)}*a^{(5/3)}*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]))]$

**Rubi [A]** time = 0.894718, antiderivative size = 578, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{5\sqrt{2}b \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \mid -7 + 4\sqrt{3} \right)}{9\sqrt[3]{3}a^{5/3}x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}}$$

$$\frac{5\sqrt{2 + \sqrt{3}}b \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \mid -7 + 4\sqrt{3} \right)}{6 \cdot 3^{3/4} a^{5/3} x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}}}$$

$$+ \frac{5b^2x}{9a^2 \left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)} + \frac{5b(a + bx^2)^{2/3}}{9a^2x} - \frac{(a + bx^2)^{2/3}}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a + b\*x^2)^(1/3)), x]

[Out]  $-(a + b*x^2)^{(2/3)}/(3*a*x^3) + (5*b*(a + b*x^2)^{(2/3)})/(9*a^2*x) + (5*b^2*x)/(9*a^2*((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})) -$

$$\begin{aligned} & (5\sqrt{2 + \sqrt{3}})b(a^{1/3} - (a + b^2x^2)^{1/3})\sqrt{(a^{2/3} + a^{1/3}(a + b^2x^2)^{1/3} + (a + b^2x^2)^{2/3})/((1 - \sqrt{3})a^{1/3} - (a + b^2x^2)^{1/3})^2} \\ & \text{EllipticE}\left[\text{ArcSin}\left(\frac{(1 + \sqrt{3})a^{1/3} - (a + b^2x^2)^{1/3}}{(1 - \sqrt{3})a^{1/3} - (a + b^2x^2)^{1/3}}\right), -7 + 4\sqrt{3}\right] \\ & \text{EllipticF}\left[\text{ArcSin}\left(\frac{(1 + \sqrt{3})a^{1/3} - (a + b^2x^2)^{1/3}}{(1 - \sqrt{3})a^{1/3} - (a + b^2x^2)^{1/3}}\right), -7 + 4\sqrt{3}\right] \\ & \text{EllipticF}\left[\text{ArcSin}\left(\frac{(1 + \sqrt{3})a^{1/3} - (a + b^2x^2)^{1/3}}{(1 - \sqrt{3})a^{1/3} - (a + b^2x^2)^{1/3}}\right), -7 + 4\sqrt{3}\right] \end{aligned}$$

**Rubi in Sympy [A]** time = 37.0958, size = 476, normalized size = 0.82

$$\begin{aligned} & \frac{(a + bx^2)^{2/3}}{3ax^3} - \frac{5b^2x}{9a^2(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})} + \frac{5b(a + bx^2)^{2/3}}{9a^2x} \\ & \frac{5\sqrt[3]{3}b \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}} \sqrt{\sqrt{3} + 2} (\sqrt[3]{a} - \sqrt[3]{a + bx^2}) E\left(\text{asin}\left(\frac{\sqrt[3]{a}(1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}{-\sqrt[3]{a}(-1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}\right)\right) \Big|_{-7 + 4\sqrt{3}}}{18a^{5/3}x \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{a + bx^2})}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}}} \\ & \frac{5\sqrt{2} \cdot 3^{3/4}b \sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}} (\sqrt[3]{a} - \sqrt[3]{a + bx^2}) F\left(\text{asin}\left(\frac{\sqrt[3]{a}(1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}{-\sqrt[3]{a}(-1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}\right)\right) \Big|_{-7 + 4\sqrt{3}}}{27a^{5/3}x \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{a + bx^2})}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rub_i_integrate(1/x**4/(b*x**2+a)**(1/3),x)`

[Out]  $-(a + b^2x^2)^{2/3}/(3a^2x^3) - 5b^2x/(9a^2(a^{1/3}(-1 + \sqrt{3}) + \sqrt{3} + 2)(a + b^2x^2)^{1/3}) + 5b(a + b^2x^2)^{2/3}/(9a^2x) - 5\sqrt[3]{3}(1/4)b\sqrt{(a^{2/3} + a^{1/3}(a + b^2x^2)^{1/3} + (a + b^2x^2)^{2/3})/((1 - \sqrt{3})a^{1/3} - (a + b^2x^2)^{1/3})^2} \text{EllipticE}(\text{asin}(\frac{(1 + \sqrt{3})a^{1/3} - (a + b^2x^2)^{1/3}}{(1 - \sqrt{3})a^{1/3} - (a + b^2x^2)^{1/3}}), -7 + 4\sqrt{3})/(18a^{5/3}x\sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}} \sqrt{\sqrt{3} + 2} (\sqrt[3]{a} - \sqrt[3]{a + bx^2})) + 5\sqrt[3]{3}(1/4)b\sqrt{(a^{2/3} + a^{1/3}(a + b^2x^2)^{1/3} + (a + b^2x^2)^{2/3})/((1 - \sqrt{3})a^{1/3} - (a + b^2x^2)^{1/3})^2} \text{EllipticF}(\text{asin}(\frac{(1 + \sqrt{3})a^{1/3} - (a + b^2x^2)^{1/3}}{(1 - \sqrt{3})a^{1/3} - (a + b^2x^2)^{1/3}}), -7 + 4\sqrt{3})/(27a^{5/3}x\sqrt{\frac{a^{2/3} + \sqrt[3]{a}\sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}} (\sqrt[3]{a} - \sqrt[3]{a + bx^2}))$

**Mathematica [C]** time = 0.0514741, size = 83, normalized size = 0.14

$$\frac{-9a^2 - 5b^2x^4\sqrt{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) + 6abx^2 + 15b^2x^4}{27a^2x^3\sqrt[3]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^4*(a + b*x^2)^(1/3)),x]`



[Out]  $(-9a^2 + 6abx^2 + 15b^2x^4 - 5b^2x^4(1 + (bx^2)/a)^{1/3}) \cdot \text{Hypergeometric2F1}[1/3, 1/2, 3/2, -((bx^2)/a)] / (27a^2x^3(a + bx^2)^{1/3})$

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \sqrt[3]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(b*x^2+a)^(1/3), x)`

[Out] `int(1/x^4/(b*x^2+a)^(1/3), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{3}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/3)*x^4), x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(1/3)*x^4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{1}{3}} x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/3)*x^4), x, algorithm="fricas")`

[Out] `integral(1/((b*x^2 + a)^(1/3)*x^4), x)`

**Sympy [A]** time = 2.79462, size = 32, normalized size = 0.06

$$-\frac{{}_2F_1\left(-\frac{3}{2}, \frac{1}{3} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3\sqrt[3]{ax^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(b*x**2+a)**(1/3), x)`

[Out] `-hyper((-3/2, 1/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/3)*x**3)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{3}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(1/3)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/3)*x^4), x)
```

$$3.714 \quad \int \frac{x^7}{(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=80

$$-\frac{3a^3\sqrt[3]{a+bx^2}}{2b^4} + \frac{9a^2(a+bx^2)^{4/3}}{8b^4} + \frac{3(a+bx^2)^{10/3}}{20b^4} - \frac{9a(a+bx^2)^{7/3}}{14b^4}$$

[Out]  $(-3*a^3*(a + b*x^2)^(1/3))/(2*b^4) + (9*a^2*(a + b*x^2)^(4/3))/(8*b^4) - (9*a*(a + b*x^2)^(7/3))/(14*b^4) + (3*(a + b*x^2)^(10/3))/(20*b^4)$

**Rubi [A]** time = 0.128401, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{3a^3\sqrt[3]{a+bx^2}}{2b^4} + \frac{9a^2(a+bx^2)^{4/3}}{8b^4} + \frac{3(a+bx^2)^{10/3}}{20b^4} - \frac{9a(a+bx^2)^{7/3}}{14b^4}$$

Antiderivative was successfully verified.

[In] Int[x^7/(a + b\*x^2)^(2/3), x]

[Out]  $(-3*a^3*(a + b*x^2)^(1/3))/(2*b^4) + (9*a^2*(a + b*x^2)^(4/3))/(8*b^4) - (9*a*(a + b*x^2)^(7/3))/(14*b^4) + (3*(a + b*x^2)^(10/3))/(20*b^4)$

**Rubi in Sympy [A]** time = 15.3842, size = 75, normalized size = 0.94

$$-\frac{3a^3\sqrt[3]{a+bx^2}}{2b^4} + \frac{9a^2(a+bx^2)^{4/3}}{8b^4} - \frac{9a(a+bx^2)^{7/3}}{14b^4} + \frac{3(a+bx^2)^{10/3}}{20b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*7/(b\*x\*\*2+a)\*\*(2/3), x)

[Out]  $-3*a**3*(a + b*x**2)**(1/3)/(2*b**4) + 9*a**2*(a + b*x**2)**(4/3)/(8*b**4) - 9*a*(a + b*x**2)**(7/3)/(14*b**4) + 3*(a + b*x**2)**(10/3)/(20*b**4)$

**Mathematica [A]** time = 0.0301498, size = 50, normalized size = 0.62

$$\frac{3\sqrt[3]{a+bx^2}(-81a^3 + 27a^2bx^2 - 18ab^2x^4 + 14b^3x^6)}{280b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a + b\*x^2)^(2/3), x]

[Out]  $(3*(a + b*x^2)^(1/3)*(-81*a^3 + 27*a^2*b*x^2 - 18*a*b^2*x^4 + 14*b^3*x^6))/(280*b^4)$

**Maple [A]** time = 0.009, size = 47, normalized size = 0.6

$$-\frac{-42b^3x^6 + 54ab^2x^4 - 81a^2bx^2 + 243a^3}{280b^4} \sqrt[3]{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7/(b*x^2+a)^(2/3),x)`

[Out] 
$$-3/280*(b*x^2+a)^{(1/3)}*(-14*b^3*x^6+18*a*b^2*x^4-27*a^2*b*x^2+81*a^3)/b^4$$

**Maxima [A]** time = 1.34161, size = 86, normalized size = 1.08

$$\frac{3(bx^2+a)^{\frac{10}{3}}}{20b^4} - \frac{9(bx^2+a)^{\frac{7}{3}}a}{14b^4} + \frac{9(bx^2+a)^{\frac{4}{3}}a^2}{8b^4} - \frac{3(bx^2+a)^{\frac{1}{3}}a^3}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^2 + a)^(2/3),x, algorithm="maxima")`

[Out] 
$$3/20*(b*x^2 + a)^{(10/3)}/b^4 - 9/14*(b*x^2 + a)^{(7/3)}*a/b^4 + 9/8*(b*x^2 + a)^{(4/3)}*a^2/b^4 - 3/2*(b*x^2 + a)^{(1/3)}*a^3/b^4$$

**Fricas [A]** time = 0.214285, size = 62, normalized size = 0.78

$$\frac{3(14b^3x^6 - 18ab^2x^4 + 27a^2bx^2 - 81a^3)(bx^2 + a)^{\frac{1}{3}}}{280b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^2 + a)^(2/3),x, algorithm="fricas")`

[Out] 
$$3/280*(14*b^3*x^6 - 18*a*b^2*x^4 + 27*a^2*b*x^2 - 81*a^3)*(b*x^2 + a)^{(1/3)}/b^4$$

**Sympy [A]** time = 9.4416, size = 1690, normalized size = 21.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7/(b*x**2+a)**(2/3),x)`

[Out] 
$$\begin{aligned} & -243*a**(70/3)*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + \\ & 243*a**(70/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) - 1377*a**(67/3)*b*x**2 \\ & *(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 1458*a**(67/3) \\ & *b*x**2/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) - 3213*a**(64/3)*b**2*x**4*(1 \\ & + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 3645*a**(64/3)*b \\ & **2*x**4/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) - 3927*a**(61/3)*b**3*x**6*(1 \\ & + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + \end{aligned}$$

```

1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 4860*a**(61/3)*b
**3*x**6/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6
*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*
b**9*x**10 + 280*a**14*b**10*x**12) - 2583*a**(58/3)*b**4*x**8*(1
+ b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200
*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 +
1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 3645*a**(58/3)*b
**4*x**8/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6
*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*
b**9*x**10 + 280*a**14*b**10*x**12) - 693*a**(55/3)*b**5*x**10*(1
+ b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200
*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 +
1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 1458*a**(55/3)*b
**5*x**10/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**
6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15
*b**9*x**10 + 280*a**14*b**10*x**12) + 273*a**(52/3)*b**6*x**12*(
1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 420
0*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 +
1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 243*a**(52/3)*b
**6*x**12/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**
6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15
*b**9*x**10 + 280*a**14*b**10*x**12) + 387*a**(49/3)*b**7*x**14*(
1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 420
0*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 +
1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 198*a**(46/3)*b
**8*x**16*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5
*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*
b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 42*a
**(43/3)*b**9*x**18*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*
a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4
200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**
12)

```

---

**GIAC/XCAS [A]** time = 0.217219, size = 77, normalized size = 0.96

$$\frac{3 \left( 14 (bx^2 + a)^{\frac{10}{3}} - 60 (bx^2 + a)^{\frac{7}{3}} a + 105 (bx^2 + a)^{\frac{4}{3}} a^2 - 140 (bx^2 + a)^{\frac{1}{3}} a^3 \right)}{280 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b\*x^2 + a)^(2/3),x, algorithm="giac")

[Out] 3/280\*(14\*(b\*x^2 + a)^(10/3) - 60\*(b\*x^2 + a)^(7/3)\*a + 105\*(b\*x^2 + a)^(4/3)\*a^2 - 140\*(b\*x^2 + a)^(1/3)\*a^3)/b^4

$$3.715 \quad \int \frac{x^5}{(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=59

$$\frac{3a^2\sqrt[3]{a+bx^2}}{2b^3} + \frac{3(a+bx^2)^{7/3}}{14b^3} - \frac{3a(a+bx^2)^{4/3}}{4b^3}$$

[Out]  $(3*a^2*(a + b*x^2)^(1/3))/(2*b^3) - (3*a*(a + b*x^2)^(4/3))/(4*b^3) + (3*(a + b*x^2)^(7/3))/(14*b^3)$

**Rubi [A]** time = 0.101157, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{3a^2\sqrt[3]{a+bx^2}}{2b^3} + \frac{3(a+bx^2)^{7/3}}{14b^3} - \frac{3a(a+bx^2)^{4/3}}{4b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a + b\*x^2)^(2/3), x]

[Out]  $(3*a^2*(a + b*x^2)^(1/3))/(2*b^3) - (3*a*(a + b*x^2)^(4/3))/(4*b^3) + (3*(a + b*x^2)^(7/3))/(14*b^3)$

**Rubi in Sympy [A]** time = 11.4539, size = 54, normalized size = 0.92

$$\frac{3a^2\sqrt[3]{a+bx^2}}{2b^3} - \frac{3a(a+bx^2)^{4/3}}{4b^3} + \frac{3(a+bx^2)^{7/3}}{14b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5/(b\*x\*\*2+a)\*\*(2/3), x)

[Out]  $3*a**2*(a + b*x**2)**(1/3)/(2*b**3) - 3*a*(a + b*x**2)**(4/3)/(4*b**3) + 3*(a + b*x**2)**(7/3)/(14*b**3)$

**Mathematica [A]** time = 0.0241341, size = 39, normalized size = 0.66

$$\frac{3\sqrt[3]{a+bx^2}(9a^2 - 3abx^2 + 2b^2x^4)}{28b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a + b\*x^2)^(2/3), x]

[Out]  $(3*(a + b*x^2)^(1/3)*(9*a^2 - 3*a*b*x^2 + 2*b^2*x^4))/(28*b^3)$

**Maple [A]** time = 0.008, size = 36, normalized size = 0.6

$$\frac{6b^2x^4 - 9abx^2 + 27a^2}{28b^3} \sqrt[3]{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(b*x^2+a)^(2/3),x)`

[Out]  $3/28*(b*x^2+a)^{(1/3)}*(2*b^2*x^4-3*a*b*x^2+9*a^2)/b^3$

**Maxima [A]** time = 1.33148, size = 63, normalized size = 1.07

$$\frac{3(bx^2+a)^{\frac{7}{3}}}{14b^3} - \frac{3(bx^2+a)^{\frac{4}{3}}a}{4b^3} + \frac{3(bx^2+a)^{\frac{1}{3}}a^2}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2+a)^(2/3),x, algorithm="maxima")`

[Out]  $3/14*(b*x^2+a)^{(7/3)}/b^3 - 3/4*(b*x^2+a)^{(4/3)}*a/b^3 + 3/2*(b*x^2+a)^{(1/3)}*a^2/b^3$

**Fricas [A]** time = 0.21553, size = 47, normalized size = 0.8

$$\frac{3(2b^2x^4 - 3abx^2 + 9a^2)(bx^2 + a)^{\frac{1}{3}}}{28b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2+a)^(2/3),x, algorithm="fricas")`

[Out]  $3/28*(2*b^2*x^4 - 3*a*b*x^2 + 9*a^2)*(b*x^2 + a)^{(1/3)}/b^3$

**Sympy [A]** time = 6.04621, size = 631, normalized size = 10.69

$$\begin{aligned} & \frac{27a^{\frac{31}{3}}\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3 + 84a^7b^4x^2 + 84a^6b^5x^4 + 28a^5b^6x^6} - \frac{27a^{\frac{31}{3}}}{28a^8b^3 + 84a^7b^4x^2 + 84a^6b^5x^4 + 28a^5b^6x^6} \\ & + \frac{72a^{\frac{28}{3}}bx^2\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3 + 84a^7b^4x^2 + 84a^6b^5x^4 + 28a^5b^6x^6} - \frac{81a^{\frac{28}{3}}bx^2}{28a^8b^3 + 84a^7b^4x^2 + 84a^6b^5x^4 + 28a^5b^6x^6} \\ & + \frac{60a^{\frac{25}{3}}b^2x^4\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3 + 84a^7b^4x^2 + 84a^6b^5x^4 + 28a^5b^6x^6} - \frac{81a^{\frac{25}{3}}b^2x^4}{28a^8b^3 + 84a^7b^4x^2 + 84a^6b^5x^4 + 28a^5b^6x^6} \\ & + \frac{18a^{\frac{22}{3}}b^3x^6\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3 + 84a^7b^4x^2 + 84a^6b^5x^4 + 28a^5b^6x^6} - \frac{27a^{\frac{22}{3}}b^3x^6}{28a^8b^3 + 84a^7b^4x^2 + 84a^6b^5x^4 + 28a^5b^6x^6} \\ & + \frac{9a^{\frac{19}{3}}b^4x^8\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3 + 84a^7b^4x^2 + 84a^6b^5x^4 + 28a^5b^6x^6} + \frac{6a^{\frac{16}{3}}b^5x^{10}\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3 + 84a^7b^4x^2 + 84a^6b^5x^4 + 28a^5b^6x^6} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(b*x**2+a)**(2/3),x)`

[Out]  $27*a**(31/3)*(1+b*x**2/a)**(1/3)/(28*a**8*b**3+84*a**7*b**4*x**2+84*a**6*b**5*x**4+28*a**5*b**6*x**6) - 27*a**(31/3)/(28*a**8*b**3+84*a**7*b**4*x**2+84*a**6*b**5*x**4+28*a**5*b**6*x**6) + 72*a**(28/3)*b*x**2*(1+b*x**2/a)**(1/3)/(28*a**8*b**3+84*a**7*b**4*x**2+84*a**6*b**5*x**4+28*a**5*b**6*x**6) - 81*a**(28/3)*b*x**2/(28*a**8*b**3+84*a**7*b**4*x**2+84*a**6*b**5*x**4+28*a**5*b**6*x**6) + 60*a**(25/3)*b**2*x**4*(1+b*x**2/a)$

```

** (1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 + 2
8*a**5*b**6*x**6) - 81*a** (25/3)*b**2*x**4/(28*a**8*b**3 + 84*a**
7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*x**6) + 18*a** (22/
3)*b**3*x**6*(1 + b*x**2/a)** (1/3)/(28*a**8*b**3 + 84*a**7*b**4*x
**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*x**6) - 27*a** (22/3)*b**3*
x**6/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a
**5*b**6*x**6) + 9*a** (19/3)*b**4*x**8*(1 + b*x**2/a)** (1/3)/(28*
a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*
x**6) + 6*a** (16/3)*b**5*x**10*(1 + b*x**2/a)** (1/3)/(28*a**8*b**
3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*x**6)

```

**GIAC/XCAS [A]** time = 0.212791, size = 58, normalized size = 0.98

$$\frac{3 \left( 2 (bx^2 + a)^{\frac{7}{3}} - 7 (bx^2 + a)^{\frac{4}{3}} a + 14 (bx^2 + a)^{\frac{1}{3}} a^2 \right)}{28 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b\*x^2 + a)^(2/3),x, algorithm="giac")

[Out] 3/28\*(2\*(b\*x^2 + a)^(7/3) - 7\*(b\*x^2 + a)^(4/3)\*a + 14\*(b\*x^2 + a)^(1/3)\*a^2)/b^3



$$3.716 \quad \int \frac{x^3}{(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=38

$$\frac{3(a+bx^2)^{4/3}}{8b^2} - \frac{3a\sqrt[3]{a+bx^2}}{2b^2}$$

[Out]  $(-3*a*(a + b*x^2)^{(1/3)})/(2*b^2) + (3*(a + b*x^2)^{(4/3)})/(8*b^2)$

**Rubi [A]** time = 0.0693643, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{3(a+bx^2)^{4/3}}{8b^2} - \frac{3a\sqrt[3]{a+bx^2}}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b\*x^2)^(2/3), x]

[Out]  $(-3*a*(a + b*x^2)^{(1/3)})/(2*b^2) + (3*(a + b*x^2)^{(4/3)})/(8*b^2)$

**Rubi in Sympy [A]** time = 7.80156, size = 34, normalized size = 0.89

$$-\frac{3a\sqrt[3]{a+bx^2}}{2b^2} + \frac{3(a+bx^2)^{4/3}}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(b\*x\*\*2+a)\*\*(2/3), x)

[Out]  $-3*a*(a + b*x**2)**(1/3)/(2*b**2) + 3*(a + b*x**2)**(4/3)/(8*b**2)$

**Mathematica [A]** time = 0.0199189, size = 27, normalized size = 0.71

$$\frac{3(bx^2 - 3a)\sqrt[3]{a+bx^2}}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b\*x^2)^(2/3), x]

[Out]  $(3*(-3*a + b*x^2)*(a + b*x^2)^{(1/3)})/(8*b^2)$

**Maple [A]** time = 0.007, size = 25, normalized size = 0.7

$$-\frac{-3bx^2 + 9a}{8b^2} \sqrt[3]{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b\*x^2+a)^(2/3), x)

[Out]  $-3/8 * (b * x^2 + a)^{(1/3)} * (-b * x^2 + 3 * a) / b^2$

**Maxima [A]** time = 1.32206, size = 41, normalized size = 1.08

$$\frac{3 (bx^2 + a)^{\frac{4}{3}}}{8 b^2} - \frac{3 (bx^2 + a)^{\frac{1}{3}} a}{2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 + a)^(2/3),x, algorithm="maxima")`

[Out]  $3/8 * (b * x^2 + a)^{(4/3)} / b^2 - 3/2 * (b * x^2 + a)^{(1/3)} * a / b^2$

**Fricas [A]** time = 0.216056, size = 31, normalized size = 0.82

$$\frac{3 (bx^2 + a)^{\frac{1}{3}} (bx^2 - 3 a)}{8 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 + a)^(2/3),x, algorithm="fricas")`

[Out]  $3/8 * (b * x^2 + a)^{(1/3)} * (b * x^2 - 3 * a) / b^2$

**Sympy [A]** time = 3.91791, size = 178, normalized size = 4.68

$$-\frac{9a^{\frac{10}{3}} \sqrt[3]{1 + \frac{bx^2}{a}}}{8a^2b^2 + 8ab^3x^2} + \frac{9a^{\frac{10}{3}}}{8a^2b^2 + 8ab^3x^2} - \frac{6a^{\frac{7}{3}}bx^2 \sqrt[3]{1 + \frac{bx^2}{a}}}{8a^2b^2 + 8ab^3x^2} + \frac{9a^{\frac{7}{3}}bx^2}{8a^2b^2 + 8ab^3x^2} + \frac{3a^{\frac{4}{3}}b^2x^4 \sqrt[3]{1 + \frac{bx^2}{a}}}{8a^2b^2 + 8ab^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(b*x**2+a)**(2/3),x)`

[Out]  $-9 * a^{(10/3)} * (1 + b * x^{**2}/a)^{** (1/3)} / (8 * a^{**2} * b^{**2} + 8 * a * b^{**3} * x^{**2}) + 9 * a^{(10/3)} / (8 * a^{**2} * b^{**2} + 8 * a * b^{**3} * x^{**2}) - 6 * a^{(7/3)} * b * x^{**2} * (1 + b * x^{**2}/a)^{** (1/3)} / (8 * a^{**2} * b^{**2} + 8 * a * b^{**3} * x^{**2}) + 9 * a^{(7/3)} * b * x^{**2} / (8 * a^{**2} * b^{**2} + 8 * a * b^{**3} * x^{**2}) + 3 * a^{(4/3)} * b^{**2} * x^{**4} * (1 + b * x^{**2}/a)^{** (1/3)} / (8 * a^{**2} * b^{**2} + 8 * a * b^{**3} * x^{**2})$

**GIAC/XCAS [A]** time = 0.215036, size = 36, normalized size = 0.95

$$\frac{3 \left( (bx^2 + a)^{\frac{4}{3}} - 4 (bx^2 + a)^{\frac{1}{3}} a \right)}{8 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 + a)^(2/3),x, algorithm="giac")`

[Out]  $3/8 * ((b * x^2 + a)^{(4/3)} - 4 * (b * x^2 + a)^{(1/3)} * a) / b^2$

$$3.717 \quad \int \frac{x}{(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=18

$$\frac{3\sqrt[3]{a+bx^2}}{2b}$$

[Out]  $(3*(a + b*x^2)^{(1/3)})/(2*b)$

**Rubi [A]** time = 0.0115786, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{3\sqrt[3]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

[In] `Int[x/(a + b*x^2)^(2/3), x]`

[Out]  $(3*(a + b*x^2)^{(1/3)})/(2*b)$

**Rubi in Sympy [A]** time = 2.16491, size = 14, normalized size = 0.78

$$\frac{3\sqrt[3]{a+bx^2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x/(b*x**2+a)**(2/3), x)`

[Out]  $3*(a + b*x**2)**(1/3)/(2*b)$

**Mathematica [A]** time = 0.00478311, size = 18, normalized size = 1.

$$\frac{3\sqrt[3]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

[In] `Integrate[x/(a + b*x^2)^(2/3), x]`

[Out]  $(3*(a + b*x^2)^{(1/3)})/(2*b)$

**Maple [A]** time = 0.005, size = 15, normalized size = 0.8

$$\frac{3}{2b}\sqrt[3]{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(b*x^2+a)^(2/3), x)`

[Out]  $3/2*(b*x^2+a)^{(1/3)}/b$

---

**Maxima [A]** time = 1.32519, size = 19, normalized size = 1.06

$$\frac{3 (bx^2 + a)^{\frac{1}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^(2/3),x, algorithm="maxima")`

[Out] `3/2*(b*x^2 + a)^(1/3)/b`

---

**Fricas [A]** time = 0.213716, size = 19, normalized size = 1.06

$$\frac{3 (bx^2 + a)^{\frac{1}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^(2/3),x, algorithm="fricas")`

[Out] `3/2*(b*x^2 + a)^(1/3)/b`

---

**Sympy [A]** time = 1.65835, size = 24, normalized size = 1.33

$$\begin{cases} \frac{3\sqrt[3]{a+bx^2}}{2b} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x**2+a)**(2/3),x)`

[Out] `Piecewise((3*(a + b*x**2)**(1/3)/(2*b), Ne(b, 0)), (x**2/(2*a**(2/3)), True))`

---

**GIAC/XCAS [A]** time = 0.217242, size = 19, normalized size = 1.06

$$\frac{3 (bx^2 + a)^{\frac{1}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^(2/3),x, algorithm="giac")`

[Out] `3/2*(b*x^2 + a)^(1/3)/b`

$$3.718 \quad \int \frac{1}{x(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=86

$$\frac{3 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{4a^{2/3}} - \frac{\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}} - \frac{\log(x)}{2a^{2/3}}$$

[Out] -(Sqrt[3]\*ArcTan[(a^(1/3) + 2\*(a + b\*x^2)^(1/3))/(Sqrt[3]\*a^(1/3)])/ (2\*a^(2/3)) - Log[x]/(2\*a^(2/3)) + (3\*Log[a^(1/3) - (a + b\*x^2)^(1/3)])/(4\*a^(2/3))

**Rubi [A]** time = 0.148382, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{3 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{4a^{2/3}} - \frac{\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}} - \frac{\log(x)}{2a^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(a + b\*x^2)^(2/3)), x]

[Out] -(Sqrt[3]\*ArcTan[(a^(1/3) + 2\*(a + b\*x^2)^(1/3))/(Sqrt[3]\*a^(1/3)])/ (2\*a^(2/3)) - Log[x]/(2\*a^(2/3)) + (3\*Log[a^(1/3) - (a + b\*x^2)^(1/3)])/(4\*a^(2/3))

**Rubi in Sympy [A]** time = 8.06623, size = 80, normalized size = 0.93

$$-\frac{\log(x^2)}{4a^{2/3}} + \frac{3 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{4a^{2/3}} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} + \frac{2\sqrt[3]{a+bx^2}}{3}\right)}{\sqrt[3]{a}}\right)}{2a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(b\*x\*\*2+a)\*\*(2/3), x)

[Out] -log(x\*\*2)/(4\*a\*\*(2/3)) + 3\*log(a\*\*(1/3) - (a + b\*x\*\*2)\*\*(1/3))/(4\*a\*\*(2/3)) - sqrt(3)\*atan(sqrt(3)\*(a\*\*(1/3)/3 + 2\*(a + b\*x\*\*2)\*\*(1/3)/3)/a\*\*(1/3))/(2\*a\*\*(2/3))

**Mathematica [C]** time = 0.0349802, size = 48, normalized size = 0.56

$$\frac{3\left(\frac{a}{bx^2} + 1\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{2}{3}, \frac{5}{3}; -\frac{a}{bx^2}\right)}{4(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(a + b\*x^2)^(2/3)), x]

[Out] (-3\*(1 + a/(b\*x^2))^(2/3)\*Hypergeometric2F1[2/3, 2/3, 5/3, -(a/(b\*x^2))])/(4\*(a + b\*x^2)^(2/3))

---

**Maple [F]** time = 0.029, size = 0, normalized size = 0.

$$\int \frac{1}{x} (bx^2 + a)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(b*x^2+a)^(2/3), x)`

[Out] `int(1/x/(b*x^2+a)^(2/3), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*x), x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.225378, size = 132, normalized size = 1.53

$$\frac{2\sqrt{3} \arctan\left(\frac{\sqrt{3}(a+2(bx^2+a)^{\frac{1}{3}}(a^2)^{\frac{1}{3}})}{3a}\right) + \log\left(a^2 + (bx^2 + a)^{\frac{1}{3}}(a^2)^{\frac{1}{3}}a + (bx^2 + a)^{\frac{2}{3}}(a^2)^{\frac{2}{3}}\right) - 2 \log\left(-a + (bx^2 + a)^{\frac{1}{3}}(a^2)^{\frac{1}{3}}\right)}{4(a^2)^{\frac{1}{3}}}$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*x), x, algorithm="fricas")`

[Out] `-1/4*(2*sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b*x^2 + a)^(1/3)*(a^2)^(1/3))/a) + log(a^2 + (b*x^2 + a)^(1/3)*(a^2)^(1/3)*a + (b*x^2 + a)^(2/3)*(a^2)^(2/3)) - 2*log(-a + (b*x^2 + a)^(1/3)*(a^2)^(1/3)))/(a^2)^(1/3)`

---

**Sympy [A]** time = 3.84192, size = 41, normalized size = 0.48

$$\frac{\left(\frac{2}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{2}{3} \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{2}{3}}x^{\frac{4}{3}}\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x**2+a)**(2/3), x)`

[Out] `-gamma(2/3)*hyper((2/3, 2/3), (5/3, ), a*exp_polar(I*pi)/(b*x**2))/(2*b**(2/3)*x**(4/3)*gamma(5/3))`

---

**GIAC/XCAS [A]** time = 0.56671, size = 117, normalized size = 1.36

$$-\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{2a^{\frac{2}{3}}}-\frac{\ln\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4a^{\frac{2}{3}}}+\frac{\ln\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{2a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(2/3)\*x),x, algorithm="giac")

[Out] -1/2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(b\*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(2/3) - 1/4\*ln((b\*x^2 + a)^(2/3) + (b\*x^2 + a)^(1/3)\*a^(1/3) + a^(2/3))/a^(2/3) + 1/2\*ln(abs((b\*x^2 + a)^(1/3) - a^(1/3)))/a^(2/3)

$$3.719 \quad \int \frac{1}{x^3(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=107

$$-\frac{b \log(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{2a^{5/3}} + \frac{b \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}} + \frac{b \log(x)}{3a^{5/3}} - \frac{\sqrt[3]{a+bx^2}}{2ax^2}$$

[Out]  $-(a + b*x^2)^{(1/3)}/(2*a*x^2) + (b*ArcTan[(a^{(1/3)} + 2*(a + b*x^2)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(5/3)}) + (b*Log[x])/(3*a^{(5/3)}) - (b*Log[a^{(1/3)} - (a + b*x^2)^{(1/3)}])/(2*a^{(5/3)})$

**Rubi [A]** time = 0.17135, antiderivative size = 107, normalized size of antiderivative = 1., number of rules used = 6, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\frac{b \log(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{2a^{5/3}} + \frac{b \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}} + \frac{b \log(x)}{3a^{5/3}} - \frac{\sqrt[3]{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(a + b\*x^2)^(2/3)), x]

[Out]  $-(a + b*x^2)^{(1/3)}/(2*a*x^2) + (b*ArcTan[(a^{(1/3)} + 2*(a + b*x^2)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(5/3)}) + (b*Log[x])/(3*a^{(5/3)}) - (b*Log[a^{(1/3)} - (a + b*x^2)^{(1/3)}])/(2*a^{(5/3)})$

**Rubi in Sympy [A]** time = 11.2251, size = 100, normalized size = 0.93

$$-\frac{\sqrt[3]{a+bx^2}}{2ax^2} + \frac{b \log(x^2)}{6a^{5/3}} - \frac{b \log(\sqrt[3]{a} - \sqrt[3]{a+bx^2})}{2a^{5/3}} + \frac{\sqrt{3}b \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} + 2\frac{\sqrt[3]{a+bx^2}}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(b\*x\*\*2+a)\*\*(2/3), x)

[Out]  $-(a + b*x**2)**(1/3)/(2*a*x**2) + b*\log(x**2)/(6*a**(5/3)) - b*\log(a**(1/3) - (a + b*x**2)**(1/3))/(2*a**(5/3)) + \operatorname{sqrt}(3)*b*\operatorname{atan}(\operatorname{sqrt}(3)*(a**(1/3)/3 + 2*(a + b*x**2)**(1/3)/3)/a**(1/3))/(3*a**(5/3))$

**Mathematica [C]** time = 0.050588, size = 69, normalized size = 0.64

$$\frac{bx^2 \left(\frac{a}{bx^2} + 1\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{a}{bx^2}\right) - a - bx^2}{2ax^2 (a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a + b\*x^2)^(2/3)), x]

[Out]  $(-a - b*x^2 + b*(1 + a/(b*x^2))^{2/3})*x^2*\operatorname{Hypergeometric2F1}[2/3, 2/3, 5/3, -(a/(b*x^2))]/(2*a*x^2*(a + b*x^2)^{(2/3)})$





**GIAC/XCAS [A]** time = 0.561839, size = 147, normalized size = 1.37

$$\frac{1}{6} b \left( \frac{2 \sqrt{3} \arctan\left(\frac{\sqrt{3}(2(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}})}{3a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}} + \frac{\ln\left((bx^2+a)^{\frac{2}{3}} + (bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}} - \frac{2 \ln\left(\left|(bx^2+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}} - \frac{3(bx^2+a)^{\frac{1}{3}}}{abx^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(2/3)\*x^3),x, algorithm="giac")

[Out] 1/6\*b\*(2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(b\*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(5/3) + ln((b\*x^2 + a)^(2/3) + (b\*x^2 + a)^(1/3)\*a^(1/3) + a^(2/3))/a^(5/3) - 2\*ln(abs((b\*x^2 + a)^(1/3) - a^(1/3)))/a^(5/3) - 3\*(b\*x^2 + a)^(1/3)/(a\*b\*x^2)

$$3.720 \quad \int \frac{1}{x^5(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=138

$$\frac{5b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{12a^{8/3}} - \frac{5b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}a^{8/3}} - \frac{5b^2 \log(x)}{18a^{8/3}} + \frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2} - \frac{\sqrt[3]{a+bx^2}}{4ax^4}$$

[Out]  $-(a + b*x^2)^{(1/3)}/(4*a*x^4) + (5*b*(a + b*x^2)^{(1/3)})/(12*a^2*x^2) - (5*b^2*ArcTan[(a^{(1/3)} + 2*(a + b*x^2)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(6*Sqrt[3]*a^{(8/3)}) - (5*b^2*Log[x])/(18*a^{(8/3)}) + (5*b^2*Log[a^{(1/3)} - (a + b*x^2)^{(1/3)}])/(12*a^{(8/3)})$

**Rubi [A]** time = 0.225628, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{5b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{12a^{8/3}} - \frac{5b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}a^{8/3}} - \frac{5b^2 \log(x)}{18a^{8/3}} + \frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2} - \frac{\sqrt[3]{a+bx^2}}{4ax^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5\*(a + b\*x^2)^(2/3)), x]

[Out]  $-(a + b*x^2)^{(1/3)}/(4*a*x^4) + (5*b*(a + b*x^2)^{(1/3)})/(12*a^2*x^2) - (5*b^2*ArcTan[(a^{(1/3)} + 2*(a + b*x^2)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(6*Sqrt[3]*a^{(8/3)}) - (5*b^2*Log[x])/(18*a^{(8/3)}) + (5*b^2*Log[a^{(1/3)} - (a + b*x^2)^{(1/3)}])/(12*a^{(8/3)})$

**Rubi in Sympy [A]** time = 16.2243, size = 133, normalized size = 0.96

$$-\frac{\sqrt[3]{a+bx^2}}{4ax^4} + \frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2} - \frac{5b^2 \log(x^2)}{36a^{\frac{8}{3}}} + \frac{5b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{12a^{\frac{8}{3}}} - \frac{5\sqrt{3}b^2 \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} + 2\frac{\sqrt[3]{a+bx^2}}{3}\right)}{\sqrt[3]{a}}\right)}{18a^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*5/(b\*x\*\*2+a)\*\*(2/3), x)

[Out]  $-(a + b*x**2)**(1/3)/(4*a*x**4) + 5*b*(a + b*x**2)**(1/3)/(12*a**2*x**2) - 5*b**2*log(x**2)/(36*a**(8/3)) + 5*b**2*log(a**(1/3) - (a + b*x**2)**(1/3))/(12*a**(8/3)) - 5*sqrt(3)*b**2*atan(sqrt(3)*(a**(1/3)/3 + 2*(a + b*x**2)**(1/3)/3)/a**(1/3))/(18*a**(8/3))$

**Mathematica [C]** time = 0.052098, size = 83, normalized size = 0.6

$$\frac{-3a^2 - 5b^2x^4 \left(\frac{a}{bx^2} + 1\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{a}{bx^2}\right) + 2abx^2 + 5b^2x^4}{12a^2x^4(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5\*(a + b\*x^2)^(2/3)), x]

[Out]  $(-3a^2 + 2abx^2 + 5b^2x^4 - 5b^2(1 + a/(bx^2))^{2/3})x^4 \text{Hypergeometric2F1}[2/3, 2/3, 5/3, -(a/(bx^2))]/(12a^2x^4(a + bx^2)^{2/3})$

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{x^5} (bx^2 + a)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^5/(b*x^2+a)^(2/3), x)`

[Out] `int(1/x^5/(b*x^2+a)^(2/3), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*x^5), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.226734, size = 220, normalized size = 1.59

$$\frac{\sqrt{3} \left( 5 \sqrt{3} b^2 x^4 \log \left( a^2 + (bx^2 + a)^{\frac{1}{3}} (a^2)^{\frac{1}{3}} a + (bx^2 + a)^{\frac{2}{3}} (a^2)^{\frac{2}{3}} \right) - 10 \sqrt{3} b^2 x^4 \log \left( -a + (bx^2 + a)^{\frac{1}{3}} (a^2)^{\frac{1}{3}} \right) + 30 b^2 x^4 \arctan \left( \frac{\sqrt{3} (a^2)^{\frac{1}{3}} a^2 x^4}{108 (a^2)^{\frac{1}{3}} a^2 x^4} \right) \right)}{108 (a^2)^{\frac{1}{3}} a^2 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*x^5), x, algorithm="fricas")`

[Out]  $-1/108 \sqrt{3} (5 \sqrt{3} b^2 x^4 \log(a^2 + (bx^2 + a)^{1/3} (a^2)^{1/3} a + (bx^2 + a)^{2/3} (a^2)^{2/3}) - 10 \sqrt{3} b^2 x^4 \log(-a + (bx^2 + a)^{1/3} (a^2)^{1/3}) + 30 b^2 x^4 \arctan(1/3 (\sqrt{3} a + 2 \sqrt{3} (bx^2 + a)^{1/3} (a^2)^{1/3})/a) - 3 \sqrt{3} (5 b^2 x^4 - 3 a) (bx^2 + a)^{1/3} (a^2)^{1/3}) / ((a^2)^{1/3} a^2 x^4)$

**Sympy [A]** time = 6.2078, size = 41, normalized size = 0.3

$$\frac{\left(\frac{8}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{8}{3} \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{2}{3}} x^{\frac{16}{3}} \left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(b*x**2+a)**(2/3), x)`

[Out]  $-\text{gamma}(8/3) \text{hyper}((2/3, 8/3), (11/3, ), a \exp_{\text{polar}}(I \pi)/(b x^2)) / (2 b^{2/3} x^{16/3} \text{gamma}(11/3))$

**GIAC/XCAS [A]** time = 0.567167, size = 171, normalized size = 1.24

$$-\frac{1}{36} b^2 \left( \frac{10 \sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{8}{3}}} + \frac{5 \ln\left(\left(bx^2+a\right)^{\frac{2}{3}} + \left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{a^{\frac{8}{3}}} - \frac{10 \ln\left(\left| \left(bx^2+a\right)^{\frac{1}{3}} - a^{\frac{1}{3}} \right| \right)}{a^{\frac{8}{3}}} - \frac{3\left(5\left(bx^2+a\right)^{\frac{4}{3}} - 8\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{a^{\frac{8}{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(2/3)\*x^5),x, algorithm="giac")

[Out] -1/36\*b^2\*(10\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(b\*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(8/3) + 5\*ln((b\*x^2 + a)^(2/3) + (b\*x^2 + a)^(1/3)\*a^(1/3) + a^(2/3))/a^(8/3) - 10\*ln(abs((b\*x^2 + a)^(1/3) - a^(1/3)))/a^(8/3) - 3\*(5\*(b\*x^2 + a)^(4/3) - 8\*(b\*x^2 + a)^(1/3)\*a^(1/3))/a^(8/3)

$$3.721 \quad \int \frac{x^4}{(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=293

$$\frac{27 \cdot 3^{3/4} \sqrt{2 - \sqrt{3}} a^2 \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \right) \Big|_{-7 + 4\sqrt{3}}}{55b^3 x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} - \frac{27ax \sqrt[3]{a + bx^2}}{55b^2} + \frac{3x^3 \sqrt[3]{a + bx^2}}{11b}$$

[Out]  $(-27 * a * x * (a + b * x^2)^{(1/3)}) / (55 * b^2) + (3 * x^3 * (a + b * x^2)^{(1/3)}) / (11 * b) - (27 * 3^{(3/4)} * \text{Sqrt}[2 - \text{Sqrt}[3]] * a^2 * (a^{(1/3)} - (a + b * x^2)^{(1/3)}) * \text{Sqrt}[(a^{(2/3)} + a^{(1/3)} * (a + b * x^2)^{(1/3)} + (a + b * x^2)^{(2/3)}) / ((1 - \text{Sqrt}[3]) * a^{(1/3)} - (a + b * x^2)^{(1/3)})^2] * \text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3]) * a^{(1/3)} - (a + b * x^2)^{(1/3)}) / ((1 - \text{Sqrt}[3]) * a^{(1/3)} - (a + b * x^2)^{(1/3)})], -7 + 4 * \text{Sqrt}[3]]) / (55 * b^3 * x * \text{Sqrt}[-((a^{(1/3)} * (a^{(1/3)} - (a + b * x^2)^{(1/3)})) / ((1 - \text{Sqrt}[3]) * a^{(1/3)} - (a + b * x^2)^{(1/3)})^2)])$

**Rubi [A]** time = 0.419613, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{27 \cdot 3^{3/4} \sqrt{2 - \sqrt{3}} a^2 \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \right) \Big|_{-7 + 4\sqrt{3}}}{55b^3 x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} - \frac{27ax \sqrt[3]{a + bx^2}}{55b^2} + \frac{3x^3 \sqrt[3]{a + bx^2}}{11b}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a + b\*x^2)^(2/3), x]

[Out]  $(-27 * a * x * (a + b * x^2)^{(1/3)}) / (55 * b^2) + (3 * x^3 * (a + b * x^2)^{(1/3)}) / (11 * b) - (27 * 3^{(3/4)} * \text{Sqrt}[2 - \text{Sqrt}[3]] * a^2 * (a^{(1/3)} - (a + b * x^2)^{(1/3)}) * \text{Sqrt}[(a^{(2/3)} + a^{(1/3)} * (a + b * x^2)^{(1/3)} + (a + b * x^2)^{(2/3)}) / ((1 - \text{Sqrt}[3]) * a^{(1/3)} - (a + b * x^2)^{(1/3)})^2] * \text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3]) * a^{(1/3)} - (a + b * x^2)^{(1/3)}) / ((1 - \text{Sqrt}[3]) * a^{(1/3)} - (a + b * x^2)^{(1/3)})], -7 + 4 * \text{Sqrt}[3]]) / (55 * b^3 * x * \text{Sqrt}[-((a^{(1/3)} * (a^{(1/3)} - (a + b * x^2)^{(1/3)})) / ((1 - \text{Sqrt}[3]) * a^{(1/3)} - (a + b * x^2)^{(1/3)})^2)])$

**Rubi in Sympy [A]** time = 15.5465, size = 243, normalized size = 0.83

$$\frac{27 \cdot 3^{3/4} a^2 \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( \sqrt[3]{a} (-1 + \sqrt{3}) + \sqrt[3]{a + bx^2} \right)^2}} \sqrt{-\sqrt{3} + 2} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) F \left( \text{asin} \left( \frac{\sqrt[3]{a} (1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}{-\sqrt[3]{a} (-1 + \sqrt{3}) - \sqrt[3]{a + bx^2}} \right) \right) \Big|_{-7 + 4\sqrt{3}}}{55b^3 x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( \sqrt[3]{a} (-1 + \sqrt{3}) + \sqrt[3]{a + bx^2} \right)^2}} - \frac{27ax \sqrt[3]{a + bx^2}}{55b^2} + \frac{3x^3 \sqrt[3]{a + bx^2}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**4/(b*x**2+a)**(2/3),x)`

[Out] 
$$-27 \cdot 3^{3/4} \cdot a^{2/3} \sqrt{(a^{2/3} + a^{1/3}(a + b x^2)^{1/3} + (a + b x^2)^{2/3}) / (a^{1/3}(-1 + \sqrt{3}) + (a + b x^2)^{1/3})} + (a + b x^2)^{2/3} \sqrt{-\sqrt{3} + 2} (a^{1/3} - (a + b x^2)^{1/3}) \operatorname{elliptic}_f(\operatorname{asin}((a^{1/3}(1 + \sqrt{3}) - (a + b x^2)^{1/3}) / (-a^{1/3}(-1 + \sqrt{3}) - (a + b x^2)^{1/3})), -7 + 4\sqrt{3}) / (55 b^3 x \sqrt{-a^{1/3}(a^{1/3} - (a + b x^2)^{1/3}) / (a^{1/3}(-1 + \sqrt{3}) + (a + b x^2)^{1/3})} - 27 a x (a + b x^2)^{1/3} / (55 b^2) + 3 x^3 (a + b x^2)^{1/3} / (11 b)$$

**Mathematica [C]** time = 0.0574078, size = 79, normalized size = 0.27

$$\frac{3 \left( 9a^2x \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{1}{2}, \frac{2}{3}; \frac{3}{2}; -\frac{bx^2}{a} \right) - 9a^2x - 4abx^3 + 5b^2x^5 \right)}{55b^2(a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[x^4/(a + b*x^2)^(2/3),x]`

[Out] 
$$(3 * (-9 * a^2 * x - 4 * a * b * x^3 + 5 * b^2 * x^5 + 9 * a^2 * x * (1 + (b * x^2) / a)^{2/3}) * \operatorname{Hypergeometric2F1}[1/2, 2/3, 3/2, -((b * x^2) / a)]) / (55 * b^2 * (a + b * x^2)^{2/3})$$

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int x^4 (bx^2 + a)^{-2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(b*x^2+a)^(2/3),x)`

[Out] `int(x^4/(b*x^2+a)^(2/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2 + a)^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^(2/3),x, algorithm="maxima")`

[Out] `integrate(x^4/(b*x^2 + a)^(2/3), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{x^4}{(bx^2 + a)^{2/3}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b\*x^2 + a)^(2/3),x, algorithm="fricas")

[Out] integral(x^4/(b\*x^2 + a)^(2/3), x)

**Sympy [A]** time = 2.53838, size = 27, normalized size = 0.09

$$\frac{x^5 {}_2F_1\left(\frac{2}{3}, \frac{5}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{5a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4/(b\*x\*\*2+a)\*\*(2/3),x)

[Out] x\*\*5\*hyper((2/3, 5/2), (7/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(5\*a\*\*(2/3))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b\*x^2 + a)^(2/3),x, algorithm="giac")

[Out] integrate(x^4/(b\*x^2 + a)^(2/3), x)



$$3.722 \quad \int \frac{x^2}{(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=269

$$\frac{3 \cdot 3^{3/4} \sqrt{2 - \sqrt{3}} a \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \right) - 7 + 4\sqrt{3}}{5b^2 x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} + \frac{3x \sqrt[3]{a + bx^2}}{5b}$$

[Out] (3\*x\*(a + b\*x^2)^(1/3))/(5\*b) + (3\*3^(3/4)\*Sqrt[2 - Sqrt[3]]\*a\*(a^(1/3) - (a + b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a + b\*x^2)^(1/3) + (a + b\*x^2)^(2/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]\*EllipticF[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))], -7 + 4\*Sqrt[3]])/(5\*b^2\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a + b\*x^2)^(1/3)))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]))

**Rubi [A]** time = 0.338557, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{3 \cdot 3^{3/4} \sqrt{2 - \sqrt{3}} a \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1 + \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}}{(1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2 + a}} \right) \right) - 7 + 4\sqrt{3}}{5b^2 x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( (1 - \sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)^2}} + \frac{3x \sqrt[3]{a + bx^2}}{5b}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*x^2)^(2/3), x]

[Out] (3\*x\*(a + b\*x^2)^(1/3))/(5\*b) + (3\*3^(3/4)\*Sqrt[2 - Sqrt[3]]\*a\*(a^(1/3) - (a + b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a + b\*x^2)^(1/3) + (a + b\*x^2)^(2/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]\*EllipticF[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))], -7 + 4\*Sqrt[3]])/(5\*b^2\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a + b\*x^2)^(1/3)))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]))

**Rubi in Sympy [A]** time = 10.695, size = 219, normalized size = 0.81

$$\frac{3 \cdot 3^{3/4} a \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a + bx^2} + (a + bx^2)^{2/3}}{\left( \sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2} \right)^2}} \sqrt{-\sqrt{3} + 2} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right) F \left( \operatorname{asin} \left( \frac{\sqrt[3]{a}(1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}{-\sqrt[3]{a}(-1 + \sqrt{3}) - \sqrt[3]{a + bx^2}} \right) \right) - 7 + 4\sqrt{3}}{5b^2 x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a + bx^2} \right)}{\left( \sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2} \right)^2}} + \frac{3x \sqrt[3]{a + bx^2}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2/(b*x**2+a)**(2/3),x)`

[Out]  $3 \cdot 3^{3/4} \cdot a \cdot \sqrt{(a^{2/3} + a^{1/3} \cdot (a + b \cdot x^2)^{1/3} + (a + b \cdot x^2)^{2/3}) / (a^{1/3} \cdot (-1 + \sqrt{3}) + (a + b \cdot x^2)^{1/3})} \cdot \sqrt{-\sqrt{3} + 2} \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3}) \cdot \text{elliptic\_f}(\text{asin}((a^{1/3} \cdot (1 + \sqrt{3}) - (a + b \cdot x^2)^{1/3}) / (-a^{1/3} \cdot (-1 + \sqrt{3}) - (a + b \cdot x^2)^{1/3})), -7 + 4 \cdot \sqrt{3}) / (5 \cdot b^{2/3} \cdot x \cdot \sqrt{-a^{1/3} \cdot (a^{1/3} - (a + b \cdot x^2)^{1/3}) / (a^{1/3} \cdot (-1 + \sqrt{3}) + (a + b \cdot x^2)^{1/3})} + 3 \cdot x \cdot (a + b \cdot x^2)^{1/3} / (5 \cdot b))$

**Mathematica [C]** time = 0.0517515, size = 62, normalized size = 0.23

$$\frac{3x \left( -a \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{1}{2}, \frac{2}{3}; \frac{3}{2}; -\frac{bx^2}{a} \right) + a + bx^2 \right)}{5b(a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[x^2/(a + b*x^2)^(2/3),x]`

[Out]  $(3 \cdot x \cdot (a + b \cdot x^2 - a \cdot (1 + (b \cdot x^2)/a)^{2/3}) \cdot \text{Hypergeometric2F1}[1/2, 2/3, 3/2, -((b \cdot x^2)/a)]) / (5 \cdot b \cdot (a + b \cdot x^2)^{2/3})$

**Maple [F]** time = 0.035, size = 0, normalized size = 0.

$$\int x^2 (bx^2 + a)^{-2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*x^2+a)^(2/3),x)`

[Out] `int(x^2/(b*x^2+a)^(2/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2 + a)^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(2/3),x, algorithm="maxima")`

[Out] `integrate(x^2/(b*x^2 + a)^(2/3), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{x^2}{(bx^2 + a)^{2/3}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(2/3),x, algorithm="fricas")`

[Out] `integral(x^2/(b*x^2 + a)^(2/3), x)`

---

**Sympy** [A] time = 2.30728, size = 27, normalized size = 0.1

$$\frac{x^3 {}_2F_1\left(\frac{2}{3}, \frac{3}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**2+a)**(2/3), x)`

[Out] `x**3*hyper((2/3, 3/2), (5/2, ), b*x**2*exp_polar(I*pi)/a)/(3*a**(2/3))`

---

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(2/3), x, algorithm="giac")`

[Out] `integrate(x^2/(b*x^2 + a)^(2/3), x)`

$$3.723 \quad \int \frac{1}{(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=246

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle| -7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}$$

[Out] -((3^(3/4)\*Sqrt[2 - Sqrt[3]]\*(a^(1/3) - (a + b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a + b\*x^2)^(1/3) + (a + b\*x^2)^(2/3)]/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]\*EllipticF[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))/(1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3)], -7 + 4\*Sqrt[3]])/(b\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a + b\*x^2)^(1/3)))/(1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]))

**Rubi [A]** time = 0.236251, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle| -7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(-2/3), x]

[Out] -((3^(3/4)\*Sqrt[2 - Sqrt[3]]\*(a^(1/3) - (a + b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a + b\*x^2)^(1/3) + (a + b\*x^2)^(2/3)]/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]\*EllipticF[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))/(1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3)], -7 + 4\*Sqrt[3]])/(b\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a + b\*x^2)^(1/3)))/(1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]))

**Rubi in Sympy [A]** time = 5.75374, size = 197, normalized size = 0.8

$$\frac{3^{3/4}\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a(-1+\sqrt{3})}+\sqrt[3]{a+bx^2}\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}(1+\sqrt{3})-\sqrt[3]{a+bx^2}}{-\sqrt[3]{a(-1+\sqrt{3})}-\sqrt[3]{a+bx^2}}\right)\middle| -7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a(-1+\sqrt{3})}+\sqrt[3]{a+bx^2}\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+a)\*\*(2/3), x)

[Out] -3\*\*(3/4)\*sqrt((a\*\*(2/3) + a\*\*(1/3)\*(a + b\*x\*\*2)\*\*(1/3) + (a + b\*x\*\*2)\*\*(2/3))/(a\*\*(1/3)\*(-1 + sqrt(3)) + (a + b\*x\*\*2)\*\*(1/3))\*\*2)\*sqrt(-sqrt(3) + 2)\*(a\*\*(1/3) - (a + b\*x\*\*2)\*\*(1/3))\*elliptic\_f(a\*sin((a\*\*(1/3)\*(1 + sqrt(3)) - (a + b\*x\*\*2)\*\*(1/3))/(-a\*\*(1/3)\*(-1

+ sqrt(3)) - (a + b\*x\*\*2)\*\*(1/3)), -7 + 4\*sqrt(3))/(b\*x\*sqrt(-a  
\*\* (1/3)\*(a\*\*(1/3) - (a + b\*x\*\*2)\*\*(1/3))/(a\*\*(1/3)\*(-1 + sqrt(3))  
+ (a + b\*x\*\*2)\*\*(1/3)\*\*2))

**Mathematica [C]** time = 0.0267711, size = 47, normalized size = 0.19

$$\frac{x \left( \frac{a+bx^2}{a} \right)^{2/3} {}_2F_1 \left( \frac{1}{2}, \frac{2}{3}; \frac{3}{2}; -\frac{bx^2}{a} \right)}{(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(-2/3), x]

[Out] (x\*((a + b\*x^2)/a)^(2/3)\*Hypergeometric2F1[1/2, 2/3, 3/2, -((b\*x^2)/a)])/(a + b\*x^2)^(2/3)

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int (bx^2 + a)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+a)^(2/3), x)

[Out] int(1/(b\*x^2+a)^(2/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-2/3), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(-2/3), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{(bx^2 + a)^{\frac{2}{3}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-2/3), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(-2/3), x)

**Sympy [A]** time = 2.18783, size = 24, normalized size = 0.1

$$\frac{{}_2F_1\left(\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*2+a)\*\*(2/3), x)

[Out] x\*hyper((1/2, 2/3), (3/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/a\*\*(2/3)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-2/3), x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(-2/3), x)

$$3.724 \quad \int \frac{1}{x^2(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=265

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\right)|_{-7+4\sqrt{3}}}{\sqrt[3]{3}ax\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}-\frac{\sqrt[3]{a+bx^2}}{ax}}$$

[Out]  $-\left((a+b*x^2)^{1/3}/(a*x)\right)+\left(\text{Sqrt}[2-\text{Sqrt}[3]]*(a^{1/3})-(a+b*x^2)^{1/3}\right)*\text{Sqrt}[(a^{2/3}+a^{1/3}*(a+b*x^2)^{1/3}+(a+b*x^2)^{2/3})/((1-\text{Sqrt}[3])*a^{1/3}-(a+b*x^2)^{1/3})^2]*\text{EllipticF}[\text{ArcSin}[(1+\text{Sqrt}[3])*a^{1/3}-(a+b*x^2)^{1/3})/((1-\text{Sqrt}[3])*a^{1/3}-(a+b*x^2)^{1/3})],-7+4*\text{Sqrt}[3]]/(3^{1/4}*a*x*\text{Sqrt}[-((a^{1/3}*(a^{1/3})-(a+b*x^2)^{1/3}))/((1-\text{Sqrt}[3])*a^{1/3}-(a+b*x^2)^{1/3})^2])]$

**Rubi [A]** time = 0.335934, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\right)|_{-7+4\sqrt{3}}}{\sqrt[3]{3}ax\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}-\frac{\sqrt[3]{a+bx^2}}{ax}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a+b\*x^2)^(2/3)),x]

[Out]  $-\left((a+b*x^2)^{1/3}/(a*x)\right)+\left(\text{Sqrt}[2-\text{Sqrt}[3]]*(a^{1/3})-(a+b*x^2)^{1/3}\right)*\text{Sqrt}[(a^{2/3}+a^{1/3}*(a+b*x^2)^{1/3}+(a+b*x^2)^{2/3})/((1-\text{Sqrt}[3])*a^{1/3}-(a+b*x^2)^{1/3})^2]*\text{EllipticF}[\text{ArcSin}[(1+\text{Sqrt}[3])*a^{1/3}-(a+b*x^2)^{1/3})/((1-\text{Sqrt}[3])*a^{1/3}-(a+b*x^2)^{1/3})],-7+4*\text{Sqrt}[3]]/(3^{1/4}*a*x*\text{Sqrt}[-((a^{1/3}*(a^{1/3})-(a+b*x^2)^{1/3}))/((1-\text{Sqrt}[3])*a^{1/3}-(a+b*x^2)^{1/3})^2])]$

**Rubi in Sympy [A]** time = 10.4015, size = 211, normalized size = 0.8

$$\frac{3^{3/4}\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)F\left(\text{asin}\left(\frac{\sqrt[3]{a}(1+\sqrt{3})-\sqrt[3]{a+bx^2}}{-\sqrt[3]{a}(-1+\sqrt{3})-\sqrt[3]{a+bx^2}}\right)\right)|_{-7+4\sqrt{3}}}{3ax\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)^2}}-\frac{\sqrt[3]{a+bx^2}}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**2/(b*x**2+a)**(2/3),x)`

[Out]  $3^{3/4} \sqrt{(a^{2/3} + a^{1/3}(a + b^2 x^2)^{1/3} + (a + b^2 x^2)^{2/3}) / (a^{1/3}(-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3})^2} \sqrt{-\sqrt{3} + 2} (a^{1/3} - (a + b^2 x^2)^{1/3}) \operatorname{elliptic\_f}(\operatorname{asin}((a^{1/3}(1 + \sqrt{3}) - (a + b^2 x^2)^{1/3}) / (-a^{1/3}(-1 + \sqrt{3}) - (a + b^2 x^2)^{1/3})), -7 + 4\sqrt{3}) / (3 a x \sqrt{-a^{1/3}(a^{1/3} - (a + b^2 x^2)^{1/3}) / (a^{1/3}(-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3})^2}) - (a + b^2 x^2)^{1/3} / (a x)$

**Mathematica [C]** time = 0.0518152, size = 70, normalized size = 0.26

$$\frac{-bx^2 \left(\frac{bx^2}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{3}{2}; -\frac{bx^2}{a}\right) - 3(a + bx^2)}{3ax(a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^2*(a + b*x^2)^(2/3)),x]`

[Out]  $(-3(a + b^2 x^2) - b^2 x^2 (1 + (b^2 x^2)/a)^{2/3} \operatorname{Hypergeometric2F1}[1/2, 2/3, 3/2, -((b^2 x^2)/a)]) / (3 a^2 x (a + b^2 x^2)^{2/3})$

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (bx^2 + a)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(b*x^2+a)^(2/3),x)`

[Out] `int(1/x^2/(b*x^2+a)^(2/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{2}{3}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*x^2),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(2/3)*x^2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{(bx^2 + a)^{\frac{2}{3}} x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*x^2),x, algorithm="fricas")`



[Out] `integral(1/((b*x^2 + a)^(2/3)*x^2), x)`

---

**Sympy [A]** time = 2.64771, size = 27, normalized size = 0.1

$$\frac{{}_2F_1\left(-\frac{1}{2}, \frac{2}{3} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{2}{3}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(b*x**2+a)**(2/3), x)`

[Out] `-hyper((-1/2, 2/3), (1/2, ), b*x**2*exp_polar(I*pi)/a)/(a**(2/3)*x)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{2}{3}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*x^2), x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(2/3)*x^2), x)`

$$3.725 \quad \int \frac{1}{x^4(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=293

$$\frac{7b\sqrt[3]{a+bx^2}}{9a^2x} - \frac{7\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|_{-7+4\sqrt{3}}\right)}{9\sqrt[3]{3}a^2x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}} - \frac{\sqrt[3]{a+bx^2}}{3ax^3}$$

[Out]  $-(a + b*x^2)^{(1/3)}/(3*a*x^3) + (7*b*(a + b*x^2)^{(1/3)})/(9*a^2*x) - (7*\text{Sqrt}[2 - \text{Sqrt}[3]]*b*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(9*3^{(1/4)}*a^2*x*\text{Sqrt}[-((a^{(1/3)}*(a + b*x^2)^{(1/3)} - (a + b*x^2)^{(1/3)}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2])]$

**Rubi [A]** time = 0.405407, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{7b\sqrt[3]{a+bx^2}}{9a^2x} - \frac{7\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\middle|_{-7+4\sqrt{3}}\right)}{9\sqrt[3]{3}a^2x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}} - \frac{\sqrt[3]{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a + b\*x^2)^(2/3)), x]

[Out]  $-(a + b*x^2)^{(1/3)}/(3*a*x^3) + (7*b*(a + b*x^2)^{(1/3)})/(9*a^2*x) - (7*\text{Sqrt}[2 - \text{Sqrt}[3]]*b*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(9*3^{(1/4)}*a^2*x*\text{Sqrt}[-((a^{(1/3)}*(a + b*x^2)^{(1/3)} - (a + b*x^2)^{(1/3)}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2])]$

**Rubi in Sympy [A]** time = 14.9637, size = 240, normalized size = 0.82

$$\frac{\sqrt[3]{a+bx^2}}{3ax^3} + \frac{7 \cdot 3^{\frac{3}{4}} b \sqrt{\frac{a^{\frac{2}{3}} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}(-1+\sqrt{3}) + \sqrt[3]{a+bx^2}\right)^2}} \sqrt{-\sqrt{3}+2} \left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right) F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}(1+\sqrt{3}) - \sqrt[3]{a+bx^2}}{-\sqrt[3]{a}(-1+\sqrt{3}) - \sqrt[3]{a+bx^2}}\right)\right)}{\left(-7 + 4\sqrt{3}\right)} + \frac{27a^2x \sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}(-1+\sqrt{3}) + \sqrt[3]{a+bx^2}\right)^2}}}{9a^2x} + \frac{7b\sqrt[3]{a+bx^2}}{9a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**4/(b*x**2+a)**(2/3),x)`

[Out]  $-(a + b*x**2)**(1/3)/(3*a*x**3) - 7*3**(3/4)*b*\operatorname{sqrt}((a**(2/3) + a**(1/3)*(a + b*x**2)**(1/3) + (a + b*x**2)**(2/3))/(a**(1/3)*(-1 + \operatorname{sqrt}(3)) + (a + b*x**2)**(1/3))**2*\operatorname{sqrt}(-\operatorname{sqrt}(3) + 2)*(a**(1/3) - (a + b*x**2)**(1/3)))*\operatorname{elliptic\_f}(\operatorname{asin}((a**(1/3)*(1 + \operatorname{sqrt}(3)) - (a + b*x**2)**(1/3))/(-a**(1/3)*(-1 + \operatorname{sqrt}(3)) - (a + b*x**2)**(1/3))), -7 + 4*\operatorname{sqrt}(3))/(27*a**2*x*\operatorname{sqrt}(-a**(1/3)*(a**(1/3) - (a + b*x**2)**(1/3)))/(a**(1/3)*(-1 + \operatorname{sqrt}(3)) + (a + b*x**2)**(1/3))**2) + 7*b*(a + b*x**2)**(1/3)/(9*a**2*x)$

**Mathematica [C]** time = 0.0482272, size = 83, normalized size = 0.28

$$\frac{-9a^2 + 7b^2x^4 \left(\frac{bx^2}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{3}{2}; -\frac{bx^2}{a}\right) + 12abx^2 + 21b^2x^4}{27a^2x^3(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^4*(a + b*x^2)^(2/3)),x]`

[Out]  $(-9*a^2 + 12*a*b*x^2 + 21*b^2*x^4 + 7*b^2*x^4*(1 + (b*x^2)/a)^(2/3))*\operatorname{Hypergeometric2F1}[1/2, 2/3, 3/2, -((b*x^2)/a)]/(27*a^2*x^3*(a + b*x^2)^(2/3))$

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} (bx^2 + a)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(b*x^2+a)^(2/3),x)`

[Out] `int(1/x^4/(b*x^2+a)^(2/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{2}{3}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*x^4),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(2/3)*x^4), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{2}{3}}x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*x^4),x, algorithm="fricas")`

[Out] `integral(1/((b*x^2 + a)^(2/3)*x^4), x)`

**Sympy** [A] time = 3.43731, size = 32, normalized size = 0.11

$$-\frac{{}_2F_1\left(-\frac{3}{2}, \frac{2}{3} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(b*x**2+a)**(2/3),x)`

[Out] `-hyper((-3/2, 2/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(2/3)*x**3)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{2}{3}}x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*x^4),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(2/3)*x^4), x)`

$$3.726 \quad \int \frac{x^7}{(a+bx^2)^{4/3}} dx$$

**Optimal.** Leaf size=80

$$\frac{3a^3}{2b^4\sqrt[3]{a+bx^2}} + \frac{9a^2(a+bx^2)^{2/3}}{4b^4} - \frac{9a(a+bx^2)^{5/3}}{10b^4} + \frac{3(a+bx^2)^{8/3}}{16b^4}$$

[Out]  $(3*a^3)/(2*b^4*(a + b*x^2)^(1/3)) + (9*a^2*(a + b*x^2)^(2/3))/(4*b^4) - (9*a*(a + b*x^2)^(5/3))/(10*b^4) + (3*(a + b*x^2)^(8/3))/(16*b^4)$

**Rubi [A]** time = 0.128359, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{3a^3}{2b^4\sqrt[3]{a+bx^2}} + \frac{9a^2(a+bx^2)^{2/3}}{4b^4} - \frac{9a(a+bx^2)^{5/3}}{10b^4} + \frac{3(a+bx^2)^{8/3}}{16b^4}$$

Antiderivative was successfully verified.

[In] Int[x^7/(a + b\*x^2)^(4/3), x]

[Out]  $(3*a^3)/(2*b^4*(a + b*x^2)^(1/3)) + (9*a^2*(a + b*x^2)^(2/3))/(4*b^4) - (9*a*(a + b*x^2)^(5/3))/(10*b^4) + (3*(a + b*x^2)^(8/3))/(16*b^4)$

**Rubi in Sympy [A]** time = 15.2353, size = 75, normalized size = 0.94

$$\frac{3a^3}{2b^4\sqrt[3]{a+bx^2}} + \frac{9a^2(a+bx^2)^{2/3}}{4b^4} - \frac{9a(a+bx^2)^{5/3}}{10b^4} + \frac{3(a+bx^2)^{8/3}}{16b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*7/(b\*x\*\*2+a)\*\*(4/3), x)

[Out]  $3*a**3/(2*b**4*(a + b*x**2)**(1/3)) + 9*a**2*(a + b*x**2)**(2/3)/(4*b**4) - 9*a*(a + b*x**2)**(5/3)/(10*b**4) + 3*(a + b*x**2)**(8/3)/(16*b**4)$

**Mathematica [A]** time = 0.0352948, size = 50, normalized size = 0.62

$$\frac{3(81a^3 + 27a^2bx^2 - 9ab^2x^4 + 5b^3x^6)}{80b^4\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a + b\*x^2)^(4/3), x]

[Out]  $(3*(81*a^3 + 27*a^2*b*x^2 - 9*a*b^2*x^4 + 5*b^3*x^6))/(80*b^4*(a + b*x^2)^(1/3))$

**Maple [A]** time = 0.007, size = 47, normalized size = 0.6

$$\frac{15b^3x^6 - 27ab^2x^4 + 81a^2bx^2 + 243a^3}{80b^4} \frac{1}{\sqrt[3]{bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7/(b*x^2+a)^(4/3),x)`

[Out]  $\frac{3}{80} \frac{1}{(bx^2+a)^{1/3}} (5b^3x^6 - 9ab^2x^4 + 27a^2bx^2 + 81a^3) / b^4$

**Maxima [A]** time = 1.34345, size = 86, normalized size = 1.08

$$\frac{3(bx^2+a)^{\frac{8}{3}}}{16b^4} - \frac{9(bx^2+a)^{\frac{5}{3}}a}{10b^4} + \frac{9(bx^2+a)^{\frac{2}{3}}a^2}{4b^4} + \frac{3a^3}{2(bx^2+a)^{\frac{1}{3}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^2 + a)^(4/3),x, algorithm="maxima")`

[Out]  $\frac{3}{16} \frac{(bx^2+a)^{8/3}}{b^4} - \frac{9}{10} \frac{(bx^2+a)^{5/3}a}{b^4} + \frac{9}{4} \frac{(bx^2+a)^{2/3}a^2}{b^4} + \frac{3}{2} \frac{a^3}{(bx^2+a)^{1/3}b^4}$

**Fricas [A]** time = 0.211628, size = 62, normalized size = 0.78

$$\frac{3(5b^3x^6 - 9ab^2x^4 + 27a^2bx^2 + 81a^3)}{80(bx^2+a)^{\frac{1}{3}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(b*x^2 + a)^(4/3),x, algorithm="fricas")`

[Out]  $\frac{3}{80} \frac{(5b^3x^6 - 9ab^2x^4 + 27a^2bx^2 + 81a^3)}{(bx^2+a)^{1/3}b^4}$

**Sympy [A]** time = 9.9194, size = 1584, normalized size = 19.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7/(b*x**2+a)**(4/3),x)`

[Out]  $243a^{68/3}(1 + bx^2/a)^{2/3} / (80a^{20}b^4 + 480a^{19}b^5x^2 + 1200a^{18}b^6x^4 + 1600a^{17}b^7x^6 + 1200a^{16}b^8x^8 + 480a^{15}b^9x^{10} + 80a^{14}b^{10}x^{12}) - 243a^{65/3}(80a^{20}b^4 + 480a^{19}b^5x^2 + 1200a^{18}b^6x^4 + 1600a^{17}b^7x^6 + 1200a^{16}b^8x^8 + 480a^{15}b^9x^{10} + 80a^{14}b^{10}x^{12}) + 1296a^{65/3}b^2(1 + bx^2/a)^{2/3} / (80a^{20}b^4 + 480a^{19}b^5x^2 + 1200a^{18}b^6x^4 + 1600a^{17}b^7x^6 + 1200a^{16}b^8x^8 + 480a^{15}b^9x^{10} + 80a^{14}b^{10}x^{12}) - 1458a^{65/3}b^2x^2(1 + bx^2/a)^{2/3} / (80a^{20}b^4 + 480a^{19}b^5x^2 + 1200a^{18}b^6x^4 + 1600a^{17}b^7x^6 + 1200a^{16}b^8x^8 + 480a^{15}b^9x^{10} + 80a^{14}b^{10}x^{12}) + 2808a^{62/3}b^2x^4(1 + bx^2/a)^{2/3} / (80a^{20}b^4 + 480a^{19}b^5x^2 + 1200a^{18}b^6x^4 + 1600a^{17}b^7x^6 + 1200a^{16}b^8x^8 + 480a^{15}b^9x^{10} + 80a^{14}b^{10}x^{12}) - 3645a^{62/3}b^2x^4 / (80a^{20}b^4 + 480a^{19}b^5x^2 + 1200a^{18}b^6x^4 + 1600a^{17}b^7x^6 + 1200a^{16}b^8x^8 + 480a^{15}b^9x^{10} + 80a^{14}b^{10}x^{12}) + 3120a^{59/3}b^3x^6(1 + bx^2/a)^{2/3} / (80a^{20}b^4 + 480a^{19}b^5x^2 + 1200a^{18}b^6x^4 + 1600a^{17}b^7x^6 + 1200a^{16}b^8x^8 + 480a^{15}b^9x^{10} + 80a^{14}b^{10}x^{12}) + 3120a^{56/3}b^3x^6 / (80a^{20}b^4 + 480a^{19}b^5x^2 + 1200a^{18}b^6x^4 + 1600a^{17}b^7x^6 + 1200a^{16}b^8x^8 + 480a^{15}b^9x^{10} + 80a^{14}b^{10}x^{12})$

$17*b^{7}*x^{6} + 1200*a^{16}*b^{8}*x^{8} + 480*a^{15}*b^{9}*x^{10} + 80*a^{14}*b^{10}*x^{12} - 4860*a^{19}*b^{5}*x^{2} + 1200*a^{18}*b^{6}*x^{4} + 1600*a^{17}*b^{7}*x^{6} + 1200*a^{16}*b^{8}*x^{8} + 480*a^{15}*b^{9}*x^{10} + 80*a^{14}*b^{10}*x^{12} + 1830*a^{19}*b^{5}*x^{2} + 1200*a^{18}*b^{6}*x^{4} + 1600*a^{17}*b^{7}*x^{6} + 1200*a^{16}*b^{8}*x^{8} + 480*a^{15}*b^{9}*x^{10} + 80*a^{14}*b^{10}*x^{12} - 3645*a^{19}*b^{5}*x^{2} + 1200*a^{18}*b^{6}*x^{4} + 1600*a^{17}*b^{7}*x^{6} + 1200*a^{16}*b^{8}*x^{8} + 480*a^{15}*b^{9}*x^{10} + 80*a^{14}*b^{10}*x^{12} + 528*a^{19}*b^{5}*x^{2} + 1200*a^{18}*b^{6}*x^{4} + 1600*a^{17}*b^{7}*x^{6} + 1200*a^{16}*b^{8}*x^{8} + 480*a^{15}*b^{9}*x^{10} + 80*a^{14}*b^{10}*x^{12} - 1458*a^{19}*b^{5}*x^{2} + 1200*a^{18}*b^{6}*x^{4} + 1600*a^{17}*b^{7}*x^{6} + 1200*a^{16}*b^{8}*x^{8} + 480*a^{15}*b^{9}*x^{10} + 80*a^{14}*b^{10}*x^{12} + 96*a^{19}*b^{5}*x^{2} + 1200*a^{18}*b^{6}*x^{4} + 1600*a^{17}*b^{7}*x^{6} + 1200*a^{16}*b^{8}*x^{8} + 480*a^{15}*b^{9}*x^{10} + 80*a^{14}*b^{10}*x^{12} + 48*a^{19}*b^{5}*x^{2} + 1200*a^{18}*b^{6}*x^{4} + 1600*a^{17}*b^{7}*x^{6} + 1200*a^{16}*b^{8}*x^{8} + 480*a^{15}*b^{9}*x^{10} + 80*a^{14}*b^{10}*x^{12} + 15*a^{19}*b^{5}*x^{2} + 1200*a^{18}*b^{6}*x^{4} + 1600*a^{17}*b^{7}*x^{6} + 1200*a^{16}*b^{8}*x^{8} + 480*a^{15}*b^{9}*x^{10} + 80*a^{14}*b^{10}*x^{12}$

---

**GIAC/XCAS [A]** time = 0.213203, size = 77, normalized size = 0.96

$$\frac{3 \left( 5 (bx^2 + a)^{\frac{8}{3}} - 24 (bx^2 + a)^{\frac{5}{3}} a + 60 (bx^2 + a)^{\frac{2}{3}} a^2 + \frac{40 a^3}{(bx^2 + a)^{\frac{1}{3}}} \right)}{80 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(b\*x^2 + a)^(4/3),x, algorithm="giac")

[Out] 3/80\*(5\*(b\*x^2 + a)^(8/3) - 24\*(b\*x^2 + a)^(5/3)\*a + 60\*(b\*x^2 + a)^(2/3)\*a^2 + 40\*a^3/(b\*x^2 + a)^(1/3))/b^4

$$3.727 \quad \int \frac{x^5}{(a+bx^2)^{4/3}} dx$$

**Optimal.** Leaf size=59

$$-\frac{3a^2}{2b^3\sqrt[3]{a+bx^2}} - \frac{3a(a+bx^2)^{2/3}}{2b^3} + \frac{3(a+bx^2)^{5/3}}{10b^3}$$

[Out]  $(-3*a^2)/(2*b^3*(a+b*x^2)^{(1/3)}) - (3*a*(a+b*x^2)^{(2/3)})/(2*b^3) + (3*(a+b*x^2)^{(5/3)})/(10*b^3)$

**Rubi [A]** time = 0.0996635, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{3a^2}{2b^3\sqrt[3]{a+bx^2}} - \frac{3a(a+bx^2)^{2/3}}{2b^3} + \frac{3(a+bx^2)^{5/3}}{10b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a + b\*x^2)^(4/3), x]

[Out]  $(-3*a^2)/(2*b^3*(a+b*x^2)^{(1/3)}) - (3*a*(a+b*x^2)^{(2/3)})/(2*b^3) + (3*(a+b*x^2)^{(5/3)})/(10*b^3)$

**Rubi in Sympy [A]** time = 11.4622, size = 54, normalized size = 0.92

$$-\frac{3a^2}{2b^3\sqrt[3]{a+bx^2}} - \frac{3a(a+bx^2)^{2/3}}{2b^3} + \frac{3(a+bx^2)^{5/3}}{10b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5/(b\*x\*\*2+a)\*\*(4/3), x)

[Out]  $-3*a**2/(2*b**3*(a+b*x**2)**(1/3)) - 3*a*(a+b*x**2)**(2/3)/(2*b**3) + 3*(a+b*x**2)**(5/3)/(10*b**3)$

**Mathematica [A]** time = 0.0287818, size = 38, normalized size = 0.64

$$\frac{3(-9a^2 - 3abx^2 + b^2x^4)}{10b^3\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a + b\*x^2)^(4/3), x]

[Out]  $(3*(-9*a^2 - 3*a*b*x^2 + b^2*x^4))/(10*b^3*(a + b*x^2)^{(1/3)})$

**Maple [A]** time = 0.008, size = 36, normalized size = 0.6

$$-\frac{-3b^2x^4 + 9abx^2 + 27a^2}{10b^3} \frac{1}{\sqrt[3]{bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(x^5/(b*x^2+a)^(4/3),x)`

[Out]  $-3/10/(b*x^2+a)^{(1/3)}*(-b^2*x^4+3*a*b*x^2+9*a^2)/b^3$

**Maxima [A]** time = 1.33747, size = 63, normalized size = 1.07

$$\frac{3(bx^2+a)^{\frac{5}{3}}}{10b^3} - \frac{3(bx^2+a)^{\frac{2}{3}}a}{2b^3} - \frac{3a^2}{2(bx^2+a)^{\frac{1}{3}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2+a)^(4/3),x, algorithm="maxima")`

[Out]  $3/10*(b*x^2+a)^{(5/3)}/b^3 - 3/2*(b*x^2+a)^{(2/3)}*a/b^3 - 3/2*a^2/((b*x^2+a)^{(1/3)}*b^3)$

**Fricas [A]** time = 0.210167, size = 46, normalized size = 0.78

$$\frac{3(b^2x^4 - 3abx^2 - 9a^2)}{10(bx^2+a)^{\frac{1}{3}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(b*x^2+a)^(4/3),x, algorithm="fricas")`

[Out]  $3/10*(b^2*x^4 - 3*a*b*x^2 - 9*a^2)/((b*x^2+a)^{(1/3)}*b^3)$

**Sympy [A]** time = 6.19344, size = 561, normalized size = 9.51

$$\begin{aligned} & -\frac{27a^{\frac{29}{3}}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6} + \frac{27a^{\frac{29}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6} \\ & -\frac{63a^{\frac{26}{3}}bx^2\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6} \\ & + \frac{81a^{\frac{26}{3}}bx^2}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6} - \frac{42a^{\frac{23}{3}}b^2x^4\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6} \\ & + \frac{81a^{\frac{23}{3}}b^2x^4}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6} - \frac{3a^{\frac{20}{3}}b^3x^6\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6} \\ & + \frac{27a^{\frac{20}{3}}b^3x^6}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6} + \frac{3a^{\frac{17}{3}}b^4x^8\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(b*x**2+a)**(4/3),x)`

[Out]  $-27*a**(29/3)*(1+b*x**2/a)**(2/3)/(10*a**8*b**3+30*a**7*b**4*x**2+30*a**6*b**5*x**4+10*a**5*b**6*x**6)+27*a**(29/3)/(10*a**8*b**3+30*a**7*b**4*x**2+30*a**6*b**5*x**4+10*a**5*b**6*x**6)-63*a**(26/3)*b*x**2*(1+b*x**2/a)**(2/3)/(10*a**8*b**3+30*a**7*b**4*x**2+30*a**6*b**5*x**4+10*a**5*b**6*x**6)+81*a**(26/3)*b*x**2/(10*a**8*b**3+30*a**7*b**4*x**2+30*a**6*b**5*x**4+10*a**5*b**6*x**6)-42*a**(23/3)*b**2*x**4*(1+b*x**2/a)**(2/3)/(10*a**8*b**3+30*a**7*b**4*x**2+30*a**6*b**5*x**4+10*a**5*b**6*x**6)+81*a**(23/3)*b**2*x**4/(10*a**8*b**3+30*a**7*b**4*x**2+30*a**6*b**5*x**4+10*a**5*b**6*x**6)+3*a**(20/3)*b**3*x**6*(1+b*x**2/a)**(2/3)/(10*a**8*b**3+30*a**7*b**4*x**2+30*a**6*b**5*x**4+10*a**5*b**6*x**6)+27*a**(20/3)*b**3*x**6/(10*a**8*b**3+30*a**7*b**4*x**2+30*a**6*b**5*x**4+10*a**5*b**6*x**6)+3*a**(17/3)*b**4*x**8*(1+b*x**2/a)**(2/3)/(10*a**8*b**3+30*a**7*b**4*x**2+30*a**6*b**5*x**4+10*a**5*b**6*x**6)$

$$10*a**5*b**6*x**6) + 81*a**(23/3)*b**2*x**4/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 + 10*a**5*b**6*x**6) - 3*a**(20/3)*b**3*x**6*(1 + b*x**2/a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 + 10*a**5*b**6*x**6) + 27*a**(20/3)*b**3*x**6/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 + 10*a**5*b**6*x**6) + 3*a**(17/3)*b**4*x**8*(1 + b*x**2/a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 + 10*a**5*b**6*x**6)$$

**GIAC/XCAS [A]** time = 0.215172, size = 55, normalized size = 0.93

$$\frac{3 \left( (bx^2 + a)^{\frac{5}{3}} - 5 (bx^2 + a)^{\frac{2}{3}} a - \frac{5a^2}{(bx^2 + a)^{\frac{1}{3}}} \right)}{10b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b\*x^2 + a)^(4/3),x, algorithm="giac")

[Out] 3/10\*((b\*x^2 + a)^(5/3) - 5\*(b\*x^2 + a)^(2/3)\*a - 5\*a^2/(b\*x^2 + a)^(1/3))/b^3

$$3.728 \quad \int \frac{x^3}{(a+bx^2)^{4/3}} dx$$

Optimal. Leaf size=38

$$\frac{3a}{2b^2\sqrt[3]{a+bx^2}} + \frac{3(a+bx^2)^{2/3}}{4b^2}$$

[Out] (3\*a)/(2\*b^2\*(a + b\*x^2)^(1/3)) + (3\*(a + b\*x^2)^(2/3))/(4\*b^2)

Rubi [A] time = 0.070699, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{3a}{2b^2\sqrt[3]{a+bx^2}} + \frac{3(a+bx^2)^{2/3}}{4b^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b\*x^2)^(4/3), x]

[Out] (3\*a)/(2\*b^2\*(a + b\*x^2)^(1/3)) + (3\*(a + b\*x^2)^(2/3))/(4\*b^2)

Rubi in Sympy [A] time = 7.83016, size = 34, normalized size = 0.89

$$\frac{3a}{2b^2\sqrt[3]{a+bx^2}} + \frac{3(a+bx^2)^{2/3}}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(b\*x\*\*2+a)\*\*(4/3), x)

[Out] 3\*a/(2\*b\*\*2\*(a + b\*x\*\*2)\*\*(1/3)) + 3\*(a + b\*x\*\*2)\*\*(2/3)/(4\*b\*\*2)

Mathematica [A] time = 0.0238519, size = 27, normalized size = 0.71

$$\frac{3(3a+bx^2)}{4b^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b\*x^2)^(4/3), x]

[Out] (3\*(3\*a + b\*x^2))/(4\*b^2\*(a + b\*x^2)^(1/3))

Maple [A] time = 0.007, size = 24, normalized size = 0.6

$$\frac{3bx^2 + 9a}{4b^2} \frac{1}{\sqrt[3]{bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b\*x^2+a)^(4/3), x)

[Out]  $3/4/(b*x^2+a)^{(1/3)}*(b*x^2+3*a)/b^2$

**Maxima [A]** time = 1.47639, size = 41, normalized size = 1.08

$$\frac{3(bx^2 + a)^{\frac{2}{3}}}{4b^2} + \frac{3a}{2(bx^2 + a)^{\frac{1}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 + a)^(4/3),x, algorithm="maxima")`

[Out]  $3/4*(b*x^2 + a)^{(2/3)}/b^2 + 3/2*a/((b*x^2 + a)^{(1/3)}*b^2)$

**Fricas [A]** time = 0.214525, size = 31, normalized size = 0.82

$$\frac{3(bx^2 + 3a)}{4(bx^2 + a)^{\frac{1}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 + a)^(4/3),x, algorithm="fricas")`

[Out]  $3/4*(b*x^2 + 3*a)/((b*x^2 + a)^{(1/3)}*b^2)$

**Sympy [A]** time = 2.46287, size = 46, normalized size = 1.21

$$\begin{cases} \frac{9a}{4b^2\sqrt[3]{a+bx^2}} + \frac{3x^2}{4b\sqrt[3]{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(b*x**2+a)**(4/3),x)`

[Out] `Piecewise((9*a/(4*b**2*(a + b*x**2)**(1/3)) + 3*x**2/(4*b*(a + b*x**2)**(1/3)), Ne(b, 0)), (x**4/(4*a**(4/3)), True))`

**GIAC/XCAS [A]** time = 0.213392, size = 36, normalized size = 0.95

$$\frac{3\left((bx^2 + a)^{\frac{2}{3}} + \frac{2a}{(bx^2+a)^{\frac{1}{3}}}\right)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(b*x^2 + a)^(4/3),x, algorithm="giac")`

[Out]  $3/4*((b*x^2 + a)^{(2/3)} + 2*a/(b*x^2 + a)^{(1/3)})/b^2$

$$3.729 \quad \int \frac{x}{(a+bx^2)^{4/3}} dx$$

**Optimal.** Leaf size=18

$$-\frac{3}{2b\sqrt[3]{a+bx^2}}$$

[Out]  $-3/(2*b*(a + b*x^2)^(1/3))$

**Rubi [A]** time = 0.0118733, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{3}{2b\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x/(a + b*x^2)^(4/3), x]$

[Out]  $-3/(2*b*(a + b*x^2)^(1/3))$

**Rubi in Sympy [A]** time = 2.13948, size = 15, normalized size = 0.83

$$-\frac{3}{2b\sqrt[3]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x/(b*x**2+a)**(4/3), x)$

[Out]  $-3/(2*b*(a + b*x**2)**(1/3))$

**Mathematica [A]** time = 0.00543971, size = 18, normalized size = 1.

$$-\frac{3}{2b\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x/(a + b*x^2)^(4/3), x]$

[Out]  $-3/(2*b*(a + b*x^2)^(1/3))$

**Maple [A]** time = 0.004, size = 15, normalized size = 0.8

$$-\frac{3}{2b} \frac{1}{\sqrt[3]{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x/(b*x^2+a)^(4/3), x)$

[Out]  $-3/2/b/(b*x^2+a)^(1/3)$

---

**Maxima [A]** time = 1.35662, size = 19, normalized size = 1.06

$$-\frac{3}{2(bx^2 + a)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^(4/3),x, algorithm="maxima")`

[Out] `-3/2/((b*x^2 + a)^(1/3)*b)`

---

**Fricas [A]** time = 0.212868, size = 19, normalized size = 1.06

$$-\frac{3}{2(bx^2 + a)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^(4/3),x, algorithm="fricas")`

[Out] `-3/2/((b*x^2 + a)^(1/3)*b)`

---

**Sympy [A]** time = 2.33188, size = 26, normalized size = 1.44

$$\begin{cases} -\frac{3}{2b\sqrt[3]{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x**2+a)**(4/3),x)`

[Out] `Piecewise((-3/(2*b*(a + b*x**2)**(1/3)), Ne(b, 0)), (x**2/(2*a**(4/3)), True))`

---

**GIAC/XCAS [A]** time = 0.212069, size = 19, normalized size = 1.06

$$-\frac{3}{2(bx^2 + a)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(b*x^2 + a)^(4/3),x, algorithm="giac")`

[Out] `-3/2/((b*x^2 + a)^(1/3)*b)`

$$3.730 \quad \int \frac{1}{x(a+bx^2)^{4/3}} dx$$

**Optimal.** Leaf size=104

$$\frac{3 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{4a^{4/3}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{4/3}} - \frac{\log(x)}{2a^{4/3}} + \frac{3}{2a\sqrt[3]{a+bx^2}}$$

[Out]  $3/(2*a*(a + b*x^2)^{(1/3)}) + (\text{Sqrt}[3]*\text{ArcTan}[(a^{(1/3)} + 2*(a + b*x^2)^{(1/3)})/(\text{Sqrt}[3]*a^{(1/3)})])/(2*a^{(4/3)}) - \text{Log}[x]/(2*a^{(4/3)}) + (3*\text{Log}[a^{(1/3)} - (a + b*x^2)^{(1/3)}])/(4*a^{(4/3)})$

**Rubi [A]** time = 0.190817, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{3 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{4a^{4/3}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{4/3}} - \frac{\log(x)}{2a^{4/3}} + \frac{3}{2a\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(a + b\*x^2)^(4/3)), x]

[Out]  $3/(2*a*(a + b*x^2)^{(1/3)}) + (\text{Sqrt}[3]*\text{ArcTan}[(a^{(1/3)} + 2*(a + b*x^2)^{(1/3)})/(\text{Sqrt}[3]*a^{(1/3)})])/(2*a^{(4/3)}) - \text{Log}[x]/(2*a^{(4/3)}) + (3*\text{Log}[a^{(1/3)} - (a + b*x^2)^{(1/3)}])/(4*a^{(4/3)})$

**Rubi in Sympy [A]** time = 11.0278, size = 95, normalized size = 0.91

$$\frac{3}{2a\sqrt[3]{a+bx^2}} - \frac{\log(x^2)}{4a^{4/3}} + \frac{3 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{4a^{4/3}} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} + 2\frac{\sqrt[3]{a+bx^2}}{3}\right)}{\sqrt[3]{a}}\right)}{2a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(b\*x\*\*2+a)\*\*(4/3), x)

[Out]  $3/(2*a*(a + b*x^2)^{(1/3)}) - \log(x^2)/(4*a^{(4/3)}) + 3*\log(a^{(1/3)} - (a + b*x^2)^{(1/3)})/(4*a^{(4/3)}) + \text{sqrt}(3)*\text{atan}(\text{sqrt}(3)*(a^{(1/3)}/3 + 2*(a + b*x^2)^{(1/3)}/3)/a^{(1/3)})/(2*a^{(4/3)})$

**Mathematica [C]** time = 0.0482269, size = 55, normalized size = 0.53

$$\frac{3 - 3\sqrt[3]{\frac{a}{bx^2}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{bx^2}\right)}{2a\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(a + b\*x^2)^(4/3)), x]

[Out]  $(3 - 3*(1 + a/(b*x^2))^{(1/3)}*\text{Hypergeometric2F1}[1/3, 1/3, 4/3, -(a/(b*x^2))])/(2*a*(a + b*x^2)^{(1/3)})$

---

**Maple [F]** time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{x} (bx^2 + a)^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(b*x^2+a)^(4/3),x)`

[Out] `int(1/x/(b*x^2+a)^(4/3),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(4/3)*x),x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.22565, size = 173, normalized size = 1.66

$$\frac{2\sqrt{3}(bx^2 + a)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(2(bx^2+a)^{\frac{1}{3}}a^{\frac{2}{3}}+a\right)}{3a}\right) - (bx^2 + a)^{\frac{1}{3}} \log\left(\left(bx^2 + a\right)^{\frac{2}{3}}a^{\frac{1}{3}} + (bx^2 + a)^{\frac{1}{3}}a^{\frac{2}{3}} + a\right) + 2(bx^2 + a)^{\frac{1}{3}} \log\left(\left(bx^2 + a\right)^{\frac{1}{3}}a^{\frac{4}{3}}\right)}{4(bx^2 + a)^{\frac{1}{3}}a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(4/3)*x),x, algorithm="fricas")`

[Out] `1/4*(2*sqrt(3)*(b*x^2 + a)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3)*a^(2/3) + a)/a) - (b*x^2 + a)^(1/3)*log((b*x^2 + a)^(2/3)*a^(1/3) + (b*x^2 + a)^(1/3)*a^(2/3) + a) + 2*(b*x^2 + a)^(1/3)*log((b*x^2 + a)^(1/3)*a^(4/3))`

---

**Sympy [A]** time = 4.18553, size = 41, normalized size = 0.39

$$\frac{\left(\frac{4}{3}\right) {}_2F_1\left(\frac{4}{3}, \frac{4}{3} \mid \frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{4}{3}}x^{\frac{8}{3}}\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x**2+a)**(4/3),x)`

[Out] `-gamma(4/3)*hyper((4/3, 4/3), (7/3, ), a*exp_polar(I*pi)/(b*x**2))/(2*b**(4/3)*x**(8/3)*gamma(7/3))`

---



**GIAC/XCAS [A]** time = 0.575047, size = 136, normalized size = 1.31

$$\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{2a^{\frac{4}{3}}} - \frac{\ln\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4a^{\frac{4}{3}}} + \frac{\ln\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{2a^{\frac{4}{3}}} + \frac{3}{2\left(bx^2+a\right)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(4/3)\*x),x, algorithm="giac")

[Out] 1/2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(b\*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(4/3) - 1/4\*ln((b\*x^2 + a)^(2/3) + (b\*x^2 + a)^(1/3)\*a^(1/3) + a^(2/3))/a^(4/3) + 1/2\*ln(abs((b\*x^2 + a)^(1/3) - a^(1/3)))/a^(4/3) + 3/2/((b\*x^2 + a)^(1/3)\*a)

$$3.731 \quad \int \frac{1}{x^3(a+bx^2)^{4/3}} dx$$

**Optimal.** Leaf size=125

$$-\frac{b \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{a^{7/3}} - \frac{2b \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}} + \frac{2b \log(x)}{3a^{7/3}} - \frac{2(a+bx^2)^{2/3}}{a^2x^2} + \frac{3}{2ax^2\sqrt[3]{a+bx^2}}$$

[Out]  $3/(2*a*x^2*(a+b*x^2)^{(1/3)}) - (2*(a+b*x^2)^{(2/3)})/(a^2*x^2) - (2*b*ArcTan[(a^{(1/3)} + 2*(a+b*x^2)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(7/3)}) + (2*b*Log[x])/(3*a^{(7/3)}) - (b*Log[a^{(1/3)} - (a+b*x^2)^{(1/3)}])/a^{(7/3)}$

**Rubi [A]** time = 0.223348, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\frac{b \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{a^{7/3}} - \frac{2b \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}} + \frac{2b \log(x)}{3a^{7/3}} - \frac{2(a+bx^2)^{2/3}}{a^2x^2} + \frac{3}{2ax^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(a+b\*x^2)^(4/3)),x]

[Out]  $3/(2*a*x^2*(a+b*x^2)^{(1/3)}) - (2*(a+b*x^2)^{(2/3)})/(a^2*x^2) - (2*b*ArcTan[(a^{(1/3)} + 2*(a+b*x^2)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(7/3)}) + (2*b*Log[x])/(3*a^{(7/3)}) - (b*Log[a^{(1/3)} - (a+b*x^2)^{(1/3)}])/a^{(7/3)}$

**Rubi in Sympy [A]** time = 14.8722, size = 121, normalized size = 0.97

$$\frac{3}{2ax^2\sqrt[3]{a+bx^2}} - \frac{2(a+bx^2)^{2/3}}{a^2x^2} + \frac{b \log(x^2)}{3a^{7/3}} - \frac{b \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{a^{7/3}} - \frac{2\sqrt{3}b \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{a}}{3} + 2\frac{\sqrt[3]{a+bx^2}}{3}\right)}{\sqrt[3]{a}}\right)}{3a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(b\*x\*\*2+a)\*\*(4/3),x)

[Out]  $3/(2*a*x**2*(a+b*x**2)**(1/3)) - 2*(a+b*x**2)**(2/3)/(a**2*x**2) + b*\log(x**2)/(3*a**(7/3)) - b*\log(a**(1/3) - (a+b*x**2)**(1/3))/a**(7/3) - 2*\sqrt{3}*b*\operatorname{atan}(\sqrt{3}*(a**(1/3)/3 + 2*(a+b*x**2)**(1/3)/3))/a**(1/3)/(3*a**(7/3))$

**Mathematica [C]** time = 0.0517208, size = 70, normalized size = 0.56

$$\frac{4bx^2\sqrt[3]{\frac{a}{bx^2}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{a}{bx^2}\right) - a - 4bx^2}{2a^2x^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a+b\*x^2)^(4/3)),x]

[Out]  $(-a - 4bx^2 + 4b(1 + a/(bx^2))^{1/3})x^2 \text{Hypergeometric2F1}[1/3, 1/3, 4/3, -(a/(bx^2))]/(2a^2x^2(a + bx^2)^{1/3})$

**Maple [F]** time = 0.069, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} (bx^2 + a)^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(b*x^2+a)^(4/3), x)`

[Out] `int(1/x^3/(b*x^2+a)^(4/3), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(4/3)*x^3), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.230101, size = 246, normalized size = 1.97

$$\frac{\sqrt{3} \left( 2\sqrt{3}(bx^2 + a)^{\frac{1}{3}} bx^2 \log \left( (bx^2 + a)^{\frac{2}{3}} (-a)^{\frac{1}{3}} - (bx^2 + a)^{\frac{1}{3}} (-a)^{\frac{2}{3}} - a \right) - 4\sqrt{3}(bx^2 + a)^{\frac{1}{3}} bx^2 \log \left( (bx^2 + a)^{\frac{1}{3}} (-a)^{\frac{2}{3}} - a \right) \right)}{18(bx^2 + a)^{\frac{1}{3}} (-a)^{\frac{1}{3}} a^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(4/3)*x^3), x, algorithm="fricas")`

[Out]  $-1/18 \sqrt{3} (2 \sqrt{3} (bx^2 + a)^{1/3} bx^2 \log((bx^2 + a)^{2/3} (-a)^{1/3} - (bx^2 + a)^{1/3} (-a)^{2/3} - a) - 4 \sqrt{3} (bx^2 + a)^{1/3} bx^2 \log((bx^2 + a)^{1/3} (-a)^{2/3} - a) - 12 (bx^2 + a)^{1/3} bx^2 \arctan(1/3 (2 \sqrt{3} (bx^2 + a)^{1/3} (-a)^{2/3} + \sqrt{3} a)/a) + 3 \sqrt{3} (4 bx^2 + a) (-a)^{1/3}) / ((bx^2 + a)^{1/3} (-a)^{1/3} a^2 x^2)$

**Sympy [A]** time = 5.63797, size = 41, normalized size = 0.33

$$\frac{\left(\frac{7}{3}\right) {}_2F_1\left(\frac{4}{3}, \frac{7}{3} \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{4}{3}} x^{\frac{14}{3}} \left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(b*x**2+a)**(4/3), x)`

[Out]  $-\gamma(7/3) \text{hyper}((4/3, 7/3), (10/3, ), a \exp_{\text{polar}}(I \pi)/(b x^2)) / (2 b^{4/3} x^{14/3} \gamma(10/3))$

**GIAC/XCAS [A]** time = 0.575712, size = 171, normalized size = 1.37

$$-\frac{1}{6}b \left( \frac{4\sqrt{3} \arctan\left(\frac{\sqrt{3}(2(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}})}{3a^{\frac{1}{3}}}\right)}{a^{\frac{7}{3}}} - \frac{2\ln\left(\left(bx^2+a\right)^{\frac{2}{3}} + \left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{a^{\frac{7}{3}}} + \frac{4\ln\left(\left|\left(bx^2+a\right)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{a^{\frac{7}{3}}} + \frac{3(4(bx^2+a)^{\frac{4}{3}})}{(bx^2+a)^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(4/3)\*x^3),x, algorithm="giac")

[Out] -1/6\*b\*(4\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(b\*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(7/3) - 2\*ln((b\*x^2 + a)^(2/3) + (b\*x^2 + a)^(1/3)\*a^(1/3) + a^(2/3))/a^(7/3) + 4\*ln(abs((b\*x^2 + a)^(1/3) - a^(1/3)))/a^(7/3) + 3\*(4\*b\*x^2 + a)/((b\*x^2 + a)^(4/3) - (b\*x^2 + a)^(1/3)\*a)

$$3.732 \quad \int \frac{1}{x^5(a+bx^2)^{4/3}} dx$$

**Optimal.** Leaf size=159

$$\frac{7b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{6a^{10/3}} + \frac{7b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}} - \frac{7b^2 \log(x)}{9a^{10/3}} + \frac{7b(a+bx^2)^{2/3}}{3a^3x^2} - \frac{7(a+bx^2)^{2/3}}{4a^2x^4} + \frac{3}{2ax^4\sqrt[3]{a+bx^2}}$$

[Out]  $3/(2*a*x^4*(a+b*x^2)^{(1/3)}) - (7*(a+b*x^2)^{(2/3)})/(4*a^2*x^4) + (7*b*(a+b*x^2)^{(2/3)})/(3*a^3*x^2) + (7*b^2*ArcTan[(a^{(1/3)} + 2*(a+b*x^2)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(3*Sqrt[3]*a^{(10/3)}) - (7*b^2*Log[x])/(9*a^{(10/3)}) + (7*b^2*Log[a^{(1/3)} - (a+b*x^2)^{(1/3)}])/(6*a^{(10/3)})$

**Rubi [A]** time = 0.284372, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{7b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{6a^{10/3}} + \frac{7b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx^2} + \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}} - \frac{7b^2 \log(x)}{9a^{10/3}} + \frac{7b(a+bx^2)^{2/3}}{3a^3x^2} - \frac{7(a+bx^2)^{2/3}}{4a^2x^4} + \frac{3}{2ax^4\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5\*(a+b\*x^2)^(4/3)),x]

[Out]  $3/(2*a*x^4*(a+b*x^2)^{(1/3)}) - (7*(a+b*x^2)^{(2/3)})/(4*a^2*x^4) + (7*b*(a+b*x^2)^{(2/3)})/(3*a^3*x^2) + (7*b^2*ArcTan[(a^{(1/3)} + 2*(a+b*x^2)^{(1/3)})/(Sqrt[3]*a^{(1/3)})])/(3*Sqrt[3]*a^{(10/3)}) - (7*b^2*Log[x])/(9*a^{(10/3)}) + (7*b^2*Log[a^{(1/3)} - (a+b*x^2)^{(1/3)}])/(6*a^{(10/3)})$

**Rubi in Sympy [A]** time = 20.879, size = 155, normalized size = 0.97

$$\frac{3}{2ax^4\sqrt[3]{a+bx^2}} - \frac{7(a+bx^2)^{\frac{2}{3}}}{4a^2x^4} + \frac{7b(a+bx^2)^{\frac{2}{3}}}{3a^3x^2} - \frac{7b^2 \log(x^2)}{18a^{\frac{10}{3}}} + \frac{7b^2 \log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{6a^{\frac{10}{3}}} + \frac{7\sqrt{3}b^2 \operatorname{atan}\left(\frac{\sqrt{3}\left(\sqrt[3]{a} + 2\sqrt[3]{a+bx^2}\right)}{\sqrt[3]{a}}\right)}{9a^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*5/(b\*x\*\*2+a)\*\*(4/3),x)

[Out]  $3/(2*a*x**4*(a+b*x**2)**(1/3)) - 7*(a+b*x**2)**(2/3)/(4*a**2*x**4) + 7*b*(a+b*x**2)**(2/3)/(3*a**3*x**2) - 7*b**2*log(x**2)/(18*a**(10/3)) + 7*b**2*log(a**(1/3) - (a+b*x**2)**(1/3))/(6*a**(10/3)) + 7*sqrt(3)*b**2*atan(sqrt(3)*(a**(1/3)/3 + 2*(a+b*x**2)**(1/3)/3)/a**(1/3))/(9*a**(10/3))$

**Mathematica [C]** time = 0.0591492, size = 83, normalized size = 0.52

$$\frac{-3a^2 - 28b^2x^4 \sqrt[3]{\frac{a}{bx^2}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{a}{bx^2}\right) + 7abx^2 + 28b^2x^4}{12a^3x^4\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5\*(a + b\*x^2)^(4/3)), x]

[Out]  $(-3a^2 + 7abx^2 + 28b^2x^4 - 28b^2(1 + a/(bx^2))^{1/3}x^4 \text{Hypergeometric2F1}[1/3, 1/3, 4/3, -(a/(bx^2))]) / (12a^3x^4(a + bx^2)^{1/3})$

**Maple [F]** time = 0.077, size = 0, normalized size = 0.

$$\int \frac{1}{x^5} (bx^2 + a)^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(b\*x^2+a)^(4/3), x)

[Out] int(1/x^5/(b\*x^2+a)^(4/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(4/3)\*x^5), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.229265, size = 247, normalized size = 1.55

$$\frac{\sqrt{3}\left(14\sqrt{3}(bx^2+a)^{\frac{1}{3}}b^2x^4\log\left((bx^2+a)^{\frac{2}{3}}a^{\frac{1}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{2}{3}}+a\right)-28\sqrt{3}(bx^2+a)^{\frac{1}{3}}b^2x^4\log\left((bx^2+a)^{\frac{1}{3}}a^{\frac{2}{3}}-a\right)-84\left((bx^2+a)^{\frac{1}{3}}a^{\frac{2}{3}}+a\right)\right)}{108(bx^2+a)^{\frac{1}{3}}a^{\frac{10}{3}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(4/3)\*x^5), x, algorithm="fricas")

[Out]  $-1/108*\sqrt{3}*(14*\sqrt{3}*(b*x^2 + a)^{1/3}*b^2*x^4*\log((b*x^2 + a)^{2/3}*a^{1/3} + (b*x^2 + a)^{1/3}*a^{2/3} + a) - 28*\sqrt{3}*(b*x^2 + a)^{1/3}*b^2*x^4*\log((b*x^2 + a)^{1/3}*a^{2/3} - a) - 84*(b*x^2 + a)^{1/3}*b^2*x^4*\arctan(1/3*(2*\sqrt{3}*(b*x^2 + a)^{1/3}*a^{2/3} + \sqrt{3}*a)/a) - 3*\sqrt{3}*(28*b^2*x^4 + 7*a*b*x^2 - 3*a^2)*a^{1/3}) / ((b*x^2 + a)^{1/3}*a^{10/3}*x^4)$

**Sympy [A]** time = 7.34219, size = 41, normalized size = 0.26

$$\frac{\left(\frac{10}{3}\right) {}_2F_1\left(\frac{4}{3}, \frac{10}{3} \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{4}{3}}x^{\frac{20}{3}}\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*5/(b\*x\*\*2+a)\*\*(4/3),x)

[Out] -gamma(10/3)\*hyper((4/3, 10/3), (13/3, ), a\*exp\_polar(I\*pi)/(b\*x\*\*2))/(2\*b\*\*(4/3)\*x\*\*(20/3)\*gamma(13/3))

**GIAC/XCAS [A]** time = 0.652358, size = 190, normalized size = 1.19

$$\frac{1}{36}b^2 \left( \frac{28\sqrt{3} \arctan\left(\frac{\sqrt{3}(2(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}})}{3a^{\frac{1}{3}}}\right)}{a^{\frac{10}{3}}} - \frac{14 \ln\left((bx^2+a)^{\frac{2}{3}} + (bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{a^{\frac{10}{3}}} + \frac{28 \ln\left(\left|(bx^2+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{a^{\frac{10}{3}}} + \frac{54}{(bx^2+a)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(4/3)\*x^5),x, algorithm="giac")

[Out] 1/36\*b^2\*(28\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*(b\*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(10/3) - 14\*ln((b\*x^2 + a)^(2/3) + (b\*x^2 + a)^(1/3)\*a^(1/3) + a^(2/3))/a^(10/3) + 28\*ln(abs((b\*x^2 + a)^(1/3) - a^(1/3)))/a^(10/3) + 54/((b\*x^2 + a)^(1/3)\*a^3) + 3\*(10\*(b\*x^2 + a)^(5/3) - 13\*(b\*x^2 + a)^(2/3)\*a)/(a^3\*b^2\*x^4))

$$3.733 \quad \int \frac{x^4}{(a+bx^2)^{4/3}} dx$$

**Optimal.** Leaf size=577

$$\frac{27 \cdot 3^{3/4} a^{4/3} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{7\sqrt{2}b^3x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$\frac{81 \sqrt[4]{3} \sqrt{2 + \sqrt{3}} a^{4/3} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{28b^3x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$+ \frac{27x(a+bx^2)^{2/3}}{14b^2} + \frac{81ax}{14b^2 \left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)} - \frac{3x^3}{2b\sqrt[3]{a+bx^2}}$$

[Out]  $(-3*x^3)/(2*b*(a+b*x^2)^(1/3)) + (27*x*(a+b*x^2)^(2/3))/(14*b^2) + (81*a*x)/(14*b^2*((1-Sqrt[3])*a^(1/3)-(a+b*x^2)^(1/3))) - (81*3^(1/4)*Sqrt[2+Sqrt[3]]*a^(4/3)*(a^(1/3)-(a+b*x^2)^(1/3))*Sqrt[(a^(2/3)+a^(1/3)*(a+b*x^2)^(1/3)+(a+b*x^2)^(2/3)]/((1-Sqrt[3])*a^(1/3)-(a+b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1+Sqrt[3])*a^(1/3)-(a+b*x^2)^(1/3))/((1-Sqrt[3])*a^(1/3)-(a+b*x^2)^(1/3))], -7+4*Sqrt[3]])/(28*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3)-(a+b*x^2)^(1/3)))/((1-Sqrt[3])*a^(1/3)-(a+b*x^2)^(1/3))^2]) + (27*3^(3/4)*a^(4/3)*(a^(1/3)-(a+b*x^2)^(1/3))*Sqrt[(a^(2/3)+a^(1/3)*(a+b*x^2)^(1/3)+(a+b*x^2)^(2/3)]/((1-Sqrt[3])*a^(1/3)-(a+b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1+Sqrt[3])*a^(1/3)-(a+b*x^2)^(1/3))/((1-Sqrt[3])*a^(1/3)-(a+b*x^2)^(1/3))], -7+4*Sqrt[3]])/(7*Sqrt[2]*b^3*x*Sqrt[-((a^(1/3)*(a^(1/3)-(a+b*x^2)^(1/3)))/((1-Sqrt[3])*a^(1/3)-(a+b*x^2)^(1/3))^2])$

**Rubi [A]** time = 0.872946, antiderivative size = 577, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{27 \cdot 3^{3/4} a^{4/3} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{7\sqrt{2}b^3x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$\frac{81 \sqrt[4]{3} \sqrt{2 + \sqrt{3}} a^{4/3} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{28b^3x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$+ \frac{27x(a+bx^2)^{2/3}}{14b^2} + \frac{81ax}{14b^2 \left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)} - \frac{3x^3}{2b\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a+b\*x^2)^(4/3),x]

[Out]  $(-3*x^3)/(2*b*(a+b*x^2)^(1/3)) + (27*x*(a+b*x^2)^(2/3))/(14*b^2) + (81*a*x)/(14*b^2*((1-Sqrt[3])*a^(1/3)-(a+b*x^2)^(1/3)))$



) - (81\*3^(1/4)\*Sqrt[2 + Sqrt[3]]\*a^(4/3)\*(a^(1/3) - (a + b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a + b\*x^2)^(1/3) + (a + b\*x^2)^(2/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]\*EllipticE[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))], -7 + 4\*Sqrt[3]])/(28\*b^3\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a + b\*x^2)^(1/3)))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]) + (27\*3^(3/4)\*a^(4/3)\*(a^(1/3) - (a + b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a + b\*x^2)^(1/3) + (a + b\*x^2)^(2/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]\*EllipticF[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))], -7 + 4\*Sqrt[3]])/(7\*Sqrt[2]\*b^3\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a + b\*x^2)^(1/3)))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2])])

**Rubi in Sympy [A]** time = 38.3236, size = 478, normalized size = 0.83

$$\frac{81\sqrt[3]{3}a^{\frac{4}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt{a+bx^2}+(a+bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}(1+\sqrt{3})-\sqrt[3]{a+bx^2}}{-\sqrt[3]{a}(-1+\sqrt{3})-\sqrt[3]{a+bx^2}}\right)\right)\Big|_{-7+4\sqrt{3}}}{28b^3x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)^2}}}$$

$$+\frac{27\sqrt{2}\cdot 3^{\frac{3}{4}}a^{\frac{4}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt{a+bx^2}+(a+bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}(1+\sqrt{3})-\sqrt[3]{a+bx^2}}{-\sqrt[3]{a}(-1+\sqrt{3})-\sqrt[3]{a+bx^2}}\right)\right)\Big|_{-7+4\sqrt{3}}}{14b^3x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{81ax}{14b^2\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)}-\frac{3x^3}{2b\sqrt[3]{a+bx^2}}+\frac{27x(a+bx^2)^{\frac{2}{3}}}{14b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**4/(b*x**2+a)**(4/3),x)`

[Out] `-81*3**(1/4)*a**(4/3)*sqrt((a**(2/3) + a**(1/3)*(a + b*x**2)**(1/3) + (a + b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a + b*x**2)**(1/3))**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a + b*x**2)**(1/3))*elliptic_e(asin((a**(1/3)*(1 + sqrt(3)) - (a + b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)) - (a + b*x**2)**(1/3))), -7 + 4*sqrt(3))/(28*b**3*x*sqrt(-a**(1/3)*(a**(1/3) - (a + b*x**2)**(1/3)))/(a**(1/3)*(-1 + sqrt(3)) + (a + b*x**2)**(1/3))**2) + 27*sqrt(2)*3**(3/4)*a**(4/3)*sqrt((a**(2/3) + a**(1/3)*(a + b*x**2)**(1/3) + (a + b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a + b*x**2)**(1/3))**2)*(a**(1/3) - (a + b*x**2)**(1/3))*elliptic_f(asin((a**(1/3)*(1 + sqrt(3)) - (a + b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)) - (a + b*x**2)**(1/3))), -7 + 4*sqrt(3))/(14*b**3*x*sqrt(-a**(1/3)*(a**(1/3) - (a + b*x**2)**(1/3)))/(a**(1/3)*(-1 + sqrt(3)) + (a + b*x**2)**(1/3))**2) - 81*a*x/(14*b**2*(a**(1/3)*(-1 + sqrt(3)) + (a + b*x**2)**(1/3))) - 3*x**3/(2*b*(a + b*x**2)**(1/3)) + 27*x*(a + b*x**2)**(2/3)/(14*b**2)`

**Mathematica [C]** time = 0.0617138, size = 65, normalized size = 0.11

$$\frac{3x\left(-9a\sqrt[3]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{2}, \frac{3}{2}; -\frac{bx^2}{a}\right) + 9a + 2bx^2\right)}{14b^2\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a + b\*x^2)^(4/3), x]

[Out] (3\*x\*(9\*a + 2\*b\*x^2 - 9\*a\*(1 + (b\*x^2)/a)^(1/3)\*Hypergeometric2F1[1/3, 1/2, 3/2, -((b\*x^2)/a)]))/(14\*b^2\*(a + b\*x^2)^(1/3))

**Maple [F]** time = 0.064, size = 0, normalized size = 0.

$$\int x^4 (bx^2 + a)^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b\*x^2+a)^(4/3), x)

[Out] int(x^4/(b\*x^2+a)^(4/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2 + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b\*x^2 + a)^(4/3), x, algorithm="maxima")

[Out] integrate(x^4/(b\*x^2 + a)^(4/3), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{(bx^2 + a)^{\frac{4}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b\*x^2 + a)^(4/3), x, algorithm="fricas")

[Out] integral(x^4/(b\*x^2 + a)^(4/3), x)

**Sympy [A]** time = 2.58212, size = 27, normalized size = 0.05

$$\frac{x^5 {}_2F_1\left(\frac{4}{3}, \frac{5}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4/(b\*x\*\*2+a)\*\*(4/3), x)

[Out] x\*\*5\*hyper((4/3, 5/2), (7/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(5\*a\*\*(4/3))

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2 + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2 + a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(4/3), x)
```

$$3.734 \quad \int \frac{x^2}{(a+bx^2)^{4/3}} dx$$

**Optimal.** Leaf size=553

$$\frac{3 \cdot 3^{3/4} \sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{\sqrt{2} b^2 x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$+ \frac{9 \sqrt[3]{3} \sqrt{2 + \sqrt{3}} \sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{4 b^2 x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$- \frac{3x}{2b \sqrt[3]{a+bx^2}} - \frac{9x}{2b \left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}$$

[Out]  $(-3*x)/(2*b*(a + b*x^2)^{(1/3)}) - (9*x)/(2*b*((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})) + (9*3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticE}[\text{ArcSin}(((1 + \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})), -7 + 4*\text{Sqrt}[3]])/(4*b^2*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]) - (3*3^{(3/4)}*a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}(((1 + \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})), -7 + 4*\text{Sqrt}[3]])/(\text{Sqrt}[2]*b^2*x*\text{Sqrt}[-((a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)}))/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2])$

**Rubi [A]** time = 0.748408, antiderivative size = 553, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{3 \cdot 3^{3/4} \sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{\sqrt{2} b^2 x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$+ \frac{9 \sqrt[3]{3} \sqrt{2 + \sqrt{3}} \sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{4 b^2 x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$- \frac{3x}{2b \sqrt[3]{a+bx^2}} - \frac{9x}{2b \left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2/(a + b*x^2)^{(4/3)}, x]$

[Out]  $(-3*x)/(2*b*(a + b*x^2)^{(1/3)}) - (9*x)/(2*b*((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})) + (9*3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^{(1/3)}*(a^{(1/3)} - (a + b*x^2)^{(1/3)})*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}*(a + b*x^2)^{(1/3)} + (a + b*x^2)^{(2/3)})/((1 - \text{Sqrt}[3])*a^{(1/3)} - (a + b*x^2)^{(1/3)})^2]$

$$\frac{1/3 + (a + b^2 x^2)^{2/3}}{((1 - \sqrt{3}) a^{1/3} - (a + b^2 x^2)^{1/3})^2} \text{EllipticE}\left[\text{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} - (a + b^2 x^2)^{1/3}}{(1 - \sqrt{3}) a^{1/3} - (a + b^2 x^2)^{1/3}}\right], -7 + 4\sqrt{3}\right] \\ \frac{1}{(4 b^2 x \sqrt{-((a^{1/3}) (a^{1/3}) - (a + b^2 x^2)^{1/3})) / ((1 - \sqrt{3}) a^{1/3} - (a + b^2 x^2)^{1/3})^2}} - (3^{3/4} a^{1/3} (a^{1/3} - (a + b^2 x^2)^{1/3}) \sqrt{(a^{2/3} + a^{1/3} (a + b^2 x^2)^{1/3} + (a + b^2 x^2)^{2/3})} / ((1 - \sqrt{3}) a^{1/3} - (a + b^2 x^2)^{1/3})^2) \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} - (a + b^2 x^2)^{1/3}}{(1 - \sqrt{3}) a^{1/3} - (a + b^2 x^2)^{1/3}}\right], -7 + 4\sqrt{3}\right] \\ \frac{1}{(\sqrt{2} b^2 x \sqrt{-((a^{1/3}) (a^{1/3}) - (a + b^2 x^2)^{1/3})) / ((1 - \sqrt{3}) a^{1/3} - (a + b^2 x^2)^{1/3})^2}}$$

**Rubi in Sympy [A]** time = 29.8383, size = 454, normalized size = 0.82

$$\frac{9\sqrt{3}\sqrt[3]{a} \sqrt{\frac{a^{\frac{2}{3}} + \sqrt[3]{a}\sqrt{a + bx^2} + (a + bx^2)^{\frac{2}{3}}}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}} \sqrt{\sqrt{3} + 2} (\sqrt[3]{a} - \sqrt[3]{a + bx^2}) E\left(\text{asin}\left(\frac{\sqrt[3]{a}(1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}{-\sqrt[3]{a}(-1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}\right)\right) \Big|_{-7 + 4\sqrt{3}}}{4b^2x \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{a + bx^2})}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}}} \\ \frac{3\sqrt{2} \cdot 3^{\frac{3}{4}} \sqrt[3]{a} \sqrt{\frac{a^{\frac{2}{3}} + \sqrt[3]{a}\sqrt{a + bx^2} + (a + bx^2)^{\frac{2}{3}}}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}} (\sqrt[3]{a} - \sqrt[3]{a + bx^2}) F\left(\text{asin}\left(\frac{\sqrt[3]{a}(1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}{-\sqrt[3]{a}(-1 + \sqrt{3}) - \sqrt[3]{a + bx^2}}\right)\right) \Big|_{-7 + 4\sqrt{3}}}{2b^2x \sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a} - \sqrt[3]{a + bx^2})}{(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})^2}}} \\ + \frac{9x}{2b(\sqrt[3]{a}(-1 + \sqrt{3}) + \sqrt[3]{a + bx^2})} - \frac{3x}{2b\sqrt[3]{a + bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(b\*x\*\*2+a)\*\*(4/3),x)

[Out]  $9 \cdot 3^{1/4} \cdot a^{1/3} \cdot \sqrt{(a^{2/3} + a^{1/3} (a + b^2 x^2)^{1/3}) / (a^{1/3} (-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3})^2} \cdot \sqrt{\sqrt{3} + 2} \cdot (a^{1/3} - (a + b^2 x^2)^{1/3}) \cdot \text{elliptic\_e}(\text{asin}((a^{1/3} (1 + \sqrt{3}) - (a + b^2 x^2)^{1/3}) / (-a^{1/3} (-1 + \sqrt{3}) - (a + b^2 x^2)^{1/3})), -7 + 4\sqrt{3}) / (4 b^2 x \sqrt{-a^{1/3} (a^{1/3} - (a + b^2 x^2)^{1/3}) / (a^{1/3} (-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3})^2}) - 3 \sqrt{2} \cdot 3^{3/4} \cdot a^{1/3} \sqrt{(a^{2/3} + a^{1/3} (a + b^2 x^2)^{1/3} + (a + b^2 x^2)^{2/3}) / (a^{1/3} (-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3})^2} \cdot (a^{1/3} - (a + b^2 x^2)^{1/3}) \cdot \text{elliptic\_f}(\text{asin}((a^{1/3} (1 + \sqrt{3}) - (a + b^2 x^2)^{1/3}) / (-a^{1/3} (-1 + \sqrt{3}) - (a + b^2 x^2)^{1/3})), -7 + 4\sqrt{3}) / (2 b^2 x \sqrt{-a^{1/3} (a^{1/3} - (a + b^2 x^2)^{1/3}) / (a^{1/3} (-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3})^2}) + 9 x / (2 b (a^{1/3} (-1 + \sqrt{3}) + (a + b^2 x^2)^{1/3})) - 3 x / (2 b (a + b^2 x^2)^{1/3})$

**Mathematica [C]** time = 0.0488451, size = 55, normalized size = 0.1

$$\frac{3x \left( \sqrt[3]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) - 1 \right)}{2b\sqrt[3]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*x^2)^(4/3),x]

[Out]  $(3*x*(-1 + (1 + (b*x^2)/a)^{1/3}) * \text{Hypergeometric2F1}[1/3, 1/2, 3/2, -((b*x^2)/a)]) / (2*b*(a + b*x^2)^{1/3})$

**Maple [F]** time = 0.034, size = 0, normalized size = 0.

$$\int x^2 (bx^2 + a)^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*x^2+a)^(4/3), x)`

[Out] `int(x^2/(b*x^2+a)^(4/3), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2 + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(4/3), x, algorithm="maxima")`

[Out] `integrate(x^2/(b*x^2 + a)^(4/3), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{(bx^2 + a)^{\frac{4}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(4/3), x, algorithm="fricas")`

[Out] `integral(x^2/(b*x^2 + a)^(4/3), x)`

**Sympy [A]** time = 2.49445, size = 27, normalized size = 0.05

$$\frac{x^3 {}_2F_1\left(\frac{4}{3}, \frac{3}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**2+a)**(4/3), x)`

[Out] `x**3*hyper((4/3, 3/2), (5/2, ), b*x**2*exp_polar(I*pi)/a)/(3*a**(4/3))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2 + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2 + a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(4/3), x)
```

$$3.735 \quad \int \frac{1}{(a+bx^2)^{4/3}} dx$$

**Optimal.** Leaf size=552

$$\frac{3^{3/4} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{\sqrt{2} a^{2/3} b x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$\frac{3^4 \sqrt{3} \sqrt{2 + \sqrt{3}} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{4 a^{2/3} b x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$+ \frac{3x}{2a \sqrt[3]{a+bx^2}} + \frac{3x}{2a \left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}$$

[Out] (3\*x)/(2\*a\*(a + b\*x^2)^(1/3)) + (3\*x)/(2\*a\*((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))) - (3^3^(1/4)\*Sqrt[2 + Sqrt[3]]\*(a^(1/3) - (a + b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a + b\*x^2)^(1/3) + (a + b\*x^2)^(2/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]\*EllipticE[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))], -7 + 4\*Sqrt[3]])/(4\*a^(2/3)\*b\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a + b\*x^2)^(1/3)))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2)]) + (3^(3/4)\*(a^(1/3) - (a + b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a + b\*x^2)^(1/3) + (a + b\*x^2)^(2/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]\*EllipticF[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))], -7 + 4\*Sqrt[3]])/(Sqrt[2]\*a^(2/3)\*b\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a + b\*x^2)^(1/3)))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2)])

**Rubi [A]** time = 0.757106, antiderivative size = 552, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$

$$\frac{3^{3/4} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{\sqrt{2} a^{2/3} b x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$\frac{3^4 \sqrt{3} \sqrt{2 + \sqrt{3}} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{4 a^{2/3} b x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$+ \frac{3x}{2a \sqrt[3]{a+bx^2}} + \frac{3x}{2a \left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(-4/3), x]

[Out] (3\*x)/(2\*a\*(a + b\*x^2)^(1/3)) + (3\*x)/(2\*a\*((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))) - (3^3^(1/4)\*Sqrt[2 + Sqrt[3]]\*(a^(1/3) - (a + b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a + b\*x^2)^(1/3) + (a + b\*x^2)^(2/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]\*EllipticE[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))], -7 + 4\*Sqrt[3]])/(4\*a^(2/3)\*b\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a + b\*x^2)^(1/3)))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2)]) + (3^(3/4)\*(a^(1/3) - (a + b\*x^2)^(1/3))\*Sqrt[(a^(2/3) + a^(1/3)\*(a + b\*x^2)^(1/3) + (a + b\*x^2)^(2/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2]\*EllipticF[ArcSin[((1 + Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))], -7 + 4\*Sqrt[3]])/(Sqrt[2]\*a^(2/3)\*b\*x\*Sqrt[-((a^(1/3)\*(a^(1/3) - (a + b\*x^2)^(1/3)))/((1 - Sqrt[3])\*a^(1/3) - (a + b\*x^2)^(1/3))^2)])



$$\begin{aligned} & + b^*x^2)^{(2/3)})/((1 - \text{Sqrt}[3])^*a^{(1/3)} - (a + b^*x^2)^{(1/3)})^2]^*E \\ & \text{lIpticE}[\text{ArcSin}[\frac{(1 + \text{Sqrt}[3])^*a^{(1/3)} - (a + b^*x^2)^{(1/3)}}{(1 - \text{Sqrt}[3])^*a^{(1/3)} - (a + b^*x^2)^{(1/3)}}], -7 + 4^*\text{Sqrt}[3]]]/(4^*a^{(2/3)} \\ & *b^*x^*\text{Sqrt}[-((a^{(1/3)}^*(a^{(1/3)} - (a + b^*x^2)^{(1/3)})))/(1 - \text{Sqrt}[3])^*a^{(1/3)} - (a + b^*x^2)^{(1/3)})^2]) + (3^{(3/4)}^*(a^{(1/3)} - (a + \\ & b^*x^2)^{(1/3)})^*\text{Sqrt}[(a^{(2/3)} + a^{(1/3)}^*(a + b^*x^2)^{(1/3)} + (a + b^*x^2)^{(2/3)})]/((1 - \text{Sqrt}[3])^*a^{(1/3)} - (a + b^*x^2)^{(1/3)})^2]^*Ellip \\ & \text{ticF}[\text{ArcSin}[\frac{(1 + \text{Sqrt}[3])^*a^{(1/3)} - (a + b^*x^2)^{(1/3)}}{(1 - \text{Sqrt}[3])^*a^{(1/3)} - (a + b^*x^2)^{(1/3)}}], -7 + 4^*\text{Sqrt}[3]]]/(\text{Sqrt}[2]^*a^{(2/3)} \\ & *b^*x^*\text{Sqrt}[-((a^{(1/3)}^*(a^{(1/3)} - (a + b^*x^2)^{(1/3)})))/(1 - \text{Sqrt}[3])^*a^{(1/3)} - (a + b^*x^2)^{(1/3)})^2]) \end{aligned}$$

**Rubi in Sympy [A]** time = 27.7195, size = 449, normalized size = 0.81

$$\begin{aligned} & -\frac{3x}{2a\left(\sqrt[3]{a(-1+\sqrt{3})} + \sqrt[3]{a+bx^2}\right)} + \frac{3x}{2a\sqrt[3]{a+bx^2}} \\ & \frac{3\sqrt[3]{3}\sqrt{\frac{a^{\frac{2}{3}} + \sqrt[3]{a}\sqrt[3]{a+bx^2} + (a+bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a(-1+\sqrt{3})} + \sqrt[3]{a+bx^2}\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)E\left(\text{asin}\left(\frac{\sqrt[3]{a(1+\sqrt{3})} - \sqrt[3]{a+bx^2}}{-\sqrt[3]{a(-1+\sqrt{3})} - \sqrt[3]{a+bx^2}}\right)\right)\Big|_{-7+4\sqrt{3}}}{\dots} \\ & \frac{4a^{\frac{2}{3}}bx\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a(-1+\sqrt{3})} + \sqrt[3]{a+bx^2}\right)^2}}}{\dots} \\ & + \frac{\sqrt{2} \cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}} + \sqrt[3]{a}\sqrt[3]{a+bx^2} + (a+bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a(-1+\sqrt{3})} + \sqrt[3]{a+bx^2}\right)^2}}\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)F\left(\text{asin}\left(\frac{\sqrt[3]{a(1+\sqrt{3})} - \sqrt[3]{a+bx^2}}{-\sqrt[3]{a(-1+\sqrt{3})} - \sqrt[3]{a+bx^2}}\right)\right)\Big|_{-7+4\sqrt{3}}}{\dots} \\ & + \frac{2a^{\frac{2}{3}}bx\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a(-1+\sqrt{3})} + \sqrt[3]{a+bx^2}\right)^2}}}{\dots} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(b*x**2+a)**(4/3),x)`

[Out] 
$$\begin{aligned} & -3^*x/(2^*a^*(a^{*(1/3)}*(-1 + \text{sqrt}(3)) + (a + b^*x^{**2})^{*(1/3)})) + 3^*x/ \\ & (2^*a^*(a + b^*x^{**2})^{*(1/3)} - 3^*3^{*(1/4)}*\text{sqrt}((a^{*(2/3)} + a^{*(1/3)}^* \\ & (a + b^*x^{**2})^{*(1/3)} + (a + b^*x^{**2})^{*(2/3)})/(a^{*(1/3)}*(-1 + \text{sqrt}(3) \\ & )) + (a + b^*x^{**2})^{*(1/3)})^{**2})^*\text{sqrt}(\text{sqrt}(3) + 2)^*(a^{*(1/3)} - (a + \\ & b^*x^{**2})^{*(1/3)})^*\text{elliptic}_e(\text{asin}((a^{*(1/3)}*(1 + \text{sqrt}(3)) - (a + b^* \\ & x^{**2})^{*(1/3)})/(-a^{*(1/3)}*(-1 + \text{sqrt}(3)) - (a + b^*x^{**2})^{*(1/3)})), \\ & -7 + 4^*\text{sqrt}(3))/ (4^*a^{*(2/3)}*b^*x^*\text{sqrt}(-a^{*(1/3)}*(a^{*(1/3)} - (a + b^* \\ & x^{**2})^{*(1/3)})/(a^{*(1/3)}*(-1 + \text{sqrt}(3)) + (a + b^*x^{**2})^{*(1/3)})^{**2} \\ & )) + \text{sqrt}(2)^*3^{*(3/4)}*\text{sqrt}((a^{*(2/3)} + a^{*(1/3)}^*(a + b^*x^{**2})^{*(1/3)} \\ & + (a + b^*x^{**2})^{*(2/3)})/(a^{*(1/3)}*(-1 + \text{sqrt}(3)) + (a + b^*x^{**2}) \\ & ^{*(1/3)})^{**2})^*(a^{*(1/3)} - (a + b^*x^{**2})^{*(1/3)})^*\text{elliptic}_f(\text{asin}((a^* \\ & (1/3)^*(1 + \text{sqrt}(3)) - (a + b^*x^{**2})^{*(1/3)})/(-a^{*(1/3)}*(-1 + \text{sqrt} \\ & (3)) - (a + b^*x^{**2})^{*(1/3)})), -7 + 4^*\text{sqrt}(3))/ (2^*a^{*(2/3)}*b^*x^*\text{sr} \\ & \text{t}(-a^{*(1/3)}*(a^{*(1/3)} - (a + b^*x^{**2})^{*(1/3)})/(a^{*(1/3)}*(-1 + \text{sqrt} \\ & (3)) + (a + b^*x^{**2})^{*(1/3)})^{**2})) \end{aligned}$$

**Mathematica [C]** time = 0.0375097, size = 58, normalized size = 0.11

$$\frac{3x - x\sqrt[3]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{2}, \frac{3}{2}, -\frac{bx^2}{a}\right)}{2a\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(-4/3),x]`

[Out] 
$$\frac{(3^*x - x^*(1 + (b^*x^2)/a)^{(1/3)}*\text{Hypergeometric2F1}[1/3, 1/2, 3/2, - \\ ((b^*x^2)/a)])/(2^*a^*(a + b^*x^2)^{(1/3)})$$

---

**Maple [F]** time = 0.042, size = 0, normalized size = 0.

$$\int (bx^2 + a)^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+a)^(4/3), x)

[Out] int(1/(b\*x^2+a)^(4/3), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-4/3), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(-4/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{4}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-4/3), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(-4/3), x)

---

**Sympy [A]** time = 2.43965, size = 24, normalized size = 0.04

$$\frac{x {}_2F_1\left(\frac{1}{2}, \frac{4}{3} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*2+a)\*\*(4/3), x)

[Out] x\*hyper((1/2, 4/3), (3/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/a\*\*(4/3)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(-4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-4/3), x)
```

$$3.736 \quad \int \frac{1}{x^2(a+bx^2)^{4/3}} dx$$

**Optimal.** Leaf size=571

$$\frac{5 \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{\sqrt{2} \sqrt[3]{3} a^{5/3} x \sqrt{-\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$+ \frac{5 \sqrt[3]{3} \sqrt{2 + \sqrt{3}} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{4 a^{5/3} x \sqrt{-\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$- \frac{5bx}{2a^2 \left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)} - \frac{5(a+bx^2)^{2/3}}{2a^2 x} + \frac{3}{2ax \sqrt[3]{a+bx^2}}$$

[Out]  $3/(2*a*x*(a+b*x^2)^(1/3)) - (5*(a+b*x^2)^(2/3))/(2*a^2*x) - (5*b*x)/(2*a^2*((1-Sqrt[3])*a^(1/3)-(a+b*x^2)^(1/3))) + (5^3)^(1/4)*Sqrt[2+Sqrt[3]]*(a^(1/3)-(a+b*x^2)^(1/3))*Sqrt[(a^(2/3)+a^(1/3)*(a+b*x^2)^(1/3)+(a+b*x^2)^(2/3))/((1-Sqrt[3])*a^(1/3)-(a+b*x^2)^(1/3))]^2*EllipticE[ArcSin[((1+Sqrt[3])*a^(1/3)-(a+b*x^2)^(1/3))/((1-Sqrt[3])*a^(1/3)-(a+b*x^2)^(1/3))], -7+4*Sqrt[3]]/(4*a^(5/3)*x*Sqrt[-((a^(1/3)*(a^(1/3)-(a+b*x^2)^(1/3)))/((1-Sqrt[3])*a^(1/3)-(a+b*x^2)^(1/3)))^2]) - (5*(a^(1/3)-(a+b*x^2)^(1/3))*Sqrt[(a^(2/3)+a^(1/3)*(a+b*x^2)^(1/3)+(a+b*x^2)^(2/3))/((1-Sqrt[3])*a^(1/3)-(a+b*x^2)^(1/3))]^2*EllipticF[ArcSin[((1+Sqrt[3])*a^(1/3)-(a+b*x^2)^(1/3))/((1-Sqrt[3])*a^(1/3)-(a+b*x^2)^(1/3))], -7+4*Sqrt[3]]/(Sqrt[2]*3^(1/4)*a^(5/3)*x*Sqrt[-((a^(1/3)*(a^(1/3)-(a+b*x^2)^(1/3)))/((1-Sqrt[3])*a^(1/3)-(a+b*x^2)^(1/3)))^2])]$

**Rubi [A]** time = 0.868662, antiderivative size = 571, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{5 \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{\sqrt{2} \sqrt[3]{3} a^{5/3} x \sqrt{-\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$+ \frac{5 \sqrt[3]{3} \sqrt{2 + \sqrt{3}} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{4 a^{5/3} x \sqrt{-\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$- \frac{5bx}{2a^2 \left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)} - \frac{5(a+bx^2)^{2/3}}{2a^2 x} + \frac{3}{2ax \sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a+b\*x^2)^(4/3)),x]

[Out]  $3/(2*a*x*(a+b*x^2)^(1/3)) - (5*(a+b*x^2)^(2/3))/(2*a^2*x) - (5*b*x)/(2*a^2*((1-Sqrt[3])*a^(1/3)-(a+b*x^2)^(1/3))) + (5^3)$

$$\begin{aligned} & \left( \frac{1}{4} \sqrt{2 + \sqrt{3}} \right) \left( a^{1/3} - (a + b x^2)^{1/3} \right) \sqrt{\left( a^{2/3} + a^{1/3} (a + b x^2)^{1/3} + (a + b x^2)^{2/3} \right) / \left( (1 - \sqrt{3}) a^{1/3} - (a + b x^2)^{1/3} \right)^2} \\ & \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{(1 + \sqrt{3}) a^{1/3} - (a + b x^2)^{1/3}}{(1 - \sqrt{3}) a^{1/3} - (a + b x^2)^{1/3}} \right], -7 + 4 \sqrt{3} \right] \\ & \left( \frac{1}{4} \sqrt{2 + \sqrt{3}} \right) \left( a^{5/3} x \sqrt{-\left( a^{1/3} (a + b x^2)^{1/3} - (a + b x^2)^{1/3} \right) / \left( (1 - \sqrt{3}) a^{1/3} - (a + b x^2)^{1/3} \right)^2} \right) \\ & - \left( 5 \left( a^{1/3} - (a + b x^2)^{1/3} \right) \sqrt{\left( a^{2/3} + a^{1/3} (a + b x^2)^{1/3} + (a + b x^2)^{2/3} \right) / \left( (1 - \sqrt{3}) a^{1/3} - (a + b x^2)^{1/3} \right)^2} \right) \\ & \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1 + \sqrt{3}) a^{1/3} - (a + b x^2)^{1/3}}{(1 - \sqrt{3}) a^{1/3} - (a + b x^2)^{1/3}} \right], -7 + 4 \sqrt{3} \right] \\ & \left( \frac{1}{4} \sqrt{2 + \sqrt{3}} \right) \left( \sqrt{2} \cdot 3^{1/4} a^{5/3} x \sqrt{-\left( a^{1/3} (a + b x^2)^{1/3} - (a + b x^2)^{1/3} \right) / \left( (1 - \sqrt{3}) a^{1/3} - (a + b x^2)^{1/3} \right)^2} \right) \end{aligned}$$

**Rubi in Sympy [A]** time = 36.9219, size = 469, normalized size = 0.82

$$\begin{aligned} & \frac{3}{2ax\sqrt[3]{a+bx^2}} + \frac{5bx}{2a^2\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)} - \frac{5(a+bx^2)^{\frac{2}{3}}}{2a^2x} \\ & + \frac{5\sqrt[3]{3}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a+bx^2+(a+bx^2)^{\frac{2}{3}}}}{\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)E\left(\text{asin}\left(\frac{\sqrt[3]{a}(1+\sqrt{3})-\sqrt[3]{a+bx^2}}{-\sqrt[3]{a}(-1+\sqrt{3})-\sqrt[3]{a+bx^2}}\right)\right)\Big|_{-7+4\sqrt{3}}}{4a^{\frac{5}{3}}x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)^2}}} \\ & + \frac{5\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a+bx^2+(a+bx^2)^{\frac{2}{3}}}}{\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)F\left(\text{asin}\left(\frac{\sqrt[3]{a}(1+\sqrt{3})-\sqrt[3]{a+bx^2}}{-\sqrt[3]{a}(-1+\sqrt{3})-\sqrt[3]{a+bx^2}}\right)\right)\Big|_{-7+4\sqrt{3}}}{6a^{\frac{5}{3}}x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)^2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**2/(b*x**2+a)**(4/3),x)`

[Out] 
$$\begin{aligned} & \frac{3}{2 a x \sqrt[3]{a+b x^2}} + \frac{5 b x}{2 a^2 \left( a^{1/3} (-1+\sqrt{3}) + \sqrt[3]{a+b x^2} \right)} - \frac{5 \left( a+b x^2 \right)^{2/3}}{2 a^2 x} \\ & + \frac{5 \cdot 3^{1/4} \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+b x^2 + (a+b x^2)^{2/3}}}{\left( \sqrt[3]{a} (-1+\sqrt{3}) + \sqrt[3]{a+b x^2} \right)^2}} \sqrt{\sqrt{3}+2} \left( \sqrt[3]{a} - \sqrt[3]{a+b x^2} \right) \text{elliptic}_e \left( \text{asin} \left( \frac{\sqrt[3]{a} (1+\sqrt{3}) - \sqrt[3]{a+b x^2}}{-\sqrt[3]{a} (-1+\sqrt{3}) - \sqrt[3]{a+b x^2}} \right), -7+4 \sqrt{3} \right)}{4 a^{5/3} x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+b x^2} \right)}{\left( \sqrt[3]{a} (-1+\sqrt{3}) + \sqrt[3]{a+b x^2} \right)^2}}} \\ & - \frac{5 \sqrt{2} \cdot 3^{3/4} \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+b x^2 + (a+b x^2)^{2/3}}}{\left( \sqrt[3]{a} (-1+\sqrt{3}) + \sqrt[3]{a+b x^2} \right)^2}} \left( \sqrt[3]{a} - \sqrt[3]{a+b x^2} \right) \text{elliptic}_f \left( \text{asin} \left( \frac{\sqrt[3]{a} (1+\sqrt{3}) - \sqrt[3]{a+b x^2}}{-\sqrt[3]{a} (-1+\sqrt{3}) - \sqrt[3]{a+b x^2}} \right), -7+4 \sqrt{3} \right)}{6 a^{5/3} x \sqrt{\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+b x^2} \right)}{\left( \sqrt[3]{a} (-1+\sqrt{3}) + \sqrt[3]{a+b x^2} \right)^2}}} \end{aligned}$$

**Mathematica [C]** time = 0.0520552, size = 70, normalized size = 0.12

$$\frac{5 b x^2 \sqrt[3]{\frac{b x^2}{a}} + {}_2F_1 \left( \frac{1}{3}, \frac{1}{2}; \frac{3}{2}; -\frac{b x^2}{a} \right) - 6 a - 15 b x^2}{6 a^2 x \sqrt[3]{a+b x^2}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^2*(a+b*x^2)^(4/3)),x]`

[Out]  $(-6*a - 15*b*x^2 + 5*b*x^2*(1 + (b*x^2)/a))^{1/3} \text{Hypergeometric2F1}[1/3, 1/2, 3/2, -(b*x^2)/a] / (6*a^2*x*(a + b*x^2)^{1/3})$

**Maple [F]** time = 0.062, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (bx^2 + a)^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(b*x^2+a)^(4/3), x)`

[Out] `int(1/x^2/(b*x^2+a)^(4/3), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{4}{3}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(4/3)*x^2), x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(4/3)*x^2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^4 + ax^2)(bx^2 + a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(4/3)*x^2), x, algorithm="fricas")`

[Out] `integral(1/((b*x^4 + a*x^2)*(b*x^2 + a)^(1/3)), x)`

**Sympy [A]** time = 3.34856, size = 27, normalized size = 0.05

$$\frac{{}_2F_1\left(-\frac{1}{2}, \frac{4}{3} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{4}{3}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(b*x**2+a)**(4/3), x)`

[Out] `-hyper((-1/2, 4/3), (1/2, ), b*x**2*exp_polar(I*pi)/a)/(a**(4/3)*x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{4}{3}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(4/3)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(4/3)*x^2), x)
```

$$3.737 \quad \int \frac{1}{x^4(a+bx^2)^{4/3}} dx$$

**Optimal.** Leaf size=599

$$\frac{55b \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{9\sqrt{2} \sqrt[3]{3} a^{8/3} x \sqrt{-\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$+ \frac{55\sqrt{2+\sqrt{3}}b \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{12 \cdot 3^{3/4} a^{8/3} x \sqrt{-\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$+ \frac{55b^2 x}{18a^3 \left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)} + \frac{55b(a+bx^2)^{2/3}}{18a^3 x} - \frac{11(a+bx^2)^{2/3}}{6a^2 x^3} + \frac{3}{2ax^3 \sqrt[3]{a+bx^2}}$$

[Out]  $3/(2*a*x^3*(a+b*x^2)^(1/3)) - (11*(a+b*x^2)^(2/3))/(6*a^2*x^3) + (55*b*(a+b*x^2)^(2/3))/(18*a^3*x) + (55*b^2*x)/(18*a^3*((1-\sqrt{3})*a^(1/3) - (a+b*x^2)^(1/3))) - (55*\sqrt{2+\sqrt{3}})*b*(a^(1/3) - (a+b*x^2)^(1/3))*\sqrt{(a^(2/3) + a^(1/3)*(a+b*x^2)^(1/3) + (a+b*x^2)^(2/3))/((1-\sqrt{3})*a^(1/3) - (a+b*x^2)^(1/3))^2}*\text{EllipticE}[\text{ArcSin}(((1+\sqrt{3})*a^(1/3) - (a+b*x^2)^(1/3))/((1-\sqrt{3})*a^(1/3) - (a+b*x^2)^(1/3))), -7 + 4*\sqrt{3}]/(12*3^(3/4)*a^(8/3)*x*\sqrt{-((a^(1/3)*(a^(1/3) - (a+b*x^2)^(1/3)))/((1-\sqrt{3})*a^(1/3) - (a+b*x^2)^(1/3))^2)}) + (55*b*(a^(1/3) - (a+b*x^2)^(1/3))*\sqrt{(a^(2/3) + a^(1/3)*(a+b*x^2)^(1/3) + (a+b*x^2)^(2/3))/((1-\sqrt{3})*a^(1/3) - (a+b*x^2)^(1/3))^2}*\text{EllipticF}[\text{ArcSin}(((1+\sqrt{3})*a^(1/3) - (a+b*x^2)^(1/3))/((1-\sqrt{3})*a^(1/3) - (a+b*x^2)^(1/3))), -7 + 4*\sqrt{3}]/(9*\sqrt{2+\sqrt{3}}*3^(1/4)*a^(8/3)*x*\sqrt{-((a^(1/3)*(a^(1/3) - (a+b*x^2)^(1/3)))/((1-\sqrt{3})*a^(1/3) - (a+b*x^2)^(1/3))^2)})$

**Rubi [A]** time = 1.0218, antiderivative size = 599, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{55b \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} F \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{9\sqrt{2} \sqrt[3]{3} a^{8/3} x \sqrt{-\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$+ \frac{55\sqrt{2+\sqrt{3}}b \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right) \sqrt{\frac{a^{2/3} + \sqrt[3]{a} \sqrt[3]{a+bx^2} + (a+bx^2)^{2/3}}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}} E \left( \sin^{-1} \left( \frac{(1+\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}}{(1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{bx^2+a}} \right) \mid -7 + 4\sqrt{3} \right)}{12 \cdot 3^{3/4} a^{8/3} x \sqrt{-\frac{\sqrt[3]{a} \left( \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)}{\left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)^2}}}$$

$$+ \frac{55b^2 x}{18a^3 \left( (1-\sqrt{3}) \sqrt[3]{a} - \sqrt[3]{a+bx^2} \right)} + \frac{55b(a+bx^2)^{2/3}}{18a^3 x} - \frac{11(a+bx^2)^{2/3}}{6a^2 x^3} + \frac{3}{2ax^3 \sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a+b\*x^2)^(4/3)),x]

[Out]  $3/(2*a*x^3*(a+b*x^2)^(1/3)) - (11*(a+b*x^2)^(2/3))/(6*a^2*x^3) + (55*b*(a+b*x^2)^(2/3))/(18*a^3*x) + (55*b^2*x)/(18*a^3*((1-\sqrt{3})*a^(1/3) - (a+b*x^2)^(1/3))) - (55*\sqrt{2+\sqrt{3}})*b*(a^(1/3) - (a+b*x^2)^(1/3))*\sqrt{(a^(2/3) + a^(1/3)*(a+b*x^2)^(1/3) + (a+b*x^2)^(2/3))/((1-\sqrt{3})*a^(1/3) - (a+b*x^2)^(1/3))^2}*\text{EllipticE}[\text{ArcSin}(((1+\sqrt{3})*a^(1/3) - (a+b*x^2)^(1/3))/((1-\sqrt{3})*a^(1/3) - (a+b*x^2)^(1/3))), -7 + 4*\sqrt{3}]/(12*3^(3/4)*a^(8/3)*x*\sqrt{-((a^(1/3)*(a^(1/3) - (a+b*x^2)^(1/3)))/((1-\sqrt{3})*a^(1/3) - (a+b*x^2)^(1/3))^2)}) + (55*b*(a^(1/3) - (a+b*x^2)^(1/3))*\sqrt{(a^(2/3) + a^(1/3)*(a+b*x^2)^(1/3) + (a+b*x^2)^(2/3))/((1-\sqrt{3})*a^(1/3) - (a+b*x^2)^(1/3))^2}*\text{EllipticF}[\text{ArcSin}(((1+\sqrt{3})*a^(1/3) - (a+b*x^2)^(1/3))/((1-\sqrt{3})*a^(1/3) - (a+b*x^2)^(1/3))), -7 + 4*\sqrt{3}]/(9*\sqrt{2+\sqrt{3}}*3^(1/4)*a^(8/3)*x*\sqrt{-((a^(1/3)*(a^(1/3) - (a+b*x^2)^(1/3)))/((1-\sqrt{3})*a^(1/3) - (a+b*x^2)^(1/3))^2)})$



$$\begin{aligned}
& - \sqrt[3]{3} \cdot a^{1/3} - (a + b^2 x^2)^{1/3} \Big) - (55 \sqrt[3]{2 + \sqrt[3]{3}}) \cdot \\
& b \cdot (a^{1/3} - (a + b^2 x^2)^{1/3}) \cdot \sqrt[3]{(a^{2/3} + a^{1/3} \cdot (a + b^2 x^2)^{1/3} + (a + b^2 x^2)^{2/3}) / ((1 - \sqrt[3]{3}) \cdot a^{1/3} - (a + b^2 x^2)^{1/3})^2} \\
& \cdot \text{EllipticE}\left[\text{ArcSin}\left[\frac{(1 + \sqrt[3]{3}) \cdot a^{1/3} - (a + b^2 x^2)^{1/3}}{(1 - \sqrt[3]{3}) \cdot a^{1/3} - (a + b^2 x^2)^{1/3}}\right], -7 + 4 \sqrt[3]{3}\right] \\
& \Big) / (12 \cdot 3^{3/4} \cdot a^{8/3} \cdot x \cdot \sqrt[3]{-(a^{1/3} \cdot (a^{1/3} - (a + b^2 x^2)^{1/3})) / ((1 - \sqrt[3]{3}) \cdot a^{1/3} - (a + b^2 x^2)^{1/3})^2}) + (55 \cdot \\
& b \cdot (a^{1/3} - (a + b^2 x^2)^{1/3}) \cdot \sqrt[3]{(a^{2/3} + a^{1/3} \cdot (a + b^2 x^2)^{1/3} + (a + b^2 x^2)^{2/3}) / ((1 - \sqrt[3]{3}) \cdot a^{1/3} - (a + b^2 x^2)^{1/3})^2} \\
& \cdot \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1 + \sqrt[3]{3}) \cdot a^{1/3} - (a + b^2 x^2)^{1/3}}{(1 - \sqrt[3]{3}) \cdot a^{1/3} - (a + b^2 x^2)^{1/3}}\right], -7 + 4 \sqrt[3]{3}\right] \\
& \Big) / (9 \cdot \sqrt[3]{2} \cdot 3^{1/4} \cdot a^{8/3} \cdot x \cdot \sqrt[3]{-(a^{1/3} \cdot (a^{1/3} - (a + b^2 x^2)^{1/3})) / ((1 - \sqrt[3]{3}) \cdot a^{1/3} - (a + b^2 x^2)^{1/3})^2})
\end{aligned}$$

**Rubi in Sympy [A]** time = 45.6966, size = 498, normalized size = 0.83

$$\begin{aligned}
& \frac{3}{2ax^3\sqrt[3]{a+bx^2}} - \frac{11(a+bx^2)^{\frac{2}{3}}}{6a^2x^3} - \frac{55b^2x}{18a^3\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)} + \frac{55b(a+bx^2)^{\frac{2}{3}}}{18a^3x} \\
& \frac{55\sqrt[3]{3}b\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a+bx^2+(a+bx^2)^{\frac{2}{3}}}}{\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)E\left(\text{asin}\left(\frac{\sqrt[3]{a}(1+\sqrt{3})-\sqrt[3]{a+bx^2}}{-\sqrt[3]{a}(-1+\sqrt{3})-\sqrt[3]{a+bx^2}}\right)\right)\Big|_{-7+4\sqrt{3}}}{\dots} \\
& \frac{36a^{\frac{8}{3}}x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)^2}}}{\dots} \\
& \frac{55\sqrt{2}\cdot 3^{\frac{3}{4}}b\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a+bx^2+(a+bx^2)^{\frac{2}{3}}}}{\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)F\left(\text{asin}\left(\frac{\sqrt[3]{a}(1+\sqrt{3})-\sqrt[3]{a+bx^2}}{-\sqrt[3]{a}(-1+\sqrt{3})-\sqrt[3]{a+bx^2}}\right)\right)\Big|_{-7+4\sqrt{3}}}{\dots} \\
& + \frac{54a^{\frac{8}{3}}x\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}(-1+\sqrt{3})+\sqrt[3]{a+bx^2}\right)^2}}}{\dots}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**4/(b*x**2+a)**(4/3),x)`

[Out]  $3/(2 \cdot a \cdot x^{3 \cdot (a + b^2 x^2)^{1/3}}) - 11 \cdot (a + b^2 x^2)^{2/3} / (6 \cdot a^{2 \cdot x^{3 \cdot (a + b^2 x^2)^{1/3}}}) - 55 \cdot b^2 \cdot x / (18 \cdot a^{3 \cdot (a^{1/3} \cdot (-1 + \sqrt[3]{3}) + (a + b^2 x^2)^{1/3})) + 55 \cdot b \cdot (a + b^2 x^2)^{2/3} / (18 \cdot a^{3 \cdot x}) - 55 \cdot 3^{1/4} \cdot b \cdot \sqrt[3]{(a^{2/3} + a^{1/3} \cdot (a + b^2 x^2)^{1/3} + (a + b^2 x^2)^{2/3}) / (a^{1/3} \cdot (-1 + \sqrt[3]{3}) + (a + b^2 x^2)^{1/3})^2} \cdot \sqrt[3]{(2 + \sqrt[3]{3})} \cdot (a^{1/3} - (a + b^2 x^2)^{1/3}) \cdot \text{elliptic\_e}(\text{asin}((a^{1/3} \cdot (1 + \sqrt[3]{3}) - (a + b^2 x^2)^{1/3}) / (-a^{1/3} \cdot (-1 + \sqrt[3]{3}) - (a + b^2 x^2)^{1/3})), -7 + 4 \sqrt[3]{3}) / (36 \cdot a^{8/3} \cdot x \cdot \sqrt[3]{-(a^{1/3} \cdot (a^{1/3} - (a + b^2 x^2)^{1/3})) / (a^{1/3} \cdot (-1 + \sqrt[3]{3}) + (a + b^2 x^2)^{1/3})^2}) + (55 \cdot \sqrt[3]{2} \cdot 3^{3/4} \cdot b \cdot \sqrt[3]{(a^{2/3} + a^{1/3} \cdot (a + b^2 x^2)^{1/3} + (a + b^2 x^2)^{2/3}) / (a^{1/3} \cdot (-1 + \sqrt[3]{3}) + (a + b^2 x^2)^{1/3})^2} \cdot (a^{1/3} - (a + b^2 x^2)^{1/3}) \cdot \text{elliptic\_f}(\text{asin}((a^{1/3} \cdot (1 + \sqrt[3]{3}) - (a + b^2 x^2)^{1/3}) / (-a^{1/3} \cdot (-1 + \sqrt[3]{3}) - (a + b^2 x^2)^{1/3})), -7 + 4 \sqrt[3]{3}) / (54 \cdot a^{8/3} \cdot x \cdot \sqrt[3]{-(a^{1/3} \cdot (a^{1/3} - (a + b^2 x^2)^{1/3})) / (a^{1/3} \cdot (-1 + \sqrt[3]{3}) + (a + b^2 x^2)^{1/3})^2})$

**Mathematica [C]** time = 0.0564722, size = 83, normalized size = 0.14

$$\frac{-18a^2 - 55b^2x^4\sqrt[3]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) + 66abx^2 + 165b^2x^4}{54a^3x^3\sqrt[3]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^4*(a + b*x^2)^(4/3)),x]`

[Out]  $(-18*a^2 + 66*a*b*x^2 + 165*b^2*x^4 - 55*b^2*x^4*(1 + (b*x^2)/a)^{(1/3)} * \text{Hypergeometric2F1}[1/3, 1/2, 3/2, -((b*x^2)/a)]) / (54*a^3*x^3 * (a + b*x^2)^{(1/3)})$

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} (bx^2 + a)^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(b*x^2+a)^(4/3),x)`

[Out] `int(1/x^4/(b*x^2+a)^(4/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{4}{3}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(4/3)*x^4),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(4/3)*x^4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{(bx^6 + ax^4)(bx^2 + a)^{\frac{1}{3}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(4/3)*x^4),x, algorithm="fricas")`

[Out] `integral(1/((b*x^6 + a*x^4)*(b*x^2 + a)^(1/3)), x)`

**Sympy [A]** time = 4.12367, size = 32, normalized size = 0.05

$$-\frac{{}_2F_1 \left( \begin{matrix} -\frac{3}{2}, \frac{4}{3} \\ -\frac{1}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a} \right)}{3a^{\frac{4}{3}} x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(b*x**2+a)**(4/3),x)`

[Out] `-hyper((-3/2, 4/3), (-1/2, ), b*x**2*exp_polar(I*pi)/a)/(3*a**(4/3)*x**3)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{4}{3}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(4/3)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(4/3)*x^4), x)
```

### 3.738 $\int (cx)^{13/3} \sqrt[3]{a+bx^2} dx$

**Optimal.** Leaf size=275

$$\begin{aligned} & -\frac{5a^3c^{13/3} \log\left(c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)}{162b^{8/3}} + \frac{5a^3c^{13/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{bc^{2/3}(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} + c^{4/3}\right)}{324b^{8/3}} \\ & -\frac{5a^3c^{13/3} \tan^{-1}\left(\frac{\frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} + c^{2/3}}{\sqrt[3]{3c^{2/3}}}\right)}{54\sqrt{3}b^{8/3}} - \frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} \end{aligned}$$

[Out]  $(-5*a^2*c^3*(c*x)^{(4/3)}*(a+b*x^2)^{(1/3)})/(108*b^2) + (a*c*(c*x)^{(10/3)}*(a+b*x^2)^{(1/3)})/(36*b) + ((c*x)^{(16/3)}*(a+b*x^2)^{(1/3)})/(6*c) - (5*a^3*c^{(13/3)}*ArcTan[(c^{(2/3)} + (2*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(Sqrt[3]*c^{(2/3)})])/(54*Sqrt[3]*b^{(8/3)}) - (5*a^3*c^{(13/3)}*Log[c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}])/(162*b^{(8/3)}) + (5*a^3*c^{(13/3)}*Log[c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a+b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}])/(324*b^{(8/3)})$

**Rubi [A]** time = 0.872078, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$

$$\begin{aligned} & -\frac{5a^3c^{13/3} \log\left(c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)}{162b^{8/3}} + \frac{5a^3c^{13/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{bc^{2/3}(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} + c^{4/3}\right)}{324b^{8/3}} \\ & -\frac{5a^3c^{13/3} \tan^{-1}\left(\frac{\frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} + c^{2/3}}{\sqrt[3]{3c^{2/3}}}\right)}{54\sqrt{3}b^{8/3}} - \frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*x)^{(13/3)}*(a+b*x^2)^{(1/3)}, x]$

[Out]  $(-5*a^2*c^3*(c*x)^{(4/3)}*(a+b*x^2)^{(1/3)})/(108*b^2) + (a*c*(c*x)^{(10/3)}*(a+b*x^2)^{(1/3)})/(36*b) + ((c*x)^{(16/3)}*(a+b*x^2)^{(1/3)})/(6*c) - (5*a^3*c^{(13/3)}*ArcTan[(c^{(2/3)} + (2*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(Sqrt[3]*c^{(2/3)})])/(54*Sqrt[3]*b^{(8/3)}) - (5*a^3*c^{(13/3)}*Log[c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}])/(162*b^{(8/3)}) + (5*a^3*c^{(13/3)}*Log[c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a+b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}])/(324*b^{(8/3)})$

**Rubi in Sympy [A]** time = 75.24, size = 260, normalized size = 0.95

$$\begin{aligned} & -\frac{5a^3c^{13/3} \log\left(-\frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} + c^{2/3}\right)}{162b^{8/3}} + \frac{5a^3c^{13/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{c^{4/3}(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b(cx)^{2/3}}}{c^{2/3}\sqrt[3]{a+bx^2}} + 1\right)}{324b^{8/3}} \\ & -\frac{5\sqrt{3}a^3c^{13/3} \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} + \frac{c^{2/3}}{3}\right)}{c^{2/3}}\right)}{162b^{8/3}} - \frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2} + \frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b} + \frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x)**(13/3)*(b*x**2+a)**(1/3),x)`

[Out] 
$$-5*a**3*c**(13/3)*\log(-b**(1/3)*(c*x)**(2/3)/(a+b*x**2)**(1/3)+c**(2/3))/(162*b**(8/3))+5*a**3*c**(13/3)*\log(b**(2/3)*(c*x)**(4/3)/(c**(4/3)*(a+b*x**2)**(2/3))+b**(1/3)*(c*x)**(2/3)/(c**(2/3)*(a+b*x**2)**(1/3))+1)/(324*b**(8/3))-5*\sqrt{3}*a**3*c**(13/3)*\operatorname{atan}(\sqrt{3}*(2*b**(1/3)*(c*x)**(2/3)/(3*(a+b*x**2)**(1/3))+c**(2/3)/3)/c**(2/3))/(162*b**(8/3))-5*a**2*c**3*(c*x)**(4/3)*(a+b*x**2)**(1/3)/(108*b**2)+a*c*(c*x)**(10/3)*(a+b*x**2)**(1/3)/(36*b)+(c*x)**(16/3)*(a+b*x**2)**(1/3)/(6*c)$$

**Mathematica [C]** time = 0.0690735, size = 98, normalized size = 0.36

$$\frac{c^3(cx)^{4/3} \left( 5a^3 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^2}{a} \right) - 5a^3 - 2a^2bx^2 + 21ab^2x^4 + 18b^3x^6 \right)}{108b^2(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c*x)^(13/3)*(a+b*x^2)^(1/3),x]`

[Out] 
$$(c^3*(c*x)^{4/3}*(-5*a^3-2*a^2*b*x^2+21*a*b^2*x^4+18*b^3*x^6+5*a^3*(1+(b*x^2)/a)^{2/3}*\operatorname{Hypergeometric2F1}[2/3, 2/3, 5/3, -(b*x^2)/a]))/(108*b^2*(a+b*x^2)^{2/3})$$

**Maple [F]** time = 0.044, size = 0, normalized size = 0.

$$\int (cx)^{\frac{13}{3}} \sqrt[3]{bx^2+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(13/3)*(b*x^2+a)^(1/3),x)`

[Out] `int((c*x)^(13/3)*(b*x^2+a)^(1/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^(1/3)*(c*x)^(13/3),x,algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^(1/3)*(c*x)^(13/3),x,algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(13/3)*(b*x**2+a)**(1/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{3}} (cx)^{\frac{13}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*(c*x)^(13/3),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(1/3)*(c*x)^(13/3), x)`

### 3.739 $\int (cx)^{7/3} \sqrt[3]{a + bx^2} dx$

**Optimal.** Leaf size=244

$$\frac{a^2 c^{7/3} \log\left(c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a + bx^2}}\right)}{18b^{5/3}} - \frac{a^2 c^{7/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a + bx^2}} + c^{4/3}\right)}{36b^{5/3}}$$

$$+ \frac{a^2 c^{7/3} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3} + c^{2/3}}{\sqrt[3]{a + bx^2}}}{\sqrt[3]{3}c^{2/3}}\right)}{6\sqrt[3]{3}b^{5/3}} + \frac{(cx)^{10/3}\sqrt[3]{a + bx^2}}{4c} + \frac{ac(cx)^{4/3}\sqrt[3]{a + bx^2}}{12b}$$

[Out]  $(a * c * (c * x)^{(4/3)} * (a + b * x^2)^{(1/3)}) / (12 * b) + ((c * x)^{(10/3)} * (a + b * x^2)^{(1/3)}) / (4 * c) + (a^2 * c^{(7/3)} * \text{ArcTan}[(c^{(2/3)} + (2 * b^{(1/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)}) / (\text{Sqrt}[3] * c^{(2/3)})]) / (6 * \text{Sqrt}[3] * b^{(5/3)}) + (a^2 * c^{(7/3)} * \text{Log}[c^{(2/3)} - (b^{(1/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)})] / (18 * b^{(5/3)}) - (a^2 * c^{(7/3)} * \text{Log}[c^{(4/3)} + (b^{(2/3)} * (c * x)^{(4/3)}) / (a + b * x^2)^{(2/3)} + (b^{(1/3)} * c^{(2/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)})] / (36 * b^{(5/3)})$

**Rubi [A]** time = 0.678661, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$

$$\frac{a^2 c^{7/3} \log\left(c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a + bx^2}}\right)}{18b^{5/3}} - \frac{a^2 c^{7/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a + bx^2}} + c^{4/3}\right)}{36b^{5/3}}$$

$$+ \frac{a^2 c^{7/3} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3} + c^{2/3}}{\sqrt[3]{a + bx^2}}}{\sqrt[3]{3}c^{2/3}}\right)}{6\sqrt[3]{3}b^{5/3}} + \frac{(cx)^{10/3}\sqrt[3]{a + bx^2}}{4c} + \frac{ac(cx)^{4/3}\sqrt[3]{a + bx^2}}{12b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c * x)^{(7/3)} * (a + b * x^2)^{(1/3)}, x]$

[Out]  $(a * c * (c * x)^{(4/3)} * (a + b * x^2)^{(1/3)}) / (12 * b) + ((c * x)^{(10/3)} * (a + b * x^2)^{(1/3)}) / (4 * c) + (a^2 * c^{(7/3)} * \text{ArcTan}[(c^{(2/3)} + (2 * b^{(1/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)}) / (\text{Sqrt}[3] * c^{(2/3)})]) / (6 * \text{Sqrt}[3] * b^{(5/3)}) + (a^2 * c^{(7/3)} * \text{Log}[c^{(2/3)} - (b^{(1/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)})] / (18 * b^{(5/3)}) - (a^2 * c^{(7/3)} * \text{Log}[c^{(4/3)} + (b^{(2/3)} * (c * x)^{(4/3)}) / (a + b * x^2)^{(2/3)} + (b^{(1/3)} * c^{(2/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)})] / (36 * b^{(5/3)})$

**Rubi in Sympy [A]** time = 65.5726, size = 224, normalized size = 0.92

$$\frac{a^2 c^{7/3} \log\left(-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a + bx^2}} + c^{2/3}\right)}{18b^{5/3}} - \frac{a^2 c^{7/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{c^{4/3}(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}}{c^{2/3}\sqrt[3]{a + bx^2}} + 1\right)}{36b^{5/3}}$$

$$+ \frac{\sqrt{3}a^2 c^{7/3} \text{atan}\left(\frac{\sqrt{3}\left(\frac{2\sqrt[3]{b}(cx)^{2/3} + c^{2/3}}{\sqrt[3]{a + bx^2}} + \frac{c^{2/3}}{3}\right)}{c^{2/3}}\right)}{18b^{5/3}} + \frac{ac(cx)^{4/3}\sqrt[3]{a + bx^2}}{12b} + \frac{(cx)^{10/3}\sqrt[3]{a + bx^2}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c * x)^{(7/3)} * (b * x^2 + a)^{(1/3)}, x)$

[Out]  $a^{2/3} c^{7/3} \log(-b^{1/3} (c x)^{2/3} / (a + b x^2)^{1/3} + c^{2/3}) / (18 b^{5/3}) - a^{2/3} c^{7/3} \log(b^{2/3} (c x)^{4/3} / (c^{4/3} (a + b x^2)^{2/3})) + b^{1/3} (c x)^{2/3} / (c^{2/3} (a + b x^2)^{1/3}) + 1 / (36 b^{5/3}) + \sqrt{3} a^{2/3} c^{7/3} \operatorname{atan}(\sqrt{3} (2 b^{1/3} (c x)^{2/3} / (3 (a + b x^2)^{1/3})) + c^{2/3} (2/3) / c^{2/3}) / (18 b^{5/3}) + a c (c x)^{4/3} (a + b x^2)^{1/3} / (12 b) + (c x)^{10/3} (a + b x^2)^{1/3} / (4 c)$

**Mathematica [C]** time = 0.0579528, size = 83, normalized size = 0.34

$$\frac{c(c x)^{4/3} \left( -a^2 \left( \frac{b x^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{b x^2}{a} \right) + a^2 + 4 a b x^2 + 3 b^2 x^4 \right)}{12 b (a + b x^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(7/3)\*(a + b\*x^2)^(1/3),x]

[Out]  $(c (c x)^{4/3} (a^2 + 4 a b x^2 + 3 b^2 x^4 - a^2 (1 + (b x^2)/a)^{2/3} \operatorname{Hypergeometric2F1}[2/3, 2/3, 5/3, -(b x^2)/a])) / (12 b (a + b x^2)^{2/3})$

**Maple [F]** time = 0.035, size = 0, normalized size = 0.

$$\int (c x)^{7/3} \sqrt[3]{b x^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(7/3)\*(b\*x^2+a)^(1/3),x)

[Out] int((c\*x)^(7/3)\*(b\*x^2+a)^(1/3),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)\*(c\*x)^(7/3),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)\*(c\*x)^(7/3),x, algorithm="fricas")

[Out] Timed out



**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(7/3)*(b*x**2+a)**(1/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{3}} (cx)^{\frac{7}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*(c*x)^(7/3),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(1/3)*(c*x)^(7/3), x)`

### 3.740 $\int \sqrt[3]{cx} \sqrt[3]{a + bx^2} dx$

**Optimal.** Leaf size=211

$$-\frac{a\sqrt[3]{c} \log\left(c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^{2/3}} + \frac{a\sqrt[3]{c} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}\right)}{12b^{2/3}}$$

$$-\frac{a\sqrt[3]{c} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3} + c^{2/3}}{\sqrt[3]{a+bx^2}}}{\sqrt[3]{3c^{2/3}}}\right)}{2\sqrt[3]{3}b^{2/3}} + \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}$$

[Out]  $((c*x)^{(4/3)}*(a + b*x^2)^{(1/3)})/(2*c) - (a*c^{(1/3)}*ArcTan[(c^{(2/3)} + (2*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(Sqrt[3]*c^{(2/3)})])/(2*Sqrt[3]*b^{(2/3)}) - (a*c^{(1/3)}*Log[c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})]/(6*b^{(2/3)}) + (a*c^{(1/3)}*Log[c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a + b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})]/(12*b^{(2/3)})$

**Rubi [A]** time = 0.590294, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$-\frac{a\sqrt[3]{c} \log\left(c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{6b^{2/3}} + \frac{a\sqrt[3]{c} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}\right)}{12b^{2/3}}$$

$$-\frac{a\sqrt[3]{c} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3} + c^{2/3}}{\sqrt[3]{a+bx^2}}}{\sqrt[3]{3c^{2/3}}}\right)}{2\sqrt[3]{3}b^{2/3}} + \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(1/3)\*(a + b\*x^2)^(1/3), x]

[Out]  $((c*x)^{(4/3)}*(a + b*x^2)^{(1/3)})/(2*c) - (a*c^{(1/3)}*ArcTan[(c^{(2/3)} + (2*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(Sqrt[3]*c^{(2/3)})])/(2*Sqrt[3]*b^{(2/3)}) - (a*c^{(1/3)}*Log[c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})]/(6*b^{(2/3)}) + (a*c^{(1/3)}*Log[c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a + b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})]/(12*b^{(2/3)})$

**Rubi in Sympy [A]** time = 56.9619, size = 196, normalized size = 0.93

$$-\frac{a\sqrt[3]{c} \log\left(-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{2/3}\right)}{6b^{2/3}} + \frac{a\sqrt[3]{c} \log\left(\frac{b^{2/3}(cx)^{4/3}}{c^{4/3}(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}} + 1\right)}{12b^{2/3}}$$

$$-\frac{\sqrt[3]{3}a\sqrt[3]{c} \operatorname{atan}\left(\frac{\sqrt[3]{3}\left(\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + \frac{c^{2/3}}{\sqrt[3]{3}}\right)}{c^{2/3}}\right)}{6b^{2/3}} + \frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(1/3)\*(b\*x\*\*2+a)\*\*(1/3), x)

[Out]  $-a*c^{(1/3)}*\log(-b^{(1/3)}*(c*x)^{(2/3)/(a + b*x^2)^{(1/3)} + c^{(2/3)})/(6*b^{(2/3)}) + a*c^{(1/3)}*\log(b^{(2/3)}*(c*x)^{(4/3)/(c^{(4/3)}(a+bx^2)^{(2/3)} + \frac{\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}} + 1)})/12*b^{(2/3)}$

$$3) * (a + b * x^{**2})^{** (2/3)} + b^{** (1/3)} * (c * x)^{** (2/3)} / (c^{** (2/3)} * (a + b * x^{**2})^{** (1/3)}) + 1) / (12 * b^{** (2/3)}) - \text{sqrt}(3) * a * c^{** (1/3)} * \text{atan}(\text{sqrt}(3) * (2 * b^{** (1/3)} * (c * x)^{** (2/3)} / (3 * (a + b * x^{**2})^{** (1/3)}) + c^{** (2/3)} / 3) / c^{** (2/3)}) / (6 * b^{** (2/3)}) + (c * x)^{** (4/3)} * (a + b * x^{**2})^{** (1/3)} / (2 * c)$$

**Mathematica [C]** time = 0.049813, size = 68, normalized size = 0.32

$$\frac{x \sqrt[3]{cx} \left( a \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^2}{a} \right) + 2(a + bx^2) \right)}{4(a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(1/3)\*(a + b\*x^2)^(1/3), x]

[Out] (x\*(c\*x)^(1/3)\*(2\*(a + b\*x^2) + a\*(1 + (b\*x^2)/a)^(2/3)\*Hypergeometric2F1[2/3, 2/3, 5/3, -((b\*x^2)/a)])/(4\*(a + b\*x^2)^(2/3))

**Maple [F]** time = 0.033, size = 0, normalized size = 0.

$$\int \sqrt[3]{cx} \sqrt[3]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(1/3)\*(b\*x^2+a)^(1/3), x)

[Out] int((c\*x)^(1/3)\*(b\*x^2+a)^(1/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)\*(c\*x)^(1/3), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)\*(c\*x)^(1/3), x, algorithm="fricas")

[Out] Timed out

**Sympy [A]** time = 6.71784, size = 46, normalized size = 0.22

$$\frac{\sqrt[3]{a} \sqrt[3]{cx}^{\frac{4}{3}} \left( \frac{2}{3} \right) {}_2F_1 \left( -\frac{1}{3}, \frac{2}{3} \middle| \frac{bx^2 e^{i\pi}}{a} \right)}{2 \left( \frac{5}{3} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(1/3)*(b*x**2+a)**(1/3),x)`

[Out] `a**(1/3)*c**(1/3)*x**(4/3)*gamma(2/3)*hyper((-1/3, 2/3), (5/3, ), b*x**2*exp_polar(1*pi)/a)/(2*gamma(5/3))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{3}} (cx)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*(c*x)^(1/3),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(1/3)*(c*x)^(1/3), x)`

$$3.741 \quad \int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}} dx$$

**Optimal.** Leaf size=208

$$\frac{\sqrt[3]{b} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{bc^{2/3}(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} + c^{4/3}\right)}{4c^{5/3}} - \frac{\sqrt[3]{b} \log\left(c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{5/3}}$$

$$- \frac{\sqrt{3}\sqrt[3]{b} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{2/3}}{\sqrt{3c^{2/3}}}\right)}{2c^{5/3}} - \frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}}$$

[Out]  $(-3*(a + b*x^2)^{(1/3)})/(2*c*(c*x)^{(2/3)}) - (\text{Sqrt}[3]*b^{(1/3)}*\text{ArcTan}[(c^{(2/3)} + (2*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(\text{Sqrt}[3]*c^{(2/3)})]/(2*c^{(5/3)}) - (b^{(1/3)}*\text{Log}[c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})]/(2*c^{(5/3)}) + (b^{(1/3)}*\text{Log}[c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a + b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})]/(a + b*x^2)^{(1/3)})]/(4*c^{(5/3)})$

**Rubi [A]** time = 0.59588, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$\frac{\sqrt[3]{b} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{bc^{2/3}(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} + c^{4/3}\right)}{4c^{5/3}} - \frac{\sqrt[3]{b} \log\left(c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{5/3}}$$

$$- \frac{\sqrt{3}\sqrt[3]{b} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{2/3}}{\sqrt{3c^{2/3}}}\right)}{2c^{5/3}} - \frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/3)/(c\*x)^(5/3), x]

[Out]  $(-3*(a + b*x^2)^{(1/3)})/(2*c*(c*x)^{(2/3)}) - (\text{Sqrt}[3]*b^{(1/3)}*\text{ArcTan}[(c^{(2/3)} + (2*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(\text{Sqrt}[3]*c^{(2/3)})]/(2*c^{(5/3)}) - (b^{(1/3)}*\text{Log}[c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})]/(2*c^{(5/3)}) + (b^{(1/3)}*\text{Log}[c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a + b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})]/(a + b*x^2)^{(1/3)})]/(4*c^{(5/3)})$

**Rubi in Sympy [A]** time = 57.6591, size = 192, normalized size = 0.92

$$-\frac{\sqrt[3]{b} \log\left(-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{2/3}\right)}{2c^{5/3}} + \frac{\sqrt[3]{b} \log\left(\frac{b^{2/3}(cx)^{4/3}}{c^{4/3}(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}} + 1\right)}{4c^{5/3}}$$

$$- \frac{\sqrt{3}\sqrt[3]{b} \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{2/3}\right)}{c^{2/3}}\right)}{2c^{5/3}} - \frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/3)/(c\*x)\*\*(5/3), x)

[Out]  $-b^{1/3} \log(-b^{1/3} (cx)^{2/3} / (a + bx^2)^{1/3} + c^{2/3} / 3) / (2c^{5/3}) + b^{1/3} \log(b^{2/3} (cx)^{4/3} / (c^{4/3} (a + bx^2)^{2/3})) + b^{1/3} (cx)^{2/3} / (c^{2/3} (a + bx^2)^{1/3}) + 1 / (4c^{5/3}) - \sqrt{3} b^{1/3} \operatorname{atan}(\sqrt{3} (2b^{1/3} (cx)^{2/3} / (3(a + bx^2)^{1/3}) + c^{2/3} / 3) / c^{2/3}) / (2c^{5/3}) - 3(a + bx^2)^{1/3} / (2c (cx)^{2/3})$

**Mathematica [C]** time = 0.0510811, size = 72, normalized size = 0.35

$$\frac{x \left( 3bx^2 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{bx^2}{a} \right) - 6(a + bx^2) \right)}{4(cx)^{5/3} (a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/3)/(c\*x)^(5/3), x]

[Out]  $(x * (-6 * (a + b * x^2) + 3 * b * x^2 * (1 + (b * x^2) / a)^{2/3} * \operatorname{Hypergeometric2F1}[2/3, 2/3, 5/3, -((b * x^2) / a)])) / (4 * (c * x)^{5/3} * (a + b * x^2)^{2/3})$

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int \sqrt[3]{bx^2 + a} (cx)^{-5/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(1/3)/(c\*x)^(5/3), x)

[Out] int((b\*x^2+a)^(1/3)/(c\*x)^(5/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)/(c\*x)^(5/3), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)/(c\*x)^(5/3), x, algorithm="fricas")

[Out] Timed out

**Sympy [A]** time = 26.4133, size = 49, normalized size = 0.24

$$\frac{\sqrt[3]{a} \left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2c^{\frac{5}{3}} x^{\frac{2}{3}} \left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/3)/(c\*x)\*\*(5/3), x)

[Out] a\*\*(1/3)\*gamma(-1/3)\*hyper((-1/3, -1/3), (2/3, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*c\*\*(5/3)\*x\*\*(2/3)\*gamma(2/3))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{3}}}{(cx)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)/(c\*x)^(5/3), x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(1/3)/(c\*x)^(5/3), x)

$$3.742 \quad \int \frac{\sqrt[3]{a + bx^2}}{(cx)^{11/3}} dx$$

**Optimal.** Leaf size=28

$$-\frac{3(a + bx^2)^{4/3}}{8ac(cx)^{8/3}}$$

[Out]  $(-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(8/3))$

**Rubi [A]** time = 0.0277771, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$-\frac{3(a + bx^2)^{4/3}}{8ac(cx)^{8/3}}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*x^2)^(1/3)/(c*x)^(11/3), x]`

[Out]  $(-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(8/3))$

**Rubi in Sympy [A]** time = 3.55652, size = 24, normalized size = 0.86

$$-\frac{3(a + bx^2)^{\frac{4}{3}}}{8ac(cx)^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(1/3)/(c*x)**(11/3), x)`

[Out]  $-3*(a + b*x**2)**(4/3)/(8*a*c*(c*x)**(8/3))$

**Mathematica [A]** time = 0.0316578, size = 26, normalized size = 0.93

$$-\frac{3x(a + bx^2)^{4/3}}{8a(cx)^{11/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(1/3)/(c*x)^(11/3), x]`

[Out]  $(-3*x*(a + b*x^2)^(4/3))/(8*a*(c*x)^(11/3))$

**Maple [A]** time = 0.006, size = 21, normalized size = 0.8

$$-\frac{3x}{8a}(bx^2 + a)^{\frac{4}{3}}(cx)^{-\frac{11}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/3)/(c*x)^(11/3), x)`



[Out]  $-3/8 * x * (b * x^2 + a)^{4/3} / a / (c * x)^{11/3}$

**Maxima [A]** time = 1.36025, size = 47, normalized size = 1.68

$$\frac{3 \left( b c^{\frac{1}{3}} x^3 + a c^{\frac{1}{3}} x \right) (b x^2 + a)^{\frac{1}{3}}}{8 a c^4 x^{\frac{11}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/(c*x)^(11/3), x, algorithm="maxima")`

[Out]  $-3/8 * (b * c^{1/3} * x^3 + a * c^{1/3} * x) * (b * x^2 + a)^{1/3} / (a * c^4 * x^{11/3})$

**Fricas [A]** time = 0.234378, size = 34, normalized size = 1.21

$$\frac{3 (b x^2 + a)^{\frac{4}{3}} (c x)^{\frac{1}{3}}}{8 a c^4 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/(c*x)^(11/3), x, algorithm="fricas")`

[Out]  $-3/8 * (b * x^2 + a)^{4/3} * (c * x)^{1/3} / (a * c^4 * x^3)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/3)/(c*x)**(11/3), x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b x^2 + a)^{\frac{1}{3}}}{(c x)^{\frac{11}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/(c*x)^(11/3), x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(1/3)/(c*x)^(11/3), x)`

$$3.743 \quad \int \frac{\sqrt[3]{a + bx^2}}{(cx)^{17/3}} dx$$

**Optimal.** Leaf size=57

$$\frac{9(a + bx^2)^{7/3}}{56a^2c(cx)^{14/3}} - \frac{3(a + bx^2)^{4/3}}{8ac(cx)^{14/3}}$$

[Out]  $(-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(14/3)) + (9*(a + b*x^2)^(7/3))/(56*a^2*c*(c*x)^(14/3))$

**Rubi [A]** time = 0.0581911, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{9(a + bx^2)^{7/3}}{56a^2c(cx)^{14/3}} - \frac{3(a + bx^2)^{4/3}}{8ac(cx)^{14/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/3)/(c\*x)^(17/3), x]

[Out]  $(-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(14/3)) + (9*(a + b*x^2)^(7/3))/(56*a^2*c*(c*x)^(14/3))$

**Rubi in Sympy [A]** time = 6.72123, size = 48, normalized size = 0.84

$$-\frac{3(a + bx^2)^{4/3}}{8ac(cx)^{14/3}} + \frac{9(a + bx^2)^{7/3}}{56a^2c(cx)^{14/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/3)/(c\*x)\*\*(17/3), x)

[Out]  $-3*(a + b*x**2)**(4/3)/(8*a*c*(c*x)**(14/3)) + 9*(a + b*x**2)**(7/3)/(56*a**2*c*(c*x)**(14/3))$

**Mathematica [A]** time = 0.0422864, size = 51, normalized size = 0.89

$$-\frac{3\sqrt[3]{cx}\sqrt[3]{a + bx^2}(4a^2 + abx^2 - 3b^2x^4)}{56a^2c^6x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/3)/(c\*x)^(17/3), x]

[Out]  $(-3*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(4*a^2 + a*b*x^2 - 3*b^2*x^4))/(56*a^2*c^6*x^5)$

**Maple [A]** time = 0.006, size = 31, normalized size = 0.5

$$-\frac{3x(-3bx^2 + 4a)}{56a^2} (bx^2 + a)^{4/3} (cx)^{-17/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/3)/(c*x)^(17/3),x)`

[Out] `-3/56*x*(b*x^2+a)^(4/3)*(-3*b*x^2+4*a)/a^2/(c*x)^(17/3)`

**Maxima [A]** time = 1.39592, size = 51, normalized size = 0.89

$$\frac{3 \left( \frac{7(bx^2+a)^{\frac{4}{3}}b}{x^{\frac{8}{3}}} - \frac{4(bx^2+a)^{\frac{7}{3}}}{x^{\frac{14}{3}}} \right)}{56 a^2 c^{\frac{17}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/(c*x)^(17/3),x, algorithm="maxima")`

[Out] `3/56*(7*(b*x^2 + a)^(4/3)*b/x^(8/3) - 4*(b*x^2 + a)^(7/3)/x^(14/3))/ (a^2*c^(17/3))`

**Fricas [A]** time = 0.229433, size = 62, normalized size = 1.09

$$\frac{3(3b^2x^4 - abx^2 - 4a^2)(bx^2 + a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{56a^2c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/(c*x)^(17/3),x, algorithm="fricas")`

[Out] `3/56*(3*b^2*x^4 - a*b*x^2 - 4*a^2)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/ (a^2*c^6*x^5)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/3)/(c*x)**(17/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{3}}}{(cx)^{\frac{17}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/(c*x)^(17/3),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(1/3)/(c*x)^(17/3), x)`

$$3.744 \quad \int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}} dx$$

**Optimal.** Leaf size=85

$$-\frac{27(a+bx^2)^{10/3}}{280a^3c(cx)^{20/3}} + \frac{9(a+bx^2)^{7/3}}{28a^2c(cx)^{20/3}} - \frac{3(a+bx^2)^{4/3}}{8ac(cx)^{20/3}}$$

[Out]  $(-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(20/3)) + (9*(a + b*x^2)^(7/3))/(28*a^2*c*(c*x)^(20/3)) - (27*(a + b*x^2)^(10/3))/(280*a^3*c*(c*x)^(20/3))$

**Rubi [A]** time = 0.0897418, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$-\frac{27(a+bx^2)^{10/3}}{280a^3c(cx)^{20/3}} + \frac{9(a+bx^2)^{7/3}}{28a^2c(cx)^{20/3}} - \frac{3(a+bx^2)^{4/3}}{8ac(cx)^{20/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/3)/(c\*x)^(23/3), x]

[Out]  $(-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(20/3)) + (9*(a + b*x^2)^(7/3))/(28*a^2*c*(c*x)^(20/3)) - (27*(a + b*x^2)^(10/3))/(280*a^3*c*(c*x)^(20/3))$

**Rubi in Sympy [A]** time = 10.6772, size = 73, normalized size = 0.86

$$-\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{20/3}} + \frac{9(a+bx^2)^{7/3}}{28a^2c(cx)^{20/3}} - \frac{27(a+bx^2)^{10/3}}{280a^3c(cx)^{20/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/3)/(c\*x)\*\*(23/3), x)

[Out]  $-3*(a + b*x**2)**(4/3)/(8*a*c*(c*x)**(20/3)) + 9*(a + b*x**2)**(7/3)/(28*a**2*c*(c*x)**(20/3)) - 27*(a + b*x**2)**(10/3)/(280*a**3*c*(c*x)**(20/3))$

**Mathematica [A]** time = 0.0486502, size = 63, normalized size = 0.74

$$-\frac{3\sqrt[3]{cx}\sqrt[3]{a+bx^2}(14a^3+2a^2bx^2-3ab^2x^4+9b^3x^6)}{280a^3c^8x^7}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/3)/(c\*x)^(23/3), x]

[Out]  $(-3*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(14*a^3 + 2*a^2*b*x^2 - 3*a*b^2*x^4 + 9*b^3*x^6))/(280*a^3*c^8*x^7)$

**Maple [A]** time = 0.009, size = 42, normalized size = 0.5

$$-\frac{3x(9b^2x^4 - 12abx^2 + 14a^2)}{280a^3} (bx^2 + a)^{4/3} (cx)^{-23/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/3)/(c*x)^(23/3),x)`

[Out] 
$$-3/280*x*(b*x^2+a)^{(4/3)}*(9*b^2*x^4-12*a*b*x^2+14*a^2)/a^3/(c*x)^{(23/3)}$$

**Maxima [A]** time = 1.40159, size = 74, normalized size = 0.87

$$-\frac{3\left(\frac{35(bx^2+a)^{\frac{4}{3}}b^2}{x^{\frac{8}{3}}}-\frac{40(bx^2+a)^{\frac{7}{3}}b}{x^{\frac{14}{3}}}+\frac{14(bx^2+a)^{\frac{10}{3}}}{x^{\frac{20}{3}}}\right)}{280a^3c^{\frac{23}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/(c*x)^(23/3),x, algorithm="maxima")`

[Out] 
$$-3/280*(35*(b*x^2 + a)^{(4/3)}*b^2/x^{(8/3)} - 40*(b*x^2 + a)^{(7/3)}*b/x^{(14/3)} + 14*(b*x^2 + a)^{(10/3)}/x^{(20/3)})/(a^3*c^{(23/3)})$$

**Fricas [A]** time = 0.230776, size = 77, normalized size = 0.91

$$-\frac{3(9b^3x^6 - 3ab^2x^4 + 2a^2bx^2 + 14a^3)(bx^2 + a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{280a^3c^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/(c*x)^(23/3),x, algorithm="fricas")`

[Out] 
$$-3/280*(9*b^3*x^6 - 3*a*b^2*x^4 + 2*a^2*b*x^2 + 14*a^3)*(b*x^2 + a)^{(1/3)}*(c*x)^{(1/3)}/(a^3*c^8*x^7)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/3)/(c*x)**(23/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{3}}}{(cx)^{\frac{23}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/(c*x)^(23/3),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(1/3)/(c*x)^(23/3), x)`

$$3.745 \quad \int \frac{\sqrt[3]{a+bx^2}}{(cx)^{29/3}} dx$$

**Optimal.** Leaf size=113

$$\frac{243(a+bx^2)^{13/3}}{3640a^4c(cx)^{26/3}} - \frac{81(a+bx^2)^{10/3}}{280a^3c(cx)^{26/3}} + \frac{27(a+bx^2)^{7/3}}{56a^2c(cx)^{26/3}} - \frac{3(a+bx^2)^{4/3}}{8ac(cx)^{26/3}}$$

[Out]  $(-3*(a+b*x^2)^(4/3))/(8*a*c*(c*x)^(26/3)) + (27*(a+b*x^2)^(7/3))/(56*a^2*c*(c*x)^(26/3)) - (81*(a+b*x^2)^(10/3))/(280*a^3*c*(c*x)^(26/3)) + (243*(a+b*x^2)^(13/3))/(3640*a^4*c*(c*x)^(26/3))$

**Rubi [A]** time = 0.125806, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{243(a+bx^2)^{13/3}}{3640a^4c(cx)^{26/3}} - \frac{81(a+bx^2)^{10/3}}{280a^3c(cx)^{26/3}} + \frac{27(a+bx^2)^{7/3}}{56a^2c(cx)^{26/3}} - \frac{3(a+bx^2)^{4/3}}{8ac(cx)^{26/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/3)/(c\*x)^(29/3), x]

[Out]  $(-3*(a+b*x^2)^(4/3))/(8*a*c*(c*x)^(26/3)) + (27*(a+b*x^2)^(7/3))/(56*a^2*c*(c*x)^(26/3)) - (81*(a+b*x^2)^(10/3))/(280*a^3*c*(c*x)^(26/3)) + (243*(a+b*x^2)^(13/3))/(3640*a^4*c*(c*x)^(26/3))$

**Rubi in Sympy [A]** time = 15.4148, size = 99, normalized size = 0.88

$$-\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{26/3}} + \frac{27(a+bx^2)^{7/3}}{56a^2c(cx)^{26/3}} - \frac{81(a+bx^2)^{10/3}}{280a^3c(cx)^{26/3}} + \frac{243(a+bx^2)^{13/3}}{3640a^4c(cx)^{26/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/3)/(c\*x)\*\*(29/3), x)

[Out]  $-3*(a+b*x**2)**(4/3)/(8*a*c*(c*x)**(26/3)) + 27*(a+b*x**2)**(7/3)/(56*a**2*c*(c*x)**(26/3)) - 81*(a+b*x**2)**(10/3)/(280*a**3*c*(c*x)**(26/3)) + 243*(a+b*x**2)**(13/3)/(3640*a**4*c*(c*x)**(26/3))$

**Mathematica [A]** time = 0.0554998, size = 74, normalized size = 0.65

$$\frac{\sqrt[3]{a+bx^2}(-140a^4 - 14a^3bx^2 + 18a^2b^2x^4 - 27ab^3x^6 + 81b^4x^8)}{3640a^4c^9x^8(cx)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/3)/(c\*x)^(29/3), x]

[Out]  $(3*(a+b*x^2)^(1/3)*(-140*a^4 - 14*a^3*b*x^2 + 18*a^2*b^2*x^4 - 27*a*b^3*x^6 + 81*b^4*x^8))/(3640*a^4*c^9*x^8*(c*x)^(2/3))$

**Maple [A]** time = 0.008, size = 53, normalized size = 0.5

$$-\frac{3x(-81b^3x^6 + 108ab^2x^4 - 126a^2bx^2 + 140a^3)}{3640a^4}(bx^2 + a)^{\frac{4}{3}}(cx)^{-\frac{29}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/3)/(c*x)^(29/3), x)`

[Out] `-3/3640*x*(b*x^2+a)^(4/3)*(-81*b^3*x^6+108*a*b^2*x^4-126*a^2*b*x^2+140*a^3)/a^4/(c*x)^(29/3)`

**Maxima [A]** time = 1.36927, size = 86, normalized size = 0.76

$$\frac{3(81b^4x^9 - 27ab^3x^7 + 18a^2b^2x^5 - 14a^3bx^3 - 140a^4x)(bx^2 + a)^{\frac{1}{3}}}{3640a^4c^{\frac{29}{3}}x^{\frac{29}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/(c*x)^(29/3), x, algorithm="maxima")`

[Out] `3/3640*(81*b^4*x^9 - 27*a*b^3*x^7 + 18*a^2*b^2*x^5 - 14*a^3*b*x^3 - 140*a^4*x)*(b*x^2 + a)^(1/3)/(a^4*c^(29/3)*x^(29/3))`

**Fricas [A]** time = 0.232532, size = 92, normalized size = 0.81

$$\frac{3(81b^4x^8 - 27ab^3x^6 + 18a^2b^2x^4 - 14a^3bx^2 - 140a^4)(bx^2 + a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{3640a^4c^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/(c*x)^(29/3), x, algorithm="fricas")`

[Out] `3/3640*(81*b^4*x^8 - 27*a*b^3*x^6 + 18*a^2*b^2*x^4 - 14*a^3*b*x^2 - 140*a^4)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^4*c^10*x^9)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/3)/(c*x)**(29/3), x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{3}}}{(cx)^{\frac{29}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(1/3)/(c*x)^(29/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(29/3), x)
```



### 3.746 $\int (cx)^{10/3} \sqrt[3]{a + bx^2} dx$

**Optimal.** Leaf size=451

$$\frac{14a^2c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{135b^2} + \frac{7a^2c^{7/3}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}+c^{4/3}}{(a+bx^2)^{2/3}+\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)^2}}F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}\right)\right)\left|\frac{1}{4}\right.(2+\sqrt{3})}{135\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b(cx)^{2/3}}\left(c^{2/3}-\frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

$$+ \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c} + \frac{2ac(cx)^{7/3}\sqrt[3]{a+bx^2}}{45b}$$

[Out]  $(-14*a^2*c^3*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)})/(135*b^2) + (2*a*c*(c*x)^{(7/3)}*(a+b*x^2)^{(1/3)})/(45*b) + ((c*x)^{(13/3)}*(a+b*x^2)^{(1/3)})/(5*c) + (7*a^2*c^{(7/3)}*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})*\text{Sqrt}[(c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a+b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcCos}[(c^{(2/3)} - ((1 - \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})], (2 + \text{Sqrt}[3])/4])/((135*3^{(1/4)}*b^2*\text{Sqrt}[-((b^{(1/3)}*(c*x)^{(2/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}))/((a+b*x^2)^{(1/3)}*(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2)])]$

**Rubi [A]** time = 1.88481, antiderivative size = 451, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{14a^2c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{135b^2} + \frac{7a^2c^{7/3}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}+c^{4/3}}{(a+bx^2)^{2/3}+\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)^2}}F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}\right)\right)\left|\frac{1}{4}\right.(2+\sqrt{3})}{135\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b(cx)^{2/3}}\left(c^{2/3}-\frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

$$+ \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c} + \frac{2ac(cx)^{7/3}\sqrt[3]{a+bx^2}}{45b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*x)^{(10/3)}*(a+b*x^2)^{(1/3)}, x]$

[Out]  $(-14*a^2*c^3*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)})/(135*b^2) + (2*a*c*(c*x)^{(7/3)}*(a+b*x^2)^{(1/3)})/(45*b) + ((c*x)^{(13/3)}*(a+b*x^2)^{(1/3)})/(5*c) + (7*a^2*c^{(7/3)}*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})*\text{Sqrt}[(c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a+b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcCos}[(c^{(2/3)} - ((1 - \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})], (2 + \text{Sqrt}[3])/4])/((135*3^{(1/4)}*b^2*\text{Sqrt}[-((b^{(1/3)}*(c*x)^{(2/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}))/((a+b*x^2)^{(1/3)}*(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2)])]$

$$\frac{1}{(135 \cdot 3^{1/4} \cdot b^2 \cdot \sqrt{-((b^{1/3} \cdot (c \cdot x)^{2/3}) \cdot (c^{2/3}) - (b^{1/3} \cdot (c \cdot x)^{2/3})) / (a + b \cdot x^2)^{1/3}}) / ((a + b \cdot x^2)^{1/3} \cdot (c^{2/3} - (1 + \sqrt{3})) \cdot b^{1/3} \cdot (c \cdot x)^{2/3} / (a + b \cdot x^2)^{1/3})^2)}$$

**Rubi in Sympy [A]** time = 45.3965, size = 437, normalized size = 0.97

$$\frac{7 \cdot 3^{3/4} a^3 c^{7/3} \sqrt[3]{cx} \sqrt{\frac{\frac{b^{2/3} (cx)^{4/3} + \sqrt[3]{bc} \frac{2}{3} (cx)^{2/3} + c^{4/3}}{(a+bx^2)^{2/3} + \sqrt[3]{a+bx^2}}}{\left(\frac{\sqrt[3]{b} (cx)^{2/3} (-\sqrt{3}-1) + c^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2} \left(-\frac{\sqrt[3]{b} (cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{2/3}\right)}{F\left(\arcsin\left(\frac{\frac{\sqrt[3]{b} (cx)^{2/3} (-1+\sqrt{3}) + c^{2/3}}{\sqrt[3]{a+bx^2}}}{\frac{\sqrt[3]{b} (cx)^{2/3} (-\sqrt{3}-1) + c^{2/3}}{\sqrt[3]{a+bx^2}}}\right)}{\frac{\sqrt{3}}{4} + \frac{1}{2}}\right)}}}{405b^2 \sqrt{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{b} (cx)^{2/3} \left(-\frac{\sqrt[3]{b} (cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{2/3}\right)}{\sqrt[3]{a+bx^2} \left(\frac{\sqrt[3]{b} (cx)^{2/3} (-\sqrt{3}-1) + c^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2} (a+bx^2)^{2/3} \sqrt{-\frac{bx^2}{a+bx^2} + 1}} - \frac{14a^2 c^3 \sqrt[3]{cx} \sqrt[3]{a+bx^2}}{135b^2} + \frac{2ac (cx)^{7/3} \sqrt[3]{a+bx^2}}{45b} + \frac{(cx)^{13/3} \sqrt[3]{a+bx^2}}{5c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x)**(10/3)*(b*x**2+a)**(1/3),x)`

[Out]  $7 \cdot 3^{3/4} a^3 c^{7/3} (c \cdot x)^{1/3} \sqrt{(b^{2/3} (c \cdot x)^{4/3}) / (a + b \cdot x^2)^{2/3} + b^{1/3} (c^{2/3} (c \cdot x)^{2/3}) / (a + b \cdot x^2)^{1/3} + c^{4/3}) / (b^{1/3} (c \cdot x)^{2/3} (-\sqrt{3} - 1) / (a + b \cdot x^2)^{1/3} + c^{2/3})^2 \cdot (-b^{1/3} (c \cdot x)^{2/3} / (a + b \cdot x^2)^{1/3} + c^{2/3}) \cdot \text{elliptic\_f}(\arcsin((b^{1/3} (c \cdot x)^{2/3} (-1 + \sqrt{3})) / (a + b \cdot x^2)^{1/3} + c^{2/3}) / (b^{1/3} (c \cdot x)^{2/3} (-\sqrt{3} - 1) / (a + b \cdot x^2)^{1/3} + c^{2/3})), \sqrt{3} / 4 + 1/2) / (405 \cdot b^2 \cdot \sqrt{a / (a + b \cdot x^2)} \cdot \sqrt{-b^{1/3} (c \cdot x)^{2/3} (-b^{1/3} (c \cdot x)^{2/3} / (a + b \cdot x^2)^{1/3} + c^{2/3}) / ((a + b \cdot x^2)^{1/3} (b^{1/3} (c \cdot x)^{2/3} (-\sqrt{3} - 1) / (a + b \cdot x^2)^{1/3} + c^{2/3})^2) \cdot (a + b \cdot x^2)^{2/3} \sqrt{-b \cdot x^2 / (a + b \cdot x^2) + 1}} - 14 \cdot a^2 \cdot c^3 \cdot (c \cdot x)^{7/3} \cdot (a + b \cdot x^2)^{1/3} / (135 \cdot b^2) + 2 \cdot a \cdot c \cdot (c \cdot x)^{13/3} \cdot (a + b \cdot x^2)^{1/3} / (45 \cdot b) + (c \cdot x)^{13/3} \cdot (a + b \cdot x^2)^{1/3} / (5 \cdot c)$

**Mathematica [C]** time = 0.0637099, size = 98, normalized size = 0.22

$$\frac{c^3 \sqrt[3]{cx} \left( 14a^3 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{1}{6}, \frac{2}{3}; \frac{7}{6}; -\frac{bx^2}{a} \right) - 14a^3 - 8a^2 bx^2 + 33ab^2 x^4 + 27b^3 x^6 \right)}{135b^2 (a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c*x)^(10/3)*(a + b*x^2)^(1/3),x]`

[Out]  $(c^3 (c \cdot x)^{1/3} (-14 \cdot a^3 - 8 \cdot a^2 \cdot b \cdot x^2 + 33 \cdot a \cdot b^2 \cdot x^4 + 27 \cdot b^3 \cdot x^6 + 14 \cdot a^3 \cdot (1 + (b \cdot x^2) / a)^{2/3} \text{Hypergeometric2F1}[1/6, 2/3, 7/6, -(b \cdot x^2) / a]) / (135 \cdot b^2 \cdot (a + b \cdot x^2)^{2/3})$

**Maple [F]** time = 0.04, size = 0, normalized size = 0.

$$\int (cx)^{10/3} \sqrt[3]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(10/3)*(b*x^2+a)^(1/3),x)`

[Out] `int((c*x)^(10/3)*(b*x^2+a)^(1/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{3}} (cx)^{\frac{10}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*(c*x)^(10/3),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(1/3)*(c*x)^(10/3), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^{\frac{1}{3}} (cx)^{\frac{1}{3}} c^3 x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*(c*x)^(10/3),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)*c^3*x^3, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(10/3)*(b*x**2+a)**(1/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{3}} (cx)^{\frac{10}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*(c*x)^(10/3),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(1/3)*(c*x)^(10/3), x)`

$$3.747 \quad \int (cx)^{4/3} \sqrt[3]{a + bx^2} dx$$

**Optimal.** Leaf size=418

$$\frac{a\sqrt[3]{c}\sqrt[3]{cx}\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}}{\left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right)^2}} F \left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}} \right) \Big|_{\frac{1}{4}} (2 + \sqrt{3}) \right)}{9\sqrt[3]{3}b \sqrt{\frac{\sqrt[3]{b}(cx)^{2/3} \left( c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right)^2}}}$$

$$+ \frac{(cx)^{7/3} \sqrt[3]{a+bx^2}}{3c} + \frac{2ac\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9b}$$

[Out]  $(2*a*c*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)})/(9*b) + ((c*x)^{(7/3)}*(a+b*x^2)^{(1/3)})/(3*c) - (a*c^{(1/3)}*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})*\text{Sqrt}[(c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a+b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcCos}[(c^{(2/3)} - ((1 - \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})], (2 + \text{Sqrt}[3])/4)]/(9*3^{(1/4)}*b*\text{Sqrt}[-((b^{(1/3)}*(c*x)^{(2/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}))/((a+b*x^2)^{(1/3)}*(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2)])]$

**Rubi [A]** time = 1.53783, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{a\sqrt[3]{c}\sqrt[3]{cx}\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}}{\left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right)^2}} F \left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}} \right) \Big|_{\frac{1}{4}} (2 + \sqrt{3}) \right)}{9\sqrt[3]{3}b \sqrt{\frac{\sqrt[3]{b}(cx)^{2/3} \left( c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right)^2}}}$$

$$+ \frac{(cx)^{7/3} \sqrt[3]{a+bx^2}}{3c} + \frac{2ac\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*x)^{(4/3)}*(a+b*x^2)^{(1/3)}, x]$

[Out]  $(2*a*c*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)})/(9*b) + ((c*x)^{(7/3)}*(a+b*x^2)^{(1/3)})/(3*c) - (a*c^{(1/3)}*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})*\text{Sqrt}[(c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a+b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcCos}[(c^{(2/3)} - ((1 - \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})], (2 + \text{Sqrt}[3])/4)]/(9*3^{(1/4)}*b*\text{Sqrt}[-((b^{(1/3)}*(c*x)^{(2/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}))/((a+b*x^2)^{(1/3)}*(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2)])]$

**Rubi in Sympy [A]** time = 35.618, size = 403, normalized size = 0.96

$$\frac{3^{\frac{3}{4}} a^2 \sqrt[3]{c} \sqrt[3]{cx} \sqrt{\frac{\frac{b^{\frac{2}{3}}(cx)^{\frac{4}{3}} + \sqrt[3]{bc} \frac{2}{3} (cx)^{\frac{2}{3}}}{(a+bx^2)^{\frac{2}{3}} + \sqrt[3]{a+bx^2}} + c^{\frac{4}{3}}}{\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}}\right)^2} \left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right) F\left(\arcsin\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-1+\sqrt{3})}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)\right)}{\sqrt{\frac{27b\sqrt{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}\left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)}{\sqrt[3]{a+bx^2}\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}}\right)^2} (a+bx^2)^{\frac{2}{3}} \sqrt{-\frac{bx^2}{a+bx^2} + 1}}}{9b} + \frac{(cx)^{\frac{7}{3}} \sqrt[3]{a+bx^2}}{3c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x)**(4/3)*(b*x**2+a)**(1/3),x)`

[Out] 
$$-3^{3/4} a^2 c^{1/3} (c x)^{1/3} \sqrt{(b^{2/3} (c x)^{4/3} / (a + b x^2)^{2/3} + b^{1/3} c^{2/3} (c x)^{2/3} / (a + b x^2)^{1/3} + c^{4/3}) / (b^{1/3} (c x)^{2/3} (-\sqrt{3} - 1) / (a + b x^2)^{1/3} + c^{2/3})^2} (-b^{1/3} (c x)^{2/3} / (a + b x^2)^{1/3} + c^{2/3}) \operatorname{elliptic}_f(\arcsin((b^{1/3} (c x)^{2/3} (-1 + \sqrt{3}) / (a + b x^2)^{1/3} + c^{2/3}) / (b^{1/3} (c x)^{2/3} (-\sqrt{3} - 1) / (a + b x^2)^{1/3} + c^{2/3})), \sqrt{3} / 4 + 1/2) / (27 b \sqrt{a / (a + b x^2)} \sqrt{-b^{1/3} (c x)^{2/3} (-b^{1/3} (c x)^{2/3} / (a + b x^2)^{1/3} + c^{2/3}) / ((a + b x^2)^{1/3} (b^{1/3} (c x)^{2/3} (-\sqrt{3} - 1) / (a + b x^2)^{1/3} + c^{2/3}))^2} \sqrt{-b x^2 / (a + b x^2) + 1}) + 2 a c (c x)^{1/3} (a + b x^2)^{1/3} / (9 b) + (c x)^{7/3} (a + b x^2)^{1/3} / (3 c)$$

**Mathematica [C]** time = 0.0565944, size = 85, normalized size = 0.2

$$\frac{c \sqrt[3]{cx} \left( -2a^2 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{1}{6}, \frac{2}{3}; \frac{7}{6}; -\frac{bx^2}{a} \right) + 2a^2 + 5abx^2 + 3b^2x^4 \right)}{9b(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c*x)^(4/3)*(a+b*x^2)^(1/3),x]`

[Out] 
$$(c (c x)^{1/3} (2 a^2 + 5 a b x^2 + 3 b^2 x^4 - 2 a^2 (1 + (b x^2 / a)^{2/3}) \operatorname{Hypergeometric2F1}[1/6, 2/3, 7/6, -(b x^2 / a)])) / (9 b (a + b x^2)^{2/3})$$

**Maple [F]** time = 0.034, size = 0, normalized size = 0.

$$\int (cx)^{\frac{4}{3}} \sqrt[3]{bx^2+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(4/3)*(b*x^2+a)^(1/3),x)`

[Out] `int((c*x)^(4/3)*(b*x^2+a)^(1/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{3}} (cx)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)\*(c\*x)^(4/3),x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(1/3)\*(c\*x)^(4/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^{\frac{1}{3}} (cx)^{\frac{4}{3}} cx, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)\*(c\*x)^(4/3),x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(1/3)\*(c\*x)^(1/3)\*c\*x, x)

---

**Sympy [A]** time = 149.214, size = 46, normalized size = 0.11

$$\frac{\sqrt[3]{ac^{\frac{4}{3}}x^{\frac{7}{3}}}\left(\frac{7}{6}\right) {}_2F_1\left(-\frac{1}{3}, \frac{7}{6} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(4/3)\*(b\*x\*\*2+a)\*\*(1/3),x)

[Out] a\*\*(1/3)\*c\*\*(4/3)\*x\*\*(7/3)\*gamma(7/6)\*hyper((-1/3, 7/6), (13/6, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*gamma(13/6))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{3}} (cx)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)\*(c\*x)^(4/3),x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(1/3)\*(c\*x)^(4/3), x)

$$3.748 \quad \int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}} dx$$

**Optimal.** Leaf size=381

$$\frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}}{\left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}} F \left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}} \right) \right) \Big|_{\frac{1}{4}} (2 + \sqrt{3})}{\sqrt[4]{3}c^{5/3} \sqrt{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}}$$

$$+ \frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{c}$$

[Out]  $((c*x)^{(1/3)}*(a+b*x^2)^{(1/3)})/c + ((c*x)^{(1/3)}*(a+b*x^2)^{(1/3)})*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})*\text{Sqrt}[(c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a+b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1+\text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcCos}[(c^{(2/3)} - ((1-\text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1+\text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})], (2+\text{Sqrt}[3])/4)]/(3^{(1/4)}*c^{(5/3)}*\text{Sqrt}[-((b^{(1/3)}*(c*x)^{(2/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}))/(a+b*x^2)^{(1/3)}*(c^{(2/3)} - ((1+\text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2)])]$

**Rubi [A]** time = 1.38366, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}}{\left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}} F \left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}} \right) \right) \Big|_{\frac{1}{4}} (2 + \sqrt{3})}{\sqrt[4]{3}c^{5/3} \sqrt{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}}$$

$$+ \frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{c}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a+b*x^2)^{(1/3)}/(c*x)^{(2/3)}, x]$

[Out]  $((c*x)^{(1/3)}*(a+b*x^2)^{(1/3)})/c + ((c*x)^{(1/3)}*(a+b*x^2)^{(1/3)})*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})*\text{Sqrt}[(c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a+b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1+\text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcCos}[(c^{(2/3)} - ((1-\text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1+\text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})], (2+\text{Sqrt}[3])/4)]/(3^{(1/4)}*c^{(5/3)}*\text{Sqrt}[-((b^{(1/3)}*(c*x)^{(2/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}))/(a+b*x^2)^{(1/3)}*(c^{(2/3)} - ((1+\text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2)])]$

**Rubi in Sympy [A]** time = 27.732, size = 372, normalized size = 0.98

$$\frac{3^{\frac{3}{4}} a \sqrt[3]{cx} \sqrt{\frac{\frac{b^{\frac{2}{3}}(cx)^{\frac{4}{3}} + \sqrt[3]{bc^{\frac{2}{3}}(cx)^{\frac{2}{3}} + c^{\frac{4}{3}}}{(a+bx^2)^{\frac{2}{3}} + \sqrt[3]{a+bx^2}}}{\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}}\right)^2} \left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right) F\left(\arcsin\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-1+\sqrt{3})}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)\right) \frac{\sqrt{3}}{4} + \frac{1}{2}}{\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}}\right)^2} + \frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{c}}{3c^{\frac{5}{3}} \sqrt{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}\left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)}{\sqrt[3]{a+bx^2}\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}}\right)^2} (a+bx^2)^{\frac{2}{3}} \sqrt{-\frac{bx^2}{a+bx^2} + 1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(1/3)/(c*x)**(2/3), x)`

[Out]  $3^{3/4} a (c x)^{1/3} \sqrt{(b^{2/3} (c x)^{4/3} / (a + b x^2)^{2/3} + b^{1/3} c^{2/3} (c x)^{2/3} / (a + b x^2)^{1/3} + c^{4/3} (4/3) / (b^{1/3} (c x)^{2/3} (-\sqrt{3} - 1) / (a + b x^2)^{1/3} + c^{2/3} (2/3))^{**2} (-b^{1/3} (c x)^{2/3} / (a + b x^2)^{1/3} + c^{2/3} (2/3))^{**2} \operatorname{elliptic\_f}(\arcsin((b^{1/3} (c x)^{2/3} (-1 + \sqrt{3})) / (a + b x^2)^{1/3} + c^{2/3} / (b^{1/3} (c x)^{2/3} (-\sqrt{3} - 1) / (a + b x^2)^{1/3} + c^{2/3}))), \sqrt{3} / 4 + 1/2} / (3 c^{5/3} \sqrt{a / (a + b x^2)} \sqrt{-b^{1/3} (c x)^{2/3} (-b^{1/3} (c x)^{2/3} / (a + b x^2)^{1/3} + c^{2/3} / ((a + b x^2)^{1/3} (b^{1/3} (c x)^{2/3} (-\sqrt{3} - 1) / (a + b x^2)^{1/3} + c^{2/3}))^{**2} (a + b x^2)^{2/3} \sqrt{-b x^2 / (a + b x^2) + 1}) + (c x)^{1/3} (a + b x^2)^{1/3} / c}$

**Mathematica [C]** time = 0.0428873, size = 63, normalized size = 0.17

$$\frac{x \left( 2a \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{1}{6}, \frac{2}{3}; \frac{7}{6}; -\frac{bx^2}{a} \right) + a + bx^2 \right)}{(cx)^{2/3} (a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(1/3)/(c*x)^(2/3), x]`

[Out]  $(x^*(a + b x^2 + 2 a (1 + (b x^2)/a)^{2/3} \operatorname{Hypergeometric2F1}[1/6, 2/3, 7/6, -((b x^2)/a)]) / ((c x)^{2/3} (a + b x^2)^{2/3})$

**Maple [F]** time = 0.031, size = 0, normalized size = 0.

$$\int 1 \sqrt[3]{bx^2 + a} (cx)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/3)/(c*x)^(2/3), x)`

[Out] `int((b*x^2+a)^(1/3)/(c*x)^(2/3), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{3}}}{(cx)^{\frac{2}{3}}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/(c*x)^(2/3), x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(1/3)/(c*x)^(2/3), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{1}{3}}}{(cx)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/(c*x)^(2/3), x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(1/3)/(c*x)^(2/3), x)`

**Sympy** [A] time = 5.41571, size = 46, normalized size = 0.12

$$\frac{\sqrt[3]{a}\sqrt[3]{x} \left(\frac{1}{6}\right) {}_2F_1\left(-\frac{1}{3}, \frac{1}{6} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2c^{\frac{2}{3}} \left(\frac{7}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/3)/(c*x)**(2/3), x)`

[Out] `a**(1/3)*x**(1/3)*gamma(1/6)*hyper((-1/3, 1/6), (7/6,), b*x**2*exp_polar(I*pi)/a)/(2*c**(2/3)*gamma(7/6))`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{3}}}{(cx)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/(c*x)^(2/3), x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(1/3)/(c*x)^(2/3), x)`

$$3.749 \quad \int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}} dx$$

**Optimal.** Leaf size=391

$$\frac{3^{3/4} b \sqrt[3]{cx} \sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3} + \sqrt[3]{b} c^{2/3} (cx)^{2/3} + c^{4/3}}{(a+bx^2)^{2/3} + \sqrt[3]{a+bx^2}}}{\left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}} F \left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}} \right) \middle| \frac{1}{4} (2 + \sqrt{3}) \right)}{5ac^{11/3} \sqrt{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}}$$

$$- \frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}$$

[Out]  $(-3*(a + b*x^2)^{(1/3)})/(5*c*(c*x)^{(5/3)}) + (3^{(3/4)}*b*(c*x)^{(1/3)}*(a + b*x^2)^{(1/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})*\text{Sqrt}[(c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a + b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcCos}[(c^{(2/3)} - ((1 - \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})], (2 + \text{Sqrt}[3])/4)]/(5*a*c^{(11/3)}*\text{Sqrt}[-((b^{(1/3)}*(c*x)^{(2/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)}))/((a + b*x^2)^{(1/3)}*(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)}))^2])]$

**Rubi [A]** time = 1.43553, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{3^{3/4} b \sqrt[3]{cx} \sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3} + \sqrt[3]{b} c^{2/3} (cx)^{2/3} + c^{4/3}}{(a+bx^2)^{2/3} + \sqrt[3]{a+bx^2}}}{\left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}} F \left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}} \right) \middle| \frac{1}{4} (2 + \sqrt{3}) \right)}{5ac^{11/3} \sqrt{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}}$$

$$- \frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^{(1/3)}/(c*x)^{(8/3)}, x]$

[Out]  $(-3*(a + b*x^2)^{(1/3)})/(5*c*(c*x)^{(5/3)}) + (3^{(3/4)}*b*(c*x)^{(1/3)}*(a + b*x^2)^{(1/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})*\text{Sqrt}[(c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a + b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcCos}[(c^{(2/3)} - ((1 - \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})], (2 + \text{Sqrt}[3])/4)]/(5*a*c^{(11/3)}*\text{Sqrt}[-((b^{(1/3)}*(c*x)^{(2/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)}))/((a + b*x^2)^{(1/3)}*(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)}))^2])]$

**Rubi in Sympy [A]** time = 28.3266, size = 376, normalized size = 0.96

$$\frac{3^{\frac{3}{4}} b \sqrt[3]{cx} \sqrt{\frac{\frac{b^{\frac{2}{3}}(cx)^{\frac{4}{3}} + \sqrt[3]{bc^{\frac{2}{3}}(cx)^{\frac{2}{3}} + c^{\frac{4}{3}}}{(a+bx^2)^{\frac{2}{3}} + \sqrt[3]{a+bx^2}}}{\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}}\right)^2} \left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right) F\left(\arcsin\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-1+\sqrt{3})}{\sqrt[3]{a+bx^2} + c^{\frac{2}{3}}}\right) \left|\frac{\sqrt{3}}{4} + \frac{1}{2}\right.\right)}{\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}}\right)^2} \left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right) \sqrt{\frac{a}{a+bx^2}}}{5c^{\frac{11}{3}} \sqrt[3]{a+bx^2} \sqrt{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}\left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)}{\sqrt[3]{a+bx^2}\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}}\right)^2} (a+bx^2)^{\frac{2}{3}} \sqrt{-\frac{bx^2}{a+bx^2} + 1}}}}{5c^{\frac{5}{3}}(cx)^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(1/3)/(c*x)**(8/3), x)`

[Out]  $3^{3/4} b (cx)^{1/3} \sqrt{(b^{2/3}(cx)^{4/3}/(a+b^2x^2))^{2/3} + b^{1/3} c^{2/3} (cx)^{2/3}/(a+b^2x^2)^{1/3} + c^{4/3}/(b^{1/3}(cx)^{2/3}(-\sqrt{3}-1)/(a+b^2x^2)^{1/3} + c^{2/3})^2} (-b^{1/3}(cx)^{2/3}/(a+b^2x^2)^{1/3} + c^{2/3})^{2/3} \text{elliptic}_f(\arcsin(b^{1/3}(cx)^{2/3}(-1+\sqrt{3})/(a+b^2x^2)^{1/3} + c^{2/3})/(b^{1/3}(cx)^{2/3}(-\sqrt{3}-1)/(a+b^2x^2)^{1/3} + c^{2/3})), \sqrt{3}/4 + 1/2)/(5c^{11/3} \sqrt{a/(a+b^2x^2)} \sqrt{-b^{1/3}(cx)^{2/3}(-b^{1/3}(cx)^{2/3}/(a+b^2x^2)^{1/3} + c^{2/3})/((a+b^2x^2)^{1/3}(b^{1/3}(cx)^{2/3}(-\sqrt{3}-1)/(a+b^2x^2)^{1/3} + c^{2/3}))^2} (a+b^2x^2)^{2/3} \sqrt{-bx^2/(a+bx^2) + 1}) - 3(a+b^2x^2)^{1/3}/(5c^{5/3}(cx)^{5/3})$

**Mathematica [C]** time = 0.0539847, size = 69, normalized size = 0.18

$$\frac{3x \left(-2bx^2 \left(\frac{bx^2}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{6}, \frac{2}{3}; \frac{7}{6}; -\frac{bx^2}{a}\right) + a + bx^2\right)}{5(cx)^{8/3} (a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(1/3)/(c*x)^(8/3), x]`

[Out]  $(-3*x*(a + b*x^2 - 2*b*x^2*(1 + (b*x^2)/a)^{2/3} \text{Hypergeometric2F1}[1/6, 2/3, 7/6, -((b*x^2)/a)])/(5*(c*x)^{8/3}*(a + b*x^2)^{2/3})$

**Maple [F]** time = 0.037, size = 0, normalized size = 0.

$$\int 1\sqrt[3]{bx^2+a}(cx)^{-\frac{8}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/3)/(c*x)^(8/3), x)`

[Out] `int((b*x^2+a)^(1/3)/(c*x)^(8/3), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/(c*x)^(8/3),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(1/3)/(c*x)^(8/3), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{1}{3}}}{(cx)^{\frac{2}{3}} c^2 x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/(c*x)^(8/3),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(1/3)/((c*x)^(2/3)*c^2*x^2), x)`

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/3)/(c*x)**(8/3),x)`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{3}}}{(cx)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/(c*x)^(8/3),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(1/3)/(c*x)^(8/3), x)`

$$3.750 \quad \int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}} dx$$

**Optimal.** Leaf size=422

$$\frac{3 \cdot 3^{3/4} b^2 \sqrt[3]{cx} \sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3} + \sqrt[3]{b} c^{2/3} (cx)^{2/3} + c^{4/3}}{(a+bx^2)^{2/3} + \sqrt[3]{a+bx^2}} + c^{4/3}}{\left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}} F \left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{\frac{\sqrt[3]{bx^2+a}}{c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}} \right) \right)^{\frac{1}{4}} (2 + \sqrt{3})}{55a^2 c^{17/3} \sqrt{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}}$$

$$-\frac{6b\sqrt[3]{a+bx^2}}{55ac^3(cx)^{5/3}} - \frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}}$$

[Out]  $(-3*(a + b*x^2)^{(1/3)})/(11*c*(c*x)^{(11/3)}) - (6*b*(a + b*x^2)^{(1/3)})/(55*a*c^3*(c*x)^{(5/3)}) - (3*3^{(3/4)}*b^2*(c*x)^{(1/3)}*(a + b*x^2)^{(1/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})*\text{Sqrt}[(c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a + b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcCos}[(c^{(2/3)} - ((1 - \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})], (2 + \text{Sqrt}[3])/4])/ (55*a^2*c^{(17/3)}*\text{Sqrt}[-((b^{(1/3)}*(c*x)^{(2/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)}))/((a + b*x^2)^{(1/3)}*(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)}))^2]))]$

**Rubi [A]** time = 1.55173, antiderivative size = 422, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{3 \cdot 3^{3/4} b^2 \sqrt[3]{cx} \sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3} + \sqrt[3]{b} c^{2/3} (cx)^{2/3} + c^{4/3}}{(a+bx^2)^{2/3} + \sqrt[3]{a+bx^2}} + c^{4/3}}{\left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}} F \left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{\frac{\sqrt[3]{bx^2+a}}{c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}} \right) \right)^{\frac{1}{4}} (2 + \sqrt{3})}{55a^2 c^{17/3} \sqrt{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}}$$

$$-\frac{6b\sqrt[3]{a+bx^2}}{55ac^3(cx)^{5/3}} - \frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^{(1/3)}/(c*x)^{(14/3)}, x]$

[Out]  $(-3*(a + b*x^2)^{(1/3)})/(11*c*(c*x)^{(11/3)}) - (6*b*(a + b*x^2)^{(1/3)})/(55*a*c^3*(c*x)^{(5/3)}) - (3*3^{(3/4)}*b^2*(c*x)^{(1/3)}*(a + b*x^2)^{(1/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})*\text{Sqrt}[(c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a + b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcCos}[(c^{(2/3)} - ((1 - \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})], (2 + \text{Sqrt}[3])/4])/ (55*a^2*c^{(17/3)}*\text{Sqrt}[-((b^{(1/3)}*(c*x)^{(2/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)}))/((a + b*x^2)^{(1/3)}*(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)}))^2]))]$

**Rubi in Sympy [A]** time = 37.1305, size = 410, normalized size = 0.97

$$\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{\frac{11}{3}}}$$

$$\frac{3 \cdot 3^{\frac{3}{4}} b^2 \sqrt[3]{cx} \sqrt{\frac{\frac{b^{\frac{2}{3}}(cx)^{\frac{4}{3}}}{(a+bx^2)^{\frac{2}{3}} + \frac{\sqrt[3]{b}c^{\frac{2}{3}}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{4}{3}}}{\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1)}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)^2} \left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right) F\left(\arcsin\left(\frac{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-1+\sqrt{3})}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}}{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1)}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}}\right)\right) \frac{\sqrt{3}}{4} + \frac{1}{2}}}{55ac^{\frac{17}{3}} \sqrt{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}\left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)}{\sqrt[3]{a+bx^2}\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1)}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)^2} (a+bx^2)^{\frac{2}{3}} \sqrt{-\frac{bx^2}{a+bx^2} + 1}}}{6b\sqrt[3]{a+bx^2}}}{55ac^3(cx)^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(1/3)/(c*x)**(14/3), x)`

[Out]  $-3*(a + b*x^2)^{(1/3)}/(11*c*(c*x)^{(11/3)}) - 3*3^{(3/4)}*b^{*2}*(c*x)^{(1/3)}*\sqrt{(b^{(2/3)}*(c*x)^{(4/3)}/(a + b*x^2)^{(2/3)} + b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)}/(a + b*x^2)^{(1/3)} + c^{(4/3)})}/(b^{(1/3)}*(c*x)^{(2/3)}*(-\sqrt{3} - 1)/(a + b*x^2)^{(1/3)} + c^{(2/3)})^{*2})*(-b^{(1/3)}*(c*x)^{(2/3)}/(a + b*x^2)^{(1/3)} + c^{(2/3)})^{*2})*\text{elliptic\_f}(\arcsin((b^{(1/3)}*(c*x)^{(2/3)}*(-1 + \sqrt{3}))/((a + b*x^2)^{(1/3)} + c^{(2/3)}))/((b^{(1/3)}*(c*x)^{(2/3)}*(-\sqrt{3} - 1)/(a + b*x^2)^{(1/3)} + c^{(2/3)})), \sqrt{3}/4 + 1/2)/(55*a*c^{(17/3)}*\sqrt{a/(a + b*x^2)}*\sqrt{-b^{(1/3)}*(c*x)^{(2/3)}/(a + b*x^2)^{(1/3)} + c^{(2/3)}})/((a + b*x^2)^{(1/3)}*(b^{(1/3)}*(c*x)^{(2/3)}*(-\sqrt{3} - 1)/(a + b*x^2)^{(1/3)} + c^{(2/3)})^{*2}))* (a + b*x^2)^{(2/3)}*\sqrt{-b*x^2/(a + b*x^2) + 1}) - 6*b*(a + b*x^2)^{(1/3)}/(55*a*c^{*3}*(c*x)^{(5/3)})$

**Mathematica [C]** time = 0.0783366, size = 93, normalized size = 0.22

$$\frac{3\sqrt[3]{cx} \left(5a^2 + 6b^2x^4 \left(\frac{bx^2}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{6}, \frac{2}{3}; \frac{7}{6}; -\frac{bx^2}{a}\right) + 7abx^2 + 2b^2x^4\right)}{55ac^5x^4(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(1/3)/(c*x)^(14/3), x]`

[Out]  $(-3*(c*x)^{(1/3)}*(5*a^2 + 7*a*b*x^2 + 2*b^2*x^4 + 6*b^2*x^4*(1 + (b*x^2)/a)^{(2/3)}*\text{Hypergeometric2F1}[1/6, 2/3, 7/6, -(b*x^2)/a]))/(55*a*c^5*x^4*(a + b*x^2)^{(2/3)})$

**Maple [F]** time = 0.041, size = 0, normalized size = 0.

$$\int 1\sqrt[3]{bx^2+a}(cx)^{-\frac{14}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/3)/(c*x)^(14/3), x)`

[Out] `int((b*x^2+a)^(1/3)/(c*x)^(14/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{3}}}{(cx)^{\frac{14}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/(c*x)^(14/3),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(1/3)/(c*x)^(14/3), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{1}{3}}}{(cx)^{\frac{2}{3}} c^4 x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/(c*x)^(14/3),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(1/3)/((c*x)^(2/3)*c^4*x^4), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/3)/(c*x)**(14/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{3}}}{(cx)^{\frac{14}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)/(c*x)^(14/3),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(1/3)/(c*x)^(14/3), x)`

$$3.751 \quad \int (cx)^{2/3} \sqrt[3]{a + bx^2} dx$$

**Optimal.** Leaf size=58

$$\frac{3(cx)^{5/3} \sqrt[3]{a + bx^2} {}_2F_1\left(-\frac{1}{3}, \frac{5}{6}, \frac{11}{6}, -\frac{bx^2}{a}\right)}{5c \sqrt[3]{\frac{bx^2}{a} + 1}}$$

[Out]  $(3*(c*x)^{(5/3)}*(a + b*x^2)^{(1/3)}*\text{Hypergeometric2F1}[-1/3, 5/6, 11/6, -(b*x^2)/a])/(5*c*(1 + (b*x^2)/a)^{(1/3)})$

**Rubi [A]** time = 0.0655194, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{3(cx)^{5/3} \sqrt[3]{a + bx^2} {}_2F_1\left(-\frac{1}{3}, \frac{5}{6}, \frac{11}{6}, -\frac{bx^2}{a}\right)}{5c \sqrt[3]{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(2/3)\*(a + b\*x^2)^(1/3), x]

[Out]  $(3*(c*x)^{(5/3)}*(a + b*x^2)^{(1/3)}*\text{Hypergeometric2F1}[-1/3, 5/6, 11/6, -(b*x^2)/a])/(5*c*(1 + (b*x^2)/a)^{(1/3)})$

**Rubi in Sympy [A]** time = 7.49475, size = 49, normalized size = 0.84

$$\frac{3(cx)^{5/3} \sqrt[3]{a + bx^2} {}_2F_1\left(-\frac{1}{3}, \frac{5}{6}, \frac{11}{6}, -\frac{bx^2}{a}\right)}{5c \sqrt[3]{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(2/3)\*(b\*x\*\*2+a)\*\*(1/3), x)

[Out]  $3*(c*x)**(5/3)*(a + b*x**2)**(1/3)*\text{hyper}((-1/3, 5/6), (11/6, ), -b*x**2/a)/(5*c*(1 + b*x**2/a)**(1/3))$

**Mathematica [A]** time = 0.0519905, size = 69, normalized size = 1.19

$$\frac{3x(cx)^{2/3} \left(2a \left(\frac{bx^2}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{5}{6}, \frac{11}{6}, -\frac{bx^2}{a}\right) + 5(a + bx^2)\right)}{35(a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(2/3)\*(a + b\*x^2)^(1/3), x]

[Out]  $(3*x*(c*x)^{(2/3)}*(5*(a + b*x^2) + 2*a*(1 + (b*x^2)/a)^{(2/3)}*\text{Hypergeometric2F1}[2/3, 5/6, 11/6, -(b*x^2)/a])/(35*(a + b*x^2)^{(2/3)})$



**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int (cx)^{\frac{2}{3}} \sqrt[3]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(2/3)*(b*x^2+a)^(1/3),x)`

[Out] `int((c*x)^(2/3)*(b*x^2+a)^(1/3),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{3}} (cx)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*(c*x)^(2/3),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(1/3)*(c*x)^(2/3), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^{\frac{1}{3}} (cx)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*(c*x)^(2/3),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(1/3)*(c*x)^(2/3), x)`

---

**Sympy [A]** time = 11.7127, size = 46, normalized size = 0.79

$$\frac{\sqrt[3]{ac^{\frac{2}{3}}x^{\frac{5}{3}}}\left(\frac{5}{6}\right) {}_2F_1\left(-\frac{1}{3}, \frac{5}{6} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(2/3)*(b*x**2+a)**(1/3),x)`

[Out] `a**(1/3)*c**(2/3)*x**(5/3)*gamma(5/6)*hyper((-1/3, 5/6), (11/6, ), b*x**2*exp_polar(I*pi)/a)/(2*gamma(11/6))`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{3}} (cx)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/3)*(c*x)^(2/3),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(1/3)*(c*x)^(2/3), x)`

$$3.752 \quad \int \frac{\sqrt[3]{a + bx^2}}{\sqrt[3]{cx}} dx$$

**Optimal.** Leaf size=58

$$\frac{3(cx)^{2/3} \sqrt[3]{a + bx^2} {}_2F_1\left(-\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^2}{a}\right)}{2c \sqrt[3]{\frac{bx^2}{a} + 1}}$$

[Out] (3\*(c\*x)^(2/3)\*(a + b\*x^2)^(1/3)\*Hypergeometric2F1[-1/3, 1/3, 4/3, -(b\*x^2)/a])/(2\*c\*(1 + (b\*x^2)/a)^(1/3))

**Rubi [A]** time = 0.0653917, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{3(cx)^{2/3} \sqrt[3]{a + bx^2} {}_2F_1\left(-\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^2}{a}\right)}{2c \sqrt[3]{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/3)/(c\*x)^(1/3), x]

[Out] (3\*(c\*x)^(2/3)\*(a + b\*x^2)^(1/3)\*Hypergeometric2F1[-1/3, 1/3, 4/3, -(b\*x^2)/a])/(2\*c\*(1 + (b\*x^2)/a)^(1/3))

**Rubi in Sympy [A]** time = 7.42832, size = 49, normalized size = 0.84

$$\frac{3(cx)^{2/3} \sqrt[3]{a + bx^2} {}_2F_1\left(-\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^2}{a}\right)}{2c \sqrt[3]{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/3)/(c\*x)\*\*(1/3), x)

[Out] 3\*(c\*x)\*\*(2/3)\*(a + b\*x\*\*2)\*\*(1/3)\*hyper((-1/3, 1/3), (4/3, ), -b\*x\*\*2/a)/(2\*c\*(1 + b\*x\*\*2/a)\*\*(1/3))

**Mathematica [A]** time = 0.0442485, size = 65, normalized size = 1.12

$$\frac{3x \left( a \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^2}{a}\right) + a + bx^2 \right)}{4 \sqrt[3]{cx} (a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/3)/(c\*x)^(1/3), x]

[Out] (3\*x\*(a + b\*x^2 + a\*(1 + (b\*x^2)/a)^(2/3)\*Hypergeometric2F1[1/3, 2/3, 4/3, -(b\*x^2)/a])/(4\*(c\*x)^(1/3)\*(a + b\*x^2)^(2/3))

**Maple [F]** time = 0.035, size = 0, normalized size = 0.

$$\int \sqrt[3]{bx^2 + a} \frac{1}{\sqrt[3]{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(1/3)/(c\*x)^(1/3), x)

[Out] int((b\*x^2+a)^(1/3)/(c\*x)^(1/3), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{3}}}{(cx)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)/(c\*x)^(1/3), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(1/3)/(c\*x)^(1/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{1}{3}}}{(cx)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)/(c\*x)^(1/3), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(1/3)/(c\*x)^(1/3), x)

---

**Sympy [A]** time = 3.65395, size = 46, normalized size = 0.79

$$\frac{\sqrt[3]{ax^{\frac{2}{3}} \left(\frac{1}{3}\right)} {}_2F_1\left(\left(-\frac{1}{3}, \frac{1}{3}\right) \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt[3]{c} \left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/3)/(c\*x)\*\*(1/3), x)

[Out] a\*\*(1/3)\*x\*\*(2/3)\*gamma(1/3)\*hyper((-1/3, 1/3), (4/3, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*c\*\*(1/3)\*gamma(4/3))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{3}}}{(cx)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(1/3)/(c*x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(1/3), x)
```

$$3.753 \quad \int \frac{\sqrt[3]{a + bx^2}}{(cx)^{4/3}} dx$$

**Optimal.** Leaf size=56

$$\frac{3\sqrt[3]{a + bx^2} {}_2F_1\left(-\frac{1}{3}, -\frac{1}{6}; \frac{5}{6}; -\frac{bx^2}{a}\right)}{c\sqrt[3]{cx} \sqrt[3]{\frac{bx^2}{a} + 1}}$$

[Out]  $(-3*(a + b*x^2)^(1/3)*\text{Hypergeometric2F1}[-1/3, -1/6, 5/6, -(b*x^2)/a])/(c*(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))$

**Rubi [A]** time = 0.0637845, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{3\sqrt[3]{a + bx^2} {}_2F_1\left(-\frac{1}{3}, -\frac{1}{6}; \frac{5}{6}; -\frac{bx^2}{a}\right)}{c\sqrt[3]{cx} \sqrt[3]{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^(1/3)/(c*x)^(4/3), x]$

[Out]  $(-3*(a + b*x^2)^(1/3)*\text{Hypergeometric2F1}[-1/3, -1/6, 5/6, -(b*x^2)/a])/(c*(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))$

**Rubi in Sympy [A]** time = 7.47105, size = 51, normalized size = 0.91

$$\frac{3\sqrt[3]{a + bx^2} {}_2F_1\left(-\frac{1}{3}, -\frac{1}{6}; \frac{5}{6}; -\frac{bx^2}{a}\right)}{c\sqrt[3]{cx} \sqrt[3]{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x**2+a)**(1/3)/(c*x)**(4/3), x)$

[Out]  $-3*(a + b*x**2)**(1/3)*\text{hyper}((-1/3, -1/6), (5/6, ), -b*x**2/a)/(c*(c*x)**(1/3)*(1 + b*x**2/a)**(1/3))$

**Mathematica [A]** time = 0.0510562, size = 72, normalized size = 1.29

$$\frac{x \left( 6bx^2 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{5}{6}; \frac{11}{6}; -\frac{bx^2}{a}\right) - 15(a + bx^2) \right)}{5(cx)^{4/3} (a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b*x^2)^(1/3)/(c*x)^(4/3), x]$

[Out]  $(x*(-15*(a + b*x^2) + 6*b*x^2*(1 + (b*x^2)/a)^(2/3)*\text{Hypergeometric2F1}[2/3, 5/6, 11/6, -(b*x^2)/a])/(5*(c*x)^(4/3)*(a + b*x^2)^(2/3))$

---

**Maple [F]** time = 0.034, size = 0, normalized size = 0.

$$\int \sqrt[3]{bx^2 + a} (cx)^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(1/3)/(c\*x)^(4/3), x)

[Out] int((b\*x^2+a)^(1/3)/(c\*x)^(4/3), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{3}}}{(cx)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)/(c\*x)^(4/3), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(1/3)/(c\*x)^(4/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{1}{3}}}{(cx)^{\frac{1}{3}} cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/3)/(c\*x)^(4/3), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(1/3)/((c\*x)^(1/3)\*c\*x), x)

---

**Sympy [A]** time = 12.9224, size = 49, normalized size = 0.88

$$\frac{\sqrt[3]{a} \left(-\frac{1}{6}\right) {}_2F_1\left(-\frac{1}{3}, -\frac{1}{6} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2c^{\frac{4}{3}} \sqrt[3]{x} \left(\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/3)/(c\*x)\*\*(4/3), x)

[Out] a\*\*(1/3)\*gamma(-1/6)\*hyper((-1/3, -1/6), (5/6, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*c\*\*(4/3)\*x\*\*(1/3)\*gamma(5/6))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{3}}}{(cx)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(1/3)/(c*x)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/3)/(c*x)^(4/3), x)
```

$$3.754 \quad \int (cx)^{13/3} (a + bx^2)^{4/3} dx$$

**Optimal.** Leaf size=303

$$\begin{aligned} & -\frac{5a^4c^{13/3} \log\left(c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt{a+bx^2}}\right)}{486b^{8/3}} + \frac{5a^4c^{13/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{bc^{2/3}(cx)^{2/3}}}{\sqrt{a+bx^2}} + c^{4/3}\right)}{972b^{8/3}} \\ & - \frac{5a^4c^{13/3} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b(cx)^{2/3}}}{\sqrt{a+bx^2}} + c^{2/3}}{\sqrt{3c^{2/3}}}\right)}{162\sqrt{3}b^{8/3}} - \frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2} \\ & + \frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b} + \frac{(cx)^{16/3}(a+bx^2)^{4/3}}{8c} + \frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c} \end{aligned}$$

[Out]  $(-5*a^3*c^3*(c*x)^{(4/3)}*(a+b*x^2)^{(1/3)})/(324*b^2) + (a^2*c*(c*x)^{(10/3)}*(a+b*x^2)^{(1/3)})/(108*b) + (a*(c*x)^{(16/3)}*(a+b*x^2)^{(1/3)})/(18*c) + ((c*x)^{(16/3)}*(a+b*x^2)^{(4/3)})/(8*c) - (5*a^4*c^{13/3}*ArcTan[(c^{2/3} + (2*b^{1/3}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(Sqrt[3]*c^{2/3})])/(162*Sqrt[3]*b^{8/3}) - (5*a^4*c^{13/3}*Log[c^{2/3} - (b^{1/3}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3})])/(486*b^{8/3}) + (5*a^4*c^{13/3}*Log[b^{2/3}*(c*x)^{4/3}/(a+bx^2)^{2/3} + \sqrt[3]{bc^{2/3}(cx)^{2/3}}/sqrt{a+bx^2} + c^{4/3}])/(972*b^{8/3})$

**Rubi [A]** time = 0.889575, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$

$$\begin{aligned} & -\frac{5a^4c^{13/3} \log\left(c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt{a+bx^2}}\right)}{486b^{8/3}} + \frac{5a^4c^{13/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{bc^{2/3}(cx)^{2/3}}}{\sqrt{a+bx^2}} + c^{4/3}\right)}{972b^{8/3}} \\ & - \frac{5a^4c^{13/3} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b(cx)^{2/3}}}{\sqrt{a+bx^2}} + c^{2/3}}{\sqrt{3c^{2/3}}}\right)}{162\sqrt{3}b^{8/3}} - \frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2} \\ & + \frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b} + \frac{(cx)^{16/3}(a+bx^2)^{4/3}}{8c} + \frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(13/3)\*(a+b\*x^2)^(4/3),x]

[Out]  $(-5*a^3*c^3*(c*x)^{(4/3)}*(a+b*x^2)^{(1/3)})/(324*b^2) + (a^2*c*(c*x)^{(10/3)}*(a+b*x^2)^{(1/3)})/(108*b) + (a*(c*x)^{(16/3)}*(a+b*x^2)^{(1/3)})/(18*c) + ((c*x)^{(16/3)}*(a+b*x^2)^{(4/3)})/(8*c) - (5*a^4*c^{13/3}*ArcTan[(c^{2/3} + (2*b^{1/3}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(Sqrt[3]*c^{2/3})])/(162*Sqrt[3]*b^{8/3}) - (5*a^4*c^{13/3}*Log[c^{2/3} - (b^{1/3}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3})])/(486*b^{8/3}) + (5*a^4*c^{13/3}*Log[b^{2/3}*(c*x)^{4/3}/(a+bx^2)^{2/3} + \sqrt[3]{bc^{2/3}(cx)^{2/3}}/sqrt{a+bx^2} + c^{4/3}])/(972*b^{8/3})$



**Rubi in Sympy [A]** time = 84.4022, size = 284, normalized size = 0.94

$$\begin{aligned} & \frac{5a^4 c^{\frac{13}{3}} \log\left(-\frac{\sqrt[3]{b(cx)^{\frac{2}{3}}} + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}}\right)}{486b^{\frac{8}{3}}} + \frac{5a^4 c^{\frac{13}{3}} \log\left(\frac{b^{\frac{2}{3}}(cx)^{\frac{4}{3}} + \frac{\sqrt[3]{b(cx)^{\frac{2}{3}}}}{c^{\frac{2}{3}}\sqrt[3]{a+bx^2}} + 1}{c^{\frac{4}{3}}(a+bx^2)^{\frac{2}{3}}}\right)}{972b^{\frac{8}{3}}} \\ & - \frac{5\sqrt{3}a^4 c^{\frac{13}{3}} \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{2\sqrt[3]{b(cx)^{\frac{2}{3}}} + c^{\frac{2}{3}}}{3\sqrt[3]{a+bx^2}}\right)}{c^{\frac{2}{3}}}\right)}{486b^{\frac{8}{3}}} - \frac{5a^3 c^3 (cx)^{\frac{4}{3}} \sqrt[3]{a+bx^2}}{324b^2} \\ & + \frac{a^2 c (cx)^{\frac{10}{3}} \sqrt[3]{a+bx^2}}{108b} + \frac{a (cx)^{\frac{16}{3}} \sqrt[3]{a+bx^2}}{18c} + \frac{(cx)^{\frac{16}{3}} (a+bx^2)^{\frac{4}{3}}}{8c} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x)**(13/3)*(b*x**2+a)**(4/3),x)`

[Out] `-5*a**4*c**(13/3)*log(-b**(1/3)*(c*x)**(2/3)/(a+b*x**2)**(1/3)+c**(2/3))/(486*b**(8/3))+5*a**4*c**(13/3)*log(b**(2/3)*(c*x)**(4/3)/(c**(4/3)*(a+b*x**2)**(2/3))+b**(1/3)*(c*x)**(2/3)/(c**(2/3)*(a+b*x**2)**(1/3))+1)/(972*b**(8/3))-5*sqrt(3)*a**4*c**(13/3)*atan(sqrt(3)*(2*b**(1/3)*(c*x)**(2/3)/(3*(a+b*x**2)**(1/3))+c**(2/3)/3)/c**(2/3))/(486*b**(8/3))-5*a**3*c**3*(c*x)**(4/3)*(a+b*x**2)**(1/3)/(324*b**2)+a**2*c*(c*x)**(10/3)*(a+b*x**2)**(1/3)/(108*b)+a*(c*x)**(16/3)*(a+b*x**2)**(1/3)/(18*c)+(c*x)**(16/3)*(a+b*x**2)**(4/3)/(8*c)`

**Mathematica [C]** time = 0.0811835, size = 109, normalized size = 0.36

$$\frac{c^3 (cx)^{4/3} \left( 10a^4 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^2}{a} \right) - 10a^4 - 4a^3 bx^2 + 123a^2 b^2 x^4 + 198ab^3 x^6 + 81b^4 x^8 \right)}{648b^2 (a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c*x)^(13/3)*(a+b*x^2)^(4/3),x]`

[Out] `(c^3*(c*x)^(4/3)*(-10*a^4-4*a^3*b*x^2+123*a^2*b^2*x^4+198*a*b^3*x^6+81*b^4*x^8+10*a^4*(1+(b*x^2)/a)^(2/3)*Hypergeometric2F1[2/3,2/3,5/3,-((b*x^2)/a)])/(648*b^2*(a+b*x^2)^(2/3))`

**Maple [F]** time = 0.034, size = 0, normalized size = 0.

$$\int (cx)^{\frac{13}{3}} (bx^2 + a)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(13/3)*(b*x^2+a)^(4/3),x)`

[Out] `int((c*x)^(13/3)*(b*x^2+a)^(4/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)*(c*x)^(13/3),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)*(c*x)^(13/3),x, algorithm="fricas")`

[Out] Timed out

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(13/3)*(b*x**2+a)**(4/3),x)`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{4}{3}} (cx)^{\frac{13}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)*(c*x)^(13/3),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(4/3)*(c*x)^(13/3), x)`

$$3.755 \quad \int (cx)^{7/3} (a + bx^2)^{4/3} dx$$

**Optimal.** Leaf size=272

$$\frac{2a^3 c^{7/3} \log\left(c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt{a+bx^2}}\right)}{81b^{5/3}} - \frac{a^3 c^{7/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt{a+bx^2}} + c^{4/3}\right)}{81b^{5/3}}$$

$$+ \frac{2a^3 c^{7/3} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt{a+bx^2}} + c^{2/3}}{\sqrt[3]{3}c^{2/3}}\right)}{27\sqrt[3]{3}b^{5/3}} + \frac{a^2 c (cx)^{4/3} \sqrt[3]{a+bx^2}}{27b} + \frac{(cx)^{10/3} (a+bx^2)^{4/3}}{6c} + \frac{a(cx)^{10/3} \sqrt[3]{a+bx^2}}{9c}$$

[Out]  $(a^2 * c * (c * x)^{(4/3)} * (a + b * x^2)^{(1/3)}) / (27 * b) + (a * (c * x)^{(10/3)} * (a + b * x^2)^{(1/3)}) / (9 * c) + ((c * x)^{(10/3)} * (a + b * x^2)^{(4/3)}) / (6 * c) + (2 * a^3 * c^{7/3} * \text{ArcTan}[(c^{2/3}) + (2 * b^{1/3} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)}) / (\text{Sqrt}[3] * c^{2/3})] / (27 * \text{Sqrt}[3] * b^{5/3}) + (2 * a^3 * c^{7/3} * \text{Log}[c^{2/3} - (b^{1/3} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3})]) / (81 * b^{5/3}) - (a^3 * c^{7/3} * \text{Log}[c^{4/3} + (b^{2/3} * (c * x)^{(4/3)}) / (a + b * x^2)^{(2/3)} + (b^{1/3} * c^{2/3} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3})]) / (81 * b^{5/3})$

**Rubi [A]** time = 0.745014, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$

$$\frac{2a^3 c^{7/3} \log\left(c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt{a+bx^2}}\right)}{81b^{5/3}} - \frac{a^3 c^{7/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt{a+bx^2}} + c^{4/3}\right)}{81b^{5/3}}$$

$$+ \frac{2a^3 c^{7/3} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt{a+bx^2}} + c^{2/3}}{\sqrt[3]{3}c^{2/3}}\right)}{27\sqrt[3]{3}b^{5/3}} + \frac{a^2 c (cx)^{4/3} \sqrt[3]{a+bx^2}}{27b} + \frac{(cx)^{10/3} (a+bx^2)^{4/3}}{6c} + \frac{a(cx)^{10/3} \sqrt[3]{a+bx^2}}{9c}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c * x)^{(7/3)} * (a + b * x^2)^{(4/3)}, x]$

[Out]  $(a^2 * c * (c * x)^{(4/3)} * (a + b * x^2)^{(1/3)}) / (27 * b) + (a * (c * x)^{(10/3)} * (a + b * x^2)^{(1/3)}) / (9 * c) + ((c * x)^{(10/3)} * (a + b * x^2)^{(4/3)}) / (6 * c) + (2 * a^3 * c^{7/3} * \text{ArcTan}[(c^{2/3}) + (2 * b^{1/3} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)}) / (\text{Sqrt}[3] * c^{2/3})] / (27 * \text{Sqrt}[3] * b^{5/3}) + (2 * a^3 * c^{7/3} * \text{Log}[c^{2/3} - (b^{1/3} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3})]) / (81 * b^{5/3}) - (a^3 * c^{7/3} * \text{Log}[c^{4/3} + (b^{2/3} * (c * x)^{(4/3)}) / (a + b * x^2)^{(2/3)} + (b^{1/3} * c^{2/3} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3})]) / (81 * b^{5/3})$

**Rubi in Sympy [A]** time = 73.2678, size = 252, normalized size = 0.93

$$\frac{2a^3 c^{7/3} \log\left(-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt{a+bx^2}} + c^{2/3}\right)}{81b^{5/3}} - \frac{a^3 c^{7/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{c^{4/3}(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt{a+bx^2}} + 1\right)}{81b^{5/3}}$$

$$+ \frac{2\sqrt[3]{3}a^3 c^{7/3} \text{atan}\left(\frac{\sqrt[3]{3}\left(\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt{a+bx^2}} + \frac{c^{2/3}}{3}\right)}{c^{2/3}}\right)}{81b^{5/3}} + \frac{a^2 c (cx)^{4/3} \sqrt[3]{a+bx^2}}{27b} + \frac{a(cx)^{10/3} \sqrt[3]{a+bx^2}}{9c} + \frac{(cx)^{10/3} (a+bx^2)^{4/3}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x)**(7/3)*(b*x**2+a)**(4/3),x)`

[Out]  $2*a**3*c**(7/3)*\log(-b**(1/3)*(c*x)**(2/3)/(a+b*x**2)**(1/3)+c**(2/3))/(81*b**(5/3))-a**3*c**(7/3)*\log(b**(2/3)*(c*x)**(4/3)/(c**(4/3)*(a+b*x**2)**(2/3))+b**(1/3)*(c*x)**(2/3)/(c**(2/3)*(a+b*x**2)**(1/3))+1)/(81*b**(5/3))+2*\sqrt{3}*a**3*c**(7/3)*\operatorname{atan}(\sqrt{3}*(2*b**(1/3)*(c*x)**(2/3)/(3*(a+b*x**2)**(1/3))+c**(2/3)/3)/c**(2/3))/(81*b**(5/3))+a**2*c*(c*x)**(4/3)*(a+b*x**2)**(1/3)/(27*b)+a*(c*x)**(10/3)*(a+b*x**2)**(1/3)/(9*c)+(c*x)**(10/3)*(a+b*x**2)**(4/3)/(6*c)$

**Mathematica [C]** time = 0.0694616, size = 96, normalized size = 0.35

$$\frac{c(cx)^{4/3} \left( -2a^3 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^2}{a} \right) + 2a^3 + 17a^2bx^2 + 24ab^2x^4 + 9b^3x^6 \right)}{54b(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c*x)^(7/3)*(a+b*x^2)^(4/3),x]`

[Out]  $(c*(c*x)^{4/3}*(2*a^3+17*a^2*b*x^2+24*a*b^2*x^4+9*b^3*x^6-2*a^3*(1+(b*x^2)/a)^{2/3}*\operatorname{Hypergeometric2F1}[2/3,2/3,5/3,-((b*x^2)/a)]))/(54*b*(a+b*x^2)^{2/3})$

**Maple [F]** time = 0.033, size = 0, normalized size = 0.

$$\int (cx)^{\frac{7}{3}} (bx^2+a)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(7/3)*(b*x^2+a)^(4/3),x)`

[Out] `int((c*x)^(7/3)*(b*x^2+a)^(4/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^(4/3)*(c*x)^(7/3),x,algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^(4/3)*(c*x)^(7/3),x,algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(7/3)*(b*x**2+a)**(4/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{4}{3}} (cx)^{\frac{7}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)*(c*x)^(7/3),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(4/3)*(c*x)^(7/3), x)`

$$3.756 \quad \int \sqrt[3]{cx} (a + bx^2)^{4/3} dx$$

**Optimal.** Leaf size=243

$$\begin{aligned} & -\frac{a^2 \sqrt[3]{c} \log\left(c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)}{9b^{2/3}} + \frac{a^2 \sqrt[3]{c} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{bc^{2/3}(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} + c^{4/3}\right)}{18b^{2/3}} \\ & -\frac{a^2 \sqrt[3]{c} \tan^{-1}\left(\frac{\frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} + c^{2/3}}{\sqrt[3]{c^{2/3}}}\right)}{3\sqrt[3]{b^{2/3}}} + \frac{a(cx)^{4/3} \sqrt[3]{a+bx^2}}{3c} + \frac{(cx)^{4/3} (a+bx^2)^{4/3}}{4c} \end{aligned}$$

[Out] (a\*(c\*x)^(4/3)\*(a+b\*x^2)^(1/3))/(3\*c) + ((c\*x)^(4/3)\*(a+b\*x^2)^(4/3))/(4\*c) - (a^2\*c^(1/3)\*ArcTan[(c^(2/3)+(2\*b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3)]/(Sqrt[3]\*c^(2/3)))/(3\*Sqrt[3]\*b^(2/3)) - (a^2\*c^(1/3)\*Log[c^(2/3)-(b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3)]/(9\*b^(2/3)))+(a^2\*c^(1/3)\*Log[c^(4/3)+(b^(2/3)\*(c\*x)^(4/3))/(a+b\*x^2)^(2/3)+(b^(1/3)\*c^(2/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3)]/(18\*b^(2/3))

**Rubi [A]** time = 0.6546, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$\begin{aligned} & -\frac{a^2 \sqrt[3]{c} \log\left(c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)}{9b^{2/3}} + \frac{a^2 \sqrt[3]{c} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{bc^{2/3}(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} + c^{4/3}\right)}{18b^{2/3}} \\ & -\frac{a^2 \sqrt[3]{c} \tan^{-1}\left(\frac{\frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} + c^{2/3}}{\sqrt[3]{c^{2/3}}}\right)}{3\sqrt[3]{b^{2/3}}} + \frac{a(cx)^{4/3} \sqrt[3]{a+bx^2}}{3c} + \frac{(cx)^{4/3} (a+bx^2)^{4/3}}{4c} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(1/3)\*(a+b\*x^2)^(4/3),x]

[Out] (a\*(c\*x)^(4/3)\*(a+b\*x^2)^(1/3))/(3\*c) + ((c\*x)^(4/3)\*(a+b\*x^2)^(4/3))/(4\*c) - (a^2\*c^(1/3)\*ArcTan[(c^(2/3)+(2\*b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3)]/(Sqrt[3]\*c^(2/3)))/(3\*Sqrt[3]\*b^(2/3)) - (a^2\*c^(1/3)\*Log[c^(2/3)-(b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3)]/(9\*b^(2/3)))+(a^2\*c^(1/3)\*Log[c^(4/3)+(b^(2/3)\*(c\*x)^(4/3))/(a+b\*x^2)^(2/3)+(b^(1/3)\*c^(2/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3)]/(18\*b^(2/3))

**Rubi in Sympy [A]** time = 63.639, size = 223, normalized size = 0.92

$$\begin{aligned} & -\frac{a^2 \sqrt[3]{c} \log\left(-\frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} + c^{2/3}\right)}{9b^{2/3}} + \frac{a^2 \sqrt[3]{c} \log\left(\frac{b^{2/3}(cx)^{4/3}}{c^{4/3}(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b(cx)^{2/3}}}{c^{2/3} \sqrt[3]{a+bx^2}} + 1\right)}{18b^{2/3}} \\ & -\frac{\sqrt[3]{3} a^2 \sqrt[3]{c} \operatorname{atan}\left(\frac{\sqrt[3]{\left(\frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} + c^{2/3}\right)}}{c^{2/3}}\right)}{9b^{2/3}} + \frac{a(cx)^{4/3} \sqrt[3]{a+bx^2}}{3c} + \frac{(cx)^{4/3} (a+bx^2)^{4/3}}{4c} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(1/3)\*(b\*x\*\*2+a)\*\*(4/3),x)

[Out]  $-a^{2/3}c^{1/3}\log(-b^{1/3}(cx)^{2/3}/(a+bx^2)^{1/3}) + c^{2/3}/(9b^{2/3}) + a^{2/3}c^{1/3}\log(b^{2/3}(cx)^{4/3}/(c^{4/3}(a+bx^2)^{2/3})) + b^{1/3}(cx)^{2/3}/(c^{2/3}(a+bx^2)^{1/3}) + 1/(18b^{2/3}) - \sqrt{3}a^{2/3}c^{1/3}\operatorname{atan}(\sqrt{3}(2b^{1/3}(cx)^{2/3}/(3(a+bx^2)^{1/3})) + c^{2/3}/(3c))/c^{2/3}/(9b^{2/3})) + a(cx)^{4/3}(a+bx^2)^{1/3}/(3c) + (cx)^{4/3}(a+bx^2)^{4/3}/(4c)$

**Mathematica [C]** time = 0.0660986, size = 83, normalized size = 0.34

$$\frac{\sqrt[3]{cx} \left( 2a^2x \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^2}{a} \right) + 7a^2x + 10abx^3 + 3b^2x^5 \right)}{12(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(1/3)\*(a + b\*x^2)^(4/3), x]

[Out]  $((cx)^{1/3}(7a^2x + 10a^2bx^3 + 3b^2x^5 + 2a^2x(1 + (bx^2)/a)^{2/3}\operatorname{Hypergeometric2F1}[2/3, 2/3, 5/3, -(bx^2)/a]))/(12(a+bx^2)^{2/3})$

**Maple [F]** time = 0.032, size = 0, normalized size = 0.

$$\int \sqrt[3]{cx} (bx^2 + a)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(1/3)\*(b\*x^2+a)^(4/3), x)

[Out] int((c\*x)^(1/3)\*(b\*x^2+a)^(4/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)\*(c\*x)^(1/3), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)\*(c\*x)^(1/3), x, algorithm="fricas")

[Out] Timed out

**Sympy [A]** time = 150.617, size = 46, normalized size = 0.19

$$\frac{a^{\frac{4}{3}} \sqrt[3]{cx^{\frac{4}{3}}} \left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2 \left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(1/3)\*(b\*x\*\*2+a)\*\*(4/3),x)

[Out] a\*\*(4/3)\*c\*\*(1/3)\*x\*\*(4/3)\*gamma(2/3)\*hyper((-4/3, 2/3), (5/3, ), b\*x\*\*2\*exp\_polar(1\*pi)/a)/(2\*gamma(5/3))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{4}{3}} (cx)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)\*(c\*x)^(1/3),x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(4/3)\*(c\*x)^(1/3), x)



$$3.757 \quad \int \frac{(a+bx^2)^{4/3}}{(cx)^{5/3}} dx$$

**Optimal.** Leaf size=233

$$\frac{a\sqrt[3]{b} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}\right)}{3c^{5/3}} - \frac{2a\sqrt[3]{b} \log\left(c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c^{5/3}}$$

$$- \frac{2a\sqrt[3]{b} \tan^{-1}\left(\frac{\sqrt[3]{b}(cx)^{2/3} + c^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt{3}c^{5/3}} + \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3(a+bx^2)^{4/3}}{2c(cx)^{2/3}}$$

[Out]  $(2*b*(c*x)^{(4/3)}*(a + b*x^2)^{(1/3)})/c^3 - (3*(a + b*x^2)^{(4/3)})/(2*c*(c*x)^{(2/3)}) - (2*a*b^{(1/3)}*ArcTan[(c^{(2/3)} + (2*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(Sqrt[3]*c^{(2/3)})])/(Sqrt[3]*c^{(5/3)}) - (2*a*b^{(1/3)}*Log[c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})]/(3*c^{(5/3)}) + (a*b^{(1/3)}*Log[c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a + b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})]/(3*c^{(5/3)})$

**Rubi [A]** time = 0.658101, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$

$$\frac{a\sqrt[3]{b} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}\right)}{3c^{5/3}} - \frac{2a\sqrt[3]{b} \log\left(c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{3c^{5/3}}$$

$$- \frac{2a\sqrt[3]{b} \tan^{-1}\left(\frac{\sqrt[3]{b}(cx)^{2/3} + c^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt{3}c^{5/3}} + \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3(a+bx^2)^{4/3}}{2c(cx)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(4/3)/(c\*x)^(5/3), x]

[Out]  $(2*b*(c*x)^{(4/3)}*(a + b*x^2)^{(1/3)})/c^3 - (3*(a + b*x^2)^{(4/3)})/(2*c*(c*x)^{(2/3)}) - (2*a*b^{(1/3)}*ArcTan[(c^{(2/3)} + (2*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(Sqrt[3]*c^{(2/3)})])/(Sqrt[3]*c^{(5/3)}) - (2*a*b^{(1/3)}*Log[c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})]/(3*c^{(5/3)}) + (a*b^{(1/3)}*Log[c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a + b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})]/(3*c^{(5/3)})$

**Rubi in Sympy [A]** time = 63.887, size = 224, normalized size = 0.96

$$\frac{2a\sqrt[3]{b} \log\left(-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{2/3}\right)}{3c^{5/3}} + \frac{a\sqrt[3]{b} \log\left(\frac{b^{2/3}(cx)^{4/3}}{c^{4/3}(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}} + 1\right)}{3c^{5/3}}$$

$$- \frac{2\sqrt{3}a\sqrt[3]{b} \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{2\sqrt[3]{b}(cx)^{2/3}}{3\sqrt[3]{a+bx^2}} + \frac{c^{2/3}}{3}\right)}{c^{2/3}}\right)}{3c^{5/3}} + \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3(a+bx^2)^{4/3}}{2c(cx)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(4/3)/(c\*x)\*\*(5/3), x)

[Out]  $-2ab^{1/3} \log(-b^{1/3}(cx)^{2/3}/(a+bx^2)^{1/3} + c^{2/3})/(3c^{5/3}) + ab^{1/3} \log(b^{2/3}(cx)^{4/3}/(c^{4/3}(a+bx^2)^{2/3}) + b^{1/3}(cx)^{2/3}/(c^{2/3}(a+bx^2)^{1/3}) + 1)/(3c^{5/3}) - 2\sqrt{3}ab^{1/3} \operatorname{atan}(\sqrt{3}(2b^{1/3}(cx)^{2/3}/(3(a+bx^2)^{1/3}) + c^{2/3}/3)/c^{2/3})/(3c^{5/3}) + 2b^{1/3}(cx)^{4/3}(a+bx^2)^{1/3}/c^3 - 3(a+bx^2)^{4/3}/(2c^{2/3}(cx)^{2/3})$

**Mathematica [C]** time = 0.0594999, size = 83, normalized size = 0.36

$$\frac{x \left( -3a^2 + 2abx^2 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^2}{a} \right) - 2abx^2 + b^2x^4 \right)}{2(cx)^{5/3} (a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(4/3)/(c\*x)^(5/3), x]

[Out]  $(x^* (-3*a^2 - 2*a*b*x^2 + b^2*x^4 + 2*a*b*x^2*(1 + (b*x^2)/a))^{2/3} * \operatorname{Hypergeometric2F1}[2/3, 2/3, 5/3, -((b*x^2)/a)]) / (2*(c*x)^{5/3} * (a + b*x^2)^{2/3})$

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int 1 (bx^2 + a)^{\frac{4}{3}} (cx)^{-\frac{5}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(4/3)/(c\*x)^(5/3), x)

[Out] int((b\*x^2+a)^(4/3)/(c\*x)^(5/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(5/3), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(5/3), x, algorithm="fricas")

[Out] Timed out

**Sympy [A]** time = 171.363, size = 49, normalized size = 0.21

$$\frac{a^{\frac{4}{3}} \left(-\frac{1}{3}\right) {}_2F_1\left(-\frac{4}{3}, -\frac{1}{3} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2c^{\frac{5}{3}} x^{\frac{2}{3}} \left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(4/3)/(c\*x)\*\*(5/3), x)

[Out] a\*\*(4/3)\*gamma(-1/3)\*hyper((-4/3, -1/3), (2/3, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*c\*\*(5/3)\*x\*\*(2/3)\*gamma(2/3))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(5/3), x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(5/3), x)

$$3.758 \quad \int \frac{(a+bx^2)^{4/3}}{(cx)^{11/3}} dx$$

**Optimal.** Leaf size=234

$$\begin{aligned} & -\frac{b^{4/3} \log\left(c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)}{2c^{11/3}} + \frac{b^{4/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}\right)}{4c^{11/3}} \\ & -\frac{\sqrt{3}b^{4/3} \tan^{-1}\left(\frac{\frac{2}{3}\sqrt[3]{b(cx)^{2/3}} + c^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{11/3}} - \frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} \end{aligned}$$

[Out]  $(-3*b*(a + b*x^2)^{(1/3)})/(2*c^3*(c*x)^{(2/3)}) - (3*(a + b*x^2)^{(4/3)})/(8*c*(c*x)^{(8/3)}) - (\text{Sqrt}[3]*b^{(4/3)}*\text{ArcTan}[(c^{(2/3)} + (2*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})]/(\text{Sqrt}[3]*c^{(2/3)}))/((2*c^{(11/3)}) - (b^{(4/3)}*\text{Log}[c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)}])/(2*c^{(11/3)}) + (b^{(4/3)}*\text{Log}[c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a + b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)}])/(4*c^{(11/3)})$

**Rubi [A]** time = 0.667193, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$\begin{aligned} & -\frac{b^{4/3} \log\left(c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)}{2c^{11/3}} + \frac{b^{4/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}\right)}{4c^{11/3}} \\ & -\frac{\sqrt{3}b^{4/3} \tan^{-1}\left(\frac{\frac{2}{3}\sqrt[3]{b(cx)^{2/3}} + c^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2c^{11/3}} - \frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^{(4/3)}/(c*x)^{(11/3)}, x]$

[Out]  $(-3*b*(a + b*x^2)^{(1/3)})/(2*c^3*(c*x)^{(2/3)}) - (3*(a + b*x^2)^{(4/3)})/(8*c*(c*x)^{(8/3)}) - (\text{Sqrt}[3]*b^{(4/3)}*\text{ArcTan}[(c^{(2/3)} + (2*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})]/(\text{Sqrt}[3]*c^{(2/3)}))/((2*c^{(11/3)}) - (b^{(4/3)}*\text{Log}[c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)}])/(2*c^{(11/3)}) + (b^{(4/3)}*\text{Log}[c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a + b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)}])/(4*c^{(11/3)})$

**Rubi in Sympy [A]** time = 64.6492, size = 218, normalized size = 0.93

$$\begin{aligned} & -\frac{b^{4/3} \log\left(-\frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} + c^{2/3}\right)}{2c^{11/3}} + \frac{b^{4/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{c^{4/3}(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}} + 1\right)}{4c^{11/3}} \\ & -\frac{\sqrt{3}b^{4/3} \text{atan}\left(\frac{\sqrt{3}\left(\frac{2}{3}\sqrt[3]{b(cx)^{2/3}} + c^{2/3}\right)}{c^{2/3}}\right)}{2c^{11/3}} - \frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x**2+a)**(4/3)/(c*x)**(11/3), x)$

[Out]  $-b^{4/3} \log(-b^{1/3} (c x)^{2/3} / (a + b x^2)^{1/3}) + c^{2/3} (2/3) / (2 c^{11/3}) + b^{4/3} \log(b^{2/3} (c x)^{4/3} / (c^{4/3} (a + b x^2)^{2/3})) + b^{1/3} (c x)^{2/3} / (c^{2/3} (a + b x^2)^{1/3}) + 1 / (4 c^{11/3}) - \sqrt{3} b^{4/3} \operatorname{atan}(\sqrt{3} (2 b^{1/3} (c x)^{2/3} / (3 (a + b x^2)^{1/3}) + c^{2/3} / 3) / c^{2/3}) / (2 c^{11/3}) - 3 b (a + b x^2)^{1/3} / (2 c^3 (c x)^{2/3}) - 3 (a + b x^2)^{4/3} / (8 c (c x)^{8/3})$

**Mathematica [C]** time = 0.0678655, size = 83, normalized size = 0.35

$$\frac{3x \left( a^2 - 2b^2 x^4 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^2}{a} \right) + 6abx^2 + 5b^2 x^4 \right)}{8(cx)^{11/3} (a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(4/3)/(c\*x)^(11/3), x]

[Out]  $(-3 x (a^2 + 6 a b x^2 + 5 b^2 x^4 - 2 b^2 x^4 (1 + (b x^2)/a)^{2/3} \operatorname{Hypergeometric2F1}[2/3, 2/3, 5/3, -(b x^2)/a])) / (8 (c x)^{11/3} (a + b x^2)^{2/3})$

**Maple [F]** time = 0.045, size = 0, normalized size = 0.

$$\int 1 (bx^2 + a)^{\frac{4}{3}} (cx)^{-\frac{11}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(4/3)/(c\*x)^(11/3), x)

[Out] int((b\*x^2+a)^(4/3)/(c\*x)^(11/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(11/3), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(11/3), x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(4/3)/(c*x)**(11/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{11}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)/(c*x)^(11/3),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(4/3)/(c*x)^(11/3), x)`

$$3.759 \quad \int \frac{(a+bx^2)^{4/3}}{(cx)^{17/3}} dx$$

**Optimal.** Leaf size=28

$$-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{14/3}}$$

[Out]  $(-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(14/3))$

**Rubi [A]** time = 0.0279684, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{14/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(4/3)/(c\*x)^(17/3), x]

[Out]  $(-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(14/3))$

**Rubi in Sympy [A]** time = 3.61206, size = 24, normalized size = 0.86

$$-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{14/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(4/3)/(c\*x)\*\*(17/3), x)

[Out]  $-3*(a + b*x**2)**(7/3)/(14*a*c*(c*x)**(14/3))$

**Mathematica [A]** time = 0.0441429, size = 31, normalized size = 1.11

$$-\frac{3\sqrt[3]{cx}(a+bx^2)^{7/3}}{14ac^6x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(4/3)/(c\*x)^(17/3), x]

[Out]  $(-3*(c*x)^(1/3)*(a + b*x^2)^(7/3))/(14*a*c^6*x^5)$

**Maple [A]** time = 0.007, size = 21, normalized size = 0.8

$$-\frac{3x}{14a}(bx^2+a)^{7/3}(cx)^{-17/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(4/3)/(c\*x)^(17/3), x)

[Out]  $-3/14 * x * (b * x^2 + a)^{(7/3)} / a / (c * x)^{(17/3)}$

**Maxima [A]** time = 1.39078, size = 27, normalized size = 0.96

$$-\frac{3 (bx^2 + a)^{\frac{7}{3}}}{14 ac^{\frac{17}{3}} x^{\frac{14}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)/(c*x)^(17/3),x, algorithm="maxima")`

[Out]  $-3/14 * (b * x^2 + a)^{(7/3)} / (a * c^{(17/3)} * x^{(14/3)})$

**Fricas [A]** time = 0.222927, size = 58, normalized size = 2.07

$$-\frac{3 (b^2x^4 + 2abx^2 + a^2) (bx^2 + a)^{\frac{1}{3}} (cx)^{\frac{1}{3}}}{14 ac^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)/(c*x)^(17/3),x, algorithm="fricas")`

[Out]  $-3/14 * (b^2 * x^4 + 2 * a * b * x^2 + a^2) * (b * x^2 + a)^{(1/3)} * (c * x)^{(1/3)} / (a * c^6 * x^5)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(4/3)/(c*x)**(17/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{17}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)/(c*x)^(17/3),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(4/3)/(c*x)^(17/3), x)`



$$3.760 \quad \int \frac{(a+bx^2)^{4/3}}{(cx)^{23/3}} dx$$

**Optimal.** Leaf size=57

$$\frac{9(a+bx^2)^{10/3}}{140a^2c(cx)^{20/3}} - \frac{3(a+bx^2)^{7/3}}{14ac(cx)^{20/3}}$$

[Out]  $(-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(20/3)) + (9*(a + b*x^2)^(10/3))/(140*a^2*c*(c*x)^(20/3))$

**Rubi [A]** time = 0.0578139, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{9(a+bx^2)^{10/3}}{140a^2c(cx)^{20/3}} - \frac{3(a+bx^2)^{7/3}}{14ac(cx)^{20/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(4/3)/(c\*x)^(23/3), x]

[Out]  $(-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(20/3)) + (9*(a + b*x^2)^(10/3))/(140*a^2*c*(c*x)^(20/3))$

**Rubi in Sympy [A]** time = 6.75506, size = 48, normalized size = 0.84

$$-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{20/3}} + \frac{9(a+bx^2)^{10/3}}{140a^2c(cx)^{20/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(4/3)/(c\*x)\*\*(23/3), x)

[Out]  $-3*(a + b*x**2)**(7/3)/(14*a*c*(c*x)**(20/3)) + 9*(a + b*x**2)**(10/3)/(140*a**2*c*(c*x)**(20/3))$

**Mathematica [A]** time = 0.058993, size = 41, normalized size = 0.72

$$\frac{3\sqrt[3]{cx}(a+bx^2)^{7/3}(3bx^2-7a)}{140a^2c^8x^7}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(4/3)/(c\*x)^(23/3), x]

[Out]  $(3*(c*x)^(1/3)*(a + b*x^2)^(7/3)*(-7*a + 3*b*x^2))/(140*a^2*c^8*x^7)$

**Maple [A]** time = 0.006, size = 31, normalized size = 0.5

$$-\frac{3x(-3bx^2+7a)}{140a^2}(bx^2+a)^{7/3}(cx)^{-23/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(4/3)/(c*x)^(23/3),x)`

[Out] `-3/140*x*(b*x^2+a)^(7/3)*(-3*b*x^2+7*a)/a^2/(c*x)^(23/3)`

**Maxima [A]** time = 1.41326, size = 51, normalized size = 0.89

$$\frac{3 \left( \frac{10 (bx^2+a)^{\frac{7}{3}} b}{x^{\frac{14}{3}}} - \frac{7 (bx^2+a)^{\frac{10}{3}}}{x^{\frac{20}{3}}} \right)}{140 a^2 c^{\frac{23}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)/(c*x)^(23/3),x, algorithm="maxima")`

[Out] `3/140*(10*(b*x^2 + a)^(7/3)*b/x^(14/3) - 7*(b*x^2 + a)^(10/3)/x^(20/3))/(a^2*c^(23/3))`

**Fricas [A]** time = 0.222702, size = 77, normalized size = 1.35

$$\frac{3 (3 b^3 x^6 - a b^2 x^4 - 11 a^2 b x^2 - 7 a^3) (b x^2 + a)^{\frac{1}{3}} (c x)^{\frac{1}{3}}}{140 a^2 c^8 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)/(c*x)^(23/3),x, algorithm="fricas")`

[Out] `3/140*(3*b^3*x^6 - a*b^2*x^4 - 11*a^2*b*x^2 - 7*a^3)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^2*c^8*x^7)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(4/3)/(c*x)**(23/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{23}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)/(c*x)^(23/3),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(4/3)/(c*x)^(23/3), x)`

$$3.761 \quad \int \frac{(a+bx^2)^{4/3}}{(cx)^{29/3}} dx$$

**Optimal.** Leaf size=85

$$-\frac{27(a+bx^2)^{13/3}}{910a^3c(cx)^{26/3}} + \frac{9(a+bx^2)^{10/3}}{70a^2c(cx)^{26/3}} - \frac{3(a+bx^2)^{7/3}}{14ac(cx)^{26/3}}$$

[Out]  $(-3*(a + b*x^2)^{(7/3)})/(14*a*c*(c*x)^{(26/3)}) + (9*(a + b*x^2)^{(10/3)})/(70*a^2*c*(c*x)^{(26/3)}) - (27*(a + b*x^2)^{(13/3)})/(910*a^3*c*(c*x)^{(26/3)})$

**Rubi [A]** time = 0.0888606, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$-\frac{27(a+bx^2)^{13/3}}{910a^3c(cx)^{26/3}} + \frac{9(a+bx^2)^{10/3}}{70a^2c(cx)^{26/3}} - \frac{3(a+bx^2)^{7/3}}{14ac(cx)^{26/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(4/3)/(c\*x)^(29/3), x]

[Out]  $(-3*(a + b*x^2)^{(7/3)})/(14*a*c*(c*x)^{(26/3)}) + (9*(a + b*x^2)^{(10/3)})/(70*a^2*c*(c*x)^{(26/3)}) - (27*(a + b*x^2)^{(13/3)})/(910*a^3*c*(c*x)^{(26/3)})$

**Rubi in Sympy [A]** time = 10.7469, size = 73, normalized size = 0.86

$$-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{26/3}} + \frac{9(a+bx^2)^{10/3}}{70a^2c(cx)^{26/3}} - \frac{27(a+bx^2)^{13/3}}{910a^3c(cx)^{26/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(4/3)/(c\*x)\*\*(29/3), x)

[Out]  $-3*(a + b*x**2)**(7/3)/(14*a*c*(c*x)**(26/3)) + 9*(a + b*x**2)**(10/3)/(70*a**2*c*(c*x)**(26/3)) - 27*(a + b*x**2)**(13/3)/(910*a**3*c*(c*x)**(26/3))$

**Mathematica [A]** time = 0.0635553, size = 52, normalized size = 0.61

$$\frac{3(a+bx^2)^{7/3}(35a^2 - 21abx^2 + 9b^2x^4)}{910a^3c^9x^8(cx)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(4/3)/(c\*x)^(29/3), x]

[Out]  $(-3*(a + b*x^2)^{(7/3)}*(35*a^2 - 21*a*b*x^2 + 9*b^2*x^4))/(910*a^3*c^9*x^8*(c*x)^{(2/3)})$

**Maple [A]** time = 0.007, size = 42, normalized size = 0.5

$$-\frac{3x(9b^2x^4 - 21abx^2 + 35a^2)}{910a^3} (bx^2 + a)^{7/3} (cx)^{-29/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(4/3)/(c*x)^(29/3),x)`

[Out] 
$$-3/910*x*(b*x^2+a)^{7/3}*(9*b^2*x^4-21*a*b*x^2+35*a^2)/a^3/(c*x)^{29/3}$$

**Maxima [A]** time = 1.39171, size = 74, normalized size = 0.87

$$3 \frac{\left( \frac{65 (bx^2+a)^{7/3} b^2}{x^{14/3}} - \frac{91 (bx^2+a)^{10/3} b}{x^{20/3}} + \frac{35 (bx^2+a)^{13/3}}{x^{26/3}} \right)}{910 a^3 c^{29/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)/(c*x)^(29/3),x, algorithm="maxima")`

[Out] 
$$-3/910*(65*(b*x^2 + a)^{7/3}*b^2/x^{14/3} - 91*(b*x^2 + a)^{10/3}*b/x^{20/3} + 35*(b*x^2 + a)^{13/3}/x^{26/3})/(a^3*c^{29/3})$$

**Fricas [A]** time = 0.225448, size = 92, normalized size = 1.08

$$-\frac{3(9b^4x^8 - 3ab^3x^6 + 2a^2b^2x^4 + 49a^3bx^2 + 35a^4)(bx^2 + a)^{1/3}(cx)^{1/3}}{910a^3c^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)/(c*x)^(29/3),x, algorithm="fricas")`

[Out] 
$$-3/910*(9*b^4*x^8 - 3*a*b^3*x^6 + 2*a^2*b^2*x^4 + 49*a^3*b*x^2 + 35*a^4)*(b*x^2 + a)^{1/3}*(c*x)^{1/3}/(a^3*c^{10}*x^9)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(4/3)/(c*x)**(29/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{4/3}}{(cx)^{29/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)/(c*x)^(29/3),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(4/3)/(c*x)^(29/3), x)`

$$3.762 \quad \int (cx)^{10/3} (a + bx^2)^{4/3} dx$$

**Optimal.** Leaf size=479

$$\frac{16a^3c^3\sqrt[3]{cx}\sqrt{a+bx^2}}{405b^2} + \frac{8a^3c^{7/3}\sqrt[3]{cx}\sqrt{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}+c^{4/3}}{(a+bx^2)^{2/3}+\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)^2}}F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\right)(2+\sqrt{3})}{405\sqrt[3]{3}b^2\sqrt{\frac{\sqrt[3]{b(cx)^{2/3}}\left(c^{2/3}-\frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

$$+ \frac{16a^2c(cx)^{7/3}\sqrt[3]{a+bx^2}}{945b} + \frac{(cx)^{13/3}(a+bx^2)^{4/3}}{7c} + \frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}$$

[Out]  $(-16*a^3*c^3*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)})/(405*b^2) + (16*a^2*c*(c*x)^{(7/3)}*(a+b*x^2)^{(1/3)})/(945*b) + (8*a*(c*x)^{(13/3)}*(a+b*x^2)^{(1/3)})/(105*c) + ((c*x)^{(13/3)}*(a+b*x^2)^{(4/3)})/(7*c) + (8*a^3*c^{7/3}*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)}*(c^{2/3} - (b^{1/3})*(c*x)^{(2/3)))/(a+b*x^2)^{(1/3)}) * Sqrt[(c^{4/3} + (b^{2/3}*(c*x)^{(4/3)))/(a+b*x^2)^{(2/3)} + (b^{1/3})*c^{2/3}*(c*x)^{(2/3)))/(a+b*x^2)^{(1/3)}] / (c^{2/3} - ((1 + Sqrt[3])*b^{1/3}*(c*x)^{(2/3)))/(a+b*x^2)^{(1/3)})^2 * EllipticF[ArcCos[(c^{2/3} - ((1 - Sqrt[3])*b^{1/3}*(c*x)^{(2/3)))/(a+b*x^2)^{(1/3)})/(c^{2/3} - ((1 + Sqrt[3])*b^{1/3}*(c*x)^{(2/3)))/(a+b*x^2)^{(1/3)})], (2 + Sqrt[3])/4] / (405*3^{1/4})*b^2*Sqrt[-((b^{1/3}*(c*x)^{(2/3})*c^{2/3} - (b^{1/3}*(c*x)^{(2/3)))/(a+b*x^2)^{(1/3)))/((a+b*x^2)^{(1/3})*c^{2/3} - ((1 + Sqrt[3])*b^{1/3}*(c*x)^{(2/3)))/(a+b*x^2)^{(1/3)})^2]]$

**Rubi [A]** time = 1.83371, antiderivative size = 479, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{16a^3c^3\sqrt[3]{cx}\sqrt{a+bx^2}}{405b^2} + \frac{8a^3c^{7/3}\sqrt[3]{cx}\sqrt{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}+c^{4/3}}{(a+bx^2)^{2/3}+\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)^2}}F\left(\cos^{-1}\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}\right)\middle|\frac{1}{4}\right)(2+\sqrt{3})}{405\sqrt[3]{3}b^2\sqrt{\frac{\sqrt[3]{b(cx)^{2/3}}\left(c^{2/3}-\frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

$$+ \frac{16a^2c(cx)^{7/3}\sqrt[3]{a+bx^2}}{945b} + \frac{(cx)^{13/3}(a+bx^2)^{4/3}}{7c} + \frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*x)^{(10/3)}*(a+b*x^2)^{(4/3)}, x]$

[Out]  $(-16*a^3*c^3*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)})/(405*b^2) + (16*a^2*c*(c*x)^{(7/3)}*(a+b*x^2)^{(1/3)})/(945*b) + (8*a*(c*x)^{(13/3)}*(a+b*x^2)^{(1/3)})/(105*c) + ((c*x)^{(13/3)}*(a+b*x^2)^{(4/3)})/(7*c) + (8*a^3*c^{7/3}*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)}*(c^{2/3} - (b^{1/3})*(c*x)^{(2/3)))/(a+b*x^2)^{(1/3)}) * Sqrt[(c^{4/3} + (b^{2/3}*(c*x)^{(4/3)))/(a+b*x^2)^{(2/3)} + (b^{1/3})*c^{2/3}*(c*x)^{(2/3)))/(a+b*x^2)^{(1/3)}] / (c^{2/3} - ((1 + Sqrt[3])*b^{1/3}*(c*x)^{(2/3)))/(a+b*x^2)^{(1/3)})^2 * EllipticF[ArcCos[(c^{2/3} - ((1 - Sqrt[3])*b^{1/3}*(c*x)^{(2/3)))/(a+b*x^2)^{(1/3)})/(c^{2/3} - ((1 + Sqrt[3])*b^{1/3}*(c*x)^{(2/3)))/(a+b*x^2)^{(1/3)})], (2 + Sqrt[3])/4] / (405*3^{1/4})*b^2*Sqrt[-((b^{1/3}*(c*x)^{(2/3})*c^{2/3} - (b^{1/3}*(c*x)^{(2/3)))/(a+b*x^2)^{(1/3)))/((a+b*x^2)^{(1/3})*c^{2/3} - ((1 + Sqrt[3])*b^{1/3}*(c*x)^{(2/3)))/(a+b*x^2)^{(1/3)})^2]]$

$$\frac{(c^2 x)^{2/3} / (a + b x^2)^{1/3} / (c^{2/3} - ((1 + \sqrt{3}) b^{1/3} (c^2 x)^{2/3} / (a + b x^2)^{1/3})) / (2 + \sqrt{3}) / 4]}{(405^3 3^{1/4} b^2 \sqrt{-((b^{1/3} (c^2 x)^{2/3} / (a + b x^2)^{1/3}) / (c^{2/3} - ((1 + \sqrt{3}) b^{1/3} (c^2 x)^{2/3} / (a + b x^2)^{1/3}))) / ((a + b x^2)^{1/3} (c^{2/3} - ((1 + \sqrt{3}) b^{1/3} (c^2 x)^{2/3} / (a + b x^2)^{1/3})))^2}})}{1215 b^2 \sqrt{\frac{a}{a + b x^2}} \sqrt{-\frac{\sqrt{b(c x)^{2/3} \left(-\frac{\sqrt{b(c x)^{2/3}}{\sqrt{a + b x^2}} + c^{2/3}\right)}}{\sqrt{b(c x)^{2/3} \left(-\frac{\sqrt{b(c x)^{2/3}}{\sqrt{a + b x^2}} + c^{2/3}\right)}}} (a + b x^2)^{2/3} \sqrt{-\frac{b x^2}{a + b x^2} + 1}}}$$

**Rubi in Sympy [A]** time = 54.5737, size = 462, normalized size = 0.96

$$\frac{8 \cdot 3^{\frac{3}{4}} a^4 c^{\frac{7}{3}} \sqrt{c x} \sqrt{\frac{\frac{b^{\frac{2}{3}} (c x)^{\frac{4}{3}}}{(a + b x^2)^{\frac{2}{3}}} + \frac{\sqrt{b} c^{\frac{2}{3}} (c x)^{\frac{2}{3}}}{\sqrt{a + b x^2}} + c^{\frac{4}{3}}}}{\left(\frac{\sqrt{b} (c x)^{\frac{2}{3}} (-\sqrt{3} - 1)}{\sqrt{a + b x^2}} + c^{\frac{2}{3}}\right)^2} \left(-\frac{\sqrt{b} (c x)^{\frac{2}{3}}}{\sqrt{a + b x^2}} + c^{\frac{2}{3}}\right) F\left(\arcsin\left(\frac{\sqrt[3]{b} (c x)^{\frac{2}{3}} (-1 + \sqrt{3}) + c^{\frac{2}{3}}}{\sqrt[3]{a + b x^2}}\right)}{\sqrt[3]{b} (c x)^{\frac{2}{3}} (-\sqrt{3} - 1) + c^{\frac{2}{3}}}}{\frac{\sqrt{3}}{4} + \frac{1}{2}}\right)}}{1215 b^2 \sqrt{\frac{a}{a + b x^2}} \sqrt{-\frac{\sqrt{b} (c x)^{\frac{2}{3}} \left(-\frac{\sqrt{b} (c x)^{\frac{2}{3}}}{\sqrt{a + b x^2}} + c^{\frac{2}{3}}\right)}{\sqrt{b} (c x)^{\frac{2}{3}} \left(-\frac{\sqrt{b} (c x)^{\frac{2}{3}}}{\sqrt{a + b x^2}} + c^{\frac{2}{3}}\right)}} (a + b x^2)^{\frac{2}{3}} \sqrt{-\frac{b x^2}{a + b x^2} + 1}}}$$

$$-\frac{16 a^3 c^3 \sqrt{c x} \sqrt{a + b x^2}}{405 b^2} + \frac{16 a^2 c (c x)^{\frac{7}{3}} \sqrt{a + b x^2}}{945 b} + \frac{8 a (c x)^{\frac{13}{3}} \sqrt{a + b x^2}}{105 c} + \frac{(c x)^{\frac{13}{3}} (a + b x^2)^{\frac{4}{3}}}{7 c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x)**(10/3)*(b*x**2+a)**(4/3),x)`

[Out]  $8 \cdot 3^{3/4} a^4 c^{7/3} (c x)^{1/3} \sqrt{(b^{2/3} (c x)^{4/3} / (a + b x^2)^{2/3} + b^{1/3} c^{2/3} (c x)^{2/3} / (a + b x^2)^{1/3} + c^{4/3}) / (b^{1/3} (c x)^{2/3} (-\sqrt{3} - 1) / (a + b x^2)^{1/3} + c^{2/3})} \sqrt{-b^{1/3} (c x)^{2/3} / (a + b x^2)^{1/3} + c^{2/3}} \operatorname{elliptic}_f(\arcsin((b^{1/3} (c x)^{2/3} (-1 + \sqrt{3}) / (a + b x^2)^{1/3} + c^{2/3}) / (b^{1/3} (c x)^{2/3} (-1 + \sqrt{3}) / (a + b x^2)^{1/3} + c^{2/3})), \sqrt{3} / 4 + 1/2) / (1215 b^2 \sqrt{a / (a + b x^2)} \sqrt{-b^{1/3} (c x)^{2/3} / (a + b x^2)^{1/3} + c^{2/3}} / ((a + b x^2)^{2/3} + c^{2/3})} \sqrt{b^{1/3} (c x)^{2/3} (-\sqrt{3} - 1) / (a + b x^2)^{1/3} + c^{2/3}} \sqrt{-b x^2 / (a + b x^2) + 1} - 16 a^3 c^3 (c x)^{1/3} (a + b x^2)^{1/3} / (405 b^2) + 16 a^2 c (c x)^{7/3} (a + b x^2)^{1/3} / (945 b) + 8 a (c x)^{13/3} (a + b x^2)^{1/3} / (105 c) + (c x)^{13/3} (a + b x^2)^{4/3} / (7 c)$

**Mathematica [C]** time = 0.0780247, size = 109, normalized size = 0.23

$$\frac{c^3 \sqrt{c x} \left(112 a^4 \left(\frac{b x^2}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{6}, \frac{2}{3}; \frac{7}{6}; -\frac{b x^2}{a}\right) - 112 a^4 - 64 a^3 b x^2 + 669 a^2 b^2 x^4 + 1026 a b^3 x^6 + 405 b^4 x^8\right)}{2835 b^2 (a + b x^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c*x)^(10/3)*(a + b*x^2)^(4/3),x]`

[Out]  $(c^3 (c x)^{1/3} (-112 a^4 - 64 a^3 b x^2 + 669 a^2 b^2 x^4 + 1026 a b^3 x^6 + 405 b^4 x^8 + 112 a^4 (1 + (b x^2)/a)^{2/3} \operatorname{Hypergeometric2F1}[1/6, 2/3, 7/6, -(b x^2)/a]) / (2835 b^2 (a + b x^2)^{2/3})$

**Maple [F]** time = 0.035, size = 0, normalized size = 0.

$$\int (c x)^{\frac{10}{3}} (b x^2 + a)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(10/3)*(b*x^2+a)^(4/3),x)`

[Out] `int((c*x)^(10/3)*(b*x^2+a)^(4/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{4}{3}} (cx)^{\frac{10}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)*(c*x)^(10/3),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(4/3)*(c*x)^(10/3), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bc^3x^5 + ac^3x^3\right)(bx^2 + a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)*(c*x)^(10/3),x, algorithm="fricas")`

[Out] `integral((b*c^3*x^5 + a*c^3*x^3)*(b*x^2 + a)^(1/3)*(c*x)^(1/3), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(10/3)*(b*x**2+a)**(4/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{4}{3}} (cx)^{\frac{10}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)*(c*x)^(10/3),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(4/3)*(c*x)^(10/3), x)`

### 3.763 $\int (cx)^{4/3} (a + bx^2)^{4/3} dx$

**Optimal.** Leaf size=448

$$\frac{8a^2 \sqrt[3]{c} \sqrt[3]{cx} \sqrt[3]{a + bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a + bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3} + \sqrt[3]{b} c^{2/3} (cx)^{2/3} + c^{4/3}}{(a+bx^2)^{2/3} + \sqrt[3]{a + bx^2}}}{\left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a + bx^2}} \right)^2}} F\left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2 + a}}}{c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2 + a}}} \right) \right) \frac{1}{4} (2 + \sqrt{3})}{135 \sqrt[3]{3} b \sqrt{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a + bx^2}} \right)}{\sqrt[3]{a + bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a + bx^2}} \right)^2}}}$$

$$+ \frac{16a^2 c \sqrt[3]{c} \sqrt[3]{a + bx^2}}{135b} + \frac{(cx)^{7/3} (a + bx^2)^{4/3}}{5c} + \frac{8a(cx)^{7/3} \sqrt[3]{a + bx^2}}{45c}$$

[Out]  $(16 * a^2 * c * (c * x)^{(1/3)} * (a + b * x^2)^{(1/3)}) / (135 * b) + (8 * a * (c * x)^{(7/3)} * (a + b * x^2)^{(1/3)}) / (45 * c) + ((c * x)^{(7/3)} * (a + b * x^2)^{(4/3)}) / (5 * c) - (8 * a^2 * c^{(1/3)} * (c * x)^{(1/3)} * (a + b * x^2)^{(1/3)} * (c^{(2/3)} - (b^{(1/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)}) * \text{Sqrt}[(c^{(4/3)} + (b^{(2/3)} * (c * x)^{(4/3)}) / (a + b * x^2)^{(2/3)} + (b^{(1/3)} * c^{(2/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)}) / (c^{(2/3)} - ((1 + \text{Sqrt}[3]) * b^{(1/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)})^2] * \text{EllipticF}[\text{ArcCos}[(c^{(2/3)} - ((1 - \text{Sqrt}[3]) * b^{(1/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)}) / (c^{(2/3)} - ((1 + \text{Sqrt}[3]) * b^{(1/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)})], (2 + \text{Sqrt}[3]) / 4]) / (135 * 3^{(1/4)} * b * \text{Sqrt}[-((b^{(1/3)} * (c * x)^{(2/3)} * c^{(2/3)} - (b^{(1/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)}) / ((a + b * x^2)^{(1/3)} * (c^{(2/3)} - ((1 + \text{Sqrt}[3]) * b^{(1/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)})^2)])$

**Rubi [A]** time = 1.6542, antiderivative size = 448, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{8a^2 \sqrt[3]{c} \sqrt[3]{cx} \sqrt[3]{a + bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a + bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3} + \sqrt[3]{b} c^{2/3} (cx)^{2/3} + c^{4/3}}{(a+bx^2)^{2/3} + \sqrt[3]{a + bx^2}}}{\left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a + bx^2}} \right)^2}} F\left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2 + a}}}{c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2 + a}}} \right) \right) \frac{1}{4} (2 + \sqrt{3})}{135 \sqrt[3]{3} b \sqrt{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a + bx^2}} \right)}{\sqrt[3]{a + bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a + bx^2}} \right)^2}}}$$

$$+ \frac{16a^2 c \sqrt[3]{c} \sqrt[3]{a + bx^2}}{135b} + \frac{(cx)^{7/3} (a + bx^2)^{4/3}}{5c} + \frac{8a(cx)^{7/3} \sqrt[3]{a + bx^2}}{45c}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c * x)^{(4/3)} * (a + b * x^2)^{(4/3)}, x]$

[Out]  $(16 * a^2 * c * (c * x)^{(1/3)} * (a + b * x^2)^{(1/3)}) / (135 * b) + (8 * a * (c * x)^{(7/3)} * (a + b * x^2)^{(1/3)}) / (45 * c) + ((c * x)^{(7/3)} * (a + b * x^2)^{(4/3)}) / (5 * c) - (8 * a^2 * c^{(1/3)} * (c * x)^{(1/3)} * (a + b * x^2)^{(1/3)} * (c^{(2/3)} - (b^{(1/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)}) * \text{Sqrt}[(c^{(4/3)} + (b^{(2/3)} * (c * x)^{(4/3)}) / (a + b * x^2)^{(2/3)} + (b^{(1/3)} * c^{(2/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)}) / (c^{(2/3)} - ((1 + \text{Sqrt}[3]) * b^{(1/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)})^2] * \text{EllipticF}[\text{ArcCos}[(c^{(2/3)} - ((1 - \text{Sqrt}[3]) * b^{(1/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)}) / (c^{(2/3)} - ((1 + \text{Sqrt}[3]) * b^{(1/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)})], (2 + \text{Sqrt}[3]) / 4]) / (135 * 3^{(1/4)} * b * \text{Sqrt}[-((b^{(1/3)} * (c * x)^{(2/3)} * c^{(2/3)} - (b^{(1/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)}) / ((a + b * x^2)^{(1/3)} * (c^{(2/3)} - ((1 + \text{Sqrt}[3]) * b^{(1/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)})^2)])$



**Rubi in Sympy [A]** time = 43.9759, size = 430, normalized size = 0.96

$$\frac{8 \cdot 3^{\frac{3}{4}} a^3 \sqrt[3]{c} \sqrt[3]{cx} \sqrt{\frac{\frac{b^{\frac{2}{3}}(cx)^{\frac{4}{3}}}{(a+bx^2)^{\frac{2}{3}} + \frac{\sqrt[3]{b}c^{\frac{2}{3}}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{4}{3}}}{\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1)}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)^2} \left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right) F\left(\operatorname{acos}\left(\frac{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-1+\sqrt{3})}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}}{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1)}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}}\right)\right) \frac{\sqrt{3}}{4} + \frac{1}{2}}}{405b \sqrt{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}\left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)}{\sqrt[3]{a+bx^2}\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1)}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)^2} (a+bx^2)^{\frac{2}{3}} \sqrt{-\frac{bx^2}{a+bx^2} + 1}} + \frac{16a^2c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{135b} + \frac{8a(cx)^{\frac{7}{3}}\sqrt[3]{a+bx^2}}{45c} + \frac{(cx)^{\frac{7}{3}}(a+bx^2)^{\frac{4}{3}}}{5c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x)**(4/3)*(b*x**2+a)**(4/3),x)`

[Out]  $-8 \cdot 3^{\frac{3}{4}} a^3 c^{\frac{1}{3}} (c x)^{\frac{1}{3}} \sqrt{(b^{\frac{2}{3}} (c x)^{\frac{4}{3}}) / (a + b x^2)^{\frac{2}{3}} + b^{\frac{1}{3}} c^{\frac{2}{3}} (c x)^{\frac{2}{3}} / (a + b x^2)^{\frac{2}{3}} + c^{\frac{4}{3}}) / (b^{\frac{1}{3}} (c x)^{\frac{2}{3}} (-\sqrt{3} - 1) / (a + b x^2)^{\frac{1}{3}} + c^{\frac{2}{3}})^2} (-b^{\frac{1}{3}} (c x)^{\frac{2}{3}} / (a + b x^2)^{\frac{1}{3}} + c^{\frac{2}{3}}) \operatorname{elliptic}_f(\operatorname{acos}(b^{\frac{1}{3}} (c x)^{\frac{2}{3}} (-1 + \sqrt{3}) / (a + b x^2)^{\frac{1}{3}} + c^{\frac{2}{3}}) / (b^{\frac{1}{3}} (c x)^{\frac{2}{3}} (-\sqrt{3} - 1) / (a + b x^2)^{\frac{1}{3}} + c^{\frac{2}{3}})), \sqrt{3} / 4 + 1 / 2) / (405 b \sqrt{a / (a + b x^2)} \sqrt{-b^{\frac{1}{3}} (c x)^{\frac{2}{3}} (-b^{\frac{1}{3}} (c x)^{\frac{2}{3}} / (a + b x^2)^{\frac{1}{3}} + c^{\frac{2}{3}}) / ((a + b x^2)^{\frac{1}{3}} (b^{\frac{1}{3}} (c x)^{\frac{2}{3}} (-\sqrt{3} - 1) / (a + b x^2)^{\frac{1}{3}} + c^{\frac{2}{3}})^2)} (a + b x^2)^{\frac{2}{3}} \sqrt{-b x^2 / (a + b x^2) + 1}) + 16 a^2 c \sqrt[3]{c x} (c x)^{\frac{1}{3}} (a + b x^2)^{\frac{1}{3}} / (135 b) + 8 a (c x)^{\frac{7}{3}} (a + b x^2)^{\frac{1}{3}} / (45 c) + (c x)^{\frac{7}{3}} (a + b x^2)^{\frac{4}{3}} / (5 c)$

**Mathematica [C]** time = 0.0631214, size = 96, normalized size = 0.21

$$\frac{c \sqrt[3]{cx} \left( -16a^3 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{1}{6}, \frac{2}{3}; \frac{7}{6}; -\frac{bx^2}{a} \right) + 16a^3 + 67a^2bx^2 + 78ab^2x^4 + 27b^3x^6 \right)}{135b(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c*x)^(4/3)*(a+b*x^2)^(4/3),x]`

[Out]  $(c (c x)^{\frac{1}{3}} (16 a^3 + 67 a^2 b x^2 + 78 a b^2 x^4 + 27 b^3 x^6 - 16 a^3 (1 + (b x^2)/a)^{\frac{2}{3}} \operatorname{Hypergeometric2F1}[1/6, 2/3, 7/6, -(b x^2)/a])) / (135 b (a + b x^2)^{\frac{2}{3}})$

**Maple [F]** time = 0.031, size = 0, normalized size = 0.

$$\int (cx)^{\frac{4}{3}} (bx^2 + a)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(4/3)*(b*x^2+a)^(4/3),x)`

[Out] `int((c*x)^(4/3)*(b*x^2+a)^(4/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{4}{3}} (cx)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)\*(c\*x)^(4/3),x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(4/3)\*(c\*x)^(4/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bc x^3 + ac x\right)\left(bx^2 + a\right)^{\frac{1}{3}}\left(cx\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)\*(c\*x)^(4/3),x, algorithm="fricas")

[Out] integral((b\*c\*x^3 + a\*c\*x)\*(b\*x^2 + a)^(1/3)\*(c\*x)^(1/3), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(4/3)\*(b\*x\*\*2+a)\*\*(4/3),x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{4}{3}} (cx)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)\*(c\*x)^(4/3),x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(4/3)\*(c\*x)^(4/3), x)

$$3.764 \quad \int \frac{(a+bx^2)^{4/3}}{(cx)^{2/3}} dx$$

**Optimal.** Leaf size=414

$$\frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3} + \sqrt[3]{b}c^{2/3}(cx)^{2/3} + c^{4/3}}{(a+bx^2)^{2/3} + \sqrt[3]{a+bx^2}}}{\left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}} F\left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}} \right) \middle| \frac{1}{4} (2 + \sqrt{3}) \right)}{9\sqrt[4]{3}c^{5/3} \sqrt{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}}$$

$$+ \frac{\sqrt[3]{cx} (a+bx^2)^{4/3}}{3c} + \frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9c}$$

[Out]  $(8*a*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)})/(9*c) + ((c*x)^{(1/3)}*(a+b*x^2)^{(4/3)})/(3*c) + (8*a*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})*\text{Sqrt}[(c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a+b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcCos}[(c^{(2/3)} - ((1 - \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})], (2 + \text{Sqrt}[3])/4])/(9*3^{(1/4)}*c^{(5/3)}*\text{Sqrt}[-((b^{(1/3)}*(c*x)^{(2/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})))/((a+b*x^2)^{(1/3)}*(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2)])]$

**Rubi [A]** time = 1.49301, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3} + \sqrt[3]{b}c^{2/3}(cx)^{2/3} + c^{4/3}}{(a+bx^2)^{2/3} + \sqrt[3]{a+bx^2}}}{\left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}} F\left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}} \right) \middle| \frac{1}{4} (2 + \sqrt{3}) \right)}{9\sqrt[4]{3}c^{5/3} \sqrt{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}}$$

$$+ \frac{\sqrt[3]{cx} (a+bx^2)^{4/3}}{3c} + \frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(4/3)/(c\*x)^(2/3), x]

[Out]  $(8*a*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)})/(9*c) + ((c*x)^{(1/3)}*(a+b*x^2)^{(4/3)})/(3*c) + (8*a*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})*\text{Sqrt}[(c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a+b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcCos}[(c^{(2/3)} - ((1 - \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})], (2 + \text{Sqrt}[3])/4])/(9*3^{(1/4)}*c^{(5/3)}*\text{Sqrt}[-((b^{(1/3)}*(c*x)^{(2/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})))/((a+b*x^2)^{(1/3)}*(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2)])]$

**Rubi in Sympy [A]** time = 34.7053, size = 401, normalized size = 0.97

$$\frac{8 \cdot 3^{\frac{3}{4}} a^2 \sqrt[3]{cx} \sqrt{\frac{\frac{b^{\frac{2}{3}}(cx)^{\frac{4}{3}} + \sqrt[3]{bc^{\frac{2}{3}}(cx)^{\frac{2}{3}}}}{(a+bx^2)^{\frac{2}{3}} + c^{\frac{2}{3}}} + \sqrt[3]{a+bx^2}}}{\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)^2} \left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right) F\left(\operatorname{acos}\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-1+\sqrt{3})}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)\right)}{\frac{\sqrt{3}}{4} + \frac{1}{2}}}{27c^{\frac{5}{3}} \sqrt{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}\left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)}{\sqrt[3]{a+bx^2}\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)^2}} (a+bx^2)^{\frac{2}{3}} \sqrt{-\frac{bx^2}{a+bx^2} + 1}} + \frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9c} + \frac{\sqrt[3]{cx}(a+bx^2)^{\frac{4}{3}}}{3c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(4/3)/(c*x)**(2/3), x)`

[Out]  $8 \cdot 3^{\frac{3}{4}} a^2 (c x)^{\frac{1}{3}} \sqrt{(b^{\frac{2}{3}} (c x)^{\frac{4}{3}} / (a + b^2 x^2)^{\frac{2}{3}} + b^{\frac{1}{3}} c^{\frac{2}{3}} (c x)^{\frac{2}{3}} / (a + b^2 x^2)^{\frac{1}{3}} + c^{\frac{4}{3}}) / (b^{\frac{1}{3}} (c x)^{\frac{2}{3}} (-\sqrt{3} - 1) / (a + b^2 x^2)^{\frac{1}{3}} + c^{\frac{2}{3}})^2} (-b^{\frac{1}{3}} (c x)^{\frac{2}{3}} / (a + b^2 x^2)^{\frac{1}{3}} + c^{\frac{2}{3}}) \operatorname{elliptic}_f(\operatorname{acos}((b^{\frac{1}{3}} (c x)^{\frac{2}{3}} (-1 + \sqrt{3})) / (a + b^2 x^2)^{\frac{1}{3}} + c^{\frac{2}{3}}) / (b^{\frac{1}{3}} (c x)^{\frac{2}{3}} (-\sqrt{3} - 1) / (a + b^2 x^2)^{\frac{1}{3}} + c^{\frac{2}{3}})), \sqrt{3} / 4 + 1 / 2} / (27 c^{\frac{5}{3}} \sqrt{a / (a + b^2 x^2)} \sqrt{-b^{\frac{1}{3}} (c x)^{\frac{2}{3}} (-b^{\frac{1}{3}} (c x)^{\frac{2}{3}} / (a + b^2 x^2)^{\frac{1}{3}} + c^{\frac{2}{3}}) / ((a + b^2 x^2)^{\frac{1}{3}} (b^{\frac{1}{3}} (c x)^{\frac{2}{3}} (-\sqrt{3} - 1) / (a + b^2 x^2)^{\frac{1}{3}} + c^{\frac{2}{3}})^2)} (a + b^2 x^2)^{\frac{2}{3}} \sqrt{-b^2 x^2 / (a + b^2 x^2) + 1}) + 8 a (c x)^{\frac{1}{3}} (a + b^2 x^2)^{\frac{1}{3}} / (9 c) + (c x)^{\frac{1}{3}} (a + b^2 x^2)^{\frac{4}{3}} / (3 c)$

**Mathematica [C]** time = 0.0543939, size = 83, normalized size = 0.2

$$\frac{16a^2x \left(\frac{bx^2}{a} + 1\right)^{\frac{2}{3}} {}_2F_1\left(\frac{1}{6}, \frac{2}{3}; \frac{7}{6}; -\frac{bx^2}{a}\right) + 11a^2x + 14abx^3 + 3b^2x^5}{9(cx)^{\frac{2}{3}}(a+bx^2)^{\frac{2}{3}}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(4/3)/(c*x)^(2/3), x]`

[Out]  $(11 a^2 x + 14 a b x^3 + 3 b^2 x^5 + 16 a^2 x (1 + (b x^2)/a)^{\frac{2}{3}} \operatorname{Hypergeometric2F1}[1/6, 2/3, 7/6, -((b x^2)/a)]) / (9 (c x)^{\frac{2}{3}} (a + b x^2)^{\frac{2}{3}})$

**Maple [F]** time = 0.033, size = 0, normalized size = 0.

$$\int 1 (bx^2 + a)^{\frac{4}{3}} (cx)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(4/3)/(c*x)^(2/3), x)`

[Out] `int((b*x^2+a)^(4/3)/(c*x)^(2/3), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(2/3), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(2/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(2/3), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(4/3)/(c\*x)^(2/3), x)

---

**Sympy [A]** time = 107.832, size = 46, normalized size = 0.11

$$\frac{a^{\frac{4}{3}} \sqrt[3]{x} \left(\frac{1}{6}\right) {}_2F_1\left(-\frac{4}{3}, \frac{1}{6} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2c^{\frac{2}{3}} \left(\frac{7}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(4/3)/(c\*x)\*\*(2/3), x)

[Out] a\*\*(4/3)\*x\*\*(1/3)\*gamma(1/6)\*hyper((-4/3, 1/6), (7/6,), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*c\*\*(2/3)\*gamma(7/6))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(2/3), x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(2/3), x)

$$3.765 \quad \int \frac{(a+bx^2)^{4/3}}{(cx)^{8/3}} dx$$

**Optimal.** Leaf size=414

$$\frac{8b\sqrt[3]{cx}\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}}{\left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}} F \left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}} \right) \right) \Big|_{\frac{1}{4}} (2 + \sqrt{3})}{5\sqrt[3]{3}c^{11/3} \sqrt{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}}$$

$$+ \frac{8b\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{5c^3} - \frac{3(a+bx^2)^{4/3}}{5c(cx)^{5/3}}$$

[Out]  $(8*b*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)})/(5*c^3) - (3*(a+b*x^2)^{(4/3)})/(5*c*(c*x)^{(5/3)}) + (8*b*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})*\text{Sqrt}[(c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a+b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcCos}[(c^{(2/3)} - ((1 - \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})], (2 + \text{Sqrt}[3])/4])/ (5*3^{(1/4)}*c^{(11/3)}*\text{Sqrt}[-((b^{(1/3)}*(c*x)^{(2/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}))/(a+b*x^2)^{(1/3)}*(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2)])]$

**Rubi [A]** time = 1.51277, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{8b\sqrt[3]{cx}\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}}{\left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}} F \left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}} \right) \right) \Big|_{\frac{1}{4}} (2 + \sqrt{3})}{5\sqrt[3]{3}c^{11/3} \sqrt{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}}$$

$$+ \frac{8b\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{5c^3} - \frac{3(a+bx^2)^{4/3}}{5c(cx)^{5/3}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^{(4/3)}/(c*x)^{(8/3)}, x]$

[Out]  $(8*b*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)})/(5*c^3) - (3*(a+b*x^2)^{(4/3)})/(5*c*(c*x)^{(5/3)}) + (8*b*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})*\text{Sqrt}[(c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a+b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcCos}[(c^{(2/3)} - ((1 - \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})], (2 + \text{Sqrt}[3])/4])/ (5*3^{(1/4)}*c^{(11/3)}*\text{Sqrt}[-((b^{(1/3)}*(c*x)^{(2/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}))/(a+b*x^2)^{(1/3)}*(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2)])]$

**Rubi in Sympy [A]** time = 35.4648, size = 405, normalized size = 0.98

$$\begin{aligned}
 & \frac{8 \cdot 3^{\frac{3}{4}} ab \sqrt[3]{cx} \sqrt{\frac{\frac{b^{\frac{2}{3}}(cx)^{\frac{4}{3}} + \sqrt[3]{b} c^{\frac{2}{3}} (cx)^{\frac{2}{3}} + c^{\frac{4}{3}}}{(a+bx^2)^{\frac{2}{3}} + \sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}}}{\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}}\right)^2} \left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right) F\left(\operatorname{acos}\left(\frac{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-1+\sqrt{3})}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}}{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}}}\right)}{\frac{\sqrt{3}}{4} + \frac{1}{2}}\right)}{\sqrt{\frac{15c^{\frac{11}{3}} \sqrt{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}\left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)}{\sqrt[3]{a+bx^2}\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}}\right)^2} (a+bx^2)^{\frac{2}{3}} \sqrt{-\frac{bx^2}{a+bx^2} + 1}}}{\sqrt[3]{a+bx^2}\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}}\right)^2} + \frac{8b\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{5c^3} - \frac{3(a+bx^2)^{\frac{4}{3}}}{5c(cx)^{\frac{5}{3}}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(4/3)/(c*x)**(8/3),x)`

[Out]  $8 \cdot 3^{\frac{3}{4}} a b (c x)^{\frac{1}{3}} \sqrt{(b^{\frac{2}{3}} (c x)^{\frac{4}{3}} / (a + b x^2)^{\frac{2}{3}} + b^{\frac{1}{3}} c^{\frac{2}{3}} (c x)^{\frac{2}{3}} / (a + b x^2)^{\frac{1}{3}} + c^{\frac{4}{3}}) / (b^{\frac{1}{3}} (c x)^{\frac{2}{3}} (-\sqrt{3} - 1) / (a + b x^2)^{\frac{1}{3}} + c^{\frac{2}{3}})^2} (-b^{\frac{1}{3}} (c x)^{\frac{2}{3}} / (a + b x^2)^{\frac{1}{3}} + c^{\frac{2}{3}}) \operatorname{elliptic\_f}(\operatorname{acos}((b^{\frac{1}{3}} (c x)^{\frac{2}{3}} (-1 + \sqrt{3})) / (a + b x^2)^{\frac{1}{3}} + c^{\frac{2}{3}}) / (b^{\frac{1}{3}} (c x)^{\frac{2}{3}} (-\sqrt{3} - 1) / (a + b x^2)^{\frac{1}{3}} + c^{\frac{2}{3}})), \sqrt{3} / 4 + 1 / 2) / (15 c^{\frac{11}{3}} \sqrt{a / (a + b x^2)} \sqrt{-b^{\frac{1}{3}} (c x)^{\frac{2}{3}} (-b^{\frac{1}{3}} (c x)^{\frac{2}{3}} / (a + b x^2)^{\frac{1}{3}} + c^{\frac{2}{3}}) / ((a + b x^2)^{\frac{1}{3}} (b^{\frac{1}{3}} (c x)^{\frac{2}{3}} (-\sqrt{3} - 1) / (a + b x^2)^{\frac{1}{3}} + c^{\frac{2}{3}})^2)} (a + b x^2)^{\frac{2}{3}} \sqrt{-b x^2 / (a + b x^2) + 1}) + 8 b (c x)^{\frac{1}{3}} (a + b x^2)^{\frac{1}{3}} / (5 c^3) - 3 (a + b x^2)^{\frac{4}{3}} / (5 c (c x)^{\frac{5}{3}})$

**Mathematica [C]** time = 0.059098, size = 84, normalized size = 0.2

$$\frac{x \left( -3a^2 + 16abx^2 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{1}{6}, \frac{2}{3}; \frac{7}{6}; -\frac{bx^2}{a} \right) + 2abx^2 + 5b^2x^4 \right)}{5(cx)^{8/3} (a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(4/3)/(c*x)^(8/3),x]`

[Out]  $(x^5 (-3a^2 + 2abx^2 + 5b^2x^4 + 16a^2bx^2 (1 + (bx^2)/a)^{2/3} \operatorname{Hypergeometric2F1}[1/6, 2/3, 7/6, -(bx^2)/a]) / (5(c x)^{8/3} (a + b x^2)^{2/3}))$

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int 1 (bx^2 + a)^{\frac{4}{3}} (cx)^{-\frac{8}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(4/3)/(c*x)^(8/3),x)`

[Out] `int((b*x^2+a)^(4/3)/(c*x)^(8/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(8/3),x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(8/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{2}{3}} c^2 x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(8/3),x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(4/3)/((c\*x)^(2/3)\*c^2\*x^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(4/3)/(c\*x)\*\*(8/3),x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(8/3),x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(8/3), x)



$$3.766 \quad \int \frac{(a+bx^2)^{4/3}}{(cx)^{14/3}} dx$$

**Optimal.** Leaf size=419

$$\frac{8 \cdot 3^{3/4} b^2 \sqrt[3]{cx} \sqrt{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3} + \sqrt[3]{b} c^{2/3} (cx)^{2/3} + c^{4/3}}{(a+bx^2)^{2/3} + \sqrt[3]{a+bx^2}}}{\left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}} F\left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}} \right) \middle| \frac{1}{4} (2 + \sqrt{3}) \right)}{55ac^{17/3} \sqrt{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}}$$

$$- \frac{24b\sqrt[3]{a+bx^2}}{55c^3(cx)^{5/3}} - \frac{3(a+bx^2)^{4/3}}{11c(cx)^{11/3}}$$

[Out]  $(-24*b*(a + b*x^2)^{(1/3)})/(55*c^3*(c*x)^{(5/3)}) - (3*(a + b*x^2)^{(4/3)})/(11*c*(c*x)^{(11/3)}) + (8*3^{3/4}*b^2*(c*x)^{(1/3)}*(a + b*x^2)^{(1/3)}*(c^{2/3} - (b^{1/3}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})*\text{Sqrt}[(c^{4/3} + (b^{2/3}*(c*x)^{(4/3)})/(a + b*x^2)^{(2/3)} + (b^{1/3})*c^{2/3}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(c^{2/3} - ((1 + \text{Sqrt}[3])*b^{1/3}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcCos}[(c^{2/3} - ((1 - \text{Sqrt}[3])*b^{1/3}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(c^{2/3} - ((1 + \text{Sqrt}[3])*b^{1/3}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})], (2 + \text{Sqrt}[3])/4)]/(55*a*c^{17/3}*\text{Sqrt}[-((b^{1/3}*(c*x)^{(2/3})*(c^{2/3} - (b^{1/3}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})))/((a + b*x^2)^{(1/3})* (c^{2/3} - ((1 + \text{Sqrt}[3])*b^{1/3}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)}))^2])]$

**Rubi [A]** time = 1.52411, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{8 \cdot 3^{3/4} b^2 \sqrt[3]{cx} \sqrt{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3} + \sqrt[3]{b} c^{2/3} (cx)^{2/3} + c^{4/3}}{(a+bx^2)^{2/3} + \sqrt[3]{a+bx^2}}}{\left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}} F\left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}} \right) \middle| \frac{1}{4} (2 + \sqrt{3}) \right)}{55ac^{17/3} \sqrt{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}}$$

$$- \frac{24b\sqrt[3]{a+bx^2}}{55c^3(cx)^{5/3}} - \frac{3(a+bx^2)^{4/3}}{11c(cx)^{11/3}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^{(4/3)}/(c*x)^{(14/3)}, x]$

[Out]  $(-24*b*(a + b*x^2)^{(1/3)})/(55*c^3*(c*x)^{(5/3)}) - (3*(a + b*x^2)^{(4/3)})/(11*c*(c*x)^{(11/3)}) + (8*3^{3/4}*b^2*(c*x)^{(1/3)}*(a + b*x^2)^{(1/3)}*(c^{2/3} - (b^{1/3}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})*\text{Sqrt}[(c^{4/3} + (b^{2/3}*(c*x)^{(4/3)})/(a + b*x^2)^{(2/3)} + (b^{1/3})*c^{2/3}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(c^{2/3} - ((1 + \text{Sqrt}[3])*b^{1/3}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcCos}[(c^{2/3} - ((1 - \text{Sqrt}[3])*b^{1/3}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(c^{2/3} - ((1 + \text{Sqrt}[3])*b^{1/3}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})], (2 + \text{Sqrt}[3])/4)]/(55*a*c^{17/3}*\text{Sqrt}[-((b^{1/3}*(c*x)^{(2/3})*(c^{2/3} - (b^{1/3}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})))/((a + b*x^2)^{(1/3})* (c^{2/3} - ((1 + \text{Sqrt}[3])*b^{1/3}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)}))^2])]$

**Rubi in Sympy [A]** time = 36.0744, size = 405, normalized size = 0.97

$$\frac{8 \cdot 3^{\frac{3}{4}} b^2 \sqrt[3]{cx} \sqrt{\frac{\frac{b^{\frac{2}{3}}(cx)^{\frac{4}{3}} + \sqrt[3]{b} c^{\frac{2}{3}} (cx)^{\frac{2}{3}}}{(a+bx^2)^{\frac{2}{3}} + \sqrt[3]{a+bx^2}} + c^{\frac{4}{3}}}{\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}}\right)^2} \left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right) F\left(\operatorname{acos}\left(\frac{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-1+\sqrt{3})}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}}{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}}}\right)\right)}{\frac{\sqrt{3}}{4} + \frac{1}{2}}}{55c^{\frac{17}{3}} \sqrt{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}} \left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)}{\sqrt[3]{a+bx^2} \left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}}\right)^2} (a+bx^2)^{\frac{2}{3}} \sqrt{-\frac{bx^2}{a+bx^2} + 1}}}{-\frac{24b\sqrt[3]{a+bx^2}}{55c^3(cx)^{\frac{5}{3}}} - \frac{3(a+bx^2)^{\frac{4}{3}}}{11c(cx)^{\frac{11}{3}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(4/3)/(c*x)**(14/3), x)`

[Out]  $8 \cdot 3^{\frac{3}{4}} b^2 (c x)^{\frac{4}{3}} \sqrt{(b^{\frac{2}{3}} (c x)^{\frac{4}{3}} / (a + b x^2)^{\frac{2}{3}} + b^{\frac{1}{3}} c^{\frac{2}{3}} (c x)^{\frac{2}{3}} / (a + b x^2)^{\frac{1}{3}} + c^{\frac{4}{3}}) / (b^{\frac{1}{3}} (c x)^{\frac{2}{3}} (-\sqrt{3} - 1) / (a + b x^2)^{\frac{1}{3}} + c^{\frac{2}{3}})^2 (-b^{\frac{1}{3}} (c x)^{\frac{2}{3}} / (a + b x^2)^{\frac{1}{3}} + c^{\frac{2}{3}}) \operatorname{elliptic\_f}(\operatorname{acos}(b^{\frac{1}{3}} (c x)^{\frac{2}{3}} (-1 + \sqrt{3}) / (a + b x^2)^{\frac{1}{3}} + c^{\frac{2}{3}}) / (b^{\frac{1}{3}} (c x)^{\frac{2}{3}} (-\sqrt{3} - 1) / (a + b x^2)^{\frac{1}{3}} + c^{\frac{2}{3}})), \sqrt{3} / 4 + 1 / 2} / (55 c^{\frac{17}{3}} \sqrt{a / (a + b x^2)} \sqrt{\frac{\sqrt[3]{b} (c x)^{\frac{2}{3}} \left(-\frac{\sqrt[3]{b} (c x)^{\frac{2}{3}}}{\sqrt[3]{a + b x^2}} + c^{\frac{2}{3}}\right)}{\sqrt[3]{a + b x^2} \left(\frac{\sqrt[3]{b} (c x)^{\frac{2}{3}} (-\sqrt{3} - 1) + c^{\frac{2}{3}}}{\sqrt[3]{a + b x^2}}\right)^2} (a + b x^2)^{\frac{2}{3}} \sqrt{-\frac{b x^2}{a + b x^2} + 1}} - \frac{24 b \sqrt[3]{a + b x^2}}{55 c^3 (c x)^{\frac{5}{3}}} - \frac{3 (a + b x^2)^{\frac{4}{3}}}{11 c (c x)^{\frac{11}{3}}}$

**Mathematica [C]** time = 0.0913391, size = 90, normalized size = 0.21

$$\frac{3\sqrt[3]{cx} \left(-5a^2 + 16b^2x^4 \left(\frac{bx^2}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{6}, \frac{2}{3}, \frac{7}{6}, -\frac{bx^2}{a}\right) - 18abx^2 - 13b^2x^4\right)}{55c^5x^4(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(4/3)/(c*x)^(14/3), x]`

[Out]  $(3 (c x)^{\frac{1}{3}} (-5 a^2 - 18 a b x^2 - 13 b^2 x^4 + 16 b^2 x^4 (1 + (b x^2)/a)^{\frac{2}{3}} \operatorname{Hypergeometric2F1}[1/6, 2/3, 7/6, -(b x^2)/a]) / (55 c^5 x^4 (a + b x^2)^{\frac{2}{3}})$

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int 1 (bx^2 + a)^{\frac{4}{3}} (cx)^{-\frac{14}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(4/3)/(c*x)^(14/3), x)`

[Out] `int((b*x^2+a)^(4/3)/(c*x)^(14/3), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{14}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(14/3), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(14/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{2}{3}} c^4 x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(14/3), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(4/3)/((c\*x)^(2/3)\*c^4\*x^4), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(4/3)/(c\*x)\*\*(14/3), x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{14}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(14/3), x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(14/3), x)

$$3.767 \quad \int \frac{(a+bx^2)^{4/3}}{(cx)^{20/3}} dx$$

**Optimal.** Leaf size=450

$$\frac{24 \cdot 3^{3/4} b^3 \sqrt[3]{cx} \sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3} + \sqrt[3]{b} c^{2/3} (cx)^{2/3} + c^{4/3}}{(a+bx^2)^{2/3} + \sqrt[3]{a+bx^2}}}{\left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}} F\left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}} \right) \right) \frac{1}{4} (2 + \sqrt{3})}{935 a^2 c^{23/3} \sqrt{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}}$$

$$- \frac{48 b^2 \sqrt[3]{a+bx^2}}{935 a^5 (cx)^{5/3}} - \frac{24 b \sqrt[3]{a+bx^2}}{187 c^3 (cx)^{11/3}} - \frac{3 (a+bx^2)^{4/3}}{17 c (cx)^{17/3}}$$

[Out]  $(-24*b*(a+b*x^2)^(1/3))/(187*c^3*(c*x)^(11/3)) - (48*b^2*(a+b*x^2)^(1/3))/(935*a*c^5*(c*x)^(5/3)) - (3*(a+b*x^2)^(4/3))/(17*c*(c*x)^(17/3)) - (24*3^(3/4)*b^3*(c*x)^(1/3)*(a+b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a+b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a+b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a+b*x^2)^(1/3))/(c^(2/3) - ((1+Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a+b*x^2)^(1/3))]^2)*EllipticF[ArcCos[(c^(2/3) - ((1-Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a+b*x^2)^(1/3))/(c^(2/3) - ((1+Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a+b*x^2)^(1/3))], (2+Sqrt[3])/4])/(935*a^2*c^(23/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a+b*x^2)^(1/3)))/((a+b*x^2)^(1/3)*(c^(2/3) - ((1+Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a+b*x^2)^(1/3))^2)])]$

**Rubi [A]** time = 1.67581, antiderivative size = 450, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{24 \cdot 3^{3/4} b^3 \sqrt[3]{cx} \sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3} + \sqrt[3]{b} c^{2/3} (cx)^{2/3} + c^{4/3}}{(a+bx^2)^{2/3} + \sqrt[3]{a+bx^2}}}{\left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}} F\left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}} \right) \right) \frac{1}{4} (2 + \sqrt{3})}{935 a^2 c^{23/3} \sqrt{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}}$$

$$- \frac{48 b^2 \sqrt[3]{a+bx^2}}{935 a^5 (cx)^{5/3}} - \frac{24 b \sqrt[3]{a+bx^2}}{187 c^3 (cx)^{11/3}} - \frac{3 (a+bx^2)^{4/3}}{17 c (cx)^{17/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(4/3)/(c\*x)^(20/3), x]

[Out]  $(-24*b*(a+b*x^2)^(1/3))/(187*c^3*(c*x)^(11/3)) - (48*b^2*(a+b*x^2)^(1/3))/(935*a*c^5*(c*x)^(5/3)) - (3*(a+b*x^2)^(4/3))/(17*c*(c*x)^(17/3)) - (24*3^(3/4)*b^3*(c*x)^(1/3)*(a+b*x^2)^(1/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a+b*x^2)^(1/3))*Sqrt[(c^(4/3) + (b^(2/3)*(c*x)^(4/3))/(a+b*x^2)^(2/3) + (b^(1/3)*c^(2/3)*(c*x)^(2/3))/(a+b*x^2)^(1/3))/(c^(2/3) - ((1+Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a+b*x^2)^(1/3))]^2)*EllipticF[ArcCos[(c^(2/3) - ((1-Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a+b*x^2)^(1/3))/(c^(2/3) - ((1+Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a+b*x^2)^(1/3))], (2+Sqrt[3])/4])/(935*a^2*c^(23/3)*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - (b^(1/3)*(c*x)^(2/3))/(a+b*x^2)^(1/3)))/((a+b*x^2)^(1/3)*(c^(2/3) - ((1+Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(a+b*x^2)^(1/3))^2)])]$

**Rubi in Sympy [A]** time = 46.797, size = 437, normalized size = 0.97

$$\frac{24b\sqrt[3]{a+bx^2}}{187c^3(cx)^{\frac{11}{3}}} - \frac{3(a+bx^2)^{\frac{4}{3}}}{17c(cx)^{\frac{17}{3}}}$$

$$24 \cdot 3^{\frac{3}{4}} b^3 \sqrt[3]{cx} \sqrt{\frac{\frac{b^{\frac{2}{3}}(cx)^{\frac{4}{3}} + \sqrt[3]{bc^{\frac{2}{3}}(cx)^{\frac{2}{3}}}}{(a+bx^2)^{\frac{2}{3}} + \sqrt[3]{a+bx^2}} + c^{\frac{4}{3}}}{\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1)}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)^2}} \left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right) F\left(\operatorname{acos}\left(\frac{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-1+\sqrt{3})}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}}{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1)}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}}\right)\right) \left|\frac{\sqrt{3}}{4} + \frac{1}{2}\right.$$

$$\frac{935ac^{\frac{23}{3}} \sqrt{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}} \left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)}{\sqrt[3]{a+bx^2} \left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1)}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)^2}} (a+bx^2)^{\frac{2}{3}} \sqrt{-\frac{bx^2}{a+bx^2} + 1}}{\sqrt{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}} \left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)}{\sqrt[3]{a+bx^2} \left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1)}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)^2}}}}$$

$$-\frac{48b^2\sqrt[3]{a+bx^2}}{935ac^5(cx)^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(4/3)/(c*x)**(20/3), x)`

[Out]  $-24*b*(a + b*x**2)**(1/3)/(187*c**3*(c*x)**(11/3)) - 3*(a + b*x**2)**(4/3)/(17*c*(c*x)**(17/3)) - 24*3**(3/4)*b**3*(c*x)**(1/3)*\operatorname{sqrt}((b**(2/3)*(c*x)**(4/3)/(a + b*x**2)**(2/3) + b**(1/3)*c**(2/3)*(c*x)**(2/3)/(a + b*x**2)**(1/3) + c**(4/3))/(b**(1/3)*(c*x)**(2/3)*(-\operatorname{sqrt}(3) - 1)/(a + b*x**2)**(1/3) + c**(2/3))**2)*(-b**(1/3)*(c*x)**(2/3)/(a + b*x**2)**(1/3) + c**(2/3))*\operatorname{elliptic}_f(\operatorname{acos}((b*(1/3)*(c*x)**(2/3)*(-1 + \operatorname{sqrt}(3)))/(a + b*x**2)**(1/3) + c**(2/3))/(b*(1/3)*(c*x)**(2/3)*(-\operatorname{sqrt}(3) - 1)/(a + b*x**2)**(1/3) + c**(2/3))), \operatorname{sqrt}(3)/4 + 1/2)/(935*a*c**(23/3)*\operatorname{sqrt}(a/(a + b*x**2))*\operatorname{sqrt}(-b**(1/3)*(c*x)**(2/3)*(-b**(1/3)*(c*x)**(2/3)/(a + b*x**2)**(1/3) + c**(2/3))/((a + b*x**2)**(1/3)*(b**(1/3)*(c*x)**(2/3)*(-\operatorname{sqrt}(3) - 1)/(a + b*x**2)**(1/3) + c**(2/3))**2))*(a + b*x**2)**(2/3)*\operatorname{sqrt}(-b*x**2/(a + b*x**2) + 1)) - 48*b**2*(a + b*x**2)**(1/3)/(935*a*c**5*(c*x)**(5/3))$

**Mathematica [C]** time = 0.0912035, size = 104, normalized size = 0.23

$$\frac{3\sqrt[3]{cx} \left(55a^3 + 150a^2bx^2 + 48b^3x^6 \left(\frac{bx^2}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{6}, \frac{2}{3}; \frac{7}{6}; -\frac{bx^2}{a}\right) + 111ab^2x^4 + 16b^3x^6\right)}{935ac^7x^6(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(4/3)/(c*x)^(20/3), x]`

[Out]  $(-3*(c*x)^{(1/3)}*(55*a^3 + 150*a^2*b*x^2 + 111*a*b^2*x^4 + 16*b^3*x^6 + 48*b^3*x^6*(1 + (b*x^2)/a)^{(2/3)}*\operatorname{Hypergeometric2F1}[1/6, 2/3, 7/6, -(b*x^2)/a]))/(935*a*c^7*x^6*(a + b*x^2)^{(2/3)})$

**Maple [F]** time = 0.05, size = 0, normalized size = 0.

$$\int 1 (bx^2 + a)^{\frac{4}{3}} (cx)^{-\frac{20}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(4/3)/(c*x)^(20/3), x)`

[Out] `int((b*x^2+a)^(4/3)/(c*x)^(20/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{20}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)/(c*x)^(20/3),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(4/3)/(c*x)^(20/3), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{2}{3}} c^6 x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)/(c*x)^(20/3),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(4/3)/((c*x)^(2/3)*c^6*x^6), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(4/3)/(c*x)**(20/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{20}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)/(c*x)^(20/3),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(4/3)/(c*x)^(20/3), x)`

$$3.768 \quad \int (cx)^{2/3} (a + bx^2)^{4/3} dx$$

**Optimal.** Leaf size=59

$$\frac{3a(cx)^{5/3} \sqrt[3]{a + bx^2} {}_2F_1\left(-\frac{4}{3}, \frac{5}{6}; \frac{11}{6}; -\frac{bx^2}{a}\right)}{5c \sqrt[3]{\frac{bx^2}{a} + 1}}$$

[Out] (3\*a\*(c\*x)^(5/3)\*(a + b\*x^2)^(1/3)\*Hypergeometric2F1[-4/3, 5/6, 11/6, -(b\*x^2)/a])/(5\*c\*(1 + (b\*x^2)/a)^(1/3))

**Rubi [A]** time = 0.0675187, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{3a(cx)^{5/3} \sqrt[3]{a + bx^2} {}_2F_1\left(-\frac{4}{3}, \frac{5}{6}; \frac{11}{6}; -\frac{bx^2}{a}\right)}{5c \sqrt[3]{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(2/3)\*(a + b\*x^2)^(4/3), x]

[Out] (3\*a\*(c\*x)^(5/3)\*(a + b\*x^2)^(1/3)\*Hypergeometric2F1[-4/3, 5/6, 11/6, -(b\*x^2)/a])/(5\*c\*(1 + (b\*x^2)/a)^(1/3))

**Rubi in Sympy [A]** time = 7.5422, size = 51, normalized size = 0.86

$$\frac{3a(cx)^{5/3} \sqrt[3]{a + bx^2} {}_2F_1\left(-\frac{4}{3}, \frac{5}{6}; \frac{11}{6}; -\frac{bx^2}{a}\right)}{5c \sqrt[3]{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(2/3)\*(b\*x\*\*2+a)\*\*(4/3), x)

[Out] 3\*a\*(c\*x)\*\*(5/3)\*(a + b\*x\*\*2)\*\*(1/3)\*hyper((-4/3, 5/6), (11/6, ), -b\*x\*\*2/a)/(5\*c\*(1 + b\*x\*\*2/a)\*\*(1/3))

**Mathematica [A]** time = 0.058603, size = 83, normalized size = 1.41

$$\frac{3(cx)^{2/3} \left(16a^2x \left(\frac{bx^2}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{5}{6}; \frac{11}{6}; -\frac{bx^2}{a}\right) + 75a^2x + 110abx^3 + 35b^2x^5\right)}{455(a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(2/3)\*(a + b\*x^2)^(4/3), x]

[Out] (3\*(c\*x)^(2/3)\*(75\*a^2\*x + 110\*a\*b\*x^3 + 35\*b^2\*x^5 + 16\*a^2\*x\*(1 + (b\*x^2)/a)^(2/3)\*Hypergeometric2F1[2/3, 5/6, 11/6, -(b\*x^2)/a]))/(455\*(a + b\*x^2)^(2/3))

**Maple [F]** time = 0.033, size = 0, normalized size = 0.

$$\int (cx)^{\frac{2}{3}} (bx^2 + a)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(2/3)*(b*x^2+a)^(4/3),x)`

[Out] `int((c*x)^(2/3)*(b*x^2+a)^(4/3),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{4}{3}} (cx)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)*(c*x)^(2/3),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(4/3)*(c*x)^(2/3), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^{\frac{4}{3}} (cx)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)*(c*x)^(2/3),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(4/3)*(c*x)^(2/3), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(2/3)*(b*x**2+a)**(4/3),x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{4}{3}} (cx)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(4/3)*(c*x)^(2/3),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(4/3)*(c*x)^(2/3), x)`



$$3.769 \quad \int \frac{(a+bx^2)^{4/3}}{\sqrt[3]{cx}} dx$$

**Optimal.** Leaf size=59

$$\frac{3a(cx)^{2/3} \sqrt[3]{a+bx^2} {}_2F_1\left(-\frac{4}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^2}{a}\right)}{2c \sqrt[3]{\frac{bx^2}{a} + 1}}$$

[Out] (3\*a\*(c\*x)^(2/3)\*(a + b\*x^2)^(1/3)\*Hypergeometric2F1[-4/3, 1/3, 4/3, -(b\*x^2)/a])/(2\*c\*(1 + (b\*x^2)/a)^(1/3))

**Rubi [A]** time = 0.0675049, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{3a(cx)^{2/3} \sqrt[3]{a+bx^2} {}_2F_1\left(-\frac{4}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^2}{a}\right)}{2c \sqrt[3]{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(4/3)/(c\*x)^(1/3), x]

[Out] (3\*a\*(c\*x)^(2/3)\*(a + b\*x^2)^(1/3)\*Hypergeometric2F1[-4/3, 1/3, 4/3, -(b\*x^2)/a])/(2\*c\*(1 + (b\*x^2)/a)^(1/3))

**Rubi in Sympy [A]** time = 7.56447, size = 51, normalized size = 0.86

$$\frac{3a(cx)^{\frac{2}{3}} \sqrt[3]{a+bx^2} {}_2F_1\left(-\frac{4}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^2}{a}\right)}{2c \sqrt[3]{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(4/3)/(c\*x)\*\*(1/3), x)

[Out] 3\*a\*(c\*x)\*\*(2/3)\*(a + b\*x\*\*2)\*\*(1/3)\*hyper((-4/3, 1/3), (4/3, ), -b\*x\*\*2/a)/(2\*c\*(1 + b\*x\*\*2/a)\*\*(1/3))

**Mathematica [A]** time = 0.0591085, size = 81, normalized size = 1.37

$$\frac{3x \left( 2a^2 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^2}{a}\right) + 3a^2 + 4abx^2 + b^2x^4 \right)}{10 \sqrt[3]{cx} (a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(4/3)/(c\*x)^(1/3), x]

[Out] (3\*x\*(3\*a^2 + 4\*a\*b\*x^2 + b^2\*x^4 + 2\*a^2\*(1 + (b\*x^2)/a)^(2/3)\*Hypergeometric2F1[1/3, 2/3, 4/3, -(b\*x^2)/a]))/(10\*(c\*x)^(1/3)\*(a + b\*x^2)^(2/3))

---

**Maple [F]** time = 0.032, size = 0, normalized size = 0.

$$\int 1 (bx^2 + a)^{\frac{4}{3}} \frac{1}{\sqrt[3]{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(4/3)/(c\*x)^(1/3), x)

[Out] int((b\*x^2+a)^(4/3)/(c\*x)^(1/3), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(1/3), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(1/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(1/3), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(4/3)/(c\*x)^(1/3), x)

---

**Sympy [A]** time = 68.4535, size = 46, normalized size = 0.78

$$\frac{a^{\frac{4}{3}} x^{\frac{2}{3}} \left(\frac{1}{3}\right) {}_2F_1\left(-\frac{4}{3}, \frac{1}{3} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt[3]{c} \left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(4/3)/(c\*x)\*\*(1/3), x)

[Out] a\*\*(4/3)\*x\*\*(2/3)\*gamma(1/3)\*hyper((-4/3, 1/3), (4/3,), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*c\*\*(1/3)\*gamma(4/3))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(4/3)/(c*x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(1/3), x)
```

$$3.770 \quad \int \frac{(a+bx^2)^{4/3}}{(cx)^{4/3}} dx$$

**Optimal.** Leaf size=57

$$\frac{3a\sqrt[3]{a+bx^2} {}_2F_1\left(-\frac{4}{3}, -\frac{1}{6}, \frac{5}{6}; -\frac{bx^2}{a}\right)}{c\sqrt[3]{cx} \sqrt[3]{\frac{bx^2}{a} + 1}}$$

[Out]  $(-3*a*(a + b*x^2)^(1/3)*\text{Hypergeometric2F1}[-4/3, -1/6, 5/6, -((b*x^2)/a)])/(c*(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))$

**Rubi [A]** time = 0.0681794, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{3a\sqrt[3]{a+bx^2} {}_2F_1\left(-\frac{4}{3}, -\frac{1}{6}, \frac{5}{6}; -\frac{bx^2}{a}\right)}{c\sqrt[3]{cx} \sqrt[3]{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(4/3)/(c\*x)^(4/3), x]

[Out]  $(-3*a*(a + b*x^2)^(1/3)*\text{Hypergeometric2F1}[-4/3, -1/6, 5/6, -((b*x^2)/a)])/(c*(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))$

**Rubi in Sympy [A]** time = 7.63392, size = 53, normalized size = 0.93

$$\frac{3a\sqrt[3]{a+bx^2} {}_2F_1\left(-\frac{4}{3}, -\frac{1}{6}, \frac{5}{6}; -\frac{bx^2}{a}\right)}{c\sqrt[3]{cx} \sqrt[3]{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(4/3)/(c\*x)\*\*(4/3), x)

[Out]  $-3*a*(a + b*x**2)**(1/3)*\text{hyper}((-4/3, -1/6), (5/6, ), -b*x**2/a)/(c*(c*x)**(1/3)*(1 + b*x**2/a)**(1/3))$

**Mathematica [A]** time = 0.0811199, size = 86, normalized size = 1.51

$$\frac{3x \left( 5(-7a^2 - 6abx^2 + b^2x^4) + 16abx^2 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{5}{6}, \frac{11}{6}; -\frac{bx^2}{a}\right) \right)}{35(cx)^{4/3} (a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(4/3)/(c\*x)^(4/3), x]

[Out]  $(3*x*(5*(-7*a^2 - 6*a*b*x^2 + b^2*x^4) + 16*a*b*x^2*(1 + (b*x^2)/a)^(2/3)*\text{Hypergeometric2F1}[2/3, 5/6, 11/6, -((b*x^2)/a)]))/(35*(c*x)^(4/3)*(a + b*x^2)^(2/3))$

---

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int 1 (bx^2 + a)^{\frac{4}{3}} (cx)^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(4/3)/(c\*x)^(4/3), x)

[Out] int((b\*x^2+a)^(4/3)/(c\*x)^(4/3), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(4/3), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(4/3), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{1}{3}} cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(4/3)/(c\*x)^(4/3), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(4/3)/((c\*x)^(1/3)\*c\*x), x)

---

**Sympy [A]** time = 107.735, size = 49, normalized size = 0.86

$$\frac{a^{\frac{4}{3}} \left(-\frac{1}{6}\right) {}_2F_1\left(-\frac{4}{3}, -\frac{1}{6} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2c^{\frac{4}{3}} \sqrt[3]{x} \left(\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(4/3)/(c\*x)\*\*(4/3), x)

[Out] a\*\*(4/3)\*gamma(-1/6)\*hyper((-4/3, -1/6), (5/6, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*c\*\*(4/3)\*x\*\*(1/3)\*gamma(5/6))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(4/3)/(c*x)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(4/3)/(c*x)^(4/3), x)
```

$$3.771 \quad \int \frac{(cx)^{19/3}}{(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=278

$$\begin{aligned} & \frac{20a^3c^{19/3} \log\left(c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt{a+bx^2}}\right)}{81b^{11/3}} - \frac{10a^3c^{19/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt{a+bx^2}} + c^{4/3}\right)}{81b^{11/3}} \\ & + \frac{20a^3c^{19/3} \tan^{-1}\left(\frac{\frac{2}{3}\sqrt[3]{b}(cx)^{2/3} + c^{2/3}}{\sqrt{a+bx^2}}}{\sqrt{3}c^{2/3}}\right)}{27\sqrt{3}b^{11/3}} + \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} \\ & - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} \end{aligned}$$

[Out]  $(10*a^2*c^5*(c*x)^{(4/3)}*(a+b*x^2)^{(1/3)})/(27*b^3) - (2*a*c^3*(c*x)^{(10/3)}*(a+b*x^2)^{(1/3)})/(9*b^2) + (c*(c*x)^{(16/3)}*(a+b*x^2)^{(1/3)})/(6*b) + (20*a^3*c^{19/3}*ArcTan[(c^{2/3} + (2*b^{1/3})*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}]/(Sqrt[3]*c^{2/3}))/ (27*Sqrt[3]*b^{11/3}) + (20*a^3*c^{19/3}*Log[c^{2/3} - (b^{1/3}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}])/(81*b^{11/3}) - (10*a^3*c^{19/3}*Log[c^{4/3} + (b^{2/3}*(c*x)^{(4/3)})/(a+b*x^2)^{(2/3)} + (b^{1/3}*(c*x)^{(2/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}])/(81*b^{11/3})$

**Rubi [A]** time = 0.833962, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$\begin{aligned} & \frac{20a^3c^{19/3} \log\left(c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt{a+bx^2}}\right)}{81b^{11/3}} - \frac{10a^3c^{19/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt{a+bx^2}} + c^{4/3}\right)}{81b^{11/3}} \\ & + \frac{20a^3c^{19/3} \tan^{-1}\left(\frac{\frac{2}{3}\sqrt[3]{b}(cx)^{2/3} + c^{2/3}}{\sqrt{a+bx^2}}}{\sqrt{3}c^{2/3}}\right)}{27\sqrt{3}b^{11/3}} + \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} \\ & - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(19/3)/(a + b\*x^2)^(2/3), x]

[Out]  $(10*a^2*c^5*(c*x)^{(4/3)}*(a+b*x^2)^{(1/3)})/(27*b^3) - (2*a*c^3*(c*x)^{(10/3)}*(a+b*x^2)^{(1/3)})/(9*b^2) + (c*(c*x)^{(16/3)}*(a+b*x^2)^{(1/3)})/(6*b) + (20*a^3*c^{19/3}*ArcTan[(c^{2/3} + (2*b^{1/3})*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}]/(Sqrt[3]*c^{2/3}))/ (27*Sqrt[3]*b^{11/3}) + (20*a^3*c^{19/3}*Log[c^{2/3} - (b^{1/3}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}])/(81*b^{11/3}) - (10*a^3*c^{19/3}*Log[c^{4/3} + (b^{2/3}*(c*x)^{(4/3)})/(a+b*x^2)^{(2/3)} + (b^{1/3}*(c*x)^{(2/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}])/(81*b^{11/3})$

**Rubi in Sympy [A]** time = 77.8088, size = 267, normalized size = 0.96

$$\frac{20a^3 c^{\frac{19}{3}} \log\left(-\frac{\sqrt[3]{b(cx)^{\frac{2}{3}}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)}{81b^{\frac{11}{3}}} - \frac{10a^3 c^{\frac{19}{3}} \log\left(\frac{b^{\frac{2}{3}}(cx)^{\frac{4}{3}}}{c^{\frac{4}{3}}(a+bx^2)^{\frac{2}{3}}} + \frac{\sqrt[3]{b(cx)^{\frac{2}{3}}}}{c^{\frac{2}{3}}\sqrt[3]{a+bx^2}} + 1\right)}{81b^{\frac{11}{3}}}$$

$$+ \frac{20\sqrt{3}a^3 c^{\frac{19}{3}} \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{2}{3}\sqrt[3]{b(cx)^{\frac{2}{3}}} + \frac{c^{\frac{2}{3}}}{3}\right)}{c^{\frac{2}{3}}\sqrt[3]{a+bx^2}}\right)}{81b^{\frac{11}{3}}} + \frac{10a^2 c^5 (cx)^{\frac{4}{3}} \sqrt[3]{a+bx^2}}{27b^3}$$

$$- \frac{2ac^3 (cx)^{\frac{10}{3}} \sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{\frac{16}{3}} \sqrt[3]{a+bx^2}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x)**(19/3)/(b*x**2+a)**(2/3), x)`

[Out]  $20*a^{**3}*c^{**}(19/3)*\log(-b^{**}(1/3)*(c*x)^{**}(2/3)/(a+b*x^{**2})^{**}(1/3)+c^{**}(2/3))/(81*b^{**}(11/3))-10*a^{**3}*c^{**}(19/3)*\log(b^{**}(2/3)*(c*x)^{**}(4/3)/(c^{**}(4/3)*(a+b*x^{**2})^{**}(2/3))+b^{**}(1/3)*(c*x)^{**}(2/3)/(c^{**}(2/3)*(a+b*x^{**2})^{**}(1/3))+1))/(81*b^{**}(11/3))+20*\operatorname{sqrt}(3)*a^{**3}*c^{**}(19/3)*\operatorname{atan}(\operatorname{sqrt}(3)*(2*b^{**}(1/3)*(c*x)^{**}(2/3)/(3*(a+b*x^{**2})^{**}(1/3))+c^{**}(2/3)/3)/c^{**}(2/3))/(81*b^{**}(11/3))+10*a^{**2}*c^{**5}*(c*x)^{**}(4/3)*(a+b*x^{**2})^{**}(1/3)/(27*b^{**3})-2*a*c^{**3}*(c*x)^{**}(10/3)*(a+b*x^{**2})^{**}(1/3)/(9*b^{**2})+c*(c*x)^{**}(16/3)*(a+b*x^{**2})^{**}(1/3)/(6*b)$

**Mathematica [C]** time = 0.072576, size = 98, normalized size = 0.35

$$\frac{c^5 (cx)^{4/3} \left( -20a^3 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{2}{3}, \frac{2}{3}, \frac{5}{3}; -\frac{bx^2}{a} \right) + 20a^3 + 8a^2 bx^2 - 3ab^2 x^4 + 9b^3 x^6 \right)}{54b^3 (a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c*x)^(19/3)/(a+b*x^2)^(2/3), x]`

[Out]  $(c^5*(c*x)^{4/3}*(20*a^3+8*a^2*b*x^2-3*a*b^2*x^4+9*b^3*x^6-20*a^3*(1+(b*x^2)/a)^{2/3}*\operatorname{Hypergeometric2F1}[2/3, 2/3, 5/3, -(b*x^2)/a]))/(54*b^3*(a+b*x^2)^{2/3})$

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int 1 (cx)^{\frac{19}{3}} (bx^2 + a)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(19/3)/(b*x^2+a)^(2/3), x)`

[Out] `int((c*x)^(19/3)/(b*x^2+a)^(2/3), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((c*x)^(19/3)/(b*x^2 + a)^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(19/3)/(b*x^2 + a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(19/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{19}{3}}}{(bx^2 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(19/3)/(b*x^2 + a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(19/3)/(b*x^2 + a)^(2/3), x)
```

$$3.772 \quad \int \frac{(cx)^{13/3}}{(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=247

$$\begin{aligned} & -\frac{5a^2c^{13/3} \log\left(c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt{a+bx^2}}\right)}{18b^{8/3}} + \frac{5a^2c^{13/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt{a+bx^2}} + c^{4/3}\right)}{36b^{8/3}} \\ & -\frac{5a^2c^{13/3} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt{a+bx^2}} + c^{2/3}}{\sqrt{3}c^{2/3}}\right)}{6\sqrt{3}b^{8/3}} - \frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} \end{aligned}$$

[Out]  $(-5*a*c^{13/3}*(c*x)^{(4/3)}*(a+b*x^2)^{(1/3)})/(12*b^2) + (c*(c*x)^{(10/3)}*(a+b*x^2)^{(1/3)})/(4*b) - (5*a^2*c^{13/3}*ArcTan[(c^{(2/3)} + (2*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(Sqrt[3]*c^{(2/3)})])/(6*Sqrt[3]*b^{(8/3)}) - (5*a^2*c^{13/3}*Log[c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}])/(18*b^{(8/3)}) + (5*a^2*c^{13/3}*Log[c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a+b*x^2)^{(2/3)} + (b^{(1/3)}*(c*x)^{(2/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}])/(36*b^{(8/3)})$

**Rubi [A]** time = 0.677233, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$\begin{aligned} & -\frac{5a^2c^{13/3} \log\left(c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt{a+bx^2}}\right)}{18b^{8/3}} + \frac{5a^2c^{13/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt{a+bx^2}} + c^{4/3}\right)}{36b^{8/3}} \\ & -\frac{5a^2c^{13/3} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt{a+bx^2}} + c^{2/3}}{\sqrt{3}c^{2/3}}\right)}{6\sqrt{3}b^{8/3}} - \frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(13/3)/(a+b\*x^2)^(2/3), x]

[Out]  $(-5*a*c^{13/3}*(c*x)^{(4/3)}*(a+b*x^2)^{(1/3)})/(12*b^2) + (c*(c*x)^{(10/3)}*(a+b*x^2)^{(1/3)})/(4*b) - (5*a^2*c^{13/3}*ArcTan[(c^{(2/3)} + (2*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(Sqrt[3]*c^{(2/3)})])/(6*Sqrt[3]*b^{(8/3)}) - (5*a^2*c^{13/3}*Log[c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}])/(18*b^{(8/3)}) + (5*a^2*c^{13/3}*Log[c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a+b*x^2)^{(2/3)} + (b^{(1/3)}*(c*x)^{(2/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}])/(36*b^{(8/3)})$

**Rubi in Sympy [A]** time = 67.4719, size = 236, normalized size = 0.96

$$\begin{aligned} & -\frac{5a^2c^{13/3} \log\left(-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt{a+bx^2}} + c^{2/3}\right)}{18b^{8/3}} + \frac{5a^2c^{13/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{c^{4/3}(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt{a+bx^2}} + 1\right)}{36b^{8/3}} \\ & -\frac{5\sqrt{3}a^2c^{13/3} \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt{a+bx^2}} + c^{2/3}\right)}{c^{2/3}}\right)}{18b^{8/3}} - \frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2} + \frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(13/3)/(b\*x\*\*2+a)\*\*(2/3), x)

[Out]  $-5a^{2/3}c^{13/3} \log(-b^{1/3}(cx)^{2/3}/(a+bx^2)^{1/3} + c^{2/3})/(18b^{8/3}) + 5a^{2/3}c^{13/3} \log(b^{2/3}(cx)^{4/3}/(c^{4/3}(a+bx^2)^{2/3}) + b^{1/3}(cx)^{2/3}/(c^{2/3}(a+bx^2)^{1/3}) + 1)/(36b^{8/3}) - 5\sqrt{3}a^{2/3}c^{13/3} \operatorname{atan}(\sqrt{3}(2b^{1/3}(cx)^{2/3}/(3(a+bx^2)^{1/3}) + c^{2/3}/3)/c^{2/3})/(18b^{8/3}) - 5a^{2/3}c^{13/3}(cx)^{4/3}(a+bx^2)^{1/3}/(12b^2) + c^{10/3}(a+bx^2)^{1/3}/(4b)$

**Mathematica [C]** time = 0.067104, size = 87, normalized size = 0.35

$$\frac{c^3(cx)^{4/3} \left( 5a^2 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^2}{a} \right) - 5a^2 - 2abx^2 + 3b^2x^4 \right)}{12b^2(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(13/3)/(a + b\*x^2)^(2/3), x]

[Out]  $(c^3(c*x)^{4/3}(-5a^2 - 2a*b*x^2 + 3b^2*x^4 + 5a^2*(1 + (b*x^2)/a)^{2/3} \operatorname{Hypergeometric2F1}[2/3, 2/3, 5/3, -((b*x^2)/a)]))/(12*b^2*(a + b*x^2)^{2/3})$

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int 1 (cx)^{\frac{13}{3}} (bx^2 + a)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(13/3)/(b\*x^2+a)^(2/3), x)

[Out] int((c\*x)^(13/3)/(b\*x^2+a)^(2/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(13/3)/(b\*x^2 + a)^(2/3), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(13/3)/(b\*x^2 + a)^(2/3), x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(13/3)/(b*x**2+a)**(2/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{13}{3}}}{(bx^2 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(13/3)/(b*x^2 + a)^(2/3),x, algorithm="giac")`

[Out] `integrate((c*x)^(13/3)/(b*x^2 + a)^(2/3), x)`

$$3.773 \quad \int \frac{(cx)^{7/3}}{(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=209

$$\frac{ac^{7/3} \log\left(c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt{a+bx^2}}\right)}{3b^{5/3}} - \frac{ac^{7/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt{a+bx^2}} + c^{4/3}\right)}{6b^{5/3}}$$

$$+ \frac{ac^{7/3} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt{a+bx^2}} + c^{2/3}}{\sqrt{3}c^{2/3}}\right)}{\sqrt{3}b^{5/3}} + \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}$$

[Out]  $(c*(c*x)^{(4/3)}*(a+b*x^2)^{(1/3)})/(2*b) + (a*c^{(7/3)}*ArcTan[(c^{(2/3)} + (2*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(Sqrt[3]*c^{(2/3)})])/(Sqrt[3]*b^{(5/3)} + (a*c^{(7/3)}*Log[c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})])/(3*b^{(5/3)} - (a*c^{(7/3)}*Log[c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a+b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})])/(6*b^{(5/3)})$

**Rubi [A]** time = 0.589379, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$\frac{ac^{7/3} \log\left(c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt{a+bx^2}}\right)}{3b^{5/3}} - \frac{ac^{7/3} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt{a+bx^2}} + c^{4/3}\right)}{6b^{5/3}}$$

$$+ \frac{ac^{7/3} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt{a+bx^2}} + c^{2/3}}{\sqrt{3}c^{2/3}}\right)}{\sqrt{3}b^{5/3}} + \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(7/3)/(a+b\*x^2)^(2/3),x]

[Out]  $(c*(c*x)^{(4/3)}*(a+b*x^2)^{(1/3)})/(2*b) + (a*c^{(7/3)}*ArcTan[(c^{(2/3)} + (2*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(Sqrt[3]*c^{(2/3)})])/(Sqrt[3]*b^{(5/3)} + (a*c^{(7/3)}*Log[c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})])/(3*b^{(5/3)} - (a*c^{(7/3)}*Log[c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a+b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})])/(6*b^{(5/3)})$

**Rubi in Sympy [A]** time = 58.1946, size = 197, normalized size = 0.94

$$\frac{ac^{\frac{7}{3}} \log\left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt{a+bx^2}} + c^{\frac{2}{3}}\right)}{3b^{\frac{5}{3}}} - \frac{ac^{\frac{7}{3}} \log\left(\frac{b^{\frac{2}{3}}(cx)^{\frac{4}{3}}}{c^{\frac{4}{3}}(a+bx^2)^{\frac{2}{3}}} + \frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{c^{\frac{2}{3}}\sqrt{a+bx^2}} + 1\right)}{6b^{\frac{5}{3}}}$$

$$+ \frac{\sqrt{3}ac^{\frac{7}{3}} \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{2\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt{a+bx^2}} + c^{\frac{2}{3}}\right)}{c^{\frac{2}{3}}}\right)}{3b^{\frac{5}{3}}} + \frac{c(cx)^{\frac{4}{3}}\sqrt[3]{a+bx^2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(7/3)/(b\*x\*\*2+a)\*\*(2/3),x)

[Out]  $a \cdot c^{7/3} \log(-b^{1/3} (c \cdot x)^{2/3} / (a + b \cdot x^2)^{1/3}) + c^{2/3} (2/3) / (3 \cdot b^{5/3}) - a \cdot c^{7/3} \log(b^{2/3} (c \cdot x)^{4/3} / (c^{4/3} (a + b \cdot x^2)^{2/3})) + b^{1/3} (c \cdot x)^{2/3} / (c^{2/3} (a + b \cdot x^2)^{1/3}) + 1 / (6 \cdot b^{5/3}) + \sqrt{3} \cdot a \cdot c^{7/3} \operatorname{atan}(\sqrt{3} \cdot (2 \cdot b^{1/3} (c \cdot x)^{2/3} / (3 \cdot (a + b \cdot x^2)^{1/3})) + c^{2/3} / 3) / c^{2/3} / (3 \cdot b^{5/3}) + c \cdot (c \cdot x)^{4/3} \cdot (a + b \cdot x^2)^{1/3} / (2 \cdot b)$

**Mathematica [C]** time = 0.0565378, size = 69, normalized size = 0.33

$$\frac{c(cx)^{4/3} \left( -a \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^2}{a} \right) + a + bx^2 \right)}{2b(a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(7/3)/(a + b\*x^2)^(2/3), x]

[Out]  $(c \cdot (c \cdot x)^{4/3} \cdot (a + b \cdot x^2 - a \cdot (1 + (b \cdot x^2)/a)^{2/3}) \cdot \operatorname{Hypergeometric2F1}[2/3, 2/3, 5/3, -(b \cdot x^2)/a]) / (2 \cdot b \cdot (a + b \cdot x^2)^{2/3})$

**Maple [F]** time = 0.033, size = 0, normalized size = 0.

$$\int 1 (cx)^{7/3} (bx^2 + a)^{-2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(7/3)/(b\*x^2+a)^(2/3), x)

[Out] int((c\*x)^(7/3)/(b\*x^2+a)^(2/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(7/3)/(b\*x^2 + a)^(2/3), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(7/3)/(b\*x^2 + a)^(2/3), x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/3)/(b*x**2+a)**(2/3),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{7}{3}}}{(bx^2 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/3)/(b*x^2 + a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(7/3)/(b*x^2 + a)^(2/3), x)
```

$$3.774 \quad \int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=183

$$\frac{\sqrt[3]{c} \log\left(c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2b^{2/3}} + \frac{\sqrt[3]{c} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}\right)}{4b^{2/3}} - \frac{\sqrt{3}\sqrt[3]{c} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{2/3}}{\sqrt{3}c^{2/3}}\right)}{2b^{2/3}}$$

[Out]  $-(\text{Sqrt}[3] * c^{(1/3)} * \text{ArcTan}[(c^{(2/3)} + (2 * b^{(1/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)}) / (\text{Sqrt}[3] * c^{(2/3)})]) / (2 * b^{(2/3)}) - (c^{(1/3)} * \text{Log}[c^{(2/3)} - (b^{(1/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)}]) / (2 * b^{(2/3)}) + (c^{(1/3)} * \text{Log}[c^{(4/3)} + (b^{(2/3)} * (c * x)^{(4/3)}) / (a + b * x^2)^{(2/3)} + (b^{(1/3)} * c^{(2/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)}]) / (4 * b^{(2/3)})$

**Rubi [A]** time = 0.529926, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$

$$\frac{\sqrt[3]{c} \log\left(c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{2b^{2/3}} + \frac{\sqrt[3]{c} \log\left(\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}\right)}{4b^{2/3}} - \frac{\sqrt{3}\sqrt[3]{c} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{2/3}}{\sqrt{3}c^{2/3}}\right)}{2b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(1/3)/(a + b\*x^2)^(2/3), x]

[Out]  $-(\text{Sqrt}[3] * c^{(1/3)} * \text{ArcTan}[(c^{(2/3)} + (2 * b^{(1/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)}) / (\text{Sqrt}[3] * c^{(2/3)})]) / (2 * b^{(2/3)}) - (c^{(1/3)} * \text{Log}[c^{(2/3)} - (b^{(1/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)}]) / (2 * b^{(2/3)}) + (c^{(1/3)} * \text{Log}[c^{(4/3)} + (b^{(2/3)} * (c * x)^{(4/3)}) / (a + b * x^2)^{(2/3)} + (b^{(1/3)} * c^{(2/3)} * (c * x)^{(2/3)}) / (a + b * x^2)^{(1/3)}]) / (4 * b^{(2/3)})$

**Rubi in Sympy [A]** time = 51.2281, size = 170, normalized size = 0.93

$$\frac{\sqrt[3]{c} \log\left(-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{2/3}\right)}{2b^{2/3}} + \frac{\sqrt[3]{c} \log\left(\frac{b^{2/3}(cx)^{4/3}}{c^{4/3}(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}} + 1\right)}{4b^{2/3}} - \frac{\sqrt{3}\sqrt[3]{c} \operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + \frac{c^{2/3}}{\sqrt{3}}\right)}{c^{2/3}}\right)}{2b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(1/3)/(b\*x\*\*2+a)\*\*(2/3), x)

[Out]  $-c^{(1/3)} * \log(-b^{(1/3)} * (c * x)^{(2/3)} / (a + b * x^2)^{(1/3)} + c^{(2/3)}) / (2 * b^{(2/3)}) + c^{(1/3)} * \log(b^{(2/3)} * (c * x)^{(4/3)} / (c^{(4/3)} * (a + b * x^2)^{(2/3)} + b^{(1/3)} * (c * x)^{(2/3)} / (c^{(2/3)} * (a + b * x^2)^{(1/3)} + 1)) / (4 * b^{(2/3)}) - \text{sqrt}(3) * c^{(1/3)} * \operatorname{atan}(\text{sqrt}(3) * (2 * b^{(1/3)} * (c * x)^{(2/3)} / (3 * (a + b * x^2)^{(1/3)} + c^{(2/3)}) / 3) / c^{(2/3)}) / (2 * b^{(2/3)})$

**Mathematica [C]** time = 0.0317583, size = 57, normalized size = 0.31

$$\frac{3x\sqrt[3]{cx} \left(\frac{a+bx^2}{a}\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{2}{3}; \frac{5}{3}; -\frac{bx^2}{a}\right)}{4(a+bx^2)^{2/3}}$$



Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(1/3)/(a + b\*x^2)^(2/3), x]

[Out] (3\*x\*(c\*x)^(1/3)\*((a + b\*x^2)/a)^(2/3)\*Hypergeometric2F1[2/3, 2/3, 5/3, -(b\*x^2)/a])/(4\*(a + b\*x^2)^(2/3))

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int \sqrt[3]{cx} (bx^2 + a)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(1/3)/(b\*x^2+a)^(2/3), x)

[Out] int((c\*x)^(1/3)/(b\*x^2+a)^(2/3), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(1/3)/(b\*x^2 + a)^(2/3), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(1/3)/(b\*x^2 + a)^(2/3), x, algorithm="fricas")

[Out] Timed out

**Sympy [A]** time = 6.90801, size = 44, normalized size = 0.24

$$\frac{\sqrt[3]{cx^{\frac{4}{3}}} \left(\frac{2}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{2}{3} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}} \left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(1/3)/(b\*x\*\*2+a)\*\*(2/3), x)

[Out] c\*\*(1/3)\*x\*\*(4/3)\*gamma(2/3)\*hyper((2/3, 2/3), (5/3, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*a\*\*(2/3)\*gamma(5/3))

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{1}{3}}}{(bx^2 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/3)/(b*x^2 + a)^(2/3), x, algorithm="giac")
```

```
[Out] integrate((c*x)^(1/3)/(b*x^2 + a)^(2/3), x)
```

$$3.775 \quad \int \frac{1}{(cx)^{5/3}(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=28

$$-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{2/3}}$$

[Out]  $(-3*(a + b*x^2)^{(1/3)})/(2*a*c*(c*x)^{(2/3)})$

**Rubi [A]** time = 0.0285114, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Int[1/((c*x)^(5/3)*(a + b*x^2)^(2/3)), x]`

[Out]  $(-3*(a + b*x^2)^{(1/3)})/(2*a*c*(c*x)^{(2/3)})$

**Rubi in Sympy [A]** time = 3.547, size = 24, normalized size = 0.86

$$-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(c*x)**(5/3)/(b*x**2+a)**(2/3), x)`

[Out]  $-3*(a + b*x**2)**(1/3)/(2*a*c*(c*x)**(2/3))$

**Mathematica [A]** time = 0.0192195, size = 26, normalized size = 0.93

$$-\frac{3x\sqrt[3]{a+bx^2}}{2a(cx)^{5/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/((c*x)^(5/3)*(a + b*x^2)^(2/3)), x]`

[Out]  $(-3*x*(a + b*x^2)^{(1/3)})/(2*a*(c*x)^{(5/3)})$

**Maple [A]** time = 0.006, size = 21, normalized size = 0.8

$$-\frac{3x}{2a}\sqrt[3]{bx^2+a}(cx)^{-5/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(5/3)/(b*x^2+a)^(2/3), x)`

[Out]  $-3/2 * x * (b * x^2 + a)^{(1/3)} / a / (c * x)^{(5/3)}$

**Maxima [A]** time = 1.36566, size = 47, normalized size = 1.68

$$\frac{3 \left( bc^{\frac{1}{3}} x^3 + ac^{\frac{1}{3}} x \right)}{2 (bx^2 + a)^{\frac{2}{3}} ac^2 x^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(5/3)),x, algorithm="maxima")`

[Out]  $-3/2 * (b * c^{(1/3)} * x^3 + a * c^{(1/3)} * x) / ((b * x^2 + a)^{(2/3)} * a * c^2 * x^{(5/3)})$

**Fricas [A]** time = 0.228858, size = 34, normalized size = 1.21

$$\frac{3 (bx^2 + a)^{\frac{1}{3}} (cx)^{\frac{1}{3}}}{2 ac^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(5/3)),x, algorithm="fricas")`

[Out]  $-3/2 * (b * x^2 + a)^{(1/3)} * (c * x)^{(1/3)} / (a * c^2 * x)$

**Sympy [A]** time = 54.7297, size = 36, normalized size = 1.29

$$\frac{\sqrt[3]{b} \sqrt[3]{\frac{a}{bx^2} + 1} \left(-\frac{1}{3}\right)}{2ac^{\frac{5}{3}} \left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(5/3)/(b*x**2+a)**(2/3),x)`

[Out]  $b^{(1/3)} * (a / (b * x^2) + 1)^{(1/3)} * \text{gamma}(-1/3) / (2 * a * c^{(5/3)} * \text{gamma}(2/3))$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{2}{3}} (cx)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(5/3)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(5/3)), x)`

$$3.776 \quad \int \frac{1}{(cx)^{11/3}(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=57

$$\frac{9(a+bx^2)^{4/3}}{8a^2c(cx)^{8/3}} - \frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}}$$

[Out]  $(-3*(a + b*x^2)^{(1/3)})/(2*a*c*(c*x)^{(8/3)}) + (9*(a + b*x^2)^{(4/3)})/(8*a^2*c*(c*x)^{(8/3)})$

**Rubi [A]** time = 0.0575739, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{9(a+bx^2)^{4/3}}{8a^2c(cx)^{8/3}} - \frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(11/3)\*(a + b\*x^2)^(2/3)), x]

[Out]  $(-3*(a + b*x^2)^{(1/3)})/(2*a*c*(c*x)^{(8/3)}) + (9*(a + b*x^2)^{(4/3)})/(8*a^2*c*(c*x)^{(8/3)})$

**Rubi in Sympy [A]** time = 6.6536, size = 48, normalized size = 0.84

$$-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}} + \frac{9(a+bx^2)^{4/3}}{8a^2c(cx)^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(11/3)/(b\*x\*\*2+a)\*\*(2/3), x)

[Out]  $-3*(a + b*x**2)**(1/3)/(2*a*c*(c*x)**(8/3)) + 9*(a + b*x**2)**(4/3)/(8*a**2*c*(c*x)**(8/3))$

**Mathematica [A]** time = 0.0372911, size = 34, normalized size = 0.6

$$-\frac{3x(a-3bx^2)\sqrt[3]{a+bx^2}}{8a^2(cx)^{11/3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(11/3)\*(a + b\*x^2)^(2/3)), x]

[Out]  $(-3*x*(a - 3*b*x^2)*(a + b*x^2)^{(1/3)})/(8*a^2*(c*x)^{(11/3)})$

**Maple [A]** time = 0.007, size = 29, normalized size = 0.5

$$-\frac{3x(-3bx^2+a)\sqrt[3]{bx^2+a}(cx)^{-11/3}}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x)`

[Out]  $-3/8*x*(b*x^2+a)^(1/3)*(-3*b*x^2+a)/a^2/(c*x)^(11/3)$

**Maxima [A]** time = 1.39331, size = 51, normalized size = 0.89

$$\frac{3 \left( \frac{4(bx^2+a)^{\frac{1}{3}}b}{x^{\frac{2}{3}}} - \frac{(bx^2+a)^{\frac{4}{3}}}{x^{\frac{8}{3}}} \right)}{8a^2c^{\frac{11}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(11/3)),x, algorithm="maxima")`

[Out]  $3/8*(4*(b*x^2 + a)^(1/3)*b/x^(2/3) - (b*x^2 + a)^(4/3)/x^(8/3))/(a^2*c^(11/3))$

**Fricas [A]** time = 0.229495, size = 47, normalized size = 0.82

$$\frac{3(3bx^2 - a)(bx^2 + a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{8a^2c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(11/3)),x, algorithm="fricas")`

[Out]  $3/8*(3*b*x^2 - a)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^2*c^4*x^3)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(11/3)/(b*x**2+a)**(2/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{2}{3}}(cx)^{\frac{11}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(11/3)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(11/3)), x)`

$$3.777 \quad \int \frac{1}{(cx)^{17/3}(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=85

$$-\frac{27(a+bx^2)^{7/3}}{28a^3c(cx)^{14/3}} + \frac{9(a+bx^2)^{4/3}}{4a^2c(cx)^{14/3}} - \frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}}$$

[Out]  $(-3*(a + b*x^2)^{(1/3)})/(2*a*c*(c*x)^{(14/3)}) + (9*(a + b*x^2)^{(4/3)})/(4*a^2*c*(c*x)^{(14/3)}) - (27*(a + b*x^2)^{(7/3)})/(28*a^3*c*(c*x)^{(14/3)})$

**Rubi [A]** time = 0.0894068, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$-\frac{27(a+bx^2)^{7/3}}{28a^3c(cx)^{14/3}} + \frac{9(a+bx^2)^{4/3}}{4a^2c(cx)^{14/3}} - \frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(17/3)\*(a + b\*x^2)^(2/3)), x]

[Out]  $(-3*(a + b*x^2)^{(1/3)})/(2*a*c*(c*x)^{(14/3)}) + (9*(a + b*x^2)^{(4/3)})/(4*a^2*c*(c*x)^{(14/3)}) - (27*(a + b*x^2)^{(7/3)})/(28*a^3*c*(c*x)^{(14/3)})$

**Rubi in Sympy [A]** time = 10.7068, size = 73, normalized size = 0.86

$$-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{\frac{14}{3}}} + \frac{9(a+bx^2)^{\frac{4}{3}}}{4a^2c(cx)^{\frac{14}{3}}} - \frac{27(a+bx^2)^{\frac{7}{3}}}{28a^3c(cx)^{\frac{14}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(17/3)/(b\*x\*\*2+a)\*\*(2/3), x)

[Out]  $-3*(a + b*x**2)**(1/3)/(2*a*c*(c*x)**(14/3)) + 9*(a + b*x**2)**(4/3)/(4*a**2*c*(c*x)**(14/3)) - 27*(a + b*x**2)**(7/3)/(28*a**3*c*(c*x)**(14/3))$

**Mathematica [A]** time = 0.0490489, size = 52, normalized size = 0.61

$$\frac{3\sqrt[3]{cx}\sqrt[3]{a+bx^2}(2a^2-3abx^2+9b^2x^4)}{28a^3c^6x^5}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(17/3)\*(a + b\*x^2)^(2/3)), x]

[Out]  $(-3*(c*x)^{(1/3)}*(a + b*x^2)^{(1/3)}*(2*a^2 - 3*a*b*x^2 + 9*b^2*x^4))/(28*a^3*c^6*x^5)$

**Maple [A]** time = 0.007, size = 42, normalized size = 0.5

$$-\frac{3x(9b^2x^4 - 3abx^2 + 2a^2)}{28a^3} \sqrt[3]{bx^2 + a} (cx)^{-\frac{17}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x)`

[Out]  $-3/28*x*(b*x^2+a)^{(1/3)}*(9*b^2*x^4-3*a*b*x^2+2*a^2)/a^3/(c*x)^{(17/3)}$

**Maxima [A]** time = 1.40287, size = 74, normalized size = 0.87

$$\frac{3 \left( \frac{14(bx^2+a)^{\frac{1}{3}}b^2}{x^{\frac{2}{3}}} - \frac{7(bx^2+a)^{\frac{4}{3}}b}{x^{\frac{8}{3}}} + \frac{2(bx^2+a)^{\frac{7}{3}}}{x^{\frac{14}{3}}} \right)}{28a^3c^{\frac{17}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(17/3)),x, algorithm="maxima")`

[Out]  $-3/28*(14*(b*x^2 + a)^{(1/3)}*b^2/x^{(2/3)} - 7*(b*x^2 + a)^{(4/3)}*b/x^{(8/3)} + 2*(b*x^2 + a)^{(7/3)}/x^{(14/3)})/(a^3*c^{(17/3)})$

**Fricas [A]** time = 0.229017, size = 62, normalized size = 0.73

$$\frac{3(9b^2x^4 - 3abx^2 + 2a^2)(bx^2 + a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{28a^3c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(17/3)),x, algorithm="fricas")`

[Out]  $-3/28*(9*b^2*x^4 - 3*a*b*x^2 + 2*a^2)*(b*x^2 + a)^{(1/3)}*(c*x)^{(1/3)}/(a^3*c^6*x^5)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(17/3)/(b*x**2+a)**(2/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{2}{3}}(cx)^{\frac{17}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(17/3)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(17/3)), x)`



$$3.778 \quad \int \frac{1}{(cx)^{23/3}(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=113

$$\frac{243(a+bx^2)^{10/3}}{280a^4c(cx)^{20/3}} - \frac{81(a+bx^2)^{7/3}}{28a^3c(cx)^{20/3}} + \frac{27(a+bx^2)^{4/3}}{8a^2c(cx)^{20/3}} - \frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}}$$

[Out]  $(-3*(a + b*x^2)^{(1/3)})/(2*a*c*(c*x)^{(20/3)}) + (27*(a + b*x^2)^{(4/3)})/(8*a^2*c*(c*x)^{(20/3)}) - (81*(a + b*x^2)^{(7/3)})/(28*a^3*c*(c*x)^{(20/3)}) + (243*(a + b*x^2)^{(10/3)})/(280*a^4*c*(c*x)^{(20/3)})$

**Rubi [A]** time = 0.123032, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{243(a+bx^2)^{10/3}}{280a^4c(cx)^{20/3}} - \frac{81(a+bx^2)^{7/3}}{28a^3c(cx)^{20/3}} + \frac{27(a+bx^2)^{4/3}}{8a^2c(cx)^{20/3}} - \frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(23/3)\*(a + b\*x^2)^(2/3)), x]

[Out]  $(-3*(a + b*x^2)^{(1/3)})/(2*a*c*(c*x)^{(20/3)}) + (27*(a + b*x^2)^{(4/3)})/(8*a^2*c*(c*x)^{(20/3)}) - (81*(a + b*x^2)^{(7/3)})/(28*a^3*c*(c*x)^{(20/3)}) + (243*(a + b*x^2)^{(10/3)})/(280*a^4*c*(c*x)^{(20/3)})$

**Rubi in Sympy [A]** time = 15.3909, size = 99, normalized size = 0.88

$$-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{\frac{20}{3}}} + \frac{27(a+bx^2)^{\frac{4}{3}}}{8a^2c(cx)^{\frac{20}{3}}} - \frac{81(a+bx^2)^{\frac{7}{3}}}{28a^3c(cx)^{\frac{20}{3}}} + \frac{243(a+bx^2)^{\frac{10}{3}}}{280a^4c(cx)^{\frac{20}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(23/3)/(b\*x\*\*2+a)\*\*(2/3), x)

[Out]  $-3*(a + b*x**2)**(1/3)/(2*a*c*(c*x)**(20/3)) + 27*(a + b*x**2)**(4/3)/(8*a**2*c*(c*x)**(20/3)) - 81*(a + b*x**2)**(7/3)/(28*a**3*c*(c*x)**(20/3)) + 243*(a + b*x**2)**(10/3)/(280*a**4*c*(c*x)**(20/3))$

**Mathematica [A]** time = 0.0557839, size = 63, normalized size = 0.56

$$\frac{3\sqrt[3]{cx}\sqrt[3]{a+bx^2}(-14a^3 + 18a^2bx^2 - 27ab^2x^4 + 81b^3x^6)}{280a^4c^8x^7}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(23/3)\*(a + b\*x^2)^(2/3)), x]

[Out]  $(3*(c*x)^{(1/3)}*(a + b*x^2)^{(1/3)}*(-14*a^3 + 18*a^2*b*x^2 - 27*a*b^2*x^4 + 81*b^3*x^6))/(280*a^4*c^8*x^7)$

**Maple [A]** time = 0.01, size = 53, normalized size = 0.5

$$-\frac{3x(-81b^3x^6 + 27ab^2x^4 - 18a^2bx^2 + 14a^3)\sqrt[3]{bx^2 + a}(cx)^{-\frac{23}{3}}}{280a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x)`

[Out] 
$$-3/280*x*(b*x^2+a)^(1/3)*(-81*b^3*x^6+27*a*b^2*x^4-18*a^2*b*x^2+14*a^3)/a^4/(c*x)^(23/3)$$

**Maxima [A]** time = 1.36288, size = 86, normalized size = 0.76

$$\frac{3(81b^4x^9 + 54ab^3x^7 - 9a^2b^2x^5 + 4a^3bx^3 - 14a^4x)}{280(bx^2 + a)^{\frac{2}{3}}a^4c^{\frac{23}{3}}x^{\frac{23}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(23/3)),x, algorithm="maxima")`

[Out] 
$$3/280*(81*b^4*x^9 + 54*a*b^3*x^7 - 9*a^2*b^2*x^5 + 4*a^3*b*x^3 - 14*a^4*x)/((b*x^2 + a)^(2/3)*a^4*c^(23/3)*x^(23/3))$$

**Fricas [A]** time = 0.233513, size = 77, normalized size = 0.68

$$\frac{3(81b^3x^6 - 27ab^2x^4 + 18a^2bx^2 - 14a^3)(bx^2 + a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{280a^4c^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(23/3)),x, algorithm="fricas")`

[Out] 
$$3/280*(81*b^3*x^6 - 27*a*b^2*x^4 + 18*a^2*b*x^2 - 14*a^3)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^4*c^8*x^7)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(23/3)/(b*x**2+a)**(2/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{2}{3}}(cx)^{\frac{23}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(23/3)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(23/3)), x)`

$$3.779 \quad \int \frac{(cx)^{10/3}}{(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=421

$$\frac{7ac^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9b^2} + \frac{7ac^{7/3}\sqrt[3]{cx}\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}}{\left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right)^2}}}{F \left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}} \right) \right) \left| \frac{1}{4} (2 + \sqrt{3}) \right.}}{18\sqrt[4]{3}b^2 \sqrt{\frac{\sqrt[3]{b}(cx)^{2/3} \left( c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right)^2}}} + \frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}$$

[Out]  $(-7*a*c^3*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)})/(9*b^2) + (c*(c*x)^{(7/3)}*(a+b*x^2)^{(1/3)})/(3*b) + (7*a*c^{(7/3)}*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})*\text{Sqrt}[(c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a+b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcCos}[(c^{(2/3)} - ((1 - \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})], (2 + \text{Sqrt}[3])/4)]/(18*3^{(1/4)}*b^2*\text{Sqrt}[-((b^{(1/3)}*(c*x)^{(2/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}))/(a+b*x^2)^{(1/3)}*(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2)])$

**Rubi [A]** time = 1.55699, antiderivative size = 421, normalized size of antiderivative = 1., number of rules used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{7ac^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9b^2} + \frac{7ac^{7/3}\sqrt[3]{cx}\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}}{\left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right)^2}}}{F \left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}} \right) \right) \left| \frac{1}{4} (2 + \sqrt{3}) \right.}}{18\sqrt[4]{3}b^2 \sqrt{\frac{\sqrt[3]{b}(cx)^{2/3} \left( c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right)^2}}} + \frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*x)^{(10/3)}/(a+b*x^2)^{(2/3)}, x]$

[Out]  $(-7*a*c^3*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)})/(9*b^2) + (c*(c*x)^{(7/3)}*(a+b*x^2)^{(1/3)})/(3*b) + (7*a*c^{(7/3)}*(c*x)^{(1/3)}*(a+b*x^2)^{(1/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})*\text{Sqrt}[(c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a+b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcCos}[(c^{(2/3)} - ((1 - \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})], (2 + \text{Sqrt}[3])/4)]/(18*3^{(1/4)}*b^2*\text{Sqrt}[-((b^{(1/3)}*(c*x)^{(2/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)}))/(a+b*x^2)^{(1/3)}*(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a+b*x^2)^{(1/3)})^2)])$

$$- (b^{1/3} (c x)^{2/3}) / (a + b x^2)^{1/3} / ((a + b x^2)^{1/3} (c^{2/3} - ((1 + \sqrt{3}) b^{1/3} (c x)^{2/3}) / (a + b x^2)^{1/3}))^2$$

**Rubi in Sympy [A]** time = 36.779, size = 411, normalized size = 0.98

$$\frac{7 \cdot 3^{3/4} a^2 c^{7/3} \sqrt{cx} \sqrt{\frac{\frac{b^{2/3} (cx)^{4/3} + \sqrt[3]{bc} \frac{2}{3} (cx)^{2/3} + c^{4/3}}{(a+bx^2)^{2/3}} + \sqrt[3]{a+bx^2}}{\left(\frac{\sqrt[3]{b} (cx)^{2/3} (-\sqrt{3}-1) + c^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2} \left(-\frac{\sqrt[3]{b} (cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{2/3}\right) F\left(\arcsin\left(\frac{\sqrt[3]{b} (cx)^{2/3} (-1+\sqrt{3}) + c^{2/3}}{\sqrt[3]{a+bx^2}}\right) \left|\frac{\sqrt{3}}{4} + \frac{1}{2}\right.\right)}}{54b^2 \sqrt{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{b} (cx)^{2/3} \left(-\frac{\sqrt[3]{b} (cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{2/3}\right)}{\sqrt[3]{a+bx^2} \left(\frac{\sqrt[3]{b} (cx)^{2/3} (-\sqrt{3}-1) + c^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2} (a+bx^2)^{2/3} \sqrt{-\frac{bx^2}{a+bx^2} + 1}} - \frac{7ac^3 \sqrt{cx} \sqrt{a+bx^2}}{9b^2} + \frac{c (cx)^{7/3} \sqrt{a+bx^2}}{3b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x)**(10/3)/(b*x**2+a)**(2/3), x)`

[Out]  $7 \cdot 3^{3/4} a^2 c^{7/3} (c x)^{1/3} \sqrt{(b^{2/3} (c x)^{4/3}) / (a + b x^2)^{2/3} + b^{1/3} c^{2/3} (c x)^{2/3} / (a + b x^2)^{1/3} + c^{4/3}) / (b^{1/3} (c x)^{2/3} (-\sqrt{3} - 1) / (a + b x^2)^{1/3} + c^{2/3})^2} (-b^{1/3} (c x)^{2/3} / (a + b x^2)^{1/3} + c^{2/3}) \operatorname{elliptic}_f(\arcsin((b^{1/3} (c x)^{2/3} (-1 + \sqrt{3})) / (a + b x^2)^{1/3} + c^{2/3}) / (b^{1/3} (c x)^{2/3} (-\sqrt{3} - 1) / (a + b x^2)^{1/3} + c^{2/3})), \sqrt{3} / 4 + 1 / 2) / (54 b^2 \sqrt{a / (a + b x^2)} \sqrt{-b^{1/3} (c x)^{2/3} (-b^{1/3} (c x)^{2/3} / (a + b x^2)^{1/3} + c^{2/3}) / ((a + b x^2)^{1/3} (b^{1/3} (c x)^{2/3} (-\sqrt{3} - 1) / (a + b x^2)^{1/3} + c^{2/3})^2)} (a + b x^2)^{2/3} \sqrt{-b x^2 / (a + b x^2) + 1}) - 7 a^2 c^3 (c x)^{1/3} (a + b x^2)^{1/3} / (9 b^2) + c (c x)^{7/3} (a + b x^2)^{1/3} / (3 b)$

**Mathematica [C]** time = 0.0622693, size = 87, normalized size = 0.21

$$\frac{c^3 \sqrt{cx} \left(7a^2 \left(\frac{bx^2}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{6}, \frac{2}{3}; \frac{7}{6}; -\frac{bx^2}{a}\right) - 7a^2 - 4abx^2 + 3b^2x^4\right)}{9b^2 (a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c*x)^(10/3)/(a + b*x^2)^(2/3), x]`

[Out]  $(c^3 (c x)^{1/3} (-7 a^2 - 4 a b x^2 + 3 b^2 x^4 + 7 a^2 (1 + (b x^2)/a)^{2/3} \operatorname{Hypergeometric2F1}[1/6, 2/3, 7/6, -(b x^2)/a])) / (9 b^2 (a + b x^2)^{2/3})$

**Maple [F]** time = 0.035, size = 0, normalized size = 0.

$$\int 1 (cx)^{10/3} (bx^2 + a)^{-2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(10/3)/(b*x^2+a)^(2/3), x)`

[Out] `int((c*x)^(10/3)/(b*x^2+a)^(2/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{10}{3}}}{(bx^2 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(10/3)/(b*x^2 + a)^(2/3),x, algorithm="maxima")`

[Out] `integrate((c*x)^(10/3)/(b*x^2 + a)^(2/3), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^{\frac{1}{3}} c^3 x^3}{(bx^2 + a)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(10/3)/(b*x^2 + a)^(2/3),x, algorithm="fricas")`

[Out] `integral((c*x)^(1/3)*c^3*x^3/(b*x^2 + a)^(2/3), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(10/3)/(b*x**2+a)**(2/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{10}{3}}}{(bx^2 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(10/3)/(b*x^2 + a)^(2/3),x, algorithm="giac")`

[Out] `integrate((c*x)^(10/3)/(b*x^2 + a)^(2/3), x)`

$$3.780 \quad \int \frac{(cx)^{4/3}}{(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=388

$$\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}$$

$$\sqrt[3]{c}\sqrt[3]{cx}\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}}{\left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right)^2}} F \left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}} \right) \middle| \frac{1}{4} (2 + \sqrt{3}) \right)$$


---


$$2\sqrt[4]{3}b \sqrt{\frac{\sqrt[3]{b}(cx)^{2/3} \left( c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right)^2}}$$

[Out] (c\*(c\*x)^(1/3)\*(a+b\*x^2)^(1/3))/b - (c^(1/3)\*(c\*x)^(1/3)\*(a+b\*x^2)^(1/3)\*(c^(2/3) - (b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))\*Sqrt[(c^(4/3) + (b^(2/3)\*(c\*x)^(4/3))/(a+b\*x^2)^(2/3) + (b^(1/3)\*c^(2/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))/(c^(2/3) - ((1+Sqrt[3])\*b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))^2]\*EllipticF[ArcCos[(c^(2/3) - ((1-Sqrt[3])\*b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))/(c^(2/3) - ((1+Sqrt[3])\*b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))], (2+Sqrt[3])/4])/(2\*3^(1/4)\*b\*Sqrt[-((b^(1/3)\*(c\*x)^(2/3)\*(c^(2/3) - (b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3)))/((a+b\*x^2)^(1/3)\*(c^(2/3) - ((1+Sqrt[3])\*b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))^2))])]

**Rubi [A]** time = 1.41581, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}$$

$$\sqrt[3]{c}\sqrt[3]{cx}\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}}{\left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right)^2}} F \left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{bx^2+a}}} \right) \middle| \frac{1}{4} (2 + \sqrt{3}) \right)$$


---


$$2\sqrt[4]{3}b \sqrt{\frac{\sqrt[3]{b}(cx)^{2/3} \left( c^{2/3} - \frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}} \right)^2}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(4/3)/(a+b\*x^2)^(2/3),x]

[Out] (c\*(c\*x)^(1/3)\*(a+b\*x^2)^(1/3))/b - (c^(1/3)\*(c\*x)^(1/3)\*(a+b\*x^2)^(1/3)\*(c^(2/3) - (b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))\*Sqrt[(c^(4/3) + (b^(2/3)\*(c\*x)^(4/3))/(a+b\*x^2)^(2/3) + (b^(1/3)\*c^(2/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))/(c^(2/3) - ((1+Sqrt[3])\*b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))^2]\*EllipticF[ArcCos[(c^(2/3) - ((1-Sqrt[3])\*b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))/(c^(2/3) - ((1+Sqrt[3])\*b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))], (2+Sqrt[3])/4])/(2\*3^(1/4)\*b\*Sqrt[-((b^(1/3)\*(c\*x)^(2/3)\*(c^(2/3) - (b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3)))/((a+b\*x^2)^(1/3)\*(c^(2/3) - ((1+Sqrt[3])\*b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))^2))])]

**Rubi in Sympy [A]** time = 28.0813, size = 376, normalized size = 0.97

$$\frac{3^{\frac{3}{4}} a^{\frac{3}{4}} c^{\frac{3}{4}} \sqrt{cx}}{\sqrt{\frac{\frac{b^{\frac{2}{3}}(cx)^{\frac{4}{3}} + \sqrt[3]{b} c^{\frac{2}{3}} (cx)^{\frac{2}{3}}}{(a+bx^2)^{\frac{2}{3}} + \sqrt[3]{a+bx^2}} + c^{\frac{4}{3}}}{\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1)}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)^2} \left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right) F\left(\arcsin\left(\frac{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-1+\sqrt{3})}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}}{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1)}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}}\right)\right) \left|\frac{\sqrt{3}}{4} + \frac{1}{2}\right.}}}{6b\sqrt{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}\left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)}{\sqrt[3]{a+bx^2}\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1)}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)^2} (a+bx^2)^{\frac{2}{3}} \sqrt{-\frac{bx^2}{a+bx^2} + 1}} + \frac{c^{\frac{3}{4}} \sqrt{cx} \sqrt{a+bx^2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x)**(4/3)/(b*x**2+a)**(2/3), x)`

[Out]  $-3^{3/4} a^{3/4} c^{3/4} (c x)^{1/3} (c x)^{1/3} \sqrt{(b^{2/3} (c x)^{4/3} / (a + b x^2)^{2/3} + b^{1/3} c^{2/3} (c x)^{2/3} / (a + b x^2)^{1/3} + c^{4/3}) / (b^{1/3} (c x)^{2/3} (-\sqrt{3} - 1) / (a + b x^2)^{1/3} + c^{2/3})} (-b^{1/3} (c x)^{2/3} / (a + b x^2)^{1/3} + c^{2/3}) \operatorname{elliptic}_f(\arcsin((b^{1/3} (c x)^{2/3} (-1 + \sqrt{3})) / (a + b x^2)^{1/3} + c^{2/3}) / (b^{1/3} (c x)^{2/3} (-\sqrt{3} - 1) / (a + b x^2)^{1/3} + c^{2/3})), \sqrt{3} / 4 + 1 / 2) / (6 b \sqrt{a / (a + b x^2)} \sqrt{-b^{1/3} (c x)^{2/3} (-b^{1/3} (c x)^{2/3} / (a + b x^2)^{1/3} + c^{2/3}) / ((a + b x^2)^{1/3} (b^{1/3} (c x)^{2/3} (-\sqrt{3} - 1) / (a + b x^2)^{1/3} + c^{2/3}))} (a + b x^2)^{2/3} \sqrt{-b x^2 / (a + b x^2) + 1}) + c^{3/4} \sqrt{c x} \sqrt{a + b x^2} / b$

**Mathematica [C]** time = 0.0519176, size = 66, normalized size = 0.17

$$\frac{c^{\frac{3}{4}} \sqrt{cx} \left(-a \left(\frac{bx^2}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{6}, \frac{2}{3}; \frac{7}{6}; -\frac{bx^2}{a}\right) + a + bx^2\right)}{b(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c*x)^(4/3)/(a + b*x^2)^(2/3), x]`

[Out]  $(c (c x)^{1/3} (a + b x^2 - a (1 + (b x^2)/a)^{2/3}) \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{2}{3}, \frac{7}{6}, -\frac{b x^2}{a}\right]) / (b (a + b x^2)^{2/3})$

**Maple [F]** time = 0.028, size = 0, normalized size = 0.

$$\int 1 (cx)^{\frac{4}{3}} (bx^2 + a)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(4/3)/(b*x^2+a)^(2/3), x)`

[Out] `int((c*x)^(4/3)/(b*x^2+a)^(2/3), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{4}{3}}}{(bx^2 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(4/3)/(b*x^2 + a)^(2/3),x, algorithm="maxima")`

[Out] `integrate((c*x)^(4/3)/(b*x^2 + a)^(2/3), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^{\frac{1}{3}} cx}{(bx^2 + a)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(4/3)/(b*x^2 + a)^(2/3),x, algorithm="fricas")`

[Out] `integral((c*x)^(1/3)*c*x/(b*x^2 + a)^(2/3), x)`

**Sympy** [A] time = 69.5841, size = 44, normalized size = 0.11

$$\frac{c^{\frac{4}{3}} x^{\frac{7}{3}} \left(\frac{7}{6}\right) {}_2F_1\left(\frac{2}{3}, \frac{7}{6} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}} \left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(4/3)/(b*x**2+a)**(2/3),x)`

[Out] `c**(4/3)*x**(7/3)*gamma(7/6)*hyper((2/3, 7/6), (13/6,), b*x**2*exp_polar(I*pi)/a)/(2*a**(2/3)*gamma(13/6))`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{4}{3}}}{(bx^2 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(4/3)/(b*x^2 + a)^(2/3),x, algorithm="giac")`

[Out] `integrate((c*x)^(4/3)/(b*x^2 + a)^(2/3), x)`



$$3.781 \quad \int \frac{1}{(cx)^{2/3}(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=364

$$\frac{3^{3/4} \sqrt[3]{cx} \sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b} c^{2/3} (cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}}{\left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}}{F \left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{\frac{\sqrt[3]{bx^2+a}}{c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}} \right) \right) \Big|_{\frac{1}{4}} (2 + \sqrt{3})}}{2ac^{5/3} \sqrt{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}}$$

[Out] (3^(3/4)\*(c\*x)^(1/3)\*(a+b\*x^2)^(1/3)\*(c^(2/3)-(b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))\*Sqrt[(c^(4/3)+(b^(2/3)\*(c\*x)^(4/3))/(a+b\*x^2)^(2/3)+(b^(1/3)\*c^(2/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))]/(c^(2/3)-((1+Sqrt[3])\*b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))^2]\*EllipticF[ArcCos[(c^(2/3)-((1-Sqrt[3])\*b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))/(c^(2/3)-((1+Sqrt[3])\*b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))],(2+Sqrt[3])/4]]/(2\*a\*c^(5/3)\*Sqrt[-((b^(1/3)\*(c\*x)^(2/3)\*(c^(2/3)-(b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3)))/((a+b\*x^2)^(1/3)\*(c^(2/3)-((1+Sqrt[3])\*b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))^2))]]

**Rubi [A]** time = 1.27889, antiderivative size = 364, normalized size of antiderivative = 1., number of rules used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{3^{3/4} \sqrt[3]{cx} \sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right) \sqrt{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b} c^{2/3} (cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}}{\left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}}{F \left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{\frac{\sqrt[3]{bx^2+a}}{c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}} \right) \right) \Big|_{\frac{1}{4}} (2 + \sqrt{3})}}{2ac^{5/3} \sqrt{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(2/3)\*(a+b\*x^2)^(2/3)),x]

[Out] (3^(3/4)\*(c\*x)^(1/3)\*(a+b\*x^2)^(1/3)\*(c^(2/3)-(b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))\*Sqrt[(c^(4/3)+(b^(2/3)\*(c\*x)^(4/3))/(a+b\*x^2)^(2/3)+(b^(1/3)\*c^(2/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))]/(c^(2/3)-((1+Sqrt[3])\*b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))^2]\*EllipticF[ArcCos[(c^(2/3)-((1-Sqrt[3])\*b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))/(c^(2/3)-((1+Sqrt[3])\*b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))],(2+Sqrt[3])/4]]/(2\*a\*c^(5/3)\*Sqrt[-((b^(1/3)\*(c\*x)^(2/3)\*(c^(2/3)-(b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3)))/((a+b\*x^2)^(1/3)\*(c^(2/3)-((1+Sqrt[3])\*b^(1/3)\*(c\*x)^(2/3))/(a+b\*x^2)^(1/3))^2))]]

**Rubi in Sympy [A]** time = 21.4245, size = 352, normalized size = 0.97

$$3^{\frac{3}{4}} \sqrt[3]{cx} \sqrt{\frac{\frac{b^{\frac{2}{3}}(cx)^{\frac{4}{3}}}{(a+bx^2)^{\frac{2}{3}}} + \frac{\sqrt[3]{b}c^{\frac{2}{3}}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{4}{3}}}{\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1)}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)^2} \left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right) F\left(\operatorname{acos}\left(\frac{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-1+\sqrt{3})}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}}{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1)}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}}\right)\right) \left|\frac{\sqrt{3}}{4} + \frac{1}{2}\right|}}{2c^{\frac{5}{3}} \sqrt{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}\left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)}{\sqrt[3]{a+bx^2}\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1)}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)^2} (a+bx^2)^{\frac{2}{3}} \sqrt{-\frac{bx^2}{a+bx^2} + 1}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(c*x)**(2/3)/(b*x**2+a)**(2/3), x)`

[Out] `3**(3/4)*(c*x)**(1/3)*sqrt((b**(2/3)*(c*x)**(4/3)/(a + b*x**2)**(2/3) + b**(1/3)*c**(2/3)*(c*x)**(2/3)/(a + b*x**2)**(1/3) + c**(4/3))/(b**(1/3)*(c*x)**(2/3)*(-sqrt(3) - 1)/(a + b*x**2)**(1/3) + c**(2/3)**2)*(-b**(1/3)*(c*x)**(2/3)/(a + b*x**2)**(1/3) + c**(2/3))*elliptic_f(acos((b**(1/3)*(c*x)**(2/3)*(-1 + sqrt(3)))/(a + b*x**2)**(1/3) + c**(2/3))/(b**(1/3)*(c*x)**(2/3)*(-sqrt(3) - 1)/(a + b*x**2)**(1/3) + c**(2/3))), sqrt(3)/4 + 1/2)/(2*c**(5/3)*sqrt(a/(a + b*x**2))*sqrt(-b**(1/3)*(c*x)**(2/3)*(-b**(1/3)*(c*x)**(2/3)/(a + b*x**2)**(1/3) + c**(2/3))/((a + b*x**2)**(1/3)*(b**(1/3)*(c*x)**(2/3)*(-sqrt(3) - 1)/(a + b*x**2)**(1/3) + c**(2/3))**2))*(a + b*x**2)**(2/3)*sqrt(-b*x**2/(a + b*x**2) + 1))`

**Mathematica [C]** time = 0.0323055, size = 55, normalized size = 0.15

$$\frac{3x \left(\frac{a+bx^2}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{6}, \frac{2}{3}; \frac{7}{6}; -\frac{bx^2}{a}\right)}{(cx)^{2/3} (a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/((c*x)^(2/3)*(a + b*x^2)^(2/3)), x]`

[Out] `(3*x*((a + b*x^2)/a)^(2/3)*Hypergeometric2F1[1/6, 2/3, 7/6, -(b*x^2/a)])/((c*x)^(2/3)*(a + b*x^2)^(2/3))`

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int 1 (cx)^{-\frac{2}{3}} (bx^2 + a)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(2/3)/(b*x^2+a)^(2/3), x)`

[Out] `int(1/(c*x)^(2/3)/(b*x^2+a)^(2/3), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{2}{3}} (cx)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(2/3)),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(2/3)), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{2}{3}}(cx)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(2/3)),x, algorithm="fricas")`

[Out] `integral(1/((b*x^2 + a)^(2/3)*(c*x)^(2/3)), x)`

**Sympy** [A] time = 8.97642, size = 31, normalized size = 0.09

$$-\frac{{}_2F_1\left(\frac{1}{2}, \frac{2}{3} \mid \frac{ae^{i\pi}}{bx^2}\right)}{b^{\frac{2}{3}}c^{\frac{2}{3}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(2/3)/(b*x**2+a)**(2/3),x)`

[Out] `-hyper((1/2, 2/3), (3/2, ), a*exp_polar(I*pi)/(b*x**2))/(b**(2/3)*c**(2/3)*x)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{2}{3}}(cx)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(2/3)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(2/3)), x)`

$$3.782 \quad \int \frac{1}{(cx)^{8/3}(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=394

$$\frac{3^{3/4} b \sqrt[3]{cx} \sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right) \sqrt[3]{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b} c^{2/3} (cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}}{\left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}} F \left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}} \right) \right) \Big|_{\frac{1}{4}} (2 + \sqrt{3})}{10a^2 c^{11/3} \sqrt[3]{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}}$$

$$- \frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}$$

[Out]  $(-3*(a + b*x^2)^{(1/3)})/(5*a*c*(c*x)^{(5/3)}) - (3^{3/4}*b*(c*x)^{(1/3)}*(a + b*x^2)^{(1/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3})*Sqrt[(c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a + b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + Sqrt[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})^2]*EllipticF[ArcCos[(c^{(2/3)} - ((1 - Sqrt[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + Sqrt[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})], (2 + Sqrt[3])/4])/(10*a^2*c^{(11/3)}*Sqrt[-((b^{(1/3)}*(c*x)^{(2/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})))/((a + b*x^2)^{(1/3)}*(c^{(2/3)} - ((1 + Sqrt[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)}))^2])]$

**Rubi [A]** time = 1.40602, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{3^{3/4} b \sqrt[3]{cx} \sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right) \sqrt[3]{\frac{\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}} + \frac{\sqrt[3]{b} c^{2/3} (cx)^{2/3}}{\sqrt[3]{a+bx^2}} + c^{4/3}}{\left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}} F \left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}} \right) \right) \Big|_{\frac{1}{4}} (2 + \sqrt{3})}{10a^2 c^{11/3} \sqrt[3]{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}}$$

$$- \frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(8/3)\*(a + b\*x^2)^(2/3)),x]

[Out]  $(-3*(a + b*x^2)^{(1/3)})/(5*a*c*(c*x)^{(5/3)}) - (3^{3/4}*b*(c*x)^{(1/3)}*(a + b*x^2)^{(1/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3})*Sqrt[(c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a + b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + Sqrt[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})^2]*EllipticF[ArcCos[(c^{(2/3)} - ((1 - Sqrt[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + Sqrt[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})], (2 + Sqrt[3])/4])/(10*a^2*c^{(11/3)}*Sqrt[-((b^{(1/3)}*(c*x)^{(2/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})))/((a + b*x^2)^{(1/3)}*(c^{(2/3)} - ((1 + Sqrt[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)}))^2])]$

**Rubi in Sympy [A]** time = 28.4713, size = 382, normalized size = 0.97

$$\frac{3 \cdot 3^{\frac{3}{4}} b \sqrt[3]{cx} \sqrt{\frac{\frac{b^{\frac{2}{3}}(cx)^{\frac{4}{3}} + \sqrt[3]{b} c^{\frac{2}{3}} (cx)^{\frac{2}{3}}}{(a+bx^2)^{\frac{2}{3}} + \sqrt[3]{a+bx^2}} + c^{\frac{4}{3}}}{\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1)}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)^2} \left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right) F\left(\operatorname{acos}\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-1+\sqrt{3})}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)\right)}{\frac{\sqrt{3}}{4} + \frac{1}{2}}}}{10ac^{\frac{11}{3}} \sqrt{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}} \left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)}{\sqrt[3]{a+bx^2} \left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1)}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)^2} (a+bx^2)^{\frac{2}{3}} \sqrt{-\frac{bx^2}{a+bx^2} + 1}} - \frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{\frac{5}{3}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(c*x)**(8/3)/(b*x**2+a)**(2/3), x)`

[Out]  $-3 \cdot 3^{\frac{3}{4}} \cdot b \cdot (c \cdot x)^{\frac{1}{3}} \cdot \sqrt{(b^{\frac{2}{3}} \cdot (c \cdot x)^{\frac{4}{3}} / (a + b \cdot x^2)^{\frac{2}{3}} + b^{\frac{1}{3}} \cdot c^{\frac{2}{3}} \cdot (c \cdot x)^{\frac{2}{3}} / (a + b \cdot x^2)^{\frac{1}{3}} + c^{\frac{4}{3}}) / (b^{\frac{1}{3}} \cdot (c \cdot x)^{\frac{2}{3}} \cdot (-\sqrt{3} - 1) / (a + b \cdot x^2)^{\frac{1}{3}} + c^{\frac{2}{3}})^2} \cdot (-b^{\frac{1}{3}} \cdot (c \cdot x)^{\frac{2}{3}} / (a + b \cdot x^2)^{\frac{1}{3}} + c^{\frac{2}{3}}) \cdot \operatorname{elliptic\_f}(\operatorname{acos}((b^{\frac{1}{3}} \cdot (c \cdot x)^{\frac{2}{3}} \cdot (-1 + \sqrt{3})) / (a + b \cdot x^2)^{\frac{1}{3}} + c^{\frac{2}{3}}) / (b^{\frac{1}{3}} \cdot (c \cdot x)^{\frac{2}{3}} \cdot (-\sqrt{3} - 1) / (a + b \cdot x^2)^{\frac{1}{3}} + c^{\frac{2}{3}})), \sqrt{3} / 4 + 1 / 2) / (10 \cdot a \cdot c^{\frac{11}{3}} \cdot \sqrt{a / (a + b \cdot x^2)} \cdot \sqrt{-b \cdot x^2 / (a + b \cdot x^2) + 1}) \cdot (b^{\frac{1}{3}} \cdot (c \cdot x)^{\frac{2}{3}} / (a + b \cdot x^2)^{\frac{1}{3}} + c^{\frac{2}{3}}) / ((a + b \cdot x^2)^{\frac{1}{3}} \cdot (b^{\frac{1}{3}} \cdot (c \cdot x)^{\frac{2}{3}} \cdot (-\sqrt{3} - 1) / (a + b \cdot x^2)^{\frac{1}{3}} + c^{\frac{2}{3}})^2) \cdot (a + b \cdot x^2)^{\frac{2}{3}} \cdot \sqrt{-b \cdot x^2 / (a + b \cdot x^2) + 1}) - 3 \cdot (a + b \cdot x^2)^{\frac{1}{3}} / (5 \cdot a \cdot c \cdot (c \cdot x)^{\frac{5}{3}})$

**Mathematica [C]** time = 0.058139, size = 72, normalized size = 0.18

$$\frac{3x \left( 3bx^2 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{1}{6}, \frac{2}{3}; \frac{7}{6}; -\frac{bx^2}{a} \right) + a + bx^2 \right)}{5a(cx)^{8/3} (a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/((c*x)^(8/3)*(a + b*x^2)^(2/3)), x]`

[Out]  $(-3 \cdot x \cdot (a + b \cdot x^2 + 3 \cdot b \cdot x^2 \cdot (1 + (b \cdot x^2) / a)^{2/3} \cdot \operatorname{Hypergeometric2F1}[1/6, 2/3, 7/6, -((b \cdot x^2) / a)]) / (5 \cdot a \cdot (c \cdot x)^{8/3} \cdot (a + b \cdot x^2)^{2/3})$

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int 1 (cx)^{-\frac{8}{3}} (bx^2 + a)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(8/3)/(b*x^2+a)^(2/3), x)`

[Out] `int(1/(c*x)^(8/3)/(b*x^2+a)^(2/3), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{2}{3}} (cx)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(2/3)\*(c\*x)^(8/3)),x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(2/3)\*(c\*x)^(8/3)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{2}{3}} (cx)^{\frac{2}{3}} c^2 x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(2/3)\*(c\*x)^(8/3)),x, algorithm="fricas")

[Out] integral(1/((b\*x^2 + a)^(2/3)\*(c\*x)^(2/3)\*c^2\*x^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(8/3)/(b\*x\*\*2+a)\*\*(2/3),x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{2}{3}} (cx)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(2/3)\*(c\*x)^(8/3)),x, algorithm="giac")

[Out] integrate(1/((b\*x^2 + a)^(2/3)\*(c\*x)^(8/3)), x)

$$3.783 \quad \int \frac{1}{(cx)^{14/3}(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=425

$$27 \cdot 3^{3/4} b^2 \sqrt[3]{cx} \sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right) \sqrt[3]{\frac{\frac{b^{2/3}(cx)^{4/3} + \sqrt[3]{b} c^{2/3} (cx)^{2/3} + c^{4/3}}{(a+bx^2)^{2/3} + \sqrt[3]{a+bx^2}} + c^{4/3}}{\left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}} F \left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}} \right) \right) \frac{1}{4} (2 + \sqrt{3})$$


---


$$110a^3 c^{17/3} \sqrt[3]{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}$$


---


$$+ \frac{27b \sqrt[3]{a+bx^2}}{55a^2 c^3 (cx)^{5/3}} - \frac{3 \sqrt[3]{a+bx^2}}{11ac (cx)^{11/3}}$$

[Out]  $(-3*(a + b*x^2)^{(1/3)})/(11*a*c*(c*x)^{(11/3)}) + (27*b*(a + b*x^2)^{(1/3)})/(55*a^2*c^3*(c*x)^{(5/3)}) + (27*3^{(3/4)}*b^2*(c*x)^{(1/3)}*(a + b*x^2)^{(1/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})*\text{Sqrt}[(c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a + b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcCos}[(c^{(2/3)} - ((1 - \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})], (2 + \text{Sqrt}[3])/4]/(110*a^3*c^{(17/3)}*\text{Sqrt}[-((b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)}))]/((a + b*x^2)^{(1/3)}*(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})^2))]$

**Rubi [A]** time = 1.54505, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$27 \cdot 3^{3/4} b^2 \sqrt[3]{cx} \sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right) \sqrt[3]{\frac{\frac{b^{2/3}(cx)^{4/3} + \sqrt[3]{b} c^{2/3} (cx)^{2/3} + c^{4/3}}{(a+bx^2)^{2/3} + \sqrt[3]{a+bx^2}} + c^{4/3}}{\left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}} F \left( \cos^{-1} \left( \frac{c^{2/3} - \frac{(1-\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}}{c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{bx^2+a}}} \right) \right) \frac{1}{4} (2 + \sqrt{3})$$


---


$$110a^3 c^{17/3} \sqrt[3]{\frac{\sqrt[3]{b(cx)^{2/3}} \left( c^{2/3} - \frac{\sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)}{\sqrt[3]{a+bx^2} \left( c^{2/3} - \frac{(1+\sqrt{3}) \sqrt[3]{b(cx)^{2/3}}}{\sqrt[3]{a+bx^2}} \right)^2}}$$


---


$$+ \frac{27b \sqrt[3]{a+bx^2}}{55a^2 c^3 (cx)^{5/3}} - \frac{3 \sqrt[3]{a+bx^2}}{11ac (cx)^{11/3}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(14/3)\*(a + b\*x^2)^(2/3)), x]

[Out]  $(-3*(a + b*x^2)^{(1/3)})/(11*a*c*(c*x)^{(11/3)}) + (27*b*(a + b*x^2)^{(1/3)})/(55*a^2*c^3*(c*x)^{(5/3)}) + (27*3^{(3/4)}*b^2*(c*x)^{(1/3)}*(a + b*x^2)^{(1/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})*\text{Sqrt}[(c^{(4/3)} + (b^{(2/3)}*(c*x)^{(4/3)})/(a + b*x^2)^{(2/3)} + (b^{(1/3)}*c^{(2/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})^2]*\text{EllipticF}[\text{ArcCos}[(c^{(2/3)} - ((1 - \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})/(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})], (2 + \text{Sqrt}[3])/4]/(110*a^3*c^{(17/3)}*\text{Sqrt}[-((b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)}*(c^{(2/3)} - (b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)}))]/((a + b*x^2)^{(1/3)}*(c^{(2/3)} - ((1 + \text{Sqrt}[3])*b^{(1/3)}*(c*x)^{(2/3)})/(a + b*x^2)^{(1/3)})^2))]$

**Rubi in Sympy [A]** time = 37.2542, size = 413, normalized size = 0.97

$$\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{\frac{11}{3}}} + \frac{27 \cdot 3^{\frac{3}{4}} b^2 \sqrt[3]{cx} \sqrt{\frac{\frac{b^{\frac{2}{3}}(cx)^{\frac{4}{3}} + \sqrt[3]{bc^{\frac{2}{3}}(cx)^{\frac{2}{3}} + c^{\frac{4}{3}}}{(a+bx^2)^{\frac{2}{3}} + \sqrt[3]{a+bx^2}}}{\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}}\right)^2} \left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right) F\left(\arcsin\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-1+\sqrt{3}) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}}\right)}{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}}}\right)}{\frac{\sqrt{3}}{4} + \frac{1}{2}}}}{110a^2c^{\frac{17}{3}}\sqrt{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}\left(-\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}} + c^{\frac{2}{3}}\right)}{\sqrt[3]{a+bx^2}\left(\frac{\sqrt[3]{b}(cx)^{\frac{2}{3}}(-\sqrt{3}-1) + c^{\frac{2}{3}}}{\sqrt[3]{a+bx^2}}\right)^2} (a+bx^2)^{\frac{2}{3}} \sqrt{-\frac{bx^2}{a+bx^2} + 1}}}} + \frac{27b\sqrt[3]{a+bx^2}}{55a^2c^3(cx)^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(c*x)**(14/3)/(b*x**2+a)**(2/3),x)`

[Out]  $-3 \cdot (a + b \cdot x^2)^{\frac{1}{3}} / (11 \cdot a \cdot c \cdot (c \cdot x)^{\frac{11}{3}}) + 27 \cdot 3^{\frac{3}{4}} \cdot b^2 \cdot (c \cdot x)^{\frac{1}{3}} \cdot \sqrt{(b^{\frac{2}{3}} \cdot (c \cdot x)^{\frac{4}{3}} / (a + b \cdot x^2)^{\frac{2}{3}} + b^{\frac{2}{3}} \cdot (c \cdot x)^{\frac{2}{3}} \cdot c^{\frac{2}{3}} / (a + b \cdot x^2)^{\frac{1}{3}} + c^{\frac{4}{3}})} / (b^{\frac{1}{3}} \cdot (c \cdot x)^{\frac{2}{3}} \cdot (-\sqrt{3} - 1) / (a + b \cdot x^2)^{\frac{1}{3}} + c^{\frac{2}{3}})^{\frac{1}{2}} \cdot \text{elliptic\_f}(\arcsin((b^{\frac{1}{3}} \cdot (c \cdot x)^{\frac{2}{3}} \cdot (-1 + \sqrt{3})) / (a + b \cdot x^2)^{\frac{1}{3}} + c^{\frac{2}{3}})) / (b^{\frac{1}{3}} \cdot (c \cdot x)^{\frac{2}{3}} \cdot (-\sqrt{3} - 1) / (a + b \cdot x^2)^{\frac{1}{3}} + c^{\frac{2}{3}}))$ ,  $\sqrt{3}/4 + 1/2) / (110 \cdot a^2 \cdot c^{\frac{17}{3}} \cdot \sqrt{a/(a + b \cdot x^2)} \cdot \sqrt{-b^{\frac{1}{3}} \cdot (c \cdot x)^{\frac{2}{3}} \cdot (-b^{\frac{1}{3}} \cdot (c \cdot x)^{\frac{2}{3}} / (a + b \cdot x^2)^{\frac{1}{3}} + c^{\frac{2}{3}})} / ((a + b \cdot x^2)^{\frac{1}{3}} \cdot (b^{\frac{1}{3}} \cdot (c \cdot x)^{\frac{2}{3}} \cdot (-\sqrt{3} - 1) / (a + b \cdot x^2)^{\frac{1}{3}} + c^{\frac{2}{3}})^{\frac{1}{2}}) \cdot (a + b \cdot x^2)^{\frac{2}{3}} \cdot \sqrt{-b \cdot x^2 / (a + b \cdot x^2) + 1}) + 27 \cdot b \cdot (a + b \cdot x^2)^{\frac{1}{3}} / (55 \cdot a^2 \cdot c^3 \cdot (c \cdot x)^{\frac{5}{3}})$

**Mathematica [C]** time = 0.084455, size = 93, normalized size = 0.22

$$\frac{3\sqrt[3]{cx} \left( -5a^2 + 27b^2x^4 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{1}{6}, \frac{2}{3}; \frac{7}{6}; -\frac{bx^2}{a} \right) + 4abx^2 + 9b^2x^4 \right)}{55a^2c^5x^4(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/((c*x)^(14/3)*(a+b*x^2)^(2/3)),x]`

[Out]  $(3 \cdot (c \cdot x)^{\frac{1}{3}} \cdot (-5 \cdot a^2 + 4 \cdot a \cdot b \cdot x^2 + 9 \cdot b^2 \cdot x^4 + 27 \cdot b^2 \cdot x^4 \cdot (1 + (b \cdot x^2)/a)^{\frac{2}{3}} \cdot \text{Hypergeometric2F1}[1/6, 2/3, 7/6, -(b \cdot x^2)/a]) / (55 \cdot a^2 \cdot c^5 \cdot x^4 \cdot (a + b \cdot x^2)^{\frac{2}{3}})$

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int 1(cx)^{-\frac{14}{3}} (bx^2+a)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x)`



[Out] `int(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{2}{3}} (cx)^{\frac{14}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(14/3)),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(14/3)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{2}{3}} (cx)^{\frac{2}{3}} c^4 x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(14/3)),x, algorithm="fricas")`

[Out] `integral(1/((b*x^2 + a)^(2/3)*(c*x)^(2/3)*c^4*x^4), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(14/3)/(b*x**2+a)**(2/3),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{2}{3}} (cx)^{\frac{14}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(14/3)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(14/3)), x)`

$$3.784 \quad \int \frac{(cx)^{2/3}}{(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=58

$$\frac{3(cx)^{5/3} \left(\frac{bx^2}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{5}{6}, \frac{11}{6}; -\frac{bx^2}{a}\right)}{5c(a+bx^2)^{2/3}}$$

[Out]  $(3*(c*x)^{(5/3)}*(1+(b*x^2)/a)^{(2/3)}*\text{Hypergeometric2F1}[2/3, 5/6, 11/6, -(b*x^2)/a])/(5*c*(a+b*x^2)^{(2/3)})$

**Rubi [A]** time = 0.065769, antiderivative size = 58, normalized size of antiderivative = 1., number of rules used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{3(cx)^{5/3} \left(\frac{bx^2}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{5}{6}, \frac{11}{6}; -\frac{bx^2}{a}\right)}{5c(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(2/3)/(a+b\*x^2)^(2/3),x]

[Out]  $(3*(c*x)^{(5/3)}*(1+(b*x^2)/a)^{(2/3)}*\text{Hypergeometric2F1}[2/3, 5/6, 11/6, -(b*x^2)/a])/(5*c*(a+b*x^2)^{(2/3)})$

**Rubi in Sympy [A]** time = 7.76509, size = 49, normalized size = 0.84

$$\frac{3(cx)^{\frac{5}{3}} \sqrt[3]{a+bx^2} {}_2F_1\left(\frac{2}{3}, \frac{5}{6} \middle| \frac{11}{6}; -\frac{bx^2}{a}\right)}{5ac \sqrt[3]{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(2/3)/(b\*x\*\*2+a)\*\*(2/3),x)

[Out]  $3*(c*x)**(5/3)*(a+b*x**2)**(1/3)*\text{hyper}((2/3, 5/6), (11/6, ), -b*x**2/a)/(5*a*c*(1+b*x**2/a)**(1/3))$

**Mathematica [A]** time = 0.0347646, size = 57, normalized size = 0.98

$$\frac{3x(cx)^{2/3} \left(\frac{a+bx^2}{a}\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{5}{6}, \frac{11}{6}; -\frac{bx^2}{a}\right)}{5(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(2/3)/(a+b\*x^2)^(2/3),x]

[Out]  $(3*x*(c*x)^{(2/3)}*((a+b*x^2)/a)^{(2/3)}*\text{Hypergeometric2F1}[2/3, 5/6, 11/6, -(b*x^2)/a])/(5*(a+b*x^2)^{(2/3)})$

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int 1(cx)^{\frac{2}{3}}(bx^2+a)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(2/3)/(b*x^2+a)^(2/3),x)`

[Out] `int((c*x)^(2/3)/(b*x^2+a)^(2/3),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{2}{3}}}{(bx^2 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(2/3)/(b*x^2 + a)^(2/3),x, algorithm="maxima")`

[Out] `integrate((c*x)^(2/3)/(b*x^2 + a)^(2/3), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^{\frac{2}{3}}}{(bx^2 + a)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(2/3)/(b*x^2 + a)^(2/3),x, algorithm="fricas")`

[Out] `integral((c*x)^(2/3)/(b*x^2 + a)^(2/3), x)`

**Sympy** [A] time = 7.76541, size = 44, normalized size = 0.76

$$\frac{c^{\frac{2}{3}} x^{\frac{5}{3}} \left(\frac{5}{6}\right) {}_2F_1\left(\frac{2}{3}, \frac{5}{6} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}} \left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(2/3)/(b*x**2+a)**(2/3),x)`

[Out] `c**(2/3)*x**(5/3)*gamma(5/6)*hyper((2/3, 5/6), (11/6,), b*x**2*exp_polar(I*pi)/a)/(2*a**(2/3)*gamma(11/6))`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{2}{3}}}{(bx^2 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(2/3)/(b*x^2 + a)^(2/3),x, algorithm="giac")`

[Out] `integrate((c*x)^(2/3)/(b*x^2 + a)^(2/3), x)`

$$3.785 \quad \int \frac{1}{\sqrt[3]{cx(a+bx^2)^{2/3}}} dx$$

**Optimal.** Leaf size=58

$$\frac{3(cx)^{2/3} \left(\frac{bx^2}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^2}{a}\right)}{2c(a+bx^2)^{2/3}}$$

[Out]  $(3*(c*x)^{(2/3)}*(1+(b*x^2)/a)^{(2/3)}*\text{Hypergeometric2F1}[1/3, 2/3, 4/3, -(b*x^2)/a])/(2*c*(a+b*x^2)^{(2/3)})$

**Rubi [A]** time = 0.0670153, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{3(cx)^{2/3} \left(\frac{bx^2}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^2}{a}\right)}{2c(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(1/3)\*(a+b\*x^2)^(2/3)),x]

[Out]  $(3*(c*x)^{(2/3)}*(1+(b*x^2)/a)^{(2/3)}*\text{Hypergeometric2F1}[1/3, 2/3, 4/3, -(b*x^2)/a])/(2*c*(a+b*x^2)^{(2/3)})$

**Rubi in Sympy [A]** time = 7.83487, size = 49, normalized size = 0.84

$$\frac{3(cx)^{\frac{2}{3}} \sqrt[3]{a+bx^2} {}_2F_1\left(\frac{2}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^2}{a}\right)}{2ac \sqrt[3]{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(1/3)/(b\*x\*\*2+a)\*\*(2/3),x)

[Out]  $3*(c*x)**(2/3)*(a+b*x**2)**(1/3)*\text{hyper}((2/3, 1/3), (4/3, ), -b*x**2/a)/(2*a*c*(1+b*x**2/a)**(1/3))$

**Mathematica [A]** time = 0.0319026, size = 57, normalized size = 0.98

$$\frac{3x \left(\frac{a+bx^2}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^2}{a}\right)}{2\sqrt[3]{cx}(a+bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(1/3)\*(a+b\*x^2)^(2/3)),x]

[Out]  $(3*x*((a+b*x^2)/a)^{(2/3)}*\text{Hypergeometric2F1}[1/3, 2/3, 4/3, -(b*x^2)/a])/(2*(c*x)^{(1/3)}*(a+b*x^2)^{(2/3)})$

**Maple [F]** time = 0.042, size = 0, normalized size = 0.

$$\int 1 \frac{1}{\sqrt[3]{cx}} (bx^2 + a)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x)`

[Out] `int(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{2}{3}} (cx)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(1/3)),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(1/3)), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{2}{3}} (cx)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(1/3)),x, algorithm="fricas")`

[Out] `integral(1/((b*x^2 + a)^(2/3)*(c*x)^(1/3)), x)`

**Sympy** [A] time = 5.69109, size = 46, normalized size = 0.79

$$\frac{\left(-\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{2}{3} \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{2}{3}} \sqrt[3]{cx^{\frac{2}{3}} \left(\frac{2}{3}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(1/3)/(b*x**2+a)**(2/3),x)`

[Out] `gamma(-1/3)*hyper((1/3, 2/3), (4/3, ), a*exp_polar(I*pi)/(b*x**2))/(2*b**(2/3)*c**(1/3)*x**(2/3)*gamma(2/3)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{2}{3}} (cx)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(1/3)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(1/3)), x)`

$$3.786 \quad \int \frac{1}{(cx)^{4/3}(a+bx^2)^{2/3}} dx$$

**Optimal.** Leaf size=56

$$\frac{3 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( -\frac{1}{6}, \frac{2}{3}; \frac{5}{6}; -\frac{bx^2}{a} \right)}{c \sqrt[3]{cx} (a + bx^2)^{2/3}}$$

[Out]  $(-3*(1 + (b*x^2)/a)^{(2/3)}*Hypergeometric2F1[-1/6, 2/3, 5/6, -(b*x^2)/a])/(c*(c*x)^{(1/3)}*(a + b*x^2)^{(2/3)})$

**Rubi [A]** time = 0.0669824, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{3 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( -\frac{1}{6}, \frac{2}{3}; \frac{5}{6}; -\frac{bx^2}{a} \right)}{c \sqrt[3]{cx} (a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(4/3)\*(a + b\*x^2)^(2/3)),x]

[Out]  $(-3*(1 + (b*x^2)/a)^{(2/3)}*Hypergeometric2F1[-1/6, 2/3, 5/6, -(b*x^2)/a])/(c*(c*x)^{(1/3)}*(a + b*x^2)^{(2/3)})$

**Rubi in Sympy [A]** time = 7.80874, size = 51, normalized size = 0.91

$$\frac{3 \sqrt[3]{a + bx^2} {}_2F_1 \left( \frac{2}{3}, -\frac{1}{6} \middle| -\frac{bx^2}{a} \right)}{ac \sqrt[3]{cx} \sqrt[3]{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(4/3)/(b\*x\*\*2+a)\*\*(2/3),x)

[Out]  $-3*(a + b*x**2)**(1/3)*hyper((2/3, -1/6), (5/6, ), -b*x**2/a)/(a*c*(c*x)**(1/3)*(1 + b*x**2/a)**(1/3))$

**Mathematica [A]** time = 0.0626722, size = 74, normalized size = 1.32

$$\frac{3x \left( bx^2 \left( \frac{bx^2}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{2}{3}, \frac{5}{6}; \frac{11}{6}; -\frac{bx^2}{a} \right) - 5(a + bx^2) \right)}{5a(cx)^{4/3} (a + bx^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(4/3)\*(a + b\*x^2)^(2/3)),x]

[Out]  $(3*x*(-5*(a + b*x^2) + b*x^2*(1 + (b*x^2)/a)^{(2/3)}*Hypergeometric2F1[2/3, 5/6, 11/6, -(b*x^2)/a])/(5*a*(c*x)^{(4/3)}*(a + b*x^2)^{(2/3)})$

**Maple [F]** time = 0.035, size = 0, normalized size = 0.

$$\int 1 (cx)^{-\frac{4}{3}} (bx^2 + a)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x)^(4/3)/(b\*x^2+a)^(2/3), x)

[Out] int(1/(c\*x)^(4/3)/(b\*x^2+a)^(2/3), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{2}{3}} (cx)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(2/3)\*(c\*x)^(4/3)), x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(2/3)\*(c\*x)^(4/3)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{2}{3}} (cx)^{\frac{1}{3}} cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(2/3)\*(c\*x)^(4/3)), x, algorithm="fricas")

[Out] integral(1/((b\*x^2 + a)^(2/3)\*(c\*x)^(1/3)\*c\*x), x)

---

**Sympy [A]** time = 26.7313, size = 48, normalized size = 0.86

$$\frac{\left(-\frac{1}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{6}, \frac{2}{3} \\ \frac{5}{6} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}} c^{\frac{4}{3}} \sqrt[3]{x} \left(\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(4/3)/(b\*x\*\*2+a)\*\*(2/3), x)

[Out] gamma(-1/6)\*hyper((-1/6, 2/3), (5/6, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*a\*\*(2/3)\*c\*\*(4/3)\*x\*\*(1/3)\*gamma(5/6))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{2}{3}} (cx)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(4/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(4/3)), x)
```



$$3.787 \quad \int x^4 \sqrt[4]{a + bx^2} dx$$

**Optimal.** Leaf size=121

$$\frac{8a^{7/2} \left(\frac{bx^2}{a} + 1\right)^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{77b^{5/2} (a + bx^2)^{3/4}} - \frac{4a^2 x \sqrt[4]{a + bx^2}}{77b^2} + \frac{2}{11} x^5 \sqrt[4]{a + bx^2} + \frac{2ax^3 \sqrt[4]{a + bx^2}}{77b}$$

[Out]  $(-4*a^2*x*(a + b*x^2)^{(1/4)})/(77*b^2) + (2*a*x^3*(a + b*x^2)^{(1/4)})/(77*b) + (2*x^5*(a + b*x^2)^{(1/4)})/11 + (8*a^{(7/2)}*(1 + (b*x^2)/a)^{(3/4)}*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^{(5/2)}*(a + b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.148729, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{8a^{7/2} \left(\frac{bx^2}{a} + 1\right)^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{77b^{5/2} (a + bx^2)^{3/4}} - \frac{4a^2 x \sqrt[4]{a + bx^2}}{77b^2} + \frac{2}{11} x^5 \sqrt[4]{a + bx^2} + \frac{2ax^3 \sqrt[4]{a + bx^2}}{77b}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a + b\*x^2)^(1/4), x]

[Out]  $(-4*a^2*x*(a + b*x^2)^{(1/4)})/(77*b^2) + (2*a*x^3*(a + b*x^2)^{(1/4)})/(77*b) + (2*x^5*(a + b*x^2)^{(1/4)})/11 + (8*a^{(7/2)}*(1 + (b*x^2)/a)^{(3/4)}*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^{(5/2)}*(a + b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 16.1499, size = 110, normalized size = 0.91

$$\frac{8a^{7/2} \left(1 + \frac{bx^2}{a}\right)^{3/4} F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{77b^{5/2} (a + bx^2)^{3/4}} - \frac{4a^2 x \sqrt[4]{a + bx^2}}{77b^2} + \frac{2ax^3 \sqrt[4]{a + bx^2}}{77b} + \frac{2x^5 \sqrt[4]{a + bx^2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $8*a^{(7/2)}*(1 + b*x**2/a)^{(3/4)}*elliptic\_f(\operatorname{atan}(\operatorname{sqrt}(b)*x/\operatorname{sqrt}(a))/2, 2)/(77*b^{(5/2)}*(a + b*x**2)^{(3/4)}) - 4*a^{(7/2)}*x*(a + b*x**2)^{(1/4)}/(77*b^2) + 2*a*x**3*(a + b*x**2)^{(1/4)}/(77*b) + 2*x**5*(a + b*x**2)^{(1/4)}/11$

**Mathematica [C]** time = 0.0644206, size = 89, normalized size = 0.74

$$\frac{2x \left(2a^3 \left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{bx^2}{a}\right) - 2a^3 - a^2 bx^2 + 8ab^2 x^4 + 7b^3 x^6\right)}{77b^2 (a + bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a + b\*x^2)^(1/4), x]

[Out]  $(2*x*(-2*a^3 - a^2*b*x^2 + 8*a*b^2*x^4 + 7*b^3*x^6 + 2*a^3*(1 + (b*x^2)/a)^{(3/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, -(b*x^2)/a]))/77b^2(a + b*x^2)^{3/4}$

$$(77*b^2*(a + b*x^2)^(3/4))$$

**Maple [F]** time = 0.049, size = 0, normalized size = 0.

$$\int x^4 \sqrt[4]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(b\*x^2+a)^(1/4), x)

[Out] int(x^4\*(b\*x^2+a)^(1/4), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{4}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)\*x^4, x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(1/4)\*x^4, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^{\frac{1}{4}} x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)\*x^4, x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(1/4)\*x^4, x)

**Sympy [A]** time = 3.01684, size = 29, normalized size = 0.24

$$\frac{\sqrt[4]{a} x^5 {}_2F_1\left(-\frac{1}{4}, \frac{5}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(b\*x\*\*2+a)\*\*(1/4), x)

[Out] a\*\*(1/4)\*x\*\*5\*hyper((-1/4, 5/2), (7/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/5

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{4}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(1/4)*x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*x^4, x)
```

$$3.788 \quad \int x^2 \sqrt[4]{a + bx^2} dx$$

**Optimal.** Leaf size=97

$$-\frac{4a^{5/2} \left(\frac{bx^2}{a} + 1\right)^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{21b^{3/2} (a + bx^2)^{3/4}} + \frac{2ax\sqrt[4]{a + bx^2}}{21b} + \frac{2}{7}x^3\sqrt[4]{a + bx^2}$$

[Out] (2\*a\*x\*(a + b\*x^2)^(1/4))/(21\*b) + (2\*x^3\*(a + b\*x^2)^(1/4))/7 - (4\*a^(5/2)\*(1 + (b\*x^2)/a)^(3/4)\*EllipticF[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(21\*b^(3/2)\*(a + b\*x^2)^(3/4))

**Rubi [A]** time = 0.101807, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{4a^{5/2} \left(\frac{bx^2}{a} + 1\right)^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{21b^{3/2} (a + bx^2)^{3/4}} + \frac{2ax\sqrt[4]{a + bx^2}}{21b} + \frac{2}{7}x^3\sqrt[4]{a + bx^2}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*x^2)^(1/4), x]

[Out] (2\*a\*x\*(a + b\*x^2)^(1/4))/(21\*b) + (2\*x^3\*(a + b\*x^2)^(1/4))/7 - (4\*a^(5/2)\*(1 + (b\*x^2)/a)^(3/4)\*EllipticF[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(21\*b^(3/2)\*(a + b\*x^2)^(3/4))

**Rubi in Sympy [A]** time = 11.9828, size = 87, normalized size = 0.9

$$-\frac{4a^{5/2} \left(1 + \frac{bx^2}{a}\right)^{3/4} F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{21b^{3/2} (a + bx^2)^{3/4}} + \frac{2ax\sqrt[4]{a + bx^2}}{21b} + \frac{2x^3\sqrt[4]{a + bx^2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(b\*x\*\*2+a)\*\*(1/4), x)

[Out] -4\*a\*\*(5/2)\*(1 + b\*x\*\*2/a)\*\*(3/4)\*elliptic\_f(atan(sqrt(b)\*x/sqrt(a))/2, 2)/(21\*b\*\*(3/2)\*(a + b\*x\*\*2)\*\*(3/4)) + 2\*a\*x\*(a + b\*x\*\*2)\*\*(1/4)/(21\*b) + 2\*x\*\*3\*(a + b\*x\*\*2)\*\*(1/4)/7

**Mathematica [C]** time = 0.0522142, size = 76, normalized size = 0.78

$$\frac{2x \left( -a^2 \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1 \left( \frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{bx^2}{a} \right) + a^2 + 4abx^2 + 3b^2x^4 \right)}{21b(a + bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*x^2)^(1/4), x]

[Out] (2\*x\*(a^2 + 4\*a\*b\*x^2 + 3\*b^2\*x^4 - a^2\*(1 + (b\*x^2)/a)^(3/4)\*Hypergeometric2F1[1/2, 3/4, 3/2, -(b\*x^2)/a])/(21\*b\*(a + b\*x^2)^(3/4))

---

**Maple [F]** time = 0.037, size = 0, normalized size = 0.

$$\int x^2 \sqrt[4]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^2+a)^(1/4),x)`

[Out] `int(x^2*(b*x^2+a)^(1/4),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{4}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)*x^2,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(1/4)*x^2, x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^{\frac{1}{4}} x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)*x^2,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(1/4)*x^2, x)`

---

**Sympy [A]** time = 2.54743, size = 29, normalized size = 0.3

$$\frac{\sqrt[4]{ax^3} {}_2F_1\left(-\frac{1}{4}, \frac{3}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**2+a)**(1/4),x)`

[Out] `a**(1/4)*x**3*hyper((-1/4, 3/2), (5/2, ), b*x**2*exp_polar(I*pi)/a)/3`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{4}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(1/4)*x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*x^2, x)
```

### 3.789 $\int \sqrt[4]{a + bx^2} dx$

**Optimal.** Leaf size=75

$$\frac{2a^{3/2} \left( \frac{bx^2}{a} + 1 \right)^{3/4} F \left( \frac{1}{2} \tan^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right) \middle| 2 \right)}{3\sqrt{b}(a + bx^2)^{3/4}} + \frac{2}{3} x \sqrt[4]{a + bx^2}$$

[Out] (2\*x\*(a + b\*x^2)^(1/4))/3 + (2\*a^(3/2)\*(1 + (b\*x^2)/a)^(3/4)\*EllipticF[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(3\*Sqrt[b]\*(a + b\*x^2)^(3/4))

**Rubi [A]** time = 0.0518148, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{2a^{3/2} \left( \frac{bx^2}{a} + 1 \right)^{3/4} F \left( \frac{1}{2} \tan^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right) \middle| 2 \right)}{3\sqrt{b}(a + bx^2)^{3/4}} + \frac{2}{3} x \sqrt[4]{a + bx^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/4), x]

[Out] (2\*x\*(a + b\*x^2)^(1/4))/3 + (2\*a^(3/2)\*(1 + (b\*x^2)/a)^(3/4)\*EllipticF[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(3\*Sqrt[b]\*(a + b\*x^2)^(3/4))

**Rubi in Sympy [A]** time = 5.66467, size = 66, normalized size = 0.88

$$\frac{2a^{3/2} \left( 1 + \frac{bx^2}{a} \right)^{3/4} F \left( \frac{\operatorname{atan} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right)}{2} \middle| 2 \right)}{3\sqrt{b}(a + bx^2)^{3/4}} + \frac{2x \sqrt[4]{a + bx^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/4), x)

[Out] 2\*a\*\*(3/2)\*(1 + b\*x\*\*2/a)\*\*(3/4)\*elliptic\_f(atan(sqrt(b)\*x/sqrt(a))/2, 2)/(3\*sqrt(b)\*(a + b\*x\*\*2)\*\*(3/4)) + 2\*x\*(a + b\*x\*\*2)\*\*(1/4)/3

**Mathematica [C]** time = 0.033219, size = 62, normalized size = 0.83

$$\frac{ax \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1 \left( \frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{bx^2}{a} \right) + 2x(a + bx^2)}{3(a + bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/4), x]

[Out] (2\*x\*(a + b\*x^2) + a\*x\*(1 + (b\*x^2)/a)^(3/4)\*Hypergeometric2F1[1/2, 3/4, 3/2, -(b\*x^2)/a])/(3\*(a + b\*x^2)^(3/4))

**Maple [F]** time = 0.047, size = 0, normalized size = 0.

$$\int \sqrt[4]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(1/4), x)

[Out] int((b\*x^2+a)^(1/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(1/4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^{\frac{1}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(1/4), x)

---

**Sympy [A]** time = 2.32004, size = 26, normalized size = 0.35

$$\sqrt[4]{ax^2} F_1\left(-\frac{1}{4}, \frac{1}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/4), x)

[Out] a\*\*(1/4)\*x\*hyper((-1/4, 1/2), (3/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4), x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(1/4), x)



$$3.790 \quad \int \frac{\sqrt[4]{a + bx^2}}{x^2} dx$$

**Optimal.** Leaf size=72

$$\frac{\sqrt{a}\sqrt{b} \left(\frac{bx^2}{a} + 1\right)^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{(a + bx^2)^{3/4}} - \frac{\sqrt[4]{a + bx^2}}{x}$$

[Out]  $-\left((a + b*x^2)^{(1/4)}/x\right) + \left(\text{Sqrt}[a]*\text{Sqrt}[b]*(1 + (b*x^2)/a)^{(3/4)}*\text{EllipticF}[\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2]\right)/(a + b*x^2)^{(3/4)}$

**Rubi [A]** time = 0.0633, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\sqrt{a}\sqrt{b} \left(\frac{bx^2}{a} + 1\right)^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{(a + bx^2)^{3/4}} - \frac{\sqrt[4]{a + bx^2}}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/4)/x^2, x]

[Out]  $-\left((a + b*x^2)^{(1/4)}/x\right) + \left(\text{Sqrt}[a]*\text{Sqrt}[b]*(1 + (b*x^2)/a)^{(3/4)}*\text{EllipticF}[\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2]\right)/(a + b*x^2)^{(3/4)}$

**Rubi in Sympy [A]** time = 7.64162, size = 60, normalized size = 0.83

$$\frac{\sqrt{a}\sqrt{b} \left(1 + \frac{bx^2}{a}\right)^{3/4} F\left(\frac{\text{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{(a + bx^2)^{3/4}} - \frac{\sqrt[4]{a + bx^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/4)/x\*\*2, x)

[Out]  $\text{sqrt}(a)*\text{sqrt}(b)*(1 + b*x**2/a)**(3/4)*\text{elliptic\_f}(\text{atan}(\text{sqrt}(b)*x/\text{sqrt}(a))/2, 2)/(a + b*x**2)**(3/4) - (a + b*x**2)**(1/4)/x$

**Mathematica [C]** time = 0.0399432, size = 68, normalized size = 0.94

$$\frac{bx \left(\frac{a+bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{bx^2}{a}\right)}{2(a + bx^2)^{3/4}} - \frac{\sqrt[4]{a + bx^2}}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/4)/x^2, x]

[Out]  $-\left((a + b*x^2)^{(1/4)}/x\right) + \left(b*x*\left((a + b*x^2)/a\right)^{(3/4)}*\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\left((b*x^2)/a\right)\right]\right)/(2*(a + b*x^2)^{(3/4)})$

**Maple [F]** time = 0.037, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \sqrt[4]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/4)/x^2,x)`

[Out] `int((b*x^2+a)^(1/4)/x^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{4}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)/x^2,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(1/4)/x^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{1}{4}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)/x^2,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(1/4)/x^2, x)`

**Sympy [A]** time = 2.56579, size = 29, normalized size = 0.4

$$\frac{\sqrt[4]{a} {}_2F_1\left(-\frac{1}{2}, -\frac{1}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/4)/x**2,x)`

[Out] `-a**(1/4)*hyper((-1/2, -1/4), (1/2, ), b*x**2*exp_polar(I*pi)/a)/x`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{4}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)/x^2,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(1/4)/x^2, x)`

$$3.791 \quad \int \frac{\sqrt[4]{a+bx^2}}{x^4} dx$$

**Optimal.** Leaf size=99

$$-\frac{b^{3/2} \left( \frac{bx^2}{a} + 1 \right)^{3/4} F\left( \frac{1}{2} \tan^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right) \middle| 2 \right)}{6\sqrt{a}(a+bx^2)^{3/4}} - \frac{b\sqrt[4]{a+bx^2}}{6ax} - \frac{\sqrt[4]{a+bx^2}}{3x^3}$$

[Out]  $-(a + b*x^2)^{(1/4)}/(3*x^3) - (b*(a + b*x^2)^{(1/4)})/(6*a*x) - (b^{3/2}*(1 + (b*x^2)/a)^{(3/4)}*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*Sqrt[a]*(a + b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.0953223, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{b^{3/2} \left( \frac{bx^2}{a} + 1 \right)^{3/4} F\left( \frac{1}{2} \tan^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right) \middle| 2 \right)}{6\sqrt{a}(a+bx^2)^{3/4}} - \frac{b\sqrt[4]{a+bx^2}}{6ax} - \frac{\sqrt[4]{a+bx^2}}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/4)/x^4, x]

[Out]  $-(a + b*x^2)^{(1/4)}/(3*x^3) - (b*(a + b*x^2)^{(1/4)})/(6*a*x) - (b^{3/2}*(1 + (b*x^2)/a)^{(3/4)}*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*Sqrt[a]*(a + b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 11.5719, size = 83, normalized size = 0.84

$$-\frac{\sqrt[4]{a+bx^2}}{3x^3} - \frac{b\sqrt[4]{a+bx^2}}{6ax} - \frac{b^{3/2} \left( 1 + \frac{bx^2}{a} \right)^{3/4} F\left( \frac{\text{atan}\left( \frac{\sqrt{bx}}{\sqrt{a}} \right)}{2} \middle| 2 \right)}{6\sqrt{a}(a+bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/4)/x\*\*4, x)

[Out]  $-(a + b*x**2)**(1/4)/(3*x**3) - b*(a + b*x**2)**(1/4)/(6*a*x) - b^{3/2}*(1 + b*x**2/a)**(3/4)*elliptic_f(atan(sqrt(b)*x/sqrt(a))/2, 2)/(6*sqr(a)*(a + b*x**2)**(3/4))$

**Mathematica [C]** time = 0.0452389, size = 85, normalized size = 0.86

$$\frac{-2(2a^2 + 3abx^2 + b^2x^4) - b^2x^4 \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1\left( \frac{1}{2}, \frac{3}{4}, \frac{3}{2}; -\frac{bx^2}{a} \right)}{12ax^3(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/4)/x^4, x]

[Out]  $(-2*(2*a^2 + 3*a*b*x^2 + b^2*x^4) - b^2*x^4*(1 + (b*x^2)/a)^{(3/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)])/(12*a*x^3*(a + b*x^2)^{(3/4)})$

---

**Maple [F]** time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} \sqrt[4]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(1/4)/x^4, x)

[Out] int((b\*x^2+a)^(1/4)/x^4, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{4}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)/x^4, x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(1/4)/x^4, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{1}{4}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)/x^4, x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(1/4)/x^4, x)

---

**Sympy [A]** time = 3.14243, size = 34, normalized size = 0.34

$$\frac{\sqrt[4]{a} {}_2F_1\left(-\frac{3}{2}, -\frac{1}{4} \middle| -\frac{bx^2 e^{i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/4)/x\*\*4, x)

[Out] -a\*\*(1/4)\*hyper((-3/2, -1/4), (-1/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(3\*x\*\*3)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{4}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(1/4)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/x^4, x)
```

$$3.792 \quad \int \frac{\sqrt[4]{a+bx^2}}{x^6} dx$$

**Optimal.** Leaf size=123

$$\frac{b^{5/2} \left( \frac{bx^2}{a} + 1 \right)^{3/4} F \left( \frac{1}{2} \tan^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right) \middle| 2 \right)}{12a^{3/2} (a+bx^2)^{3/4}} + \frac{b^2 \sqrt[4]{a+bx^2}}{12a^2 x} - \frac{\sqrt[4]{a+bx^2}}{5x^5} - \frac{b \sqrt[4]{a+bx^2}}{30ax^3}$$

[Out]  $-(a + b*x^2)^{(1/4)}/(5*x^5) - (b*(a + b*x^2)^{(1/4)})/(30*a*x^3) + (b^2*(a + b*x^2)^{(1/4)})/(12*a^2*x) + (b^{(5/2)}*(1 + (b*x^2)/a)^{(3/4)})*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2]/(12*a^{(3/2)}*(a + b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.136636, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{b^{5/2} \left( \frac{bx^2}{a} + 1 \right)^{3/4} F \left( \frac{1}{2} \tan^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right) \middle| 2 \right)}{12a^{3/2} (a+bx^2)^{3/4}} + \frac{b^2 \sqrt[4]{a+bx^2}}{12a^2 x} - \frac{\sqrt[4]{a+bx^2}}{5x^5} - \frac{b \sqrt[4]{a+bx^2}}{30ax^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/4)/x^6, x]

[Out]  $-(a + b*x^2)^{(1/4)}/(5*x^5) - (b*(a + b*x^2)^{(1/4)})/(30*a*x^3) + (b^2*(a + b*x^2)^{(1/4)})/(12*a^2*x) + (b^{(5/2)}*(1 + (b*x^2)/a)^{(3/4)})*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2]/(12*a^{(3/2)}*(a + b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 15.7723, size = 104, normalized size = 0.85

$$-\frac{\sqrt[4]{a+bx^2}}{5x^5} - \frac{b \sqrt[4]{a+bx^2}}{30ax^3} + \frac{b^2 \sqrt[4]{a+bx^2}}{12a^2 x} + \frac{b^{5/2} \left( 1 + \frac{bx^2}{a} \right)^{3/4} F \left( \frac{\operatorname{atan} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right)}{2} \middle| 2 \right)}{12a^{3/2} (a+bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/4)/x\*\*6, x)

[Out]  $-(a + b*x^2)^{(1/4)}/(5*x^5) - b*(a + b*x^2)^{(1/4)}/(30*a*x^3) + b^2*(a + b*x^2)^{(1/4)}/(12*a^2*x) + b^{(5/2)}*(1 + b*x^2/a)^{(3/4)}*elliptic_f(atan(sqrt(b)*x/sqrt(a))/2, 2)/(12*a^{(3/2)}*(a + b*x^2)^{(3/4)})$

**Mathematica [C]** time = 0.05344, size = 94, normalized size = 0.76

$$\frac{-24a^3 - 28a^2bx^2 + 5b^3x^6 \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1 \left( \frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{bx^2}{a} \right) + 6ab^2x^4 + 10b^3x^6}{120a^2x^5 (a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/4)/x^6, x]

[Out]  $(-24*a^3 - 28*a^2*b*x^2 + 6*a*b^2*x^4 + 10*b^3*x^6 + 5*b^3*x^6*(1 + (b*x^2)/a)^{(3/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, -(b*x^2)/a])$

)]/(120\*a^2\*x^5\*(a + b\*x^2)^(3/4))

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{x^6} \sqrt[4]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(1/4)/x^6,x)

[Out] int((b\*x^2+a)^(1/4)/x^6,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{4}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)/x^6,x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(1/4)/x^6, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{1}{4}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)/x^6,x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(1/4)/x^6, x)

**Sympy [A]** time = 4.01253, size = 34, normalized size = 0.28

$$\frac{\sqrt[4]{a} {}_2F_1\left(-\frac{5}{2}, -\frac{1}{4} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/4)/x\*\*6,x)

[Out] -a\*\*(1/4)\*hyper((-5/2, -1/4), (-3/2,), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(5\*x\*\*5)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{4}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(1/4)/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/x^6, x)
```



### 3.793 $\int x^4 \sqrt[4]{a - bx^2} dx$

**Optimal.** Leaf size=126

$$\frac{8a^{7/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{77b^{5/2} (a - bx^2)^{3/4}} - \frac{4a^2 x \sqrt[4]{a - bx^2}}{77b^2} + \frac{2}{11} x^5 \sqrt[4]{a - bx^2} - \frac{2ax^3 \sqrt[4]{a - bx^2}}{77b}$$

[Out]  $(-4*a^2*x*(a - b*x^2)^{(1/4)})/(77*b^2) - (2*a*x^3*(a - b*x^2)^{(1/4)})/(77*b) + (2*x^5*(a - b*x^2)^{(1/4)})/11 + (8*a^{(7/2)}*(1 - (b*x^2)/a)^{(3/4)}*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^{(5/2)}*(a - b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.146479, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{8a^{7/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{77b^{5/2} (a - bx^2)^{3/4}} - \frac{4a^2 x \sqrt[4]{a - bx^2}}{77b^2} + \frac{2}{11} x^5 \sqrt[4]{a - bx^2} - \frac{2ax^3 \sqrt[4]{a - bx^2}}{77b}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a - b\*x^2)^(1/4), x]

[Out]  $(-4*a^2*x*(a - b*x^2)^{(1/4)})/(77*b^2) - (2*a*x^3*(a - b*x^2)^{(1/4)})/(77*b) + (2*x^5*(a - b*x^2)^{(1/4)})/11 + (8*a^{(7/2)}*(1 - (b*x^2)/a)^{(3/4)}*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^{(5/2)}*(a - b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 18.9377, size = 110, normalized size = 0.87

$$\frac{8a^{7/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{77b^{5/2} (a - bx^2)^{3/4}} - \frac{4a^2 x \sqrt[4]{a - bx^2}}{77b^2} - \frac{2ax^3 \sqrt[4]{a - bx^2}}{77b} + \frac{2x^5 \sqrt[4]{a - bx^2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(-b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $8*a^{(7/2)}*(1 - b*x**2/a)**(3/4)*elliptic_f(asin(sqrt(b)*x/sqrt(a))/2, 2)/(77*b^{(5/2)}*(a - b*x**2)**(3/4)) - 4*a**2*x*(a - b*x**2)**(1/4)/(77*b**2) - 2*a*x**3*(a - b*x**2)**(1/4)/(77*b) + 2*x**5*(a - b*x**2)**(1/4)/11$

**Mathematica [C]** time = 0.0881803, size = 89, normalized size = 0.71

$$\frac{2x \left(2a^3 \left(1 - \frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}, \frac{bx^2}{a}\right) - 2a^3 + a^2 bx^2 + 8ab^2 x^4 - 7b^3 x^6\right)}{77b^2 (a - bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a - b\*x^2)^(1/4), x]

[Out]  $(2*x*(-2*a^3 + a^2*b*x^2 + 8*a*b^2*x^4 - 7*b^3*x^6 + 2*a^3*(1 - (b*x^2)/a))^{(3/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a])/(77$

$$*b^2*(a - b*x^2)^(3/4)$$

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int x^4 \sqrt[4]{-bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(-b\*x^2+a)^(1/4),x)

[Out] int(x^4\*(-b\*x^2+a)^(1/4),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{\frac{1}{4}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4)\*x^4,x, algorithm="maxima")

[Out] integrate((-b\*x^2 + a)^(1/4)\*x^4, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((-bx^2 + a)^{\frac{1}{4}} x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4)\*x^4,x, algorithm="fricas")

[Out] integral((-b\*x^2 + a)^(1/4)\*x^4, x)

**Sympy [A]** time = 3.0579, size = 31, normalized size = 0.25

$$\frac{\sqrt[4]{ax^5} {}_2F_1\left(-\frac{1}{4}, \frac{5}{2} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(-b\*x\*\*2+a)\*\*(1/4),x)

[Out] a\*\*(1/4)\*x\*\*5\*hyper((-1/4, 5/2), (7/2, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/5

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{\frac{1}{4}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2 + a)^(1/4)*x^4,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*x^4, x)
```

### 3.794 $\int x^2 \sqrt[4]{a - bx^2} dx$

**Optimal.** Leaf size=101

$$\frac{4a^{5/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{21b^{3/2} (a - bx^2)^{3/4}} - \frac{2ax\sqrt{a - bx^2}}{21b} + \frac{2}{7}x^3\sqrt[4]{a - bx^2}$$

[Out]  $(-2*a*x*(a - b*x^2)^{(1/4)})/(21*b) + (2*x^3*(a - b*x^2)^{(1/4)})/7 + (4*a^{(5/2)}*(1 - (b*x^2)/a)^{(3/4)}*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*b^{(3/2)}*(a - b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.102623, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{4a^{5/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{21b^{3/2} (a - bx^2)^{3/4}} - \frac{2ax\sqrt{a - bx^2}}{21b} + \frac{2}{7}x^3\sqrt[4]{a - bx^2}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a - b\*x^2)^(1/4), x]

[Out]  $(-2*a*x*(a - b*x^2)^{(1/4)})/(21*b) + (2*x^3*(a - b*x^2)^{(1/4)})/7 + (4*a^{(5/2)}*(1 - (b*x^2)/a)^{(3/4)}*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*b^{(3/2)}*(a - b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 14.465, size = 87, normalized size = 0.86

$$\frac{4a^{5/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{21b^{3/2} (a - bx^2)^{3/4}} - \frac{2ax\sqrt{a - bx^2}}{21b} + \frac{2x^3\sqrt[4]{a - bx^2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(-b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $4*a^{(5/2)}*(1 - b*x**2/a)**(3/4)*elliptic_f(\text{asin}(\text{sqrt}(b)*x/\text{sqrt}(a)))/2, 2)/(21*b^{(3/2)}*(a - b*x**2)**(3/4)) - 2*a*x*(a - b*x**2)**(1/4)/(21*b) + 2*x**3*(a - b*x**2)**(1/4)/7$

**Mathematica [C]** time = 0.0711751, size = 79, normalized size = 0.78

$$\frac{2 \left( a^2 x \left( 1 - \frac{bx^2}{a} \right)^{3/4} {}_2F_1 \left( \frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{bx^2}{a} \right) - a^2 x + 4abx^3 - 3b^2x^5 \right)}{21b(a - bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a - b\*x^2)^(1/4), x]

[Out]  $(2*(-(a^2*x) + 4*a*b*x^3 - 3*b^2*x^5 + a^2*x*(1 - (b*x^2)/a)^{(3/4)})*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a])/(21*b*(a - b*x^2)^{(3/4)})$

---

**Maple [F]** time = 0.027, size = 0, normalized size = 0.

$$\int x^2 \sqrt[4]{-bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(-b*x^2+a)^(1/4),x)`

[Out] `int(x^2*(-b*x^2+a)^(1/4),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{\frac{1}{4}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)*x^2,x, algorithm="maxima")`

[Out] `integrate((-b*x^2 + a)^(1/4)*x^2, x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((-bx^2 + a)^{\frac{1}{4}} x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)*x^2,x, algorithm="fricas")`

[Out] `integral((-b*x^2 + a)^(1/4)*x^2, x)`

---

**Sympy [A]** time = 2.54451, size = 31, normalized size = 0.31

$$\frac{\sqrt[4]{ax^3} {}_2F_1\left(-\frac{1}{4}, \frac{3}{2} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(-b*x**2+a)**(1/4),x)`

[Out] `a**(1/4)*x**3*hyper((-1/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi/a)/3)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{\frac{1}{4}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2 + a)^(1/4)*x^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*x^2, x)
```

### 3.795 $\int \sqrt[4]{a - bx^2} dx$

**Optimal.** Leaf size=78

$$\frac{2a^{3/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{3\sqrt{b}(a - bx^2)^{3/4}} + \frac{2}{3}x\sqrt[4]{a - bx^2}$$

[Out] (2\*x\*(a - b\*x^2)^(1/4))/3 + (2\*a^(3/2)\*(1 - (b\*x^2)/a)^(3/4)\*EllipticF[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(3\*Sqrt[b]\*(a - b\*x^2)^(3/4))

**Rubi [A]** time = 0.0561887, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{2a^{3/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{3\sqrt{b}(a - bx^2)^{3/4}} + \frac{2}{3}x\sqrt[4]{a - bx^2}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(1/4), x]

[Out] (2\*x\*(a - b\*x^2)^(1/4))/3 + (2\*a^(3/2)\*(1 - (b\*x^2)/a)^(3/4)\*EllipticF[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(3\*Sqrt[b]\*(a - b\*x^2)^(3/4))

**Rubi in Sympy [A]** time = 7.52594, size = 66, normalized size = 0.85

$$\frac{2a^{3/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{\arcsin\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{3\sqrt{b}(a - bx^2)^{3/4}} + \frac{2x\sqrt[4]{a - bx^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-b\*x\*\*2+a)\*\*(1/4), x)

[Out] 2\*a\*\*(3/2)\*(1 - b\*x\*\*2/a)\*\*(3/4)\*elliptic\_f(asin(sqrt(b)\*x/sqrt(a))/2, 2)/(3\*sqrt(b)\*(a - b\*x\*\*2)\*\*(3/4)) + 2\*x\*(a - b\*x\*\*2)\*\*(1/4)/3

**Mathematica [C]** time = 0.0397393, size = 63, normalized size = 0.81

$$\frac{ax \left(1 - \frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{bx^2}{a}\right) + 2ax - 2bx^3}{3(a - bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(1/4), x]

[Out] (2\*a\*x - 2\*b\*x^3 + a\*x\*(1 - (b\*x^2)/a)^(3/4)\*Hypergeometric2F1[1/2, 3/4, 3/2, (b\*x^2)/a])/(3\*(a - b\*x^2)^(3/4))

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \sqrt[4]{-bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^2+a)^(1/4), x)

[Out] int((-b\*x^2+a)^(1/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4), x, algorithm="maxima")

[Out] integrate((-b\*x^2 + a)^(1/4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((-bx^2 + a)^{\frac{1}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4), x, algorithm="fricas")

[Out] integral((-b\*x^2 + a)^(1/4), x)

---

**Sympy [A]** time = 2.37684, size = 27, normalized size = 0.35

$$\sqrt[4]{ax} {}_2F_1\left(-\frac{1}{4}, \frac{1}{2} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*2+a)\*\*(1/4), x)

[Out] a\*\*(1/4)\*x\*hyper((-1/4, 1/2), (3/2,), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4), x, algorithm="giac")

[Out] integrate((-b\*x^2 + a)^(1/4), x)



$$3.796 \quad \int \frac{\sqrt[4]{a - bx^2}}{x^2} dx$$

**Optimal.** Leaf size=76

$$-\frac{\sqrt[4]{a - bx^2}}{x} - \frac{\sqrt{a}\sqrt{b} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{(a - bx^2)^{3/4}}$$

[Out]  $-\left((a - b*x^2)^{(1/4)}/x\right) - (\text{Sqrt}[a]*\text{Sqrt}[b]*(1 - (b*x^2)/a)^{(3/4)}*\text{EllipticF}[\text{ArcSin}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(a - b*x^2)^{(3/4)}$

**Rubi [A]** time = 0.0676009, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$-\frac{\sqrt[4]{a - bx^2}}{x} - \frac{\sqrt{a}\sqrt{b} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{(a - bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(1/4)/x^2, x]

[Out]  $-\left((a - b*x^2)^{(1/4)}/x\right) - (\text{Sqrt}[a]*\text{Sqrt}[b]*(1 - (b*x^2)/a)^{(3/4)}*\text{EllipticF}[\text{ArcSin}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(a - b*x^2)^{(3/4)}$

**Rubi in Sympy [A]** time = 9.62842, size = 61, normalized size = 0.8

$$-\frac{\sqrt{a}\sqrt{b} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{(a - bx^2)^{3/4}} - \frac{\sqrt[4]{a - bx^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-b\*x\*\*2+a)\*\*(1/4)/x\*\*2, x)

[Out]  $-\text{sqrt}(a)*\text{sqrt}(b)*(1 - b*x**2/a)**(3/4)*\text{elliptic\_f}(\text{asin}(\text{sqrt}(b)*x/\text{sqrt}(a))/2, 2)/(a - b*x**2)**(3/4) - (a - b*x**2)**(1/4)/x$

**Mathematica [C]** time = 0.0426733, size = 70, normalized size = 0.92

$$-\frac{bx \left(\frac{a-bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{bx^2}{a}\right)}{2(a - bx^2)^{3/4}} - \frac{\sqrt[4]{a - bx^2}}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(1/4)/x^2, x]

[Out]  $-\left((a - b*x^2)^{(1/4)}/x\right) - (b*x*((a - b*x^2)/a)^{(3/4)}*\text{Hypergeometric2F1}[1/2, 3/4, 3/2, (b*x^2)/a])/(2*(a - b*x^2)^{(3/4)})$

**Maple [F]** time = 0.031, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \sqrt[4]{-bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^2+a)^(1/4)/x^2,x)`

[Out] `int((-b*x^2+a)^(1/4)/x^2,x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{\frac{1}{4}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)/x^2,x, algorithm="maxima")`

[Out] `integrate((-b*x^2 + a)^(1/4)/x^2, x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-bx^2 + a)^{\frac{1}{4}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)/x^2,x, algorithm="fricas")`

[Out] `integral((-b*x^2 + a)^(1/4)/x^2, x)`

**Sympy** [A] time = 2.63101, size = 31, normalized size = 0.41

$$-\frac{\sqrt[4]{a} {}_2F_1\left(-\frac{1}{2}, -\frac{1}{4} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**2+a)**(1/4)/x**2,x)`

[Out] `-a**(1/4)*hyper((-1/2, -1/4), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/x`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{\frac{1}{4}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)/x^2,x, algorithm="giac")`

[Out] `integrate((-b*x^2 + a)^(1/4)/x^2, x)`

$$3.797 \quad \int \frac{\sqrt[4]{a - bx^2}}{x^4} dx$$

**Optimal.** Leaf size=103

$$-\frac{b^{3/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{6\sqrt{a}(a - bx^2)^{3/4}} + \frac{b\sqrt[4]{a - bx^2}}{6ax} - \frac{\sqrt[4]{a - bx^2}}{3x^3}$$

[Out]  $-(a - b*x^2)^{(1/4)}/(3*x^3) + (b*(a - b*x^2)^{(1/4)})/(6*a*x) - (b^{3/2}*(1 - (b*x^2)/a)^{(3/4)}*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*Sqrt[a]*(a - b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.110913, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{b^{3/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{6\sqrt{a}(a - bx^2)^{3/4}} + \frac{b\sqrt[4]{a - bx^2}}{6ax} - \frac{\sqrt[4]{a - bx^2}}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(1/4)/x^4, x]

[Out]  $-(a - b*x^2)^{(1/4)}/(3*x^3) + (b*(a - b*x^2)^{(1/4)})/(6*a*x) - (b^{3/2}*(1 - (b*x^2)/a)^{(3/4)}*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*Sqrt[a]*(a - b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 14.0851, size = 82, normalized size = 0.8

$$-\frac{\sqrt[4]{a - bx^2}}{3x^3} + \frac{b\sqrt[4]{a - bx^2}}{6ax} - \frac{b^{3/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{6\sqrt{a}(a - bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-b\*x\*\*2+a)\*\*(1/4)/x\*\*4, x)

[Out]  $-(a - b*x**2)**(1/4)/(3*x**3) + b*(a - b*x**2)**(1/4)/(6*a*x) - b^{3/2}*(1 - b*x**2/a)**(3/4)*elliptic_f(asin(sqrt(b)*x/sqrt(a))/2, 2)/(6*sqr(a)*(a - b*x**2)**(3/4))$

**Mathematica [C]** time = 0.0509637, size = 84, normalized size = 0.82

$$\frac{-4a^2 - b^2x^4 \left(1 - \frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{bx^2}{a}\right) + 6abx^2 - 2b^2x^4}{12ax^3(a - bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(1/4)/x^4, x]

[Out]  $(-4*a^2 + 6*a*b*x^2 - 2*b^2*x^4 - b^2*x^4*(1 - (b*x^2)/a)^{(3/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a])/(12*a*x^3*(a - b*x^2)^{(3/4)})$

---

**Maple [F]** time = 0.032, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} \sqrt[4]{-bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^2+a)^(1/4)/x^4, x)

[Out] int((-b\*x^2+a)^(1/4)/x^4, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{\frac{1}{4}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4)/x^4, x, algorithm="maxima")

[Out] integrate((-b\*x^2 + a)^(1/4)/x^4, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-bx^2 + a)^{\frac{1}{4}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4)/x^4, x, algorithm="fricas")

[Out] integral((-b\*x^2 + a)^(1/4)/x^4, x)

---

**Sympy [A]** time = 3.13489, size = 36, normalized size = 0.35

$$\frac{\sqrt[4]{a} {}_2F_1\left(-\frac{3}{2}, -\frac{1}{4} \middle| -\frac{bx^2 e^{2i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*2+a)\*\*(1/4)/x\*\*4, x)

[Out] -a\*\*(1/4)\*hyper((-3/2, -1/4), (-1/2, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/(3\*x\*\*3)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{\frac{1}{4}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2 + a)^(1/4)/x^4,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/x^4, x)
```

$$3.798 \quad \int \frac{\sqrt[4]{a - bx^2}}{x^6} dx$$

**Optimal.** Leaf size=128

$$-\frac{b^{5/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{12a^{3/2} (a - bx^2)^{3/4}} + \frac{b^2 \sqrt[4]{a - bx^2}}{12a^2 x} - \frac{\sqrt[4]{a - bx^2}}{5x^5} + \frac{b \sqrt[4]{a - bx^2}}{30ax^3}$$

[Out]  $-(a - b*x^2)^{(1/4)}/(5*x^5) + (b*(a - b*x^2)^{(1/4)})/(30*a*x^3) + (b^2*(a - b*x^2)^{(1/4)})/(12*a^2*x) - (b^{(5/2)}*(1 - (b*x^2)/a)^{(3/4)})*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2]/(12*a^{(3/2)}*(a - b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.141045, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{b^{5/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{12a^{3/2} (a - bx^2)^{3/4}} + \frac{b^2 \sqrt[4]{a - bx^2}}{12a^2 x} - \frac{\sqrt[4]{a - bx^2}}{5x^5} + \frac{b \sqrt[4]{a - bx^2}}{30ax^3}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(1/4)/x^6, x]

[Out]  $-(a - b*x^2)^{(1/4)}/(5*x^5) + (b*(a - b*x^2)^{(1/4)})/(30*a*x^3) + (b^2*(a - b*x^2)^{(1/4)})/(12*a^2*x) - (b^{(5/2)}*(1 - (b*x^2)/a)^{(3/4)})*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2]/(12*a^{(3/2)}*(a - b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 18.5699, size = 104, normalized size = 0.81

$$-\frac{\sqrt[4]{a - bx^2}}{5x^5} + \frac{b \sqrt[4]{a - bx^2}}{30ax^3} + \frac{b^2 \sqrt[4]{a - bx^2}}{12a^2 x} - \frac{b^{5/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{12a^{3/2} (a - bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-b\*x\*\*2+a)\*\*(1/4)/x\*\*6, x)

[Out]  $-(a - b*x^2)**(1/4)/(5*x^5) + b*(a - b*x^2)**(1/4)/(30*a*x^3) + b^2*(a - b*x^2)**(1/4)/(12*a^2*x) - b^{(5/2)}*(1 - b*x^2/a)**(3/4)*elliptic_f(asin(sqrt(b)*x/sqrt(a))/2, 2)/(12*a^{(3/2)}*(a - b*x^2)**(3/4))$

**Mathematica [C]** time = 0.0517221, size = 95, normalized size = 0.74

$$\frac{-24a^3 + 28a^2bx^2 - 5b^3x^6 \left(1 - \frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{bx^2}{a}\right) + 6ab^2x^4 - 10b^3x^6}{120a^2x^5 (a - bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(1/4)/x^6, x]

[Out]  $(-24*a^3 + 28*a^2*b*x^2 + 6*a*b^2*x^4 - 10*b^3*x^6 - 5*b^3*x^6*(1 - (b*x^2)/a)^{(3/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a])/$

$(120*a^2*x^5*(a - b*x^2)^(3/4))$

**Maple [F]** time = 0.034, size = 0, normalized size = 0.

$$\int \frac{1}{x^6} \sqrt[4]{-bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^2+a)^(1/4)/x^6,x)`

[Out] `int((-b*x^2+a)^(1/4)/x^6,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{\frac{1}{4}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)/x^6,x, algorithm="maxima")`

[Out] `integrate((-b*x^2 + a)^(1/4)/x^6, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-bx^2 + a)^{\frac{1}{4}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)/x^6,x, algorithm="fricas")`

[Out] `integral((-b*x^2 + a)^(1/4)/x^6, x)`

**Sympy [A]** time = 4.04602, size = 36, normalized size = 0.28

$$\frac{\sqrt[4]{a} {}_2F_1\left(-\frac{5}{2}, -\frac{1}{4} \mid \frac{bx^2 e^{2i\pi}}{a}\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**2+a)**(1/4)/x**6,x)`

[Out] `-a**(1/4)*hyper((-5/2, -1/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*x**5)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{\frac{1}{4}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2 + a)^(1/4)/x^6,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/x^6, x)
```



$$3.799 \quad \int x^4 (a + bx^2)^{3/4} dx$$

**Optimal.** Leaf size=143

$$\frac{8a^{7/2} \sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{65b^{5/2} \sqrt[4]{a+bx^2}} + \frac{8a^3x}{65b^2 \sqrt[4]{a+bx^2}} - \frac{4a^2x(a+bx^2)^{3/4}}{65b^2} + \frac{2}{13}x^5(a+bx^2)^{3/4} + \frac{2ax^3(a+bx^2)^{3/4}}{39b}$$

[Out]  $(8*a^3*x)/(65*b^2*(a+b*x^2)^(1/4)) - (4*a^2*x*(a+b*x^2)^(3/4))/(65*b^2) + (2*a*x^3*(a+b*x^2)^(3/4))/(39*b) + (2*x^5*(a+b*x^2)^(3/4))/13 - (8*a^(7/2)*(1+(b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(65*b^(5/2)*(a+b*x^2)^(1/4))$

**Rubi [A]** time = 0.161711, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{8a^{7/2} \sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{65b^{5/2} \sqrt[4]{a+bx^2}} + \frac{8a^3x}{65b^2 \sqrt[4]{a+bx^2}} - \frac{4a^2x(a+bx^2)^{3/4}}{65b^2} + \frac{2}{13}x^5(a+bx^2)^{3/4} + \frac{2ax^3(a+bx^2)^{3/4}}{39b}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a+b\*x^2)^(3/4),x]

[Out]  $(8*a^3*x)/(65*b^2*(a+b*x^2)^(1/4)) - (4*a^2*x*(a+b*x^2)^(3/4))/(65*b^2) + (2*a*x^3*(a+b*x^2)^(3/4))/(39*b) + (2*x^5*(a+b*x^2)^(3/4))/13 - (8*a^(7/2)*(1+(b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(65*b^(5/2)*(a+b*x^2)^(1/4))$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{4a^3 \int \frac{1}{\sqrt[4]{a+bx^2}} dx}{65b^2} - \frac{4a^2x(a+bx^2)^{3/4}}{65b^2} + \frac{2ax^3(a+bx^2)^{3/4}}{39b} + \frac{2x^5(a+bx^2)^{3/4}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(b\*x\*\*2+a)\*\*(3/4),x)

[Out]  $4*a**3*Integral((a+b*x**2)**(-1/4),x)/(65*b**2) - 4*a**2*x*(a+b*x**2)**(3/4)/(65*b**2) + 2*a*x**3*(a+b*x**2)**(3/4)/(39*b) + 2*x**5*(a+b*x**2)**(3/4)/13$

**Mathematica [C]** time = 0.0787801, size = 89, normalized size = 0.62

$$\frac{2x \left( 6a^3 \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) - 6a^3 - a^2bx^2 + 20ab^2x^4 + 15b^3x^6 \right)}{195b^2 \sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a+b\*x^2)^(3/4),x]

[Out]  $(2*x*(-6*a^3 - a^2*b*x^2 + 20*a*b^2*x^4 + 15*b^3*x^6 + 6*a^3*(1 + (b*x^2)/a)^{1/4}) * \text{Hypergeometric2F1}[1/4, 1/2, 3/2, -((b*x^2)/a)]) / (195*b^2*(a + b*x^2)^{1/4})$

**Maple [F]** time = 0.041, size = 0, normalized size = 0.

$$\int x^4 (bx^2 + a)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(b*x^2+a)^(3/4),x)`

[Out] `int(x^4*(b*x^2+a)^(3/4),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{3}{4}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/4)*x^4,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(3/4)*x^4, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^{\frac{3}{4}} x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/4)*x^4,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(3/4)*x^4, x)`

**Sympy [A]** time = 4.9493, size = 29, normalized size = 0.2

$$\frac{a^{\frac{3}{4}} x^5 {}_2F_1\left(-\frac{3}{4}, \frac{5}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(b*x**2+a)**(3/4),x)`

[Out] `a**(3/4)*x**5*hyper((-3/4, 5/2), (7/2, ), b*x**2*exp_polar(I*pi)/a)/5`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/4)*x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.800 \quad \int x^2 (a + bx^2)^{3/4} dx$$

**Optimal.** Leaf size=119

$$\frac{4a^{5/2} \sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{15b^{3/2} \sqrt[4]{a + bx^2}} - \frac{4a^2 x}{15b \sqrt[4]{a + bx^2}} + \frac{2ax (a + bx^2)^{3/4}}{15b} + \frac{2}{9} x^3 (a + bx^2)^{3/4}$$

[Out]  $(-4*a^{5/2}*x)/(15*b*(a + b*x^2)^{(1/4)}) + (2*a*x*(a + b*x^2)^{(3/4)})/(15*b) + (2*x^3*(a + b*x^2)^{(3/4)})/9 + (4*a^{5/2}*(1 + (b*x^2)/a)^{(1/4)}*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^{3/2}*(a + b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.114739, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{4a^{5/2} \sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{15b^{3/2} \sqrt[4]{a + bx^2}} - \frac{4a^2 x}{15b \sqrt[4]{a + bx^2}} + \frac{2ax (a + bx^2)^{3/4}}{15b} + \frac{2}{9} x^3 (a + bx^2)^{3/4}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*x^2)^(3/4), x]

[Out]  $(-4*a^{5/2}*x)/(15*b*(a + b*x^2)^{(1/4)}) + (2*a*x*(a + b*x^2)^{(3/4)})/(15*b) + (2*x^3*(a + b*x^2)^{(3/4)})/9 + (4*a^{5/2}*(1 + (b*x^2)/a)^{(1/4)}*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^{3/2}*(a + b*x^2)^{(1/4)})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2a^3 \int \frac{1}{(a+bx^2)^{5/4}} dx}{15b} - \frac{4a^2 x}{15b \sqrt[4]{a + bx^2}} + \frac{2ax (a + bx^2)^{3/4}}{15b} + \frac{2x^3 (a + bx^2)^{3/4}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $2*a^{3/2}*Integral((a + b*x**2)**(-5/4), x)/(15*b) - 4*a^{5/2}*x/(15*b*(a + b*x**2)**(1/4)) + 2*a*x*(a + b*x**2)**(3/4)/(15*b) + 2*x**3*(a + b*x**2)**(3/4)/9$

**Mathematica [C]** time = 0.0582782, size = 78, normalized size = 0.66

$$\frac{2x \left( -3a^2 \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) + 3a^2 + 8abx^2 + 5b^2x^4 \right)}{45b \sqrt[4]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*x^2)^(3/4), x]

[Out]  $(2*x*(3*a^2 + 8*a*b*x^2 + 5*b^2*x^4 - 3*a^2*(1 + (b*x^2)/a)^{(1/4)}*Hypergeometric2F1[1/4, 1/2, 3/2, -(b*x^2)/a]))/(45*b*(a + b*x^2)^{(1/4)})$

$2)^{(1/4)})$

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int x^2 (bx^2 + a)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(b\*x^2+a)^(3/4), x)

[Out] int(x^2\*(b\*x^2+a)^(3/4), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{3}{4}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/4)\*x^2, x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(3/4)\*x^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^{\frac{3}{4}} x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/4)\*x^2, x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(3/4)\*x^2, x)

**Sympy [A]** time = 3.71148, size = 29, normalized size = 0.24

$$\frac{a^{\frac{3}{4}} x^3 {}_2F_1\left(-\frac{3}{4}, \frac{3}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(b\*x\*\*2+a)\*\*(3/4), x)

[Out] a\*\*(3/4)\*x\*\*3\*hyper((-3/4, 3/2), (5/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/3

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/4)*x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

### 3.801 $\int (a + bx^2)^{3/4} dx$

**Optimal.** Leaf size=92

$$-\frac{6a^{3/2}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\sqrt[4]{a+bx^2}} + \frac{6ax}{5\sqrt[4]{a+bx^2}} + \frac{2}{5}x(a+bx^2)^{3/4}$$

[Out]  $(6*a*x)/(5*(a+b*x^2)^(1/4)) + (2*x*(a+b*x^2)^(3/4))/5 - (6*a^(3/2)*(1+(b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*Sqrt[b]*(a+b*x^2)^(1/4))$

**Rubi [A]** time = 0.0652625, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$-\frac{6a^{3/2}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\sqrt[4]{a+bx^2}} + \frac{6ax}{5\sqrt[4]{a+bx^2}} + \frac{2}{5}x(a+bx^2)^{3/4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(3/4), x]

[Out]  $(6*a*x)/(5*(a+b*x^2)^(1/4)) + (2*x*(a+b*x^2)^(3/4))/5 - (6*a^(3/2)*(1+(b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*Sqrt[b]*(a+b*x^2)^(1/4))$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{3a \int \frac{1}{\sqrt[4]{a+bx^2}} dx}{5} + \frac{2x(a+bx^2)^{3/4}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $3*a*Integral((a+b*x**2)**(-1/4), x)/5 + 2*x*(a+b*x**2)**(3/4)/5$

**Mathematica [C]** time = 0.0355616, size = 63, normalized size = 0.68

$$\frac{3ax\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) + 2x(a+bx^2)}{5\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(3/4), x]

[Out]  $(2*x*(a+b*x^2) + 3*a*x*(1+(b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)])/(5*(a+b*x^2)^(1/4))$

**Maple [F]** time = 0.042, size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(3/4), x)

[Out] int((b\*x^2+a)^(3/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/4), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(3/4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^{\frac{3}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/4), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(3/4), x)

---

**Sympy [A]** time = 2.95917, size = 26, normalized size = 0.28

$$a^{\frac{3}{4}} x {}_2F_1\left(-\frac{3}{4}, \frac{1}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(3/4), x)

[Out] a\*\*(3/4)\*x\*hyper((-3/4, 1/2), (3/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/4), x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(3/4), x)



$$3.802 \quad \int \frac{(a+bx^2)^{3/4}}{x^2} dx$$

**Optimal.** Leaf size=88

$$\frac{3bx}{\sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{x} - \frac{3\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\right)\Big|_2}{\sqrt[4]{a+bx^2}}$$

[Out]  $(3*b*x)/(a + b*x^2)^{(1/4)} - (a + b*x^2)^{(3/4)}/x - (3*\text{Sqrt}[a]*\text{Sqrt}[b]*(1 + (b*x^2)/a)^{(1/4)}*\text{EllipticE}[\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(a + b*x^2)^{(1/4)}$

**Rubi [A]** time = 0.0798857, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{3bx}{\sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{x} - \frac{3\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\right)\Big|_2}{\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(3/4)/x^2, x]

[Out]  $(3*b*x)/(a + b*x^2)^{(1/4)} - (a + b*x^2)^{(3/4)}/x - (3*\text{Sqrt}[a]*\text{Sqrt}[b]*(1 + (b*x^2)/a)^{(1/4)}*\text{EllipticE}[\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(a + b*x^2)^{(1/4)}$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{3b \int \frac{1}{\sqrt[4]{a+bx^2}} dx}{2} - \frac{(a+bx^2)^{3/4}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(3/4)/x\*\*2, x)

[Out]  $3*b*\text{Integral}((a + b*x**2)**(-1/4), x)/2 - (a + b*x**2)**(3/4)/x$

**Mathematica [C]** time = 0.0403463, size = 68, normalized size = 0.77

$$\frac{3bx\sqrt[4]{\frac{a+bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right)}{2\sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(3/4)/x^2, x]

[Out]  $-((a + b*x^2)^{(3/4)}/x) + (3*b*x*((a + b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/4, 1/2, 3/2, -((b*x^2)/a)])/(2*(a + b*x^2)^{(1/4)})$

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (bx^2 + a)^{3/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(3/4)/x^2,x)`

[Out] `int((b*x^2+a)^(3/4)/x^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{3}{4}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/4)/x^2,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(3/4)/x^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{3}{4}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/4)/x^2,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(3/4)/x^2, x)`

**Sympy [A]** time = 3.53234, size = 29, normalized size = 0.33

$$\frac{a^{\frac{3}{4}} {}_2F_1\left(-\frac{3}{4}, -\frac{1}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(3/4)/x**2,x)`

[Out] `-a**(3/4)*hyper((-3/4, -1/2), (1/2, ), b*x**2*exp_polar(I*pi)/a)/x`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/4)/x^2,x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.803 \quad \int \frac{(a+bx^2)^{3/4}}{x^4} dx$$

**Optimal.** Leaf size=121

$$-\frac{b^{3/2}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\sqrt[4]{a+bx^2}} + \frac{b^2x}{2a\sqrt[4]{a+bx^2}} - \frac{b(a+bx^2)^{3/4}}{2ax} - \frac{(a+bx^2)^{3/4}}{3x^3}$$

[Out] (b^2\*x)/(2\*a\*(a + b\*x^2)^(1/4)) - (a + b\*x^2)^(3/4)/(3\*x^3) - (b\*(a + b\*x^2)^(3/4))/(2\*a\*x) - (b^(3/2)\*(1 + (b\*x^2)/a)^(1/4)\*EllipticE[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(2\*Sqrt[a]\*(a + b\*x^2)^(1/4))

**Rubi [A]** time = 0.114921, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{b^{3/2}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\sqrt[4]{a+bx^2}} + \frac{b^2x}{2a\sqrt[4]{a+bx^2}} - \frac{b(a+bx^2)^{3/4}}{2ax} - \frac{(a+bx^2)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(3/4)/x^4, x]

[Out] (b^2\*x)/(2\*a\*(a + b\*x^2)^(1/4)) - (a + b\*x^2)^(3/4)/(3\*x^3) - (b\*(a + b\*x^2)^(3/4))/(2\*a\*x) - (b^(3/2)\*(1 + (b\*x^2)/a)^(1/4)\*EllipticE[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(2\*Sqrt[a]\*(a + b\*x^2)^(1/4))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{b^2 \int \frac{1}{(a+bx^2)^{5/4}} dx}{4} - \frac{(a+bx^2)^{3/4}}{3x^3} + \frac{b^2x}{2a\sqrt[4]{a+bx^2}} - \frac{b(a+bx^2)^{3/4}}{2ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(3/4)/x\*\*4, x)

[Out] -b\*\*2\*Integral((a + b\*x\*\*2)\*\*(-5/4), x)/4 - (a + b\*x\*\*2)\*\*(3/4)/(3\*x\*\*3) + b\*\*2\*x/(2\*a\*(a + b\*x\*\*2)\*\*(1/4)) - b\*(a + b\*x\*\*2)\*\*(3/4)/(2\*a\*x)

**Mathematica [C]** time = 0.046641, size = 88, normalized size = 0.73

$$\frac{b^2x^4\sqrt[4]{\frac{a+bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right)}{4a\sqrt[4]{a+bx^2}} + \left(-\frac{b}{2ax} - \frac{1}{3x^3}\right)(a+bx^2)^{3/4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(3/4)/x^4, x]

[Out] (-1/(3\*x^3) - b/(2\*a\*x))\*(a + b\*x^2)^(3/4) + (b^2\*x\*((a + b\*x^2)/a)^(1/4)\*Hypergeometric2F1[1/4, 1/2, 3/2, -(b\*x^2)/a])/(4\*a\*(a

+  $b \cdot x^2)^{1/4}$ )

**Maple [F]** time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} (bx^2 + a)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(3/4)/x^4, x)`

[Out] `int((b*x^2+a)^(3/4)/x^4, x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{3}{4}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/4)/x^4, x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(3/4)/x^4, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{3}{4}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(3/4)/x^4, x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(3/4)/x^4, x)`

**Sympy [A]** time = 4.03735, size = 34, normalized size = 0.28

$$\frac{a^{\frac{3}{4}} {}_2F_1\left(-\frac{3}{2}, -\frac{3}{4} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(3/4)/x**4, x)`

[Out] `-a**(3/4)*hyper((-3/2, -3/4), (-1/2, ), b*x**2*exp_polar(I*pi)/a)/(3*x**3)`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/4)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.804 \quad \int \frac{(a+bx^2)^{3/4}}{x^6} dx$$

**Optimal.** Leaf size=145

$$\frac{3b^{5/2}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{20a^{3/2}\sqrt[4]{a+bx^2}} - \frac{3b^3x}{20a^2\sqrt[4]{a+bx^2}} + \frac{3b^2(a+bx^2)^{3/4}}{20a^2x} - \frac{(a+bx^2)^{3/4}}{5x^5} - \frac{b(a+bx^2)^{3/4}}{10ax^3}$$

[Out]  $(-3*b^3*x)/(20*a^2*(a+b*x^2)^(1/4)) - (a+b*x^2)^(3/4)/(5*x^5) - (b*(a+b*x^2)^(3/4))/(10*a*x^3) + (3*b^2*(a+b*x^2)^(3/4))/(20*a^2*x) + (3*b^(5/2)*(1+(b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^(3/2)*(a+b*x^2)^(1/4))$

**Rubi [A]** time = 0.154334, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{3b^{5/2}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{20a^{3/2}\sqrt[4]{a+bx^2}} - \frac{3b^3x}{20a^2\sqrt[4]{a+bx^2}} + \frac{3b^2(a+bx^2)^{3/4}}{20a^2x} - \frac{(a+bx^2)^{3/4}}{5x^5} - \frac{b(a+bx^2)^{3/4}}{10ax^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(3/4)/x^6, x]

[Out]  $(-3*b^3*x)/(20*a^2*(a+b*x^2)^(1/4)) - (a+b*x^2)^(3/4)/(5*x^5) - (b*(a+b*x^2)^(3/4))/(10*a*x^3) + (3*b^2*(a+b*x^2)^(3/4))/(20*a^2*x) + (3*b^(5/2)*(1+(b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^(3/2)*(a+b*x^2)^(1/4))$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{(a+bx^2)^{3/4}}{5x^5} - \frac{b(a+bx^2)^{3/4}}{10ax^3} - \frac{3b^3 \int \frac{1}{\sqrt[4]{a+bx^2}} dx}{40a^2} + \frac{3b^2(a+bx^2)^{3/4}}{20a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(3/4)/x\*\*6, x)

[Out]  $-(a+b*x^2)**(3/4)/(5*x^5) - b*(a+b*x^2)**(3/4)/(10*a*x^3) - 3*b^3*Integral((a+b*x^2)**(-1/4), x)/(40*a^2) + 3*b^2*(a+b*x^2)**(3/4)/(20*a^2*x)$

**Mathematica [C]** time = 0.0495263, size = 94, normalized size = 0.65

$$\frac{-8a^3 - 12a^2bx^2 - 3b^3x^6\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) + 2ab^2x^4 + 6b^3x^6}{40a^2x^5\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(3/4)/x^6, x]

[Out]  $(-8*a^3 - 12*a^2*b*x^2 + 2*a*b^2*x^4 + 6*b^3*x^6 - 3*b^3*x^6*(1+(b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -(b*x^2)/a])/(40*a^2*x^5*(a+b*x^2)^(1/4))$

---

**Maple [F]** time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{x^6} (bx^2 + a)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(3/4)/x^6, x)

[Out] int((b\*x^2+a)^(3/4)/x^6, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{3}{4}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/4)/x^6, x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(3/4)/x^6, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{3}{4}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(3/4)/x^6, x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(3/4)/x^6, x)

---

**Sympy [A]** time = 5.12632, size = 34, normalized size = 0.23

$$\frac{a^{\frac{3}{4}} {}_2F_1\left(-\frac{5}{2}, -\frac{3}{4} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(3/4)/x\*\*6, x)

[Out] -a\*\*(3/4)\*hyper((-5/2, -3/4), (-3/2,), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(5\*x\*\*5)

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/4)/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```



### 3.805 $\int x^4 (a - bx^2)^{3/4} dx$

**Optimal.** Leaf size=126

$$\frac{8a^{7/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{65b^{5/2} \sqrt[4]{a - bx^2}} - \frac{4a^2 x (a - bx^2)^{3/4}}{65b^2} + \frac{2}{13} x^5 (a - bx^2)^{3/4} - \frac{2ax^3 (a - bx^2)^{3/4}}{39b}$$

[Out]  $(-4*a^2*x*(a - b*x^2)^{(3/4)})/(65*b^2) - (2*a*x^3*(a - b*x^2)^{(3/4)})/(39*b) + (2*x^5*(a - b*x^2)^{(3/4)})/13 + (8*a^{(7/2)}*(1 - (b*x^2)/a)^{(1/4)}*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(65*b^{(5/2)}*(a - b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.146628, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{8a^{7/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{65b^{5/2} \sqrt[4]{a - bx^2}} - \frac{4a^2 x (a - bx^2)^{3/4}}{65b^2} + \frac{2}{13} x^5 (a - bx^2)^{3/4} - \frac{2ax^3 (a - bx^2)^{3/4}}{39b}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a - b\*x^2)^(3/4), x]

[Out]  $(-4*a^2*x*(a - b*x^2)^{(3/4)})/(65*b^2) - (2*a*x^3*(a - b*x^2)^{(3/4)})/(39*b) + (2*x^5*(a - b*x^2)^{(3/4)})/13 + (8*a^{(7/2)}*(1 - (b*x^2)/a)^{(1/4)}*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(65*b^{(5/2)}*(a - b*x^2)^{(1/4)})$

**Rubi in Sympy [A]** time = 18.993, size = 110, normalized size = 0.87

$$\frac{8a^{7/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{65b^{5/2} \sqrt[4]{a - bx^2}} - \frac{4a^2 x (a - bx^2)^{3/4}}{65b^2} - \frac{2ax^3 (a - bx^2)^{3/4}}{39b} + \frac{2x^5 (a - bx^2)^{3/4}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(-b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $8*a^{(7/2)}*(1 - b*x**2/a)**(1/4)*elliptic_e(\text{asin}(\text{sqrt}(b)*x/\text{sqrt}(a))/2, 2)/(65*b^{(5/2)}*(a - b*x**2)**(1/4)) - 4*a**2*x*(a - b*x**2)**(3/4)/(65*b**2) - 2*a*x**3*(a - b*x**2)**(3/4)/(39*b) + 2*x**5*(a - b*x**2)**(3/4)/13$

**Mathematica [C]** time = 0.0763012, size = 89, normalized size = 0.71

$$\frac{2x \left( 6a^3 \sqrt[4]{1 - \frac{bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{bx^2}{a}\right) - 6a^3 + a^2 bx^2 + 20ab^2 x^4 - 15b^3 x^6 \right)}{195b^2 \sqrt[4]{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a - b\*x^2)^(3/4), x]

[Out]  $(2*x*(-6*a^3 + a^2*b*x^2 + 20*a*b^2*x^4 - 15*b^3*x^6 + 6*a^3*(1 - (b*x^2)/a)^{(1/4)}*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a]))/($

$$195*b^2*(a - b*x^2)^(1/4)$$

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int x^4 (-bx^2 + a)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(-b\*x^2+a)^(3/4),x)

[Out] int(x^4\*(-b\*x^2+a)^(3/4),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{\frac{3}{4}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(3/4)\*x^4,x, algorithm="maxima")

[Out] integrate((-b\*x^2 + a)^(3/4)\*x^4, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((-bx^2 + a)^{\frac{3}{4}} x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(3/4)\*x^4,x, algorithm="fricas")

[Out] integral((-b\*x^2 + a)^(3/4)\*x^4, x)

**Sympy [A]** time = 5.08682, size = 31, normalized size = 0.25

$$\frac{a^{\frac{3}{4}} x^5 {}_2F_1\left(-\frac{3}{4}, \frac{5}{2} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(-b\*x\*\*2+a)\*\*(3/4),x)

[Out] a\*\*(3/4)\*x\*\*5\*hyper((-3/4, 5/2), (7/2, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/5

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2 + a)^(3/4)*x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

### 3.806 $\int x^2 (a - bx^2)^{3/4} dx$

**Optimal.** Leaf size=101

$$\frac{4a^{5/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{15b^{3/2} \sqrt[4]{a - bx^2}} - \frac{2ax(a - bx^2)^{3/4}}{15b} + \frac{2}{9} x^3 (a - bx^2)^{3/4}$$

[Out]  $(-2*a*x*(a - b*x^2)^{(3/4)})/(15*b) + (2*x^3*(a - b*x^2)^{(3/4)})/9 + (4*a^{(5/2)}*(1 - (b*x^2)/a)^{(1/4)}*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^{(3/2)}*(a - b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.104744, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{4a^{5/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{15b^{3/2} \sqrt[4]{a - bx^2}} - \frac{2ax(a - bx^2)^{3/4}}{15b} + \frac{2}{9} x^3 (a - bx^2)^{3/4}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a - b\*x^2)^(3/4), x]

[Out]  $(-2*a*x*(a - b*x^2)^{(3/4)})/(15*b) + (2*x^3*(a - b*x^2)^{(3/4)})/9 + (4*a^{(5/2)}*(1 - (b*x^2)/a)^{(1/4)}*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^{(3/2)}*(a - b*x^2)^{(1/4)})$

**Rubi in Sympy [A]** time = 14.633, size = 87, normalized size = 0.86

$$\frac{4a^{5/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{15b^{3/2} \sqrt[4]{a - bx^2}} - \frac{2ax(a - bx^2)^{3/4}}{15b} + \frac{2x^3(a - bx^2)^{3/4}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(-b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $4*a^{(5/2)}*(1 - b*x**2/a)**(1/4)*elliptic_e(\text{asin}(\text{sqrt}(b)*x/\text{sqrt}(a)))/2, 2)/(15*b^{(3/2)}*(a - b*x**2)**(1/4)) - 2*a*x*(a - b*x**2)**(3/4)/(15*b) + 2*x**3*(a - b*x**2)**(3/4)/9$

**Mathematica [C]** time = 0.0652625, size = 80, normalized size = 0.79

$$\frac{2\left(3a^2x\sqrt[4]{1 - \frac{bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{bx^2}{a}\right) - 3a^2x + 8abx^3 - 5b^2x^5\right)}{45b\sqrt[4]{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a - b\*x^2)^(3/4), x]

[Out]  $(2*(-3*a^2*x + 8*a*b*x^3 - 5*b^2*x^5 + 3*a^2*x*(1 - (b*x^2)/a))^{(1/4)}*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a])/(45*b*(a - b*x^2)^{(1/4)})$

---

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int x^2 (-bx^2 + a)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(-b*x^2+a)^(3/4),x)`

[Out] `int(x^2*(-b*x^2+a)^(3/4),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{\frac{3}{4}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(3/4)*x^2,x, algorithm="maxima")`

[Out] `integrate((-b*x^2 + a)^(3/4)*x^2, x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((-bx^2 + a)^{\frac{3}{4}} x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(3/4)*x^2,x, algorithm="fricas")`

[Out] `integral((-b*x^2 + a)^(3/4)*x^2, x)`

---

**Sympy [A]** time = 3.75254, size = 31, normalized size = 0.31

$$\frac{a^{\frac{3}{4}} x^3 {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(-b*x**2+a)**(3/4),x)`

[Out] `a**(3/4)*x**3*hyper((-3/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi/a)/3)`

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2 + a)^(3/4)*x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

### 3.807 $\int (a - bx^2)^{3/4} dx$

**Optimal.** Leaf size=78

$$\frac{6a^{3/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{5\sqrt{b}\sqrt[4]{a - bx^2}} + \frac{2}{5}x(a - bx^2)^{3/4}$$

[Out] (2\*x\*(a - b\*x^2)^(3/4))/5 + (6\*a^(3/2)\*(1 - (b\*x^2)/a)^(1/4)\*EllipticE[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(5\*Sqrt[b]\*(a - b\*x^2)^(1/4))

**Rubi [A]** time = 0.0567359, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{6a^{3/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{5\sqrt{b}\sqrt[4]{a - bx^2}} + \frac{2}{5}x(a - bx^2)^{3/4}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(3/4), x]

[Out] (2\*x\*(a - b\*x^2)^(3/4))/5 + (6\*a^(3/2)\*(1 - (b\*x^2)/a)^(1/4)\*EllipticE[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(5\*Sqrt[b]\*(a - b\*x^2)^(1/4))

**Rubi in Sympy [A]** time = 7.51637, size = 66, normalized size = 0.85

$$\frac{6a^{\frac{3}{2}} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{\operatorname{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{5\sqrt{b}\sqrt[4]{a - bx^2}} + \frac{2x(a - bx^2)^{\frac{3}{4}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-b\*x\*\*2+a)\*\*(3/4), x)

[Out] 6\*a\*\*(3/2)\*(1 - b\*x\*\*2/a)\*\*(1/4)\*elliptic\_e(asin(sqrt(b)\*x/sqrt(a))/2, 2)/(5\*sqrt(b)\*(a - b\*x\*\*2)\*\*(1/4)) + 2\*x\*(a - b\*x\*\*2)\*\*(3/4)/5

**Mathematica [C]** time = 0.039948, size = 64, normalized size = 0.82

$$\frac{3ax \sqrt[4]{1 - \frac{bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{bx^2}{a}\right) + 2ax - 2bx^3}{5\sqrt[4]{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(3/4), x]

[Out] (2\*a\*x - 2\*b\*x^3 + 3\*a\*x\*(1 - (b\*x^2)/a)^(1/4)\*Hypergeometric2F1[1/4, 1/2, 3/2, (b\*x^2)/a])/(5\*(a - b\*x^2)^(1/4))

**Maple [F]** time = 0.045, size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^2+a)^(3/4),x)`

[Out] `int((-b*x^2+a)^(3/4),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(3/4),x, algorithm="maxima")`

[Out] `integrate((-b*x^2 + a)^(3/4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((-bx^2 + a)^{\frac{3}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(3/4),x, algorithm="fricas")`

[Out] `integral((-b*x^2 + a)^(3/4), x)`

---

**Sympy [A]** time = 3.03445, size = 27, normalized size = 0.35

$$a^{\frac{3}{4}} x {}_2F_1\left(-\frac{3}{4}, \frac{1}{2} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**2+a)**(3/4),x)`

[Out] `a**(3/4)*x*hyper((-3/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(3/4),x, algorithm="giac")`

[Out] `integrate((-b*x^2 + a)^(3/4), x)`



$$3.808 \quad \int \frac{(a-bx^2)^{3/4}}{x^2} dx$$

**Optimal.** Leaf size=76

$$-\frac{(a-bx^2)^{3/4}}{x} - \frac{3\sqrt{a}\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a-bx^2}}$$

[Out]  $-\left((a - b*x^2)^{(3/4)}/x\right) - \left(3*\text{Sqrt}[a]*\text{Sqrt}[b]*(1 - (b*x^2)/a)^{(1/4)}*\text{EllipticE}[\text{ArcSin}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2]\right)/(a - b*x^2)^{(1/4)}$

**Rubi [A]** time = 0.0669219, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$-\frac{(a-bx^2)^{3/4}}{x} - \frac{3\sqrt{a}\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(3/4)/x^2, x]

[Out]  $-\left((a - b*x^2)^{(3/4)}/x\right) - \left(3*\text{Sqrt}[a]*\text{Sqrt}[b]*(1 - (b*x^2)/a)^{(1/4)}*\text{EllipticE}[\text{ArcSin}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2]\right)/(a - b*x^2)^{(1/4)}$

**Rubi in Sympy [A]** time = 9.75461, size = 63, normalized size = 0.83

$$-\frac{3\sqrt{a}\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2}\middle|2\right)}{\sqrt[4]{a-bx^2}} - \frac{(a-bx^2)^{3/4}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-b\*x\*\*2+a)\*\*(3/4)/x\*\*2, x)

[Out]  $-3*\text{sqrt}(a)*\text{sqrt}(b)*(1 - b*x**2/a)**(1/4)*\text{elliptic}_e(\text{asin}(\text{sqrt}(b)*x/\text{sqrt}(a))/2, 2)/(a - b*x**2)**(1/4) - (a - b*x**2)**(3/4)/x$

**Mathematica [C]** time = 0.0409985, size = 70, normalized size = 0.92

$$-\frac{3bx\sqrt[4]{\frac{a-bx^2}{a}}{}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{bx^2}{a}\right)}{2\sqrt[4]{a-bx^2}} - \frac{(a-bx^2)^{3/4}}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(3/4)/x^2, x]

[Out]  $-\left((a - b*x^2)^{(3/4)}/x\right) - \left(3*b*x*((a - b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, (b*x^2)/a\right]\right)/(2*(a - b*x^2)^{(1/4)})$

**Maple [F]** time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (-bx^2 + a)^{3/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^2+a)^(3/4)/x^2,x)`

[Out] `int((-b*x^2+a)^(3/4)/x^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{\frac{3}{4}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(3/4)/x^2,x, algorithm="maxima")`

[Out] `integrate((-b*x^2 + a)^(3/4)/x^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-bx^2 + a)^{\frac{3}{4}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(3/4)/x^2,x, algorithm="fricas")`

[Out] `integral((-b*x^2 + a)^(3/4)/x^2, x)`

**Sympy [A]** time = 3.55102, size = 31, normalized size = 0.41

$$\frac{a^{\frac{3}{4}} {}_2F_1\left(-\frac{3}{4}, -\frac{1}{2} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**2+a)**(3/4)/x**2,x)`

[Out] `-a**(3/4)*hyper((-3/4, -1/2), (1/2, ), b*x**2*exp_polar(2*I*pi)/a)/x`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(3/4)/x^2,x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.809 \quad \int \frac{(a-bx^2)^{3/4}}{x^4} dx$$

**Optimal.** Leaf size=103

$$\frac{b^{3/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{2\sqrt{a} \sqrt[4]{a-bx^2}} + \frac{b(a-bx^2)^{3/4}}{2ax} - \frac{(a-bx^2)^{3/4}}{3x^3}$$

[Out]  $-(a - b*x^2)^{(3/4)}/(3*x^3) + (b*(a - b*x^2)^{(3/4)})/(2*a*x) + (b^{3/2}*(1 - (b*x^2)/a)^{(1/4)}*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*Sqrt[a]*(a - b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.100093, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{b^{3/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{2\sqrt{a} \sqrt[4]{a-bx^2}} + \frac{b(a-bx^2)^{3/4}}{2ax} - \frac{(a-bx^2)^{3/4}}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(3/4)/x^4, x]

[Out]  $-(a - b*x^2)^{(3/4)}/(3*x^3) + (b*(a - b*x^2)^{(3/4)})/(2*a*x) + (b^{3/2}*(1 - (b*x^2)/a)^{(1/4)}*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*Sqrt[a]*(a - b*x^2)^{(1/4)})$

**Rubi in Sympy [A]** time = 13.8268, size = 82, normalized size = 0.8

$$-\frac{(a-bx^2)^{3/4}}{3x^3} + \frac{b(a-bx^2)^{3/4}}{2ax} + \frac{b^{3/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{2\sqrt{a} \sqrt[4]{a-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-b\*x\*\*2+a)\*\*(3/4)/x\*\*4, x)

[Out]  $-(a - b*x**2)**(3/4)/(3*x**3) + b*(a - b*x**2)**(3/4)/(2*a*x) + b^{3/2}*(1 - b*x**2/a)**(1/4)*elliptic_e(asin(sqrt(b)*x/sqrt(a))/2, 2)/(2*sqrt(a)*(a - b*x**2)**(1/4))$

**Mathematica [C]** time = 0.0481872, size = 84, normalized size = 0.82

$$\frac{-4a^2 + 3b^2x^4 \sqrt[4]{1 - \frac{bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{bx^2}{a}\right) + 10abx^2 - 6b^2x^4}{12ax^3 \sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(3/4)/x^4, x]

[Out]  $(-4*a^2 + 10*a*b*x^2 - 6*b^2*x^4 + 3*b^2*x^4*(1 - (b*x^2)/a)^{(1/4)})*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a]/(12*a*x^3*(a - b*x^2)^{(1/4)})$

---

**Maple [F]** time = 0.049, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} (-bx^2 + a)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^2+a)^(3/4)/x^4, x)

[Out] int((-b\*x^2+a)^(3/4)/x^4, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{\frac{3}{4}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(3/4)/x^4, x, algorithm="maxima")

[Out] integrate((-b\*x^2 + a)^(3/4)/x^4, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-bx^2 + a)^{\frac{3}{4}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(3/4)/x^4, x, algorithm="fricas")

[Out] integral((-b\*x^2 + a)^(3/4)/x^4, x)

---

**Sympy [A]** time = 4.04885, size = 36, normalized size = 0.35

$$\frac{a^{\frac{3}{4}} {}_2F_1\left(-\frac{3}{2}, -\frac{3}{4} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*2+a)\*\*(3/4)/x\*\*4, x)

[Out] -a\*\*(3/4)\*hyper((-3/2, -3/4), (-1/2,), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/(3\*x\*\*3)

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2 + a)^(3/4)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.810 \quad \int \frac{(a-bx^2)^{3/4}}{x^6} dx$$

**Optimal.** Leaf size=128

$$\frac{3b^{5/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{20a^{3/2} \sqrt[4]{a - bx^2}} + \frac{3b^2 (a - bx^2)^{3/4}}{20a^2 x} - \frac{(a - bx^2)^{3/4}}{5x^5} + \frac{b (a - bx^2)^{3/4}}{10ax^3}$$

[Out]  $-(a - b*x^2)^{(3/4)}/(5*x^5) + (b*(a - b*x^2)^{(3/4)})/(10*a*x^3) + (3*b^2*(a - b*x^2)^{(3/4)})/(20*a^2*x) + (3*b^{5/2}*(1 - (b*x^2)/a)^{(1/4)}*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^{(3/2)}*(a - b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.142305, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{3b^{5/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{20a^{3/2} \sqrt[4]{a - bx^2}} + \frac{3b^2 (a - bx^2)^{3/4}}{20a^2 x} - \frac{(a - bx^2)^{3/4}}{5x^5} + \frac{b (a - bx^2)^{3/4}}{10ax^3}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(3/4)/x^6, x]

[Out]  $-(a - b*x^2)^{(3/4)}/(5*x^5) + (b*(a - b*x^2)^{(3/4)})/(10*a*x^3) + (3*b^2*(a - b*x^2)^{(3/4)})/(20*a^2*x) + (3*b^{5/2}*(1 - (b*x^2)/a)^{(1/4)}*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^{(3/2)}*(a - b*x^2)^{(1/4)})$

**Rubi in Sympy [A]** time = 18.3247, size = 107, normalized size = 0.84

$$-\frac{(a - bx^2)^{3/4}}{5x^5} + \frac{b(a - bx^2)^{3/4}}{10ax^3} + \frac{3b^2(a - bx^2)^{3/4}}{20a^2x} + \frac{3b^{5/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{20a^{3/2} \sqrt[4]{a - bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-b\*x\*\*2+a)\*\*(3/4)/x\*\*6, x)

[Out]  $-(a - b*x^2)^{(3/4)}/(5*x^5) + b*(a - b*x^2)^{(3/4)}/(10*a*x^3) + 3*b^2*(a - b*x^2)^{(3/4)}/(20*a^2*x) + 3*b^{5/2}*(1 - b*x^2/a)^{(1/4)}*elliptic_e(asin(sqrt(b)*x/sqrt(a))/2, 2)/(20*a^{(3/2)}*(a - b*x^2)^{(1/4)})$

**Mathematica [C]** time = 0.055487, size = 95, normalized size = 0.74

$$\frac{-8a^3 + 12a^2bx^2 + 3b^3x^6 \sqrt[4]{1 - \frac{bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{bx^2}{a}\right) + 2ab^2x^4 - 6b^3x^6}{40a^2x^5 \sqrt[4]{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(3/4)/x^6, x]

[Out]  $(-8*a^3 + 12*a^2*b*x^2 + 2*a*b^2*x^4 - 6*b^3*x^6 + 3*b^3*x^6*(1 - (b*x^2)/a)^{(1/4)}*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a])/(40*a^2*x^5*\sqrt[4]{a - bx^2})$

$$0 * a^2 * x^5 * (a - b * x^2)^{(1/4)}$$

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x^6} (-bx^2 + a)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^2+a)^(3/4)/x^6, x)

[Out] int((-b\*x^2+a)^(3/4)/x^6, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{\frac{3}{4}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(3/4)/x^6, x, algorithm="maxima")

[Out] integrate((-b\*x^2 + a)^(3/4)/x^6, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-bx^2 + a)^{\frac{3}{4}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(3/4)/x^6, x, algorithm="fricas")

[Out] integral((-b\*x^2 + a)^(3/4)/x^6, x)

**Sympy [A]** time = 5.2162, size = 36, normalized size = 0.28

$$\frac{a^{\frac{3}{4}} {}_2F_1\left(-\frac{5}{2}, -\frac{3}{4} \mid \frac{bx^2 e^{2i\pi}}{a}\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*2+a)\*\*(3/4)/x\*\*6, x)

[Out] -a\*\*(3/4)\*hyper((-5/2, -3/4), (-3/2, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/(5\*x\*\*5)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2 + a)^(3/4)/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```



### 3.811 $\int (a + bx^2)^{5/4} dx$

**Optimal.** Leaf size=92

$$\frac{10a^{5/2} \left( \frac{bx^2}{a} + 1 \right)^{3/4} F \left( \frac{1}{2} \tan^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right) \middle| 2 \right)}{21\sqrt{b}(a + bx^2)^{3/4}} + \frac{10}{21} ax \sqrt[4]{a + bx^2} + \frac{2}{7} x (a + bx^2)^{5/4}$$

[Out]  $(10*a*x*(a + b*x^2)^{(1/4)})/21 + (2*x*(a + b*x^2)^{(5/4)})/7 + (10*a^{5/2}*(1 + (b*x^2)/a)^{(3/4)}*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*Sqrt[b]*(a + b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.0717012, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{10a^{5/2} \left( \frac{bx^2}{a} + 1 \right)^{3/4} F \left( \frac{1}{2} \tan^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right) \middle| 2 \right)}{21\sqrt{b}(a + bx^2)^{3/4}} + \frac{10}{21} ax \sqrt[4]{a + bx^2} + \frac{2}{7} x (a + bx^2)^{5/4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(5/4), x]

[Out]  $(10*a*x*(a + b*x^2)^{(1/4)})/21 + (2*x*(a + b*x^2)^{(5/4)})/7 + (10*a^{5/2}*(1 + (b*x^2)/a)^{(3/4)}*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*Sqrt[b]*(a + b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 7.27992, size = 83, normalized size = 0.9

$$\frac{10a^{5/2} \left( 1 + \frac{bx^2}{a} \right)^{3/4} F \left( \frac{\text{atan} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right)}{2} \middle| 2 \right)}{21\sqrt{b}(a + bx^2)^{3/4}} + \frac{10ax \sqrt[4]{a + bx^2}}{21} + \frac{2x(a + bx^2)^{5/4}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(5/4), x)

[Out]  $10*a^{5/2}*(1 + b*x**2/a)**(3/4)*elliptic_f(\text{atan}(\text{sqrt}(b)*x/\text{sqrt}(a))/2, 2)/(21*\text{sqrt}(b)*(a + b*x**2)**(3/4)) + 10*a*x*(a + b*x**2)**(1/4)/21 + 2*x*(a + b*x**2)**(5/4)/7$

**Mathematica [C]** time = 0.0492204, size = 76, normalized size = 0.83

$$\frac{5a^2x \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1 \left( \frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{bx^2}{a} \right) + 16a^2x + 22abx^3 + 6b^2x^5}{21(a + bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(5/4), x]

[Out]  $(16*a^2*x + 22*a*b*x^3 + 6*b^2*x^5 + 5*a^2*x*(1 + (b*x^2)/a)^{(3/4)})*Hypergeometric2F1[1/2, 3/4, 3/2, -(b*x^2)/a]/(21*(a + b*x^2)^{(3/4)})$

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(5/4), x)`

[Out] `int((b*x^2+a)^(5/4), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/4), x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(5/4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^{\frac{5}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/4), x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(5/4), x)`

---

**Sympy [A]** time = 5.2428, size = 26, normalized size = 0.28

$$a^{\frac{5}{4}} x {}_2F_1\left(-\frac{5}{4}, \frac{1}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(5/4), x)`

[Out] `a**(5/4)*x*hyper((-5/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(5/4), x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(5/4), x)`

### 3.812 $\int (a - bx^2)^{5/4} dx$

**Optimal.** Leaf size=96

$$\frac{10a^{5/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{21\sqrt{b}(a - bx^2)^{3/4}} + \frac{10}{21}ax\sqrt[4]{a - bx^2} + \frac{2}{7}x(a - bx^2)^{5/4}$$

[Out] (10\*a\*x\*(a - b\*x^2)^(1/4))/21 + (2\*x\*(a - b\*x^2)^(5/4))/7 + (10\*a^(5/2)\*(1 - (b\*x^2)/a)^(3/4)\*EllipticF[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(21\*Sqrt[b]\*(a - b\*x^2)^(3/4))

**Rubi [A]** time = 0.0739049, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{10a^{5/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{21\sqrt{b}(a - bx^2)^{3/4}} + \frac{10}{21}ax\sqrt[4]{a - bx^2} + \frac{2}{7}x(a - bx^2)^{5/4}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(5/4), x]

[Out] (10\*a\*x\*(a - b\*x^2)^(1/4))/21 + (2\*x\*(a - b\*x^2)^(5/4))/7 + (10\*a^(5/2)\*(1 - (b\*x^2)/a)^(3/4)\*EllipticF[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(21\*Sqrt[b]\*(a - b\*x^2)^(3/4))

**Rubi in Sympy [A]** time = 9.13217, size = 83, normalized size = 0.86

$$\frac{10a^{5/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{21\sqrt{b}(a - bx^2)^{3/4}} + \frac{10ax\sqrt[4]{a - bx^2}}{21} + \frac{2x(a - bx^2)^{5/4}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-b\*x\*\*2+a)\*\*(5/4), x)

[Out] 10\*a\*\*(5/2)\*(1 - b\*x\*\*2/a)\*\*(3/4)\*elliptic\_f(asin(sqrt(b)\*x/sqrt(a))/2, 2)/(21\*sqrt(b)\*(a - b\*x\*\*2)\*\*(3/4)) + 10\*a\*x\*(a - b\*x\*\*2)\*\*(1/4)/21 + 2\*x\*(a - b\*x\*\*2)\*\*(5/4)/7

**Mathematica [C]** time = 0.0490576, size = 77, normalized size = 0.8

$$\frac{5a^2x \left(1 - \frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{bx^2}{a}\right) + 16a^2x - 22abx^3 + 6b^2x^5}{21(a - bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(5/4), x]

[Out] (16\*a^2\*x - 22\*a\*b\*x^3 + 6\*b^2\*x^5 + 5\*a^2\*x\*(1 - (b\*x^2)/a)^(3/4))\*Hypergeometric2F1[1/2, 3/4, 3/2, (b\*x^2)/a]/(21\*(a - b\*x^2)^(3/4))

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^2+a)^(5/4),x)`

[Out] `int((-b*x^2+a)^(5/4),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(5/4),x, algorithm="maxima")`

[Out] `integrate((-b*x^2 + a)^(5/4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((-bx^2 + a)^{\frac{5}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(5/4),x, algorithm="fricas")`

[Out] `integral((-b*x^2 + a)^(5/4), x)`

---

**Sympy [A]** time = 5.30618, size = 27, normalized size = 0.28

$$a^{\frac{5}{4}} x {}_2F_1\left(-\frac{5}{4}, \frac{1}{2} \mid \frac{bx^2 e^{2i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**2+a)**(5/4),x)`

[Out] `a**(5/4)*x*hyper((-5/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(5/4),x, algorithm="giac")`

[Out] `integrate((-b*x^2 + a)^(5/4), x)`

### 3.813 $\int (a + bx^2)^{7/4} dx$

**Optimal.** Leaf size=111

$$-\frac{14a^{5/2}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{b}\sqrt[4]{a+bx^2}} + \frac{14a^2x}{15\sqrt[4]{a+bx^2}} + \frac{14}{45}ax(a+bx^2)^{3/4} + \frac{2}{9}x(a+bx^2)^{7/4}$$

[Out] (14\*a^2\*x)/(15\*(a + b\*x^2)^(1/4)) + (14\*a\*x\*(a + b\*x^2)^(3/4))/45 + (2\*x\*(a + b\*x^2)^(7/4))/9 - (14\*a^(5/2)\*(1 + (b\*x^2)/a)^(1/4)\*EllipticE[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(15\*Sqrt[b]\*(a + b\*x^2)^(1/4))

**Rubi [A]** time = 0.0861093, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$-\frac{14a^{5/2}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{15\sqrt{b}\sqrt[4]{a+bx^2}} + \frac{14a^2x}{15\sqrt[4]{a+bx^2}} + \frac{14}{45}ax(a+bx^2)^{3/4} + \frac{2}{9}x(a+bx^2)^{7/4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(7/4), x]

[Out] (14\*a^2\*x)/(15\*(a + b\*x^2)^(1/4)) + (14\*a\*x\*(a + b\*x^2)^(3/4))/45 + (2\*x\*(a + b\*x^2)^(7/4))/9 - (14\*a^(5/2)\*(1 + (b\*x^2)/a)^(1/4)\*EllipticE[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(15\*Sqrt[b]\*(a + b\*x^2)^(1/4))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{7a^3 \int \frac{1}{(a+bx^2)^5} dx}{15} + \frac{14a^2x}{15\sqrt[4]{a+bx^2}} + \frac{14ax(a+bx^2)^{3/4}}{45} + \frac{2x(a+bx^2)^{7/4}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(7/4), x)

[Out] -7\*a\*\*3\*Integral((a+ b\*x\*\*2)\*\*(-5/4), x)/15 + 14\*a\*\*2\*x/(15\*(a + b\*x\*\*2)\*\*(1/4)) + 14\*a\*x\*(a + b\*x\*\*2)\*\*(3/4)/45 + 2\*x\*(a + b\*x\*\*2)\*\*(7/4)/9

**Mathematica [C]** time = 0.0478442, size = 76, normalized size = 0.68

$$\frac{21a^2x\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) + 24a^2x + 34abx^3 + 10b^2x^5}{45\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(7/4), x]

[Out] (24\*a^2\*x + 34\*a\*b\*x^3 + 10\*b^2\*x^5 + 21\*a^2\*x\*(1 + (b\*x^2)/a)^(1/4)\*Hypergeometric2F1[1/4, 1/2, 3/2, -(b\*x^2)/a])/(45\*(a + b\*x^2)^(1/4))

---

**Maple [F]** time = 0.044, size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(7/4), x)

[Out] int((b\*x^2+a)^(7/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(7/4), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(7/4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^{\frac{7}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(7/4), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(7/4), x)

---

**Sympy [A]** time = 10.2539, size = 26, normalized size = 0.23

$$a^{\frac{7}{4}} x {}_2F_1\left(-\frac{7}{4}, \frac{1}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(7/4), x)

[Out] a\*\*(7/4)\*x\*hyper((-7/4, 1/2), (3/2,), b\*x\*\*2\*exp\_polar(I\*pi)/a)

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(7/4), x, algorithm="giac")

[Out] Exception raised: TypeError

### 3.814 $\int (a - bx^2)^{7/4} dx$

**Optimal.** Leaf size=96

$$\frac{14a^{5/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{15\sqrt{b} \sqrt[4]{a - bx^2}} + \frac{14}{45} ax (a - bx^2)^{3/4} + \frac{2}{9} x (a - bx^2)^{7/4}$$

[Out] (14\*a\*x\*(a - b\*x^2)^(3/4))/45 + (2\*x\*(a - b\*x^2)^(7/4))/9 + (14\*a^(5/2)\*(1 - (b\*x^2)/a)^(1/4)\*EllipticE[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(15\*Sqrt[b]\*(a - b\*x^2)^(1/4))

**Rubi [A]** time = 0.0746524, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{14a^{5/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{15\sqrt{b} \sqrt[4]{a - bx^2}} + \frac{14}{45} ax (a - bx^2)^{3/4} + \frac{2}{9} x (a - bx^2)^{7/4}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(7/4), x]

[Out] (14\*a\*x\*(a - b\*x^2)^(3/4))/45 + (2\*x\*(a - b\*x^2)^(7/4))/9 + (14\*a^(5/2)\*(1 - (b\*x^2)/a)^(1/4)\*EllipticE[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(15\*Sqrt[b]\*(a - b\*x^2)^(1/4))

**Rubi in Sympy [A]** time = 9.24125, size = 83, normalized size = 0.86

$$\frac{14a^{5/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{\operatorname{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{15\sqrt{b} \sqrt[4]{a - bx^2}} + \frac{14ax (a - bx^2)^{3/4}}{45} + \frac{2x (a - bx^2)^{7/4}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-b\*x\*\*2+a)\*\*(7/4), x)

[Out] 14\*a\*\*(5/2)\*(1 - b\*x\*\*2/a)\*\*(1/4)\*elliptic\_e(asin(sqrt(b)\*x/sqrt(a))/2, 2)/(15\*sqrt(b)\*(a - b\*x\*\*2)\*\*(1/4)) + 14\*a\*x\*(a - b\*x\*\*2)\*\*(3/4)/45 + 2\*x\*(a - b\*x\*\*2)\*\*(7/4)/9

**Mathematica [C]** time = 0.0501919, size = 77, normalized size = 0.8

$$\frac{21a^2x \sqrt[4]{1 - \frac{bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{bx^2}{a}\right) + 24a^2x - 34abx^3 + 10b^2x^5}{45\sqrt[4]{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(7/4), x]

[Out] (24\*a^2\*x - 34\*a\*b\*x^3 + 10\*b^2\*x^5 + 21\*a^2\*x\*(1 - (b\*x^2)/a)^(1/4)\*Hypergeometric2F1[1/4, 1/2, 3/2, (b\*x^2)/a])/(45\*(a - b\*x^2)^(1/4))

---

**Maple [F]** time = 0.045, size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^2+a)^(7/4),x)`

[Out] `int((-b*x^2+a)^(7/4),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(7/4),x, algorithm="maxima")`

[Out] `integrate((-b*x^2 + a)^(7/4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((-bx^2 + a)^{\frac{7}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(7/4),x, algorithm="fricas")`

[Out] `integral((-b*x^2 + a)^(7/4), x)`

---

**Sympy [A]** time = 10.2884, size = 27, normalized size = 0.28

$$a^{\frac{7}{4}} x {}_2F_1\left(-\frac{7}{4}, \frac{1}{2} \mid \frac{bx^2 e^{2i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**2+a)**(7/4),x)`

[Out] `a**(7/4)*x*hyper((-7/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)`

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(7/4),x, algorithm="giac")`

[Out] Exception raised: TypeError



$$3.815 \quad \int \frac{x^6}{\sqrt[4]{a+bx^2}} dx$$

**Optimal.** Leaf size=146

$$\frac{16a^{7/2} \sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{39b^{7/2} \sqrt[4]{a+bx^2}} - \frac{16a^3 x}{39b^3 \sqrt[4]{a+bx^2}} + \frac{8a^2 x (a+bx^2)^{3/4}}{39b^3} - \frac{20ax^3 (a+bx^2)^{3/4}}{117b^2} + \frac{2x^5 (a+bx^2)^{3/4}}{13b}$$

[Out]  $(-16*a^3*x)/(39*b^3*(a+b*x^2)^(1/4)) + (8*a^2*x*(a+b*x^2)^(3/4))/(39*b^3) - (20*a*x^3*(a+b*x^2)^(3/4))/(117*b^2) + (2*x^5*(a+b*x^2)^(3/4))/(13*b) + (16*a^(7/2)*(1+(b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(39*b^(7/2)*(a+b*x^2)^(1/4))$

**Rubi [A]** time = 0.165702, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{16a^{7/2} \sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{39b^{7/2} \sqrt[4]{a+bx^2}} - \frac{16a^3 x}{39b^3 \sqrt[4]{a+bx^2}} + \frac{8a^2 x (a+bx^2)^{3/4}}{39b^3} - \frac{20ax^3 (a+bx^2)^{3/4}}{117b^2} + \frac{2x^5 (a+bx^2)^{3/4}}{13b}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a + b\*x^2)^(1/4), x]

[Out]  $(-16*a^3*x)/(39*b^3*(a+b*x^2)^(1/4)) + (8*a^2*x*(a+b*x^2)^(3/4))/(39*b^3) - (20*a*x^3*(a+b*x^2)^(3/4))/(117*b^2) + (2*x^5*(a+b*x^2)^(3/4))/(13*b) + (16*a^(7/2)*(1+(b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(39*b^(7/2)*(a+b*x^2)^(1/4))$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{8a^3 \int \frac{1}{\sqrt[4]{a+bx^2}} dx}{39b^3} + \frac{8a^2 x (a+bx^2)^{3/4}}{39b^3} - \frac{20ax^3 (a+bx^2)^{3/4}}{117b^2} + \frac{2x^5 (a+bx^2)^{3/4}}{13b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6/(b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $-8*a**3*Integral((a+b*x**2)**(-1/4), x)/(39*b**3) + 8*a**2*x*(a+b*x**2)**(3/4)/(39*b**3) - 20*a*x**3*(a+b*x**2)**(3/4)/(117*b**2) + 2*x**5*(a+b*x**2)**(3/4)/(13*b)$

**Mathematica [C]** time = 0.0757217, size = 90, normalized size = 0.62

$$\frac{2 \left( -12a^3 x \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) + 12a^3 x + 2a^2 b x^3 - ab^2 x^5 + 9b^3 x^7 \right)}{117b^3 \sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a + b\*x^2)^(1/4), x]

[Out] (2\*(12\*a^3\*x + 2\*a^2\*b\*x^3 - a\*b^2\*x^5 + 9\*b^3\*x^7 - 12\*a^3\*x\*(1 + (b\*x^2)/a)^(1/4)\*Hypergeometric2F1[1/4, 1/2, 3/2, -((b\*x^2)/a)])/(117\*b^3\*(a + b\*x^2)^(1/4))

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int x^6 \frac{1}{\sqrt[4]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(b\*x^2+a)^(1/4), x)

[Out] int(x^6/(b\*x^2+a)^(1/4), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b\*x^2 + a)^(1/4), x, algorithm="maxima")

[Out] integrate(x^6/(b\*x^2 + a)^(1/4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{(bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b\*x^2 + a)^(1/4), x, algorithm="fricas")

[Out] integral(x^6/(b\*x^2 + a)^(1/4), x)

**Sympy [A]** time = 3.09115, size = 27, normalized size = 0.18

$$\frac{x^7 {}_2F_1\left(\frac{1}{4}, \frac{7}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{7\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*6/(b\*x\*\*2+a)\*\*(1/4), x)

[Out] x\*\*7\*hyper((1/4, 7/2), (9/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(7\*a\*\*(1/4))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^2 + a)^(1/4), x, algorithm="giac")`

[Out] `integrate(x^6/(b*x^2 + a)^(1/4), x)`

$$3.816 \quad \int \frac{x^4}{\sqrt[4]{a + bx^2}} dx$$

**Optimal.** Leaf size=122

$$-\frac{8a^{5/2}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\right)\Big|_2}{15b^{5/2}\sqrt[4]{a + bx^2}} + \frac{8a^2x}{15b^2\sqrt[4]{a + bx^2}} - \frac{4ax(a + bx^2)^{3/4}}{15b^2} + \frac{2x^3(a + bx^2)^{3/4}}{9b}$$

[Out]  $(8*a^2*x)/(15*b^2*(a + b*x^2)^{(1/4)}) - (4*a*x*(a + b*x^2)^{(3/4)})/(15*b^2) + (2*x^3*(a + b*x^2)^{(3/4)})/(9*b) - (8*a^{(5/2)}*(1 + (b*x^2)/a)^{(1/4)}*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^{(5/2)}*(a + b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.121214, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$-\frac{8a^{5/2}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\right)\Big|_2}{15b^{5/2}\sqrt[4]{a + bx^2}} + \frac{8a^2x}{15b^2\sqrt[4]{a + bx^2}} - \frac{4ax(a + bx^2)^{3/4}}{15b^2} + \frac{2x^3(a + bx^2)^{3/4}}{9b}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a + b\*x^2)^(1/4), x]

[Out]  $(8*a^2*x)/(15*b^2*(a + b*x^2)^{(1/4)}) - (4*a*x*(a + b*x^2)^{(3/4)})/(15*b^2) + (2*x^3*(a + b*x^2)^{(3/4)})/(9*b) - (8*a^{(5/2)}*(1 + (b*x^2)/a)^{(1/4)}*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^{(5/2)}*(a + b*x^2)^{(1/4)})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{4a^3 \int \frac{1}{(a+bx^2)^{5/4}} dx}{15b^2} + \frac{8a^2x}{15b^2\sqrt[4]{a + bx^2}} - \frac{4ax(a + bx^2)^{3/4}}{15b^2} + \frac{2x^3(a + bx^2)^{3/4}}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $-4*a^3*Integral((a + b*x**2)**(-5/4), x)/(15*b**2) + 8*a^2*x/(15*b**2*(a + b*x**2)**(1/4)) - 4*a*x*(a + b*x**2)**(3/4)/(15*b**2) + 2*x^3*(a + b*x**2)**(3/4)/(9*b)$

**Mathematica [C]** time = 0.056046, size = 79, normalized size = 0.65

$$\frac{2\left(6a^2x\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) - 6a^2x - abx^3 + 5b^2x^5\right)}{45b^2\sqrt[4]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a + b\*x^2)^(1/4), x]

[Out]  $(2*(-6*a^2*x - a*b*x^3 + 5*b^2*x^5 + 6*a^2*x*(1 + (b*x^2)/a)^{(1/4)})*Hypergeometric2F1[1/4, 1/2, 3/2, -(b*x^2)/a])/(45*b^2*(a + b$

$*x^2)^{(1/4)}$

**Maple [F]** time = 0.04, size = 0, normalized size = 0.

$$\int x^4 \frac{1}{\sqrt[4]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(b*x^2+a)^(1/4), x)`

[Out] `int(x^4/(b*x^2+a)^(1/4), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^(1/4), x, algorithm="maxima")`

[Out] `integrate(x^4/(b*x^2 + a)^(1/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{(bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^(1/4), x, algorithm="fricas")`

[Out] `integral(x^4/(b*x^2 + a)^(1/4), x)`

**Sympy [A]** time = 2.57385, size = 27, normalized size = 0.22

$$\frac{x^5 {}_2F_1\left(\frac{1}{4}, \frac{5}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{5\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(b*x**2+a)**(1/4), x)`

[Out] `x**5*hyper((1/4, 5/2), (7/2, ), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/4))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2 + a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(1/4), x)
```

$$3.817 \quad \int \frac{x^2}{\sqrt[4]{a+bx^2}} dx$$

**Optimal.** Leaf size=98

$$\frac{4a^{3/2} \sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{5b^{3/2} \sqrt[4]{a+bx^2}} - \frac{4ax}{5b \sqrt[4]{a+bx^2}} + \frac{2x(a+bx^2)^{3/4}}{5b}$$

[Out]  $(-4*a*x)/(5*b*(a+b*x^2)^{(1/4)}) + (2*x*(a+b*x^2)^{(3/4)})/(5*b) + (4*a^{(3/2)}*(1+(b*x^2)/a)^{(1/4)}*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b^{(3/2)}*(a+b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.078983, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{4a^{3/2} \sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{5b^{3/2} \sqrt[4]{a+bx^2}} - \frac{4ax}{5b \sqrt[4]{a+bx^2}} + \frac{2x(a+bx^2)^{3/4}}{5b}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*x^2)^(1/4), x]

[Out]  $(-4*a*x)/(5*b*(a+b*x^2)^{(1/4)}) + (2*x*(a+b*x^2)^{(3/4)})/(5*b) + (4*a^{(3/2)}*(1+(b*x^2)/a)^{(1/4)}*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b^{(3/2)}*(a+b*x^2)^{(1/4)})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{2a \int \frac{1}{\sqrt[4]{a+bx^2}} dx}{5b} + \frac{2x(a+bx^2)^{3/4}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $-2*a*Integral((a+b*x**2)**(-1/4), x)/(5*b) + 2*x*(a+b*x**2)**(3/4)/(5*b)$

**Mathematica [C]** time = 0.0456888, size = 62, normalized size = 0.63

$$\frac{2x \left( -a \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) + a + bx^2 \right)}{5b \sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*x^2)^(1/4), x]

[Out]  $(2*x*(a+b*x^2 - a*(1+(b*x^2)/a)^{(1/4)}*Hypergeometric2F1[1/4, 1/2, 3/2, -(b*x^2)/a])/(5*b*(a+b*x^2)^{(1/4)})$

**Maple [F]** time = 0.034, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt[4]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*x^2+a)^(1/4), x)`

[Out] `int(x^2/(b*x^2+a)^(1/4), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(1/4), x, algorithm="maxima")`

[Out] `integrate(x^2/(b*x^2 + a)^(1/4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{(bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(1/4), x, algorithm="fricas")`

[Out] `integral(x^2/(b*x^2 + a)^(1/4), x)`

---

**Sympy [A]** time = 2.37673, size = 27, normalized size = 0.28

$$\frac{x^3 {}_2F_1\left(\frac{1}{4}, \frac{3}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{3\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**2+a)**(1/4), x)`

[Out] `x**3*hyper((1/4, 3/2), (5/2, ), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/4))`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(x^2/(b*x^2 + a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(1/4), x)
```

$$3.818 \quad \int \frac{1}{\sqrt[4]{a+bx^2}} dx$$

**Optimal.** Leaf size=71

$$\frac{2x}{\sqrt[4]{a+bx^2}} - \frac{2\sqrt{a}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^2}}$$

[Out] (2\*x)/(a + b\*x^2)^(1/4) - (2\*Sqrt[a]\*(1 + (b\*x^2)/a)^(1/4)\*EllipticE[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(Sqrt[b]\*(a + b\*x^2)^(1/4))

**Rubi [A]** time = 0.0467527, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{2x}{\sqrt[4]{a+bx^2}} - \frac{2\sqrt{a}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(-1/4), x]

[Out] (2\*x)/(a + b\*x^2)^(1/4) - (2\*Sqrt[a]\*(1 + (b\*x^2)/a)^(1/4)\*EllipticE[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(Sqrt[b]\*(a + b\*x^2)^(1/4))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-a \int \frac{1}{(a+bx^2)^{5/4}} dx + \frac{2x}{\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+a)\*\*(1/4), x)

[Out] -a\*Integral((a + b\*x\*\*2)\*\*(-5/4), x) + 2\*x/(a + b\*x\*\*2)\*\*(1/4)

**Mathematica [C]** time = 0.0241523, size = 47, normalized size = 0.66

$$\frac{x\sqrt[4]{\frac{a+bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{bx^2}{a}\right)}{\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(-1/4), x]

[Out] (x\*((a + b\*x^2)/a)^(1/4)\*Hypergeometric2F1[1/4, 1/2, 3/2, -((b\*x^2)/a)])/(a + b\*x^2)^(1/4)

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{bx^2+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^2+a)^(1/4),x)`

[Out] `int(1/(b*x^2+a)^(1/4),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(-1/4),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(-1/4), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(-1/4),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(-1/4), x)`

**Sympy** [A] time = 2.28094, size = 24, normalized size = 0.34

$$\frac{x {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**2+a)**(1/4),x)`

[Out] `x*hyper((1/4, 1/2), (3/2, ), b*x**2*exp_polar(I*pi)/a)/a**(1/4)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(-1/4),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(-1/4), x)`

$$3.819 \quad \int \frac{1}{x^2 \sqrt[4]{a + bx^2}} dx$$

**Optimal.** Leaf size=93

$$\frac{bx}{a\sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{ax} - \frac{\sqrt{b}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\right)\bigg|_2}{\sqrt{a}\sqrt[4]{a+bx^2}}$$

[Out] (b\*x)/(a\*(a + b\*x^2)^(1/4)) - (a + b\*x^2)^(3/4)/(a\*x) - (Sqrt[b] \* (1 + (b\*x^2)/a)^(1/4)\*EllipticE[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2] )/(Sqrt[a]\*(a + b\*x^2)^(1/4))

**Rubi [A]** time = 0.0779815, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{bx}{a\sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{ax} - \frac{\sqrt{b}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\right)\bigg|_2}{\sqrt{a}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a + b\*x^2)^(1/4)), x]

[Out] (b\*x)/(a\*(a + b\*x^2)^(1/4)) - (a + b\*x^2)^(3/4)/(a\*x) - (Sqrt[b] \* (1 + (b\*x^2)/a)^(1/4)\*EllipticE[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2] )/(Sqrt[a]\*(a + b\*x^2)^(1/4))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{b \int \frac{1}{\sqrt[4]{a+bx^2}} dx}{2a} - \frac{(a+bx^2)^{3/4}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(b\*x\*\*2+a)\*\*(1/4), x)

[Out] b\*Integral((a + b\*x\*\*2)\*\*(-1/4), x)/(2\*a) - (a + b\*x\*\*2)\*\*(3/4)/(a\*x)

**Mathematica [C]** time = 0.0462353, size = 69, normalized size = 0.74

$$\frac{bx^2\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) - 2(a+bx^2)}{2ax\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a + b\*x^2)^(1/4)), x]

[Out] (-2\*(a + b\*x^2) + b\*x^2\*(1 + (b\*x^2)/a)^(1/4)\*Hypergeometric2F1[1/4, 1/2, 3/2, -(b\*x^2)/a])/(2\*a\*x\*(a + b\*x^2)^(1/4))

**Maple [F]** time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt[4]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b\*x^2+a)^(1/4), x)

[Out] int(1/x^2/(b\*x^2+a)^(1/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/4)\*x^2), x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(1/4)\*x^2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{1}{4}} x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/4)\*x^2), x, algorithm="fricas")

[Out] integral(1/((b\*x^2 + a)^(1/4)\*x^2), x)

---

**Sympy [A]** time = 2.67383, size = 27, normalized size = 0.29

$$-\frac{{}_2F_1\left(-\frac{1}{2}, \frac{1}{4} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{\sqrt[4]{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(b\*x\*\*2+a)\*\*(1/4), x)

[Out] -hyper((-1/2, 1/4), (1/2,), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(a\*\*(1/4)\*x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(1/4)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^2), x)
```

$$3.820 \quad \int \frac{1}{x^4 \sqrt[4]{a + bx^2}} dx$$

**Optimal.** Leaf size=124

$$\frac{b^{3/2} \sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{2a^{3/2} \sqrt[4]{a + bx^2}} - \frac{b^2 x}{2a^2 \sqrt[4]{a + bx^2}} + \frac{b(a + bx^2)^{3/4}}{2a^2 x} - \frac{(a + bx^2)^{3/4}}{3ax^3}$$

[Out]  $-(b^2 x)/(2 a^2 (a + b x^2)^{1/4}) - (a + b x^2)^{3/4}/(3 a x^3) + (b (a + b x^2)^{3/4})/(2 a^2 x) + (b^{3/2} (1 + (b x^2)/a)^{1/4}) * \text{EllipticE}[\text{ArcTan}[\text{Sqrt}[b] x]/\text{Sqrt}[a]/2, 2]/(2 a^{3/2} (a + b x^2)^{1/4})$

**Rubi [A]** time = 0.115833, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{b^{3/2} \sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{2a^{3/2} \sqrt[4]{a + bx^2}} - \frac{b^2 x}{2a^2 \sqrt[4]{a + bx^2}} + \frac{b(a + bx^2)^{3/4}}{2a^2 x} - \frac{(a + bx^2)^{3/4}}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a + b\*x^2)^(1/4)), x]

[Out]  $-(b^2 x)/(2 a^2 (a + b x^2)^{1/4}) - (a + b x^2)^{3/4}/(3 a x^3) + (b (a + b x^2)^{3/4})/(2 a^2 x) + (b^{3/2} (1 + (b x^2)/a)^{1/4}) * \text{EllipticE}[\text{ArcTan}[\text{Sqrt}[b] x]/\text{Sqrt}[a]/2, 2]/(2 a^{3/2} (a + b x^2)^{1/4})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{b^2 \int \frac{1}{(a+bx^2)^{5/4}} dx}{4a} - \frac{(a + bx^2)^{3/4}}{3ax^3} - \frac{b^2 x}{2a^2 \sqrt[4]{a + bx^2}} + \frac{b(a + bx^2)^{3/4}}{2a^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $b^2 * \text{Integral}((a + b*x^2)^{-5/4}, x)/(4*a) - (a + b*x^2)^{3/4}/(3*a*x^3) - b^2*x/(2*a^2*(a + b*x^2)^{1/4}) + b*(a + b*x^2)^{3/4}/(2*a^2*x)$

**Mathematica [C]** time = 0.0498697, size = 83, normalized size = 0.67

$$\frac{-4a^2 - 3b^2 x^4 \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) + 2abx^2 + 6b^2 x^4}{12a^2 x^3 \sqrt[4]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a + b\*x^2)^(1/4)), x]

[Out]  $(-4*a^2 + 2*a*b*x^2 + 6*b^2*x^4 - 3*b^2*x^4*(1 + (b*x^2)/a)^{1/4}) * \text{Hypergeometric2F1}[1/4, 1/2, 3/2, -(b*x^2)/a]/(12*a^2*x^3*(a +$

$$b \cdot x^2)^{(1/4)})$$

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \sqrt[4]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b\*x^2+a)^(1/4), x)

[Out] int(1/x^4/(b\*x^2+a)^(1/4), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/4)\*x^4), x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(1/4)\*x^4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{1}{4}} x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/4)\*x^4), x, algorithm="fricas")

[Out] integral(1/((b\*x^2 + a)^(1/4)\*x^4), x)

**Sympy [A]** time = 3.21458, size = 32, normalized size = 0.26

$$-\frac{{}_2F_1\left(-\frac{3}{2}, \frac{1}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3\sqrt[4]{ax^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(b\*x\*\*2+a)\*\*(1/4), x)

[Out] -hyper((-3/2, 1/4), (-1/2,), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(3\*a\*\*(1/4)\*x\*\*3)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}} x^4} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(1/4)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^4), x)
```

$$3.821 \quad \int \frac{1}{x^6 \sqrt[4]{a + bx^2}} dx$$

**Optimal.** Leaf size=148

$$\frac{7b^{5/2} \sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\right) \Big|_2}{20a^{5/2} \sqrt[4]{a + bx^2}} + \frac{7b^3 x}{20a^3 \sqrt[4]{a + bx^2}} - \frac{7b^2 (a + bx^2)^{3/4}}{20a^3 x} + \frac{7b (a + bx^2)^{3/4}}{30a^2 x^3} - \frac{(a + bx^2)^{3/4}}{5ax^5}$$

[Out]  $(7*b^3*x)/(20*a^3*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(5*a*x^5) + (7*b*(a + b*x^2)^(3/4))/(30*a^2*x^3) - (7*b^2*(a + b*x^2)^(3/4))/(20*a^3*x) - (7*b^(5/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^(5/2)*(a + b*x^2)^(1/4))$

**Rubi [A]** time = 0.159011, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{7b^{5/2} \sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\right) \Big|_2}{20a^{5/2} \sqrt[4]{a + bx^2}} + \frac{7b^3 x}{20a^3 \sqrt[4]{a + bx^2}} - \frac{7b^2 (a + bx^2)^{3/4}}{20a^3 x} + \frac{7b (a + bx^2)^{3/4}}{30a^2 x^3} - \frac{(a + bx^2)^{3/4}}{5ax^5}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(a + b\*x^2)^(1/4)), x]

[Out]  $(7*b^3*x)/(20*a^3*(a + b*x^2)^(1/4)) - (a + b*x^2)^(3/4)/(5*a*x^5) + (7*b*(a + b*x^2)^(3/4))/(30*a^2*x^3) - (7*b^2*(a + b*x^2)^(3/4))/(20*a^3*x) - (7*b^(5/2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^(5/2)*(a + b*x^2)^(1/4))$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{(a + bx^2)^{3/4}}{5ax^5} + \frac{7b(a + bx^2)^{3/4}}{30a^2x^3} + \frac{7b^3 \int \frac{1}{\sqrt[4]{a + bx^2}} dx}{40a^3} - \frac{7b^2(a + bx^2)^{3/4}}{20a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*6/(b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $-(a + b*x^2)^(3/4)/(5*a*x^5) + 7*b*(a + b*x^2)^(3/4)/(30*a^2*x^3) + 7*b^3*Integral((a + b*x^2)^(3/4)*(-1/4), x)/(40*a^3) - 7*b^2*(a + b*x^2)^(3/4)/(20*a^3*x)$

**Mathematica [C]** time = 0.0596042, size = 94, normalized size = 0.64

$$\frac{-24a^3 + 4a^2bx^2 + 21b^3x^6 \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) - 14ab^2x^4 - 42b^3x^6}{120a^3x^5 \sqrt[4]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6\*(a + b\*x^2)^(1/4)), x]

[Out]  $(-24*a^3 + 4*a^2*b*x^2 - 14*a*b^2*x^4 - 42*b^3*x^6 + 21*b^3*x^6*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -(b*x^2)/a])/(120*a^3*x^5*(a + b*x^2)^(1/4))$

---

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{x^6 \sqrt[4]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^6/(b*x^2+a)^(1/4), x)`

[Out] `int(1/x^6/(b*x^2+a)^(1/4), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/4)*x^6), x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(1/4)*x^6), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{1}{4}} x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/4)*x^6), x, algorithm="fricas")`

[Out] `integral(1/((b*x^2 + a)^(1/4)*x^6), x)`

---

**Sympy [A]** time = 4.10449, size = 32, normalized size = 0.22

$$-\frac{{}_2F_1\left(-\frac{5}{2}, \frac{1}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5\sqrt[4]{ax^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**6/(b*x**2+a)**(1/4), x)`

[Out] `-hyper((-5/2, 1/4), (-3/2, ), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/4)*x**5)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(1/4)*x^6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*x^6), x)
```

$$3.822 \quad \int \frac{x^6}{\sqrt[4]{a - bx^2}} dx$$

**Optimal.** Leaf size=129

$$\frac{16a^{7/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{39b^{7/2} \sqrt[4]{a - bx^2}} - \frac{8a^2 x (a - bx^2)^{3/4}}{39b^3} - \frac{20ax^3 (a - bx^2)^{3/4}}{117b^2} - \frac{2x^5 (a - bx^2)^{3/4}}{13b}$$

[Out]  $(-8*a^2*x*(a - b*x^2)^{(3/4)})/(39*b^3) - (20*a*x^3*(a - b*x^2)^{(3/4)})/(117*b^2) - (2*x^5*(a - b*x^2)^{(3/4)})/(13*b) + (16*a^{(7/2)}*(1 - (b*x^2)/a)^{(1/4)}*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(39*b^{(7/2)}*(a - b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.146651, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{16a^{7/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{39b^{7/2} \sqrt[4]{a - bx^2}} - \frac{8a^2 x (a - bx^2)^{3/4}}{39b^3} - \frac{20ax^3 (a - bx^2)^{3/4}}{117b^2} - \frac{2x^5 (a - bx^2)^{3/4}}{13b}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a - b\*x^2)^(1/4), x]

[Out]  $(-8*a^2*x*(a - b*x^2)^{(3/4)})/(39*b^3) - (20*a*x^3*(a - b*x^2)^{(3/4)})/(117*b^2) - (2*x^5*(a - b*x^2)^{(3/4)})/(13*b) + (16*a^{(7/2)}*(1 - (b*x^2)/a)^{(1/4)}*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(39*b^{(7/2)}*(a - b*x^2)^{(1/4)})$

**Rubi in Sympy [A]** time = 19.2399, size = 114, normalized size = 0.88

$$\frac{16a^{7/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{39b^{7/2} \sqrt[4]{a - bx^2}} - \frac{8a^2 x (a - bx^2)^{3/4}}{39b^3} - \frac{20ax^3 (a - bx^2)^{3/4}}{117b^2} - \frac{2x^5 (a - bx^2)^{3/4}}{13b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6/(-b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $16*a^{(7/2)}*(1 - b*x**2/a)^{(1/4)}*elliptic_e(\text{asin}(\text{sqrt}(b)*x/\text{sqrt}(a))/2, 2)/(39*b^{(7/2)}*(a - b*x**2)^{(1/4)}) - 8*a^{(7/2)}*x*(a - b*x**2)^{(3/4)}/(39*b^3) - 20*a*x^3*(a - b*x**2)^{(3/4)}/(117*b^2) - 2*x^5*(a - b*x**2)^{(3/4)}/(13*b)$

**Mathematica [C]** time = 0.0930933, size = 89, normalized size = 0.69

$$\frac{2x \left( 12a^3 \sqrt[4]{1 - \frac{bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{bx^2}{a}\right) - 12a^3 + 2a^2bx^2 + ab^2x^4 + 9b^3x^6 \right)}{117b^3 \sqrt[4]{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a - b\*x^2)^(1/4), x]

[Out]  $(2*x*(-12*a^3 + 2*a^2*b*x^2 + a*b^2*x^4 + 9*b^3*x^6 + 12*a^3*(1 - (b*x^2)/a)^{1/4}) * \text{Hypergeometric2F1}[1/4, 1/2, 3/2, (b*x^2)/a]) / (117*b^3*(a - b*x^2)^{1/4})$

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int x^6 \frac{1}{\sqrt[4]{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(-b*x^2+a)^(1/4), x)`

[Out] `int(x^6/(-b*x^2+a)^(1/4), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(-bx^2 + a)^{1/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(-b*x^2 + a)^(1/4), x, algorithm="maxima")`

[Out] `integrate(x^6/(-b*x^2 + a)^(1/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{(-bx^2 + a)^{1/4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(-b*x^2 + a)^(1/4), x, algorithm="fricas")`

[Out] `integral(x^6/(-b*x^2 + a)^(1/4), x)`

**Sympy [A]** time = 3.28356, size = 29, normalized size = 0.22

$$\frac{x^7 {}_2F_1\left(\frac{1}{4}, \frac{7}{2} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{7\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(-b*x**2+a)**(1/4), x)`

[Out] `x**7*hyper((1/4, 7/2), (9/2, ), b*x**2*exp_polar(2*I*pi)/a)/(7*a**(1/4))`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(-bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(-b*x^2 + a)^(1/4),x, algorithm="giac")`

[Out] `integrate(x^6/(-b*x^2 + a)^(1/4), x)`

$$3.823 \quad \int \frac{x^4}{\sqrt[4]{a - bx^2}} dx$$

**Optimal.** Leaf size=104

$$\frac{8a^{5/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{15b^{5/2} \sqrt[4]{a - bx^2}} - \frac{4ax(a - bx^2)^{3/4}}{15b^2} - \frac{2x^3(a - bx^2)^{3/4}}{9b}$$

[Out]  $(-4*a*x*(a - b*x^2)^{(3/4)})/(15*b^2) - (2*x^3*(a - b*x^2)^{(3/4)})/(9*b) + (8*a^{(5/2)}*(1 - (b*x^2)/a)^{(1/4)}*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^{(5/2)}*(a - b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.106829, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{8a^{5/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{15b^{5/2} \sqrt[4]{a - bx^2}} - \frac{4ax(a - bx^2)^{3/4}}{15b^2} - \frac{2x^3(a - bx^2)^{3/4}}{9b}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a - b\*x^2)^(1/4), x]

[Out]  $(-4*a*x*(a - b*x^2)^{(3/4)})/(15*b^2) - (2*x^3*(a - b*x^2)^{(3/4)})/(9*b) + (8*a^{(5/2)}*(1 - (b*x^2)/a)^{(1/4)}*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b^{(5/2)}*(a - b*x^2)^{(1/4)})$

**Rubi in Sympy [A]** time = 14.5722, size = 90, normalized size = 0.87

$$\frac{8a^{\frac{5}{2}} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{15b^{\frac{5}{2}} \sqrt[4]{a - bx^2}} - \frac{4ax(a - bx^2)^{\frac{3}{4}}}{15b^2} - \frac{2x^3(a - bx^2)^{\frac{3}{4}}}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(-b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $8*a^{(5/2)}*(1 - b*x^2/a)^{(1/4)}*elliptic_e(\text{asin}(\text{sqrt}(b)*x/\text{sqrt}(a)))/2, 2)/(15*b^{(5/2)}*(a - b*x^2)^{(1/4)}) - 4*a*x*(a - b*x^2)^{(3/4)}/(15*b^2) - 2*x^3*(a - b*x^2)^{(3/4)}/(9*b)$

**Mathematica [C]** time = 0.0608393, size = 79, normalized size = 0.76

$$\frac{2 \left( 6a^2 x^4 \sqrt[4]{1 - \frac{bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{bx^2}{a}\right) - 6a^2 x + abx^3 + 5b^2 x^5 \right)}{45b^2 \sqrt[4]{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a - b\*x^2)^(1/4), x]

[Out]  $(2*(-6*a^2*x + a*b*x^3 + 5*b^2*x^5 + 6*a^2*x*(1 - (b*x^2)/a)^{(1/4)})*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a])/(45*b^2*(a - b*x^2)^{(1/4)})$



---

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int x^4 \frac{1}{\sqrt[4]{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(-b\*x^2+a)^(1/4), x)

[Out] int(x^4/(-b\*x^2+a)^(1/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(-bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-b\*x^2 + a)^(1/4), x, algorithm="maxima")

[Out] integrate(x^4/(-b\*x^2 + a)^(1/4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{(-bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-b\*x^2 + a)^(1/4), x, algorithm="fricas")

[Out] integral(x^4/(-b\*x^2 + a)^(1/4), x)

---

**Sympy [A]** time = 2.8812, size = 29, normalized size = 0.28

$$\frac{x^5 {}_2F_1\left(\frac{1}{4}, \frac{5}{2} \mid \frac{bx^2 e^{2i\pi}}{a}\right)}{5\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4/(-b\*x\*\*2+a)\*\*(1/4), x)

[Out] x\*\*5\*hyper((1/4, 5/2), (7/2, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/(5\*a\*\*(1/4))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(-bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2 + a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-b*x^2 + a)^(1/4), x)
```

$$3.824 \quad \int \frac{x^2}{\sqrt[4]{a - bx^2}} dx$$

**Optimal.** Leaf size=81

$$\frac{4a^{3/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{5b^{3/2} \sqrt[4]{a - bx^2}} - \frac{2x(a - bx^2)^{3/4}}{5b}$$

[Out]  $(-2*x*(a - b*x^2)^{(3/4)})/(5*b) + (4*a^{(3/2)}*(1 - (b*x^2)/a)^{(1/4)} * \text{EllipticE}[\text{ArcSin}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(5*b^{(3/2)}*(a - b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.0732192, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{4a^{3/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{5b^{3/2} \sqrt[4]{a - bx^2}} - \frac{2x(a - bx^2)^{3/4}}{5b}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a - b\*x^2)^(1/4), x]

[Out]  $(-2*x*(a - b*x^2)^{(3/4)})/(5*b) + (4*a^{(3/2)}*(1 - (b*x^2)/a)^{(1/4)} * \text{EllipticE}[\text{ArcSin}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(5*b^{(3/2)}*(a - b*x^2)^{(1/4)})$

**Rubi in Sympy [A]** time = 10.6412, size = 68, normalized size = 0.84

$$\frac{4a^{\frac{3}{2}} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{5b^{\frac{3}{2}} \sqrt[4]{a - bx^2}} - \frac{2x(a - bx^2)^{\frac{3}{4}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(-b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $4*a^{(3/2)}*(1 - b*x**2/a)^{(1/4)}*\text{elliptic\_e}(\text{asin}(\text{sqrt}(b)*x/\text{sqrt}(a))/2, 2)/(5*b^{(3/2)}*(a - b*x**2)^{(1/4)}) - 2*x*(a - b*x**2)^{(3/4)}/(5*b)$

**Mathematica [C]** time = 0.0551187, size = 64, normalized size = 0.79

$$\frac{2x \left( a \sqrt[4]{1 - \frac{bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{bx^2}{a}\right) - a + bx^2 \right)}{5b \sqrt[4]{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a - b\*x^2)^(1/4), x]

[Out]  $(2*x*(-a + b*x^2 + a*(1 - (b*x^2)/a)^{(1/4)} * \text{Hypergeometric2F1}[1/4, 1/2, 3/2, (b*x^2)/a])/(5*b*(a - b*x^2)^{(1/4)})$

---

**Maple [F]** time = 0.035, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt[4]{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-b\*x^2+a)^(1/4), x)

[Out] int(x^2/(-b\*x^2+a)^(1/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-b\*x^2 + a)^(1/4), x, algorithm="maxima")

[Out] integrate(x^2/(-b\*x^2 + a)^(1/4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{(-bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-b\*x^2 + a)^(1/4), x, algorithm="fricas")

[Out] integral(x^2/(-b\*x^2 + a)^(1/4), x)

---

**Sympy [A]** time = 2.48831, size = 29, normalized size = 0.36

$$\frac{x^3 {}_2F_1\left(\frac{1}{4}, \frac{3}{2} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{3\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(-b\*x\*\*2+a)\*\*(1/4), x)

[Out] x\*\*3\*hyper((1/4, 3/2), (5/2, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/(3\*a\*\*(1/4))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2 + a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-b*x^2 + a)^(1/4), x)
```

$$3.825 \quad \int \frac{1}{\sqrt[4]{a - bx^2}} dx$$

**Optimal.** Leaf size=58

$$\frac{2\sqrt{a}\sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{b}\sqrt[4]{a - bx^2}}$$

[Out] (2\*Sqrt[a]\*(1 - (b\*x^2)/a)^(1/4)\*EllipticE[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(Sqrt[b]\*(a - b\*x^2)^(1/4))

**Rubi [A]** time = 0.0374412, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{2\sqrt{a}\sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{b}\sqrt[4]{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(-1/4), x]

[Out] (2\*Sqrt[a]\*(1 - (b\*x^2)/a)^(1/4)\*EllipticE[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(Sqrt[b]\*(a - b\*x^2)^(1/4))

**Rubi in Sympy [A]** time = 6.15118, size = 49, normalized size = 0.84

$$\frac{2\sqrt{a}\sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{\sqrt{b}\sqrt[4]{a - bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-b\*x\*\*2+a)\*\*(1/4), x)

[Out] 2\*sqrt(a)\*(1 - b\*x\*\*2/a)\*\*(1/4)\*elliptic\_e(asin(sqrt(b)\*x/sqrt(a))/2, 2)/(sqrt(b)\*(a - b\*x\*\*2)\*\*(1/4))

**Mathematica [C]** time = 0.0288289, size = 48, normalized size = 0.83

$$\frac{x\sqrt[4]{\frac{a - bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \frac{bx^2}{a}\right)}{\sqrt[4]{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(-1/4), x]

[Out] (x\*((a - b\*x^2)/a)^(1/4)\*Hypergeometric2F1[1/4, 1/2, 3/2, (b\*x^2)/a])/(a - b\*x^2)^(1/4)

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-b*x^2+a)^(1/4),x)`

[Out] `int(1/(-b*x^2+a)^(1/4),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(-1/4),x, algorithm="maxima")`

[Out] `integrate((-b*x^2 + a)^(-1/4), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(-1/4),x, algorithm="fricas")`

[Out] `integral((-b*x^2 + a)^(-1/4), x)`

**Sympy** [A] time = 2.39791, size = 26, normalized size = 0.45

$$\frac{{}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{\sqrt[4]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**2+a)**(1/4),x)`

[Out] `x*hyper((1/4, 1/2), (3/2, ), b*x**2*exp_polar(2*I*pi)/a)/a**(1/4)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(-1/4),x, algorithm="giac")`

[Out] `integrate((-b*x^2 + a)^(-1/4), x)`

$$3.826 \quad \int \frac{1}{x^2 \sqrt[4]{a - bx^2}} dx$$

**Optimal.** Leaf size=79

$$-\frac{(a - bx^2)^{3/4}}{ax} - \frac{\sqrt{b} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{a} \sqrt[4]{a - bx^2}}$$

[Out] -((a - b\*x^2)^(3/4)/(a\*x)) - (Sqrt[b]\*(1 - (b\*x^2)/a)^(1/4)\*EllipticE[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(Sqrt[a]\*(a - b\*x^2)^(1/4))

**Rubi [A]** time = 0.0690984, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$-\frac{(a - bx^2)^{3/4}}{ax} - \frac{\sqrt{b} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{a} \sqrt[4]{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a - b\*x^2)^(1/4)), x]

[Out] -((a - b\*x^2)^(3/4)/(a\*x)) - (Sqrt[b]\*(1 - (b\*x^2)/a)^(1/4)\*EllipticE[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(Sqrt[a]\*(a - b\*x^2)^(1/4))

**Rubi in Sympy [A]** time = 10.0581, size = 63, normalized size = 0.8

$$-\frac{(a - bx^2)^{\frac{3}{4}}}{ax} - \frac{\sqrt{b} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{\sqrt{a} \sqrt[4]{a - bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(-b\*x\*\*2+a)\*\*(1/4), x)

[Out] -(a - b\*x\*\*2)\*\*(3/4)/(a\*x) - sqrt(b)\*(1 - b\*x\*\*2/a)\*\*(1/4)\*elliptic\_e(asin(sqrt(b)\*x/sqrt(a))/2, 2)/(sqrt(a)\*(a - b\*x\*\*2)\*\*(1/4))

**Mathematica [C]** time = 0.0491759, size = 71, normalized size = 0.9

$$\frac{-bx^2 \sqrt[4]{1 - \frac{bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{bx^2}{a}\right) - 2a + 2bx^2}{2ax \sqrt[4]{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a - b\*x^2)^(1/4)), x]

[Out] (-2\*a + 2\*b\*x^2 - b\*x^2\*(1 - (b\*x^2)/a)^(1/4)\*Hypergeometric2F1[1/4, 1/2, 3/2, (b\*x^2)/a])/(2\*a\*x\*(a - b\*x^2)^(1/4))



**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt[4]{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(-b\*x^2+a)^(1/4), x)

[Out] int(1/x^2/(-b\*x^2+a)^(1/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{1}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(1/4)\*x^2), x, algorithm="maxima")

[Out] integrate(1/((-b\*x^2 + a)^(1/4)\*x^2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-bx^2 + a)^{\frac{1}{4}} x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(1/4)\*x^2), x, algorithm="fricas")

[Out] integral(1/((-b\*x^2 + a)^(1/4)\*x^2), x)

---

**Sympy [A]** time = 2.75678, size = 29, normalized size = 0.37

$$-\frac{{}_2F_1\left(-\frac{1}{2}, \frac{1}{4} \mid \frac{bx^2 e^{2i\pi}}{a}\right)}{\sqrt[4]{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(-b\*x\*\*2+a)\*\*(1/4), x)

[Out] -hyper((-1/2, 1/4), (1/2, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/(a\*\*(1/4)\*x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{1}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^2 + a)^(1/4)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^2), x)
```

$$3.827 \quad \int \frac{1}{x^4 \sqrt[4]{a - bx^2}} dx$$

**Optimal.** Leaf size=106

$$-\frac{b^{3/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{2a^{3/2} \sqrt[4]{a - bx^2}} - \frac{b(a - bx^2)^{3/4}}{2a^2 x} - \frac{(a - bx^2)^{3/4}}{3ax^3}$$

[Out]  $-(a - b*x^2)^{(3/4)}/(3*a*x^3) - (b*(a - b*x^2)^{(3/4)})/(2*a^2*x) - (b^{(3/2)}*(1 - (b*x^2)/a)^{(1/4)}*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*a^{(3/2)}*(a - b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.105164, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$-\frac{b^{3/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{2a^{3/2} \sqrt[4]{a - bx^2}} - \frac{b(a - bx^2)^{3/4}}{2a^2 x} - \frac{(a - bx^2)^{3/4}}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a - b\*x^2)^(1/4)), x]

[Out]  $-(a - b*x^2)^{(3/4)}/(3*a*x^3) - (b*(a - b*x^2)^{(3/4)})/(2*a^2*x) - (b^{(3/2)}*(1 - (b*x^2)/a)^{(1/4)}*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*a^{(3/2)}*(a - b*x^2)^{(1/4)})$

**Rubi in Sympy [A]** time = 13.9601, size = 87, normalized size = 0.82

$$\frac{(a - bx^2)^{3/4}}{3ax^3} - \frac{b(a - bx^2)^{3/4}}{2a^2 x} - \frac{b^{3/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{2a^{3/2} \sqrt[4]{a - bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(-b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $-(a - b*x^2)^{(3/4)}/(3*a*x^3) - b*(a - b*x^2)^{(3/4)}/(2*a^2*x) - b^{(3/2)}*(1 - b*x^2/a)^{(1/4)}*elliptic_e(\text{asin}(\text{sqrt}(b)*x/\text{sqrt}(a))/2, 2)/(2*a^{(3/2)}*(a - b*x^2)^{(1/4)})$

**Mathematica [C]** time = 0.0575566, size = 84, normalized size = 0.79

$$\frac{-4a^2 - 3b^2 x^4 \sqrt[4]{1 - \frac{bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{bx^2}{a}\right) - 2abx^2 + 6b^2 x^4}{12a^2 x^3 \sqrt[4]{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a - b\*x^2)^(1/4)), x]

[Out]  $(-4*a^2 - 2*a*b*x^2 + 6*b^2*x^4 - 3*b^2*x^4*(1 - (b*x^2)/a)^{(1/4)}*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a])/(12*a^2*x^3*(a - b*x^2)^{(1/4)})$

---

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} \frac{1}{\sqrt[4]{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(-b*x^2+a)^(1/4), x)`

[Out] `int(1/x^4/(-b*x^2+a)^(1/4), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{1}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(1/4)*x^4), x, algorithm="maxima")`

[Out] `integrate(1/((-b*x^2 + a)^(1/4)*x^4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-bx^2 + a)^{\frac{1}{4}} x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(1/4)*x^4), x, algorithm="fricas")`

[Out] `integral(1/((-b*x^2 + a)^(1/4)*x^4), x)`

---

**Sympy [A]** time = 3.3383, size = 34, normalized size = 0.32

$$\frac{{}_2F_1\left(-\frac{3}{2}, \frac{1}{4} \mid \frac{bx^2 e^{2i\pi}}{a}\right)}{3\sqrt[4]{ax^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(-b*x**2+a)**(1/4), x)`

[Out] `-hyper((-3/2, 1/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(1/4)*x**3)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{1}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^2 + a)^(1/4)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^4), x)
```

$$3.828 \quad \int \frac{1}{x^6 \sqrt[4]{a - bx^2}} dx$$

**Optimal.** Leaf size=131

$$\frac{7b^{5/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{20a^{5/2} \sqrt[4]{a - bx^2}} - \frac{7b^2 (a - bx^2)^{3/4}}{20a^3 x} - \frac{7b (a - bx^2)^{3/4}}{30a^2 x^3} - \frac{(a - bx^2)^{3/4}}{5ax^5}$$

[Out]  $-(a - b*x^2)^{(3/4)}/(5*a*x^5) - (7*b*(a - b*x^2)^{(3/4)})/(30*a^2*x^3) - (7*b^2*(a - b*x^2)^{(3/4)})/(20*a^3*x) - (7*b^{(5/2)}*(1 - (b*x^2)/a)^{(1/4)}*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^{(5/2)}*(a - b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.144018, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{7b^{5/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{20a^{5/2} \sqrt[4]{a - bx^2}} - \frac{7b^2 (a - bx^2)^{3/4}}{20a^3 x} - \frac{7b (a - bx^2)^{3/4}}{30a^2 x^3} - \frac{(a - bx^2)^{3/4}}{5ax^5}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(a - b\*x^2)^(1/4)), x]

[Out]  $-(a - b*x^2)^{(3/4)}/(5*a*x^5) - (7*b*(a - b*x^2)^{(3/4)})/(30*a^2*x^3) - (7*b^2*(a - b*x^2)^{(3/4)})/(20*a^3*x) - (7*b^{(5/2)}*(1 - (b*x^2)/a)^{(1/4)}*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^{(5/2)}*(a - b*x^2)^{(1/4)})$

**Rubi in Sympy [A]** time = 18.6961, size = 114, normalized size = 0.87

$$\frac{(a - bx^2)^{3/4}}{5ax^5} - \frac{7b(a - bx^2)^{3/4}}{30a^2x^3} - \frac{7b^2(a - bx^2)^{3/4}}{20a^3x} - \frac{7b^{5/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{20a^{5/2} \sqrt[4]{a - bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*6/(-b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $-(a - b*x**2)**(3/4)/(5*a*x**5) - 7*b*(a - b*x**2)**(3/4)/(30*a**2*x**3) - 7*b**2*(a - b*x**2)**(3/4)/(20*a**3*x) - 7*b**2*(5/2)*(1 - b*x**2/a)**(1/4)*elliptic_e(asin(sqrt(b)*x/sqrt(a))/2, 2)/(20*a**2*(5/2)*(a - b*x**2)**(1/4))$

**Mathematica [C]** time = 0.0682722, size = 95, normalized size = 0.73

$$\frac{-24a^3 - 4a^2bx^2 - 21b^3x^6 \sqrt[4]{1 - \frac{bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{bx^2}{a}\right) - 14ab^2x^4 + 42b^3x^6}{120a^3x^5 \sqrt[4]{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6\*(a - b\*x^2)^(1/4)), x]

[Out]  $(-24*a^3 - 4*a^2*b*x^2 - 14*a*b^2*x^4 + 42*b^3*x^6 - 21*b^3*x^6*(1 - (b*x^2)/a)^{(1/4)}*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a])$

$/(120*a^3*x^5*(a - b*x^2)^(1/4))$

**Maple [F]** time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{x^6 \sqrt[4]{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^6/(-b*x^2+a)^(1/4), x)`

[Out] `int(1/x^6/(-b*x^2+a)^(1/4), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{1}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(1/4)*x^6), x, algorithm="maxima")`

[Out] `integrate(1/((-b*x^2 + a)^(1/4)*x^6), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-bx^2 + a)^{\frac{1}{4}} x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(1/4)*x^6), x, algorithm="fricas")`

[Out] `integral(1/((-b*x^2 + a)^(1/4)*x^6), x)`

**Sympy [A]** time = 4.22515, size = 34, normalized size = 0.26

$$\frac{{}_2F_1\left(-\frac{5}{2}, \frac{1}{4} \mid \frac{bx^2 e^{2i\pi}}{a}\right)}{5\sqrt[4]{ax^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**6/(-b*x**2+a)**(1/4), x)`

[Out] `-hyper((-5/2, 1/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(1/4)*x**5)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{1}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^2 + a)^(1/4)*x^6),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*x^6), x)
```



$$3.829 \quad \int \frac{x^6}{(a+bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=124

$$-\frac{80a^{7/2} \left(\frac{bx^2}{a} + 1\right)^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{77b^{7/2} (a+bx^2)^{3/4}} + \frac{40a^2 x \sqrt[4]{a+bx^2}}{77b^3} - \frac{20ax^3 \sqrt[4]{a+bx^2}}{77b^2} + \frac{2x^5 \sqrt[4]{a+bx^2}}{11b}$$

[Out]  $(40*a^2*x*(a+b*x^2)^{(1/4)})/(77*b^3) - (20*a*x^3*(a+b*x^2)^{(1/4)})/(77*b^2) + (2*x^5*(a+b*x^2)^{(1/4)})/(11*b) - (80*a^{(7/2)}*(1+(b*x^2)/a)^{(3/4)}*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^{(7/2)}*(a+b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.138766, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{80a^{7/2} \left(\frac{bx^2}{a} + 1\right)^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{77b^{7/2} (a+bx^2)^{3/4}} + \frac{40a^2 x \sqrt[4]{a+bx^2}}{77b^3} - \frac{20ax^3 \sqrt[4]{a+bx^2}}{77b^2} + \frac{2x^5 \sqrt[4]{a+bx^2}}{11b}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a + b\*x^2)^(3/4), x]

[Out]  $(40*a^2*x*(a+b*x^2)^{(1/4)})/(77*b^3) - (20*a*x^3*(a+b*x^2)^{(1/4)})/(77*b^2) + (2*x^5*(a+b*x^2)^{(1/4)})/(11*b) - (80*a^{(7/2)}*(1+(b*x^2)/a)^{(3/4)}*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^{(7/2)}*(a+b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 16.3914, size = 114, normalized size = 0.92

$$-\frac{80a^{7/2} \left(1 + \frac{bx^2}{a}\right)^{3/4} F\left(\frac{\text{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{77b^{7/2} (a+bx^2)^{3/4}} + \frac{40a^2 x \sqrt[4]{a+bx^2}}{77b^3} - \frac{20ax^3 \sqrt[4]{a+bx^2}}{77b^2} + \frac{2x^5 \sqrt[4]{a+bx^2}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6/(b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $-80*a^{(7/2)}*(1+b*x^2/a)^{(3/4)}*elliptic\_f(\text{atan}(\text{sqrt}(b)*x/\text{sqrt}(a))/2, 2)/(77*b^{(7/2)}*(a+b*x^2)^{(3/4)}) + 40*a^2*x*(a+b*x^2)^{(1/4)}/(77*b^3) - 20*a*x^3*(a+b*x^2)^{(1/4)}/(77*b^2) + 2*x^5*(a+b*x^2)^{(1/4)}/(11*b)$

**Mathematica [C]** time = 0.0763399, size = 90, normalized size = 0.73

$$\frac{2 \left( -20a^3 x \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1 \left( \frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{bx^2}{a} \right) + 20a^3 x + 10a^2 b x^3 - 3ab^2 x^5 + 7b^3 x^7 \right)}{77b^3 (a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a + b\*x^2)^(3/4), x]

[Out]  $(2*(20*a^3*x + 10*a^2*b*x^3 - 3*a*b^2*x^5 + 7*b^3*x^7 - 20*a^3*x*(1+(b*x^2)/a)^{(3/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, -(b*x^2)/a])/(77*b^3*(a+b*x^2)^{(3/4)})$

a)))/(77\*b^3\*(a + b\*x^2)^(3/4))

**Maple [F]** time = 0.037, size = 0, normalized size = 0.

$$\int x^6 (bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(b\*x^2+a)^(3/4), x)

[Out] int(x^6/(b\*x^2+a)^(3/4), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b\*x^2 + a)^(3/4), x, algorithm="maxima")

[Out] integrate(x^6/(b\*x^2 + a)^(3/4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{(bx^2 + a)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b\*x^2 + a)^(3/4), x, algorithm="fricas")

[Out] integral(x^6/(b\*x^2 + a)^(3/4), x)

**Sympy [A]** time = 3.11487, size = 27, normalized size = 0.22

$$\frac{x^7 {}_2F_1\left(\frac{3}{4}, \frac{7}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{7a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*6/(b\*x\*\*2+a)\*\*(3/4), x)

[Out] x\*\*7\*hyper((3/4, 7/2), (9/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(7\*a\*\*(3/4))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2 + a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(3/4), x)
```

$$3.830 \quad \int \frac{x^4}{(a+bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=100

$$\frac{8a^{5/2} \left( \frac{bx^2}{a} + 1 \right)^{3/4} F \left( \frac{1}{2} \tan^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right) \middle| 2 \right)}{7b^{5/2} (a + bx^2)^{3/4}} - \frac{4ax\sqrt[4]{a + bx^2}}{7b^2} + \frac{2x^3\sqrt[4]{a + bx^2}}{7b}$$

[Out]  $(-4*a*x*(a + b*x^2)^{(1/4)})/(7*b^2) + (2*x^3*(a + b*x^2)^{(1/4)})/(7*b) + (8*a^{(5/2)}*(1 + (b*x^2)/a)^{(3/4)}*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(7*b^{(5/2)}*(a + b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.0997195, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{8a^{5/2} \left( \frac{bx^2}{a} + 1 \right)^{3/4} F \left( \frac{1}{2} \tan^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right) \middle| 2 \right)}{7b^{5/2} (a + bx^2)^{3/4}} - \frac{4ax\sqrt[4]{a + bx^2}}{7b^2} + \frac{2x^3\sqrt[4]{a + bx^2}}{7b}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a + b\*x^2)^(3/4), x]

[Out]  $(-4*a*x*(a + b*x^2)^{(1/4)})/(7*b^2) + (2*x^3*(a + b*x^2)^{(1/4)})/(7*b) + (8*a^{(5/2)}*(1 + (b*x^2)/a)^{(3/4)}*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(7*b^{(5/2)}*(a + b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 12.0548, size = 90, normalized size = 0.9

$$\frac{8a^{5/2} \left( 1 + \frac{bx^2}{a} \right)^{3/4} F \left( \frac{\operatorname{atan} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right)}{2} \middle| 2 \right)}{7b^{5/2} (a + bx^2)^{3/4}} - \frac{4ax\sqrt[4]{a + bx^2}}{7b^2} + \frac{2x^3\sqrt[4]{a + bx^2}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $8*a^{(5/2)}*(1 + b*x^2/a)^{(3/4)}*elliptic\_f(\operatorname{atan}(\operatorname{sqrt}(b)*x/\operatorname{sqrt}(a)))/2, 2)/(7*b^{(5/2)}*(a + b*x^2)^{(3/4)}) - 4*a*x*(a + b*x^2)^{(1/4)}/(7*b^2) + 2*x^3*(a + b*x^2)^{(1/4)}/(7*b)$

**Mathematica [C]** time = 0.062158, size = 78, normalized size = 0.78

$$\frac{2 \left( 2a^2x \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1 \left( \frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{bx^2}{a} \right) - 2a^2x - abx^3 + b^2x^5 \right)}{7b^2 (a + bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a + b\*x^2)^(3/4), x]

[Out]  $(2*(-2*a^2*x - a*b*x^3 + b^2*x^5 + 2*a^2*x*(1 + (b*x^2)/a)^{(3/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, -(b*x^2)/a]))/(7*b^2*(a + b*x^2)^{(3/4)})$

---

**Maple [F]** time = 0.037, size = 0, normalized size = 0.

$$\int x^4 (bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(b*x^2+a)^(3/4),x)`

[Out] `int(x^4/(b*x^2+a)^(3/4),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^(3/4),x, algorithm="maxima")`

[Out] `integrate(x^4/(b*x^2 + a)^(3/4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{(bx^2 + a)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^(3/4),x, algorithm="fricas")`

[Out] `integral(x^4/(b*x^2 + a)^(3/4), x)`

---

**Sympy [A]** time = 2.6327, size = 27, normalized size = 0.27

$$\frac{x^5 {}_2F_1\left(\frac{3}{4}, \frac{5}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{5a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(b*x**2+a)**(3/4),x)`

[Out] `x**5*hyper((3/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(3/4))`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2 + a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(3/4), x)
```

$$3.831 \quad \int \frac{x^2}{(a+bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=78

$$\frac{2x\sqrt[4]{a+bx^2}}{3b} - \frac{4a^{3/2} \left(\frac{bx^2}{a} + 1\right)^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{3b^{3/2}(a+bx^2)^{3/4}}$$

[Out]  $(2*x*(a + b*x^2)^{(1/4)})/(3*b) - (4*a^{(3/2)}*(1 + (b*x^2)/a)^{(3/4)}*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^{(3/2)}*(a + b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.0674675, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{2x\sqrt[4]{a+bx^2}}{3b} - \frac{4a^{3/2} \left(\frac{bx^2}{a} + 1\right)^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{3b^{3/2}(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*x^2)^(3/4), x]

[Out]  $(2*x*(a + b*x^2)^{(1/4)})/(3*b) - (4*a^{(3/2)}*(1 + (b*x^2)/a)^{(3/4)}*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^{(3/2)}*(a + b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 8.35396, size = 68, normalized size = 0.87

$$-\frac{4a^{\frac{3}{2}} \left(1 + \frac{bx^2}{a}\right)^{\frac{3}{4}} F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{3b^{\frac{3}{2}}(a+bx^2)^{\frac{3}{4}}} + \frac{2x\sqrt[4]{a+bx^2}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $-4*a^{(3/2)}*(1 + b*x**2/a)**(3/4)*elliptic_f(\operatorname{atan}(\operatorname{sqrt}(b)*x/\operatorname{sqrt}(a))/2, 2)/(3*b^{(3/2)}*(a + b*x**2)**(3/4)) + 2*x*(a + b*x**2)**(1/4)/(3*b)$

**Mathematica [C]** time = 0.0446747, size = 62, normalized size = 0.79

$$\frac{2x \left(-a \left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{bx^2}{a}\right) + a + bx^2\right)}{3b(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*x^2)^(3/4), x]

[Out]  $(2*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^{(3/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, -(b*x^2)/a])/(3*b*(a + b*x^2)^{(3/4)})$

**Maple [F]** time = 0.035, size = 0, normalized size = 0.

$$\int x^2 (bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*x^2+a)^(3/4),x)`

[Out] `int(x^2/(b*x^2+a)^(3/4),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(3/4),x, algorithm="maxima")`

[Out] `integrate(x^2/(b*x^2 + a)^(3/4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{(bx^2 + a)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(3/4),x, algorithm="fricas")`

[Out] `integral(x^2/(b*x^2 + a)^(3/4), x)`

---

**Sympy [A]** time = 2.44848, size = 27, normalized size = 0.35

$$\frac{x^3 {}_2F_1\left(\frac{3}{4}, \frac{3}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**2+a)**(3/4),x)`

[Out] `x**3*hyper((3/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(3/4))`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(x^2/(b*x^2 + a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(3/4), x)
```

$$3.832 \quad \int \frac{1}{(a+bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=56

$$\frac{2\sqrt{a} \left( \frac{bx^2}{a} + 1 \right)^{3/4} F\left( \frac{1}{2} \tan^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right) \middle| 2 \right)}{\sqrt{b} (a + bx^2)^{3/4}}$$

[Out] (2\*Sqrt[a]\*(1 + (b\*x^2)/a)^(3/4)\*EllipticF[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(Sqrt[b]\*(a + b\*x^2)^(3/4))

**Rubi [A]** time = 0.0339307, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{2\sqrt{a} \left( \frac{bx^2}{a} + 1 \right)^{3/4} F\left( \frac{1}{2} \tan^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right) \middle| 2 \right)}{\sqrt{b} (a + bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(-3/4), x]

[Out] (2\*Sqrt[a]\*(1 + (b\*x^2)/a)^(3/4)\*EllipticF[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(Sqrt[b]\*(a + b\*x^2)^(3/4))

**Rubi in Sympy [A]** time = 4.31457, size = 49, normalized size = 0.88

$$\frac{2\sqrt{a} \left( 1 + \frac{bx^2}{a} \right)^{3/4} F\left( \frac{\operatorname{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2 \right)}{\sqrt{b} (a + bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+a)\*\*(3/4), x)

[Out] 2\*sqrt(a)\*(1 + b\*x\*\*2/a)\*\*(3/4)\*elliptic\_f(atan(sqrt(b)\*x/sqrt(a))/2, 2)/(sqrt(b)\*(a + b\*x\*\*2)\*\*(3/4))

**Mathematica [C]** time = 0.0235443, size = 47, normalized size = 0.84

$$\frac{x \left( \frac{a+bx^2}{a} \right)^{3/4} {}_2F_1\left( \frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{bx^2}{a} \right)}{(a + bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(-3/4), x]

[Out] (x\*((a + b\*x^2)/a)^(3/4)\*Hypergeometric2F1[1/2, 3/4, 3/2, -(b\*x^2)/a])/(a + b\*x^2)^(3/4)

**Maple [F]** time = 0.04, size = 0, normalized size = 0.

$$\int (bx^2 + a)^{-3/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^2+a)^(3/4),x)`

[Out] `int(1/(b*x^2+a)^(3/4),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(-3/4),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(-3/4), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(-3/4),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(-3/4), x)`

**Sympy** [A] time = 2.35777, size = 24, normalized size = 0.43

$$\frac{{}_2F_1\left(\frac{1}{2}, \frac{3}{4} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**2+a)**(3/4),x)`

[Out] `x*hyper((1/2, 3/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(3/4)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(-3/4),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(-3/4), x)`

$$3.833 \quad \int \frac{1}{x^2(a+bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=76

$$-\frac{\sqrt[4]{a+bx^2}}{ax} - \frac{\sqrt{b} \left( \frac{bx^2}{a} + 1 \right)^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{a}(a+bx^2)^{3/4}}$$

[Out] -((a + b\*x^2)^(1/4)/(a\*x)) - (Sqrt[b]\*(1 + (b\*x^2)/a)^(3/4)\*EllipticF[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(Sqrt[a]\*(a + b\*x^2)^(3/4))

**Rubi [A]** time = 0.065673, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{\sqrt[4]{a+bx^2}}{ax} - \frac{\sqrt{b} \left( \frac{bx^2}{a} + 1 \right)^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{a}(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a + b\*x^2)^(3/4)), x]

[Out] -((a + b\*x^2)^(1/4)/(a\*x)) - (Sqrt[b]\*(1 + (b\*x^2)/a)^(3/4)\*EllipticF[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(Sqrt[a]\*(a + b\*x^2)^(3/4))

**Rubi in Sympy [A]** time = 7.93875, size = 63, normalized size = 0.83

$$-\frac{\sqrt[4]{a+bx^2}}{ax} - \frac{\sqrt{b} \left( 1 + \frac{bx^2}{a} \right)^{3/4} F\left(\frac{\text{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{\sqrt{a}(a+bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(b\*x\*\*2+a)\*\*(3/4), x)

[Out] -(a + b\*x\*\*2)\*\*(1/4)/(a\*x) - sqrt(b)\*(1 + b\*x\*\*2/a)\*\*(3/4)\*elliptic\_f(atan(sqrt(b)\*x/sqrt(a))/2, 2)/(sqrt(a)\*(a + b\*x\*\*2)\*\*(3/4))

**Mathematica [C]** time = 0.0446168, size = 70, normalized size = 0.92

$$\frac{-bx^2 \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{bx^2}{a}\right) - 2(a+bx^2)}{2ax(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a + b\*x^2)^(3/4)), x]

[Out] (-2\*(a + b\*x^2) - b\*x^2\*(1 + (b\*x^2)/a)^(3/4)\*Hypergeometric2F1[1/2, 3/4, 3/2, -(b\*x^2)/a])/(2\*a\*x\*(a + b\*x^2)^(3/4))

**Maple [F]** time = 0.037, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(b*x^2+a)^(3/4), x)`

[Out] `int(1/x^2/(b*x^2+a)^(3/4), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/4)*x^2), x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(3/4)*x^2), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{3}{4}} x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/4)*x^2), x, algorithm="fricas")`

[Out] `integral(1/((b*x^2 + a)^(3/4)*x^2), x)`

---

**Sympy [A]** time = 3.12821, size = 27, normalized size = 0.36

$$-\frac{{}_2F_1\left(-\frac{1}{2}, \frac{3}{4} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{3}{4}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(b*x**2+a)**(3/4), x)`

[Out] `-hyper((-1/2, 3/4), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(3/4)*x)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(3/4)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^2), x)
```

$$3.834 \quad \int \frac{1}{x^4(a+bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=102

$$\frac{5b^{3/2} \left( \frac{bx^2}{a} + 1 \right)^{3/4} F \left( \frac{1}{2} \tan^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right) \middle| 2 \right)}{6a^{3/2} (a + bx^2)^{3/4}} + \frac{5b\sqrt[4]{a+bx^2}}{6a^2x} - \frac{\sqrt[4]{a+bx^2}}{3ax^3}$$

[Out]  $-(a + b*x^2)^{(1/4)}/(3*a*x^3) + (5*b*(a + b*x^2)^{(1/4)})/(6*a^2*x) + (5*b^{(3/2)}*(1 + (b*x^2)/a)^{(3/4)}*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*a^{(3/2)}*(a + b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.0965869, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{5b^{3/2} \left( \frac{bx^2}{a} + 1 \right)^{3/4} F \left( \frac{1}{2} \tan^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right) \middle| 2 \right)}{6a^{3/2} (a + bx^2)^{3/4}} + \frac{5b\sqrt[4]{a+bx^2}}{6a^2x} - \frac{\sqrt[4]{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a + b\*x^2)^(3/4)), x]

[Out]  $-(a + b*x^2)^{(1/4)}/(3*a*x^3) + (5*b*(a + b*x^2)^{(1/4)})/(6*a^2*x) + (5*b^{(3/2)}*(1 + (b*x^2)/a)^{(3/4)}*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*a^{(3/2)}*(a + b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 11.5788, size = 88, normalized size = 0.86

$$-\frac{\sqrt[4]{a+bx^2}}{3ax^3} + \frac{5b\sqrt[4]{a+bx^2}}{6a^2x} + \frac{5b^{3/2} \left( 1 + \frac{bx^2}{a} \right)^{3/4} F \left( \frac{\text{atan} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right)}{2} \middle| 2 \right)}{6a^{3/2} (a + bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $-(a + b*x**2)**(1/4)/(3*a*x**3) + 5*b*(a + b*x**2)**(1/4)/(6*a**2*x) + 5*b**(3/2)*(1 + b*x**2/a)**(3/4)*elliptic_f(atan(sqrt(b)*x/sqrt(a))/2, 2)/(6*a**(3/2)*(a + b*x**2)**(3/4))$

**Mathematica [C]** time = 0.053337, size = 83, normalized size = 0.81

$$\frac{-4a^2 + 5b^2x^4 \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1 \left( \frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{bx^2}{a} \right) + 6abx^2 + 10b^2x^4}{12a^2x^3 (a + bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a + b\*x^2)^(3/4)), x]

[Out]  $(-4*a^2 + 6*a*b*x^2 + 10*b^2*x^4 + 5*b^2*x^4*(1 + (b*x^2)/a)^{(3/4)})*Hypergeometric2F1[1/2, 3/4, 3/2, -(b*x^2)/a])/(12*a^2*x^3*(a + b*x^2)^{(3/4)})$

---

**Maple [F]** time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} (bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b\*x^2+a)^(3/4), x)

[Out] int(1/x^4/(b\*x^2+a)^(3/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(3/4)\*x^4), x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(3/4)\*x^4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{3}{4}} x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(3/4)\*x^4), x, algorithm="fricas")

[Out] integral(1/((b\*x^2 + a)^(3/4)\*x^4), x)

---

**Sympy [A]** time = 3.8991, size = 32, normalized size = 0.31

$$-\frac{{}_2F_1\left(-\frac{3}{2}, \frac{3}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{3}{4}} x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(b\*x\*\*2+a)\*\*(3/4), x)

[Out] -hyper((-3/2, 3/4), (-1/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(3\*a\*\*(3/4)\*x\*\*3)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{4}} x^4} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(3/4)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^4), x)
```

$$3.835 \quad \int \frac{1}{x^6(a+bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=126

$$-\frac{3b^{5/2} \left( \frac{bx^2}{a} + 1 \right)^{3/4} F \left( \frac{1}{2} \tan^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right) \middle| 2 \right)}{4a^{5/2} (a + bx^2)^{3/4}} - \frac{3b^2 \sqrt[4]{a + bx^2}}{4a^3 x} + \frac{3b \sqrt[4]{a + bx^2}}{10a^2 x^3} - \frac{\sqrt[4]{a + bx^2}}{5ax^5}$$

[Out]  $-(a + b*x^2)^{(1/4)}/(5*a*x^5) + (3*b*(a + b*x^2)^{(1/4)})/(10*a^2*x^3) - (3*b^2*(a + b*x^2)^{(1/4)})/(4*a^3*x) - (3*b^{(5/2)}*(1 + (b*x^2)/a)^{(3/4)}*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(4*a^{(5/2)}*(a + b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.133766, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{3b^{5/2} \left( \frac{bx^2}{a} + 1 \right)^{3/4} F \left( \frac{1}{2} \tan^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right) \middle| 2 \right)}{4a^{5/2} (a + bx^2)^{3/4}} - \frac{3b^2 \sqrt[4]{a + bx^2}}{4a^3 x} + \frac{3b \sqrt[4]{a + bx^2}}{10a^2 x^3} - \frac{\sqrt[4]{a + bx^2}}{5ax^5}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(a + b\*x^2)^(3/4)), x]

[Out]  $-(a + b*x^2)^{(1/4)}/(5*a*x^5) + (3*b*(a + b*x^2)^{(1/4)})/(10*a^2*x^3) - (3*b^2*(a + b*x^2)^{(1/4)})/(4*a^3*x) - (3*b^{(5/2)}*(1 + (b*x^2)/a)^{(3/4)}*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(4*a^{(5/2)}*(a + b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 15.9738, size = 112, normalized size = 0.89

$$-\frac{\sqrt[4]{a + bx^2}}{5ax^5} + \frac{3b \sqrt[4]{a + bx^2}}{10a^2 x^3} - \frac{3b^2 \sqrt[4]{a + bx^2}}{4a^3 x} - \frac{3b^{5/2} \left( 1 + \frac{bx^2}{a} \right)^{3/4} F \left( \frac{\operatorname{atan} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right)}{2} \middle| 2 \right)}{4a^{5/2} (a + bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*6/(b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $-(a + b*x**2)**(1/4)/(5*a*x**5) + 3*b*(a + b*x**2)**(1/4)/(10*a**2*x**3) - 3*b**2*(a + b*x**2)**(1/4)/(4*a**3*x) - 3*b**5/2*(1 + b*x**2/a)**(3/4)*elliptic_f(atan(sqrt(b)*x/sqrt(a))/2, 2)/(4*a**5/2*(a + b*x**2)**(3/4))$

**Mathematica [C]** time = 0.0631611, size = 94, normalized size = 0.75

$$\frac{-8a^3 + 4a^2bx^2 - 15b^3x^6 \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1 \left( \frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{bx^2}{a} \right) - 18ab^2x^4 - 30b^3x^6}{40a^3x^5 (a + bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6\*(a + b\*x^2)^(3/4)), x]

[Out]  $(-8*a^3 + 4*a^2*b*x^2 - 18*a*b^2*x^4 - 30*b^3*x^6 - 15*b^3*x^6*(1 + (b*x^2)/a)^{(3/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, -(b*x^2)/a])$

)]/(40\*a^3\*x^5\*(a + b\*x^2)^(3/4))

**Maple [F]** time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{x^6} (bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(b\*x^2+a)^(3/4), x)

[Out] int(1/x^6/(b\*x^2+a)^(3/4), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(3/4)\*x^6), x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(3/4)\*x^6), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{3}{4}} x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(3/4)\*x^6), x, algorithm="fricas")

[Out] integral(1/((b\*x^2 + a)^(3/4)\*x^6), x)

**Sympy [A]** time = 5.07331, size = 32, normalized size = 0.25

$$\frac{{}_2F_1\left(-\frac{5}{2}, \frac{3}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5a^{\frac{3}{4}} x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*6/(b\*x\*\*2+a)\*\*(3/4), x)

[Out] -hyper((-5/2, 3/4), (-3/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(5\*a\*\*(3/4)\*x\*\*5)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(3/4)*x^6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*x^6), x)
```

$$3.836 \quad \int \frac{x^6}{(a-bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=129

$$\frac{80a^{7/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{77b^{7/2} (a-bx^2)^{3/4}} - \frac{40a^2 x \sqrt[4]{a-bx^2}}{77b^3} - \frac{20ax^3 \sqrt[4]{a-bx^2}}{77b^2} - \frac{2x^5 \sqrt[4]{a-bx^2}}{11b}$$

[Out]  $(-40*a^2*x*(a-b*x^2)^{(1/4)})/(77*b^3) - (20*a*x^3*(a-b*x^2)^{(1/4)})/(77*b^2) - (2*x^5*(a-b*x^2)^{(1/4)})/(11*b) + (80*a^{(7/2)}*(1 - (b*x^2)/a)^{(3/4)}*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^{(7/2)}*(a-b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.143882, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{80a^{7/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{77b^{7/2} (a-bx^2)^{3/4}} - \frac{40a^2 x \sqrt[4]{a-bx^2}}{77b^3} - \frac{20ax^3 \sqrt[4]{a-bx^2}}{77b^2} - \frac{2x^5 \sqrt[4]{a-bx^2}}{11b}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a - b\*x^2)^(3/4), x]

[Out]  $(-40*a^2*x*(a-b*x^2)^{(1/4)})/(77*b^3) - (20*a*x^3*(a-b*x^2)^{(1/4)})/(77*b^2) - (2*x^5*(a-b*x^2)^{(1/4)})/(11*b) + (80*a^{(7/2)}*(1 - (b*x^2)/a)^{(3/4)}*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*b^{(7/2)}*(a-b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 19.1014, size = 114, normalized size = 0.88

$$\frac{80a^{7/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{77b^{7/2} (a-bx^2)^{3/4}} - \frac{40a^2 x \sqrt[4]{a-bx^2}}{77b^3} - \frac{20ax^3 \sqrt[4]{a-bx^2}}{77b^2} - \frac{2x^5 \sqrt[4]{a-bx^2}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6/(-b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $80*a^{(7/2)}*(1 - b*x**2/a)**(3/4)*elliptic_f(asin(sqrt(b)*x/sqrt(a))/2, 2)/(77*b^{(7/2)}*(a - b*x**2)**(3/4)) - 40*a**2*x*(a - b*x**2)**(1/4)/(77*b**3) - 20*a*x**3*(a - b*x**2)**(1/4)/(77*b**2) - 2*x**5*(a - b*x**2)**(1/4)/(11*b)$

**Mathematica [C]** time = 0.0822427, size = 91, normalized size = 0.71

$$\frac{2 \left( 20a^3 x \left(1 - \frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{bx^2}{a}\right) - 20a^3 x + 10a^2 bx^3 + 3ab^2 x^5 + 7b^3 x^7 \right)}{77b^3 (a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a - b\*x^2)^(3/4), x]

[Out]  $(2*(-20*a^3*x + 10*a^2*b*x^3 + 3*a*b^2*x^5 + 7*b^3*x^7 + 20*a^3*x*(1 - (b*x^2)/a)^{(3/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a])/(77*b^3*(a-b*x^2)^{(3/4)})$

]))/(77\*b^3\*(a - b\*x^2)^(3/4))

**Maple [F]** time = 0.028, size = 0, normalized size = 0.

$$\int x^6 (-bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(-b\*x^2+a)^(3/4), x)

[Out] int(x^6/(-b\*x^2+a)^(3/4), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(-bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(-b\*x^2 + a)^(3/4), x, algorithm="maxima")

[Out] integrate(x^6/(-b\*x^2 + a)^(3/4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{(-bx^2 + a)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(-b\*x^2 + a)^(3/4), x, algorithm="fricas")

[Out] integral(x^6/(-b\*x^2 + a)^(3/4), x)

**Sympy [A]** time = 3.19207, size = 29, normalized size = 0.22

$$\frac{x^7 {}_2F_1\left(\frac{3}{4}, \frac{7}{2} \mid \frac{bx^2 e^{2i\pi}}{a}\right)}{7a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*6/(-b\*x\*\*2+a)\*\*(3/4), x)

[Out] x\*\*7\*hyper((3/4, 7/2), (9/2, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/(7\*a\*\*(3/4))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(-bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2 + a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(-b*x^2 + a)^(3/4), x)
```

$$3.837 \quad \int \frac{x^4}{(a-bx^2)^{3/4}} dx$$

Optimal. Leaf size=104

$$\frac{8a^{5/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{7b^{5/2} (a-bx^2)^{3/4}} - \frac{4ax\sqrt[4]{a-bx^2}}{7b^2} - \frac{2x^3\sqrt[4]{a-bx^2}}{7b}$$

[Out]  $(-4*a*x*(a - b*x^2)^{(1/4)})/(7*b^2) - (2*x^3*(a - b*x^2)^{(1/4)})/(7*b) + (8*a^{(5/2)}*(1 - (b*x^2)/a)^{(3/4)}*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(7*b^{(5/2)}*(a - b*x^2)^{(3/4)})$

Rubi [A] time = 0.106016, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{8a^{5/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{7b^{5/2} (a-bx^2)^{3/4}} - \frac{4ax\sqrt[4]{a-bx^2}}{7b^2} - \frac{2x^3\sqrt[4]{a-bx^2}}{7b}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a - b\*x^2)^(3/4), x]

[Out]  $(-4*a*x*(a - b*x^2)^{(1/4)})/(7*b^2) - (2*x^3*(a - b*x^2)^{(1/4)})/(7*b) + (8*a^{(5/2)}*(1 - (b*x^2)/a)^{(3/4)}*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(7*b^{(5/2)}*(a - b*x^2)^{(3/4)})$

Rubi in Sympy [A] time = 14.5026, size = 90, normalized size = 0.87

$$\frac{8a^{5/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{7b^{5/2} (a-bx^2)^{3/4}} - \frac{4ax\sqrt[4]{a-bx^2}}{7b^2} - \frac{2x^3\sqrt[4]{a-bx^2}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(-b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $8*a^{(5/2)}*(1 - b*x**2/a)^{(3/4)}*elliptic\_f(\text{asin}(\text{sqrt}(b)*x/\text{sqrt}(a))/2, 2)/(7*b^{(5/2)}*(a - b*x**2)^{(3/4)}) - 4*a*x*(a - b*x**2)^{(1/4)}/(7*b**2) - 2*x**3*(a - b*x**2)^{(1/4)}/(7*b)$

Mathematica [C] time = 0.0633182, size = 77, normalized size = 0.74

$$\frac{2x \left(2a^2 \left(1 - \frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{bx^2}{a}\right) - 2a^2 + abx^2 + b^2x^4\right)}{7b^2 (a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a - b\*x^2)^(3/4), x]

[Out]  $(2*x*(-2*a^2 + a*b*x^2 + b^2*x^4 + 2*a^2*(1 - (b*x^2)/a)^{(3/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a]))/(7*b^2*(a - b*x^2)^{(3/4)})$



---

**Maple [F]** time = 0.028, size = 0, normalized size = 0.

$$\int x^4 (-bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(-b*x^2+a)^(3/4),x)`

[Out] `int(x^4/(-b*x^2+a)^(3/4),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(-bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(-b*x^2 + a)^(3/4),x, algorithm="maxima")`

[Out] `integrate(x^4/(-b*x^2 + a)^(3/4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{(-bx^2 + a)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(-b*x^2 + a)^(3/4),x, algorithm="fricas")`

[Out] `integral(x^4/(-b*x^2 + a)^(3/4), x)`

---

**Sympy [A]** time = 2.69107, size = 29, normalized size = 0.28

$$\frac{x^5 {}_2F_1\left(\frac{3}{4}, \frac{5}{2} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{5a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(-b*x**2+a)**(3/4),x)`

[Out] `x**5*hyper((3/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(3/4))`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(-bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2 + a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-b*x^2 + a)^(3/4), x)
```

$$3.838 \quad \int \frac{x^2}{(a-bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=81

$$\frac{4a^{3/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{3b^{3/2} (a-bx^2)^{3/4}} - \frac{2x\sqrt{a-bx^2}}{3b}$$

[Out]  $(-2*x*(a - b*x^2)^{(1/4)})/(3*b) + (4*a^{(3/2)}*(1 - (b*x^2)/a)^{(3/4)}*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^{(3/2)}*(a - b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.0700971, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{4a^{3/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{3b^{3/2} (a-bx^2)^{3/4}} - \frac{2x\sqrt{a-bx^2}}{3b}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a - b\*x^2)^(3/4), x]

[Out]  $(-2*x*(a - b*x^2)^{(1/4)})/(3*b) + (4*a^{(3/2)}*(1 - (b*x^2)/a)^{(3/4)}*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^{(3/2)}*(a - b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 10.6095, size = 68, normalized size = 0.84

$$\frac{4a^{\frac{3}{2}} \left(1 - \frac{bx^2}{a}\right)^{\frac{3}{4}} F\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{3b^{\frac{3}{2}} (a-bx^2)^{\frac{3}{4}}} - \frac{2x\sqrt{a-bx^2}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(-b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $4*a^{(3/2)}*(1 - b*x**2/a)**(3/4)*elliptic_f(\text{asin}(\text{sqrt}(b)*x/\text{sqrt}(a))/2, 2)/(3*b^{(3/2)}*(a - b*x**2)**(3/4)) - 2*x*(a - b*x**2)**(1/4)/(3*b)$

**Mathematica [C]** time = 0.0510693, size = 64, normalized size = 0.79

$$\frac{2x \left( a \left( 1 - \frac{bx^2}{a} \right)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{bx^2}{a}\right) - a + bx^2 \right)}{3b(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a - b\*x^2)^(3/4), x]

[Out]  $(2*x*(-a + b*x^2 + a*(1 - (b*x^2)/a)^{(3/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a])/(3*b*(a - b*x^2)^{(3/4)})$

**Maple [F]** time = 0.028, size = 0, normalized size = 0.

$$\int x^2 (-bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(-b*x^2+a)^(3/4),x)`

[Out] `int(x^2/(-b*x^2+a)^(3/4),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-b*x^2 + a)^(3/4),x, algorithm="maxima")`

[Out] `integrate(x^2/(-b*x^2 + a)^(3/4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{(-bx^2 + a)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-b*x^2 + a)^(3/4),x, algorithm="fricas")`

[Out] `integral(x^2/(-b*x^2 + a)^(3/4), x)`

---

**Sympy [A]** time = 2.45568, size = 29, normalized size = 0.36

$$\frac{x^3 {}_2F_1\left(\frac{3}{4}, \frac{3}{2} \mid \frac{bx^2 e^{2i\pi}}{a}\right)}{3a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(-b*x**2+a)**(3/4),x)`

[Out] `x**3*hyper((3/4, 3/2), (5/2, ), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(3/4))`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2 + a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-b*x^2 + a)^(3/4), x)
```

$$3.839 \quad \int \frac{1}{(a-bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=58

$$\frac{2\sqrt{a}\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{b}(a-bx^2)^{3/4}}$$

[Out] (2\*Sqrt[a]\*(1-(b\*x^2)/a)^(3/4)\*EllipticF[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(Sqrt[b]\*(a-b\*x^2)^(3/4))

**Rubi [A]** time = 0.0363235, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{2\sqrt{a}\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{b}(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(-3/4), x]

[Out] (2\*Sqrt[a]\*(1-(b\*x^2)/a)^(3/4)\*EllipticF[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(Sqrt[b]\*(a-b\*x^2)^(3/4))

**Rubi in Sympy [A]** time = 6.04105, size = 49, normalized size = 0.84

$$\frac{2\sqrt{a}\left(1-\frac{bx^2}{a}\right)^{3/4} F\left(\frac{\operatorname{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{\sqrt{b}(a-bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-b\*x\*\*2+a)\*\*(3/4), x)

[Out] 2\*sqrt(a)\*(1-b\*x\*\*2/a)\*\*(3/4)\*elliptic\_f(asin(sqrt(b)\*x/sqrt(a))/2, 2)/(sqrt(b)\*(a-b\*x\*\*2)\*\*(3/4))

**Mathematica [C]** time = 0.025166, size = 48, normalized size = 0.83

$$\frac{x\left(\frac{a-bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{bx^2}{a}\right)}{(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(-3/4), x]

[Out] (x\*((a - b\*x^2)/a)^(3/4)\*Hypergeometric2F1[1/2, 3/4, 3/2, (b\*x^2)/a])/(a - b\*x^2)^(3/4)

**Maple [F]** time = 0.041, size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{-3/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-b*x^2+a)^(3/4),x)`

[Out] `int(1/(-b*x^2+a)^(3/4),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(-3/4),x, algorithm="maxima")`

[Out] `integrate((-b*x^2 + a)^(-3/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-bx^2 + a)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(-3/4),x, algorithm="fricas")`

[Out] `integral((-b*x^2 + a)^(-3/4), x)`

**Sympy [A]** time = 2.36092, size = 26, normalized size = 0.45

$$\frac{{}_2F_1\left(\frac{1}{2}, \frac{3}{4} \mid \frac{bx^2 e^{2i\pi}}{a}\right)}{a^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**2+a)**(3/4),x)`

[Out] `x*hyper((1/2, 3/4), (3/2, ), b*x**2*exp_polar(2*I*pi)/a)/a**(3/4)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(-3/4),x, algorithm="giac")`

[Out] `integrate((-b*x^2 + a)^(-3/4), x)`

$$3.840 \quad \int \frac{1}{x^2(a-bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=78

$$\frac{\sqrt{b} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{a}(a-bx^2)^{3/4}} - \frac{\sqrt[4]{a-bx^2}}{ax}$$

[Out] -((a - b\*x^2)^(1/4)/(a\*x)) + (Sqrt[b]\*(1 - (b\*x^2)/a)^(3/4)\*EllipticF[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(Sqrt[a]\*(a - b\*x^2)^(3/4))

**Rubi [A]** time = 0.0676902, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{\sqrt{b} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{a}(a-bx^2)^{3/4}} - \frac{\sqrt[4]{a-bx^2}}{ax}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a - b\*x^2)^(3/4)), x]

[Out] -((a - b\*x^2)^(1/4)/(a\*x)) + (Sqrt[b]\*(1 - (b\*x^2)/a)^(3/4)\*EllipticF[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(Sqrt[a]\*(a - b\*x^2)^(3/4))

**Rubi in Sympy [A]** time = 10.1922, size = 61, normalized size = 0.78

$$-\frac{\sqrt[4]{a-bx^2}}{ax} + \frac{\sqrt{b} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{\sqrt{a}(a-bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(-b\*x\*\*2+a)\*\*(3/4), x)

[Out] -(a - b\*x\*\*2)\*\*(1/4)/(a\*x) + sqrt(b)\*(1 - b\*x\*\*2/a)\*\*(3/4)\*elliptic\_f(asin(sqrt(b)\*x/sqrt(a))/2, 2)/(sqrt(a)\*(a - b\*x\*\*2)\*\*(3/4))

**Mathematica [C]** time = 0.0468164, size = 70, normalized size = 0.9

$$\frac{bx^2 \left(1 - \frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{bx^2}{a}\right) - 2a + 2bx^2}{2ax(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a - b\*x^2)^(3/4)), x]

[Out] (-2\*a + 2\*b\*x^2 + b\*x^2\*(1 - (b\*x^2)/a)^(3/4)\*Hypergeometric2F1[1/2, 3/4, 3/2, (b\*x^2)/a])/(2\*a\*x\*(a - b\*x^2)^(3/4))



**Maple [F]** time = 0.029, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (-bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(-b\*x^2+a)^(3/4), x)

[Out] int(1/x^2/(-b\*x^2+a)^(3/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{3}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(3/4)\*x^2), x, algorithm="maxima")

[Out] integrate(1/((-b\*x^2 + a)^(3/4)\*x^2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-bx^2 + a)^{\frac{3}{4}} x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(3/4)\*x^2), x, algorithm="fricas")

[Out] integral(1/((-b\*x^2 + a)^(3/4)\*x^2), x)

---

**Sympy [A]** time = 3.10566, size = 29, normalized size = 0.37

$$-\frac{{}_2F_1\left(-\frac{1}{2}, \frac{3}{4} \mid \frac{bx^2 e^{2i\pi}}{a}\right)}{a^{\frac{3}{4}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(-b\*x\*\*2+a)\*\*(3/4), x)

[Out] -hyper((-1/2, 3/4), (1/2, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/(a\*\*(3/4)\*x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{3}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^2 + a)^(3/4)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^2), x)
```

$$3.841 \quad \int \frac{1}{x^4(a-bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=106

$$\frac{5b^{3/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{6a^{3/2}(a-bx^2)^{3/4}} - \frac{5b\sqrt[4]{a-bx^2}}{6a^2x} - \frac{\sqrt[4]{a-bx^2}}{3ax^3}$$

[Out]  $-(a - b*x^2)^{(1/4)}/(3*a*x^3) - (5*b*(a - b*x^2)^{(1/4)})/(6*a^2*x) + (5*b^{(3/2)}*(1 - (b*x^2)/a)^{(3/4)}*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*a^{(3/2)}*(a - b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.101055, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{5b^{3/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{6a^{3/2}(a-bx^2)^{3/4}} - \frac{5b\sqrt[4]{a-bx^2}}{6a^2x} - \frac{\sqrt[4]{a-bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a - b\*x^2)^(3/4)), x]

[Out]  $-(a - b*x^2)^{(1/4)}/(3*a*x^3) - (5*b*(a - b*x^2)^{(1/4)})/(6*a^2*x) + (5*b^{(3/2)}*(1 - (b*x^2)/a)^{(3/4)}*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*a^{(3/2)}*(a - b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 14.1587, size = 88, normalized size = 0.83

$$-\frac{\sqrt[4]{a-bx^2}}{3ax^3} - \frac{5b\sqrt[4]{a-bx^2}}{6a^2x} + \frac{5b^{3/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{6a^{3/2}(a-bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(-b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $-(a - b*x**2)**(1/4)/(3*a*x**3) - 5*b*(a - b*x**2)**(1/4)/(6*a**2*x) + 5*b** (3/2)*(1 - b*x**2/a)**(3/4)*elliptic_f(asin(sqrt(b)*x/sqrt(a))/2, 2)/(6*a** (3/2)*(a - b*x**2)**(3/4))$

**Mathematica [C]** time = 0.0567551, size = 84, normalized size = 0.79

$$\frac{-4a^2 + 5b^2x^4 \left(1 - \frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{bx^2}{a}\right) - 6abx^2 + 10b^2x^4}{12a^2x^3(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a - b\*x^2)^(3/4)), x]

[Out]  $(-4*a^2 - 6*a*b*x^2 + 10*b^2*x^4 + 5*b^2*x^4*(1 - (b*x^2)/a)^{(3/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a])/(12*a^2*x^3*(a - b*x^2)^{(3/4)})$

---

**Maple [F]** time = 0.03, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} (-bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(-b\*x^2+a)^(3/4), x)

[Out] int(1/x^4/(-b\*x^2+a)^(3/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{3}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(3/4)\*x^4), x, algorithm="maxima")

[Out] integrate(1/((-b\*x^2 + a)^(3/4)\*x^4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-bx^2 + a)^{\frac{3}{4}} x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(3/4)\*x^4), x, algorithm="fricas")

[Out] integral(1/((-b\*x^2 + a)^(3/4)\*x^4), x)

---

**Sympy [A]** time = 3.94931, size = 34, normalized size = 0.32

$$\frac{{}_2F_1\left(-\frac{3}{2}, \frac{3}{4} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{3a^{\frac{3}{4}} x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(-b\*x\*\*2+a)\*\*(3/4), x)

[Out] -hyper((-3/2, 3/4), (-1/2, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/(3\*a\*\*(3/4)\*x\*\*3)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{3}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^2 + a)^(3/4)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^4), x)
```

$$3.842 \quad \int \frac{1}{x^6(a-bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=131

$$\frac{3b^{5/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{4a^{5/2}(a-bx^2)^{3/4}} - \frac{3b^2\sqrt[4]{a-bx^2}}{4a^3x} - \frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3} - \frac{\sqrt[4]{a-bx^2}}{5ax^5}$$

[Out]  $-(a - b*x^2)^{(1/4)}/(5*a*x^5) - (3*b*(a - b*x^2)^{(1/4)})/(10*a^2*x^3) - (3*b^2*(a - b*x^2)^{(1/4)})/(4*a^3*x) + (3*b^{(5/2)}*(1 - (b*x^2)/a)^{(3/4)}*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(4*a^{(5/2)}*(a - b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.140269, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{3b^{5/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{4a^{5/2}(a-bx^2)^{3/4}} - \frac{3b^2\sqrt[4]{a-bx^2}}{4a^3x} - \frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3} - \frac{\sqrt[4]{a-bx^2}}{5ax^5}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(a - b\*x^2)^(3/4)), x]

[Out]  $-(a - b*x^2)^{(1/4)}/(5*a*x^5) - (3*b*(a - b*x^2)^{(1/4)})/(10*a^2*x^3) - (3*b^2*(a - b*x^2)^{(1/4)})/(4*a^3*x) + (3*b^{(5/2)}*(1 - (b*x^2)/a)^{(3/4)}*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(4*a^{(5/2)}*(a - b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 18.7282, size = 112, normalized size = 0.85

$$-\frac{\sqrt[4]{a-bx^2}}{5ax^5} - \frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3} - \frac{3b^2\sqrt[4]{a-bx^2}}{4a^3x} + \frac{3b^{5/2} \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{\operatorname{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{4a^{5/2}(a-bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*6/(-b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $-(a - b*x**2)**(1/4)/(5*a*x**5) - 3*b*(a - b*x**2)**(1/4)/(10*a**2*x**3) - 3*b**2*(a - b*x**2)**(1/4)/(4*a**3*x) + 3*b**5/2*(1 - b*x**2/a)**(3/4)*elliptic_f(asin(sqrt(b)*x/sqrt(a))/2, 2)/(4*a**5/2*(a - b*x**2)**(3/4))$

**Mathematica [C]** time = 0.0624818, size = 95, normalized size = 0.73

$$\frac{-8a^3 - 4a^2bx^2 + 15b^3x^6 \left(1 - \frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{bx^2}{a}\right) - 18ab^2x^4 + 30b^3x^6}{40a^3x^5(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6\*(a - b\*x^2)^(3/4)), x]

[Out]  $(-8*a^3 - 4*a^2*b*x^2 - 18*a*b^2*x^4 + 30*b^3*x^6 + 15*b^3*x^6*(1 - (b*x^2)/a)^{(3/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a])/$

$$(40*a^3*x^5*(a - b*x^2)^(3/4))$$

**Maple [F]** time = 0.031, size = 0, normalized size = 0.

$$\int \frac{1}{x^6} (-bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(-b\*x^2+a)^(3/4), x)

[Out] int(1/x^6/(-b\*x^2+a)^(3/4), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{3}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(3/4)\*x^6), x, algorithm="maxima")

[Out] integrate(1/((-b\*x^2 + a)^(3/4)\*x^6), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-bx^2 + a)^{\frac{3}{4}} x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(3/4)\*x^6), x, algorithm="fricas")

[Out] integral(1/((-b\*x^2 + a)^(3/4)\*x^6), x)

**Sympy [A]** time = 5.19717, size = 34, normalized size = 0.26

$$\frac{{}_2F_1\left(-\frac{5}{2}, \frac{3}{4} \mid \frac{bx^2 e^{2i\pi}}{a}\right)}{5a^{\frac{3}{4}} x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*6/(-b\*x\*\*2+a)\*\*(3/4), x)

[Out] -hyper((-5/2, 3/4), (-3/2, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/(5\*a\*\*(3/4)\*x\*\*5)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{3}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^2 + a)^(3/4)*x^6),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*x^6), x)
```



$$3.843 \quad \int \frac{x^6}{(a+bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=124

$$-\frac{16a^{5/2}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a+bx^2}} + \frac{8a^2x}{3b^3\sqrt[4]{a+bx^2}} - \frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}} + \frac{2x^5}{9b\sqrt[4]{a+bx^2}}$$

[Out]  $(8*a^2*x)/(3*b^3*(a+b*x^2)^(1/4)) - (4*a*x^3)/(9*b^2*(a+b*x^2)^(1/4)) + (2*x^5)/(9*b*(a+b*x^2)^(1/4)) - (16*a^(5/2)*(1+(b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(7/2)*(a+b*x^2)^(1/4))$

**Rubi [A]** time = 0.143559, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{16a^{5/2}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a+bx^2}} + \frac{8a^2x}{3b^3\sqrt[4]{a+bx^2}} - \frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}} + \frac{2x^5}{9b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a + b\*x^2)^(5/4), x]

[Out]  $(8*a^2*x)/(3*b^3*(a+b*x^2)^(1/4)) - (4*a*x^3)/(9*b^2*(a+b*x^2)^(1/4)) + (2*x^5)/(9*b*(a+b*x^2)^(1/4)) - (16*a^(5/2)*(1+(b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(7/2)*(a+b*x^2)^(1/4))$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{8a^3 \int \frac{1}{(a+bx^2)^{5/4}} dx}{3b^3} + \frac{8a^2x}{3b^3\sqrt[4]{a+bx^2}} - \frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}} + \frac{2x^5}{9b\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6/(b\*x\*\*2+a)\*\*(5/4), x)

[Out]  $-8*a**3*Integral((a+b*x**2)**(-5/4), x)/(3*b**3) + 8*a**2*x/(3*b**3*(a+b*x**2)**(1/4)) - 4*a*x**3/(9*b**2*(a+b*x**2)**(1/4)) + 2*x**5/(9*b*(a+b*x**2)**(1/4))$

**Mathematica [C]** time = 0.0736454, size = 78, normalized size = 0.63

$$\frac{2\left(12a^2x\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) - 12a^2x - 2abx^3 + b^2x^5\right)}{9b^3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a + b\*x^2)^(5/4), x]

[Out]  $(2*(-12*a^2*x - 2*a*b*x^3 + b^2*x^5 + 12*a^2*x*(1+(b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -(b*x^2)/a]))/(9*b^3*(a +$

$$b \cdot x^2)^{(1/4)}$$

**Maple [F]** time = 0.065, size = 0, normalized size = 0.

$$\int x^6 (bx^2 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(b*x^2+a)^(5/4), x)`

[Out] `int(x^6/(b*x^2+a)^(5/4), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^2 + a)^(5/4), x, algorithm="maxima")`

[Out] `integrate(x^6/(b*x^2 + a)^(5/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{(bx^2 + a)^{\frac{5}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^2 + a)^(5/4), x, algorithm="fricas")`

[Out] `integral(x^6/(b*x^2 + a)^(5/4), x)`

**Sympy [A]** time = 3.2613, size = 27, normalized size = 0.22

$$\frac{x^7 {}_2F_1\left(\frac{5}{4}, \frac{7}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{7a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(b*x**2+a)**(5/4), x)`

[Out] `x**7*hyper((5/4, 7/2), (9/2, ), b*x**2*exp_polar(I*pi)/a)/(7*a**(5/4))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^2 + a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^2 + a)^(5/4), x)
```

$$3.844 \quad \int \frac{x^4}{(a+bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=100

$$\frac{24a^{3/2} \sqrt{\frac{bx^2}{a}} + 1E\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{5b^{5/2} \sqrt[4]{a+bx^2}} - \frac{12ax}{5b^2 \sqrt[4]{a+bx^2}} + \frac{2x^3}{5b \sqrt[4]{a+bx^2}}$$

[Out]  $(-12*a*x)/(5*b^2*(a + b*x^2)^{(1/4)}) + (2*x^3)/(5*b*(a + b*x^2)^{(1/4)}) + (24*a^{(3/2)}*(1 + (b*x^2)/a)^{(1/4)}*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b^{(5/2)}*(a + b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.100283, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{24a^{3/2} \sqrt{\frac{bx^2}{a}} + 1E\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{5b^{5/2} \sqrt[4]{a+bx^2}} - \frac{12ax}{5b^2 \sqrt[4]{a+bx^2}} + \frac{2x^3}{5b \sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a + b\*x^2)^(5/4), x]

[Out]  $(-12*a*x)/(5*b^2*(a + b*x^2)^{(1/4)}) + (2*x^3)/(5*b*(a + b*x^2)^{(1/4)}) + (24*a^{(3/2)}*(1 + (b*x^2)/a)^{(1/4)}*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b^{(5/2)}*(a + b*x^2)^{(1/4)})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{12ax}{5b^2 \sqrt[4]{a+bx^2}} - \frac{12a \int \frac{1}{\sqrt[4]{a+bx^2}} dx}{5b^2} + \frac{2x^3}{5b \sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(b\*x\*\*2+a)\*\*(5/4), x)

[Out]  $12*a*x/(5*b^2*(a + b*x^2)^{(1/4)}) - 12*a*Integral((a + b*x^2)^*(-1/4), x)/(5*b^2) + 2*x^3/(5*b*(a + b*x^2)^{(1/4)})$

**Mathematica [C]** time = 0.0542896, size = 64, normalized size = 0.64

$$\frac{2x \left( -6a \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) + 6a + bx^2 \right)}{5b^2 \sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a + b\*x^2)^(5/4), x]

[Out]  $(2*x*(6*a + b*x^2 - 6*a*(1 + (b*x^2)/a)^{(1/4)}*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)])/(5*b^2*(a + b*x^2)^{(1/4)})$

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int x^4 (bx^2 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(b*x^2+a)^(5/4),x)`

[Out] `int(x^4/(b*x^2+a)^(5/4),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^(5/4),x, algorithm="maxima")`

[Out] `integrate(x^4/(b*x^2 + a)^(5/4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{(bx^2 + a)^{\frac{5}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^(5/4),x, algorithm="fricas")`

[Out] `integral(x^4/(b*x^2 + a)^(5/4), x)`

---

**Sympy [A]** time = 2.89027, size = 27, normalized size = 0.27

$$\frac{x^5 {}_2F_1\left(\frac{5}{4}, \frac{5}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{5a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(b*x**2+a)**(5/4),x)`

[Out] `x**5*hyper((5/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/4))`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2 + a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^2 + a)^(5/4), x)
```

$$3.845 \quad \int \frac{x^2}{(a+bx^2)^{5/4}} dx$$

Optimal. Leaf size=74

$$\frac{2x}{b\sqrt[4]{a+bx^2}} - \frac{4\sqrt{a}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\right)\Big|_2}{b^{3/2}\sqrt[4]{a+bx^2}}$$

[Out] (2\*x)/(b\*(a + b\*x^2)^(1/4)) - (4\*Sqrt[a]\*(1 + (b\*x^2)/a)^(1/4)\*EllipticE[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(b^(3/2)\*(a + b\*x^2)^(1/4))

Rubi [A] time = 0.0671062, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{2x}{b\sqrt[4]{a+bx^2}} - \frac{4\sqrt{a}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\right)\Big|_2}{b^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*x^2)^(5/4), x]

[Out] (2\*x)/(b\*(a + b\*x^2)^(1/4)) - (4\*Sqrt[a]\*(1 + (b\*x^2)/a)^(1/4)\*EllipticE[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(b^(3/2)\*(a + b\*x^2)^(1/4))

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2a \int \frac{1}{(a+bx^2)^{5/4}} dx}{b} + \frac{2x}{b\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(b\*x\*\*2+a)\*\*(5/4), x)

[Out] -2\*a\*Integral((a + b\*x\*\*2)\*\*(-5/4), x)/b + 2\*x/(b\*(a + b\*x\*\*2)\*\*(1/4))

Mathematica [C] time = 0.0481904, size = 53, normalized size = 0.72

$$\frac{2x \left( \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) - 1 \right)}{b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*x^2)^(5/4), x]

[Out] (2\*x\*(-1 + (1 + (b\*x^2)/a)^(1/4)\*Hypergeometric2F1[1/4, 1/2, 3/2, -(b\*x^2)/a]))/(b\*(a + b\*x^2)^(1/4))

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int x^2 (bx^2 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b\*x^2+a)^(5/4), x)

[Out] int(x^2/(b\*x^2+a)^(5/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b\*x^2 + a)^(5/4), x, algorithm="maxima")

[Out] integrate(x^2/(b\*x^2 + a)^(5/4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{(bx^2 + a)^{\frac{5}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b\*x^2 + a)^(5/4), x, algorithm="fricas")

[Out] integral(x^2/(b\*x^2 + a)^(5/4), x)

---

**Sympy [A]** time = 2.82698, size = 27, normalized size = 0.36

$$\frac{x^3 {}_2F_1\left(\frac{5}{4}, \frac{3}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(b\*x\*\*2+a)\*\*(5/4), x)

[Out] x\*\*3\*hyper((5/4, 3/2), (5/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(3\*a\*\*(5/4))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(x^2/(b*x^2 + a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^2 + a)^(5/4), x)
```

$$3.846 \quad \int \frac{1}{(a+bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=56

$$\frac{2\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\right) \Big|_2}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^2}}$$

[Out]  $(2*(1 + (b*x^2)/a)^{(1/4)}*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*Sqrt[b]*(a + b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.0350618, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{2\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\right) \Big|_2}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(-5/4), x]

[Out]  $(2*(1 + (b*x^2)/a)^{(1/4)}*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*Sqrt[b]*(a + b*x^2)^{(1/4)})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2x}{a\sqrt[4]{a+bx^2}} - \frac{\int \frac{1}{\sqrt[4]{a+bx^2}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+a)\*\*(5/4), x)

[Out]  $2*x/(a*(a + b*x**2)**(1/4)) - \text{Integral}((a + b*x**2)**(-1/4), x)/a$

**Mathematica [C]** time = 0.035376, size = 55, normalized size = 0.98

$$\frac{2x - x\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right)}{a\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(-5/4), x]

[Out]  $(2*x - x*(1 + (b*x^2)/a)^{(1/4)}*Hypergeometric2F1[1/4, 1/2, 3/2, -(b*x^2)/a])/(a*(a + b*x^2)^{(1/4)})$

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int (bx^2 + a)^{-5/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^2+a)^(5/4),x)`

[Out] `int(1/(b*x^2+a)^(5/4),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(-5/4),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(-5/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{5}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(-5/4),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(-5/4), x)`

**Sympy [A]** time = 2.77653, size = 24, normalized size = 0.43

$$\frac{{}_2F_1\left(\frac{1}{2}, \frac{5}{4} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**2+a)**(5/4),x)`

[Out] `x*hyper((1/2, 5/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/4)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(-5/4),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(-5/4), x)`

$$3.847 \quad \int \frac{1}{x^2(a+bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=76

$$-\frac{3\sqrt{b}\sqrt{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\right)\Big|_2}{a^{3/2}\sqrt[4]{a+bx^2}} - \frac{1}{ax\sqrt[4]{a+bx^2}}$$

[Out]  $-(1/(a*x*(a+b*x^2)^(1/4))) - (3*\text{Sqrt}[b]*(1+(b*x^2)/a)^(1/4)*\text{EllipticE}[\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(a^(3/2)*(a+b*x^2)^(1/4))$

**Rubi [A]** time = 0.0676975, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{3\sqrt{b}\sqrt{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\right)\Big|_2}{a^{3/2}\sqrt[4]{a+bx^2}} - \frac{1}{ax\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^2*(a+b*x^2)^(5/4)), x]$

[Out]  $-(1/(a*x*(a+b*x^2)^(1/4))) - (3*\text{Sqrt}[b]*(1+(b*x^2)/a)^(1/4)*\text{EllipticE}[\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(a^(3/2)*(a+b*x^2)^(1/4))$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{3b \int \frac{1}{(a+bx^2)^{5/4}} dx}{2a} - \frac{1}{ax\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x**2/(b*x**2+a)**(5/4), x)$

[Out]  $-3*b*\text{Integral}((a+b*x**2)**(-5/4), x)/(2*a) - 1/(a*x*(a+b*x**2)**(1/4))$

**Mathematica [C]** time = 0.0490377, size = 71, normalized size = 0.93

$$\frac{3bx^2\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) - 2(a+3bx^2)}{2a^2x\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/(x^2*(a+b*x^2)^(5/4)), x]$

[Out]  $(-2*(a+3*b*x^2) + 3*b*x^2*(1+(b*x^2)/a)^(1/4)*\text{Hypergeometric2F1}[1/4, 1/2, 3/2, -(b*x^2)/a])/(2*a^2*x*(a+b*x^2)^(1/4))$

**Maple [F]** time = 0.062, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (bx^2 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b\*x^2+a)^(5/4), x)

[Out] int(1/x^2/(b\*x^2+a)^(5/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/4)\*x^2), x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(5/4)\*x^2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^4 + ax^2)(bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/4)\*x^2), x, algorithm="fricas")

[Out] integral(1/((b\*x^4 + a\*x^2)\*(b\*x^2 + a)^(1/4)), x)

---

**Sympy [A]** time = 4.08966, size = 27, normalized size = 0.36

$$-\frac{{}_2F_1\left(-\frac{1}{2}, \frac{5}{4} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{5}{4}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(b\*x\*\*2+a)\*\*(5/4), x)

[Out] -hyper((-1/2, 5/4), (1/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(a\*\*(5/4)\*x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(5/4)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^2), x)
```

$$3.848 \quad \int \frac{1}{x^4(a+bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=102

$$\frac{7b^{3/2} \sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{2a^{5/2} \sqrt[4]{a+bx^2}} + \frac{7b}{6a^2 x \sqrt[4]{a+bx^2}} - \frac{1}{3ax^3 \sqrt[4]{a+bx^2}}$$

[Out]  $-1/(3*a*x^3*(a+b*x^2)^(1/4)) + (7*b)/(6*a^2*x*(a+b*x^2)^(1/4)) + (7*b^(3/2)*(1+(b*x^2)/a)^(1/4)*\text{EllipticE}[\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(2*a^(5/2)*(a+b*x^2)^(1/4))$

**Rubi [A]** time = 0.098558, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{7b^{3/2} \sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{2a^{5/2} \sqrt[4]{a+bx^2}} + \frac{7b}{6a^2 x \sqrt[4]{a+bx^2}} - \frac{1}{3ax^3 \sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a+b\*x^2)^(5/4)),x]

[Out]  $-1/(3*a*x^3*(a+b*x^2)^(1/4)) + (7*b)/(6*a^2*x*(a+b*x^2)^(1/4)) + (7*b^(3/2)*(1+(b*x^2)/a)^(1/4)*\text{EllipticE}[\text{ArcTan}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(2*a^(5/2)*(a+b*x^2)^(1/4))$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3ax^3 \sqrt[4]{a+bx^2}} + \frac{7b}{6a^2 x \sqrt[4]{a+bx^2}} + \frac{7b^2 x}{2a^3 \sqrt[4]{a+bx^2}} - \frac{7b^2 \int \frac{1}{\sqrt[4]{a+bx^2}} dx}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(b\*x\*\*2+a)\*\*(5/4),x)

[Out]  $-1/(3*a*x**3*(a+b*x**2)**(1/4)) + 7*b/(6*a**2*x*(a+b*x**2)**(1/4)) + 7*b**2*x/(2*a**3*(a+b*x**2)**(1/4)) - 7*b**2*\text{Integral}((a+b*x**2)**(-1/4), x)/(4*a**3)$

**Mathematica [C]** time = 0.058609, size = 83, normalized size = 0.81

$$\frac{-4a^2 - 21b^2 x^4 \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) + 14abx^2 + 42b^2 x^4}{12a^3 x^3 \sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a+b\*x^2)^(5/4)),x]

[Out]  $(-4*a^2 + 14*a*b*x^2 + 42*b^2*x^4 - 21*b^2*x^4*(1+(b*x^2)/a)^(1/4)*\text{Hypergeometric2F1}[1/4, 1/2, 3/2, -(b*x^2)/a])/(12*a^3*x^3*(a+b*x^2)^(1/4))$

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} (bx^2 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b\*x^2+a)^(5/4), x)

[Out] int(1/x^4/(b\*x^2+a)^(5/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/4)\*x^4), x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(5/4)\*x^4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^6 + ax^4)(bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/4)\*x^4), x, algorithm="fricas")

[Out] integral(1/((b\*x^6 + a\*x^4)\*(b\*x^2 + a)^(1/4)), x)

---

**Sympy [A]** time = 5.27541, size = 32, normalized size = 0.31

$$-\frac{{}_2F_1\left(-\frac{3}{2}, \frac{5}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{5}{4}} x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(b\*x\*\*2+a)\*\*(5/4), x)

[Out] -hyper((-3/2, 5/4), (-1/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(3\*a\*\*(5/4)\*x\*\*3)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(1/((b*x^2 + a)^(5/4)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^4), x)
```

$$3.849 \quad \int \frac{1}{x^6(a+bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=126

$$-\frac{77b^{5/2}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{20a^{7/2}\sqrt[4]{a+bx^2}} - \frac{77b^2}{60a^3x\sqrt[4]{a+bx^2}} + \frac{11b}{30a^2x^3\sqrt[4]{a+bx^2}} - \frac{1}{5ax^5\sqrt[4]{a+bx^2}}$$

[Out]  $-1/(5*a*x^5*(a+b*x^2)^(1/4)) + (11*b)/(30*a^2*x^3*(a+b*x^2)^(1/4)) - (77*b^2)/(60*a^3*x*(a+b*x^2)^(1/4)) - (77*b^(5/2)*(1+(b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^(7/2)*(a+b*x^2)^(1/4))$

**Rubi [A]** time = 0.13732, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{77b^{5/2}\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{20a^{7/2}\sqrt[4]{a+bx^2}} - \frac{77b^2}{60a^3x\sqrt[4]{a+bx^2}} + \frac{11b}{30a^2x^3\sqrt[4]{a+bx^2}} - \frac{1}{5ax^5\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(a+b\*x^2)^(5/4)),x]

[Out]  $-1/(5*a*x^5*(a+b*x^2)^(1/4)) + (11*b)/(30*a^2*x^3*(a+b*x^2)^(1/4)) - (77*b^2)/(60*a^3*x*(a+b*x^2)^(1/4)) - (77*b^(5/2)*(1+(b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^(7/2)*(a+b*x^2)^(1/4))$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{5ax^5\sqrt[4]{a+bx^2}} + \frac{11b}{30a^2x^3\sqrt[4]{a+bx^2}} - \frac{77b^3 \int \frac{1}{(a+bx^2)^{5/4}} dx}{40a^3} - \frac{77b^2}{60a^3x\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*6/(b\*x\*\*2+a)\*\*(5/4),x)

[Out]  $-1/(5*a*x**5*(a+b*x**2)**(1/4)) + 11*b/(30*a**2*x**3*(a+b*x**2)**(1/4)) - 77*b**3*Integral((a+b*x**2)**(-5/4),x)/(40*a**3) - 77*b**2/(60*a**3*x*(a+b*x**2)**(1/4))$

**Mathematica [C]** time = 0.065434, size = 94, normalized size = 0.75

$$\frac{-24a^3 + 44a^2bx^2 + 231b^3x^6\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) - 154ab^2x^4 - 462b^3x^6}{120a^4x^5\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6\*(a+b\*x^2)^(5/4)),x]

[Out]  $(-24*a^3 + 44*a^2*b*x^2 - 154*a*b^2*x^4 - 462*b^3*x^6 + 231*b^3*x^6*(1+(b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -(b*x^2)/a])/(120*a^4*x^5*(a+b*x^2)^(1/4))$

---

**Maple [F]** time = 0.07, size = 0, normalized size = 0.

$$\int \frac{1}{x^6} (bx^2 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(b\*x^2+a)^(5/4), x)

[Out] int(1/x^6/(b\*x^2+a)^(5/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/4)\*x^6), x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(5/4)\*x^6), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^8 + ax^6)(bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/4)\*x^6), x, algorithm="fricas")

[Out] integral(1/((b\*x^8 + a\*x^6)\*(b\*x^2 + a)^(1/4)), x)

---

**Sympy [A]** time = 7.07982, size = 32, normalized size = 0.25

$$-\frac{{}_2F_1\left(-\frac{5}{2}, \frac{5}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5a^{\frac{5}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*6/(b\*x\*\*2+a)\*\*(5/4), x)

[Out] -hyper((-5/2, 5/4), (-3/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(5\*a\*\*(5/4)\*x\*\*5)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(5/4)*x^6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*x^6), x)
```

$$3.850 \quad \int \frac{x^6}{(a-bx^2)^{5/4}} dx$$

Optimal. Leaf size=124

$$-\frac{16a^{5/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{3b^{7/2} \sqrt[4]{a-bx^2}} + \frac{8ax(a-bx^2)^{3/4}}{3b^3} + \frac{20x^3(a-bx^2)^{3/4}}{9b^2} + \frac{2x^5}{b\sqrt[4]{a-bx^2}}$$

[Out]  $(2*x^5)/(b*(a - b*x^2)^(1/4)) + (8*a*x*(a - b*x^2)^(3/4))/(3*b^3) + (20*x^3*(a - b*x^2)^(3/4))/(9*b^2) - (16*a^(5/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(7/2)*(a - b*x^2)^(1/4))$

Rubi [A] time = 0.145109, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{16a^{5/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{3b^{7/2} \sqrt[4]{a-bx^2}} + \frac{8ax(a-bx^2)^{3/4}}{3b^3} + \frac{20x^3(a-bx^2)^{3/4}}{9b^2} + \frac{2x^5}{b\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a - b\*x^2)^(5/4), x]

[Out]  $(2*x^5)/(b*(a - b*x^2)^(1/4)) + (8*a*x*(a - b*x^2)^(3/4))/(3*b^3) + (20*x^3*(a - b*x^2)^(3/4))/(9*b^2) - (16*a^(5/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(7/2)*(a - b*x^2)^(1/4))$

Rubi in Sympy [A] time = 19.4584, size = 109, normalized size = 0.88

$$-\frac{16a^{5/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{3b^{7/2} \sqrt[4]{a-bx^2}} + \frac{8ax(a-bx^2)^{3/4}}{3b^3} + \frac{2x^5}{b\sqrt[4]{a-bx^2}} + \frac{20x^3(a-bx^2)^{3/4}}{9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6/(-b\*x\*\*2+a)\*\*(5/4), x)

[Out]  $-16*a**(5/2)*(1 - b*x**2/a)**(1/4)*elliptic_e(\text{asin}(\text{sqrt}(b)*x/\text{sqrt}(a))/2, 2)/(3*b**(7/2)*(a - b*x**2)**(1/4)) + 8*a*x*(a - b*x**2)**(3/4)/(3*b**3) + 2*x**5/(b*(a - b*x**2)**(1/4)) + 20*x**3*(a - b*x**2)**(3/4)/(9*b**2)$

Mathematica [C] time = 0.0756344, size = 78, normalized size = 0.63

$$\frac{2x \left( 12a^2 \sqrt[4]{1 - \frac{bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}, \frac{bx^2}{a}\right) - 12a^2 + 2abx^2 + b^2x^4 \right)}{9b^3 \sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a - b\*x^2)^(5/4), x]

[Out]  $(-2*x*(-12*a^2 + 2*a*b*x^2 + b^2*x^4 + 12*a^2*(1 - (b*x^2)/a))^{1/4} * \text{Hypergeometric2F1}[1/4, 1/2, 3/2, (b*x^2)/a]) / (9*b^3*(a - b*x^2)^{1/4})$

**Maple [F]** time = 0.074, size = 0, normalized size = 0.

$$\int x^6 (-bx^2 + a)^{-5/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(-b*x^2+a)^(5/4),x)`

[Out] `int(x^6/(-b*x^2+a)^(5/4),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(-bx^2 + a)^{5/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(-b*x^2 + a)^(5/4),x, algorithm="maxima")`

[Out] `integrate(x^6/(-b*x^2 + a)^(5/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^6}{(bx^2 - a)(-bx^2 + a)^{1/4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(-b*x^2 + a)^(5/4),x, algorithm="fricas")`

[Out] `integral(-x^6/((b*x^2 - a)*(-b*x^2 + a)^(1/4)), x)`

**Sympy [A]** time = 3.28378, size = 29, normalized size = 0.23

$$\frac{x^7 {}_2F_1\left(\frac{5}{4}, \frac{7}{2} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{7a^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(-b*x**2+a)**(5/4),x)`

[Out] `x**7*hyper((5/4, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/(7*a**(5/4))`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(-bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-b*x^2 + a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(-b*x^2 + a)^(5/4), x)
```

$$3.851 \quad \int \frac{x^4}{(a-bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=101

$$-\frac{24a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a-bx^2}} + \frac{12x(a-bx^2)^{3/4}}{5b^2} + \frac{2x^3}{b\sqrt[4]{a-bx^2}}$$

[Out]  $(2*x^3)/(b*(a-b*x^2)^(1/4)) + (12*x*(a-b*x^2)^(3/4))/(5*b^2) - (24*a^(3/2)*(1-(b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b^(5/2)*(a-b*x^2)^(1/4))$

**Rubi [A]** time = 0.104501, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{24a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a-bx^2}} + \frac{12x(a-bx^2)^{3/4}}{5b^2} + \frac{2x^3}{b\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a - b\*x^2)^(5/4), x]

[Out]  $(2*x^3)/(b*(a-b*x^2)^(1/4)) + (12*x*(a-b*x^2)^(3/4))/(5*b^2) - (24*a^(3/2)*(1-(b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b^(5/2)*(a-b*x^2)^(1/4))$

**Rubi in Sympy [A]** time = 15.0392, size = 87, normalized size = 0.86

$$-\frac{24a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2}\middle|2\right)}{5b^{5/2}\sqrt[4]{a-bx^2}} + \frac{2x^3}{b\sqrt[4]{a-bx^2}} + \frac{12x(a-bx^2)^{3/4}}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(-b\*x\*\*2+a)\*\*(5/4), x)

[Out]  $-24*a**(3/2)*(1-b*x**2/a)**(1/4)*elliptic_e(\text{asin}(\text{sqrt}(b)*x/\text{sqrt}(a))/2, 2)/(5*b**(5/2)*(a-b*x**2)**(1/4)) + 2*x**3/(b*(a-b*x**2)**(1/4)) + 12*x*(a-b*x**2)**(3/4)/(5*b**2)$

**Mathematica [C]** time = 0.0564367, size = 65, normalized size = 0.64

$$-\frac{2x\left(6a\sqrt[4]{1-\frac{bx^2}{a}}{}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{bx^2}{a}\right) - 6a + bx^2\right)}{5b^2\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a - b\*x^2)^(5/4), x]

[Out]  $(-2*x*(-6*a + b*x^2 + 6*a*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a]))/(5*b^2*(a - b*x^2)^(1/4))$



---

**Maple [F]** time = 0.068, size = 0, normalized size = 0.

$$\int x^4 (-bx^2 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(-b*x^2+a)^(5/4),x)`

[Out] `int(x^4/(-b*x^2+a)^(5/4),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(-bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(-b*x^2 + a)^(5/4),x, algorithm="maxima")`

[Out] `integrate(x^4/(-b*x^2 + a)^(5/4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^4}{(bx^2 - a)(-bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(-b*x^2 + a)^(5/4),x, algorithm="fricas")`

[Out] `integral(-x^4/((b*x^2 - a)*(-b*x^2 + a)^(1/4)), x)`

---

**Sympy [A]** time = 2.99925, size = 29, normalized size = 0.29

$$\frac{x^5 {}_2F_1\left(\frac{5}{4}, \frac{5}{2} \mid \frac{bx^2 e^{2i\pi}}{a}\right)}{5a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(-b*x**2+a)**(5/4),x)`

[Out] `x**5*hyper((5/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(5/4))`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(-bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-b*x^2 + a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-b*x^2 + a)^(5/4), x)
```

$$3.852 \quad \int \frac{x^2}{(a-bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=77

$$\frac{2x}{b\sqrt[4]{a-bx^2}} - \frac{4\sqrt{a}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a-bx^2}}$$

[Out] (2\*x)/(b\*(a - b\*x^2)^(1/4)) - (4\*Sqrt[a]\*(1 - (b\*x^2)/a)^(1/4)\*EllipticE[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(b^(3/2)\*(a - b\*x^2)^(1/4))

**Rubi [A]** time = 0.0709114, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{2x}{b\sqrt[4]{a-bx^2}} - \frac{4\sqrt{a}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a - b\*x^2)^(5/4), x]

[Out] (2\*x)/(b\*(a - b\*x^2)^(1/4)) - (4\*Sqrt[a]\*(1 - (b\*x^2)/a)^(1/4)\*EllipticE[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(b^(3/2)\*(a - b\*x^2)^(1/4))

**Rubi in Sympy [A]** time = 10.4808, size = 65, normalized size = 0.84

$$-\frac{4\sqrt{a}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2}\middle|2\right)}{b^{\frac{3}{2}}\sqrt[4]{a-bx^2}} + \frac{2x}{b\sqrt[4]{a-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(-b\*x\*\*2+a)\*\*(5/4), x)

[Out] -4\*srt(a)\*(1 - b\*x\*\*2/a)\*\*(1/4)\*elliptic\_e(asin(sqrt(b)\*x/sqrt(a))/2, 2)/(b\*\*(3/2)\*(a - b\*x\*\*2)\*\*(1/4)) + 2\*x/(b\*(a - b\*x\*\*2)\*\*(1/4))

**Mathematica [C]** time = 0.0519592, size = 54, normalized size = 0.7

$$\frac{2x\left(\sqrt[4]{1-\frac{bx^2}{a}}{}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{bx^2}{a}\right) - 1\right)}{b\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a - b\*x^2)^(5/4), x]

[Out] (-2\*x\*(-1 + (1 - (b\*x^2)/a)^(1/4))\*Hypergeometric2F1[1/4, 1/2, 3/2, (b\*x^2)/a])/(b\*(a - b\*x^2)^(1/4))

---

**Maple [F]** time = 0.037, size = 0, normalized size = 0.

$$\int x^2 (-bx^2 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(-b*x^2+a)^(5/4),x)`

[Out] `int(x^2/(-b*x^2+a)^(5/4),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-b*x^2 + a)^(5/4),x, algorithm="maxima")`

[Out] `integrate(x^2/(-b*x^2 + a)^(5/4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^2}{(bx^2 - a)(-bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-b*x^2 + a)^(5/4),x, algorithm="fricas")`

[Out] `integral(-x^2/((b*x^2 - a)*(-b*x^2 + a)^(1/4)), x)`

---

**Sympy [A]** time = 2.90959, size = 29, normalized size = 0.38

$$\frac{x^3 {}_2F_1\left(\frac{5}{4}, \frac{3}{2} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{3a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(-b*x**2+a)**(5/4),x)`

[Out] `x**3*hyper((5/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(5/4))`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2 + a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-b*x^2 + a)^(5/4), x)
```

$$3.853 \quad \int \frac{1}{(a-bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=77

$$\frac{2x}{a\sqrt[4]{a-bx^2}} - \frac{2\sqrt[4]{1-\frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a-bx^2}}$$

[Out] (2\*x)/(a\*(a - b\*x^2)^(1/4)) - (2\*(1 - (b\*x^2)/a)^(1/4)\*EllipticE[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(Sqrt[a]\*Sqrt[b]\*(a - b\*x^2)^(1/4))

**Rubi [A]** time = 0.0566402, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{2x}{a\sqrt[4]{a-bx^2}} - \frac{2\sqrt[4]{1-\frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(-5/4), x]

[Out] (2\*x)/(a\*(a - b\*x^2)^(1/4)) - (2\*(1 - (b\*x^2)/a)^(1/4)\*EllipticE[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(Sqrt[a]\*Sqrt[b]\*(a - b\*x^2)^(1/4))

**Rubi in Sympy [A]** time = 7.92424, size = 65, normalized size = 0.84

$$\frac{2x}{a\sqrt[4]{a-bx^2}} - \frac{2\sqrt[4]{1-\frac{bx^2}{a}} E\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-b\*x\*\*2+a)\*\*(5/4), x)

[Out] 2\*x/(a\*(a - b\*x\*\*2)\*\*(1/4)) - 2\*(1 - b\*x\*\*2/a)\*\*(1/4)\*elliptic\_e(asin(sqrt(b)\*x/sqrt(a))/2, 2)/(sqrt(a)\*sqrt(b)\*(a - b\*x\*\*2)\*\*(1/4))

**Mathematica [C]** time = 0.0416333, size = 54, normalized size = 0.7

$$\frac{x \left( \sqrt[4]{1-\frac{bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \frac{bx^2}{a}\right) - 2 \right)}{a\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(-5/4), x]

[Out] -((x\*(-2 + (1 - (b\*x^2)/a)^(1/4)\*Hypergeometric2F1[1/4, 1/2, 3/2, (b\*x^2)/a]))/(a\*(a - b\*x^2)^(1/4)))

---

**Maple [F]** time = 0.044, size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^2+a)^(5/4), x)

[Out] int(1/(-b\*x^2+a)^(5/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(-5/4), x, algorithm="maxima")

[Out] integrate((-b\*x^2 + a)^(-5/4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{(bx^2 - a)(-bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(-5/4), x, algorithm="fricas")

[Out] integral(-1/((b\*x^2 - a)\*(-b\*x^2 + a)^(1/4)), x)

---

**Sympy [A]** time = 2.88285, size = 26, normalized size = 0.34

$$\frac{x {}_2F_1\left(\frac{1}{2}, \frac{5}{4} \mid \frac{bx^2 e^{2i\pi}}{a}\right)}{a^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x\*\*2+a)\*\*(5/4), x)

[Out] x\*hyper((1/2, 5/4), (3/2, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/a\*\*(5/4)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2 + a)^(-5/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(-5/4), x)
```



$$3.854 \quad \int \frac{1}{x^2(a-bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=99

$$-\frac{3\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a-bx^2}} - \frac{3(a-bx^2)^{3/4}}{a^2x} + \frac{2}{ax\sqrt[4]{a-bx^2}}$$

[Out]  $2/(a*x*(a-b*x^2)^(1/4)) - (3*(a-b*x^2)^(3/4))/(a^2*x) - (3*\text{Sqrt}[b]*(1-(b*x^2)/a)^(1/4)*\text{EllipticE}[\text{ArcSin}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(a^(3/2)*(a-b*x^2)^(1/4))$

**Rubi [A]** time = 0.101465, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{3\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a-bx^2}} - \frac{3(a-bx^2)^{3/4}}{a^2x} + \frac{2}{ax\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a-b\*x^2)^(5/4)), x]

[Out]  $2/(a*x*(a-b*x^2)^(1/4)) - (3*(a-b*x^2)^(3/4))/(a^2*x) - (3*\text{Sqrt}[b]*(1-(b*x^2)/a)^(1/4)*\text{EllipticE}[\text{ArcSin}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(a^(3/2)*(a-b*x^2)^(1/4))$

**Rubi in Sympy [A]** time = 14.4233, size = 82, normalized size = 0.83

$$\frac{2}{ax\sqrt[4]{a-bx^2}} - \frac{3(a-bx^2)^{3/4}}{a^2x} - \frac{3\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2}\middle|2\right)}{a^{3/2}\sqrt[4]{a-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(-b\*x\*\*2+a)\*\*(5/4), x)

[Out]  $2/(a*x*(a-b*x**2)**(1/4)) - 3*(a-b*x**2)**(3/4)/(a**2*x) - 3*\text{sqrt}(b)*(1-b*x**2/a)**(1/4)*\text{elliptic\_e}(\text{asin}(\text{sqrt}(b)*x/\text{sqrt}(a))/2, 2)/(a**(3/2)*(a-b*x**2)**(1/4))$

**Mathematica [C]** time = 0.0513109, size = 71, normalized size = 0.72

$$\frac{-3bx^2\sqrt[4]{1-\frac{bx^2}{a}}{}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \frac{bx^2}{a}\right) - 2a + 6bx^2}{2a^2x\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a-b\*x^2)^(5/4)), x]

[Out]  $(-2*a + 6*b*x^2 - 3*b*x^2*(1-(b*x^2)/a)^(1/4)*\text{Hypergeometric2F1}[1/4, 1/2, 3/2, (b*x^2)/a])/(2*a^2*x*(a-b*x^2)^(1/4))$

**Maple [F]** time = 0.07, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (-bx^2 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(-b\*x^2+a)^(5/4), x)

[Out] int(1/x^2/(-b\*x^2+a)^(5/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{5}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(5/4)\*x^2), x, algorithm="maxima")

[Out] integrate(1/((-b\*x^2 + a)^(5/4)\*x^2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{(bx^4 - ax^2)(-bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(5/4)\*x^2), x, algorithm="fricas")

[Out] integral(-1/((b\*x^4 - a\*x^2)\*(-b\*x^2 + a)^(1/4)), x)

---

**Sympy [A]** time = 4.00131, size = 29, normalized size = 0.29

$$-\frac{{}_2F_1\left(-\frac{1}{2}, \frac{5}{4} \mid \frac{bx^2 e^{2i\pi}}{a}\right)}{a^{\frac{5}{4}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(-b\*x\*\*2+a)\*\*(5/4), x)

[Out] -hyper((-1/2, 5/4), (1/2,), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/(a\*\*(5/4)\*x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{5}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^2 + a)^(5/4)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(5/4)*x^2), x)
```

$$3.855 \quad \int \frac{1}{x^4(a-bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=126

$$-\frac{7b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a-bx^2}} - \frac{7b(a-bx^2)^{3/4}}{2a^3x} - \frac{7(a-bx^2)^{3/4}}{3a^2x^3} + \frac{2}{ax^3\sqrt[4]{a-bx^2}}$$

[Out]  $2/(a*x^3*(a-b*x^2)^(1/4)) - (7*(a-b*x^2)^(3/4))/(3*a^2*x^3) - (7*b*(a-b*x^2)^(3/4))/(2*a^3*x) - (7*b^(3/2)*(1-(b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*a^(5/2)*(a-b*x^2)^(1/4))$

**Rubi [A]** time = 0.14064, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{7b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a-bx^2}} - \frac{7b(a-bx^2)^{3/4}}{2a^3x} - \frac{7(a-bx^2)^{3/4}}{3a^2x^3} + \frac{2}{ax^3\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a-b\*x^2)^(5/4)),x]

[Out]  $2/(a*x^3*(a-b*x^2)^(1/4)) - (7*(a-b*x^2)^(3/4))/(3*a^2*x^3) - (7*b*(a-b*x^2)^(3/4))/(2*a^3*x) - (7*b^(3/2)*(1-(b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*a^(5/2)*(a-b*x^2)^(1/4))$

**Rubi in Sympy [A]** time = 18.7665, size = 109, normalized size = 0.87

$$\frac{2}{ax^3\sqrt[4]{a-bx^2}} - \frac{7(a-bx^2)^{3/4}}{3a^2x^3} - \frac{7b(a-bx^2)^{3/4}}{2a^3x} - \frac{7b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2}\middle|2\right)}{2a^{5/2}\sqrt[4]{a-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(-b\*x\*\*2+a)\*\*(5/4),x)

[Out]  $2/(a*x**3*(a-b*x**2)**(1/4)) - 7*(a-b*x**2)**(3/4)/(3*a**2*x**3) - 7*b*(a-b*x**2)**(3/4)/(2*a**3*x) - 7*b**(3/2)*(1-b*x**2/a)**(1/4)*elliptic_e(asin(sqrt(b)*x/sqrt(a))/2, 2)/(2*a**(5/2)*(a-b*x**2)**(1/4))$

**Mathematica [C]** time = 0.0589073, size = 84, normalized size = 0.67

$$\frac{-4a^2 - 21b^2x^4\sqrt[4]{1-\frac{bx^2}{a}}{}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{bx^2}{a}\right) - 14abx^2 + 42b^2x^4}{12a^3x^3\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a-b\*x^2)^(5/4)),x]

[Out]  $(-4*a^2 - 14*a*b*x^2 + 42*b^2*x^4 - 21*b^2*x^4*(1-(b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a])/(12*a^3*x^3*(a-b*x^2)^(1/4))$

$$b \cdot x^2)^{(1/4)}$$

**Maple [F]** time = 0.073, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} (-bx^2 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(-b\*x^2+a)^(5/4), x)

[Out] int(1/x^4/(-b\*x^2+a)^(5/4), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{5}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(5/4)\*x^4), x, algorithm="maxima")

[Out] integrate(1/((-b\*x^2 + a)^(5/4)\*x^4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{(bx^6 - ax^4)(-bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(5/4)\*x^4), x, algorithm="fricas")

[Out] integral(-1/((b\*x^6 - a\*x^4)\*(-b\*x^2 + a)^(1/4)), x)

**Sympy [A]** time = 5.24561, size = 34, normalized size = 0.27

$$\frac{{}_2F_1\left(-\frac{3}{2}, \frac{5}{4} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{3a^{\frac{5}{4}} x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(-b\*x\*\*2+a)\*\*(5/4), x)

[Out] -hyper((-3/2, 5/4), (-1/2, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/(3\*a\*\*(5/4)\*x\*\*3)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{5}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^2 + a)^(5/4)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(5/4)*x^4), x)
```

$$3.856 \quad \int \frac{1}{x^6(a-bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=151

$$\frac{77b^{5/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{20a^{7/2} \sqrt[4]{a-bx^2}} - \frac{77b^2 (a-bx^2)^{3/4}}{20a^4 x} - \frac{77b (a-bx^2)^{3/4}}{30a^3 x^3} - \frac{11 (a-bx^2)^{3/4}}{5a^2 x^5} + \frac{2}{ax^5 \sqrt[4]{a-bx^2}}$$

[Out]  $2/(a*x^5*(a-b*x^2)^(1/4)) - (11*(a-b*x^2)^(3/4))/(5*a^2*x^5) - (77*b*(a-b*x^2)^(3/4))/(30*a^3*x^3) - (77*b^2*(a-b*x^2)^(3/4))/(20*a^4*x) - (77*b^(5/2)*(1-(b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^(7/2)*(a-b*x^2)^(1/4))$

**Rubi [A]** time = 0.181405, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{77b^{5/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{20a^{7/2} \sqrt[4]{a-bx^2}} - \frac{77b^2 (a-bx^2)^{3/4}}{20a^4 x} - \frac{77b (a-bx^2)^{3/4}}{30a^3 x^3} - \frac{11 (a-bx^2)^{3/4}}{5a^2 x^5} + \frac{2}{ax^5 \sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(a-b\*x^2)^(5/4)),x]

[Out]  $2/(a*x^5*(a-b*x^2)^(1/4)) - (11*(a-b*x^2)^(3/4))/(5*a^2*x^5) - (77*b*(a-b*x^2)^(3/4))/(30*a^3*x^3) - (77*b^2*(a-b*x^2)^(3/4))/(20*a^4*x) - (77*b^(5/2)*(1-(b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(20*a^(7/2)*(a-b*x^2)^(1/4))$

**Rubi in Sympy [A]** time = 24.2878, size = 133, normalized size = 0.88

$$\frac{2}{ax^5 \sqrt[4]{a-bx^2}} - \frac{11 (a-bx^2)^{3/4}}{5a^2 x^5} - \frac{77b (a-bx^2)^{3/4}}{30a^3 x^3} - \frac{77b^2 (a-bx^2)^{3/4}}{20a^4 x} - \frac{77b^{5/2} \sqrt[4]{1 - \frac{bx^2}{a}} E\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{20a^{7/2} \sqrt[4]{a-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*6/(-b\*x\*\*2+a)\*\*(5/4),x)

[Out]  $2/(a*x**5*(a-b*x**2)**(1/4)) - 11*(a-b*x**2)**(3/4)/(5*a**2*x**5) - 77*b*(a-b*x**2)**(3/4)/(30*a**3*x**3) - 77*b**2*(a-b*x**2)**(3/4)/(20*a**4*x) - 77*b**5/2*(1-b*x**2/a)**(1/4)*elliptic_e(asin(sqrt(b)*x/sqrt(a))/2, 2)/(20*a**7/2*(a-b*x**2)**(1/4))$

**Mathematica [C]** time = 0.0704395, size = 95, normalized size = 0.63

$$\frac{-24a^3 - 44a^2bx^2 - 231b^3x^6 \sqrt[4]{1 - \frac{bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{bx^2}{a}\right) - 154ab^2x^4 + 462b^3x^6}{120a^4x^5 \sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6\*(a - b\*x^2)^(5/4)),x]

[Out]  $(-24*a^3 - 44*a^2*b*x^2 - 154*a*b^2*x^4 + 462*b^3*x^6 - 231*b^3*x^6*(1 - (b*x^2)/a)^{1/4} \text{Hypergeometric2F1}[1/4, 1/2, 3/2, (b*x^2)/a]) / (120*a^4*x^5*(a - b*x^2)^{1/4})$

**Maple [F]** time = 0.085, size = 0, normalized size = 0.

$$\int \frac{1}{x^6} (-bx^2 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(-b\*x^2+a)^(5/4),x)

[Out] int(1/x^6/(-b\*x^2+a)^(5/4),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{5}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(5/4)\*x^6),x, algorithm="maxima")

[Out] integrate(1/((-b\*x^2 + a)^(5/4)\*x^6), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^8 - ax^6)(-bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(5/4)\*x^6),x, algorithm="fricas")

[Out] integral(-1/((b\*x^8 - a\*x^6)\*(-b\*x^2 + a)^(1/4)), x)

**Sympy [A]** time = 7.19762, size = 34, normalized size = 0.23

$$-\frac{{}_2F_1\left(-\frac{5}{2}, \frac{5}{4} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{5a^{\frac{5}{4}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*6/(-b\*x\*\*2+a)\*\*(5/4),x)

[Out] -hyper((-5/2, 5/4), (-3/2,), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/(5\*a\*\*(5/4)\*x\*\*5)



---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{5}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(5/4)*x^6),x, algorithm="giac")`

[Out] `integrate(1/((-b*x^2 + a)^(5/4)*x^6), x)`

$$3.857 \quad \int \frac{1}{(a+bx^2)^{7/4}} dx$$

**Optimal.** Leaf size=78

$$\frac{2x}{3a(a+bx^2)^{3/4}} + \frac{2\left(\frac{bx^2}{a} + 1\right)^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{3\sqrt{a}\sqrt{b}(a+bx^2)^{3/4}}$$

[Out] (2\*x)/(3\*a\*(a + b\*x^2)^(3/4)) + (2\*(1 + (b\*x^2)/a)^(3/4)\*EllipticF[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(3\*Sqrt[a]\*Sqrt[b]\*(a + b\*x^2)^(3/4))

**Rubi [A]** time = 0.0533191, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{2x}{3a(a+bx^2)^{3/4}} + \frac{2\left(\frac{bx^2}{a} + 1\right)^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{3\sqrt{a}\sqrt{b}(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(-7/4), x]

[Out] (2\*x)/(3\*a\*(a + b\*x^2)^(3/4)) + (2\*(1 + (b\*x^2)/a)^(3/4)\*EllipticF[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(3\*Sqrt[a]\*Sqrt[b]\*(a + b\*x^2)^(3/4))

**Rubi in Sympy [A]** time = 5.97665, size = 68, normalized size = 0.87

$$\frac{2x}{3a(a+bx^2)^{3/4}} + \frac{2\left(1 + \frac{bx^2}{a}\right)^{3/4} F\left(\frac{\text{atan}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{3\sqrt{a}\sqrt{b}(a+bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+a)\*\*(7/4), x)

[Out] 2\*x/(3\*a\*(a + b\*x\*\*2)\*\*(3/4)) + 2\*(1 + b\*x\*\*2/a)\*\*(3/4)\*elliptic\_f(atan(sqrt(b)\*x/sqrt(a))/2, 2)/(3\*sqrt(a)\*sqrt(b)\*(a + b\*x\*\*2)\*\*(3/4))

**Mathematica [C]** time = 0.0478071, size = 55, normalized size = 0.71

$$\frac{x\left(\left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{bx^2}{a}\right) + 2\right)}{3a(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(-7/4), x]

[Out] (x\*(2 + (1 + (b\*x^2)/a)^(3/4)\*Hypergeometric2F1[1/2, 3/4, 3/2, -(b\*x^2)/a]))/(3\*a\*(a + b\*x^2)^(3/4))

**Maple [F]** time = 0.044, size = 0, normalized size = 0.

$$\int (bx^2 + a)^{-\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+a)^(7/4), x)

[Out] int(1/(b\*x^2+a)^(7/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{7}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-7/4), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(-7/4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{7}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-7/4), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(-7/4), x)

---

**Sympy [A]** time = 4.04126, size = 24, normalized size = 0.31

$$\frac{x {}_2F_1\left(\frac{1}{2}, \frac{7}{4} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*2+a)\*\*(7/4), x)

[Out] x\*hyper((1/2, 7/4), (3/2,), b\*x\*\*2\*exp\_polar(I\*pi)/a)/a\*\*(7/4)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{7}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(-7/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-7/4), x)
```

$$3.858 \quad \int \frac{1}{(a+bx^2)^{9/4}} dx$$

**Optimal.** Leaf size=78

$$\frac{6\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a+bx^2}} + \frac{2x}{5a(a+bx^2)^{5/4}}$$

[Out] (2\*x)/(5\*a\*(a + b\*x^2)^(5/4)) + (6\*(1 + (b\*x^2)/a)^(1/4)\*EllipticE[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(5\*a^(3/2)\*Sqrt[b]\*(a + b\*x^2)^(1/4))

**Rubi [A]** time = 0.0557094, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{6\sqrt[4]{\frac{bx^2}{a}} + 1E\left(\frac{1}{2} \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a+bx^2}} + \frac{2x}{5a(a+bx^2)^{5/4}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(-9/4), x]

[Out] (2\*x)/(5\*a\*(a + b\*x^2)^(5/4)) + (6\*(1 + (b\*x^2)/a)^(1/4)\*EllipticE[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(5\*a^(3/2)\*Sqrt[b]\*(a + b\*x^2)^(1/4))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2x}{5a(a+bx^2)^{5/4}} + \frac{3 \int \frac{1}{(a+bx^2)^{5/4}} dx}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+a)\*\*(9/4), x)

[Out] 2\*x/(5\*a\*(a + b\*x\*\*2)\*\*(5/4)) + 3\*Integral((a + b\*x\*\*2)\*\*(-5/4), x)/(5\*a)

**Mathematica [C]** time = 0.089354, size = 72, normalized size = 0.92

$$\frac{-3x(a+bx^2)\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) + 8ax + 6bx^3}{5a^2(a+bx^2)^{5/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(-9/4), x]

[Out] (8\*a\*x + 6\*b\*x^3 - 3\*x\*(a + b\*x^2)\*(1 + (b\*x^2)/a)^(1/4)\*Hypergeometric2F1[1/4, 1/2, 3/2, -(b\*x^2)/a])/(5\*a^2\*(a + b\*x^2)^(5/4))

**Maple [F]** time = 0.048, size = 0, normalized size = 0.

$$\int (bx^2 + a)^{-\frac{9}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+a)^(9/4), x)

[Out] int(1/(b\*x^2+a)^(9/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{9}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-9/4), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(-9/4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(b^2x^4 + 2abx^2 + a^2)(bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-9/4), x, algorithm="fricas")

[Out] integral(1/((b^2\*x^4 + 2\*a\*b\*x^2 + a^2)\*(b\*x^2 + a)^(1/4)), x)

---

**Sympy [A]** time = 7.24556, size = 24, normalized size = 0.31

$$\frac{x {}_2F_1\left(\frac{1}{2}, \frac{9}{4} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*2+a)\*\*(9/4), x)

[Out] x\*hyper((1/2, 9/4), (3/2,), b\*x\*\*2\*exp\_polar(I\*pi)/a)/a\*\*(9/4)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{9}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(-9/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-9/4), x)
```

$$3.859 \quad \int \frac{1}{(a+bx^2)^{11/4}} dx$$

**Optimal.** Leaf size=97

$$\frac{10 \left( \frac{bx^2}{a} + 1 \right)^{3/4} F \left( \frac{1}{2} \tan^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right) \middle| 2 \right)}{21a^{3/2} \sqrt{b} (a+bx^2)^{3/4}} + \frac{10x}{21a^2 (a+bx^2)^{3/4}} + \frac{2x}{7a (a+bx^2)^{7/4}}$$

[Out] (2\*x)/(7\*a\*(a+b\*x^2)^(7/4)) + (10\*x)/(21\*a^2\*(a+b\*x^2)^(3/4)) + (10\*(1+(b\*x^2)/a)^(3/4)\*EllipticF[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(21\*a^(3/2)\*Sqrt[b]\*(a+b\*x^2)^(3/4))

**Rubi [A]** time = 0.0739135, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{10 \left( \frac{bx^2}{a} + 1 \right)^{3/4} F \left( \frac{1}{2} \tan^{-1} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right) \middle| 2 \right)}{21a^{3/2} \sqrt{b} (a+bx^2)^{3/4}} + \frac{10x}{21a^2 (a+bx^2)^{3/4}} + \frac{2x}{7a (a+bx^2)^{7/4}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(-11/4), x]

[Out] (2\*x)/(7\*a\*(a+b\*x^2)^(7/4)) + (10\*x)/(21\*a^2\*(a+b\*x^2)^(3/4)) + (10\*(1+(b\*x^2)/a)^(3/4)\*EllipticF[ArcTan[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(21\*a^(3/2)\*Sqrt[b]\*(a+b\*x^2)^(3/4))

**Rubi in Sympy [A]** time = 8.11637, size = 87, normalized size = 0.9

$$\frac{2x}{7a(a+bx^2)^{7/4}} + \frac{10x}{21a^2(a+bx^2)^{3/4}} + \frac{10 \left( 1 + \frac{bx^2}{a} \right)^{3/4} F \left( \frac{\operatorname{atan} \left( \frac{\sqrt{bx}}{\sqrt{a}} \right)}{2} \middle| 2 \right)}{21a^{3/2} \sqrt{b} (a+bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+a)\*\*(11/4), x)

[Out] 2\*x/(7\*a\*(a+b\*x\*\*2)\*\*(7/4)) + 10\*x/(21\*a\*\*2\*(a+b\*x\*\*2)\*\*(3/4)) + 10\*(1+b\*x\*\*2/a)\*\*(3/4)\*elliptic\_f(atan(sqrt(b)\*x/sqrt(a))/2, 2)/(21\*a\*\*(3/2)\*sqrt(b)\*(a+b\*x\*\*2)\*\*(3/4))

**Mathematica [C]** time = 0.0807333, size = 75, normalized size = 0.77

$$\frac{5x(a+bx^2) \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1 \left( \frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{bx^2}{a} \right) + 2x(8a+5bx^2)}{21a^2(a+bx^2)^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(-11/4), x]

[Out] (2\*x\*(8\*a + 5\*b\*x^2) + 5\*x\*(a + b\*x^2)\*(1 + (b\*x^2)/a)^(3/4)\*Hypergeometric2F1[1/2, 3/4, 3/2, -((b\*x^2)/a)])/(21\*a^2\*(a + b\*x^2)^(7/4))



---

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int (bx^2 + a)^{-\frac{11}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+a)^(11/4), x)

[Out] int(1/(b\*x^2+a)^(11/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{11}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-11/4), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(-11/4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(b^2x^4 + 2abx^2 + a^2)(bx^2 + a)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-11/4), x, algorithm="fricas")

[Out] integral(1/((b^2\*x^4 + 2\*a\*b\*x^2 + a^2)\*(b\*x^2 + a)^(3/4)), x)

---

**Sympy [A]** time = 15.3863, size = 24, normalized size = 0.25

$$\frac{x {}_2F_1\left(\frac{1}{2}, \frac{11}{4} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{11}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*2+a)\*\*(11/4), x)

[Out] x\*hyper((1/2, 11/4), (3/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/a\*\*(11/4)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{11}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(-11/4),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(-11/4), x)
```

$$3.860 \quad \int \frac{1}{(a-bx^2)^{7/4}} dx$$

**Optimal.** Leaf size=81

$$\frac{2x}{3a(a-bx^2)^{3/4}} + \frac{2\left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{3\sqrt{a}\sqrt{b}(a-bx^2)^{3/4}}$$

[Out] (2\*x)/(3\*a\*(a - b\*x^2)^(3/4)) + (2\*(1 - (b\*x^2)/a)^(3/4)\*EllipticF[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(3\*Sqrt[a]\*Sqrt[b]\*(a - b\*x^2)^(3/4))

**Rubi [A]** time = 0.0587764, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{2x}{3a(a-bx^2)^{3/4}} + \frac{2\left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{3\sqrt{a}\sqrt{b}(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(-7/4), x]

[Out] (2\*x)/(3\*a\*(a - b\*x^2)^(3/4)) + (2\*(1 - (b\*x^2)/a)^(3/4)\*EllipticF[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(3\*Sqrt[a]\*Sqrt[b]\*(a - b\*x^2)^(3/4))

**Rubi in Sympy [A]** time = 7.79778, size = 68, normalized size = 0.84

$$\frac{2x}{3a(a-bx^2)^{3/4}} + \frac{2\left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{3\sqrt{a}\sqrt{b}(a-bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-b\*x\*\*2+a)\*\*(7/4), x)

[Out] 2\*x/(3\*a\*(a - b\*x\*\*2)\*\*(3/4)) + 2\*(1 - b\*x\*\*2/a)\*\*(3/4)\*elliptic\_f(asin(sqrt(b)\*x/sqrt(a))/2, 2)/(3\*sqrt(a)\*sqrt(b)\*(a - b\*x\*\*2)\*\*(3/4))

**Mathematica [C]** time = 0.0484678, size = 56, normalized size = 0.69

$$\frac{x\left(\left(1 - \frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{bx^2}{a}\right) + 2\right)}{3a(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(-7/4), x]

[Out] (x\*(2 + (1 - (b\*x^2)/a)^(3/4)\*Hypergeometric2F1[1/2, 3/4, 3/2, (b\*x^2)/a]))/(3\*a\*(a - b\*x^2)^(3/4))

**Maple [F]** time = 0.052, size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{-\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-b*x^2+a)^(7/4), x)`

[Out] `int(1/(-b*x^2+a)^(7/4), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{7}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(-7/4), x, algorithm="maxima")`

[Out] `integrate((-b*x^2 + a)^(-7/4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{(bx^2 - a)(-bx^2 + a)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(-7/4), x, algorithm="fricas")`

[Out] `integral(-1/((b*x^2 - a)*(-b*x^2 + a)^(3/4)), x)`

---

**Sympy [A]** time = 4.046, size = 26, normalized size = 0.32

$$\frac{x {}_2F_1\left(\frac{1}{2}, \frac{7}{4} \mid \frac{bx^2 e^{2i\pi}}{a}\right)}{a^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**2+a)**(7/4), x)`

[Out] `x*hyper((1/2, 7/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(7/4)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{7}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2 + a)^(-7/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(-7/4), x)
```

$$3.861 \quad \int \frac{1}{(a-bx^2)^{9/4}} dx$$

**Optimal.** Leaf size=101

$$-\frac{6\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a-bx^2}} + \frac{6x}{5a^2\sqrt[4]{a-bx^2}} + \frac{2x}{5a(a-bx^2)^{5/4}}$$

[Out]  $(2*x)/(5*a*(a - b*x^2)^{(5/4)}) + (6*x)/(5*a^2*(a - b*x^2)^{(1/4)}) - (6*(1 - (b*x^2)/a)^{(1/4)}*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^{(3/2)}*Sqrt[b]*(a - b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.0785731, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{6\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a-bx^2}} + \frac{6x}{5a^2\sqrt[4]{a-bx^2}} + \frac{2x}{5a(a-bx^2)^{5/4}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(-9/4), x]

[Out]  $(2*x)/(5*a*(a - b*x^2)^{(5/4)}) + (6*x)/(5*a^2*(a - b*x^2)^{(1/4)}) - (6*(1 - (b*x^2)/a)^{(1/4)}*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^{(3/2)}*Sqrt[b]*(a - b*x^2)^{(1/4)})$

**Rubi in Sympy [A]** time = 10.1304, size = 87, normalized size = 0.86

$$\frac{2x}{5a(a-bx^2)^{5/4}} + \frac{6x}{5a^2\sqrt[4]{a-bx^2}} - \frac{6\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2}\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-b\*x\*\*2+a)\*\*(9/4), x)

[Out]  $2*x/(5*a*(a - b*x**2)**(5/4)) + 6*x/(5*a**2*(a - b*x**2)**(1/4)) - 6*(1 - b*x**2/a)**(1/4)*elliptic_e(asin(sqrt(b)*x/sqrt(a))/2, 2)/(5*a**(3/2)*sqrt(b)*(a - b*x**2)**(1/4))$

**Mathematica [C]** time = 0.0925906, size = 74, normalized size = 0.73

$$\frac{-3x(a-bx^2)\sqrt[4]{1-\frac{bx^2}{a}}{}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{bx^2}{a}\right) + 8ax - 6bx^3}{5a^2(a-bx^2)^{5/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(-9/4), x]

[Out]  $(8*a*x - 6*b*x^3 - 3*x*(a - b*x^2)*(1 - (b*x^2)/a)^{(1/4)}*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a])/(5*a^2*(a - b*x^2)^{(5/4)})$

---

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{-\frac{9}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^2+a)^(9/4), x)

[Out] int(1/(-b\*x^2+a)^(9/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{9}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(-9/4), x, algorithm="maxima")

[Out] integrate((-b\*x^2 + a)^(-9/4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(b^2x^4 - 2abx^2 + a^2)(-bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(-9/4), x, algorithm="fricas")

[Out] integral(1/((b^2\*x^4 - 2\*a\*b\*x^2 + a^2)\*(-b\*x^2 + a)^(1/4)), x)

---

**Sympy [A]** time = 7.43424, size = 26, normalized size = 0.26

$$\frac{x {}_2F_1\left(\frac{1}{2}, \frac{9}{4} \mid \frac{bx^2 e^{2i\pi}}{a}\right)}{a^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x\*\*2+a)\*\*(9/4), x)

[Out] x\*hyper((1/2, 9/4), (3/2, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/a\*\*(9/4)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{9}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2 + a)^(-9/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(-9/4), x)
```



$$3.862 \quad \int \frac{1}{(a-bx^2)^{11/4}} dx$$

**Optimal.** Leaf size=101

$$\frac{10 \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{21a^{3/2}\sqrt{b}(a-bx^2)^{3/4}} + \frac{10x}{21a^2(a-bx^2)^{3/4}} + \frac{2x}{7a(a-bx^2)^{7/4}}$$

[Out] (2\*x)/(7\*a\*(a - b\*x^2)^(7/4)) + (10\*x)/(21\*a^2\*(a - b\*x^2)^(3/4)) + (10\*(1 - (b\*x^2)/a)^(3/4)\*EllipticF[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(21\*a^(3/2)\*Sqrt[b]\*(a - b\*x^2)^(3/4))

**Rubi [A]** time = 0.0770378, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{10 \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{21a^{3/2}\sqrt{b}(a-bx^2)^{3/4}} + \frac{10x}{21a^2(a-bx^2)^{3/4}} + \frac{2x}{7a(a-bx^2)^{7/4}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(-11/4), x]

[Out] (2\*x)/(7\*a\*(a - b\*x^2)^(7/4)) + (10\*x)/(21\*a^2\*(a - b\*x^2)^(3/4)) + (10\*(1 - (b\*x^2)/a)^(3/4)\*EllipticF[ArcSin[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(21\*a^(3/2)\*Sqrt[b]\*(a - b\*x^2)^(3/4))

**Rubi in Sympy [A]** time = 9.91093, size = 87, normalized size = 0.86

$$\frac{2x}{7a(a-bx^2)^{7/4}} + \frac{10x}{21a^2(a-bx^2)^{3/4}} + \frac{10 \left(1 - \frac{bx^2}{a}\right)^{3/4} F\left(\frac{\text{asin}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2} \middle| 2\right)}{21a^{3/2}\sqrt{b}(a-bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-b\*x\*\*2+a)\*\*(11/4), x)

[Out] 2\*x/(7\*a\*(a - b\*x\*\*2)\*\*(7/4)) + 10\*x/(21\*a\*\*2\*(a - b\*x\*\*2)\*\*(3/4)) + 10\*(1 - b\*x\*\*2/a)\*\*(3/4)\*elliptic\_f(asin(sqrt(b)\*x/sqrt(a))/2, 2)/(21\*a\*\*(3/2)\*sqrt(b)\*(a - b\*x\*\*2)\*\*(3/4))

**Mathematica [C]** time = 0.0948414, size = 77, normalized size = 0.76

$$\frac{5x(a-bx^2) \left(1 - \frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{bx^2}{a}\right) + 2x(8a - 5bx^2)}{21a^2(a-bx^2)^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(-11/4), x]

[Out] (2\*x\*(8\*a - 5\*b\*x^2) + 5\*x\*(a - b\*x^2)\*(1 - (b\*x^2)/a)^(3/4)\*Hypergeometric2F1[1/2, 3/4, 3/2, (b\*x^2)/a])/(21\*a^2\*(a - b\*x^2)^(7/4))

---

**Maple [F]** time = 0.051, size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{-\frac{11}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^2+a)^(11/4), x)

[Out] int(1/(-b\*x^2+a)^(11/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{11}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(-11/4), x, algorithm="maxima")

[Out] integrate((-b\*x^2 + a)^(-11/4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(b^2x^4 - 2abx^2 + a^2)(-bx^2 + a)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(-11/4), x, algorithm="fricas")

[Out] integral(1/((b^2\*x^4 - 2\*a\*b\*x^2 + a^2)\*(-b\*x^2 + a)^(3/4)), x)

---

**Sympy [A]** time = 15.5896, size = 26, normalized size = 0.26

$$\frac{x {}_2F_1\left(\frac{1}{2}, \frac{11}{4} \mid \frac{bx^2 e^{2i\pi}}{a}\right)}{a^{\frac{11}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x\*\*2+a)\*\*(11/4), x)

[Out] x\*hyper((1/2, 11/4), (3/2, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/a\*\*(11/4)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{11}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2 + a)^(-11/4),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(-11/4), x)
```

$$3.863 \quad \int \frac{x^6}{\sqrt[4]{2+3x^2}} dx$$

**Optimal.** Leaf size=99

$$\frac{32(3x^2+2)^{3/4}x}{1053} - \frac{128x}{1053\sqrt[4]{3x^2+2}} + \frac{2}{39}(3x^2+2)^{3/4}x^5 - \frac{40(3x^2+2)^{3/4}x^3}{1053} + \frac{128\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{1053\sqrt{3}}$$

[Out]  $(-128*x)/(1053*(2+3*x^2)^{(1/4)}) + (32*x*(2+3*x^2)^{(3/4)})/1053 - (40*x^3*(2+3*x^2)^{(3/4)})/1053 + (2*x^5*(2+3*x^2)^{(3/4)})/39 + (128*2^{(1/4)}*EllipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/(1053*Sqrt[3])$

**Rubi [A]** time = 0.0938667, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{32(3x^2+2)^{3/4}x}{1053} - \frac{128x}{1053\sqrt[4]{3x^2+2}} + \frac{2}{39}(3x^2+2)^{3/4}x^5 - \frac{40(3x^2+2)^{3/4}x^3}{1053} + \frac{128\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{1053\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(2+3\*x^2)^(1/4), x]

[Out]  $(-128*x)/(1053*(2+3*x^2)^{(1/4)}) + (32*x*(2+3*x^2)^{(3/4)})/1053 - (40*x^3*(2+3*x^2)^{(3/4)})/1053 + (2*x^5*(2+3*x^2)^{(3/4)})/39 + (128*2^{(1/4)}*EllipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/(1053*Sqrt[3])$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2x^5(3x^2+2)^{3/4}}{39} - \frac{40x^3(3x^2+2)^{3/4}}{1053} + \frac{32x(3x^2+2)^{3/4}}{1053} - \frac{64\int\frac{1}{\sqrt[4]{3x^2+2}}dx}{1053}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6/(3\*x\*\*2+2)\*\*(1/4), x)

[Out]  $2*x**5*(3*x**2+2)**(3/4)/39 - 40*x**3*(3*x**2+2)**(3/4)/1053 + 32*x*(3*x**2+2)**(3/4)/1053 - 64*Integral((3*x**2+2)**(-1/4), x)/1053$

**Mathematica [C]** time = 0.0498438, size = 54, normalized size = 0.55

$$\frac{2x\left((3x^2+2)^{3/4}(27x^4-20x^2+16) - 16\cdot 2^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right)\right)}{1053}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(2+3\*x^2)^(1/4), x]

[Out]  $(2*x*((2+3*x^2)^{(3/4)}*(16-20*x^2+27*x^4) - 16*2^{(3/4)}*Hypergeometric2F1[1/4, 1/2, 3/2, (-3*x^2)/2]))/1053$

**Maple [C]** time = 0.053, size = 43, normalized size = 0.4

$$\frac{2x(27x^4 - 20x^2 + 16)}{1053} (3x^2 + 2)^{\frac{3}{4}} - \frac{32 \cdot 2^{3/4} x}{1053} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(3*x^2+2)^(1/4), x)`

[Out] `2/1053*x*(27*x^4-20*x^2+16)*(3*x^2+2)^(3/4)-32/1053*2^(3/4)*x*hypergeom([1/4, 1/2], [3/2], -3/2*x^2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(3x^2 + 2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(3*x^2 + 2)^(1/4), x, algorithm="maxima")`

[Out] `integrate(x^6/(3*x^2 + 2)^(1/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{(3x^2 + 2)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(3*x^2 + 2)^(1/4), x, algorithm="fricas")`

[Out] `integral(x^6/(3*x^2 + 2)^(1/4), x)`

**Sympy [A]** time = 2.83117, size = 27, normalized size = 0.27

$$\frac{2^{\frac{3}{4}} x^7 {}_2F_1\left(\frac{1}{4}, \frac{7}{2} \middle| \frac{3x^2 e^{i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(3*x**2+2)**(1/4), x)`

[Out] `2**(3/4)*x**7*hyper((1/4, 7/2), (9/2, ), 3*x**2*exp_polar(I*pi)/2)/14`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(3x^2 + 2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2 + 2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(3*x^2 + 2)^(1/4), x)
```

$$3.864 \quad \int \frac{x^4}{\sqrt[4]{2+3x^2}} dx$$

**Optimal.** Leaf size=81

$$-\frac{8}{135} (3x^2+2)^{3/4} x + \frac{32x}{135\sqrt[4]{3x^2+2}} + \frac{2}{27} (3x^2+2)^{3/4} x^3 - \frac{32\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{135\sqrt{3}}$$

[Out] (32\*x)/(135\*(2+3\*x^2)^(1/4)) - (8\*x\*(2+3\*x^2)^(3/4))/135 + (2\*x^3\*(2+3\*x^2)^(3/4))/27 - (32\*2^(1/4)\*EllipticE[ArcTan[Sqrt[3/2]\*x]/2, 2])/(135\*Sqrt[3])

**Rubi [A]** time = 0.0665603, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{8}{135} (3x^2+2)^{3/4} x + \frac{32x}{135\sqrt[4]{3x^2+2}} + \frac{2}{27} (3x^2+2)^{3/4} x^3 - \frac{32\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{135\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(2+3\*x^2)^(1/4), x]

[Out] (32\*x)/(135\*(2+3\*x^2)^(1/4)) - (8\*x\*(2+3\*x^2)^(3/4))/135 + (2\*x^3\*(2+3\*x^2)^(3/4))/27 - (32\*2^(1/4)\*EllipticE[ArcTan[Sqrt[3/2]\*x]/2, 2])/(135\*Sqrt[3])

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2x^3(3x^2+2)^{3/4}}{27} - \frac{8x(3x^2+2)^{3/4}}{135} + \frac{32x}{135\sqrt[4]{3x^2+2}} - \frac{32\int\frac{1}{(3x^2+2)^{5/4}}dx}{135}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(3\*x\*\*2+2)\*\*(1/4), x)

[Out] 2\*x\*\*3\*(3\*x\*\*2+2)\*\*(3/4)/27 - 8\*x\*(3\*x\*\*2+2)\*\*(3/4)/135 + 32\*x/(135\*(3\*x\*\*2+2)\*\*(1/4)) - 32\*Integral((3\*x\*\*2+2)\*\*(-5/4), x)/135

**Mathematica [C]** time = 0.0377081, size = 49, normalized size = 0.6

$$\frac{2}{135}x\left(4\,{}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right) + (3x^2+2)^{3/4}(5x^2-4)\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(2+3\*x^2)^(1/4), x]

[Out] (2\*x\*((2+3\*x^2)^(3/4)\*(-4+5\*x^2) + 4\*2^(3/4)\*Hypergeometric2F1[1/4, 1/2, 3/2, (-3\*x^2)/2]))/135

**Maple [C]** time = 0.03, size = 38, normalized size = 0.5

$$\frac{2x(5x^2-4)}{135}(3x^2+2)^{\frac{3}{4}} + \frac{8 \cdot 2^{3/4}x}{135} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(3*x^2+2)^(1/4), x)`

[Out] `2/135*x*(5*x^2-4)*(3*x^2+2)^(3/4)+8/135*2^(3/4)*x*hypergeom([1/4, 1/2], [3/2], -3/2*x^2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(3x^2+2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(3*x^2+2)^(1/4), x, algorithm="maxima")`

[Out] `integrate(x^4/(3*x^2+2)^(1/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{(3x^2+2)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(3*x^2+2)^(1/4), x, algorithm="fricas")`

[Out] `integral(x^4/(3*x^2+2)^(1/4), x)`

**Sympy [A]** time = 2.32393, size = 27, normalized size = 0.33

$$\frac{2^{\frac{3}{4}}x^5 {}_2F_1\left(\frac{1}{4}, \frac{5}{2}; \frac{7}{2}; \frac{3x^2 e^{i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(3*x**2+2)**(1/4), x)`

[Out] `2**(3/4)*x**5*hyper((1/4, 5/2), (7/2, ), 3*x**2*exp_polar(I*pi)/2)/10`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(3x^2+2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(x^4/(3*x^2 + 2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(3*x^2 + 2)^(1/4), x)
```

$$3.865 \quad \int \frac{x^2}{\sqrt[4]{2+3x^2}} dx$$

**Optimal.** Leaf size=63

$$\frac{2}{15} (3x^2 + 2)^{3/4} x - \frac{8x}{15\sqrt[4]{3x^2 + 2}} + \frac{8\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}$$

[Out]  $(-8*x)/(15*(2+3*x^2)^{(1/4)}) + (2*x*(2+3*x^2)^{(3/4)})/15 + (8*2^{1/4}*EllipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/(15*Sqrt[3])$

**Rubi [A]** time = 0.0454174, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{2}{15} (3x^2 + 2)^{3/4} x - \frac{8x}{15\sqrt[4]{3x^2 + 2}} + \frac{8\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(2+3\*x^2)^(1/4), x]

[Out]  $(-8*x)/(15*(2+3*x^2)^{(1/4)}) + (2*x*(2+3*x^2)^{(3/4)})/15 + (8*2^{1/4}*EllipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/(15*Sqrt[3])$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2x(3x^2+2)^{3/4}}{15} - \frac{4\int\frac{1}{\sqrt[4]{3x^2+2}}dx}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(3\*x\*\*2+2)\*\*(1/4), x)

[Out]  $2*x*(3*x**2+2)**(3/4)/15 - 4*Integral((3*x**2+2)**(-1/4), x)/15$

**Mathematica [C]** time = 0.0245888, size = 41, normalized size = 0.65

$$\frac{2}{15}x\left((3x^2+2)^{3/4} - 2^{3/4}{}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(2+3\*x^2)^(1/4), x]

[Out]  $(2*x*((2+3*x^2)^{(3/4)} - 2^{(3/4)}*Hypergeometric2F1[1/4, 1/2, 3/2, (-3*x^2)/2]))/15$

**Maple [C]** time = 0.028, size = 31, normalized size = 0.5

$$\frac{2x}{15}(3x^2+2)^{3/4} - \frac{2^{3/4}x}{15}{}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(3*x^2+2)^(1/4),x)`

[Out]  $2/15*x*(3*x^2+2)^(3/4)-2/15*2^(3/4)*x*\text{hypergeom}([1/4, 1/2], [3/2], -3/2*x^2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(3x^2+2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(3*x^2+2)^(1/4),x, algorithm="maxima")`

[Out] `integrate(x^2/(3*x^2+2)^(1/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{(3x^2+2)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(3*x^2+2)^(1/4),x, algorithm="fricas")`

[Out] `integral(x^2/(3*x^2+2)^(1/4), x)`

**Sympy [A]** time = 2.1087, size = 27, normalized size = 0.43

$$\frac{2^{\frac{3}{4}}x^3 {}_2F_1\left(\frac{1}{4}, \frac{3}{2} \mid \frac{3x^2 e^{i\pi}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(3*x**2+2)**(1/4),x)`

[Out]  $2^{3/4}*x^3*\text{hyper}((1/4, 3/2), (5/2, ), 3*x^2*\text{exp\_polar}(I*\text{pi})/2)/6$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(3x^2+2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(3*x^2+2)^(1/4),x, algorithm="giac")`

[Out] `integrate(x^2/(3*x^2+2)^(1/4), x)`

$$3.866 \quad \int \frac{1}{\sqrt[4]{2+3x^2}} dx$$

**Optimal.** Leaf size=43

$$\frac{2x}{\sqrt[4]{3x^2+2}} - \frac{2\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

[Out] (2\*x)/(2 + 3\*x^2)^(1/4) - (2\*2^(1/4)\*EllipticE[ArcTan[Sqrt[3/2]\*x]/2, 2])/Sqrt[3]

**Rubi [A]** time = 0.0246636, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{2x}{\sqrt[4]{3x^2+2}} - \frac{2\sqrt[4]{2}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 3\*x^2)^(-1/4), x]

[Out] (2\*x)/(2 + 3\*x^2)^(1/4) - (2\*2^(1/4)\*EllipticE[ArcTan[Sqrt[3/2]\*x]/2, 2])/Sqrt[3]

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2x}{\sqrt[4]{3x^2+2}} - 2 \int \frac{1}{(3x^2+2)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(3\*x\*\*2+2)\*\*(1/4), x)

[Out] 2\*x/(3\*x\*\*2 + 2)\*\*(1/4) - 2\*Integral((3\*x\*\*2 + 2)\*\*(-5/4), x)

**Mathematica [C]** time = 0.0147944, size = 24, normalized size = 0.56

$$\frac{x {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right)}{\sqrt[4]{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3\*x^2)^(-1/4), x]

[Out] (x\*Hypergeometric2F1[1/4, 1/2, 3/2, (-3\*x^2)/2])/2^(1/4)

**Maple [C]** time = 0.013, size = 18, normalized size = 0.4

$$\frac{2^{\frac{3}{4}}x}{2} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*x^2+2)^(1/4),x)`

[Out] `1/2*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],-3/2*x^2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 + 2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2 + 2)^(-1/4),x, algorithm="maxima")`

[Out] `integrate((3*x^2 + 2)^(-1/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(3x^2 + 2)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2 + 2)^(-1/4),x, algorithm="fricas")`

[Out] `integral((3*x^2 + 2)^(-1/4), x)`

**Sympy [A]** time = 2.0383, size = 26, normalized size = 0.6

$$\frac{2^{\frac{3}{4}} x {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{3x^2 e^{i\pi}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3*x**2+2)**(1/4),x)`

[Out] `2**(3/4)*x*hyper((1/4, 1/2), (3/2, ), 3*x**2*exp_polar(I*pi)/2)/2`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 + 2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2 + 2)^(-1/4),x, algorithm="giac")`

[Out] `integrate((3*x^2 + 2)^(-1/4), x)`

$$3.867 \quad \int \frac{1}{x^2 \sqrt[4]{2+3x^2}} dx$$

**Optimal.** Leaf size=63

$$\frac{3x}{2\sqrt[4]{3x^2+2}} - \frac{(3x^2+2)^{3/4}}{2x} - \frac{\sqrt{3}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2^{3/4}}$$

[Out]  $(3*x)/(2*(2+3*x^2)^{(1/4)}) - (2+3*x^2)^{(3/4)}/(2*x) - (\text{Sqrt}[3]*\text{EllipticE}[\text{ArcTan}[\text{Sqrt}[3/2]*x]/2, 2])/2^{(3/4)}$

**Rubi [A]** time = 0.0445234, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{3x}{2\sqrt[4]{3x^2+2}} - \frac{(3x^2+2)^{3/4}}{2x} - \frac{\sqrt{3}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(2+3\*x^2)^(1/4)),x]

[Out]  $(3*x)/(2*(2+3*x^2)^{(1/4)}) - (2+3*x^2)^{(3/4)}/(2*x) - (\text{Sqrt}[3]*\text{EllipticE}[\text{ArcTan}[\text{Sqrt}[3/2]*x]/2, 2])/2^{(3/4)}$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{3 \int \frac{1}{\sqrt[4]{3x^2+2}} dx}{4} - \frac{(3x^2+2)^{3/4}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(3\*x\*\*2+2)\*\*(1/4),x)

[Out]  $3*\text{Integral}((3*x**2+2)**(-1/4),x)/4 - (3*x**2+2)**(3/4)/(2*x)$

**Mathematica [C]** time = 0.0319561, size = 46, normalized size = 0.73

$$\frac{3x {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right)}{4\sqrt[4]{2}} - \frac{(3x^2+2)^{3/4}}{2x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(2+3\*x^2)^(1/4)),x]

[Out]  $-(2+3*x^2)^{(3/4)}/(2*x) + (3*x*\text{Hypergeometric2F1}[1/4, 1/2, 3/2, (-3*x^2)/2])/ (4*2^{(1/4)})$

**Maple [C]** time = 0.032, size = 33, normalized size = 0.5

$$-\frac{1}{2x}(3x^2+2)^{3/4} + \frac{32^{3/4}x}{8} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(3*x^2+2)^(1/4),x)`

[Out]  $-1/2*(3*x^2+2)^(3/4)/x+3/8*2^(3/4)*x*\text{hypergeom}([1/4, 1/2], [3/2], -3/2*x^2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 + 2)^{\frac{1}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 + 2)^(1/4)*x^2),x, algorithm="maxima")`

[Out] `integrate(1/((3*x^2 + 2)^(1/4)*x^2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(3x^2 + 2)^{\frac{1}{4}} x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 + 2)^(1/4)*x^2),x, algorithm="fricas")`

[Out] `integral(1/((3*x^2 + 2)^(1/4)*x^2), x)`

**Sympy [A]** time = 2.32325, size = 29, normalized size = 0.46

$$-\frac{{}_2F_1\left(-\frac{1}{2}, \frac{1}{4} \middle| \frac{3x^2 e^{i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(3*x**2+2)**(1/4),x)`

[Out]  $-2**(3/4)*\text{hyper}((-1/2, 1/4), (1/2, ), 3*x**2*\text{exp\_polar}(I*\text{pi})/2)/(2*x)$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 + 2)^{\frac{1}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 + 2)^(1/4)*x^2),x, algorithm="giac")`

[Out] `integrate(1/((3*x^2 + 2)^(1/4)*x^2), x)`

$$3.868 \quad \int \frac{1}{x^4 \sqrt[4]{2+3x^2}} dx$$

**Optimal.** Leaf size=83

$$-\frac{9x}{8\sqrt[4]{3x^2+2}} + \frac{3(3x^2+2)^{3/4}}{8x} - \frac{(3x^2+2)^{3/4}}{6x^3} + \frac{3\sqrt{3}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\cdot 2^{3/4}}$$

[Out]  $(-9*x)/(8*(2+3*x^2)^{(1/4)}) - (2+3*x^2)^{(3/4)}/(6*x^3) + (3*(2+3*x^2)^{(3/4)})/(8*x) + (3*\text{Sqrt}[3]*\text{EllipticE}[\text{ArcTan}[\text{Sqrt}[3/2]*x]/2, 2])/(4*2^{(3/4)})$

**Rubi [A]** time = 0.0648532, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{9x}{8\sqrt[4]{3x^2+2}} + \frac{3(3x^2+2)^{3/4}}{8x} - \frac{(3x^2+2)^{3/4}}{6x^3} + \frac{3\sqrt{3}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\cdot 2^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(2+3\*x^2)^(1/4)),x]

[Out]  $(-9*x)/(8*(2+3*x^2)^{(1/4)}) - (2+3*x^2)^{(3/4)}/(6*x^3) + (3*(2+3*x^2)^{(3/4)})/(8*x) + (3*\text{Sqrt}[3]*\text{EllipticE}[\text{ArcTan}[\text{Sqrt}[3/2]*x]/2, 2])/(4*2^{(3/4)})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{9x}{8\sqrt[4]{3x^2+2}} + \frac{9\int \frac{1}{(3x^2+2)^{5/4}} dx}{8} + \frac{3(3x^2+2)^{3/4}}{8x} - \frac{(3x^2+2)^{3/4}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(3\*x\*\*2+2)\*\*(1/4),x)

[Out]  $-9*x/(8*(3*x**2+2)**(1/4)) + 9*\text{Integral}((3*x**2+2)**(-5/4),x)/8 + 3*(3*x**2+2)**(3/4)/(8*x) - (3*x**2+2)**(3/4)/(6*x**3)$

**Mathematica [C]** time = 0.0425078, size = 55, normalized size = 0.66

$$\left(\frac{3}{8x} - \frac{1}{6x^3}\right)(3x^2+2)^{3/4} - \frac{9x {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{3x^2}{2}\right)}{16\sqrt[4]{2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(2+3\*x^2)^(1/4)),x]

[Out]  $(-1/(6*x^3) + 3/(8*x))*(2+3*x^2)^{(3/4)} - (9*x*\text{Hypergeometric2F1}[1/4, 1/2, 3/2, (-3*x^2)/2])/(16*2^{(1/4)})$



**Maple [C]** time = 0.032, size = 45, normalized size = 0.5

$$\frac{27x^4 + 6x^2 - 8}{24x^3} \frac{1}{\sqrt[4]{3x^2 + 2}} - \frac{9 \cdot 2^{3/4} x}{32} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(3*x^2+2)^(1/4), x)`

[Out] `1/24*(27*x^4+6*x^2-8)/x^3/(3*x^2+2)^(1/4)-9/32*2^(3/4)*x*hypergeom([1/4, 1/2], [3/2], -3/2*x^2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 + 2)^{\frac{1}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 + 2)^(1/4)*x^4), x, algorithm="maxima")`

[Out] `integrate(1/((3*x^2 + 2)^(1/4)*x^4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(3x^2 + 2)^{\frac{1}{4}} x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 + 2)^(1/4)*x^4), x, algorithm="fricas")`

[Out] `integral(1/((3*x^2 + 2)^(1/4)*x^4), x)`

**Sympy [A]** time = 2.86666, size = 32, normalized size = 0.39

$$-\frac{2^{\frac{3}{4}} {}_2F_1\left(-\frac{3}{2}, \frac{1}{4} \middle| \frac{3x^2 e^{i\pi}}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(3*x**2+2)**(1/4), x)`

[Out] `-2**(3/4)*hyper((-3/2, 1/4), (-1/2,), 3*x**2*exp_polar(I*pi)/2)/(6*x**3)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 + 2)^{\frac{1}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 + 2)^(1/4)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 2)^(1/4)*x^4), x)
```

$$3.869 \quad \int \frac{1}{x^6 \sqrt[4]{2+3x^2}} dx$$

**Optimal.** Leaf size=101

$$\frac{189x}{160\sqrt[4]{3x^2+2}} - \frac{63(3x^2+2)^{3/4}}{160x} - \frac{(3x^2+2)^{3/4}}{10x^5} + \frac{7(3x^2+2)^{3/4}}{40x^3} - \frac{63\sqrt{3}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{80 \cdot 2^{3/4}}$$

[Out] (189\*x)/(160\*(2+3\*x^2)^(1/4)) - (2+3\*x^2)^(3/4)/(10\*x^5) + (7\*(2+3\*x^2)^(3/4))/(40\*x^3) - (63\*(2+3\*x^2)^(3/4))/(160\*x) - (63\*Sqrt[3]\*EllipticE[ArcTan[Sqrt[3/2]\*x]/2, 2])/(80\*2^(3/4))

**Rubi [A]** time = 0.0883239, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{189x}{160\sqrt[4]{3x^2+2}} - \frac{63(3x^2+2)^{3/4}}{160x} - \frac{(3x^2+2)^{3/4}}{10x^5} + \frac{7(3x^2+2)^{3/4}}{40x^3} - \frac{63\sqrt{3}E\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{80 \cdot 2^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(2+3\*x^2)^(1/4)), x]

[Out] (189\*x)/(160\*(2+3\*x^2)^(1/4)) - (2+3\*x^2)^(3/4)/(10\*x^5) + (7\*(2+3\*x^2)^(3/4))/(40\*x^3) - (63\*(2+3\*x^2)^(3/4))/(160\*x) - (63\*Sqrt[3]\*EllipticE[ArcTan[Sqrt[3/2]\*x]/2, 2])/(80\*2^(3/4))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{189 \int \frac{1}{\sqrt[4]{3x^2+2}} dx}{320} - \frac{63(3x^2+2)^{3/4}}{160x} + \frac{7(3x^2+2)^{3/4}}{40x^3} - \frac{(3x^2+2)^{3/4}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*6/(3\*x\*\*2+2)\*\*(1/4), x)

[Out] 189\*Integral((3\*x\*\*2+2)\*\*(-1/4), x)/320 - 63\*(3\*x\*\*2+2)\*\*(3/4)/(160\*x) + 7\*(3\*x\*\*2+2)\*\*(3/4)/(40\*x\*\*3) - (3\*x\*\*2+2)\*\*(3/4)/(10\*x\*\*5)

**Mathematica [C]** time = 0.0333601, size = 62, normalized size = 0.61

$$\frac{189x {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right)}{320\sqrt{2}} + (3x^2+2)^{3/4} \left(-\frac{1}{10x^5} + \frac{7}{40x^3} - \frac{63}{160x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6\*(2+3\*x^2)^(1/4)), x]

[Out] (-1/(10\*x^5) + 7/(40\*x^3) - 63/(160\*x))\*(2+3\*x^2)^(3/4) + (189\*x\*Hypergeometric2F1[1/4, 1/2, 3/2, (-3\*x^2)/2])/(320\*2^(1/4))

**Maple [C]** time = 0.033, size = 50, normalized size = 0.5

$$-\frac{189x^6 + 42x^4 - 8x^2 + 32}{160x^5} \frac{1}{\sqrt[4]{3x^2 + 2}} + \frac{189 \cdot 2^{3/4} x}{640} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^6/(3*x^2+2)^(1/4), x)`

[Out] `-1/160*(189*x^6+42*x^4-8*x^2+32)/x^5/(3*x^2+2)^(1/4)+189/640*2^(3/4)*x*hypergeom([1/4, 1/2], [3/2], -3/2*x^2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 + 2)^{\frac{1}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 + 2)^(1/4)*x^6), x, algorithm="maxima")`

[Out] `integrate(1/((3*x^2 + 2)^(1/4)*x^6), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(3x^2 + 2)^{\frac{1}{4}} x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 + 2)^(1/4)*x^6), x, algorithm="fricas")`

[Out] `integral(1/((3*x^2 + 2)^(1/4)*x^6), x)`

**Sympy [A]** time = 3.70262, size = 32, normalized size = 0.32

$$-\frac{2^{\frac{3}{4}} {}_2F_1\left(-\frac{5}{2}, \frac{1}{4} \middle| \frac{3x^2 e^{i\pi}}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**6/(3*x**2+2)**(1/4), x)`

[Out] `-2**(3/4)*hyper((-5/2, 1/4), (-3/2,), 3*x**2*exp_polar(I*pi)/2)/(10*x**5)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 + 2)^{\frac{1}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 + 2)^(1/4)*x^6),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 2)^(1/4)*x^6), x)
```

$$3.870 \quad \int \frac{x^6}{\sqrt[4]{2-3x^2}} dx$$

**Optimal.** Leaf size=83

$$-\frac{32(2-3x^2)^{3/4}x}{1053} - \frac{2}{39}(2-3x^2)^{3/4}x^5 - \frac{40(2-3x^2)^{3/4}x^3}{1053} + \frac{128\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\right)}{1053\sqrt{3}}$$

[Out]  $(-32*x*(2-3*x^2)^(3/4))/1053 - (40*x^3*(2-3*x^2)^(3/4))/1053 - (2*x^5*(2-3*x^2)^(3/4))/39 + (128*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(1053*Sqrt[3])$

**Rubi [A]** time = 0.0776138, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{32(2-3x^2)^{3/4}x}{1053} - \frac{2}{39}(2-3x^2)^{3/4}x^5 - \frac{40(2-3x^2)^{3/4}x^3}{1053} + \frac{128\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\right)}{1053\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(2-3\*x^2)^(1/4),x]

[Out]  $(-32*x*(2-3*x^2)^(3/4))/1053 - (40*x^3*(2-3*x^2)^(3/4))/1053 - (2*x^5*(2-3*x^2)^(3/4))/39 + (128*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(1053*Sqrt[3])$

**Rubi in Sympy [A]** time = 7.01607, size = 75, normalized size = 0.9

$$-\frac{2x^5(-3x^2+2)^{3/4}}{39} - \frac{40x^3(-3x^2+2)^{3/4}}{1053} - \frac{32x(-3x^2+2)^{3/4}}{1053} + \frac{128\sqrt[4]{2}\sqrt{3}E\left(\frac{\text{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2}\right)}{3159}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6/(-3\*x\*\*2+2)\*\*(1/4),x)

[Out]  $-2*x**5*(-3*x**2+2)**(3/4)/39 - 40*x**3*(-3*x**2+2)**(3/4)/1053 - 32*x*(-3*x**2+2)**(3/4)/1053 + 128*2**(1/4)*sqrt(3)*elliptic_e(\text{asin}(\text{sqrt}(6)*x/2)/2, 2)/3159$

**Mathematica [C]** time = 0.0609184, size = 55, normalized size = 0.66

$$\frac{2x\left(16\ 2^{3/4}\ {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right) - (2-3x^2)^{3/4}(27x^4+20x^2+16)\right)}{1053}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(2-3\*x^2)^(1/4),x]

[Out]  $(2*x*(-((2-3*x^2)^(3/4)*(16+20*x^2+27*x^4)) + 16*2^(3/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (3*x^2)/2]))/1053$

**Maple [C]** time = 0.056, size = 50, normalized size = 0.6

$$\frac{2x(27x^4 + 20x^2 + 16)(3x^2 - 2)}{1053} \frac{1}{\sqrt[4]{-3x^2 + 2}} + \frac{32 \cdot 2^{3/4} x}{1053} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(-3*x^2+2)^(1/4), x)`

[Out] `2/1053*x*(27*x^4+20*x^2+16)*(3*x^2-2)/(-3*x^2+2)^(1/4)+32/1053*2^(3/4)*x*hypergeom([1/4, 1/2], [3/2], 3/2*x^2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(-3x^2 + 2)^{1/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(-3*x^2 + 2)^(1/4), x, algorithm="maxima")`

[Out] `integrate(x^6/(-3*x^2 + 2)^(1/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{(-3x^2 + 2)^{1/4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(-3*x^2 + 2)^(1/4), x, algorithm="fricas")`

[Out] `integral(x^6/(-3*x^2 + 2)^(1/4), x)`

**Sympy [A]** time = 2.87001, size = 29, normalized size = 0.35

$$\frac{2^{3/4} x^7 {}_2F_1\left(\frac{1}{4}, \frac{7}{2} \middle| \frac{3x^2 e^{2i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(-3*x**2+2)**(1/4), x)`

[Out] `2**(3/4)*x**7*hyper((1/4, 7/2), (9/2, ), 3*x**2*exp_polar(2*I*pi)/2)/14`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(-3x^2 + 2)^{1/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2 + 2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(-3*x^2 + 2)^(1/4), x)
```



$$3.871 \quad \int \frac{x^4}{\sqrt[4]{2-3x^2}} dx$$

**Optimal.** Leaf size=65

$$-\frac{8}{135} (2-3x^2)^{3/4} x - \frac{2}{27} (2-3x^2)^{3/4} x^3 + \frac{32\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{135\sqrt{3}}$$

[Out]  $(-8*x*(2-3*x^2)^{(3/4)})/135 - (2*x^3*(2-3*x^2)^{(3/4)})/27 + (32*2^{1/4}*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(135*Sqrt[3])$

**Rubi [A]** time = 0.0565701, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{8}{135} (2-3x^2)^{3/4} x - \frac{2}{27} (2-3x^2)^{3/4} x^3 + \frac{32\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{135\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(2-3\*x^2)^(1/4), x]

[Out]  $(-8*x*(2-3*x^2)^{(3/4)})/135 - (2*x^3*(2-3*x^2)^{(3/4)})/27 + (32*2^{1/4}*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(135*Sqrt[3])$

**Rubi in Sympy [A]** time = 5.2437, size = 58, normalized size = 0.89

$$\frac{2x^3(-3x^2+2)^{3/4}}{27} - \frac{8x(-3x^2+2)^{3/4}}{135} + \frac{32\sqrt[4]{2}\sqrt{3}E\left(\frac{\text{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2}\middle|2\right)}{405}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(-3\*x\*\*2+2)\*\*(1/4), x)

[Out]  $-2*x**3*(-3*x**2+2)**(3/4)/27 - 8*x*(-3*x**2+2)**(3/4)/135 + 32*2**(1/4)*sqrt(3)*elliptic_e(asin(sqrt(6)*x/2)/2, 2)/405$

**Mathematica [C]** time = 0.0498281, size = 50, normalized size = 0.77

$$\frac{2}{135}x\left(4\sqrt[4]{2}{}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right) - (2-3x^2)^{3/4}(5x^2+4)\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(2-3\*x^2)^(1/4), x]

[Out]  $(2*x*(-((2-3*x^2)^{(3/4})*(4+5*x^2)) + 4*2^{3/4}*Hypergeometric2F1[1/4, 1/2, 3/2, (3*x^2)/2]))/135$

**Maple [C]** time = 0.039, size = 45, normalized size = 0.7

$$\frac{2x(5x^2+4)(3x^2-2)}{135} \frac{1}{\sqrt[4]{-3x^2+2}} + \frac{8\sqrt[4]{2}x}{135} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(-3*x^2+2)^(1/4),x)`

[Out] `2/135*x*(5*x^2+4)*(3*x^2-2)/(-3*x^2+2)^(1/4)+8/135*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],3/2*x^2)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(-3x^2 + 2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(-3*x^2 + 2)^(1/4),x, algorithm="maxima")`

[Out] `integrate(x^4/(-3*x^2 + 2)^(1/4), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{(-3x^2 + 2)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(-3*x^2 + 2)^(1/4),x, algorithm="fricas")`

[Out] `integral(x^4/(-3*x^2 + 2)^(1/4), x)`

**Sympy** [A] time = 2.39721, size = 29, normalized size = 0.45

$$\frac{2^{\frac{3}{4}}x^5 {}_2F_1\left(\frac{1}{4}, \frac{5}{2} \middle| \frac{3x^2 e^{2i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(-3*x**2+2)**(1/4),x)`

[Out] `2**(3/4)*x**5*hyper((1/4, 5/2), (7/2, ), 3*x**2*exp_polar(2*I*pi)/2)/10`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(-3x^2 + 2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(-3*x^2 + 2)^(1/4),x, algorithm="giac")`

[Out] `integrate(x^4/(-3*x^2 + 2)^(1/4), x)`

$$3.872 \quad \int \frac{x^2}{\sqrt[4]{2-3x^2}} dx$$

**Optimal.** Leaf size=47

$$\frac{8\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}} - \frac{2}{15}x(2-3x^2)^{3/4}$$

[Out]  $(-2*x*(2-3*x^2)^{(3/4)})/15 + (8*2^{(1/4)}*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(15*Sqrt[3])$

**Rubi [A]** time = 0.0355987, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{8\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}} - \frac{2}{15}x(2-3x^2)^{3/4}$$

Antiderivative was successfully verified.

[In] Int[x^2/(2-3\*x^2)^(1/4),x]

[Out]  $(-2*x*(2-3*x^2)^{(3/4)})/15 + (8*2^{(1/4)}*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(15*Sqrt[3])$

**Rubi in Sympy [A]** time = 3.64129, size = 41, normalized size = 0.87

$$-\frac{2x(-3x^2+2)^{3/4}}{15} + \frac{8\sqrt[4]{2}\sqrt{3}E\left(\frac{\text{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2}\middle|2\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(-3\*x\*\*2+2)\*\*(1/4),x)

[Out]  $-2*x*(-3*x**2+2)**(3/4)/15 + 8*2**(1/4)*sqrt(3)*elliptic_e(\text{asin}(\text{sqrt}(6)*x/2)/2, 2)/45$

**Mathematica [C]** time = 0.0243942, size = 41, normalized size = 0.87

$$-\frac{2}{15}x\left((2-3x^2)^{3/4} - 2^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(2-3\*x^2)^(1/4),x]

[Out]  $(-2*x*((2-3*x^2)^{(3/4)} - 2^{(3/4)}*Hypergeometric2F1[1/4, 1/2, 3/2, (3*x^2)/2]))/15$

**Maple [C]** time = 0.039, size = 38, normalized size = 0.8

$$\frac{2x(3x^2-2)}{15} \frac{1}{\sqrt[4]{-3x^2+2}} + \frac{2^{3/4}x}{15} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(-3*x^2+2)^(1/4),x)`

[Out]  $2/15*x*(3*x^2-2)/(-3*x^2+2)^{1/4}+2/15*2^{3/4}*x*\text{hypergeom}([1/4, 1/2], [3/2], 3/2*x^2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-3x^2+2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-3*x^2+2)^(1/4),x, algorithm="maxima")`

[Out] `integrate(x^2/(-3*x^2+2)^(1/4),x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{(-3x^2+2)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-3*x^2+2)^(1/4),x, algorithm="fricas")`

[Out] `integral(x^2/(-3*x^2+2)^(1/4),x)`

**Sympy [A]** time = 2.12904, size = 29, normalized size = 0.62

$$\frac{2^{\frac{3}{4}}x^3{}_2F_1\left(\frac{1}{4}, \frac{3}{2} \middle| \frac{3x^2e^{2i\pi}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(-3*x**2+2)**(1/4),x)`

[Out]  $2^{3/4}*x^3*\text{hyper}((1/4, 3/2), (5/2,)), 3*x**2*\text{exp\_polar}(2*I*pi)/2)/6$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-3x^2+2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-3*x^2+2)^(1/4),x, algorithm="giac")`

[Out] `integrate(x^2/(-3*x^2+2)^(1/4),x)`

$$3.873 \quad \int \frac{1}{\sqrt[4]{2-3x^2}} dx$$

**Optimal.** Leaf size=28

$$\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

[Out] (2\*2^(1/4)\*EllipticE[ArcSin[Sqrt[3/2]\*x]/2, 2])/Sqrt[3]

**Rubi [A]** time = 0.0148942, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2 - 3\*x^2)^(-1/4), x]

[Out] (2\*2^(1/4)\*EllipticE[ArcSin[Sqrt[3/2]\*x]/2, 2])/Sqrt[3]

**Rubi in Sympy [A]** time = 1.16436, size = 26, normalized size = 0.93

$$\frac{2\sqrt[4]{2}\sqrt{3}E\left(\frac{\text{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2}\middle|2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-3\*x\*\*2+2)\*\*(1/4), x)

[Out] 2\*2\*\*(1/4)\*sqrt(3)\*elliptic\_e(asin(sqrt(6)\*x/2)/2, 2)/3

**Mathematica [C]** time = 0.0145797, size = 24, normalized size = 0.86

$$\frac{x {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \frac{3x^2}{2}\right)}{\sqrt[4]{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - 3\*x^2)^(-1/4), x]

[Out] (x\*Hypergeometric2F1[1/4, 1/2, 3/2, (3\*x^2)/2])/2^(1/4)

**Maple [C]** time = 0.024, size = 18, normalized size = 0.6

$$\frac{2^{\frac{3}{4}}x}{2} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-3*x^2+2)^(1/4),x)`

[Out] `1/2*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],3/2*x^2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 + 2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3*x^2 + 2)^(-1/4),x, algorithm="maxima")`

[Out] `integrate((-3*x^2 + 2)^(-1/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-3x^2 + 2)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3*x^2 + 2)^(-1/4),x, algorithm="fricas")`

[Out] `integral((-3*x^2 + 2)^(-1/4), x)`

**Sympy [A]** time = 2.07144, size = 27, normalized size = 0.96

$$\frac{2^{\frac{3}{4}} x {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{3x^2 e^{2i\pi}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3*x**2+2)**(1/4),x)`

[Out] `2**(3/4)*x*hyper((1/4, 1/2), (3/2,), 3*x**2*exp_polar(2*I*pi)/2)/2`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 + 2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3*x^2 + 2)^(-1/4),x, algorithm="giac")`

[Out] `integrate((-3*x^2 + 2)^(-1/4), x)`

$$3.874 \quad \int \frac{1}{x^2 \sqrt[4]{2-3x^2}} dx$$

**Optimal.** Leaf size=47

$$-\frac{(2-3x^2)^{3/4}}{2x} - \frac{\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2^{3/4}}$$

[Out]  $-(2-3x^2)^{3/4}/(2x) - (\text{Sqrt}[3]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[3/2]*x]/2, 2])/2^{3/4}$

**Rubi [A]** time = 0.0337604, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{(2-3x^2)^{3/4}}{2x} - \frac{\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(2-3\*x^2)^(1/4)),x]

[Out]  $-(2-3x^2)^{3/4}/(2x) - (\text{Sqrt}[3]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[3/2]*x]/2, 2])/2^{3/4}$

**Rubi in Sympy [A]** time = 3.44161, size = 39, normalized size = 0.83

$$-\frac{\sqrt[4]{2}\sqrt{3}E\left(\frac{\text{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2}\middle|2\right)}{2} - \frac{(-3x^2+2)^{3/4}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(-3\*x\*\*2+2)\*\*(1/4),x)

[Out]  $-2^{1/4}*\text{sqrt}(3)*\text{elliptic}_e(\text{asin}(\text{sqrt}(6)*x/2)/2, 2)/2 - (-3*x**2+2)**(3/4)/(2*x)$

**Mathematica [C]** time = 0.0283924, size = 46, normalized size = 0.98

$$-\frac{3x {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right)}{4\sqrt[4]{2}} - \frac{(2-3x^2)^{3/4}}{2x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(2-3\*x^2)^(1/4)),x]

[Out]  $-(2-3x^2)^{3/4}/(2x) - (3*x*\text{Hypergeometric2F1}[1/4, 1/2, 3/2, (3*x^2)/2])/((4*2^{1/4}))$

**Maple [C]** time = 0.041, size = 40, normalized size = 0.9

$$\frac{3x^2-2}{2x} \frac{1}{\sqrt[4]{-3x^2+2}} - \frac{3 \cdot 2^{3/4} x}{8} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(-3*x^2+2)^(1/4),x)`

[Out]  $\frac{1}{2} \cdot (3x^2 - 2) / x / (-3x^2 + 2)^{1/4} - 3/8 \cdot 2^{3/4} \cdot x \cdot \text{hypergeom}([1/4, 1/2], [3/2], 3/2 \cdot x^2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 + 2)^{1/4} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-3*x^2 + 2)^(1/4)*x^2),x, algorithm="maxima")`

[Out] `integrate(1/((-3*x^2 + 2)^(1/4)*x^2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-3x^2 + 2)^{1/4} x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-3*x^2 + 2)^(1/4)*x^2),x, algorithm="fricas")`

[Out] `integral(1/((-3*x^2 + 2)^(1/4)*x^2), x)`

**Sympy [A]** time = 2.38581, size = 31, normalized size = 0.66

$$-\frac{{}_2F_1\left(-\frac{1}{2}, \frac{1}{4} \mid \frac{3x^2 e^{2i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(-3*x**2+2)**(1/4),x)`

[Out]  $-2^{3/4} \cdot \text{hyper}((-1/2, 1/4), (1/2, ), 3x^2 \cdot \exp(\text{polar}(2 \cdot I \cdot \pi)/2)) / (2x)$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 + 2)^{1/4} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-3*x^2 + 2)^(1/4)*x^2),x, algorithm="giac")`

[Out] `integrate(1/((-3*x^2 + 2)^(1/4)*x^2), x)`



$$3.875 \quad \int \frac{1}{x^4 \sqrt[4]{2-3x^2}} dx$$

**Optimal.** Leaf size=67

$$-\frac{3(2-3x^2)^{3/4}}{8x} - \frac{(2-3x^2)^{3/4}}{6x^3} - \frac{3\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4 \cdot 2^{3/4}}$$

[Out]  $-(2-3x^2)^{3/4}/(6x^3) - (3(2-3x^2)^{3/4})/(8x) - (3\sqrt{3}\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[3/2]*x]/2, 2])/(4 \cdot 2^{3/4})$

**Rubi [A]** time = 0.0535511, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{3(2-3x^2)^{3/4}}{8x} - \frac{(2-3x^2)^{3/4}}{6x^3} - \frac{3\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4 \cdot 2^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(2-3\*x^2)^(1/4)),x]

[Out]  $-(2-3x^2)^{3/4}/(6x^3) - (3(2-3x^2)^{3/4})/(8x) - (3\sqrt{3}\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[3/2]*x]/2, 2])/(4 \cdot 2^{3/4})$

**Rubi in Sympy [A]** time = 5.03987, size = 58, normalized size = 0.87

$$-\frac{3\sqrt[4]{2}\sqrt{3}E\left(\frac{\text{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2}\middle|2\right)}{8} - \frac{3(-3x^2+2)^{3/4}}{8x} - \frac{(-3x^2+2)^{3/4}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(-3\*x\*\*2+2)\*\*(1/4),x)

[Out]  $-3^{1/4}\sqrt[4]{2}\sqrt{3}\text{elliptic}_e(\text{asin}(\text{sqrt}(6)*x/2)/2, 2)/8 - 3*(-3x^2+2)^{3/4}/(8x) - (-3x^2+2)^{3/4}/(6x^3)$

**Mathematica [C]** time = 0.0413255, size = 55, normalized size = 0.82

$$\left(-\frac{1}{6x^3} - \frac{3}{8x}\right)(2-3x^2)^{3/4} - \frac{9x {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \frac{3x^2}{2}\right)}{16\sqrt[4]{2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(2-3\*x^2)^(1/4)),x]

[Out]  $(-1/(6x^3) - 3/(8x))*(2-3x^2)^{3/4} - (9x \text{Hypergeometric2F1}[1/4, 1/2, 3/2, (3x^2)/2])/(16 \cdot 2^{1/4})$

**Maple [C]** time = 0.042, size = 45, normalized size = 0.7

$$\frac{27x^4 - 6x^2 - 8}{24x^3} \frac{1}{\sqrt[4]{-3x^2+2}} - \frac{9 \cdot 2^{3/4}x}{32} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(-3*x^2+2)^(1/4),x)`

[Out]  $\frac{1}{24} \cdot (27x^4 - 6x^2 - 8) / x^3 / (-3x^2 + 2)^{1/4} - 9/32 \cdot 2^{3/4} \cdot x \cdot \text{hypergeom}([1/4, 1/2], [3/2], 3/2 \cdot x^2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 + 2)^{1/4} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-3*x^2 + 2)^(1/4)*x^4),x, algorithm="maxima")`

[Out] `integrate(1/((-3*x^2 + 2)^(1/4)*x^4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-3x^2 + 2)^{1/4} x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-3*x^2 + 2)^(1/4)*x^4),x, algorithm="fricas")`

[Out] `integral(1/((-3*x^2 + 2)^(1/4)*x^4), x)`

**Sympy [A]** time = 2.94196, size = 34, normalized size = 0.51

$$\frac{{}_2F_1\left(-\frac{3}{2}, \frac{1}{4} \mid \frac{3x^2 e^{2i\pi}}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(-3*x**2+2)**(1/4),x)`

[Out]  $-2^{3/4} \cdot \text{hyper}((-3/2, 1/4), (-1/2, ), 3x^2 \cdot \exp_{\text{polar}}(2 \cdot i \cdot \pi) / 2) / (6x^3)$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 + 2)^{1/4} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-3*x^2 + 2)^(1/4)*x^4),x, algorithm="giac")`

[Out] `integrate(1/((-3*x^2 + 2)^(1/4)*x^4), x)`

$$3.876 \quad \int \frac{1}{x^6 \sqrt[4]{2-3x^2}} dx$$

**Optimal.** Leaf size=85

$$-\frac{63(2-3x^2)^{3/4}}{160x} - \frac{(2-3x^2)^{3/4}}{10x^5} - \frac{7(2-3x^2)^{3/4}}{40x^3} - \frac{63\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{80 \cdot 2^{3/4}}$$

[Out]  $-(2-3x^2)^{3/4}/(10x^5) - (7(2-3x^2)^{3/4})/(40x^3) - (63(2-3x^2)^{3/4})/(160x) - (63\sqrt{3}E[\text{ArcSin}[\text{Sqrt}[3/2]*x]/2, 2])/(80 \cdot 2^{3/4})$

**Rubi [A]** time = 0.0747803, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{63(2-3x^2)^{3/4}}{160x} - \frac{(2-3x^2)^{3/4}}{10x^5} - \frac{7(2-3x^2)^{3/4}}{40x^3} - \frac{63\sqrt{3}E\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{80 \cdot 2^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(2-3\*x^2)^(1/4)),x]

[Out]  $-(2-3x^2)^{3/4}/(10x^5) - (7(2-3x^2)^{3/4})/(40x^3) - (63(2-3x^2)^{3/4})/(160x) - (63\sqrt{3}E[\text{ArcSin}[\text{Sqrt}[3/2]*x]/2, 2])/(80 \cdot 2^{3/4})$

**Rubi in Sympy [A]** time = 6.77728, size = 75, normalized size = 0.88

$$-\frac{63\sqrt[4]{2}\sqrt{3}E\left(\frac{\text{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2}\middle|2\right)}{160} - \frac{63(-3x^2+2)^{3/4}}{160x} - \frac{7(-3x^2+2)^{3/4}}{40x^3} - \frac{(-3x^2+2)^{3/4}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*6/(-3\*x\*\*2+2)\*\*(1/4),x)

[Out]  $-63 \cdot 2^{1/4} \cdot \text{sqrt}(3) \cdot \text{elliptic\_e}(\text{asin}(\text{sqrt}(6) \cdot x/2)/2, 2)/160 - 63 \cdot (-3x^2+2)^{3/4}/(160x) - 7 \cdot (-3x^2+2)^{3/4}/(40x^3) - (-3x^2+2)^{3/4}/(10x^5)$

**Mathematica [C]** time = 0.0473031, size = 62, normalized size = 0.73

$$\left(-\frac{1}{10x^5} - \frac{7}{40x^3} - \frac{63}{160x}\right)(2-3x^2)^{3/4} - \frac{189x {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, \frac{3x^2}{2}\right)}{320\sqrt[4]{2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6\*(2-3\*x^2)^(1/4)),x]

[Out]  $(-1/(10x^5) - 7/(40x^3) - 63/(160x)) \cdot (2-3x^2)^{3/4} - (189x \cdot \text{Hypergeometric2F1}[1/4, 1/2, 3/2, (3x^2)/2])/(320 \cdot 2^{1/4})$

**Maple [C]** time = 0.045, size = 50, normalized size = 0.6

$$\frac{189x^6 - 42x^4 - 8x^2 - 32}{160x^5} \frac{1}{\sqrt[4]{-3x^2 + 2}} - \frac{189 \cdot 2^{3/4} x}{640} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^6/(-3*x^2+2)^(1/4), x)`

[Out] `1/160*(189*x^6-42*x^4-8*x^2-32)/x^5/(-3*x^2+2)^(1/4)-189/640*2^(3/4)*x*hypergeom([1/4, 1/2], [3/2], 3/2*x^2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 + 2)^{\frac{1}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-3*x^2 + 2)^(1/4)*x^6), x, algorithm="maxima")`

[Out] `integrate(1/((-3*x^2 + 2)^(1/4)*x^6), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-3x^2 + 2)^{\frac{1}{4}} x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-3*x^2 + 2)^(1/4)*x^6), x, algorithm="fricas")`

[Out] `integral(1/((-3*x^2 + 2)^(1/4)*x^6), x)`

**Sympy [A]** time = 3.6624, size = 34, normalized size = 0.4

$$-\frac{2^{\frac{3}{4}} {}_2F_1\left(-\frac{5}{2}, \frac{1}{4} \middle| \frac{3x^2 e^{2i\pi}}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**6/(-3*x**2+2)**(1/4), x)`

[Out] `-2**(3/4)*hyper((-5/2, 1/4), (-3/2,), 3*x**2*exp_polar(2*I*pi)/2)/(10*x**5)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 + 2)^{\frac{1}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-3*x^2 + 2)^(1/4)*x^6),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(1/4)*x^6), x)
```

$$3.877 \quad \int \frac{x^6}{(2+3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=83

$$\frac{160\sqrt[4]{3x^2+2x}}{2079} + \frac{2}{33}\sqrt[4]{3x^2+2x^5} - \frac{40}{693}\sqrt[4]{3x^2+2x^3} - \frac{320 \cdot 2^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\sqrt{\frac{3}{2}x}\right) \middle| 2\right)}{2079\sqrt{3}}$$

[Out] (160\*x\*(2 + 3\*x^2)^(1/4))/2079 - (40\*x^3\*(2 + 3\*x^2)^(1/4))/693 + (2\*x^5\*(2 + 3\*x^2)^(1/4))/33 - (320\*2^(3/4)\*EllipticF[ArcTan[Sqrt[3/2]\*x]/2, 2])/(2079\*Sqrt[3])

**Rubi [A]** time = 0.0773639, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{160\sqrt[4]{3x^2+2x}}{2079} + \frac{2}{33}\sqrt[4]{3x^2+2x^5} - \frac{40}{693}\sqrt[4]{3x^2+2x^3} - \frac{320 \cdot 2^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\sqrt{\frac{3}{2}x}\right) \middle| 2\right)}{2079\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(2 + 3\*x^2)^(3/4), x]

[Out] (160\*x\*(2 + 3\*x^2)^(1/4))/2079 - (40\*x^3\*(2 + 3\*x^2)^(1/4))/693 + (2\*x^5\*(2 + 3\*x^2)^(1/4))/33 - (320\*2^(3/4)\*EllipticF[ArcTan[Sqrt[3/2]\*x]/2, 2])/(2079\*Sqrt[3])

**Rubi in Sympy [A]** time = 6.89445, size = 75, normalized size = 0.9

$$\frac{2x^5\sqrt[4]{3x^2+2}}{33} - \frac{40x^3\sqrt[4]{3x^2+2}}{693} + \frac{160x\sqrt[4]{3x^2+2}}{2079} - \frac{320 \cdot 2^{3/4} \sqrt{3} F\left(\frac{\text{atan}\left(\frac{\sqrt{6}x}{2}\right)}{2} \middle| 2\right)}{6237}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6/(3\*x\*\*2+2)\*\*(3/4), x)

[Out] 2\*x\*\*5\*(3\*x\*\*2 + 2)\*\*(1/4)/33 - 40\*x\*\*3\*(3\*x\*\*2 + 2)\*\*(1/4)/693 + 160\*x\*(3\*x\*\*2 + 2)\*\*(1/4)/2079 - 320\*2\*\*(3/4)\*sqrt(3)\*elliptic\_f(atan(sqrt(6)\*x/2)/2, 2)/6237

**Mathematica [C]** time = 0.0427894, size = 54, normalized size = 0.65

$$\frac{2x \left( \sqrt[4]{3x^2+2} (63x^4 - 60x^2 + 80) - 80\sqrt{2} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{3x^2}{2}\right) \right)}{2079}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(2 + 3\*x^2)^(3/4), x]

[Out] (2\*x\*((2 + 3\*x^2)^(1/4)\*(80 - 60\*x^2 + 63\*x^4) - 80\*2^(1/4)\*Hypergeometric2F1[1/2, 3/4, 3/2, (-3\*x^2)/2]))/2079

**Maple [C]** time = 0.046, size = 43, normalized size = 0.5

$$\frac{2x(63x^4 - 60x^2 + 80)}{2079} \sqrt[4]{3x^2 + 2} - \frac{160\sqrt[4]{2}x}{2079} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(3*x^2+2)^(3/4), x)`

[Out] `2/2079*x*(63*x^4-60*x^2+80)*(3*x^2+2)^(1/4)-160/2079*2^(1/4)*x*hypergeom([1/2, 3/4], [3/2], -3/2*x^2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(3x^2 + 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(3*x^2 + 2)^(3/4), x, algorithm="maxima")`

[Out] `integrate(x^6/(3*x^2 + 2)^(3/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{(3x^2 + 2)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(3*x^2 + 2)^(3/4), x, algorithm="fricas")`

[Out] `integral(x^6/(3*x^2 + 2)^(3/4), x)`

**Sympy [A]** time = 2.72742, size = 27, normalized size = 0.33

$$\frac{\sqrt[4]{2}x^7 {}_2F_1\left(\frac{3}{4}, \frac{7}{2}; \frac{9}{2}; \frac{3x^2 e^{i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(3*x**2+2)**(3/4), x)`

[Out] `2**(1/4)*x**7*hyper((3/4, 7/2), (9/2, ), 3*x**2*exp_polar(I*pi)/2)/14`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(3x^2 + 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2 + 2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(3*x^2 + 2)^(3/4), x)
```



$$3.878 \quad \int \frac{x^4}{(2+3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=65

$$-\frac{8}{63}\sqrt[4]{3x^2+2x} + \frac{2}{21}\sqrt[4]{3x^2+2x^3} + \frac{16 \cdot 2^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\sqrt{\frac{3}{2}}x\right) \middle| 2\right)}{63\sqrt{3}}$$

[Out]  $(-8*x*(2+3*x^2)^{(1/4)})/63 + (2*x^3*(2+3*x^2)^{(1/4)})/21 + (16*2^{(3/4)}*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(63*Sqrt[3])$

**Rubi [A]** time = 0.054103, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{8}{63}\sqrt[4]{3x^2+2x} + \frac{2}{21}\sqrt[4]{3x^2+2x^3} + \frac{16 \cdot 2^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\sqrt{\frac{3}{2}}x\right) \middle| 2\right)}{63\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(2+3\*x^2)^(3/4), x]

[Out]  $(-8*x*(2+3*x^2)^{(1/4)})/63 + (2*x^3*(2+3*x^2)^{(1/4)})/21 + (16*2^{(3/4)}*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(63*Sqrt[3])$

**Rubi in Sympy [A]** time = 5.15494, size = 58, normalized size = 0.89

$$\frac{2x^3\sqrt[4]{3x^2+2}}{21} - \frac{8x\sqrt[4]{3x^2+2}}{63} + \frac{16 \cdot 2^{3/4} \sqrt{3} F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{2} \middle| 2\right)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(3\*x\*\*2+2)\*\*(3/4), x)

[Out]  $2*x**3*(3*x**2+2)**(1/4)/21 - 8*x*(3*x**2+2)**(1/4)/63 + 16*2**(3/4)*sqrt(3)*elliptic_f(\operatorname{atan}(\operatorname{sqrt}(6)*x/2)/2, 2)/189$

**Mathematica [C]** time = 0.0360237, size = 49, normalized size = 0.75

$$\frac{2}{63}x \left( 4\sqrt[4]{2} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{3x^2}{2}\right) + \sqrt[4]{3x^2+2} (3x^2-4) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(2+3\*x^2)^(3/4), x]

[Out]  $(2*x*((-4+3*x^2)*(2+3*x^2)^{(1/4)} + 4*2^{(1/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, (-3*x^2)/2]))/63$

**Maple [C]** time = 0.03, size = 38, normalized size = 0.6

$$\frac{2x(3x^2-4)}{63}\sqrt[4]{3x^2+2} + \frac{8\sqrt[4]{2}x}{63} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(3*x^2+2)^(3/4),x)`

[Out] `2/63*x*(3*x^2-4)*(3*x^2+2)^(1/4)+8/63*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],-3/2*x^2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(3x^2 + 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(3*x^2 + 2)^(3/4),x, algorithm="maxima")`

[Out] `integrate(x^4/(3*x^2 + 2)^(3/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{(3x^2 + 2)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(3*x^2 + 2)^(3/4),x, algorithm="fricas")`

[Out] `integral(x^4/(3*x^2 + 2)^(3/4), x)`

**Sympy [A]** time = 2.3332, size = 27, normalized size = 0.42

$$\frac{\sqrt[4]{2}x^5 {}_2F_1\left(\frac{3}{4}, \frac{5}{2} \mid \frac{3x^2 e^{i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(3*x**2+2)**(3/4),x)`

[Out] `2**(1/4)*x**5*hyper((3/4, 5/2), (7/2, ), 3*x**2*exp_polar(I*pi)/2)/10`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(3x^2 + 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(3*x^2 + 2)^(3/4),x, algorithm="giac")`

[Out] `integrate(x^4/(3*x^2 + 2)^(3/4), x)`

$$3.879 \quad \int \frac{x^2}{(2+3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=47

$$\frac{2}{9}x\sqrt[4]{3x^2+2} - \frac{4 \cdot 2^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\sqrt{\frac{3}{2}}x\right) \middle| 2\right)}{9\sqrt{3}}$$

[Out] (2\*x\*(2 + 3\*x^2)^(1/4))/9 - (4\*2^(3/4)\*EllipticF[ArcTan[Sqrt[3/2]\*x]/2, 2])/(9\*Sqrt[3])

**Rubi [A]** time = 0.0334056, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{2}{9}x\sqrt[4]{3x^2+2} - \frac{4 \cdot 2^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\sqrt{\frac{3}{2}}x\right) \middle| 2\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(2 + 3\*x^2)^(3/4), x]

[Out] (2\*x\*(2 + 3\*x^2)^(1/4))/9 - (4\*2^(3/4)\*EllipticF[ArcTan[Sqrt[3/2]\*x]/2, 2])/(9\*Sqrt[3])

**Rubi in Sympy [A]** time = 3.58923, size = 41, normalized size = 0.87

$$\frac{2x\sqrt[4]{3x^2+2}}{9} - \frac{4 \cdot 2^{3/4} \sqrt{3} F\left(\frac{\text{atan}\left(\frac{\sqrt{6}x}{2}\right)}{2} \middle| 2\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(3\*x\*\*2+2)\*\*(3/4), x)

[Out] 2\*x\*(3\*x\*\*2 + 2)\*\*(1/4)/9 - 4\*2\*\*(3/4)\*sqrt(3)\*elliptic\_f(atan(sqrt(6)\*x/2)/2, 2)/27

**Mathematica [C]** time = 0.0207826, size = 41, normalized size = 0.87

$$\frac{2}{9}x \left( \sqrt[4]{3x^2+2} - \sqrt[4]{2} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{3x^2}{2}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(2 + 3\*x^2)^(3/4), x]

[Out] (2\*x\*((2 + 3\*x^2)^(1/4) - 2^(1/4)\*Hypergeometric2F1[1/2, 3/4, 3/2, (-3\*x^2)/2]))/9

**Maple [C]** time = 0.029, size = 31, normalized size = 0.7

$$\frac{2x}{9}\sqrt[4]{3x^2+2} - \frac{2\sqrt[4]{2}x}{9} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(3*x^2+2)^(3/4),x)`

[Out]  $2/9*x*(3*x^2+2)^{1/4}-2/9*2^{1/4}*x*\text{hypergeom}([1/2,3/4],[3/2],-3/2*x^2)$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(3x^2+2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(3*x^2+2)^(3/4),x,algorithm="maxima")`

[Out] `integrate(x^2/(3*x^2+2)^(3/4),x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{(3x^2+2)^{\frac{3}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(3*x^2+2)^(3/4),x,algorithm="fricas")`

[Out] `integral(x^2/(3*x^2+2)^(3/4),x)`

**Sympy** [A] time = 2.12111, size = 27, normalized size = 0.57

$$\frac{\sqrt[4]{2}x^3 {}_2F_1\left(\frac{3}{4}, \frac{3}{2} \mid \frac{3x^2 e^{i\pi}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(3*x**2+2)**(3/4),x)`

[Out]  $2^{**}(1/4)*x^{**3}\text{hyper}((3/4,3/2),(5/2,)),3*x^{**2}\text{exp\_polar}(I*\text{pi})/2)/6$

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(3x^2+2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(3*x^2+2)^(3/4),x,algorithm="giac")`

[Out] `integrate(x^2/(3*x^2+2)^(3/4),x)`

$$3.880 \quad \int \frac{1}{(2+3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=27

$$\frac{2^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\sqrt{\frac{3}{2}}x\right) \middle| 2\right)}{\sqrt{3}}$$

[Out]  $(2^{(3/4)} * \text{EllipticF}[\text{ArcTan}[\text{Sqrt}[3/2]*x]/2, 2]) / \text{Sqrt}[3]$

**Rubi [A]** time = 0.0138725, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{2^{3/4} F\left(\frac{1}{2} \tan^{-1}\left(\sqrt{\frac{3}{2}}x\right) \middle| 2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(2 + 3*x^2)^{-3/4}, x]$

[Out]  $(2^{(3/4)} * \text{EllipticF}[\text{ArcTan}[\text{Sqrt}[3/2]*x]/2, 2]) / \text{Sqrt}[3]$

**Rubi in Sympy [A]** time = 1.07586, size = 24, normalized size = 0.89

$$\frac{2^{3/4} \sqrt{3} F\left(\frac{\text{atan}\left(\frac{\sqrt{6}x}{2}\right)}{2} \middle| 2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/(3*x**2+2)**(3/4), x)$

[Out]  $2^{(3/4)} * \text{sqrt}(3) * \text{elliptic\_f}(\text{atan}(\text{sqrt}(6)*x/2)/2, 2)/3$

**Mathematica [C]** time = 0.0123072, size = 24, normalized size = 0.89

$$\frac{x {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{3x^2}{2}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(2 + 3*x^2)^{-3/4}, x]$

[Out]  $(x * \text{Hypergeometric2F1}[1/2, 3/4, 3/2, (-3*x^2)/2]) / 2^{(3/4)}$

**Maple [C]** time = 0.014, size = 18, normalized size = 0.7

$$\frac{\sqrt[4]{2}x}{2} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*x^2+2)^(3/4),x)`

[Out] `1/2*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],-3/2*x^2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 + 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2 + 2)^(-3/4),x, algorithm="maxima")`

[Out] `integrate((3*x^2 + 2)^(-3/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(3x^2 + 2)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2 + 2)^(-3/4),x, algorithm="fricas")`

[Out] `integral((3*x^2 + 2)^(-3/4), x)`

**Sympy [A]** time = 2.00819, size = 26, normalized size = 0.96

$$\frac{\sqrt[4]{2}x {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{3x^2 e^{i\pi}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3*x**2+2)**(3/4),x)`

[Out] `2**(1/4)*x*hyper((1/2, 3/4), (3/2, ), 3*x**2*exp_polar(I*pi)/2)/2`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 + 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2 + 2)^(-3/4),x, algorithm="giac")`

[Out] `integrate((3*x^2 + 2)^(-3/4), x)`

$$3.881 \quad \int \frac{1}{x^2(2+3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=49

$$-\frac{\sqrt[4]{3x^2+2}}{2x} - \frac{\sqrt{3}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2\sqrt[4]{2}}$$

[Out]  $-(2 + 3*x^2)^{(1/4)}/(2*x) - (\text{Sqrt}[3]*\text{EllipticF}[\text{ArcTan}[\text{Sqrt}[3/2]*x]/2, 2])/(2*2^{(1/4)})$

**Rubi [A]** time = 0.0328475, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{\sqrt[4]{3x^2+2}}{2x} - \frac{\sqrt{3}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2\sqrt[4]{2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(2 + 3\*x^2)^(3/4)), x]

[Out]  $-(2 + 3*x^2)^{(1/4)}/(2*x) - (\text{Sqrt}[3]*\text{EllipticF}[\text{ArcTan}[\text{Sqrt}[3/2]*x]/2, 2])/(2*2^{(1/4)})$

**Rubi in Sympy [A]** time = 3.37698, size = 39, normalized size = 0.8

$$-\frac{2^{3/4}\sqrt{3}F\left(\frac{\text{atan}\left(\frac{\sqrt{6}x}{2}\right)}{2}\middle|2\right)}{4} - \frac{\sqrt[4]{3x^2+2}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(3\*x\*\*2+2)\*\*(3/4), x)

[Out]  $-2^{(3/4)}*\text{sqrt}(3)*\text{elliptic\_f}(\text{atan}(\text{sqrt}(6)*x/2)/2, 2)/4 - (3*x**2 + 2)^{(1/4)}/(2*x)$

**Mathematica [C]** time = 0.0255852, size = 46, normalized size = 0.94

$$-\frac{3x {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{3x^2}{2}\right)}{4 \cdot 2^{3/4}} - \frac{\sqrt[4]{3x^2+2}}{2x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(2 + 3\*x^2)^(3/4)), x]

[Out]  $-(2 + 3*x^2)^{(1/4)}/(2*x) - (3*x*\text{Hypergeometric2F1}[1/2, 3/4, 3/2, (-3*x^2)/2])/(4*2^{(3/4)})$

**Maple [C]** time = 0.033, size = 33, normalized size = 0.7

$$-\frac{1}{2x}\sqrt[4]{3x^2+2} - \frac{3\sqrt[4]{2}x}{8} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(3*x^2+2)^(3/4),x)`

[Out]  $-1/2*(3*x^2+2)^(1/4)/x-3/8*2^(1/4)*x*\text{hypergeom}([1/2, 3/4], [3/2], -3/2*x^2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 + 2)^{\frac{3}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 + 2)^(3/4)*x^2),x, algorithm="maxima")`

[Out] `integrate(1/((3*x^2 + 2)^(3/4)*x^2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(3x^2 + 2)^{\frac{3}{4}} x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 + 2)^(3/4)*x^2),x, algorithm="fricas")`

[Out] `integral(1/((3*x^2 + 2)^(3/4)*x^2), x)`

**Sympy [A]** time = 2.75271, size = 29, normalized size = 0.59

$$-\frac{\sqrt[4]{2} {}_2F_1\left(-\frac{1}{2}, \frac{3}{4} \mid \frac{3x^2 e^{i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(3*x**2+2)**(3/4),x)`

[Out]  $-2**(1/4)*\text{hyper}((-1/2, 3/4), (1/2, ), 3*x**2*\text{exp\_polar}(I*\text{pi})/2)/(2*x)$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 + 2)^{\frac{3}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 + 2)^(3/4)*x^2),x, algorithm="giac")`

[Out] `integrate(1/((3*x^2 + 2)^(3/4)*x^2), x)`



$$3.882 \quad \int \frac{1}{x^4(2+3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=67

$$\frac{5\sqrt[4]{3x^2+2}}{8x} - \frac{\sqrt[4]{3x^2+2}}{6x^3} + \frac{5\sqrt{3}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{8\sqrt[4]{2}}$$

[Out]  $-(2 + 3*x^2)^{(1/4)}/(6*x^3) + (5*(2 + 3*x^2)^{(1/4)})/(8*x) + (5*\text{Sqrt}[3]*\text{EllipticF}[\text{ArcTan}[\text{Sqrt}[3/2]*x]/2, 2])/(8*2^{(1/4)})$

**Rubi [A]** time = 0.0528519, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{5\sqrt[4]{3x^2+2}}{8x} - \frac{\sqrt[4]{3x^2+2}}{6x^3} + \frac{5\sqrt{3}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{8\sqrt[4]{2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^4*(2 + 3*x^2)^{(3/4)}), x]$

[Out]  $-(2 + 3*x^2)^{(1/4)}/(6*x^3) + (5*(2 + 3*x^2)^{(1/4)})/(8*x) + (5*\text{Sqrt}[3]*\text{EllipticF}[\text{ArcTan}[\text{Sqrt}[3/2]*x]/2, 2])/(8*2^{(1/4)})$

**Rubi in Sympy [A]** time = 5.35768, size = 56, normalized size = 0.84

$$\frac{5 \cdot 2^{\frac{3}{4}} \sqrt{3} F\left(\frac{\text{atan}\left(\frac{\sqrt{6}x}{2}\right)}{2}\middle|2\right)}{16} + \frac{5\sqrt[4]{3x^2+2}}{8x} - \frac{\sqrt[4]{3x^2+2}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{**4}/(3*x^{**2}+2)^{(3/4)}, x)$

[Out]  $5*2^{** (3/4)}*\text{sqrt}(3)*\text{elliptic\_f}(\text{atan}(\text{sqrt}(6)*x/2)/2, 2)/16 + 5*(3*x^{**2} + 2)^{(1/4)}/(8*x) - (3*x^{**2} + 2)^{(1/4)}/(6*x^{**3})$

**Mathematica [C]** time = 0.0258037, size = 55, normalized size = 0.82

$$\frac{15x {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{3x^2}{2}\right)}{16 \cdot 2^{3/4}} + \sqrt[4]{3x^2+2} \left(\frac{5}{8x} - \frac{1}{6x^3}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/(x^4*(2 + 3*x^2)^{(3/4)}), x]$

[Out]  $(-1/(6*x^3) + 5/(8*x))*(2 + 3*x^2)^{(1/4)} + (15*x*\text{Hypergeometric2F1}[1/2, 3/4, 3/2, (-3*x^2)/2])/(16*2^{(3/4)})$

**Maple [C]** time = 0.033, size = 45, normalized size = 0.7

$$\frac{45x^4 + 18x^2 - 8}{24x^3} (3x^2 + 2)^{-\frac{3}{4}} + \frac{15\sqrt[4]{2}x}{32} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(3*x^2+2)^(3/4),x)`

[Out]  $\frac{1}{24} \cdot (45x^4 + 18x^2 - 8) / x^3 / (3x^2 + 2)^{3/4} + 15/32 \cdot 2^{1/4} \cdot x \cdot \text{hypergeom}([1/2, 3/4], [3/2], -3/2 \cdot x^2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 + 2)^{3/4} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 + 2)^(3/4)*x^4),x, algorithm="maxima")`

[Out] `integrate(1/((3*x^2 + 2)^(3/4)*x^4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(3x^2 + 2)^{3/4} x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 + 2)^(3/4)*x^4),x, algorithm="fricas")`

[Out] `integral(1/((3*x^2 + 2)^(3/4)*x^4), x)`

**Sympy [A]** time = 3.36039, size = 32, normalized size = 0.48

$$-\frac{\sqrt[4]{2} F_1\left(-\frac{3}{2}, \frac{3}{4} \middle| \frac{3x^2 e^{i\pi}}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(3*x**2+2)**(3/4),x)`

[Out]  $-2^{1/4} \cdot \text{hyper}((-3/2, 3/4), (-1/2, ), 3x^2 \exp(\pi i) / 2) / (6x^3)$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 + 2)^{3/4} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 + 2)^(3/4)*x^4),x, algorithm="giac")`

[Out] `integrate(1/((3*x^2 + 2)^(3/4)*x^4), x)`

$$3.883 \quad \int \frac{1}{x^6(2+3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=85

$$-\frac{27\sqrt[4]{3x^2+2}}{32x} - \frac{\sqrt[4]{3x^2+2}}{10x^5} + \frac{9\sqrt[4]{3x^2+2}}{40x^3} - \frac{27\sqrt{3}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{32\sqrt[4]{2}}$$

[Out]  $-(2 + 3*x^2)^{(1/4)}/(10*x^5) + (9*(2 + 3*x^2)^{(1/4)})/(40*x^3) - (27*(2 + 3*x^2)^{(1/4)})/(32*x) - (27*\text{Sqrt}[3]*\text{EllipticF}[\text{ArcTan}[\text{Sqrt}[3/2]*x]/2, 2])/(32*2^{(1/4)})$

**Rubi [A]** time = 0.0730115, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{27\sqrt[4]{3x^2+2}}{32x} - \frac{\sqrt[4]{3x^2+2}}{10x^5} + \frac{9\sqrt[4]{3x^2+2}}{40x^3} - \frac{27\sqrt{3}F\left(\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{32\sqrt[4]{2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(2 + 3\*x^2)^(3/4)), x]

[Out]  $-(2 + 3*x^2)^{(1/4)}/(10*x^5) + (9*(2 + 3*x^2)^{(1/4)})/(40*x^3) - (27*(2 + 3*x^2)^{(1/4)})/(32*x) - (27*\text{Sqrt}[3]*\text{EllipticF}[\text{ArcTan}[\text{Sqrt}[3/2]*x]/2, 2])/(32*2^{(1/4)})$

**Rubi in Sympy [A]** time = 6.67816, size = 73, normalized size = 0.86

$$-\frac{27 \cdot 2^{\frac{3}{4}} \sqrt{3} F\left(\frac{\text{atan}\left(\frac{\sqrt{6}x}{2}\right)}{2}\middle|2\right)}{64} - \frac{27\sqrt[4]{3x^2+2}}{32x} + \frac{9\sqrt[4]{3x^2+2}}{40x^3} - \frac{\sqrt[4]{3x^2+2}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*6/(3\*x\*\*2+2)\*\*(3/4), x)

[Out]  $-27*2^{(3/4)}*\text{sqrt}(3)*\text{elliptic\_f}(\text{atan}(\text{sqrt}(6)*x/2)/2, 2)/64 - 27*(3*x**2 + 2)**(1/4)/(32*x) + 9*(3*x**2 + 2)**(1/4)/(40*x**3) - (3*x**2 + 2)**(1/4)/(10*x**5)$

**Mathematica [C]** time = 0.0490028, size = 58, normalized size = 0.68

$$-\frac{81x {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{3x^2}{2}\right)}{64 \cdot 2^{3/4}} - \frac{\sqrt[4]{3x^2+2} (135x^4 - 36x^2 + 16)}{160x^5}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6\*(2 + 3\*x^2)^(3/4)), x]

[Out]  $-((2 + 3*x^2)^{(1/4)}*(16 - 36*x^2 + 135*x^4))/(160*x^5) - (81*x*\text{Hypergeometric2F1}[1/2, 3/4, 3/2, (-3*x^2)/2])/(64*2^{(3/4)})$

**Maple [C]** time = 0.034, size = 50, normalized size = 0.6

$$-\frac{405x^6 + 162x^4 - 24x^2 + 32}{160x^5} (3x^2 + 2)^{-\frac{3}{4}} - \frac{81\sqrt[4]{2}x}{128} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^6/(3*x^2+2)^(3/4), x)`

[Out] `-1/160*(405*x^6+162*x^4-24*x^2+32)/x^5/(3*x^2+2)^(3/4)-81/128*2^(1/4)*x*hypergeom([1/2, 3/4], [3/2], -3/2*x^2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 + 2)^{\frac{3}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 + 2)^(3/4)*x^6), x, algorithm="maxima")`

[Out] `integrate(1/((3*x^2 + 2)^(3/4)*x^6), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(3x^2 + 2)^{\frac{3}{4}} x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 + 2)^(3/4)*x^6), x, algorithm="fricas")`

[Out] `integral(1/((3*x^2 + 2)^(3/4)*x^6), x)`

**Sympy [A]** time = 4.30507, size = 32, normalized size = 0.38

$$-\frac{\sqrt[4]{2} {}_2F_1\left(-\frac{5}{2}, \frac{3}{4}; -\frac{3}{2}; \frac{3x^2 e^{i\pi}}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**6/(3*x**2+2)**(3/4), x)`

[Out] `-2**(1/4)*hyper((-5/2, 3/4), (-3/2,), 3*x**2*exp_polar(I*pi)/2)/(10*x**5)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 + 2)^{\frac{3}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 + 2)^(3/4)*x^6),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 2)^(3/4)*x^6), x)
```

$$3.884 \quad \int \frac{x^6}{(2-3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=83

$$-\frac{160\sqrt[4]{2-3x^2}x}{2079} - \frac{2}{33}\sqrt[4]{2-3x^2}x^5 - \frac{40}{693}\sqrt[4]{2-3x^2}x^3 + \frac{320 \cdot 2^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle| 2\right)}{2079\sqrt{3}}$$

[Out]  $(-160*x*(2 - 3*x^2)^{(1/4)})/2079 - (40*x^3*(2 - 3*x^2)^{(1/4)})/693 - (2*x^5*(2 - 3*x^2)^{(1/4)})/33 + (320*2^{(3/4)}*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(2079*Sqrt[3])$

**Rubi [A]** time = 0.0769585, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{160\sqrt[4]{2-3x^2}x}{2079} - \frac{2}{33}\sqrt[4]{2-3x^2}x^5 - \frac{40}{693}\sqrt[4]{2-3x^2}x^3 + \frac{320 \cdot 2^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle| 2\right)}{2079\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(2 - 3\*x^2)^(3/4), x]

[Out]  $(-160*x*(2 - 3*x^2)^{(1/4)})/2079 - (40*x^3*(2 - 3*x^2)^{(1/4)})/693 - (2*x^5*(2 - 3*x^2)^{(1/4)})/33 + (320*2^{(3/4)}*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(2079*Sqrt[3])$

**Rubi in Sympy [A]** time = 7.00676, size = 75, normalized size = 0.9

$$-\frac{2x^5\sqrt[4]{-3x^2+2}}{33} - \frac{40x^3\sqrt[4]{-3x^2+2}}{693} - \frac{160x\sqrt[4]{-3x^2+2}}{2079} + \frac{320 \cdot 2^{3/4} \sqrt{3} F\left(\frac{\text{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2}\middle| 2\right)}{6237}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6/(-3\*x\*\*2+2)\*\*(3/4), x)

[Out]  $-2*x**5*(-3*x**2 + 2)**(1/4)/33 - 40*x**3*(-3*x**2 + 2)**(1/4)/693 - 160*x*(-3*x**2 + 2)**(1/4)/2079 + 320*2**(3/4)*sqrt(3)*elliptic_f(asin(sqrt(6)*x/2)/2, 2)/6237$

**Mathematica [C]** time = 0.0529975, size = 55, normalized size = 0.66

$$\frac{2x \left( 80\sqrt{2} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{3x^2}{2}\right) - \sqrt[4]{2-3x^2} (63x^4 + 60x^2 + 80) \right)}{2079}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(2 - 3\*x^2)^(3/4), x]

[Out]  $(2*x*(-((2 - 3*x^2)^{(1/4)}*(80 + 60*x^2 + 63*x^4)) + 80*2^{(1/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, (3*x^2)/2]))/2079$

**Maple [F]** time = 0.033, size = 0, normalized size = 0.

$$\int x^6 (-3x^2 + 2)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(-3*x^2+2)^(3/4),x)`

[Out] `int(x^6/(-3*x^2+2)^(3/4),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(-3x^2 + 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(-3*x^2 + 2)^(3/4),x, algorithm="maxima")`

[Out] `integrate(x^6/(-3*x^2 + 2)^(3/4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{(-3x^2 + 2)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(-3*x^2 + 2)^(3/4),x, algorithm="fricas")`

[Out] `integral(x^6/(-3*x^2 + 2)^(3/4), x)`

---

**Sympy [A]** time = 2.8204, size = 29, normalized size = 0.35

$$\frac{\sqrt[4]{2}x^7 {}_2F_1\left(\frac{3}{4}, \frac{7}{2} \middle| \frac{3x^2 e^{2i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(-3*x**2+2)**(3/4),x)`

[Out] `2**(1/4)*x**7*hyper((3/4, 7/2), (9/2, ), 3*x**2*exp_polar(2*I*pi)/2)/14`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(-3x^2 + 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-3*x^2 + 2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^6/(-3*x^2 + 2)^(3/4), x)
```



$$3.885 \quad \int \frac{x^4}{(2-3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=65

$$-\frac{8}{63}\sqrt[4]{2-3x^2}x - \frac{2}{21}\sqrt[4]{2-3x^2}x^3 + \frac{16 \cdot 2^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle| 2\right)}{63\sqrt{3}}$$

[Out]  $(-8*x*(2-3*x^2)^{(1/4)})/63 - (2*x^3*(2-3*x^2)^{(1/4)})/21 + (16*2^{(3/4)}*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(63*Sqrt[3])$

**Rubi [A]** time = 0.0565224, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{8}{63}\sqrt[4]{2-3x^2}x - \frac{2}{21}\sqrt[4]{2-3x^2}x^3 + \frac{16 \cdot 2^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle| 2\right)}{63\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(2-3\*x^2)^(3/4), x]

[Out]  $(-8*x*(2-3*x^2)^{(1/4)})/63 - (2*x^3*(2-3*x^2)^{(1/4)})/21 + (16*2^{(3/4)}*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(63*Sqrt[3])$

**Rubi in Sympy [A]** time = 5.26723, size = 58, normalized size = 0.89

$$-\frac{2x^3\sqrt[4]{-3x^2+2}}{21} - \frac{8x\sqrt[4]{-3x^2+2}}{63} + \frac{16 \cdot 2^{3/4}\sqrt{3}F\left(\frac{\text{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2}\middle| 2\right)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(-3\*x\*\*2+2)\*\*(3/4), x)

[Out]  $-2*x**3*(-3*x**2+2)**(1/4)/21 - 8*x*(-3*x**2+2)**(1/4)/63 + 16*2**(3/4)*sqrt(3)*elliptic_f(asin(sqrt(6)*x/2)/2, 2)/189$

**Mathematica [C]** time = 0.0402868, size = 50, normalized size = 0.77

$$\frac{2}{63}x \left( 4\sqrt[4]{2} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{3x^2}{2}\right) - \sqrt[4]{2-3x^2} (3x^2+4) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(2-3\*x^2)^(3/4), x]

[Out]  $(2*x*(-((2-3*x^2)^{(1/4)}*(4+3*x^2)) + 4*2^{(1/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, (3*x^2)/2]))/63$

**Maple [F]** time = 0.022, size = 0, normalized size = 0.

$$\int x^4 (-3x^2 + 2)^{-3/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(-3*x^2+2)^(3/4),x)`

[Out] `int(x^4/(-3*x^2+2)^(3/4),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(-3x^2 + 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(-3*x^2 + 2)^(3/4),x, algorithm="maxima")`

[Out] `integrate(x^4/(-3*x^2 + 2)^(3/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{(-3x^2 + 2)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(-3*x^2 + 2)^(3/4),x, algorithm="fricas")`

[Out] `integral(x^4/(-3*x^2 + 2)^(3/4), x)`

**Sympy [A]** time = 2.35074, size = 29, normalized size = 0.45

$$\frac{\sqrt[4]{2}x^5 {}_2F_1\left(\frac{3}{4}, \frac{5}{2} \middle| \frac{3x^2 e^{2i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(-3*x**2+2)**(3/4),x)`

[Out] `2**(1/4)*x**5*hyper((3/4, 5/2), (7/2, ), 3*x**2*exp_polar(2*I*pi)/2)/10`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(-3x^2 + 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(-3*x^2 + 2)^(3/4),x, algorithm="giac")`

[Out] `integrate(x^4/(-3*x^2 + 2)^(3/4), x)`

$$3.886 \quad \int \frac{x^2}{(2-3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=47

$$\frac{4 \cdot 2^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\sqrt{\frac{3}{2}}x\right) \middle| 2\right)}{9\sqrt{3}} - \frac{2}{9}x\sqrt{2-3x^2}$$

[Out]  $(-2*x*(2 - 3*x^2)^{(1/4)})/9 + (4*2^{(3/4)}*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(9*Sqrt[3])$

**Rubi [A]** time = 0.0350858, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{4 \cdot 2^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\sqrt{\frac{3}{2}}x\right) \middle| 2\right)}{9\sqrt{3}} - \frac{2}{9}x\sqrt{2-3x^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/(2 - 3\*x^2)^(3/4), x]

[Out]  $(-2*x*(2 - 3*x^2)^{(1/4)})/9 + (4*2^{(3/4)}*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(9*Sqrt[3])$

**Rubi in Sympy [A]** time = 3.6455, size = 41, normalized size = 0.87

$$-\frac{2x\sqrt[4]{-3x^2+2}}{9} + \frac{4 \cdot 2^{3/4} \sqrt{3} F\left(\frac{\text{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2} \middle| 2\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(-3\*x\*\*2+2)\*\*(3/4), x)

[Out]  $-2*x*(-3*x**2 + 2)**(1/4)/9 + 4*2**(3/4)*sqrt(3)*elliptic_f(asin(sqrt(6)*x/2)/2, 2)/27$

**Mathematica [C]** time = 0.0205282, size = 41, normalized size = 0.87

$$-\frac{2}{9}x\left(\sqrt[4]{2-3x^2} - \sqrt{2} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{3x^2}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(2 - 3\*x^2)^(3/4), x]

[Out]  $(-2*x*((2 - 3*x^2)^{(1/4)} - 2^{(1/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, (3*x^2)/2]))/9$

**Maple [F]** time = 0.02, size = 0, normalized size = 0.

$$\int x^2 (-3x^2 + 2)^{-3/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(-3*x^2+2)^(3/4),x)`

[Out] `int(x^2/(-3*x^2+2)^(3/4),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-3x^2 + 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-3*x^2 + 2)^(3/4),x, algorithm="maxima")`

[Out] `integrate(x^2/(-3*x^2 + 2)^(3/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{(-3x^2 + 2)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-3*x^2 + 2)^(3/4),x, algorithm="fricas")`

[Out] `integral(x^2/(-3*x^2 + 2)^(3/4), x)`

**Sympy [A]** time = 2.19613, size = 29, normalized size = 0.62

$$\frac{\sqrt[4]{2}x^3 {}_2F_1\left(\frac{3}{4}, \frac{3}{2} \middle| \frac{3x^2 e^{2i\pi}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(-3*x**2+2)**(3/4),x)`

[Out] `2**(1/4)*x**3*hyper((3/4, 3/2), (5/2, ), 3*x**2*exp_polar(2*I*pi)/2)/6`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-3x^2 + 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-3*x^2 + 2)^(3/4),x, algorithm="giac")`

[Out] `integrate(x^2/(-3*x^2 + 2)^(3/4), x)`

$$3.887 \quad \int \frac{1}{(2-3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=27

$$\frac{2^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\sqrt{\frac{3}{2}}x\right) \middle| 2\right)}{\sqrt{3}}$$

[Out] (2^(3/4)\*EllipticF[ArcSin[Sqrt[3/2]\*x]/2, 2])/Sqrt[3]

**Rubi [A]** time = 0.0155976, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{2^{3/4} F\left(\frac{1}{2} \sin^{-1}\left(\sqrt{\frac{3}{2}}x\right) \middle| 2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2 - 3\*x^2)^(-3/4), x]

[Out] (2^(3/4)\*EllipticF[ArcSin[Sqrt[3/2]\*x]/2, 2])/Sqrt[3]

**Rubi in Sympy [A]** time = 1.17345, size = 24, normalized size = 0.89

$$\frac{2^{3/4} \sqrt{3} F\left(\frac{\text{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2} \middle| 2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-3\*x\*\*2+2)\*\*(3/4), x)

[Out] 2\*\*(3/4)\*sqrt(3)\*elliptic\_f(asin(sqrt(6)\*x/2)/2, 2)/3

**Mathematica [C]** time = 0.0116496, size = 24, normalized size = 0.89

$$\frac{x {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{3x^2}{2}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - 3\*x^2)^(-3/4), x]

[Out] (x\*Hypergeometric2F1[1/2, 3/4, 3/2, (3\*x^2)/2])/2^(3/4)

**Maple [C]** time = 0.028, size = 18, normalized size = 0.7

$$\frac{\sqrt[4]{2}x}{2} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-3*x^2+2)^(3/4),x)`

[Out] `1/2*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],3/2*x^2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 + 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3*x^2 + 2)^(-3/4),x, algorithm="maxima")`

[Out] `integrate((-3*x^2 + 2)^(-3/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-3x^2 + 2)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3*x^2 + 2)^(-3/4),x, algorithm="fricas")`

[Out] `integral((-3*x^2 + 2)^(-3/4), x)`

**Sympy [A]** time = 2.14339, size = 27, normalized size = 1.

$$\frac{\sqrt[4]{2}x {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{3x^2 e^{2i\pi}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3*x**2+2)**(3/4),x)`

[Out] `2**(1/4)*x*hyper((1/2, 3/4), (3/2, ), 3*x**2*exp_polar(2*I*pi)/2)/2`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 + 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3*x^2 + 2)^(-3/4),x, algorithm="giac")`

[Out] `integrate((-3*x^2 + 2)^(-3/4), x)`

$$3.888 \quad \int \frac{1}{x^2(2-3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=49

$$\frac{\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2\sqrt[4]{2}} - \frac{\sqrt[4]{2-3x^2}}{2x}$$

[Out]  $-(2 - 3*x^2)^{(1/4)}/(2*x) + (\text{Sqrt}[3]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[3/2]*x]/2, 2])/(2*2^{(1/4)})$

**Rubi [A]** time = 0.0345063, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2\sqrt[4]{2}} - \frac{\sqrt[4]{2-3x^2}}{2x}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(2 - 3\*x^2)^(3/4)), x]

[Out]  $-(2 - 3*x^2)^{(1/4)}/(2*x) + (\text{Sqrt}[3]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[3/2]*x]/2, 2])/(2*2^{(1/4)})$

**Rubi in Sympy [A]** time = 3.44901, size = 37, normalized size = 0.76

$$\frac{2^{3/4}\sqrt{3}F\left(\frac{\text{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2}\middle|2\right)}{4} - \frac{\sqrt[4]{-3x^2+2}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(-3\*x\*\*2+2)\*\*(3/4), x)

[Out]  $2^{(3/4)}*\text{sqrt}(3)*\text{elliptic\_f}(\text{asin}(\text{sqrt}(6)*x/2)/2, 2)/4 - (-3*x**2 + 2)^{(1/4)}/(2*x)$

**Mathematica [C]** time = 0.0292384, size = 46, normalized size = 0.94

$$\frac{3x {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{3x^2}{2}\right)}{4 \cdot 2^{3/4}} - \frac{\sqrt[4]{2-3x^2}}{2x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(2 - 3\*x^2)^(3/4)), x]

[Out]  $-(2 - 3*x^2)^{(1/4)}/(2*x) + (3*x*\text{Hypergeometric2F1}[1/2, 3/4, 3/2, (3*x^2)/2])/(4*2^{(3/4)})$

**Maple [F]** time = 0.024, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (-3x^2 + 2)^{-3/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(-3*x^2+2)^(3/4),x)`

[Out] `int(1/x^2/(-3*x^2+2)^(3/4),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 + 2)^{\frac{3}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-3*x^2 + 2)^(3/4)*x^2),x, algorithm="maxima")`

[Out] `integrate(1/((-3*x^2 + 2)^(3/4)*x^2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-3x^2 + 2)^{\frac{3}{4}} x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-3*x^2 + 2)^(3/4)*x^2),x, algorithm="fricas")`

[Out] `integral(1/((-3*x^2 + 2)^(3/4)*x^2), x)`

**Sympy [A]** time = 2.72694, size = 31, normalized size = 0.63

$$-\frac{\sqrt[4]{2} {}_2F_1\left(-\frac{1}{2}, \frac{3}{4} \mid \frac{3x^2 e^{2i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(-3*x**2+2)**(3/4),x)`

[Out] `-2**(1/4)*hyper((-1/2, 3/4), (1/2,), 3*x**2*exp_polar(2*I*pi)/2)/(2*x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 + 2)^{\frac{3}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-3*x^2 + 2)^(3/4)*x^2),x, algorithm="giac")`

[Out] `integrate(1/((-3*x^2 + 2)^(3/4)*x^2), x)`



$$3.889 \quad \int \frac{1}{x^4(2-3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=67

$$-\frac{5\sqrt[4]{2-3x^2}}{8x} - \frac{\sqrt[4]{2-3x^2}}{6x^3} + \frac{5\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{8\sqrt[4]{2}}$$

[Out]  $-(2 - 3*x^2)^{(1/4)}/(6*x^3) - (5*(2 - 3*x^2)^{(1/4)})/(8*x) + (5*\text{Sqrt}[3]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[3/2]*x]/2, 2])/(8*2^{(1/4)})$

**Rubi [A]** time = 0.0553558, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{5\sqrt[4]{2-3x^2}}{8x} - \frac{\sqrt[4]{2-3x^2}}{6x^3} + \frac{5\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{8\sqrt[4]{2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(2 - 3\*x^2)^(3/4)), x]

[Out]  $-(2 - 3*x^2)^{(1/4)}/(6*x^3) - (5*(2 - 3*x^2)^{(1/4)})/(8*x) + (5*\text{Sqrt}[3]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[3/2]*x]/2, 2])/(8*2^{(1/4)})$

**Rubi in Sympy [A]** time = 5.11973, size = 56, normalized size = 0.84

$$\frac{5 \cdot 2^{\frac{3}{4}} \sqrt{3} F\left(\frac{\text{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2}\middle|2\right)}{16} - \frac{5\sqrt[4]{-3x^2+2}}{8x} - \frac{\sqrt[4]{-3x^2+2}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(-3\*x\*\*2+2)\*\*(3/4), x)

[Out]  $5*2^{(3/4)}*\text{sqrt}(3)*\text{elliptic\_f}(\text{asin}(\text{sqrt}(6)*x/2)/2, 2)/16 - 5*(-3*x**2 + 2)**(1/4)/(8*x) - (-3*x**2 + 2)**(1/4)/(6*x**3)$

**Mathematica [C]** time = 0.0299766, size = 55, normalized size = 0.82

$$\frac{15x {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{3x^2}{2}\right)}{16 \cdot 2^{3/4}} + \sqrt[4]{2-3x^2} \left(-\frac{1}{6x^3} - \frac{5}{8x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(2 - 3\*x^2)^(3/4)), x]

[Out]  $(-1/(6*x^3) - 5/(8*x))*(2 - 3*x^2)^{(1/4)} + (15*x*\text{Hypergeometric2F1}[1/2, 3/4, 3/2, (3*x^2)/2])/(16*2^{(3/4)})$

**Maple [F]** time = 0.025, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} (-3x^2 + 2)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(-3*x^2+2)^(3/4),x)`

[Out] `int(1/x^4/(-3*x^2+2)^(3/4),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 + 2)^{\frac{3}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-3*x^2 + 2)^(3/4)*x^4),x, algorithm="maxima")`

[Out] `integrate(1/((-3*x^2 + 2)^(3/4)*x^4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-3x^2 + 2)^{\frac{3}{4}} x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-3*x^2 + 2)^(3/4)*x^4),x, algorithm="fricas")`

[Out] `integral(1/((-3*x^2 + 2)^(3/4)*x^4), x)`

**Sympy [A]** time = 3.43907, size = 34, normalized size = 0.51

$$-\frac{\sqrt[4]{2} {}_2F_1\left(-\frac{3}{2}, \frac{3}{4} \mid \frac{3x^2 e^{2i\pi}}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(-3*x**2+2)**(3/4),x)`

[Out] `-2**(1/4)*hyper((-3/2, 3/4), (-1/2,), 3*x**2*exp_polar(2*I*pi)/2)/(6*x**3)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 + 2)^{\frac{3}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-3*x^2 + 2)^(3/4)*x^4),x, algorithm="giac")`

[Out] `integrate(1/((-3*x^2 + 2)^(3/4)*x^4), x)`

$$3.890 \quad \int \frac{1}{x^6(2-3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=85

$$-\frac{27\sqrt[4]{2-3x^2}}{32x} - \frac{\sqrt[4]{2-3x^2}}{10x^5} - \frac{9\sqrt[4]{2-3x^2}}{40x^3} + \frac{27\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{32\sqrt[4]{2}}$$

[Out]  $-(2-3x^2)^{1/4}/(10x^5) - (9(2-3x^2)^{1/4})/(40x^3) - (27(2-3x^2)^{1/4})/(32x) + (27\sqrt{3}\text{EllipticF}[\text{ArcSin}[\sqrt{3/2}x]/2, 2])/(32\sqrt[4]{2})$

**Rubi [A]** time = 0.0754834, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{27\sqrt[4]{2-3x^2}}{32x} - \frac{\sqrt[4]{2-3x^2}}{10x^5} - \frac{9\sqrt[4]{2-3x^2}}{40x^3} + \frac{27\sqrt{3}F\left(\frac{1}{2}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{32\sqrt[4]{2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(2-3\*x^2)^(3/4)),x]

[Out]  $-(2-3x^2)^{1/4}/(10x^5) - (9(2-3x^2)^{1/4})/(40x^3) - (27(2-3x^2)^{1/4})/(32x) + (27\sqrt{3}\text{EllipticF}[\text{ArcSin}[\sqrt{3/2}x]/2, 2])/(32\sqrt[4]{2})$

**Rubi in Sympy [A]** time = 6.82349, size = 73, normalized size = 0.86

$$\frac{27 \cdot 2^{3/4} \sqrt{3} F\left(\frac{\text{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2}\middle|2\right)}{64} - \frac{27\sqrt[4]{-3x^2+2}}{32x} - \frac{9\sqrt[4]{-3x^2+2}}{40x^3} - \frac{\sqrt[4]{-3x^2+2}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*6/(-3\*x\*\*2+2)\*\*(3/4),x)

[Out]  $27*2^{3/4}*sqrt(3)*elliptic\_f(\text{asin}(\text{sqrt}(6)*x/2)/2, 2)/64 - 27*(-3*x**2+2)**(1/4)/(32*x) - 9*(-3*x**2+2)**(1/4)/(40*x**3) - (-3*x**2+2)**(1/4)/(10*x**5)$

**Mathematica [C]** time = 0.0480826, size = 58, normalized size = 0.68

$$\frac{81x {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{3x^2}{2}\right)}{64 \cdot 2^{3/4}} - \frac{\sqrt[4]{2-3x^2}(135x^4+36x^2+16)}{160x^5}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6\*(2-3\*x^2)^(3/4)),x]

[Out]  $-((2-3x^2)^{1/4}*(16+36x^2+135x^4))/(160x^5) + (81x \text{Hypergeometric2F1}[1/2, 3/4, 3/2, (3x^2)/2])/(64\sqrt[4]{2})$

**Maple [F]** time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{x^6} (-3x^2 + 2)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^6/(-3*x^2+2)^(3/4), x)`

[Out] `int(1/x^6/(-3*x^2+2)^(3/4), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 + 2)^{\frac{3}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-3*x^2 + 2)^(3/4)*x^6), x, algorithm="maxima")`

[Out] `integrate(1/((-3*x^2 + 2)^(3/4)*x^6), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-3x^2 + 2)^{\frac{3}{4}} x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-3*x^2 + 2)^(3/4)*x^6), x, algorithm="fricas")`

[Out] `integral(1/((-3*x^2 + 2)^(3/4)*x^6), x)`

---

**Sympy [A]** time = 4.38869, size = 34, normalized size = 0.4

$$\frac{\sqrt[4]{2} {}_2F_1\left(-\frac{5}{2}, \frac{3}{4} \mid \frac{3x^2 e^{2i\pi}}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**6/(-3*x**2+2)**(3/4), x)`

[Out] `-2**(1/4)*hyper((-5/2, 3/4), (-3/2, ), 3*x**2*exp_polar(2*I*pi)/2)/(10*x**5)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 + 2)^{\frac{3}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-3*x^2 + 2)^(3/4)*x^6),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 + 2)^(3/4)*x^6), x)
```

$$3.891 \quad \int \frac{x^6}{\sqrt[4]{-2 + 3x^2}} dx$$

**Optimal.** Leaf size=258

$$\begin{aligned} & \frac{32(3x^2 - 2)^{3/4} x}{1053} + \frac{128\sqrt[4]{3x^2 - 2} x}{1053(\sqrt{3x^2 - 2} + \sqrt{2})} \\ & + \frac{64\sqrt[4]{2} \sqrt{\frac{x^2}{(\sqrt{3x^2 - 2} + \sqrt{2})^2}} (\sqrt{3x^2 - 2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2 - 2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{1053\sqrt{3}x} \\ & - \frac{128\sqrt[4]{2} \sqrt{\frac{x^2}{(\sqrt{3x^2 - 2} + \sqrt{2})^2}} (\sqrt{3x^2 - 2} + \sqrt{2}) E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2 - 2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{1053\sqrt{3}x} \\ & + \frac{2}{39} (3x^2 - 2)^{3/4} x^5 + \frac{40(3x^2 - 2)^{3/4} x^3}{1053} \end{aligned}$$

[Out] (32\*x\*(-2 + 3\*x^2)^(3/4))/1053 + (40\*x^3\*(-2 + 3\*x^2)^(3/4))/1053 + (2\*x^5\*(-2 + 3\*x^2)^(3/4))/39 + (128\*x\*(-2 + 3\*x^2)^(1/4))/(1053\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])) - (128\*2^(1/4)\*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3\*x^2])^2]\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])\*EllipticE[2\*ArcTan[(-2 + 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(1053\*Sqrt[3]\*x) + (64\*2^(1/4)\*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3\*x^2])^2]\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])\*EllipticF[2\*ArcTan[(-2 + 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(1053\*Sqrt[3]\*x)

**Rubi [A]** time = 0.337327, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\begin{aligned} & \frac{32(3x^2 - 2)^{3/4} x}{1053} + \frac{128\sqrt[4]{3x^2 - 2} x}{1053(\sqrt{3x^2 - 2} + \sqrt{2})} \\ & + \frac{64\sqrt[4]{2} \sqrt{\frac{x^2}{(\sqrt{3x^2 - 2} + \sqrt{2})^2}} (\sqrt{3x^2 - 2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2 - 2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{1053\sqrt{3}x} \\ & - \frac{128\sqrt[4]{2} \sqrt{\frac{x^2}{(\sqrt{3x^2 - 2} + \sqrt{2})^2}} (\sqrt{3x^2 - 2} + \sqrt{2}) E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2 - 2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{1053\sqrt{3}x} \\ & + \frac{2}{39} (3x^2 - 2)^{3/4} x^5 + \frac{40(3x^2 - 2)^{3/4} x^3}{1053} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^6/(-2 + 3\*x^2)^(1/4), x]

[Out] (32\*x\*(-2 + 3\*x^2)^(3/4))/1053 + (40\*x^3\*(-2 + 3\*x^2)^(3/4))/1053 + (2\*x^5\*(-2 + 3\*x^2)^(3/4))/39 + (128\*x\*(-2 + 3\*x^2)^(1/4))/(1053\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])) - (128\*2^(1/4)\*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3\*x^2])^2]\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])\*EllipticE[2\*ArcTan[(-2 + 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(1053\*Sqrt[3]\*x) + (64\*2^(1/4)\*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3\*x^2])^2]\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])\*EllipticF[2\*ArcTan[(-2 + 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(1053\*Sqrt[3]\*x)

**Rubi in Sympy [A]** time = 7.81062, size = 92, normalized size = 0.36

$$\frac{2x^5(3x^2 - 2)^{\frac{3}{4}}}{39} + \frac{40x^3(3x^2 - 2)^{\frac{3}{4}}}{1053} + \frac{32x(3x^2 - 2)^{\frac{3}{4}}}{1053} + \frac{128\sqrt[4]{6}\sqrt{-\frac{3x^2}{2}} + 1E\left(\frac{\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2}\right)}{3159\sqrt[4]{3x^2 - 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**6/(3*x**2-2)**(1/4),x)`

[Out]  $2x^{5/4}(3x^2-2)^{3/4}/39 + 40x^{3/4}(3x^2-2)^{3/4}/1053 + 32x(3x^2-2)^{3/4}/1053 + 128\sqrt{6}(-3x^{2/2}+1)^{1/4}\text{elliptic}_e(\text{asin}(\sqrt{6}x/2)/2, 2)/(3159(3x^2-2)^{1/4})$

**Mathematica [C]** time = 0.0434361, size = 68, normalized size = 0.26

$$\frac{2x \left( 16 \sqrt[4]{2-3x^2} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right) + 81x^6 + 6x^4 + 8x^2 - 32 \right)}{1053 \sqrt[4]{3x^2-2}}$$

Antiderivative was successfully verified.

[In] `Integrate[x^6/(-2 + 3*x^2)^(1/4),x]`

[Out]  $(2x^{5/4}(-32 + 8x^2 + 6x^4 + 81x^6 + 16 \cdot 2^{3/4} (2 - 3x^2)^{1/4}) \text{Hypergeometric2F1}[1/4, 1/2, 3/2, (3x^2)/2]) / (1053 (-2 + 3x^2)^{1/4})$

**Maple [C]** time = 0.06, size = 65, normalized size = 0.3

$$\frac{2x(27x^4 + 20x^2 + 16)}{1053} (3x^2 - 2)^{3/4} + \frac{32 \cdot 2^{3/4} x}{1053} \sqrt[4]{-\text{signum}\left(-1 + \frac{3x^2}{2}\right)} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right) \frac{1}{\sqrt[4]{\text{signum}\left(-1 + \frac{3x^2}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(3*x^2-2)^(1/4),x)`

[Out]  $2/1053 x^{5/4} (27x^4 + 20x^2 + 16) (3x^2 - 2)^{3/4} + 32/1053 \cdot 2^{3/4} / \text{signum}(-1 + 3/2 x^2)^{1/4} \cdot (-\text{signum}(-1 + 3/2 x^2))^{1/4} x \cdot \text{hypergeom}([1/4, 1/2], [3/2], 3/2 x^2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(3x^2 - 2)^{1/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(3*x^2 - 2)^(1/4),x, algorithm="maxima")`

[Out] `integrate(x^6/(3*x^2 - 2)^(1/4), x)`

**Ericsas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{(3x^2 - 2)^{1/4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(3*x^2 - 2)^(1/4),x, algorithm="fricas")`

[Out] `integral(x^6/(3*x^2 - 2)^(1/4), x)`

**Sympy [A]** time = 2.86186, size = 31, normalized size = 0.12

$$\frac{2^{\frac{3}{4}} x^7 e^{\frac{15i\pi}{4}} {}_2F_1\left(\frac{1}{4}, \frac{7}{2} \middle| \frac{3x^2}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(3*x**2-2)**(1/4),x)`

[Out] `2**(3/4)*x**7*exp(15*I*pi/4)*hyper((1/4, 7/2), (9/2,), 3*x**2/2)/14`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(3x^2 - 2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(3*x^2 - 2)^(1/4),x, algorithm="giac")`

[Out] `integrate(x^6/(3*x^2 - 2)^(1/4), x)`



$$3.892 \quad \int \frac{x^4}{\sqrt[4]{-2 + 3x^2}} dx$$

**Optimal.** Leaf size=240

$$\begin{aligned} & \frac{8}{135} (3x^2 - 2)^{3/4} x + \frac{32\sqrt[4]{3x^2 - 2}x}{135(\sqrt{3x^2 - 2} + \sqrt{2})} \\ & + \frac{16\sqrt{2} \sqrt{\frac{x^2}{(\sqrt{3x^2 - 2} + \sqrt{2})^2}} (\sqrt{3x^2 - 2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2 - 2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{135\sqrt{3}x} \\ & - \frac{32\sqrt{2} \sqrt{\frac{x^2}{(\sqrt{3x^2 - 2} + \sqrt{2})^2}} (\sqrt{3x^2 - 2} + \sqrt{2}) E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2 - 2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{135\sqrt{3}x} + \frac{2}{27} (3x^2 - 2)^{3/4} x^3 \end{aligned}$$

[Out]  $(8*x*(-2 + 3*x^2)^{(3/4)})/135 + (2*x^3*(-2 + 3*x^2)^{(3/4)})/27 + (3*2*x*(-2 + 3*x^2)^{(1/4)})/(135*(\text{Sqrt}[2] + \text{Sqrt}[-2 + 3*x^2])) - (32*2^{(1/4)}*\text{Sqrt}[x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 + 3*x^2])^2]*(\text{Sqrt}[2] + \text{Sqrt}[-2 + 3*x^2]))*\text{EllipticE}[2*\text{ArcTan}[(-2 + 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2])/(135*\text{Sqrt}[3]*x) + (16*2^{(1/4)}*\text{Sqrt}[x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 + 3*x^2])^2]*(\text{Sqrt}[2] + \text{Sqrt}[-2 + 3*x^2]))*\text{EllipticF}[2*\text{ArcTan}[(-2 + 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2])/(135*\text{Sqrt}[3]*x)$

**Rubi [A]** time = 0.282993, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\begin{aligned} & \frac{8}{135} (3x^2 - 2)^{3/4} x + \frac{32\sqrt[4]{3x^2 - 2}x}{135(\sqrt{3x^2 - 2} + \sqrt{2})} \\ & + \frac{16\sqrt{2} \sqrt{\frac{x^2}{(\sqrt{3x^2 - 2} + \sqrt{2})^2}} (\sqrt{3x^2 - 2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2 - 2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{135\sqrt{3}x} \\ & - \frac{32\sqrt{2} \sqrt{\frac{x^2}{(\sqrt{3x^2 - 2} + \sqrt{2})^2}} (\sqrt{3x^2 - 2} + \sqrt{2}) E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2 - 2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{135\sqrt{3}x} + \frac{2}{27} (3x^2 - 2)^{3/4} x^3 \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^4/(-2 + 3*x^2)^{(1/4)}, x]$

[Out]  $(8*x*(-2 + 3*x^2)^{(3/4)})/135 + (2*x^3*(-2 + 3*x^2)^{(3/4)})/27 + (3*2*x*(-2 + 3*x^2)^{(1/4)})/(135*(\text{Sqrt}[2] + \text{Sqrt}[-2 + 3*x^2])) - (32*2^{(1/4)}*\text{Sqrt}[x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 + 3*x^2])^2]*(\text{Sqrt}[2] + \text{Sqrt}[-2 + 3*x^2]))*\text{EllipticE}[2*\text{ArcTan}[(-2 + 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2])/(135*\text{Sqrt}[3]*x) + (16*2^{(1/4)}*\text{Sqrt}[x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 + 3*x^2])^2]*(\text{Sqrt}[2] + \text{Sqrt}[-2 + 3*x^2]))*\text{EllipticF}[2*\text{ArcTan}[(-2 + 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2])/(135*\text{Sqrt}[3]*x)$

**Rubi in Sympy [A]** time = 5.85185, size = 75, normalized size = 0.31

$$\frac{2x^3(3x^2 - 2)^{\frac{3}{4}}}{27} + \frac{8x(3x^2 - 2)^{\frac{3}{4}}}{135} + \frac{32\sqrt{6}\sqrt[4]{-\frac{3x^2}{2}} + 1E\left(\frac{\text{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2}\right)}{405\sqrt[4]{3x^2 - 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**4}/(3*x^{**2}-2)^{(1/4)}, x)$

[Out]  $2^3 x^3 (3x^2 - 2)^{3/4} / 27 + 8x (3x^2 - 2)^{3/4} / 135 + 32 \sqrt{6} (-3x^2/2 + 1)^{1/4} \text{elliptic}_e(\text{asin}(\sqrt{6}x/2)/2, 2) / (405 (3x^2 - 2)^{1/4})$

**Mathematica [C]** time = 0.0357802, size = 63, normalized size = 0.26

$$\frac{2x \left( 4 \cdot 2^{3/4} \sqrt{2 - 3x^2} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right) + 15x^4 + 2x^2 - 8 \right)}{135 \sqrt[4]{3x^2 - 2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(-2 + 3\*x^2)^(1/4), x]

[Out]  $(2^3 x^3 (-8 + 2x^2 + 15x^4 + 4 \cdot 2^{3/4}) (2 - 3x^2)^{1/4} \text{Hypergeometric2F1}[1/4, 1/2, 3/2, (3x^2)/2]) / (135 (-2 + 3x^2)^{1/4})$

**Maple [C]** time = 0.051, size = 60, normalized size = 0.3

$$\frac{2x(5x^2 + 4)}{135} (3x^2 - 2)^{3/4} + \frac{8 \cdot 2^{3/4} x}{135} \sqrt[4]{-\text{signum}\left(-1 + \frac{3x^2}{2}\right)} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right) \frac{1}{\sqrt[4]{\text{signum}\left(-1 + \frac{3x^2}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(3\*x^2-2)^(1/4), x)

[Out]  $2/135 x^3 (5x^2 + 4) (3x^2 - 2)^{3/4} + 8/135 \cdot 2^{3/4} / \text{signum}(-1 + 3/2 x^2)^{1/4} (-\text{signum}(-1 + 3/2 x^2))^{1/4} x \text{hypergeom}([1/4, 1/2], [3/2], 3/2 x^2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(3x^2 - 2)^{1/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(3\*x^2 - 2)^(1/4), x, algorithm="maxima")

[Out] integrate(x^4/(3\*x^2 - 2)^(1/4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{(3x^2 - 2)^{1/4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(3\*x^2 - 2)^(1/4), x, algorithm="fricas")

[Out] integral(x^4/(3\*x^2 - 2)^(1/4), x)

---

**Sympy [A]** time = 2.46017, size = 31, normalized size = 0.13

$$\frac{2^{\frac{3}{4}} x^5 e^{\frac{15i\pi}{4}} {}_2F_1\left(\frac{1}{4}, \frac{5}{2} \middle| \frac{3x^2}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4/(3\*x\*\*2-2)\*\*(1/4), x)

[Out] 2\*\*(3/4)\*x\*\*5\*exp(15\*I\*pi/4)\*hyper((1/4, 5/2), (7/2,), 3\*x\*\*2/2)/10

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(3x^2 - 2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(3\*x^2 - 2)^(1/4), x, algorithm="giac")

[Out] integrate(x^4/(3\*x^2 - 2)^(1/4), x)

$$3.893 \quad \int \frac{x^2}{\sqrt[4]{-2 + 3x^2}} dx$$

**Optimal.** Leaf size=222

$$\begin{aligned} & \frac{2}{15} (3x^2 - 2)^{3/4} x + \frac{8\sqrt[4]{3x^2 - 2} x}{15 (\sqrt{3x^2 - 2} + \sqrt{2})} \\ & + \frac{4\sqrt[4]{2} \sqrt{\frac{x^2}{(\sqrt{3x^2 - 2} + \sqrt{2})^2}} (\sqrt{3x^2 - 2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2 - 2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{15\sqrt{3}x} \\ & - \frac{8\sqrt[4]{2} \sqrt{\frac{x^2}{(\sqrt{3x^2 - 2} + \sqrt{2})^2}} (\sqrt{3x^2 - 2} + \sqrt{2}) E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2 - 2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{15\sqrt{3}x} \end{aligned}$$

[Out] (2\*x\*(-2 + 3\*x^2)^(3/4))/15 + (8\*x\*(-2 + 3\*x^2)^(1/4))/(15\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])) - (8\*2^(1/4)\*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3\*x^2])]^2\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])\*EllipticE[2\*ArcTan[(-2 + 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(15\*Sqrt[3]\*x) + (4\*2^(1/4)\*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3\*x^2])]^2\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])\*EllipticF[2\*ArcTan[(-2 + 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(15\*Sqrt[3]\*x)

**Rubi [A]** time = 0.24071, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\begin{aligned} & \frac{2}{15} (3x^2 - 2)^{3/4} x + \frac{8\sqrt[4]{3x^2 - 2} x}{15 (\sqrt{3x^2 - 2} + \sqrt{2})} \\ & + \frac{4\sqrt[4]{2} \sqrt{\frac{x^2}{(\sqrt{3x^2 - 2} + \sqrt{2})^2}} (\sqrt{3x^2 - 2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2 - 2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{15\sqrt{3}x} \\ & - \frac{8\sqrt[4]{2} \sqrt{\frac{x^2}{(\sqrt{3x^2 - 2} + \sqrt{2})^2}} (\sqrt{3x^2 - 2} + \sqrt{2}) E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2 - 2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{15\sqrt{3}x} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^2/(-2 + 3\*x^2)^(1/4), x]

[Out] (2\*x\*(-2 + 3\*x^2)^(3/4))/15 + (8\*x\*(-2 + 3\*x^2)^(1/4))/(15\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])) - (8\*2^(1/4)\*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3\*x^2])]^2\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])\*EllipticE[2\*ArcTan[(-2 + 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(15\*Sqrt[3]\*x) + (4\*2^(1/4)\*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3\*x^2])]^2\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])\*EllipticF[2\*ArcTan[(-2 + 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(15\*Sqrt[3]\*x)

**Rubi in Sympy [A]** time = 4.17174, size = 58, normalized size = 0.26

$$\frac{2x(3x^2 - 2)^{3/4}}{15} + \frac{8\sqrt{6}\sqrt[4]{-\frac{3x^2}{2}} + 1E\left(\frac{\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2}\right)}{45\sqrt[4]{3x^2 - 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(3\*x\*\*2-2)\*\*(1/4), x)

[Out]  $2x(3x^2 - 2)^{3/4}/15 + 8\sqrt{6}(-3x^2/2 + 1)^{1/4}\text{elliptic}_e(\text{asin}(\sqrt{6}x/2)/2, 2)/(45(3x^2 - 2)^{1/4})$

**Mathematica [C]** time = 0.0306128, size = 57, normalized size = 0.26

$$\frac{2x \left( 2^{3/4} \sqrt[4]{2 - 3x^2} {}_2F_1 \left( \frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2} \right) + 3x^2 - 2 \right)}{15 \sqrt[4]{3x^2 - 2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(-2 + 3\*x^2)^(1/4), x]

[Out]  $(2x(-2 + 3x^2 + 2^{3/4}(2 - 3x^2)^{1/4})\text{Hypergeometric2F1}[1/4, 1/2, 3/2, (3x^2)/2])/(15(-2 + 3x^2)^{1/4})$

**Maple [C]** time = 0.052, size = 53, normalized size = 0.2

$$\frac{2x}{15} (3x^2 - 2)^{3/4} + \frac{2 \cdot 2^{3/4} x}{15} \sqrt[4]{-\text{signum} \left( -1 + \frac{3x^2}{2} \right)} {}_2F_1 \left( \frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2} \right) \frac{1}{\sqrt[4]{\text{signum} \left( -1 + \frac{3x^2}{2} \right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(3\*x^2-2)^(1/4), x)

[Out]  $2/15x(3x^2-2)^{3/4} + 2/15 \cdot 2^{3/4} / \text{signum}(-1+3/2x^2)^{1/4} \cdot (-\text{signum}(-1+3/2x^2))^{1/4} \cdot x \cdot \text{hypergeom}([1/4, 1/2], [3/2], 3/2x^2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(3x^2 - 2)^{1/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(3\*x^2 - 2)^(1/4), x, algorithm="maxima")

[Out] integrate(x^2/(3\*x^2 - 2)^(1/4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{x^2}{(3x^2 - 2)^{1/4}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(3\*x^2 - 2)^(1/4), x, algorithm="fricas")

[Out] integral(x^2/(3\*x^2 - 2)^(1/4), x)

**Sympy [A]** time = 2.19942, size = 31, normalized size = 0.14

$$\frac{2^{\frac{3}{4}} x^3 e^{\frac{7i\pi}{4}} {}_2F_1\left(\frac{1}{4}, \frac{3}{2} \middle| \frac{3x^2}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(3\*x\*\*2-2)\*\*(1/4), x)

[Out] 2\*\*(3/4)\*x\*\*3\*exp(7\*I\*pi/4)\*hyper((1/4, 3/2), (5/2, ), 3\*x\*\*2/2)/6

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(3x^2 - 2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(3\*x^2 - 2)^(1/4), x, algorithm="giac")

[Out] integrate(x^2/(3\*x^2 - 2)^(1/4), x)

$$3.894 \quad \int \frac{1}{\sqrt[4]{-2 + 3x^2}} dx$$

**Optimal.** Leaf size=199

$$\frac{2\sqrt[4]{3x^2 - 2x}}{\sqrt{3x^2 - 2} + \sqrt{2}} + \frac{\sqrt[4]{2} \sqrt{\frac{x^2}{(\sqrt{3x^2 - 2} + \sqrt{2})^2}} (\sqrt{3x^2 - 2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2 - 2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{\sqrt{3x}}$$

$$- \frac{2\sqrt[4]{2} \sqrt{\frac{x^2}{(\sqrt{3x^2 - 2} + \sqrt{2})^2}} (\sqrt{3x^2 - 2} + \sqrt{2}) E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2 - 2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{\sqrt{3x}}$$

[Out] (2\*x\*(-2 + 3\*x^2)^(1/4))/(Sqrt[2] + Sqrt[-2 + 3\*x^2]) - (2\*2^(1/4)\*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3\*x^2])^2]\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])\*EllipticE[2\*ArcTan[(-2 + 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(Sqrt[3]\*x) + (2^(1/4)\*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3\*x^2])^2]\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])\*EllipticF[2\*ArcTan[(-2 + 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(Sqrt[3]\*x)

**Rubi [A]** time = 0.206198, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$\frac{2\sqrt[4]{3x^2 - 2x}}{\sqrt{3x^2 - 2} + \sqrt{2}} + \frac{\sqrt[4]{2} \sqrt{\frac{x^2}{(\sqrt{3x^2 - 2} + \sqrt{2})^2}} (\sqrt{3x^2 - 2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2 - 2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{\sqrt{3x}}$$

$$- \frac{2\sqrt[4]{2} \sqrt{\frac{x^2}{(\sqrt{3x^2 - 2} + \sqrt{2})^2}} (\sqrt{3x^2 - 2} + \sqrt{2}) E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2 - 2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{\sqrt{3x}}$$

Antiderivative was successfully verified.

[In] Int[(-2 + 3\*x^2)^(-1/4), x]

[Out] (2\*x\*(-2 + 3\*x^2)^(1/4))/(Sqrt[2] + Sqrt[-2 + 3\*x^2]) - (2\*2^(1/4)\*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3\*x^2])^2]\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])\*EllipticE[2\*ArcTan[(-2 + 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(Sqrt[3]\*x) + (2^(1/4)\*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3\*x^2])^2]\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])\*EllipticF[2\*ArcTan[(-2 + 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(Sqrt[3]\*x)

**Rubi in Sympy [A]** time = 1.55473, size = 42, normalized size = 0.21

$$\frac{2\sqrt[4]{6}\sqrt{-\frac{3x^2}{2} + 1}E\left(\frac{\operatorname{asin}\left(\frac{\sqrt{6x}}{2}\right)}{2} \middle| 2\right)}{3\sqrt[4]{3x^2 - 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(3\*x\*\*2-2)\*\*(1/4), x)

[Out] 2\*sqrt(6)\*(-3\*x\*\*2/2 + 1)\*\*(1/4)\*elliptic\_e(asin(sqrt(6)\*x/2)/2, 2)/(3\*(3\*x\*\*2 - 2)\*\*(1/4))

**Mathematica [C]** time = 0.0190982, size = 41, normalized size = 0.21

$$\frac{x\sqrt[4]{2 - 3x^2} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}, \frac{3x^2}{2}\right)}{\sqrt[4]{6x^2 - 4}}$$

Antiderivative was successfully verified.

[In] Integrate[(-2 + 3\*x^2)^(-1/4), x]

[Out] (x\*(2 - 3\*x^2)^(1/4)\*Hypergeometric2F1[1/4, 1/2, 3/2, (3\*x^2)/2]) / (-4 + 6\*x^2)^(1/4)

**Maple [C]** time = 0.04, size = 40, normalized size = 0.2

$$\frac{2^{\frac{3}{4}}x}{2} \sqrt[4]{-\text{signum}\left(-1 + \frac{3x^2}{2}\right)} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right) \frac{1}{\sqrt[4]{\text{signum}\left(-1 + \frac{3x^2}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3\*x^2-2)^(1/4), x)

[Out] 1/2\*2^(3/4)/signum(-1+3/2\*x^2)^(1/4)\*(-signum(-1+3/2\*x^2))^(1/4)\*x\*hypergeom([1/4, 1/2], [3/2], 3/2\*x^2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 - 2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2 - 2)^(-1/4), x, algorithm="maxima")

[Out] integrate((3\*x^2 - 2)^(-1/4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(3x^2 - 2)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2 - 2)^(-1/4), x, algorithm="fricas")

[Out] integral((3\*x^2 - 2)^(-1/4), x)

**Sympy [A]** time = 2.14467, size = 29, normalized size = 0.15

$$\frac{2^{\frac{3}{4}}xe^{\frac{7i\pi}{4}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3\*x\*\*2-2)\*\*(1/4), x)



[Out]  $2^{3/4} x \exp(7i\pi/4) \operatorname{hyper}((1/4, 1/2), (3/2, ), 3x^{2/2})/2$

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 - 2)^{1/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2 - 2)^(-1/4),x, algorithm="giac")`

[Out] `integrate((3*x^2 - 2)^(-1/4), x)`

$$3.895 \quad \int \frac{1}{x^2 \sqrt[4]{-2 + 3x^2}} dx$$

**Optimal.** Leaf size=221

$$\begin{aligned} & -\frac{3\sqrt[4]{3x^2-2}x}{2(\sqrt{3x^2-2}+\sqrt{2})} + \frac{(3x^2-2)^{3/4}}{2x} \\ & -\frac{\sqrt{3}\sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}}(\sqrt{3x^2-2}+\sqrt{2})F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt{2}}\right)\middle|\frac{1}{2}\right)}{2^{2^{3/4}}x} \\ & +\frac{\sqrt{3}\sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}}(\sqrt{3x^2-2}+\sqrt{2})E\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt{2}}\right)\middle|\frac{1}{2}\right)}{2^{3/4}x} \end{aligned}$$

[Out]  $(-2 + 3x^2)^{3/4}/(2x) - (3x(-2 + 3x^2)^{1/4})/(2(\sqrt{2} + \sqrt{-2 + 3x^2})) + (\sqrt{3}\sqrt{x^2/(\sqrt{2} + \sqrt{-2 + 3x^2})^2}(\sqrt{2} + \sqrt{-2 + 3x^2})^2)^{1/4}/2^{1/4} \operatorname{EllipticE}[2 \operatorname{ArcTan}((-2 + 3x^2)^{1/4}/2^{1/4}), 1/2]/(2^{3/4}x) - (\sqrt{3}\sqrt{x^2/(\sqrt{2} + \sqrt{-2 + 3x^2})^2}(\sqrt{2} + \sqrt{-2 + 3x^2})^2)^{1/4}/2^{1/4} \operatorname{EllipticF}[2 \operatorname{ArcTan}((-2 + 3x^2)^{1/4}/2^{1/4}), 1/2]/(2^{3/4}x)$

**Rubi [A]** time = 0.242435, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\begin{aligned} & -\frac{3\sqrt[4]{3x^2-2}x}{2(\sqrt{3x^2-2}+\sqrt{2})} + \frac{(3x^2-2)^{3/4}}{2x} \\ & -\frac{\sqrt{3}\sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}}(\sqrt{3x^2-2}+\sqrt{2})F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt{2}}\right)\middle|\frac{1}{2}\right)}{2^{2^{3/4}}x} \\ & +\frac{\sqrt{3}\sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}}(\sqrt{3x^2-2}+\sqrt{2})E\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt{2}}\right)\middle|\frac{1}{2}\right)}{2^{3/4}x} \end{aligned}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^2*(-2 + 3*x^2)^(1/4)), x]`

[Out]  $(-2 + 3x^2)^{3/4}/(2x) - (3x(-2 + 3x^2)^{1/4})/(2(\sqrt{2} + \sqrt{-2 + 3x^2})) + (\sqrt{3}\sqrt{x^2/(\sqrt{2} + \sqrt{-2 + 3x^2})^2}(\sqrt{2} + \sqrt{-2 + 3x^2})^2)^{1/4}/2^{1/4} \operatorname{EllipticE}[2 \operatorname{ArcTan}((-2 + 3x^2)^{1/4}/2^{1/4}), 1/2]/(2^{3/4}x) - (\sqrt{3}\sqrt{x^2/(\sqrt{2} + \sqrt{-2 + 3x^2})^2}(\sqrt{2} + \sqrt{-2 + 3x^2})^2)^{1/4}/2^{1/4} \operatorname{EllipticF}[2 \operatorname{ArcTan}((-2 + 3x^2)^{1/4}/2^{1/4}), 1/2]/(2^{3/4}x)$

**Rubi in Sympy [A]** time = 3.93418, size = 54, normalized size = 0.24

$$-\frac{\sqrt{6}\sqrt[4]{-\frac{3x^2}{2}+1}E\left(\frac{\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2}\middle|2\right)}{2\sqrt[4]{3x^2-2}} + \frac{(3x^2-2)^{3/4}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**2/(3*x**2-2)**(1/4), x)`

[Out]  $-\sqrt{6}\sqrt[4]{-3x^2/2 + 1} \operatorname{elliptic}_e(\operatorname{asin}(\sqrt{6}x/2)/2, 2)/(2(3x^2 - 2)^{1/4}) + (3x^2 - 2)^{3/4}/(2x)$

---

**Mathematica [C]** time = 0.0263848, size = 63, normalized size = 0.29

$$\frac{-3 \cdot 2^{3/4} \sqrt[4]{2 - 3x^2} x^2 {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right) + 12x^2 - 8}{8x \sqrt[4]{3x^2 - 2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(-2 + 3\*x^2)^(1/4)),x]

[Out] (-8 + 12\*x^2 - 3\*2^(3/4)\*x^2\*(2 - 3\*x^2)^(1/4)\*Hypergeometric2F1[1/4, 1/2, 3/2, (3\*x^2)/2])/(8\*x\*(2 + 3\*x^2)^(1/4))

---

**Maple [C]** time = 0.057, size = 55, normalized size = 0.3

$$\frac{1}{2x} (3x^2 - 2)^{\frac{3}{4}} - \frac{3 \cdot 2^{3/4} x}{8} \sqrt[4]{-\text{signum}\left(-1 + \frac{3x^2}{2}\right)} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right) \frac{1}{\sqrt[4]{\text{signum}\left(-1 + \frac{3x^2}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(3\*x^2-2)^(1/4),x)

[Out] 1/2\*(3\*x^2-2)^(3/4)/x-3/8\*2^(3/4)/signum(-1+3/2\*x^2)^(1/4)\*(-signum(-1+3/2\*x^2))^(1/4)\*x\*hypergeom([1/4, 1/2],[3/2],3/2\*x^2)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 - 2)^{\frac{1}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((3\*x^2 - 2)^(1/4)\*x^2),x, algorithm="maxima")

[Out] integrate(1/((3\*x^2 - 2)^(1/4)\*x^2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(3x^2 - 2)^{\frac{1}{4}} x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((3\*x^2 - 2)^(1/4)\*x^2),x, algorithm="fricas")

[Out] integral(1/((3\*x^2 - 2)^(1/4)\*x^2), x)

---

**Sympy [A]** time = 2.49537, size = 31, normalized size = 0.14

$$\frac{2^{\frac{3}{4}} e^{\frac{3i\pi}{4}} {}_2F_1\left(-\frac{1}{2}, \frac{1}{4}; \frac{3}{2}; \frac{3x^2}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(3*x**2-2)**(1/4),x)`

[Out] `2**(3/4)*exp(3*I*pi/4)*hyper((-1/2, 1/4), (1/2,), 3*x**2/2)/(2*x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 - 2)^{\frac{1}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 - 2)^(1/4)*x^2),x, algorithm="giac")`

[Out] `integrate(1/((3*x^2 - 2)^(1/4)*x^2), x)`

$$3.896 \quad \int \frac{1}{x^4 \sqrt[4]{-2 + 3x^2}} dx$$

**Optimal.** Leaf size=242

$$\begin{aligned} & -\frac{9\sqrt[4]{3x^2-2}x}{8(\sqrt{3x^2-2}+\sqrt{2})} + \frac{3(3x^2-2)^{3/4}}{8x} \\ & - \frac{3\sqrt{3}\sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}}(\sqrt{3x^2-2}+\sqrt{2})F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt{2}}\right)\middle|\frac{1}{2}\right)}{8\cdot 2^{3/4}x} \\ & + \frac{3\sqrt{3}\sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}}(\sqrt{3x^2-2}+\sqrt{2})E\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt{2}}\right)\middle|\frac{1}{2}\right)}{4\cdot 2^{3/4}x} + \frac{(3x^2-2)^{3/4}}{6x^3} \end{aligned}$$

[Out]  $(-2 + 3x^2)^{3/4}/(6x^3) + (3(-2 + 3x^2)^{3/4})/(8x) - (9x^4(-2 + 3x^2)^{1/4})/(8(\sqrt{2} + \sqrt{-2 + 3x^2})) + (3\sqrt{3}\sqrt{x^2/(\sqrt{2} + \sqrt{-2 + 3x^2})^2}(\sqrt{2} + \sqrt{-2 + 3x^2}))\text{EllipticE}[2\text{ArcTan}[-2 + 3x^2]^{1/4}/2^{1/4}, 1/2]/(4\cdot 2^{3/4}x) - (3\sqrt{3}\sqrt{x^2/(\sqrt{2} + \sqrt{-2 + 3x^2})^2}(\sqrt{2} + \sqrt{-2 + 3x^2}))\text{EllipticF}[2\text{ArcTan}[-2 + 3x^2]^{1/4}/2^{1/4}, 1/2]/(8\cdot 2^{3/4}x)$

**Rubi [A]** time = 0.282946, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\begin{aligned} & -\frac{9\sqrt[4]{3x^2-2}x}{8(\sqrt{3x^2-2}+\sqrt{2})} + \frac{3(3x^2-2)^{3/4}}{8x} \\ & - \frac{3\sqrt{3}\sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}}(\sqrt{3x^2-2}+\sqrt{2})F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt{2}}\right)\middle|\frac{1}{2}\right)}{8\cdot 2^{3/4}x} \\ & + \frac{3\sqrt{3}\sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}}(\sqrt{3x^2-2}+\sqrt{2})E\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt{2}}\right)\middle|\frac{1}{2}\right)}{4\cdot 2^{3/4}x} + \frac{(3x^2-2)^{3/4}}{6x^3} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(-2 + 3\*x^2)^(1/4)), x]

[Out]  $(-2 + 3x^2)^{3/4}/(6x^3) + (3(-2 + 3x^2)^{3/4})/(8x) - (9x^4(-2 + 3x^2)^{1/4})/(8(\sqrt{2} + \sqrt{-2 + 3x^2})) + (3\sqrt{3}\sqrt{x^2/(\sqrt{2} + \sqrt{-2 + 3x^2})^2}(\sqrt{2} + \sqrt{-2 + 3x^2}))\text{EllipticE}[2\text{ArcTan}[-2 + 3x^2]^{1/4}/2^{1/4}, 1/2]/(4\cdot 2^{3/4}x) - (3\sqrt{3}\sqrt{x^2/(\sqrt{2} + \sqrt{-2 + 3x^2})^2}(\sqrt{2} + \sqrt{-2 + 3x^2}))\text{EllipticF}[2\text{ArcTan}[-2 + 3x^2]^{1/4}/2^{1/4}, 1/2]/(8\cdot 2^{3/4}x)$

**Rubi in Sympy [A]** time = 5.71533, size = 73, normalized size = 0.3

$$-\frac{3\sqrt{6}\sqrt[4]{-\frac{3x^2}{2}} + 1E\left(\frac{\text{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2}\middle|2\right)}{8\sqrt[4]{3x^2-2}} + \frac{3(3x^2-2)^{3/4}}{8x} + \frac{(3x^2-2)^{3/4}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(3\*x\*\*2-2)\*\*(1/4), x)

[Out]  $-3\sqrt{6}(-3x^{2/2} + 1)^{(1/4)}\text{elliptic}_e(\text{asin}(\sqrt{6}x/2)/2, 2)/(8(3x^2 - 2)^{(1/4)}) + 3(3x^2 - 2)^{(3/4)}/(8x) + (3x^2 - 2)^{(3/4)}/(6x^3)$

**Mathematica [C]** time = 0.034633, size = 71, normalized size = 0.29

$$\frac{4(27x^4 - 6x^2 - 8) - 27 \cdot 2^{3/4} x^4 \sqrt[4]{2 - 3x^2} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right)}{96x^3 \sqrt[4]{3x^2 - 2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(-2 + 3\*x^2)^(1/4)), x]

[Out]  $(4(-8 - 6x^2 + 27x^4) - 27 \cdot 2^{3/4} x^4 (2 - 3x^2)^{(1/4)} \text{Hypergeometric2F1}[1/4, 1/2, 3/2, (3x^2)/2]) / (96x^3 (-2 + 3x^2)^{(1/4)})$

**Maple [C]** time = 0.062, size = 67, normalized size = 0.3

$$\frac{27x^4 - 6x^2 - 8}{24x^3} \frac{1}{\sqrt[4]{3x^2 - 2}} - \frac{9 \cdot 2^{3/4} x}{32} \sqrt[4]{-\text{signum}\left(-1 + \frac{3x^2}{2}\right)} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right) \frac{1}{\sqrt[4]{\text{signum}\left(-1 + \frac{3x^2}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(3\*x^2-2)^(1/4), x)

[Out]  $1/24 \cdot (27x^4 - 6x^2 - 8) / x^3 / (3x^2 - 2)^{(1/4)} - 9/32 \cdot 2^{3/4} / \text{signum}(-1 + 3/2x^2)^{(1/4)} \cdot (-\text{signum}(-1 + 3/2x^2))^{(1/4)} \cdot x \cdot \text{hypergeom}([1/4, 1/2], [3/2], 3/2x^2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 - 2)^{\frac{1}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((3\*x^2 - 2)^(1/4)\*x^4), x, algorithm="maxima")

[Out] integrate(1/((3\*x^2 - 2)^(1/4)\*x^4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(3x^2 - 2)^{\frac{1}{4}} x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((3\*x^2 - 2)^(1/4)\*x^4), x, algorithm="fricas")

[Out] integral(1/((3\*x^2 - 2)^(1/4)\*x^4), x)

---

**Sympy [A]** time = 2.93385, size = 34, normalized size = 0.14

$$\frac{2^{\frac{3}{4}} e^{-\frac{5i\pi}{4}} {}_2F_1\left(-\frac{3}{2}, \frac{1}{4} \middle| \frac{3x^2}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(3\*x\*\*2-2)\*\*(1/4), x)

[Out] 2\*\*(3/4)\*exp(-5\*I\*pi/4)\*hyper((-3/2, 1/4), (-1/2, ), 3\*x\*\*2/2)/(6\*x\*\*3)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 - 2)^{\frac{1}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((3\*x^2 - 2)^(1/4)\*x^4), x, algorithm="giac")

[Out] integrate(1/((3\*x^2 - 2)^(1/4)\*x^4), x)

$$3.897 \quad \int \frac{1}{x^6 \sqrt[4]{-2 + 3x^2}} dx$$

**Optimal.** Leaf size=260

$$\begin{aligned} & -\frac{189\sqrt[4]{3x^2-2}x}{160(\sqrt{3x^2-2}+\sqrt{2})} + \frac{63(3x^2-2)^{3/4}}{160x} \\ & -\frac{63\sqrt{3}\sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}}(\sqrt{3x^2-2}+\sqrt{2})F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt{2}}\right)\middle|\frac{1}{2}\right)}{160\cdot 2^{3/4}x} \\ & +\frac{63\sqrt{3}\sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}}(\sqrt{3x^2-2}+\sqrt{2})E\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt{2}}\right)\middle|\frac{1}{2}\right)}{80\cdot 2^{3/4}x} + \frac{(3x^2-2)^{3/4}}{10x^5} + \frac{7(3x^2-2)^{3/4}}{40x^3} \end{aligned}$$

[Out]  $(-2 + 3x^2)^{3/4}/(10x^5) + (7(-2 + 3x^2)^{3/4})/(40x^3) + (63(-2 + 3x^2)^{3/4})/(160x) - (189x(-2 + 3x^2)^{1/4})/(160(\sqrt{2} + \sqrt{-2 + 3x^2})) + (63\sqrt{3}\sqrt{x^2/(\sqrt{2} + \sqrt{-2 + 3x^2})^2}(\sqrt{2} + \sqrt{-2 + 3x^2})\text{EllipticE}[2\text{ArcTan}[-2 + 3x^2]^{1/4}/2^{1/4}], 1/2))/(80\cdot 2^{3/4}x) - (63\sqrt{3}\sqrt{x^2/(\sqrt{2} + \sqrt{-2 + 3x^2})^2}(\sqrt{2} + \sqrt{-2 + 3x^2})\text{EllipticF}[2\text{ArcTan}[-2 + 3x^2]^{1/4}/2^{1/4}], 1/2))/(160\cdot 2^{3/4}x)$

**Rubi [A]** time = 0.327219, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\begin{aligned} & -\frac{189\sqrt[4]{3x^2-2}x}{160(\sqrt{3x^2-2}+\sqrt{2})} + \frac{63(3x^2-2)^{3/4}}{160x} \\ & -\frac{63\sqrt{3}\sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}}(\sqrt{3x^2-2}+\sqrt{2})F\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt{2}}\right)\middle|\frac{1}{2}\right)}{160\cdot 2^{3/4}x} \\ & +\frac{63\sqrt{3}\sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}}(\sqrt{3x^2-2}+\sqrt{2})E\left(2\tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt{2}}\right)\middle|\frac{1}{2}\right)}{80\cdot 2^{3/4}x} + \frac{(3x^2-2)^{3/4}}{10x^5} + \frac{7(3x^2-2)^{3/4}}{40x^3} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(-2 + 3\*x^2)^(1/4)), x]

[Out]  $(-2 + 3x^2)^{3/4}/(10x^5) + (7(-2 + 3x^2)^{3/4})/(40x^3) + (63(-2 + 3x^2)^{3/4})/(160x) - (189x(-2 + 3x^2)^{1/4})/(160(\sqrt{2} + \sqrt{-2 + 3x^2})) + (63\sqrt{3}\sqrt{x^2/(\sqrt{2} + \sqrt{-2 + 3x^2})^2}(\sqrt{2} + \sqrt{-2 + 3x^2})\text{EllipticE}[2\text{ArcTan}[-2 + 3x^2]^{1/4}/2^{1/4}], 1/2))/(80\cdot 2^{3/4}x) - (63\sqrt{3}\sqrt{x^2/(\sqrt{2} + \sqrt{-2 + 3x^2})^2}(\sqrt{2} + \sqrt{-2 + 3x^2})\text{EllipticF}[2\text{ArcTan}[-2 + 3x^2]^{1/4}/2^{1/4}], 1/2))/(160\cdot 2^{3/4}x)$

**Rubi in Sympy [A]** time = 7.42916, size = 90, normalized size = 0.35

$$-\frac{63\sqrt{6}\sqrt[4]{-\frac{3x^2}{2}+1}E\left(\frac{\text{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2}\middle|2\right)}{160\sqrt[4]{3x^2-2}} + \frac{63(3x^2-2)^{3/4}}{160x} + \frac{7(3x^2-2)^{3/4}}{40x^3} + \frac{(3x^2-2)^{3/4}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*6/(3\*x\*\*2-2)\*\*(1/4), x)



[Out]  $-63 \sqrt{6} (-3x^{2/2} + 1)^{1/4} \text{elliptic}_e(\text{asin}(\sqrt{6}x/2)/2, 2)/(160(3x^{2/2} - 2)^{1/4}) + 63(3x^{2/2} - 2)^{3/4}/(160x) + 7(3x^{2/2} - 2)^{3/4}/(40x^3) + (3x^{2/2} - 2)^{3/4}/(10x^5)$

**Mathematica [C]** time = 0.0372006, size = 76, normalized size = 0.29

$$\frac{4(189x^6 - 42x^4 - 8x^2 - 32) - 189 \cdot 2^{3/4} x^6 \sqrt[4]{2 - 3x^2} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right)}{640x^5 \sqrt[4]{3x^2 - 2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6\*(-2 + 3\*x^2)^(1/4)), x]

[Out]  $(4(-32 - 8x^2 - 42x^4 + 189x^6) - 189 \cdot 2^{3/4} x^6 (2 - 3x^2)^{1/4} \text{Hypergeometric2F1}[1/4, 1/2, 3/2, (3x^2)/2]) / (640x^5 (-2 + 3x^2)^{1/4})$

**Maple [C]** time = 0.053, size = 72, normalized size = 0.3

$$\frac{189x^6 - 42x^4 - 8x^2 - 32}{160x^5} \frac{1}{\sqrt[4]{3x^2 - 2}} - \frac{189 \cdot 2^{3/4} x}{640} \sqrt[4]{-\text{signum}\left(-1 + \frac{3x^2}{2}\right)} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2}{2}\right) \frac{1}{\sqrt[4]{\text{signum}\left(-1 + \frac{3x^2}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(3\*x^2-2)^(1/4), x)

[Out]  $1/160 * (189 * x^6 - 42 * x^4 - 8 * x^2 - 32) / x^5 / (3 * x^2 - 2)^{1/4} - 189 / 640 * 2^{3/4} / \text{signum}(-1 + 3/2 * x^2)^{1/4} * (-\text{signum}(-1 + 3/2 * x^2))^{1/4} * x * \text{hypergeom}([1/4, 1/2], [3/2], 3/2 * x^2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 - 2)^{1/4} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((3\*x^2 - 2)^(1/4)\*x^6), x, algorithm="maxima")

[Out] integrate(1/((3\*x^2 - 2)^(1/4)\*x^6), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(3x^2 - 2)^{1/4} x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((3\*x^2 - 2)^(1/4)\*x^6), x, algorithm="fricas")

[Out] `integral(1/((3*x^2 - 2)^(1/4)*x^6), x)`

**Sympy [A]** time = 3.77551, size = 34, normalized size = 0.13

$$\frac{2^{\frac{3}{4}} e^{-\frac{5i\pi}{4}} {}_2F_1\left(-\frac{5}{2}, \frac{1}{4} \middle| -\frac{3}{2} \middle| \frac{3x^2}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**6/(3*x**2-2)**(1/4), x)`

[Out] `2**(3/4)*exp(-5*I*pi/4)*hyper((-5/2, 1/4), (-3/2,), 3*x**2/2)/(10*x**5)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 - 2)^{\frac{1}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 - 2)^(1/4)*x^6), x, algorithm="giac")`

[Out] `integrate(1/((3*x^2 - 2)^(1/4)*x^6), x)`

$$3.898 \quad \int \frac{x^6}{\sqrt[4]{-2-3x^2}} dx$$

**Optimal.** Leaf size=260

$$\begin{aligned} & -\frac{32(-3x^2-2)^{3/4}x}{1053} - \frac{128\sqrt[4]{-3x^2-2}x}{1053(\sqrt{-3x^2-2}+\sqrt{2})} \\ & + \frac{64\sqrt[4]{2}\sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}}(\sqrt{-3x^2-2}+\sqrt{2})F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)\middle|\frac{1}{2}\right)}{1053\sqrt{3}x} \\ & - \frac{128\sqrt[4]{2}\sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}}(\sqrt{-3x^2-2}+\sqrt{2})E\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)\middle|\frac{1}{2}\right)}{1053\sqrt{3}x} \\ & - \frac{2}{39}(-3x^2-2)^{3/4}x^5 + \frac{40(-3x^2-2)^{3/4}x^3}{1053} \end{aligned}$$

[Out]  $(-32*x*(-2-3*x^2)^(3/4))/1053 + (40*x^3*(-2-3*x^2)^(3/4))/1053 - (2*x^5*(-2-3*x^2)^(3/4))/39 - (128*x*(-2-3*x^2)^(1/4))/(1053*(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])) - (128*2^(1/4)*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])^2))*(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])*EllipticE[2*\text{ArcTan}[(-2-3*x^2)^(1/4)/2^(1/4)], 1/2])/(1053*\text{Sqrt}[3]*x) + (64*2^(1/4)*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])^2))*(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])*EllipticF[2*\text{ArcTan}[(-2-3*x^2)^(1/4)/2^(1/4)], 1/2])/(1053*\text{Sqrt}[3]*x)$

**Rubi [A]** time = 0.343778, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\begin{aligned} & -\frac{32(-3x^2-2)^{3/4}x}{1053} - \frac{128\sqrt[4]{-3x^2-2}x}{1053(\sqrt{-3x^2-2}+\sqrt{2})} \\ & + \frac{64\sqrt[4]{2}\sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}}(\sqrt{-3x^2-2}+\sqrt{2})F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)\middle|\frac{1}{2}\right)}{1053\sqrt{3}x} \\ & - \frac{128\sqrt[4]{2}\sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}}(\sqrt{-3x^2-2}+\sqrt{2})E\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)\middle|\frac{1}{2}\right)}{1053\sqrt{3}x} \\ & - \frac{2}{39}(-3x^2-2)^{3/4}x^5 + \frac{40(-3x^2-2)^{3/4}x^3}{1053} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^6/(-2-3\*x^2)^(1/4),x]

[Out]  $(-32*x*(-2-3*x^2)^(3/4))/1053 + (40*x^3*(-2-3*x^2)^(3/4))/1053 - (2*x^5*(-2-3*x^2)^(3/4))/39 - (128*x*(-2-3*x^2)^(1/4))/(1053*(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])) - (128*2^(1/4)*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])^2))*(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])*EllipticE[2*\text{ArcTan}[(-2-3*x^2)^(1/4)/2^(1/4)], 1/2])/(1053*\text{Sqrt}[3]*x) + (64*2^(1/4)*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])^2))*(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])*EllipticF[2*\text{ArcTan}[(-2-3*x^2)^(1/4)/2^(1/4)], 1/2])/(1053*\text{Sqrt}[3]*x)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{2x^5(-3x^2-2)^{3/4}}{39} + \frac{40x^3(-3x^2-2)^{3/4}}{1053} - \frac{32x(-3x^2-2)^{3/4}}{1053} - \frac{64\int\frac{1}{\sqrt[4]{-3x^2-2}}dx}{1053}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**6/(-3*x**2-2)**(1/4),x)`

[Out]  $-2*x**5*(-3*x**2-2)**(3/4)/39 + 40*x**3*(-3*x**2-2)**(3/4)/1053 - 32*x*(-3*x**2-2)**(3/4)/1053 - 64*Integral((-3*x**2-2)**(-1/4),x)/1053$

**Mathematica [C]** time = 0.0432041, size = 68, normalized size = 0.26

$$\frac{2x \left( -16 \sqrt[4]{3x^2 + 2} {}_2F_1 \left( \frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2} \right) + 81x^6 - 6x^4 + 8x^2 + 32 \right)}{1053 \sqrt[4]{-3x^2 - 2}}$$

Antiderivative was successfully verified.

[In] `Integrate[x^6/(-2-3*x^2)^(1/4),x]`

[Out]  $(2*x*(32 + 8*x^2 - 6*x^4 + 81*x^6 - 16*2^(3/4)*(2 + 3*x^2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (-3*x^2)/2]))/(1053*(-2 - 3*x^2)^(1/4))$

**Maple [C]** time = 0.033, size = 53, normalized size = 0.2

$$\frac{2x(27x^4 - 20x^2 + 16)(3x^2 + 2)}{1053} \frac{1}{\sqrt[4]{-3x^2 - 2}} + \frac{32(-1)^{3/4}2^{3/4}x}{1053} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(-3*x^2-2)^(1/4),x)`

[Out]  $2/1053*x*(27*x^4-20*x^2+16)*(3*x^2+2)/(-3*x^2-2)^(1/4)+32/1053*(-1)^(3/4)*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],-3/2*x^2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(-3x^2 - 2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(-3*x^2-2)^(1/4),x,algorithm="maxima")`

[Out] `integrate(x^6/(-3*x^2-2)^(1/4),x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{3159 \operatorname{xintegral}\left(\frac{256(-3x^2-2)^{\frac{3}{4}}}{3159(3x^4+2x^2)}, x\right) - 2(81x^6 - 60x^4 + 48x^2 - 64)(-3x^2 - 2)^{\frac{3}{4}}}{3159x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(-3*x^2-2)^(1/4),x,algorithm="fricas")`

[Out]  $\frac{1}{3159} (3159x \operatorname{integral}(256/3159(-3x^2 - 2)^{3/4}/(3x^4 + 2x^2), x) - 2(81x^6 - 60x^4 + 48x^2 - 64)(-3x^2 - 2)^{3/4})/x$

**Sympy [A]** time = 2.8078, size = 34, normalized size = 0.13

$$\frac{2^{\frac{3}{4}} x^7 e^{-\frac{i\pi}{4}} {}_2F_1\left(\frac{1}{4}, \frac{7}{2} \middle| \frac{3x^2 e^{i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(-3*x**2-2)**(1/4),x)`

[Out]  $2^{3/4} x^{7/2} \exp(-I\pi/4) \operatorname{hyper}((1/4, 7/2), (9/2, ), 3x^2 \exp_p \operatorname{olar}(I\pi)/2)/14$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(-3x^2 - 2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(-3*x^2 - 2)^(1/4),x, algorithm="giac")`

[Out] `integrate(x^6/(-3*x^2 - 2)^(1/4), x)`

$$3.899 \quad \int \frac{x^4}{\sqrt[4]{-2-3x^2}} dx$$

**Optimal.** Leaf size=242

$$\begin{aligned} & \frac{8}{135} (-3x^2 - 2)^{3/4} x + \frac{32\sqrt[4]{-3x^2 - 2}x}{135(\sqrt{-3x^2 - 2} + \sqrt{2})} \\ & - \frac{16\sqrt[4]{2} \sqrt{-\frac{x^2}{(\sqrt{-3x^2 - 2} + \sqrt{2})^2}} (\sqrt{-3x^2 - 2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2 - 2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{135\sqrt{3}x} \\ & + \frac{32\sqrt[4]{2} \sqrt{-\frac{x^2}{(\sqrt{-3x^2 - 2} + \sqrt{2})^2}} (\sqrt{-3x^2 - 2} + \sqrt{2}) E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2 - 2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{135\sqrt{3}x} - \frac{2}{27} (-3x^2 - 2)^{3/4} x^3 \end{aligned}$$

[Out] (8\*x\*(-2 - 3\*x^2)^(3/4))/135 - (2\*x^3\*(-2 - 3\*x^2)^(3/4))/27 + (3\*2\*x\*(-2 - 3\*x^2)^(1/4))/(135\*(Sqrt[2] + Sqrt[-2 - 3\*x^2])) + (32\*2^(1/4)\*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3\*x^2]))^2])\*(Sqrt[2] + Sqrt[-2 - 3\*x^2])\*EllipticE[2\*ArcTan[(-2 - 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(135\*Sqrt[3]\*x) - (16\*2^(1/4)\*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3\*x^2]))^2])\*(Sqrt[2] + Sqrt[-2 - 3\*x^2])\*EllipticF[2\*ArcTan[(-2 - 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(135\*Sqrt[3]\*x)

**Rubi [A]** time = 0.289821, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\begin{aligned} & \frac{8}{135} (-3x^2 - 2)^{3/4} x + \frac{32\sqrt[4]{-3x^2 - 2}x}{135(\sqrt{-3x^2 - 2} + \sqrt{2})} \\ & - \frac{16\sqrt[4]{2} \sqrt{-\frac{x^2}{(\sqrt{-3x^2 - 2} + \sqrt{2})^2}} (\sqrt{-3x^2 - 2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2 - 2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{135\sqrt{3}x} \\ & + \frac{32\sqrt[4]{2} \sqrt{-\frac{x^2}{(\sqrt{-3x^2 - 2} + \sqrt{2})^2}} (\sqrt{-3x^2 - 2} + \sqrt{2}) E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2 - 2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{135\sqrt{3}x} - \frac{2}{27} (-3x^2 - 2)^{3/4} x^3 \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^4/(-2 - 3\*x^2)^(1/4), x]

[Out] (8\*x\*(-2 - 3\*x^2)^(3/4))/135 - (2\*x^3\*(-2 - 3\*x^2)^(3/4))/27 + (3\*2\*x\*(-2 - 3\*x^2)^(1/4))/(135\*(Sqrt[2] + Sqrt[-2 - 3\*x^2])) + (32\*2^(1/4)\*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3\*x^2]))^2])\*(Sqrt[2] + Sqrt[-2 - 3\*x^2])\*EllipticE[2\*ArcTan[(-2 - 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(135\*Sqrt[3]\*x) - (16\*2^(1/4)\*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3\*x^2]))^2])\*(Sqrt[2] + Sqrt[-2 - 3\*x^2])\*EllipticF[2\*ArcTan[(-2 - 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(135\*Sqrt[3]\*x)

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{2x^3(-3x^2-2)^{\frac{3}{4}}}{27} + \frac{8x(-3x^2-2)^{\frac{3}{4}}}{135} + \frac{32x}{135\sqrt[4]{-3x^2-2}} + \frac{32 \int \frac{1}{(-3x^2-2)^{\frac{5}{4}}} dx}{135}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(-3\*x\*\*2-2)\*\*(1/4), x)

[Out]  $-2x^{3/4}(-3x^2 - 2)^{3/4}/27 + 8x(-3x^2 - 2)^{3/4}/135 + 32x/(135(-3x^2 - 2)^{1/4}) + 32 \operatorname{Integral}((-3x^2 - 2)^{-5/4}, x)/135$

**Mathematica [C]** time = 0.0374198, size = 63, normalized size = 0.26

$$\frac{2x \left( 4 \cdot 2^{3/4} \sqrt[4]{3x^2 + 2} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right) + 15x^4 - 2x^2 - 8 \right)}{135 \sqrt[4]{-3x^2 - 2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(-2 - 3\*x^2)^(1/4), x]

[Out]  $(2x^4(-8 - 2x^2 + 15x^4 + 4 \cdot 2^{3/4} \sqrt[4]{3x^2 + 2}) \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, (-3x^2)/2\right]) / (135(-2 - 3x^2)^{1/4})$

**Maple [C]** time = 0.028, size = 48, normalized size = 0.2

$$\frac{2x(5x^2 - 4)(3x^2 + 2)}{135} \frac{1}{\sqrt[4]{-3x^2 - 2}} - \frac{8(-1)^{3/4} 2^{3/4} x}{135} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(-3\*x^2-2)^(1/4), x)

[Out]  $2/135x^4(5x^2-4)(3x^2+2)/(-3x^2-2)^{1/4} - 8/135(-1)^{3/4} 2^{3/4} x \operatorname{hypergeom}\left([\frac{1}{4}, \frac{1}{2}], [\frac{3}{2}], -3/2x^2\right)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(-3x^2 - 2)^{1/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-3\*x^2 - 2)^(1/4), x, algorithm="maxima")

[Out] integrate(x^4/(-3\*x^2 - 2)^(1/4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{405 \operatorname{xintegral}\left(-\frac{64(-3x^2-2)^{3/4}}{405(3x^4+2x^2)}, x\right) - 2(15x^4 - 12x^2 + 16)(-3x^2 - 2)^{3/4}}{405x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-3\*x^2 - 2)^(1/4), x, algorithm="fricas")

[Out]  $1/405(405x^4 \operatorname{integral}(-64/405(-3x^2 - 2)^{3/4}/(3x^4 + 2x^2), x) - 2(15x^4 - 12x^2 + 16)(-3x^2 - 2)^{3/4})/x$

**Sympy [A]** time = 2.34293, size = 34, normalized size = 0.14

$$\frac{2^{\frac{3}{4}} x^5 e^{-\frac{i\pi}{4}} {}_2F_1\left(\frac{1}{4}, \frac{5}{2} \middle| \frac{3x^2 e^{i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4/(-3\*x\*\*2-2)\*\*(1/4), x)

[Out] 2\*\*(3/4)\*x\*\*5\*exp(-I\*pi/4)\*hyper((1/4, 5/2), (7/2, ), 3\*x\*\*2\*exp\_polar(I\*pi)/2)/10

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(-3x^2 - 2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-3\*x^2 - 2)^(1/4), x, algorithm="giac")

[Out] integrate(x^4/(-3\*x^2 - 2)^(1/4), x)



$$3.900 \quad \int \frac{x^2}{\sqrt[4]{-2-3x^2}} dx$$

**Optimal.** Leaf size=224

$$\begin{aligned} & -\frac{2}{15}(-3x^2-2)^{3/4}x - \frac{8\sqrt[4]{-3x^2-2}x}{15(\sqrt{-3x^2-2}+\sqrt{2})} \\ & + \frac{4\sqrt[4]{2}\sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}}(\sqrt{-3x^2-2}+\sqrt{2})F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)\middle|\frac{1}{2}\right)}{15\sqrt{3}x} \\ & - \frac{8\sqrt[4]{2}\sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}}(\sqrt{-3x^2-2}+\sqrt{2})E\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)\middle|\frac{1}{2}\right)}{15\sqrt{3}x} \end{aligned}$$

[Out]  $(-2*x*(-2-3*x^2)^{(3/4)})/15 - (8*x*(-2-3*x^2)^{(1/4)})/(15*(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])) - (8*2^{(1/4)}*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])^2)])*(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])* \text{EllipticE}[2*\text{ArcTan}[-(2-3*x^2)^{(1/4)}/2^{(1/4)}], 1/2])/(15*\text{Sqrt}[3]*x) + (4*2^{(1/4)}*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])^2)])*(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])* \text{EllipticF}[2*\text{ArcTan}[-(2-3*x^2)^{(1/4)}/2^{(1/4)}], 1/2])/(15*\text{Sqrt}[3]*x)$

**Rubi [A]** time = 0.247308, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\begin{aligned} & -\frac{2}{15}(-3x^2-2)^{3/4}x - \frac{8\sqrt[4]{-3x^2-2}x}{15(\sqrt{-3x^2-2}+\sqrt{2})} \\ & + \frac{4\sqrt[4]{2}\sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}}(\sqrt{-3x^2-2}+\sqrt{2})F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)\middle|\frac{1}{2}\right)}{15\sqrt{3}x} \\ & - \frac{8\sqrt[4]{2}\sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}}(\sqrt{-3x^2-2}+\sqrt{2})E\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right)\middle|\frac{1}{2}\right)}{15\sqrt{3}x} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^2/(-2-3\*x^2)^(1/4),x]

[Out]  $(-2*x*(-2-3*x^2)^{(3/4)})/15 - (8*x*(-2-3*x^2)^{(1/4)})/(15*(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])) - (8*2^{(1/4)}*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])^2)])*(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])* \text{EllipticE}[2*\text{ArcTan}[-(2-3*x^2)^{(1/4)}/2^{(1/4)}], 1/2])/(15*\text{Sqrt}[3]*x) + (4*2^{(1/4)}*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])^2)])*(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])* \text{EllipticF}[2*\text{ArcTan}[-(2-3*x^2)^{(1/4)}/2^{(1/4)}], 1/2])/(15*\text{Sqrt}[3]*x)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2x(-3x^2-2)^{3/4}}{15} - \frac{4\int\frac{1}{\sqrt[4]{-3x^2-2}}dx}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(-3\*x\*\*2-2)\*\*(1/4),x)

[Out]  $-2*x*(-3*x^2 - 2)^{(3/4)}/15 - 4*\text{Integral}((-3*x^2 - 2)^{(-1/4)}, x)/15$

**Mathematica [C]** time = 0.0334917, size = 58, normalized size = 0.26

$$\frac{2x \left( -2^{3/4} \sqrt[4]{3x^2 + 2} {}_2F_1 \left( \frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2} \right) + 3x^2 + 2 \right)}{15 \sqrt[4]{-3x^2 - 2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(-2 - 3\*x^2)^(1/4), x]

[Out]  $(2*x*(2 + 3*x^2 - 2^{(3/4)}*(2 + 3*x^2)^{(1/4)}*\text{Hypergeometric2F1}[1/4, 1/2, 3/2, (-3*x^2)/2])/(15*(-2 - 3*x^2)^{(1/4)})$

**Maple [C]** time = 0.029, size = 41, normalized size = 0.2

$$\frac{2x(3x^2 + 2)}{15} \frac{1}{\sqrt[4]{-3x^2 - 2}} + \frac{2(-1)^{3/4} 2^{3/4} x}{15} {}_2F_1 \left( \frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-3\*x^2-2)^(1/4), x)

[Out]  $2/15*x*(3*x^2+2)/(-3*x^2-2)^{(1/4)}+2/15*(-1)^{(3/4)}*2^{(3/4)}*x*\text{hypergeom}([1/4, 1/2], [3/2], -3/2*x^2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-3x^2 - 2)^{1/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-3\*x^2 - 2)^(1/4), x, algorithm="maxima")

[Out] integrate(x^2/(-3\*x^2 - 2)^(1/4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{45x \text{integral} \left( \frac{16(-3x^2-2)^{3/4}}{45(3x^2+2x^2)}, x \right) - 2(3x^2 - 4)(-3x^2 - 2)^{3/4}}{45x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-3\*x^2 - 2)^(1/4), x, algorithm="fricas")

[Out]  $1/45*(45*x*\text{integral}(16/45*(-3*x^2 - 2)^{(3/4)}/(3*x^4 + 2*x^2), x) - 2*(3*x^2 - 4)*(-3*x^2 - 2)^{(3/4)})/x$

**Sympy [A]** time = 2.1867, size = 34, normalized size = 0.15

$$\frac{2^{\frac{3}{4}} x^3 e^{-\frac{i\pi}{4}} {}_2F_1\left(\frac{1}{4}, \frac{3}{2} \middle| \frac{3x^2 e^{i\pi}}{2} \right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(-3\*x\*\*2-2)\*\*(1/4), x)

[Out] 2\*\*(3/4)\*x\*\*3\*exp(-I\*pi/4)\*hyper((1/4, 3/2), (5/2, ), 3\*x\*\*2\*exp\_polar(I\*pi)/2)/6

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-3x^2 - 2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-3\*x^2 - 2)^(1/4), x, algorithm="giac")

[Out] integrate(x^2/(-3\*x^2 - 2)^(1/4), x)

$$3.901 \quad \int \frac{1}{\sqrt[4]{-2-3x^2}} dx$$

**Optimal.** Leaf size=202

$$\frac{2\sqrt[4]{-3x^2-2x}}{\sqrt{-3x^2-2}+\sqrt{2}} - \frac{\sqrt[4]{2} \sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2}+\sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{\sqrt{3}x} + \frac{2\sqrt[4]{2} \sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2}+\sqrt{2}) E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{\sqrt{3}x}$$

[Out]  $(2*x*(-2-3*x^2)^{(1/4)})/(\text{Sqrt}[2]+\text{Sqrt}[-2-3*x^2])+(2*2^{(1/4)})*\text{Sqrt}[-(x^2/(\text{Sqrt}[2]+\text{Sqrt}[-2-3*x^2])^2)]*(\text{Sqrt}[2]+\text{Sqrt}[-2-3*x^2])* \text{EllipticE}[2*\text{ArcTan}[(-2-3*x^2)^{(1/4)}/2^{(1/4)}],1/2])/(\text{Sqrt}[3]*x)-(2^{(1/4)})*\text{Sqrt}[-(x^2/(\text{Sqrt}[2]+\text{Sqrt}[-2-3*x^2])^2)]*(\text{Sqrt}[2]+\text{Sqrt}[-2-3*x^2])* \text{EllipticF}[2*\text{ArcTan}[(-2-3*x^2)^{(1/4)}/2^{(1/4)}],1/2])/(\text{Sqrt}[3]*x)$

**Rubi [A]** time = 0.20747, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$\frac{2\sqrt[4]{-3x^2-2x}}{\sqrt{-3x^2-2}+\sqrt{2}} - \frac{\sqrt[4]{2} \sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2}+\sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{\sqrt{3}x} + \frac{2\sqrt[4]{2} \sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2}+\sqrt{2}) E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{\sqrt{3}x}$$

Antiderivative was successfully verified.

[In] Int[(-2-3\*x^2)^(-1/4),x]

[Out]  $(2*x*(-2-3*x^2)^{(1/4)})/(\text{Sqrt}[2]+\text{Sqrt}[-2-3*x^2])+(2*2^{(1/4)})*\text{Sqrt}[-(x^2/(\text{Sqrt}[2]+\text{Sqrt}[-2-3*x^2])^2)]*(\text{Sqrt}[2]+\text{Sqrt}[-2-3*x^2])* \text{EllipticE}[2*\text{ArcTan}[(-2-3*x^2)^{(1/4)}/2^{(1/4)}],1/2])/(\text{Sqrt}[3]*x)-(2^{(1/4)})*\text{Sqrt}[-(x^2/(\text{Sqrt}[2]+\text{Sqrt}[-2-3*x^2])^2)]*(\text{Sqrt}[2]+\text{Sqrt}[-2-3*x^2])* \text{EllipticF}[2*\text{ArcTan}[(-2-3*x^2)^{(1/4)}/2^{(1/4)}],1/2])/(\text{Sqrt}[3]*x)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2x}{\sqrt[4]{-3x^2-2}} + 2 \int \frac{1}{(-3x^2-2)^{5/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-3\*x\*\*2-2)\*\*(1/4),x)

[Out]  $2*x/(-3*x**2-2)**(1/4)+2*\text{Integral}((-3*x**2-2)**(-5/4),x)$

**Mathematica [C]** time = 0.0190902, size = 41, normalized size = 0.2

$$\frac{x\sqrt[4]{3x^2+2} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{2}; -\frac{3x^2}{2}\right)}{\sqrt[4]{-6x^2-4}}$$

Antiderivative was successfully verified.

[In] Integrate[(-2 - 3\*x^2)^(-1/4), x]

[Out] (x\*(2 + 3\*x^2)^(1/4)\*Hypergeometric2F1[1/4, 1/2, 3/2, (-3\*x^2)/2])/(-4 - 6\*x^2)^(1/4)

**Maple [C]** time = 0.014, size = 21, normalized size = 0.1

$$-\frac{(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} x}{2} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-3\*x^2-2)^(1/4), x)

[Out] -1/2\*(-1)^(3/4)\*2^(3/4)\*x\*hypergeom([1/4, 1/2], [3/2], -3/2\*x^2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 - 2)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3\*x^2 - 2)^(-1/4), x, algorithm="maxima")

[Out] integrate((-3\*x^2 - 2)^(-1/4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{3x \operatorname{integral}\left(-\frac{4(-3x^2-2)^{\frac{3}{4}}}{3(3x^4+2x^2)}, x\right) - 2(-3x^2-2)^{\frac{3}{4}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3\*x^2 - 2)^(-1/4), x, algorithm="fricas")

[Out] 1/3\*(3\*x\*integral(-4/3\*(-3\*x^2 - 2)^(3/4)/(3\*x^4 + 2\*x^2), x) - 2\*(-3\*x^2 - 2)^(3/4))/x

**Sympy [A]** time = 2.01644, size = 32, normalized size = 0.16

$$\frac{2^{\frac{3}{4}} x e^{-\frac{i\pi}{4}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{3x^2 e^{i\pi}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3\*x\*\*2-2)\*\*(1/4), x)

[Out]  $2^{3/4} x \exp(-i\pi/4) \operatorname{hyper}((1/4, 1/2), (3/2, ), 3x^2) \exp(i\pi/2) / 2$

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 - 2)^{1/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3*x^2 - 2)^(-1/4), x, algorithm="giac")`

[Out] `integrate((-3*x^2 - 2)^(-1/4), x)`

$$3.902 \quad \int \frac{1}{x^2 \sqrt[4]{-2-3x^2}} dx$$

**Optimal.** Leaf size=223

$$\begin{aligned} & \frac{3\sqrt[4]{-3x^2-2x}}{2(\sqrt{-3x^2-2}+\sqrt{2})} + \frac{(-3x^2-2)^{3/4}}{2x} \\ & - \frac{\sqrt{3} \sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2}+\sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{2 \cdot 2^{3/4} x} \\ & + \frac{\sqrt{3} \sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2}+\sqrt{2}) E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{2^{3/4} x} \end{aligned}$$

[Out]  $(-2 - 3*x^2)^{(3/4)}/(2*x) + (3*x*(-2 - 3*x^2)^{(1/4)})/(2*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])) + (\text{Sqrt}[3]*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2]))^2])*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])* \text{EllipticE}[2*\text{ArcTan}[(-2 - 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2])/(2^{(3/4)}*x) - (\text{Sqrt}[3]*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2]))^2])*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])* \text{EllipticF}[2*\text{ArcTan}[(-2 - 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2])/(2*2^{(3/4)}*x)$

**Rubi [A]** time = 0.24765, antiderivative size = 223, normalized size of antiderivative = 1., number of rules used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\begin{aligned} & \frac{3\sqrt[4]{-3x^2-2x}}{2(\sqrt{-3x^2-2}+\sqrt{2})} + \frac{(-3x^2-2)^{3/4}}{2x} \\ & - \frac{\sqrt{3} \sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2}+\sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{2 \cdot 2^{3/4} x} \\ & + \frac{\sqrt{3} \sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2}+\sqrt{2}) E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{2^{3/4} x} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(-2-3\*x^2)^(1/4)),x]

[Out]  $(-2 - 3*x^2)^{(3/4)}/(2*x) + (3*x*(-2 - 3*x^2)^{(1/4)})/(2*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])) + (\text{Sqrt}[3]*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2]))^2])*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])* \text{EllipticE}[2*\text{ArcTan}[(-2 - 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2])/(2^{(3/4)}*x) - (\text{Sqrt}[3]*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2]))^2])*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])* \text{EllipticF}[2*\text{ArcTan}[(-2 - 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2])/(2*2^{(3/4)}*x)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{3 \int \frac{1}{\sqrt[4]{-3x^2-2}} dx}{4} + \frac{(-3x^2-2)^{3/4}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(-3\*x\*\*2-2)\*\*(1/4),x)

[Out]  $3*\text{Integral}((-3*x**2 - 2)**(-1/4), x)/4 + (-3*x**2 - 2)**(3/4)/(2*x)$

---

**Mathematica [C]** time = 0.0288769, size = 63, normalized size = 0.28

$$\frac{3 \cdot 2^{3/4} \sqrt[4]{3x^2 + 2} x^2 {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right) - 12x^2 - 8}{8x \sqrt[4]{-3x^2 - 2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(-2 - 3\*x^2)^(1/4)), x]

[Out] (-8 - 12\*x^2 + 3\*2^(3/4)\*x^2\*(2 + 3\*x^2)^(1/4)\*Hypergeometric2F1[1/4, 1/2, 3/2, (-3\*x^2)/2])/(8\*x\*(-2 - 3\*x^2)^(1/4))

---

**Maple [C]** time = 0.031, size = 43, normalized size = 0.2

$$-\frac{3x^2 + 2}{2x} \frac{1}{\sqrt[4]{-3x^2 - 2}} - \frac{3(-1)^{3/4} 2^{3/4} x}{8} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(-3\*x^2-2)^(1/4), x)

[Out] -1/2\*(3\*x^2+2)/x/(-3\*x^2-2)^(1/4)-3/8\*(-1)^(3/4)\*2^(3/4)\*x\*hypergeom([1/4, 1/2], [3/2], -3/2\*x^2)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 - 2)^{1/4} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-3\*x^2 - 2)^(1/4)\*x^2), x, algorithm="maxima")

[Out] integrate(1/((-3\*x^2 - 2)^(1/4)\*x^2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2x \operatorname{xintegral}\left(-\frac{3(-3x^2-2)^{3/4}}{4(3x^2+2)}, x\right) + (-3x^2-2)^{3/4}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-3\*x^2 - 2)^(1/4)\*x^2), x, algorithm="fricas")

[Out] 1/2\*(2\*x\*integral(-3/4\*(-3\*x^2 - 2)^(3/4)/(3\*x^2 + 2), x) + (-3\*x^2 - 2)^(3/4))/x

---

**Sympy [A]** time = 2.29205, size = 36, normalized size = 0.16

$$\frac{2^{3/4} e^{3i\pi/4} {}_2F_1\left(-\frac{1}{2}, \frac{1}{4}; \frac{1}{2}; \frac{3x^2 e^{i\pi}}{2}\right)}{2x}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(-3\*x\*\*2-2)\*\*(1/4),x)

[Out] 2\*\*(3/4)\*exp(3\*I\*pi/4)\*hyper((-1/2, 1/4), (1/2,), 3\*x\*\*2\*exp\_polar(I\*pi)/2)/(2\*x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 - 2)^{\frac{1}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-3\*x^2 - 2)^(1/4)\*x^2),x, algorithm="giac")

[Out] integrate(1/((-3\*x^2 - 2)^(1/4)\*x^2), x)

$$3.903 \quad \int \frac{1}{x^4 \sqrt[4]{-2-3x^2}} dx$$

**Optimal.** Leaf size=244

$$\begin{aligned} & \frac{9\sqrt[4]{-3x^2-2}x}{8(\sqrt{-3x^2-2}+\sqrt{2})} - \frac{3(-3x^2-2)^{3/4}}{8x} \\ & + \frac{3\sqrt{3}\sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}}(\sqrt{-3x^2-2}+\sqrt{2})F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt{2}}\right)\middle|\frac{1}{2}\right)}{8\cdot 2^{3/4}x} \\ & - \frac{3\sqrt{3}\sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}}(\sqrt{-3x^2-2}+\sqrt{2})E\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt{2}}\right)\middle|\frac{1}{2}\right)}{4\cdot 2^{3/4}x} + \frac{(-3x^2-2)^{3/4}}{6x^3} \end{aligned}$$

[Out]  $(-2 - 3*x^2)^{(3/4)}/(6*x^3) - (3*(-2 - 3*x^2)^{(3/4)})/(8*x) - (9*x*(-2 - 3*x^2)^{(1/4)})/(8*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])) - (3*\text{Sqrt}[3]*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])^2)]*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2]))*\text{EllipticE}[2*\text{ArcTan}[(-2 - 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2]/(4*2^{(3/4)*x}) + (3*\text{Sqrt}[3]*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])^2)]*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2]))*\text{EllipticF}[2*\text{ArcTan}[(-2 - 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2]/(8*2^{(3/4)*x})$

**Rubi [A]** time = 0.292479, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\begin{aligned} & \frac{9\sqrt[4]{-3x^2-2}x}{8(\sqrt{-3x^2-2}+\sqrt{2})} - \frac{3(-3x^2-2)^{3/4}}{8x} \\ & + \frac{3\sqrt{3}\sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}}(\sqrt{-3x^2-2}+\sqrt{2})F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt{2}}\right)\middle|\frac{1}{2}\right)}{8\cdot 2^{3/4}x} \\ & - \frac{3\sqrt{3}\sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}}(\sqrt{-3x^2-2}+\sqrt{2})E\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt{2}}\right)\middle|\frac{1}{2}\right)}{4\cdot 2^{3/4}x} + \frac{(-3x^2-2)^{3/4}}{6x^3} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(-2-3\*x^2)^(1/4)),x]

[Out]  $(-2 - 3*x^2)^{(3/4)}/(6*x^3) - (3*(-2 - 3*x^2)^{(3/4)})/(8*x) - (9*x*(-2 - 3*x^2)^{(1/4)})/(8*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])) - (3*\text{Sqrt}[3]*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])^2)]*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2]))*\text{EllipticE}[2*\text{ArcTan}[(-2 - 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2]/(4*2^{(3/4)*x}) + (3*\text{Sqrt}[3]*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])^2)]*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2]))*\text{EllipticF}[2*\text{ArcTan}[(-2 - 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2]/(8*2^{(3/4)*x})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{9x}{8\sqrt[4]{-3x^2-2}} - \frac{9\int\frac{1}{(-3x^2-2)^{5/4}}dx}{8} - \frac{3(-3x^2-2)^{3/4}}{8x} + \frac{(-3x^2-2)^{3/4}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(-3\*x\*\*2-2)\*\*(1/4),x)

[Out]  $-9*x/(8*(-3*x^2-2)^{(1/4)}) - 9*\text{Integral}((-3*x^2-2)^{(-5/4)},x)/8 - 3*(-3*x^2-2)^{(3/4)}/(8*x) + (-3*x^2-2)^{(3/4)}/(6*x^3)$

\* 3)

---

**Mathematica [C]** time = 0.0364797, size = 71, normalized size = 0.29

$$\frac{4(27x^4 + 6x^2 - 8) - 27 \cdot 2^{3/4} x^4 \sqrt[4]{3x^2 + 2} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right)}{96x^3 \sqrt[4]{-3x^2 - 2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(-2 - 3\*x^2)^(1/4)), x]

[Out] (4\*(-8 + 6\*x^2 + 27\*x^4) - 27\*2^(3/4)\*x^4\*(2 + 3\*x^2)^(1/4)\*Hypergeometric2F1[1/4, 1/2, 3/2, (-3\*x^2)/2])/(96\*x^3\*(-2 - 3\*x^2)^(1/4))

---

**Maple [C]** time = 0.033, size = 48, normalized size = 0.2

$$\frac{27x^4 + 6x^2 - 8}{24x^3} \frac{1}{\sqrt[4]{-3x^2 - 2}} + \frac{9(-1)^{3/4} 2^{3/4} x}{32} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(-3\*x^2-2)^(1/4), x)

[Out] 1/24\*(27\*x^4+6\*x^2-8)/x^3/(-3\*x^2-2)^(1/4)+9/32\*(-1)^(3/4)\*2^(3/4)\*x\*hypergeom([1/4, 1/2], [3/2], -3/2\*x^2)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 - 2)^{\frac{1}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-3\*x^2 - 2)^(1/4)\*x^4), x, algorithm="maxima")

[Out] integrate(1/((-3\*x^2 - 2)^(1/4)\*x^4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{24x^3 \operatorname{integral}\left(\frac{9(-3x^2-2)^{\frac{3}{4}}}{16(3x^2+2)}, x\right) - (9x^2 - 4)(-3x^2 - 2)^{\frac{3}{4}}}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-3\*x^2 - 2)^(1/4)\*x^4), x, algorithm="fricas")

[Out] 1/24\*(24\*x^3\*integral(9/16\*(-3\*x^2 - 2)^(3/4)/(3\*x^2 + 2), x) - (9\*x^2 - 4)\*(-3\*x^2 - 2)^(3/4))/x^3

---

**Sympy [A]** time = 2.77158, size = 39, normalized size = 0.16

$$\frac{2^{\frac{3}{4}} e^{\frac{3i\pi}{4}} {}_2F_1\left(-\frac{3}{2}, \frac{1}{4} \middle| \frac{3x^2 e^{i\pi}}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(-3\*x\*\*2-2)\*\*(1/4),x)

[Out] 2\*\*(3/4)\*exp(3\*I\*pi/4)\*hyper((-3/2, 1/4), (-1/2, ), 3\*x\*\*2\*exp\_polar(I\*pi)/2)/(6\*x\*\*3)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 - 2)^{\frac{1}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-3\*x^2 - 2)^(1/4)\*x^4),x, algorithm="giac")

[Out] integrate(1/((-3\*x^2 - 2)^(1/4)\*x^4), x)

$$3.904 \quad \int \frac{1}{x^6 \sqrt[4]{-2-3x^2}} dx$$

**Optimal.** Leaf size=262

$$\begin{aligned} & \frac{189\sqrt[4]{-3x^2-2}x}{160(\sqrt{-3x^2-2}+\sqrt{2})} + \frac{63(-3x^2-2)^{3/4}}{160x} \\ & - \frac{63\sqrt{3}\sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}}(\sqrt{-3x^2-2}+\sqrt{2})F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt{2}}\right)\middle|\frac{1}{2}\right)}{160\cdot 2^{3/4}x} \\ & + \frac{63\sqrt{3}\sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}}(\sqrt{-3x^2-2}+\sqrt{2})E\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt{2}}\right)\middle|\frac{1}{2}\right)}{80\cdot 2^{3/4}x} \\ & + \frac{(-3x^2-2)^{3/4}}{10x^5} - \frac{7(-3x^2-2)^{3/4}}{40x^3} \end{aligned}$$

[Out]  $(-2 - 3*x^2)^{(3/4)}/(10*x^5) - (7*(-2 - 3*x^2)^{(3/4)})/(40*x^3) + (63*(-2 - 3*x^2)^{(3/4)})/(160*x) + (189*x*(-2 - 3*x^2)^{(1/4)})/(160*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])) + (63*\text{Sqrt}[3]*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])^2)]*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])*EllipticE[2*\text{ArcTan}[(-2 - 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2])/(80*2^{(3/4)}*x) - (63*\text{Sqrt}[3]*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])^2)]*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])*EllipticF[2*\text{ArcTan}[(-2 - 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2])/(160*2^{(3/4)}*x)$

**Rubi [A]** time = 0.337877, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\begin{aligned} & \frac{189\sqrt[4]{-3x^2-2}x}{160(\sqrt{-3x^2-2}+\sqrt{2})} + \frac{63(-3x^2-2)^{3/4}}{160x} \\ & - \frac{63\sqrt{3}\sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}}(\sqrt{-3x^2-2}+\sqrt{2})F\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt{2}}\right)\middle|\frac{1}{2}\right)}{160\cdot 2^{3/4}x} \\ & + \frac{63\sqrt{3}\sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}}(\sqrt{-3x^2-2}+\sqrt{2})E\left(2\tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt{2}}\right)\middle|\frac{1}{2}\right)}{80\cdot 2^{3/4}x} \\ & + \frac{(-3x^2-2)^{3/4}}{10x^5} - \frac{7(-3x^2-2)^{3/4}}{40x^3} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(-2-3\*x^2)^(1/4)),x]

[Out]  $(-2 - 3*x^2)^{(3/4)}/(10*x^5) - (7*(-2 - 3*x^2)^{(3/4)})/(40*x^3) + (63*(-2 - 3*x^2)^{(3/4)})/(160*x) + (189*x*(-2 - 3*x^2)^{(1/4)})/(160*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])) + (63*\text{Sqrt}[3]*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])^2)]*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])*EllipticE[2*\text{ArcTan}[(-2 - 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2])/(80*2^{(3/4)}*x) - (63*\text{Sqrt}[3]*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])^2)]*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])*EllipticF[2*\text{ArcTan}[(-2 - 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2])/(160*2^{(3/4)}*x)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{189 \int \frac{1}{\sqrt[4]{-3x^2-2}} dx}{320} + \frac{63(-3x^2-2)^{\frac{3}{4}}}{160x} - \frac{7(-3x^2-2)^{\frac{3}{4}}}{40x^3} + \frac{(-3x^2-2)^{\frac{3}{4}}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**6/(-3*x**2-2)**(1/4),x)`

[Out]  $189 \cdot \text{Integral}((-3x^2 - 2)^{-1/4}, x)/320 + 63 \cdot (-3x^2 - 2)^{3/4}/(160x) - 7 \cdot (-3x^2 - 2)^{3/4}/(40x^3) + (-3x^2 - 2)^{3/4}/(10x^5)$

**Mathematica [C]** time = 0.0418448, size = 76, normalized size = 0.29

$$\frac{189 \cdot 2^{3/4} x^6 \sqrt[4]{3x^2 + 2} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right) - 4(189x^6 + 42x^4 - 8x^2 + 32)}{640x^5 \sqrt[4]{-3x^2 - 2}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^6*(-2-3*x^2)^(1/4)),x]`

[Out]  $(-4(32 - 8x^2 + 42x^4 + 189x^6) + 189 \cdot 2^{3/4} x^6 (2 + 3x^2)^{1/4}) \cdot \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, (-3x^2)/2\right] / (640x^5 (-2 - 3x^2)^{1/4})$

**Maple [C]** time = 0.034, size = 53, normalized size = 0.2

$$-\frac{189x^6 + 42x^4 - 8x^2 + 32}{160x^5} \frac{1}{\sqrt[4]{-3x^2 - 2}} - \frac{189(-1)^{3/4} 2^{3/4} x}{640} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^6/(-3*x^2-2)^(1/4),x)`

[Out]  $-1/160 \cdot (189x^6 + 42x^4 - 8x^2 + 32)/x^5 / (-3x^2 - 2)^{1/4} - 189/640 \cdot (-1)^{3/4} \cdot 2^{3/4} \cdot x \cdot \text{hypergeom}\left[\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{3}{2}\right], -\frac{3}{2}x^2\right]$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 - 2)^{1/4} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-3*x^2-2)^(1/4)*x^6),x, algorithm="maxima")`

[Out] `integrate(1/((-3*x^2-2)^(1/4)*x^6),x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{160x^5 \text{integral}\left(-\frac{189(-3x^2-2)^{3/4}}{320(3x^2+2)}, x\right) + (63x^4 - 28x^2 + 16)(-3x^2 - 2)^{3/4}}{160x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-3*x^2-2)^(1/4)*x^6),x, algorithm="fricas")`

[Out]  $\frac{1}{160} \cdot (160 \cdot x^5 \cdot \text{integral}(-189/320 \cdot (-3 \cdot x^2 - 2)^{3/4} / (3 \cdot x^2 + 2), x) + (63 \cdot x^4 - 28 \cdot x^2 + 16) \cdot (-3 \cdot x^2 - 2)^{3/4}) / x^5$

**Sympy [A]** time = 3.62297, size = 39, normalized size = 0.15

$$\frac{2^{\frac{3}{4}} e^{\frac{3i\pi}{4}} {}_2F_1\left(-\frac{5}{2}, \frac{1}{4} \middle| \frac{3x^2 e^{i\pi}}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**6/(-3*x**2-2)**(1/4),x)`

[Out]  $2^{3/4} \exp(3 \cdot I \cdot \pi / 4) \cdot \text{hyper}((-5/2, 1/4), (-3/2, ), 3 \cdot x^{2} \cdot \exp(\text{pol}(\text{ar}(I \cdot \pi) / 2)) / (10 \cdot x^5))$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 - 2)^{\frac{1}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-3*x^2 - 2)^(1/4)*x^6),x, algorithm="giac")`

[Out] `integrate(1/((-3*x^2 - 2)^(1/4)*x^6), x)`

$$3.905 \quad \int \frac{x^6}{(-2+3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=138

$$\frac{160\sqrt[4]{3x^2-2x}}{2079} + \frac{160 \cdot 2^{3/4} \sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}} (\sqrt{3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{2079\sqrt{3}x} + \frac{2}{33}\sqrt[4]{3x^2-2}x^5 + \frac{40}{693}\sqrt[4]{3x^2-2}x^3$$

[Out] (160\*x\*(-2 + 3\*x^2)^(1/4))/2079 + (40\*x^3\*(-2 + 3\*x^2)^(1/4))/693 + (2\*x^5\*(-2 + 3\*x^2)^(1/4))/33 + (160\*2^(3/4)\*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3\*x^2])^2]\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])\*EllipticF[2\*ArcTan[(-2 + 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(2079\*Sqrt[3]\*x)

**Rubi [A]** time = 0.157635, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{160\sqrt[4]{3x^2-2x}}{2079} + \frac{160 \cdot 2^{3/4} \sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}} (\sqrt{3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{2079\sqrt{3}x} + \frac{2}{33}\sqrt[4]{3x^2-2}x^5 + \frac{40}{693}\sqrt[4]{3x^2-2}x^3$$

Antiderivative was successfully verified.

[In] Int[x^6/(-2 + 3\*x^2)^(3/4), x]

[Out] (160\*x\*(-2 + 3\*x^2)^(1/4))/2079 + (40\*x^3\*(-2 + 3\*x^2)^(1/4))/693 + (2\*x^5\*(-2 + 3\*x^2)^(1/4))/33 + (160\*2^(3/4)\*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3\*x^2])^2]\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])\*EllipticF[2\*ArcTan[(-2 + 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(2079\*Sqrt[3]\*x)

**Rubi in Sympy [A]** time = 7.69618, size = 92, normalized size = 0.67

$$\frac{2x^5\sqrt[4]{3x^2-2}}{33} + \frac{40x^3\sqrt[4]{3x^2-2}}{693} + \frac{160x\sqrt[4]{3x^2-2}}{2079} + \frac{640\sqrt{6} \left(-\frac{3x^2}{2} + 1\right)^{\frac{3}{4}} F\left(\frac{\arcsin\left(\frac{\sqrt{6}x}{2}\right)}{2} \middle| 2\right)}{6237(3x^2-2)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6/(3\*x\*\*2-2)\*\*(3/4), x)

[Out] 2\*x\*\*5\*(3\*x\*\*2 - 2)\*\*(1/4)/33 + 40\*x\*\*3\*(3\*x\*\*2 - 2)\*\*(1/4)/693 + 160\*x\*(3\*x\*\*2 - 2)\*\*(1/4)/2079 + 640\*sqrt(6)\*(-3\*x\*\*2/2 + 1)\*\*(3/4)\*elliptic\_f(asin(sqrt(6)\*x/2)/2, 2)/(6237\*(3\*x\*\*2 - 2)\*\*(3/4))

**Mathematica [C]** time = 0.0424723, size = 68, normalized size = 0.49

$$\frac{2x \left(80\sqrt[4]{2} (2 - 3x^2)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{3x^2}{2}\right) + 189x^6 + 54x^4 + 120x^2 - 160\right)}{2079(3x^2 - 2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(-2 + 3\*x^2)^(3/4), x]



[Out]  $(2*x*(-160 + 120*x^2 + 54*x^4 + 189*x^6 + 80*2^{(1/4)}*(2 - 3*x^2)^{(3/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, (3*x^2)/2]))/(2079*(-2 + 3*x^2)^{(3/4)})$

**Maple [C]** time = 0.06, size = 65, normalized size = 0.5

$$\frac{2x(63x^4 + 60x^2 + 80)\sqrt[4]{3x^2 - 2}}{2079} + \frac{160\sqrt[4]{2}x}{2079} \left( -\operatorname{signum}\left(-1 + \frac{3x^2}{2}\right) \right)^{\frac{3}{4}} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{3x^2}{2}\right) \left( \operatorname{signum}\left(-1 + \frac{3x^2}{2}\right) \right)^{-\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(3*x^2-2)^(3/4), x)`

[Out]  $2/2079*x*(63*x^4+60*x^2+80)*(3*x^2-2)^{(1/4)}+160/2079*2^{(1/4)}/\operatorname{signum}(-1+3/2*x^2)^{(3/4)}*(-\operatorname{signum}(-1+3/2*x^2))^{(3/4)}*x*\operatorname{hypergeom}([1/2, 3/4], [3/2], 3/2*x^2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(3x^2 - 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(3*x^2 - 2)^(3/4), x, algorithm="maxima")`

[Out] `integrate(x^6/(3*x^2 - 2)^(3/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^6}{(3x^2 - 2)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(3*x^2 - 2)^(3/4), x, algorithm="fricas")`

[Out] `integral(x^6/(3*x^2 - 2)^(3/4), x)`

**Sympy [A]** time = 2.72882, size = 31, normalized size = 0.22

$$\frac{\sqrt[4]{2}x^7 e^{\frac{13iz}{4}} {}_2F_1\left(\frac{3}{4}, \frac{7}{2}; \frac{9}{2}; \frac{3x^2}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(3*x**2-2)**(3/4), x)`

[Out]  $2^{(1/4)}*x**7*\exp(13*I*pi/4)*\operatorname{hyper}((3/4, 7/2), (9/2, ), 3*x**2/2)/14$

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(3x^2 - 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(3*x^2 - 2)^(3/4), x, algorithm="giac")`

[Out] `integrate(x^6/(3*x^2 - 2)^(3/4), x)`

$$3.906 \quad \int \frac{x^4}{(-2+3x^2)^{3/4}} dx$$

Optimal. Leaf size=120

$$\frac{8}{63} \sqrt[4]{3x^2-2} + \frac{8 \cdot 2^{3/4} \sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}} (\sqrt{3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{63\sqrt{3}x} + \frac{2}{21} \sqrt[4]{3x^2-2} x^3$$

[Out] (8\*x\*(-2 + 3\*x^2)^(1/4))/63 + (2\*x^3\*(-2 + 3\*x^2)^(1/4))/21 + (8\*2^(3/4)\*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3\*x^2])^2]\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])\*EllipticF[2\*ArcTan[(-2 + 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(63\*Sqrt[3]\*x)

Rubi [A] time = 0.126044, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{8}{63} \sqrt[4]{3x^2-2} + \frac{8 \cdot 2^{3/4} \sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}} (\sqrt{3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{63\sqrt{3}x} + \frac{2}{21} \sqrt[4]{3x^2-2} x^3$$

Antiderivative was successfully verified.

[In] Int[x^4/(-2 + 3\*x^2)^(3/4), x]

[Out] (8\*x\*(-2 + 3\*x^2)^(1/4))/63 + (2\*x^3\*(-2 + 3\*x^2)^(1/4))/21 + (8\*2^(3/4)\*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3\*x^2])^2]\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])\*EllipticF[2\*ArcTan[(-2 + 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(63\*Sqrt[3]\*x)

Rubi in Sympy [A] time = 5.82716, size = 75, normalized size = 0.62

$$\frac{2x^3 \sqrt[4]{3x^2-2}}{21} + \frac{8x \sqrt[4]{3x^2-2}}{63} + \frac{32\sqrt{6} \left(-\frac{3x^2}{2} + 1\right)^{3/4} F\left(\frac{\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2} \middle| 2\right)}{189(3x^2-2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(3\*x\*\*2-2)\*\*(3/4), x)

[Out] 2\*x\*\*3\*(3\*x\*\*2 - 2)\*\*(1/4)/21 + 8\*x\*(3\*x\*\*2 - 2)\*\*(1/4)/63 + 32\*sqr(6)\*(-3\*x\*\*2/2 + 1)\*\*(3/4)\*elliptic\_f(asin(sqrt(6)\*x/2)/2, 2)/(189\*(3\*x\*\*2 - 2)\*\*(3/4))

Mathematica [C] time = 0.0352269, size = 63, normalized size = 0.52

$$\frac{2x \left(4\sqrt[4]{2} (2 - 3x^2)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{3x^2}{2}\right) + 9x^4 + 6x^2 - 8\right)}{63(3x^2 - 2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(-2 + 3\*x^2)^(3/4), x]

[Out] (2\*x\*(-8 + 6\*x^2 + 9\*x^4 + 4\*2^(1/4)\*(2 - 3\*x^2)^(3/4)\*Hypergeometric2F1[1/2, 3/4, 3/2, (3\*x^2)/2])/(63\*(-2 + 3\*x^2)^(3/4))

---

**Maple [C]** time = 0.051, size = 60, normalized size = 0.5

$$\frac{2x(3x^2+4)}{63}\sqrt[4]{3x^2-2} + \frac{8\sqrt[4]{2}x}{63}\left(-\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{3x^2}{2}\right)\left(\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{-\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(3\*x^2-2)^(3/4), x)

[Out] 2/63\*x\*(3\*x^2+4)\*(3\*x^2-2)^(1/4)+8/63\*2^(1/4)/signum(-1+3/2\*x^2)^(3/4)\*(-signum(-1+3/2\*x^2))^(3/4)\*x\*hypergeom([1/2, 3/4], [3/2], 3/2\*x^2)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(3x^2-2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(3\*x^2 - 2)^(3/4), x, algorithm="maxima")

[Out] integrate(x^4/(3\*x^2 - 2)^(3/4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^4}{(3x^2-2)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(3\*x^2 - 2)^(3/4), x, algorithm="fricas")

[Out] integral(x^4/(3\*x^2 - 2)^(3/4), x)

---

**Sympy [A]** time = 2.30073, size = 31, normalized size = 0.26

$$\frac{\sqrt[4]{2}x^5 e^{\frac{13i\pi}{4}} {}_2F_1\left(\frac{3}{4}, \frac{5}{2}; \frac{7}{2}; \frac{3x^2}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4/(3\*x\*\*2-2)\*\*(3/4), x)

[Out] 2\*\*(1/4)\*x\*\*5\*exp(13\*I\*pi/4)\*hyper((3/4, 5/2), (7/2,), 3\*x\*\*2/2)/10

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(3x^2-2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(3*x^2 - 2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(3*x^2 - 2)^(3/4), x)
```

$$3.907 \quad \int \frac{x^2}{(-2+3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=102

$$\frac{2}{9} \sqrt[4]{3x^2-2} + \frac{2 \cdot 2^{3/4} \sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}} (\sqrt{3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{9\sqrt{3}x}$$

[Out] (2\*x\*(-2 + 3\*x^2)^(1/4))/9 + (2\*2^(3/4)\*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3\*x^2])^2]\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])\*EllipticF[2\*ArcTan[( -2 + 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(9\*Sqrt[3]\*x)

**Rubi [A]** time = 0.103013, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{2}{9} \sqrt[4]{3x^2-2} + \frac{2 \cdot 2^{3/4} \sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}} (\sqrt{3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{9\sqrt{3}x}$$

Antiderivative was successfully verified.

[In] Int[x^2/(-2 + 3\*x^2)^(3/4), x]

[Out] (2\*x\*(-2 + 3\*x^2)^(1/4))/9 + (2\*2^(3/4)\*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3\*x^2])^2]\*(Sqrt[2] + Sqrt[-2 + 3\*x^2])\*EllipticF[2\*ArcTan[( -2 + 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(9\*Sqrt[3]\*x)

**Rubi in Sympy [A]** time = 4.14287, size = 58, normalized size = 0.57

$$\frac{2x\sqrt[4]{3x^2-2}}{9} + \frac{8\sqrt{6}\left(-\frac{3x^2}{2} + 1\right)^{3/4} F\left(\frac{\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2} \middle| 2\right)}{27(3x^2-2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(3\*x\*\*2-2)\*\*(3/4), x)

[Out] 2\*x\*(3\*x\*\*2 - 2)\*\*(1/4)/9 + 8\*sqrt(6)\*(-3\*x\*\*2/2 + 1)\*\*(3/4)\*elliptic\_f(asin(sqrt(6)\*x/2)/2, 2)/(27\*(3\*x\*\*2 - 2)\*\*(3/4))

**Mathematica [C]** time = 0.0298899, size = 57, normalized size = 0.56

$$\frac{2x \left( \sqrt[4]{2} (2 - 3x^2)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{3x^2}{2}\right) + 3x^2 - 2 \right)}{9(3x^2 - 2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(-2 + 3\*x^2)^(3/4), x]

[Out] (2\*x\*(-2 + 3\*x^2 + 2^(1/4)\*(2 - 3\*x^2)^(3/4))\*Hypergeometric2F1[1/2, 3/4, 3/2, (3\*x^2)/2])/(9\*(-2 + 3\*x^2)^(3/4))

**Maple [C]** time = 0.051, size = 53, normalized size = 0.5

$$\frac{2x\sqrt[4]{3x^2-2}}{9} + \frac{2\sqrt[4]{2}x}{9} \left( -\operatorname{signum}\left(-1 + \frac{3x^2}{2}\right) \right)^{\frac{3}{4}} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{3x^2}{2}\right) \left( \operatorname{signum}\left(-1 + \frac{3x^2}{2}\right) \right)^{-\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(3*x^2-2)^(3/4), x)`

[Out] `2/9*x*(3*x^2-2)^(1/4)+2/9*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)*x*hypergeom([1/2, 3/4], [3/2], 3/2*x^2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(3x^2-2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(3*x^2-2)^(3/4), x, algorithm="maxima")`

[Out] `integrate(x^2/(3*x^2-2)^(3/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^2}{(3x^2-2)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(3*x^2-2)^(3/4), x, algorithm="fricas")`

[Out] `integral(x^2/(3*x^2-2)^(3/4), x)`

**Sympy [A]** time = 2.11622, size = 31, normalized size = 0.3

$$\frac{\sqrt[4]{2}x^3 e^{\frac{5i\pi}{4}} {}_2F_1\left(\frac{3}{4}, \frac{3}{2}; \frac{5}{2}; \frac{3x^2}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(3*x**2-2)**(3/4), x)`

[Out] `2**(1/4)*x**3*exp(5*I*pi/4)*hyper((3/4, 3/2), (5/2, ), 3*x**2/2)/6`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(3x^2-2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2 - 2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(3*x^2 - 2)^(3/4), x)
```



$$3.908 \quad \int \frac{1}{(-2+3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=82

$$\frac{\sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}} (\sqrt{3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3x}}$$

[Out] (Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3\*x^2])^2]\*(Sqrt[2] + Sqrt[-2 + 3\*x^2]))\*EllipticF[2\*ArcTan[(-2 + 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(1/4)\*Sqrt[3]\*x)

**Rubi [A]** time = 0.0685567, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{\sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}} (\sqrt{3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3x}}$$

Antiderivative was successfully verified.

[In] Int[(-2 + 3\*x^2)^(-3/4), x]

[Out] (Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3\*x^2])^2]\*(Sqrt[2] + Sqrt[-2 + 3\*x^2]))\*EllipticF[2\*ArcTan[(-2 + 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(1/4)\*Sqrt[3]\*x)

**Rubi in Sympy [A]** time = 1.53955, size = 42, normalized size = 0.51

$$\frac{2\sqrt{6} \left(-\frac{3x^2}{2} + 1\right)^{\frac{3}{4}} F\left(\frac{\arcsin\left(\frac{\sqrt{6}x}{2}\right)}{2} \middle| 2\right)}{3(3x^2 - 2)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(3\*x\*\*2-2)\*\*(3/4), x)

[Out] 2\*sqrt(6)\*(-3\*x\*\*2/2 + 1)\*\*(3/4)\*elliptic\_f(asin(sqrt(6)\*x/2)/2, 2)/(3\*(3\*x\*\*2 - 2)\*\*(3/4))

**Mathematica [C]** time = 0.0184317, size = 41, normalized size = 0.5

$$\frac{x(2-3x^2)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{3x^2}{2}\right)}{(6x^2-4)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(-2 + 3\*x^2)^(-3/4), x]

[Out] (x\*(2 - 3\*x^2)^(3/4)\*Hypergeometric2F1[1/2, 3/4, 3/2, (3\*x^2)/2])/(-4 + 6\*x^2)^(3/4)

**Maple [C]** time = 0.036, size = 40, normalized size = 0.5

$$\frac{\sqrt[4]{2}x}{2} \left( -\operatorname{signum} \left( -1 + \frac{3x^2}{2} \right) \right)^{\frac{3}{4}} {}_2F_1 \left( \frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{3x^2}{2} \right) \left( \operatorname{signum} \left( -1 + \frac{3x^2}{2} \right) \right)^{-\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3\*x^2-2)^(3/4), x)

[Out] 1/2\*2^(1/4)/signum(-1+3/2\*x^2)^(3/4)\*(-signum(-1+3/2\*x^2))^(3/4)\*x\*hypergeom([1/2, 3/4], [3/2], 3/2\*x^2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 - 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2 - 2)^(-3/4), x, algorithm="maxima")

[Out] integrate((3\*x^2 - 2)^(-3/4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{1}{(3x^2 - 2)^{\frac{3}{4}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3\*x^2 - 2)^(-3/4), x, algorithm="fricas")

[Out] integral((3\*x^2 - 2)^(-3/4), x)

**Sympy [A]** time = 2.04208, size = 29, normalized size = 0.35

$$\frac{\sqrt[4]{2}x e^{\frac{5i\pi}{4}} {}_2F_1 \left( \frac{1}{2}, \frac{3}{4} \middle| \frac{3x^2}{2} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3\*x\*\*2-2)\*\*(3/4), x)

[Out] 2\*\*(1/4)\*x\*exp(5\*I\*pi/4)\*hyper((1/2, 3/4), (3/2, ), 3\*x\*\*2/2)/2

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 - 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 - 2)^(-3/4),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 - 2)^(-3/4), x)
```

$$3.909 \quad \int \frac{1}{x^2(-2+3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=104

$$\frac{\sqrt[4]{3x^2-2}}{2x} + \frac{\sqrt{3} \sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}} (\sqrt{3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{4\sqrt[4]{2}x}$$

[Out]  $(-2 + 3*x^2)^{(1/4)}/(2*x) + (\text{Sqrt}[3]*\text{Sqrt}[x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 + 3*x^2])^2]*(\text{Sqrt}[2] + \text{Sqrt}[-2 + 3*x^2])*\text{EllipticF}[2*\text{ArcTan}[(-2 + 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2])/(4*2^{(1/4)}*x)$

**Rubi [A]** time = 0.0977231, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\sqrt[4]{3x^2-2}}{2x} + \frac{\sqrt{3} \sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}} (\sqrt{3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{4\sqrt[4]{2}x}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(-2 + 3\*x^2)^(3/4)), x]

[Out]  $(-2 + 3*x^2)^{(1/4)}/(2*x) + (\text{Sqrt}[3]*\text{Sqrt}[x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 + 3*x^2])^2]*(\text{Sqrt}[2] + \text{Sqrt}[-2 + 3*x^2])*\text{EllipticF}[2*\text{ArcTan}[(-2 + 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2])/(4*2^{(1/4)}*x)$

**Rubi in Sympy [A]** time = 3.88935, size = 54, normalized size = 0.52

$$\frac{\sqrt{6} \left(-\frac{3x^2}{2} + 1\right)^{\frac{3}{4}} F\left(\frac{\text{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2} \middle| 2\right)}{2(3x^2-2)^{\frac{3}{4}}} + \frac{\sqrt[4]{3x^2-2}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(3\*x\*\*2-2)\*\*(3/4), x)

[Out]  $\text{sqrt}(6)*(-3*x**2/2 + 1)**(3/4)*\text{elliptic\_f}(\text{asin}(\text{sqrt}(6)*x/2)/2, 2)/(2*(3*x**2 - 2)**(3/4)) + (3*x**2 - 2)**(1/4)/(2*x)$

**Mathematica [C]** time = 0.0244137, size = 63, normalized size = 0.61

$$\frac{3\sqrt[4]{2}(2-3x^2)^{3/4} x^2 {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{3x^2}{2}\right) + 12x^2 - 8}{8x(3x^2-2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(-2 + 3\*x^2)^(3/4)), x]

[Out]  $(-8 + 12*x^2 + 3*2^{(1/4)}*x^2*(2 - 3*x^2)^{(3/4)}*\text{Hypergeometric2F1}[1/2, 3/4, 3/2, (3*x^2)/2])/(8*x*(-2 + 3*x^2)^{(3/4)})$

**Maple [C]** time = 0.056, size = 55, normalized size = 0.5

$$\frac{1}{2x} \sqrt[4]{3x^2-2} + \frac{3\sqrt[4]{2}x}{8} \left( -\operatorname{signum}\left(-1 + \frac{3x^2}{2}\right) \right)^{\frac{3}{4}} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{3x^2}{2}\right) \left( \operatorname{signum}\left(-1 + \frac{3x^2}{2}\right) \right)^{-\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(3*x^2-2)^(3/4), x)`

[Out] `1/2*(3*x^2-2)^(1/4)/x+3/8*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)*x*hypergeom([1/2, 3/4], [3/2], 3/2*x^2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2-2)^{\frac{3}{4}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2-2)^(3/4)*x^2), x, algorithm="maxima")`

[Out] `integrate(1/((3*x^2-2)^(3/4)*x^2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{(3x^2-2)^{\frac{3}{4}}x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2-2)^(3/4)*x^2), x, algorithm="fricas")`

[Out] `integral(1/((3*x^2-2)^(3/4)*x^2), x)`

**Sympy [A]** time = 2.68562, size = 29, normalized size = 0.28

$$\frac{\sqrt[4]{2}e^{\frac{i\pi}{4}} {}_2F_1\left(-\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{3x^2}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(3*x**2-2)**(3/4), x)`

[Out] `2**(1/4)*exp(I*pi/4)*hyper((-1/2, 3/4), (1/2,), 3*x**2/2)/(2*x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2-2)^{\frac{3}{4}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 - 2)^(3/4)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 2)^(3/4)*x^2), x)
```

$$3.910 \quad \int \frac{1}{x^4(-2+3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=122

$$\frac{5\sqrt[4]{3x^2-2}}{8x} + \frac{5\sqrt{3} \sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}} (\sqrt{3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{16\sqrt[4]{2}x} + \frac{\sqrt[4]{3x^2-2}}{6x^3}$$

[Out]  $(-2 + 3*x^2)^{(1/4)}/(6*x^3) + (5*(-2 + 3*x^2)^{(1/4)})/(8*x) + (5*\text{Sqrt}[3]*\text{Sqrt}[x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 + 3*x^2])^2]*(\text{Sqrt}[2] + \text{Sqrt}[-2 + 3*x^2]))*\text{EllipticF}[2*\text{ArcTan}[(-2 + 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2)]/(16*2^{(1/4)*x})$

**Rubi [A]** time = 0.125933, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{5\sqrt[4]{3x^2-2}}{8x} + \frac{5\sqrt{3} \sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}} (\sqrt{3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{16\sqrt[4]{2}x} + \frac{\sqrt[4]{3x^2-2}}{6x^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(-2 + 3\*x^2)^(3/4)), x]

[Out]  $(-2 + 3*x^2)^{(1/4)}/(6*x^3) + (5*(-2 + 3*x^2)^{(1/4)})/(8*x) + (5*\text{Sqrt}[3]*\text{Sqrt}[x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 + 3*x^2])^2]*(\text{Sqrt}[2] + \text{Sqrt}[-2 + 3*x^2]))*\text{EllipticF}[2*\text{ArcTan}[(-2 + 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2)]/(16*2^{(1/4)*x})$

**Rubi in Sympy [A]** time = 5.64207, size = 73, normalized size = 0.6

$$\frac{5\sqrt{6} \left(-\frac{3x^2}{2} + 1\right)^{\frac{3}{4}} F\left(\frac{\text{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2} \middle| 2\right)}{8(3x^2-2)^{\frac{3}{4}}} + \frac{5\sqrt[4]{3x^2-2}}{8x} + \frac{\sqrt[4]{3x^2-2}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(3\*x\*\*2-2)\*\*(3/4), x)

[Out]  $5*\text{sqrt}(6)*(-3*x**2/2 + 1)**(3/4)*\text{elliptic\_f}(\text{asin}(\text{sqrt}(6)*x/2)/2, 2)/(8*(3*x**2 - 2)**(3/4)) + 5*(3*x**2 - 2)**(1/4)/(8*x) + (3*x**2 - 2)**(1/4)/(6*x**3)$

**Mathematica [C]** time = 0.0293591, size = 68, normalized size = 0.56

$$\frac{45\sqrt[4]{2} (2 - 3x^2)^{3/4} x^4 {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{3x^2}{2}\right) + 180x^4 - 72x^2 - 32}{96x^3 (3x^2 - 2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(-2 + 3\*x^2)^(3/4)), x]

[Out]  $(-32 - 72*x^2 + 180*x^4 + 45*2^{(1/4)}*x^4*(2 - 3*x^2)^{(3/4)}*\text{Hypergeometric2F1}[1/2, 3/4, 3/2, (3*x^2)/2])/(96*x^3*(-2 + 3*x^2)^{(3/4)})$

)

---

**Maple [C]** time = 0.059, size = 67, normalized size = 0.6

$$\frac{45x^4 - 18x^2 - 8}{24x^3} (3x^2 - 2)^{-\frac{3}{4}} + \frac{15\sqrt[4]{2}x}{32} \left( -\operatorname{signum}\left(-1 + \frac{3x^2}{2}\right) \right)^{\frac{3}{4}} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{3x^2}{2}\right) \left( \operatorname{signum}\left(-1 + \frac{3x^2}{2}\right) \right)^{-\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(3*x^2-2)^(3/4), x)`

[Out] `1/24*(45*x^4-18*x^2-8)/x^3/(3*x^2-2)^(3/4)+15/32*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)*x*hypergeom([1/2, 3/4], [3/2], 3/2*x^2)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 - 2)^{\frac{3}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 - 2)^(3/4)*x^4), x, algorithm="maxima")`

[Out] `integrate(1/((3*x^2 - 2)^(3/4)*x^4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{(3x^2 - 2)^{\frac{3}{4}} x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 - 2)^(3/4)*x^4), x, algorithm="fricas")`

[Out] `integral(1/((3*x^2 - 2)^(3/4)*x^4), x)`

---

**Sympy [A]** time = 3.33944, size = 34, normalized size = 0.28

$$\frac{\sqrt[4]{2}e^{-\frac{7ix}{4}} {}_2F_1\left(-\frac{3}{2}, \frac{3}{4}; -\frac{1}{2}; \frac{3x^2}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(3*x**2-2)**(3/4), x)`

[Out] `2**(1/4)*exp(-7*I*pi/4)*hyper((-3/2, 3/4), (-1/2, ), 3*x**2/2)/(6*x**3)`



---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 - 2)^{\frac{3}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 - 2)^(3/4)*x^4),x, algorithm="giac")`

[Out] `integrate(1/((3*x^2 - 2)^(3/4)*x^4), x)`

$$3.911 \quad \int \frac{1}{x^6(-2+3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=140

$$\frac{27\sqrt[4]{3x^2-2}}{32x} + \frac{27\sqrt{3} \sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}} (\sqrt{3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{64\sqrt[4]{2}x} + \frac{\sqrt[4]{3x^2-2}}{10x^5} + \frac{9\sqrt[4]{3x^2-2}}{40x^3}$$

[Out]  $(-2 + 3x^2)^{1/4}/(10x^5) + (9(-2 + 3x^2)^{1/4})/(40x^3) + (27(-2 + 3x^2)^{1/4})/(32x) + (27\sqrt{3}\sqrt{x^2/(\sqrt{2} + \sqrt{3x^2-2})^2}(\sqrt{3x^2-2} + \sqrt{2})\text{EllipticF}[2\text{ArcTan}[-2 + 3x^2]^{1/4}/2^{1/4}], 1/2))/(64\sqrt[4]{2}x)$

**Rubi [A]** time = 0.15497, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{27\sqrt[4]{3x^2-2}}{32x} + \frac{27\sqrt{3} \sqrt{\frac{x^2}{(\sqrt{3x^2-2}+\sqrt{2})^2}} (\sqrt{3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{64\sqrt[4]{2}x} + \frac{\sqrt[4]{3x^2-2}}{10x^5} + \frac{9\sqrt[4]{3x^2-2}}{40x^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(-2 + 3\*x^2)^(3/4)), x]

[Out]  $(-2 + 3x^2)^{1/4}/(10x^5) + (9(-2 + 3x^2)^{1/4})/(40x^3) + (27(-2 + 3x^2)^{1/4})/(32x) + (27\sqrt{3}\sqrt{x^2/(\sqrt{2} + \sqrt{3x^2-2})^2}(\sqrt{3x^2-2} + \sqrt{2})\text{EllipticF}[2\text{ArcTan}[-2 + 3x^2]^{1/4}/2^{1/4}], 1/2))/(64\sqrt[4]{2}x)$

**Rubi in Sympy [A]** time = 7.44869, size = 90, normalized size = 0.64

$$\frac{27\sqrt{6} \left(-\frac{3x^2}{2} + 1\right)^{3/4} F\left(\frac{\text{asin}\left(\frac{\sqrt{6}x}{2}\right)}{2} \middle| 2\right)}{32(3x^2-2)^{3/4}} + \frac{27\sqrt[4]{3x^2-2}}{32x} + \frac{9\sqrt[4]{3x^2-2}}{40x^3} + \frac{\sqrt[4]{3x^2-2}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*6/(3\*x\*\*2-2)\*\*(3/4), x)

[Out]  $27\sqrt{6}(-3x^2/2 + 1)^{3/4}\text{elliptic\_f}(\text{asin}(\sqrt{6}x/2)/2, 2)/(32(3x^2-2)^{3/4}) + 27(3x^2-2)^{1/4}/(32x) + 9(3x^2-2)^{1/4}/(40x^3) + (3x^2-2)^{1/4}/(10x^5)$

**Mathematica [C]** time = 0.0295156, size = 73, normalized size = 0.52

$$\frac{405\sqrt[4]{2} (2-3x^2)^{3/4} x^6 {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{3x^2}{2}\right) + 1620x^6 - 648x^4 - 96x^2 - 128}{640x^5 (3x^2-2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6\*(-2 + 3\*x^2)^(3/4)), x]

[Out]  $(-128 - 96x^2 - 648x^4 + 1620x^6 + 405\sqrt[4]{2}x^6(2 - 3x^2)^{3/4})\text{Hypergeometric2F1}[1/2, 3/4, 3/2, (3x^2)/2]/(640x^5(-2$

+  $3x^2)^{3/4}$ )

**Maple [C]** time = 0.056, size = 72, normalized size = 0.5

$$\frac{405x^6 - 162x^4 - 24x^2 - 32}{160x^5} (3x^2 - 2)^{-\frac{3}{4}} + \frac{81\sqrt[4]{2}x}{128} \left(-\operatorname{signum}\left(-1 + \frac{3x^2}{2}\right)\right)^{\frac{3}{4}} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{3x^2}{2}\right) \left(\operatorname{signum}\left(-1 + \frac{3x^2}{2}\right)\right)^{-\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^6/(3*x^2-2)^(3/4), x)`

[Out] `1/160*(405*x^6-162*x^4-24*x^2-32)/x^5/(3*x^2-2)^(3/4)+81/128*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)*x*hypergeom([1/2, 3/4], [3/2], 3/2*x^2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 - 2)^{\frac{3}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 - 2)^(3/4)*x^6), x, algorithm="maxima")`

[Out] `integrate(1/((3*x^2 - 2)^(3/4)*x^6), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{(3x^2 - 2)^{\frac{3}{4}} x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 - 2)^(3/4)*x^6), x, algorithm="fricas")`

[Out] `integral(1/((3*x^2 - 2)^(3/4)*x^6), x)`

**Sympy [A]** time = 4.36326, size = 34, normalized size = 0.24

$$\frac{\sqrt[4]{2}e^{-\frac{7ix}{4}} {}_2F_1\left(-\frac{5}{2}, \frac{3}{4}; -\frac{3}{2}; \frac{3x^2}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**6/(3*x**2-2)**(3/4), x)`

[Out] `2**(1/4)*exp(-7*I*pi/4)*hyper((-5/2, 3/4), (-3/2,), 3*x**2/2)/(10*x**5)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2 - 2)^{\frac{3}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((3*x^2 - 2)^(3/4)*x^6),x, algorithm="giac")`

[Out] `integrate(1/((3*x^2 - 2)^(3/4)*x^6), x)`

$$3.912 \quad \int \frac{x^6}{(-2-3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=139

$$-\frac{160\sqrt[4]{-3x^2-2x}}{2079} + \frac{160 \cdot 2^{3/4} \sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{2079\sqrt{3}x} - \frac{2}{33}\sqrt[4]{-3x^2-2}x^5 + \frac{40}{693}\sqrt[4]{-3x^2-2}x^3$$

[Out]  $(-160*x*(-2-3*x^2)^{(1/4)})/2079 + (40*x^3*(-2-3*x^2)^{(1/4)})/693 - (2*x^5*(-2-3*x^2)^{(1/4)})/33 + (160*2^{(3/4)}*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2]))^2])*(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])*\text{EllipticF}[2*\text{ArcTan}[(-2-3*x^2)^{(1/4)}/2^{(1/4)}], 1/2)]/(2079*\text{Sqrt}[3]*x)$

**Rubi [A]** time = 0.161821, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{160\sqrt[4]{-3x^2-2x}}{2079} + \frac{160 \cdot 2^{3/4} \sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{2079\sqrt{3}x} - \frac{2}{33}\sqrt[4]{-3x^2-2}x^5 + \frac{40}{693}\sqrt[4]{-3x^2-2}x^3$$

Antiderivative was successfully verified.

[In] `Int[x^6/(-2-3*x^2)^(3/4),x]`

[Out]  $(-160*x*(-2-3*x^2)^{(1/4)})/2079 + (40*x^3*(-2-3*x^2)^{(1/4)})/693 - (2*x^5*(-2-3*x^2)^{(1/4)})/33 + (160*2^{(3/4)}*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2]))^2])*(\text{Sqrt}[2] + \text{Sqrt}[-2-3*x^2])*\text{EllipticF}[2*\text{ArcTan}[(-2-3*x^2)^{(1/4)}/2^{(1/4)}], 1/2)]/(2079*\text{Sqrt}[3]*x)$

**Rubi in Sympy [A]** time = 7.52345, size = 99, normalized size = 0.71

$$-\frac{2x^5\sqrt[4]{-3x^2-2}}{33} + \frac{40x^3\sqrt[4]{-3x^2-2}}{693} - \frac{160x\sqrt[4]{-3x^2-2}}{2079} - \frac{640\sqrt{6}\left(\frac{3x^2}{2}+1\right)^{3/4} F\left(\frac{\text{atan}\left(\frac{\sqrt{6}x}{2}\right)}{2} \middle| 2\right)}{6237(-3x^2-2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**6/(-3*x**2-2)**(3/4),x)`

[Out]  $-2*x**5*(-3*x**2-2)**(1/4)/33 + 40*x**3*(-3*x**2-2)**(1/4)/693 - 160*x*(-3*x**2-2)**(1/4)/2079 - 640*\text{sqrt}(6)*(3*x**2/2+1)**(3/4)*\text{elliptic\_f}(\text{atan}(\text{sqrt}(6)*x/2)/2, 2)/(6237*(-3*x**2-2)**(3/4))$

**Mathematica [C]** time = 0.0389768, size = 68, normalized size = 0.49

$$\frac{2x\left(-80\sqrt[4]{2}(3x^2+2)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{3x^2}{2}\right) + 189x^6 - 54x^4 + 120x^2 + 160\right)}{2079(-3x^2-2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(-2 - 3\*x^2)^(3/4),x]

[Out] (2\*x\*(160 + 120\*x^2 - 54\*x^4 + 189\*x^6 - 80\*2^(1/4)\*(2 + 3\*x^2)^(3/4)\*Hypergeometric2F1[1/2, 3/4, 3/2, (-3\*x^2)/2]))/(2079\*(-2 - 3\*x^2)^(3/4))

**Maple [F]** time = 0.032, size = 0, normalized size = 0.

$$\int x^6 (-3x^2 - 2)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(-3\*x^2-2)^(3/4),x)

[Out] int(x^6/(-3\*x^2-2)^(3/4),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(-3x^2 - 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(-3\*x^2 - 2)^(3/4),x, algorithm="maxima")

[Out] integrate(x^6/(-3\*x^2 - 2)^(3/4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{2}{2079} (63x^5 - 60x^3 + 80x) (-3x^2 - 2)^{\frac{1}{4}} + \text{integral} \left( \frac{320(-3x^2 - 2)^{\frac{1}{4}}}{2079(3x^2 + 2)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(-3\*x^2 - 2)^(3/4),x, algorithm="fricas")

[Out] -2/2079\*(63\*x^5 - 60\*x^3 + 80\*x)\*(-3\*x^2 - 2)^(1/4) + integral(320/2079\*(-3\*x^2 - 2)^(1/4)/(3\*x^2 + 2), x)

**Sympy [A]** time = 2.71848, size = 36, normalized size = 0.26

$$\frac{\sqrt[4]{2}x^7 e^{-\frac{3i\pi}{4}} {}_2F_1\left(\frac{3}{4}, \frac{7}{2} \middle| \frac{3x^2 e^{i\pi}}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*6/(-3\*x\*\*2-2)\*\*(3/4),x)

[Out] 2\*\*(1/4)\*x\*\*7\*exp(-3\*I\*pi/4)\*hyper((3/4, 7/2), (9/2, ), 3\*x\*\*2\*exp\_polar(I\*pi)/2)/14

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(-3x^2 - 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(-3*x^2 - 2)^(3/4),x, algorithm="giac")`

[Out] `integrate(x^6/(-3*x^2 - 2)^(3/4), x)`

$$3.913 \quad \int \frac{x^4}{(-2-3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=121

$$\frac{8}{63} \sqrt[4]{-3x^2-2} - \frac{8 \cdot 2^{3/4} \sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2}+\sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{63\sqrt{3}x} - \frac{2}{21} \sqrt[4]{-3x^2-2} x^3$$

[Out] (8\*x\*(-2 - 3\*x^2)^(1/4))/63 - (2\*x^3\*(-2 - 3\*x^2)^(1/4))/21 - (8\*2^(3/4)\*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3\*x^2])^2)]\*(Sqrt[2] + Sqrt[-2 - 3\*x^2]))\*EllipticF[2\*ArcTan[(-2 - 3\*x^2)^(1/4)/2^(1/4)], 1/2]/(63\*Sqrt[3]\*x)

**Rubi [A]** time = 0.133272, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{8}{63} \sqrt[4]{-3x^2-2} - \frac{8 \cdot 2^{3/4} \sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2}+\sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{63\sqrt{3}x} - \frac{2}{21} \sqrt[4]{-3x^2-2} x^3$$

Antiderivative was successfully verified.

[In] Int[x^4/(-2 - 3\*x^2)^(3/4), x]

[Out] (8\*x\*(-2 - 3\*x^2)^(1/4))/63 - (2\*x^3\*(-2 - 3\*x^2)^(1/4))/21 - (8\*2^(3/4)\*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3\*x^2])^2)]\*(Sqrt[2] + Sqrt[-2 - 3\*x^2]))\*EllipticF[2\*ArcTan[(-2 - 3\*x^2)^(1/4)/2^(1/4)], 1/2]/(63\*Sqrt[3]\*x)

**Rubi in Sympy [A]** time = 5.7245, size = 80, normalized size = 0.66

$$-\frac{2x^3\sqrt[4]{-3x^2-2}}{21} + \frac{8x\sqrt[4]{-3x^2-2}}{63} + \frac{32\sqrt{6}\left(\frac{3x^2}{2}+1\right)^{3/4} F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{2} \middle| 2\right)}{189(-3x^2-2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(-3\*x\*\*2-2)\*\*(3/4), x)

[Out] -2\*x\*\*3\*(-3\*x\*\*2 - 2)\*\*(1/4)/21 + 8\*x\*(-3\*x\*\*2 - 2)\*\*(1/4)/63 + 32\*sqrt(6)\*(3\*x\*\*2/2 + 1)\*\*(3/4)\*elliptic\_f(atan(sqrt(6)\*x/2)/2, 2)/(189\*(-3\*x\*\*2 - 2)\*\*(3/4))

**Mathematica [C]** time = 0.0350301, size = 63, normalized size = 0.52

$$\frac{2x \left(4\sqrt[4]{2} (3x^2 + 2)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{3x^2}{2}\right) + 9x^4 - 6x^2 - 8\right)}{63(-3x^2 - 2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(-2 - 3\*x^2)^(3/4), x]

[Out] (2\*x\*(-8 - 6\*x^2 + 9\*x^4 + 4\*2^(1/4)\*(2 + 3\*x^2)^(3/4))\*Hypergeometric2F1[1/2, 3/4, 3/2, (-3\*x^2)/2])/63\*(-2 - 3\*x^2)^(3/4)



---

**Maple [F]** time = 0.021, size = 0, normalized size = 0.

$$\int x^4 (-3x^2 - 2)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(-3*x^2-2)^(3/4),x)`

[Out] `int(x^4/(-3*x^2-2)^(3/4),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(-3x^2 - 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(-3*x^2 - 2)^(3/4),x, algorithm="maxima")`

[Out] `integrate(x^4/(-3*x^2 - 2)^(3/4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{2}{63} (3x^3 - 4x) (-3x^2 - 2)^{\frac{1}{4}} + \text{integral} \left( -\frac{16(-3x^2 - 2)^{\frac{1}{4}}}{63(3x^2 + 2)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(-3*x^2 - 2)^(3/4),x, algorithm="fricas")`

[Out] `-2/63*(3*x^3 - 4*x)*(-3*x^2 - 2)^(1/4) + integral(-16/63*(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x)`

---

**Sympy [A]** time = 2.26923, size = 36, normalized size = 0.3

$$\frac{\sqrt[4]{2}x^5 e^{-\frac{3i\pi}{4}} {}_2F_1\left(\frac{3}{4}, \frac{5}{2} \middle| \frac{3x^2 e^{i\pi}}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(-3*x**2-2)**(3/4),x)`

[Out] `2**(1/4)*x**5*exp(-3*I*pi/4)*hyper((3/4, 5/2), (7/2, ), 3*x**2*exp_polar(I*pi)/2)/10`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(-3x^2 - 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-3*x^2 - 2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^4/(-3*x^2 - 2)^(3/4), x)
```

$$3.914 \quad \int \frac{x^2}{(-2-3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=103

$$\frac{2^{2^{3/4}} \sqrt{\frac{-x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{9\sqrt{3}x} - \frac{2}{9}x\sqrt[4]{-3x^2-2}$$

[Out]  $(-2*x*(-2 - 3*x^2)^{(1/4)})/9 + (2*2^{(3/4)}*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2]))^2])*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])*\text{EllipticF}[2*\text{ArcTan}[(-2 - 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2)]/(9*\text{Sqrt}[3]*x)$

**Rubi [A]** time = 0.103762, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{2^{2^{3/4}} \sqrt{\frac{-x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{9\sqrt{3}x} - \frac{2}{9}x\sqrt[4]{-3x^2-2}$$

Antiderivative was successfully verified.

[In] Int[x^2/(-2 - 3\*x^2)^(3/4), x]

[Out]  $(-2*x*(-2 - 3*x^2)^{(1/4)})/9 + (2*2^{(3/4)}*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2]))^2])*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])*\text{EllipticF}[2*\text{ArcTan}[(-2 - 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2)]/(9*\text{Sqrt}[3]*x)$

**Rubi in Sympy [A]** time = 4.03814, size = 63, normalized size = 0.61

$$\frac{2x\sqrt[4]{-3x^2-2}}{9} - \frac{8\sqrt{6}\left(\frac{3x^2}{2} + 1\right)^{3/4} F\left(\frac{\text{atan}\left(\frac{\sqrt{6}x}{2}\right)}{2} \middle| 2\right)}{27(-3x^2-2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(-3\*x\*\*2-2)\*\*(3/4), x)

[Out]  $-2*x*(-3*x**2 - 2)**(1/4)/9 - 8*\text{sqrt}(6)*(3*x**2/2 + 1)**(3/4)*\text{elliptic\_f}(\text{atan}(\text{sqrt}(6)*x/2)/2, 2)/(27*(-3*x**2 - 2)**(3/4))$

**Mathematica [C]** time = 0.0294007, size = 58, normalized size = 0.56

$$\frac{2x\left(-\sqrt[4]{2}(3x^2+2)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}; -\frac{3x^2}{2}\right) + 3x^2 + 2\right)}{9(-3x^2-2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(-2 - 3\*x^2)^(3/4), x]

[Out]  $(2*x*(2 + 3*x^2 - 2^{(1/4)}*(2 + 3*x^2)^{(3/4)}*\text{Hypergeometric2F1}[1/2, 3/4, 3/2, (-3*x^2)/2])/(9*(-2 - 3*x^2)^{(3/4)})$

**Maple [F]** time = 0.02, size = 0, normalized size = 0.

$$\int x^2 (-3x^2 - 2)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(-3*x^2-2)^(3/4),x)`

[Out] `int(x^2/(-3*x^2-2)^(3/4),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-3x^2 - 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-3*x^2 - 2)^(3/4),x, algorithm="maxima")`

[Out] `integrate(x^2/(-3*x^2 - 2)^(3/4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{2}{9} (-3x^2 - 2)^{\frac{1}{4}} x + \text{integral}\left(\frac{4(-3x^2 - 2)^{\frac{1}{4}}}{9(3x^2 + 2)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-3*x^2 - 2)^(3/4),x, algorithm="fricas")`

[Out] `-2/9*(-3*x^2 - 2)^(1/4)*x + integral(4/9*(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x)`

---

**Sympy [A]** time = 2.10707, size = 36, normalized size = 0.35

$$\frac{\sqrt[4]{2}x^3 e^{-\frac{3i\pi}{4}} {}_2F_1\left(\frac{3}{4}, \frac{3}{2} \middle| \frac{3x^2 e^{i\pi}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(-3*x**2-2)**(3/4),x)`

[Out] `2**(1/4)*x**3*exp(-3*I*pi/4)*hyper((3/4, 3/2), (5/2, ), 3*x**2*exp_polar(I*pi)/2)/6`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-3x^2 - 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2 - 2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-3*x^2 - 2)^(3/4), x)
```

$$3.915 \quad \int \frac{1}{(-2-3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=84

$$\frac{\sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}x}$$

[Out] -((Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3\*x^2])^2)]\*(Sqrt[2] + Sqrt[-2 - 3\*x^2]))\*EllipticF[2\*ArcTan[(-2 - 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(1/4)\*Sqrt[3]\*x)

**Rubi [A]** time = 0.0756504, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{\sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt{2}}\right) \middle| \frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}x}$$

Antiderivative was successfully verified.

[In] Int[(-2 - 3\*x^2)^(-3/4), x]

[Out] -((Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3\*x^2])^2)]\*(Sqrt[2] + Sqrt[-2 - 3\*x^2]))\*EllipticF[2\*ArcTan[(-2 - 3\*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(1/4)\*Sqrt[3]\*x)

**Rubi in Sympy [A]** time = 1.47773, size = 44, normalized size = 0.52

$$\frac{2\sqrt{6} \left(\frac{3x^2}{2} + 1\right)^{\frac{3}{4}} F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{2} \middle| 2\right)}{3(-3x^2 - 2)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-3\*x\*\*2-2)\*\*(3/4), x)

[Out] 2\*sqrt(6)\*(3\*x\*\*2/2 + 1)\*\*(3/4)\*elliptic\_f(atan(sqrt(6)\*x/2)/2, 2)/(3\*(-3\*x\*\*2 - 2)\*\*(3/4))

**Mathematica [C]** time = 0.0177082, size = 41, normalized size = 0.49

$$\frac{x(3x^2 + 2)^{3/4} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, -\frac{3x^2}{2}\right)}{(-6x^2 - 4)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(-2 - 3\*x^2)^(-3/4), x]

[Out] (x\*(2 + 3\*x^2)^(3/4)\*Hypergeometric2F1[1/2, 3/4, 3/2, (-3\*x^2)/2])/(-4 - 6\*x^2)^(3/4)

**Maple [C]** time = 0.014, size = 21, normalized size = 0.3

$$-\frac{\sqrt[4]{-1}\sqrt[4]{2}x}{2} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{3x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-3\*x^2-2)^(3/4), x)

[Out] -1/2\*(-1)^(1/4)\*2^(1/4)\*x\*hypergeom([1/2, 3/4], [3/2], -3/2\*x^2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 - 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3\*x^2 - 2)^(-3/4), x, algorithm="maxima")

[Out] integrate((-3\*x^2 - 2)^(-3/4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(-3x^2 - 2)^{\frac{1}{4}}}{3x^2 + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3\*x^2 - 2)^(-3/4), x, algorithm="fricas")

[Out] integral(-(-3\*x^2 - 2)^(1/4)/(3\*x^2 + 2), x)

**Sympy [A]** time = 1.99472, size = 34, normalized size = 0.4

$$\frac{\sqrt[4]{2}x e^{-\frac{3i\pi}{4}} {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; \frac{3x^2 e^{i\pi}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3\*x\*\*2-2)\*\*(3/4), x)

[Out] 2\*\*(1/4)\*x\*exp(-3\*I\*pi/4)\*hyper((1/2, 3/4), (3/2, ), 3\*x\*\*2\*exp\_polar(I\*pi)/2)/2

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 - 2)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2 - 2)^(-3/4),x, algorithm="giac")
```

```
[Out] integrate((-3*x^2 - 2)^(-3/4), x)
```



$$3.916 \quad \int \frac{1}{x^2(-2-3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=105

$$\frac{\sqrt[4]{-3x^2-2}}{2x} + \frac{\sqrt{3} \sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{4\sqrt[4]{2}x}$$

[Out]  $(-2 - 3*x^2)^{(1/4)}/(2*x) + (\text{Sqrt}[3]*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2]))^2])*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])*\text{EllipticF}[2*\text{ArcTan}[( - 2 - 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2)]/(4*2^{(1/4)}*x)$

**Rubi [A]** time = 0.103299, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\sqrt[4]{-3x^2-2}}{2x} + \frac{\sqrt{3} \sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{4\sqrt[4]{2}x}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(-2 - 3\*x^2)^(3/4)), x]

[Out]  $(-2 - 3*x^2)^{(1/4)}/(2*x) + (\text{Sqrt}[3]*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2]))^2])*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])*\text{EllipticF}[2*\text{ArcTan}[( - 2 - 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2)]/(4*2^{(1/4)}*x)$

**Rubi in Sympy [A]** time = 3.82858, size = 58, normalized size = 0.55

$$-\frac{\sqrt{6} \left(\frac{3x^2}{2} + 1\right)^{3/4} F\left(\frac{\text{atan}\left(\frac{\sqrt{6}x}{2}\right)}{2} \middle| 2\right)}{2(-3x^2-2)^{3/4}} + \frac{\sqrt[4]{-3x^2-2}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(-3\*x\*\*2-2)\*\*(3/4), x)

[Out]  $-\text{sqrt}(6)*(3*x**2/2 + 1)**(3/4)*\text{elliptic\_f}(\text{atan}(\text{sqrt}(6)*x/2)/2, 2)/(2*(-3*x**2 - 2)**(3/4)) + (-3*x**2 - 2)**(1/4)/(2*x)$

**Mathematica [C]** time = 0.0259372, size = 63, normalized size = 0.6

$$\frac{-3\sqrt[4]{2}(3x^2+2)^{3/4} x^2 {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{3x^2}{2}\right) - 12x^2 - 8}{8x(-3x^2-2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(-2 - 3\*x^2)^(3/4)), x]

[Out]  $(-8 - 12*x^2 - 3*2^{(1/4)}*x^2*(2 + 3*x^2)^{(3/4)}*\text{Hypergeometric2F1}[1/2, 3/4, 3/2, (-3*x^2)/2])/(8*x*(-2 - 3*x^2)^{(3/4)})$

**Maple [F]** time = 0.025, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (-3x^2 - 2)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(-3\*x^2-2)^(3/4), x)

[Out] int(1/x^2/(-3\*x^2-2)^(3/4), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 - 2)^{\frac{3}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-3\*x^2 - 2)^(3/4)\*x^2), x, algorithm="maxima")

[Out] integrate(1/((-3\*x^2 - 2)^(3/4)\*x^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2x \operatorname{xintegral}\left(\frac{3(-3x^2-2)^{\frac{1}{4}}}{4(3x^2+2)}, x\right) + (-3x^2 - 2)^{\frac{1}{4}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-3\*x^2 - 2)^(3/4)\*x^2), x, algorithm="fricas")

[Out] 1/2\*(2\*x\*integral(3/4\*(-3\*x^2 - 2)^(1/4)/(3\*x^2 + 2), x) + (-3\*x^2 - 2)^(1/4))/x

**Sympy [A]** time = 2.6721, size = 34, normalized size = 0.32

$$\frac{\sqrt[4]{2} e^{\frac{i\pi}{4}} {}_2F_1\left(-\frac{1}{2}, \frac{3}{4} \middle| \frac{3x^2 e^{i\pi}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(-3\*x\*\*2-2)\*\*(3/4), x)

[Out] 2\*\*(1/4)\*exp(I\*pi/4)\*hyper((-1/2, 3/4), (1/2,), 3\*x\*\*2\*exp\_polar(I\*pi)/2)/(2\*x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 - 2)^{\frac{3}{4}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-3*x^2 - 2)^(3/4)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(3/4)*x^2), x)
```

$$3.917 \quad \int \frac{1}{x^4(-2-3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=123

$$-\frac{5\sqrt[4]{-3x^2-2}}{8x} - \frac{5\sqrt{3} \sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{16\sqrt[4]{2}x} + \frac{\sqrt[4]{-3x^2-2}}{6x^3}$$

[Out]  $(-2 - 3*x^2)^{(1/4)}/(6*x^3) - (5*(-2 - 3*x^2)^{(1/4)})/(8*x) - (5*\text{Sqrt}[3]*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2]))^2])*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])*\text{EllipticF}[2*\text{ArcTan}[(-2 - 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2]/(16*2^{(1/4)}*x)$

**Rubi [A]** time = 0.130296, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{5\sqrt[4]{-3x^2-2}}{8x} - \frac{5\sqrt{3} \sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{16\sqrt[4]{2}x} + \frac{\sqrt[4]{-3x^2-2}}{6x^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^4*(-2 - 3*x^2)^{(3/4)}), x]$

[Out]  $(-2 - 3*x^2)^{(1/4)}/(6*x^3) - (5*(-2 - 3*x^2)^{(1/4)})/(8*x) - (5*\text{Sqrt}[3]*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2]))^2])*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])*\text{EllipticF}[2*\text{ArcTan}[(-2 - 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2]/(16*2^{(1/4)}*x)$

**Rubi in Sympy [A]** time = 5.54847, size = 78, normalized size = 0.63

$$\frac{5\sqrt{6} \left(\frac{3x^2}{2} + 1\right)^{3/4} F\left(\frac{\text{atan}\left(\frac{\sqrt{6}x}{2}\right)}{2} \middle| 2\right)}{8(-3x^2-2)^{3/4}} - \frac{5\sqrt[4]{-3x^2-2}}{8x} + \frac{\sqrt[4]{-3x^2-2}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{**4}/(-3*x^{**2}-2)^{(3/4)}, x)$

[Out]  $5*\text{sqrt}(6)*\left(\frac{3*x^{**2}}{2} + 1\right)^{(3/4)}*\text{elliptic\_f}(\text{atan}(\text{sqrt}(6)*x/2)/2, 2)/(8*(-3*x^{**2}-2)^{(3/4)}) - 5*(-3*x^{**2}-2)^{(1/4)}/(8*x) + (-3*x^{**2}-2)^{(1/4)}/(6*x^{**3})$

**Mathematica [C]** time = 0.029352, size = 68, normalized size = 0.55

$$\frac{45\sqrt[4]{2} (3x^2 + 2)^{3/4} x^4 {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{3x^2}{2}\right) + 180x^4 + 72x^2 - 32}{96x^3(-3x^2-2)^{3/4}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/(x^4*(-2 - 3*x^2)^{(3/4)}), x]$

[Out]  $(-32 + 72*x^2 + 180*x^4 + 45*2^{(1/4)}*x^4*(2 + 3*x^2)^{(3/4)}*\text{Hypergeometric2F1}[1/2, 3/4, 3/2, (-3*x^2)/2])/(96*x^3*(-2 - 3*x^2)^{(3/4)}$

))

---

**Maple [F]** time = 0.024, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} (-3x^2 - 2)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(-3\*x^2-2)^(3/4), x)

[Out] int(1/x^4/(-3\*x^2-2)^(3/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 - 2)^{\frac{3}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-3\*x^2 - 2)^(3/4)\*x^4), x, algorithm="maxima")

[Out] integrate(1/((-3\*x^2 - 2)^(3/4)\*x^4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{24x^3 \operatorname{integral}\left(-\frac{15(-3x^2-2)^{\frac{1}{4}}}{16(3x^2+2)}, x\right) - (15x^2 - 4)(-3x^2 - 2)^{\frac{1}{4}}}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-3\*x^2 - 2)^(3/4)\*x^4), x, algorithm="fricas")

[Out] 1/24\*(24\*x^3\*integral(-15/16\*(-3\*x^2 - 2)^(1/4)/(3\*x^2 + 2), x) - (15\*x^2 - 4)\*(-3\*x^2 - 2)^(1/4))/x^3

---

**Sympy [A]** time = 3.36678, size = 37, normalized size = 0.3

$$\frac{\sqrt[4]{2} e^{\frac{i\pi}{4}} {}_2F_1\left(-\frac{3}{2}, \frac{3}{4} \middle| \frac{3x^2 e^{i\pi}}{2}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(-3\*x\*\*2-2)\*\*(3/4), x)

[Out] 2\*\*(1/4)\*exp(I\*pi/4)\*hyper((-3/2, 3/4), (-1/2, ), 3\*x\*\*2\*exp\_polar(I\*pi)/2)/(6\*x\*\*3)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 - 2)^{\frac{3}{4}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-3*x^2 - 2)^(3/4)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/((-3*x^2 - 2)^(3/4)*x^4), x)
```

$$3.918 \quad \int \frac{1}{x^6(-2-3x^2)^{3/4}} dx$$

**Optimal.** Leaf size=141

$$\frac{27\sqrt[4]{-3x^2-2}}{32x} + \frac{27\sqrt{3} \sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{64\sqrt[4]{2}x} + \frac{\sqrt[4]{-3x^2-2}}{10x^5} - \frac{9\sqrt[4]{-3x^2-2}}{40x^3}$$

[Out]  $(-2 - 3*x^2)^{(1/4)}/(10*x^5) - (9*(-2 - 3*x^2)^{(1/4)})/(40*x^3) + (27*(-2 - 3*x^2)^{(1/4)})/(32*x) + (27*\text{Sqrt}[3]*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2]))^2])*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])*\text{EllipticF}[2*\text{ArcTan}[(-2 - 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2])/(64*2^{(1/4)}*x)$

**Rubi [A]** time = 0.160488, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{27\sqrt[4]{-3x^2-2}}{32x} + \frac{27\sqrt{3} \sqrt{-\frac{x^2}{(\sqrt{-3x^2-2}+\sqrt{2})^2}} (\sqrt{-3x^2-2} + \sqrt{2}) F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right) \middle| \frac{1}{2}\right)}{64\sqrt[4]{2}x} + \frac{\sqrt[4]{-3x^2-2}}{10x^5} - \frac{9\sqrt[4]{-3x^2-2}}{40x^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(-2-3\*x^2)^(3/4)),x]

[Out]  $(-2 - 3*x^2)^{(1/4)}/(10*x^5) - (9*(-2 - 3*x^2)^{(1/4)})/(40*x^3) + (27*(-2 - 3*x^2)^{(1/4)})/(32*x) + (27*\text{Sqrt}[3]*\text{Sqrt}[-(x^2/(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2]))^2])*(\text{Sqrt}[2] + \text{Sqrt}[-2 - 3*x^2])*\text{EllipticF}[2*\text{ArcTan}[(-2 - 3*x^2)^{(1/4)}/2^{(1/4)}], 1/2])/(64*2^{(1/4)}*x)$

**Rubi in Sympy [A]** time = 7.34439, size = 97, normalized size = 0.69

$$-\frac{27\sqrt{6} \left(\frac{3x^2}{2} + 1\right)^{3/4} F\left(\frac{\text{atan}\left(\frac{\sqrt{6}x}{2}\right)}{2} \middle| 2\right)}{32(-3x^2-2)^{3/4}} + \frac{27\sqrt[4]{-3x^2-2}}{32x} - \frac{9\sqrt[4]{-3x^2-2}}{40x^3} + \frac{\sqrt[4]{-3x^2-2}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*6/(-3\*x\*\*2-2)\*\*(3/4),x)

[Out]  $-27*\text{sqrt}(6)*(3*x**2/2 + 1)**(3/4)*\text{elliptic\_f}(\text{atan}(\text{sqrt}(6)*x/2)/2, 2)/(32*(-3*x**2 - 2)**(3/4)) + 27*(-3*x**2 - 2)**(1/4)/(32*x) - 9*(-3*x**2 - 2)**(1/4)/(40*x**3) + (-3*x**2 - 2)**(1/4)/(10*x**5)$

**Mathematica [C]** time = 0.0344535, size = 76, normalized size = 0.54

$$\frac{-405\sqrt[4]{2} (3x^2 + 2)^{3/4} x^6 {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; -\frac{3x^2}{2}\right) - 4(405x^6 + 162x^4 - 24x^2 + 32)}{640x^5(-3x^2 - 2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6\*(-2 - 3\*x^2)^(3/4)),x]

[Out]  $(-4*(32 - 24*x^2 + 162*x^4 + 405*x^6) - 405*2^{(1/4)}*x^6*(2 + 3*x^2)^{(3/4)}*Hypergeometric2F1[1/2, 3/4, 3/2, (-3*x^2)/2])/(640*x^5*(-2 - 3*x^2)^{(3/4)})$

**Maple [F]** time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{x^6} (-3x^2 - 2)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(-3\*x^2-2)^(3/4),x)

[Out] int(1/x^6/(-3\*x^2-2)^(3/4),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 - 2)^{\frac{3}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-3\*x^2 - 2)^(3/4)\*x^6),x, algorithm="maxima")

[Out] integrate(1/((-3\*x^2 - 2)^(3/4)\*x^6), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{160 x^5 \operatorname{integral}\left(\frac{81(-3x^2-2)^{\frac{1}{4}}}{64(3x^2+2)}, x\right) + (135x^4 - 36x^2 + 16)(-3x^2 - 2)^{\frac{1}{4}}}{160x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-3\*x^2 - 2)^(3/4)\*x^6),x, algorithm="fricas")

[Out]  $1/160*(160*x^5*\operatorname{integral}(81/64*(-3*x^2 - 2)^{(1/4)}/(3*x^2 + 2), x) + (135*x^4 - 36*x^2 + 16)*(-3*x^2 - 2)^{(1/4)})/x^5$

**Sympy [A]** time = 4.29307, size = 37, normalized size = 0.26

$$\frac{\sqrt[4]{2} e^{\frac{i\pi}{4}} {}_2F_1\left(-\frac{5}{2}, \frac{3}{4} \middle| \frac{3x^2 e^{i\pi}}{2}\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*6/(-3\*x\*\*2-2)\*\*(3/4),x)

[Out]  $2^{**}(1/4)*\exp(I*\pi/4)*\operatorname{hyper}((-5/2, 3/4), (-3/2, ), 3*x**2*\exp\_polar(I*\pi)/2)/(10*x**5)$



---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-3x^2 - 2)^{\frac{3}{4}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-3*x^2 - 2)^(3/4)*x^6),x, algorithm="giac")`

[Out] `integrate(1/((-3*x^2 - 2)^(3/4)*x^6), x)`

### 3.919 $\int (cx)^{7/2} \sqrt[4]{a + bx^2} dx$

**Optimal.** Leaf size=152

$$\frac{a^{5/2} c^2 (cx)^{3/2} \left( \frac{a}{bx^2} + 1 \right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{12b^{3/2} (a + bx^2)^{3/4}} - \frac{a^2 c^3 \sqrt{cx} \sqrt[4]{a + bx^2}}{12b^2} + \frac{(cx)^{9/2} \sqrt[4]{a + bx^2}}{5c} + \frac{ac(cx)^{5/2} \sqrt[4]{a + bx^2}}{30b}$$

[Out]  $-(a^2 c^3 \sqrt{cx} (a + b x^2)^{1/4}) / (12 b^2) + (a c (c x)^{5/2} (a + b x^2)^{1/4}) / (30 b) + ((c x)^{9/2} (a + b x^2)^{1/4}) / (5 c) - (a^{5/2} c^2 (1 + a / (b x^2))^{3/4} (c x)^{3/2} \text{EllipticF}[\text{ArcCot}[(\sqrt{b} x) / \sqrt{a}] / 2, 2]) / (12 b^{3/2} (a + b x^2)^{3/4})$

**Rubi [A]** time = 0.311851, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$\frac{a^{5/2} c^2 (cx)^{3/2} \left( \frac{a}{bx^2} + 1 \right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{12b^{3/2} (a + bx^2)^{3/4}} - \frac{a^2 c^3 \sqrt{cx} \sqrt[4]{a + bx^2}}{12b^2} + \frac{(cx)^{9/2} \sqrt[4]{a + bx^2}}{5c} + \frac{ac(cx)^{5/2} \sqrt[4]{a + bx^2}}{30b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c x)^{(7/2)} (a + b x^2)^{(1/4)}, x]$

[Out]  $-(a^2 c^3 \sqrt{cx} (a + b x^2)^{1/4}) / (12 b^2) + (a c (c x)^{5/2} (a + b x^2)^{1/4}) / (30 b) + ((c x)^{9/2} (a + b x^2)^{1/4}) / (5 c) - (a^{5/2} c^2 (1 + a / (b x^2))^{3/4} (c x)^{3/2} \text{EllipticF}[\text{ArcCot}[(\sqrt{b} x) / \sqrt{a}] / 2, 2]) / (12 b^{3/2} (a + b x^2)^{3/4})$

**Rubi in Sympy [A]** time = 33.2618, size = 133, normalized size = 0.88

$$\frac{a^{5/2} c^2 (cx)^{3/2} \left( \frac{a}{bx^2} + 1 \right)^{3/4} F\left(\frac{\text{atan}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{12b^{3/2} (a + bx^2)^{3/4}} - \frac{a^2 c^3 \sqrt{cx} \sqrt[4]{a + bx^2}}{12b^2} + \frac{ac(cx)^{5/2} \sqrt[4]{a + bx^2}}{30b} + \frac{(cx)^{9/2} \sqrt[4]{a + bx^2}}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c x)^{(7/2)} (b x^2 + a)^{(1/4)}, x)$

[Out]  $-a^{5/2} c^2 (c x)^{3/2} (a / (b x^2) + 1)^{3/4} \text{elliptic\_f}(\text{atan}(\sqrt{a} / (\sqrt{b} x)) / 2, 2) / (12 b^{3/2} (a + b x^2)^{3/4}) - a^2 c^3 \sqrt{c x} (a + b x^2)^{1/4} / (12 b^2) + a c (c x)^{5/2} (a + b x^2)^{1/4} / (30 b) + (c x)^{9/2} (a + b x^2)^{1/4} / (5 c)$

**Mathematica [C]** time = 0.0680213, size = 98, normalized size = 0.64

$$\frac{c^3 \sqrt{cx} \left( 5a^3 \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{5}{4}; -\frac{bx^2}{a}\right) - 5a^3 - 3a^2 bx^2 + 14ab^2 x^4 + 12b^3 x^6 \right)}{60b^2 (a + bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(c x)^{(7/2)} (a + b x^2)^{(1/4)}, x]$

[Out]  $(c^3 \sqrt{cx} (-5 a^3 - 3 a^2 b x^2 + 14 a b^2 x^4 + 12 b^3 x^6 + 5 a^3 (1 + (b x^2) / a)^{3/4}) \text{Hypergeometric2F1}[1/4, 3/4, 5/4, -])$

$$(b \cdot x^2/a)])) / (60 \cdot b^2 \cdot (a + b \cdot x^2)^{3/4})$$

**Maple [F]** time = 0.049, size = 0, normalized size = 0.

$$\int (cx)^{\frac{7}{2}} \sqrt[4]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(7/2)\*(b\*x^2+a)^(1/4),x)

[Out] int((c\*x)^(7/2)\*(b\*x^2+a)^(1/4),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)\*(c\*x)^(7/2),x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(1/4)\*(c\*x)^(7/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^{\frac{1}{4}} \sqrt{cx} c^3 x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)\*(c\*x)^(7/2),x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(1/4)\*sqrt(c\*x)\*c^3\*x^3, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(7/2)\*(b\*x\*\*2+a)\*\*(1/4),x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(1/4)*(c*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(7/2), x)
```

### 3.920 $\int (cx)^{3/2} \sqrt[4]{a + bx^2} dx$

**Optimal.** Leaf size=118

$$\frac{a^{3/2}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{6\sqrt{b}(a + bx^2)^{3/4}} + \frac{(cx)^{5/2} \sqrt[4]{a + bx^2}}{3c} + \frac{ac\sqrt{cx} \sqrt[4]{a + bx^2}}{6b}$$

[Out] (a\*c\*Sqrt[c\*x]\*(a + b\*x^2)^(1/4))/(6\*b) + ((c\*x)^(5/2)\*(a + b\*x^2)^(1/4))/(3\*c) + (a^(3/2)\*(1 + a/(b\*x^2))^(3/4)\*(c\*x)^(3/2)\*EllipticF[ArcCot[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(6\*Sqrt[b]\*(a + b\*x^2)^(3/4))

**Rubi [A]** time = 0.242395, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$\frac{a^{3/2}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{6\sqrt{b}(a + bx^2)^{3/4}} + \frac{(cx)^{5/2} \sqrt[4]{a + bx^2}}{3c} + \frac{ac\sqrt{cx} \sqrt[4]{a + bx^2}}{6b}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(3/2)\*(a + b\*x^2)^(1/4), x]

[Out] (a\*c\*Sqrt[c\*x]\*(a + b\*x^2)^(1/4))/(6\*b) + ((c\*x)^(5/2)\*(a + b\*x^2)^(1/4))/(3\*c) + (a^(3/2)\*(1 + a/(b\*x^2))^(3/4)\*(c\*x)^(3/2)\*EllipticF[ArcCot[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(6\*Sqrt[b]\*(a + b\*x^2)^(3/4))

**Rubi in Sympy [A]** time = 26.628, size = 100, normalized size = 0.85

$$\frac{a^{3/2}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{6\sqrt{b}(a + bx^2)^{3/4}} + \frac{ac\sqrt{cx} \sqrt[4]{a + bx^2}}{6b} + \frac{(cx)^{5/2} \sqrt[4]{a + bx^2}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(3/2)\*(b\*x\*\*2+a)\*\*(1/4), x)

[Out] a\*\*(3/2)\*(c\*x)\*\*(3/2)\*(a/(b\*x\*\*2) + 1)\*\*(3/4)\*elliptic\_f(atan(sqrt(a)/(sqrt(b)\*x))/2, 2)/(6\*sqrt(b)\*(a + b\*x\*\*2)\*\*(3/4)) + a\*c\*sqrt(c\*x)\*(a + b\*x\*\*2)\*\*(1/4)/(6\*b) + (c\*x)\*\*(5/2)\*(a + b\*x\*\*2)\*\*(1/4)/(3\*c)

**Mathematica [C]** time = 0.0535818, size = 83, normalized size = 0.7

$$\frac{c\sqrt{cx} \left(-a^2 \left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{5}{4}; -\frac{bx^2}{a}\right) + a^2 + 3abx^2 + 2b^2x^4\right)}{6b(a + bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(3/2)\*(a + b\*x^2)^(1/4), x]

[Out] (c\*Sqrt[c\*x]\*(a^2 + 3\*a\*b\*x^2 + 2\*b^2\*x^4 - a^2\*(1 + (b\*x^2)/a)^(3/4)\*Hypergeometric2F1[1/4, 3/4, 5/4, -(b\*x^2)/a])/(6\*b\*(a + b

$*x^2)^{(3/4)}$

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int (cx)^{\frac{3}{2}} \sqrt[4]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(3/2)*(b*x^2+a)^(1/4),x)`

[Out] `int((c*x)^(3/2)*(b*x^2+a)^(1/4),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)*(c*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(1/4)*(c*x)^(3/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^{\frac{1}{4}} \sqrt{cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)*(c*x)^(3/2),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(1/4)*sqrt(c*x)*c*x, x)`

**Sympy [A]** time = 22.9869, size = 46, normalized size = 0.39

$$\frac{\sqrt[4]{ac^{\frac{3}{2}}x^{\frac{5}{2}} \left(\frac{5}{4}\right) {}_2F_1\left(-\frac{1}{4}, \frac{5}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2 \left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(3/2)*(b*x**2+a)**(1/4),x)`

[Out] `a**(1/4)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/4, 5/4), (9/4, ), b*x**2*exp_polar(I*pi)/a)/(2*gamma(9/4))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(1/4)*(c*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)*(c*x)^(3/2), x)
```

$$3.921 \quad \int \frac{\sqrt[4]{a+bx^2}}{\sqrt{cx}} dx$$

**Optimal.** Leaf size=89

$$\frac{\sqrt{cx}\sqrt[4]{a+bx^2}}{c} - \frac{\sqrt{a}\sqrt{b}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{c^2 (a+bx^2)^{3/4}}$$

[Out] (Sqrt[c\*x]\*(a + b\*x^2)^(1/4))/c - (Sqrt[a]\*Sqrt[b]\*(1 + a/(b\*x^2))^(3/4)\*(c\*x)^(3/2)\*EllipticF[ArcCot[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/ (c^2\*(a + b\*x^2)^(3/4))

**Rubi [A]** time = 0.20655, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$\frac{\sqrt{cx}\sqrt[4]{a+bx^2}}{c} - \frac{\sqrt{a}\sqrt{b}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{c^2 (a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/4)/Sqrt[c\*x], x]

[Out] (Sqrt[c\*x]\*(a + b\*x^2)^(1/4))/c - (Sqrt[a]\*Sqrt[b]\*(1 + a/(b\*x^2))^(3/4)\*(c\*x)^(3/2)\*EllipticF[ArcCot[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/ (c^2\*(a + b\*x^2)^(3/4))

**Rubi in Sympy [A]** time = 23.029, size = 76, normalized size = 0.85

$$\frac{\sqrt{a}\sqrt{b}(cx)^{\frac{3}{2}} \left(\frac{a}{bx^2} + 1\right)^{\frac{3}{4}} F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{c^2 (a+bx^2)^{\frac{3}{4}}} + \frac{\sqrt{cx}\sqrt[4]{a+bx^2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(1/2), x)

[Out] -sqrt(a)\*sqrt(b)\*(c\*x)\*\*(3/2)\*(a/(b\*x\*\*2) + 1)\*\*(3/4)\*elliptic\_f(atan(sqrt(a)/(sqrt(b)\*x))/2, 2)/(c\*\*2\*(a + b\*x\*\*2)\*\*(3/4)) + sqrt(c\*x)\*(a + b\*x\*\*2)\*\*(1/4)/c

**Mathematica [C]** time = 0.041432, size = 62, normalized size = 0.7

$$\frac{x \left( a \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1 \left( \frac{1}{4}, \frac{3}{4}; \frac{5}{4}; -\frac{bx^2}{a} \right) + a + bx^2 \right)}{\sqrt{cx} (a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/4)/Sqrt[c\*x], x]

[Out] (x\*(a + b\*x^2 + a\*(1 + (b\*x^2)/a)^(3/4)\*Hypergeometric2F1[1/4, 3/4, 5/4, -(b\*x^2)/a]))/(Sqrt[c\*x]\*(a + b\*x^2)^(3/4))



---

**Maple [F]** time = 0.037, size = 0, normalized size = 0.

$$\int \sqrt[4]{bx^2 + a} \frac{1}{\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(1/4)/(c\*x)^(1/2), x)

[Out] int((b\*x^2+a)^(1/4)/(c\*x)^(1/2), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{4}}}{\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)/sqrt(c\*x), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(1/4)/sqrt(c\*x), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{1}{4}}}{\sqrt{cx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)/sqrt(c\*x), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(1/4)/sqrt(c\*x), x)

---

**Sympy [A]** time = 5.12633, size = 46, normalized size = 0.52

$$\frac{\sqrt[4]{a}\sqrt{x} \left(\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{4}, \frac{1}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt{c} \left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(1/2), x)

[Out] a\*\*(1/4)\*sqrt(x)\*gamma(1/4)\*hyper((-1/4, 1/4), (5/4, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*sqrt(c)\*gamma(5/4))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{4}}}{\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(1/4)/sqrt(c*x),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/sqrt(c*x), x)
```

$$3.922 \quad \int \frac{\sqrt[4]{a+bx^2}}{(cx)^{5/2}} dx$$

**Optimal.** Leaf size=94

$$-\frac{2b^{3/2}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{3\sqrt{ac^4}(a+bx^2)^{3/4}} - \frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}$$

[Out]  $(-2*(a + b*x^2)^{(1/4)})/(3*c*(c*x)^{(3/2)}) - (2*b^{(3/2)}*(1 + a/(b*x^2))^{(3/4)}*(c*x)^{(3/2)}*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/ (3*Sqrt[a]*c^4*(a + b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.207308, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$-\frac{2b^{3/2}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{3\sqrt{ac^4}(a+bx^2)^{3/4}} - \frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/4)/(c\*x)^(5/2), x]

[Out]  $(-2*(a + b*x^2)^{(1/4)})/(3*c*(c*x)^{(3/2)}) - (2*b^{(3/2)}*(1 + a/(b*x^2))^{(3/4)}*(c*x)^{(3/2)}*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/ (3*Sqrt[a]*c^4*(a + b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 23.5084, size = 85, normalized size = 0.9

$$\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}} - \frac{2b^{3/2}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{3\sqrt{ac^4}(a+bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(5/2), x)

[Out]  $-2*(a + b*x^2)^{(1/4)}/(3*c*(c*x)^{(3/2)}) - 2*b^{(3/2)}*(c*x)^{(3/2)}*(a/(b*x^2) + 1)^{(3/4)}*elliptic_f(\operatorname{atan}(\operatorname{sqrt}(a)/(\operatorname{sqrt}(b)*x)))/2, 2)/(3*\operatorname{sqrt}(a)*c^4*(a + b*x^2)^{(3/4)})$

**Mathematica [C]** time = 0.0528007, size = 69, normalized size = 0.73

$$\frac{2x \left(-bx^2 \left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{5}{4}; -\frac{bx^2}{a}\right) + a + bx^2\right)}{3(cx)^{5/2}(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/4)/(c\*x)^(5/2), x]

[Out]  $(-2*x*(a + b*x^2 - b*x^2*(1 + (b*x^2)/a)^{(3/4)}*Hypergeometric2F1[1/4, 3/4, 5/4, -(b*x^2)/a]))/(3*(c*x)^{(5/2)}*(a + b*x^2)^{(3/4)})$

---

**Maple [F]** time = 0.042, size = 0, normalized size = 0.

$$\int \sqrt[4]{bx^2 + a} (cx)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(1/4)/(c\*x)^(5/2), x)

[Out] int((b\*x^2+a)^(1/4)/(c\*x)^(5/2), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)/(c\*x)^(5/2), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(1/4)/(c\*x)^(5/2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{1}{4}}}{\sqrt{cx}c^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)/(c\*x)^(5/2), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(1/4)/(sqrt(c\*x)\*c^2\*x^2), x)

---

**Sympy [A]** time = 56.3209, size = 32, normalized size = 0.34

$$\frac{\sqrt[4]{b} {}_2F_1\left(-\frac{1}{4}, \frac{1}{2} \mid \frac{ae^{i\pi}}{bx^2}\right)}{c^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(5/2), x)

[Out] -b\*\*(1/4)\*hyper((-1/4, 1/2), (3/2,), a\*exp\_polar(I\*pi)/(b\*x\*\*2))/(c\*\*(5/2)\*x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(1/4)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(5/2), x)
```

$$3.923 \quad \int \frac{\sqrt[4]{a+bx^2}}{(cx)^{9/2}} dx$$

**Optimal.** Leaf size=123

$$\frac{4b^{5/2}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{21a^{3/2}c^6(a+bx^2)^{3/4}} - \frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}} - \frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}$$

[Out]  $(-2*(a + b*x^2)^{(1/4)})/(7*c*(c*x)^{(7/2)}) - (2*b*(a + b*x^2)^{(1/4)})/(21*a*c^3*(c*x)^{(3/2)}) + (4*b^{5/2}*(1 + a/(b*x^2))^{3/4}*(c*x)^{(3/2)}*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^{3/2}*c^6*(a + b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.256944, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$\frac{4b^{5/2}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{21a^{3/2}c^6(a+bx^2)^{3/4}} - \frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}} - \frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/4)/(c\*x)^(9/2), x]

[Out]  $(-2*(a + b*x^2)^{(1/4)})/(7*c*(c*x)^{(7/2)}) - (2*b*(a + b*x^2)^{(1/4)})/(21*a*c^3*(c*x)^{(3/2)}) + (4*b^{5/2}*(1 + a/(b*x^2))^{3/4}*(c*x)^{(3/2)}*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^{3/2}*c^6*(a + b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 28.7322, size = 110, normalized size = 0.89

$$-\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}} - \frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}} + \frac{4b^{5/2}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{21a^{3/2}c^6(a+bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(9/2), x)

[Out]  $-2*(a + b*x**2)**(1/4)/(7*c*(c*x)**(7/2)) - 2*b*(a + b*x**2)**(1/4)/(21*a*c**3*(c*x)**(3/2)) + 4*b**(5/2)*(c*x)**(3/2)*(a/(b*x**2) + 1)**(3/4)*elliptic_f(atan(sqrt(a)/(sqrt(b)*x))/2, 2)/(21*a**(3/2)*c**6*(a + b*x**2)**(3/4))$

**Mathematica [C]** time = 0.068166, size = 92, normalized size = 0.75

$$\frac{2\sqrt{cx} \left(3a^2 + 2b^2x^4 \left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{5}{4}; -\frac{bx^2}{a}\right) + 4abx^2 + b^2x^4\right)}{21ac^5x^4(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/4)/(c\*x)^(9/2), x]

[Out]  $(-2*\sqrt{c*x}*(3*a^2 + 4*a*b*x^2 + b^2*x^4 + 2*b^2*x^4*(1 + (b*x^2)/a)^{(3/4)}*Hypergeometric2F1[1/4, 3/4, 5/4, -(b*x^2)/a]))/(21*$

$$a * c^5 * x^4 * (a + b * x^2)^{(3/4)}$$

**Maple [F]** time = 0.047, size = 0, normalized size = 0.

$$\int \sqrt[4]{bx^2 + a} (cx)^{-\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(1/4)/(c\*x)^(9/2), x)

[Out] int((b\*x^2+a)^(1/4)/(c\*x)^(9/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)/(c\*x)^(9/2), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(1/4)/(c\*x)^(9/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{1}{4}}}{\sqrt{cx^4}x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)/(c\*x)^(9/2), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(1/4)/(sqrt(c\*x)\*c^4\*x^4), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(9/2), x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(1/4)/(c*x)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(9/2), x)
```



$$3.924 \quad \int \frac{\sqrt[4]{a+bx^2}}{(cx)^{13/2}} dx$$

**Optimal.** Leaf size=154

$$-\frac{8b^{7/2}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{77a^{5/2}c^8(a+bx^2)^{3/4}} + \frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}} - \frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}} - \frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}$$

[Out]  $(-2*(a + b*x^2)^{(1/4)})/(11*c*(c*x)^{(11/2)}) - (2*b*(a + b*x^2)^{(1/4)})/(77*a*c^3*(c*x)^{(7/2)}) + (4*b^2*(a + b*x^2)^{(1/4)})/(77*a^2*c^5*(c*x)^{(3/2)}) - (8*b^{(7/2)}*(1 + a/(b*x^2))^{(3/4)}*(c*x)^{(3/2)}*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*a^{(5/2)}*c^8*(a + b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.301736, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$-\frac{8b^{7/2}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{77a^{5/2}c^8(a+bx^2)^{3/4}} + \frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}} - \frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}} - \frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/4)/(c\*x)^(13/2), x]

[Out]  $(-2*(a + b*x^2)^{(1/4)})/(11*c*(c*x)^{(11/2)}) - (2*b*(a + b*x^2)^{(1/4)})/(77*a*c^3*(c*x)^{(7/2)}) + (4*b^2*(a + b*x^2)^{(1/4)})/(77*a^2*c^5*(c*x)^{(3/2)}) - (8*b^{(7/2)}*(1 + a/(b*x^2))^{(3/4)}*(c*x)^{(3/2)}*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(77*a^{(5/2)}*c^8*(a + b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 35.7106, size = 141, normalized size = 0.92

$$-\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}} - \frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}} - \frac{8b^{7/2}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{77a^{5/2}c^8(a+bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(13/2), x)

[Out]  $-2*(a + b*x^2)^{(1/4)}/(11*c*(c*x)^{(11/2)}) - 2*b*(a + b*x^2)^{(1/4)}/(77*a*c^3*(c*x)^{(7/2)}) + 4*b^2*(a + b*x^2)^{(1/4)}/(77*a^2*c^5*(c*x)^{(3/2)}) - 8*b^{(7/2)}*(c*x)^{(3/2)}*(a/(b*x^2) + 1)^{(3/4)}*elliptic_f(\operatorname{atan}(\operatorname{sqrt}(a)/(\operatorname{sqrt}(b)*x)))/2, 2)/(77*a^{(5/2)}*c^8*(a + b*x^2)^{(3/4)})$

**Mathematica [C]** time = 0.0787363, size = 103, normalized size = 0.67

$$\frac{2\sqrt{cx} \left(-7a^3 - 8a^2bx^2 + 4b^3x^6 \left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{5}{4}; -\frac{bx^2}{a}\right) + ab^2x^4 + 2b^3x^6\right)}{77a^2c^7x^6(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/4)/(c\*x)^(13/2), x]

[Out]  $(2*\text{Sqrt}[c*x]*(-7*a^3 - 8*a^2*b*x^2 + a*b^2*x^4 + 2*b^3*x^6 + 4*b^3*x^6*(1 + (b*x^2)/a)^{3/4}*\text{Hypergeometric2F1}[1/4, 3/4, 5/4, -((b*x^2)/a)]))/((77*a^2*c^7*x^6*(a + b*x^2)^{3/4}))$

**Maple [F]** time = 0.052, size = 0, normalized size = 0.

$$\int 1\sqrt[4]{bx^2 + a}(cx)^{-\frac{13}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/4)/(c*x)^(13/2), x)`

[Out] `int((b*x^2+a)^(1/4)/(c*x)^(13/2), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)/(c*x)^(13/2), x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(1/4)/(c*x)^(13/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{1}{4}}}{\sqrt{cx}c^6x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)/(c*x)^(13/2), x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(1/4)/(sqrt(c*x)*c^6*x^6), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/4)/(c*x)**(13/2), x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(1/4)/(c*x)^(13/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(c*x)^(13/2), x)
```

### 3.925 $\int (cx)^{5/2} \sqrt[4]{a+bx^2} dx$

**Optimal.** Leaf size=147

$$\frac{3a^2c^{5/2} \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}} - \frac{3a^2c^{5/2} \tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}} + \frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c} + \frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}$$

[Out]  $(a*c*(c*x)^{(3/2)*(a+b*x^2)^{(1/4)}}/(16*b) + ((c*x)^{(7/2)*(a+b*x^2)^{(1/4)}}/(4*c) + (3*a^2*c^{(5/2)*ArcTan[(b^{(1/4)*Sqrt[c*x]}/(Sqrt[c]*(a+b*x^2)^{(1/4)})])/(32*b^{(7/4)}) - (3*a^2*c^{(5/2)*ArcTanh[(b^{(1/4)*Sqrt[c*x]}/(Sqrt[c]*(a+b*x^2)^{(1/4)})])/(32*b^{(7/4)})$

**Rubi [A]** time = 0.256895, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$\frac{3a^2c^{5/2} \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}} - \frac{3a^2c^{5/2} \tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}} + \frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c} + \frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(5/2)\*(a+b\*x^2)^(1/4),x]

[Out]  $(a*c*(c*x)^{(3/2)*(a+b*x^2)^{(1/4)}}/(16*b) + ((c*x)^{(7/2)*(a+b*x^2)^{(1/4)}}/(4*c) + (3*a^2*c^{(5/2)*ArcTan[(b^{(1/4)*Sqrt[c*x]}/(Sqrt[c]*(a+b*x^2)^{(1/4)})])/(32*b^{(7/4)}) - (3*a^2*c^{(5/2)*ArcTanh[(b^{(1/4)*Sqrt[c*x]}/(Sqrt[c]*(a+b*x^2)^{(1/4)})])/(32*b^{(7/4)})$

**Rubi in Sympy [A]** time = 29.4515, size = 134, normalized size = 0.91

$$\frac{3a^2c^{5/2} \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}} - \frac{3a^2c^{5/2} \operatorname{atanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}} + \frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(5/2)\*(b\*x\*\*2+a)\*\*(1/4),x)

[Out]  $3*a**2*c** (5/2)*atan(b**(1/4)*sqrt(c*x)/(sqrt(c)*(a+b*x**2)**(1/4)))/(32*b**(7/4)) - 3*a**2*c** (5/2)*atanh(b**(1/4)*sqrt(c*x)/(sqrt(c)*(a+b*x**2)**(1/4)))/(32*b**(7/4)) + a*c*(c*x)**(3/2)*(a+b*x**2)**(1/4)/(16*b) + (c*x)**(7/2)*(a+b*x**2)**(1/4)/(4*c)$

**Mathematica [C]** time = 0.0608316, size = 83, normalized size = 0.56

$$\frac{c(cx)^{3/2} \left( -a^2 \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1 \left( \frac{3}{4}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^2}{a} \right) + a^2 + 5abx^2 + 4b^2x^4 \right)}{16b(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(5/2)\*(a+b\*x^2)^(1/4),x]

[Out]  $(c*(c*x)^{(3/2)*(a^2+5*a*b*x^2+4*b^2*x^4-a^2*(1+(b*x^2)/a)})^{(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, -(b*x^2)/a]})/(16*b*(a$

+ b\*x^2)^(3/4))

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int (cx)^{\frac{5}{2}} \sqrt[4]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(5/2)\*(b\*x^2+a)^(1/4),x)

[Out] int((c\*x)^(5/2)\*(b\*x^2+a)^(1/4),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)\*(c\*x)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)\*(c\*x)^(5/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(5/2)\*(b\*x\*\*2+a)\*\*(1/4),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.262233, size = 570, normalized size = 3.88

$$-\frac{1}{128} a^2 c^6 \left( \frac{6 \sqrt{2} (-b)^{\frac{1}{4}} \sqrt{|c|} \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} (-b)^{\frac{1}{4}} \sqrt{|c|} + \frac{2 (bc^2 x^2 + ac^2)^{\frac{1}{4}} \sqrt{|c|}}{\sqrt{cx}} \right)}{2 (-b)^{\frac{1}{4}} \sqrt{|c|}} \right)}{b^2 c^4} + \frac{6 \sqrt{2} (-b)^{\frac{1}{4}} \sqrt{|c|} \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} (-b)^{\frac{1}{4}} \sqrt{|c|} - \frac{2 (bc^2 x^2 + ac^2)^{\frac{1}{4}} \sqrt{|c|}}{\sqrt{cx}} \right)}{2 (-b)^{\frac{1}{4}} \sqrt{|c|}} \right)}{b^2 c^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)\*(c\*x)^(5/2),x, algorithm="giac")

[Out] 
$$-1/128*a^2*c^6*(6*\sqrt{2}*(-b)^{1/4}*\sqrt{\text{abs}(c)}*\arctan(1/2*\sqrt{2}*\sqrt{2}*(\sqrt{2}*(-b)^{1/4}*\sqrt{\text{abs}(c)} + 2*(b*c^2*x^2 + a*c^2)^{1/4})*\sqrt{\text{abs}(c)}/\sqrt{c*x})/((-b)^{1/4}*\sqrt{\text{abs}(c)}))/b^2*c^4 + 6*\sqrt{2}*(-b)^{1/4}*\sqrt{\text{abs}(c)}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(-b)^{1/4}*\sqrt{\text{abs}(c)} - 2*(b*c^2*x^2 + a*c^2)^{1/4})*\sqrt{\text{abs}(c)}/\sqrt{c*x})/((-b)^{1/4}*\sqrt{\text{abs}(c)}))/b^2*c^4 + 3*\sqrt{2}*(-b)^{1/4}*\sqrt{\text{abs}(c)}*\ln(\sqrt{2}*(b*c^2*x^2 + a*c^2)^{1/4}*(-b)^{1/4}*\text{abs}(c)/\sqrt{c*x} + \sqrt{-b}*\text{abs}(c) + \sqrt{b*c^2*x^2 + a*c^2}*\text{abs}(c)/(c*x))/b^2*c^4 - 3*\sqrt{2}*(-b)^{1/4}*\sqrt{\text{abs}(c)}*\ln(-\sqrt{2}*(b*c^2*x^2 + a*c^2)^{1/4}*(-b)^{1/4}*\text{abs}(c)/\sqrt{c*x} + \sqrt{-b}*\text{abs}(c) + \sqrt{b*c^2*x^2 + a*c^2}*\text{abs}(c)/(c*x))/b^2*c^4 - 8*(3*(b*c^2*x^2 + a*c^2)^{1/4}*b*c^2*\sqrt{\text{abs}(c)}/\sqrt{c*x} + (b*c^2*x^2 + a*c^2)^{1/4}*(b*c^2 + a*c^2/x^2)*\sqrt{\text{abs}(c)}/\sqrt{c*x})*x^4/(a^2*b*c^6)$$

### 3.926 $\int \sqrt{cx} \sqrt[4]{a + bx^2} dx$

**Optimal.** Leaf size=116

$$-\frac{a\sqrt{c} \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}} + \frac{a\sqrt{c} \tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}} + \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}$$

[Out]  $((c*x)^{(3/2)}*(a + b*x^2)^{(1/4)})/(2*c) - (a*\text{Sqrt}[c]*\text{ArcTan}[(b^{(1/4)})*\text{Sqrt}[c*x])/(\text{Sqrt}[c]*(a + b*x^2)^{(1/4)})]/(4*b^{(3/4)}) + (a*\text{Sqrt}[c]*\text{ArcTanh}[(b^{(1/4)})*\text{Sqrt}[c*x])/(\text{Sqrt}[c]*(a + b*x^2)^{(1/4)})]/(4*b^{(3/4)})$

**Rubi [A]** time = 0.194926, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$-\frac{a\sqrt{c} \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}} + \frac{a\sqrt{c} \tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}} + \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c\*x]\*(a + b\*x^2)^(1/4), x]

[Out]  $((c*x)^{(3/2)}*(a + b*x^2)^{(1/4)})/(2*c) - (a*\text{Sqrt}[c]*\text{ArcTan}[(b^{(1/4)})*\text{Sqrt}[c*x])/(\text{Sqrt}[c]*(a + b*x^2)^{(1/4)})]/(4*b^{(3/4)}) + (a*\text{Sqrt}[c]*\text{ArcTanh}[(b^{(1/4)})*\text{Sqrt}[c*x])/(\text{Sqrt}[c]*(a + b*x^2)^{(1/4)})]/(4*b^{(3/4)})$

**Rubi in Sympy [A]** time = 23.2935, size = 104, normalized size = 0.9

$$-\frac{a\sqrt{c} \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}} + \frac{a\sqrt{c} \operatorname{atanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}} + \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(1/2)\*(b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $-a*\text{sqrt}(c)*\text{atan}(b^{(1/4)}*\text{sqrt}(c*x)/(\text{sqrt}(c)*(a + b*x**2)**(1/4)))/(4*b^{(3/4)}) + a*\text{sqrt}(c)*\text{atanh}(b^{(1/4)}*\text{sqrt}(c*x)/(\text{sqrt}(c)*(a + b*x**2)**(1/4)))/(4*b^{(3/4)}) + (c*x)**(3/2)*(a + b*x**2)**(1/4)/(2*c)$

**Mathematica [C]** time = 0.0489449, size = 68, normalized size = 0.59

$$\frac{x\sqrt{cx} \left( a \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1 \left( \frac{3}{4}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^2}{a} \right) + 3(a + bx^2) \right)}{6(a + bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c\*x]\*(a + b\*x^2)^(1/4), x]

[Out]  $(x*\text{Sqrt}[c*x]*(3*(a + b*x^2) + a*(1 + (b*x^2)/a)^{(3/4)}*\text{Hypergeometric2F1}[3/4, 3/4, 7/4, -(b*x^2)/a])/(6*(a + b*x^2)^{(3/4)})$

---

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int \sqrt{cx} \sqrt[4]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(1/2)\*(b\*x^2+a)^(1/4),x)

[Out] int((c\*x)^(1/2)\*(b\*x^2+a)^(1/4),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)\*sqrt(c\*x),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)\*sqrt(c\*x),x, algorithm="fricas")

[Out] Timed out

---

**Sympy [A]** time = 9.16064, size = 46, normalized size = 0.4

$$\frac{\sqrt[4]{a}\sqrt{cx}^{\frac{3}{2}} \left(\frac{3}{4}\right) {}_2F_1\left(-\frac{1}{4}, \frac{3}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2 \left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(1/2)\*(b\*x\*\*2+a)\*\*(1/4),x)

[Out] a\*\*(1/4)\*sqrt(c)\*x\*\*(3/2)\*gamma(3/4)\*hyper((-1/4, 3/4), (7/4, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*gamma(7/4))

---



GIAC/XCAS [A] time = 0.261059, size = 498, normalized size = 4.29

$$\frac{1}{16} ac^2 \left( \frac{8 (bc^2x^2 + ac^2)^{\frac{1}{4}} x^2 \sqrt{|c|}}{\sqrt{c} x a c^2} + \frac{2 \sqrt{2} (-b)^{\frac{1}{4}} \sqrt{|c|} \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} (-b)^{\frac{1}{4}} \sqrt{|c|} + \frac{2 (bc^2x^2 + ac^2)^{\frac{1}{4}} \sqrt{|c|}}{\sqrt{c} x} \right)}{2 (-b)^{\frac{1}{4}} \sqrt{|c|}} \right)}{bc^2} + \frac{2 \sqrt{2} (-b)^{\frac{1}{4}} \sqrt{|c|} \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} (-b)^{\frac{1}{4}} \sqrt{|c|} - \frac{2 (bc^2x^2 + ac^2)^{\frac{1}{4}} \sqrt{|c|}}{\sqrt{c} x} \right)}{2 (-b)^{\frac{1}{4}} \sqrt{|c|}} \right)}{bc^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)\*sqrt(c\*x),x, algorithm="giac")

[Out] 1/16\*a\*c^2\*(8\*(b\*c^2\*x^2 + a\*c^2)^(1/4)\*x^2\*sqrt(abs(c))/(sqrt(c\*x)\*a\*c^2) + 2\*sqrt(2)\*(-b)^(1/4)\*sqrt(abs(c))\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(-b)^(1/4)\*sqrt(abs(c)) + 2\*(b\*c^2\*x^2 + a\*c^2)^(1/4)\*sqrt(abs(c))/sqrt(c\*x))/((-b)^(1/4)\*sqrt(abs(c))))/(b\*c^2) + 2\*sqrt(2)\*(-b)^(1/4)\*sqrt(abs(c))\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(-b)^(1/4)\*sqrt(abs(c)) - 2\*(b\*c^2\*x^2 + a\*c^2)^(1/4)\*sqrt(abs(c))/sqrt(c\*x))/((-b)^(1/4)\*sqrt(abs(c))))/(b\*c^2) + sqrt(2)\*(-b)^(1/4)\*sqrt(abs(c))\*ln(sqrt(2)\*(b\*c^2\*x^2 + a\*c^2)^(1/4)\*(-b)^(1/4)\*abs(c)/sqrt(c\*x) + sqrt(-b)\*abs(c) + sqrt(b\*c^2\*x^2 + a\*c^2)\*abs(c)/(c\*x))/(b\*c^2) - sqrt(2)\*(-b)^(1/4)\*sqrt(abs(c))\*ln(-sqrt(2)\*(b\*c^2\*x^2 + a\*c^2)^(1/4)\*(-b)^(1/4)\*abs(c)/sqrt(c\*x) + sqrt(-b)\*abs(c) + sqrt(b\*c^2\*x^2 + a\*c^2)\*abs(c)/(c\*x))/(b\*c^2))

$$3.927 \quad \int \frac{\sqrt[4]{a+bx^2}}{(cx)^{3/2}} dx$$

**Optimal.** Leaf size=107

$$-\frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}} + \frac{\sqrt[4]{b} \tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}} - \frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}$$

[Out]  $(-2*(a + b*x^2)^{(1/4)})/(c*\text{Sqrt}[c*x]) - (b^{(1/4)}*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])]/(\text{Sqrt}[c]*(a + b*x^2)^{(1/4)}))/c^{(3/2)} + (b^{(1/4)}*\text{ArcTan}h[(b^{(1/4)}*\text{Sqrt}[c*x])]/(\text{Sqrt}[c]*(a + b*x^2)^{(1/4)}))/c^{(3/2)}$

**Rubi [A]** time = 0.195411, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$-\frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}} + \frac{\sqrt[4]{b} \tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}} - \frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/4)/(c\*x)^(3/2), x]

[Out]  $(-2*(a + b*x^2)^{(1/4)})/(c*\text{Sqrt}[c*x]) - (b^{(1/4)}*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])]/(\text{Sqrt}[c]*(a + b*x^2)^{(1/4)}))/c^{(3/2)} + (b^{(1/4)}*\text{ArcTan}h[(b^{(1/4)}*\text{Sqrt}[c*x])]/(\text{Sqrt}[c]*(a + b*x^2)^{(1/4)}))/c^{(3/2)}$

**Rubi in Sympy [A]** time = 24.0209, size = 97, normalized size = 0.91

$$-\frac{\sqrt[4]{b} \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{\frac{3}{2}}} + \frac{\sqrt[4]{b} \operatorname{atanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{\frac{3}{2}}} - \frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(3/2), x)

[Out]  $-b^{(1/4)}*\operatorname{atan}(b^{(1/4)}*\operatorname{sqrt}(c*x)/(\operatorname{sqrt}(c)*(a + b*x**2)**(1/4)))/c^{(3/2)} + b^{(1/4)}*\operatorname{atanh}(b^{(1/4)}*\operatorname{sqrt}(c*x)/(\operatorname{sqrt}(c)*(a + b*x**2)**(1/4)))/c^{(3/2)} - 2*(a + b*x**2)**(1/4)/(c*\operatorname{sqrt}(c*x))$

**Mathematica [C]** time = 0.0527502, size = 72, normalized size = 0.67

$$\frac{x \left( 2bx^2 \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1 \left( \frac{3}{4}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^2}{a} \right) - 6(a + bx^2) \right)}{3(cx)^{3/2} (a + bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/4)/(c\*x)^(3/2), x]

[Out]  $(x*(-6*(a + b*x^2) + 2*b*x^2*(1 + (b*x^2)/a)^{(3/4)}*\text{Hypergeometric}2F1[3/4, 3/4, 7/4, -(b*x^2)/a]))/(3*(c*x)^{(3/2)}*(a + b*x^2)^{(3/4)})$

---

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int \sqrt[4]{bx^2 + a} (cx)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(1/4)/(c\*x)^(3/2), x)

[Out] int((b\*x^2+a)^(1/4)/(c\*x)^(3/2), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)/(c\*x)^(3/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/4)/(c\*x)^(3/2), x, algorithm="fricas")

[Out] Timed out

---

**Sympy [A]** time = 14.7899, size = 49, normalized size = 0.46

$$\frac{\sqrt[4]{a} \left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2c^{\frac{3}{2}} \sqrt{x} \left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(3/2), x)

[Out] a\*\*(1/4)\*gamma(-1/4)\*hyper((-1/4, -1/4), (3/4, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*c\*\*(3/2)\*sqrt(x)\*gamma(3/4))

---

**GIAC/XCAS [A]** time = 0.25454, size = 452, normalized size = 4.22

$$2\sqrt{2}(-b)^{\frac{1}{4}}\sqrt{|c|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}\sqrt{|c|}+\frac{2(bc^2x^2+ac^2)^{\frac{1}{4}}\sqrt{|c|}}{\sqrt{cx}}\right)}{2(-b)^{\frac{1}{4}}\sqrt{|c|}}\right)+2\sqrt{2}(-b)^{\frac{1}{4}}\sqrt{|c|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-b)^{\frac{1}{4}}\sqrt{|c|}-\frac{2(bc^2x^2+ac^2)^{\frac{1}{4}}\sqrt{|c|}}{\sqrt{cx}}\right)}{2(-b)^{\frac{1}{4}}\sqrt{|c|}}\right)+\sqrt{2}$$


---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(1/4)/(c*x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*(-b)^(1/4)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(sqrt(2)
)*(-b)^(1/4)*sqrt(abs(c)) + 2*(b*c^2*x^2 + a*c^2)^(1/4)*sqrt(abs(
c))/sqrt(c*x))/((-b)^(1/4)*sqrt(abs(c)))) + 2*sqrt(2)*(-b)^(1/4)*
sqrt(abs(c))*arctan(-1/2*sqrt(2)*(sqrt(2)*(-b)^(1/4)*sqrt(abs(c))
- 2*(b*c^2*x^2 + a*c^2)^(1/4)*sqrt(abs(c))/sqrt(c*x))/((-b)^(1/4
)*sqrt(abs(c)))) + sqrt(2)*(-b)^(1/4)*sqrt(abs(c))*ln(sqrt(2)*(b*
c^2*x^2 + a*c^2)^(1/4)*(-b)^(1/4)*abs(c)/sqrt(c*x) + sqrt(-b)*abs
(c) + sqrt(b*c^2*x^2 + a*c^2)*abs(c)/(c*x)) - sqrt(2)*(-b)^(1/4)*
sqrt(abs(c))*ln(-sqrt(2)*(b*c^2*x^2 + a*c^2)^(1/4)*(-b)^(1/4)*abs
(c)/sqrt(c*x) + sqrt(-b)*abs(c) + sqrt(b*c^2*x^2 + a*c^2)*abs(c)/
(c*x)) - 8*(b*c^2*x^2 + a*c^2)^(1/4)*sqrt(abs(c))/sqrt(c*x)/c^2
```

$$3.928 \quad \int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}} dx$$

**Optimal.** Leaf size=28

$$-\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{5/2}}$$

[Out]  $(-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(5/2))$

**Rubi [A]** time = 0.0271234, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$-\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/4)/(c\*x)^(7/2), x]

[Out]  $(-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(5/2))$

**Rubi in Sympy [A]** time = 3.61159, size = 24, normalized size = 0.86

$$-\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(7/2), x)

[Out]  $-2*(a + b*x**2)**(5/4)/(5*a*c*(c*x)**(5/2))$

**Mathematica [A]** time = 0.0305315, size = 26, normalized size = 0.93

$$-\frac{2x(a+bx^2)^{5/4}}{5a(cx)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/4)/(c\*x)^(7/2), x]

[Out]  $(-2*x*(a + b*x^2)^(5/4))/(5*a*(c*x)^(7/2))$

**Maple [A]** time = 0.007, size = 21, normalized size = 0.8

$$-\frac{2x}{5a}(bx^2+a)^{5/4}(cx)^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^(1/4)/(c\*x)^(7/2), x)

[Out]  $-2/5 * x * (b * x^2 + a)^{(5/4)} / a / (c * x)^{(7/2)}$

**Maxima [A]** time = 1.38559, size = 27, normalized size = 0.96

$$-\frac{2 (bx^2 + a)^{\frac{5}{4}}}{5 ac^{\frac{7}{2}} x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)/(c*x)^(7/2), x, algorithm="maxima")`

[Out]  $-2/5 * (b * x^2 + a)^{(5/4)} / (a * c^{(7/2)} * x^{(5/2)})$

**Fricas [A]** time = 0.267596, size = 34, normalized size = 1.21

$$-\frac{2 (bx^2 + a)^{\frac{5}{4}} \sqrt{cx}}{5 ac^4 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)/(c*x)^(7/2), x, algorithm="fricas")`

[Out]  $-2/5 * (b * x^2 + a)^{(5/4)} * \text{sqrt}(c * x) / (a * c^4 * x^3)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/4)/(c*x)**(7/2), x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.234208, size = 58, normalized size = 2.07

$$-\frac{2 (bc^4 x^2 + ac^4)^{\frac{1}{4}} \left( bc^2 + \frac{ac^2}{x^2} \right)}{5 \sqrt{cx} ac^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)/(c*x)^(7/2), x, algorithm="giac")`

[Out]  $-2/5 * (b * c^4 * x^2 + a * c^4)^{(1/4)} * (b * c^2 + a * c^2 / x^2) / (\text{sqrt}(c * x) * a * c^6)$

$$3.929 \quad \int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}} dx$$

**Optimal.** Leaf size=57

$$\frac{8(a+bx^2)^{9/4}}{45a^2c(cx)^{9/2}} - \frac{2(a+bx^2)^{5/4}}{5ac(cx)^{9/2}}$$

[Out]  $(-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(9/2)) + (8*(a + b*x^2)^(9/4))/(45*a^2*c*(c*x)^(9/2))$

**Rubi [A]** time = 0.056541, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{8(a+bx^2)^{9/4}}{45a^2c(cx)^{9/2}} - \frac{2(a+bx^2)^{5/4}}{5ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/4)/(c\*x)^(11/2), x]

[Out]  $(-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(9/2)) + (8*(a + b*x^2)^(9/4))/(45*a^2*c*(c*x)^(9/2))$

**Rubi in Sympy [A]** time = 6.69595, size = 48, normalized size = 0.84

$$-\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{9/2}} + \frac{8(a+bx^2)^{9/4}}{45a^2c(cx)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(11/2), x)

[Out]  $-2*(a + b*x**2)**(5/4)/(5*a*c*(c*x)**(9/2)) + 8*(a + b*x**2)**(9/4)/(45*a**2*c*(c*x)**(9/2))$

**Mathematica [A]** time = 0.0373974, size = 51, normalized size = 0.89

$$-\frac{2\sqrt{cx}\sqrt[4]{a+bx^2}(5a^2+abx^2-4b^2x^4)}{45a^2c^6x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/4)/(c\*x)^(11/2), x]

[Out]  $(-2*\text{Sqrt}[c*x]*(a + b*x^2)^(1/4)*(5*a^2 + a*b*x^2 - 4*b^2*x^4))/(45*a^2*c^6*x^5)$

**Maple [A]** time = 0.007, size = 31, normalized size = 0.5

$$-\frac{2x(-4bx^2+5a)}{45a^2}(bx^2+a)^{5/4}(cx)^{-11/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/4)/(c*x)^(11/2),x)`

[Out]  $-2/45*x*(b*x^2+a)^{5/4}*(-4*b*x^2+5*a)/a^2/(c*x)^{11/2}$

**Maxima [A]** time = 1.38785, size = 51, normalized size = 0.89

$$\frac{2\left(\frac{9(bx^2+a)^{\frac{5}{4}}b}{x^{\frac{5}{2}}} - \frac{5(bx^2+a)^{\frac{9}{4}}}{x^{\frac{9}{2}}}\right)}{45a^2c^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)/(c*x)^(11/2),x, algorithm="maxima")`

[Out]  $2/45*(9*(b*x^2 + a)^{5/4}*b/x^{5/2} - 5*(b*x^2 + a)^{9/4}/x^{9/2})/(a^2*c^{11/2})$

**Fricas [A]** time = 0.256954, size = 62, normalized size = 1.09

$$\frac{2(4b^2x^4 - abx^2 - 5a^2)(bx^2 + a)^{\frac{1}{4}}\sqrt{cx}}{45a^2c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)/(c*x)^(11/2),x, algorithm="fricas")`

[Out]  $2/45*(4*b^2*x^4 - a*b*x^2 - 5*a^2)*(b*x^2 + a)^{1/4}*sqrt(c*x)/(a^2*c^6*x^5)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/4)/(c*x)**(11/2),x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.228822, size = 144, normalized size = 2.53

$$\frac{2\left(\frac{9(bc^4x^2+ac^4)^{\frac{1}{4}}\left(bc^2+\frac{ac^2}{x^2}\right)bc^2}{\sqrt{cx}} - \frac{5(b^2c^8x^4+2abc^8x^2+a^2c^8)(bc^4x^2+ac^4)^{\frac{1}{4}}}{\sqrt{cx}c^4x^4}\right)}{45a^2c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)/(c*x)^(11/2),x, algorithm="giac")`

[Out]  $2/45*(9*(b*c^4*x^2 + a*c^4)^{1/4}*(b*c^2 + a*c^2/x^2)*b*c^2/sqrt(c*x) - 5*(b^2*c^8*x^4 + 2*a*b*c^8*x^2 + a^2*c^8)*(b*c^4*x^2 + a*c^4)^{1/4}/(sqrt(c*x)*c^4*x^4))/(a^2*c^{10})$



$$3.930 \quad \int \frac{\sqrt[4]{a+bx^2}}{(cx)^{15/2}} dx$$

**Optimal.** Leaf size=85

$$-\frac{64(a+bx^2)^{13/4}}{585a^3c(cx)^{13/2}} + \frac{16(a+bx^2)^{9/4}}{45a^2c(cx)^{13/2}} - \frac{2(a+bx^2)^{5/4}}{5ac(cx)^{13/2}}$$

[Out]  $(-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(13/2)) + (16*(a + b*x^2)^(9/4))/(45*a^2*c*(c*x)^(13/2)) - (64*(a + b*x^2)^(13/4))/(585*a^3*c*(c*x)^(13/2))$

**Rubi [A]** time = 0.0877553, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$-\frac{64(a+bx^2)^{13/4}}{585a^3c(cx)^{13/2}} + \frac{16(a+bx^2)^{9/4}}{45a^2c(cx)^{13/2}} - \frac{2(a+bx^2)^{5/4}}{5ac(cx)^{13/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/4)/(c\*x)^(15/2), x]

[Out]  $(-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(13/2)) + (16*(a + b*x^2)^(9/4))/(45*a^2*c*(c*x)^(13/2)) - (64*(a + b*x^2)^(13/4))/(585*a^3*c*(c*x)^(13/2))$

**Rubi in Sympy [A]** time = 10.6371, size = 73, normalized size = 0.86

$$-\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{13/2}} + \frac{16(a+bx^2)^{9/4}}{45a^2c(cx)^{13/2}} - \frac{64(a+bx^2)^{13/4}}{585a^3c(cx)^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(15/2), x)

[Out]  $-2*(a + b*x**2)**(5/4)/(5*a*c*(c*x)**(13/2)) + 16*(a + b*x**2)**(9/4)/(45*a**2*c*(c*x)**(13/2)) - 64*(a + b*x**2)**(13/4)/(585*a**3*c*(c*x)**(13/2))$

**Mathematica [A]** time = 0.0411853, size = 63, normalized size = 0.74

$$\frac{2\sqrt{cx}\sqrt[4]{a+bx^2}(45a^3+5a^2bx^2-8ab^2x^4+32b^3x^6)}{585a^3c^8x^7}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/4)/(c\*x)^(15/2), x]

[Out]  $(-2*\text{Sqrt}[c*x]*(a + b*x^2)^(1/4)*(45*a^3 + 5*a^2*b*x^2 - 8*a*b^2*x^4 + 32*b^3*x^6))/(585*a^3*c^8*x^7)$

**Maple [A]** time = 0.009, size = 42, normalized size = 0.5

$$-\frac{2x(32b^2x^4 - 40abx^2 + 45a^2)}{585a^3} (bx^2 + a)^{5/4} (cx)^{-15/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/4)/(c*x)^(15/2),x)`

[Out] 
$$-2/585 * x * (b * x^2 + a)^{5/4} * (32 * b^2 * x^4 - 40 * a * b * x^2 + 45 * a^2) / a^3 / (c * x)^{15/2}$$

**Maxima [A]** time = 1.38679, size = 74, normalized size = 0.87

$$-\frac{2 \left( \frac{117 (bx^2+a)^{\frac{5}{4}} b^2}{x^{\frac{5}{2}}} - \frac{130 (bx^2+a)^{\frac{9}{4}} b}{x^{\frac{9}{2}}} + \frac{45 (bx^2+a)^{\frac{13}{4}}}{x^{\frac{13}{2}}} \right)}{585 a^3 c^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)/(c*x)^(15/2),x, algorithm="maxima")`

[Out] 
$$-2/585 * (117 * (b * x^2 + a)^{5/4} * b^2 / x^{5/2} - 130 * (b * x^2 + a)^{9/4} * b / x^{9/2} + 45 * (b * x^2 + a)^{13/4} / x^{13/2}) / (a^3 * c^{15/2})$$

**Fricas [A]** time = 0.257262, size = 77, normalized size = 0.91

$$-\frac{2 (32 b^3 x^6 - 8 a b^2 x^4 + 5 a^2 b x^2 + 45 a^3) (b x^2 + a)^{\frac{1}{4}} \sqrt{c x}}{585 a^3 c^8 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)/(c*x)^(15/2),x, algorithm="fricas")`

[Out] 
$$-2/585 * (32 * b^3 * x^6 - 8 * a * b^2 * x^4 + 5 * a^2 * b * x^2 + 45 * a^3) * (b * x^2 + a)^{1/4} * \sqrt{c * x} / (a^3 * c^8 * x^7)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/4)/(c*x)**(15/2),x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.23394, size = 244, normalized size = 2.87

$$-\frac{2 \left( \frac{117 (bc^4x^2+ac^4)^{\frac{1}{4}} \left( bc^2 + \frac{ac^2}{x^2} \right) b^2 c^4}{\sqrt{cx}} - \frac{130 (b^2c^8x^4+2abc^8x^2+a^2c^8)(bc^4x^2+ac^4)^{\frac{1}{4}} b}{\sqrt{cx}c^2x^4} + \frac{45 (b^3c^{12}x^6+3ab^2c^{12}x^4+3a^2bc^{12}x^2+a^3c^{12})(bc^4x^2+ac^4)^{\frac{1}{4}}}{\sqrt{cx}c^6x^6} \right)}{585 a^3 c^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)/(c*x)^(15/2),x, algorithm="giac")`

[Out] 
$$-2/585 * (117 * (b * c^4 * x^2 + a * c^4)^{1/4} * (b * c^2 + a * c^2 / x^2) * b^2 * c^4 / \sqrt{c * x} - 130 * (b^2 * c^8 * x^4 + 2 * a * b * c^8 * x^2 + a^2 * c^8) * (b * c^4 * x^2 + a * c^4)^{1/4} * b / \sqrt{c * x} * c^2 * x^4 + 45 * (b^3 * c^{12} * x^6 + 3 * a * b^2 * c^{12} * x^4 + 3 * a^2 * b * c^{12} * x^2 + a^3 * c^{12}) * (b * c^4 * x^2 + a * c^4)^{1/4} / \sqrt{c * x} * c^6 * x^6)$$

$$\frac{(a^2 + a^3 c^4)^{1/4} b \sqrt{c x} c^2 x^4 + 45 (b^3 c^{12} x^6 + 3 a b^2 c^{12} x^4 + 3 a^2 b c^{12} x^2 + a^3 c^{12}) (b c^4 x^2 + a c^4)^{1/4} \sqrt{c x} c^6 x^6}{a^3 c^{14}}$$

$$3.931 \quad \int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}} dx$$

**Optimal.** Leaf size=113

$$\frac{256(a+bx^2)^{17/4}}{3315a^4c(cx)^{17/2}} - \frac{64(a+bx^2)^{13/4}}{195a^3c(cx)^{17/2}} + \frac{8(a+bx^2)^{9/4}}{15a^2c(cx)^{17/2}} - \frac{2(a+bx^2)^{5/4}}{5ac(cx)^{17/2}}$$

[Out]  $(-2*(a+b*x^2)^(5/4))/(5*a*c*(c*x)^(17/2)) + (8*(a+b*x^2)^(9/4))/(15*a^2*c*(c*x)^(17/2)) - (64*(a+b*x^2)^(13/4))/(195*a^3*c*(c*x)^(17/2)) + (256*(a+b*x^2)^(17/4))/(3315*a^4*c*(c*x)^(17/2))$

**Rubi [A]** time = 0.125991, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{256(a+bx^2)^{17/4}}{3315a^4c(cx)^{17/2}} - \frac{64(a+bx^2)^{13/4}}{195a^3c(cx)^{17/2}} + \frac{8(a+bx^2)^{9/4}}{15a^2c(cx)^{17/2}} - \frac{2(a+bx^2)^{5/4}}{5ac(cx)^{17/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/4)/(c\*x)^(19/2), x]

[Out]  $(-2*(a+b*x^2)^(5/4))/(5*a*c*(c*x)^(17/2)) + (8*(a+b*x^2)^(9/4))/(15*a^2*c*(c*x)^(17/2)) - (64*(a+b*x^2)^(13/4))/(195*a^3*c*(c*x)^(17/2)) + (256*(a+b*x^2)^(17/4))/(3315*a^4*c*(c*x)^(17/2))$

**Rubi in Sympy [A]** time = 15.3173, size = 99, normalized size = 0.88

$$-\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{17/2}} + \frac{8(a+bx^2)^{9/4}}{15a^2c(cx)^{17/2}} - \frac{64(a+bx^2)^{13/4}}{195a^3c(cx)^{17/2}} + \frac{256(a+bx^2)^{17/4}}{3315a^4c(cx)^{17/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(19/2), x)

[Out]  $-2*(a+b*x**2)**(5/4)/(5*a*c*(c*x)**(17/2)) + 8*(a+b*x**2)**(9/4)/(15*a**2*c*(c*x)**(17/2)) - 64*(a+b*x**2)**(13/4)/(195*a**3*c*(c*x)**(17/2)) + 256*(a+b*x**2)**(17/4)/(3315*a**4*c*(c*x)**(17/2))$

**Mathematica [A]** time = 0.047713, size = 74, normalized size = 0.65

$$\frac{2\sqrt[4]{a+bx^2}(-195a^4-15a^3bx^2+20a^2b^2x^4-32ab^3x^6+128b^4x^8)}{3315a^4c^9x^8\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(1/4)/(c\*x)^(19/2), x]

[Out]  $(2*(a+b*x^2)^(1/4)*(-195*a^4-15*a^3*b*x^2+20*a^2*b^2*x^4-32*a*b^3*x^6+128*b^4*x^8))/(3315*a^4*c^9*x^8*\text{Sqrt}[c*x])$

**Maple [A]** time = 0.009, size = 53, normalized size = 0.5

$$-\frac{2x(-128b^3x^6+160ab^2x^4-180a^2bx^2+195a^3)}{3315a^4}(bx^2+a)^{5/4}(cx)^{-19/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/4)/(c*x)^(19/2),x)`

[Out] 
$$-2/3315*x*(b*x^2+a)^{5/4}*(-128*b^3*x^6+160*a*b^2*x^4-180*a^2*b*x^2+195*a^3)/a^4/(c*x)^{19/2}$$

**Maxima [A]** time = 1.38554, size = 97, normalized size = 0.86

$$\frac{2 \left( \frac{663 (bx^2+a)^{\frac{5}{4}} b^3}{x^{\frac{5}{2}}} - \frac{1105 (bx^2+a)^{\frac{9}{4}} b^2}{x^{\frac{9}{2}}} + \frac{765 (bx^2+a)^{\frac{13}{4}} b}{x^{\frac{13}{2}}} - \frac{195 (bx^2+a)^{\frac{17}{4}}}{x^{\frac{17}{2}}} \right)}{3315 a^4 c^{\frac{19}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)/(c*x)^(19/2),x, algorithm="maxima")`

[Out] 
$$2/3315*(663*(b*x^2 + a)^{5/4}*b^3/x^{5/2} - 1105*(b*x^2 + a)^{9/4}) * b^2/x^{9/2} + 765*(b*x^2 + a)^{13/4}*b/x^{13/2} - 195*(b*x^2 + a)^{17/4}/x^{17/2})/(a^4*c^{19/2})$$

**Fricas [A]** time = 0.238607, size = 92, normalized size = 0.81

$$\frac{2 (128 b^4 x^8 - 32 a b^3 x^6 + 20 a^2 b^2 x^4 - 15 a^3 b x^2 - 195 a^4) (b x^2 + a)^{\frac{1}{4}} \sqrt{c x}}{3315 a^4 c^{10} x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)/(c*x)^(19/2),x, algorithm="fricas")`

[Out] 
$$2/3315*(128*b^4*x^8 - 32*a*b^3*x^6 + 20*a^2*b^2*x^4 - 15*a^3*b*x^2 - 195*a^4)*(b*x^2 + a)^{1/4}*sqrt(c*x)/(a^4*c^{10}*x^9)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/4)/(c*x)**(19/2),x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.236024, size = 359, normalized size = 3.18

$$\frac{2 \left( \frac{663 (bc^4x^2+ac^4)^{\frac{1}{4}} \left( bc^2 + \frac{ac^2}{x^2} \right) b^3 c^6}{\sqrt{cx}} - \frac{1105 (b^2c^8x^4+2abc^8x^2+a^2c^8) (bc^4x^2+ac^4)^{\frac{1}{4}} b^2}{\sqrt{cx}x^4} + \frac{765 (b^3c^{12}x^6+3ab^2c^{12}x^4+3a^2bc^{12}x^2+a^3c^{12}) (bc^4x^2+ac^4)^{\frac{1}{4}} b}{\sqrt{cx}c^4x^6} \right)}{3315 a^4 c^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/4)/(c*x)^(19/2),x, algorithm="giac")`

```
[Out] 2/3315*(663*(b*c^4*x^2 + a*c^4)^(1/4)*(b*c^2 + a*c^2/x^2)*b^3*c^6
/sqrt(c*x) - 1105*(b^2*c^8*x^4 + 2*a*b*c^8*x^2 + a^2*c^8)*(b*c^4*
x^2 + a*c^4)^(1/4)*b^2/(sqrt(c*x)*x^4) + 765*(b^3*c^12*x^6 + 3*a*
b^2*c^12*x^4 + 3*a^2*b*c^12*x^2 + a^3*c^12)*(b*c^4*x^2 + a*c^4)^(
1/4)*b/(sqrt(c*x)*c^4*x^6) - 195*(b^4*c^16*x^8 + 4*a*b^3*c^16*x^6
+ 6*a^2*b^2*c^16*x^4 + 4*a^3*b*c^16*x^2 + a^4*c^16)*(b*c^4*x^2 +
a*c^4)^(1/4)/(sqrt(c*x)*c^8*x^8))/(a^4*c^18)
```

### 3.932 $\int (cx)^{3/2} \sqrt[4]{a - bx^2} dx$

**Optimal.** Leaf size=122

$$-\frac{a^{3/2}(cx)^{3/2} \left(1 - \frac{a}{bx^2}\right)^{3/4} F\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{6\sqrt{b}(a - bx^2)^{3/4}} + \frac{(cx)^{5/2} \sqrt[4]{a - bx^2}}{3c} - \frac{ac\sqrt{cx} \sqrt[4]{a - bx^2}}{6b}$$

[Out]  $-(a*c*\operatorname{Sqrt}[c*x]*(a - b*x^2)^{(1/4)})/(6*b) + ((c*x)^{(5/2)}*(a - b*x^2)^{(1/4)})/(3*c) - (a^{(3/2)}*(1 - a/(b*x^2))^{(3/4)}*(c*x)^{(3/2)}*\operatorname{EllipticF}[\operatorname{ArcCsc}[(\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]/2, 2])/(6*\operatorname{Sqrt}[b]*(a - b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.257454, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$-\frac{a^{3/2}(cx)^{3/2} \left(1 - \frac{a}{bx^2}\right)^{3/4} F\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{6\sqrt{b}(a - bx^2)^{3/4}} + \frac{(cx)^{5/2} \sqrt[4]{a - bx^2}}{3c} - \frac{ac\sqrt{cx} \sqrt[4]{a - bx^2}}{6b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c*x)^{(3/2)}*(a - b*x^2)^{(1/4)}, x]$

[Out]  $-(a*c*\operatorname{Sqrt}[c*x]*(a - b*x^2)^{(1/4)})/(6*b) + ((c*x)^{(5/2)}*(a - b*x^2)^{(1/4)})/(3*c) - (a^{(3/2)}*(1 - a/(b*x^2))^{(3/4)}*(c*x)^{(3/2)}*\operatorname{EllipticF}[\operatorname{ArcCsc}[(\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]/2, 2])/(6*\operatorname{Sqrt}[b]*(a - b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 36.1715, size = 100, normalized size = 0.82

$$-\frac{a^{\frac{3}{2}}(cx)^{\frac{3}{2}} \left(-\frac{a}{bx^2} + 1\right)^{\frac{3}{4}} F\left(\frac{\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{6\sqrt{b}(a - bx^2)^{\frac{3}{4}}} - \frac{ac\sqrt{cx} \sqrt[4]{a - bx^2}}{6b} + \frac{(cx)^{\frac{5}{2}} \sqrt[4]{a - bx^2}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}((c*x)**(3/2)*(-b*x**2+a)**(1/4), x)$

[Out]  $-a**(3/2)*(c*x)**(3/2)*(-a/(b*x**2) + 1)**(3/4)*\operatorname{elliptic\_f}(\operatorname{asin}(\operatorname{sqrt}(a)/(\operatorname{sqrt}(b)*x))/2, 2)/(6*\operatorname{sqrt}(b)*(a - b*x**2)**(3/4)) - a*c*\operatorname{sqrt}(c*x)*(a - b*x**2)**(1/4)/(6*b) + (c*x)**(5/2)*(a - b*x**2)**(1/4)/(3*c)$

**Mathematica [C]** time = 0.0646903, size = 84, normalized size = 0.69

$$\frac{c\sqrt{cx} \left(-a^2 \left(1 - \frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{5}{4}; \frac{bx^2}{a}\right) + a^2 - 3abx^2 + 2b^2x^4\right)}{6b(a - bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[(c*x)^{(3/2)}*(a - b*x^2)^{(1/4)}, x]$

[Out]  $-(c*\operatorname{Sqrt}[c*x]*(a^2 - 3*a*b*x^2 + 2*b^2*x^4 - a^2*(1 - (b*x^2)/a))^{(3/4)}*\operatorname{Hypergeometric2F1}[1/4, 3/4, 5/4, (b*x^2)/a])/ (6*b*(a - b*x$

$$^2)^{(3/4)})$$

**Maple [F]** time = 0.056, size = 0, normalized size = 0.

$$\int (cx)^{\frac{3}{2}} \sqrt[4]{-bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(3/2)\*(-b\*x^2+a)^(1/4),x)

[Out] int((c\*x)^(3/2)\*(-b\*x^2+a)^(1/4),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4)\*(c\*x)^(3/2),x, algorithm="maxima")

[Out] integrate((-b\*x^2 + a)^(1/4)\*(c\*x)^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-bx^2 + a\right)^{\frac{1}{4}} \sqrt{cx}cx, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4)\*(c\*x)^(3/2),x, algorithm="fricas")

[Out] integral((-b\*x^2 + a)^(1/4)\*sqrt(c\*x)\*c\*x, x)

**Sympy [A]** time = 24.1505, size = 48, normalized size = 0.39

$$\frac{\sqrt[4]{ac^{\frac{3}{2}}x^{\frac{5}{2}}}\left(\frac{5}{4}\right) {}_2F_1\left(-\frac{1}{4}, \frac{5}{4} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{2\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(3/2)\*(-b\*x\*\*2+a)\*\*(1/4),x)

[Out] a\*\*(1/4)\*c\*\*(3/2)\*x\*\*(5/2)\*gamma(5/4)\*hyper((-1/4, 5/4), (9/4, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/(2\*gamma(9/4))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (-bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{3}{2}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2 + a)^(1/4)*(c*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)*(c*x)^(3/2), x)
```

$$3.933 \quad \int \frac{\sqrt[4]{a - bx^2}}{\sqrt{cx}} dx$$

**Optimal.** Leaf size=92

$$\frac{\sqrt{cx}\sqrt[4]{a - bx^2}}{c} - \frac{\sqrt{a}\sqrt{b}(cx)^{3/2} \left(1 - \frac{a}{bx^2}\right)^{3/4} F\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{c^2 (a - bx^2)^{3/4}}$$

[Out] (Sqrt[c\*x]\*(a - b\*x^2)^(1/4))/c - (Sqrt[a]\*Sqrt[b]\*(1 - a/(b\*x^2))^(3/4)\*(c\*x)^(3/2)\*EllipticF[ArcCsc[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/ (c^2\*(a - b\*x^2)^(3/4))

**Rubi [A]** time = 0.215925, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{\sqrt{cx}\sqrt[4]{a - bx^2}}{c} - \frac{\sqrt{a}\sqrt{b}(cx)^{3/2} \left(1 - \frac{a}{bx^2}\right)^{3/4} F\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{c^2 (a - bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(1/4)/Sqrt[c\*x], x]

[Out] (Sqrt[c\*x]\*(a - b\*x^2)^(1/4))/c - (Sqrt[a]\*Sqrt[b]\*(1 - a/(b\*x^2))^(3/4)\*(c\*x)^(3/2)\*EllipticF[ArcCsc[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/ (c^2\*(a - b\*x^2)^(3/4))

**Rubi in Sympy [A]** time = 32.3137, size = 76, normalized size = 0.83

$$-\frac{\sqrt{a}\sqrt{b}(cx)^{\frac{3}{2}} \left(-\frac{a}{bx^2} + 1\right)^{\frac{3}{4}} F\left(\frac{\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{c^2 (a - bx^2)^{\frac{3}{4}}} + \frac{\sqrt{cx}\sqrt[4]{a - bx^2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(1/2), x)

[Out] -sqrt(a)\*sqrt(b)\*(c\*x)\*\*(3/2)\*(-a/(b\*x\*\*2) + 1)\*\*(3/4)\*elliptic\_f(asin(sqrt(a)/(sqrt(b)\*x))/2, 2)/(c\*\*2\*(a - b\*x\*\*2)\*\*(3/4)) + sqrt(c\*x)\*(a - b\*x\*\*2)\*\*(1/4)/c

**Mathematica [C]** time = 0.0468663, size = 66, normalized size = 0.72

$$\frac{ax \left(1 - \frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{bx^2}{a}\right) + ax - bx^3}{\sqrt{cx}(a - bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(1/4)/Sqrt[c\*x], x]

[Out] (a\*x - b\*x^3 + a\*x\*(1 - (b\*x^2)/a)^(3/4)\*Hypergeometric2F1[1/4, 3/4, 5/4, (b\*x^2)/a])/(Sqrt[c\*x]\*(a - b\*x^2)^(3/4))

**Maple [F]** time = 0.047, size = 0, normalized size = 0.

$$\int \sqrt[4]{-bx^2 + a} \frac{1}{\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^2+a)^(1/4)/(c\*x)^(1/2), x)

[Out] int((-b\*x^2+a)^(1/4)/(c\*x)^(1/2), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{\frac{1}{4}}}{\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4)/sqrt(c\*x), x, algorithm="maxima")

[Out] integrate((-b\*x^2 + a)^(1/4)/sqrt(c\*x), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-bx^2 + a)^{\frac{1}{4}}}{\sqrt{cx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4)/sqrt(c\*x), x, algorithm="fricas")

[Out] integral((-b\*x^2 + a)^(1/4)/sqrt(c\*x), x)

---

**Sympy [A]** time = 5.77888, size = 39, normalized size = 0.42

$$-\frac{i\sqrt[4]{bx}e^{\frac{3i\pi}{4}}{}_2F_1\left(-\frac{1}{2}, -\frac{1}{4}\middle|\frac{a}{bx^2}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(1/2), x)

[Out] -I\*b\*\*(1/4)\*x\*exp(3\*I\*pi/4)\*hyper((-1/2, -1/4), (1/2,), a/(b\*x\*\*2))/sqrt(c)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{\frac{1}{4}}}{\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2 + a)^(1/4)/sqrt(c*x),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/sqrt(c*x), x)
```

$$3.934 \quad \int \frac{\sqrt[4]{a - bx^2}}{(cx)^{5/2}} dx$$

**Optimal.** Leaf size=97

$$\frac{2b^{3/2}(cx)^{3/2} \left(1 - \frac{a}{bx^2}\right)^{3/4} F\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{3\sqrt{ac^4}(a - bx^2)^{3/4}} - \frac{2\sqrt[4]{a - bx^2}}{3c(cx)^{3/2}}$$

[Out]  $(-2*(a - b*x^2)^{(1/4)})/(3*c*(c*x)^{(3/2)}) + (2*b^{(3/2)}*(1 - a/(b*x^2))^{(3/4)}*(c*x)^{(3/2)}*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/ (3*Sqrt[a]*c^4*(a - b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.215662, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{2b^{3/2}(cx)^{3/2} \left(1 - \frac{a}{bx^2}\right)^{3/4} F\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{3\sqrt{ac^4}(a - bx^2)^{3/4}} - \frac{2\sqrt[4]{a - bx^2}}{3c(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(1/4)/(c\*x)^(5/2), x]

[Out]  $(-2*(a - b*x^2)^{(1/4)})/(3*c*(c*x)^{(3/2)}) + (2*b^{(3/2)}*(1 - a/(b*x^2))^{(3/4)}*(c*x)^{(3/2)}*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/ (3*Sqrt[a]*c^4*(a - b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 32.6221, size = 83, normalized size = 0.86

$$-\frac{2\sqrt[4]{a - bx^2}}{3c(cx)^{3/2}} + \frac{2b^{3/2}(cx)^{3/2} \left(-\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{3\sqrt{ac^4}(a - bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(5/2), x)

[Out]  $-2*(a - b*x^2)^{(1/4)}/(3*c*(c*x)^{(3/2)}) + 2*b^{(3/2)}*(c*x)^{(3/2)}*(-a/(b*x^2) + 1)^{(3/4)}*elliptic_f(asin(sqrt(a)/(sqrt(b)*x))/2, 2)/(3*sqrt(a)*c^4*(a - b*x^2)^{(3/4)})$

**Mathematica [C]** time = 0.0561065, size = 70, normalized size = 0.72

$$\frac{2x \left( bx^2 \left( 1 - \frac{bx^2}{a} \right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{5}{4}; \frac{bx^2}{a}\right) + a - bx^2 \right)}{3(cx)^{5/2}(a - bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(1/4)/(c\*x)^(5/2), x]

[Out]  $(-2*x*(a - b*x^2 + b*x^2*(1 - (b*x^2)/a))^{(3/4)}*Hypergeometric2F1[1/4, 3/4, 5/4, (b*x^2)/a])/ (3*(c*x)^{(5/2)}*(a - b*x^2)^{(3/4)})$

---

**Maple [F]** time = 0.053, size = 0, normalized size = 0.

$$\int \sqrt[4]{-bx^2 + a} (cx)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^2+a)^(1/4)/(c\*x)^(5/2), x)

[Out] int((-b\*x^2+a)^(1/4)/(c\*x)^(5/2), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4)/(c\*x)^(5/2), x, algorithm="maxima")

[Out] integrate((-b\*x^2 + a)^(1/4)/(c\*x)^(5/2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-bx^2 + a)^{\frac{1}{4}}}{\sqrt{cx^2x^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4)/(c\*x)^(5/2), x, algorithm="fricas")

[Out] integral((-b\*x^2 + a)^(1/4)/(sqrt(c\*x)\*c^2\*x^2), x)

---

**Sympy [A]** time = 58.3374, size = 37, normalized size = 0.38

$$\frac{i\sqrt[4]{be^{\frac{7i\pi}{4}}} {}_2F_1\left(-\frac{1}{4}, \frac{1}{2} \middle| \frac{a}{bx^2}\right)}{c^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(5/2), x)

[Out] -I\*b\*\*(1/4)\*exp(7\*I\*pi/4)\*hyper((-1/4, 1/2), (3/2,), a/(b\*x\*\*2))/(c\*\*(5/2)\*x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2 + a)^(1/4)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(5/2), x)
```

$$3.935 \quad \int \frac{\sqrt[4]{a - bx^2}}{(cx)^{9/2}} dx$$

**Optimal.** Leaf size=127

$$\frac{4b^{5/2}(cx)^{3/2} \left(1 - \frac{a}{bx^2}\right)^{3/4} F\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{21a^{3/2}c^6(a - bx^2)^{3/4}} + \frac{2b\sqrt[4]{a - bx^2}}{21ac^3(cx)^{3/2}} - \frac{2\sqrt[4]{a - bx^2}}{7c(cx)^{7/2}}$$

[Out]  $(-2*(a - b*x^2)^{(1/4)})/(7*c*(c*x)^{(7/2)}) + (2*b*(a - b*x^2)^{(1/4)})/(21*a*c^3*(c*x)^{(3/2)}) + (4*b^{5/2}*(1 - a/(b*x^2))^{3/4}*(c*x)^{(3/2)}*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^{3/2}*c^6*(a - b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.262285, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$\frac{4b^{5/2}(cx)^{3/2} \left(1 - \frac{a}{bx^2}\right)^{3/4} F\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{21a^{3/2}c^6(a - bx^2)^{3/4}} + \frac{2b\sqrt[4]{a - bx^2}}{21ac^3(cx)^{3/2}} - \frac{2\sqrt[4]{a - bx^2}}{7c(cx)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(1/4)/(c\*x)^(9/2), x]

[Out]  $(-2*(a - b*x^2)^{(1/4)})/(7*c*(c*x)^{(7/2)}) + (2*b*(a - b*x^2)^{(1/4)})/(21*a*c^3*(c*x)^{(3/2)}) + (4*b^{5/2}*(1 - a/(b*x^2))^{3/4}*(c*x)^{(3/2)}*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^{3/2}*c^6*(a - b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 38.4704, size = 110, normalized size = 0.87

$$-\frac{2\sqrt[4]{a - bx^2}}{7c(cx)^{7/2}} + \frac{2b\sqrt[4]{a - bx^2}}{21ac^3(cx)^{3/2}} + \frac{4b^{5/2}(cx)^{3/2} \left(-\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{21a^{3/2}c^6(a - bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(9/2), x)

[Out]  $-2*(a - b*x**2)**(1/4)/(7*c*(c*x)**(7/2)) + 2*b*(a - b*x**2)**(1/4)/(21*a*c**3*(c*x)**(3/2)) + 4*b**(5/2)*(c*x)**(3/2)*(-a/(b*x**2) + 1)**(3/4)*elliptic_f(asin(sqrt(a)/(sqrt(b)*x))/2, 2)/(21*a**3/2*c**6*(a - b*x**2)**(3/4))$

**Mathematica [C]** time = 0.0840992, size = 93, normalized size = 0.73

$$\frac{2\sqrt{cx} \left(3a^2 + 2b^2x^4 \left(1 - \frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{bx^2}{a}\right) - 4abx^2 + b^2x^4\right)}{21ac^5x^4(a - bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(1/4)/(c\*x)^(9/2), x]

[Out]  $(-2*\sqrt{c*x}*(3*a^2 - 4*a*b*x^2 + b^2*x^4 + 2*b^2*x^4*(1 - (b*x^2)/a)^{3/4})*Hypergeometric2F1[1/4, 3/4, 5/4, (b*x^2)/a])/(21*a*c$



$$x^5 \cdot x^4 \cdot (a - b \cdot x^2)^{3/4}$$

**Maple [F]** time = 0.057, size = 0, normalized size = 0.

$$\int \sqrt[4]{-bx^2 + a} (cx)^{-\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^2+a)^(1/4)/(c\*x)^(9/2), x)

[Out] int((-b\*x^2+a)^(1/4)/(c\*x)^(9/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4)/(c\*x)^(9/2), x, algorithm="maxima")

[Out] integrate((-b\*x^2 + a)^(1/4)/(c\*x)^(9/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-bx^2 + a)^{\frac{1}{4}}}{\sqrt{cx}c^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4)/(c\*x)^(9/2), x, algorithm="fricas")

[Out] integral((-b\*x^2 + a)^(1/4)/(sqrt(c\*x)\*c^4\*x^4), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(9/2), x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2 + a)^(1/4)/(c*x)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(9/2), x)
```

$$3.936 \quad \int \frac{\sqrt[4]{a - bx^2}}{(cx)^{13/2}} dx$$

**Optimal.** Leaf size=159

$$\frac{8b^{7/2}(cx)^{3/2} \left(1 - \frac{a}{bx^2}\right)^{3/4} F\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{77a^{5/2}c^8(a - bx^2)^{3/4}} + \frac{4b^2\sqrt[4]{a - bx^2}}{77a^2c^5(cx)^{3/2}} + \frac{2b\sqrt[4]{a - bx^2}}{77ac^3(cx)^{7/2}} - \frac{2\sqrt[4]{a - bx^2}}{11c(cx)^{11/2}}$$

[Out]  $(-2*(a - b*x^2)^{(1/4)})/(11*c*(c*x)^{(11/2)}) + (2*b*(a - b*x^2)^{(1/4)})/(77*a*c^3*(c*x)^{(7/2)}) + (4*b^2*(a - b*x^2)^{(1/4)})/(77*a^2*c^5*(c*x)^{(3/2)}) + (8*b^{(7/2)}*(1 - a/(b*x^2))^{(3/4)}*(c*x)^{(3/2)})*\operatorname{EllipticF}[\operatorname{ArcCsc}[(\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]/2, 2]/(77*a^{(5/2)}*c^8*(a - b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.315112, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$\frac{8b^{7/2}(cx)^{3/2} \left(1 - \frac{a}{bx^2}\right)^{3/4} F\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{77a^{5/2}c^8(a - bx^2)^{3/4}} + \frac{4b^2\sqrt[4]{a - bx^2}}{77a^2c^5(cx)^{3/2}} + \frac{2b\sqrt[4]{a - bx^2}}{77ac^3(cx)^{7/2}} - \frac{2\sqrt[4]{a - bx^2}}{11c(cx)^{11/2}}$$

Antiderivative was successfully verified.

[In] `Int[(a - b*x^2)^(1/4)/(c*x)^(13/2), x]`

[Out]  $(-2*(a - b*x^2)^{(1/4)})/(11*c*(c*x)^{(11/2)}) + (2*b*(a - b*x^2)^{(1/4)})/(77*a*c^3*(c*x)^{(7/2)}) + (4*b^2*(a - b*x^2)^{(1/4)})/(77*a^2*c^5*(c*x)^{(3/2)}) + (8*b^{(7/2)}*(1 - a/(b*x^2))^{(3/4)}*(c*x)^{(3/2)})*\operatorname{EllipticF}[\operatorname{ArcCsc}[(\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]/2, 2]/(77*a^{(5/2)}*c^8*(a - b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 45.5805, size = 141, normalized size = 0.89

$$-\frac{2\sqrt[4]{a - bx^2}}{11c(cx)^{11/2}} + \frac{2b\sqrt[4]{a - bx^2}}{77ac^3(cx)^{7/2}} + \frac{4b^2\sqrt[4]{a - bx^2}}{77a^2c^5(cx)^{3/2}} + \frac{8b^{7/2}(cx)^{3/2} \left(-\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{77a^{5/2}c^8(a - bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((-b*x**2+a)**(1/4)/(c*x)**(13/2), x)`

[Out]  $-2*(a - b*x^2)**(1/4)/(11*c*(c*x)**(11/2)) + 2*b*(a - b*x^2)**(1/4)/(77*a*c^3*(c*x)**(7/2)) + 4*b^2*(a - b*x^2)**(1/4)/(77*a^2*c^5*(c*x)**(3/2)) + 8*b^{(7/2)}*(c*x)**(3/2)*(-a/(b*x^2) + 1)**(3/4)*\operatorname{elliptic_f}(\operatorname{asin}(\operatorname{sqrt}(a)/(\operatorname{sqrt}(b)*x))/2, 2)/(77*a^{(5/2)}*c^8*(a - b*x^2)**(3/4))$

**Mathematica [C]** time = 0.105193, size = 105, normalized size = 0.66

$$\frac{2\sqrt{cx} \left(7a^3 - 8a^2bx^2 + 4b^3x^6 \left(1 - \frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{5}{4}; \frac{bx^2}{a}\right) - ab^2x^4 + 2b^3x^6\right)}{77a^2c^7x^6(a - bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a - b*x^2)^(1/4)/(c*x)^(13/2), x]`

[Out]  $(-2\sqrt{c^2x} (7a^3 - 8a^2bx^2 - ab^2x^4 + 2b^3x^6 + 4b^3x^6(1 - (bx^2)/a)^{3/4}) \text{Hypergeometric2F1}[1/4, 3/4, 5/4, (bx^2)/a]) / (77a^2c^7x^6(a - bx^2)^{3/4})$

**Maple [F]** time = 0.057, size = 0, normalized size = 0.

$$\int 1\sqrt[4]{-bx^2 + a} (cx)^{-\frac{13}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^2+a)^(1/4)/(c*x)^(13/2), x)`

[Out] `int((-b*x^2+a)^(1/4)/(c*x)^(13/2), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)/(c*x)^(13/2), x, algorithm="maxima")`

[Out] `integrate((-b*x^2 + a)^(1/4)/(c*x)^(13/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-bx^2 + a)^{\frac{1}{4}}}{\sqrt{cx}c^6x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)/(c*x)^(13/2), x, algorithm="fricas")`

[Out] `integral((-b*x^2 + a)^(1/4)/(sqrt(c*x)*c^6*x^6), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**2+a)**(1/4)/(c*x)**(13/2), x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-bx^2 + a)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2 + a)^(1/4)/(c*x)^(13/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(1/4)/(c*x)^(13/2), x)
```

### 3.937 $\int (cx)^{5/2} \sqrt[4]{a - bx^2} dx$

**Optimal.** Leaf size=343

$$\begin{aligned} & \frac{3a^2c^{5/2} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{64\sqrt{2}b^{7/4}} - \frac{3a^2c^{5/2} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{64\sqrt{2}b^{7/4}} \\ & - \frac{3a^2c^{5/2} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{32\sqrt{2}b^{7/4}} + \frac{3a^2c^{5/2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{32\sqrt{2}b^{7/4}} \\ & + \frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c} - \frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b} \end{aligned}$$

[Out]  $-(a*c*(c*x)^{(3/2)}*(a - b*x^2)^{(1/4)})/(16*b) + ((c*x)^{(7/2)}*(a - b*x^2)^{(1/4)})/(4*c) - (3*a^2*c^{(5/2)}*ArcTan[1 - (Sqrt[2]*b^{(1/4)}*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^{(1/4)})])/(32*Sqrt[2]*b^{(7/4)}) + (3*a^2*c^{(5/2)}*ArcTan[1 + (Sqrt[2]*b^{(1/4)}*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^{(1/4)})])/(32*Sqrt[2]*b^{(7/4)}) + (3*a^2*c^{(5/2)}*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] - (Sqrt[2]*b^{(1/4)}*Sqrt[c*x])/(a - b*x^2)^{(1/4)})]/(64*Sqrt[2]*b^{(7/4)}) - (3*a^2*c^{(5/2)}*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b^{(1/4)}*Sqrt[c*x])/(a - b*x^2)^{(1/4)})]/(64*Sqrt[2]*b^{(7/4)})$

**Rubi [A]** time = 0.845926, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\begin{aligned} & \frac{3a^2c^{5/2} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{64\sqrt{2}b^{7/4}} - \frac{3a^2c^{5/2} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{64\sqrt{2}b^{7/4}} \\ & - \frac{3a^2c^{5/2} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{32\sqrt{2}b^{7/4}} + \frac{3a^2c^{5/2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{32\sqrt{2}b^{7/4}} \\ & + \frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c} - \frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(5/2)\*(a - b\*x^2)^(1/4), x]

[Out]  $-(a*c*(c*x)^{(3/2)}*(a - b*x^2)^{(1/4)})/(16*b) + ((c*x)^{(7/2)}*(a - b*x^2)^{(1/4)})/(4*c) - (3*a^2*c^{(5/2)}*ArcTan[1 - (Sqrt[2]*b^{(1/4)}*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^{(1/4)})])/(32*Sqrt[2]*b^{(7/4)}) + (3*a^2*c^{(5/2)}*ArcTan[1 + (Sqrt[2]*b^{(1/4)}*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^{(1/4)})])/(32*Sqrt[2]*b^{(7/4)}) + (3*a^2*c^{(5/2)}*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] - (Sqrt[2]*b^{(1/4)}*Sqrt[c*x])/(a - b*x^2)^{(1/4)})]/(64*Sqrt[2]*b^{(7/4)}) - (3*a^2*c^{(5/2)}*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a - b*x^2] + (Sqrt[2]*b^{(1/4)}*Sqrt[c*x])/(a - b*x^2)^{(1/4)})]/(64*Sqrt[2]*b^{(7/4)})$

**Rubi in Sympy [A]** time = 76.9713, size = 311, normalized size = 0.91

$$\begin{aligned} & \frac{3\sqrt{2}a^2c^{\frac{5}{2}} \log\left(-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \frac{\sqrt{b}cx}{\sqrt{a-bx^2}} + c\right)}{128b^{\frac{7}{4}}} - \frac{3\sqrt{2}a^2c^{\frac{5}{2}} \log\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \frac{\sqrt{b}cx}{\sqrt{a-bx^2}} + c\right)}{128b^{\frac{7}{4}}} \\ & + \frac{3\sqrt{2}a^2c^{\frac{5}{2}} \operatorname{atan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} - 1\right)}{64b^{\frac{7}{4}}} + \frac{3\sqrt{2}a^2c^{\frac{5}{2}} \operatorname{atan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{64b^{\frac{7}{4}}} \\ & - \frac{ac(cx)^{\frac{3}{2}}\sqrt[4]{a-bx^2}}{16b} + \frac{(cx)^{\frac{7}{2}}\sqrt[4]{a-bx^2}}{4c} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x)**(5/2)*(-b*x**2+a)**(1/4),x)`

[Out] 
$$3\sqrt{2}a^{3/2}c^{5/2}\log(-\sqrt{2}b^{1/4}\sqrt{c}\sqrt{cx}/(a-bx^2)^{1/4} + \sqrt{b}cx/\sqrt{a-bx^2} + c)/(128b^{7/4}) - 3\sqrt{2}a^{3/2}c^{5/2}\log(\sqrt{2}b^{1/4}\sqrt{c}\sqrt{cx}/(a-bx^2)^{1/4} + \sqrt{b}cx/\sqrt{a-bx^2} + c)/(128b^{7/4}) + 3\sqrt{2}a^{3/2}c^{5/2}\operatorname{atan}(\sqrt{2}b^{1/4}\sqrt{cx}/(\sqrt{c}(a-bx^2)^{1/4}) - 1)/(64b^{7/4}) + 3\sqrt{2}a^{3/2}c^{5/2}\operatorname{atan}(\sqrt{2}b^{1/4}\sqrt{cx}/(\sqrt{c}(a-bx^2)^{1/4}) + 1)/(64b^{7/4}) - ac^{3/2}(cx)^{3/2}(a-bx^2)^{1/4}/(16b) + (cx)^{7/2}(a-bx^2)^{1/4}/(4c)$$

**Mathematica [C]** time = 0.0671904, size = 84, normalized size = 0.24

$$\frac{c(cx)^{3/2} \left( -a^2 \left( 1 - \frac{bx^2}{a} \right)^{3/4} {}_2F_1 \left( \frac{3}{4}, \frac{3}{4}; \frac{7}{4}; \frac{bx^2}{a} \right) + a^2 - 5abx^2 + 4b^2x^4 \right)}{16b(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c*x)^(5/2)*(a-b*x^2)^(1/4),x]`

[Out] 
$$-(c^{3/2}(cx)^{3/2}(a^2 - 5abx^2 + 4b^2x^4 - a^2(1 - (bx^2)/a)^{3/4})\operatorname{Hypergeometric2F1}[3/4, 3/4, 7/4, (bx^2)/a])/(16b(a - bx^2)^{3/4})$$

**Maple [F]** time = 0.053, size = 0, normalized size = 0.

$$\int (cx)^{5/2} \sqrt[4]{-bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(5/2)*(-b*x^2+a)^(1/4),x)`

[Out] `int((c*x)^(5/2)*(-b*x^2+a)^(1/4),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)*(c*x)^(5/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4)\*(c\*x)^(5/2),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(5/2)\*(-b\*x\*\*2+a)\*\*(1/4),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.26256, size = 533, normalized size = 1.55

$$-\frac{1}{128}a^2c^6 \left( \frac{6\sqrt{2}\sqrt{|c|} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}\sqrt{|c|} + \frac{2(-bc^2x^2+ac^2)^{\frac{1}{4}}\sqrt{|c|}}{\sqrt{cx}}\right)}{2b^{\frac{1}{4}}\sqrt{|c|}}\right)}{b^{\frac{7}{4}}c^4} + \frac{6\sqrt{2}\sqrt{|c|} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}\sqrt{|c|} - \frac{2(-bc^2x^2+ac^2)^{\frac{1}{4}}\sqrt{|c|}}{\sqrt{cx}}\right)}{2b^{\frac{1}{4}}\sqrt{|c|}}\right)}{b^{\frac{7}{4}}c^4} + \frac{3\sqrt{2}\sqrt{|c|}}{b^{\frac{7}{4}}c^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4)\*(c\*x)^(5/2),x, algorithm="giac")

[Out] 
$$-1/128*a^2*c^6*(6*\sqrt{2}*\sqrt{\text{abs}(c)}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*b^{1/4}*\sqrt{\text{abs}(c)} + 2*(-b*c^2*x^2 + a*c^2)^{1/4}*\sqrt{\text{abs}(c)})/\sqrt{c*x})/(b^{1/4}*\sqrt{\text{abs}(c)}))/b^{7/4}*c^4 + 6*\sqrt{2}*\sqrt{\text{abs}(c)}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*b^{1/4}*\sqrt{\text{abs}(c)} - 2*(-b*c^2*x^2 + a*c^2)^{1/4}*\sqrt{\text{abs}(c)})/\sqrt{c*x})/(b^{1/4}*\sqrt{\text{abs}(c)}))/b^{7/4}*c^4 + 3*\sqrt{2}*\sqrt{\text{abs}(c)}*\ln(\sqrt{2}*(-b*c^2*x^2 + a*c^2)^{1/4}*b^{1/4}*\text{abs}(c)/\sqrt{c*x} + \sqrt{b}*\text{abs}(c) + \sqrt{-b*c^2*x^2 + a*c^2}*\text{abs}(c)/(c*x))/b^{7/4}*c^4 - 3*\sqrt{2}*\sqrt{\text{abs}(c)}*\ln(-\sqrt{2}*(-b*c^2*x^2 + a*c^2)^{1/4}*b^{1/4}*\text{abs}(c)/\sqrt{c*x} + \sqrt{b}*\text{abs}(c) + \sqrt{-b*c^2*x^2 + a*c^2}*\text{abs}(c)/(c*x))/b^{7/4}*c^4 - 8*(3*(-b*c^2*x^2 + a*c^2)^{1/4}*b*c^2*\sqrt{\text{abs}(c)}/\sqrt{c*x} + (-b*c^2*x^2 + a*c^2)^{1/4}*(b*c^2 - a*c^2/x^2))*\sqrt{\text{abs}(c)}/\sqrt{c*x})*x^4/(a^2*b*c^6)$$



### 3.938 $\int \sqrt{cx} \sqrt[4]{a - bx^2} dx$

**Optimal.** Leaf size=307

$$\frac{a\sqrt{c} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} - \frac{\sqrt[4]{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{8\sqrt{2}b^{3/4}} - \frac{a\sqrt{c} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} + \frac{\sqrt[4]{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{8\sqrt{2}b^{3/4}} \\ - \frac{a\sqrt{c} \tan^{-1}\left(1 - \frac{\sqrt[4]{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{3/4}} + \frac{a\sqrt{c} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{4\sqrt{2}b^{3/4}} + \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c}$$

[Out]  $((c*x)^{(3/2)}*(a - b*x^2)^{(1/4)})/(2*c) - (a*\text{Sqrt}[c]*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(\text{Sqrt}[c]*(a - b*x^2)^{(1/4)})])/(4*\text{Sqrt}[2]*b^{(3/4)}) + (a*\text{Sqrt}[c]*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(\text{Sqrt}[c]*(a - b*x^2)^{(1/4)})])/(4*\text{Sqrt}[2]*b^{(3/4)}) + (a*\text{Sqrt}[c]*\text{Log}[\text{Sqrt}[c] + (\text{Sqrt}[b]*\text{Sqrt}[c]*x)/\text{Sqrt}[a - b*x^2] - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(a - b*x^2)^{(1/4)}])/(8*\text{Sqrt}[2]*b^{(3/4)}) - (a*\text{Sqrt}[c]*\text{Log}[\text{Sqrt}[c] + (\text{Sqrt}[b]*\text{Sqrt}[c]*x)/\text{Sqrt}[a - b*x^2] + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(a - b*x^2)^{(1/4)}])/(8*\text{Sqrt}[2]*b^{(3/4)})$

**Rubi [A]** time = 0.650293, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$

$$\frac{a\sqrt{c} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} - \frac{\sqrt[4]{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{8\sqrt{2}b^{3/4}} - \frac{a\sqrt{c} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} + \frac{\sqrt[4]{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{8\sqrt{2}b^{3/4}} \\ - \frac{a\sqrt{c} \tan^{-1}\left(1 - \frac{\sqrt[4]{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{3/4}} + \frac{a\sqrt{c} \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{4\sqrt{2}b^{3/4}} + \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[c*x]*(a - b*x^2)^{(1/4)}, x]$

[Out]  $((c*x)^{(3/2)}*(a - b*x^2)^{(1/4)})/(2*c) - (a*\text{Sqrt}[c]*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(\text{Sqrt}[c]*(a - b*x^2)^{(1/4)})])/(4*\text{Sqrt}[2]*b^{(3/4)}) + (a*\text{Sqrt}[c]*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(\text{Sqrt}[c]*(a - b*x^2)^{(1/4)})])/(4*\text{Sqrt}[2]*b^{(3/4)}) + (a*\text{Sqrt}[c]*\text{Log}[\text{Sqrt}[c] + (\text{Sqrt}[b]*\text{Sqrt}[c]*x)/\text{Sqrt}[a - b*x^2] - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(a - b*x^2)^{(1/4)}])/(8*\text{Sqrt}[2]*b^{(3/4)}) - (a*\text{Sqrt}[c]*\text{Log}[\text{Sqrt}[c] + (\text{Sqrt}[b]*\text{Sqrt}[c]*x)/\text{Sqrt}[a - b*x^2] + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(a - b*x^2)^{(1/4)}])/(8*\text{Sqrt}[2]*b^{(3/4)})$

**Rubi in Sympy [A]** time = 66.2793, size = 274, normalized size = 0.89

$$\frac{\sqrt{2}a\sqrt{c} \log\left(-\frac{\sqrt[4]{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt[4]{a-bx^2}} + \frac{\sqrt{bcx}}{\sqrt{a-bx^2}} + c\right)}{16b^{3/4}} - \frac{\sqrt{2}a\sqrt{c} \log\left(\frac{\sqrt[4]{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt[4]{a-bx^2}} + \frac{\sqrt{bcx}}{\sqrt{a-bx^2}} + c\right)}{16b^{3/4}} \\ + \frac{\sqrt{2}a\sqrt{c} \text{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} - 1\right)}{8b^{3/4}} + \frac{\sqrt{2}a\sqrt{c} \text{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{8b^{3/4}} + \frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c*x)**(1/2)*(-b*x**2+a)**(1/4), x)$

[Out]  $\text{sqrt}(2)*a*\text{sqrt}(c)*\log(-\text{sqrt}(2)*b**(1/4)*\text{sqrt}(c)*\text{sqrt}(c*x)/(a - b*x**2)**(1/4) + \text{sqrt}(b)*c*x/\text{sqrt}(a - b*x**2) + c)/(16*b**(3/4)) -$

$$\begin{aligned} & \sqrt{2} * a * \sqrt{c} * \log(\sqrt{2} * b^{1/4} * \sqrt{c} * \sqrt{c * x} / (a - b * x^{1/2}))^{1/4} + \sqrt{b} * c * x / \sqrt{a - b * x^{1/2} + c} / (16 * b^{3/4}) + \sqrt{2} * a * \sqrt{c} * \operatorname{atan}(\sqrt{2} * b^{1/4} * \sqrt{c * x} / (\sqrt{c} * (a - b * x^{1/2})^{1/4})) - 1) / (8 * b^{3/4}) + \sqrt{2} * a * \sqrt{c} * \operatorname{atan}(\sqrt{2} * b^{1/4} * \sqrt{c * x} / (\sqrt{c} * (a - b * x^{1/2})^{1/4})) + 1) / (8 * b^{3/4}) \\ & + (c * x)^{3/2} * (a - b * x^{1/2})^{1/4} / (2 * c) \end{aligned}$$

**Mathematica [C]** time = 0.0659322, size = 69, normalized size = 0.22

$$\frac{x\sqrt{cx} \left( a \left( 1 - \frac{bx^2}{a} \right)^{3/4} {}_2F_1 \left( \frac{3}{4}, \frac{3}{4}; \frac{7}{4}; \frac{bx^2}{a} \right) + 3a - 3bx^2 \right)}{6(a - bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c\*x]\*(a - b\*x^2)^(1/4),x]

[Out] (x\*Sqrt[c\*x]\*(3\*a - 3\*b\*x^2 + a\*(1 - (b\*x^2)/a)^(3/4)\*Hypergeometric2F1[3/4, 3/4, 7/4, (b\*x^2)/a]))/(6\*(a - b\*x^2)^(3/4))

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int \sqrt{cx} \sqrt[4]{-bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(1/2)\*(-b\*x^2+a)^(1/4),x)

[Out] int((c\*x)^(1/2)\*(-b\*x^2+a)^(1/4),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4)\*sqrt(c\*x),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4)\*sqrt(c\*x),x, algorithm="fricas")

[Out] Timed out

**Sympy [A]** time = 9.44392, size = 48, normalized size = 0.16

$$\frac{\sqrt[4]{a}\sqrt{c}x^{\frac{3}{2}}\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{3}{4} \\ \frac{7}{4} \end{matrix} \middle| \frac{bx^2e^{2i\pi}}{a}\right)}{2\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(1/2)\*(-b\*x\*\*2+a)\*\*(1/4), x)

[Out] a\*\*(1/4)\*sqrt(c)\*x\*\*(3/2)\*gamma(3/4)\*hyper((-1/4, 3/4), (7/4, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/(2\*gamma(7/4))

**GIAC/XCAS [A]** time = 0.263031, size = 459, normalized size = 1.5

$$\frac{1}{16}ac^2 \left( \frac{8(-bc^2x^2 + ac^2)^{\frac{1}{4}}x^2\sqrt{|c|}}{\sqrt{c}xac^2} - \frac{2\sqrt{2}\sqrt{|c|} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}\sqrt{|c|} + \frac{2(-bc^2x^2 + ac^2)^{\frac{1}{4}}\sqrt{|c|}}{\sqrt{c}x}\right)}{2b^{\frac{1}{4}}\sqrt{|c|}}\right)}{b^{\frac{3}{4}}c^2} - \frac{2\sqrt{2}\sqrt{|c|} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}\sqrt{|c|} - \frac{2(-bc^2x^2 + ac^2)^{\frac{1}{4}}\sqrt{|c|}}{\sqrt{c}x}\right)}{2b^{\frac{1}{4}}\sqrt{|c|}}\right)}{b^{\frac{3}{4}}c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4)\*sqrt(c\*x), x, algorithm="giac")

[Out] 1/16\*a\*c^2\*(8\*(-b\*c^2\*x^2 + a\*c^2)^(1/4)\*x^2\*sqrt(abs(c))/(sqrt(c\*x)\*a\*c^2) - 2\*sqrt(2)\*sqrt(abs(c))\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*b^(1/4)\*sqrt(abs(c)) + 2\*(-b\*c^2\*x^2 + a\*c^2)^(1/4)\*sqrt(abs(c))/sqrt(c\*x))/(b^(1/4)\*sqrt(abs(c))))/(b^(3/4)\*c^2) - 2\*sqrt(2)\*sqrt(abs(c))\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*b^(1/4)\*sqrt(abs(c)) - 2\*(-b\*c^2\*x^2 + a\*c^2)^(1/4)\*sqrt(abs(c))/sqrt(c\*x))/(b^(1/4)\*sqrt(abs(c))))/(b^(3/4)\*c^2) - sqrt(2)\*sqrt(abs(c))\*ln(sqrt(2)\*(-b\*c^2\*x^2 + a\*c^2)^(1/4)\*b^(1/4)\*abs(c)/sqrt(c\*x) + sqrt(b)\*abs(c) + sqrt(-b\*c^2\*x^2 + a\*c^2)\*abs(c)/(c\*x))/(b^(3/4)\*c^2) + sqrt(2)\*sqrt(abs(c))\*ln(-sqrt(2)\*(-b\*c^2\*x^2 + a\*c^2)^(1/4)\*b^(1/4)\*abs(c)/sqrt(c\*x) + sqrt(b)\*abs(c) + sqrt(-b\*c^2\*x^2 + a\*c^2)\*abs(c)/(c\*x))/(b^(3/4)\*c^2))

$$3.939 \quad \int \frac{\sqrt[4]{a - bx^2}}{(cx)^{3/2}} dx$$

**Optimal.** Leaf size=296

$$\begin{aligned} & -\frac{\sqrt[4]{b} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{2\sqrt{2}c^{3/2}} + \frac{\sqrt[4]{b} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{2\sqrt{2}c^{3/2}} \\ & + \frac{\sqrt[4]{b} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}c^{3/2}} - \frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{\sqrt{2}c^{3/2}} - \frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} \end{aligned}$$

[Out]  $(-2*(a - b*x^2)^{(1/4)})/(c*\text{Sqrt}[c*x]) + (b^{(1/4)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(\text{Sqrt}[c]*(a - b*x^2)^{(1/4)})]) / (\text{Sqrt}[2]*c^{(3/2)}) - (b^{(1/4)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(\text{Sqrt}[c]*(a - b*x^2)^{(1/4)})]) / (\text{Sqrt}[2]*c^{(3/2)}) - (b^{(1/4)}*\text{Log}[\text{Sqrt}[c] + (\text{Sqrt}[b]*\text{Sqrt}[c]*x)/\text{Sqrt}[a - b*x^2] - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(a - b*x^2)^{(1/4)}]) / (2*\text{Sqrt}[2]*c^{(3/2)}) + (b^{(1/4)}*\text{Log}[\text{Sqrt}[c] + (\text{Sqrt}[b]*\text{Sqrt}[c]*x)/\text{Sqrt}[a - b*x^2] + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(a - b*x^2)^{(1/4)}]) / (2*\text{Sqrt}[2]*c^{(3/2)})$

**Rubi [A]** time = 0.614862, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$

$$\begin{aligned} & -\frac{\sqrt[4]{b} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{2\sqrt{2}c^{3/2}} + \frac{\sqrt[4]{b} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{2\sqrt{2}c^{3/2}} \\ & + \frac{\sqrt[4]{b} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}c^{3/2}} - \frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{\sqrt{2}c^{3/2}} - \frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a - b*x^2)^{(1/4)}/(c*x)^{(3/2)}, x]$

[Out]  $(-2*(a - b*x^2)^{(1/4)})/(c*\text{Sqrt}[c*x]) + (b^{(1/4)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(\text{Sqrt}[c]*(a - b*x^2)^{(1/4)})]) / (\text{Sqrt}[2]*c^{(3/2)}) - (b^{(1/4)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(\text{Sqrt}[c]*(a - b*x^2)^{(1/4)})]) / (\text{Sqrt}[2]*c^{(3/2)}) - (b^{(1/4)}*\text{Log}[\text{Sqrt}[c] + (\text{Sqrt}[b]*\text{Sqrt}[c]*x)/\text{Sqrt}[a - b*x^2] - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(a - b*x^2)^{(1/4)}]) / (2*\text{Sqrt}[2]*c^{(3/2)}) + (b^{(1/4)}*\text{Log}[\text{Sqrt}[c] + (\text{Sqrt}[b]*\text{Sqrt}[c]*x)/\text{Sqrt}[a - b*x^2] + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(a - b*x^2)^{(1/4)}]) / (2*\text{Sqrt}[2]*c^{(3/2)})$

**Rubi in Sympy [A]** time = 66.3904, size = 267, normalized size = 0.9

$$\begin{aligned} & -\frac{\sqrt{2}\sqrt[4]{b} \log\left(-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \frac{\sqrt{bcx}}{\sqrt{a-bx^2}} + c\right)}{4c^{\frac{3}{2}}} + \frac{\sqrt{2}\sqrt[4]{b} \log\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \frac{\sqrt{bcx}}{\sqrt{a-bx^2}} + c\right)}{4c^{\frac{3}{2}}} \\ & - \frac{\sqrt{2}\sqrt[4]{b} \text{atan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} - 1\right)}{2c^{\frac{3}{2}}} - \frac{\sqrt{2}\sqrt[4]{b} \text{atan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{2c^{\frac{3}{2}}} - \frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-b*x**2+a)**(1/4)/(c*x)**(3/2), x)$

[Out]  $-\text{sqrt}(2)*b**(1/4)*\log(-\text{sqrt}(2)*b**(1/4)*\text{sqrt}(c)*\text{sqrt}(c*x)/(a - b*x**2)**(1/4) + \text{sqrt}(b)*c*x/\text{sqrt}(a - b*x**2) + c)/(4*c**(3/2)) + s$

$$\sqrt[2]{2} b^{1/4} \log(\sqrt[2]{2} b^{1/4} \sqrt{c} \sqrt{cx}) / (a - b x^2)^{1/4} + \sqrt{b} c x / \sqrt{a - b x^2} + c / (4 c^{3/2}) - \sqrt[2]{2} b^{1/4} \operatorname{atan}(\sqrt[2]{2} b^{1/4} \sqrt{cx}) / (\sqrt{c} (a - b x^2)^{1/4}) - 1 / (2 c^{3/2}) - \sqrt[2]{2} b^{1/4} \operatorname{atan}(\sqrt[2]{2} b^{1/4} \sqrt{cx}) / (\sqrt{c} (a - b x^2)^{1/4}) + 1 / (2 c^{3/2}) - 2 (a - b x^2)^{1/4} / (c \sqrt{cx})$$

**Mathematica [C]** time = 0.0646625, size = 72, normalized size = 0.24

$$\frac{2x \left( bx^2 \left( 1 - \frac{bx^2}{a} \right)^{3/4} {}_2F_1 \left( \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{bx^2}{a} \right) + 3a - 3bx^2 \right)}{3(cx)^{3/2} (a - bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(1/4)/(c\*x)^(3/2), x]

[Out] (-2\*x\*(3\*a - 3\*b\*x^2 + b\*x^2\*(1 - (b\*x^2)/a))^(3/4)\*Hypergeometric2F1[3/4, 3/4, 7/4, (b\*x^2)/a])/(3\*(c\*x)^(3/2)\*(a - b\*x^2)^(3/4))

**Maple [F]** time = 0.05, size = 0, normalized size = 0.

$$\int 1 \sqrt[4]{-bx^2 + a} (cx)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^2+a)^(1/4)/(c\*x)^(3/2), x)

[Out] int((-b\*x^2+a)^(1/4)/(c\*x)^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4)/(c\*x)^(3/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4)/(c\*x)^(3/2), x, algorithm="fricas")

[Out] Timed out

**Sympy [A]** time = 14.9041, size = 51, normalized size = 0.17

$$\frac{\sqrt[4]{a} \left(-\frac{1}{4}\right) {}_2F_1\left(-\frac{1}{4}, -\frac{1}{4} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{2c^{\frac{3}{2}} \sqrt{x} \left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(3/2), x)

[Out] a\*\*(1/4)\*gamma(-1/4)\*hyper((-1/4, -1/4), (3/4, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/(2\*c\*\*(3/2)\*sqrt(x)\*gamma(3/4))

**GIAC/XCAS [A]** time = 0.253768, size = 429, normalized size = 1.45

$$2\sqrt{2}b^{\frac{1}{4}}\sqrt{|c|} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}\sqrt{|c|} + \frac{2(-bc^2x^2+ac^2)^{\frac{1}{4}}\sqrt{|c|}}{\sqrt{cx}}\right)}{2b^{\frac{1}{4}}\sqrt{|c|}}\right) + 2\sqrt{2}b^{\frac{1}{4}}\sqrt{|c|} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}\sqrt{|c|} - \frac{2(-bc^2x^2+ac^2)^{\frac{1}{4}}\sqrt{|c|}}{\sqrt{cx}}\right)}{2b^{\frac{1}{4}}\sqrt{|c|}}\right) + \sqrt{2}b^{\frac{1}{4}}\sqrt{|c|} \ln\left(\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^2 + a)^(1/4)/(c\*x)^(3/2), x, algorithm="giac")

[Out] 1/4\*(2\*sqrt(2)\*b^(1/4)\*sqrt(abs(c))\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*b^(1/4)\*sqrt(abs(c)) + 2\*(-b\*c^2\*x^2 + a\*c^2)^(1/4)\*sqrt(abs(c))/sqrt(c\*x))/(b^(1/4)\*sqrt(abs(c)))) + 2\*sqrt(2)\*b^(1/4)\*sqrt(abs(c))\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*b^(1/4)\*sqrt(abs(c)) - 2\*(-b\*c^2\*x^2 + a\*c^2)^(1/4)\*sqrt(abs(c))/sqrt(c\*x))/(b^(1/4)\*sqrt(abs(c)))) + sqrt(2)\*b^(1/4)\*sqrt(abs(c))\*ln(sqrt(2)\*(-b\*c^2\*x^2 + a\*c^2)^(1/4)\*b^(1/4)\*abs(c)/sqrt(c\*x) + sqrt(b)\*abs(c) + sqrt(-b\*c^2\*x^2 + a\*c^2)\*abs(c)/(c\*x)) - sqrt(2)\*b^(1/4)\*sqrt(abs(c))\*ln(-sqrt(2)\*(-b\*c^2\*x^2 + a\*c^2)^(1/4)\*b^(1/4)\*abs(c)/sqrt(c\*x) + sqrt(b)\*abs(c) + sqrt(-b\*c^2\*x^2 + a\*c^2)\*abs(c)/(c\*x)) - 8\*(-b\*c^2\*x^2 + a\*c^2)^(1/4)\*sqrt(abs(c))/sqrt(c\*x))/c^2

$$3.940 \quad \int \frac{\sqrt[4]{a - bx^2}}{(cx)^{7/2}} dx$$

**Optimal.** Leaf size=29

$$-\frac{2(a - bx^2)^{5/4}}{5ac(cx)^{5/2}}$$

[Out]  $(-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(5/2))$

**Rubi [A]** time = 0.0278724, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$-\frac{2(a - bx^2)^{5/4}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(1/4)/(c\*x)^(7/2), x]

[Out]  $(-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(5/2))$

**Rubi in Sympy [A]** time = 3.84825, size = 24, normalized size = 0.83

$$-\frac{2(a - bx^2)^{5/4}}{5ac(cx)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(7/2), x)

[Out]  $-2*(a - b*x**2)**(5/4)/(5*a*c*(c*x)**(5/2))$

**Mathematica [A]** time = 0.033771, size = 27, normalized size = 0.93

$$-\frac{2x(a - bx^2)^{5/4}}{5a(cx)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(1/4)/(c\*x)^(7/2), x]

[Out]  $(-2*x*(a - b*x^2)^(5/4))/(5*a*(c*x)^(7/2))$

**Maple [A]** time = 0.006, size = 22, normalized size = 0.8

$$-\frac{2x}{5a}(-bx^2 + a)^{5/4}(cx)^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^2+a)^(1/4)/(c\*x)^(7/2), x)

[Out]  $-2/5 * x * (-b * x^2 + a)^{5/4} / a / (c * x)^{7/2}$

**Maxima [A]** time = 1.40177, size = 28, normalized size = 0.97

$$\frac{2(-bx^2 + a)^{5/4}}{5ac^{7/2}x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)/(c*x)^(7/2), x, algorithm="maxima")`

[Out]  $-2/5 * (-b * x^2 + a)^{5/4} / (a * c^{7/2} * x^{5/2})$

**Fricas [A]** time = 0.222881, size = 47, normalized size = 1.62

$$\frac{2(bx^2 - a)(-bx^2 + a)^{1/4}\sqrt{cx}}{5ac^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)/(c*x)^(7/2), x, algorithm="fricas")`

[Out]  $2/5 * (b * x^2 - a) * (-b * x^2 + a)^{1/4} * \text{sqrt}(c * x) / (a * c^4 * x^3)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**2+a)**(1/4)/(c*x)**(7/2), x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.228974, size = 61, normalized size = 2.1

$$\frac{2(-bc^4x^2 + ac^4)^{1/4}\left(bc^2 - \frac{ac^2}{x^2}\right)}{5\sqrt{cx}ac^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)/(c*x)^(7/2), x, algorithm="giac")`

[Out]  $2/5 * (-b * c^4 * x^2 + a * c^4)^{1/4} * (b * c^2 - a * c^2 / x^2) / (\text{sqrt}(c * x) * a * c^6)$



$$3.941 \quad \int \frac{\sqrt[4]{a - bx^2}}{(cx)^{11/2}} dx$$

**Optimal.** Leaf size=59

$$\frac{8(a - bx^2)^{9/4}}{45a^2c(cx)^{9/2}} - \frac{2(a - bx^2)^{5/4}}{5ac(cx)^{9/2}}$$

[Out]  $(-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(9/2)) + (8*(a - b*x^2)^(9/4))/(45*a^2*c*(c*x)^(9/2))$

**Rubi [A]** time = 0.0573557, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{8(a - bx^2)^{9/4}}{45a^2c(cx)^{9/2}} - \frac{2(a - bx^2)^{5/4}}{5ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(1/4)/(c\*x)^(11/2), x]

[Out]  $(-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(9/2)) + (8*(a - b*x^2)^(9/4))/(45*a^2*c*(c*x)^(9/2))$

**Rubi in Sympy [A]** time = 7.26311, size = 48, normalized size = 0.81

$$-\frac{2(a - bx^2)^{5/4}}{5ac(cx)^{9/2}} + \frac{8(a - bx^2)^{9/4}}{45a^2c(cx)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(11/2), x)

[Out]  $-2*(a - b*x**2)**(5/4)/(5*a*c*(c*x)**(9/2)) + 8*(a - b*x**2)**(9/4)/(45*a**2*c*(c*x)**(9/2))$

**Mathematica [A]** time = 0.0351386, size = 52, normalized size = 0.88

$$\frac{2\sqrt{cx}\sqrt[4]{a - bx^2}(-5a^2 + abx^2 + 4b^2x^4)}{45a^2c^6x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(1/4)/(c\*x)^(11/2), x]

[Out]  $(2*\text{Sqrt}[c*x]*(a - b*x^2)^(1/4)*(-5*a^2 + a*b*x^2 + 4*b^2*x^4))/(45*a^2*c^6*x^5)$

**Maple [A]** time = 0.008, size = 32, normalized size = 0.5

$$-\frac{2x(4bx^2 + 5a)}{45a^2}(-bx^2 + a)^{5/4}(cx)^{-11/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^2+a)^(1/4)/(c*x)^(11/2),x)`

[Out]  $-2/45*x*(-b*x^2+a)^{5/4}*(4*b*x^2+5*a)/a^2/(c*x)^{11/2}$

**Maxima [A]** time = 1.40597, size = 54, normalized size = 0.92

$$-\frac{2\left(\frac{9(-bx^2+a)^{\frac{5}{4}}b}{x^{\frac{5}{2}}} + \frac{5(-bx^2+a)^{\frac{9}{4}}}{x^{\frac{9}{2}}}\right)}{45a^2c^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)/(c*x)^(11/2),x, algorithm="maxima")`

[Out]  $-2/45*(9*(-b*x^2 + a)^{5/4}*b/x^{5/2} + 5*(-b*x^2 + a)^{9/4}/x^{9/2})/(a^2*c^{11/2})$

**Fricas [A]** time = 0.224112, size = 62, normalized size = 1.05

$$\frac{2(4b^2x^4 + abx^2 - 5a^2)(-bx^2 + a)^{\frac{1}{4}}\sqrt{cx}}{45a^2c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)/(c*x)^(11/2),x, algorithm="fricas")`

[Out]  $2/45*(4*b^2*x^4 + a*b*x^2 - 5*a^2)*(-b*x^2 + a)^{1/4}*sqrt(c*x)/(a^2*c^6*x^5)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**2+a)**(1/4)/(c*x)**(11/2),x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.232338, size = 149, normalized size = 2.53

$$\frac{2\left(\frac{9(-bc^4x^2+ac^4)^{\frac{1}{4}}\left(bc^2-\frac{ac^2}{x^2}\right)bc^2}{\sqrt{cx}} - \frac{5(b^2c^8x^4-2abc^8x^2+a^2c^8)(-bc^4x^2+ac^4)^{\frac{1}{4}}}{\sqrt{cx}c^4x^4}\right)}{45a^2c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)/(c*x)^(11/2),x, algorithm="giac")`

[Out]  $2/45*(9*(-b*c^4*x^2 + a*c^4)^{1/4}*(b*c^2 - a*c^2/x^2)*b*c^2/sqrt(c*x) - 5*(b^2*c^8*x^4 - 2*a*b*c^8*x^2 + a^2*c^8)*(-b*c^4*x^2 + a*c^4)^{1/4}/(sqrt(c*x)*c^4*x^4))/(a^2*c^{10})$

$$3.942 \quad \int \frac{\sqrt[4]{a - bx^2}}{(cx)^{15/2}} dx$$

**Optimal.** Leaf size=88

$$-\frac{64(a - bx^2)^{13/4}}{585a^3c(cx)^{13/2}} + \frac{16(a - bx^2)^{9/4}}{45a^2c(cx)^{13/2}} - \frac{2(a - bx^2)^{5/4}}{5ac(cx)^{13/2}}$$

[Out]  $(-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(13/2)) + (16*(a - b*x^2)^(9/4))/(45*a^2*c*(c*x)^(13/2)) - (64*(a - b*x^2)^(13/4))/(585*a^3*c*(c*x)^(13/2))$

**Rubi [A]** time = 0.0907427, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{64(a - bx^2)^{13/4}}{585a^3c(cx)^{13/2}} + \frac{16(a - bx^2)^{9/4}}{45a^2c(cx)^{13/2}} - \frac{2(a - bx^2)^{5/4}}{5ac(cx)^{13/2}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(1/4)/(c\*x)^(15/2), x]

[Out]  $(-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(13/2)) + (16*(a - b*x^2)^(9/4))/(45*a^2*c*(c*x)^(13/2)) - (64*(a - b*x^2)^(13/4))/(585*a^3*c*(c*x)^(13/2))$

**Rubi in Sympy [A]** time = 11.5074, size = 73, normalized size = 0.83

$$-\frac{2(a - bx^2)^{5/4}}{5ac(cx)^{13/2}} + \frac{16(a - bx^2)^{9/4}}{45a^2c(cx)^{13/2}} - \frac{64(a - bx^2)^{13/4}}{585a^3c(cx)^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(15/2), x)

[Out]  $-2*(a - b*x^2)^(5/4)/(5*a*c*(c*x)^(13/2)) + 16*(a - b*x^2)^(9/4)/(45*a^2*c*(c*x)^(13/2)) - 64*(a - b*x^2)^(13/4)/(585*a^3*c*(c*x)^(13/2))$

**Mathematica [A]** time = 0.0413316, size = 64, normalized size = 0.73

$$\frac{2\sqrt{cx}\sqrt[4]{a - bx^2}(-45a^3 + 5a^2bx^2 + 8ab^2x^4 + 32b^3x^6)}{585a^3c^8x^7}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(1/4)/(c\*x)^(15/2), x]

[Out]  $(2*\text{Sqrt}[c*x]*(a - b*x^2)^(1/4)*(-45*a^3 + 5*a^2*b*x^2 + 8*a*b^2*x^4 + 32*b^3*x^6))/(585*a^3*c^8*x^7)$

**Maple [A]** time = 0.007, size = 43, normalized size = 0.5

$$-\frac{2x(32b^2x^4 + 40abx^2 + 45a^2)}{585a^3}(-bx^2 + a)^{5/4}(cx)^{-15/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^2+a)^(1/4)/(c*x)^(15/2),x)`

[Out]  $-2/585*x*(-b*x^2+a)^{5/4}*(32*b^2*x^4+40*a*b*x^2+45*a^2)/a^3/(c*x)^{15/2}$

**Maxima [A]** time = 1.3957, size = 78, normalized size = 0.89

$$2 \left( \frac{117(-bx^2+a)^{\frac{5}{4}}b^2}{x^{\frac{5}{2}}} + \frac{130(-bx^2+a)^{\frac{9}{4}}b}{x^{\frac{9}{2}}} + \frac{45(-bx^2+a)^{\frac{13}{4}}}{x^{\frac{13}{2}}} \right) \\ \hline 585 a^3 c^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)/(c*x)^(15/2),x, algorithm="maxima")`

[Out]  $-2/585*(117*(-b*x^2 + a)^{5/4}*b^2/x^{5/2} + 130*(-b*x^2 + a)^{9/4}*b/x^{9/2} + 45*(-b*x^2 + a)^{13/4}/x^{13/2})/(a^3*c^{15/2})$

**Fricas [A]** time = 0.219121, size = 78, normalized size = 0.89

$$\frac{2(32b^3x^6 + 8ab^2x^4 + 5a^2bx^2 - 45a^3)(-bx^2 + a)^{\frac{1}{4}}\sqrt{cx}}{585a^3c^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)/(c*x)^(15/2),x, algorithm="fricas")`

[Out]  $2/585*(32*b^3*x^6 + 8*a*b^2*x^4 + 5*a^2*b*x^2 - 45*a^3)*(-b*x^2 + a)^{1/4}*sqrt(c*x)/(a^3*c^8*x^7)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**2+a)**(1/4)/(c*x)**(15/2),x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.236179, size = 251, normalized size = 2.85

$$2 \left( \frac{117(-bc^4x^2+ac^4)^{\frac{1}{4}}(bc^2-\frac{ac^2}{x^2})b^2c^4}{\sqrt{cx}} - \frac{130(b^2c^8x^4-2abc^8x^2+a^2c^8)(-bc^4x^2+ac^4)^{\frac{1}{4}}b}{\sqrt{cx}c^2x^4} + \frac{45(b^3c^{12}x^6-3ab^2c^{12}x^4+3a^2bc^{12}x^2-a^3c^{12})(-bc^4x^2+ac^4)^{\frac{1}{4}}}{\sqrt{cx}c^6x^6} \right) \\ \hline 585 a^3 c^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)/(c*x)^(15/2),x, algorithm="giac")`

[Out]  $2/585*(117*(-b*c^4*x^2 + a*c^4)^{1/4}*(b*c^2 - a*c^2/x^2)*b^2*c^4/sqrt(c*x) - 130*(b^2*c^8*x^4 - 2*a*b*c^8*x^2 + a^2*c^8)*(-b*c^4*x^2 + a*c^4)^{1/4}/sqrt(c*x) - 45*(b^3*c^{12}*x^6 - 3*a*b^2*c^{12}*x^4 + 3*a^2*b*c^{12}*x^2 - a^3*c^{12})*(-b*c^4*x^2 + a*c^4)^{1/4}/sqrt(c*x)/c^6)$

$$\frac{(x^2 + a^2 c^4)^{1/4} b \sqrt{c x} c^2 x^4 + 45 (b^3 c^{12} x^6 - 3 a^2 b^2 c^{12} x^4 + 3 a^2 b c^{12} x^2 - a^3 c^{12}) (-b c^4 x^2 + a^2 c^4)^{1/4} \sqrt{c x} c^6 x^6}{a^3 c^{14}}$$

$$3.943 \quad \int \frac{\sqrt[4]{a - bx^2}}{(cx)^{19/2}} dx$$

**Optimal.** Leaf size=117

$$\frac{256(a - bx^2)^{17/4}}{3315a^4c(cx)^{17/2}} - \frac{64(a - bx^2)^{13/4}}{195a^3c(cx)^{17/2}} + \frac{8(a - bx^2)^{9/4}}{15a^2c(cx)^{17/2}} - \frac{2(a - bx^2)^{5/4}}{5ac(cx)^{17/2}}$$

[Out]  $(-2*(a - b*x^2)^{(5/4)})/(5*a*c*(c*x)^{(17/2)}) + (8*(a - b*x^2)^{(9/4)})/(15*a^2*c*(c*x)^{(17/2)}) - (64*(a - b*x^2)^{(13/4)})/(195*a^3*c*(c*x)^{(17/2)}) + (256*(a - b*x^2)^{(17/4)})/(3315*a^4*c*(c*x)^{(17/2)})$

**Rubi [A]** time = 0.130522, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{256(a - bx^2)^{17/4}}{3315a^4c(cx)^{17/2}} - \frac{64(a - bx^2)^{13/4}}{195a^3c(cx)^{17/2}} + \frac{8(a - bx^2)^{9/4}}{15a^2c(cx)^{17/2}} - \frac{2(a - bx^2)^{5/4}}{5ac(cx)^{17/2}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^2)^(1/4)/(c\*x)^(19/2), x]

[Out]  $(-2*(a - b*x^2)^{(5/4)})/(5*a*c*(c*x)^{(17/2)}) + (8*(a - b*x^2)^{(9/4)})/(15*a^2*c*(c*x)^{(17/2)}) - (64*(a - b*x^2)^{(13/4)})/(195*a^3*c*(c*x)^{(17/2)}) + (256*(a - b*x^2)^{(17/4)})/(3315*a^4*c*(c*x)^{(17/2)})$

**Rubi in Sympy [A]** time = 16.4889, size = 99, normalized size = 0.85

$$-\frac{2(a - bx^2)^{\frac{5}{4}}}{5ac(cx)^{\frac{17}{2}}} + \frac{8(a - bx^2)^{\frac{9}{4}}}{15a^2c(cx)^{\frac{17}{2}}} - \frac{64(a - bx^2)^{\frac{13}{4}}}{195a^3c(cx)^{\frac{17}{2}}} + \frac{256(a - bx^2)^{\frac{17}{4}}}{3315a^4c(cx)^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-b\*x\*\*2+a)\*\*(1/4)/(c\*x)\*\*(19/2), x)

[Out]  $-2*(a - b*x**2)**(5/4)/(5*a*c*(c*x)**(17/2)) + 8*(a - b*x**2)**(9/4)/(15*a**2*c*(c*x)**(17/2)) - 64*(a - b*x**2)**(13/4)/(195*a**3*c*(c*x)**(17/2)) + 256*(a - b*x**2)**(17/4)/(3315*a**4*c*(c*x)**(17/2))$

**Mathematica [A]** time = 0.0446802, size = 75, normalized size = 0.64

$$\frac{2\sqrt[4]{a - bx^2}(-195a^4 + 15a^3bx^2 + 20a^2b^2x^4 + 32ab^3x^6 + 128b^4x^8)}{3315a^4c^9x^8\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^2)^(1/4)/(c\*x)^(19/2), x]

[Out]  $(2*(a - b*x^2)^{(1/4)}*(-195*a^4 + 15*a^3*b*x^2 + 20*a^2*b^2*x^4 + 32*a*b^3*x^6 + 128*b^4*x^8))/(3315*a^4*c^9*x^8*\text{Sqrt}[c*x])$

**Maple [A]** time = 0.009, size = 54, normalized size = 0.5

$$-\frac{2x(128b^3x^6 + 160ab^2x^4 + 180a^2bx^2 + 195a^3)}{3315a^4}(-bx^2 + a)^{\frac{5}{4}}(cx)^{-\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^2+a)^(1/4)/(c*x)^(19/2),x)`

[Out] 
$$-2/3315*x*(-b*x^2+a)^{5/4}*(128*b^3*x^6+160*a*b^2*x^4+180*a^2*b*x^2+195*a^3)/a^4/(c*x)^{19/2}$$

**Maxima [A]** time = 1.40495, size = 103, normalized size = 0.88

$$\frac{2 \left( \frac{663(-bx^2+a)^{\frac{5}{4}}b^3}{x^{\frac{5}{2}}} + \frac{1105(-bx^2+a)^{\frac{9}{4}}b^2}{x^{\frac{9}{2}}} + \frac{765(-bx^2+a)^{\frac{13}{4}}b}{x^{\frac{13}{2}}} + \frac{195(-bx^2+a)^{\frac{17}{4}}}{x^{\frac{17}{2}}} \right)}{3315 a^4 c^{\frac{19}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)/(c*x)^(19/2),x, algorithm="maxima")`

[Out] 
$$-2/3315*(663*(-b*x^2 + a)^{5/4}*b^3/x^{5/2} + 1105*(-b*x^2 + a)^{9/4}*b^2/x^{9/2} + 765*(-b*x^2 + a)^{13/4}*b/x^{13/2} + 195*(-b*x^2 + a)^{17/4}/x^{17/2})/(a^4*c^{19/2})$$

**Fricas [A]** time = 0.21698, size = 93, normalized size = 0.79

$$\frac{2(128b^4x^8 + 32ab^3x^6 + 20a^2b^2x^4 + 15a^3bx^2 - 195a^4)(-bx^2 + a)^{\frac{1}{4}}\sqrt{cx}}{3315 a^4 c^{10} x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)/(c*x)^(19/2),x, algorithm="fricas")`

[Out] 
$$2/3315*(128*b^4*x^8 + 32*a*b^3*x^6 + 20*a^2*b^2*x^4 + 15*a^3*b*x^2 - 195*a^4)*(-b*x^2 + a)^{1/4}*sqrt(c*x)/(a^4*c^{10}*x^9)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**2+a)**(1/4)/(c*x)**(19/2),x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.235678, size = 367, normalized size = 3.14

$$\frac{2 \left( \frac{663(-bc^4x^2+ac^4)^{\frac{1}{4}} \left( bc^2 - \frac{ac^2}{x^2} \right) b^3 c^6}{\sqrt{cx}} - \frac{1105(b^2c^8x^4 - 2abc^8x^2 + a^2c^8)(-bc^4x^2+ac^4)^{\frac{1}{4}}b^2}{\sqrt{c}xx^4} + \frac{765(b^3c^{12}x^6 - 3ab^2c^{12}x^4 + 3a^2bc^{12}x^2 - a^3c^{12})(-bc^4x^2+ac^4)^{\frac{1}{4}}}{\sqrt{cx}c^4x^6} \right)}{3315 a^4 c^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^2 + a)^(1/4)/(c*x)^(19/2),x, algorithm="giac")`

```
[Out] 2/3315*(663*(-b*c^4*x^2 + a*c^4)^(1/4)*(b*c^2 - a*c^2/x^2)*b^3*c^
6/sqrt(c*x) - 1105*(b^2*c^8*x^4 - 2*a*b*c^8*x^2 + a^2*c^8)*(-b*c^
4*x^2 + a*c^4)^(1/4)*b^2/(sqrt(c*x)*x^4) + 765*(b^3*c^12*x^6 - 3*
a*b^2*c^12*x^4 + 3*a^2*b*c^12*x^2 - a^3*c^12)*(-b*c^4*x^2 + a*c^4
)^(1/4)*b/(sqrt(c*x)*c^4*x^6) - 195*(b^4*c^16*x^8 - 4*a*b^3*c^16*
x^6 + 6*a^2*b^2*c^16*x^4 - 4*a^3*b*c^16*x^2 + a^4*c^16)*(-b*c^4*x
^2 + a*c^4)^(1/4)/(sqrt(c*x)*c^8*x^8))/(a^4*c^18)
```



$$3.944 \quad \int \frac{(cx)^{3/2}}{\sqrt[4]{a+bx^2}} dx$$

**Optimal.** Leaf size=117

$$-\frac{ac^{3/2} \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}} - \frac{ac^{3/2} \tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}} + \frac{c\sqrt{cx}(a+bx^2)^{3/4}}{2b}$$

[Out] (c\*Sqrt[c\*x]\*(a+b\*x^2)^(3/4))/(2\*b) - (a\*c^(3/2)\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a+b\*x^2)^(1/4))])/(4\*b^(5/4)) - (a\*c^(3/2)\*ArcTanh[(b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a+b\*x^2)^(1/4))])/(4\*b^(5/4))

**Rubi [A]** time = 0.160331, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$-\frac{ac^{3/2} \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}} - \frac{ac^{3/2} \tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}} + \frac{c\sqrt{cx}(a+bx^2)^{3/4}}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(3/2)/(a+b\*x^2)^(1/4),x]

[Out] (c\*Sqrt[c\*x]\*(a+b\*x^2)^(3/4))/(2\*b) - (a\*c^(3/2)\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a+b\*x^2)^(1/4))])/(4\*b^(5/4)) - (a\*c^(3/2)\*ArcTanh[(b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a+b\*x^2)^(1/4))])/(4\*b^(5/4))

**Rubi in Sympy [A]** time = 20.8795, size = 105, normalized size = 0.9

$$-\frac{ac^{\frac{3}{2}} \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{\frac{5}{4}}} - \frac{ac^{\frac{3}{2}} \operatorname{atanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{\frac{5}{4}}} + \frac{c\sqrt{cx}(a+bx^2)^{\frac{3}{4}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(3/2)/(b\*x\*\*2+a)\*\*(1/4),x)

[Out] -a\*c\*\*(3/2)\*atan(b\*\*(1/4)\*sqrt(c\*x)/(sqrt(c)\*(a+b\*x\*\*2)\*\*(1/4)))/(4\*b\*\*(5/4)) - a\*c\*\*(3/2)\*atanh(b\*\*(1/4)\*sqrt(c\*x)/(sqrt(c)\*(a+b\*x\*\*2)\*\*(1/4)))/(4\*b\*\*(5/4)) + c\*sqrt(c\*x)\*(a+b\*x\*\*2)\*\*(3/4)/(2\*b)

**Mathematica [C]** time = 0.0612729, size = 69, normalized size = 0.59

$$\frac{c\sqrt{cx}\left(-a\sqrt{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{4}; \frac{5}{4}; -\frac{bx^2}{a}\right) + a + bx^2\right)}{2b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(3/2)/(a+b\*x^2)^(1/4),x]

[Out]  $(c \sqrt{c x} (a + b x^2 - a (1 + (b x^2)/a)^{1/4}) \text{Hypergeometric2F1}[1/4, 1/4, 5/4, -((b x^2)/a)]) / (2 b (a + b x^2)^{1/4})$

**Maple [F]** time = 0.055, size = 0, normalized size = 0.

$$\int 1 (cx)^{\frac{3}{2}} \frac{1}{\sqrt[4]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(3/2)/(b*x^2+a)^(1/4),x)`

[Out] `int((c*x)^(3/2)/(b*x^2+a)^(1/4),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(3/2)/(b*x^2 + a)^(1/4),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.240898, size = 382, normalized size = 3.26

$$\frac{4 (bx^2 + a)^{\frac{3}{4}} \sqrt{cx} + 4 \left( \frac{a^4 c^6}{b^5} \right)^{\frac{1}{4}} b \arctan \left( \frac{\left( \frac{a^4 c^6}{b^5} \right)^{\frac{1}{4}} (b^2 x^2 + ab)}{(bx^2 + a)^{\frac{3}{4}} \sqrt{cx} + (bx^2 + a) \sqrt{\frac{\sqrt{bx^2 + a} a^2 c^3 x + \sqrt{\frac{a^4 c^6}{b^5}} (b^3 x^2 + ab^2)}}}{bx^2 + a}} \right) - \left( \frac{a^4 c^6}{b^5} \right)^{\frac{1}{4}} b \log \left( \frac{(bx^2 + a)^{\frac{3}{4}} \sqrt{cx} + (bx^2 + a) \sqrt{\frac{\sqrt{bx^2 + a} a^2 c^3 x + \sqrt{\frac{a^4 c^6}{b^5}} (b^3 x^2 + ab^2)}}}{bx^2 + a}} \right)}{8 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(3/2)/(b*x^2 + a)^(1/4),x, algorithm="fricas")`

[Out]  $\frac{1}{8} (4 (b x^2 + a)^{3/4} \sqrt{c x} + 4 (a^4 c^6 / b^5)^{1/4} b \arctan((a^4 c^6 / b^5)^{1/4} (b^2 x^2 + a b) / ((b x^2 + a)^{3/4} \sqrt{c x} + (b x^2 + a) \sqrt{(a^2 c^3 x + \sqrt{a^4 c^6 / b^5} (b^3 x^2 + a b^2)) / (b x^2 + a)})) - (a^4 c^6 / b^5)^{1/4} b \log(((b x^2 + a)^{3/4} \sqrt{c x} + (a^4 c^6 / b^5)^{1/4} (b^2 x^2 + a b) / (b x^2 + a)) + (a^4 c^6 / b^5)^{1/4} b \log(((b x^2 + a)^{3/4} \sqrt{c x} + (a^4 c^6 / b^5)^{1/4} (b^2 x^2 + a b) / (b x^2 + a)) - (a^4 c^6 / b^5)^{1/4} (b^2 x^2 + a b) / (b x^2 + a)) / b$

**Sympy [A]** time = 13.9146, size = 44, normalized size = 0.38

$$\frac{c^{\frac{3}{2}} x^{\frac{5}{2}} \left( \frac{5}{4} \right) {}_2F_1 \left( \frac{1}{4}, \frac{5}{4} \middle| \frac{bx^2 e^{i\pi}}{a} \right)}{2 \sqrt[4]{a} \left( \frac{9}{4} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(3/2)/(b*x**2+a)**(1/4),x)`

[Out]  $c^{3/2} x^{5/2} \gamma(5/4) \operatorname{hyper}((1/4, 5/4), (9/4, ), b x^2 \exp_{\text{polar}}(I \pi) / a) / (2 a^{1/4} \gamma(9/4))$

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(3/2)/(b*x^2 + a)^(1/4),x, algorithm="giac")`

[Out] `integrate((c*x)^(3/2)/(b*x^2 + a)^(1/4), x)`

$$3.945 \quad \int \frac{1}{\sqrt{cx} \sqrt[4]{a + bx^2}} dx$$

**Optimal.** Leaf size=83

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}}$$

[Out] ArcTan[(b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a + b\*x^2)^(1/4))]/(b^(1/4)\*Sqrt[c]) + ArcTanh[(b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a + b\*x^2)^(1/4))]/(b^(1/4)\*Sqrt[c])

**Rubi [A]** time = 0.119057, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[c\*x]\*(a + b\*x^2)^(1/4)),x]

[Out] ArcTan[(b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a + b\*x^2)^(1/4))]/(b^(1/4)\*Sqrt[c]) + ArcTanh[(b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a + b\*x^2)^(1/4))]/(b^(1/4)\*Sqrt[c])

**Rubi in Sympy [A]** time = 15.833, size = 76, normalized size = 0.92

$$\frac{\text{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}} + \frac{\text{atanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(1/2)/(b\*x\*\*2+a)\*\*(1/4),x)

[Out] atan(b\*\*(1/4)\*sqrt(c\*x)/(sqrt(c)\*(a + b\*x\*\*2)\*\*(1/4)))/(b\*\*(1/4)\*sqrt(c)) + atanh(b\*\*(1/4)\*sqrt(c\*x)/(sqrt(c)\*(a + b\*x\*\*2)\*\*(1/4)))/(b\*\*(1/4)\*sqrt(c))

**Mathematica [C]** time = 0.0294468, size = 55, normalized size = 0.66

$$\frac{2x\sqrt[4]{\frac{a+bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{4}; \frac{5}{4}; -\frac{bx^2}{a}\right)}{\sqrt{cx}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[c\*x]\*(a + b\*x^2)^(1/4)),x]

[Out] (2\*x\*((a + b\*x^2)/a)^(1/4)\*Hypergeometric2F1[1/4, 1/4, 5/4, -(b\*x^2)/a])/(Sqrt[c\*x]\*(a + b\*x^2)^(1/4))

**Maple [F]** time = 0.044, size = 0, normalized size = 0.

$$\int 1 \frac{1}{\sqrt{cx}} \frac{1}{\sqrt[4]{bx^2+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(1/2)/(b*x^2+a)^(1/4), x)`

[Out] `int(1/(c*x)^(1/2)/(b*x^2+a)^(1/4), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/4)*sqrt(c*x)), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.236945, size = 304, normalized size = 3.66

$$\begin{aligned} & -2 \left( \frac{1}{bc^2} \right)^{\frac{1}{4}} \arctan \left( \frac{(bcx^2 + ac) \left( \frac{1}{bc^2} \right)^{\frac{1}{4}}}{(bx^2 + a) \sqrt{\frac{\sqrt{bx^2+acx+(bc^2x^2+ac^2)}\sqrt{\frac{1}{bc^2}}}{bx^2+a}} + (bx^2 + a)^{\frac{3}{4}} \sqrt{cx}} \right) \\ & + \frac{1}{2} \left( \frac{1}{bc^2} \right)^{\frac{1}{4}} \log \left( \frac{(bx^2 + a)^{\frac{3}{4}} \sqrt{cx} + (bcx^2 + ac) \left( \frac{1}{bc^2} \right)^{\frac{1}{4}}}{bx^2 + a} \right) \\ & - \frac{1}{2} \left( \frac{1}{bc^2} \right)^{\frac{1}{4}} \log \left( \frac{(bx^2 + a)^{\frac{3}{4}} \sqrt{cx} - (bcx^2 + ac) \left( \frac{1}{bc^2} \right)^{\frac{1}{4}}}{bx^2 + a} \right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/4)*sqrt(c*x)), x, algorithm="fricas")`

[Out] `-2*(1/(b*c^2))^(1/4)*arctan((b*c*x^2 + a*c)*(1/(b*c^2))^(1/4)/((b*x^2 + a)*sqrt((sqrt(b*x^2 + a)*c*x + (b*c^2*x^2 + a*c^2)*sqrt(1/(b*c^2)))/(b*x^2 + a)) + (b*x^2 + a)^(3/4)*sqrt(c*x))) + 1/2*(1/(b*c^2))^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(c*x) + (b*c*x^2 + a*c)*(1/(b*c^2))^(1/4))/(b*x^2 + a)) - 1/2*(1/(b*c^2))^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(c*x) - (b*c*x^2 + a*c)*(1/(b*c^2))^(1/4))/(b*x^2 + a))`

**Sympy [A]** time = 6.58939, size = 44, normalized size = 0.53

$$\frac{\sqrt{x} \left( \frac{1}{4} \right) {}_2F_1 \left( \frac{1}{4}, \frac{1}{4} \middle| \frac{bx^2 e^{i\pi}}{a} \right)}{2\sqrt[4]{a}\sqrt{c} \left( \frac{5}{4} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(1/2)/(b*x**2+a)**(1/4),x)`

[Out] `sqrt(x)*gamma(1/4)*hyper((1/4, 1/4), (5/4, ), b*x**2*exp_polar(I*pi)/a)/(2*a**(1/4)*sqrt(c)*gamma(5/4))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}} \sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/4)*sqrt(c*x)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(1/4)*sqrt(c*x)), x)`

$$3.946 \quad \int \frac{1}{(cx)^{5/2} \sqrt[4]{a + bx^2}} dx$$

**Optimal.** Leaf size=28

$$-\frac{2(a + bx^2)^{3/4}}{3ac(cx)^{3/2}}$$

[Out]  $(-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(3/2))$

**Rubi [A]** time = 0.0281755, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$-\frac{2(a + bx^2)^{3/4}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[1/((c*x)^(5/2)*(a + b*x^2)^(1/4)), x]`

[Out]  $(-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(3/2))$

**Rubi in Sympy [A]** time = 3.59146, size = 24, normalized size = 0.86

$$-\frac{2(a + bx^2)^{\frac{3}{4}}}{3ac(cx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(c*x)**(5/2)/(b*x**2+a)**(1/4), x)`

[Out]  $-2*(a + b*x**2)**(3/4)/(3*a*c*(c*x)**(3/2))$

**Mathematica [A]** time = 0.0198367, size = 26, normalized size = 0.93

$$-\frac{2x(a + bx^2)^{3/4}}{3a(cx)^{5/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(1/4)), x]`

[Out]  $(-2*x*(a + b*x^2)^(3/4))/(3*a*(c*x)^(5/2))$

**Maple [A]** time = 0.008, size = 21, normalized size = 0.8

$$-\frac{2x}{3a}(bx^2 + a)^{\frac{3}{4}}(cx)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(5/2)/(b*x^2+a)^(1/4), x)`

[Out]  $-2/3 * x * (b * x^2 + a)^{(3/4)} / a / (c * x)^{(5/2)}$

**Maxima [A]** time = 1.39588, size = 27, normalized size = 0.96

$$-\frac{2 (bx^2 + a)^{\frac{3}{4}}}{3 ac^{\frac{5}{2}} x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/2)),x, algorithm="maxima")`

[Out]  $-2/3 * (b * x^2 + a)^{(3/4)} / (a * c^{(5/2)} * x^{(3/2)})$

**Fricas [A]** time = 0.214576, size = 34, normalized size = 1.21

$$-\frac{2 (bx^2 + a)^{\frac{3}{4}}}{3 \sqrt{cx} ac^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/2)),x, algorithm="fricas")`

[Out]  $-2/3 * (b * x^2 + a)^{(3/4)} / (\text{sqrt}(c * x) * a * c^2 * x)$

**Sympy [A]** time = 57.5585, size = 36, normalized size = 1.29

$$\frac{b^{\frac{3}{4}} \left( \frac{a}{bx^2} + 1 \right)^{\frac{3}{4}} \left( -\frac{3}{4} \right)}{2ac^{\frac{5}{2}} \left( \frac{1}{4} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(5/2)/(b*x**2+a)**(1/4),x)`

[Out]  $b^{(3/4)} * (a / (b * x^2) + 1)^{(3/4)} * \text{gamma}(-3/4) / (2 * a * c^{(5/2)} * \text{gamma}(1/4))$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/2)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)`



$$3.947 \quad \int \frac{1}{(cx)^{9/2} \sqrt[4]{a + bx^2}} dx$$

**Optimal.** Leaf size=57

$$\frac{8(a + bx^2)^{7/4}}{21a^2c(cx)^{7/2}} - \frac{2(a + bx^2)^{3/4}}{3ac(cx)^{7/2}}$$

[Out]  $(-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(7/2)) + (8*(a + b*x^2)^(7/4))/(21*a^2*c*(c*x)^(7/2))$

**Rubi [A]** time = 0.0574699, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{8(a + bx^2)^{7/4}}{21a^2c(cx)^{7/2}} - \frac{2(a + bx^2)^{3/4}}{3ac(cx)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(9/2)\*(a + b\*x^2)^(1/4)), x]

[Out]  $(-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(7/2)) + (8*(a + b*x^2)^(7/4))/(21*a^2*c*(c*x)^(7/2))$

**Rubi in Sympy [A]** time = 6.77211, size = 48, normalized size = 0.84

$$-\frac{2(a + bx^2)^{3/4}}{3ac(cx)^{7/2}} + \frac{8(a + bx^2)^{7/4}}{21a^2c(cx)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(9/2)/(b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $-2*(a + b*x**2)**(3/4)/(3*a*c*(c*x)**(7/2)) + 8*(a + b*x**2)**(7/4)/(21*a**2*c*(c*x)**(7/2))$

**Mathematica [A]** time = 0.0344929, size = 41, normalized size = 0.72

$$\frac{2\sqrt{cx}(a + bx^2)^{3/4}(4bx^2 - 3a)}{21a^2c^5x^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(9/2)\*(a + b\*x^2)^(1/4)), x]

[Out]  $(2*\text{Sqrt}[c*x]*(a + b*x^2)^(3/4)*(-3*a + 4*b*x^2))/(21*a^2*c^5*x^4)$

**Maple [A]** time = 0.007, size = 31, normalized size = 0.5

$$-\frac{2x(-4bx^2 + 3a)}{21a^2}(bx^2 + a)^{3/4}(cx)^{-9/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x)`

[Out] `-2/21*x*(b*x^2+a)^(3/4)*(-4*b*x^2+3*a)/a^2/(c*x)^(9/2)`

**Maxima [A]** time = 1.39905, size = 51, normalized size = 0.89

$$\frac{2 \left( \frac{7(bx^2+a)^{\frac{3}{4}}b}{x^{\frac{3}{2}}} - \frac{3(bx^2+a)^{\frac{7}{4}}}{x^{\frac{7}{2}}} \right)}{21a^2c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(9/2)),x, algorithm="maxima")`

[Out] `2/21*(7*(b*x^2 + a)^(3/4)*b/x^(3/2) - 3*(b*x^2 + a)^(7/4)/x^(7/2))/(a^2*c^(9/2))`

**Fricas [A]** time = 0.214624, size = 61, normalized size = 1.07

$$\frac{2(4b^2x^4 + abx^2 - 3a^2)}{21(bx^2 + a)^{\frac{1}{4}}\sqrt{cx}a^2c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(9/2)),x, algorithm="fricas")`

[Out] `2/21*(4*b^2*x^4 + a*b*x^2 - 3*a^2)/((b*x^2 + a)^(1/4)*sqrt(c*x)*a^2*c^4*x^3)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(9/2)/(b*x**2+a)**(1/4),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}}(cx)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(9/2)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)`

$$3.948 \quad \int \frac{1}{(cx)^{13/2} \sqrt[4]{a + bx^2}} dx$$

**Optimal.** Leaf size=85

$$-\frac{64(a+bx^2)^{11/4}}{231a^3c(cx)^{11/2}} + \frac{16(a+bx^2)^{7/4}}{21a^2c(cx)^{11/2}} - \frac{2(a+bx^2)^{3/4}}{3ac(cx)^{11/2}}$$

[Out]  $(-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(11/2)) + (16*(a + b*x^2)^(7/4))/(21*a^2*c*(c*x)^(11/2)) - (64*(a + b*x^2)^(11/4))/(231*a^3*c*(c*x)^(11/2))$

**Rubi [A]** time = 0.0874795, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$-\frac{64(a+bx^2)^{11/4}}{231a^3c(cx)^{11/2}} + \frac{16(a+bx^2)^{7/4}}{21a^2c(cx)^{11/2}} - \frac{2(a+bx^2)^{3/4}}{3ac(cx)^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(13/2)\*(a + b\*x^2)^(1/4)), x]

[Out]  $(-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(11/2)) + (16*(a + b*x^2)^(7/4))/(21*a^2*c*(c*x)^(11/2)) - (64*(a + b*x^2)^(11/4))/(231*a^3*c*(c*x)^(11/2))$

**Rubi in Sympy [A]** time = 10.7774, size = 73, normalized size = 0.86

$$-\frac{2(a+bx^2)^{3/4}}{3ac(cx)^{11/2}} + \frac{16(a+bx^2)^{7/4}}{21a^2c(cx)^{11/2}} - \frac{64(a+bx^2)^{11/4}}{231a^3c(cx)^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(13/2)/(b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $-2*(a + b*x**2)**(3/4)/(3*a*c*(c*x)**(11/2)) + 16*(a + b*x**2)**(7/4)/(21*a**2*c*(c*x)**(11/2)) - 64*(a + b*x**2)**(11/4)/(231*a**3*c*(c*x)**(11/2))$

**Mathematica [A]** time = 0.0436354, size = 52, normalized size = 0.61

$$\frac{2\sqrt{cx}(a+bx^2)^{3/4}(21a^2-24abx^2+32b^2x^4)}{231a^3c^7x^6}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(13/2)\*(a + b\*x^2)^(1/4)), x]

[Out]  $(-2*\text{Sqrt}[c*x]*(a + b*x^2)^(3/4)*(21*a^2 - 24*a*b*x^2 + 32*b^2*x^4))/(231*a^3*c^7*x^6)$

**Maple [A]** time = 0.008, size = 42, normalized size = 0.5

$$-\frac{2x(32b^2x^4 - 24abx^2 + 21a^2)}{231a^3} (bx^2 + a)^{3/4} (cx)^{-13/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x)`

[Out]  $-2/231*x*(b*x^2+a)^(3/4)*(32*b^2*x^4-24*a*b*x^2+21*a^2)/a^3/(c*x)^(13/2)$

**Maxima [A]** time = 1.40343, size = 74, normalized size = 0.87

$$\frac{2 \left( \frac{77 (bx^2+a)^{\frac{3}{4}} b^2}{x^{\frac{3}{2}}} - \frac{66 (bx^2+a)^{\frac{7}{4}} b}{x^{\frac{7}{2}}} + \frac{21 (bx^2+a)^{\frac{11}{4}}}{x^{\frac{11}{2}}} \right)}{231 a^3 c^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(13/2)),x, algorithm="maxima")`

[Out]  $-2/231*(77*(b*x^2 + a)^(3/4)*b^2/x^(3/2) - 66*(b*x^2 + a)^(7/4)*b/x^(7/2) + 21*(b*x^2 + a)^(11/4)/x^(11/2))/(a^3*c^(13/2))$

**Fricas [A]** time = 0.213683, size = 77, normalized size = 0.91

$$\frac{2(32b^3x^6 + 8ab^2x^4 - 3a^2bx^2 + 21a^3)}{231(bx^2 + a)^{\frac{1}{4}}\sqrt{c}xa^3c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(13/2)),x, algorithm="fricas")`

[Out]  $-2/231*(32*b^3*x^6 + 8*a*b^2*x^4 - 3*a^2*b*x^2 + 21*a^3)/((b*x^2 + a)^(1/4)*sqrt(c*x)*a^3*c^6*x^5)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(13/2)/(b*x**2+a)**(1/4),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}}(cx)^{\frac{13}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(13/2)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)`

$$3.949 \quad \int \frac{(cx)^{9/2}}{\sqrt[4]{a+bx^2}} dx$$

**Optimal.** Leaf size=156

$$\frac{7a^{5/2}c^4\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{20b^{5/2}\sqrt[4]{a+bx^2}} + \frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}} - \frac{7ac^3(cx)^{3/2}(a+bx^2)^{3/4}}{30b^2} + \frac{c(cx)^{7/2}(a+bx^2)^{3/4}}{5b}$$

[Out]  $(7*a^2*c^4*x*\text{Sqrt}[c*x])/(20*b^2*(a+b*x^2)^{(1/4)}) - (7*a*c^3*(c*x)^{(3/2)}*(a+b*x^2)^{(3/4)})/(30*b^2) + (c*(c*x)^{(7/2)}*(a+b*x^2)^{(3/4)})/(5*b) + (7*a^{(5/2)}*c^4*(1+a/(b*x^2))^{(1/4)}*\text{Sqrt}[c*x]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(20*b^{(5/2)}*(a+b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.204499, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{7a^{5/2}c^4\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{20b^{5/2}\sqrt[4]{a+bx^2}} + \frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}} - \frac{7ac^3(cx)^{3/2}(a+bx^2)^{3/4}}{30b^2} + \frac{c(cx)^{7/2}(a+bx^2)^{3/4}}{5b}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(9/2)/(a+b\*x^2)^(1/4),x]

[Out]  $(7*a^2*c^4*x*\text{Sqrt}[c*x])/(20*b^2*(a+b*x^2)^{(1/4)}) - (7*a*c^3*(c*x)^{(3/2)}*(a+b*x^2)^{(3/4)})/(30*b^2) + (c*(c*x)^{(7/2)}*(a+b*x^2)^{(3/4)})/(5*b) + (7*a^{(5/2)}*c^4*(1+a/(b*x^2))^{(1/4)}*\text{Sqrt}[c*x]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(20*b^{(5/2)}*(a+b*x^2)^{(1/4)})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{7a^3c^4\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1 \int^{\frac{1}{x}} \frac{1}{\left(\frac{ax^2}{b}+1\right)^{\frac{5}{4}}} dx}{40b^3\sqrt[4]{a+bx^2}} + \frac{7a^2c^4x\sqrt{cx}}{20b^2\sqrt[4]{a+bx^2}} - \frac{7ac^3(cx)^{\frac{3}{2}}(a+bx^2)^{\frac{3}{4}}}{30b^2} + \frac{c(cx)^{\frac{7}{2}}(a+bx^2)^{\frac{3}{4}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(9/2)/(b\*x\*\*2+a)\*\*(1/4),x)

[Out]  $7*a**3*c**4*\text{sqrt}(c*x)*(a/(b*x**2)+1)**(1/4)*\text{Integral}((a*x**2/b+1)**(-5/4),(x,1/x))/(40*b**3*(a+b*x**2)**(1/4)) + 7*a**2*c**4*x*\text{sqrt}(c*x)/(20*b**2*(a+b*x**2)**(1/4)) - 7*a*c**3*(c*x)**(3/2)*(a+b*x**2)**(3/4)/(30*b**2) + c*(c*x)**(7/2)*(a+b*x**2)**(3/4)/(5*b)$

**Mathematica [C]** time = 0.0767201, size = 87, normalized size = 0.56

$$\frac{c^3(cx)^{3/2} \left( 7a^2 \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^2}{a}\right) - 7a^2 - abx^2 + 6b^2x^4 \right)}{30b^2\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(9/2)/(a + b\*x^2)^(1/4), x]

[Out] (c^3\*(c\*x)^(3/2)\*(-7\*a^2 - a\*b\*x^2 + 6\*b^2\*x^4 + 7\*a^2\*(1 + (b\*x^2)/a)^(1/4)\*Hypergeometric2F1[1/4, 3/4, 7/4, -((b\*x^2)/a)]))/(30\*b^2\*(a + b\*x^2)^(1/4))

**Maple [F]** time = 0.06, size = 0, normalized size = 0.

$$\int 1 (cx)^{\frac{9}{2}} \frac{1}{\sqrt[4]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(9/2)/(b\*x^2+a)^(1/4), x)

[Out] int((c\*x)^(9/2)/(b\*x^2+a)^(1/4), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{9}{2}}}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(9/2)/(b\*x^2 + a)^(1/4), x, algorithm="maxima")

[Out] integrate((c\*x)^(9/2)/(b\*x^2 + a)^(1/4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}c^4x^4}{(bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(9/2)/(b\*x^2 + a)^(1/4), x, algorithm="fricas")

[Out] integral(sqrt(c\*x)\*c^4\*x^4/(b\*x^2 + a)^(1/4), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(9/2)/(b\*x\*\*2+a)\*\*(1/4), x)

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{9}{2}}}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(9/2)/(b*x^2 + a)^(1/4), x, algorithm="giac")
```

```
[Out] integrate((c*x)^(9/2)/(b*x^2 + a)^(1/4), x)
```

$$3.950 \quad \int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}} dx$$

**Optimal.** Leaf size=125

$$-\frac{a^{3/2}c^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^2}} - \frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}} + \frac{c(cx)^{3/2}(a+bx^2)^{3/4}}{3b}$$

[Out]  $-(a*c^2*x*\text{Sqrt}[c*x])/(2*b*(a+b*x^2)^{(1/4)}) + (c*(c*x)^{(3/2)}*(a+b*x^2)^{(3/4)})/(3*b) - (a^{(3/2)}*c^2*(1+a/(b*x^2))^{(1/4)}*\text{Sqrt}[c*x]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(2*b^{(3/2)}*(a+b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.153705, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$-\frac{a^{3/2}c^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^2}} - \frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}} + \frac{c(cx)^{3/2}(a+bx^2)^{3/4}}{3b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*x)^{(5/2)}/(a+b*x^2)^{(1/4)}, x]$

[Out]  $-(a*c^2*x*\text{Sqrt}[c*x])/(2*b*(a+b*x^2)^{(1/4)}) + (c*(c*x)^{(3/2)}*(a+b*x^2)^{(3/4)})/(3*b) - (a^{(3/2)}*c^2*(1+a/(b*x^2))^{(1/4)}*\text{Sqrt}[c*x]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(2*b^{(3/2)}*(a+b*x^2)^{(1/4)})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a^2c^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1\int^{\frac{1}{x}} \frac{1}{\sqrt[4]{\frac{ax^2}{b}+1}} dx}{4b^2\sqrt[4]{a+bx^2}} - \frac{a^2c^2\sqrt{cx}}{2b^2x\sqrt[4]{a+bx^2}} - \frac{ac^2x\sqrt{cx}}{2b\sqrt[4]{a+bx^2}} + \frac{c(cx)^{3/2}(a+bx^2)^{3/4}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c*x)**(5/2)/(b*x**2+a)**(1/4), x)$

[Out]  $a**2*c**2*\text{sqrt}(c*x)*(a/(b*x**2)+1)**(1/4)*\text{Integral}((a*x**2/b+1)**(-1/4), (x, 1/x))/(4*b**2*(a+b*x**2)**(1/4)) - a**2*c**2*\text{sqrt}(c*x)/(2*b**2*x*(a+b*x**2)**(1/4)) - a*c**2*x*\text{sqrt}(c*x)/(2*b*(a+b*x**2)**(1/4)) + c*(c*x)**(3/2)*(a+b*x**2)**(3/4)/(3*b)$

**Mathematica [C]** time = 0.0634539, size = 69, normalized size = 0.55

$$\frac{c(cx)^{3/2}\left(-a\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^2}{a}\right) + a + bx^2\right)}{3b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(c*x)^{(5/2)}/(a+b*x^2)^{(1/4)}, x]$



[Out]  $(c*(c*x)^{(3/2)}*(a + b*x^2 - a*(1 + (b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/4, 3/4, 7/4, -((b*x^2)/a)])/(3*b*(a + b*x^2)^{(1/4)})$

**Maple [F]** time = 0.049, size = 0, normalized size = 0.

$$\int 1 (cx)^{\frac{5}{2}} \frac{1}{\sqrt[4]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(5/2)/(b*x^2+a)^(1/4), x)`

[Out] `int((c*x)^(5/2)/(b*x^2+a)^(1/4), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(5/2)/(b*x^2 + a)^(1/4), x, algorithm="maxima")`

[Out] `integrate((c*x)^(5/2)/(b*x^2 + a)^(1/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}c^2x^2}{(bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(5/2)/(b*x^2 + a)^(1/4), x, algorithm="fricas")`

[Out] `integral(sqrt(c*x)*c^2*x^2/(b*x^2 + a)^(1/4), x)`

**Sympy [A]** time = 150.86, size = 44, normalized size = 0.35

$$\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\left(\frac{7}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{7}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(5/2)/(b*x**2+a)**(1/4), x)`

[Out] `c**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/4, 7/4), (11/4, ), b*x**2*exp_polar(I*pi)/a)/(2*a**(1/4)*gamma(11/4))`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(b*x^2 + a)^(1/4), x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(1/4), x)
```

$$3.951 \quad \int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}} dx$$

**Optimal.** Leaf size=83

$$\frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}} + \frac{\sqrt{a}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^2}}$$

[Out] (x\*Sqrt[c\*x])/(a + b\*x^2)^(1/4) + (Sqrt[a]\*(1 + a/(b\*x^2))^(1/4)\*Sqrt[c\*x]\*EllipticE[ArcCot[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(Sqrt[b]\*(a + b\*x^2)^(1/4))

**Rubi [A]** time = 0.106534, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}} + \frac{\sqrt{a}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c\*x]/(a + b\*x^2)^(1/4), x]

[Out] (x\*Sqrt[c\*x])/(a + b\*x^2)^(1/4) + (Sqrt[a]\*(1 + a/(b\*x^2))^(1/4)\*Sqrt[c\*x]\*EllipticE[ArcCot[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(Sqrt[b]\*(a + b\*x^2)^(1/4))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{a\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int^{\frac{1}{x}}\frac{1}{\left(\frac{ax^2}{b}+1\right)^{\frac{5}{4}}}dx}{2b\sqrt[4]{a+bx^2}} + \frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(1/2)/(b\*x\*\*2+a)\*\*(1/4), x)

[Out] a\*sqrt(c\*x)\*(a/(b\*x\*\*2) + 1)\*\*(1/4)\*Integral((a\*x\*\*2/b + 1)\*\*(-5/4), (x, 1/x))/(2\*b\*(a + b\*x\*\*2)\*\*(1/4)) + x\*sqrt(c\*x)/(a + b\*x\*\*2)\*\*(1/4)

**Mathematica [C]** time = 0.0323954, size = 57, normalized size = 0.69

$$\frac{2x\sqrt{cx}\sqrt[4]{\frac{a+bx^2}{a}}{}_2F_1\left(\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{bx^2}{a}\right)}{3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c\*x]/(a + b\*x^2)^(1/4), x]

[Out] (2\*x\*Sqrt[c\*x]\*((a + b\*x^2)/a)^(1/4)\*Hypergeometric2F1[1/4, 3/4, 7/4, -(b\*x^2)/a])/(3\*(a + b\*x^2)^(1/4))

---

**Maple [F]** time = 0.035, size = 0, normalized size = 0.

$$\int \sqrt{cx} \frac{1}{\sqrt[4]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(1/2)/(b*x^2+a)^(1/4), x)`

[Out] `int((c*x)^(1/2)/(b*x^2+a)^(1/4), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x)/(b*x^2 + a)^(1/4), x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x)/(b*x^2 + a)^(1/4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}}{(bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x)/(b*x^2 + a)^(1/4), x, algorithm="fricas")`

[Out] `integral(sqrt(c*x)/(b*x^2 + a)^(1/4), x)`

---

**Sympy [A]** time = 3.46851, size = 44, normalized size = 0.53

$$\frac{\sqrt{cx}^{\frac{3}{2}} \left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{3}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt[4]{a} \left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(1/2)/(b*x**2+a)**(1/4), x)`

[Out] `sqrt(c)*x**(3/2)*gamma(3/4)*hyper((1/4, 3/4), (7/4, ), b*x**2*exp_polar(I*pi)/a)/(2*a**(1/4)*gamma(7/4))`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x)/(b*x^2 + a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(1/4), x)
```

$$3.952 \quad \int \frac{1}{(cx)^{3/2} \sqrt[4]{a + bx^2}} dx$$

**Optimal.** Leaf size=90

$$\frac{2\sqrt{b}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2} + 1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{ac^2}\sqrt[4]{a + bx^2}} - \frac{2}{c\sqrt{cx}\sqrt[4]{a + bx^2}}$$

[Out]  $-2/(c*\text{Sqrt}[c*x]*(a + b*x^2)^{(1/4)}) + (2*\text{Sqrt}[b]*(1 + a/(b*x^2)))^{(1/4)}*\text{Sqrt}[c*x]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(\text{Sqrt}[a]*c^2*(a + b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.114408, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{2\sqrt{b}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2} + 1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{ac^2}\sqrt[4]{a + bx^2}} - \frac{2}{c\sqrt{cx}\sqrt[4]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(3/2)\*(a + b\*x^2)^(1/4)),x]

[Out]  $-2/(c*\text{Sqrt}[c*x]*(a + b*x^2)^{(1/4)}) + (2*\text{Sqrt}[b]*(1 + a/(b*x^2)))^{(1/4)}*\text{Sqrt}[c*x]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(\text{Sqrt}[a]*c^2*(a + b*x^2)^{(1/4)})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{2}{c\sqrt{cx}\sqrt[4]{a + bx^2}} + \frac{\sqrt{cx}\sqrt[4]{\frac{a}{bx^2} + 1} \int^{\frac{1}{x}} \frac{1}{\left(\frac{ax^2}{b} + 1\right)^{\frac{5}{4}}} dx}{c^2\sqrt[4]{a + bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(3/2)/(b\*x\*\*2+a)\*\*(1/4),x)

[Out]  $-2/(c*\text{sqrt}(c*x)*(a + b*x**2)**(1/4)) + \text{sqrt}(c*x)*(a/(b*x**2) + 1)**(1/4)*\text{Integral}((a*x**2/b + 1)**(-5/4), (x, 1/x))/(c**2*(a + b*x**2)**(1/4))$

**Mathematica [C]** time = 0.0577323, size = 75, normalized size = 0.83

$$\frac{x \left( 4bx^2 \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^2}{a}\right) - 6(a + bx^2) \right)}{3a(cx)^{3/2}\sqrt[4]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(3/2)\*(a + b\*x^2)^(1/4)),x]

[Out]  $(x*(-6*(a + b*x^2) + 4*b*x^2*(1 + (b*x^2)/a)^{(1/4)}*\text{Hypergeometric}2F1[1/4, 3/4, 7/4, -(b*x^2)/a]))/(3*a*(c*x)^{(3/2)}*(a + b*x^2)^{(1/4)})$

1/4))

---

**Maple [F]** time = 0.053, size = 0, normalized size = 0.

$$\int 1 (cx)^{-\frac{3}{2}} \frac{1}{\sqrt[4]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x)^(3/2)/(b\*x^2+a)^(1/4), x)

[Out] int(1/(c\*x)^(3/2)/(b\*x^2+a)^(1/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/4)\*(c\*x)^(3/2)), x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(1/4)\*(c\*x)^(3/2)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{1}{4}} \sqrt{cxcx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/4)\*(c\*x)^(3/2)), x, algorithm="fricas")

[Out] integral(1/((b\*x^2 + a)^(1/4)\*sqrt(c\*x)\*c\*x), x)

---

**Sympy [A]** time = 14.2025, size = 31, normalized size = 0.34

$$-\frac{{}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{ae^{i\pi}}{bx^2}\right)}{\sqrt[4]{bc^{\frac{3}{2}}x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(3/2)/(b\*x\*\*2+a)\*\*(1/4), x)

[Out] -hyper((1/4, 1/2), (3/2, ), a\*exp\_polar(I\*pi)/(b\*x\*\*2))/(b\*\*(1/4)\*c\*\*(3/2)\*x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(3/2)), x)
```



$$3.953 \quad \int \frac{1}{(cx)^{7/2} \sqrt[4]{a+bx^2}} dx$$

**Optimal.** Leaf size=126

$$\frac{4b^{3/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a+bx^2}} + \frac{4b}{5ac^3\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2(a+bx^2)^{3/4}}{5ac(cx)^{5/2}}$$

[Out] (4\*b)/(5\*a\*c^3\*Sqrt[c\*x]\*(a+b\*x^2)^(1/4)) - (2\*(a+b\*x^2)^(3/4))/(5\*a\*c\*(c\*x)^(5/2)) - (4\*b^(3/2)\*(1+a/(b\*x^2))^(1/4)\*Sqrt[c\*x]\*EllipticE[ArcCot[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(5\*a^(3/2)\*c^4\*(a+b\*x^2)^(1/4))

**Rubi [A]** time = 0.158869, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{4b^{3/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a+bx^2}} + \frac{4b}{5ac^3\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2(a+bx^2)^{3/4}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(7/2)\*(a+b\*x^2)^(1/4)),x]

[Out] (4\*b)/(5\*a\*c^3\*Sqrt[c\*x]\*(a+b\*x^2)^(1/4)) - (2\*(a+b\*x^2)^(3/4))/(5\*a\*c\*(c\*x)^(5/2)) - (4\*b^(3/2)\*(1+a/(b\*x^2))^(1/4)\*Sqrt[c\*x]\*EllipticE[ArcCot[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(5\*a^(3/2)\*c^4\*(a+b\*x^2)^(1/4))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{4b}{5ac^3\sqrt{cx}\sqrt[4]{a+bx^2}} + \frac{2b\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1 \int^{\frac{1}{x}} \frac{1}{\sqrt[4]{\frac{ax^2}{b} + 1}} dx}{5ac^4\sqrt[4]{a+bx^2}} - \frac{4b\sqrt{cx}}{5ac^4x\sqrt[4]{a+bx^2}} - \frac{2(a+bx^2)^{3/4}}{5ac(cx)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(7/2)/(b\*x\*\*2+a)\*\*(1/4),x)

[Out] 4\*b/(5\*a\*c\*\*3\*sqrt(c\*x)\*(a+b\*x\*\*2)\*\*(1/4)) + 2\*b\*sqrt(c\*x)\*(a/(b\*x\*\*2)+1)\*\*(1/4)\*Integral((a\*x\*\*2/b+1)\*\*(-1/4),(x,1/x))/(5\*a\*c\*\*4\*(a+b\*x\*\*2)\*\*(1/4)) - 4\*b\*sqrt(c\*x)/(5\*a\*c\*\*4\*x\*(a+b\*x\*\*2)\*\*(1/4)) - 2\*(a+b\*x\*\*2)\*\*(3/4)/(5\*a\*c\*(c\*x)\*\*(5/2))

**Mathematica [C]** time = 0.0714954, size = 88, normalized size = 0.7

$$\frac{x\left(-6a^2 - 8b^2x^4\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^2}{a}\right) + 6abx^2 + 12b^2x^4\right)}{15a^2(cx)^{7/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(7/2)\*(a+b\*x^2)^(1/4)),x]

[Out]  $(x^*(-6*a^2 + 6*a*b*x^2 + 12*b^2*x^4 - 8*b^2*x^4*(1 + (b*x^2)/a))^{1/4} * \text{Hypergeometric2F1}[1/4, 3/4, 7/4, -((b*x^2)/a)]) / (15*a^2*(c*x)^{7/2}*(a + b*x^2)^{1/4})$

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int 1 (cx)^{-\frac{7}{2}} \frac{1}{\sqrt[4]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(7/2)/(b*x^2+a)^(1/4), x)`

[Out] `int(1/(c*x)^(7/2)/(b*x^2+a)^(1/4), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(7/2)), x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(7/2)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{1}{4}} \sqrt{cx} c^3 x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(7/2)), x, algorithm="fricas")`

[Out] `integral(1/((b*x^2 + a)^(1/4)*sqrt(c*x)*c^3*x^3), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(7/2)/(b*x**2+a)**(1/4), x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(7/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(7/2)), x)
```

$$3.954 \quad \int \frac{1}{(cx)^{11/2} \sqrt[4]{a+bx^2}} dx$$

**Optimal.** Leaf size=157

$$\frac{8b^{5/2} \sqrt{cx} \sqrt[4]{\frac{a}{bx^2}} + 1E\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{15a^{5/2} c^6 \sqrt[4]{a+bx^2}} - \frac{8b^2}{15a^2 c^5 \sqrt{cx} \sqrt[4]{a+bx^2}} + \frac{4b(a+bx^2)^{3/4}}{15a^2 c^3 (cx)^{5/2}} - \frac{2(a+bx^2)^{3/4}}{9ac(cx)^{9/2}}$$

[Out]  $(-8*b^2)/(15*a^2*c^5*\text{Sqrt}[c*x]*(a+b*x^2)^{(1/4)}) - (2*(a+b*x^2)^{(3/4)})/(9*a*c*(c*x)^{(9/2)}) + (4*b*(a+b*x^2)^{(3/4)})/(15*a^2*c^3*(c*x)^{(5/2)}) + (8*b^{(5/2)}*(1+a/(b*x^2))^{(1/4)}*\text{Sqrt}[c*x]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(15*a^{(5/2)}*c^6*(a+b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.210187, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{8b^{5/2} \sqrt{cx} \sqrt[4]{\frac{a}{bx^2}} + 1E\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{15a^{5/2} c^6 \sqrt[4]{a+bx^2}} - \frac{8b^2}{15a^2 c^5 \sqrt{cx} \sqrt[4]{a+bx^2}} + \frac{4b(a+bx^2)^{3/4}}{15a^2 c^3 (cx)^{5/2}} - \frac{2(a+bx^2)^{3/4}}{9ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(11/2)\*(a+b\*x^2)^(1/4)),x]

[Out]  $(-8*b^2)/(15*a^2*c^5*\text{Sqrt}[c*x]*(a+b*x^2)^{(1/4)}) - (2*(a+b*x^2)^{(3/4)})/(9*a*c*(c*x)^{(9/2)}) + (4*b*(a+b*x^2)^{(3/4)})/(15*a^2*c^3*(c*x)^{(5/2)}) + (8*b^{(5/2)}*(1+a/(b*x^2))^{(1/4)}*\text{Sqrt}[c*x]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(15*a^{(5/2)}*c^6*(a+b*x^2)^{(1/4)})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{2(a+bx^2)^{\frac{3}{4}}}{9ac(cx)^{\frac{9}{2}}} - \frac{8b^2}{15a^2c^5\sqrt{cx}\sqrt[4]{a+bx^2}} + \frac{4b^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1\int^{\frac{1}{x}} \frac{1}{\left(\frac{ax^2}{b}+1\right)^{\frac{5}{4}}} dx}{15a^2c^6\sqrt[4]{a+bx^2}} + \frac{4b(a+bx^2)^{\frac{3}{4}}}{15a^2c^3(cx)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(11/2)/(b\*x\*\*2+a)\*\*(1/4),x)

[Out]  $-2*(a+b*x^2)^{(3/4)}/(9*a*c*(c*x)^{(9/2)}) - 8*b^2/(15*a^2*c^5*\text{sqrt}(c*x)*(a+b*x^2)^{(1/4)}) + 4*b^2*\text{sqrt}(c*x)*(a/(b*x^2)+1)^{(1/4)}*\text{Integral}((a*x^2/b+1)^{(-5/4)},(x,1/x))/(15*a^2*c^6*(a+b*x^2)^{(1/4)}) + 4*b*(a+b*x^2)^{(3/4)}/(15*a^2*c^3*(c*x)^{(5/2)})$

**Mathematica [C]** time = 0.09282, size = 103, normalized size = 0.66

$$\frac{2\sqrt{cx} \left( -5a^3 + a^2bx^2 + 8b^3x^6 \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^2}{a}\right) - 6ab^2x^4 - 12b^3x^6 \right)}{45a^3c^6x^5\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(11/2)\*(a + b\*x^2)^(1/4)),x]

[Out] (2\*sqrt[c\*x]\*(-5\*a^3 + a^2\*b\*x^2 - 6\*a\*b^2\*x^4 - 12\*b^3\*x^6 + 8\*b^3\*x^6\*(1 + (b\*x^2)/a)^(1/4)\*Hypergeometric2F1[1/4, 3/4, 7/4, -((b\*x^2)/a)]))/((45\*a^3\*c^6\*x^5\*(a + b\*x^2)^(1/4)))

**Maple [F]** time = 0.061, size = 0, normalized size = 0.

$$\int 1 (cx)^{-\frac{11}{2}} \frac{1}{\sqrt[4]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x)^(11/2)/(b\*x^2+a)^(1/4),x)

[Out] int(1/(c\*x)^(11/2)/(b\*x^2+a)^(1/4),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/4)\*(c\*x)^(11/2)),x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(1/4)\*(c\*x)^(11/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{1}{4}} \sqrt{cx} c^5 x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/4)\*(c\*x)^(11/2)),x, algorithm="fricas")

[Out] integral(1/((b\*x^2 + a)^(1/4)\*sqrt(c\*x)\*c^5\*x^5), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(11/2)/(b\*x\*\*2+a)\*\*(1/4),x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(11/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(11/2)), x)
```

$$3.955 \quad \int \frac{(cx)^{3/2}}{\sqrt[4]{a-bx^2}} dx$$

**Optimal.** Leaf size=308

$$\begin{aligned} & -\frac{ac^{3/2} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{8\sqrt{2}b^{5/4}} + \frac{ac^{3/2} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{8\sqrt{2}b^{5/4}} \\ & -\frac{ac^{3/2} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{5/4}} + \frac{ac^{3/2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{4\sqrt{2}b^{5/4}} - \frac{c\sqrt{cx}(a-bx^2)^{3/4}}{2b} \end{aligned}$$

[Out]  $-(c*\text{Sqrt}[c*x]*(a-b*x^2)^{(3/4)})/(2*b) - (a*c^{(3/2)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(\text{Sqrt}[c]*(a-b*x^2)^{(1/4)})])/(4*\text{Sqrt}[2]*b^{(5/4)}) + (a*c^{(3/2)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(\text{Sqrt}[c]*(a-b*x^2)^{(1/4)})])/(4*\text{Sqrt}[2]*b^{(5/4)}) - (a*c^{(3/2)}*\text{Log}[\text{Sqrt}[c] + (\text{Sqrt}[b]*\text{Sqrt}[c]*x)/\text{Sqrt}[a-b*x^2] - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(a-b*x^2)^{(1/4)})]/(8*\text{Sqrt}[2]*b^{(5/4)}) + (a*c^{(3/2)}*\text{Log}[\text{Sqrt}[c] + (\text{Sqrt}[b]*\text{Sqrt}[c]*x)/\text{Sqrt}[a-b*x^2] + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(a-b*x^2)^{(1/4)})]/(8*\text{Sqrt}[2]*b^{(5/4)})$

**Rubi [A]** time = 0.639237, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$

$$\begin{aligned} & -\frac{ac^{3/2} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{8\sqrt{2}b^{5/4}} + \frac{ac^{3/2} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{8\sqrt{2}b^{5/4}} \\ & -\frac{ac^{3/2} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{5/4}} + \frac{ac^{3/2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{4\sqrt{2}b^{5/4}} - \frac{c\sqrt{cx}(a-bx^2)^{3/4}}{2b} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*x)^{(3/2)}/(a-b*x^2)^{(1/4)}, x]$

[Out]  $-(c*\text{Sqrt}[c*x]*(a-b*x^2)^{(3/4)})/(2*b) - (a*c^{(3/2)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(\text{Sqrt}[c]*(a-b*x^2)^{(1/4)})])/(4*\text{Sqrt}[2]*b^{(5/4)}) + (a*c^{(3/2)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(\text{Sqrt}[c]*(a-b*x^2)^{(1/4)})])/(4*\text{Sqrt}[2]*b^{(5/4)}) - (a*c^{(3/2)}*\text{Log}[\text{Sqrt}[c] + (\text{Sqrt}[b]*\text{Sqrt}[c]*x)/\text{Sqrt}[a-b*x^2] - (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(a-b*x^2)^{(1/4)})]/(8*\text{Sqrt}[2]*b^{(5/4)}) + (a*c^{(3/2)}*\text{Log}[\text{Sqrt}[c] + (\text{Sqrt}[b]*\text{Sqrt}[c]*x)/\text{Sqrt}[a-b*x^2] + (\text{Sqrt}[2]*b^{(1/4)}*\text{Sqrt}[c*x])/(a-b*x^2)^{(1/4)})]/(8*\text{Sqrt}[2]*b^{(5/4)})$

**Rubi in Sympy [A]** time = 66.8372, size = 275, normalized size = 0.89

$$\begin{aligned} & -\frac{\sqrt{2}ac^{\frac{3}{2}} \log\left(-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \frac{\sqrt{bcx}}{\sqrt{a-bx^2}} + c\right)}{16b^{\frac{5}{4}}} + \frac{\sqrt{2}ac^{\frac{3}{2}} \log\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \frac{\sqrt{bcx}}{\sqrt{a-bx^2}} + c\right)}{16b^{\frac{5}{4}}} \\ & + \frac{\sqrt{2}ac^{\frac{3}{2}} \text{atan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} - 1\right)}{8b^{\frac{5}{4}}} + \frac{\sqrt{2}ac^{\frac{3}{2}} \text{atan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{8b^{\frac{5}{4}}} - \frac{c\sqrt{cx}(a-bx^2)^{\frac{3}{4}}}{2b} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c*x)^{(3/2)}/(-b*x^2+a)^{(1/4)}, x)$

[Out]  $-\text{sqrt}(2)*a*c^{(3/2)}*\log(-\text{sqrt}(2)*b^{(1/4)}*\text{sqrt}(c)*\text{sqrt}(c*x)/(a-b*x^2)^{(1/4)} + \text{sqrt}(b)*c*x/\text{sqrt}(a-b*x^2) + c)/(16*b^{(5/4)})$

+ sqrt(2)\*a\*c\*\*(3/2)\*log(sqrt(2)\*b\*\*(1/4)\*sqrt(c)\*sqrt(c\*x)/(a - b\*x\*\*2)\*\*(1/4) + sqrt(b)\*c\*x/sqrt(a - b\*x\*\*2) + c)/(16\*b\*\*(5/4)) + sqrt(2)\*a\*c\*\*(3/2)\*atan(sqrt(2)\*b\*\*(1/4)\*sqrt(c\*x)/(sqrt(c)\*(a - b\*x\*\*2)\*\*(1/4)) - 1)/(8\*b\*\*(5/4)) + sqrt(2)\*a\*c\*\*(3/2)\*atan(sqrt(2)\*b\*\*(1/4)\*sqrt(c\*x)/(sqrt(c)\*(a - b\*x\*\*2)\*\*(1/4)) + 1)/(8\*b\*\*(5/4)) - c\*sqrt(c\*x)\*(a - b\*x\*\*2)\*\*(3/4)/(2\*b)

**Mathematica [C]** time = 0.0673919, size = 71, normalized size = 0.23

$$\frac{c\sqrt{cx} \left( a\sqrt[4]{1 - \frac{bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{4}; \frac{5}{4}, \frac{bx^2}{a}\right) - a + bx^2 \right)}{2b\sqrt[4]{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(3/2)/(a - b\*x^2)^(1/4), x]

[Out] (c\*Sqrt[c\*x]\*(-a + b\*x^2 + a\*(1 - (b\*x^2)/a)^(1/4))\*Hypergeometric2F1[1/4, 1/4, 5/4, (b\*x^2)/a])/(2\*b\*(a - b\*x^2)^(1/4))

**Maple [F]** time = 0.054, size = 0, normalized size = 0.

$$\int 1 (cx)^{\frac{3}{2}} \frac{1}{\sqrt[4]{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(3/2)/(-b\*x^2+a)^(1/4), x)

[Out] int((c\*x)^(3/2)/(-b\*x^2+a)^(1/4), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(3/2)/(-b\*x^2 + a)^(1/4), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.241103, size = 417, normalized size = 1.35

$$4(-bx^2 + a)^{\frac{3}{4}}\sqrt{cx}c - 4\left(-\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}b \arctan\left(\frac{\left(-\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}(b^2x^2-ab)}{(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}ac+(bx^2-a)\sqrt{-\frac{\sqrt{-bx^2+aa^2c^3x}-\sqrt{-\frac{a^4c^6}{b^5}}(b^3x^2-ab^2)}}{bx^2-a}}}\right) + \left(-\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}b \log\left(\frac{(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}c - 4\left(-\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}b \arctan\left(\frac{\left(-\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}(b^2x^2-ab)}{(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}ac+(bx^2-a)\sqrt{-\frac{\sqrt{-bx^2+aa^2c^3x}-\sqrt{-\frac{a^4c^6}{b^5}}(b^3x^2-ab^2)}}{bx^2-a}}}\right)}{(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}c - 4\left(-\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}b \arctan\left(\frac{\left(-\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}(b^2x^2-ab)}{(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}ac+(bx^2-a)\sqrt{-\frac{\sqrt{-bx^2+aa^2c^3x}-\sqrt{-\frac{a^4c^6}{b^5}}(b^3x^2-ab^2)}}{bx^2-a}}}\right)}\right)$$

8 b

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(3/2)/(-b\*x^2 + a)^(1/4), x, algorithm="fricas")

[Out] -1/8\*(4\*(-b\*x^2 + a)^(3/4)\*sqrt(c\*x)\*c - 4\*(-a^4\*c^6/b^5)^(1/4)\*b\*arctan((-a^4\*c^6/b^5)^(1/4)\*(b^2\*x^2 - a\*b)/((-b\*x^2 + a)^(3/4))



$$\frac{\sqrt{cx} \cdot a \cdot c + (bx^2 - a) \cdot \sqrt{-(\sqrt{-bx^2 + a} \cdot a^2 \cdot c^3 \cdot x - \sqrt{-a^4 \cdot c^6 / b^5} \cdot (b^3 \cdot x^2 - a \cdot b^2)) / (bx^2 - a)}}{(-a^4 \cdot c^6 / b^5)^{1/4} \cdot b \cdot \log\left(\frac{(-bx^2 + a)^{3/4} \cdot \sqrt{cx} \cdot a \cdot c + (-a^4 \cdot c^6 / b^5)^{1/4} \cdot (b^2 \cdot x^2 - a \cdot b)}{bx^2 - a}\right) - (-a^4 \cdot c^6 / b^5)^{1/4} \cdot b \cdot \log\left(\frac{(-bx^2 + a)^{3/4} \cdot \sqrt{cx} \cdot a \cdot c - (-a^4 \cdot c^6 / b^5)^{1/4} \cdot (b^2 \cdot x^2 - a \cdot b)}{bx^2 - a}\right)}{b}$$

**Sympy [A]** time = 13.7604, size = 46, normalized size = 0.15

$$\frac{c^{\frac{3}{2}} x^{\frac{5}{2}} \left(\frac{5}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{5}{4} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{2\sqrt[4]{a} \left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(3/2)/(-b\*x\*\*2+a)\*\*(1/4), x)

[Out] c\*\*(3/2)\*x\*\*(5/2)\*gamma(5/4)\*hyper((1/4, 5/4), (9/4, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/(2\*a\*\*(1/4)\*gamma(9/4))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{(-bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(3/2)/(-b\*x^2 + a)^(1/4), x, algorithm="giac")

[Out] integrate((c\*x)^(3/2)/(-b\*x^2 + a)^(1/4), x)

$$3.956 \quad \int \frac{1}{\sqrt{cx} \sqrt[4]{a-bx^2}} dx$$

**Optimal.** Leaf size=272

$$\begin{aligned} & -\frac{\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} - \frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}} + \frac{\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} + \frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}} \\ & -\frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}} \end{aligned}$$

[Out] -(ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a - b\*x^2)^(1/4))]/(Sqrt[2]\*b^(1/4)\*Sqrt[c])) + ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a - b\*x^2)^(1/4))]/(Sqrt[2]\*b^(1/4)\*Sqrt[c]) - Log[Sqrt[c] + (Sqrt[b]\*Sqrt[c]\*x)/Sqrt[a - b\*x^2] - (Sqrt[2]\*b^(1/4)\*Sqrt[c\*x])/(a - b\*x^2)^(1/4)]/(2\*Sqrt[2]\*b^(1/4)\*Sqrt[c]) + Log[Sqrt[c] + (Sqrt[b]\*Sqrt[c]\*x)/Sqrt[a - b\*x^2] + (Sqrt[2]\*b^(1/4)\*Sqrt[c\*x])/(a - b\*x^2)^(1/4)]/(2\*Sqrt[2]\*b^(1/4)\*Sqrt[c])

**Rubi [A]** time = 0.502195, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\begin{aligned} & -\frac{\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} - \frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}} + \frac{\log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} + \frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}} \\ & -\frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[c\*x]\*(a - b\*x^2)^(1/4)),x]

[Out] -(ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a - b\*x^2)^(1/4))]/(Sqrt[2]\*b^(1/4)\*Sqrt[c])) + ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a - b\*x^2)^(1/4))]/(Sqrt[2]\*b^(1/4)\*Sqrt[c]) - Log[Sqrt[c] + (Sqrt[b]\*Sqrt[c]\*x)/Sqrt[a - b\*x^2] - (Sqrt[2]\*b^(1/4)\*Sqrt[c\*x])/(a - b\*x^2)^(1/4)]/(2\*Sqrt[2]\*b^(1/4)\*Sqrt[c]) + Log[Sqrt[c] + (Sqrt[b]\*Sqrt[c]\*x)/Sqrt[a - b\*x^2] + (Sqrt[2]\*b^(1/4)\*Sqrt[c\*x])/(a - b\*x^2)^(1/4)]/(2\*Sqrt[2]\*b^(1/4)\*Sqrt[c])

**Rubi in Sympy [A]** time = 57.8948, size = 246, normalized size = 0.9

$$\begin{aligned} & \frac{\sqrt{2} \log\left(-\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt[4]{a-bx^2}} + \frac{\sqrt{bcx}}{\sqrt{a-bx^2}} + c\right)}{4\sqrt[4]{b}\sqrt{c}} + \frac{\sqrt{2} \log\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt[4]{a-bx^2}} + \frac{\sqrt{bcx}}{\sqrt{a-bx^2}} + c\right)}{4\sqrt[4]{b}\sqrt{c}} \\ & + \frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} - 1\right)}{2\sqrt[4]{b}\sqrt{c}} + \frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{2\sqrt[4]{b}\sqrt{c}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(1/2)/(-b\*x\*\*2+a)\*\*(1/4),x)

[Out] -sqrt(2)\*log(-sqrt(2)\*b\*\*(1/4)\*sqrt(c)\*sqrt(c\*x)/(a - b\*x\*\*2)\*\*(1/4) + sqrt(b)\*c\*x/sqrt(a - b\*x\*\*2) + c)/(4\*b\*\*(1/4)\*sqrt(c)) + sqrt(2)\*log(sqrt(2)\*b\*\*(1/4)\*sqrt(c)\*sqrt(c\*x)/(a - b\*x\*\*2)\*\*(1/4)

$$+ \sqrt{b} \cdot c \cdot x / \sqrt{a - b \cdot x^2} + c) / (4 \cdot b^{1/4} \cdot \sqrt{c}) + \sqrt{2} \cdot \operatorname{atan}(\sqrt{2} \cdot b^{1/4} \cdot \sqrt{c \cdot x} / (\sqrt{c} \cdot (a - b \cdot x^2)^{1/4})) - 1) / (2 \cdot b^{1/4} \cdot \sqrt{c}) + \sqrt{2} \cdot \operatorname{atan}(\sqrt{2} \cdot b^{1/4} \cdot \sqrt{c \cdot x} / (\sqrt{c} \cdot (a - b \cdot x^2)^{1/4})) + 1) / (2 \cdot b^{1/4} \cdot \sqrt{c})$$

**Mathematica [C]** time = 0.0376406, size = 56, normalized size = 0.21

$$\frac{2x \sqrt[4]{\frac{a-bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{1}{4}; \frac{5}{4}; \frac{bx^2}{a}\right)}{\sqrt{cx} \sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[c\*x]\*(a - b\*x^2)^(1/4)), x]

[Out] (2\*x\*((a - b\*x^2)/a)^(1/4)\*Hypergeometric2F1[1/4, 1/4, 5/4, (b\*x^2)/a])/(Sqrt[c\*x]\*(a - b\*x^2)^(1/4))

**Maple [F]** time = 0.049, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx}} \frac{1}{\sqrt[4]{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x)^(1/2)/(-b\*x^2+a)^(1/4), x)

[Out] int(1/(c\*x)^(1/2)/(-b\*x^2+a)^(1/4), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(1/4)\*sqrt(c\*x)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.238066, size = 338, normalized size = 1.24

$$2 \left(-\frac{1}{bc^2}\right)^{\frac{1}{4}} \arctan \left( \frac{(bcx^2 - ac) \left(-\frac{1}{bc^2}\right)^{\frac{1}{4}}}{(bx^2 - a) \sqrt{-\frac{\sqrt{-bx^2+acx-(bc^2x^2-ac^2)}\sqrt{-\frac{1}{bc^2}}}{bx^2-a}} + (-bx^2 + a)^{\frac{3}{4}} \sqrt{cx}} \right) - \frac{1}{2} \left(-\frac{1}{bc^2}\right)^{\frac{1}{4}} \log \left( \frac{(-bx^2 + a)^{\frac{3}{4}} \sqrt{cx} + (bcx^2 - ac) \left(-\frac{1}{bc^2}\right)^{\frac{1}{4}}}{bx^2 - a} \right) + \frac{1}{2} \left(-\frac{1}{bc^2}\right)^{\frac{1}{4}} \log \left( \frac{(-bx^2 + a)^{\frac{3}{4}} \sqrt{cx} - (bcx^2 - ac) \left(-\frac{1}{bc^2}\right)^{\frac{1}{4}}}{bx^2 - a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(1/4)\*sqrt(c\*x)),x, algorithm="fricas")

[Out]  $2 \cdot (-1/(b \cdot c^2))^{1/4} \cdot \arctan\left(\frac{(b \cdot c \cdot x^2 - a \cdot c) \cdot (-1/(b \cdot c^2))^{1/4}}{(b \cdot x^2 - a) \cdot \sqrt{-(\sqrt{-b \cdot x^2 + a} \cdot c \cdot x - (b \cdot c^2 \cdot x^2 - a \cdot c^2) \cdot \sqrt{-1/(b \cdot c^2)})}}\right) / (b \cdot x^2 - a) + (-b \cdot x^2 + a)^{3/4} \cdot \sqrt{c \cdot x} - 1/2 \cdot (-1/(b \cdot c^2))^{1/4} \cdot \log\left(\frac{(-b \cdot x^2 + a)^{3/4} \cdot \sqrt{c \cdot x} + (b \cdot c \cdot x^2 - a \cdot c) \cdot (-1/(b \cdot c^2))^{1/4}}{(b \cdot x^2 - a)}\right) + 1/2 \cdot (-1/(b \cdot c^2))^{1/4} \cdot \log\left(\frac{(-b \cdot x^2 + a)^{3/4} \cdot \sqrt{c \cdot x} - (b \cdot c \cdot x^2 - a \cdot c) \cdot (-1/(b \cdot c^2))^{1/4}}{(b \cdot x^2 - a)}\right)$

**Sympy [A]** time = 6.61258, size = 46, normalized size = 0.17

$$\frac{\sqrt{x} \left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{1}{4} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}\sqrt{c} \left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(1/2)/(-b\*x\*\*2+a)\*\*(1/4),x)

[Out]  $\sqrt{x} \cdot \gamma(1/4) \cdot \text{hyper}\left(\left(\frac{1}{4}, \frac{1}{4}\right), \left(\frac{5}{4}, \right), \frac{b \cdot x^{3/2} \cdot \exp_{\text{polar}}(2 \cdot i \cdot \pi)/a}{2 \cdot a^{1/4} \cdot \sqrt{c}}\right) \cdot \gamma(5/4)$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{1/4} \sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(1/4)\*sqrt(c\*x)),x, algorithm="giac")

[Out] integrate(1/((-b\*x^2 + a)^(1/4)\*sqrt(c\*x)), x)

$$3.957 \quad \int \frac{1}{(cx)^{5/2} \sqrt[4]{a - bx^2}} dx$$

**Optimal.** Leaf size=29

$$-\frac{2(a - bx^2)^{3/4}}{3ac(cx)^{3/2}}$$

[Out]  $(-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(3/2))$

**Rubi [A]** time = 0.0292436, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$-\frac{2(a - bx^2)^{3/4}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[1/((c*x)^(5/2)*(a - b*x^2)^(1/4)), x]`

[Out]  $(-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(3/2))$

**Rubi in Sympy [A]** time = 3.88862, size = 24, normalized size = 0.83

$$-\frac{2(a - bx^2)^{\frac{3}{4}}}{3ac(cx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(c*x)**(5/2)/(-b*x**2+a)**(1/4), x)`

[Out]  $-2*(a - b*x**2)**(3/4)/(3*a*c*(c*x)**(3/2))$

**Mathematica [A]** time = 0.0245757, size = 27, normalized size = 0.93

$$-\frac{2x(a - bx^2)^{3/4}}{3a(cx)^{5/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/((c*x)^(5/2)*(a - b*x^2)^(1/4)), x]`

[Out]  $(-2*x*(a - b*x^2)^(3/4))/(3*a*(c*x)^(5/2))$

**Maple [A]** time = 0.007, size = 22, normalized size = 0.8

$$-\frac{2x}{3a}(-bx^2 + a)^{\frac{3}{4}}(cx)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4), x)`

[Out]  $-2/3 * x * (-b * x^2 + a)^{3/4} / a / (c * x)^{5/2}$

**Maxima [A]** time = 1.39802, size = 28, normalized size = 0.97

$$\frac{2(-bx^2 + a)^{\frac{3}{4}}}{3ac^{\frac{5}{2}}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(5/2)),x, algorithm="maxima")`

[Out]  $-2/3 * (-b * x^2 + a)^{3/4} / (a * c^{5/2} * x^{3/2})$

**Fricas [A]** time = 0.211975, size = 47, normalized size = 1.62

$$\frac{2(bx^2 - a)}{3(-bx^2 + a)^{\frac{1}{4}}\sqrt{c}xac^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(5/2)),x, algorithm="fricas")`

[Out]  $2/3 * (b * x^2 - a) / ((-b * x^2 + a)^{1/4} * \text{sqrt}(c * x) * a * c^2 * x)$

**Sympy [A]** time = 55.6909, size = 90, normalized size = 3.1

$$\begin{cases} \frac{b^{\frac{3}{4}} \left(\frac{a}{bx^2} - 1\right)^{\frac{3}{4}} \left(-\frac{3}{4}\right)}{2ac^{\frac{5}{2}} \left(\frac{1}{4}\right)} & \text{for } \left|\frac{a}{bx^2}\right| > 1 \\ -\frac{b^{\frac{3}{4}} \left(-\frac{a}{bx^2} + 1\right)^{\frac{3}{4}} e^{\frac{7i\pi}{4}} \left(-\frac{3}{4}\right)}{2ac^{\frac{5}{2}} \left(\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(5/2)/(-b*x**2+a)**(1/4),x)`

[Out] `Piecewise((b**(3/4)*(a/(b*x**2) - 1)**(3/4)*gamma(-3/4)/(2*a*c**(5/2)*gamma(1/4)), Abs(a/(b*x**2)) > 1), (-b**(3/4)*(-a/(b*x**2) + 1)**(3/4)*exp(7*I*pi/4)*gamma(-3/4)/(2*a*c**(5/2)*gamma(1/4)), True)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(5/2)),x, algorithm="giac")`

[Out] `integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)`

$$3.958 \quad \int \frac{1}{(cx)^{9/2} \sqrt[4]{a - bx^2}} dx$$

**Optimal.** Leaf size=59

$$\frac{8(a - bx^2)^{7/4}}{21a^2c(cx)^{7/2}} - \frac{2(a - bx^2)^{3/4}}{3ac(cx)^{7/2}}$$

[Out]  $(-2*(a - b*x^2)^{(3/4)})/(3*a*c*(c*x)^{(7/2)}) + (8*(a - b*x^2)^{(7/4)})/(21*a^2*c*(c*x)^{(7/2)})$

**Rubi [A]** time = 0.0598506, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{8(a - bx^2)^{7/4}}{21a^2c(cx)^{7/2}} - \frac{2(a - bx^2)^{3/4}}{3ac(cx)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(9/2)\*(a - b\*x^2)^(1/4)), x]

[Out]  $(-2*(a - b*x^2)^{(3/4)})/(3*a*c*(c*x)^{(7/2)}) + (8*(a - b*x^2)^{(7/4)})/(21*a^2*c*(c*x)^{(7/2)})$

**Rubi in Sympy [A]** time = 7.37526, size = 48, normalized size = 0.81

$$-\frac{2(a - bx^2)^{3/4}}{3ac(cx)^{7/2}} + \frac{8(a - bx^2)^{7/4}}{21a^2c(cx)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(9/2)/(-b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $-2*(a - b*x**2)**(3/4)/(3*a*c*(c*x)**(7/2)) + 8*(a - b*x**2)**(7/4)/(21*a**2*c*(c*x)**(7/2))$

**Mathematica [A]** time = 0.0376486, size = 42, normalized size = 0.71

$$\frac{2\sqrt{cx}(a - bx^2)^{3/4}(3a + 4bx^2)}{21a^2c^5x^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(9/2)\*(a - b\*x^2)^(1/4)), x]

[Out]  $(-2*\text{Sqrt}[c*x]*(a - b*x^2)^{(3/4)*(3*a + 4*b*x^2)})/(21*a^2*c^5*x^4)$

**Maple [A]** time = 0.007, size = 32, normalized size = 0.5

$$-\frac{2x(4bx^2 + 3a)}{21a^2} (-bx^2 + a)^{3/4} (cx)^{-9/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x)`

[Out] `-2/21*x*(-b*x^2+a)^(3/4)*(4*b*x^2+3*a)/a^2/(c*x)^(9/2)`

**Maxima [A]** time = 1.39583, size = 54, normalized size = 0.92

$$-\frac{2\left(\frac{7(-bx^2+a)^{\frac{3}{4}}b}{x^{\frac{3}{2}}} + \frac{3(-bx^2+a)^{\frac{7}{4}}}{x^{\frac{7}{2}}}\right)}{21a^2c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(9/2)),x, algorithm="maxima")`

[Out] `-2/21*(7*(-b*x^2 + a)^(3/4)*b/x^(3/2) + 3*(-b*x^2 + a)^(7/4)/x^(7/2))/(a^2*c^(9/2))`

**Fricas [A]** time = 0.21291, size = 63, normalized size = 1.07

$$\frac{2(4b^2x^4 - abx^2 - 3a^2)}{21(-bx^2 + a)^{\frac{1}{4}}\sqrt{c}xa^2c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(9/2)),x, algorithm="fricas")`

[Out] `2/21*(4*b^2*x^4 - a*b*x^2 - 3*a^2)/((-b*x^2 + a)^(1/4)*sqrt(c*x)*a^2*c^4*x^3)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(9/2)/(-b*x**2+a)**(1/4),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{1}{4}}(cx)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(9/2)),x, algorithm="giac")`

[Out] `integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)`



$$3.959 \quad \int \frac{1}{(cx)^{13/2} \sqrt[4]{a - bx^2}} dx$$

**Optimal.** Leaf size=88

$$-\frac{64(a - bx^2)^{11/4}}{231a^3c(cx)^{11/2}} + \frac{16(a - bx^2)^{7/4}}{21a^2c(cx)^{11/2}} - \frac{2(a - bx^2)^{3/4}}{3ac(cx)^{11/2}}$$

[Out]  $(-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(11/2)) + (16*(a - b*x^2)^(7/4))/(21*a^2*c*(c*x)^(11/2)) - (64*(a - b*x^2)^(11/4))/(231*a^3*c*(c*x)^(11/2))$

**Rubi [A]** time = 0.094138, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{64(a - bx^2)^{11/4}}{231a^3c(cx)^{11/2}} + \frac{16(a - bx^2)^{7/4}}{21a^2c(cx)^{11/2}} - \frac{2(a - bx^2)^{3/4}}{3ac(cx)^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(13/2)\*(a - b\*x^2)^(1/4)), x]

[Out]  $(-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(11/2)) + (16*(a - b*x^2)^(7/4))/(21*a^2*c*(c*x)^(11/2)) - (64*(a - b*x^2)^(11/4))/(231*a^3*c*(c*x)^(11/2))$

**Rubi in Sympy [A]** time = 11.5989, size = 73, normalized size = 0.83

$$-\frac{2(a - bx^2)^{3/4}}{3ac(cx)^{11/2}} + \frac{16(a - bx^2)^{7/4}}{21a^2c(cx)^{11/2}} - \frac{64(a - bx^2)^{11/4}}{231a^3c(cx)^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(13/2)/(-b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $-2*(a - b*x**2)**(3/4)/(3*a*c*(c*x)**(11/2)) + 16*(a - b*x**2)**(7/4)/(21*a**2*c*(c*x)**(11/2)) - 64*(a - b*x**2)**(11/4)/(231*a**3*c*(c*x)**(11/2))$

**Mathematica [A]** time = 0.0520158, size = 53, normalized size = 0.6

$$\frac{2\sqrt{cx}(a - bx^2)^{3/4}(21a^2 + 24abx^2 + 32b^2x^4)}{231a^3c^7x^6}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(13/2)\*(a - b\*x^2)^(1/4)), x]

[Out]  $(-2*\text{Sqrt}[c*x]*(a - b*x^2)^(3/4)*(21*a^2 + 24*a*b*x^2 + 32*b^2*x^4))/(231*a^3*c^7*x^6)$

**Maple [A]** time = 0.008, size = 43, normalized size = 0.5

$$-\frac{2x(32b^2x^4 + 24abx^2 + 21a^2)}{231a^3}(-bx^2 + a)^{3/4}(cx)^{-13/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x)`

[Out]  $-2/231*x*(-b*x^2+a)^{3/4}*(32*b^2*x^4+24*a*b*x^2+21*a^2)/a^3/(c*x)^{13/2}$

**Maxima [A]** time = 1.40843, size = 78, normalized size = 0.89

$$\frac{2 \left( \frac{77(-bx^2+a)^{\frac{3}{4}}b^2}{x^{\frac{3}{2}}} + \frac{66(-bx^2+a)^{\frac{7}{4}}b}{x^{\frac{7}{2}}} + \frac{21(-bx^2+a)^{\frac{11}{4}}}{x^{\frac{11}{2}}} \right)}{231 a^3 c^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(13/2)),x, algorithm="maxima")`

[Out]  $-2/231*(77*(-b*x^2 + a)^{3/4}*b^2/x^{3/2} + 66*(-b*x^2 + a)^{7/4}*b/x^{7/2} + 21*(-b*x^2 + a)^{11/4}/x^{11/2})/(a^3*c^{13/2})$

**Fricas [A]** time = 0.213623, size = 78, normalized size = 0.89

$$\frac{2(32b^3x^6 - 8ab^2x^4 - 3a^2bx^2 - 21a^3)}{231(-bx^2 + a)^{\frac{1}{4}}\sqrt{c}xa^3c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(13/2)),x, algorithm="fricas")`

[Out]  $2/231*(32*b^3*x^6 - 8*a*b^2*x^4 - 3*a^2*b*x^2 - 21*a^3)/((-b*x^2 + a)^{1/4}*sqrt(c*x)*a^3*c^6*x^5)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(13/2)/(-b*x**2+a)**(1/4),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{1}{4}}(cx)^{\frac{13}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(13/2)),x, algorithm="giac")`

[Out] `integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)`

$$3.960 \quad \int \frac{(cx)^{5/2}}{\sqrt[4]{a - bx^2}} dx$$

**Optimal.** Leaf size=128

$$\frac{a^{3/2}c^2\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a-bx^2}} - \frac{ac^3(a-bx^2)^{3/4}}{2b^2\sqrt{cx}} - \frac{c(cx)^{3/2}(a-bx^2)^{3/4}}{3b}$$

[Out]  $-(a*c^3*(a - b*x^2)^{(3/4)})/(2*b^2*\operatorname{Sqrt}[c*x]) - (c*(c*x)^{(3/2)}*(a - b*x^2)^{(3/4)})/(3*b) + (a^{(3/2)}*c^2*(1 - a/(b*x^2))^{(1/4)}*\operatorname{Sqrt}[c*x]*\operatorname{EllipticE}[\operatorname{ArcCsc}[(\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]/2, 2])/(2*b^{(3/2)}*(a - b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.168212, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{a^{3/2}c^2\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a-bx^2}} - \frac{ac^3(a-bx^2)^{3/4}}{2b^2\sqrt{cx}} - \frac{c(cx)^{3/2}(a-bx^2)^{3/4}}{3b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c*x)^{(5/2)}/(a - b*x^2)^{(1/4)}, x]$

[Out]  $-(a*c^3*(a - b*x^2)^{(3/4)})/(2*b^2*\operatorname{Sqrt}[c*x]) - (c*(c*x)^{(3/2)}*(a - b*x^2)^{(3/4)})/(3*b) + (a^{(3/2)}*c^2*(1 - a/(b*x^2))^{(1/4)}*\operatorname{Sqrt}[c*x]*\operatorname{EllipticE}[\operatorname{ArcCsc}[(\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]/2, 2])/(2*b^{(3/2)}*(a - b*x^2)^{(1/4)})$

**Rubi in Sympy [A]** time = 21.0657, size = 109, normalized size = 0.85

$$\frac{a^{3/2}c^2\sqrt{cx}\sqrt[4]{-\frac{a}{bx^2}+1}E\left(\frac{\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2}\middle|2\right)}{2b^{3/2}\sqrt[4]{a-bx^2}} - \frac{ac^3(a-bx^2)^{3/4}}{2b^2\sqrt{cx}} - \frac{c(cx)^{3/2}(a-bx^2)^{3/4}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}((c*x)**(5/2)/(-b*x**2+a)**(1/4), x)$

[Out]  $a^{(3/2)}*c^{(2)}*\operatorname{sqrt}(c*x)*(-a/(b*x**2) + 1)^{(1/4)}*\operatorname{elliptic\_e}(\operatorname{asin}(\operatorname{sqrt}(a)/(\operatorname{sqrt}(b)*x))/2, 2)/(2*b^{(3/2)}*(a - b*x**2)**(1/4)) - a*c^{(3)}*(a - b*x**2)^{(3/4)}/(2*b^{(2)}*\operatorname{sqrt}(c*x)) - c*(c*x)**(3/2)*(a - b*x**2)^{(3/4)}/(3*b)$

**Mathematica [C]** time = 0.0658317, size = 71, normalized size = 0.55

$$\frac{c(cx)^{3/2}\left(a\sqrt[4]{1-\frac{bx^2}{a}}{}_2F_1\left(\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{bx^2}{a}\right) - a + bx^2\right)}{3b\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[(c*x)^{(5/2)}/(a - b*x^2)^{(1/4)}, x]$

[Out]  $(c \cdot (c \cdot x)^{3/2} \cdot (-a + b \cdot x^2 + a \cdot (1 - (b \cdot x^2)/a)^{1/4}) \cdot \text{Hypergeometric2F1}[1/4, 3/4, 7/4, (b \cdot x^2)/a]) / (3 \cdot b \cdot (a - b \cdot x^2)^{1/4})$

**Maple [F]** time = 0.057, size = 0, normalized size = 0.

$$\int 1 (cx)^{\frac{5}{2}} \frac{1}{\sqrt[4]{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(5/2)/(-b*x^2+a)^(1/4), x)`

[Out] `int((c*x)^(5/2)/(-b*x^2+a)^(1/4), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{(-bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(5/2)/(-b*x^2 + a)^(1/4), x, algorithm="maxima")`

[Out] `integrate((c*x)^(5/2)/(-b*x^2 + a)^(1/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}c^2x^2}{(-bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(5/2)/(-b*x^2 + a)^(1/4), x, algorithm="fricas")`

[Out] `integral(sqrt(c*x)*c^2*x^2/(-b*x^2 + a)^(1/4), x)`

**Sympy [A]** time = 150.516, size = 46, normalized size = 0.36

$$\frac{c^{\frac{5}{2}} x^{\frac{7}{2}} \left(\frac{7}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{7}{4} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{2\sqrt[4]{a} \left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(5/2)/(-b*x**2+a)**(1/4), x)`

[Out] `c**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/4, 7/4), (11/4,), b*x**2*exp_polar(2*I*pi)/a)/(2*a**(1/4)*gamma(11/4))`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{(-bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(5/2)/(-b*x^2 + a)^(1/4), x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(-b*x^2 + a)^(1/4), x)
```

$$3.961 \quad \int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}} dx$$

**Optimal.** Leaf size=90

$$\frac{\sqrt{a}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}} - \frac{c(a-bx^2)^{3/4}}{b\sqrt{cx}}$$

[Out]  $-\left(\frac{c(a-bx^2)^{3/4}}{b\sqrt{cx}}\right) + \left(\frac{\sqrt{a}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}}\right)$

**Rubi [A]** time = 0.120699, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\sqrt{a}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}} - \frac{c(a-bx^2)^{3/4}}{b\sqrt{cx}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c\*x]/(a - b\*x^2)^(1/4), x]

[Out]  $-\left(\frac{c(a-bx^2)^{3/4}}{b\sqrt{cx}}\right) + \left(\frac{\sqrt{a}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}}\right)$

**Rubi in Sympy [A]** time = 15.2928, size = 75, normalized size = 0.83

$$\frac{\sqrt{a}\sqrt{cx}\sqrt[4]{-\frac{a}{bx^2}+1}E\left(\frac{\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2}\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}} - \frac{c(a-bx^2)^{3/4}}{b\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(1/2)/(-b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $\sqrt{a}\sqrt{cx}\sqrt[4]{-\frac{a}{bx^2}+1}E\left(\frac{\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2}\middle|2\right) - \frac{c(a-bx^2)^{3/4}}{b\sqrt{cx}}$

**Mathematica [C]** time = 0.0367187, size = 58, normalized size = 0.64

$$\frac{2x\sqrt{cx}\sqrt[4]{\frac{a-bx^2}{a}}{}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{7}{4}; \frac{bx^2}{a}\right)}{3\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c\*x]/(a - b\*x^2)^(1/4), x]

[Out]  $\frac{2x\sqrt{cx}\sqrt[4]{\frac{a-bx^2}{a}}{}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{7}{4}; \frac{bx^2}{a}\right)}{3\sqrt[4]{a-bx^2}}$

---

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int 1\sqrt{cx} \frac{1}{\sqrt[4]{-bx^2+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(1/2)/(-b\*x^2+a)^(1/4), x)

[Out] int((c\*x)^(1/2)/(-b\*x^2+a)^(1/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{(-bx^2+a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x)/(-b\*x^2 + a)^(1/4), x, algorithm="maxima")

[Out] integrate(sqrt(c\*x)/(-b\*x^2 + a)^(1/4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}}{(-bx^2+a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x)/(-b\*x^2 + a)^(1/4), x, algorithm="fricas")

[Out] integral(sqrt(c\*x)/(-b\*x^2 + a)^(1/4), x)

---

**Sympy [A]** time = 3.56457, size = 46, normalized size = 0.51

$$\frac{\sqrt{cx}^{\frac{3}{2}} \left(\frac{3}{4}\right) {}_2F_1\left(\frac{1}{4}, \frac{3}{4} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{2\sqrt[4]{a} \left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(1/2)/(-b\*x\*\*2+a)\*\*(1/4), x)

[Out] sqrt(c)\*x\*\*(3/2)\*gamma(3/4)\*hyper((1/4, 3/4), (7/4, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/(2\*a\*\*(1/4)\*gamma(7/4))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{(-bx^2+a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x)/(-b*x^2 + a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(-b*x^2 + a)^(1/4), x)
```



$$3.962 \quad \int \frac{1}{(cx)^{3/2} \sqrt[4]{a - bx^2}} dx$$

**Optimal.** Leaf size=68

$$\frac{2\sqrt{b}\sqrt{cx}\sqrt[4]{1 - \frac{a}{bx^2}} E\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{ac^2}\sqrt[4]{a - bx^2}}$$

[Out]  $(-2*\operatorname{Sqrt}[b]*(1 - a/(b*x^2))^{1/4}*\operatorname{Sqrt}[c*x]*\operatorname{EllipticE}[\operatorname{ArcCsc}[(\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]/2, 2])/(\operatorname{Sqrt}[a]*c^2*(a - b*x^2)^{1/4})$

**Rubi [A]** time = 0.0813035, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{2\sqrt{b}\sqrt{cx}\sqrt[4]{1 - \frac{a}{bx^2}} E\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{ac^2}\sqrt[4]{a - bx^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[1/((c*x)^{3/2}*(a - b*x^2)^{1/4}), x]$

[Out]  $(-2*\operatorname{Sqrt}[b]*(1 - a/(b*x^2))^{1/4}*\operatorname{Sqrt}[c*x]*\operatorname{EllipticE}[\operatorname{ArcCsc}[(\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]/2, 2])/(\operatorname{Sqrt}[a]*c^2*(a - b*x^2)^{1/4})$

**Rubi in Sympy [A]** time = 11.3844, size = 61, normalized size = 0.9

$$\frac{2\sqrt{b}\sqrt{cx}\sqrt[4]{-\frac{a}{bx^2} + 1} E\left(\frac{\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{\sqrt{ac^2}\sqrt[4]{a - bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}(1/(c*x)^{3/2}/(-b*x^2+a)^{1/4}, x)$

[Out]  $-2*\operatorname{sqrt}(b)*\operatorname{sqrt}(c*x)*(-a/(b*x^2) + 1)^{1/4}*\operatorname{elliptic\_e}(\operatorname{asin}(\operatorname{sqrt}(a)/(\operatorname{sqrt}(b)*x))/2, 2)/(\operatorname{sqrt}(a)*c^2*(a - b*x^2)^{1/4})$

**Mathematica [C]** time = 0.0560265, size = 76, normalized size = 1.12

$$\frac{x\left(-4bx^2\sqrt[4]{1 - \frac{bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{7}{4}; \frac{bx^2}{a}\right) - 6a + 6bx^2\right)}{3a(cx)^{3/2}\sqrt[4]{a - bx^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[1/((c*x)^{3/2}*(a - b*x^2)^{1/4}), x]$

[Out]  $(x*(-6*a + 6*b*x^2 - 4*b*x^2*(1 - (b*x^2)/a)^{1/4}*\operatorname{Hypergeometric2F1}[1/4, 3/4, 7/4, (b*x^2)/a])/((3*a*(c*x)^{3/2}*(a - b*x^2)^{1/4}))$

**Maple [F]** time = 0.06, size = 0, normalized size = 0.

$$\int 1 (cx)^{-\frac{3}{2}} \frac{1}{\sqrt[4]{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x)^(3/2)/(-b\*x^2+a)^(1/4), x)

[Out] int(1/(c\*x)^(3/2)/(-b\*x^2+a)^(1/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(1/4)\*(c\*x)^(3/2)), x, algorithm="maxima")

[Out] integrate(1/((-b\*x^2 + a)^(1/4)\*(c\*x)^(3/2)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-bx^2 + a)^{\frac{1}{4}} \sqrt{cxcx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(1/4)\*(c\*x)^(3/2)), x, algorithm="fricas")

[Out] integral(1/((-b\*x^2 + a)^(1/4)\*sqrt(c\*x)\*c\*x), x)

---

**Sympy [A]** time = 13.7787, size = 32, normalized size = 0.47

$$\frac{ie^{\frac{i\pi}{4}} {}_2F_1\left(\frac{1}{4}, \frac{1}{2} \middle| \frac{a}{bx^2}\right)}{\sqrt[4]{bc^{\frac{3}{2}}x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(3/2)/(-b\*x\*\*2+a)\*\*(1/4), x)

[Out] I\*exp(I\*pi/4)\*hyper((1/4, 1/2), (3/2, ), a/(b\*x\*\*2))/(b\*\*(1/4)\*c\*\*(3/2)\*x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(3/2)), x)
```

$$3.963 \quad \int \frac{1}{(cx)^{7/2} \sqrt[4]{a - bx^2}} dx$$

**Optimal.** Leaf size=100

$$-\frac{4b^{3/2}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a-bx^2}} - \frac{2(a-bx^2)^{3/4}}{5ac(cx)^{5/2}}$$

[Out]  $(-2*(a - b*x^2)^{(3/4)})/(5*a*c*(c*x)^{(5/2)}) - (4*b^{(3/2)}*(1 - a/(b*x^2))^{(1/4)}*\text{Sqrt}[c*x]*\text{EllipticE}[\text{ArcCsc}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/ (5*a^{(3/2)}*c^4*(a - b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.121854, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{4b^{3/2}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a-bx^2}} - \frac{2(a-bx^2)^{3/4}}{5ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(7/2)\*(a - b\*x^2)^(1/4)), x]

[Out]  $(-2*(a - b*x^2)^{(3/4)})/(5*a*c*(c*x)^{(5/2)}) - (4*b^{(3/2)}*(1 - a/(b*x^2))^{(1/4)}*\text{Sqrt}[c*x]*\text{EllipticE}[\text{ArcCsc}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/ (5*a^{(3/2)}*c^4*(a - b*x^2)^{(1/4)})$

**Rubi in Sympy [A]** time = 16.1951, size = 87, normalized size = 0.87

$$-\frac{2(a-bx^2)^{\frac{3}{4}}}{5ac(cx)^{\frac{5}{2}}} - \frac{4b^{\frac{3}{2}}\sqrt{cx}\sqrt[4]{-\frac{a}{bx^2}} + 1E\left(\frac{\text{asin}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2}\middle|2\right)}{5a^{\frac{3}{2}}c^4\sqrt[4]{a-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(7/2)/(-b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $-2*(a - b*x**2)**(3/4)/(5*a*c*(c*x)**(5/2)) - 4*b**(3/2)*\text{sqrt}(c*x)*(-a/(b*x**2) + 1)**(1/4)*\text{elliptic\_e}(\text{asin}(\text{sqrt}(a)/(\text{sqrt}(b)*x)))/2, 2)/(5*a**(3/2)*c**4*(a - b*x**2)**(1/4))$

**Mathematica [C]** time = 0.0830733, size = 89, normalized size = 0.89

$$\frac{x\left(-6(a^2 + abx^2 - 2b^2x^4) - 8b^2x^4\sqrt[4]{1-\frac{bx^2}{a}}{}_2F_1\left(\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{bx^2}{a}\right)\right)}{15a^2(cx)^{7/2}\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(7/2)\*(a - b\*x^2)^(1/4)), x]

[Out]  $(x*(-6*(a^2 + a*b*x^2 - 2*b^2*x^4) - 8*b^2*x^4*(1 - (b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/4, 3/4, 7/4, (b*x^2)/a]))/(15*a^2*(c*x)^{(7/2)}*\sqrt[4]{a-bx^2})$

$$7/2) * (a - b * x^2)^{(1/4)}$$

**Maple [F]** time = 0.074, size = 0, normalized size = 0.

$$\int 1 (cx)^{-7/2} \frac{1}{\sqrt[4]{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x)^(7/2)/(-b\*x^2+a)^(1/4), x)

[Out] int(1/(c\*x)^(7/2)/(-b\*x^2+a)^(1/4), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{1/4} (cx)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(1/4) \* (c\*x)^(7/2)), x, algorithm="maxima")

[Out] integrate(1/((-b\*x^2 + a)^(1/4) \* (c\*x)^(7/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-bx^2 + a)^{1/4} \sqrt{cx} c^3 x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(1/4) \* (c\*x)^(7/2)), x, algorithm="fricas")

[Out] integral(1/((-b\*x^2 + a)^(1/4) \* sqrt(c\*x) \* c^3 \* x^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(7/2)/(-b\*x\*\*2+a)\*\*(1/4), x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{1/4} (cx)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(7/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(7/2)), x)
```

$$3.964 \quad \int \frac{1}{(cx)^{11/2} \sqrt[4]{a - bx^2}} dx$$

**Optimal.** Leaf size=130

$$-\frac{8b^{5/2}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}c^6\sqrt[4]{a-bx^2}} - \frac{4b(a-bx^2)^{3/4}}{15a^2c^3(cx)^{5/2}} - \frac{2(a-bx^2)^{3/4}}{9ac(cx)^{9/2}}$$

[Out]  $(-2*(a - b*x^2)^{(3/4)})/(9*a*c*(c*x)^{(9/2)}) - (4*b*(a - b*x^2)^{(3/4)})/(15*a^2*c^3*(c*x)^{(5/2)}) - (8*b^{(5/2)}*(1 - a/(b*x^2))^{(1/4)}*Sqrt[c*x]*EllipticE[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*a^{(5/2)}*c^6*(a - b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.171698, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{8b^{5/2}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}c^6\sqrt[4]{a-bx^2}} - \frac{4b(a-bx^2)^{3/4}}{15a^2c^3(cx)^{5/2}} - \frac{2(a-bx^2)^{3/4}}{9ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(11/2)\*(a - b\*x^2)^(1/4)), x]

[Out]  $(-2*(a - b*x^2)^{(3/4)})/(9*a*c*(c*x)^{(9/2)}) - (4*b*(a - b*x^2)^{(3/4)})/(15*a^2*c^3*(c*x)^{(5/2)}) - (8*b^{(5/2)}*(1 - a/(b*x^2))^{(1/4)}*Sqrt[c*x]*EllipticE[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*a^{(5/2)}*c^6*(a - b*x^2)^{(1/4)})$

**Rubi in Sympy [A]** time = 22.2144, size = 116, normalized size = 0.89

$$-\frac{2(a-bx^2)^{\frac{3}{4}}}{9ac(cx)^{\frac{9}{2}}} - \frac{4b(a-bx^2)^{\frac{3}{4}}}{15a^2c^3(cx)^{\frac{5}{2}}} - \frac{8b^{\frac{5}{2}}\sqrt{cx}\sqrt[4]{-\frac{a}{bx^2}+1}E\left(\frac{\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2}\middle|2\right)}{15a^{\frac{5}{2}}c^6\sqrt[4]{a-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(11/2)/(-b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $-2*(a - b*x**2)**(3/4)/(9*a*c*(c*x)**(9/2)) - 4*b*(a - b*x**2)**(3/4)/(15*a**2*c**3*(c*x)**(5/2)) - 8*b**(5/2)*sqrt(c*x)*(-a/(b*x**2) + 1)**(1/4)*elliptic_e(asin(sqrt(a)/(sqrt(b)*x))/2, 2)/(15*a**(5/2)*c**6*(a - b*x**2)**(1/4))$

**Mathematica [C]** time = 0.0993832, size = 104, normalized size = 0.8

$$-\frac{2\sqrt{cx}\left(5a^3 + a^2bx^2 + 8b^3x^6\sqrt[4]{1-\frac{bx^2}{a}}{}_2F_1\left(\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{bx^2}{a}\right) + 6ab^2x^4 - 12b^3x^6\right)}{45a^3c^6x^5\sqrt[4]{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(11/2)\*(a - b\*x^2)^(1/4)), x]

[Out]  $(-2\sqrt{c^2x} (5a^3 + a^2bx^2 + 6a^2b^2x^4 - 12b^3x^6 + 8b^3x^6(1 - (bx^2)/a)^{1/4}) \text{Hypergeometric2F1}[1/4, 3/4, 7/4, (bx^2)/a]) / (45a^3c^6x^5(a - bx^2)^{1/4})$

**Maple [F]** time = 0.074, size = 0, normalized size = 0.

$$\int 1(cx)^{-\frac{11}{2}} \frac{1}{\sqrt[4]{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4), x)`

[Out] `int(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(11/2)), x, algorithm="maxima")`

[Out] `integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(11/2)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-bx^2 + a)^{\frac{1}{4}} \sqrt{cx^5x^5}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(11/2)), x, algorithm="fricas")`

[Out] `integral(1/((-b*x^2 + a)^(1/4)*sqrt(c*x)*c^5*x^5), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(11/2)/(-b*x**2+a)**(1/4), x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{11}{2}}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(11/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(11/2)), x)
```

$$3.965 \quad \int \frac{(cx)^{3/2}}{(a+bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=86

$$\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b} + \frac{\sqrt{a}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{b}(a+bx^2)^{3/4}}$$

[Out] (c\*Sqrt[c\*x]\*(a + b\*x^2)^(1/4))/b + (Sqrt[a]\*(1 + a/(b\*x^2))^(3/4))\* (c\*x)^(3/2)\*EllipticF[ArcCot[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(Sqrt[b]\*(a + b\*x^2)^(3/4))

**Rubi [A]** time = 0.202774, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b} + \frac{\sqrt{a}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{b}(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(3/2)/(a + b\*x^2)^(3/4), x]

[Out] (c\*Sqrt[c\*x]\*(a + b\*x^2)^(1/4))/b + (Sqrt[a]\*(1 + a/(b\*x^2))^(3/4))\* (c\*x)^(3/2)\*EllipticF[ArcCot[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(Sqrt[b]\*(a + b\*x^2)^(3/4))

**Rubi in Sympy [A]** time = 23.0573, size = 75, normalized size = 0.87

$$\frac{\sqrt{a}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{\sqrt{b}(a+bx^2)^{3/4}} + \frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(3/2)/(b\*x\*\*2+a)\*\*(3/4), x)

[Out] sqrt(a)\*(c\*x)\*\*(3/2)\*(a/(b\*x\*\*2) + 1)\*\*(3/4)\*elliptic\_f(atan(sqrt(a)/(sqrt(b)\*x))/2, 2)/(sqrt(b)\*(a + b\*x\*\*2)\*\*(3/4)) + c\*sqrt(c\*x)\*(a + b\*x\*\*2)\*\*(1/4)/b

**Mathematica [C]** time = 0.0562057, size = 66, normalized size = 0.77

$$\frac{c\sqrt{cx} \left(-a \left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{5}{4}; -\frac{bx^2}{a}\right) + a + bx^2\right)}{b(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(3/2)/(a + b\*x^2)^(3/4), x]

[Out] (c\*Sqrt[c\*x]\*(a + b\*x^2 - a\*(1 + (b\*x^2)/a)^(3/4)\*Hypergeometric2F1[1/4, 3/4, 5/4, -(b\*x^2)/a])/(b\*(a + b\*x^2)^(3/4))

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int 1 (cx)^{\frac{3}{2}} (bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(3/2)/(b\*x^2+a)^(3/4), x)

[Out] int((c\*x)^(3/2)/(b\*x^2+a)^(3/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{(bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(3/2)/(b\*x^2 + a)^(3/4), x, algorithm="maxima")

[Out] integrate((c\*x)^(3/2)/(b\*x^2 + a)^(3/4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{c}cx}{(bx^2 + a)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(3/2)/(b\*x^2 + a)^(3/4), x, algorithm="fricas")

[Out] integral(sqrt(c\*x)\*c\*x/(b\*x^2 + a)^(3/4), x)

---

**Sympy [A]** time = 13.5383, size = 44, normalized size = 0.51

$$\frac{c^{\frac{3}{2}} x^{\frac{5}{2}} \left(\frac{5}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{5}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}} \left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(3/2)/(b\*x\*\*2+a)\*\*(3/4), x)

[Out] c\*\*(3/2)\*x\*\*(5/2)\*gamma(5/4)\*hyper((3/4, 5/4), (9/4, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*a\*\*(3/4)\*gamma(9/4))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{(bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(b*x^2 + a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/(b*x^2 + a)^(3/4), x)
```

$$3.966 \quad \int \frac{1}{\sqrt{cx}(a+bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=66

$$\frac{2\sqrt{b}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{ac^2}(a+bx^2)^{3/4}}$$

[Out]  $(-2*\text{Sqrt}[b]*(1 + a/(b*x^2))^{3/4}*(c*x)^{3/2}*\text{EllipticF}[\text{ArcCot}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(\text{Sqrt}[a]*c^2*(a + b*x^2)^{3/4})$

**Rubi [A]** time = 0.165701, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{2\sqrt{b}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{ac^2}(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[c\*x]\*(a + b\*x^2)^(3/4)), x]

[Out]  $(-2*\text{Sqrt}[b]*(1 + a/(b*x^2))^{3/4}*(c*x)^{3/2}*\text{EllipticF}[\text{ArcCot}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(\text{Sqrt}[a]*c^2*(a + b*x^2)^{3/4})$

**Rubi in Sympy [A]** time = 19.5147, size = 61, normalized size = 0.92

$$\frac{2\sqrt{b}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{\text{atan}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{\sqrt{ac^2}(a+bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(1/2)/(b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $-2*\text{sqrt}(b)*(c*x)**(3/2)*(a/(b*x**2) + 1)**(3/4)*\text{elliptic\_f}(\text{atan}(\text{sqrt}(a)/(\text{sqrt}(b)*x))/2, 2)/(\text{sqrt}(a)*c**2*(a + b*x**2)**(3/4))$

**Mathematica [C]** time = 0.0278824, size = 55, normalized size = 0.83

$$\frac{2x \left(\frac{a+bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{bx^2}{a}\right)}{\sqrt{cx}(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[c\*x]\*(a + b\*x^2)^(3/4)), x]

[Out]  $(2*x*((a + b*x^2)/a)^{3/4}*\text{Hypergeometric2F1}[1/4, 3/4, 5/4, -(b*x^2)/a])/(\text{Sqrt}[c*x]*(a + b*x^2)^{3/4})$

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int 1 \frac{1}{\sqrt{cx}} (bx^2 + a)^{-3/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x)`

[Out] `int(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{4}} \sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/4)*sqrt(c*x)),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(3/4)*sqrt(c*x)), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{3}{4}} \sqrt{cx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/4)*sqrt(c*x)),x, algorithm="fricas")`

[Out] `integral(1/((b*x^2 + a)^(3/4)*sqrt(c*x)), x)`

**Sympy** [A] time = 9.17152, size = 31, normalized size = 0.47

$$\frac{{}_2F_1\left(\frac{1}{2}, \frac{3}{4} \mid \frac{ae^{i\pi}}{bx^2}\right)}{b^{\frac{3}{4}} \sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(1/2)/(b*x**2+a)**(3/4),x)`

[Out] `-hyper((1/2, 3/4), (3/2, ), a*exp_polar(I*pi)/(b*x**2))/(b**(3/4)*sqrt(c)*x)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{4}} \sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/4)*sqrt(c*x)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(3/4)*sqrt(c*x)), x)`

$$3.967 \quad \int \frac{1}{(cx)^{5/2}(a+bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=97

$$\frac{4b^{3/2}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{3a^{3/2}c^4(a+bx^2)^{3/4}} - \frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}$$

[Out]  $(-2*(a + b*x^2)^{(1/4)})/(3*a*c*(c*x)^{(3/2)}) + (4*b^{(3/2)}*(1 + a/(b*x^2))^{(3/4)}*(c*x)^{(3/2)}*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*a^{(3/2)}*c^4*(a + b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.202989, antiderivative size = 97, normalized size of antiderivative = 1., number of rules used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$\frac{4b^{3/2}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{3a^{3/2}c^4(a+bx^2)^{3/4}} - \frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(5/2)\*(a + b\*x^2)^(3/4)), x]

[Out]  $(-2*(a + b*x^2)^{(1/4)})/(3*a*c*(c*x)^{(3/2)}) + (4*b^{(3/2)}*(1 + a/(b*x^2))^{(3/4)}*(c*x)^{(3/2)}*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*a^{(3/2)}*c^4*(a + b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 23.6327, size = 85, normalized size = 0.88

$$-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}} + \frac{4b^{3/2}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{\text{atan}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{3a^{3/2}c^4(a+bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(5/2)/(b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $-2*(a + b*x^2)^{(1/4)}/(3*a*c*(c*x)^{(3/2)}) + 4*b^{(3/2)}*(c*x)^{(3/2)}*(a/(b*x^2) + 1)^{(3/4)}*elliptic_f(\text{atan}(\text{sqrt}(a)/(\text{sqrt}(b)*x)), 2)/(3*a^{(3/2)}*c^4*(a + b*x^2)^{(3/4)})$

**Mathematica [C]** time = 0.0554217, size = 72, normalized size = 0.74

$$\frac{2x \left(2bx^2 \left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{5}{4}; -\frac{bx^2}{a}\right) + a + bx^2\right)}{3a(cx)^{5/2}(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(5/2)\*(a + b\*x^2)^(3/4)), x]

[Out]  $(-2*x*(a + b*x^2 + 2*b*x^2*(1 + (b*x^2)/a)^{(3/4)}*Hypergeometric2F1[1/4, 3/4, 5/4, -(b*x^2)/a]))/(3*a*(c*x)^{(5/2)}*(a + b*x^2)^{(3/4)})$

---

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int 1 (cx)^{-\frac{5}{2}} (bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(5/2)/(b*x^2+a)^(3/4), x)`

[Out] `int(1/(c*x)^(5/2)/(b*x^2+a)^(3/4), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{4}} (cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(5/2)), x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(5/2)), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{3}{4}} \sqrt{cx} c^2 x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(5/2)), x, algorithm="fricas")`

[Out] `integral(1/((b*x^2 + a)^(3/4)*sqrt(c*x)*c^2*x^2), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(5/2)/(b*x**2+a)**(3/4), x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{4}} (cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(5/2)), x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(5/2)), x)`



$$3.968 \quad \int \frac{1}{(cx)^{9/2}(a+bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=126

$$-\frac{8b^{5/2}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{7a^{5/2}c^6(a+bx^2)^{3/4}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} - \frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}$$

[Out]  $(-2*(a + b*x^2)^{(1/4)})/(7*a*c*(c*x)^{(7/2)}) + (4*b*(a + b*x^2)^{(1/4)})/(7*a^2*c^3*(c*x)^{(3/2)}) - (8*b^{(5/2)}*(1 + a/(b*x^2))^{(3/4)}*(c*x)^{(3/2)}*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(7*a^{(5/2)}*c^6*(a + b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.24402, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$-\frac{8b^{5/2}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{7a^{5/2}c^6(a+bx^2)^{3/4}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} - \frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(9/2)\*(a + b\*x^2)^(3/4)), x]

[Out]  $(-2*(a + b*x^2)^{(1/4)})/(7*a*c*(c*x)^{(7/2)}) + (4*b*(a + b*x^2)^{(1/4)})/(7*a^2*c^3*(c*x)^{(3/2)}) - (8*b^{(5/2)}*(1 + a/(b*x^2))^{(3/4)}*(c*x)^{(3/2)}*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(7*a^{(5/2)}*c^6*(a + b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 29.2175, size = 114, normalized size = 0.9

$$-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}} + \frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} - \frac{8b^{5/2}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{\text{atan}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{7a^{5/2}c^6(a+bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(9/2)/(b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $-2*(a + b*x**2)**(1/4)/(7*a*c*(c*x)**(7/2)) + 4*b*(a + b*x**2)**(1/4)/(7*a**2*c**3*(c*x)**(3/2)) - 8*b**(5/2)*(c*x)**(3/2)*(a/(b*x**2) + 1)**(3/4)*elliptic_f(atan(sqrt(a)/(sqrt(b)*x))/2, 2)/(7*a**(5/2)*c**6*(a + b*x**2)**(3/4))$

**Mathematica [C]** time = 0.0818712, size = 92, normalized size = 0.73

$$\frac{2\sqrt{cx} \left(-a^2 + 4b^2x^4 \left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{5}{4}; -\frac{bx^2}{a}\right) + abx^2 + 2b^2x^4\right)}{7a^2c^5x^4(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(9/2)\*(a + b\*x^2)^(3/4)), x]

[Out]  $(2*\text{Sqrt}[c*x]*(-a^2 + a*b*x^2 + 2*b^2*x^4 + 4*b^2*x^4*(1 + (b*x^2)/a)^{(3/4)}*\text{Hypergeometric2F1}[1/4, 3/4, 5/4, -(b*x^2)/a]))/(7*a^2$

$*c^5*x^4*(a + b*x^2)^(3/4)$

**Maple [F]** time = 0.047, size = 0, normalized size = 0.

$$\int 1 (cx)^{-\frac{9}{2}} (bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x)^(9/2)/(b\*x^2+a)^(3/4), x)

[Out] int(1/(c\*x)^(9/2)/(b\*x^2+a)^(3/4), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{4}} (cx)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(3/4)\*(c\*x)^(9/2)), x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(3/4)\*(c\*x)^(9/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{3}{4}} \sqrt{cx} c^4 x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(3/4)\*(c\*x)^(9/2)), x, algorithm="fricas")

[Out] integral(1/((b\*x^2 + a)^(3/4)\*sqrt(c\*x)\*c^4\*x^4), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(9/2)/(b\*x\*\*2+a)\*\*(3/4), x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{4}} (cx)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(9/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(9/2)), x)
```

$$3.969 \quad \int \frac{1}{(cx)^{13/2}(a+bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=157

$$\frac{80b^{7/2}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{77a^{7/2}c^8(a+bx^2)^{3/4}} - \frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}} + \frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}$$

[Out]  $(-2*(a + b*x^2)^{(1/4)})/(11*a*c*(c*x)^{(11/2)}) + (20*b*(a + b*x^2)^{(1/4)})/(77*a^2*c^3*(c*x)^{(7/2)}) - (40*b^2*(a + b*x^2)^{(1/4)})/(77*a^3*c^5*(c*x)^{(3/2)}) + (80*b^{(7/2)}*(1 + a/(b*x^2))^{(3/4)}*(c*x)^{(3/2)})*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2]/(77*a^{(7/2)}*c^8*(a + b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.299122, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$\frac{80b^{7/2}(cx)^{3/2} \left(\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{77a^{7/2}c^8(a+bx^2)^{3/4}} - \frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}} + \frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(13/2)\*(a + b\*x^2)^(3/4)), x]

[Out]  $(-2*(a + b*x^2)^{(1/4)})/(11*a*c*(c*x)^{(11/2)}) + (20*b*(a + b*x^2)^{(1/4)})/(77*a^2*c^3*(c*x)^{(7/2)}) - (40*b^2*(a + b*x^2)^{(1/4)})/(77*a^3*c^5*(c*x)^{(3/2)}) + (80*b^{(7/2)}*(1 + a/(b*x^2))^{(3/4)}*(c*x)^{(3/2)})*EllipticF[ArcCot[(Sqrt[b]*x)/Sqrt[a]]/2, 2]/(77*a^{(7/2)}*c^8*(a + b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 36.4682, size = 144, normalized size = 0.92

$$-\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{\frac{11}{2}}} + \frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{\frac{7}{2}}} - \frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{\frac{3}{2}}} + \frac{80b^{\frac{7}{2}}(cx)^{\frac{3}{2}} \left(\frac{a}{bx^2} + 1\right)^{\frac{3}{4}} F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{77a^{\frac{7}{2}}c^8(a+bx^2)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(13/2)/(b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $-2*(a + b*x^2)^{(1/4)}/(11*a*c*(c*x)^{(11/2)}) + 20*b*(a + b*x^2)^{(1/4)}/(77*a^2*c^3*(c*x)^{(7/2)}) - 40*b^2*(a + b*x^2)^{(1/4)}/(77*a^3*c^5*(c*x)^{(3/2)}) + 80*b^{(7/2)}*(c*x)^{(3/2)}*(a/(b*x^2) + 1)^{(3/4)}*elliptic_f(\operatorname{atan}(\operatorname{sqrt}(a)/(\operatorname{sqrt}(b)*x))/2, 2)/(77*a^{(7/2)}*c^8*(a + b*x^2)^{(3/4)})$

**Mathematica [C]** time = 0.0899799, size = 104, normalized size = 0.66

$$\frac{2\sqrt{cx} \left(7a^3 - 3a^2bx^2 + 40b^3x^6 \left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, -\frac{bx^2}{a}\right) + 10ab^2x^4 + 20b^3x^6\right)}{77a^3c^7x^6(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(13/2)\*(a + b\*x^2)^(3/4)), x]

[Out]  $(-2\sqrt{c x} (7 a^3 - 3 a^2 b x^2 + 10 a b^2 x^4 + 20 b^3 x^6 + 40 b^3 x^6 (1 + (b x^2)/a)^{3/4} \text{Hypergeometric2F1}[1/4, 3/4, 5/4, -(b x^2)/a])) / (77 a^3 c^7 x^6 (a + b x^2)^{3/4})$

**Maple [F]** time = 0.049, size = 0, normalized size = 0.

$$\int 1 (c x)^{-\frac{13}{2}} (b x^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(13/2)/(b*x^2+a)^(3/4), x)`

[Out] `int(1/(c*x)^(13/2)/(b*x^2+a)^(3/4), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b x^2 + a)^{\frac{3}{4}} (c x)^{\frac{13}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(13/2)), x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(13/2)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(b x^2 + a)^{\frac{3}{4}} \sqrt{c x} c^6 x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(13/2)), x, algorithm="fricas")`

[Out] `integral(1/((b*x^2 + a)^(3/4)*sqrt(c*x)*c^6*x^6), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(13/2)/(b*x**2+a)**(3/4), x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b x^2 + a)^{\frac{3}{4}} (c x)^{\frac{13}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(13/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(13/2)), x)
```

$$3.970 \quad \int \frac{(cx)^{5/2}}{(a+bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=117

$$\frac{3ac^{5/2} \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}} - \frac{3ac^{5/2} \tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}} + \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}$$

[Out] (c\*(c\*x)^(3/2)\*(a + b\*x^2)^(1/4))/(2\*b) + (3\*a\*c^(5/2)\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a + b\*x^2)^(1/4))])/(4\*b^(7/4)) - (3\*a\*c^(5/2)\*ArcTanh[(b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a + b\*x^2)^(1/4))])/(4\*b^(7/4))

**Rubi [A]** time = 0.198013, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$\frac{3ac^{5/2} \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}} - \frac{3ac^{5/2} \tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}} + \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(5/2)/(a + b\*x^2)^(3/4), x]

[Out] (c\*(c\*x)^(3/2)\*(a + b\*x^2)^(1/4))/(2\*b) + (3\*a\*c^(5/2)\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a + b\*x^2)^(1/4))])/(4\*b^(7/4)) - (3\*a\*c^(5/2)\*ArcTanh[(b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a + b\*x^2)^(1/4))])/(4\*b^(7/4))

**Rubi in Sympy [A]** time = 24.2134, size = 109, normalized size = 0.93

$$\frac{3ac^{\frac{5}{2}} \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{\frac{7}{4}}} - \frac{3ac^{\frac{5}{2}} \operatorname{atanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{\frac{7}{4}}} + \frac{c(cx)^{\frac{3}{2}}\sqrt[4]{a+bx^2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(5/2)/(b\*x\*\*2+a)\*\*(3/4), x)

[Out] 3\*a\*c\*\*(5/2)\*atan(b\*\*(1/4)\*sqrt(c\*x)/(sqrt(c)\*(a + b\*x\*\*2)\*\*(1/4)))/(4\*b\*\*(7/4)) - 3\*a\*c\*\*(5/2)\*atanh(b\*\*(1/4)\*sqrt(c\*x)/(sqrt(c)\*(a + b\*x\*\*2)\*\*(1/4)))/(4\*b\*\*(7/4)) + c\*(c\*x)\*\*(3/2)\*(a + b\*x\*\*2)\*\*(1/4)/(2\*b)

**Mathematica [C]** time = 0.0581489, size = 69, normalized size = 0.59

$$\frac{c(cx)^{3/2} \left( -a \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1 \left( \frac{3}{4}, \frac{3}{4}, \frac{7}{4}; -\frac{bx^2}{a} \right) + a + bx^2 \right)}{2b(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(5/2)/(a + b\*x^2)^(3/4), x]

[Out] (c\*(c\*x)^(3/2)\*(a + b\*x^2 - a\*(1 + (b\*x^2)/a)^(3/4)\*Hypergeometric2F1[3/4, 3/4, 7/4, -(b\*x^2)/a])/(2\*b\*(a + b\*x^2)^(3/4))

---

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int 1 (cx)^{\frac{5}{2}} (bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(5/2)/(b*x^2+a)^(3/4), x)`

[Out] `int((c*x)^(5/2)/(b*x^2+a)^(3/4), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(5/2)/(b*x^2 + a)^(3/4), x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(5/2)/(b*x^2 + a)^(3/4), x, algorithm="fricas")`

[Out] Timed out

---

**Sympy [A]** time = 176.672, size = 44, normalized size = 0.38

$$\frac{c^{\frac{5}{2}} x^{\frac{7}{2}} \left(\frac{7}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{7}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}} \left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(5/2)/(b*x**2+a)**(3/4), x)`

[Out] `c**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/4, 7/4), (11/4, ), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/4)*gamma(11/4))`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{(bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((c*x)^(5/2)/(b*x^2 + a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(5/2)/(b*x^2 + a)^(3/4), x)
```

$$3.971 \quad \int \frac{\sqrt{cx}}{(a+bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=84

$$\frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}} - \frac{\sqrt{c} \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}$$

[Out] -((Sqrt[c]\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a+b\*x^2)^(1/4))])/b^(3/4)) + (Sqrt[c]\*ArcTanh[(b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a+b\*x^2)^(1/4))])/b^(3/4)

**Rubi [A]** time = 0.158434, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}} - \frac{\sqrt{c} \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c\*x]/(a+b\*x^2)^(3/4),x]

[Out] -((Sqrt[c]\*ArcTan[(b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a+b\*x^2)^(1/4))])/b^(3/4)) + (Sqrt[c]\*ArcTanh[(b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a+b\*x^2)^(1/4))])/b^(3/4)

**Rubi in Sympy [A]** time = 19.2544, size = 76, normalized size = 0.9

$$-\frac{\sqrt{c} \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}} + \frac{\sqrt{c} \operatorname{atanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(1/2)/(b\*x\*\*2+a)\*\*(3/4),x)

[Out] -sqrt(c)\*atan(b\*\*(1/4)\*sqrt(c\*x)/(sqrt(c)\*(a+b\*x\*\*2)\*\*(1/4)))/b\*\*(3/4) + sqrt(c)\*atanh(b\*\*(1/4)\*sqrt(c\*x)/(sqrt(c)\*(a+b\*x\*\*2)\*\*(1/4)))/b\*\*(3/4)

**Mathematica [C]** time = 0.030846, size = 57, normalized size = 0.68

$$\frac{2x\sqrt{cx}\left(\frac{a+bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{3}{4}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^2}{a}\right)}{3(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c\*x]/(a+b\*x^2)^(3/4),x]

[Out] (2\*x\*Sqrt[c\*x]\*((a+b\*x^2)/a)^(3/4)\*Hypergeometric2F1[3/4, 3/4, 7/4, -(b\*x^2)/a])/(3\*(a+b\*x^2)^(3/4))

**Maple [F]** time = 0.037, size = 0, normalized size = 0.

$$\int \sqrt{cx} (bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(1/2)/(b*x^2+a)^(3/4), x)`

[Out] `int((c*x)^(1/2)/(b*x^2+a)^(3/4), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x)/(b*x^2 + a)^(3/4), x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x)/(b*x^2 + a)^(3/4), x, algorithm="fricas")`

[Out] Timed out

---

**Sympy [A]** time = 6.82615, size = 44, normalized size = 0.52

$$\frac{\sqrt{cx}^{\frac{3}{2}} \left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{3}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}} \left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(1/2)/(b*x**2+a)**(3/4), x)`

[Out] `sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4, ), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/4)*gamma(7/4))`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{(bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x)/(b*x^2 + a)^(3/4), x, algorithm="giac")`

[Out] `integrate(sqrt(c*x)/(b*x^2 + a)^(3/4), x)`

$$3.972 \quad \int \frac{1}{(cx)^{3/2}(a+bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=26

$$-\frac{2\sqrt[4]{a+bx^2}}{ac\sqrt{cx}}$$

[Out]  $(-2*(a + b*x^2)^{(1/4)})/(a*c*\text{Sqrt}[c*x])$

**Rubi [A]** time = 0.0282875, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$-\frac{2\sqrt[4]{a+bx^2}}{ac\sqrt{cx}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/((c*x)^{(3/2)}*(a + b*x^2)^{(3/4)}), x]$

[Out]  $(-2*(a + b*x^2)^{(1/4)})/(a*c*\text{Sqrt}[c*x])$

**Rubi in Sympy [A]** time = 3.58504, size = 22, normalized size = 0.85

$$-\frac{2\sqrt[4]{a+bx^2}}{ac\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/(c*x)^{(3/2)}/(b*x^2+a)^{(3/4)}, x)$

[Out]  $-2*(a + b*x^2)^{(1/4)}/(a*c*\text{sqrt}(c*x))$

**Mathematica [A]** time = 0.0187059, size = 24, normalized size = 0.92

$$-\frac{2x\sqrt[4]{a+bx^2}}{a(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/((c*x)^{(3/2)}*(a + b*x^2)^{(3/4)}), x]$

[Out]  $(-2*x*(a + b*x^2)^{(1/4)})/(a*(c*x)^{(3/2)})$

**Maple [A]** time = 0.007, size = 21, normalized size = 0.8

$$-2\frac{x\sqrt[4]{bx^2+a}}{a(cx)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/(c*x)^{(3/2)}/(b*x^2+a)^{(3/4)}, x)$

[Out]  $-2*x*(b*x^2+a)^{(1/4)}/a/(c*x)^{(3/2)}$

**Maxima [A]** time = 1.40327, size = 27, normalized size = 1.04

$$-\frac{2(bx^2+a)^{\frac{1}{4}}}{ac^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(3/2)),x, algorithm="maxima")`

[Out]  $-2*(b*x^2 + a)^{(1/4)}/(a*c^{(3/2)}*sqrt(x))$

**Fricas [A]** time = 0.233551, size = 34, normalized size = 1.31

$$-\frac{2(bx^2+a)^{\frac{1}{4}}\sqrt{cx}}{ac^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(3/2)),x, algorithm="fricas")`

[Out]  $-2*(b*x^2 + a)^{(1/4)}*sqrt(c*x)/(a*c^2*x)$

**Sympy [A]** time = 28.685, size = 36, normalized size = 1.38

$$\frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^2} + 1}(-\frac{1}{4})}{2ac^{\frac{3}{2}}(\frac{3}{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(3/2)/(b*x**2+a)**(3/4),x)`

[Out]  $b^{(1/4)}*(a/(b*x**2) + 1)^{(1/4)}*\gamma(-1/4)/(2*a*c^{(3/2)}*\gamma(3/4))$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{4}}(cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(3/2)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)`

$$3.973 \quad \int \frac{1}{(cx)^{7/2}(a+bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=55

$$\frac{8(a+bx^2)^{5/4}}{5a^2c(cx)^{5/2}} - \frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}}$$

[Out]  $(-2*(a + b*x^2)^(1/4))/(a*c*(c*x)^(5/2)) + (8*(a + b*x^2)^(5/4))/(5*a^2*c*(c*x)^(5/2))$

**Rubi [A]** time = 0.0570024, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{8(a+bx^2)^{5/4}}{5a^2c(cx)^{5/2}} - \frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(7/2)\*(a + b\*x^2)^(3/4)), x]

[Out]  $(-2*(a + b*x^2)^(1/4))/(a*c*(c*x)^(5/2)) + (8*(a + b*x^2)^(5/4))/(5*a^2*c*(c*x)^(5/2))$

**Rubi in Sympy [A]** time = 6.74941, size = 46, normalized size = 0.84

$$-\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}} + \frac{8(a+bx^2)^{5/4}}{5a^2c(cx)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(7/2)/(b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $-2*(a + b*x**2)**(1/4)/(a*c*(c*x)**(5/2)) + 8*(a + b*x**2)**(5/4)/(5*a**2*c*(c*x)**(5/2))$

**Mathematica [A]** time = 0.0342663, size = 34, normalized size = 0.62

$$-\frac{2x(a-4bx^2)\sqrt[4]{a+bx^2}}{5a^2(cx)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(7/2)\*(a + b\*x^2)^(3/4)), x]

[Out]  $(-2*x*(a - 4*b*x^2)*(a + b*x^2)^(1/4))/(5*a^2*(c*x)^(7/2))$

**Maple [A]** time = 0.007, size = 29, normalized size = 0.5

$$-\frac{2x(-4bx^2+a)\sqrt[4]{bx^2+a}(cx)^{-7/2}}{5a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x)`

[Out]  $-2/5*x*(b*x^2+a)^{(1/4)}*(-4*b*x^2+a)/a^2/(c*x)^{(7/2)}$

**Maxima [A]** time = 1.41582, size = 51, normalized size = 0.93

$$\frac{2 \left( \frac{5 (bx^2+a)^{\frac{1}{4}} b}{\sqrt{x}} - \frac{(bx^2+a)^{\frac{5}{4}}}{x^{\frac{5}{2}}} \right)}{5 a^2 c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(7/2)),x, algorithm="maxima")`

[Out]  $2/5*(5*(b*x^2 + a)^{(1/4)}*b/\sqrt{x} - (b*x^2 + a)^{(5/4)}/x^{(5/2)})/(a^2*c^{(7/2)})$

**Fricas [A]** time = 0.228188, size = 47, normalized size = 0.85

$$\frac{2(4bx^2 - a)(bx^2 + a)^{\frac{1}{4}}\sqrt{cx}}{5a^2c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(7/2)),x, algorithm="fricas")`

[Out]  $2/5*(4*b*x^2 - a)*(b*x^2 + a)^{(1/4)}*\sqrt{c*x}/(a^2*c^4*x^3)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(7/2)/(b*x**2+a)**(3/4),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{4}}(cx)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(7/2)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)`

$$3.974 \quad \int \frac{1}{(cx)^{11/2}(a+bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=83

$$-\frac{64(a+bx^2)^{9/4}}{45a^3c(cx)^{9/2}} + \frac{16(a+bx^2)^{5/4}}{5a^2c(cx)^{9/2}} - \frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}}$$

[Out]  $(-2*(a + b*x^2)^(1/4))/(a*c*(c*x)^(9/2)) + (16*(a + b*x^2)^(5/4))/(5*a^2*c*(c*x)^(9/2)) - (64*(a + b*x^2)^(9/4))/(45*a^3*c*(c*x)^(9/2))$

**Rubi [A]** time = 0.087363, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$-\frac{64(a+bx^2)^{9/4}}{45a^3c(cx)^{9/2}} + \frac{16(a+bx^2)^{5/4}}{5a^2c(cx)^{9/2}} - \frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(11/2)\*(a + b\*x^2)^(3/4)), x]

[Out]  $(-2*(a + b*x^2)^(1/4))/(a*c*(c*x)^(9/2)) + (16*(a + b*x^2)^(5/4))/(5*a^2*c*(c*x)^(9/2)) - (64*(a + b*x^2)^(9/4))/(45*a^3*c*(c*x)^(9/2))$

**Rubi in Sympy [A]** time = 10.8237, size = 71, normalized size = 0.86

$$-\frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{\frac{9}{2}}} + \frac{16(a+bx^2)^{\frac{5}{4}}}{5a^2c(cx)^{\frac{9}{2}}} - \frac{64(a+bx^2)^{\frac{9}{4}}}{45a^3c(cx)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(11/2)/(b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $-2*(a + b*x**2)**(1/4)/(a*c*(c*x)**(9/2)) + 16*(a + b*x**2)**(5/4)/(5*a**2*c*(c*x)**(9/2)) - 64*(a + b*x**2)**(9/4)/(45*a**3*c*(c*x)**(9/2))$

**Mathematica [A]** time = 0.0405127, size = 52, normalized size = 0.63

$$-\frac{2\sqrt{cx}\sqrt[4]{a+bx^2}(5a^2-8abx^2+32b^2x^4)}{45a^3c^6x^5}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(11/2)\*(a + b\*x^2)^(3/4)), x]

[Out]  $(-2*\text{Sqrt}[c*x]*(a + b*x^2)^(1/4)*(5*a^2 - 8*a*b*x^2 + 32*b^2*x^4))/(45*a^3*c^6*x^5)$

**Maple [A]** time = 0.007, size = 42, normalized size = 0.5

$$-\frac{2x(32b^2x^4 - 8abx^2 + 5a^2)\sqrt[4]{bx^2 + a}(cx)^{-\frac{11}{2}}}{45a^3}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x)`

[Out]  $-2/45*x*(b*x^2+a)^{1/4}*(32*b^2*x^4-8*a*b*x^2+5*a^2)/a^3/(c*x)^{11/2}$

**Maxima [A]** time = 1.39171, size = 74, normalized size = 0.89

$$\frac{2 \left( \frac{45 (bx^2+a)^{\frac{1}{4}} b^2}{\sqrt{x}} - \frac{18 (bx^2+a)^{\frac{5}{4}} b}{x^{\frac{5}{2}}} + \frac{5 (bx^2+a)^{\frac{9}{4}}}{x^{\frac{9}{2}}} \right)}{45 a^3 c^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(11/2)),x, algorithm="maxima")`

[Out]  $-2/45*(45*(b*x^2 + a)^{1/4}*b^2/\text{sqrt}(x) - 18*(b*x^2 + a)^{5/4}*b/x^{5/2} + 5*(b*x^2 + a)^{9/4}/x^{9/2})/(a^3*c^{11/2})$

**Fricas [A]** time = 0.230961, size = 62, normalized size = 0.75

$$\frac{2(32b^2x^4 - 8abx^2 + 5a^2)(bx^2 + a)^{\frac{1}{4}}\sqrt{cx}}{45a^3c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(11/2)),x, algorithm="fricas")`

[Out]  $-2/45*(32*b^2*x^4 - 8*a*b*x^2 + 5*a^2)*(b*x^2 + a)^{1/4}*\text{sqrt}(c*x)/(a^3*c^6*x^5)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(11/2)/(b*x**2+a)**(3/4),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{3}{4}}(cx)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(11/2)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)`

$$3.975 \quad \int \frac{(cx)^{3/2}}{(a-bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=91

$$-\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b} - \frac{\sqrt{a}(cx)^{3/2} \left(1 - \frac{a}{bx^2}\right)^{3/4} F\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{b}(a-bx^2)^{3/4}}$$

[Out]  $-\left(\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}\right) - \left(\frac{\sqrt{a}(cx)^{3/2} \operatorname{EllipticF}\left[\operatorname{ArcCsc}\left[\frac{\sqrt{bx}}{\sqrt{a}}\right]/2, 2\right]}{\sqrt{b}(a-bx^2)^{3/4}}\right)$

**Rubi [A]** time = 0.208862, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b} - \frac{\sqrt{a}(cx)^{3/2} \left(1 - \frac{a}{bx^2}\right)^{3/4} F\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{b}(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] `Int[(c*x)^(3/2)/(a - b*x^2)^(3/4), x]`

[Out]  $-\left(\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}\right) - \left(\frac{\sqrt{a}(cx)^{3/2} \operatorname{EllipticF}\left[\operatorname{ArcCsc}\left[\frac{\sqrt{bx}}{\sqrt{a}}\right]/2, 2\right]}{\sqrt{b}(a-bx^2)^{3/4}}\right)$

**Rubi in Sympy [A]** time = 32.4337, size = 76, normalized size = 0.84

$$-\frac{\sqrt{a}(cx)^{\frac{3}{2}} \left(-\frac{a}{bx^2} + 1\right)^{\frac{3}{4}} F\left(\frac{\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{\sqrt{b}(a-bx^2)^{\frac{3}{4}}} - \frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x)**(3/2)/(-b*x**2+a)**(3/4), x)`

[Out]  $-\sqrt{a}(cx)^{3/2} \left(-\frac{a}{bx^2} + 1\right)^{3/4} \operatorname{elliptic\_f}\left(\frac{\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2}, 2\right) / \left(\sqrt{b}(a-bx^2)^{3/4}\right) - \frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}$

**Mathematica [C]** time = 0.0601226, size = 68, normalized size = 0.75

$$\frac{c\sqrt{cx} \left(a \left(1 - \frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{5}{4}; \frac{bx^2}{a}\right) - a + bx^2\right)}{b(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] `Integrate[(c*x)^(3/2)/(a - b*x^2)^(3/4), x]`

[Out]  $\frac{c\sqrt{cx} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{bx^2}{a}\right]}{b(a-bx^2)^{3/4}}$

**Maple [F]** time = 0.045, size = 0, normalized size = 0.

$$\int 1 (cx)^{\frac{3}{2}} (-bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(3/2)/(-b*x^2+a)^(3/4), x)`

[Out] `int((c*x)^(3/2)/(-b*x^2+a)^(3/4), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{(-bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(3/2)/(-b*x^2 + a)^(3/4), x, algorithm="maxima")`

[Out] `integrate((c*x)^(3/2)/(-b*x^2 + a)^(3/4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}cx}{(-bx^2 + a)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(3/2)/(-b*x^2 + a)^(3/4), x, algorithm="fricas")`

[Out] `integral(sqrt(c*x)*c*x/(-b*x^2 + a)^(3/4), x)`

---

**Sympy [A]** time = 13.4456, size = 46, normalized size = 0.51

$$\frac{c^{\frac{3}{2}} x^{\frac{5}{2}} \left(\frac{5}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{5}{4} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{2a^{\frac{3}{4}} \left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(3/2)/(-b*x**2+a)**(3/4), x)`

[Out] `c**(3/2)*x**(5/2)*gamma(5/4)*hyper((3/4, 5/4), (9/4, ), b*x**2*exp_polar(2*I*pi)/a)/(2*a**(3/4)*gamma(9/4))`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{(-bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(3/2)/(-b*x^2 + a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2)/(-b*x^2 + a)^(3/4), x)
```

$$3.976 \quad \int \frac{1}{\sqrt{cx}(a-bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=68

$$\frac{2\sqrt{b}(cx)^{3/2} \left(1 - \frac{a}{bx^2}\right)^{3/4} F\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{ac^2}(a-bx^2)^{3/4}}$$

[Out]  $(-2*\operatorname{Sqrt}[b]*(1 - a/(b*x^2))^{3/4}*(c*x)^{3/2}*\operatorname{EllipticF}[\operatorname{ArcCsc}[(\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]/2, 2])/(\operatorname{Sqrt}[a]*c^2*(a - b*x^2)^{3/4})$

**Rubi [A]** time = 0.178406, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{2\sqrt{b}(cx)^{3/2} \left(1 - \frac{a}{bx^2}\right)^{3/4} F\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{\sqrt{ac^2}(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[1/(\operatorname{Sqrt}[c*x]*(a - b*x^2)^{3/4}), x]$

[Out]  $(-2*\operatorname{Sqrt}[b]*(1 - a/(b*x^2))^{3/4}*(c*x)^{3/2}*\operatorname{EllipticF}[\operatorname{ArcCsc}[(\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]/2, 2])/(\operatorname{Sqrt}[a]*c^2*(a - b*x^2)^{3/4})$

**Rubi in Sympy [A]** time = 28.3793, size = 61, normalized size = 0.9

$$\frac{2\sqrt{b}(cx)^{3/2} \left(-\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{\sqrt{ac^2}(a-bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}(1/(c*x)^{(1/2)}/(-b*x^2+a)^{(3/4)}, x)$

[Out]  $-2*\operatorname{sqrt}(b)*(c*x)^{(3/2)}*(-a/(b*x^2) + 1)^{(3/4)}*\operatorname{elliptic\_f}(\operatorname{asin}(\operatorname{sqrt}(a)/(\operatorname{sqrt}(b)*x))/2, 2)/(\operatorname{sqrt}(a)*c^2*(a - b*x^2)^{(3/4)})$

**Mathematica [C]** time = 0.0316166, size = 56, normalized size = 0.82

$$\frac{2x \left(\frac{a-bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{bx^2}{a}\right)}{\sqrt{cx}(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[1/(\operatorname{Sqrt}[c*x]*(a - b*x^2)^{3/4}), x]$

[Out]  $(2*x*((a - b*x^2)/a)^{3/4}*\operatorname{Hypergeometric2F1}[1/4, 3/4, 5/4, (b*x^2)/a])/(\operatorname{Sqrt}[c*x]*(a - b*x^2)^{3/4})$

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int 1 \frac{1}{\sqrt{cx}} (-bx^2 + a)^{-3/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x)`

[Out] `int(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{3}{4}} \sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(3/4)*sqrt(c*x)),x, algorithm="maxima")`

[Out] `integrate(1/((-b*x^2 + a)^(3/4)*sqrt(c*x)), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-bx^2 + a)^{\frac{3}{4}} \sqrt{cx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(3/4)*sqrt(c*x)),x, algorithm="fricas")`

[Out] `integral(1/((-b*x^2 + a)^(3/4)*sqrt(c*x)), x)`

**Sympy** [A] time = 9.16059, size = 32, normalized size = 0.47

$$\frac{ie^{-\frac{i\pi}{4}} {}_2F_1\left(\frac{1}{2}, \frac{3}{4} \middle| \frac{a}{bx^2}\right)}{b^{\frac{3}{4}} \sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(1/2)/(-b*x**2+a)**(3/4),x)`

[Out] `I*exp(-I*pi/4)*hyper((1/2, 3/4), (3/2,), a/(b*x**2))/(b**(3/4)*sqrt(c)*x)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{3}{4}} \sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(3/4)*sqrt(c*x)),x, algorithm="giac")`

[Out] `integrate(1/((-b*x^2 + a)^(3/4)*sqrt(c*x)), x)`

$$3.977 \quad \int \frac{1}{(cx)^{5/2}(a-bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=100

$$-\frac{4b^{3/2}(cx)^{3/2} \left(1 - \frac{a}{bx^2}\right)^{3/4} F\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{3a^{3/2}c^4(a-bx^2)^{3/4}} - \frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}$$

[Out]  $(-2*(a - b*x^2)^{(1/4)})/(3*a*c*(c*x)^{(3/2)}) - (4*b^{(3/2)}*(1 - a/(b*x^2))^{(3/4)}*(c*x)^{(3/2)}*\operatorname{EllipticF}[\operatorname{ArcCsc}[(\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]/2, 2])/(3*a^{(3/2)}*c^4*(a - b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.220016, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{4b^{3/2}(cx)^{3/2} \left(1 - \frac{a}{bx^2}\right)^{3/4} F\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{3a^{3/2}c^4(a-bx^2)^{3/4}} - \frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[1/((c*x)^(5/2)*(a - b*x^2)^(3/4)), x]`

[Out]  $(-2*(a - b*x^2)^{(1/4)})/(3*a*c*(c*x)^{(3/2)}) - (4*b^{(3/2)}*(1 - a/(b*x^2))^{(3/4)}*(c*x)^{(3/2)}*\operatorname{EllipticF}[\operatorname{ArcCsc}[(\operatorname{Sqrt}[b]*x)/\operatorname{Sqrt}[a]]/2, 2])/(3*a^{(3/2)}*c^4*(a - b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 33.2305, size = 87, normalized size = 0.87

$$-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}} - \frac{4b^{3/2}(cx)^{3/2} \left(-\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{3a^{3/2}c^4(a-bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(c*x)**(5/2)/(-b*x**2+a)**(3/4), x)`

[Out]  $-2*(a - b*x**2)**(1/4)/(3*a*c*(c*x)**(3/2)) - 4*b**(3/2)*(c*x)**(3/2)*(-a/(b*x**2) + 1)**(3/4)*\operatorname{elliptic\_f}(\operatorname{asin}(\operatorname{sqrt}(a)/(\operatorname{sqrt}(b)*x))/2, 2)/(3*a**(3/2)*c**4*(a - b*x**2)**(3/4))$

**Mathematica [C]** time = 0.0648014, size = 76, normalized size = 0.76

$$\frac{x \left(4bx^2 \left(1 - \frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{5}{4}; \frac{bx^2}{a}\right) - 2a + 2bx^2\right)}{3a(cx)^{5/2}(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/((c*x)^(5/2)*(a - b*x^2)^(3/4)), x]`

[Out]  $(x*(-2*a + 2*b*x^2 + 4*b*x^2*(1 - (b*x^2)/a)^{(3/4)}*\operatorname{Hypergeometric}2F1[1/4, 3/4, 5/4, (b*x^2)/a]))/(3*a*(c*x)^{(5/2)}*(a - b*x^2)^{(3/4)})$

---

**Maple [F]** time = 0.054, size = 0, normalized size = 0.

$$\int 1 (cx)^{-\frac{5}{2}} (-bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4), x)`

[Out] `int(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{3}{4}} (cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(5/2)), x, algorithm="maxima")`

[Out] `integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(5/2)), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-bx^2 + a)^{\frac{3}{4}} \sqrt{cx} c^2 x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(5/2)), x, algorithm="fricas")`

[Out] `integral(1/((-b*x^2 + a)^(3/4)*sqrt(c*x)*c^2*x^2), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(5/2)/(-b*x**2+a)**(3/4), x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{3}{4}} (cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(5/2)), x, algorithm="giac")`

[Out] `integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(5/2)), x)`



$$3.978 \quad \int \frac{1}{(cx)^{9/2}(a-bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=130

$$-\frac{8b^{5/2}(cx)^{3/2} \left(1 - \frac{a}{bx^2}\right)^{3/4} F\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{7a^{5/2}c^6(a-bx^2)^{3/4}} - \frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}} - \frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}$$

[Out]  $(-2*(a - b*x^2)^{(1/4)})/(7*a*c*(c*x)^{(7/2)}) - (4*b*(a - b*x^2)^{(1/4)})/(7*a^2*c^3*(c*x)^{(3/2)}) - (8*b^{(5/2)}*(1 - a/(b*x^2))^{(3/4)}*(c*x)^{(3/2)}*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(7*a^{(5/2)}*c^6*(a - b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.261049, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{8b^{5/2}(cx)^{3/2} \left(1 - \frac{a}{bx^2}\right)^{3/4} F\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{7a^{5/2}c^6(a-bx^2)^{3/4}} - \frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}} - \frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(9/2)\*(a - b\*x^2)^(3/4)), x]

[Out]  $(-2*(a - b*x^2)^{(1/4)})/(7*a*c*(c*x)^{(7/2)}) - (4*b*(a - b*x^2)^{(1/4)})/(7*a^2*c^3*(c*x)^{(3/2)}) - (8*b^{(5/2)}*(1 - a/(b*x^2))^{(3/4)}*(c*x)^{(3/2)}*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(7*a^{(5/2)}*c^6*(a - b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 39.261, size = 116, normalized size = 0.89

$$-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}} - \frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}} - \frac{8b^{5/2}(cx)^{3/2} \left(-\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{7a^{5/2}c^6(a-bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(9/2)/(-b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $-2*(a - b*x**2)**(1/4)/(7*a*c*(c*x)**(7/2)) - 4*b*(a - b*x**2)**(1/4)/(7*a**2*c**3*(c*x)**(3/2)) - 8*b**(5/2)*(c*x)**(3/2)*(-a/(b*x**2) + 1)**(3/4)*elliptic_f(asin(sqrt(a)/(sqrt(b)*x))/2, 2)/(7*a**(5/2)*c**6*(a - b*x**2)**(3/4))$

**Mathematica [C]** time = 0.0978652, size = 94, normalized size = 0.72

$$\frac{\sqrt{cx} \left(8b^2x^4 \left(1 - \frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{5}{4}; \frac{bx^2}{a}\right) - 2(a^2 + abx^2 - 2b^2x^4)\right)}{7a^2c^5x^4(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(9/2)\*(a - b\*x^2)^(3/4)), x]

[Out]  $(\operatorname{Sqrt}[c*x]*(-2*(a^2 + a*b*x^2 - 2*b^2*x^4) + 8*b^2*x^4*(1 - (b*x^2)/a)^{(3/4)}*Hypergeometric2F1[1/4, 3/4, 5/4, (b*x^2)/a]))/(7*a^2*$

$$c^5 x^4 (a - b x^2)^{3/4}$$

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int 1 (cx)^{-\frac{9}{2}} (-bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x)^(9/2)/(-b\*x^2+a)^(3/4), x)

[Out] int(1/(c\*x)^(9/2)/(-b\*x^2+a)^(3/4), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{3}{4}} (cx)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(3/4)\*(c\*x)^(9/2)), x, algorithm="maxima")

[Out] integrate(1/((-b\*x^2 + a)^(3/4)\*(c\*x)^(9/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-bx^2 + a)^{\frac{3}{4}} \sqrt{cx} c^4 x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^2 + a)^(3/4)\*(c\*x)^(9/2)), x, algorithm="fricas")

[Out] integral(1/((-b\*x^2 + a)^(3/4)\*sqrt(c\*x)\*c^4\*x^4), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(9/2)/(-b\*x\*\*2+a)\*\*(3/4), x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{3}{4}} (cx)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(9/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(9/2)), x)
```

$$3.979 \quad \int \frac{1}{(cx)^{13/2}(a-bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=162

$$-\frac{80b^{7/2}(cx)^{3/2} \left(1 - \frac{a}{bx^2}\right)^{3/4} F\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{77a^{7/2}c^8(a-bx^2)^{3/4}} - \frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}} - \frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}$$

[Out]  $(-2*(a - b*x^2)^{(1/4)})/(11*a*c*(c*x)^{(11/2)}) - (20*b*(a - b*x^2)^{(1/4)})/(77*a^2*c^3*(c*x)^{(7/2)}) - (40*b^2*(a - b*x^2)^{(1/4)})/(77*a^3*c^5*(c*x)^{(3/2)}) - (80*b^{7/2}*(1 - a/(b*x^2))^{3/4}*(c*x)^{(3/2)})*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2]/(77*a^{7/2}*c^8*(a - b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.315127, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{80b^{7/2}(cx)^{3/2} \left(1 - \frac{a}{bx^2}\right)^{3/4} F\left(\frac{1}{2} \operatorname{csc}^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right) \middle| 2\right)}{77a^{7/2}c^8(a-bx^2)^{3/4}} - \frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}} - \frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(13/2)\*(a - b\*x^2)^(3/4)), x]

[Out]  $(-2*(a - b*x^2)^{(1/4)})/(11*a*c*(c*x)^{(11/2)}) - (20*b*(a - b*x^2)^{(1/4)})/(77*a^2*c^3*(c*x)^{(7/2)}) - (40*b^2*(a - b*x^2)^{(1/4)})/(77*a^3*c^5*(c*x)^{(3/2)}) - (80*b^{7/2}*(1 - a/(b*x^2))^{3/4}*(c*x)^{(3/2)})*EllipticF[ArcCsc[(Sqrt[b]*x)/Sqrt[a]]/2, 2]/(77*a^{7/2}*c^8*(a - b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 46.8419, size = 146, normalized size = 0.9

$$-\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}} - \frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}} - \frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}} - \frac{80b^{7/2}(cx)^{3/2} \left(-\frac{a}{bx^2} + 1\right)^{3/4} F\left(\frac{\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{bx}}\right)}{2} \middle| 2\right)}{77a^{7/2}c^8(a-bx^2)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(13/2)/(-b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $-2*(a - b*x^2)^{(1/4)}/(11*a*c*(c*x)^{(11/2)}) - 20*b*(a - b*x^2)^{(1/4)}/(77*a^2*c^3*(c*x)^{(7/2)}) - 40*b^2*(a - b*x^2)^{(1/4)}/(77*a^3*c^5*(c*x)^{(3/2)}) - 80*b^{7/2}*(c*x)^{(3/2)}*(-a/(b*x^2) + 1)^{(3/4)}*elliptic_f(asin(sqrt(a)/(sqrt(b)*x))/2, 2)/(77*a^{7/2}*c^8*(a - b*x^2)^{(3/4)})$

**Mathematica [C]** time = 0.108471, size = 105, normalized size = 0.65

$$\frac{2\sqrt{cx} \left(-7a^3 - 3a^2bx^2 + 40b^3x^6 \left(1 - \frac{bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{bx^2}{a}\right) - 10ab^2x^4 + 20b^3x^6\right)}{77a^3c^7x^6(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(13/2)\*(a - b\*x^2)^(3/4)), x]

[Out]  $(2\sqrt{c^2x}(-7a^3 - 3a^2bx^2 - 10ab^2x^4 + 20b^3x^6 + 40b^3x^6(1 - (bx^2)/a)^{3/4})\text{Hypergeometric2F1}[1/4, 3/4, 5/4, (bx^2)/a]) / (77a^3c^7x^6(a - bx^2)^{3/4})$

**Maple [F]** time = 0.06, size = 0, normalized size = 0.

$$\int 1(cx)^{-\frac{13}{2}}(-bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4), x)`

[Out] `int(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{3}{4}}(cx)^{\frac{13}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(13/2)), x, algorithm="maxima")`

[Out] `integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(13/2)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(-bx^2 + a)^{\frac{3}{4}}\sqrt{cxc^6x^6}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(13/2)), x, algorithm="fricas")`

[Out] `integral(1/((-b*x^2 + a)^(3/4)*sqrt(c*x)*c^6*x^6), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(13/2)/(-b*x**2+a)**(3/4), x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{3}{4}}(cx)^{\frac{13}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(13/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(13/2)), x)
```

$$3.980 \quad \int \frac{(cx)^{5/2}}{(a-bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=308

$$\frac{3ac^{5/2} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{8\sqrt{2}b^{7/4}} - \frac{3ac^{5/2} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{8\sqrt{2}b^{7/4}} \\ - \frac{3ac^{5/2} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{7/4}} + \frac{3ac^{5/2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{4\sqrt{2}b^{7/4}} - \frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}$$

[Out]  $-(c*(c*x)^{(3/2)}*(a-b*x^2)^{(1/4)})/(2*b) - (3*a*c^{(5/2)}*ArcTan[1 - (Sqrt[2]*b^{(1/4)}*Sqrt[c*x])/(Sqrt[c]*(a-b*x^2)^{(1/4)})])/(4*Sqrt[2]*b^{(7/4)}) + (3*a*c^{(5/2)}*ArcTan[1 + (Sqrt[2]*b^{(1/4)}*Sqrt[c*x])/(Sqrt[c]*(a-b*x^2)^{(1/4)})])/(4*Sqrt[2]*b^{(7/4)}) + (3*a*c^{(5/2)}*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a-b*x^2] - (Sqrt[2]*b^{(1/4)}*Sqrt[c*x])/(a-b*x^2)^{(1/4)})]/(8*Sqrt[2]*b^{(7/4)}) - (3*a*c^{(5/2)}*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a-b*x^2] + (Sqrt[2]*b^{(1/4)}*Sqrt[c*x])/(a-b*x^2)^{(1/4)})]/(8*Sqrt[2]*b^{(7/4)})$

**Rubi [A]** time = 0.67004, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$

$$\frac{3ac^{5/2} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{8\sqrt{2}b^{7/4}} - \frac{3ac^{5/2} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{8\sqrt{2}b^{7/4}} \\ - \frac{3ac^{5/2} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{7/4}} + \frac{3ac^{5/2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{4\sqrt{2}b^{7/4}} - \frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(5/2)/(a-b\*x^2)^(3/4), x]

[Out]  $-(c*(c*x)^{(3/2)}*(a-b*x^2)^{(1/4)})/(2*b) - (3*a*c^{(5/2)}*ArcTan[1 - (Sqrt[2]*b^{(1/4)}*Sqrt[c*x])/(Sqrt[c]*(a-b*x^2)^{(1/4)})])/(4*Sqrt[2]*b^{(7/4)}) + (3*a*c^{(5/2)}*ArcTan[1 + (Sqrt[2]*b^{(1/4)}*Sqrt[c*x])/(Sqrt[c]*(a-b*x^2)^{(1/4)})])/(4*Sqrt[2]*b^{(7/4)}) + (3*a*c^{(5/2)}*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a-b*x^2] - (Sqrt[2]*b^{(1/4)}*Sqrt[c*x])/(a-b*x^2)^{(1/4)})]/(8*Sqrt[2]*b^{(7/4)}) - (3*a*c^{(5/2)}*Log[Sqrt[c] + (Sqrt[b]*Sqrt[c]*x)/Sqrt[a-b*x^2] + (Sqrt[2]*b^{(1/4)}*Sqrt[c*x])/(a-b*x^2)^{(1/4)})]/(8*Sqrt[2]*b^{(7/4)})$

**Rubi in Sympy [A]** time = 68.0484, size = 282, normalized size = 0.92

$$\frac{3\sqrt{2}ac^{\frac{5}{2}} \log\left(-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \frac{\sqrt{bcx}}{\sqrt{a-bx^2}} + c\right)}{16b^{\frac{7}{4}}} - \frac{3\sqrt{2}ac^{\frac{5}{2}} \log\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \frac{\sqrt{bcx}}{\sqrt{a-bx^2}} + c\right)}{16b^{\frac{7}{4}}} \\ + \frac{3\sqrt{2}ac^{\frac{5}{2}} \operatorname{atan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} - 1\right)}{8b^{\frac{7}{4}}} + \frac{3\sqrt{2}ac^{\frac{5}{2}} \operatorname{atan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{8b^{\frac{7}{4}}} - \frac{c(cx)^{\frac{3}{2}}\sqrt[4]{a-bx^2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(5/2)/(-b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $3*\sqrt{2}*a*c^{(5/2)}*\log(-\sqrt{2}*b^{(1/4)}*\sqrt{c}*\sqrt{c*x})/(a-b*x^{*2})^{(1/4)} + \sqrt{c}*c*x/\sqrt{a-b*x^{*2}} + c)/(16*b^{(7/4)})$

$$- 3\sqrt{2} a c^{5/2} \log(\sqrt{2} b^{1/4} \sqrt{c} \sqrt{c x}) / (a - b x^2)^{1/4} + \sqrt{b} c x / \sqrt{a - b x^2} + c) / (16 b^{7/4}) + 3\sqrt{2} a c^{5/2} \operatorname{atan}(\sqrt{2} b^{1/4} \sqrt{c x}) / (\sqrt{c} (a - b x^2)^{1/4}) - 1) / (8 b^{7/4}) + 3\sqrt{2} a c^{5/2} a \tan(\sqrt{2} b^{1/4} \sqrt{c x}) / (\sqrt{c} (a - b x^2)^{1/4}) + 1) / (8 b^{7/4}) - c (c x)^{3/2} (a - b x^2)^{1/4} / (2 b)$$

**Mathematica [C]** time = 0.0629717, size = 71, normalized size = 0.23

$$\frac{c(c x)^{3/2} \left( a \left( 1 - \frac{b x^2}{a} \right)^{3/4} {}_2F_1 \left( \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b x^2}{a} \right) - a + b x^2 \right)}{2 b (a - b x^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(5/2)/(a - b\*x^2)^(3/4), x]

[Out] (c\*(c\*x)^(3/2)\*(-a + b\*x^2 + a\*(1 - (b\*x^2)/a)^(3/4)\*Hypergeometric2F1[3/4, 3/4, 7/4, (b\*x^2)/a]))/(2\*b\*(a - b\*x^2)^(3/4))

**Maple [F]** time = 0.05, size = 0, normalized size = 0.

$$\int 1 (c x)^{\frac{5}{2}} (-b x^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(5/2)/(-b\*x^2+a)^(3/4), x)

[Out] int((c\*x)^(5/2)/(-b\*x^2+a)^(3/4), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(5/2)/(-b\*x^2 + a)^(3/4), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(5/2)/(-b\*x^2 + a)^(3/4), x, algorithm="fricas")

[Out] Timed out



**Sympy [A]** time = 175.969, size = 46, normalized size = 0.15

$$\frac{c^{\frac{5}{2}} x^{\frac{7}{2}} \left(\frac{7}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{7}{4} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{2a^{\frac{3}{4}} \left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(5/2)/(-b\*x\*\*2+a)\*\*(3/4), x)

[Out] c\*\*(5/2)\*x\*\*(7/2)\*gamma(7/4)\*hyper((3/4, 7/4), (11/4, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/(2\*a\*\*(3/4)\*gamma(11/4))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{(-bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(5/2)/(-b\*x^2 + a)^(3/4), x, algorithm="giac")

[Out] integrate((c\*x)^(5/2)/(-b\*x^2 + a)^(3/4), x)

$$3.981 \quad \int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=272

$$\frac{\sqrt{c} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{2\sqrt{2}b^{3/4}} - \frac{\sqrt{c} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{2\sqrt{2}b^{3/4}} \\ - \frac{\sqrt{c} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}b^{3/4}} + \frac{\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{\sqrt{2}b^{3/4}}$$

[Out] -((Sqrt[c]\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a - b\*x^2)^(1/4))])/(Sqrt[2]\*b^(3/4))) + (Sqrt[c]\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a - b\*x^2)^(1/4))])/(Sqrt[2]\*b^(3/4)) + (Sqrt[c]\*Log[Sqrt[c] + (Sqrt[b]\*Sqrt[c]\*x)/Sqrt[a - b\*x^2] - (Sqrt[2]\*b^(1/4)\*Sqrt[c\*x])/(a - b\*x^2)^(1/4)])/(2\*Sqrt[2]\*b^(3/4)) - (Sqrt[c]\*Log[Sqrt[c] + (Sqrt[b]\*Sqrt[c]\*x)/Sqrt[a - b\*x^2] + (Sqrt[2]\*b^(1/4)\*Sqrt[c\*x])/(a - b\*x^2)^(1/4)])/(2\*Sqrt[2]\*b^(3/4))

**Rubi [A]** time = 0.547199, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{\sqrt{c} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{2\sqrt{2}b^{3/4}} - \frac{\sqrt{c} \log\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}} + \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \sqrt{c}\right)}{2\sqrt{2}b^{3/4}} \\ - \frac{\sqrt{c} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}b^{3/4}} + \frac{\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{\sqrt{2}b^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c\*x]/(a - b\*x^2)^(3/4), x]

[Out] -((Sqrt[c]\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a - b\*x^2)^(1/4))])/(Sqrt[2]\*b^(3/4))) + (Sqrt[c]\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*Sqrt[c\*x])/(Sqrt[c]\*(a - b\*x^2)^(1/4))])/(Sqrt[2]\*b^(3/4)) + (Sqrt[c]\*Log[Sqrt[c] + (Sqrt[b]\*Sqrt[c]\*x)/Sqrt[a - b\*x^2] - (Sqrt[2]\*b^(1/4)\*Sqrt[c\*x])/(a - b\*x^2)^(1/4)])/(2\*Sqrt[2]\*b^(3/4)) - (Sqrt[c]\*Log[Sqrt[c] + (Sqrt[b]\*Sqrt[c]\*x)/Sqrt[a - b\*x^2] + (Sqrt[2]\*b^(1/4)\*Sqrt[c\*x])/(a - b\*x^2)^(1/4)])/(2\*Sqrt[2]\*b^(3/4))

**Rubi in Sympy [A]** time = 59.8517, size = 246, normalized size = 0.9

$$\frac{\sqrt{2}\sqrt{c} \log\left(-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \frac{\sqrt{bcx}}{\sqrt{a-bx^2}} + c\right)}{4b^{3/4}} - \frac{\sqrt{2}\sqrt{c} \log\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}} + \frac{\sqrt{bcx}}{\sqrt{a-bx^2}} + c\right)}{4b^{3/4}} \\ + \frac{\sqrt{2}\sqrt{c} \operatorname{atan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} - 1\right)}{2b^{3/4}} + \frac{\sqrt{2}\sqrt{c} \operatorname{atan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}} + 1\right)}{2b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(1/2)/(-b\*x\*\*2+a)\*\*(3/4), x)

[Out] sqrt(2)\*sqrt(c)\*log(-sqrt(2)\*b\*\*(1/4)\*sqrt(c)\*sqrt(c\*x)/(a - b\*x\*\*2)\*\*(1/4) + sqrt(b)\*c\*x/sqrt(a - b\*x\*\*2) + c)/(4\*b\*\*(3/4)) - sqrt

$$t(2) \cdot \sqrt{c} \cdot \log(\sqrt{2} \cdot b^{1/4} \cdot \sqrt{c} \cdot \sqrt{c \cdot x} / (a - b \cdot x^2)^{1/4} + \sqrt{b} \cdot c \cdot x / \sqrt{a - b \cdot x^2} + c) / (4 \cdot b^{3/4}) + \sqrt{2} \cdot \sqrt{c} \cdot \operatorname{atan}(\sqrt{2} \cdot b^{1/4} \cdot \sqrt{c \cdot x} / (\sqrt{c} \cdot (a - b \cdot x^2)^{1/4}) - 1) / (2 \cdot b^{3/4}) + \sqrt{2} \cdot \sqrt{c} \cdot \operatorname{atan}(\sqrt{2} \cdot b^{1/4} \cdot \sqrt{c \cdot x} / (\sqrt{c} \cdot (a - b \cdot x^2)^{1/4}) + 1) / (2 \cdot b^{3/4})$$

**Mathematica [C]** time = 0.0333451, size = 58, normalized size = 0.21

$$\frac{2x\sqrt{cx} \left(\frac{a-bx^2}{a}\right)^{3/4} {}_2F_1\left(\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{bx^2}{a}\right)}{3(a-bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c\*x]/(a - b\*x^2)^(3/4), x]

[Out] (2\*x\*Sqrt[c\*x]\*((a - b\*x^2)/a)^(3/4)\*Hypergeometric2F1[3/4, 3/4, 7/4, (b\*x^2)/a])/(3\*(a - b\*x^2)^(3/4))

**Maple [F]** time = 0.047, size = 0, normalized size = 0.

$$\int 1\sqrt{cx} (-bx^2 + a)^{-\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(1/2)/(-b\*x^2+a)^(3/4), x)

[Out] int((c\*x)^(1/2)/(-b\*x^2+a)^(3/4), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x)/(-b\*x^2 + a)^(3/4), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x)/(-b\*x^2 + a)^(3/4), x, algorithm="fricas")

[Out] Timed out

**Sympy [A]** time = 6.87762, size = 46, normalized size = 0.17

$$\frac{\sqrt{cx}^{\frac{3}{2}} \left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{3}{4} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{2a^{\frac{3}{4}} \left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(1/2)/(-b\*x\*\*2+a)\*\*(3/4),x)

[Out] sqrt(c)\*x\*\*(3/2)\*gamma(3/4)\*hyper((3/4, 3/4), (7/4, ), b\*x\*\*2\*exp\_polar(2\*I\*pi)/a)/(2\*a\*\*(3/4)\*gamma(7/4))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{(-bx^2 + a)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x)/(-b\*x^2 + a)^(3/4),x, algorithm="giac")

[Out] integrate(sqrt(c\*x)/(-b\*x^2 + a)^(3/4), x)

$$3.982 \quad \int \frac{1}{(cx)^{3/2}(a-bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=27

$$-\frac{2\sqrt[4]{a-bx^2}}{ac\sqrt{cx}}$$

[Out]  $(-2*(a - b*x^2)^{(1/4)})/(a*c*\text{Sqrt}[c*x])$

**Rubi [A]** time = 0.0296647, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$-\frac{2\sqrt[4]{a-bx^2}}{ac\sqrt{cx}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/((c*x)^{(3/2)}*(a - b*x^2)^{(3/4)}), x]$

[Out]  $(-2*(a - b*x^2)^{(1/4)})/(a*c*\text{Sqrt}[c*x])$

**Rubi in Sympy [A]** time = 3.93105, size = 22, normalized size = 0.81

$$-\frac{2\sqrt[4]{a-bx^2}}{ac\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/(c*x)^{(3/2)}/(-b*x^2+a)^{(3/4)}, x)$

[Out]  $-2*(a - b*x^2)^{(1/4)}/(a*c*\text{sqrt}(c*x))$

**Mathematica [A]** time = 0.0194153, size = 25, normalized size = 0.93

$$-\frac{2x\sqrt[4]{a-bx^2}}{a(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/((c*x)^{(3/2)}*(a - b*x^2)^{(3/4)}), x]$

[Out]  $(-2*x*(a - b*x^2)^{(1/4)})/(a*(c*x)^{(3/2)})$

**Maple [A]** time = 0.006, size = 22, normalized size = 0.8

$$-2 \frac{x\sqrt[4]{-bx^2+a}}{a(cx)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/(c*x)^{(3/2)}/(-b*x^2+a)^{(3/4)}, x)$

[Out]  $-2*x*(-b*x^2+a)^{(1/4)}/a/(c*x)^{(3/2)}$

**Maxima [A]** time = 1.39217, size = 28, normalized size = 1.04

$$\frac{2(-bx^2 + a)^{\frac{1}{4}}}{ac^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(3/2)),x, algorithm="maxima")`

[Out]  $-2*(-b*x^2 + a)^{(1/4)}/(a*c^{(3/2)}*\text{sqrt}(x))$

**Fricas [A]** time = 0.229225, size = 35, normalized size = 1.3

$$-\frac{2(-bx^2 + a)^{\frac{1}{4}}\sqrt{cx}}{ac^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(3/2)),x, algorithm="fricas")`

[Out]  $-2*(-b*x^2 + a)^{(1/4)}*\text{sqrt}(c*x)/(a*c^2*x)$

**Sympy [A]** time = 28.4942, size = 90, normalized size = 3.33

$$\begin{cases} \frac{\sqrt[4]{b}^4 \sqrt{\frac{a}{bx^2} - 1}^{(-\frac{1}{4})}}{2ac^{\frac{3}{2}}(\frac{3}{4})} & \text{for } \left| \frac{a}{bx^2} \right| > 1 \\ -\frac{\sqrt[4]{b}^4 \sqrt{-\frac{a}{bx^2} + 1} e^{\frac{5i\pi}{4}(-\frac{1}{4})}}{2ac^{\frac{3}{2}}(\frac{3}{4})} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(3/2)/(-b*x**2+a)**(3/4),x)`

[Out] `Piecewise((b**(1/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-1/4)/(2*a*c**(3/2)*gamma(3/4)), Abs(a/(b*x**2)) > 1), (-b**(1/4)*(-a/(b*x**2) + 1)**(1/4)*exp(5*I*pi/4)*gamma(-1/4)/(2*a*c**(3/2)*gamma(3/4)), True)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{3}{4}}(cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(3/2)),x, algorithm="giac")`

[Out] `integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)`

$$3.983 \quad \int \frac{1}{(cx)^{7/2}(a-bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=57

$$\frac{8(a-bx^2)^{5/4}}{5a^2c(cx)^{5/2}} - \frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{5/2}}$$

[Out]  $(-2*(a - b*x^2)^{(1/4)})/(a*c*(c*x)^{(5/2)}) + (8*(a - b*x^2)^{(5/4)})/(5*a^2*c*(c*x)^{(5/2)})$

**Rubi [A]** time = 0.0599123, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{8(a-bx^2)^{5/4}}{5a^2c(cx)^{5/2}} - \frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(7/2)\*(a - b\*x^2)^(3/4)), x]

[Out]  $(-2*(a - b*x^2)^{(1/4)})/(a*c*(c*x)^{(5/2)}) + (8*(a - b*x^2)^{(5/4)})/(5*a^2*c*(c*x)^{(5/2)})$

**Rubi in Sympy [A]** time = 7.67457, size = 46, normalized size = 0.81

$$-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{\frac{5}{2}}} + \frac{8(a-bx^2)^{\frac{5}{4}}}{5a^2c(cx)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(7/2)/(-b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $-2*(a - b*x**2)**(1/4)/(a*c*(c*x)**(5/2)) + 8*(a - b*x**2)**(5/4)/(5*a**2*c*(c*x)**(5/2))$

**Mathematica [A]** time = 0.0352682, size = 35, normalized size = 0.61

$$-\frac{2x\sqrt[4]{a-bx^2}(a+4bx^2)}{5a^2(cx)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(7/2)\*(a - b\*x^2)^(3/4)), x]

[Out]  $(-2*x*(a - b*x^2)^{(1/4)*(a + 4*b*x^2)})/(5*a^2*(c*x)^{(7/2)})$

**Maple [A]** time = 0.007, size = 30, normalized size = 0.5

$$-\frac{2x(4bx^2+a)\sqrt[4]{-bx^2+a}(cx)^{-\frac{7}{2}}}{5a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x)`

[Out]  $-2/5*x*(-b*x^2+a)^{(1/4)}*(4*b*x^2+a)/a^2/(c*x)^{(7/2)}$

**Maxima [A]** time = 1.40225, size = 53, normalized size = 0.93

$$-\frac{2\left(\frac{5(-bx^2+a)^{\frac{1}{4}}b}{\sqrt{x}} + \frac{(-bx^2+a)^{\frac{5}{4}}}{x^{\frac{5}{2}}}\right)}{5a^2c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(7/2)),x, algorithm="maxima")`

[Out]  $-2/5*(5*(-b*x^2 + a)^{(1/4)}*b/\text{sqrt}(x) + (-b*x^2 + a)^{(5/4)}/x^{(5/2)})/(a^2*c^{(7/2)})$

**Fricas [A]** time = 0.226388, size = 46, normalized size = 0.81

$$-\frac{2(4bx^2 + a)(-bx^2 + a)^{\frac{1}{4}}\sqrt{cx}}{5a^2c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(7/2)),x, algorithm="fricas")`

[Out]  $-2/5*(4*b*x^2 + a)*(-b*x^2 + a)^{(1/4)}*\text{sqrt}(c*x)/(a^2*c^4*x^3)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(7/2)/(-b*x**2+a)**(3/4),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{3}{4}}(cx)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(7/2)),x, algorithm="giac")`

[Out] `integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)`



$$3.984 \quad \int \frac{1}{(cx)^{11/2}(a-bx^2)^{3/4}} dx$$

**Optimal.** Leaf size=86

$$-\frac{64(a-bx^2)^{9/4}}{45a^3c(cx)^{9/2}} + \frac{16(a-bx^2)^{5/4}}{5a^2c(cx)^{9/2}} - \frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}}$$

[Out]  $(-2*(a - b*x^2)^{(1/4)})/(a*c*(c*x)^{(9/2)}) + (16*(a - b*x^2)^{(5/4)})/(5*a^2*c*(c*x)^{(9/2)}) - (64*(a - b*x^2)^{(9/4)})/(45*a^3*c*(c*x)^{(9/2)})$

**Rubi [A]** time = 0.094076, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{64(a-bx^2)^{9/4}}{45a^3c(cx)^{9/2}} + \frac{16(a-bx^2)^{5/4}}{5a^2c(cx)^{9/2}} - \frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(11/2)\*(a - b\*x^2)^(3/4)), x]

[Out]  $(-2*(a - b*x^2)^{(1/4)})/(a*c*(c*x)^{(9/2)}) + (16*(a - b*x^2)^{(5/4)})/(5*a^2*c*(c*x)^{(9/2)}) - (64*(a - b*x^2)^{(9/4)})/(45*a^3*c*(c*x)^{(9/2)})$

**Rubi in Sympy [A]** time = 11.7116, size = 71, normalized size = 0.83

$$-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}} + \frac{16(a-bx^2)^{5/4}}{5a^2c(cx)^{9/2}} - \frac{64(a-bx^2)^{9/4}}{45a^3c(cx)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(11/2)/(-b\*x\*\*2+a)\*\*(3/4), x)

[Out]  $-2*(a - b*x**2)**(1/4)/(a*c*(c*x)**(9/2)) + 16*(a - b*x**2)**(5/4)/(5*a**2*c*(c*x)**(9/2)) - 64*(a - b*x**2)**(9/4)/(45*a**3*c*(c*x)**(9/2))$

**Mathematica [A]** time = 0.0429216, size = 53, normalized size = 0.62

$$-\frac{2\sqrt{cx}\sqrt[4]{a-bx^2}(5a^2+8abx^2+32b^2x^4)}{45a^3c^6x^5}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(11/2)\*(a - b\*x^2)^(3/4)), x]

[Out]  $(-2*\text{Sqrt}[c*x]*(a - b*x^2)^{(1/4)}*(5*a^2 + 8*a*b*x^2 + 32*b^2*x^4))/(45*a^3*c^6*x^5)$

**Maple [A]** time = 0.007, size = 43, normalized size = 0.5

$$-\frac{2x(32b^2x^4+8abx^2+5a^2)\sqrt[4]{-bx^2+a}(cx)^{-\frac{11}{2}}}{45a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x)`

[Out]  $-2/45*x*(-b*x^2+a)^{1/4}*(32*b^2*x^4+8*a*b*x^2+5*a^2)/a^3/(c*x)^{11/2}$

**Maxima [A]** time = 1.4063, size = 78, normalized size = 0.91

$$\frac{2 \left( \frac{45(-bx^2+a)^{\frac{1}{4}}b^2}{\sqrt{x}} + \frac{18(-bx^2+a)^{\frac{5}{4}}b}{x^{\frac{5}{2}}} + \frac{5(-bx^2+a)^{\frac{9}{4}}}{x^{\frac{9}{2}}} \right)}{45a^3c^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(11/2)),x, algorithm="maxima")`

[Out]  $-2/45*(45*(-b*x^2 + a)^{1/4}*b^2/\text{sqrt}(x) + 18*(-b*x^2 + a)^{5/4}*b/x^{5/2} + 5*(-b*x^2 + a)^{9/4}/x^{9/2})/(a^3*c^{11/2})$

**Fricas [A]** time = 0.228456, size = 63, normalized size = 0.73

$$\frac{2(32b^2x^4 + 8abx^2 + 5a^2)(-bx^2 + a)^{\frac{1}{4}}\sqrt{cx}}{45a^3c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(11/2)),x, algorithm="fricas")`

[Out]  $-2/45*(32*b^2*x^4 + 8*a*b*x^2 + 5*a^2)*(-b*x^2 + a)^{1/4}*sqrt(c*x)/(a^3*c^6*x^5)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(11/2)/(-b*x**2+a)**(3/4),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2 + a)^{\frac{3}{4}}(cx)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(11/2)),x, algorithm="giac")`

[Out] `integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)`

$$3.985 \quad \int \frac{(cx)^{7/2}}{(a+bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=146

$$-\frac{5ac^{7/2} \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}} - \frac{5ac^{7/2} \tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}} + \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}}$$

[Out]  $(5*a*c^3*\text{Sqrt}[c*x])/(2*b^2*(a + b*x^2)^{(1/4)}) + (c*(c*x)^{(5/2)})/(2*b*(a + b*x^2)^{(1/4)}) - (5*a*c^{(7/2)}*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(\text{Sqrt}[c]*(a + b*x^2)^{(1/4)})])/(4*b^{(9/4)}) - (5*a*c^{(7/2)}*\text{ArcTanh}[(b^{(1/4)}*\text{Sqrt}[c*x])/(\text{Sqrt}[c]*(a + b*x^2)^{(1/4)})])/(4*b^{(9/4)})$

**Rubi [A]** time = 0.210285, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$-\frac{5ac^{7/2} \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}} - \frac{5ac^{7/2} \tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}} + \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(7/2)/(a + b\*x^2)^(5/4), x]

[Out]  $(5*a*c^3*\text{Sqrt}[c*x])/(2*b^2*(a + b*x^2)^{(1/4)}) + (c*(c*x)^{(5/2)})/(2*b*(a + b*x^2)^{(1/4)}) - (5*a*c^{(7/2)}*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])/(\text{Sqrt}[c]*(a + b*x^2)^{(1/4)})])/(4*b^{(9/4)}) - (5*a*c^{(7/2)}*\text{ArcTanh}[(b^{(1/4)}*\text{Sqrt}[c*x])/(\text{Sqrt}[c]*(a + b*x^2)^{(1/4)})])/(4*b^{(9/4)})$

**Rubi in Sympy [A]** time = 28.723, size = 138, normalized size = 0.95

$$\frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} - \frac{5ac^{7/2} \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}} - \frac{5ac^{7/2} \operatorname{atanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(7/2)/(b\*x\*\*2+a)\*\*(5/4), x)

[Out]  $5*a*c^3*\text{sqrt}(c*x)/(2*b^2*(a + b*x^2)^{(1/4)}) - 5*a*c^{(7/2)}*\text{atan}(b^{(1/4)}*\text{sqrt}(c*x)/(\text{sqrt}(c)*(a + b*x^2)^{(1/4)}))/(4*b^{(9/4)}) - 5*a*c^{(7/2)}*\text{atanh}(b^{(1/4)}*\text{sqrt}(c*x)/(\text{sqrt}(c)*(a + b*x^2)^{(1/4)}))/(4*b^{(9/4)}) + c*(c*x)^{(5/2)}/(2*b*(a + b*x^2)^{(1/4)})$

**Mathematica [C]** time = 0.0687922, size = 73, normalized size = 0.5

$$\frac{c^3\sqrt{cx} \left( -5a\sqrt{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{4}; \frac{5}{4}; -\frac{bx^2}{a}\right) + 5a + bx^2 \right)}{2b^2\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(7/2)/(a + b\*x^2)^(5/4), x]

[Out]  $(c^3 \sqrt{c x} (5 a + b x^2 - 5 a (1 + (b x^2)/a)^{1/4}) \text{Hypergeometric2F1}[1/4, 1/4, 5/4, -((b x^2)/a)]) / (2 b^2 (a + b x^2)^{1/4})$

**Maple [F]** time = 0.087, size = 0, normalized size = 0.

$$\int 1 (c x)^{7/2} (b x^2 + a)^{-5/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(7/2)/(b*x^2+a)^(5/4),x)`

[Out] `int((c*x)^(7/2)/(b*x^2+a)^(5/4),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(7/2)/(b*x^2 + a)^(5/4),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.261313, size = 486, normalized size = 3.33

$$4 (b c^3 x^2 + 5 a c^3) (b x^2 + a)^{3/4} \sqrt{c x} + 20 \left( \frac{a^4 c^{14}}{b^9} \right)^{1/4} (b^3 x^2 + a b^2) \arctan \left( \frac{\left( \frac{a^4 c^{14}}{b^9} \right)^{1/4} (b^3 x^2 + a b^2)}{(b x^2 + a)^{3/4} \sqrt{c x} a c^3 + (b x^2 + a) \sqrt{\frac{a^4 c^{14}}{b^9} (b^5 x^2 + a b^4)}} \right) - 5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(7/2)/(b*x^2 + a)^(5/4),x, algorithm="fricas")`

[Out]  $1/8 * (4 * (b^3 c^3 x^2 + 5 a^2 c^3) * (b x^2 + a)^{3/4} * \sqrt{c x} + 20 * (a^4 c^{14}/b^9)^{1/4} * (b^3 x^2 + a b^2) * \arctan((a^4 c^{14}/b^9)^{1/4} * (b^3 x^2 + a b^2) / ((b x^2 + a)^{3/4} * \sqrt{c x} * a c^3 + (b x^2 + a) * \sqrt{(a^4 c^{14}/b^9) * (b^5 x^2 + a b^4)})) - 5 * (a^4 c^{14}/b^9)^{1/4} * (b^3 x^2 + a b^2) * \log(5 * ((b x^2 + a)^{3/4} * \sqrt{c x} * a c^3 + (a^4 c^{14}/b^9)^{1/4} * (b^3 x^2 + a b^2)) / (b x^2 + a)) + 5 * (a^4 c^{14}/b^9)^{1/4} * (b^3 x^2 + a b^2) * \log(5 * ((b x^2 + a)^{3/4} * \sqrt{c x} * a c^3 - (a^4 c^{14}/b^9)^{1/4} * (b^3 x^2 + a b^2)) / (b x^2 + a)) / (b^3 x^2 + a b^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(7/2)/(b*x**2+a)**(5/4),x)`

[Out] Timed out

---

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{7}{2}}}{(bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(7/2)/(b\*x^2 + a)^(5/4), x, algorithm="giac")

[Out] integrate((c\*x)^(7/2)/(b\*x^2 + a)^(5/4), x)

$$3.986 \quad \int \frac{(cx)^{3/2}}{(a+bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=107

$$\frac{c^{3/2} \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}} + \frac{c^{3/2} \tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}} - \frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}$$

[Out]  $(-2*c*\text{Sqrt}[c*x])/(b*(a + b*x^2)^{(1/4)}) + (c^{(3/2)}*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])]/(\text{Sqrt}[c]*(a + b*x^2)^{(1/4)}))/b^{(5/4)} + (c^{(3/2)}*\text{ArcTanh}[(b^{(1/4)}*\text{Sqrt}[c*x])]/(\text{Sqrt}[c]*(a + b*x^2)^{(1/4)}))/b^{(5/4)}$

**Rubi [A]** time = 0.155266, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$\frac{c^{3/2} \tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}} + \frac{c^{3/2} \tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}} - \frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(3/2)/(a + b\*x^2)^(5/4), x]

[Out]  $(-2*c*\text{Sqrt}[c*x])/(b*(a + b*x^2)^{(1/4)}) + (c^{(3/2)}*\text{ArcTan}[(b^{(1/4)}*\text{Sqrt}[c*x])]/(\text{Sqrt}[c]*(a + b*x^2)^{(1/4)}))/b^{(5/4)} + (c^{(3/2)}*\text{ArcTanh}[(b^{(1/4)}*\text{Sqrt}[c*x])]/(\text{Sqrt}[c]*(a + b*x^2)^{(1/4)}))/b^{(5/4)}$

**Rubi in Sympy [A]** time = 22.3747, size = 99, normalized size = 0.93

$$-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}} + \frac{c^{3/2} \operatorname{atan}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}} + \frac{c^{3/2} \operatorname{atanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(3/2)/(b\*x\*\*2+a)\*\*(5/4), x)

[Out]  $-2*c*\text{sqrt}(c*x)/(b*(a + b*x**2)**(1/4)) + c**(3/2)*\text{atan}(b**(1/4)*\text{sqrt}(c*x)/(\text{sqrt}(c)*(a + b*x**2)**(1/4)))/b**(5/4) + c**(3/2)*\text{atanh}(b**(1/4)*\text{sqrt}(c*x)/(\text{sqrt}(c)*(a + b*x**2)**(1/4)))/b**(5/4)$

**Mathematica [C]** time = 0.0528439, size = 60, normalized size = 0.56

$$\frac{2c\sqrt{cx} \left( \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{1}{4}; \frac{5}{4}; -\frac{bx^2}{a}\right) - 1 \right)}{b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(3/2)/(a + b\*x^2)^(5/4), x]

[Out]  $(2*c*\text{Sqrt}[c*x]*(-1 + (1 + (b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/4, 1/4, 5/4, -(b*x^2)/a]))/(b*(a + b*x^2)^{(1/4)})$

**Maple [F]** time = 0.04, size = 0, normalized size = 0.

$$\int 1 (cx)^{\frac{3}{2}} (bx^2 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(3/2)/(b*x^2+a)^(5/4), x)`

[Out] `int((c*x)^(3/2)/(b*x^2+a)^(5/4), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(3/2)/(b*x^2 + a)^(5/4), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.232755, size = 400, normalized size = 3.74

$$\frac{4 (bx^2 + a)^{\frac{3}{4}} \sqrt{cx} + 4 (b^2x^2 + ab) \left(\frac{c^6}{b^5}\right)^{\frac{1}{4}} \arctan\left(\frac{(b^2x^2+ab)\left(\frac{c^6}{b^5}\right)^{\frac{1}{4}}}{(bx^2+a)^{\frac{3}{4}}\sqrt{cx}+(bx^2+a)\sqrt{\frac{\sqrt{bx^2+ac^3x+(b^3x^2+ab^2)}\sqrt{\frac{c^6}{b^5}}}{bx^2+a}}}\right) - (b^2x^2 + ab) \left(\frac{c^6}{b^5}\right)^{\frac{1}{4}} \log\left(\frac{\dots}{2(b^2x^2 + ab)}\right)}{2(b^2x^2 + ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(3/2)/(b*x^2 + a)^(5/4), x, algorithm="fricas")`

[Out] `-1/2*(4*(b*x^2 + a)^(3/4)*sqrt(c*x)*c + 4*(b^2*x^2 + a*b)*(c^6/b^5)^(1/4)*arctan((b^2*x^2 + a*b)*(c^6/b^5)^(1/4)/((b*x^2 + a)^(3/4)*sqrt(c*x)*c + (b*x^2 + a)*sqrt((sqrt(b*x^2 + a)*c^3*x + (b^3*x^2 + a*b^2)*sqrt(c^6/b^5))/(b*x^2 + a)))) - (b^2*x^2 + a*b)*(c^6/b^5)^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(c*x)*c + (b^2*x^2 + a*b)*(c^6/b^5)^(1/4))/(b*x^2 + a)) + (b^2*x^2 + a*b)*(c^6/b^5)^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(c*x)*c - (b^2*x^2 + a*b)*(c^6/b^5)^(1/4))/(b*x^2 + a)))/(b^2*x^2 + a*b)`

**Sympy [A]** time = 60.1419, size = 44, normalized size = 0.41

$$\frac{c^{\frac{3}{2}} x^{\frac{5}{2}} \left(\frac{5}{4}\right) {}_2F_1\left(\frac{5}{4}, \frac{5}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}} \left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(3/2)/(b*x**2+a)**(5/4), x)`

[Out] `c**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 5/4), (9/4, ), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/4)*gamma(9/4))`

---

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}}}{(bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(3/2)/(b\*x^2 + a)^(5/4), x, algorithm="giac")

[Out] integrate((c\*x)^(3/2)/(b\*x^2 + a)^(5/4), x)



$$3.987 \quad \int \frac{1}{\sqrt{cx}(a+bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=26

$$\frac{2\sqrt{cx}}{ac\sqrt[4]{a+bx^2}}$$

[Out] (2\*Sqrt[c\*x])/(a\*c\*(a + b\*x^2)^(1/4))

**Rubi [A]** time = 0.0282011, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{2\sqrt{cx}}{ac\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[c\*x]\*(a + b\*x^2)^(5/4)), x]

[Out] (2\*Sqrt[c\*x])/(a\*c\*(a + b\*x^2)^(1/4))

**Rubi in Sympy [A]** time = 3.70404, size = 20, normalized size = 0.77

$$\frac{2\sqrt{cx}}{ac\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(1/2)/(b\*x\*\*2+a)\*\*(5/4), x)

[Out] 2\*sqrt(c\*x)/(a\*c\*(a + b\*x\*\*2)\*\*(1/4))

**Mathematica [A]** time = 0.0180883, size = 24, normalized size = 0.92

$$\frac{2x}{a\sqrt{cx}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[c\*x]\*(a + b\*x^2)^(5/4)), x]

[Out] (2\*x)/(a\*Sqrt[c\*x]\*(a + b\*x^2)^(1/4))

**Maple [A]** time = 0.006, size = 21, normalized size = 0.8

$$2 \frac{x}{\sqrt[4]{bx^2 + aa}\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x)^(1/2)/(b\*x^2+a)^(5/4), x)

[Out]  $2*x/(b*x^2+a)^{(1/4)}/a/(c*x)^{(1/2)}$

**Maxima [A]** time = 1.39872, size = 27, normalized size = 1.04

$$\frac{2\sqrt{x}}{(bx^2 + a)^{\frac{1}{4}}a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/4)*sqrt(c*x)),x, algorithm="maxima")`

[Out]  $2*\sqrt{x}/((b*x^2 + a)^{(1/4)}*a*\sqrt{c})$

**Fricas [A]** time = 0.209055, size = 42, normalized size = 1.62

$$\frac{2(bx^2 + a)^{\frac{3}{4}}\sqrt{cx}}{abcx^2 + a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/4)*sqrt(c*x)),x, algorithm="fricas")`

[Out]  $2*(b*x^2 + a)^{(3/4)}*\sqrt{c*x}/(a*b*c*x^2 + a^2*c)$

**Sympy [A]** time = 28.2004, size = 34, normalized size = 1.31

$$\frac{\left(\frac{1}{4}\right)}{2a\sqrt[4]{b}\sqrt[4]{c}\sqrt[4]{\frac{a}{bx^2} + 1}\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(1/2)/(b*x**2+a)**(5/4),x)`

[Out]  $\text{gamma}(1/4)/(2*a*b^{(1/4)}*\sqrt{c}*(a/(b*x**2) + 1)^{(1/4)}*\text{gamma}(5/4))$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}}\sqrt{cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/4)*sqrt(c*x)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(5/4)*sqrt(c*x)), x)`

$$3.988 \quad \int \frac{1}{(cx)^{5/2}(a+bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=55

$$\frac{2}{ac(cx)^{3/2}\sqrt[4]{a+bx^2}} - \frac{8(a+bx^2)^{3/4}}{3a^2c(cx)^{3/2}}$$

[Out]  $2/(a*c*(c*x)^{(3/2)*(a+b*x^2)^{(1/4)}) - (8*(a+b*x^2)^{(3/4)})/(3*a^2*c*(c*x)^{(3/2)})$

**Rubi [A]** time = 0.0569096, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{2}{ac(cx)^{3/2}\sqrt[4]{a+bx^2}} - \frac{8(a+bx^2)^{3/4}}{3a^2c(cx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(5/2)\*(a+b\*x^2)^(5/4)),x]

[Out]  $2/(a*c*(c*x)^{(3/2)*(a+b*x^2)^{(1/4)}) - (8*(a+b*x^2)^{(3/4)})/(3*a^2*c*(c*x)^{(3/2)})$

**Rubi in Sympy [A]** time = 6.76421, size = 46, normalized size = 0.84

$$\frac{2}{ac(cx)^{\frac{3}{2}}\sqrt[4]{a+bx^2}} - \frac{8(a+bx^2)^{\frac{3}{4}}}{3a^2c(cx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(5/2)/(b\*x\*\*2+a)\*\*(5/4),x)

[Out]  $2/(a*c*(c*x)**(3/2)*(a+b*x**2)**(1/4)) - 8*(a+b*x**2)**(3/4)/(3*a**2*c*(c*x)**(3/2))$

**Mathematica [A]** time = 0.0355492, size = 34, normalized size = 0.62

$$-\frac{2x(a+4bx^2)}{3a^2(cx)^{5/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(5/2)\*(a+b\*x^2)^(5/4)),x]

[Out]  $(-2*x*(a+4*b*x^2))/(3*a^2*(c*x)^(5/2)*(a+b*x^2)^(1/4))$

**Maple [A]** time = 0.008, size = 29, normalized size = 0.5

$$-\frac{2x(4bx^2+a)}{3a^2}(cx)^{-\frac{5}{2}}\frac{1}{\sqrt[4]{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x)`

[Out] `-2/3*x*(4*b*x^2+a)/(b*x^2+a)^(1/4)/a^2/(c*x)^(5/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}} (cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(5/2)),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(5/2)), x)`

**Fricas [A]** time = 0.210495, size = 45, normalized size = 0.82

$$-\frac{2(4bx^2 + a)}{3(bx^2 + a)^{\frac{1}{4}}\sqrt{cxa^2c^2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(5/2)),x, algorithm="fricas")`

[Out] `-2/3*(4*b*x^2 + a)/((b*x^2 + a)^(1/4)*sqrt(c*x)*a^2*c^2*x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(5/2)/(b*x**2+a)**(5/4),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}} (cx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(5/2)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(5/2)), x)`

$$3.989 \quad \int \frac{1}{(cx)^{9/2}(a+bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=83

$$\frac{64(a+bx^2)^{7/4}}{21a^3c(cx)^{7/2}} - \frac{16(a+bx^2)^{3/4}}{3a^2c(cx)^{7/2}} + \frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}}$$

[Out] 2/(a\*c\*(c\*x)^(7/2)\*(a+b\*x^2)^(1/4)) - (16\*(a+b\*x^2)^(3/4))/(3\*a^2\*c\*(c\*x)^(7/2)) + (64\*(a+b\*x^2)^(7/4))/(21\*a^3\*c\*(c\*x)^(7/2))

**Rubi [A]** time = 0.0886737, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{64(a+bx^2)^{7/4}}{21a^3c(cx)^{7/2}} - \frac{16(a+bx^2)^{3/4}}{3a^2c(cx)^{7/2}} + \frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(9/2)\*(a+b\*x^2)^(5/4)),x]

[Out] 2/(a\*c\*(c\*x)^(7/2)\*(a+b\*x^2)^(1/4)) - (16\*(a+b\*x^2)^(3/4))/(3\*a^2\*c\*(c\*x)^(7/2)) + (64\*(a+b\*x^2)^(7/4))/(21\*a^3\*c\*(c\*x)^(7/2))

**Rubi in Sympy [A]** time = 10.8446, size = 71, normalized size = 0.86

$$\frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}} - \frac{16(a+bx^2)^{3/4}}{3a^2c(cx)^{7/2}} + \frac{64(a+bx^2)^{7/4}}{21a^3c(cx)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(9/2)/(b\*x\*\*2+a)\*\*(5/4),x)

[Out] 2/(a\*c\*(c\*x)\*\*(7/2)\*(a+b\*x\*\*2)\*\*(1/4)) - 16\*(a+b\*x\*\*2)\*\*(3/4)/(3\*a\*\*2\*c\*(c\*x)\*\*(7/2)) + 64\*(a+b\*x\*\*2)\*\*(7/4)/(21\*a\*\*3\*c\*(c\*x)\*\*(7/2))

**Mathematica [A]** time = 0.0490694, size = 52, normalized size = 0.63

$$\frac{2\sqrt{cx}(-3a^2+8abx^2+32b^2x^4)}{21a^3c^5x^4\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(9/2)\*(a+b\*x^2)^(5/4)),x]

[Out] (2\*Sqrt[c\*x]\*(-3\*a^2+8\*a\*b\*x^2+32\*b^2\*x^4))/(21\*a^3\*c^5\*x^4\*(a+b\*x^2)^(1/4))

**Maple [A]** time = 0.008, size = 42, normalized size = 0.5

$$-\frac{2x(-32b^2x^4-8abx^2+3a^2)}{21a^3}\frac{1}{\sqrt[4]{bx^2+a}}(cx)^{-9/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x)`

[Out]  $-2/21*x*(-32*b^2*x^4-8*a*b*x^2+3*a^2)/(b*x^2+a)^(1/4)/a^3/(c*x)^(9/2)$

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}} (cx)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(9/2)),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(9/2)), x)`

**Fricas** [A] time = 0.211325, size = 62, normalized size = 0.75

$$\frac{2(32b^2x^4 + 8abx^2 - 3a^2)}{21(bx^2 + a)^{\frac{1}{4}}\sqrt{c}xa^3c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(9/2)),x, algorithm="fricas")`

[Out]  $2/21*(32*b^2*x^4 + 8*a*b*x^2 - 3*a^2)/((b*x^2 + a)^(1/4)*\sqrt{c*x})^3$

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(9/2)/(b*x**2+a)**(5/4),x)`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}} (cx)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(9/2)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(9/2)), x)`

$$3.990 \quad \int \frac{1}{(cx)^{13/2}(a+bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=109

$$-\frac{256(a+bx^2)^{11/4}}{77a^4c(cx)^{11/2}} + \frac{64(a+bx^2)^{7/4}}{7a^3c(cx)^{11/2}} - \frac{8(a+bx^2)^{3/4}}{a^2c(cx)^{11/2}} + \frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}}$$

[Out]  $2/(a*c*(c*x)^{(11/2)}*(a+b*x^2)^{(1/4)}) - (8*(a+b*x^2)^{(3/4)})/(a^2*c*(c*x)^{(11/2)}) + (64*(a+b*x^2)^{(7/4)})/(7*a^3*c*(c*x)^{(11/2)}) - (256*(a+b*x^2)^{(11/4)})/(77*a^4*c*(c*x)^{(11/2)})$

**Rubi [A]** time = 0.126085, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$-\frac{256(a+bx^2)^{11/4}}{77a^4c(cx)^{11/2}} + \frac{64(a+bx^2)^{7/4}}{7a^3c(cx)^{11/2}} - \frac{8(a+bx^2)^{3/4}}{a^2c(cx)^{11/2}} + \frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(13/2)\*(a+b\*x^2)^(5/4)),x]

[Out]  $2/(a*c*(c*x)^{(11/2)}*(a+b*x^2)^{(1/4)}) - (8*(a+b*x^2)^{(3/4)})/(a^2*c*(c*x)^{(11/2)}) + (64*(a+b*x^2)^{(7/4)})/(7*a^3*c*(c*x)^{(11/2)}) - (256*(a+b*x^2)^{(11/4)})/(77*a^4*c*(c*x)^{(11/2)})$

**Rubi in Sympy [A]** time = 15.7196, size = 95, normalized size = 0.87

$$\frac{2}{ac(cx)^{\frac{11}{2}}\sqrt[4]{a+bx^2}} - \frac{8(a+bx^2)^{\frac{3}{4}}}{a^2c(cx)^{\frac{11}{2}}} + \frac{64(a+bx^2)^{\frac{7}{4}}}{7a^3c(cx)^{\frac{11}{2}}} - \frac{256(a+bx^2)^{\frac{11}{4}}}{77a^4c(cx)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(13/2)/(b\*x\*\*2+a)\*\*(5/4),x)

[Out]  $2/(a*c*(c*x)**(11/2)*(a+b*x**2)**(1/4)) - 8*(a+b*x**2)**(3/4)/(a**2*c*(c*x)**(11/2)) + 64*(a+b*x**2)**(7/4)/(7*a**3*c*(c*x)**(11/2)) - 256*(a+b*x**2)**(11/4)/(77*a**4*c*(c*x)**(11/2))$

**Mathematica [A]** time = 0.0624786, size = 63, normalized size = 0.58

$$-\frac{2\sqrt{cx}(7a^3-12a^2bx^2+32ab^2x^4+128b^3x^6)}{77a^4c^7x^6\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(13/2)\*(a+b\*x^2)^(5/4)),x]

[Out]  $(-2*\text{Sqrt}[c*x]*(7*a^3-12*a^2*b*x^2+32*a*b^2*x^4+128*b^3*x^6))/(77*a^4*c^7*x^6*(a+b*x^2)^(1/4))$

**Maple [A]** time = 0.008, size = 53, normalized size = 0.5

$$-\frac{2x(128b^3x^6+32ab^2x^4-12a^2bx^2+7a^3)}{77a^4} \frac{1}{\sqrt[4]{bx^2+a}} (cx)^{-\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x)`

[Out] 
$$-2/77*x*(128*b^3*x^6+32*a*b^2*x^4-12*a^2*b*x^2+7*a^3)/(b*x^2+a)^(1/4)/a^4/(c*x)^(13/2)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}} (cx)^{\frac{13}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(13/2)),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(13/2)), x)`

**Fricas [A]** time = 0.20905, size = 77, normalized size = 0.71

$$-\frac{2(128b^3x^6 + 32ab^2x^4 - 12a^2bx^2 + 7a^3)}{77(bx^2 + a)^{\frac{1}{4}}\sqrt{cxa^4c^6x^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(13/2)),x, algorithm="fricas")`

[Out] 
$$-2/77*(128*b^3*x^6 + 32*a*b^2*x^4 - 12*a^2*b*x^2 + 7*a^3)/((b*x^2 + a)^(1/4)*sqrt(c*x)*a^4*c^6*x^5)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(13/2)/(b*x**2+a)**(5/4),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}} (cx)^{\frac{13}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(13/2)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(13/2)), x)`



$$3.991 \quad \int \frac{(cx)^{13/2}}{(a+bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=155

$$\frac{77a^{5/2}c^6\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{20b^{7/2}\sqrt[4]{a+bx^2}} + \frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} - \frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}}$$

[Out] (77\*a^2\*c^5\*(c\*x)^(3/2))/(60\*b^3\*(a+b\*x^2)^(1/4)) - (11\*a\*c^3\*(c\*x)^(7/2))/(30\*b^2\*(a+b\*x^2)^(1/4)) + (c\*(c\*x)^(11/2))/(5\*b\*(a+b\*x^2)^(1/4)) + (77\*a^(5/2)\*c^6\*(1+a/(b\*x^2))^(1/4)\*Sqrt[c\*x]\*EllipticE[ArcCot[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(20\*b^(7/2)\*(a+b\*x^2)^(1/4))

**Rubi [A]** time = 0.213203, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{77a^{5/2}c^6\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{20b^{7/2}\sqrt[4]{a+bx^2}} + \frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} - \frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(13/2)/(a+b\*x^2)^(5/4), x]

[Out] (77\*a^2\*c^5\*(c\*x)^(3/2))/(60\*b^3\*(a+b\*x^2)^(1/4)) - (11\*a\*c^3\*(c\*x)^(7/2))/(30\*b^2\*(a+b\*x^2)^(1/4)) + (c\*(c\*x)^(11/2))/(5\*b\*(a+b\*x^2)^(1/4)) + (77\*a^(5/2)\*c^6\*(1+a/(b\*x^2))^(1/4)\*Sqrt[c\*x]\*EllipticE[ArcCot[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/(20\*b^(7/2)\*(a+b\*x^2)^(1/4))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{77a^3c^6\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1\int^{\frac{1}{x}}\frac{1}{\left(\frac{ax^2}{b}+1\right)^{\frac{5}{4}}}dx}{40b^4\sqrt[4]{a+bx^2}} + \frac{77a^2c^5(cx)^{\frac{3}{2}}}{60b^3\sqrt[4]{a+bx^2}} - \frac{11ac^3(cx)^{\frac{7}{2}}}{30b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{\frac{11}{2}}}{5b\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(13/2)/(b\*x\*\*2+a)\*\*(5/4), x)

[Out] 77\*a\*\*3\*c\*\*6\*sqrt(c\*x)\*(a/(b\*x\*\*2)+1)\*\*(1/4)\*Integral((a\*x\*\*2/b+1)\*\*(-5/4),(x,1/x))/(40\*b\*\*4\*(a+b\*x\*\*2)\*\*(1/4))+77\*a\*\*2\*c\*\*5\*(c\*x)\*\*(3/2)/(60\*b\*\*3\*(a+b\*x\*\*2)\*\*(1/4))-11\*a\*c\*\*3\*(c\*x)\*\*(7/2)/(30\*b\*\*2\*(a+b\*x\*\*2)\*\*(1/4))+c\*(c\*x)\*\*(11/2)/(5\*b\*(a+b\*x\*\*2)\*\*(1/4))

**Mathematica [C]** time = 0.079182, size = 87, normalized size = 0.56

$$\frac{c^5(cx)^{3/2}\left(77a^2\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^2}{a}\right) - 77a^2 - 11abx^2 + 6b^2x^4\right)}{30b^3\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(13/2)/(a + b\*x^2)^(5/4), x]

[Out] (c^5\*(c\*x)^(3/2)\*(-77\*a^2 - 11\*a\*b\*x^2 + 6\*b^2\*x^4 + 77\*a^2\*(1 + (b\*x^2)/a)^(1/4)\*Hypergeometric2F1[1/4, 3/4, 7/4, -((b\*x^2)/a)])) / (30\*b^3\*(a + b\*x^2)^(1/4))

**Maple [F]** time = 0.087, size = 0, normalized size = 0.

$$\int 1 (cx)^{\frac{13}{2}} (bx^2 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(13/2)/(b\*x^2+a)^(5/4), x)

[Out] int((c\*x)^(13/2)/(b\*x^2+a)^(5/4), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{13}{2}}}{(bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(13/2)/(b\*x^2 + a)^(5/4), x, algorithm="maxima")

[Out] integrate((c\*x)^(13/2)/(b\*x^2 + a)^(5/4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}c^6x^6}{(bx^2 + a)^{\frac{5}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(13/2)/(b\*x^2 + a)^(5/4), x, algorithm="fricas")

[Out] integral(sqrt(c\*x)\*c^6\*x^6/(b\*x^2 + a)^(5/4), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(13/2)/(b\*x\*\*2+a)\*\*(5/4), x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{13}{2}}}{(bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(13/2)/(b*x^2 + a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(13/2)/(b*x^2 + a)^(5/4), x)
```

$$3.992 \quad \int \frac{(cx)^{9/2}}{(a+bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=124

$$\frac{7a^{3/2}c^4\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{2b^{5/2}\sqrt[4]{a+bx^2}} - \frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}}$$

[Out]  $(-7*a*c^3*(c*x)^{(3/2)})/(6*b^2*(a+b*x^2)^{(1/4)}) + (c*(c*x)^{(7/2)})/(3*b*(a+b*x^2)^{(1/4)}) - (7*a^{(3/2)}*c^4*(1+a/(b*x^2))^{(1/4)}*\text{Sqrt}[c*x]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(2*b^{(5/2)}*(a+b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.15563, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{7a^{3/2}c^4\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{2b^{5/2}\sqrt[4]{a+bx^2}} - \frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(9/2)/(a + b\*x^2)^(5/4), x]

[Out]  $(-7*a*c^3*(c*x)^{(3/2)})/(6*b^2*(a+b*x^2)^{(1/4)}) + (c*(c*x)^{(7/2)})/(3*b*(a+b*x^2)^{(1/4)}) - (7*a^{(3/2)}*c^4*(1+a/(b*x^2))^{(1/4)}*\text{Sqrt}[c*x]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(2*b^{(5/2)}*(a+b*x^2)^{(1/4)})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{7a^2c^4\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int^{\frac{1}{x}}\frac{1}{\sqrt[4]{\frac{ax^2}{b}+1}}dx}{4b^3\sqrt[4]{a+bx^2}} - \frac{7a^2c^4\sqrt{cx}}{2b^3x\sqrt[4]{a+bx^2}} - \frac{7ac^3(cx)^{\frac{3}{2}}}{6b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{\frac{7}{2}}}{3b\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(9/2)/(b\*x\*\*2+a)\*\*(5/4), x)

[Out]  $7*a**2*c**4*\text{sqrt}(c*x)*(a/(b*x**2)+1)**(1/4)*\text{Integral}((a*x**2/b+1)**(-1/4),(x,1/x))/(4*b**3*(a+b*x**2)**(1/4)) - 7*a**2*c**4*\text{sqrt}(c*x)/(2*b**3*x*(a+b*x**2)**(1/4)) - 7*a*c**3*(c*x)**(3/2)/(6*b**2*(a+b*x**2)**(1/4)) + c*(c*x)**(7/2)/(3*b*(a+b*x**2)**(1/4))$

**Mathematica [C]** time = 0.069475, size = 73, normalized size = 0.59

$$\frac{c^3(cx)^{3/2}\left(-7a\sqrt[4]{\frac{bx^2}{a}}+{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+7a+bx^2\right)}{3b^2\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(9/2)/(a + b\*x^2)^(5/4), x]

[Out]  $(c^3 (c x)^{3/2} (7 a + b x^2 - 7 a (1 + (b x^2)/a)^{1/4}) \text{Hypergeometric2F1}[1/4, 3/4, 7/4, -((b x^2)/a)]) / (3 b^2 (a + b x^2)^{1/4})$

**Maple [F]** time = 0.077, size = 0, normalized size = 0.

$$\int 1 (cx)^{\frac{9}{2}} (bx^2 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(9/2)/(b*x^2+a)^(5/4), x)`

[Out] `int((c*x)^(9/2)/(b*x^2+a)^(5/4), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{9}{2}}}{(bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(9/2)/(b*x^2 + a)^(5/4), x, algorithm="maxima")`

[Out] `integrate((c*x)^(9/2)/(b*x^2 + a)^(5/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}c^4x^4}{(bx^2 + a)^{\frac{5}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(9/2)/(b*x^2 + a)^(5/4), x, algorithm="fricas")`

[Out] `integral(sqrt(c*x)*c^4*x^4/(b*x^2 + a)^(5/4), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(9/2)/(b*x**2+a)**(5/4), x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{9}{2}}}{(bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(9/2)/(b*x^2 + a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(9/2)/(b*x^2 + a)^(5/4), x)
```

$$3.993 \quad \int \frac{(cx)^{5/2}}{(a+bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=90

$$\frac{3\sqrt{ac^2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}} + \frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}$$

[Out] (c\*(c\*x)^(3/2))/(b\*(a + b\*x^2)^(1/4)) + (3\*Sqrt[a]\*c^2\*(1 + a/(b\*x^2))^(1/4)\*Sqrt[c\*x]\*EllipticE[ArcCot[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/ (b^(3/2)\*(a + b\*x^2)^(1/4))

**Rubi [A]** time = 0.114394, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{3\sqrt{ac^2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}} + \frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(5/2)/(a + b\*x^2)^(5/4), x]

[Out] (c\*(c\*x)^(3/2))/(b\*(a + b\*x^2)^(1/4)) + (3\*Sqrt[a]\*c^2\*(1 + a/(b\*x^2))^(1/4)\*Sqrt[c\*x]\*EllipticE[ArcCot[(Sqrt[b]\*x)/Sqrt[a]]/2, 2])/ (b^(3/2)\*(a + b\*x^2)^(1/4))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{3ac^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1\int^{\frac{1}{x}} \frac{1}{\left(\frac{ax^2}{b}+1\right)^{5/4}} dx}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(5/2)/(b\*x\*\*2+a)\*\*(5/4), x)

[Out] 3\*a\*c\*\*2\*sqrt(c\*x)\*(a/(b\*x\*\*2) + 1)\*\*(1/4)\*Integral((a\*x\*\*2/b + 1)\*\*(-5/4), (x, 1/x))/(2\*b\*\*2\*(a + b\*x\*\*2)\*\*(1/4)) + c\*(c\*x)\*\*(3/2)/(b\*(a + b\*x\*\*2)\*\*(1/4))

**Mathematica [C]** time = 0.0578222, size = 60, normalized size = 0.67

$$\frac{2c(cx)^{3/2}\left(\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^2}{a}\right) - 1\right)}{b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(5/2)/(a + b\*x^2)^(5/4), x]

[Out] (2\*c\*(c\*x)^(3/2)\*(-1 + (1 + (b\*x^2)/a)^(1/4)\*Hypergeometric2F1[1/4, 3/4, 7/4, -(b\*x^2)/a]))/(b\*(a + b\*x^2)^(1/4))

---

**Maple [F]** time = 0.047, size = 0, normalized size = 0.

$$\int 1 (cx)^{\frac{5}{2}} (bx^2 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(5/2)/(b*x^2+a)^(5/4), x)`

[Out] `int((c*x)^(5/2)/(b*x^2+a)^(5/4), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{(bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(5/2)/(b*x^2 + a)^(5/4), x, algorithm="maxima")`

[Out] `integrate((c*x)^(5/2)/(b*x^2 + a)^(5/4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}c^2x^2}{(bx^2 + a)^{\frac{5}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(5/2)/(b*x^2 + a)^(5/4), x, algorithm="fricas")`

[Out] `integral(sqrt(c*x)*c^2*x^2/(b*x^2 + a)^(5/4), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(5/2)/(b*x**2+a)**(5/4), x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{2}}}{(bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(5/2)/(b*x^2 + a)^(5/4), x, algorithm="giac")`

[Out] `integrate((c*x)^(5/2)/(b*x^2 + a)^(5/4), x)`



$$3.994 \quad \int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=63

$$\frac{2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\right)\Big|_2}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^2}}$$

[Out]  $(-2*(1 + a/(b*x^2))^{(1/4)}*\text{Sqrt}[c*x]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(\text{Sqrt}[a]*\text{Sqrt}[b]*(a + b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.0742066, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\right)\Big|_2}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[c*x]/(a + b*x^2)^{(5/4)}, x]$

[Out]  $(-2*(1 + a/(b*x^2))^{(1/4)}*\text{Sqrt}[c*x]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(\text{Sqrt}[a]*\text{Sqrt}[b]*(a + b*x^2)^{(1/4)})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1 \int^{\frac{1}{x}} \frac{1}{\sqrt[4]{\frac{ax^2}{b} + 1}} dx}{b^4\sqrt[4]{a+bx^2}} - \frac{2\sqrt{cx}}{bx^4\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c*x)**(1/2)/(b*x**2+a)**(5/4), x)$

[Out]  $\text{sqrt}(c*x)*(a/(b*x**2) + 1)**(1/4)*\text{Integral}((a*x**2/b + 1)**(-1/4), (x, 1/x))/(b*(a + b*x**2)**(1/4)) - 2*\text{sqrt}(c*x)/(b*x*(a + b*x**2)**(1/4))$

**Mathematica [C]** time = 0.0503388, size = 63, normalized size = 1.

$$\frac{2x\sqrt{cx}\left(2\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{3}{4}, \frac{7}{4}; -\frac{bx^2}{a}\right) - 3\right)}{3a\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Sqrt}[c*x]/(a + b*x^2)^{(5/4)}, x]$

[Out]  $(-2*x*\text{Sqrt}[c*x]*(-3 + 2*(1 + (b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/4, 3/4, 7/4, -((b*x^2)/a)])/(3*a*(a + b*x^2)^{(1/4)})$

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int \sqrt{cx} (bx^2 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(1/2)/(b*x^2+a)^(5/4), x)`

[Out] `int((c*x)^(1/2)/(b*x^2+a)^(5/4), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{(bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x)/(b*x^2 + a)^(5/4), x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x)/(b*x^2 + a)^(5/4), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx}}{(bx^2 + a)^{\frac{5}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x)/(b*x^2 + a)^(5/4), x, algorithm="fricas")`

[Out] `integral(sqrt(c*x)/(b*x^2 + a)^(5/4), x)`

---

**Sympy [A]** time = 14.1861, size = 44, normalized size = 0.7

$$\frac{\sqrt{cx}^{\frac{3}{2}} \left(\frac{3}{4}\right) {}_2F_1\left(\frac{3}{4}, \frac{5}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}} \left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(1/2)/(b*x**2+a)**(5/4), x)`

[Out] `sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 5/4), (7/4, ), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/4)*gamma(7/4))`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{(bx^2 + a)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x)/(b*x^2 + a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(b*x^2 + a)^(5/4), x)
```

$$3.995 \quad \int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=93

$$\frac{4\sqrt{b}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2} + 1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}c^2\sqrt[4]{a+bx^2}} - \frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}$$

[Out]  $-2/(a*c*\text{Sqrt}[c*x]*(a + b*x^2)^{(1/4)}) + (4*\text{Sqrt}[b]*(1 + a/(b*x^2))^{(1/4)}*\text{Sqrt}[c*x]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(a^{(3/2)}*c^2*(a + b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.118814, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{4\sqrt{b}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2} + 1}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}c^2\sqrt[4]{a+bx^2}} - \frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(3/2)\*(a + b\*x^2)^(5/4)), x]

[Out]  $-2/(a*c*\text{Sqrt}[c*x]*(a + b*x^2)^{(1/4)}) + (4*\text{Sqrt}[b]*(1 + a/(b*x^2))^{(1/4)}*\text{Sqrt}[c*x]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(a^{(3/2)}*c^2*(a + b*x^2)^{(1/4)})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}} + \frac{2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2} + 1} \int^{\frac{1}{x}} \frac{1}{\left(\frac{ax^2}{b} + 1\right)^{\frac{5}{4}}} dx}{ac^2\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(3/2)/(b\*x\*\*2+a)\*\*(5/4), x)

[Out]  $-2/(a*c*\text{sqrt}(c*x)*(a + b*x^2)^{(1/4)}) + 2*\text{sqrt}(c*x)*(a/(b*x^2) + 1)^{(1/4)}*\text{Integral}((a*x^2/b + 1)^{(-5/4)}, (x, 1/x))/(a*c^2*(a + b*x^2)^{(1/4)})$

**Mathematica [C]** time = 0.0656208, size = 76, normalized size = 0.82

$$\frac{x\left(8bx^2\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{bx^2}{a}\right) - 6(a + 2bx^2)\right)}{3a^2(cx)^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(3/2)\*(a + b\*x^2)^(5/4)), x]

[Out]  $(x*(-6*(a + 2*b*x^2) + 8*b*x^2*(1 + (b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/4, 3/4, 7/4, -(b*x^2)/a])/(3*a^2*(c*x)^{(3/2)}*(a + b*x^2)^{(1/4)})$

$2)^{(1/4)})$

**Maple [F]** time = 0.082, size = 0, normalized size = 0.

$$\int 1 (cx)^{-\frac{3}{2}} (bx^2 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(3/2)/(b*x^2+a)^(5/4), x)`

[Out] `int(1/(c*x)^(3/2)/(b*x^2+a)^(5/4), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}} (cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(3/2)), x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(3/2)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bcx^3 + acx)(bx^2 + a)^{\frac{1}{4}}\sqrt{cx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(3/2)), x, algorithm="fricas")`

[Out] `integral(1/((b*c*x^3 + a*c*x)*(b*x^2 + a)^(1/4)*sqrt(c*x)), x)`

**Sympy [A]** time = 132.303, size = 48, normalized size = 0.52

$$\frac{\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{5}{4} \\ \frac{3}{4} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}c^{\frac{3}{2}}\sqrt{x}\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(3/2)/(b*x**2+a)**(5/4), x)`

[Out] `gamma(-1/4)*hyper((-1/4, 5/4), (3/4, ), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/4)*c**(3/2)*sqrt(x)*gamma(3/4)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}} (cx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(3/2)), x)
```

$$3.996 \quad \int \frac{1}{(cx)^{7/2}(a+bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=126

$$-\frac{24b^{3/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\right)\Big|_2}{5a^{5/2}c^4\sqrt[4]{a+bx^2}} + \frac{12b}{5a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}}$$

[Out]  $-2/(5*a*c*(c*x)^{(5/2)*(a+b*x^2)^{(1/4)}) + (12*b)/(5*a^2*c^3*\text{Sqrt}[c*x]*(a+b*x^2)^{(1/4)}) - (24*b^{(3/2)*(1+a/(b*x^2))}^{(1/4)*\text{Sqrt}[c*x]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(5*a^{(5/2)*c^4*(a+b*x^2)^{(1/4)})}$

**Rubi [A]** time = 0.16101, antiderivative size = 126, normalized size of antiderivative = 1., number of rules used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$-\frac{24b^{3/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\right)\Big|_2}{5a^{5/2}c^4\sqrt[4]{a+bx^2}} + \frac{12b}{5a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(7/2)\*(a+b\*x^2)^(5/4)),x]

[Out]  $-2/(5*a*c*(c*x)^{(5/2)*(a+b*x^2)^{(1/4)}) + (12*b)/(5*a^2*c^3*\text{Sqrt}[c*x]*(a+b*x^2)^{(1/4)}) - (24*b^{(3/2)*(1+a/(b*x^2))}^{(1/4)*\text{Sqrt}[c*x]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(5*a^{(5/2)*c^4*(a+b*x^2)^{(1/4)})}$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}} + \frac{12b}{5a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}} + \frac{12b\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1\int^{\frac{1}{x}} \frac{1}{\sqrt[4]{ax^2} + 1} dx}{5a^2c^4\sqrt[4]{a+bx^2}} - \frac{24b\sqrt{cx}}{5a^2c^4x\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(7/2)/(b\*x\*\*2+a)\*\*(5/4),x)

[Out]  $-2/(5*a*c*(c*x)^{(5/2)*(a+b*x^2)^{(1/4)}) + 12*b/(5*a^2*c^3*\text{sqrt}(c*x)*(a+b*x^2)^{(1/4)}) + 12*b*\text{sqrt}(c*x)*(a/(b*x^2)+1)^{(1/4)*\text{Integral}((a*x^2/b+1)^{(-1/4)},(x,1/x))/(5*a^2*c^4*(a+b*x^2)^{(1/4)}) - 24*b*\text{sqrt}(c*x)/(5*a^2*c^4*x*(a+b*x^2)^{(1/4)})}$

**Mathematica [C]** time = 0.0821361, size = 86, normalized size = 0.68

$$-\frac{2x\left(a^2 + 8b^2x^4\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{7}{4}; -\frac{bx^2}{a}\right) - 6abx^2 - 12b^2x^4\right)}{5a^3(cx)^{7/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(7/2)\*(a+b\*x^2)^(5/4)),x]

[Out]  $(-2*x*(a^2 - 6*a*b*x^2 - 12*b^2*x^4 + 8*b^2*x^4*(1 + (b*x^2)/a))^{1/4} * \text{Hypergeometric2F1}[1/4, 3/4, 7/4, -((b*x^2)/a)]) / (5*a^3*(c*x)^{7/2}*(a + b*x^2)^{1/4})$

**Maple [F]** time = 0.086, size = 0, normalized size = 0.

$$\int 1 (cx)^{-\frac{7}{2}} (bx^2 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(7/2)/(b*x^2+a)^(5/4), x)`

[Out] `int(1/(c*x)^(7/2)/(b*x^2+a)^(5/4), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}} (cx)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(7/2)), x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(7/2)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bc^3x^5 + ac^3x^3)(bx^2 + a)^{\frac{1}{4}}\sqrt{cx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(7/2)), x, algorithm="fricas")`

[Out] `integral(1/((b*c^3*x^5 + a*c^3*x^3)*(b*x^2 + a)^(1/4)*sqrt(c*x)), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(7/2)/(b*x**2+a)**(5/4), x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}} (cx)^{\frac{7}{2}}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(7/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(7/2)), x)
```

$$3.997 \quad \int \frac{1}{(cx)^{11/2}(a+bx^2)^{5/4}} dx$$

**Optimal.** Leaf size=157

$$\frac{16b^{5/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{3a^{7/2}c^6\sqrt[4]{a+bx^2}} - \frac{8b^2}{3a^3c^5\sqrt{cx}\sqrt[4]{a+bx^2}}$$

$$+ \frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}} - \frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}$$

[Out]  $-2/(9*a*c*(c*x)^{(9/2)}*(a+b*x^2)^{(1/4)}) + (4*b)/(9*a^2*c^3*(c*x)^{(5/2)}*(a+b*x^2)^{(1/4)}) - (8*b^2)/(3*a^3*c^5*\text{Sqrt}[c*x]*(a+b*x^2)^{(1/4)}) + (16*b^{(5/2)}*(1+a/(b*x^2))^{(1/4)}*\text{Sqrt}[c*x]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(3*a^{(7/2)}*c^6*(a+b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.215549, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{16b^{5/2}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\middle|2\right)}{3a^{7/2}c^6\sqrt[4]{a+bx^2}} - \frac{8b^2}{3a^3c^5\sqrt{cx}\sqrt[4]{a+bx^2}}$$

$$+ \frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}} - \frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(11/2)\*(a+b\*x^2)^(5/4)),x]

[Out]  $-2/(9*a*c*(c*x)^{(9/2)}*(a+b*x^2)^{(1/4)}) + (4*b)/(9*a^2*c^3*(c*x)^{(5/2)}*(a+b*x^2)^{(1/4)}) - (8*b^2)/(3*a^3*c^5*\text{Sqrt}[c*x]*(a+b*x^2)^{(1/4)}) + (16*b^{(5/2)}*(1+a/(b*x^2))^{(1/4)}*\text{Sqrt}[c*x]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[b]*x)/\text{Sqrt}[a]]/2, 2])/(3*a^{(7/2)}*c^6*(a+b*x^2)^{(1/4)})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{2}{9ac(cx)^{\frac{9}{2}}\sqrt[4]{a+bx^2}} + \frac{4b}{9a^2c^3(cx)^{\frac{5}{2}}\sqrt[4]{a+bx^2}} - \frac{8b^2}{3a^3c^5\sqrt{cx}\sqrt[4]{a+bx^2}} + \frac{8b^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}} + 1\int^{\frac{1}{x}}\frac{1}{\left(\frac{ax^2}{b}+1\right)^{\frac{5}{4}}}dx}{3a^3c^6\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(11/2)/(b\*x\*\*2+a)\*\*(5/4),x)

[Out]  $-2/(9*a*c*(c*x)**(9/2)*(a+b*x**2)**(1/4)) + 4*b/(9*a**2*c**3*(c*x)**(5/2)*(a+b*x**2)**(1/4)) - 8*b**2/(3*a**3*c**5*\text{sqrt}(c*x)*(a+b*x**2)**(1/4)) + 8*b**2*\text{sqrt}(c*x)*(a/(b*x**2)+1)**(1/4)*\text{Integral}((a*x**2/b+1)**(-5/4),(x,1/x))/(3*a**3*c**6*(a+b*x**2)**(1/4))$

**Mathematica [C]** time = 0.122338, size = 105, normalized size = 0.67

$$\frac{\sqrt{cx}\left(32b^3x^6\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, -\frac{bx^2}{a}\right) - 2(a^3 - 2a^2bx^2 + 12ab^2x^4 + 24b^3x^6)\right)}{9a^4c^6x^5\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(11/2)\*(a + b\*x^2)^(5/4)),x]

[Out] (Sqrt[c\*x]\*(-2\*(a^3 - 2\*a^2\*b\*x^2 + 12\*a\*b^2\*x^4 + 24\*b^3\*x^6) + 32\*b^3\*x^6\*(1 + (b\*x^2)/a)^(1/4)\*Hypergeometric2F1[1/4, 3/4, 7/4, -(b\*x^2)/a]))/(9\*a^4\*c^6\*x^5\*(a + b\*x^2)^(1/4))

**Maple [F]** time = 0.092, size = 0, normalized size = 0.

$$\int 1 (cx)^{-\frac{11}{2}} (bx^2 + a)^{-\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x)^(11/2)/(b\*x^2+a)^(5/4),x)

[Out] int(1/(c\*x)^(11/2)/(b\*x^2+a)^(5/4),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}} (cx)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/4)\*(c\*x)^(11/2)),x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(5/4)\*(c\*x)^(11/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bc^5x^7 + ac^5x^5)(bx^2 + a)^{\frac{1}{4}}\sqrt{cx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(5/4)\*(c\*x)^(11/2)),x, algorithm="fricas")

[Out] integral(1/((b\*c^5\*x^7 + a\*c^5\*x^5)\*(b\*x^2 + a)^(1/4)\*sqrt(c\*x)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(11/2)/(b\*x\*\*2+a)\*\*(5/4),x)

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}} (cx)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(11/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(11/2)), x)
```

$$3.998 \quad \int \frac{(cx)^{5/4}}{\sqrt[4]{a+bx^2}} dx$$

**Optimal.** Leaf size=58

$$\frac{4(cx)^{9/4} \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{9}{8}, \frac{17}{8}; -\frac{bx^2}{a}\right)}{9c\sqrt[4]{a+bx^2}}$$

[Out]  $(4*(c*x)^{(9/4)}*(1+(b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/4, 9/8, 17/8, -(b*x^2)/a])/(9*c*(a+b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.0673859, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{4(cx)^{9/4} \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{9}{8}, \frac{17}{8}; -\frac{bx^2}{a}\right)}{9c\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(5/4)/(a+b\*x^2)^(1/4),x]

[Out]  $(4*(c*x)^{(9/4)}*(1+(b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/4, 9/8, 17/8, -(b*x^2)/a])/(9*c*(a+b*x^2)^{(1/4)})$

**Rubi in Sympy [A]** time = 8.07022, size = 49, normalized size = 0.84

$$\frac{4(cx)^{\frac{9}{4}}(a+bx^2)^{\frac{3}{4}} {}_2F_1\left(\frac{1}{4}, \frac{9}{8} \middle| \frac{17}{8}; -\frac{bx^2}{a}\right)}{9ac\left(1+\frac{bx^2}{a}\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(5/4)/(b\*x\*\*2+a)\*\*(1/4),x)

[Out]  $4*(c*x)**(9/4)*(a+b*x**2)**(3/4)*\text{hyper}((1/4, 9/8), (17/8, ), -b*x**2/a)/(9*a*c*(1+b*x**2/a)**(3/4))$

**Mathematica [A]** time = 0.0578219, size = 69, normalized size = 1.19

$$\frac{4c\sqrt[4]{cx}\left(-a\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{8}, \frac{1}{4}, \frac{9}{8}; -\frac{bx^2}{a}\right) + a + bx^2\right)}{7b\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(5/4)/(a+b\*x^2)^(1/4),x]

[Out]  $(4*c*(c*x)^{(1/4)}*(a+b*x^2-a*(1+(b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/8, 1/4, 9/8, -(b*x^2)/a])/(7*b*(a+b*x^2)^{(1/4)})$

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int 1 (cx)^{\frac{5}{4}} \frac{1}{\sqrt[4]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(5/4)/(b\*x^2+a)^(1/4), x)

[Out] int((c\*x)^(5/4)/(b\*x^2+a)^(1/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{4}}}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(5/4)/(b\*x^2 + a)^(1/4), x, algorithm="maxima")

[Out] integrate((c\*x)^(5/4)/(b\*x^2 + a)^(1/4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^{\frac{1}{4}} cx}{(bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(5/4)/(b\*x^2 + a)^(1/4), x, algorithm="fricas")

[Out] integral((c\*x)^(1/4)\*c\*x/(b\*x^2 + a)^(1/4), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(5/4)/(b\*x\*\*2+a)\*\*(1/4), x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{4}}}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(5/4)/(b\*x^2 + a)^(1/4), x, algorithm="giac")

[Out] integrate((c\*x)^(5/4)/(b\*x^2 + a)^(1/4), x)

$$3.999 \quad \int \frac{(cx)^{3/4}}{\sqrt[4]{a+bx^2}} dx$$

**Optimal.** Leaf size=58

$$\frac{4(cx)^{7/4} \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{7}{8}, \frac{15}{8}, -\frac{bx^2}{a}\right)}{7c\sqrt[4]{a+bx^2}}$$

[Out]  $(4*(c*x)^{(7/4)}*(1+(b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/4, 7/8, 15/8, -(b*x^2)/a])/(7*c*(a+b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.0663859, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{4(cx)^{7/4} \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{7}{8}, \frac{15}{8}, -\frac{bx^2}{a}\right)}{7c\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*x)^{(3/4)}/(a+b*x^2)^{(1/4)}, x]$

[Out]  $(4*(c*x)^{(7/4)}*(1+(b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/4, 7/8, 15/8, -(b*x^2)/a])/(7*c*(a+b*x^2)^{(1/4)})$

**Rubi in Sympy [A]** time = 8.05192, size = 49, normalized size = 0.84

$$\frac{4(cx)^{\frac{7}{4}}(a+bx^2)^{\frac{3}{4}} {}_2F_1\left(\frac{1}{4}, \frac{7}{8}, \frac{15}{8}, -\frac{bx^2}{a}\right)}{7ac\left(1+\frac{bx^2}{a}\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c*x)**(3/4)/(b*x**2+a)**(1/4), x)$

[Out]  $4*(c*x)**(7/4)*(a+b*x**2)**(3/4)*\text{hyper}((1/4, 7/8), (15/8, ), -b*x**2/a)/(7*a*c*(1+b*x**2/a)**(3/4))$

**Mathematica [A]** time = 0.0380991, size = 57, normalized size = 0.98

$$\frac{4x(cx)^{3/4} \sqrt[4]{\frac{a+bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{7}{8}, \frac{15}{8}, -\frac{bx^2}{a}\right)}{7\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(c*x)^{(3/4)}/(a+b*x^2)^{(1/4)}, x]$

[Out]  $(4*x*(c*x)^{(3/4)}*((a+b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/4, 7/8, 15/8, -(b*x^2)/a])/(7*(a+b*x^2)^{(1/4)})$

**Maple [F]** time = 0.042, size = 0, normalized size = 0.

$$\int 1 (cx)^{\frac{3}{4}} \frac{1}{\sqrt[4]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(3/4)/(b\*x^2+a)^(1/4), x)

[Out] int((c\*x)^(3/4)/(b\*x^2+a)^(1/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{4}}}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(3/4)/(b\*x^2 + a)^(1/4), x, algorithm="maxima")

[Out] integrate((c\*x)^(3/4)/(b\*x^2 + a)^(1/4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^{\frac{3}{4}}}{(bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(3/4)/(b\*x^2 + a)^(1/4), x, algorithm="fricas")

[Out] integral((c\*x)^(3/4)/(b\*x^2 + a)^(1/4), x)

---

**Sympy [A]** time = 40.0906, size = 44, normalized size = 0.76

$$\frac{c^{\frac{3}{4}} x^{\frac{7}{4}} \left(\frac{7}{8}\right) {}_2F_1\left(\frac{1}{4}, \frac{7}{8} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt[4]{a} \left(\frac{15}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(3/4)/(b\*x\*\*2+a)\*\*(1/4), x)

[Out] c\*\*(3/4)\*x\*\*(7/4)\*gamma(7/8)\*hyper((1/4, 7/8), (15/8,), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*a\*\*(1/4)\*gamma(15/8))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{4}}}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((c*x)^(3/4)/(b*x^2 + a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/4)/(b*x^2 + a)^(1/4), x)
```

$$3.1000 \quad \int \frac{\sqrt[4]{cx}}{\sqrt[4]{a+bx^2}} dx$$

**Optimal.** Leaf size=58

$$\frac{4(cx)^{5/4} \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{5}{8}, \frac{13}{8}; -\frac{bx^2}{a}\right)}{5c\sqrt[4]{a+bx^2}}$$

[Out]  $(4*(c*x)^{(5/4)}*(1+(b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/4, 5/8, 13/8, -(b*x^2)/a])/(5*c*(a+b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.066728, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{4(cx)^{5/4} \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{5}{8}, \frac{13}{8}; -\frac{bx^2}{a}\right)}{5c\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(1/4)/(a+b\*x^2)^(1/4),x]

[Out]  $(4*(c*x)^{(5/4)}*(1+(b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/4, 5/8, 13/8, -(b*x^2)/a])/(5*c*(a+b*x^2)^{(1/4)})$

**Rubi in Sympy [A]** time = 8.0807, size = 49, normalized size = 0.84

$$\frac{4(cx)^{5/4} (a+bx^2)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{5}{8}, \frac{13}{8}; -\frac{bx^2}{a}\right)}{5ac \left(1 + \frac{bx^2}{a}\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(1/4)/(b\*x\*\*2+a)\*\*(1/4),x)

[Out]  $4*(c*x)**(5/4)*(a+b*x**2)**(3/4)*\text{hyper}((1/4, 5/8), (13/8, ), -b*x**2/a)/(5*a*c*(1+b*x**2/a)**(3/4))$

**Mathematica [A]** time = 0.0356906, size = 57, normalized size = 0.98

$$\frac{4x\sqrt[4]{cx}\sqrt[4]{\frac{a+bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{5}{8}, \frac{13}{8}; -\frac{bx^2}{a}\right)}{5\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(1/4)/(a+b\*x^2)^(1/4),x]

[Out]  $(4*x*(c*x)^{(1/4)}*((a+b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/4, 5/8, 13/8, -(b*x^2)/a])/(5*(a+b*x^2)^{(1/4)})$

**Maple [F]** time = 0.042, size = 0, normalized size = 0.

$$\int \sqrt[4]{cx} \frac{1}{\sqrt[4]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(1/4)/(b\*x^2+a)^(1/4), x)

[Out] int((c\*x)^(1/4)/(b\*x^2+a)^(1/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{1}{4}}}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(1/4)/(b\*x^2 + a)^(1/4), x, algorithm="maxima")

[Out] integrate((c\*x)^(1/4)/(b\*x^2 + a)^(1/4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^{\frac{1}{4}}}{(bx^2 + a)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(1/4)/(b\*x^2 + a)^(1/4), x, algorithm="fricas")

[Out] integral((c\*x)^(1/4)/(b\*x^2 + a)^(1/4), x)

---

**Sympy [A]** time = 4.93563, size = 44, normalized size = 0.76

$$\frac{\sqrt[4]{c} x^{\frac{5}{4}} \left(\frac{5}{8}\right) {}_2F_1\left(\frac{1}{4}, \frac{5}{8} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt[4]{a} \left(\frac{13}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(1/4)/(b\*x\*\*2+a)\*\*(1/4), x)

[Out] c\*\*(1/4)\*x\*\*(5/4)\*gamma(5/8)\*hyper((1/4, 5/8), (13/8,), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*a\*\*(1/4)\*gamma(13/8))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{1}{4}}}{(bx^2 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/4)/(b*x^2 + a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(1/4)/(b*x^2 + a)^(1/4), x)
```

$$3.1001 \quad \int \frac{1}{\sqrt[4]{cx} \sqrt[4]{a + bx^2}} dx$$

**Optimal.** Leaf size=58

$$\frac{4(cx)^{3/4} \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{3}{8}, \frac{11}{8}; -\frac{bx^2}{a}\right)}{3c \sqrt[4]{a + bx^2}}$$

[Out]  $(4*(c*x)^{(3/4)}*(1 + (b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/4, 3/8, 11/8, -((b*x^2)/a)])/(3*c*(a + b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.0670611, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{4(cx)^{3/4} \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{3}{8}, \frac{11}{8}; -\frac{bx^2}{a}\right)}{3c \sqrt[4]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(1/4)\*(a + b\*x^2)^(1/4)),x]

[Out]  $(4*(c*x)^{(3/4)}*(1 + (b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/4, 3/8, 11/8, -((b*x^2)/a)])/(3*c*(a + b*x^2)^{(1/4)})$

**Rubi in Sympy [A]** time = 8.08754, size = 49, normalized size = 0.84

$$\frac{4(cx)^{\frac{3}{4}}(a + bx^2)^{\frac{3}{4}} {}_2F_1\left(\frac{1}{4}, \frac{3}{8} \middle| \frac{11}{8}; -\frac{bx^2}{a}\right)}{3ac \left(1 + \frac{bx^2}{a}\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(1/4)/(b\*x\*\*2+a)\*\*(1/4),x)

[Out]  $4*(c*x)**(3/4)*(a + b*x**2)**(3/4)*\text{hyper}((1/4, 3/8), (11/8, ), -b*x**2/a)/(3*a*c*(1 + b*x**2/a)**(3/4))$

**Mathematica [A]** time = 0.0375967, size = 57, normalized size = 0.98

$$\frac{4x \sqrt[4]{\frac{a + bx^2}{a}} {}_2F_1\left(\frac{1}{4}, \frac{3}{8}, \frac{11}{8}; -\frac{bx^2}{a}\right)}{3 \sqrt[4]{cx} \sqrt[4]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(1/4)\*(a + b\*x^2)^(1/4)),x]

[Out]  $(4*x*((a + b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/4, 3/8, 11/8, -((b*x^2)/a)])/(3*(c*x)^{(1/4)}*(a + b*x^2)^{(1/4)})$

**Maple [F]** time = 0.044, size = 0, normalized size = 0.

$$\int 1 \frac{1}{\sqrt[4]{cx}} \frac{1}{\sqrt[4]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x)`

[Out] `int(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(1/4)),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(1/4)), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(1/4)),x, algorithm="fricas")`

[Out] `integral(1/((b*x^2 + a)^(1/4)*(c*x)^(1/4)), x)`

---

**Sympy [A]** time = 5.30653, size = 44, normalized size = 0.76

$$\frac{x^{\frac{3}{4}} \left(\frac{3}{8}\right) {}_2F_1\left(\frac{1}{4}, \frac{3}{8} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\sqrt[4]{c} \left(\frac{11}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(1/4)/(b*x**2+a)**(1/4),x)`

[Out] `x**(3/4)*gamma(3/8)*hyper((1/4, 3/8), (11/8, ), b*x**2*exp_polar(I*pi)/a)/(2*a**(1/4)*c**(1/4)*gamma(11/8))`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(1/4)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(1/4)), x)
```

$$3.1002 \quad \int \frac{1}{(cx)^{3/4} \sqrt[4]{a + bx^2}} dx$$

**Optimal.** Leaf size=56

$$\frac{4\sqrt[4]{cx} \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{8}, \frac{1}{4}; \frac{9}{8}; -\frac{bx^2}{a}\right)}{c\sqrt[4]{a + bx^2}}$$

[Out]  $(4*(c*x)^{(1/4)}*(1 + (b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/8, 1/4, 9/8, -((b*x^2)/a)])/(c*(a + b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.0661366, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{4\sqrt[4]{cx} \sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{8}, \frac{1}{4}; \frac{9}{8}; -\frac{bx^2}{a}\right)}{c\sqrt[4]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(3/4)\*(a + b\*x^2)^(1/4)), x]

[Out]  $(4*(c*x)^{(1/4)}*(1 + (b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/8, 1/4, 9/8, -((b*x^2)/a)])/(c*(a + b*x^2)^{(1/4)})$

**Rubi in Sympy [A]** time = 8.09243, size = 48, normalized size = 0.86

$$\frac{4\sqrt[4]{cx} (a + bx^2)^{\frac{3}{4}} {}_2F_1\left(\frac{1}{4}, \frac{1}{8}; \frac{9}{8}; -\frac{bx^2}{a}\right)}{ac \left(1 + \frac{bx^2}{a}\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(3/4)/(b\*x\*\*2+a)\*\*(1/4), x)

[Out]  $4*(c*x)**(1/4)*(a + b*x**2)**(3/4)*\text{hyper}((1/4, 1/8), (9/8, ), -b*x**2/a)/(a*c*(1 + b*x**2/a)**(3/4))$

**Mathematica [A]** time = 0.0291351, size = 55, normalized size = 0.98

$$\frac{4x \sqrt[4]{\frac{a + bx^2}{a}} {}_2F_1\left(\frac{1}{8}, \frac{1}{4}; \frac{9}{8}; -\frac{bx^2}{a}\right)}{(cx)^{3/4} \sqrt[4]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(3/4)\*(a + b\*x^2)^(1/4)), x]

[Out]  $(4*x*((a + b*x^2)/a)^{(1/4)}*\text{Hypergeometric2F1}[1/8, 1/4, 9/8, -((b*x^2)/a)])/((c*x)^{(3/4)}*(a + b*x^2)^{(1/4)})$



**Maple [F]** time = 0.045, size = 0, normalized size = 0.

$$\int 1 (cx)^{-\frac{3}{4}} \frac{1}{\sqrt[4]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x)^(3/4)/(b\*x^2+a)^(1/4), x)

[Out] int(1/(c\*x)^(3/4)/(b\*x^2+a)^(1/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/4)\*(c\*x)^(3/4)), x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(1/4)\*(c\*x)^(3/4)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/4)\*(c\*x)^(3/4)), x, algorithm="fricas")

[Out] integral(1/((b\*x^2 + a)^(1/4)\*(c\*x)^(3/4)), x)

---

**Sympy [A]** time = 13.7278, size = 44, normalized size = 0.79

$$\frac{\sqrt[4]{x} \left(\frac{1}{8}\right) {}_2F_1\left(\frac{1}{8}, \frac{1}{4} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt[4]{ac} \left(\frac{9}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(3/4)/(b\*x\*\*2+a)\*\*(1/4), x)

[Out] x\*\*(1/4)\*gamma(1/8)\*hyper((1/8, 1/4), (9/8, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*a\*\*(1/4)\*c\*\*(3/4)\*gamma(9/8))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(3/4)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(3/4)), x)
```

$$3.1003 \quad \int \frac{1}{(cx)^{5/4} \sqrt[4]{a + bx^2}} dx$$

**Optimal.** Leaf size=56

$$-\frac{4\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(-\frac{1}{8}, \frac{1}{4}; \frac{7}{8}; -\frac{bx^2}{a}\right)}{c\sqrt[4]{cx}\sqrt[4]{a + bx^2}}$$

[Out]  $(-4*(1 + (b*x^2)/a)^{(1/4)}*Hypergeometric2F1[-1/8, 1/4, 7/8, -(b*x^2)/a])/(c*(c*x)^{(1/4)}*(a + b*x^2)^{(1/4)})$

**Rubi [A]** time = 0.0656608, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$-\frac{4\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(-\frac{1}{8}, \frac{1}{4}; \frac{7}{8}; -\frac{bx^2}{a}\right)}{c\sqrt[4]{cx}\sqrt[4]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(5/4)\*(a + b\*x^2)^(1/4)),x]

[Out]  $(-4*(1 + (b*x^2)/a)^{(1/4)}*Hypergeometric2F1[-1/8, 1/4, 7/8, -(b*x^2)/a])/(c*(c*x)^{(1/4)}*(a + b*x^2)^{(1/4)})$

**Rubi in Sympy [A]** time = 8.08151, size = 51, normalized size = 0.91

$$-\frac{4(a + bx^2)^{\frac{3}{4}} {}_2F_1\left(\frac{1}{4}, -\frac{1}{8}; \frac{7}{8}; -\frac{bx^2}{a}\right)}{ac\sqrt[4]{cx}\left(1 + \frac{bx^2}{a}\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(5/4)/(b\*x\*\*2+a)\*\*(1/4),x)

[Out]  $-4*(a + b*x**2)**(3/4)*hyper((1/4, -1/8), (7/8, ), -b*x**2/a)/(a*c*(c*x)**(1/4)*(1 + b*x**2/a)**(3/4))$

**Mathematica [A]** time = 0.091343, size = 75, normalized size = 1.34

$$\frac{4x\left(5bx^2\sqrt[4]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{4}, \frac{7}{8}; \frac{15}{8}; -\frac{bx^2}{a}\right) - 7(a + bx^2)\right)}{7a(cx)^{5/4}\sqrt[4]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(5/4)\*(a + b\*x^2)^(1/4)),x]

[Out]  $(4*x*(-7*(a + b*x^2) + 5*b*x^2*(1 + (b*x^2)/a)^{(1/4)}*Hypergeometric2F1[1/4, 7/8, 15/8, -(b*x^2)/a])/(7*a*(c*x)^{(5/4)}*(a + b*x^2)^{(1/4)})$

**Maple [F]** time = 0.053, size = 0, normalized size = 0.

$$\int 1 (cx)^{-\frac{5}{4}} \frac{1}{\sqrt[4]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x)^(5/4)/(b\*x^2+a)^(1/4), x)

[Out] int(1/(c\*x)^(5/4)/(b\*x^2+a)^(1/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/4)\*(c\*x)^(5/4)), x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(1/4)\*(c\*x)^(5/4)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{1}{4}} cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/4)\*(c\*x)^(5/4)), x, algorithm="fricas")

[Out] integral(1/((b\*x^2 + a)^(1/4)\*(c\*x)^(1/4)\*c\*x), x)

---

**Sympy [A]** time = 57.2572, size = 48, normalized size = 0.86

$$\frac{\left(-\frac{1}{8}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{8}, \frac{1}{4} \\ \frac{7}{8} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt[4]{ac} \sqrt[4]{x} \left(\frac{7}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x)\*\*(5/4)/(b\*x\*\*2+a)\*\*(1/4), x)

[Out] gamma(-1/8)\*hyper((-1/8, 1/4), (7/8, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(2\*a\*\*(1/4)\*c\*\*(5/4)\*x\*\*(1/4)\*gamma(7/8))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}} (cx)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/4)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/4)), x)
```

$$3.1004 \quad \int \frac{(cx)^{5/4}}{(a+bx^2)^{7/4}} dx$$

**Optimal.** Leaf size=61

$$\frac{4(cx)^{9/4} \left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{9}{8}, \frac{7}{4}, \frac{17}{8}, -\frac{bx^2}{a}\right)}{9ac(a+bx^2)^{3/4}}$$

[Out]  $(4*(c*x)^{(9/4)}*(1+(b*x^2)/a)^{(3/4)}*Hypergeometric2F1[9/8, 7/4, 17/8, -(b*x^2)/a])/(9*a*c*(a+b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.0674959, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{4(cx)^{9/4} \left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{9}{8}, \frac{7}{4}, \frac{17}{8}, -\frac{bx^2}{a}\right)}{9ac(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(5/4)/(a+b\*x^2)^(7/4),x]

[Out]  $(4*(c*x)^{(9/4)}*(1+(b*x^2)/a)^{(3/4)}*Hypergeometric2F1[9/8, 7/4, 17/8, -(b*x^2)/a])/(9*a*c*(a+b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 8.1488, size = 51, normalized size = 0.84

$$\frac{4(cx)^{\frac{9}{4}} \sqrt[4]{a+bx^2} {}_2F_1\left(\frac{7}{4}, \frac{9}{8}, \frac{17}{8}, -\frac{bx^2}{a}\right)}{9a^2c \sqrt[4]{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(5/4)/(b\*x\*\*2+a)\*\*(7/4),x)

[Out]  $4*(c*x)**(9/4)*(a+b*x**2)**(1/4)*hyper((7/4, 9/8), (17/8, ), -b*x**2/a)/(9*a**2*c*(1+b*x**2/a)**(1/4))$

**Mathematica [A]** time = 0.0594784, size = 62, normalized size = 1.02

$$\frac{2c\sqrt[4]{cx} \left(\left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{1}{8}, \frac{3}{4}, \frac{9}{8}, -\frac{bx^2}{a}\right) - 1\right)}{3b(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(5/4)/(a+b\*x^2)^(7/4),x]

[Out]  $(2*c*(c*x)^{(1/4)}*(-1+(1+(b*x^2)/a)^{(3/4)}*Hypergeometric2F1[1/8, 3/4, 9/8, -(b*x^2)/a]))/(3*b*(a+b*x^2)^{(3/4)})$

**Maple [F]** time = 0.055, size = 0, normalized size = 0.

$$\int 1 (cx)^{\frac{5}{4}} (bx^2 + a)^{-\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(5/4)/(b*x^2+a)^(7/4),x)`

[Out] `int((c*x)^(5/4)/(b*x^2+a)^(7/4),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{4}}}{(bx^2 + a)^{\frac{7}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(5/4)/(b*x^2 + a)^(7/4),x, algorithm="maxima")`

[Out] `integrate((c*x)^(5/4)/(b*x^2 + a)^(7/4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^{\frac{1}{4}} cx}{(bx^2 + a)^{\frac{7}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(5/4)/(b*x^2 + a)^(7/4),x, algorithm="fricas")`

[Out] `integral((c*x)^(1/4)*c*x/(b*x^2 + a)^(7/4), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(5/4)/(b*x**2+a)**(7/4),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{5}{4}}}{(bx^2 + a)^{\frac{7}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(5/4)/(b*x^2 + a)^(7/4),x, algorithm="giac")`

[Out] `integrate((c*x)^(5/4)/(b*x^2 + a)^(7/4), x)`

$$3.1005 \quad \int \frac{(cx)^{3/4}}{(a+bx^2)^{7/4}} dx$$

**Optimal.** Leaf size=61

$$\frac{4(cx)^{7/4} \left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{7}{8}, \frac{7}{4}, \frac{15}{8}; -\frac{bx^2}{a}\right)}{7ac(a+bx^2)^{3/4}}$$

[Out]  $(4*(c*x)^{(7/4)}*(1+(b*x^2)/a)^{(3/4)}*Hypergeometric2F1[7/8, 7/4, 15/8, -(b*x^2)/a])/(7*a*c*(a+b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.0666797, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{4(cx)^{7/4} \left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{7}{8}, \frac{7}{4}, \frac{15}{8}; -\frac{bx^2}{a}\right)}{7ac(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^(3/4)/(a+b\*x^2)^(7/4),x]

[Out]  $(4*(c*x)^{(7/4)}*(1+(b*x^2)/a)^{(3/4)}*Hypergeometric2F1[7/8, 7/4, 15/8, -(b*x^2)/a])/(7*a*c*(a+b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 8.12389, size = 51, normalized size = 0.84

$$\frac{4(cx)^{\frac{7}{4}} \sqrt[4]{a+bx^2} {}_2F_1\left(\frac{7}{4}, \frac{7}{8} \middle| \frac{15}{8}; -\frac{bx^2}{a}\right)}{7a^2c \sqrt[4]{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*(3/4)/(b\*x\*\*2+a)\*\*(7/4),x)

[Out]  $4*(c*x)**(7/4)*(a+b*x**2)**(1/4)*hyper((7/4, 7/8), (15/8, ), -b*x**2/a)/(7*a**2*c*(1+b*x**2/a)**(1/4))$

**Mathematica [A]** time = 0.0512133, size = 62, normalized size = 1.02

$$\frac{2x(cx)^{3/4} \left(\left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{3}{4}, \frac{7}{8}, \frac{15}{8}; -\frac{bx^2}{a}\right) - 7\right)}{21a(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^(3/4)/(a+b\*x^2)^(7/4),x]

[Out]  $(-2*x*(c*x)^{(3/4)}*(-7+(1+(b*x^2)/a)^{(3/4)}*Hypergeometric2F1[3/4, 7/8, 15/8, -(b*x^2)/a]))/(21*a*(a+b*x^2)^{(3/4)})$

**Maple [F]** time = 0.053, size = 0, normalized size = 0.

$$\int 1 (cx)^{\frac{3}{4}} (bx^2 + a)^{-\frac{7}{4}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^(3/4)/(b*x^2+a)^(7/4),x)`

[Out] `int((c*x)^(3/4)/(b*x^2+a)^(7/4),x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{4}}}{(bx^2 + a)^{\frac{7}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(3/4)/(b*x^2 + a)^(7/4),x, algorithm="maxima")`

[Out] `integrate((c*x)^(3/4)/(b*x^2 + a)^(7/4), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^{\frac{3}{4}}}{(bx^2 + a)^{\frac{7}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(3/4)/(b*x^2 + a)^(7/4),x, algorithm="fricas")`

[Out] `integral((c*x)^(3/4)/(b*x^2 + a)^(7/4), x)`

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**(3/4)/(b*x**2+a)**(7/4),x)`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{4}}}{(bx^2 + a)^{\frac{7}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)^(3/4)/(b*x^2 + a)^(7/4),x, algorithm="giac")`

[Out] `integrate((c*x)^(3/4)/(b*x^2 + a)^(7/4), x)`

$$3.1006 \quad \int \frac{\sqrt[4]{cx}}{(a+bx^2)^{7/4}} dx$$

**Optimal.** Leaf size=61

$$\frac{4(cx)^{5/4} \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1 \left( \frac{5}{8}, \frac{7}{4}, \frac{13}{8}; -\frac{bx^2}{a} \right)}{5ac(a+bx^2)^{3/4}}$$

[Out]  $(4*(c*x)^{(5/4)}*(1+(b*x^2)/a)^{(3/4)}*\text{Hypergeometric2F1}[5/8, 7/4, 13/8, -(b*x^2)/a])/(5*a*c*(a+b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.0655268, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{4(cx)^{5/4} \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1 \left( \frac{5}{8}, \frac{7}{4}, \frac{13}{8}; -\frac{bx^2}{a} \right)}{5ac(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*x)^{(1/4)}/(a+b*x^2)^{(7/4)}, x]$

[Out]  $(4*(c*x)^{(5/4)}*(1+(b*x^2)/a)^{(3/4)}*\text{Hypergeometric2F1}[5/8, 7/4, 13/8, -(b*x^2)/a])/(5*a*c*(a+b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 8.19319, size = 51, normalized size = 0.84

$$\frac{4(cx)^{5/4} \sqrt[4]{a+bx^2} {}_2F_1 \left( \frac{7}{4}, \frac{5}{8}; \frac{13}{8}; -\frac{bx^2}{a} \right)}{5a^2 c \sqrt[4]{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c*x)**(1/4)/(b*x**2+a)**(7/4), x)$

[Out]  $4*(c*x)**(5/4)*(a+b*x**2)**(1/4)*\text{hyper}((7/4, 5/8), (13/8, ), -b*x**2/a)/(5*a**2*c*(1+b*x**2/a)**(1/4))$

**Mathematica [A]** time = 0.0488793, size = 62, normalized size = 1.02

$$\frac{2x\sqrt[4]{cx} \left( \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1 \left( \frac{5}{8}, \frac{3}{4}, \frac{13}{8}; -\frac{bx^2}{a} \right) + 5 \right)}{15a(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(c*x)^{(1/4)}/(a+b*x^2)^{(7/4)}, x]$

[Out]  $(2*x*(c*x)^{(1/4)}*(5+(1+(b*x^2)/a)^{(3/4)}*\text{Hypergeometric2F1}[5/8, 3/4, 13/8, -(b*x^2)/a]))/(15*a*(a+b*x^2)^{(3/4)})$

**Maple [F]** time = 0.05, size = 0, normalized size = 0.

$$\int \sqrt[4]{cx} (bx^2 + a)^{-\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x)^(1/4)/(b\*x^2+a)^(7/4), x)

[Out] int((c\*x)^(1/4)/(b\*x^2+a)^(7/4), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{1}{4}}}{(bx^2 + a)^{\frac{7}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(1/4)/(b\*x^2 + a)^(7/4), x, algorithm="maxima")

[Out] integrate((c\*x)^(1/4)/(b\*x^2 + a)^(7/4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^{\frac{1}{4}}}{(bx^2 + a)^{\frac{7}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(1/4)/(b\*x^2 + a)^(7/4), x, algorithm="fricas")

[Out] integral((c\*x)^(1/4)/(b\*x^2 + a)^(7/4), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)\*\*(1/4)/(b\*x\*\*2+a)\*\*(7/4), x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{1}{4}}}{(bx^2 + a)^{\frac{7}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x)^(1/4)/(b\*x^2 + a)^(7/4), x, algorithm="giac")

[Out] integrate((c\*x)^(1/4)/(b\*x^2 + a)^(7/4), x)

$$3.1007 \quad \int \frac{1}{\sqrt[4]{cx(a+bx^2)^{7/4}}} dx$$

**Optimal.** Leaf size=61

$$\frac{4(cx)^{3/4} \left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{3}{8}, \frac{7}{4}, \frac{11}{8}, -\frac{bx^2}{a}\right)}{3ac(a+bx^2)^{3/4}}$$

[Out]  $(4*(c*x)^{(3/4)}*(1+(b*x^2)/a)^{(3/4)}*\text{Hypergeometric2F1}[3/8, 7/4, 11/8, -(b*x^2)/a])/(3*a*c*(a+b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.0661082, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{4(cx)^{3/4} \left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{3}{8}, \frac{7}{4}, \frac{11}{8}, -\frac{bx^2}{a}\right)}{3ac(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(1/4)\*(a+b\*x^2)^(7/4)),x]

[Out]  $(4*(c*x)^{(3/4)}*(1+(b*x^2)/a)^{(3/4)}*\text{Hypergeometric2F1}[3/8, 7/4, 11/8, -(b*x^2)/a])/(3*a*c*(a+b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 8.31156, size = 51, normalized size = 0.84

$$\frac{4(cx)^{\frac{3}{4}} \sqrt[4]{a+bx^2} {}_2F_1\left(\frac{7}{4}, \frac{3}{8}, \frac{11}{8}, -\frac{bx^2}{a}\right)}{3a^2c\sqrt[4]{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(1/4)/(b\*x\*\*2+a)\*\*(7/4),x)

[Out]  $4*(c*x)**(3/4)*(a+b*x**2)**(1/4)*\text{hyper}((7/4, 3/8), (11/8, ), -b*x**2/a)/(3*a**2*c*(1+b*x**2/a)**(1/4))$

**Mathematica [A]** time = 0.0471261, size = 62, normalized size = 1.02

$$\frac{2\left(x\left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{3}{8}, \frac{3}{4}, \frac{11}{8}, -\frac{bx^2}{a}\right) + x\right)}{3a\sqrt[4]{cx}(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(1/4)\*(a+b\*x^2)^(7/4)),x]

[Out]  $(2*(x+x*(1+(b*x^2)/a)^{(3/4)}*\text{Hypergeometric2F1}[3/8, 3/4, 11/8, -(b*x^2)/a])/(3*a*(c*x)^{(1/4)}*(a+b*x^2)^{(3/4)})$

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{cx}} (bx^2 + a)^{-\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(1/4)/(b*x^2+a)^(7/4), x)`

[Out] `int(1/(c*x)^(1/4)/(b*x^2+a)^(7/4), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{7}{4}} (cx)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(1/4)), x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(1/4)), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{7}{4}} (cx)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(1/4)), x, algorithm="fricas")`

[Out] `integral(1/((b*x^2 + a)^(7/4)*(c*x)^(1/4)), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(1/4)/(b*x**2+a)**(7/4), x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{7}{4}} (cx)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(1/4)), x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(1/4)), x)`

$$3.1008 \quad \int \frac{1}{(cx)^{3/4}(a+bx^2)^{7/4}} dx$$

**Optimal.** Leaf size=59

$$\frac{4\sqrt[4]{cx} \left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{1}{8}, \frac{7}{4}; \frac{9}{8}; -\frac{bx^2}{a}\right)}{ac(a+bx^2)^{3/4}}$$

[Out]  $(4*(c*x)^{(1/4)}*(1+(b*x^2)/a)^{(3/4)}*\text{Hypergeometric2F1}[1/8, 7/4, 9/8, -(b*x^2)/a])/(a*c*(a+b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.0668793, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{4\sqrt[4]{cx} \left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{1}{8}, \frac{7}{4}; \frac{9}{8}; -\frac{bx^2}{a}\right)}{ac(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(3/4)\*(a+b\*x^2)^(7/4)),x]

[Out]  $(4*(c*x)^{(1/4)}*(1+(b*x^2)/a)^{(3/4)}*\text{Hypergeometric2F1}[1/8, 7/4, 9/8, -(b*x^2)/a])/(a*c*(a+b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 8.32128, size = 49, normalized size = 0.83

$$\frac{4\sqrt[4]{cx}\sqrt[4]{a+bx^2}{}_2F_1\left(\frac{7}{4}, \frac{1}{8} \middle| \frac{9}{8}; -\frac{bx^2}{a}\right)}{a^2c^4\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(3/4)/(b\*x\*\*2+a)\*\*(7/4),x)

[Out]  $4*(c*x)**(1/4)*(a+b*x**2)**(1/4)*\text{hyper}((7/4, 1/8), (9/8, ), -b*x**2/a)/(a**2*c*(1+b*x**2/a)**(1/4))$

**Mathematica [A]** time = 0.045259, size = 63, normalized size = 1.07

$$\frac{2\left(5x\left(\frac{bx^2}{a} + 1\right)^{3/4} {}_2F_1\left(\frac{1}{8}, \frac{3}{4}; \frac{9}{8}; -\frac{bx^2}{a}\right) + x\right)}{3a(cx)^{3/4}(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(3/4)\*(a+b\*x^2)^(7/4)),x]

[Out]  $(2*(x+5*x*(1+(b*x^2)/a)^{(3/4)}*\text{Hypergeometric2F1}[1/8, 3/4, 9/8, -(b*x^2)/a])/(3*a*(c*x)^{(3/4)}*(a+b*x^2)^{(3/4)})$

**Maple [F]** time = 0.057, size = 0, normalized size = 0.

$$\int 1 (cx)^{-\frac{3}{4}} (bx^2 + a)^{-\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x)`

[Out] `int(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{7}{4}} (cx)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(3/4)),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(3/4)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{7}{4}} (cx)^{\frac{3}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(3/4)),x, algorithm="fricas")`

[Out] `integral(1/((b*x^2 + a)^(7/4)*(c*x)^(3/4)), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(3/4)/(b*x**2+a)**(7/4),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{7}{4}} (cx)^{\frac{3}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(3/4)),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(3/4)), x)`

$$3.1009 \quad \int \frac{1}{(cx)^{5/4}(a+bx^2)^{7/4}} dx$$

**Optimal.** Leaf size=59

$$\frac{4 \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1 \left( -\frac{1}{8}, \frac{7}{4}; \frac{7}{8}; -\frac{bx^2}{a} \right)}{ac\sqrt[4]{cx} (a + bx^2)^{3/4}}$$

[Out]  $(-4*(1 + (b*x^2)/a)^{(3/4)}*Hypergeometric2F1[-1/8, 7/4, 7/8, -(b*x^2)/a])/(a*c*(c*x)^{(1/4)}*(a + b*x^2)^{(3/4)})$

**Rubi [A]** time = 0.0661165, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{4 \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1 \left( -\frac{1}{8}, \frac{7}{4}; \frac{7}{8}; -\frac{bx^2}{a} \right)}{ac\sqrt[4]{cx} (a + bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[1/((c\*x)^(5/4)\*(a + b\*x^2)^(7/4)),x]

[Out]  $(-4*(1 + (b*x^2)/a)^{(3/4)}*Hypergeometric2F1[-1/8, 7/4, 7/8, -(b*x^2)/a])/(a*c*(c*x)^{(1/4)}*(a + b*x^2)^{(3/4)})$

**Rubi in Sympy [A]** time = 8.34575, size = 53, normalized size = 0.9

$$\frac{4\sqrt[4]{a + bx^2} {}_2F_1 \left( \frac{7}{4}, -\frac{1}{8} \middle| -\frac{bx^2}{a} \right)}{a^2 c \sqrt[4]{cx} \sqrt{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(c\*x)\*\*(5/4)/(b\*x\*\*2+a)\*\*(7/4),x)

[Out]  $-4*(a + b*x**2)**(1/4)*hyper((7/4, -1/8), (7/8, ), -b*x**2/a)/(a**2*c*(c*x)**(1/4)*(1 + b*x**2/a)**(1/4))$

**Mathematica [A]** time = 0.0613923, size = 74, normalized size = 1.25

$$\frac{2x \left( bx^2 \left( \frac{bx^2}{a} + 1 \right)^{3/4} {}_2F_1 \left( \frac{3}{4}, \frac{7}{8}; \frac{15}{8}; -\frac{bx^2}{a} \right) - 6a - 7bx^2 \right)}{3a^2(cx)^{5/4} (a + bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*x)^(5/4)\*(a + b\*x^2)^(7/4)),x]

[Out]  $(2*x*(-6*a - 7*b*x^2 + b*x^2*(1 + (b*x^2)/a)^{(3/4)}*Hypergeometric2F1[3/4, 7/8, 15/8, -(b*x^2)/a])/(3*a^2*(c*x)^{(5/4)}*(a + b*x^2)^{(3/4)})$



**Maple [F]** time = 0.078, size = 0, normalized size = 0.

$$\int 1 (cx)^{-\frac{5}{4}} (bx^2 + a)^{-\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x)^(5/4)/(b*x^2+a)^(7/4), x)`

[Out] `int(1/(c*x)^(5/4)/(b*x^2+a)^(7/4), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{7}{4}} (cx)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(5/4)), x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(5/4)), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bcx^3 + acx)(bx^2 + a)^{\frac{3}{4}} (cx)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(5/4)), x, algorithm="fricas")`

[Out] `integral(1/((b*c*x^3 + a*c*x)*(b*x^2 + a)^(3/4)*(c*x)^(1/4)), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x)**(5/4)/(b*x**2+a)**(7/4), x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{7}{4}} (cx)^{\frac{5}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(5/4)), x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(5/4)), x)`

### 3.1010 $\int x^6 \sqrt[6]{a + bx^2} dx$

**Optimal.** Leaf size=345

$$\begin{aligned}
 & 81 \cdot 3^{3/4} \sqrt{2 - \sqrt{3}} a^4 \sqrt[6]{a + bx^2} \left( 1 - \sqrt[3]{\frac{a}{a + bx^2}} \right) \sqrt{\frac{\left(\frac{a}{a + bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a + bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a + bx^2}} - \sqrt{3} + 1\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a + bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a + bx^2}} - \sqrt{3} + 1}\right) \middle| -7 + 4\sqrt{3}\right) \\
 & \frac{2816b^4 x^3 \sqrt[3]{\frac{a}{a + bx^2}} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a + bx^2}}}{\left(-\sqrt[3]{\frac{a}{a + bx^2}} - \sqrt{3} + 1\right)^2}}}{2816b^3} - \frac{9a^2 x^3 \sqrt[6]{a + bx^2}}{704b^2} + \frac{3}{22} x^7 \sqrt[6]{a + bx^2} + \frac{3ax^5 \sqrt[6]{a + bx^2}}{352b}
 \end{aligned}$$

[Out]  $(81 \cdot a^3 \cdot x \cdot (a + b \cdot x^2)^{(1/6)}) / (2816 \cdot b^3) - (9 \cdot a^2 \cdot x^3 \cdot (a + b \cdot x^2)^{(1/6)}) / (704 \cdot b^2) + (3 \cdot a \cdot x^5 \cdot (a + b \cdot x^2)^{(1/6)}) / (352 \cdot b) + (3 \cdot x^7 \cdot (a + b \cdot x^2)^{(1/6)}) / 22 - (81 \cdot 3^{(3/4)} \cdot \text{Sqrt}[2 - \text{Sqrt}[3]] \cdot a^4 \cdot (a + b \cdot x^2)^{(1/6)} \cdot (1 - (a / (a + b \cdot x^2))^{(1/3)}) \cdot \text{Sqrt}[(1 + (a / (a + b \cdot x^2))^{(1/3)} + (a / (a + b \cdot x^2))^{(2/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})]^2 \cdot \text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})], -7 + 4 \cdot \text{Sqrt}[3]]) / (2816 \cdot b^4 \cdot x \cdot (a / (a + b \cdot x^2))^{(1/3)} \cdot \text{Sqrt}[-((1 - (a / (a + b \cdot x^2))^{(1/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})^2)])$

**Rubi [A]** time = 0.890334, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\begin{aligned}
 & 81 \cdot 3^{3/4} \sqrt{2 - \sqrt{3}} a^4 \sqrt[6]{a + bx^2} \left( 1 - \sqrt[3]{\frac{a}{a + bx^2}} \right) \sqrt{\frac{\left(\frac{a}{a + bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a + bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a + bx^2}} - \sqrt{3} + 1\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a + bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a + bx^2}} - \sqrt{3} + 1}\right) \middle| -7 + 4\sqrt{3}\right) \\
 & \frac{2816b^4 x^3 \sqrt[3]{\frac{a}{a + bx^2}} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a + bx^2}}}{\left(-\sqrt[3]{\frac{a}{a + bx^2}} - \sqrt{3} + 1\right)^2}}}{2816b^3} - \frac{9a^2 x^3 \sqrt[6]{a + bx^2}}{704b^2} + \frac{3}{22} x^7 \sqrt[6]{a + bx^2} + \frac{3ax^5 \sqrt[6]{a + bx^2}}{352b}
 \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^6 \cdot (a + b \cdot x^2)^{(1/6)}, x]$

[Out]  $(81 \cdot a^3 \cdot x \cdot (a + b \cdot x^2)^{(1/6)}) / (2816 \cdot b^3) - (9 \cdot a^2 \cdot x^3 \cdot (a + b \cdot x^2)^{(1/6)}) / (704 \cdot b^2) + (3 \cdot a \cdot x^5 \cdot (a + b \cdot x^2)^{(1/6)}) / (352 \cdot b) + (3 \cdot x^7 \cdot (a + b \cdot x^2)^{(1/6)}) / 22 - (81 \cdot 3^{(3/4)} \cdot \text{Sqrt}[2 - \text{Sqrt}[3]] \cdot a^4 \cdot (a + b \cdot x^2)^{(1/6)} \cdot (1 - (a / (a + b \cdot x^2))^{(1/3)}) \cdot \text{Sqrt}[(1 + (a / (a + b \cdot x^2))^{(1/3)} + (a / (a + b \cdot x^2))^{(2/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})]^2 \cdot \text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})], -7 + 4 \cdot \text{Sqrt}[3]]) / (2816 \cdot b^4 \cdot x \cdot (a / (a + b \cdot x^2))^{(1/3)} \cdot \text{Sqrt}[-((1 - (a / (a + b \cdot x^2))^{(1/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})^2)])$

**Rubi in Sympy [A]** time = 29.8853, size = 320, normalized size = 0.93

$$81 \cdot 3^{\frac{3}{4}} a^4 \sqrt{\frac{\left(-\frac{bx^2}{a+bx^2}+1\right)^{\frac{2}{3}} + \sqrt{-\frac{bx^2}{a+bx^2}+1} + 1}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2}} \sqrt{-\sqrt{3}+2} \sqrt[6]{a+bx^2} \left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1\right) F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+\sqrt{3}}}{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}}\right)\right) - \frac{2816b^4x^3\sqrt{\frac{a}{a+bx^2}}\sqrt{\frac{\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2}}}{2816b^3} - \frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2} + \frac{3ax^5\sqrt[6]{a+bx^2}}{352b} + \frac{3x^7\sqrt[6]{a+bx^2}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**6*(b*x**2+a)**(1/6),x)`

[Out]  $-81 \cdot 3^{3/4} \cdot a^{4/3} \cdot \sqrt{\left(\left(-b \cdot x^{2/3} / (a + b \cdot x^{2/3}) + 1\right)^{2/3} + \left(-b \cdot x^{2/3} / (a + b \cdot x^{2/3}) + 1\right)^{1/3} + 1\right) / \left(-\left(-b \cdot x^{2/3} / (a + b \cdot x^{2/3}) + 1\right)^{1/3} - \sqrt{3} + 1\right)^2} \cdot \sqrt{-\sqrt{3} + 2} \cdot (a + b \cdot x^{2/3})^{1/6} \cdot \left(-\left(-b \cdot x^{2/3} / (a + b \cdot x^{2/3}) + 1\right)^{1/3} + 1\right) \cdot \operatorname{elliptic\_f}\left(\operatorname{asin}\left(\frac{-\left(-b \cdot x^{2/3} / (a + b \cdot x^{2/3}) + 1\right)^{1/3} + 1 + \sqrt{3}}{-\left(-b \cdot x^{2/3} / (a + b \cdot x^{2/3}) + 1\right)^{1/3} - \sqrt{3} + 1}\right)\right) / \left(-\left(-b \cdot x^{2/3} / (a + b \cdot x^{2/3}) + 1\right)^{1/3} - \sqrt{3} + 1\right) - 7 + 4 \cdot \sqrt{3}}{2816 \cdot b^4 \cdot x^3 \cdot (a / (a + b \cdot x^{2/3}))^{1/3} \cdot \sqrt{\left(\left(-b \cdot x^{2/3} / (a + b \cdot x^{2/3}) + 1\right)^{1/3} - 1\right) / \left(-\left(-b \cdot x^{2/3} / (a + b \cdot x^{2/3}) + 1\right)^{1/3} - \sqrt{3} + 1\right)^2}} + 81 \cdot a^{3/3} \cdot x^3 \cdot (a + b \cdot x^{2/3})^{1/6} / (2816 \cdot b^3) - 9 \cdot a^{2/3} \cdot x^3 \cdot (a + b \cdot x^{2/3})^{1/6} / (704 \cdot b^2) + 3 \cdot a^{1/3} \cdot x^5 \cdot (a + b \cdot x^{2/3})^{1/6} / (352 \cdot b) + 3 \cdot x^7 \cdot (a + b \cdot x^{2/3})^{1/6} / 22$

**Mathematica [C]** time = 0.0790908, size = 101, normalized size = 0.29

$$\frac{3 \left( -27a^4x \left( \frac{bx^2}{a} + 1 \right)^{5/6} {}_2F_1 \left( \frac{1}{2}, \frac{5}{6}; \frac{3}{2}; -\frac{bx^2}{a} \right) + 27a^4x + 15a^3bx^3 - 4a^2b^2x^5 + 136ab^3x^7 + 128b^4x^9 \right)}{2816b^3(a+bx^2)^{5/6}}$$

Antiderivative was successfully verified.

[In] `Integrate[x^6*(a+b*x^2)^(1/6),x]`

[Out]  $(3 \cdot (27 \cdot a^4 \cdot x + 15 \cdot a^3 \cdot b \cdot x^3 - 4 \cdot a^2 \cdot b^2 \cdot x^5 + 136 \cdot a \cdot b^3 \cdot x^7 + 128 \cdot b^4 \cdot x^9 - 27 \cdot a^4 \cdot x \cdot (1 + (b \cdot x^2)/a)^{5/6} \cdot \operatorname{Hypergeometric2F1}[1/2, 5/6, 3/2, -(b \cdot x^2)/a])) / (2816 \cdot b^3 \cdot (a + b \cdot x^2)^{5/6})$

**Maple [F]** time = 0.049, size = 0, normalized size = 0.

$$\int x^6 \sqrt[6]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*(b*x^2+a)^(1/6),x)`

[Out] `int(x^6*(b*x^2+a)^(1/6),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{6}} x^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/6)*x^6,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(1/6)*x^6, x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^{\frac{1}{6}}x^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/6)*x^6,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(1/6)*x^6, x)`

**Sympy** [A] time = 5.17152, size = 29, normalized size = 0.08

$$\frac{\sqrt[6]{ax^7} {}_2F_1\left(-\frac{1}{6}, \frac{7}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6*(b*x**2+a)**(1/6), x)`

[Out] `a**(1/6)*x**7*hyper((-1/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/7`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{6}} x^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/6)*x^6,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(1/6)*x^6, x)`

### 3.1011 $\int x^4 \sqrt[6]{a + bx^2} dx$

**Optimal.** Leaf size=321

$$\frac{27 \cdot 3^{3/4} \sqrt{2 - \sqrt{3}} a^3 \sqrt[6]{a + bx^2} \left(1 - \sqrt[3]{\frac{a}{a + bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a + bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a + bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a + bx^2} - \sqrt{3} + 1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a + bx^2} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{a + bx^2} - \sqrt{3} + 1}}\right) \middle| -7 + 4\sqrt{3}\right)}{640b^3 x^3 \sqrt[3]{\frac{a}{a + bx^2}} \sqrt{-\frac{1 - \sqrt[3]{\frac{a}{a + bx^2}}}{\left(-\sqrt[3]{\frac{a}{a + bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$- \frac{27a^2 x \sqrt[6]{a + bx^2}}{640b^2} + \frac{3}{16} x^5 \sqrt[6]{a + bx^2} + \frac{3ax^3 \sqrt[6]{a + bx^2}}{160b}$$

[Out]  $(-27 \cdot a^2 \cdot x \cdot (a + b \cdot x^2)^{(1/6)}) / (640 \cdot b^2) + (3 \cdot a \cdot x^3 \cdot (a + b \cdot x^2)^{(1/6)}) / (160 \cdot b) + (3 \cdot x^5 \cdot (a + b \cdot x^2)^{(1/6)}) / 16 + (27 \cdot 3^{(3/4)} \cdot \text{Sqrt}[2 - \text{Sqrt}[3]] \cdot a^3 \cdot (a + b \cdot x^2)^{(1/6)} \cdot (1 - (a / (a + b \cdot x^2))^{(1/3)}) \cdot \text{Sqrt}[(1 + (a / (a + b \cdot x^2))^{(1/3)} + (a / (a + b \cdot x^2))^{(2/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})^2] \cdot \text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})], -7 + 4 \cdot \text{Sqrt}[3]]) / (640 \cdot b^3 \cdot x^3 \cdot (a / (a + b \cdot x^2))^{(1/3)} \cdot \text{Sqrt}[-((1 - (a / (a + b \cdot x^2))^{(1/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})^2)])$

**Rubi [A]** time = 0.668398, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{27 \cdot 3^{3/4} \sqrt{2 - \sqrt{3}} a^3 \sqrt[6]{a + bx^2} \left(1 - \sqrt[3]{\frac{a}{a + bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a + bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a + bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a + bx^2} - \sqrt{3} + 1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a + bx^2} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{a + bx^2} - \sqrt{3} + 1}}\right) \middle| -7 + 4\sqrt{3}\right)}{640b^3 x^3 \sqrt[3]{\frac{a}{a + bx^2}} \sqrt{-\frac{1 - \sqrt[3]{\frac{a}{a + bx^2}}}{\left(-\sqrt[3]{\frac{a}{a + bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$- \frac{27a^2 x \sqrt[6]{a + bx^2}}{640b^2} + \frac{3}{16} x^5 \sqrt[6]{a + bx^2} + \frac{3ax^3 \sqrt[6]{a + bx^2}}{160b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^4 \cdot (a + b \cdot x^2)^{(1/6)}, x]$

[Out]  $(-27 \cdot a^2 \cdot x \cdot (a + b \cdot x^2)^{(1/6)}) / (640 \cdot b^2) + (3 \cdot a \cdot x^3 \cdot (a + b \cdot x^2)^{(1/6)}) / (160 \cdot b) + (3 \cdot x^5 \cdot (a + b \cdot x^2)^{(1/6)}) / 16 + (27 \cdot 3^{(3/4)} \cdot \text{Sqrt}[2 - \text{Sqrt}[3]] \cdot a^3 \cdot (a + b \cdot x^2)^{(1/6)} \cdot (1 - (a / (a + b \cdot x^2))^{(1/3)}) \cdot \text{Sqrt}[(1 + (a / (a + b \cdot x^2))^{(1/3)} + (a / (a + b \cdot x^2))^{(2/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})^2] \cdot \text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})], -7 + 4 \cdot \text{Sqrt}[3]]) / (640 \cdot b^3 \cdot x^3 \cdot (a / (a + b \cdot x^2))^{(1/3)} \cdot \text{Sqrt}[-((1 - (a / (a + b \cdot x^2))^{(1/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})^2)])$

**Rubi in Sympy [A]** time = 23.4581, size = 296, normalized size = 0.92

$$27 \cdot 3^{\frac{3}{4}} a^3 \sqrt{\frac{\left(-\frac{bx^2}{a+bx^2}+1\right)^{\frac{2}{3}} + \sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} - \sqrt{3} + 1\right)^2}} \sqrt{-\sqrt{3} + 2} \sqrt[6]{a+bx^2} \left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1\right) F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1 + \sqrt{3}}{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} - \sqrt{3} + 1}\right)\right) - 7$$


---


$$640b^3x^3\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} - 1}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} - \sqrt{3} + 1\right)^2}}$$

$$-\frac{27a^2x\sqrt[6]{a+bx^2}}{640b^2} + \frac{3ax^3\sqrt[6]{a+bx^2}}{160b} + \frac{3x^5\sqrt[6]{a+bx^2}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**4*(b*x**2+a)**(1/6),x)`

[Out]  $27 \cdot 3^{3/4} \cdot a^3 \cdot \sqrt{\left(\left(-b \cdot x^2 / (a + b \cdot x^2) + 1\right)^{2/3} + \left(-b \cdot x^2 / (a + b \cdot x^2) + 1\right)^{1/3} + 1\right) / \left(-\left(-b \cdot x^2 / (a + b \cdot x^2) + 1\right)^{1/3} - \sqrt{3} + 1\right)^2} \cdot \sqrt{-\sqrt{3} + 2} \cdot (a + b \cdot x^2)^{1/6} \cdot \left(-\left(-b \cdot x^2 / (a + b \cdot x^2) + 1\right)^{1/3} + 1\right) \cdot \operatorname{elliptic\_f}\left(\operatorname{asin}\left(\frac{-\left(-b \cdot x^2 / (a + b \cdot x^2) + 1\right)^{1/3} + 1 + \sqrt{3}}{-\left(-b \cdot x^2 / (a + b \cdot x^2) + 1\right)^{1/3} - \sqrt{3} + 1}\right), -7 + 4 \cdot \sqrt{3}\right) / \left(640 \cdot b^3 \cdot x^3 \cdot (a / (a + b \cdot x^2))^{1/3} \cdot \sqrt{\left(\left(-b \cdot x^2 / (a + b \cdot x^2) + 1\right)^{1/3} - 1\right) / \left(-\left(-b \cdot x^2 / (a + b \cdot x^2) + 1\right)^{1/3} - \sqrt{3} + 1\right)^2}\right) - 27 \cdot a^2 \cdot x \cdot (a + b \cdot x^2)^{1/6} / \left(640 \cdot b^2\right) + 3 \cdot a \cdot x^3 \cdot (a + b \cdot x^2)^{1/6} / \left(160 \cdot b\right) + 3 \cdot x^5 \cdot (a + b \cdot x^2)^{1/6} / 16$

**Mathematica [C]** time = 0.0537392, size = 90, normalized size = 0.28

$$\frac{3 \left( 9a^3x \left( \frac{bx^2}{a} + 1 \right)^{5/6} {}_2F_1 \left( \frac{1}{2}, \frac{5}{6}; \frac{3}{2}; -\frac{bx^2}{a} \right) - 9a^3x - 5a^2bx^3 + 44ab^2x^5 + 40b^3x^7 \right)}{640b^2(a+bx^2)^{5/6}}$$

Antiderivative was successfully verified.

[In] `Integrate[x^4*(a+b*x^2)^(1/6),x]`

[Out]  $(3 \cdot (-9 \cdot a^3 \cdot x - 5 \cdot a^2 \cdot b \cdot x^3 + 44 \cdot a \cdot b^2 \cdot x^5 + 40 \cdot b^3 \cdot x^7 + 9 \cdot a^3 \cdot x \cdot (1 + (b \cdot x^2) / a)^{5/6}) \cdot \operatorname{Hypergeometric2F1}\left[1/2, 5/6, 3/2, -((b \cdot x^2) / a)\right]) / (640 \cdot b^2 \cdot (a + b \cdot x^2)^{5/6})$

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int x^4 \sqrt[6]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(b*x^2+a)^(1/6),x)`

[Out] `int(x^4*(b*x^2+a)^(1/6),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{6}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/6)*x^4,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(1/6)*x^4, x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^{\frac{1}{6}}x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/6)*x^4,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(1/6)*x^4, x)`

**Sympy** [A] time = 3.81465, size = 29, normalized size = 0.09

$$\frac{\sqrt[6]{ax^5} {}_2F_1\left(-\frac{1}{6}, \frac{5}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(b*x**2+a)**(1/6), x)`

[Out] `a**(1/6)*x**5*hyper((-1/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{6}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/6)*x^4,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(1/6)*x^4, x)`

### 3.1012 $\int x^2 \sqrt[6]{a + bx^2} dx$

**Optimal.** Leaf size=297

$$\begin{aligned}
 & 3 \cdot 3^{3/4} \sqrt{2 - \sqrt{3}} a^2 \sqrt[6]{a + bx^2} \left( 1 - \sqrt[3]{\frac{a}{a + bx^2}} \right) \sqrt[3]{\frac{\left(\frac{a}{a + bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a + bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a + bx^2}} - \sqrt{3} + 1\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2 + a}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{bx^2 + a}} - \sqrt{3} + 1}\right) \mid -7 + 4\sqrt{3}\right) \\
 & \frac{40b^2 x^3 \sqrt[3]{\frac{a}{a + bx^2}} \sqrt[3]{\frac{1 - \sqrt[3]{\frac{a}{a + bx^2}}}{\left(-\sqrt[3]{\frac{a}{a + bx^2}} - \sqrt{3} + 1\right)^2}}}{\sqrt[3]{\left(-\sqrt[3]{\frac{a}{a + bx^2}} - \sqrt{3} + 1\right)^2}} \\
 & + \frac{3ax \sqrt[6]{a + bx^2}}{40b} + \frac{3}{10} x^3 \sqrt[6]{a + bx^2}
 \end{aligned}$$

[Out] (3\*a\*x\*(a + b\*x^2)^(1/6))/(40\*b) + (3\*x^3\*(a + b\*x^2)^(1/6))/10 - (3\*3^(3/4)\*Sqrt[2 - Sqrt[3]]\*a^2\*(a + b\*x^2)^(1/6)\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))^2]\*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(40\*b^2\*x\*(a/(a + b\*x^2))^(1/3)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))^2)])

**Rubi [A]** time = 0.561333, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\begin{aligned}
 & 3 \cdot 3^{3/4} \sqrt{2 - \sqrt{3}} a^2 \sqrt[6]{a + bx^2} \left( 1 - \sqrt[3]{\frac{a}{a + bx^2}} \right) \sqrt[3]{\frac{\left(\frac{a}{a + bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a + bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a + bx^2}} - \sqrt{3} + 1\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2 + a}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{bx^2 + a}} - \sqrt{3} + 1}\right) \mid -7 + 4\sqrt{3}\right) \\
 & \frac{40b^2 x^3 \sqrt[3]{\frac{a}{a + bx^2}} \sqrt[3]{\frac{1 - \sqrt[3]{\frac{a}{a + bx^2}}}{\left(-\sqrt[3]{\frac{a}{a + bx^2}} - \sqrt{3} + 1\right)^2}}}{\sqrt[3]{\left(-\sqrt[3]{\frac{a}{a + bx^2}} - \sqrt{3} + 1\right)^2}} \\
 & + \frac{3ax \sqrt[6]{a + bx^2}}{40b} + \frac{3}{10} x^3 \sqrt[6]{a + bx^2}
 \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*x^2)^(1/6), x]

[Out] (3\*a\*x\*(a + b\*x^2)^(1/6))/(40\*b) + (3\*x^3\*(a + b\*x^2)^(1/6))/10 - (3\*3^(3/4)\*Sqrt[2 - Sqrt[3]]\*a^2\*(a + b\*x^2)^(1/6)\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))^2]\*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(40\*b^2\*x\*(a/(a + b\*x^2))^(1/3)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))^2)])



**Rubi in Sympy [A]** time = 17.8313, size = 272, normalized size = 0.92

$$3 \cdot 3^{\frac{3}{4}} a^2 \sqrt{\frac{\left(-\frac{bx^2}{a+bx^2}+1\right)^{\frac{2}{3}} + \sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2}} \sqrt{-\sqrt{3}+2\sqrt{a+bx^2}} \left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1}\right) F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1+\sqrt{3}}}{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}}\right)\right) - 7$$


---


$$40b^2 x \sqrt[3]{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2}}$$

$$+ \frac{3ax\sqrt[6]{a+bx^2}}{40b} + \frac{3x^3\sqrt[6]{a+bx^2}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2*(b*x**2+a)**(1/6),x)`

[Out]  $-3 \cdot 3^{3/4} \cdot a^{2/3} \cdot \sqrt{\left(\left(-\frac{bx^2}{a+bx^2}+1\right)^{2/3} + \left(-\frac{bx^2}{a+bx^2}+1\right)^{1/3} + 1\right) / \left(-\left(-\frac{bx^2}{a+bx^2}+1\right)^{1/3} - \sqrt{3} + 1\right)^2} \cdot \sqrt{-\sqrt{3}+2\sqrt{a+bx^2}} \cdot \left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1}\right) \cdot F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1+\sqrt{3}}}{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}}\right)\right) - 7 + 4\sqrt{3} \cdot \sqrt[3]{\frac{a}{a+bx^2}} \cdot \sqrt{\frac{\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2}} + 3ax\sqrt[6]{a+bx^2} + 3x^3\sqrt[6]{a+bx^2}$

**Mathematica [C]** time = 0.0580667, size = 76, normalized size = 0.26

$$\frac{3x \left( -a^2 \left( \frac{bx^2}{a} + 1 \right)^{5/6} {}_2F_1 \left( \frac{1}{2}, \frac{5}{6}; \frac{3}{2}; -\frac{bx^2}{a} \right) + a^2 + 5abx^2 + 4b^2x^4 \right)}{40b(a+bx^2)^{5/6}}$$

Antiderivative was successfully verified.

[In] `Integrate[x^2*(a+b*x^2)^(1/6),x]`

[Out]  $(3x^3(a^2 + 5abx^2 + 4b^2x^4) - a^2(1 + (bx^2)/a)^{5/6} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{3}{2}, -\left(\frac{bx^2}{a}\right)\right]) / (40b(a+bx^2)^{5/6})$

**Maple [F]** time = 0.037, size = 0, normalized size = 0.

$$\int x^2 \sqrt[6]{bx^2+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^2+a)^(1/6),x)`

[Out] `int(x^2*(b*x^2+a)^(1/6),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2+a)^{\frac{1}{6}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/6)*x^2,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(1/6)*x^2, x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^{\frac{1}{6}}x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/6)*x^2,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(1/6)*x^2, x)`

**Sympy** [A] time = 3.02929, size = 29, normalized size = 0.1

$$\frac{\sqrt[6]{ax^3} {}_2F_1\left(-\frac{1}{6}, \frac{3}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**2+a)**(1/6), x)`

[Out] `a**(1/6)*x**3*hyper((-1/6, 3/2), (5/2, ), b*x**2*exp_polar(I*pi)/a)/3`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{6}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/6)*x^2,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(1/6)*x^2, x)`

### 3.1013 $\int \sqrt[6]{a + bx^2} dx$

**Optimal.** Leaf size=273

$$\frac{3}{4}x\sqrt[6]{a + bx^2}$$

$$\frac{3^{3/4}\sqrt{2 - \sqrt{3}}a\sqrt[6]{a + bx^2}\left(1 - \sqrt[3]{\frac{a}{a + bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a + bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a + bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a + bx^2}} - \sqrt{3} + 1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a + bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a + bx^2}} - \sqrt{3} + 1}\right)\right)|_{-7 + 4\sqrt{3}}}{4bx\sqrt[3]{\frac{a}{a + bx^2}}\sqrt{\frac{1 - \sqrt[3]{\frac{a}{a + bx^2}}}{\left(-\sqrt[3]{\frac{a}{a + bx^2}} - \sqrt{3} + 1\right)^2}}}$$

[Out] (3\*x\*(a + b\*x^2)^(1/6))/4 + (3^(3/4)\*Sqrt[2 - Sqrt[3]]\*a\*(a + b\*x^2)^(1/6)\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))]^2\*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]]/(4\*b\*x\*(a/(a + b\*x^2))^(1/3)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3)))^2]))

**Rubi [A]** time = 0.470247, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$\frac{3}{4}x\sqrt[6]{a + bx^2}$$

$$\frac{3^{3/4}\sqrt{2 - \sqrt{3}}a\sqrt[6]{a + bx^2}\left(1 - \sqrt[3]{\frac{a}{a + bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a + bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a + bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a + bx^2}} - \sqrt{3} + 1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a + bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a + bx^2}} - \sqrt{3} + 1}\right)\right)|_{-7 + 4\sqrt{3}}}{4bx\sqrt[3]{\frac{a}{a + bx^2}}\sqrt{\frac{1 - \sqrt[3]{\frac{a}{a + bx^2}}}{\left(-\sqrt[3]{\frac{a}{a + bx^2}} - \sqrt{3} + 1\right)^2}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/6), x]

[Out] (3\*x\*(a + b\*x^2)^(1/6))/4 + (3^(3/4)\*Sqrt[2 - Sqrt[3]]\*a\*(a + b\*x^2)^(1/6)\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))]^2\*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]]/(4\*b\*x\*(a/(a + b\*x^2))^(1/3)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3)))^2]))

**Rubi in Sympy [A]** time = 10.0109, size = 246, normalized size = 0.9

$$\frac{3^{\frac{3}{4}} a \sqrt{\frac{\left(-\frac{bx^2}{a+bx^2}+1\right)^{\frac{2}{3}} + \sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2} \sqrt{-\sqrt{3}+2} \sqrt[6]{a+bx^2} \left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1}\right) F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1+\sqrt{3}}}{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}}\right)\right) - 7 + 4\sqrt{3}}{4bx^3 \sqrt[3]{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2}} + \frac{3x\sqrt[6]{a+bx^2}}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(1/6),x)`

[Out]  $3^{3/4} a \sqrt{\left(\left(-b x^2 / (a + b x^2) + 1\right)^{2/3} + \left(-b x^2 / (a + b x^2) + 1\right)^{1/3} + 1\right) / \left(-\left(-b x^2 / (a + b x^2) + 1\right)^{1/3} - \sqrt{3} + 1\right)^2} \sqrt{-\sqrt{3} + 2} (a + b x^2)^{1/6} \left(-\left(-b x^2 / (a + b x^2) + 1\right)^{1/3} + 1\right) \operatorname{elliptic}_f\left(\operatorname{asin}\left(\frac{-\left(-b x^2 / (a + b x^2) + 1\right)^{1/3} + 1 + \sqrt{3}}{-\left(-b x^2 / (a + b x^2) + 1\right)^{1/3} - \sqrt{3} + 1}\right), -7 + 4 \sqrt{3}\right) / \left(4 b x^3 (a / (a + b x^2))^{1/3} \sqrt{\left(\left(-b x^2 / (a + b x^2) + 1\right)^{1/3} - 1\right) / \left(-\left(-b x^2 / (a + b x^2) + 1\right)^{1/3} - \sqrt{3} + 1\right)^2}\right) + 3 x (a + b x^2)^{1/6} / 4$

**Mathematica [C]** time = 0.0332939, size = 62, normalized size = 0.23

$$\frac{ax \left(\frac{bx^2}{a} + 1\right)^{5/6} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; -\frac{bx^2}{a}\right) + 3x(a + bx^2)}{4(a + bx^2)^{5/6}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(1/6),x]`

[Out]  $(3 x (a + b x^2) + a x (1 + (b x^2) / a)^{5/6} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{3}{2}, -\frac{(b x^2)}{a}\right]) / (4 (a + b x^2)^{5/6})$

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int \sqrt[6]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/6),x)`

[Out] `int((b*x^2+a)^(1/6),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/6),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(1/6), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^{\frac{1}{6}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/6),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(1/6), x)`

**Sympy** [A] time = 2.68838, size = 26, normalized size = 0.1

$$\sqrt[6]{ax} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/6),x)`

[Out] `a**(1/6)*x*hyper((-1/6, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^{\frac{1}{6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/6),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(1/6), x)`

$$3.1014 \quad \int \frac{\sqrt[6]{a+bx^2}}{x^2} dx$$

**Optimal.** Leaf size=266

$$\frac{\sqrt{2-\sqrt{3}}\sqrt[6]{a+bx^2} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}+1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}+\sqrt{3}+1}}{-\sqrt[3]{\frac{a}{bx^2+a}-\sqrt{3}+1}}\right) \mid -7+4\sqrt{3}\right)}{\sqrt[3]{3}x\sqrt[3]{\frac{a}{a+bx^2}} \sqrt{\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}}}$$

$$-\frac{\sqrt[6]{a+bx^2}}{x}$$

[Out] -((a + b\*x^2)^(1/6)/x) + (Sqrt[2 - Sqrt[3]]\*(a + b\*x^2)^(1/6)\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))^2]\*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(3^(1/4)\*x\*(a/(a + b\*x^2))^(1/3)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))^2)])

**Rubi [A]** time = 0.460453, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{\sqrt{2-\sqrt{3}}\sqrt[6]{a+bx^2} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}+1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}+\sqrt{3}+1}}{-\sqrt[3]{\frac{a}{bx^2+a}-\sqrt{3}+1}}\right) \mid -7+4\sqrt{3}\right)}{\sqrt[3]{3}x\sqrt[3]{\frac{a}{a+bx^2}} \sqrt{\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}}}$$

$$-\frac{\sqrt[6]{a+bx^2}}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/6)/x^2, x]

[Out] -((a + b\*x^2)^(1/6)/x) + (Sqrt[2 - Sqrt[3]]\*(a + b\*x^2)^(1/6)\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))^2]\*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(3^(1/4)\*x\*(a/(a + b\*x^2))^(1/3)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))^2)])

**Rubi in Sympy [A]** time = 11.8613, size = 240, normalized size = 0.9

$$\frac{\sqrt[6]{a+bx^2}}{x} + \frac{3^{\frac{3}{4}} \sqrt{\frac{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1}\right)^2 + \sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2}} \sqrt{-\sqrt{3}+2} \sqrt[6]{a+bx^2} \left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1}\right) F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1+\sqrt{3}}}{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}}\right)\right)}{\sqrt[6]{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(1/6)/x**2,x)`

[Out]  $-(a + b*x**2)**(1/6)/x + 3**(3/4)*\operatorname{sqrt}\left(\left(-b*x**2/(a + b*x**2) + 1\right)**(2/3) + (-b*x**2/(a + b*x**2) + 1)**(1/3) + 1\right)/\left(-(-b*x**2/(a + b*x**2) + 1)**(1/3) - \operatorname{sqrt}(3) + 1\right)**2*\operatorname{sqrt}(-\operatorname{sqrt}(3) + 2)*(a + b*x**2)**(1/6)*\left(-(-b*x**2/(a + b*x**2) + 1)**(1/3) + 1\right)*\operatorname{elliptic\_f}\left(\operatorname{asin}\left(\frac{-(-b*x**2/(a + b*x**2) + 1)**(1/3) + 1 + \operatorname{sqrt}(3))}{(-b*x**2/(a + b*x**2) + 1)**(1/3) - \operatorname{sqrt}(3) + 1}\right), -7 + 4*\operatorname{sqrt}(3)\right)/(3*x*(a/(a + b*x**2))**(1/3)*\operatorname{sqrt}\left(\left(-b*x**2/(a + b*x**2) + 1\right)**(1/3) - 1\right)/\left(-(-b*x**2/(a + b*x**2) + 1)**(1/3) - \operatorname{sqrt}(3) + 1\right)**2)\right)$

**Mathematica [C]** time = 0.0409511, size = 68, normalized size = 0.26

$$\frac{bx \left(\frac{a+bx^2}{a}\right)^{5/6} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; -\frac{bx^2}{a}\right)}{3(a+bx^2)^{5/6}} - \frac{\sqrt[6]{a+bx^2}}{x}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(1/6)/x^2,x]`

[Out]  $-\left((a + b*x^2)^{1/6}/x\right) + (b*x*((a + b*x^2)/a)^{5/6}*\operatorname{Hypergeometric2F1}\left[1/2, 5/6, 3/2, -((b*x^2)/a)\right])/(3*(a + b*x^2)^{5/6})$

**Maple [F]** time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \sqrt[6]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/6)/x^2,x)`

[Out] `int((b*x^2+a)^(1/6)/x^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{6}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/6)/x^2,x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(1/6)/x^2, x)

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{1}{6}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/6)/x^2,x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(1/6)/x^2, x)

**Sympy** [A] time = 3.18897, size = 29, normalized size = 0.11

$$\frac{\sqrt[6]{a} {}_2F_1\left(-\frac{1}{2}, -\frac{1}{6} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/6)/x\*\*2,x)

[Out] -a\*\*(1/6)\*hyper((-1/2, -1/6), (1/2,), b\*x\*\*2\*exp\_polar(I\*pi)/a)/x

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{6}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/6)/x^2,x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(1/6)/x^2, x)



$$3.1015 \quad \int \frac{\sqrt[6]{a+bx^2}}{x^4} dx$$

**Optimal.** Leaf size=297

$$\frac{b\sqrt[6]{a+bx^2}}{9ax} - \frac{2\sqrt{2-\sqrt{3}}b\sqrt[6]{a+bx^2} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt[3]{1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}} + \sqrt[3]{1}}{-\sqrt[3]{\frac{a}{bx^2+a}} - \sqrt[3]{1}}\right) \mid -7 + 4\sqrt{3}\right)}{9\sqrt[4]{3}ax\sqrt[3]{\frac{a}{a+bx^2}} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt[3]{1}\right)^2}}}$$

$$- \frac{\sqrt[6]{a+bx^2}}{3x^3}$$

[Out]  $-(a + b*x^2)^{(1/6)}/(3*x^3) - (b*(a + b*x^2)^{(1/6)})/(9*a*x) - (2*\text{Sqrt}[2 - \text{Sqrt}[3]]*b*(a + b*x^2)^{(1/6)}*(1 - (a/(a + b*x^2))^{(1/3)}))*\text{Sqrt}[(1 + (a/(a + b*x^2))^{(1/3)} + (a/(a + b*x^2))^{(2/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})], -7 + 4*\text{Sqrt}[3]]/(9*3^{(1/4)}*a*x*(a/(a + b*x^2))^{(1/3)}*\text{Sqrt}[-((1 - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2))])$

**Rubi [A]** time = 0.558486, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{b\sqrt[6]{a+bx^2}}{9ax} - \frac{2\sqrt{2-\sqrt{3}}b\sqrt[6]{a+bx^2} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt[3]{1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}} + \sqrt[3]{1}}{-\sqrt[3]{\frac{a}{bx^2+a}} - \sqrt[3]{1}}\right) \mid -7 + 4\sqrt{3}\right)}{9\sqrt[4]{3}ax\sqrt[3]{\frac{a}{a+bx^2}} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt[3]{1}\right)^2}}}$$

$$- \frac{\sqrt[6]{a+bx^2}}{3x^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^{(1/6)}/x^4, x]$

[Out]  $-(a + b*x^2)^{(1/6)}/(3*x^3) - (b*(a + b*x^2)^{(1/6)})/(9*a*x) - (2*\text{Sqrt}[2 - \text{Sqrt}[3]]*b*(a + b*x^2)^{(1/6)}*(1 - (a/(a + b*x^2))^{(1/3)}))*\text{Sqrt}[(1 + (a/(a + b*x^2))^{(1/3)} + (a/(a + b*x^2))^{(2/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})], -7 + 4*\text{Sqrt}[3]]/(9*3^{(1/4)}*a*x*(a/(a + b*x^2))^{(1/3)}*\text{Sqrt}[-((1 - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2))])$

**Rubi in Sympy [A]** time = 17.2399, size = 267, normalized size = 0.9

$$\frac{\sqrt[6]{a+bx^2}}{3x^3} - \frac{b\sqrt[6]{a+bx^2}}{9ax} + \frac{2 \cdot 3^{\frac{3}{4}} b \sqrt{\frac{\left(-\frac{bx^2}{a+bx^2}+1\right)^{\frac{2}{3}} + \sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2}} \sqrt{-\sqrt{3}+2}\sqrt[6]{a+bx^2} \left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1}\right) F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1+\sqrt{3}}}{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}}\right)\right)}{\sqrt{\frac{\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(1/6)/x**4,x)`

[Out]  $-(a + b*x**2)**(1/6)/(3*x**3) - b*(a + b*x**2)**(1/6)/(9*a*x) - 2*3**(3/4)*b*\sqrt{\left(\left(-b*x**2/(a + b*x**2) + 1\right)**(2/3) + \left(-b*x**2/(a + b*x**2) + 1\right)**(1/3) + 1\right)/\left(-\left(-b*x**2/(a + b*x**2) + 1\right)**(1/3) - \sqrt{3} + 1\right)**2}*\sqrt{-\sqrt{3} + 2}*(a + b*x**2)**(1/6)*\left(-\left(-b*x**2/(a + b*x**2) + 1\right)**(1/3) + 1\right)*\operatorname{elliptic\_f}\left(\operatorname{asin}\left(\frac{-\left(-b*x**2/(a + b*x**2) + 1\right)**(1/3) + 1 + \sqrt{3}}{-\left(-b*x**2/(a + b*x**2) + 1\right)**(1/3) - \sqrt{3} + 1}\right), -7 + 4*\sqrt{3}\right)/(27*a*x*(a/(a + b*x**2))**(1/3)*\sqrt{\left(\left(-b*x**2/(a + b*x**2) + 1\right)**(1/3) - 1\right)/\left(-\left(-b*x**2/(a + b*x**2) + 1\right)**(1/3) - \sqrt{3} + 1\right)**2})}$

**Mathematica [C]** time = 0.0504287, size = 85, normalized size = 0.29

$$\frac{-3(3a^2 + 4abx^2 + b^2x^4) - 2b^2x^4\left(\frac{bx^2}{a} + 1\right)^{5/6} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; -\frac{bx^2}{a}\right)}{27ax^3(a+bx^2)^{5/6}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(1/6)/x^4,x]`

[Out]  $(-3*(3*a^2 + 4*a*b*x^2 + b^2*x^4) - 2*b^2*x^4*(1 + (b*x^2)/a)^(5/6)*\operatorname{Hypergeometric2F1}\left[1/2, 5/6, 3/2, -((b*x^2)/a)\right])/(27*a*x^3*(a + b*x^2)^(5/6))$

**Maple [F]** time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} \sqrt[6]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/6)/x^4,x)`

[Out] `int((b*x^2+a)^(1/6)/x^4,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{6}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/6)/x^4,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(1/6)/x^4, x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{1}{6}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/6)/x^4,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(1/6)/x^4, x)`

**Sympy** [A] time = 4.05234, size = 34, normalized size = 0.11

$$\frac{\sqrt[3]{a} {}_2F_1\left(-\frac{3}{2}, -\frac{1}{6} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/6)/x**4,x)`

[Out] `-a**(1/6)*hyper((-3/2, -1/6), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{6}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/6)/x^4,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(1/6)/x^4, x)`

$$3.1016 \quad \int \frac{\sqrt[6]{a+bx^2}}{x^6} dx$$

**Optimal.** Leaf size=323

$$\frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x} + \frac{16\sqrt{2-\sqrt{3}}b^2\sqrt[6]{a+bx^2} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}+1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}+\sqrt{3}+1}}{-\sqrt[3]{\frac{a}{bx^2+a}-\sqrt{3}+1}}\right) \middle| -7+4\sqrt{3}\right)}{135\sqrt[4]{3}a^2x^3\sqrt[3]{\frac{a}{a+bx^2}} \sqrt{\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}}} - \frac{\sqrt[6]{a+bx^2}}{5x^5} - \frac{b\sqrt[6]{a+bx^2}}{45ax^3}$$

[Out]  $-(a + b*x^2)^{(1/6)}/(5*x^5) - (b*(a + b*x^2)^{(1/6)})/(45*a*x^3) + (8*b^2*(a + b*x^2)^{(1/6)})/(135*a^2*x) + (16*\text{Sqrt}[2 - \text{Sqrt}[3]]*b^2*(a + b*x^2)^{(1/6)}*(1 - (a/(a + b*x^2))^{(1/3)})*\text{Sqrt}[(1 + (a/(a + b*x^2))^{(1/3)} + (a/(a + b*x^2))^{(2/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(135*3^{(1/4)}*a^2*x*(a/(a + b*x^2))^{(1/3)}*\text{Sqrt}[-((1 - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2)])$

**Rubi [A]** time = 0.645614, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x} + \frac{16\sqrt{2-\sqrt{3}}b^2\sqrt[6]{a+bx^2} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}+1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}+\sqrt{3}+1}}{-\sqrt[3]{\frac{a}{bx^2+a}-\sqrt{3}+1}}\right) \middle| -7+4\sqrt{3}\right)}{135\sqrt[4]{3}a^2x^3\sqrt[3]{\frac{a}{a+bx^2}} \sqrt{\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}}} - \frac{\sqrt[6]{a+bx^2}}{5x^5} - \frac{b\sqrt[6]{a+bx^2}}{45ax^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^{(1/6)}/x^6, x]$

[Out]  $-(a + b*x^2)^{(1/6)}/(5*x^5) - (b*(a + b*x^2)^{(1/6)})/(45*a*x^3) + (8*b^2*(a + b*x^2)^{(1/6)})/(135*a^2*x) + (16*\text{Sqrt}[2 - \text{Sqrt}[3]]*b^2*(a + b*x^2)^{(1/6)}*(1 - (a/(a + b*x^2))^{(1/3)})*\text{Sqrt}[(1 + (a/(a + b*x^2))^{(1/3)} + (a/(a + b*x^2))^{(2/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(135*3^{(1/4)}*a^2*x*(a/(a + b*x^2))^{(1/3)}*\text{Sqrt}[-((1 - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2)])$

**Rubi in Sympy [A]** time = 22.8995, size = 292, normalized size = 0.9

$$\frac{\sqrt[6]{a+bx^2}}{5x^5} - \frac{b\sqrt[6]{a+bx^2}}{45ax^3} + \frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x} + \frac{16 \cdot 3^{\frac{3}{4}} b^2 \sqrt{\frac{\left(-\frac{bx^2}{a+bx^2}+1\right)^{\frac{2}{3}} + \sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2} \sqrt{-\sqrt{3}+2}\sqrt[6]{a+bx^2} \left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1}\right) F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1+\sqrt{3}}}{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}}\right)\right)}{405a^2x^3\sqrt{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(1/6)/x**6,x)`

[Out]  $-(a + b*x**2)**(1/6)/(5*x**5) - b*(a + b*x**2)**(1/6)/(45*a*x**3) + 8*b**2*(a + b*x**2)**(1/6)/(135*a**2*x) + 16*3**(3/4)*b**2*\operatorname{sqrt}\left(\frac{\left(-\frac{b*x**2}{a + b*x**2} + 1\right)**(2/3) + \left(-\frac{b*x**2}{a + b*x**2} + 1\right)**(1/3) + 1}{\left(-\frac{b*x**2}{a + b*x**2} + 1\right)**(1/3) - \operatorname{sqrt}(3) + 1}\right)**2*\operatorname{sqrt}(-\operatorname{sqrt}(3) + 2)*(a + b*x**2)**(1/6)*\left(-\frac{b*x**2}{a + b*x**2} + 1\right)**(1/3) + 1*\operatorname{elliptic\_f}\left(\operatorname{asin}\left(\frac{\left(-\frac{b*x**2}{a + b*x**2} + 1\right)**(1/3) + 1 + \operatorname{sqrt}(3)}{\left(-\frac{b*x**2}{a + b*x**2} + 1\right)**(1/3) - \operatorname{sqrt}(3) + 1}\right), -7 + 4*\operatorname{sqrt}(3)\right)/(405*a**2*x*(a/(a + b*x**2))**(1/3)*\operatorname{sqrt}\left(\frac{\left(-\frac{b*x**2}{a + b*x**2} + 1\right)**(1/3) - 1}{\left(-\frac{b*x**2}{a + b*x**2} + 1\right)**(1/3) - \operatorname{sqrt}(3) + 1}\right)**2\right)$

**Mathematica [C]** time = 0.0596304, size = 94, normalized size = 0.29

$$\frac{-81a^3 - 90a^2bx^2 + 16b^3x^6 \left(\frac{bx^2}{a} + 1\right)^{5/6} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}, \frac{3}{2}, -\frac{bx^2}{a}\right) + 15ab^2x^4 + 24b^3x^6}{405a^2x^5(a+bx^2)^{5/6}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(1/6)/x^6,x]`

[Out]  $(-81*a^3 - 90*a^2*b*x^2 + 15*a*b^2*x^4 + 24*b^3*x^6 + 16*b^3*x^6*(1 + (b*x^2)/a)^{(5/6)}*\operatorname{Hypergeometric2F1}\left[1/2, 5/6, 3/2, -((b*x^2)/a)\right])/(405*a^2*x^5*(a + b*x^2)^{(5/6)})$

**Maple [F]** time = 0.045, size = 0, normalized size = 0.

$$\int \frac{1}{x^6} \sqrt[6]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/6)/x^6,x)`

[Out] `int((b*x^2+a)^(1/6)/x^6,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{6}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/6)/x^6,x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(1/6)/x^6, x)

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{1}{6}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/6)/x^6,x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(1/6)/x^6, x)

**Sympy** [A] time = 5.26074, size = 34, normalized size = 0.11

$$\frac{\sqrt[6]{a} {}_2F_1\left(-\frac{5}{2}, -\frac{1}{6} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*(1/6)/x\*\*6,x)

[Out] -a\*\*(1/6)\*hyper((-5/2, -1/6), (-3/2,), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(5\*x\*\*5)

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{6}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(1/6)/x^6,x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(1/6)/x^6, x)

$$3.1017 \quad \int \frac{\sqrt[6]{a+bx^2}}{x^8} dx$$

**Optimal.** Leaf size=347

$$\frac{16b^3\sqrt[6]{a+bx^2}}{405a^3x} + \frac{32\sqrt{2-\sqrt{3}}b^3\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}+1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}+\sqrt{3}+1}}{-\sqrt[3]{\frac{a}{bx^2+a}-\sqrt{3}+1}}\right)\right)}{\sqrt{\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}}}$$


---


$$405\sqrt[4]{3}a^3x^3\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}}$$


---


$$+\frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3}-\frac{\sqrt[6]{a+bx^2}}{7x^7}-\frac{b\sqrt[6]{a+bx^2}}{105ax^5}$$

[Out]  $-(a+b*x^2)^{(1/6)}/(7*x^7) - (b*(a+b*x^2)^{(1/6)})/(105*a*x^5) + (2*b^2*(a+b*x^2)^{(1/6)})/(135*a^2*x^3) - (16*b^3*(a+b*x^2)^{(1/6)})/(405*a^3*x) - (32*sqrt[2 - sqrt[3]]*b^3*(a+b*x^2)^{(1/6)}*(1 - (a/(a+b*x^2))^{(1/3)})*sqrt[(1 + (a/(a+b*x^2))^{(1/3)} + (a/(a+b*x^2))^{(2/3)})/(1 - sqrt[3] - (a/(a+b*x^2))^{(1/3)})^2]*EllipticF[ArcSin[(1 + sqrt[3] - (a/(a+b*x^2))^{(1/3)})/(1 - sqrt[3] - (a/(a+b*x^2))^{(1/3)})], -7 + 4*sqrt[3]])/(405*3^{(1/4)}*a^3*x*(a/(a+b*x^2))^{(1/3)}*sqrt[-((1 - (a/(a+b*x^2))^{(1/3)})/(1 - sqrt[3] - (a/(a+b*x^2))^{(1/3)})^2)])$

**Rubi [A]** time = 0.756328, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{16b^3\sqrt[6]{a+bx^2}}{405a^3x} + \frac{32\sqrt{2-\sqrt{3}}b^3\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}+1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}+\sqrt{3}+1}}{-\sqrt[3]{\frac{a}{bx^2+a}-\sqrt{3}+1}}\right)\right)}{\sqrt{\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}}}$$


---


$$405\sqrt[4]{3}a^3x^3\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}}$$


---


$$+\frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3}-\frac{\sqrt[6]{a+bx^2}}{7x^7}-\frac{b\sqrt[6]{a+bx^2}}{105ax^5}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(1/6)/x^8, x]

[Out]  $-(a+b*x^2)^{(1/6)}/(7*x^7) - (b*(a+b*x^2)^{(1/6)})/(105*a*x^5) + (2*b^2*(a+b*x^2)^{(1/6)})/(135*a^2*x^3) - (16*b^3*(a+b*x^2)^{(1/6)})/(405*a^3*x) - (32*sqrt[2 - sqrt[3]]*b^3*(a+b*x^2)^{(1/6)}*(1 - (a/(a+b*x^2))^{(1/3)})*sqrt[(1 + (a/(a+b*x^2))^{(1/3)} + (a/(a+b*x^2))^{(2/3)})/(1 - sqrt[3] - (a/(a+b*x^2))^{(1/3)})^2]*EllipticF[ArcSin[(1 + sqrt[3] - (a/(a+b*x^2))^{(1/3)})/(1 - sqrt[3] - (a/(a+b*x^2))^{(1/3)})], -7 + 4*sqrt[3]])/(405*3^{(1/4)}*a^3*x*(a/(a+b*x^2))^{(1/3)}*sqrt[-((1 - (a/(a+b*x^2))^{(1/3)})/(1 - sqrt[3] - (a/(a+b*x^2))^{(1/3)})^2)])$

**Rubi in Sympy [A]** time = 29.8831, size = 316, normalized size = 0.91

$$\frac{\sqrt[6]{a+bx^2}}{7x^7} - \frac{b\sqrt[6]{a+bx^2}}{105ax^5} + \frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3} - \frac{16b^3\sqrt[6]{a+bx^2}}{405a^3x}$$

$$- \frac{32 \cdot 3^{\frac{3}{4}} b^3 \sqrt{\frac{\left(-\frac{bx^2}{a+bx^2}+1\right)^{\frac{2}{3}} + \sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2} \sqrt{-\sqrt{3}+2\sqrt[6]{a+bx^2}} \left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1\right) F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1+\sqrt{3}}}{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}}\right)\right)}{\sqrt{\frac{1215a^3x\sqrt[3]{\frac{a}{a+bx^2}}}{\sqrt{\frac{\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2}}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**(1/6)/x**8,x)`

[Out]  $-(a + b*x**2)**(1/6)/(7*x**7) - b*(a + b*x**2)**(1/6)/(105*a*x**5) + 2*b**2*(a + b*x**2)**(1/6)/(135*a**2*x**3) - 16*b**3*(a + b*x**2)**(1/6)/(405*a**3*x) - 32*3**(3/4)*b**3*\sqrt{\left(\left(-b*x**2/(a + b*x**2) + 1\right)**(2/3) + \sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1\right)**(1/3) + 1}/\left(-\left(-b*x**2/(a + b*x**2) + 1\right)**(1/3) - \sqrt{3} + 1\right)**2*\sqrt{-\sqrt{3} + 2*\sqrt[6]{a+bx^2}}*\operatorname{asin}\left(\frac{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1+\sqrt{3}}}{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}}\right)$

**Mathematica [C]** time = 0.0641691, size = 108, normalized size = 0.31

$$\frac{-3(405a^4 + 432a^3bx^2 - 15a^2b^2x^4 + 70ab^3x^6 + 112b^4x^8) - 224b^4x^8\left(\frac{bx^2}{a} + 1\right)^{5/6} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}, \frac{3}{2}, -\frac{bx^2}{a}\right)}{8505a^3x^7(a+bx^2)^{5/6}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(1/6)/x^8,x]`

[Out]  $(-3*(405*a^4 + 432*a^3*b*x^2 - 15*a^2*b^2*x^4 + 70*a*b^3*x^6 + 112*b^4*x^8) - 224*b^4*x^8*(1 + (b*x^2)/a)^{5/6}*\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{3}{2}, -\left(\frac{bx^2}{a}\right)\right])/(8505*a^3*x^7*(a + b*x^2)^{5/6})$

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x^8} \sqrt[6]{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^(1/6)/x^8,x)`

[Out] `int((b*x^2+a)^(1/6)/x^8,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{6}}}{x^8} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/6)/x^8,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(1/6)/x^8, x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^{\frac{1}{6}}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/6)/x^8,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^(1/6)/x^8, x)`

**Sympy** [A] time = 7.45747, size = 34, normalized size = 0.1

$$\frac{\sqrt[6]{a} {}_2F_1\left(-\frac{7}{2}, -\frac{1}{6} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**(1/6)/x**8,x)`

[Out] `-a**(1/6)*hyper((-7/2, -1/6), (-5/2,), b*x**2*exp_polar(I*pi)/a)/(7*x**7)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{1}{6}}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(1/6)/x^8,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(1/6)/x^8, x)`

$$3.1018 \quad \int \frac{x^6}{\sqrt[6]{a+bx^2}} dx$$

**Optimal.** Leaf size=659

$$\frac{81 \cdot 3^{3/4} a^4 \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \middle| -7 + 4\sqrt{3}\right)}{448\sqrt{2}b^4x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{-\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$\frac{243\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^4 \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \middle| -7 + 4\sqrt{3}\right)}{1792b^4x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{-\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$\frac{243a^4x}{896b^3 \left(\frac{a}{a+bx^2}\right)^{2/3} (a+bx^2)^{7/6} \left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)} - \frac{243a^3x}{896b^3 \sqrt[6]{a+bx^2}}$$

$$+ \frac{81a^2x (a+bx^2)^{5/6}}{448b^3} - \frac{9ax^3 (a+bx^2)^{5/6}}{56b^2} + \frac{3x^5 (a+bx^2)^{5/6}}{20b}$$

[Out]  $(-243*a^3*x)/(896*b^3*(a+b*x^2)^(1/6)) + (81*a^2*x*(a+b*x^2)^(5/6))/(448*b^3) - (9*a*x^3*(a+b*x^2)^(5/6))/(56*b^2) + (3*x^5*(a+b*x^2)^(5/6))/(20*b) - (243*a^4*x)/(896*b^3*(a/(a+b*x^2))^(2/3)*(a+b*x^2)^(7/6)*(1-Sqrt[3]-(a/(a+b*x^2))^(1/3))) - (243*3^(1/4)*Sqrt[2+Sqrt[3]]*a^4*(1-(a/(a+b*x^2))^(1/3))*Sqrt[(1+(a/(a+b*x^2))^(1/3)+(a/(a+b*x^2))^(2/3))/(1-Sqrt[3]-(a/(a+b*x^2))^(1/3))]^2*EllipticE[ArcSin[(1+Sqrt[3]-(a/(a+b*x^2))^(1/3))/(1-Sqrt[3]-(a/(a+b*x^2))^(1/3))], -7+4*Sqrt[3]])/(1792*b^4*x*(a/(a+b*x^2))^(2/3)*(a+b*x^2)^(1/6)*Sqrt[-((1-(a/(a+b*x^2))^(1/3))/(1-Sqrt[3]-(a/(a+b*x^2))^(1/3)))^2]) + (81*3^(3/4)*a^4*(1-(a/(a+b*x^2))^(1/3))*Sqrt[(1+(a/(a+b*x^2))^(1/3)+(a/(a+b*x^2))^(2/3))/(1-Sqrt[3]-(a/(a+b*x^2))^(1/3))]^2*EllipticF[ArcSin[(1+Sqrt[3]-(a/(a+b*x^2))^(1/3))/(1-Sqrt[3]-(a/(a+b*x^2))^(1/3))], -7+4*Sqrt[3]])/(448*Sqrt[2]*b^4*x*(a/(a+b*x^2))^(2/3)*(a+b*x^2)^(1/6)*Sqrt[-((1-(a/(a+b*x^2))^(1/3))/(1-Sqrt[3]-(a/(a+b*x^2))^(1/3)))^2])$

**Rubi [A]** time = 1.54479, antiderivative size = 659, normalized size of antiderivative = 1., number of

steps used = 9, number of rules used = 7, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$\frac{81 \cdot 3^{3/4} \cdot a^4 \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \middle| -7 + 4\sqrt{3}\right)}{448\sqrt{2}b^4x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$\frac{243\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^4 \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \middle| -7 + 4\sqrt{3}\right)}{1792b^4x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$\frac{243a^4x}{896b^3 \left(\frac{a}{a+bx^2}\right)^{2/3} (a+bx^2)^{7/6} \left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)} - \frac{243a^3x}{896b^3 \sqrt[6]{a+bx^2}}$$

$$+ \frac{81a^2x (a+bx^2)^{5/6}}{448b^3} - \frac{9ax^3 (a+bx^2)^{5/6}}{56b^2} + \frac{3x^5 (a+bx^2)^{5/6}}{20b}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a + b\*x^2)^(1/6), x]

[Out]  $(-243 \cdot a^3 \cdot x) / (896 \cdot b^3 \cdot (a + b \cdot x^2)^{(1/6)}) + (81 \cdot a^2 \cdot x \cdot (a + b \cdot x^2)^{(5/6)}) / (448 \cdot b^3) - (9 \cdot a \cdot x^3 \cdot (a + b \cdot x^2)^{(5/6)}) / (56 \cdot b^2) + (3 \cdot x^5 \cdot (a + b \cdot x^2)^{(5/6)}) / (20 \cdot b) - (243 \cdot a^4 \cdot x) / (896 \cdot b^3 \cdot (a / (a + b \cdot x^2))^{2/3} \cdot (a + b \cdot x^2)^{(7/6)} \cdot (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})) - (243 \cdot 3^{1/4} \cdot \text{Sqrt}[2 + \text{Sqrt}[3]] \cdot a^4 \cdot (1 - (a / (a + b \cdot x^2))^{(1/3)})) \cdot \text{Sqrt}[(1 + (a / (a + b \cdot x^2))^{(1/3)} + (a / (a + b \cdot x^2))^{(2/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})^2] \cdot \text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})], -7 + 4 \cdot \text{Sqrt}[3]]) / (1792 \cdot b^4 \cdot x \cdot (a / (a + b \cdot x^2))^{2/3} \cdot (a + b \cdot x^2)^{(1/6)} \cdot \text{Sqrt}[-((1 - (a / (a + b \cdot x^2))^{(1/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})^2)]) + (81 \cdot 3^{3/4} \cdot a^4 \cdot (1 - (a / (a + b \cdot x^2))^{(1/3)}) \cdot \text{Sqrt}[(1 + (a / (a + b \cdot x^2))^{(1/3)} + (a / (a + b \cdot x^2))^{(2/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})^2] \cdot \text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})], -7 + 4 \cdot \text{Sqrt}[3]]) / (448 \cdot \text{Sqrt}[2] \cdot b^4 \cdot x \cdot (a / (a + b \cdot x^2))^{2/3} \cdot (a + b \cdot x^2)^{(1/6)} \cdot \text{Sqrt}[-((1 - (a / (a + b \cdot x^2))^{(1/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})^2)])$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{81a^3 \int \frac{1}{\sqrt[6]{a+bx^2}} dx}{448b^3} + \frac{81a^2x (a+bx^2)^{5/6}}{448b^3} - \frac{9ax^3 (a+bx^2)^{5/6}}{56b^2} + \frac{3x^5 (a+bx^2)^{5/6}}{20b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6/(b\*x\*\*2+a)\*\*(1/6), x)

[Out]  $-81 \cdot a^{**3} \cdot \text{Integral}((a + b \cdot x^{**2})^{**}(-1/6), x) / (448 \cdot b^{**3}) + 81 \cdot a^{**2} \cdot x \cdot (a + b \cdot x^{**2})^{**}(5/6) / (448 \cdot b^{**3}) - 9 \cdot a \cdot x^{**3} \cdot (a + b \cdot x^{**2})^{**}(5/6) / (56 \cdot b^{**2}) + 3 \cdot x^{**5} \cdot (a + b \cdot x^{**2})^{**}(5/6) / (20 \cdot b)$

---

**Mathematica [C]** time = 0.0820299, size = 90, normalized size = 0.14

$$\frac{3 \left( -135a^3x^6 \sqrt{\frac{bx^2}{a}} + {}_2F_1 \left( \frac{1}{6}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a} \right) + 135a^3x + 15a^2bx^3 - 8ab^2x^5 + 112b^3x^7 \right)}{2240b^3 \sqrt[6]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a + b\*x^2)^(1/6), x]

[Out] (3\*(135\*a^3\*x + 15\*a^2\*b\*x^3 - 8\*a\*b^2\*x^5 + 112\*b^3\*x^7 - 135\*a^3\*x\*(1 + (b\*x^2)/a)^(1/6)\*Hypergeometric2F1[1/6, 1/2, 3/2, -(b\*x^2)/a]))/(2240\*b^3\*(a + b\*x^2)^(1/6))

---

**Maple [F]** time = 0.04, size = 0, normalized size = 0.

$$\int x^6 \frac{1}{\sqrt[6]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(b\*x^2+a)^(1/6), x)

[Out] int(x^6/(b\*x^2+a)^(1/6), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(bx^2 + a)^{\frac{1}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b\*x^2 + a)^(1/6), x, algorithm="maxima")

[Out] integrate(x^6/(b\*x^2 + a)^(1/6), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{x^6}{(bx^2 + a)^{\frac{1}{6}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b\*x^2 + a)^(1/6), x, algorithm="fricas")

[Out] integral(x^6/(b\*x^2 + a)^(1/6), x)

---

**Sympy [A]** time = 3.9778, size = 27, normalized size = 0.04

$$\frac{x^7 {}_2F_1 \left( \frac{1}{6}, \frac{7}{2} \middle| \frac{bx^2 e^{i\pi}}{a} \right)}{7 \sqrt[6]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(b*x**2+a)**(1/6),x)`

[Out] `x**7*hyper((1/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(1/6))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(bx^2 + a)^{\frac{1}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^2 + a)^(1/6),x, algorithm="giac")`

[Out] `integrate(x^6/(b*x^2 + a)^(1/6), x)`

$$3.1019 \quad \int \frac{x^4}{\sqrt[6]{a+bx^2}} dx$$

**Optimal.** Leaf size=635

$$\frac{27 \cdot 3^{3/4} a^3 \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \mid -7 + 4\sqrt{3}\right)}{112\sqrt{2}b^3x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$+ \frac{81\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^3 \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \mid -7 + 4\sqrt{3}\right)}{448b^3x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$+ \frac{81a^3x}{224b^2 \left(\frac{a}{a+bx^2}\right)^{2/3} (a+bx^2)^{7/6} \left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)}$$

$$+ \frac{81a^2x}{224b^2\sqrt[6]{a+bx^2}} - \frac{27ax(a+bx^2)^{5/6}}{112b^2} + \frac{3x^3(a+bx^2)^{5/6}}{14b}$$

[Out] (81\*a^2\*x)/(224\*b^2\*(a+b\*x^2)^(1/6)) - (27\*a\*x\*(a+b\*x^2)^(5/6))/(112\*b^2) + (3\*x^3\*(a+b\*x^2)^(5/6))/(14\*b) + (81\*a^3\*x)/(224\*b^2\*(a/(a+b\*x^2))^(2/3)\*(a+b\*x^2)^(7/6)\*(1-Sqrt[3]-(a/(a+b\*x^2))^(1/3))) + (81\*3^(1/4)\*Sqrt[2+Sqrt[3]]\*a^3\*(1-(a/(a+b\*x^2))^(1/3))\*Sqrt[(1+(a/(a+b\*x^2))^(1/3)+(a/(a+b\*x^2))^(2/3))/(1-Sqrt[3]-(a/(a+b\*x^2))^(1/3))^2]\*EllipticE[ArcSin[(1+Sqrt[3]-(a/(a+b\*x^2))^(1/3))/(1-Sqrt[3]-(a/(a+b\*x^2))^(1/3))], -7+4\*Sqrt[3]])/(448\*b^3\*x\*(a/(a+b\*x^2))^(2/3)\*(a+b\*x^2)^(1/6)\*Sqrt[-((1-(a/(a+b\*x^2))^(1/3))/(1-Sqrt[3]-(a/(a+b\*x^2))^(1/3))^2)]) - (27\*3^(3/4)\*a^3\*(1-(a/(a+b\*x^2))^(1/3))\*Sqrt[(1+(a/(a+b\*x^2))^(1/3)+(a/(a+b\*x^2))^(2/3))/(1-Sqrt[3]-(a/(a+b\*x^2))^(1/3))^2]\*EllipticF[ArcSin[(1+Sqrt[3]-(a/(a+b\*x^2))^(1/3))/(1-Sqrt[3]-(a/(a+b\*x^2))^(1/3))], -7+4\*Sqrt[3]])/(112\*Sqrt[2]\*b^3\*x\*(a/(a+b\*x^2))^(2/3)\*(a+b\*x^2)^(1/6)\*Sqrt[-((1-(a/(a+b\*x^2))^(1/3))/(1-Sqrt[3]-(a/(a+b\*x^2))^(1/3))^2)])]

**Rubi [A]** time = 1.36469, antiderivative size = 635, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 7, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$\begin{aligned}
 & \frac{27 \cdot 3^{3/4} a^3 \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \mid -7 + 4\sqrt{3}\right)}{112\sqrt{2}b^3x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{-\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}} \\
 & + \frac{81\sqrt[4]{3}\sqrt{2 + \sqrt{3}}a^3 \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \mid -7 + 4\sqrt{3}\right)}{448b^3x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{-\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}} \\
 & + \frac{81a^3x}{224b^2 \left(\frac{a}{a+bx^2}\right)^{2/3} (a+bx^2)^{7/6} \left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)} \\
 & + \frac{81a^2x}{224b^2 \sqrt[6]{a+bx^2}} - \frac{27ax(a+bx^2)^{5/6}}{112b^2} + \frac{3x^3(a+bx^2)^{5/6}}{14b}
 \end{aligned}$$

Warning: Unable to verify antiderivative.

[In] Int[x^4/(a + b\*x^2)^(1/6), x]

[Out]  $(81 \cdot a^2 \cdot x) / (224 \cdot b^2 \cdot (a + b \cdot x^2)^{1/6}) - (27 \cdot a \cdot x \cdot (a + b \cdot x^2)^{5/6}) / (112 \cdot b^2) + (3 \cdot x^3 \cdot (a + b \cdot x^2)^{5/6}) / (14 \cdot b) + (81 \cdot a^3 \cdot x) / (224 \cdot b^2 \cdot (a / (a + b \cdot x^2))^{2/3} \cdot (a + b \cdot x^2)^{7/6} \cdot (1 - \sqrt{3} - (a / (a + b \cdot x^2))^{1/3})) + (81 \cdot 3^{1/4} \cdot \sqrt{2 + \sqrt{3}} \cdot a^3 \cdot (1 - (a / (a + b \cdot x^2))^{1/3})) \cdot \sqrt{(1 + (a / (a + b \cdot x^2))^{1/3} + (a / (a + b \cdot x^2))^{2/3}) / (1 - \sqrt{3} - (a / (a + b \cdot x^2))^{1/3})^2} \cdot \text{EllipticE}[\text{ArcSin}[(1 + \sqrt{3} - (a / (a + b \cdot x^2))^{1/3}) / (1 - \sqrt{3} - (a / (a + b \cdot x^2))^{1/3})], -7 + 4 \cdot \sqrt{3}]) / (448 \cdot b^3 \cdot x \cdot (a / (a + b \cdot x^2))^{2/3} \cdot (a + b \cdot x^2)^{5/6} \cdot \sqrt{-(1 - (a / (a + b \cdot x^2))^{1/3}) / (1 - \sqrt{3} - (a / (a + b \cdot x^2))^{1/3})^2}) - (27 \cdot 3^{3/4} \cdot a^3 \cdot (1 - (a / (a + b \cdot x^2))^{1/3})) \cdot \sqrt{(1 + (a / (a + b \cdot x^2))^{1/3} + (a / (a + b \cdot x^2))^{2/3}) / (1 - \sqrt{3} - (a / (a + b \cdot x^2))^{1/3})^2} \cdot \text{EllipticF}[\text{ArcSin}[(1 + \sqrt{3} - (a / (a + b \cdot x^2))^{1/3}) / (1 - \sqrt{3} - (a / (a + b \cdot x^2))^{1/3})], -7 + 4 \cdot \sqrt{3}]) / (112 \cdot \sqrt{2} \cdot b^3 \cdot x \cdot (a / (a + b \cdot x^2))^{2/3} \cdot (a + b \cdot x^2)^{5/6} \cdot \sqrt{-(1 - (a / (a + b \cdot x^2))^{1/3}) / (1 - \sqrt{3} - (a / (a + b \cdot x^2))^{1/3})^2})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{27a^3 \int \frac{1}{(a+bx^2)^{7/6}} dx}{224b^2} + \frac{81a^2x}{224b^2 \sqrt[6]{a+bx^2}} - \frac{27ax(a+bx^2)^{5/6}}{112b^2} + \frac{3x^3(a+bx^2)^{5/6}}{14b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(b\*x\*\*2+a)\*\*(1/6), x)

[Out]  $-27 \cdot a^{**3} \cdot \text{Integral}((a + b \cdot x^{**2})^{**(-7/6)}, x) / (224 \cdot b^{**2}) + 81 \cdot a^{**2} \cdot x / (224 \cdot b^{**2} \cdot (a + b \cdot x^{**2})^{** (1/6)}) - 27 \cdot a \cdot x \cdot (a + b \cdot x^{**2})^{** (5/6)} / (112 \cdot b^{**2}) + 3 \cdot x^{**3} \cdot (a + b \cdot x^{**2})^{** (5/6)} / (14 \cdot b)$

---

**Mathematica [C]** time = 0.0614175, size = 79, normalized size = 0.12

$$\frac{3 \left( 9a^2 x \sqrt{\frac{bx^2}{a}} + {}_2F_1 \left( \frac{1}{6}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a} \right) - 9a^2 x - abx^3 + 8b^2 x^5 \right)}{112b^2 \sqrt[6]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a + b\*x^2)^(1/6), x]

[Out] (3\*(-9\*a^2\*x - a\*b\*x^3 + 8\*b^2\*x^5 + 9\*a^2\*x\*(1 + (b\*x^2)/a)^(1/6))\*Hypergeometric2F1[1/6, 1/2, 3/2, -(b\*x^2)/a])/(112\*b^2\*(a + b\*x^2)^(1/6))

---

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int x^4 \frac{1}{\sqrt[6]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b\*x^2+a)^(1/6), x)

[Out] int(x^4/(b\*x^2+a)^(1/6), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2 + a)^{\frac{1}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b\*x^2 + a)^(1/6), x, algorithm="maxima")

[Out] integrate(x^4/(b\*x^2 + a)^(1/6), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{x^4}{(bx^2 + a)^{\frac{1}{6}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b\*x^2 + a)^(1/6), x, algorithm="fricas")

[Out] integral(x^4/(b\*x^2 + a)^(1/6), x)

---

**Sympy [A]** time = 3.15297, size = 27, normalized size = 0.04

$$\frac{x^5 {}_2F_1 \left( \frac{1}{6}, \frac{5}{2} \middle| \frac{bx^2 e^{i\pi}}{a} \right)}{5\sqrt[6]{a}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(b*x**2+a)**(1/6),x)`

[Out] `x**5*hyper((1/6, 5/2), (7/2, ), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/6))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2 + a)^{\frac{1}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^(1/6),x, algorithm="giac")`

[Out] `integrate(x^4/(b*x^2 + a)^(1/6), x)`

$$3.1020 \quad \int \frac{x^2}{\sqrt[6]{a+bx^2}} dx$$

**Optimal.** Leaf size=611

$$\frac{3 \cdot 3^{3/4} a^2 \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \middle| -7 + 4\sqrt{3}\right)}{8\sqrt{2}b^2x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^2 \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \middle| -7 + 4\sqrt{3}\right)}{32b^2x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$\frac{9a^2x}{16b \left(\frac{a}{a+bx^2}\right)^{2/3} (a+bx^2)^{7/6} \left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)} - \frac{9ax}{16b\sqrt[6]{a+bx^2}} + \frac{3x(a+bx^2)^{5/6}}{8b}$$

[Out]  $(-9*a*x)/(16*b*(a+b*x^2)^(1/6)) + (3*x*(a+b*x^2)^(5/6))/(8*b) - (9*a^2*x)/(16*b*(a/(a+b*x^2))^(2/3)*(a+b*x^2)^(7/6)*(1 - \text{Sqrt}[3] - (a/(a+b*x^2))^(1/3))) - (9*3^(1/4)*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^2*(1 - (a/(a+b*x^2))^(1/3))*\text{Sqrt}[(1 + (a/(a+b*x^2))^(1/3) + (a/(a+b*x^2))^(2/3))/(1 - \text{Sqrt}[3] - (a/(a+b*x^2))^(1/3))]^2)*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a/(a+b*x^2))^(1/3))/(1 - \text{Sqrt}[3] - (a/(a+b*x^2))^(1/3))], -7 + 4*\text{Sqrt}[3]])/(32*b^2*x*(a/(a+b*x^2))^(2/3)*(a+b*x^2)^(1/6)*\text{Sqrt}[-((1 - (a/(a+b*x^2))^(1/3))/(1 - \text{Sqrt}[3] - (a/(a+b*x^2))^(1/3)))^2]) + (3*3^(3/4)*a^2*(1 - (a/(a+b*x^2))^(1/3))*\text{Sqrt}[(1 + (a/(a+b*x^2))^(1/3) + (a/(a+b*x^2))^(2/3))/(1 - \text{Sqrt}[3] - (a/(a+b*x^2))^(1/3))]^2)*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a/(a+b*x^2))^(1/3))/(1 - \text{Sqrt}[3] - (a/(a+b*x^2))^(1/3))], -7 + 4*\text{Sqrt}[3]])/(8*\text{Sqrt}[2]*b^2*x*(a/(a+b*x^2))^(2/3)*(a+b*x^2)^(1/6)*\text{Sqrt}[-((1 - (a/(a+b*x^2))^(1/3))/(1 - \text{Sqrt}[3] - (a/(a+b*x^2))^(1/3)))^2])$

**Rubi [A]** time = 1.23319, antiderivative size = 611, normalized size of antiderivative = 1., number of

steps used = 7, number of rules used = 7, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$\frac{3 \cdot 3^{3/4} a^2 \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \middle| -7 + 4\sqrt{3}\right)}{8\sqrt{2}b^2x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$- \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^2 \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \middle| -7 + 4\sqrt{3}\right)}{32b^2x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$- \frac{9a^2x}{16b \left(\frac{a}{a+bx^2}\right)^{2/3} (a+bx^2)^{7/6} \left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)} - \frac{9ax}{16b\sqrt[6]{a+bx^2}} + \frac{3x(a+bx^2)^{5/6}}{8b}$$

Warning: Unable to verify antiderivative.

[In] Int[x^2/(a + b\*x^2)^(1/6), x]

[Out]  $(-9*a*x)/(16*b*(a + b*x^2)^{(1/6)}) + (3*x*(a + b*x^2)^{(5/6)})/(8*b) - (9*a^2*x)/(16*b*(a/(a + b*x^2))^{(2/3)}*(a + b*x^2)^{(7/6)}*(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})) - (9*3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^2*(1 - (a/(a + b*x^2))^{(1/3)})*\text{Sqrt}[(1 + (a/(a + b*x^2))^{(1/3)} + (a/(a + b*x^2))^{(2/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(32*b^2*x*(a/(a + b*x^2))^{(2/3)}*(a + b*x^2)^{(1/6)}*\text{Sqrt}[-((1 - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2)]) + (3*3^{(3/4)}*a^2*(1 - (a/(a + b*x^2))^{(1/3)})*\text{Sqrt}[(1 + (a/(a + b*x^2))^{(1/3)} + (a/(a + b*x^2))^{(2/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(8*\text{Sqrt}[2]*b^2*x*(a/(a + b*x^2))^{(2/3)}*(a + b*x^2)^{(1/6)}*\text{Sqrt}[-((1 - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2)])$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{3a \int \frac{1}{\sqrt[6]{a+bx^2}} dx}{8b} + \frac{3x(a+bx^2)^{5/6}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(b\*x\*\*2+a)\*\*(1/6), x)

[Out]  $-3*a*\text{Integral}((a + b*x^2)**(-1/6), x)/(8*b) + 3*x*(a + b*x^2)**(5/6)/(8*b)$

**Mathematica [C]** time = 0.0494524, size = 62, normalized size = 0.1

$$\frac{3x \left( -a \sqrt[6]{\frac{bx^2}{a}} + {}_2F_1 \left( \frac{1}{6}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a} \right) + a + bx^2 \right)}{8b \sqrt[6]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*x^2)^(1/6), x]

[Out] (3\*x\*(a + b\*x^2 - a\*(1 + (b\*x^2)/a)^(1/6)\*Hypergeometric2F1[1/6, 1/2, 3/2, -(b\*x^2)/a]))/(8\*b\*(a + b\*x^2)^(1/6))

**Maple [F]** time = 0.035, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt[6]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b\*x^2+a)^(1/6), x)

[Out] int(x^2/(b\*x^2+a)^(1/6), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2 + a)^{\frac{1}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b\*x^2 + a)^(1/6), x, algorithm="maxima")

[Out] integrate(x^2/(b\*x^2 + a)^(1/6), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{x^2}{(bx^2 + a)^{\frac{1}{6}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b\*x^2 + a)^(1/6), x, algorithm="fricas")

[Out] integral(x^2/(b\*x^2 + a)^(1/6), x)

**Sympy [A]** time = 2.65635, size = 27, normalized size = 0.04

$$\frac{x^3 {}_2F_1 \left( \frac{1}{6}, \frac{3}{2} \middle| \frac{bx^2 e^{i\pi}}{a} \right)}{3 \sqrt[6]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**2+a)**(1/6),x)`

[Out] `x**3*hyper((1/6, 3/2), (5/2, ), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/6))`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2 + a)^{\frac{1}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(1/6),x, algorithm="giac")`

[Out] `integrate(x^2/(b*x^2 + a)^(1/6), x)`

$$3.1021 \quad \int \frac{1}{\sqrt[6]{a+bx^2}} dx$$

**Optimal.** Leaf size=577

$$\frac{\frac{3x}{2\sqrt[6]{a+bx^2}} + \frac{3ax}{2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)}}{3^{3/4}a\left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1}\right) \middle| -7 + 4\sqrt{3}\right)} - \frac{\sqrt{2}bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}{3^{4/3}\sqrt{2 + \sqrt{3}}a\left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1}\right) \middle| -7 + 4\sqrt{3}\right)} + \frac{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}{}$$

[Out] (3\*x)/(2\*(a + b\*x^2)^(1/6)) + (3\*a\*x)/(2\*(a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(7/6)\*(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))) + (3^3^(1/4)\*Sqrt[2 + Sqrt[3]]\*a\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))]^2\*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(4\*b\*x\*(a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(1/6)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3)))^2]) - (3^(3/4)\*a\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))]^2\*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(Sqrt[2]\*b\*x\*(a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(1/6)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3)))^2])

**Rubi [A]** time = 1.0618, antiderivative size = 577, normalized size of antiderivative = 1., number of

steps used = 6, number of rules used = 6, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$

$$\frac{3x}{2\sqrt[6]{a+bx^2}} + \frac{3ax}{2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)}$$

$$- \frac{3^{3/4}a\left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1}\right) \middle| -7 + 4\sqrt{3}\right)}{\sqrt{2}bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}$$

$$+ \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}a\left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1}\right) \middle| -7 + 4\sqrt{3}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(-1/6), x]

[Out] (3\*x)/(2\*(a + b\*x^2)^(1/6)) + (3\*a\*x)/(2\*(a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(7/6)\*(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))) + (3\*3^(1/4)\*Sqrt[2 + Sqrt[3]]\*a\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))]^2\*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(4\*b\*x\*(a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(1/6)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3)))^2]) - (3^(3/4)\*a\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))]^2\*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(Sqrt[2]\*b\*x\*(a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(1/6)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3)))^2])

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{a \int \frac{1}{(a+bx^2)^{7/6}} dx}{2} + \frac{3x}{2\sqrt[6]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+a)\*\*(1/6), x)

[Out] -a\*Integral((a + b\*x\*\*2)\*\*(-7/6), x)/2 + 3\*x/(2\*(a + b\*x\*\*2)\*\*(1/6))

**Mathematica [C]** time = 0.0251743, size = 47, normalized size = 0.08

$$\frac{x\sqrt[6]{\frac{a+bx^2}{a}} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, -\frac{bx^2}{a}\right)}{\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(-1/6), x]

[Out] (x\*((a + b\*x^2)/a)^(1/6)\*Hypergeometric2F1[1/6, 1/2, 3/2, -((b\*x^2)/a)])/(a + b\*x^2)^(1/6)

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[6]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+a)^(1/6), x)

[Out] int(1/(b\*x^2+a)^(1/6), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-1/6), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(-1/6), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{1}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-1/6), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(-1/6), x)

**Sympy [A]** time = 2.57766, size = 24, normalized size = 0.04

$$\frac{x_2F_1\left(\frac{1}{6}, \frac{1}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{\sqrt[6]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*2+a)\*\*(1/6), x)

[Out] x\*hyper((1/6, 1/2), (3/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/a\*\*(1/6)



---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-1/6),x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(-1/6), x)

$$3.1022 \quad \int \frac{1}{x^2 \sqrt[6]{a+bx^2}} dx$$

**Optimal.** Leaf size=586

$$\frac{\frac{bx}{a\sqrt[6]{a+bx^2}} + \frac{bx}{\left(\frac{a}{a+bx^2}\right)^{2/3} (a+bx^2)^{7/6} \left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)} - \frac{(a+bx^2)^{5/6}}{ax}}{\sqrt{2} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1}\right) \middle| -7 + 4\sqrt{3}\right)}} + \frac{\sqrt[4]{3} \sqrt{2 + \sqrt{3}} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1}\right) \middle| -7 + 4\sqrt{3}\right)}}{2x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}$$

[Out] (b\*x)/(a\*(a+b\*x^2)^(1/6)) - (a+b\*x^2)^(5/6)/(a\*x) + (b\*x)/((a/(a+b\*x^2))^(2/3)\*(a+b\*x^2)^(7/6)\*(1-Sqrt[3]-(a/(a+b\*x^2))^(1/3))) + (3^(1/4)\*Sqrt[2+Sqrt[3]]\*(1-(a/(a+b\*x^2))^(1/3))\*Sqrt[(1+(a/(a+b\*x^2))^(1/3)+(a/(a+b\*x^2))^(2/3))/(1-Sqrt[3]-(a/(a+b\*x^2))^(1/3))]^2\*EllipticE[ArcSin[(1+Sqrt[3]-a/(a+b\*x^2)^(1/3))/(1-Sqrt[3]-a/(a+b\*x^2)^(1/3))], -7+4\*Sqrt[3]])/(2\*x\*(a/(a+b\*x^2))^(2/3)\*(a+b\*x^2)^(1/6)\*Sqrt[-((1-(a/(a+b\*x^2))^(1/3))/(1-Sqrt[3]-a/(a+b\*x^2)^(1/3)))^2]) - (Sqrt[2]\*(1-(a/(a+b\*x^2))^(1/3))\*Sqrt[(1+(a/(a+b\*x^2))^(1/3)+(a/(a+b\*x^2))^(2/3))/(1-Sqrt[3]-a/(a+b\*x^2)^(1/3))]^2\*EllipticF[ArcSin[(1+Sqrt[3]-a/(a+b\*x^2)^(1/3))/(1-Sqrt[3]-a/(a+b\*x^2)^(1/3))], -7+4\*Sqrt[3]])/(3^(1/4)\*x\*(a/(a+b\*x^2))^(2/3)\*(a+b\*x^2)^(1/6)\*Sqrt[-((1-(a/(a+b\*x^2))^(1/3))/(1-Sqrt[3]-a/(a+b\*x^2)^(1/3)))^2])

**Rubi [A]** time = 1.17905, antiderivative size = 586, normalized size of antiderivative = 1., number of

steps used = 7, number of rules used = 7, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$\frac{bx}{a\sqrt[6]{a+bx^2}} + \frac{bx}{\left(\frac{a}{a+bx^2}\right)^{2/3} (a+bx^2)^{7/6} \left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)} - \frac{(a+bx^2)^{5/6}}{ax}$$

$$- \frac{\sqrt{2} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1}\right) \middle| -7 + 4\sqrt{3}\right)}{\sqrt[4]{3}x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{-\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}$$

$$+ \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1}\right) \middle| -7 + 4\sqrt{3}\right)}{2x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{-\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a + b\*x^2)^(1/6)), x]

[Out] (b\*x)/(a\*(a + b\*x^2)^(1/6)) - (a + b\*x^2)^(5/6)/(a\*x) + (b\*x)/((a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(7/6)\*(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))) + (3^(1/4)\*Sqrt[2 + Sqrt[3]]\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))]^2\*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(2\*x\*(a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(1/6)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3)))^2]) - (Sqrt[2]\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))]^2\*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(3^(1/4)\*x\*(a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(1/6)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3)))^2])

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2b \int \frac{1}{\sqrt[6]{a+bx^2}} dx}{3a} - \frac{(a+bx^2)^{5/6}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(b\*x\*\*2+a)\*\*(1/6), x)

[Out] 2\*b\*Integral((a + b\*x\*\*2)\*\*(-1/6), x)/(3\*a) - (a + b\*x\*\*2)\*\*(5/6)/(a\*x)

**Mathematica [C]** time = 0.0487849, size = 70, normalized size = 0.12

$$\frac{2bx^2 \sqrt[6]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) - 3(a+bx^2)}{3ax\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a + b\*x^2)^(1/6)),x]

[Out]  $(-3*(a + b*x^2) + 2*b*x^2*(1 + (b*x^2)/a)^(1/6)*\text{Hypergeometric2F1}[1/6, 1/2, 3/2, -((b*x^2)/a)])/(3*a*x*(a + b*x^2)^(1/6))$

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt[6]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b\*x^2+a)^(1/6),x)

[Out] int(1/x^2/(b\*x^2+a)^(1/6),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{6}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/6)\*x^2),x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(1/6)\*x^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{1}{6}} x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/6)\*x^2),x, algorithm="fricas")

[Out] integral(1/((b\*x^2 + a)^(1/6)\*x^2), x)

**Sympy [A]** time = 3.12632, size = 27, normalized size = 0.05

$$-\frac{{}_2F_1\left(-\frac{1}{2}, \frac{1}{6} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{\sqrt[6]{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(b\*x\*\*2+a)\*\*(1/6),x)

[Out] -hyper((-1/2, 1/6), (1/2,), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(a\*\*(1/6)\*x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{6}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/6)*x^2),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(1/6)*x^2), x)`

$$3.1023 \quad \int \frac{1}{x^4 \sqrt[6]{a + bx^2}} dx$$

**Optimal.** Leaf size=633

$$\begin{aligned} & -\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}} + \frac{4b(a+bx^2)^{5/6}}{9a^2x} - \frac{4b^2x}{9a\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)} \\ & + \frac{4\sqrt{2}b\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)\middle| -7+4\sqrt{3}\right)}{\sqrt{\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}} \\ & + \frac{9\sqrt[4]{3}ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}{\sqrt{\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}} \\ & + \frac{2\sqrt{2+\sqrt{3}}b\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{bx^2+a}}-\sqrt{3}+1}\right)\middle| -7+4\sqrt{3}\right)}{\sqrt{\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}} \\ & + \frac{3\cdot 3^{3/4}ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}}{\sqrt{\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}} \\ & - \frac{(a+bx^2)^{5/6}}{3ax^3} \end{aligned}$$

[Out]  $(-4*b^2*x)/(9*a^2*(a+b*x^2)^(1/6)) - (a+b*x^2)^(5/6)/(3*a*x^3) + (4*b*(a+b*x^2)^(5/6))/(9*a^2*x) - (4*b^2*x)/(9*a*(a/(a+b*x^2))^(2/3)*(a+b*x^2)^(7/6)*(1-\sqrt{3}-(a/(a+b*x^2))^(1/3)))) - (2*\sqrt{2+\sqrt{3}}*b*(1-(a/(a+b*x^2))^(1/3))*\sqrt{(1+(a/(a+b*x^2))^(1/3)+(a/(a+b*x^2))^(2/3))/(1-\sqrt{3}-(a/(a+b*x^2))^(1/3))^2}*\text{EllipticE}[\text{ArcSin}[(1+\sqrt{3}-(a/(a+b*x^2))^(1/3))/(1-\sqrt{3}-(a/(a+b*x^2))^(1/3))], -7+4*\sqrt{3}])/(3*3^(3/4)*a*x*(a/(a+b*x^2))^(2/3)*(a+b*x^2)^(1/6)*\sqrt{-((1-(a/(a+b*x^2))^(1/3))/(1-\sqrt{3}-(a/(a+b*x^2))^(1/3))^2)}) + (4*\sqrt{2}*b*(1-(a/(a+b*x^2))^(1/3))*\sqrt{(1+(a/(a+b*x^2))^(1/3)+(a/(a+b*x^2))^(2/3))/(1-\sqrt{3}-(a/(a+b*x^2))^(1/3))^2}*\text{EllipticF}[\text{ArcSin}[(1+\sqrt{3}-(a/(a+b*x^2))^(1/3))/(1-\sqrt{3}-(a/(a+b*x^2))^(1/3))], -7+4*\sqrt{3}])/(9*3^(1/4)*a*x*(a/(a+b*x^2))^(2/3)*(a+b*x^2)^(1/6)*\sqrt{-((1-(a/(a+b*x^2))^(1/3))/(1-\sqrt{3}-(a/(a+b*x^2))^(1/3))^2)})$

**Rubi [A]** time = 1.34764, antiderivative size = 633, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 7, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$\begin{aligned}
 & -\frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}} + \frac{4b(a+bx^2)^{5/6}}{9a^2x} - \frac{4b^2x}{9a\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)} \\
 & + \frac{4\sqrt{2}b\left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{bx^2+a}} - \sqrt{3} + 1}\right) \mid -7 + 4\sqrt{3}\right)}{\sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}} \\
 & + \frac{9\sqrt[4]{3}ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}{\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{bx^2+a}} - \sqrt{3} + 1}\right) \mid -7 + 4\sqrt{3}\right)} \\
 & - \frac{3^{3/4}ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}{\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{bx^2+a}} - \sqrt{3} + 1}\right) \mid -7 + 4\sqrt{3}\right)} \\
 & - \frac{(a+bx^2)^{5/6}}{3ax^3}
 \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a + b\*x^2)^(1/6)), x]

[Out]  $(-4*b^2*x)/(9*a^2*(a + b*x^2)^{(1/6)}) - (a + b*x^2)^{(5/6)}/(3*a*x^3) + (4*b*(a + b*x^2)^{(5/6)})/(9*a^2*x) - (4*b^2*x)/(9*a*(a/(a + b*x^2))^{(2/3)}*(a + b*x^2)^{(7/6)}*(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})) - (2*\text{Sqrt}[2 + \text{Sqrt}[3]]*b*(1 - (a/(a + b*x^2))^{(1/3)})*\text{Sqrt}[(1 + (a/(a + b*x^2))^{(1/3)} + (a/(a + b*x^2))^{(2/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(3*3^{(3/4)}*a*x*(a/(a + b*x^2))^{(2/3)}*(a + b*x^2)^{(1/6)}*\text{Sqrt}[-((1 - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2)]) + (4*\text{Sqrt}[2]*b*(1 - (a/(a + b*x^2))^{(1/3)})*\text{Sqrt}[(1 + (a/(a + b*x^2))^{(1/3)} + (a/(a + b*x^2))^{(2/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(9*3^{(1/4)}*a*x*(a/(a + b*x^2))^{(2/3)}*(a + b*x^2)^{(1/6)}*\text{Sqrt}[-((1 - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2)])$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{4b^2 \int \frac{1}{(a+bx^2)^{7/6}} dx}{27a} - \frac{(a+bx^2)^{5/6}}{3ax^3} - \frac{4b^2x}{9a^2\sqrt[6]{a+bx^2}} + \frac{4b(a+bx^2)^{5/6}}{9a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(b\*x\*\*2+a)\*\*(1/6), x)

[Out]  $4*b**2*Integral((a + b*x**2)**(-7/6), x)/(27*a) - (a + b*x**2)**(5/6)/(3*a*x**3) - 4*b**2*x/(9*a**2*(a + b*x**2)**(1/6)) + 4*b*(a + b*x**2)**(5/6)/(9*a**2*x)$

---

**Mathematica [C]** time = 0.0548304, size = 83, normalized size = 0.13

$$\frac{-9a^2 - 8b^2x^4\sqrt{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, -\frac{bx^2}{a}\right) + 3abx^2 + 12b^2x^4}{27a^2x^3\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a + b\*x^2)^(1/6)), x]

[Out] (-9\*a^2 + 3\*a\*b\*x^2 + 12\*b^2\*x^4 - 8\*b^2\*x^4\*(1 + (b\*x^2)/a)^(1/6))\*Hypergeometric2F1[1/6, 1/2, 3/2, -(b\*x^2)/a]/(27\*a^2\*x^3\*(a + b\*x^2)^(1/6))

---

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 \sqrt[6]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b\*x^2+a)^(1/6), x)

[Out] int(1/x^4/(b\*x^2+a)^(1/6), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{6}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/6)\*x^4), x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(1/6)\*x^4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{1}{6}} x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/6)\*x^4), x, algorithm="fricas")

[Out] integral(1/((b\*x^2 + a)^(1/6)\*x^4), x)

---

**Sympy [A]** time = 4.06412, size = 32, normalized size = 0.05

$$-\frac{{}_2F_1\left(-\frac{3}{2}, \frac{1}{6} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3\sqrt[6]{ax^3}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(b*x**2+a)**(1/6),x)`

[Out] `-hyper((-3/2, 1/6), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/6)*x**3)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{6}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/6)*x^4),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(1/6)*x^4), x)`

$$3.1024 \quad \int \frac{1}{x^6 \sqrt[6]{a + bx^2}} dx$$

**Optimal.** Leaf size=661

$$\begin{aligned} & \frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}} - \frac{8b^2(a+bx^2)^{5/6}}{27a^3x} + \frac{8b^3x}{27a^2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)} \\ & - \frac{8\sqrt{2}b^2\left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}+1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \mid -7 + 4\sqrt{3}\right)}{27\sqrt[4]{3}a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}} \\ & + \frac{4\sqrt{2 + \sqrt{3}}b^2\left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}+1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \mid -7 + 4\sqrt{3}\right)}{9 \cdot 3^{3/4}a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}} \\ & + \frac{2b(a+bx^2)^{5/6}}{9a^2x^3} - \frac{(a+bx^2)^{5/6}}{5ax^5} \end{aligned}$$

[Out]  $(8*b^3*x)/(27*a^3*(a + b*x^2)^(1/6)) - (a + b*x^2)^(5/6)/(5*a*x^5) + (2*b*(a + b*x^2)^(5/6))/(9*a^2*x^3) - (8*b^2*(a + b*x^2)^(5/6))/(27*a^3*x) + (8*b^3*x)/(27*a^2*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (4*Sqrt[2 + Sqrt[3]]*b^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))]^2*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(3/4)*a^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3)))^2]) - (8*Sqrt[2]*b^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))]^2*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(27*3^(1/4)*a^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3)))^2])$

**Rubi [A]** time = 1.50293, antiderivative size = 661, normalized size of antiderivative = 1., number of

steps used = 9, number of rules used = 7, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$\frac{\frac{8b^3x}{27a^3\sqrt[6]{a+bx^2}} - \frac{8b^2(a+bx^2)^{5/6}}{27a^3x} + \frac{8b^3x}{27a^2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)}}{8\sqrt{2}b^2\left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{bx^2+a}} - \sqrt{3} + 1}\right)\right)|_{-7+4\sqrt{3}}}$$

$$- \frac{27\sqrt[4]{3}a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}{4\sqrt{2+\sqrt{3}}b^2\left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{bx^2+a}} - \sqrt{3} + 1}\right)\right)|_{-7+4\sqrt{3}}}$$

$$+ \frac{9\cdot 3^{3/4}a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}{\frac{2b(a+bx^2)^{5/6}}{9a^2x^3} - \frac{(a+bx^2)^{5/6}}{5ax^5}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(a + b\*x^2)^(1/6)),x]

[Out]  $(8*b^3*x)/(27*a^3*(a + b*x^2)^(1/6)) - (a + b*x^2)^(5/6)/(5*a*x^5) + (2*b*(a + b*x^2)^(5/6))/(9*a^2*x^3) - (8*b^2*(a + b*x^2)^(5/6))/(27*a^3*x) + (8*b^3*x)/(27*a^2*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (4*Sqrt[2 + Sqrt[3]]*b^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))]^2)*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(3/4)*a^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3)))^2]) - (8*Sqrt[2]*b^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))]^2)*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(27*3^(1/4)*a^2*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3)))^2])]$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{(a+bx^2)^{5/6}}{5ax^5} + \frac{2b(a+bx^2)^{5/6}}{9a^2x^3} + \frac{16b^3 \int \frac{1}{\sqrt[6]{a+bx^2}} dx}{81a^3} - \frac{8b^2(a+bx^2)^{5/6}}{27a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*6/(b\*x\*\*2+a)\*\*(1/6),x)

[Out]  $-(a + b*x^2)^(5/6)/(5*a*x^5) + 2*b*(a + b*x^2)^(5/6)/(9*a^2*x^3) + 16*b^3*Integral((a + b*x^2)^(5/6), x)/(81*a^3) - 8*b^2*(a + b*x^2)^(5/6)/(27*a^3*x)$

**Mathematica [C]** time = 0.0606179, size = 94, normalized size = 0.14

$$\frac{-81a^3 + 9a^2bx^2 + 80b^3x^6 \sqrt[6]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{6}, \frac{1}{2}, \frac{3}{2}; -\frac{bx^2}{a}\right) - 30ab^2x^4 - 120b^3x^6}{405a^3x^5 \sqrt[6]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6\*(a + b\*x^2)^(1/6)), x]

[Out] (-81\*a^3 + 9\*a^2\*b\*x^2 - 30\*a\*b^2\*x^4 - 120\*b^3\*x^6 + 80\*b^3\*x^6\*(1 + (b\*x^2)/a)^(1/6)\*Hypergeometric2F1[1/6, 1/2, 3/2, -(b\*x^2)/a])/(405\*a^3\*x^5\*(a + b\*x^2)^(1/6))

**Maple [F]** time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{x^6 \sqrt[6]{bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(b\*x^2+a)^(1/6), x)

[Out] int(1/x^6/(b\*x^2+a)^(1/6), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{6}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/6)\*x^6), x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(1/6)\*x^6), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{1}{6}} x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(1/6)\*x^6), x, algorithm="fricas")

[Out] integral(1/((b\*x^2 + a)^(1/6)\*x^6), x)

**Sympy [A]** time = 5.34235, size = 32, normalized size = 0.05

$$\frac{{}_2F_1\left(-\frac{5}{2}, \frac{1}{6} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5\sqrt[6]{ax^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**6/(b*x**2+a)**(1/6),x)`

[Out] `-hyper((-5/2, 1/6), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/6)*x**5)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{6}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(1/6)*x^6),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(1/6)*x^6), x)`

$$3.1025 \quad \int \frac{x^6}{(a+bx^2)^{5/6}} dx$$

Optimal. Leaf size=324

$$\frac{81 \cdot 3^{3/4} \sqrt{2 - \sqrt{3}} a^3 \sqrt[6]{a+bx^2} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \mid -7 + 4\sqrt{3}\right)}{128b^4 x^3 \sqrt{\frac{a}{a+bx^2}} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$+ \frac{81a^2 x \sqrt[6]{a+bx^2}}{128b^3} - \frac{9ax^3 \sqrt[6]{a+bx^2}}{32b^2} + \frac{3x^5 \sqrt[6]{a+bx^2}}{16b}$$

[Out] (81\*a^2\*x\*(a + b\*x^2)^(1/6))/(128\*b^3) - (9\*a\*x^3\*(a + b\*x^2)^(1/6))/(32\*b^2) + (3\*x^5\*(a + b\*x^2)^(1/6))/(16\*b) - (81\*3^(3/4)\*Sqrt[2 - Sqrt[3]]\*a^3\*(a + b\*x^2)^(1/6)\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))]^2)\*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(128\*b^4\*x\*(a/(a + b\*x^2))^(1/3)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3)))^2]))

**Rubi [A]** time = 0.68738, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{81 \cdot 3^{3/4} \sqrt{2 - \sqrt{3}} a^3 \sqrt[6]{a+bx^2} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \mid -7 + 4\sqrt{3}\right)}{128b^4 x^3 \sqrt{\frac{a}{a+bx^2}} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$+ \frac{81a^2 x \sqrt[6]{a+bx^2}}{128b^3} - \frac{9ax^3 \sqrt[6]{a+bx^2}}{32b^2} + \frac{3x^5 \sqrt[6]{a+bx^2}}{16b}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a + b\*x^2)^(5/6), x]

[Out] (81\*a^2\*x\*(a + b\*x^2)^(1/6))/(128\*b^3) - (9\*a\*x^3\*(a + b\*x^2)^(1/6))/(32\*b^2) + (3\*x^5\*(a + b\*x^2)^(1/6))/(16\*b) - (81\*3^(3/4)\*Sqrt[2 - Sqrt[3]]\*a^3\*(a + b\*x^2)^(1/6)\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))]^2)\*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(128\*b^4\*x\*(a/(a + b\*x^2))^(1/3)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3)))^2]))

**Rubi in Sympy [A]** time = 23.5635, size = 299, normalized size = 0.92

$$81 \cdot 3^{\frac{3}{4}} a^3 \sqrt{\frac{\left(-\frac{bx^2}{a+bx^2}+1\right)^{\frac{2}{3}} + \sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1 - \sqrt{3} + 1\right)^2}} \sqrt{-\sqrt{3} + 2\sqrt[6]{a+bx^2}} \left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1\right) F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1 + \sqrt{3}}{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1 - \sqrt{3} + 1}\right)\right)$$


---


$$128b^4x^3\sqrt{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} - 1}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1 - \sqrt{3} + 1\right)^2}}$$

$$+ \frac{81a^2x\sqrt[6]{a+bx^2}}{128b^3} - \frac{9ax^3\sqrt[6]{a+bx^2}}{32b^2} + \frac{3x^5\sqrt[6]{a+bx^2}}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**6/(b*x**2+a)**(5/6),x)`

[Out]  $-81 \cdot 3^{3/4} \cdot a^3 \cdot \sqrt{\left(\left(-b \cdot x^2 / (a + b \cdot x^2) + 1\right)^{2/3} + \left(-b \cdot x^2 / (a + b \cdot x^2) + 1\right)^{1/3} + 1\right) / \left(-\left(-b \cdot x^2 / (a + b \cdot x^2) + 1\right)^{1/3} - \sqrt{3} + 1\right)^2} \cdot \sqrt{-\sqrt{3} + 2} \cdot (a + b \cdot x^2)^{1/6} \cdot \left(-\left(-b \cdot x^2 / (a + b \cdot x^2) + 1\right)^{1/3} + 1 + \sqrt{3}\right) / \left(-\left(-b \cdot x^2 / (a + b \cdot x^2) + 1\right)^{1/3} - \sqrt{3} + 1\right)} \cdot \operatorname{elliptic\_f}\left(\operatorname{asin}\left(\frac{-\left(-b \cdot x^2 / (a + b \cdot x^2) + 1\right)^{1/3} + 1 + \sqrt{3}}{-\left(-b \cdot x^2 / (a + b \cdot x^2) + 1\right)^{1/3} - \sqrt{3} + 1}\right), -7 + 4 \cdot \sqrt{3}\right) / \left(128 \cdot b^4 \cdot x \cdot (a / (a + b \cdot x^2))^{1/3} \cdot \sqrt{\left(\left(-b \cdot x^2 / (a + b \cdot x^2) + 1\right)^{1/3} - 1\right) / \left(-\left(-b \cdot x^2 / (a + b \cdot x^2) + 1\right)^{1/3} - \sqrt{3} + 1\right)^2}\right) + 81 \cdot a^2 \cdot x^2 \cdot (a + b \cdot x^2)^{1/6} / \left(128 \cdot b^3\right) - 9 \cdot a \cdot x^3 \cdot (a + b \cdot x^2)^{1/6} / \left(32 \cdot b^2\right) + 3 \cdot x^5 \cdot (a + b \cdot x^2)^{1/6} / \left(16 \cdot b\right)$

**Mathematica [C]** time = 0.0661533, size = 89, normalized size = 0.27

$$\frac{3x \left( -27a^3 \left( \frac{bx^2}{a} + 1 \right)^{5/6} {}_2F_1 \left( \frac{1}{2}, \frac{5}{6}; \frac{3}{2}; -\frac{bx^2}{a} \right) + 27a^3 + 15a^2bx^2 - 4ab^2x^4 + 8b^3x^6 \right)}{128b^3(a+bx^2)^{5/6}}$$

Antiderivative was successfully verified.

[In] `Integrate[x^6/(a + b*x^2)^(5/6),x]`

[Out]  $(3 \cdot x \cdot (27 \cdot a^3 + 15 \cdot a^2 \cdot b \cdot x^2 - 4 \cdot a \cdot b^2 \cdot x^4 + 8 \cdot b^3 \cdot x^6 - 27 \cdot a^3 \cdot (1 + (b \cdot x^2) / a)^{5/6}) \cdot \operatorname{Hypergeometric2F1}\left[1/2, 5/6, 3/2, -((b \cdot x^2) / a)\right]) / (128 \cdot b^3 \cdot (a + b \cdot x^2)^{5/6})$

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int x^6 (bx^2 + a)^{-5/6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(b*x^2+a)^(5/6),x)`

[Out] `int(x^6/(b*x^2+a)^(5/6),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(bx^2 + a)^{5/6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^2 + a)^(5/6),x, algorithm="maxima")`

[Out] `integrate(x^6/(b*x^2 + a)^(5/6), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{(bx^2 + a)^{\frac{5}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^2 + a)^(5/6),x, algorithm="fricas")`

[Out] `integral(x^6/(b*x^2 + a)^(5/6), x)`

**Sympy** [A] time = 3.16743, size = 27, normalized size = 0.08

$$\frac{x^7 {}_2F_1\left(\frac{5}{6}, \frac{7}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{7a^{\frac{5}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(b*x**2+a)**(5/6),x)`

[Out] `x**7*hyper((5/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(5/6))`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(bx^2 + a)^{\frac{5}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^2 + a)^(5/6),x, algorithm="giac")`

[Out] `integrate(x^6/(b*x^2 + a)^(5/6), x)`



$$3.1026 \quad \int \frac{x^4}{(a+bx^2)^{5/6}} dx$$

Optimal. Leaf size=300

$$\frac{27 \cdot 3^{3/4} \sqrt{2 - \sqrt{3}} a^2 \sqrt[6]{a + bx^2} \left(1 - \sqrt[3]{\frac{a}{a + bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a + bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a + bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a + bx^2} - \sqrt{3} + 1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2 + a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2 + a} - \sqrt{3} + 1}}\right) \middle| -7 + 4\sqrt{3}\right)}{40b^3 x \sqrt[3]{\frac{a}{a + bx^2}} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a + bx^2}}}{\left(-\sqrt[3]{\frac{a}{a + bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$- \frac{27ax\sqrt[6]{a + bx^2}}{40b^2} + \frac{3x^3\sqrt[6]{a + bx^2}}{10b}$$

[Out]  $(-27 \cdot a \cdot x \cdot (a + b \cdot x^2)^{(1/6)}) / (40 \cdot b^2) + (3 \cdot x^3 \cdot (a + b \cdot x^2)^{(1/6)}) / (10 \cdot b) + (27 \cdot 3^{3/4} \cdot \text{Sqrt}[2 - \text{Sqrt}[3]] \cdot a^2 \cdot (a + b \cdot x^2)^{(1/6)} \cdot (1 - (a / (a + b \cdot x^2))^{(1/3)}) \cdot \text{Sqrt}[(1 + (a / (a + b \cdot x^2))^{(1/3)} + (a / (a + b \cdot x^2))^{(2/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})^2] \cdot \text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})], -7 + 4 \cdot \text{Sqrt}[3]]) / (40 \cdot b^3 \cdot x \cdot (a / (a + b \cdot x^2))^{(1/3)} \cdot \text{Sqrt}[-((1 - (a / (a + b \cdot x^2))^{(1/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})^2])]$

**Rubi [A]** time = 0.575122, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{27 \cdot 3^{3/4} \sqrt{2 - \sqrt{3}} a^2 \sqrt[6]{a + bx^2} \left(1 - \sqrt[3]{\frac{a}{a + bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a + bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a + bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a + bx^2} - \sqrt{3} + 1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2 + a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2 + a} - \sqrt{3} + 1}}\right) \middle| -7 + 4\sqrt{3}\right)}{40b^3 x \sqrt[3]{\frac{a}{a + bx^2}} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a + bx^2}}}{\left(-\sqrt[3]{\frac{a}{a + bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$- \frac{27ax\sqrt[6]{a + bx^2}}{40b^2} + \frac{3x^3\sqrt[6]{a + bx^2}}{10b}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a + b\*x^2)^(5/6), x]

[Out]  $(-27 \cdot a \cdot x \cdot (a + b \cdot x^2)^{(1/6)}) / (40 \cdot b^2) + (3 \cdot x^3 \cdot (a + b \cdot x^2)^{(1/6)}) / (10 \cdot b) + (27 \cdot 3^{3/4} \cdot \text{Sqrt}[2 - \text{Sqrt}[3]] \cdot a^2 \cdot (a + b \cdot x^2)^{(1/6)} \cdot (1 - (a / (a + b \cdot x^2))^{(1/3)}) \cdot \text{Sqrt}[(1 + (a / (a + b \cdot x^2))^{(1/3)} + (a / (a + b \cdot x^2))^{(2/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})^2] \cdot \text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})], -7 + 4 \cdot \text{Sqrt}[3]]) / (40 \cdot b^3 \cdot x \cdot (a / (a + b \cdot x^2))^{(1/3)} \cdot \text{Sqrt}[-((1 - (a / (a + b \cdot x^2))^{(1/3)}) / (1 - \text{Sqrt}[3] - (a / (a + b \cdot x^2))^{(1/3)})^2])]$

**Rubi in Sympy [A]** time = 18.1516, size = 275, normalized size = 0.92

$$27 \cdot 3^{\frac{3}{4}} a^2 \frac{\sqrt{\frac{\left(-\frac{bx^2}{a+bx^2}+1\right)^{\frac{2}{3}} + \sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2} \sqrt{-\sqrt{3}+2} \sqrt[6]{a+bx^2} \left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1}\right) F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1+\sqrt{3}}}{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}}\right)\right)}{\sqrt{\frac{\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2}}}$$

$$- \frac{27ax\sqrt[6]{a+bx^2}}{40b^2} + \frac{3x^3\sqrt[6]{a+bx^2}}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**4/(b*x**2+a)**(5/6),x)`

[Out]  $27 \cdot 3^{3/4} \cdot a^{2/3} \cdot \sqrt{\left(\left(-\frac{bx^2}{a+bx^2}+1\right)^{2/3} + \sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1\right) / \left(-\left(-\frac{bx^2}{a+bx^2}+1\right)^{1/3} - \sqrt{3} + 1\right)^2} \cdot \sqrt{-\sqrt{3}+2} \cdot \sqrt[6]{a+bx^2} \cdot \left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1}\right) \cdot F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1+\sqrt{3}}}{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}}\right)\right) - \frac{27ax\sqrt[6]{a+bx^2}}{40b^2} + \frac{3x^3\sqrt[6]{a+bx^2}}{10b}$

**Mathematica [C]** time = 0.0552159, size = 79, normalized size = 0.26

$$\frac{3 \left( 9a^2x \left( \frac{bx^2}{a} + 1 \right)^{5/6} {}_2F_1 \left( \frac{1}{2}, \frac{5}{6}; \frac{3}{2}; -\frac{bx^2}{a} \right) - 9a^2x - 5abx^3 + 4b^2x^5 \right)}{40b^2 (a + bx^2)^{5/6}}$$

Antiderivative was successfully verified.

[In] `Integrate[x^4/(a + b*x^2)^(5/6),x]`

[Out]  $(3 \cdot (-9a^2x - 5abx^3 + 4b^2x^5 + 9a^2x \cdot (1 + (bx^2)/a)^{5/6}) \cdot \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{3}{2}, -\frac{bx^2}{a}\right]) / (40b^2(a + bx^2)^{5/6})$

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int x^4 (bx^2 + a)^{-5/6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(b*x^2+a)^(5/6),x)`

[Out] `int(x^4/(b*x^2+a)^(5/6),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2 + a)^{5/6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^(5/6),x, algorithm="maxima")`

[Out] `integrate(x^4/(b*x^2 + a)^(5/6), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{(bx^2 + a)^{\frac{5}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^(5/6),x, algorithm="fricas")`

[Out] `integral(x^4/(b*x^2 + a)^(5/6), x)`

**Sympy** [A] time = 2.7677, size = 27, normalized size = 0.09

$$\frac{x^5 {}_2F_1\left(\frac{5}{6}, \frac{5}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{5a^{\frac{5}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(b*x**2+a)**(5/6),x)`

[Out] `x**5*hyper((5/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/6))`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2 + a)^{\frac{5}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^(5/6),x, algorithm="giac")`

[Out] `integrate(x^4/(b*x^2 + a)^(5/6), x)`

$$3.1027 \quad \int \frac{x^2}{(a+bx^2)^{5/6}} dx$$

**Optimal.** Leaf size=276

$$\frac{3x\sqrt[6]{a+bx^2}}{4b} - \frac{3 \cdot 3^{3/4} \sqrt{2-\sqrt{3}} a \sqrt[6]{a+bx^2} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}+1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}+\sqrt{3}+1}}{-\sqrt[3]{\frac{a}{bx^2+a}-\sqrt{3}+1}}\right) \mid -7+4\sqrt{3}\right)}{4b^2 x \sqrt[3]{\frac{a}{a+bx^2}} \sqrt{\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}}}$$

[Out] (3\*x\*(a + b\*x^2)^(1/6))/(4\*b) - (3\*3^(3/4)\*Sqrt[2 - Sqrt[3]]\*a\*(a + b\*x^2)^(1/6)\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))]^2\*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(4\*b^2\*x\*(a/(a + b\*x^2))^(1/3)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3)))^2])

**Rubi [A]** time = 0.486809, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{3x\sqrt[6]{a+bx^2}}{4b} - \frac{3 \cdot 3^{3/4} \sqrt{2-\sqrt{3}} a \sqrt[6]{a+bx^2} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}+1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}+\sqrt{3}+1}}{-\sqrt[3]{\frac{a}{bx^2+a}-\sqrt{3}+1}}\right) \mid -7+4\sqrt{3}\right)}{4b^2 x \sqrt[3]{\frac{a}{a+bx^2}} \sqrt{\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*x^2)^(5/6), x]

[Out] (3\*x\*(a + b\*x^2)^(1/6))/(4\*b) - (3\*3^(3/4)\*Sqrt[2 - Sqrt[3]]\*a\*(a + b\*x^2)^(1/6)\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))]^2\*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(4\*b^2\*x\*(a/(a + b\*x^2))^(1/3)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3)))^2])

**Rubi in Sympy [A]** time = 12.5027, size = 252, normalized size = 0.91

$$\frac{3 \cdot 3^{\frac{3}{4}} a \sqrt{\frac{\left(-\frac{bx^2}{a+bx^2}+1\right)^{\frac{2}{3}}+\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2} \sqrt{-\sqrt{3}+2} \sqrt[6]{a+bx^2} \left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1}\right) F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1+\sqrt{3}}}{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}}\right)\right)}{4b^2 x \sqrt[3]{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2}} + \frac{3x \sqrt[6]{a+bx^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2/(b*x**2+a)**(5/6),x)`

[Out]  $-3 \cdot 3^{3/4} \cdot a \cdot \sqrt{\left(\left(-\frac{b x^2}{a+b x^2}+1\right)^{2/3}+\left(-\frac{b x^2}{a+b x^2}+1\right)^{1/3}+1\right) / \left(-\left(-\frac{b x^2}{a+b x^2}+1\right)^{1/3}-\sqrt{3}+1\right)^2} \cdot \sqrt{-\sqrt{3}+2} \cdot \sqrt[6]{a+b x^2} \cdot \left(-\sqrt[3]{-\frac{b x^2}{a+b x^2}+1+1}\right) \cdot F\left(\operatorname{asin}\left(\frac{-\left(-\frac{b x^2}{a+b x^2}+1\right)^{1/3}+1+\sqrt{3}}{-\left(-\frac{b x^2}{a+b x^2}+1\right)^{1/3}-\sqrt{3}+1}\right)\right)$   
 $+ \frac{3 x \sqrt[6]{a+b x^2}}{4 b}$

**Mathematica [C]** time = 0.0463243, size = 62, normalized size = 0.22

$$\frac{3x \left(-a \left(\frac{bx^2}{a} + 1\right)^{5/6} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; -\frac{bx^2}{a}\right) + a + bx^2\right)}{4b(a+bx^2)^{5/6}}$$

Antiderivative was successfully verified.

[In] `Integrate[x^2/(a + b*x^2)^(5/6),x]`

[Out]  $(3 x^2 (a + b x^2 - a (1 + (b x^2)/a)^{5/6}) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{3}{2}, -\frac{b x^2}{a}\right]) / (4 b (a + b x^2)^{5/6})$

**Maple [F]** time = 0.037, size = 0, normalized size = 0.

$$\int x^2 (bx^2 + a)^{-5/6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(b*x^2+a)^(5/6),x)`

[Out] `int(x^2/(b*x^2+a)^(5/6),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2 + a)^{5/6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(5/6),x, algorithm="maxima")`

[Out] `integrate(x^2/(b*x^2 + a)^(5/6), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{(bx^2 + a)^{\frac{5}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(5/6),x, algorithm="fricas")`

[Out] `integral(x^2/(b*x^2 + a)^(5/6), x)`

**Sympy** [A] time = 2.75535, size = 27, normalized size = 0.1

$$\frac{x^3 {}_2F_1\left(\frac{5}{6}, \frac{3}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{5}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(b*x**2+a)**(5/6), x)`

[Out] `x**3*hyper((5/6, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/6))`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2 + a)^{\frac{5}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(5/6),x, algorithm="giac")`

[Out] `integrate(x^2/(b*x^2 + a)^(5/6), x)`

$$3.1028 \quad \int \frac{1}{(a+bx^2)^{5/6}} dx$$

**Optimal.** Leaf size=252

$$\frac{3^{3/4} \sqrt{2 - \sqrt{3}} \sqrt[6]{a + bx^2} \left(1 - \sqrt[3]{\frac{a}{a + bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a + bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a + bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a + bx^2} - \sqrt{3} + 1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2 + a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2 + a} - \sqrt{3} + 1}}\right) \mid -7 + 4\sqrt{3}\right)}{bx \sqrt[3]{\frac{a}{a + bx^2}} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a + bx^2}}}{\left(-\sqrt[3]{\frac{a}{a + bx^2} - \sqrt{3} + 1}\right)^2}}}$$

[Out] (3^(3/4)\*Sqrt[2 - Sqrt[3]]\*(a + b\*x^2)^(1/6)\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))^2]\*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(b\*x\*(a/(a + b\*x^2))^(1/3)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))^2)])

**Rubi [A]** time = 0.389205, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{3^{3/4} \sqrt{2 - \sqrt{3}} \sqrt[6]{a + bx^2} \left(1 - \sqrt[3]{\frac{a}{a + bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a + bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a + bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a + bx^2} - \sqrt{3} + 1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2 + a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2 + a} - \sqrt{3} + 1}}\right) \mid -7 + 4\sqrt{3}\right)}{bx \sqrt[3]{\frac{a}{a + bx^2}} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a + bx^2}}}{\left(-\sqrt[3]{\frac{a}{a + bx^2} - \sqrt{3} + 1}\right)^2}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(-5/6), x]

[Out] (3^(3/4)\*Sqrt[2 - Sqrt[3]]\*(a + b\*x^2)^(1/6)\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))^2]\*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(b\*x\*(a/(a + b\*x^2))^(1/3)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))^2)])

**Rubi in Sympy [A]** time = 7.23833, size = 228, normalized size = 0.9

$$\frac{3^{3/4} \sqrt{\frac{\left(-\frac{bx^2}{a+bx^2} + 1\right)^{2/3} + \sqrt[3]{-\frac{bx^2}{a+bx^2} + 1} + 1}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2} + 1 - \sqrt{3} + 1}\right)^2}} \sqrt{-\sqrt{3} + 2} \sqrt[6]{a + bx^2} \left(-\sqrt[3]{-\frac{bx^2}{a + bx^2} + 1} + 1\right) F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-\frac{bx^2}{a + bx^2} + 1 + \sqrt{3}}}{-\sqrt[3]{-\frac{bx^2}{a + bx^2} + 1 - \sqrt{3} + 1}}\right) \mid -7 + 4\sqrt{3}\right)}{bx \sqrt[3]{\frac{a}{a + bx^2}} \sqrt{\frac{\sqrt[3]{-\frac{bx^2}{a + bx^2} + 1} - 1}{\left(-\sqrt[3]{-\frac{bx^2}{a + bx^2} + 1 - \sqrt{3} + 1}\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(b*x**2+a)**(5/6),x)`

[Out]  $3^{3/4} \sqrt{\left(\frac{-bx^2}{a+bx^2} + 1\right)^{2/3} + \frac{-bx^2}{a+bx^2} + 1} \sqrt{\frac{1}{3} + 1} / \left(-\left(\frac{-bx^2}{a+bx^2} + 1\right)^{1/3} - \sqrt{3} + 1\right)^{1/2} \sqrt{-\sqrt{3} + 2} (a+bx^2)^{1/6} \left(-\left(\frac{-bx^2}{a+bx^2} + 1\right)^{1/3} + 1\right) \text{elliptic\_f}\left(\text{asin}\left(\frac{-\left(\frac{-bx^2}{a+bx^2} + 1\right)^{1/3} + 1 + \sqrt{3}}{-\left(\frac{-bx^2}{a+bx^2} + 1\right)^{1/3} - \sqrt{3} + 1}\right), -7 + 4\sqrt{3}\right) / (bx^2(a/(a+bx^2))^{1/3}) \sqrt{\left(\frac{-bx^2}{a+bx^2} + 1\right)^{1/3} - 1} / \left(-\left(\frac{-bx^2}{a+bx^2} + 1\right)^{1/3} - \sqrt{3} + 1\right)^{1/2}$

**Mathematica [C]** time = 0.0222846, size = 47, normalized size = 0.19

$$\frac{x \left(\frac{a+bx^2}{a}\right)^{5/6} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; -\frac{bx^2}{a}\right)}{(a+bx^2)^{5/6}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x^2)^(-5/6), x]`

[Out]  $(x^* ((a + b*x^2)/a)^{(5/6)} * \text{Hypergeometric2F1}[1/2, 5/6, 3/2, -((b*x^2)/a)]) / (a + b*x^2)^{(5/6)}$

**Maple [F]** time = 0.039, size = 0, normalized size = 0.

$$\int (bx^2 + a)^{-5/6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^2+a)^(5/6),x)`

[Out] `int(1/(b*x^2+a)^(5/6),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{5/6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(-5/6),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^(-5/6), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{5/6}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(-5/6),x, algorithm="fricas")`



[Out] `integral((b*x^2 + a)^(-5/6), x)`

---

**Sympy [A]** time = 2.75853, size = 24, normalized size = 0.1

$$\frac{x {}_2F_1\left(\frac{1}{2}, \frac{5}{6} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{5}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**2+a)**(5/6), x)`

[Out] `x*hyper((1/2, 5/6), (3/2, ), b*x**2*exp_polar(I*pi)/a)/a**(5/6)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^(-5/6), x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^(-5/6), x)`

$$3.1029 \quad \int \frac{1}{x^2(a+bx^2)^{5/6}} dx$$

**Optimal.** Leaf size=273

$$\frac{\sqrt[6]{a+bx^2}}{ax} - \frac{2\sqrt{2-\sqrt{3}}\sqrt[6]{a+bx^2} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt[3]{1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt[3]{1}}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt[3]{1}}\right) \middle| -7 + 4\sqrt{3}\right)}{\sqrt[4]{3}ax \sqrt[3]{\frac{a}{a+bx^2}} \sqrt{-\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt[3]{1}\right)^2}}}$$

[Out]  $-\left(\frac{a+bx^2}{a}\right)^{1/6} - \frac{2\sqrt{2-\sqrt{3}}(a+bx^2)^{1/6} \left(1 - \left(\frac{a}{a+bx^2}\right)^{1/3}\right) \sqrt{\left(1 + \left(\frac{a}{a+bx^2}\right)^{1/3} + \left(\frac{a}{a+bx^2}\right)^{2/3}\right) / \left(1 - \sqrt{3} - \left(\frac{a}{a+bx^2}\right)^{1/3}\right)^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1 + \sqrt{3} - \left(\frac{a}{a+bx^2}\right)^{1/3}}{1 - \sqrt{3} - \left(\frac{a}{a+bx^2}\right)^{1/3}}\right] - \left(\frac{a}{a+bx^2}\right)^{1/3}\right], -7 + 4\sqrt{3}}{\left(3^{1/4} a x \left(\frac{a}{a+bx^2}\right)^{1/3} \sqrt{-\left(1 - \left(\frac{a}{a+bx^2}\right)^{1/3}\right) / \left(1 - \sqrt{3} - \left(\frac{a}{a+bx^2}\right)^{1/3}\right)^2}\right)}$

**Rubi [A]** time = 0.480879, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{\sqrt[6]{a+bx^2}}{ax} - \frac{2\sqrt{2-\sqrt{3}}\sqrt[6]{a+bx^2} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt[3]{1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt[3]{1}}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt[3]{1}}\right) \middle| -7 + 4\sqrt{3}\right)}{\sqrt[4]{3}ax \sqrt[3]{\frac{a}{a+bx^2}} \sqrt{-\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt[3]{1}\right)^2}}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\frac{1}{x^2(a+bx^2)^{5/6}}, x\right]$

[Out]  $-\left(\frac{a+bx^2}{a}\right)^{1/6} - \frac{2\sqrt{2-\sqrt{3}}(a+bx^2)^{1/6} \left(1 - \left(\frac{a}{a+bx^2}\right)^{1/3}\right) \sqrt{\left(1 + \left(\frac{a}{a+bx^2}\right)^{1/3} + \left(\frac{a}{a+bx^2}\right)^{2/3}\right) / \left(1 - \sqrt{3} - \left(\frac{a}{a+bx^2}\right)^{1/3}\right)^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1 + \sqrt{3} - \left(\frac{a}{a+bx^2}\right)^{1/3}}{1 - \sqrt{3} - \left(\frac{a}{a+bx^2}\right)^{1/3}}\right] - \left(\frac{a}{a+bx^2}\right)^{1/3}\right], -7 + 4\sqrt{3}}{\left(3^{1/4} a x \left(\frac{a}{a+bx^2}\right)^{1/3} \sqrt{-\left(1 - \left(\frac{a}{a+bx^2}\right)^{1/3}\right) / \left(1 - \sqrt{3} - \left(\frac{a}{a+bx^2}\right)^{1/3}\right)^2}\right)}$

**Rubi in Sympy [A]** time = 12.1184, size = 246, normalized size = 0.9

$$\frac{\sqrt[6]{a+bx^2}}{ax} \cdot \frac{2 \cdot 3^{\frac{3}{4}} \sqrt{\frac{\left(-\frac{bx^2}{a+bx^2}+1\right)^{\frac{2}{3}} + \sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2}} \sqrt{-\sqrt{3}+2} \sqrt[6]{a+bx^2} \left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1}\right) F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1+\sqrt{3}}}{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}}\right)\right)}{\sqrt[3]{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**2/(b*x**2+a)**(5/6),x)`

[Out]  $-(a + b*x**2)**(1/6)/(a*x) - 2*3**(3/4)*\sqrt{(((-b*x**2/(a + b*x**2) + 1)**(2/3) + (-b*x**2/(a + b*x**2) + 1)**(1/3) + 1)/(-(-b*x**2/(a + b*x**2) + 1)**(1/3) - \sqrt{3} + 1)**2)*\sqrt{(-\sqrt{3} + 2)*(a + b*x**2)**(1/6)*(-(-b*x**2/(a + b*x**2) + 1)**(1/3) + 1)*\operatorname{elliptic\_f}(\operatorname{asin}((-(-b*x**2/(a + b*x**2) + 1)**(1/3) + 1 + \sqrt{3}))/(-(-b*x**2/(a + b*x**2) + 1)**(1/3) - \sqrt{3} + 1)), -7 + 4*\sqrt{3})/(3*a*x*(a/(a + b*x**2))**(1/3)*\sqrt{(((-b*x**2/(a + b*x**2) + 1)**(1/3) - 1)/(-(-b*x**2/(a + b*x**2) + 1)**(1/3) - \sqrt{3} + 1)**2))}$

**Mathematica [C]** time = 0.0511403, size = 70, normalized size = 0.26

$$\frac{-2bx^2 \left(\frac{bx^2}{a} + 1\right)^{5/6} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; -\frac{bx^2}{a}\right) - 3(a + bx^2)}{3ax(a + bx^2)^{5/6}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^2*(a + b*x^2)^(5/6)),x]`

[Out]  $(-3*(a + b*x^2) - 2*b*x^2*(1 + (b*x^2)/a)^(5/6)*\operatorname{Hypergeometric2F1}[1/2, 5/6, 3/2, -((b*x^2)/a)]/(3*a*x*(a + b*x^2)^(5/6))$

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (bx^2 + a)^{-\frac{5}{6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(b*x^2+a)^(5/6),x)`

[Out] `int(1/x^2/(b*x^2+a)^(5/6),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{6}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/6)*x^2),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(5/6)*x^2), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{5}{6}}x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/6)*x^2),x, algorithm="fricas")`

[Out] `integral(1/((b*x^2 + a)^(5/6)*x^2), x)`

**Sympy** [A] time = 3.98095, size = 27, normalized size = 0.1

$$-\frac{{}_2F_1\left(-\frac{1}{2}, \frac{5}{6} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{5}{6}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(b*x**2+a)**(5/6),x)`

[Out] `-hyper((-1/2, 5/6), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(5/6)*x)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{6}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/6)*x^2),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(5/6)*x^2), x)`

$$3.1030 \quad \int \frac{1}{x^4(a+bx^2)^{5/6}} dx$$

**Optimal.** Leaf size=300

$$\frac{8b\sqrt[6]{a+bx^2}}{9a^2x} + \frac{16\sqrt{2-\sqrt{3}}b\sqrt[6]{a+bx^2} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}+1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}+\sqrt{3}+1}}{-\sqrt[3]{\frac{a}{bx^2+a}-\sqrt{3}+1}}\right) \mid -7+4\sqrt{3}\right)}{9\sqrt[4]{3}a^2x\sqrt[3]{\frac{a}{a+bx^2}} \sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}}} - \frac{\sqrt[6]{a+bx^2}}{3ax^3}$$

[Out]  $-(a + b*x^2)^{(1/6)}/(3*a*x^3) + (8*b*(a + b*x^2)^{(1/6)})/(9*a^2*x) + (16*\text{Sqrt}[2 - \text{Sqrt}[3]]*b*(a + b*x^2)^{(1/6)}*(1 - (a/(a + b*x^2))^{(1/3)})*\text{Sqrt}[(1 + (a/(a + b*x^2))^{(1/3)} + (a/(a + b*x^2))^{(2/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(9*3^{(1/4)}*a^2*x*(a/(a + b*x^2))^{(1/3)}*\text{Sqrt}[-((1 - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2)])$

**Rubi [A]** time = 0.563093, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{8b\sqrt[6]{a+bx^2}}{9a^2x} + \frac{16\sqrt{2-\sqrt{3}}b\sqrt[6]{a+bx^2} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}+1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}+\sqrt{3}+1}}{-\sqrt[3]{\frac{a}{bx^2+a}-\sqrt{3}+1}}\right) \mid -7+4\sqrt{3}\right)}{9\sqrt[4]{3}a^2x\sqrt[3]{\frac{a}{a+bx^2}} \sqrt{-\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}}} - \frac{\sqrt[6]{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^4*(a + b*x^2)^{(5/6)}), x]$

[Out]  $-(a + b*x^2)^{(1/6)}/(3*a*x^3) + (8*b*(a + b*x^2)^{(1/6)})/(9*a^2*x) + (16*\text{Sqrt}[2 - \text{Sqrt}[3]]*b*(a + b*x^2)^{(1/6)}*(1 - (a/(a + b*x^2))^{(1/3)})*\text{Sqrt}[(1 + (a/(a + b*x^2))^{(1/3)} + (a/(a + b*x^2))^{(2/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(9*3^{(1/4)}*a^2*x*(a/(a + b*x^2))^{(1/3)}*\text{Sqrt}[-((1 - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2)])$

**Rubi in Sympy [A]** time = 16.8969, size = 272, normalized size = 0.91

$$\begin{aligned}
 & -\frac{\sqrt[6]{a+bx^2}}{3ax^3} + \frac{8b\sqrt[6]{a+bx^2}}{9a^2x} \\
 & + \frac{16 \cdot 3^{\frac{3}{4}} b \sqrt{\frac{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1}\right)^{\frac{2}{3}} + \sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2}} \sqrt{-\sqrt{3}+2\sqrt[6]{a+bx^2}} \left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1}\right) F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1+1+\sqrt{3}}}{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}}\right)\right)}{\sqrt{\frac{27a^2x\sqrt[3]{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2}}}}{27a^2x\sqrt[3]{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-1}}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1-\sqrt{3}+1}\right)^2}}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**4/(b*x**2+a)**(5/6),x)`

[Out]  $-(a + b^2x^2)^{1/6}/(3a^2x^3) + 8b(a + b^2x^2)^{1/6}/(9a^2x) + 16 \cdot 3^{3/4} b \sqrt{\left(\left(-\frac{bx^2}{a + b^2x^2} + 1\right)^{2/3} + \sqrt[3]{-\frac{bx^2}{a + b^2x^2} + 1 + 1}\right) \sqrt{-\sqrt{3} + 2\sqrt[6]{a + b^2x^2}} \left(-\sqrt[3]{-\frac{bx^2}{a + b^2x^2} + 1 + 1}\right) F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-\frac{bx^2}{a + b^2x^2} + 1 + 1 + \sqrt{3}}}{-\sqrt[3]{-\frac{bx^2}{a + b^2x^2} + 1 - \sqrt{3} + 1}}\right)\right)}$

**Mathematica [C]** time = 0.0471329, size = 83, normalized size = 0.28

$$\frac{-9a^2 + 16b^2x^4 \left(\frac{bx^2}{a} + 1\right)^{5/6} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}, \frac{3}{2}; -\frac{bx^2}{a}\right) + 15abx^2 + 24b^2x^4}{27a^2x^3(a + bx^2)^{5/6}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^4*(a + b*x^2)^(5/6)),x]`

[Out]  $(-9a^2 + 15abx^2 + 24b^2x^4 + 16b^2x^4(1 + (bx^2)/a)^{5/6}) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{3}{2}, -\frac{bx^2}{a}\right]/(27a^2x^3(a + b^2x^2)^{5/6})$

**Maple [F]** time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} (bx^2 + a)^{-\frac{5}{6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(b*x^2+a)^(5/6),x)`

[Out] `int(1/x^4/(b*x^2+a)^(5/6),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{6}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/6)*x^4),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(5/6)*x^4), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{5}{6}}x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/6)*x^4),x, algorithm="fricas")`

[Out] `integral(1/((b*x^2 + a)^(5/6)*x^4), x)`

**Sympy** [A] time = 5.08114, size = 32, normalized size = 0.11

$$-\frac{{}_2F_1\left(-\frac{3}{2}, \frac{5}{6} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{5}{6}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(b*x**2+a)**(5/6),x)`

[Out] `-hyper((-3/2, 5/6), (-1/2, ), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/6)*x**3)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{6}}x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/6)*x^4),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(5/6)*x^4), x)`

$$3.1031 \quad \int \frac{1}{x^6(a+bx^2)^{5/6}} dx$$

**Optimal.** Leaf size=326

$$\frac{112b^2\sqrt[6]{a+bx^2}}{135a^3x} + \frac{224\sqrt{2-\sqrt{3}}b^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}+1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}+\sqrt{3}+1}}{-\sqrt[3]{\frac{a}{bx^2+a}-\sqrt{3}+1}}\right)\right)}{\sqrt{\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}}}$$

$$+ \frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3} - \frac{\sqrt[6]{a+bx^2}}{5ax^5}$$

[Out]  $-(a + b*x^2)^{(1/6)}/(5*a*x^5) + (14*b*(a + b*x^2)^{(1/6)})/(45*a^2*x^3) - (112*b^2*(a + b*x^2)^{(1/6)})/(135*a^3*x) - (224*sqrt[2 - sqrt[3]]*b^2*(a + b*x^2)^{(1/6)}*(1 - (a/(a + b*x^2))^{(1/3)})*sqrt[(1 + (a/(a + b*x^2))^{(1/3)} + (a/(a + b*x^2))^{(2/3)})/(1 - sqrt[3] - (a/(a + b*x^2))^{(1/3)})^2]*EllipticF[ArcSin[(1 + sqrt[3] - (a/(a + b*x^2))^{(1/3)})/(1 - sqrt[3] - (a/(a + b*x^2))^{(1/3)})], -7 + 4*sqrt[3]])/(135*3^{(1/4)}*a^3*x*(a/(a + b*x^2))^{(1/3)}*sqrt[-((1 - (a/(a + b*x^2))^{(1/3)})/(1 - sqrt[3] - (a/(a + b*x^2))^{(1/3)})^2)])$

**Rubi [A]** time = 0.65442, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{112b^2\sqrt[6]{a+bx^2}}{135a^3x} + \frac{224\sqrt{2-\sqrt{3}}b^2\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}+1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}+\sqrt{3}+1}}{-\sqrt[3]{\frac{a}{bx^2+a}-\sqrt{3}+1}}\right)\right)}{\sqrt{\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}-\sqrt{3}+1}\right)^2}}}$$

$$+ \frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3} - \frac{\sqrt[6]{a+bx^2}}{5ax^5}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(a + b\*x^2)^(5/6)),x]

[Out]  $-(a + b*x^2)^{(1/6)}/(5*a*x^5) + (14*b*(a + b*x^2)^{(1/6)})/(45*a^2*x^3) - (112*b^2*(a + b*x^2)^{(1/6)})/(135*a^3*x) - (224*sqrt[2 - sqrt[3]]*b^2*(a + b*x^2)^{(1/6)}*(1 - (a/(a + b*x^2))^{(1/3)})*sqrt[(1 + (a/(a + b*x^2))^{(1/3)} + (a/(a + b*x^2))^{(2/3)})/(1 - sqrt[3] - (a/(a + b*x^2))^{(1/3)})^2]*EllipticF[ArcSin[(1 + sqrt[3] - (a/(a + b*x^2))^{(1/3)})/(1 - sqrt[3] - (a/(a + b*x^2))^{(1/3)})], -7 + 4*sqrt[3]])/(135*3^{(1/4)}*a^3*x*(a/(a + b*x^2))^{(1/3)}*sqrt[-((1 - (a/(a + b*x^2))^{(1/3)})/(1 - sqrt[3] - (a/(a + b*x^2))^{(1/3)})^2)])$



**Rubi in Sympy [A]** time = 22.5946, size = 298, normalized size = 0.91

$$\frac{\sqrt[6]{a+bx^2}}{5ax^5} + \frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3} - \frac{112b^2\sqrt[6]{a+bx^2}}{135a^3x}$$

$$224 \cdot 3^{\frac{3}{4}} b^2 \sqrt{\frac{\left(-\frac{bx^2}{a+bx^2}+1\right)^{\frac{2}{3}} + \sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1\right)^2}} \sqrt{-\sqrt{3} + 2\sqrt[6]{a+bx^2}} \left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1\right) F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1 + \sqrt{3}}{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1 - \sqrt{3} + 1}\right)\right)$$

$$405a^3x\sqrt[3]{\frac{a}{a+bx^2}} \sqrt{\frac{\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} - 1}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**6/(b*x**2+a)**(5/6),x)`

[Out]  $-(a + b*x^2)^{(1/6)}/(5*a*x^5) + 14*b*(a + b*x^2)^{(1/6)}/(45*a^2*x^3) - 112*b^2*(a + b*x^2)^{(1/6)}/(135*a^3*x) - 224*3^{3/4}*b^2*\sqrt{\frac{\left(-\frac{bx^2}{a+bx^2}+1\right)^{\frac{2}{3}} + \sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1\right)^2}}*\sqrt{-\sqrt{3} + 2\sqrt[6]{a+bx^2}}*\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1\right)*F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1 + \sqrt{3}}{-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1 - \sqrt{3} + 1}\right)\right) - 405*a^3*x*\sqrt[3]{\frac{a}{a+bx^2}}*\sqrt{\frac{\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} - 1}{\left(-\sqrt[3]{-\frac{bx^2}{a+bx^2}+1} + 1\right)^2}}$

**Mathematica [C]** time = 0.0546838, size = 94, normalized size = 0.29

$$\frac{-81a^3 + 45a^2bx^2 - 224b^3x^6 \left(\frac{bx^2}{a} + 1\right)^{5/6} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; -\frac{bx^2}{a}\right) - 210ab^2x^4 - 336b^3x^6}{405a^3x^5 (a + bx^2)^{5/6}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^6*(a + b*x^2)^(5/6)),x]`

[Out]  $(-81*a^3 + 45*a^2*b*x^2 - 210*a*b^2*x^4 - 336*b^3*x^6 - 224*b^3*x^6*(1 + (b*x^2)/a)^{5/6}*\operatorname{Hypergeometric2F1}\left[1/2, 5/6, 3/2, -((b*x^2)/a)\right])/ (405*a^3*x^5*(a + b*x^2)^{5/6})$

**Maple [F]** time = 0.042, size = 0, normalized size = 0.

$$\int \frac{1}{x^6} (bx^2 + a)^{-\frac{5}{6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^6/(b*x^2+a)^(5/6),x)`

[Out] `int(1/x^6/(b*x^2+a)^(5/6),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{6}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/6)*x^6),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^2 + a)^(5/6)*x^6), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{5}{6}}x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/6)*x^6),x, algorithm="fricas")`

[Out] `integral(1/((b*x^2 + a)^(5/6)*x^6), x)`

**Sympy** [A] time = 7.05922, size = 32, normalized size = 0.1

$$-\frac{{}_2F_1\left(-\frac{5}{2}, \frac{5}{6} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5a^{\frac{5}{6}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**6/(b*x**2+a)**(5/6),x)`

[Out] `-hyper((-5/2, 5/6), (-3/2, ), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/6)*x**5)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{6}}x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(5/6)*x^6),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(5/6)*x^6), x)`

$$3.1032 \quad \int \frac{x^6}{(a+bx^2)^{7/6}} dx$$

**Optimal.** Leaf size=654

$$\frac{405 \cdot 3^{3/4} a^3 \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \mid -7 + 4\sqrt{3}\right)}{112\sqrt{2}b^4x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{-\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$+ \frac{1215\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^3 \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \mid -7 + 4\sqrt{3}\right)}{448b^4x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{-\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$+ \frac{1215a^3x}{224b^3 \left(\frac{a}{a+bx^2}\right)^{2/3} (a+bx^2)^{7/6} \left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)} + \frac{1215a^2x}{224b^3 \sqrt[6]{a+bx^2}}$$

$$- \frac{405ax(a+bx^2)^{5/6}}{112b^3} + \frac{45x^3(a+bx^2)^{5/6}}{14b^2} - \frac{3x^5}{b\sqrt[6]{a+bx^2}}$$

[Out] (1215\*a^2\*x)/(224\*b^3\*(a + b\*x^2)^(1/6)) - (3\*x^5)/(b\*(a + b\*x^2)^(1/6)) - (405\*a\*x\*(a + b\*x^2)^(5/6))/(112\*b^3) + (45\*x^3\*(a + b\*x^2)^(5/6))/(14\*b^2) + (1215\*a^3\*x)/(224\*b^3\*(a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(7/6)\*(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))) + (1215\*3^(1/4)\*Sqrt[2 + Sqrt[3]]\*a^3\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))]^2\*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(448\*b^4\*x\*(a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(1/6)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3)))^2]) - (405\*3^(3/4)\*a^3\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))]^2\*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(112\*Sqrt[2]\*b^4\*x\*(a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(1/6)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3)))^2]))

**Rubi [A]** time = 1.517, antiderivative size = 654, normalized size of antiderivative = 1., number of

steps used = 9, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$

$$\begin{aligned}
 & \frac{405 \cdot 3^{3/4} a^3 \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \middle| -7 + 4\sqrt{3}\right)}{112\sqrt{2}b^4x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{-\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}} \\
 & + \frac{1215\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^3 \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \middle| -7 + 4\sqrt{3}\right)}{448b^4x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{-\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}} \\
 & + \frac{1215a^3x}{224b^3 \left(\frac{a}{a+bx^2}\right)^{2/3} (a+bx^2)^{7/6} \left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)} + \frac{1215a^2x}{224b^3 \sqrt[6]{a+bx^2}} \\
 & - \frac{405ax (a+bx^2)^{5/6}}{112b^3} + \frac{45x^3 (a+bx^2)^{5/6}}{14b^2} - \frac{3x^5}{b \sqrt[6]{a+bx^2}}
 \end{aligned}$$

Warning: Unable to verify antiderivative.

[In] Int[x^6/(a + b\*x^2)^(7/6), x]

[Out] (1215\*a^2\*x)/(224\*b^3\*(a + b\*x^2)^(1/6)) - (3\*x^5)/(b\*(a + b\*x^2)^(1/6)) - (405\*a\*x\*(a + b\*x^2)^(5/6))/(112\*b^3) + (45\*x^3\*(a + b\*x^2)^(5/6))/(14\*b^2) + (1215\*a^3\*x)/(224\*b^3\*(a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(7/6)\*(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))) + (1215\*3^(1/4)\*Sqrt[2 + Sqrt[3]]\*a^3\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))^2]\*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(448\*b^4\*x\*(a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(1/6)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3)))^2]) - (405\*3^(3/4)\*a^3\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))^2]\*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(112\*Sqrt[2]\*b^4\*x\*(a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(1/6)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3)))^2])]

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{405a^2 \int \frac{1}{\sqrt[6]{a+bx^2}} dx}{112b^3} - \frac{405ax (a+bx^2)^{5/6}}{112b^3} - \frac{3x^5}{b \sqrt[6]{a+bx^2}} + \frac{45x^3 (a+bx^2)^{5/6}}{14b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6/(b\*x\*\*2+a)\*\*(7/6), x)

[Out] 405\*a\*\*2\*Integral((a + b\*x\*\*2)\*\*(-1/6), x)/(112\*b\*\*3) - 405\*a\*x\*(a + b\*x\*\*2)\*\*(5/6)/(112\*b\*\*3) - 3\*x\*\*5/(b\*(a + b\*x\*\*2)\*\*(1/6)) + 45\*x\*\*3\*(a + b\*x\*\*2)\*\*(5/6)/(14\*b\*\*2)

---

**Mathematica [C]** time = 0.0710612, size = 79, normalized size = 0.12

$$\frac{3 \left( 135a^2x \sqrt[6]{\frac{bx^2}{a}} + {}_2F_1 \left( \frac{1}{6}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a} \right) - 135a^2x - 15abx^3 + 8b^2x^5 \right)}{112b^3 \sqrt[6]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a + b\*x^2)^(7/6), x]

[Out] (3\*(-135\*a^2\*x - 15\*a\*b\*x^3 + 8\*b^2\*x^5 + 135\*a^2\*x\*(1 + (b\*x^2)/a)^(1/6)\*Hypergeometric2F1[1/6, 1/2, 3/2, -(b\*x^2)/a]))/(112\*b^3\*(a + b\*x^2)^(1/6))

---

**Maple [F]** time = 0.064, size = 0, normalized size = 0.

$$\int x^6 (bx^2 + a)^{-\frac{7}{6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(b\*x^2+a)^(7/6), x)

[Out] int(x^6/(b\*x^2+a)^(7/6), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(bx^2 + a)^{\frac{7}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b\*x^2 + a)^(7/6), x, algorithm="maxima")

[Out] integrate(x^6/(b\*x^2 + a)^(7/6), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{x^6}{(bx^2 + a)^{\frac{7}{6}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b\*x^2 + a)^(7/6), x, algorithm="fricas")

[Out] integral(x^6/(b\*x^2 + a)^(7/6), x)

---

**Sympy [A]** time = 3.49725, size = 27, normalized size = 0.04

$$\frac{x^7 {}_2F_1 \left( \frac{7}{6}, \frac{7}{2} \middle| \frac{bx^2 e^{i\pi}}{a} \right)}{7a^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6/(b*x**2+a)**(7/6),x)`

[Out] `x**7*hyper((7/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(7/6))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(bx^2 + a)^{\frac{7}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(b*x^2 + a)^(7/6),x, algorithm="giac")`

[Out] `integrate(x^6/(b*x^2 + a)^(7/6), x)`

$$3.1033 \quad \int \frac{x^4}{(a+bx^2)^{7/6}} dx$$

Optimal. Leaf size=630

$$\frac{27 \cdot 3^{3/4} a^2 \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \mid -7 + 4\sqrt{3}\right)}{8\sqrt{2}b^3x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$\frac{81 \sqrt[4]{3} \sqrt{2 + \sqrt{3}} a^2 \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \mid -7 + 4\sqrt{3}\right)}{32b^3x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$\frac{81a^2x}{16b^2 \left(\frac{a}{a+bx^2}\right)^{2/3} (a+bx^2)^{7/6} \left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)} + \frac{27x(a+bx^2)^{5/6}}{8b^2} - \frac{81ax}{16b^2 \sqrt[6]{a+bx^2}} - \frac{3x^3}{b \sqrt[6]{a+bx^2}}$$

[Out]  $(-81*a*x)/(16*b^2*(a+b*x^2)^(1/6)) - (3*x^3)/(b*(a+b*x^2)^(1/6)) + (27*x*(a+b*x^2)^(5/6))/(8*b^2) - (81*a^2*x)/(16*b^2*(a/(a+b*x^2))^(2/3)*(a+b*x^2)^(7/6)*(1-Sqrt[3]-(a/(a+b*x^2))^(1/3))) - (81*3^(1/4)*Sqrt[2+Sqrt[3]]*a^2*(1-(a/(a+b*x^2))^(1/3))*Sqrt[(1+(a/(a+b*x^2))^(1/3)+(a/(a+b*x^2))^(2/3))/(1-Sqrt[3]-(a/(a+b*x^2))^(1/3))]^2*EllipticE[ArcSin[(1+Sqrt[3]-(a/(a+b*x^2))^(1/3))/(1-Sqrt[3]-(a/(a+b*x^2))^(1/3))], -7+4*Sqrt[3]])/(32*b^3*x*(a/(a+b*x^2))^(2/3)*(a+b*x^2)^(1/6)*Sqrt[-((1-(a/(a+b*x^2))^(1/3))/(1-Sqrt[3]-(a/(a+b*x^2))^(1/3)))^2]) + (27*3^(3/4)*a^2*(1-(a/(a+b*x^2))^(1/3))*Sqrt[(1+(a/(a+b*x^2))^(1/3)+(a/(a+b*x^2))^(2/3))/(1-Sqrt[3]-(a/(a+b*x^2))^(1/3))]^2*EllipticF[ArcSin[(1+Sqrt[3]-(a/(a+b*x^2))^(1/3))/(1-Sqrt[3]-(a/(a+b*x^2))^(1/3))], -7+4*Sqrt[3]])/(8*Sqrt[2]*b^3*x*(a/(a+b*x^2))^(2/3)*(a+b*x^2)^(1/6)*Sqrt[-((1-(a/(a+b*x^2))^(1/3))/(1-Sqrt[3]-(a/(a+b*x^2))^(1/3)))^2])$

Rubi [A] time = 1.3758, antiderivative size = 630, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$

$$\frac{27 \cdot 3^{3/4} a^2 \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \middle| -7 + 4\sqrt{3}\right)}{8\sqrt{2}b^3x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$- \frac{81\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^2 \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \middle| -7 + 4\sqrt{3}\right)}{32b^3x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}}$$

$$- \frac{81a^2x}{16b^2 \left(\frac{a}{a+bx^2}\right)^{2/3} (a+bx^2)^{7/6} \left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)} + \frac{27x(a+bx^2)^{5/6}}{8b^2} - \frac{81ax}{16b^2\sqrt[6]{a+bx^2}} - \frac{3x^3}{b\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a + b\*x^2)^(7/6), x]

[Out]  $(-81*a*x)/(16*b^2*(a + b*x^2)^{(1/6)}) - (3*x^3)/(b*(a + b*x^2)^{(1/6)}) + (27*x*(a + b*x^2)^{(5/6)})/(8*b^2) - (81*a^2*x)/(16*b^2*(a/(a + b*x^2))^{(2/3)}*(a + b*x^2)^{(7/6)}*(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})) - (81*3^{(1/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*a^2*(1 - (a/(a + b*x^2))^{(1/3)})*\text{Sqrt}[(1 + (a/(a + b*x^2))^{(1/3)} + (a/(a + b*x^2))^{(2/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(32*b^3*x*(a/(a + b*x^2))^{(2/3)}*(a + b*x^2)^{(1/6)}*\text{Sqrt}[-((1 - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2)] + (27*3^{(3/4)}*a^2*(1 - (a/(a + b*x^2))^{(1/3)})*\text{Sqrt}[(1 + (a/(a + b*x^2))^{(1/3)} + (a/(a + b*x^2))^{(2/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(8*\text{Sqrt}[2]*b^3*x*(a/(a + b*x^2))^{(2/3)}*(a + b*x^2)^{(1/6)}*\text{Sqrt}[-((1 - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2)])$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{27a^2 \int \frac{1}{(a+bx^2)^{7/6}} dx}{16b^2} - \frac{81ax}{16b^2\sqrt[6]{a+bx^2}} - \frac{3x^3}{b\sqrt[6]{a+bx^2}} + \frac{27x(a+bx^2)^{5/6}}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(b\*x\*\*2+a)\*\*(7/6), x)

[Out]  $27*a**2*Integral((a + b*x**2)**(-7/6), x)/(16*b**2) - 81*a*x/(16*b**2*(a + b*x**2)**(1/6)) - 3*x**3/(b*(a + b*x**2)**(1/6)) + 27*x*(a + b*x**2)**(5/6)/(8*b**2)$



**Mathematica [C]** time = 0.0548339, size = 64, normalized size = 0.1

$$\frac{3x \left( -9a \sqrt[6]{\frac{bx^2}{a}} + {}_2F_1 \left( \frac{1}{6}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a} \right) + 9a + bx^2 \right)}{8b^2 \sqrt[6]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a + b\*x^2)^(7/6), x]

[Out] (3\*x\*(9\*a + b\*x^2 - 9\*a\*(1 + (b\*x^2)/a)^(1/6)\*Hypergeometric2F1[1/6, 1/2, 3/2, -(b\*x^2)/a]))/(8\*b^2\*(a + b\*x^2)^(1/6))

**Maple [F]** time = 0.062, size = 0, normalized size = 0.

$$\int x^4 (bx^2 + a)^{-\frac{7}{6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b\*x^2+a)^(7/6), x)

[Out] int(x^4/(b\*x^2+a)^(7/6), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2 + a)^{\frac{7}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b\*x^2 + a)^(7/6), x, algorithm="maxima")

[Out] integrate(x^4/(b\*x^2 + a)^(7/6), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{x^4}{(bx^2 + a)^{\frac{7}{6}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b\*x^2 + a)^(7/6), x, algorithm="fricas")

[Out] integral(x^4/(b\*x^2 + a)^(7/6), x)

**Sympy [A]** time = 3.53157, size = 27, normalized size = 0.04

$$\frac{x^5 {}_2F_1 \left( \frac{7}{6}, \frac{5}{2} \middle| \frac{bx^2 e^{i\pi}}{a} \right)}{5a^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(b*x**2+a)**(7/6),x)`

[Out] `x**5*hyper((7/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(7/6))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(bx^2 + a)^{\frac{7}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(b*x^2 + a)^(7/6),x, algorithm="giac")`

[Out] `integrate(x^4/(b*x^2 + a)^(7/6), x)`

$$3.1034 \quad \int \frac{x^2}{(a+bx^2)^{7/6}} dx$$

Optimal. Leaf size=583

$$\begin{aligned} & \frac{3 \cdot 3^{3/4} a \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \mid -7 + 4\sqrt{3}\right)}{\sqrt{2} b^2 x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}} \\ & + \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}a \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2} + 1}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a} + \sqrt{3} + 1}}{-\sqrt[3]{\frac{a}{bx^2+a} - \sqrt{3} + 1}}\right) \mid -7 + 4\sqrt{3}\right)}{4b^2 x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)^2}}} \\ & + \frac{3x}{2b\sqrt[6]{a+bx^2}} + \frac{9ax}{2b\left(\frac{a}{a+bx^2}\right)^{2/3} (a+bx^2)^{7/6} \left(-\sqrt[3]{\frac{a}{a+bx^2} - \sqrt{3} + 1}\right)} \end{aligned}$$

[Out] (3\*x)/(2\*b\*(a + b\*x^2)^(1/6)) + (9\*a\*x)/(2\*b\*(a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(7/6)\*(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))) + (9\*3^(1/4)\*Sqrt[2 + Sqrt[3]]\*a\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))]^2\*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(4\*b^2\*x\*(a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(1/6)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3)))^2]) - (3\*3^(3/4)\*a\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))]^2\*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]])/(Sqrt[2]\*b^2\*x\*(a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(1/6)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3)))^2])

Rubi [A] time = 1.06485, antiderivative size = 583, normalized size of antiderivative = 1., number of

steps used = 7, number of rules used = 7, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$

$$\begin{aligned}
 & \frac{3 \cdot 3^{3/4} a \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt[3]{1}\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}} + \sqrt[3]{1}}{-\sqrt[3]{\frac{a}{bx^2+a}} - \sqrt[3]{1}}\right) \mid -7 + 4\sqrt{3}\right)}{\sqrt{2b^2x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt[3]{1}\right)^2}}} \\
 & + \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}a \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt[3]{1}\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}} + \sqrt[3]{1}}{-\sqrt[3]{\frac{a}{bx^2+a}} - \sqrt[3]{1}}\right) \mid -7 + 4\sqrt{3}\right)}{4b^2x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt[3]{1}\right)^2}}} \\
 & + \frac{3x}{2b\sqrt[6]{a+bx^2}} + \frac{9ax}{2b \left(\frac{a}{a+bx^2}\right)^{2/3} (a+bx^2)^{7/6} \left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt[3]{1}\right)}
 \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*x^2)^(7/6), x]

[Out]  $(3x)/(2b(a + b^2x^2)^{1/6}) + (9a^2x)/(2b(a/(a + b^2x^2))^{2/3}(a + b^2x^2)^{7/6}(1 - \sqrt{3} - (a/(a + b^2x^2))^{1/3})) + (9^3)^{1/4} \sqrt{2 + \sqrt{3}} a (1 - (a/(a + b^2x^2))^{1/3}) \sqrt{(1 + (a/(a + b^2x^2))^{1/3} + (a/(a + b^2x^2))^{2/3})/(1 - \sqrt{3} - (a/(a + b^2x^2))^{1/3})^2} \text{EllipticE}[\text{ArcSin}[(1 + \sqrt{3} - (a/(a + b^2x^2))^{1/3})/(1 - \sqrt{3} - (a/(a + b^2x^2))^{1/3})], -7 + 4\sqrt{3}]/(4b^2x^2(a/(a + b^2x^2))^{2/3}(a + b^2x^2)^{1/6} \sqrt{-(1 - (a/(a + b^2x^2))^{1/3})/(1 - \sqrt{3} - (a/(a + b^2x^2))^{1/3})^2}) - (3^3)^{3/4} a (1 - (a/(a + b^2x^2))^{1/3}) \sqrt{(1 + (a/(a + b^2x^2))^{1/3} + (a/(a + b^2x^2))^{2/3})/(1 - \sqrt{3} - (a/(a + b^2x^2))^{1/3})^2} \text{EllipticF}[\text{ArcSin}[(1 + \sqrt{3} - (a/(a + b^2x^2))^{1/3})/(1 - \sqrt{3} - (a/(a + b^2x^2))^{1/3})], -7 + 4\sqrt{3}]/(\sqrt{2} b^2 x^2 (a/(a + b^2x^2))^{2/3} (a + b^2x^2)^{1/6} \sqrt{-(1 - (a/(a + b^2x^2))^{1/3})/(1 - \sqrt{3} - (a/(a + b^2x^2))^{1/3})^2})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{3x}{b\sqrt[6]{a+bx^2}} + \frac{3 \int \frac{1}{\sqrt[6]{a+bx^2}} dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(b\*x\*\*2+a)\*\*(7/6), x)

[Out]  $-3x/(b(a + b^2x^2)^{1/6}) + 3 \text{Integral}((a + b^2x^2)^{-1/6}, x)/b$

**Mathematica [C]** time = 0.0485312, size = 53, normalized size = 0.09

$$\frac{3x \left( \sqrt[6]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) - 1 \right)}{b\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*x^2)^(7/6), x]

[Out] (3\*x\*(-1 + (1 + (b\*x^2)/a)^(1/6)\*Hypergeometric2F1[1/6, 1/2, 3/2, -(b\*x^2)/a]))/(b\*(a + b\*x^2)^(1/6))

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int x^2 (bx^2 + a)^{-\frac{7}{6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b\*x^2+a)^(7/6), x)

[Out] int(x^2/(b\*x^2+a)^(7/6), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2 + a)^{\frac{7}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b\*x^2 + a)^(7/6), x, algorithm="maxima")

[Out] integrate(x^2/(b\*x^2 + a)^(7/6), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{(bx^2 + a)^{\frac{7}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b\*x^2 + a)^(7/6), x, algorithm="fricas")

[Out] integral(x^2/(b\*x^2 + a)^(7/6), x)

**Sympy [A]** time = 3.49827, size = 27, normalized size = 0.05

$$\frac{x^3 {}_2F_1\left(\frac{7}{6}, \frac{3}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(b\*x\*\*2+a)\*\*(7/6), x)

[Out] x\*\*3\*hyper((7/6, 3/2), (5/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(3\*a\*\*(7/6))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(bx^2 + a)^{\frac{7}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(b*x^2 + a)^(7/6), x, algorithm="giac")`

[Out] `integrate(x^2/(b*x^2 + a)^(7/6), x)`

$$3.1035 \quad \int \frac{1}{(a+bx^2)^{7/6}} dx$$

**Optimal.** Leaf size=555

$$\frac{3x}{\left(\frac{a}{a+bx^2}\right)^{2/3} (a+bx^2)^{7/6} \left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)} + \frac{\sqrt{23}^{3/4} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1}\right) \middle| -7 + 4\sqrt{3}\right)}{bx \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}} + \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1}\right) \middle| -7 + 4\sqrt{3}\right)}{2bx \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}$$

[Out]  $(-3*x)/((a/(a + b*x^2))^{2/3} * (a + b*x^2)^{7/6} * (1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{1/3})) - (3*3^{1/4} * \text{Sqrt}[2 + \text{Sqrt}[3]] * (1 - (a/(a + b*x^2))^{1/3})) * \text{Sqrt}[(1 + (a/(a + b*x^2))^{1/3}) + (a/(a + b*x^2))^{2/3}]/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{1/3})^2] * \text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a/(a + b*x^2))^{1/3})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{1/3})], -7 + 4*\text{Sqrt}[3]]/(2*b*x*(a/(a + b*x^2))^{2/3} * (a + b*x^2)^{1/6} * \text{Sqrt}[-((1 - (a/(a + b*x^2))^{1/3})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{1/3}))^2]) + (\text{Sqrt}[2]*3^{3/4} * (1 - (a/(a + b*x^2))^{1/3})) * \text{Sqrt}[(1 + (a/(a + b*x^2))^{1/3}) + (a/(a + b*x^2))^{2/3}]/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{1/3})^2] * \text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a/(a + b*x^2))^{1/3})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{1/3})], -7 + 4*\text{Sqrt}[3]]/(b*x*(a/(a + b*x^2))^{2/3} * (a + b*x^2)^{1/6} * \text{Sqrt}[-((1 - (a/(a + b*x^2))^{1/3})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{1/3}))^2])$

**Rubi [A]** time = 0.821051, antiderivative size = 555, normalized size of antiderivative = 1., number

of steps used = 5, number of rules used = 5, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$

$$\frac{3x}{\left(\frac{a}{a+bx^2}\right)^{2/3} (a+bx^2)^{7/6} \left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)} + \frac{\sqrt{23}^{3/4} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1}\right) \middle| -7 + 4\sqrt{3}\right)}{bx \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}} + \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}} \left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1}\right) \middle| -7 + 4\sqrt{3}\right)}{2bx \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^(-7/6), x]

[Out]  $(-3*x)/((a/(a+b*x^2))^{2/3}*(a+b*x^2)^{7/6}*(1-\text{Sqrt}[3]-(a/(a+b*x^2))^{1/3})) - (3*3^{1/4}*\text{Sqrt}[2+\text{Sqrt}[3]]*(1-(a/(a+b*x^2))^{1/3}))*\text{Sqrt}[(1+(a/(a+b*x^2))^{1/3}+(a/(a+b*x^2))^{2/3})/(1-\text{Sqrt}[3]-(a/(a+b*x^2))^{1/3})^2]*\text{EllipticE}[\text{ArcSin}[(1+\text{Sqrt}[3]-(a/(a+b*x^2))^{1/3})/(1-\text{Sqrt}[3]-(a/(a+b*x^2))^{1/3})], -7+4*\text{Sqrt}[3]])/(2*b*x*(a/(a+b*x^2))^{2/3}*(a+b*x^2)^{1/6}*\text{Sqrt}[-((1-(a/(a+b*x^2))^{1/3})/(1-\text{Sqrt}[3]-(a/(a+b*x^2))^{1/3}))^2]) + (\text{Sqrt}[2]*3^{3/4}*(1-(a/(a+b*x^2))^{1/3}))*\text{Sqrt}[(1+(a/(a+b*x^2))^{1/3}+(a/(a+b*x^2))^{2/3})/(1-\text{Sqrt}[3]-(a/(a+b*x^2))^{1/3})^2]*\text{EllipticF}[\text{ArcSin}[(1+\text{Sqrt}[3]-(a/(a+b*x^2))^{1/3})/(1-\text{Sqrt}[3]-(a/(a+b*x^2))^{1/3})], -7+4*\text{Sqrt}[3]])/(b*x*(a/(a+b*x^2))^{2/3}*(a+b*x^2)^{1/6}*\text{Sqrt}[-((1-(a/(a+b*x^2))^{1/3})/(1-\text{Sqrt}[3]-(a/(a+b*x^2))^{1/3}))^2])$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{3x}{a\sqrt[6]{a+bx^2}} - \frac{2 \int \frac{1}{\sqrt[6]{a+bx^2}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(b\*x\*\*2+a)\*\*(7/6), x)

[Out]  $3*x/(a*(a+b*x^2)^{1/6}) - 2*\text{Integral}((a+b*x^2)^{-1/6}, x)/a$

**Mathematica [C]** time = 0.0374374, size = 55, normalized size = 0.1

$$\frac{3x - 2x\sqrt[6]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right)}{a\sqrt[6]{a+bx^2}}$$



Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^(-7/6), x]

[Out] (3\*x - 2\*x\*(1 + (b\*x^2)/a)^(1/6)\*Hypergeometric2F1[1/6, 1/2, 3/2, -((b\*x^2)/a)])/(a\*(a + b\*x^2)^(1/6))

**Maple [F]** time = 0.047, size = 0, normalized size = 0.

$$\int (bx^2 + a)^{-\frac{7}{6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^2+a)^(7/6), x)

[Out] int(1/(b\*x^2+a)^(7/6), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{7}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-7/6), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^(-7/6), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2 + a)^{\frac{7}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-7/6), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^(-7/6), x)

**Sympy [A]** time = 3.33929, size = 24, normalized size = 0.04

$$\frac{x_2F_1\left(\frac{1}{2}, \frac{7}{6} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*2+a)\*\*(7/6), x)

[Out] x\*hyper((1/2, 7/6), (3/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/a\*\*(7/6)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{7}{6}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^(-7/6),x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^(-7/6), x)

$$3.1036 \quad \int \frac{1}{x^2(a+bx^2)^{7/6}} dx$$

**Optimal.** Leaf size=614

$$\frac{4bx}{a^2\sqrt[6]{a+bx^2}} - \frac{4(a+bx^2)^{5/6}}{a^2x} + \frac{4bx}{a\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)} + \frac{3}{ax\sqrt[6]{a+bx^2}}$$

$$+ \frac{4\sqrt{2}\left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1}\right) \mid -7 + 4\sqrt{3}\right)}{\sqrt[6]{3ax}\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}$$

$$+ \frac{2\sqrt[4]{3}\sqrt{2 + \sqrt{3}}\left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1}\right) \mid -7 + 4\sqrt{3}\right)}{ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}$$

[Out] 3/(a\*x\*(a + b\*x^2)^(1/6)) + (4\*b\*x)/(a^2\*(a + b\*x^2)^(1/6)) - (4\*(a + b\*x^2)^(5/6))/(a^2\*x) + (4\*b\*x)/(a\*(a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(7/6)\*(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))) + (2\*3^(1/4))\*Sqrt[2 + Sqrt[3]]\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))]^2\*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]]/(a\*x\*(a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(1/6)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3)))^2]) - (4\*Sqrt[2]\*(1 - (a/(a + b\*x^2))^(1/3))\*Sqrt[(1 + (a/(a + b\*x^2))^(1/3) + (a/(a + b\*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))]^2)\*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3))], -7 + 4\*Sqrt[3]]/(3^(1/4)\*a\*x\*(a/(a + b\*x^2))^(2/3)\*(a + b\*x^2)^(1/6)\*Sqrt[-((1 - (a/(a + b\*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b\*x^2))^(1/3)))^2])]

**Rubi [A]** time = 1.17509, antiderivative size = 614, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$

$$\frac{4bx}{a^2\sqrt[6]{a+bx^2}} - \frac{4(a+bx^2)^{5/6}}{a^2x} + \frac{4bx}{a\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)} + \frac{3}{ax\sqrt[6]{a+bx^2}}$$

$$+ \frac{4\sqrt{2}\left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1}\right) \middle| -7 + 4\sqrt{3}\right)}{\sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}$$

$$+ \frac{2\sqrt[4]{3}\sqrt{2 + \sqrt{3}}\left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right) \sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1}\right) \middle| -7 + 4\sqrt{3}\right)}{\sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}$$

$$+ \frac{ax\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}{\sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a + b\*x^2)^(7/6)), x]

[Out]  $\frac{3}{a^2} \frac{1}{(a + b x^2)^{1/6}} + \frac{4 b x}{a^2 (a + b x^2)^{1/6}} - \frac{4 (a + b x^2)^{5/6}}{a^2 x} + \frac{4 b x}{a (a + b x^2)^{2/3} (a + b x^2)^{7/6} (1 - \sqrt{3} - (a/(a + b x^2))^{1/3})} + \frac{2 \cdot 3^{1/4} \sqrt{2 + \sqrt{3}} \sqrt{1 - (a/(a + b x^2))^{1/3}} \sqrt{1 + (a/(a + b x^2))^{1/3}} \sqrt{1 + (a/(a + b x^2))^{2/3}}}{(1 - \sqrt{3} - (a/(a + b x^2))^{1/3})^2} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1 + \sqrt{3} - (a/(a + b x^2))^{1/3}}{1 - \sqrt{3} - (a/(a + b x^2))^{1/3}}\right], -7 + 4 \sqrt{3}\right] \frac{1}{(a + b x^2)^{1/6} \sqrt{1 - ((1 - (a/(a + b x^2))^{1/3})^2)}}$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{4b \int \frac{1}{(a+bx^2)^{7/6}} dx}{3a} + \frac{3}{ax\sqrt[6]{a+bx^2}} + \frac{4bx}{a^2\sqrt[6]{a+bx^2}} - \frac{4(a+bx^2)^{5/6}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(b\*x\*\*2+a)\*\*(7/6), x)

[Out]  $-4*b \text{Integral}((a + b*x**2)**(-7/6), x)/(3*a) + 3/(a*x*(a + b*x**2)**(1/6)) + 4*b*x/(a**2*(a + b*x**2)**(1/6)) - 4*(a + b*x**2)**(5/6)/(a**2*x)$

**Mathematica [C]** time = 0.0543126, size = 71, normalized size = 0.12

$$\frac{8bx^2\sqrt[6]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) - 3(a + 4bx^2)}{3a^2x\sqrt[6]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a + b\*x^2)^(7/6)), x]

[Out] (-3\*(a + 4\*b\*x^2) + 8\*b\*x^2\*(1 + (b\*x^2)/a)^(1/6)\*Hypergeometric2F1[1/6, 1/2, 3/2, -(b\*x^2)/a])/(3\*a^2\*x\*(a + b\*x^2)^(1/6))

**Maple [F]** time = 0.083, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (bx^2 + a)^{-\frac{7}{6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b\*x^2+a)^(7/6), x)

[Out] int(1/x^2/(b\*x^2+a)^(7/6), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{7}{6}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(7/6)\*x^2), x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(7/6)\*x^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^4 + ax^2)(bx^2 + a)^{\frac{1}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(7/6)\*x^2), x, algorithm="fricas")

[Out] integral(1/((b\*x^4 + a\*x^2)\*(b\*x^2 + a)^(1/6)), x)

**Sympy [A]** time = 5.1765, size = 27, normalized size = 0.04

$$-\frac{{}_2F_1\left(-\frac{1}{2}, \frac{7}{6} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{7}{6}} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(b\*x\*\*2+a)\*\*(7/6),x)

[Out] -hyper((-1/2, 7/6), (1/2,), b\*x\*\*2\*exp\_polar(I\*pi)/a)/(a\*\*(7/6)\*x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{7}{6}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(7/6)\*x^2),x, algorithm="giac")

[Out] integrate(1/((b\*x^2 + a)^(7/6)\*x^2), x)

$$3.1037 \quad \int \frac{1}{x^4(a+bx^2)^{7/6}} dx$$

**Optimal.** Leaf size=652

$$\begin{aligned} & -\frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}} + \frac{40b(a+bx^2)^{5/6}}{9a^3x} - \frac{40b^2x}{9a^2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)} \\ & + \frac{40\sqrt{2}b\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right)\right)|_{-7+4\sqrt{3}}}{9\sqrt[4]{3}a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}} \\ & + \frac{20\sqrt{2+\sqrt{3}}b\left(1-\sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3}+\sqrt[3]{\frac{a}{a+bx^2}}+1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}}+\sqrt{3}+1}{-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1}\right)\right)|_{-7+4\sqrt{3}}}{3\cdot 3^{3/4}a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{\frac{1-\sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}}-\sqrt{3}+1\right)^2}}} \\ & - \frac{10(a+bx^2)^{5/6}}{3a^2x^3} + \frac{3}{ax^3\sqrt[6]{a+bx^2}} \end{aligned}$$

```
[Out] 3/(a*x^3*(a + b*x^2)^(1/6)) - (40*b^2*x)/(9*a^3*(a + b*x^2)^(1/6))
) - (10*(a + b*x^2)^(5/6))/(3*a^2*x^3) + (40*b*(a + b*x^2)^(5/6))
/(9*a^3*x) - (40*b^2*x)/(9*a^2*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(
7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) - (20*Sqrt[2 + Sqrt[
3]]*b*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3)
+ (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))]^2
]*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqr
t[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]]]/(3*3^(3/4)*a^2*x
*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^
2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3)))^2]) + (40*Sqrt[
2]*b*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3)
+ (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))]^2
]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqr
t[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]]]/(9*3^(1/4)*a^2*x*
(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2)
))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))]^2])
```

**Rubi [A]** time = 1.34077, antiderivative size = 652, normalized size of antiderivative = 1., number of

steps used = 9, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$

$$\begin{aligned}
 & -\frac{40b^2x}{9a^3\sqrt[6]{a+bx^2}} + \frac{40b(a+bx^2)^{5/6}}{9a^3x} - \frac{40b^2x}{9a^2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)} \\
 & + \frac{40\sqrt{2}b\left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1}\right) \middle| -7 + 4\sqrt{3}\right)}{\sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}} \\
 & + \frac{9\sqrt[4]{3}a^2x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}{\sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}} \\
 & - \frac{20\sqrt{2 + \sqrt{3}}b\left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1}\right) \middle| -7 + 4\sqrt{3}\right)}{\sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}} \\
 & - \frac{3 \cdot 3^{3/4} a^2 x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}{\sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}} \\
 & - \frac{10(a+bx^2)^{5/6}}{3a^2x^3} + \frac{3}{ax^3\sqrt[6]{a+bx^2}}
 \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a + b\*x^2)^(7/6)), x]

[Out]  $3/(a*x^3*(a + b*x^2)^{(1/6)}) - (40*b^2*x)/(9*a^3*(a + b*x^2)^{(1/6)}) - (10*(a + b*x^2)^{(5/6)})/(3*a^2*x^3) + (40*b*(a + b*x^2)^{(5/6)})/(9*a^3*x) - (40*b^2*x)/(9*a^2*(a/(a + b*x^2))^{(2/3)}*(a + b*x^2)^{(7/6)}*(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})) - (20*\text{Sqrt}[2 + \text{Sqrt}[3]]*b*(1 - (a/(a + b*x^2))^{(1/3)})*\text{Sqrt}[(1 + (a/(a + b*x^2))^{(1/3)} + (a/(a + b*x^2))^{(2/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(3*3^{(3/4)}*a^2*x*(a/(a + b*x^2))^{(2/3)}*(a + b*x^2)^{(1/6)}*\text{Sqrt}[-((1 - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2)]) + (40*\text{Sqrt}[2]*b*(1 - (a/(a + b*x^2))^{(1/3)})*\text{Sqrt}[(1 + (a/(a + b*x^2))^{(1/3)} + (a/(a + b*x^2))^{(2/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})], -7 + 4*\text{Sqrt}[3]])/(9*3^{(1/4)}*a^2*x*(a/(a + b*x^2))^{(2/3)}*(a + b*x^2)^{(1/6)}*\text{Sqrt}[-((1 - (a/(a + b*x^2))^{(1/3)})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{(1/3)})^2)])$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{10(a+bx^2)^{5/6}}{3a^2x^3} - \frac{80b^2 \int \frac{1}{\sqrt[6]{a+bx^2}} dx}{27a^3} + \frac{40b(a+bx^2)^{5/6}}{9a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*4/(b\*x\*\*2+a)\*\*(7/6), x)

[Out]  $3/(a*x^3*(a + b*x^2)^{(1/6)}) - 10*(a + b*x^2)^{(5/6)}/(3*a^2*x^3) - 80*b^2*Integral((a + b*x^2)^{(-1/6)}, x)/(27*a^3) + 40*b*(a + b*x^2)^{(5/6)}/(9*a^3*x)$



---

**Mathematica [C]** time = 0.0583911, size = 83, normalized size = 0.13

$$\frac{-9a^2 - 80b^2x^4\sqrt[6]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) + 30abx^2 + 120b^2x^4}{27a^3x^3\sqrt[6]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a + b\*x^2)^(7/6)), x]

[Out]  $(-9a^2 + 30abx^2 + 120b^2x^4 - 80b^2x^4(1 + (bx^2)/a)^{(1/6)} \text{Hypergeometric2F1}[1/6, 1/2, 3/2, -(bx^2)/a]) / (27a^3x^3(a + bx^2)^{(1/6)})$

---

**Maple [F]** time = 0.075, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} (bx^2 + a)^{-\frac{7}{6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b\*x^2+a)^(7/6), x)

[Out] int(1/x^4/(b\*x^2+a)^(7/6), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{7}{6}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(7/6)\*x^4), x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(7/6)\*x^4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^6 + ax^4)(bx^2 + a)^{\frac{1}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(7/6)\*x^4), x, algorithm="fricas")

[Out] integral(1/((b\*x^6 + a\*x^4)\*(b\*x^2 + a)^(1/6)), x)

---

**Sympy [A]** time = 7.12878, size = 32, normalized size = 0.05

$$\frac{{}_2F_1\left(-\frac{3}{2}, \frac{7}{6} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{7}{6}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(b*x**2+a)**(7/6),x)`

[Out] `-hyper((-3/2, 7/6), (-1/2, ), b*x**2*exp_polar(I*pi)/a)/(3*a**(7/6)*x**3)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{7}{6}} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(7/6)*x^4),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(7/6)*x^4), x)`

$$3.1038 \quad \int \frac{1}{x^6(a+bx^2)^{7/6}} dx$$

**Optimal.** Leaf size=680

$$\frac{\frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}} - \frac{128b^2(a+bx^2)^{5/6}}{27a^4x} + \frac{128b^3x}{27a^3\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)}}{128\sqrt{2}b^2\left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{bx^2+a}} - \sqrt{3} + 1}\right) \mid -7 + 4\sqrt{3}\right)}$$


---


$$\frac{27\sqrt[4]{3}a^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}{64\sqrt{2 + \sqrt{3}}b^2\left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{bx^2+a}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{bx^2+a}} - \sqrt{3} + 1}\right) \mid -7 + 4\sqrt{3}\right)}$$


---


$$+ \frac{9 \cdot 3^{3/4} a^3 x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}{\frac{32b(a+bx^2)^{5/6}}{9a^3x^3} - \frac{16(a+bx^2)^{5/6}}{5a^2x^5} + \frac{3}{ax^5\sqrt[6]{a+bx^2}}}$$

```
[Out] 3/(a*x^5*(a + b*x^2)^(1/6)) + (128*b^3*x)/(27*a^4*(a + b*x^2)^(1/6)) - (16*(a + b*x^2)^(5/6))/(5*a^2*x^5) + (32*b*(a + b*x^2)^(5/6))/(9*a^3*x^3) - (128*b^2*(a + b*x^2)^(5/6))/(27*a^4*x) + (128*b^3*x)/(27*a^3*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(7/6)*(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))) + (64*Sqrt[2 + Sqrt[3]]*b^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(3/4)*a^3*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)] - (128*Sqrt[2]*b^2*(1 - (a/(a + b*x^2))^(1/3))*Sqrt[(1 + (a/(a + b*x^2))^(1/3) + (a/(a + b*x^2))^(2/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(27*3^(1/4)*a^3*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)*Sqrt[-((1 - (a/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (a/(a + b*x^2))^(1/3))^2)])
```

**Rubi [A]** time = 1.47851, antiderivative size = 680, normalized size of antiderivative = 1., number of

steps used = 10, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$

$$\frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}} - \frac{128b^2(a+bx^2)^{5/6}}{27a^4x} + \frac{128b^3x}{27a^3\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{7/6}\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)}$$

$$\frac{128\sqrt{2}b^2\left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1}\right)\right) |_{-7 + 4\sqrt{3}}}{27\sqrt[4]{3}a^3x\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}$$

$$+ \frac{64\sqrt{2 + \sqrt{3}}b^2\left(1 - \sqrt[3]{\frac{a}{a+bx^2}}\right)\sqrt{\frac{\left(\frac{a}{a+bx^2}\right)^{2/3} + \sqrt[3]{\frac{a}{a+bx^2}} + 1}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}} E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{a}{a+bx^2}} + \sqrt{3} + 1}{-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1}\right)\right) |_{-7 + 4\sqrt{3}}}{9 \cdot 3^{3/4} a^3 x \left(\frac{a}{a+bx^2}\right)^{2/3} \sqrt[6]{a+bx^2} \sqrt{\frac{1 - \sqrt[3]{\frac{a}{a+bx^2}}}{\left(-\sqrt[3]{\frac{a}{a+bx^2}} - \sqrt{3} + 1\right)^2}}}$$

$$+ \frac{32b(a+bx^2)^{5/6}}{9a^3x^3} - \frac{16(a+bx^2)^{5/6}}{5a^2x^5} + \frac{3}{ax^5\sqrt[6]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6\*(a + b\*x^2)^(7/6)), x]

[Out]  $3/(a*x^5*(a + b*x^2)^{(1/6)}) + (128*b^3*x)/(27*a^4*(a + b*x^2)^{(1/6)}) - (16*(a + b*x^2)^{(5/6)})/(5*a^2*x^5) + (32*b*(a + b*x^2)^{(5/6)})/(9*a^3*x^3) - (128*b^2*(a + b*x^2)^{(5/6)})/(27*a^4*x) + (128*b^3*x)/(27*a^3*(a/(a + b*x^2))^{2/3}*(a + b*x^2)^{7/6}*(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{1/3})) + (64*\text{Sqrt}[2 + \text{Sqrt}[3]]*b^2*(1 - (a/(a + b*x^2))^{1/3}))*\text{Sqrt}[(1 + (a/(a + b*x^2))^{1/3}) + (a/(a + b*x^2))^{2/3}]/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{1/3})^2]*\text{EllipticE}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a/(a + b*x^2))^{1/3})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{1/3})], -7 + 4*\text{Sqrt}[3]]/(9*3^{3/4}*a^3*x*(a/(a + b*x^2))^{2/3}*(a + b*x^2)^{1/6}*\text{Sqrt}[-((1 - (a/(a + b*x^2))^{1/3})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{1/3})^2)] - (128*\text{Sqrt}[2]*b^2*(1 - (a/(a + b*x^2))^{1/3}))*\text{Sqrt}[(1 + (a/(a + b*x^2))^{1/3}) + (a/(a + b*x^2))^{2/3}]/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{1/3})^2]*\text{EllipticF}[\text{ArcSin}[(1 + \text{Sqrt}[3] - (a/(a + b*x^2))^{1/3})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{1/3})], -7 + 4*\text{Sqrt}[3]]/(27*3^{1/4}*a^3*x*(a/(a + b*x^2))^{2/3}*(a + b*x^2)^{1/6}*\text{Sqrt}[-((1 - (a/(a + b*x^2))^{1/3})/(1 - \text{Sqrt}[3] - (a/(a + b*x^2))^{1/3})^2)])$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{3}{ax^5\sqrt[6]{a+bx^2}} - \frac{16(a+bx^2)^{5/6}}{5a^2x^5} - \frac{128b^3\int\frac{1}{(a+bx^2)^{7/6}}dx}{81a^3} + \frac{32b(a+bx^2)^{5/6}}{9a^3x^3} + \frac{128b^3x}{27a^4\sqrt[6]{a+bx^2}} - \frac{128b^2(a+bx^2)^{5/6}}{27a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*6/(b\*x\*\*2+a)\*\*(7/6), x)

[Out]  $3/(a*x**5*(a + b*x**2)**(1/6)) - 16*(a + b*x**2)**(5/6)/(5*a**2*x**5) - 128*b**3*Integral((a + b*x**2)**(-7/6), x)/(81*a**3) + 32*b*(a + b*x**2)**(5/6)/(9*a**3*x**3) + 128*b**3*x/(27*a**4*(a + b$

$$x^{**2}**(1/6)) - 128*b^{**2}*(a + b*x^{**2})^{**}(5/6)/(27*a^{**4}*x)$$

**Mathematica [C]** time = 0.0670537, size = 97, normalized size = 0.14

$$\frac{1280b^3x^6\sqrt[6]{\frac{bx^2}{a}} + {}_2F_1\left(\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, -\frac{bx^2}{a}\right) - 3(27a^3 - 48a^2bx^2 + 160ab^2x^4 + 640b^3x^6)}{405a^4x^5\sqrt[6]{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^6\*(a + b\*x^2)^(7/6)), x]

[Out] (-3\*(27\*a^3 - 48\*a^2\*b\*x^2 + 160\*a\*b^2\*x^4 + 640\*b^3\*x^6) + 1280\*b^3\*x^6\*(1 + (b\*x^2)/a)^(1/6)\*Hypergeometric2F1[1/6, 1/2, 3/2, -(b\*x^2)/a])/(405\*a^4\*x^5\*(a + b\*x^2)^(1/6))

**Maple [F]** time = 0.075, size = 0, normalized size = 0.

$$\int \frac{1}{x^6} (bx^2 + a)^{-\frac{7}{6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^6/(b\*x^2+a)^(7/6), x)

[Out] int(1/x^6/(b\*x^2+a)^(7/6), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{7}{6}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(7/6)\*x^6), x, algorithm="maxima")

[Out] integrate(1/((b\*x^2 + a)^(7/6)\*x^6), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^8 + ax^6)(bx^2 + a)^{\frac{1}{6}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b\*x^2 + a)^(7/6)\*x^6), x, algorithm="fricas")

[Out] integral(1/((b\*x^8 + a\*x^6)\*(b\*x^2 + a)^(1/6)), x)

**Sympy [A]** time = 10.2058, size = 32, normalized size = 0.05

$$\frac{{}_2F_1\left(-\frac{5}{2}, \frac{7}{6} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5a^{\frac{7}{6}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**6/(b*x**2+a)**(7/6),x)`

[Out] `-hyper((-5/2, 7/6), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(7/6)*x**5)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{7}{6}} x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x^2 + a)^(7/6)*x^6),x, algorithm="giac")`

[Out] `integrate(1/((b*x^2 + a)^(7/6)*x^6), x)`

### 3.1039 $\int x^7 (a + bx^2)^p dx$

**Optimal.** Leaf size=100

$$-\frac{a^3 (a + bx^2)^{p+1}}{2b^4(p+1)} + \frac{3a^2 (a + bx^2)^{p+2}}{2b^4(p+2)} - \frac{3a (a + bx^2)^{p+3}}{2b^4(p+3)} + \frac{(a + bx^2)^{p+4}}{2b^4(p+4)}$$

[Out]  $-(a^3*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) + (3*a^2*(a + b*x^2)^(2 + p))/(2*b^4*(2 + p)) - (3*a*(a + b*x^2)^(3 + p))/(2*b^4*(3 + p)) + (a + b*x^2)^(4 + p)/(2*b^4*(4 + p))$

**Rubi [A]** time = 0.13883, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{a^3 (a + bx^2)^{p+1}}{2b^4(p+1)} + \frac{3a^2 (a + bx^2)^{p+2}}{2b^4(p+2)} - \frac{3a (a + bx^2)^{p+3}}{2b^4(p+3)} + \frac{(a + bx^2)^{p+4}}{2b^4(p+4)}$$

Antiderivative was successfully verified.

[In] Int[x^7\*(a + b\*x^2)^p, x]

[Out]  $-(a^3*(a + b*x^2)^(1 + p))/(2*b^4*(1 + p)) + (3*a^2*(a + b*x^2)^(2 + p))/(2*b^4*(2 + p)) - (3*a*(a + b*x^2)^(3 + p))/(2*b^4*(3 + p)) + (a + b*x^2)^(4 + p)/(2*b^4*(4 + p))$

**Rubi in Sympy [A]** time = 24.7299, size = 85, normalized size = 0.85

$$-\frac{a^3 (a + bx^2)^{p+1}}{2b^4 (p + 1)} + \frac{3a^2 (a + bx^2)^{p+2}}{2b^4 (p + 2)} - \frac{3a (a + bx^2)^{p+3}}{2b^4 (p + 3)} + \frac{(a + bx^2)^{p+4}}{2b^4 (p + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*7\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $-a**3*(a + b*x**2)**(p + 1)/(2*b**4*(p + 1)) + 3*a**2*(a + b*x**2)**(p + 2)/(2*b**4*(p + 2)) - 3*a*(a + b*x**2)**(p + 3)/(2*b**4*(p + 3)) + (a + b*x**2)**(p + 4)/(2*b**4*(p + 4))$

**Mathematica [A]** time = 0.0654368, size = 93, normalized size = 0.93

$$\frac{(a + bx^2)^{p+1} (-6a^3 + 6a^2b(p+1)x^2 - 3ab^2(p^2 + 3p + 2)x^4 + b^3(p^3 + 6p^2 + 11p + 6)x^6)}{2b^4(p+1)(p+2)(p+3)(p+4)}$$

Antiderivative was successfully verified.

[In] Integrate[x^7\*(a + b\*x^2)^p, x]

[Out]  $((a + b*x^2)^(1 + p)*(-6*a^3 + 6*a^2*b*(1 + p)*x^2 - 3*a*b^2*(2 + 3*p + p^2)*x^4 + b^3*(6 + 11*p + 6*p^2 + p^3)*x^6))/(2*b^4*(1 + p)*(2 + p)*(3 + p)*(4 + p))$

**Maple [A]** time = 0.009, size = 132, normalized size = 1.3

$$\frac{(bx^2 + a)^{1+p} (-b^3p^3x^6 - 6b^3p^2x^6 - 11b^3px^6 + 3ab^2p^2x^4 - 6b^3x^6 + 9ab^2px^4 + 6ab^2x^4 - 6a^2bpx^2 - 6a^2bx^2 + 6a^3)}{2b^4(p^4 + 10p^3 + 35p^2 + 50p + 24)}$$





```

*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) - 6*b**3*x**6/(12*a**3*b**
4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6), Eq(p, -4)
), (-6*a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5
*x**2 + 4*b**6*x**4) - 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**
2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 9*a**3/(4*a**2*b**4 + 8*a
*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x**2*log(-I*sqrt(a)*sqrt(1/
b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x
**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4
*b**6*x**4) - 12*a**2*b*x**2/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**
6*x**4) - 6*a*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**
4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 6*a*b**2*x**4*log(I*sqrt(a)*sq
rt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) + 2*b**3
*x**6/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4), Eq(p, -3)), (6
*a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2) + 6*
a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2) + 6*a
**3/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(-I*sqrt(a)*sqrt(1
/b) + x)/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(I*sqrt(a)*s
qrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2) - 3*a*b**2*x**4/(4*a*b**4
+ 4*b**5*x**2) + b**3*x**6/(4*a*b**4 + 4*b**5*x**2), Eq(p, -2)),
(-a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**4) - a**3*log(I*sqrt(a)
)*sqrt(1/b) + x)/(2*b**4) + a**2*x**2/(2*b**3) - a*x**4/(4*b**2)
+ x**6/(6*b), Eq(p, -1)), (-6*a**4*(a + b*x**2)**p/(2*b**4*p**4 +
20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*a**3*b*p
*x**2*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2
+ 100*b**4*p + 48*b**4) - 3*a**2*b**2*p**2*x**4*(a + b*x**2)**p/(
2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4)
- 3*a**2*b**2*p*x**4*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3
+ 70*b**4*p**2 + 100*b**4*p + 48*b**4) + a*b**3*p**3*x**6*(a + b
*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p
+ 48*b**4) + 3*a*b**3*p**2*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 2
0*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 2*a*b**3*p*x
**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 +
100*b**4*p + 48*b**4) + b**4*p**3*x**8*(a + b*x**2)**p/(2*b**4*p
**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*b**4
*p**2*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*
p**2 + 100*b**4*p + 48*b**4) + 11*b**4*p*x**8*(a + b*x**2)**p/(2*
b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) +
6*b**4*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**
4*p**2 + 100*b**4*p + 48*b**4), True))

```

---

**GIAC/XCAS [A]** time = 0.237967, size = 597, normalized size = 5.97

$$(bx^2 + a)^4 p^3 e^{p \ln(bx^2 + a)} - 3(bx^2 + a)^3 a p^3 e^{p \ln(bx^2 + a)} + 3(bx^2 + a)^2 a^2 p^3 e^{p \ln(bx^2 + a)} - (bx^2 + a) a^3 p^3 e^{p \ln(bx^2 + a)} + 6(bx^2 + a)^4 p^3 e^{p \ln(bx^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p\*x^7,x, algorithm="giac")

[Out] 
$$\frac{1}{2} \left( (bx^2 + a)^4 p^3 e^{p \ln(bx^2 + a)} - 3(bx^2 + a)^3 a p^3 e^{p \ln(bx^2 + a)} + 3(bx^2 + a)^2 a^2 p^3 e^{p \ln(bx^2 + a)} - (bx^2 + a) a^3 p^3 e^{p \ln(bx^2 + a)} + 6(bx^2 + a)^4 p^3 e^{p \ln(bx^2 + a)} - 21(bx^2 + a)^3 a p^2 e^{p \ln(bx^2 + a)} + 24(bx^2 + a)^2 a^2 p^2 e^{p \ln(bx^2 + a)} - 9(bx^2 + a) a^3 p^2 e^{p \ln(bx^2 + a)} + 11(bx^2 + a)^4 p e^{p \ln(bx^2 + a)} - 42(bx^2 + a)^3 a p e^{p \ln(bx^2 + a)} + 57(bx^2 + a)^2 a^2 p e^{p \ln(bx^2 + a)} - 26(bx^2 + a) a^3 p e^{p \ln(bx^2 + a)} + 6(bx^2 + a)^4 e^{p \ln(bx^2 + a)} - 24(bx^2 + a)^3 a e^{p \ln(bx^2 + a)} + 36(bx^2 + a)^2 a^2 e^{p \ln(bx^2 + a)} - 24(bx^2 + a) a^3 e^{p \ln(bx^2 + a)} \right) / ((b^3 p^4 + 10 b^3 p^3 + 35 b^3 p^2 + 50 b^3 p + 24 b^3) b)$$

### 3.1040 $\int x^5 (a + bx^2)^p dx$

**Optimal.** Leaf size=72

$$\frac{a^2 (a + bx^2)^{p+1}}{2b^3(p+1)} - \frac{a (a + bx^2)^{p+2}}{b^3(p+2)} + \frac{(a + bx^2)^{p+3}}{2b^3(p+3)}$$

[Out]  $(a^2*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) - (a*(a + b*x^2)^(2 + p))/(b^3*(2 + p)) + (a + b*x^2)^(3 + p)/(2*b^3*(3 + p))$

**Rubi [A]** time = 0.0926792, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{a^2 (a + bx^2)^{p+1}}{2b^3(p+1)} - \frac{a (a + bx^2)^{p+2}}{b^3(p+2)} + \frac{(a + bx^2)^{p+3}}{2b^3(p+3)}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(a + b\*x^2)^p, x]

[Out]  $(a^2*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) - (a*(a + b*x^2)^(2 + p))/(b^3*(2 + p)) + (a + b*x^2)^(3 + p)/(2*b^3*(3 + p))$

**Rubi in Sympy [A]** time = 17.6882, size = 58, normalized size = 0.81

$$\frac{a^2 (a + bx^2)^{p+1}}{2b^3(p+1)} - \frac{a (a + bx^2)^{p+2}}{b^3(p+2)} + \frac{(a + bx^2)^{p+3}}{2b^3(p+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $a**2*(a + b*x**2)**(p + 1)/(2*b**3*(p + 1)) - a*(a + b*x**2)**(p + 2)/(b**3*(p + 2)) + (a + b*x**2)**(p + 3)/(2*b**3*(p + 3))$

**Mathematica [A]** time = 0.0435254, size = 64, normalized size = 0.89

$$\frac{(a + bx^2)^{p+1} (2a^2 - 2ab(p+1)x^2 + b^2(p^2 + 3p + 2)x^4)}{2b^3(p+1)(p+2)(p+3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*x^2)^p, x]

[Out]  $((a + b*x^2)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p + p^2)*x^4))/(2*b^3*(1 + p)*(2 + p)*(3 + p))$

**Maple [A]** time = 0.008, size = 80, normalized size = 1.1

$$\frac{(bx^2 + a)^{1+p} (b^2p^2x^4 + 3b^2px^4 + 2b^2x^4 - 2abpx^2 - 2abx^2 + 2a^2)}{2b^3(p^3 + 6p^2 + 11p + 6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(b*x^2+a)^p,x)`

[Out]  $\frac{1}{2} \cdot (b \cdot x^2 + a)^{(1+p)} \cdot (b^2 \cdot p^2 \cdot x^4 + 3 \cdot b^2 \cdot p \cdot x^4 + 2 \cdot b^2 \cdot x^4 - 2 \cdot a \cdot b \cdot p \cdot x^2 - 2 \cdot a \cdot b \cdot x^2 + 2 \cdot a^2) / b^3 / (p^3 + 6 \cdot p^2 + 11 \cdot p + 6)$

**Maxima [A]** time = 1.36643, size = 99, normalized size = 1.38

$$\frac{((p^2 + 3p + 2)b^3x^6 + (p^2 + p)ab^2x^4 - 2a^2bpx^2 + 2a^3)(bx^2 + a)^p}{2(p^3 + 6p^2 + 11p + 6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^5,x, algorithm="maxima")`

[Out]  $\frac{1}{2} \cdot ((p^2 + 3 \cdot p + 2) \cdot b^3 \cdot x^6 + (p^2 + p) \cdot a \cdot b^2 \cdot x^4 - 2 \cdot a^2 \cdot b \cdot p \cdot x^2 + 2 \cdot a^3) \cdot (b \cdot x^2 + a)^p / ((p^3 + 6 \cdot p^2 + 11 \cdot p + 6) \cdot b^3)$

**Fricas [A]** time = 0.219338, size = 132, normalized size = 1.83

$$\frac{((b^3p^2 + 3b^3p + 2b^3)x^6 - 2a^2bpx^2 + (ab^2p^2 + ab^2p)x^4 + 2a^3)(bx^2 + a)^p}{2(b^3p^3 + 6b^3p^2 + 11b^3p + 6b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^5,x, algorithm="fricas")`

[Out]  $\frac{1}{2} \cdot ((b^3 \cdot p^2 + 3 \cdot b^3 \cdot p + 2 \cdot b^3) \cdot x^6 - 2 \cdot a^2 \cdot b \cdot p \cdot x^2 + (a \cdot b^2 \cdot p^2 + a \cdot b^2 \cdot p) \cdot x^4 + 2 \cdot a^3) \cdot (b \cdot x^2 + a)^p / (b^3 \cdot p^3 + 6 \cdot b^3 \cdot p^2 + 11 \cdot b^3 \cdot p + 6 \cdot b^3)$

**Sympy [A]** time = 15.8674, size = 981, normalized size = 13.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(b*x**2+a)**p,x)`

[Out]  $\text{Piecewise}((a \cdot p \cdot x^{6/6}, \text{Eq}(b, 0)), (2 \cdot a^{**2} \cdot \log(-I \cdot \sqrt{a}) \cdot \sqrt{1/b} + x) / (4 \cdot a^{**2} \cdot b^{**3} + 8 \cdot a \cdot b^{**4} \cdot x^{**2} + 4 \cdot b^{**5} \cdot x^{**4}) + 2 \cdot a^{**2} \cdot \log(I \cdot \sqrt{a}) \cdot \sqrt{1/b} + x) / (4 \cdot a^{**2} \cdot b^{**3} + 8 \cdot a \cdot b^{**4} \cdot x^{**2} + 4 \cdot b^{**5} \cdot x^{**4}) + 3 \cdot a^{**2} / (4 \cdot a^{**2} \cdot b^{**3} + 8 \cdot a \cdot b^{**4} \cdot x^{**2} + 4 \cdot b^{**5} \cdot x^{**4}) + 4 \cdot a \cdot b^{**2} \cdot \log(-I \cdot \sqrt{a}) \cdot \sqrt{1/b} + x) / (4 \cdot a^{**2} \cdot b^{**3} + 8 \cdot a \cdot b^{**4} \cdot x^{**2} + 4 \cdot b^{**5} \cdot x^{**4}) + 4 \cdot a \cdot b^{**5} \cdot x^{**4} + 4 \cdot a \cdot b^{**2} \cdot \log(I \cdot \sqrt{a}) \cdot \sqrt{1/b} + x) / (4 \cdot a^{**2} \cdot b^{**3} + 8 \cdot a \cdot b^{**4} \cdot x^{**2} + 4 \cdot b^{**5} \cdot x^{**4}) + 4 \cdot a \cdot b^{**2} \cdot x^{**2} / (4 \cdot a^{**2} \cdot b^{**3} + 8 \cdot a \cdot b^{**4} \cdot x^{**2} + 4 \cdot b^{**5} \cdot x^{**4}) + 2 \cdot b^{**2} \cdot x^{**4} \cdot \log(-I \cdot \sqrt{a}) \cdot \sqrt{1/b} + x) / (4 \cdot a^{**2} \cdot b^{**3} + 8 \cdot a \cdot b^{**4} \cdot x^{**2} + 4 \cdot b^{**5} \cdot x^{**4}) + 2 \cdot b^{**2} \cdot x^{**4} \cdot \log(I \cdot \sqrt{a}) \cdot \sqrt{1/b} + x) / (4 \cdot a^{**2} \cdot b^{**3} + 8 \cdot a \cdot b^{**4} \cdot x^{**2} + 4 \cdot b^{**5} \cdot x^{**4}), \text{Eq}(p, -3)), (-2 \cdot a^{**2} \cdot \log(-I \cdot \sqrt{a}) \cdot \sqrt{1/b} + x) / (2 \cdot a \cdot b^{**3} + 2 \cdot b^{**4} \cdot x^{**2}) - 2 \cdot a^{**2} \cdot \log(I \cdot \sqrt{a}) \cdot \sqrt{1/b} + x) / (2 \cdot a \cdot b^{**3} + 2 \cdot b^{**4} \cdot x^{**2}) - 2 \cdot a \cdot b^{**2} \cdot \log(-I \cdot \sqrt{a}) \cdot \sqrt{1/b} + x) / (2 \cdot a \cdot b^{**3} + 2 \cdot b^{**4} \cdot x^{**2}) - 2 \cdot a \cdot b^{**2} \cdot \log(I \cdot \sqrt{a}) \cdot \sqrt{1/b} + x) / (2 \cdot a \cdot b^{**3} + 2 \cdot b^{**4} \cdot x^{**2}) + b^{**2} \cdot x^{**4} / (2 \cdot a \cdot b^{**3} + 2 \cdot b^{**4} \cdot x^{**2}), \text{Eq}(p, -2)), (a^{**2} \cdot \log(-I \cdot \sqrt{a}) \cdot \sqrt{1/b} + x) / (2 \cdot b^{**3}) + a^{**2} \cdot \log(I \cdot \sqrt{a}) \cdot \sqrt{1/b} + x) / (2 \cdot b^{**3}) - a \cdot x^{**2} / (2 \cdot b^{**2}) + x^{**4} / (4 \cdot b), \text{Eq}(p, -1)), (2 \cdot a^{**3} \cdot (a + b \cdot x^{**2}) \cdot p / (2 \cdot b^{**3} \cdot p^{**3} + 12 \cdot b^{**3} \cdot p^{**2} + 22 \cdot b^{**3} \cdot p + 12 \cdot b^{**3}) - 2 \cdot a^{**2} \cdot b \cdot p \cdot x^{**2} \cdot (a + b \cdot x^{**2}) \cdot p / (2 \cdot b^{**3} \cdot p^{**3} + 12 \cdot b^{**3} \cdot p^{**2} + 22 \cdot b^{**3} \cdot p + 12 \cdot b^{**3}) + a \cdot b^{**2} \cdot p^{**2} \cdot x^{**4} \cdot (a + b \cdot x^{**2}) \cdot p / (2 \cdot b^{**3} \cdot p^{**3} + 12 \cdot b^{**3} \cdot p^{**2} + 22 \cdot b^{**3} \cdot p + 12 \cdot b^{**3}))$

```
* 3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2
*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6
*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**
3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 +
22*b**3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 +
12*b**3*p**2 + 22*b**3*p + 12*b**3), True))
```

**GIAC/XCAS [A]** time = 0.248286, size = 336, normalized size = 4.67

$$\frac{(bx^2 + a)^3 p^2 e^{p \ln(bx^2 + a)} - 2 (bx^2 + a)^2 a p^2 e^{p \ln(bx^2 + a)} + (bx^2 + a) a^2 p^2 e^{p \ln(bx^2 + a)} + 3 (bx^2 + a)^3 p e^{p \ln(bx^2 + a)} - 8 (bx^2 + a)^2 a p e^{p \ln(bx^2 + a)} + 6 (bx^2 + a) a^2 p e^{p \ln(bx^2 + a)}}{2(b^2 p^3 + 6 b^2 p^2 + 11 b^2 p + 6 b^2) b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^p*x^5,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)^3*p^2*e^(p*ln(b*x^2 + a)) - 2*(b*x^2 + a)^2*a*p^
2*e^(p*ln(b*x^2 + a)) + (b*x^2 + a)*a^2*p^2*e^(p*ln(b*x^2 + a)) +
3*(b*x^2 + a)^3*p*e^(p*ln(b*x^2 + a)) - 8*(b*x^2 + a)^2*a*p*e^(p
*ln(b*x^2 + a)) + 5*(b*x^2 + a)*a^2*p*e^(p*ln(b*x^2 + a)) + 2*(b*
x^2 + a)^3*e^(p*ln(b*x^2 + a)) - 6*(b*x^2 + a)^2*a*e^(p*ln(b*x^2
+ a)) + 6*(b*x^2 + a)*a^2*e^(p*ln(b*x^2 + a)))/((b^2*p^3 + 6*b^2*
p^2 + 11*b^2*p + 6*b^2)*b)
```

### 3.1041 $\int x^3 (a + bx^2)^p dx$

**Optimal.** Leaf size=48

$$\frac{(a + bx^2)^{p+2}}{2b^2(p+2)} - \frac{a(a + bx^2)^{p+1}}{2b^2(p+1)}$$

[Out]  $-(a*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (a + b*x^2)^(2 + p)/(2*b^2*(2 + p))$

**Rubi [A]** time = 0.0629979, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{(a + bx^2)^{p+2}}{2b^2(p+2)} - \frac{a(a + bx^2)^{p+1}}{2b^2(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*x^2)^p, x]

[Out]  $-(a*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (a + b*x^2)^(2 + p)/(2*b^2*(2 + p))$

**Rubi in Sympy [A]** time = 11.053, size = 37, normalized size = 0.77

$$-\frac{a(a + bx^2)^{p+1}}{2b^2(p+1)} + \frac{(a + bx^2)^{p+2}}{2b^2(p+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $-a*(a + b*x**2)**(p + 1)/(2*b**2*(p + 1)) + (a + b*x**2)**(p + 2)/(2*b**2*(p + 2))$

**Mathematica [A]** time = 0.0291495, size = 40, normalized size = 0.83

$$\frac{(a + bx^2)^{p+1} (b(p+1)x^2 - a)}{2b^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*x^2)^p, x]

[Out]  $((a + b*x^2)^(1 + p)*(-a + b*(1 + p)*x^2))/(2*b^2*(1 + p)*(2 + p))$

**Maple [A]** time = 0.007, size = 42, normalized size = 0.9

$$-\frac{(bx^2 + a)^{1+p} (-x^2pb - bx^2 + a)}{2b^2(p^2 + 3p + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(b*x^2+a)^p,x)`

[Out]  $-1/2*(b*x^2+a)^{(1+p)}*(-b*p*x^2-b*x^2+a)/b^2/(p^2+3*p+2)$

**Maxima [A]** time = 1.36255, size = 63, normalized size = 1.31

$$\frac{(b^2(p+1)x^4 + abpx^2 - a^2)(bx^2 + a)^p}{2(p^2 + 3p + 2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^3,x, algorithm="maxima")`

[Out]  $1/2*(b^2*(p+1)*x^4 + a*b*p*x^2 - a^2)*(b*x^2 + a)^p/((p^2 + 3*p + 2)*b^2)$

**Fricas [A]** time = 0.219296, size = 78, normalized size = 1.62

$$\frac{(abpx^2 + (b^2p + b^2)x^4 - a^2)(bx^2 + a)^p}{2(b^2p^2 + 3b^2p + 2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^3,x, algorithm="fricas")`

[Out]  $1/2*(a*b*p*x^2 + (b^2*p + b^2)*x^4 - a^2)*(b*x^2 + a)^p/(b^2*p^2 + 3*b^2*p + 2*b^2)$

**Sympy [A]** time = 5.83407, size = 364, normalized size = 7.58

$$\begin{cases} \frac{a^p x^4}{4} & \text{for } b = 0 \\ \frac{a \log(-i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2ab^2+2b^3x^2} + \frac{a \log(i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2ab^2+2b^3x^2} + \frac{a}{2ab^2+2b^3x^2} + \frac{bx^2 \log(-i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2ab^2+2b^3x^2} + \frac{bx^2 \log(i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2ab^2+2b^3x^2} & \text{for } p = -2 \\ -\frac{a \log(-i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2b^2} - \frac{a \log(i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2b^2} + \frac{x^2}{2b} & \text{for } p = -1 \\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**2+a)**p,x)`

[Out] `Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2), True))`

**GIAC/XCAS [A]** time = 0.240953, size = 138, normalized size = 2.88

$$\frac{(bx^2 + a)^2 pe^{p \ln(bx^2+a)} - (bx^2 + a) a p e^{p \ln(bx^2+a)} + (bx^2 + a)^2 e^{p \ln(bx^2+a)} - 2(bx^2 + a) a e^{p \ln(bx^2+a)}}{2(p^2 + 3p + 2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^p*x^3,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)^2*p*e^(p*ln(b*x^2 + a)) - (b*x^2 + a)*a*p*e^(p*ln(b*x^2 + a)) + (b*x^2 + a)^2*e^(p*ln(b*x^2 + a)) - 2*(b*x^2 + a)*a*e^(p*ln(b*x^2 + a)))/((p^2 + 3*p + 2)*b^2)
```

### 3.1042 $\int x (a + bx^2)^p dx$

**Optimal.** Leaf size=23

$$\frac{(a + bx^2)^{p+1}}{2b(p+1)}$$

[Out]  $(a + b*x^2)^{(1 + p)}/(2*b*(1 + p))$

**Rubi [A]** time = 0.0156318, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{(a + bx^2)^{p+1}}{2b(p+1)}$$

Antiderivative was successfully verified.

[In] `Int[x*(a + b*x^2)^p, x]`

[Out]  $(a + b*x^2)^{(1 + p)}/(2*b*(1 + p))$

**Rubi in Sympy [A]** time = 2.6632, size = 15, normalized size = 0.65

$$\frac{(a + bx^2)^{p+1}}{2b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x*(b*x**2+a)**p, x)`

[Out]  $(a + b*x**2)**(p + 1)/(2*b*(p + 1))$

**Mathematica [A]** time = 0.00581633, size = 22, normalized size = 0.96

$$\frac{(a + bx^2)^{p+1}}{2bp + 2b}$$

Antiderivative was successfully verified.

[In] `Integrate[x*(a + b*x^2)^p, x]`

[Out]  $(a + b*x^2)^{(1 + p)}/(2*b + 2*b*p)$

**Maple [A]** time = 0.003, size = 22, normalized size = 1.

$$\frac{(bx^2 + a)^{1+p}}{2b(1+p)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x^2+a)^p, x)`



[Out]  $1/2 * (b * x^2 + a)^{(1+p)} / b / (1+p)$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x,x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.216218, size = 34, normalized size = 1.48

$$\frac{(bx^2 + a)(bx^2 + a)^p}{2(bp + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x,x, algorithm="fricas")`

[Out]  $1/2 * (b * x^2 + a) * (b * x^2 + a)^p / (b * p + b)$

---

**Sympy [A]** time = 2.13288, size = 97, normalized size = 4.22

$$\begin{cases} \frac{x^2}{2a} & \text{for } b = 0 \wedge p = -1 \\ \frac{a^p x^2}{2} & \text{for } b = 0 \\ \frac{\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}+x}\right)}{2b} + \frac{\log\left(i\sqrt{a}\sqrt{\frac{1}{b}+x}\right)}{2b} & \text{for } p = -1 \\ \frac{a(a+bx^2)^p}{2bp+2b} + \frac{bx^2(a+bx^2)^p}{2bp+2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x**2+a)**p,x)`

[Out] `Piecewise((x**2/(2*a), Eq(b, 0) & Eq(p, -1)), (a**p*x**2/2, Eq(b, 0)), (log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b) + log(I*sqrt(a)*sqrt(1/b) + x)/(2*b), Eq(p, -1)), (a*(a + b*x**2)**p/(2*b*p + 2*b) + b*x**2*(a + b*x**2)**p/(2*b*p + 2*b), True))`

---

**GIAC/XCAS [A]** time = 0.227714, size = 28, normalized size = 1.22

$$\frac{(bx^2 + a)^{p+1}}{2b(p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x,x, algorithm="giac")`

[Out]  $1/2 * (b * x^2 + a)^{(p + 1)} / (b * (p + 1))$

$$3.1043 \quad \int \frac{(a+bx^2)^p}{x} dx$$

**Optimal.** Leaf size=41

$$-\frac{(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{2a(p+1)}$$

[Out]  $-\left((a + b*x^2)^{(1 + p)} * \text{Hypergeometric2F1}[1, 1 + p, 2 + p, 1 + (b*x^2)/a]\right) / (2*a*(1 + p))$

**Rubi [A]** time = 0.049823, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^p/x, x]

[Out]  $-\left((a + b*x^2)^{(1 + p)} * \text{Hypergeometric2F1}[1, 1 + p, 2 + p, 1 + (b*x^2)/a]\right) / (2*a*(1 + p))$

**Rubi in Sympy [A]** time = 6.06637, size = 31, normalized size = 0.76

$$-\frac{(a+bx^2)^{p+1} {}_2F_1\left(1, p+1 \middle| 1 + \frac{bx^2}{a}\right)}{2a(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*p/x, x)

[Out]  $-(a + b*x**2)**(p + 1) * \text{hyper}((1, p + 1), (p + 2, ), 1 + b*x**2/a) / (2*a*(p + 1))$

**Mathematica [A]** time = 0.0249347, size = 51, normalized size = 1.24

$$\frac{\left(\frac{a}{bx^2} + 1\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, -p; 1-p; -\frac{a}{bx^2}\right)}{2p}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^p/x, x]

[Out]  $\left((a + b*x^2)^p * \text{Hypergeometric2F1}[-p, -p, 1 - p, -(a/(b*x^2))]\right) / (2*p*(1 + a/(b*x^2))^p)$

**Maple [F]** time = 0.036, size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^p/x, x)`

[Out] `int((b*x^2+a)^p/x, x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x, x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p/x, x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x, x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p/x, x)`

**Sympy** [A] time = 11.0402, size = 39, normalized size = 0.95

$$-\frac{b^p x^{2p} (-p) {}_2F_1\left(-p, -p \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2(-p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**p/x, x)`

[Out] `-b**p*x**(2*p)*gamma(-p)*hyper((-p, -p), (-p + 1, ), a*exp_polar(I*pi)/(b*x**2))/(2*gamma(-p + 1))`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x, x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p/x, x)`

$$3.1044 \quad \int \frac{(a+bx^2)^p}{x^3} dx$$

**Optimal.** Leaf size=42

$$\frac{b(a+bx^2)^{p+1} {}_2F_1\left(2, p+1; p+2; \frac{bx^2}{a} + 1\right)}{2a^2(p+1)}$$

[Out] (b\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b\*x^2)/a])/(2\*a^2\*(1 + p))

**Rubi [A]** time = 0.0517038, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{b(a+bx^2)^{p+1} {}_2F_1\left(2, p+1; p+2; \frac{bx^2}{a} + 1\right)}{2a^2(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^p/x^3, x]

[Out] (b\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b\*x^2)/a])/(2\*a^2\*(1 + p))

**Rubi in Sympy [A]** time = 6.54817, size = 32, normalized size = 0.76

$$\frac{b(a+bx^2)^{p+1} {}_2F_1\left(2, p+1 \middle| 1 + \frac{bx^2}{a}\right)}{2a^2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*p/x\*\*3, x)

[Out] b\*(a + b\*x\*\*2)\*\*(p + 1)\*hyper((2, p + 1), (p + 2, ), 1 + b\*x\*\*2/a)/(2\*a\*\*2\*(p + 1))

**Mathematica [A]** time = 0.0276001, size = 58, normalized size = 1.38

$$\frac{\left(\frac{a}{bx^2} + 1\right)^{-p} (a+bx^2)^p {}_2F_1\left(1-p, -p; 2-p; -\frac{a}{bx^2}\right)}{2(p-1)x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^p/x^3, x]

[Out] ((a + b\*x^2)^p\*Hypergeometric2F1[1 - p, -p, 2 - p, -(a/(b\*x^2))])/(2\*(-1 + p)\*(1 + a/(b\*x^2))^p\*x^2)

**Maple [F]** time = 0.048, size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^p/x^3,x)`

[Out] `int((b*x^2+a)^p/x^3,x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x^3,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p/x^3, x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x^3,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p/x^3, x)`

**Sympy** [A] time = 24.7464, size = 42, normalized size = 1.

$$\frac{b^p x^{2p} (-p + 1) {}_2F_1\left(-p, -p + 1 \mid \frac{ae^{i\pi}}{bx^2}\right)}{2x^2 (-p + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**p/x**3,x)`

[Out] `-b**p*x**(2*p)*gamma(-p + 1)*hyper((-p, -p + 1), (-p + 2, ), a*exp_polar(I*pi)/(b*x**2))/(2*x**2*gamma(-p + 2))`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x^3,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p/x^3, x)`

### 3.1045 $\int x^6 (a + bx^2)^p dx$

**Optimal.** Leaf size=40

$$\frac{x^7 (a + bx^2)^{p+1} {}_2F_1\left(1, p + \frac{9}{2}; \frac{9}{2}; -\frac{bx^2}{a}\right)}{7a}$$

[Out]  $(x^{7*(a + b*x^2)^{(1 + p)*Hypergeometric2F1[1, 9/2 + p, 9/2, -((b*x^2)/a)]})/(7*a)$

**Rubi [A]** time = 0.0413751, antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{1}{7}x^7 (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{7}{2}, -p; \frac{9}{2}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[x^6\*(a + b\*x^2)^p, x]

[Out]  $(x^{7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)]})/(7*(1 + (b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 7.94813, size = 37, normalized size = 0.92

$$\frac{x^7 \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, \frac{7}{2}; \frac{9}{2}; -\frac{bx^2}{a}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $x^{7*(1 + b*x^2/a)**(-p)*(a + b*x^2)**p*hyper((-p, 7/2), (9/2, ), -b*x^2/a)/7$

**Mathematica [A]** time = 0.0290004, size = 49, normalized size = 1.22

$$\frac{1}{7}x^7 (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{7}{2}, -p; \frac{9}{2}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^6\*(a + b\*x^2)^p, x]

[Out]  $(x^{7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)]})/(7*(1 + (b*x^2)/a)^p)$

**Maple [F]** time = 0.076, size = 0, normalized size = 0.

$$\int x^6 (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*(b*x^2+a)^p,x)`

[Out] `int(x^6*(b*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^6,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^6, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^p x^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^6,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^6, x)`

**Sympy [A]** time = 59.0101, size = 26, normalized size = 0.65

$$\frac{a^p x^7 {}_2F_1\left(\frac{7}{2}, -p \mid \frac{bx^2 e^{i\pi}}{a}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6*(b*x**2+a)**p,x)`

[Out] `a**p*x**7*hyper((7/2, -p), (9/2, ), b*x**2*exp_polar(I*pi)/a)/7`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^6,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^6, x)`

### 3.1046 $\int x^4 (a + bx^2)^p dx$

**Optimal.** Leaf size=40

$$\frac{x^5 (a + bx^2)^{p+1} {}_2F_1\left(1, p + \frac{7}{2}; \frac{7}{2}; -\frac{bx^2}{a}\right)}{5a}$$

[Out]  $(x^5*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 7/2 + p, 7/2, -(b*x^2)/a])/(5*a)$

**Rubi [A]** time = 0.0403486, antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{1}{5}x^5 (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a + b\*x^2)^p, x]

[Out]  $(x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -(b*x^2)/a])/(5*(1 + (b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 7.88686, size = 37, normalized size = 0.92

$$\frac{x^5 \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, \frac{5}{2}; \frac{7}{2}; -\frac{bx^2}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $x**5*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*hyper((-p, 5/2), (7/2, ), -b*x**2/a)/5$

**Mathematica [A]** time = 0.021955, size = 49, normalized size = 1.22

$$\frac{1}{5}x^5 (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a + b\*x^2)^p, x]

[Out]  $(x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -(b*x^2)/a])/(5*(1 + (b*x^2)/a)^p)$

**Maple [F]** time = 0.051, size = 0, normalized size = 0.

$$\int x^4 (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(x^4*(b*x^2+a)^p,x)`

[Out] `int(x^4*(b*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^4,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^4, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^p x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^4,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^4, x)`

**Sympy [A]** time = 29.7026, size = 26, normalized size = 0.65

$$\frac{a^p x^5 {}_2F_1\left(\frac{5}{2}, -p \mid \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(b*x**2+a)**p,x)`

[Out] `a**p*x**5*hyper((5/2, -p), (7/2, ), b*x**2*exp_polar(I*pi)/a)/5`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^4,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^4, x)`

### 3.1047 $\int x^2 (a + bx^2)^p dx$

**Optimal.** Leaf size=40

$$\frac{x^3 (a + bx^2)^{p+1} {}_2F_1\left(1, p + \frac{5}{2}; \frac{5}{2}; -\frac{bx^2}{a}\right)}{3a}$$

[Out]  $(x^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 5/2 + p, 5/2, -(b*x^2)/a])/(3*a)$

**Rubi [A]** time = 0.0407342, antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{1}{3}x^3 (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*x^2)^p, x]

[Out]  $(x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -(b*x^2)/a])/(3*(1 + (b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 8.28579, size = 37, normalized size = 0.92

$$\frac{x^3 \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, \frac{3}{2}; \frac{5}{2}; -\frac{bx^2}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $x**3*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*hyper((-p, 3/2), (5/2, ), -b*x**2/a)/3$

**Mathematica [A]** time = 0.0191014, size = 49, normalized size = 1.22

$$\frac{1}{3}x^3 (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*x^2)^p, x]

[Out]  $(x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -(b*x^2)/a])/(3*(1 + (b*x^2)/a)^p)$

**Maple [F]** time = 0.045, size = 0, normalized size = 0.

$$\int x^2 (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^2+a)^p,x)`

[Out] `int(x^2*(b*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^2,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^2,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^2, x)`

**Sympy [A]** time = 15.692, size = 26, normalized size = 0.65

$$\frac{a^p x^3 {}_2F_1\left(\frac{3}{2}, -p \mid \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**2+a)**p,x)`

[Out] `a**p*x**3*hyper((3/2, -p), (5/2, ), b*x**2*exp_polar(I*pi)/a)/3`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^2,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^2, x)`

### 3.1048 $\int (a + bx^2)^p dx$

**Optimal.** Leaf size=35

$$\frac{x (a + bx^2)^{p+1} {}_2F_1\left(1, p + \frac{3}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right)}{a}$$

[Out] (x\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 3/2 + p, 3/2, -(b\*x^2/a)])/a

**Rubi [A]** time = 0.0249465, antiderivative size = 44, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$x (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^p, x]

[Out] (x\*(a + b\*x^2)^p\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2/a)])/(1 + (b\*x^2)/a)^p

**Rubi in Sympy [A]** time = 4.75008, size = 34, normalized size = 0.97

$$x \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, \frac{1}{2} \middle| \frac{3}{2}; -\frac{bx^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*p, x)

[Out] x\*(1 + b\*x\*\*2/a)\*\*(-p)\*(a + b\*x\*\*2)\*\*p\*hyper((-p, 1/2), (3/2, ), -b\*x\*\*2/a)

**Mathematica [A]** time = 0.0142636, size = 44, normalized size = 1.26

$$x (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^p, x]

[Out] (x\*(a + b\*x^2)^p\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2/a)])/(1 + (b\*x^2)/a)^p

**Maple [F]** time = 0.033, size = 0, normalized size = 0.

$$\int (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^p,x)`

[Out] `int((b*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p, x)`

**Sympy [A]** time = 8.78634, size = 22, normalized size = 0.63

$$a^p x {}_2F_1\left(\frac{1}{2}, -p \mid \frac{bx^2 e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**p,x)`

[Out] `a**p*x*hyper((1/2, -p), (3/2, ), b*x**2*exp_polar(I*pi)/a)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p, x)`

$$3.1049 \quad \int \frac{(a+bx^2)^p}{x^2} dx$$

**Optimal.** Leaf size=38

$$-\frac{(a+bx^2)^{p+1} {}_2F_1\left(1, p+\frac{1}{2}; \frac{1}{2}; -\frac{bx^2}{a}\right)}{ax}$$

[Out] -(((a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1/2 + p, 1/2, -(b\*x^2)/a]))/(a\*x)

**Rubi [A]** time = 0.0381746, antiderivative size = 47, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; -\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^p/x^2, x]

[Out] -(((a + b\*x^2)^p\*Hypergeometric2F1[-1/2, -p, 1/2, -(b\*x^2)/a]))/(x\*(1 + (b\*x^2)/a)^p)

**Rubi in Sympy [A]** time = 7.84178, size = 37, normalized size = 0.97

$$-\frac{\left(1 + \frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, -\frac{1}{2}; \frac{1}{2}; -\frac{bx^2}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*p/x\*\*2, x)

[Out] -(1 + b\*x\*\*2/a)\*\*(-p)\*(a + b\*x\*\*2)\*\*p\*hyper((-p, -1/2), (1/2, ), -b\*x\*\*2/a)/x

**Mathematica [A]** time = 0.0173732, size = 47, normalized size = 1.24

$$-\frac{(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; -\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^p/x^2, x]

[Out] -(((a + b\*x^2)^p\*Hypergeometric2F1[-1/2, -p, 1/2, -(b\*x^2)/a]))/(x\*(1 + (b\*x^2)/a)^p)

**Maple [F]** time = 0.045, size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^p/x^2,x)`

[Out] `int((b*x^2+a)^p/x^2,x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x^2,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p/x^2, x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x^2,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p/x^2, x)`

**Sympy** [A] time = 16.7445, size = 26, normalized size = 0.68

$$\frac{a^p {}_2F_1\left(-\frac{1}{2}, -p \mid \frac{bx^2 e^{i\pi}}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**p/x**2,x)`

[Out] `-a**p*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x^2,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p/x^2, x)`

### 3.1050 $\int x^{7/2} (a + bx^2)^p dx$

**Optimal.** Leaf size=42

$$\frac{2x^{9/2} (a + bx^2)^{p+1} {}_2F_1\left(1, p + \frac{13}{4}; \frac{13}{4}; -\frac{bx^2}{a}\right)}{9a}$$

[Out]  $(2*x^{9/2}*(a + b*x^2)^{(1 + p)}*Hypergeometric2F1[1, 13/4 + p, 13/4, -(b*x^2)/a])/ (9*a)$

**Rubi [A]** time = 0.037775, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{2}{9}x^{9/2} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{9}{4}, -p; \frac{13}{4}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[x^(7/2)\*(a + b\*x^2)^p, x]

[Out]  $(2*x^{9/2}*(a + b*x^2)^p*Hypergeometric2F1[9/4, -p, 13/4, -(b*x^2)/a])/ (9*(1 + (b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 8.04621, size = 41, normalized size = 0.98

$$\frac{2x^{\frac{9}{2}} \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, \frac{9}{4}; \frac{13}{4}; -\frac{bx^2}{a}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(7/2)\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $2*x^{9/2}*(1 + b*x^2/a)**(-p)*(a + b*x^2)**p*hyper((-p, 9/4), (13/4, ), -b*x^2/a)/9$

**Mathematica [A]** time = 0.0249318, size = 51, normalized size = 1.21

$$\frac{2}{9}x^{9/2} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{9}{4}, -p; \frac{13}{4}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(7/2)\*(a + b\*x^2)^p, x]

[Out]  $(2*x^{9/2}*(a + b*x^2)^p*Hypergeometric2F1[9/4, -p, 13/4, -(b*x^2)/a])/ (9*(1 + (b*x^2)/a)^p)$

**Maple [F]** time = 0.026, size = 0, normalized size = 0.

$$\int x^{\frac{7}{2}} (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(x^(7/2)*(b*x^2+a)^p,x)`

[Out] `int(x^(7/2)*(b*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(7/2),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^(7/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^p x^{\frac{7}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(7/2),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^(7/2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(7/2)*(b*x**2+a)**p,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(7/2),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^(7/2), x)`

### 3.1051 $\int x^{5/2} (a + bx^2)^p dx$

**Optimal.** Leaf size=42

$$\frac{2x^{7/2} (a + bx^2)^{p+1} {}_2F_1\left(1, p + \frac{11}{4}; \frac{11}{4}; -\frac{bx^2}{a}\right)}{7a}$$

[Out]  $(2*x^{(7/2)}*(a + b*x^2)^(1 + p)*\text{Hypergeometric2F1}[1, 11/4 + p, 11/4, -(b*x^2)/a])/ (7*a)$

**Rubi [A]** time = 0.0386261, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{2}{7}x^{7/2} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{7}{4}, -p; \frac{11}{4}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)\*(a + b\*x^2)^p, x]

[Out]  $(2*x^{(7/2)}*(a + b*x^2)^p*\text{Hypergeometric2F1}[7/4, -p, 11/4, -(b*x^2)/a])/ (7*(1 + (b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 7.73296, size = 41, normalized size = 0.98

$$\frac{2x^{\frac{7}{2}} \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, \frac{7}{4}; \frac{11}{4}; -\frac{bx^2}{a}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(5/2)\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $2*x^{(7/2)}*(1 + b*x^2/a)**(-p)*(a + b*x^2)**p*\text{hyper}((-p, 7/4), (11/4, ), -b*x^2/a)/7$

**Mathematica [A]** time = 0.0228433, size = 51, normalized size = 1.21

$$\frac{2}{7}x^{7/2} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{7}{4}, -p; \frac{11}{4}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2)\*(a + b\*x^2)^p, x]

[Out]  $(2*x^{(7/2)}*(a + b*x^2)^p*\text{Hypergeometric2F1}[7/4, -p, 11/4, -(b*x^2)/a])/ (7*(1 + (b*x^2)/a)^p)$

**Maple [F]** time = 0.027, size = 0, normalized size = 0.

$$\int x^{\frac{5}{2}} (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(5/2)*(b*x^2+a)^p,x)`

[Out] `int(x^(5/2)*(b*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(5/2),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^(5/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + a\right)^p x^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(5/2),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^(5/2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(5/2)*(b*x**2+a)**p,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(5/2),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^(5/2), x)`

### 3.1052 $\int x^{3/2} (a + bx^2)^p dx$

**Optimal.** Leaf size=42

$$\frac{2x^{5/2} (a + bx^2)^{p+1} {}_2F_1\left(1, p + \frac{9}{4}; \frac{9}{4}; -\frac{bx^2}{a}\right)}{5a}$$

[Out]  $(2*x^{5/2}*(a + b*x^2)^{(1 + p)}*Hypergeometric2F1[1, 9/4 + p, 9/4, -(b*x^2)/a])/(5*a)$

**Rubi [A]** time = 0.037469, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{2}{5}x^{5/2} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{5}{4}, -p; \frac{9}{4}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)\*(a + b\*x^2)^p, x]

[Out]  $(2*x^{5/2}*(a + b*x^2)^p*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^2)/a])/(5*(1 + (b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 7.83742, size = 41, normalized size = 0.98

$$\frac{2x^{\frac{5}{2}} \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, \frac{5}{4}; \frac{9}{4}; -\frac{bx^2}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(3/2)\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $2*x^{5/2}*(1 + b*x^2/a)**(-p)*(a + b*x^2)**p*hyper((-p, 5/4), (9/4, ), -b*x^2/a)/5$

**Mathematica [A]** time = 0.0231924, size = 51, normalized size = 1.21

$$\frac{2}{5}x^{5/2} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{5}{4}, -p; \frac{9}{4}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)\*(a + b\*x^2)^p, x]

[Out]  $(2*x^{5/2}*(a + b*x^2)^p*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^2)/a])/(5*(1 + (b*x^2)/a)^p)$

**Maple [F]** time = 0.026, size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}} (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3/2)*(b*x^2+a)^p,x)`

[Out] `int(x^(3/2)*(b*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^(3/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^p x^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(3/2),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^(3/2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(3/2)*(b*x**2+a)**p,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(3/2),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^(3/2), x)`

### 3.1053 $\int \sqrt{x} (a + bx^2)^p dx$

**Optimal.** Leaf size=42

$$\frac{2x^{3/2} (a + bx^2)^{p+1} {}_2F_1\left(1, p + \frac{7}{4}; \frac{7}{4}; -\frac{bx^2}{a}\right)}{3a}$$

[Out]  $(2*x^{3/2}*(a + b*x^2)^{(1 + p)}*Hypergeometric2F1[1, 7/4 + p, 7/4, -(b*x^2)/a])/(3*a)$

**Rubi [A]** time = 0.0358842, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{2}{3}x^{3/2} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]\*(a + b\*x^2)^p, x]

[Out]  $(2*x^{3/2}*(a + b*x^2)^p*Hypergeometric2F1[3/4, -p, 7/4, -(b*x^2)/a])/(3*(1 + (b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 7.67244, size = 41, normalized size = 0.98

$$\frac{2x^{\frac{3}{2}} \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, \frac{3}{4}; \frac{7}{4}; -\frac{bx^2}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(1/2)\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $2*x^{3/2}*(1 + b*x^2/a)**(-p)*(a + b*x^2)**p*hyper((-p, 3/4), (7/4, ), -b*x^2/a)/3$

**Mathematica [A]** time = 0.0211915, size = 51, normalized size = 1.21

$$\frac{2}{3}x^{3/2} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{4}, -p; \frac{7}{4}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]\*(a + b\*x^2)^p, x]

[Out]  $(2*x^{3/2}*(a + b*x^2)^p*Hypergeometric2F1[3/4, -p, 7/4, -(b*x^2)/a])/(3*(1 + (b*x^2)/a)^p)$

**Maple [F]** time = 0.027, size = 0, normalized size = 0.

$$\int \sqrt{x} (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*(b*x^2+a)^p,x)`

[Out] `int(x^(1/2)*(b*x^2+a)^p,x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p \sqrt{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*sqrt(x),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*sqrt(x), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^p \sqrt{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*sqrt(x),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*sqrt(x), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1/2)*(b*x**2+a)**p,x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p \sqrt{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*sqrt(x),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*sqrt(x), x)`

$$3.1054 \quad \int \frac{(a+bx^2)^p}{\sqrt{x}} dx$$

**Optimal.** Leaf size=40

$$\frac{2\sqrt{x} (a + bx^2)^{p+1} {}_2F_1\left(1, p + \frac{5}{4}; \frac{5}{4}; -\frac{bx^2}{a}\right)}{a}$$

[Out] (2\*Sqrt[x]\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 5/4 + p, 5/4, -(b\*x^2)/a])/a

**Rubi [A]** time = 0.0359386, antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$2\sqrt{x} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^p/Sqrt[x], x]

[Out] (2\*Sqrt[x]\*(a + b\*x^2)^p\*Hypergeometric2F1[1/4, -p, 5/4, -(b\*x^2)/a])/(1 + (b\*x^2)/a)^p

**Rubi in Sympy [A]** time = 7.93054, size = 39, normalized size = 0.98

$$2\sqrt{x} \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, \frac{1}{4}; \frac{5}{4}; -\frac{bx^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*p/x\*\*(1/2), x)

[Out] 2\*sqrt(x)\*(1 + b\*x\*\*2/a)\*\*(-p)\*(a + b\*x\*\*2)\*\*p\*hyper((-p, 1/4), (5/4, ), -b\*x\*\*2/a)

**Mathematica [A]** time = 0.0211848, size = 49, normalized size = 1.22

$$2\sqrt{x} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{4}, -p; \frac{5}{4}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^p/Sqrt[x], x]

[Out] (2\*Sqrt[x]\*(a + b\*x^2)^p\*Hypergeometric2F1[1/4, -p, 5/4, -(b\*x^2)/a])/(1 + (b\*x^2)/a)^p

**Maple [F]** time = 0.028, size = 0, normalized size = 0.

$$\int (bx^2 + a)^p \frac{1}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((b*x^2+a)^p/x^(1/2),x)`

[Out] `int((b*x^2+a)^p/x^(1/2),x)`

---

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/sqrt(x),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p/sqrt(x), x)`

---

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p}{\sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/sqrt(x),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p/sqrt(x), x)`

---

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**p/x**(1/2),x)`

[Out] Timed out

---

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/sqrt(x),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p/sqrt(x), x)`

$$3.1055 \quad \int \frac{(a+bx^2)^p}{x^{3/2}} dx$$

**Optimal.** Leaf size=40

$$\frac{2(a+bx^2)^{p+1} {}_2F_1\left(1, p + \frac{3}{4}; \frac{3}{4}; -\frac{bx^2}{a}\right)}{a\sqrt{x}}$$

[Out]  $(-2*(a + b*x^2)^(1 + p)*\text{Hypergeometric2F1}[1, 3/4 + p, 3/4, -((b*x^2)/a)])/(a*\text{Sqrt}[x])$

**Rubi [A]** time = 0.0369526, antiderivative size = 49, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{2(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(-\frac{1}{4}, -p; \frac{3}{4}; -\frac{bx^2}{a}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^p/x^(3/2), x]

[Out]  $(-2*(a + b*x^2)^p*\text{Hypergeometric2F1}[-1/4, -p, 3/4, -((b*x^2)/a)])/(\text{Sqrt}[x]*(1 + (b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 7.70457, size = 42, normalized size = 1.05

$$\frac{2\left(1 + \frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, -\frac{1}{4}; \frac{3}{4}; -\frac{bx^2}{a}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*p/x\*\*(3/2), x)

[Out]  $-2*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*\text{hyper}((-p, -1/4), (3/4, ), -b*x**2/a)/\text{sqrt}(x)$

**Mathematica [A]** time = 0.0251321, size = 49, normalized size = 1.22

$$\frac{2(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(-\frac{1}{4}, -p; \frac{3}{4}; -\frac{bx^2}{a}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^p/x^(3/2), x]

[Out]  $(-2*(a + b*x^2)^p*\text{Hypergeometric2F1}[-1/4, -p, 3/4, -((b*x^2)/a)])/(\text{Sqrt}[x]*(1 + (b*x^2)/a)^p)$

**Maple [F]** time = 0.027, size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^p/x^(3/2),x)`

[Out] `int((b*x^2+a)^p/x^(3/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p/x^(3/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p}{x^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x^(3/2),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p/x^(3/2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**p/x**(3/2),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x^(3/2),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p/x^(3/2), x)`

$$3.1056 \quad \int \frac{(a+bx^2)^p}{x^{5/2}} dx$$

**Optimal.** Leaf size=42

$$\frac{2(a+bx^2)^{p+1} {}_2F_1\left(1, p + \frac{1}{4}; \frac{1}{4}; -\frac{bx^2}{a}\right)}{3ax^{3/2}}$$

[Out]  $(-2*(a + b*x^2)^(1 + p)*\text{Hypergeometric2F1}[1, 1/4 + p, 1/4, -((b*x^2)/a)])/(3*a*x^(3/2))$

**Rubi [A]** time = 0.037119, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{2(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(-\frac{3}{4}, -p; \frac{1}{4}; -\frac{bx^2}{a}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^p/x^(5/2), x]

[Out]  $(-2*(a + b*x^2)^p*\text{Hypergeometric2F1}[-3/4, -p, 1/4, -((b*x^2)/a)])/(3*x^(3/2)*(1 + (b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 7.49264, size = 44, normalized size = 1.05

$$\frac{2\left(1 + \frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, -\frac{3}{4}; \frac{1}{4}; -\frac{bx^2}{a}\right)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*p/x\*\*(5/2), x)

[Out]  $-2*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*\text{hyper}((-p, -3/4), (1/4, ), -b*x**2/a)/(3*x**(3/2))$

**Mathematica [A]** time = 0.0277752, size = 51, normalized size = 1.21

$$\frac{2(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(-\frac{3}{4}, -p; \frac{1}{4}; -\frac{bx^2}{a}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^p/x^(5/2), x]

[Out]  $(-2*(a + b*x^2)^p*\text{Hypergeometric2F1}[-3/4, -p, 1/4, -((b*x^2)/a)])/(3*x^(3/2)*(1 + (b*x^2)/a)^p)$

**Maple [F]** time = 0.027, size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^p/x^(5/2),x)`

[Out] `int((b*x^2+a)^p/x^(5/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x^(5/2),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p/x^(5/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p}{x^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x^(5/2),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p/x^(5/2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**p/x**(5/2),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x^(5/2),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p/x^(5/2), x)`

$$3.1057 \quad \int \frac{(a+bx^2)^p}{x^{7/2}} dx$$

**Optimal.** Leaf size=42

$$\frac{2(a+bx^2)^{p+1} {}_2F_1\left(1, p-\frac{1}{4}; -\frac{1}{4}; -\frac{bx^2}{a}\right)}{5ax^{5/2}}$$

[Out]  $(-2*(a + b*x^2)^(1 + p)*\text{Hypergeometric2F1}[1, -1/4 + p, -1/4, -(b*x^2)/a])/(5*a*x^(5/2))$

**Rubi [A]** time = 0.0370707, antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{2(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(-\frac{5}{4}, -p; -\frac{1}{4}; -\frac{bx^2}{a}\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^p/x^(7/2), x]

[Out]  $(-2*(a + b*x^2)^p*\text{Hypergeometric2F1}[-5/4, -p, -1/4, -(b*x^2)/a])/(5*x^(5/2)*(1 + (b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 8.63025, size = 46, normalized size = 1.1

$$\frac{2\left(1 + \frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, -\frac{5}{4}; -\frac{1}{4}; -\frac{bx^2}{a}\right)}{5x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*p/x\*\*(7/2), x)

[Out]  $-2*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*\text{hyper}((-p, -5/4), (-1/4, ), -b*x**2/a)/(5*x**(5/2))$

**Mathematica [A]** time = 0.0313561, size = 51, normalized size = 1.21

$$\frac{2(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(-\frac{5}{4}, -p; -\frac{1}{4}; -\frac{bx^2}{a}\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^p/x^(7/2), x]

[Out]  $(-2*(a + b*x^2)^p*\text{Hypergeometric2F1}[-5/4, -p, -1/4, -(b*x^2)/a])/(5*x^(5/2)*(1 + (b*x^2)/a)^p)$

**Maple [F]** time = 0.029, size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^p/x^(7/2),x)`

[Out] `int((b*x^2+a)^p/x^(7/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x^(7/2),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p/x^(7/2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p}{x^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x^(7/2),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p/x^(7/2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**p/x**(7/2),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x^(7/2),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p/x^(7/2), x)`

### 3.1058 $\int x^m (a + bx^2)^p dx$

**Optimal.** Leaf size=53

$$\frac{x^{m+1} (a + bx^2)^{p+1} {}_2F_1\left(1, \frac{1}{2}(m + 2p + 3); \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{a(m+1)}$$

[Out]  $(x^{(1+m)}*(a+b*x^2)^{(1+p)}*Hypergeometric2F1[1, (3+m+2*p)/2, (3+m)/2, -((b*x^2)/a)])/(a*(1+m))$

**Rubi [A]** time = 0.0458078, antiderivative size = 61, normalized size of antiderivative = 1.15, number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{x^{m+1} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

[In] Int[x^m\*(a + b\*x^2)^p, x]

[Out]  $(x^{(1+m)}*(a+b*x^2)^p*Hypergeometric2F1[(1+m)/2, -p, (3+m)/2, -((b*x^2)/a)])/((1+m)*(1+(b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 9.21851, size = 48, normalized size = 0.91

$$\frac{x^{m+1} \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, \frac{m}{2} + \frac{1}{2}; \frac{m}{2} + \frac{3}{2}; -\frac{bx^2}{a}\right)}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*m\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $x^{(m+1)}*(1+b*x**2/a)**(-p)*(a+b*x**2)**p*hyper((-p, m/2+1/2), (m/2+3/2, ), -b*x**2/a)/(m+1)$

**Mathematica [A]** time = 0.0487225, size = 63, normalized size = 1.19

$$\frac{x^{m+1} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+1}{2} + 1; -\frac{bx^2}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*(a + b\*x^2)^p, x]

[Out]  $(x^{(1+m)}*(a+b*x^2)^p*Hypergeometric2F1[(1+m)/2, -p, 1+(1+m)/2, -((b*x^2)/a)])/((1+m)*(1+(b*x^2)/a)^p)$

**Maple [F]** time = 0.08, size = 0, normalized size = 0.

$$\int x^m (bx^2 + a)^p dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*(b*x^2+a)^p,x)`

[Out] `int(x^m*(b*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^m,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^m, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^p x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^m,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^m, x)`

**Sympy [A]** time = 111.17, size = 51, normalized size = 0.96

$$\frac{a^p x x^m \left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(-p, \frac{m}{2} + \frac{1}{2} \mid \frac{bx^2 e^{i\pi}}{a}\right)}{2 \left(\frac{m}{2} + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*(b*x**2+a)**p,x)`

[Out] `a**p*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2, ), b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^m,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^m, x)`

### 3.1059 $\int (cx)^m (a + bx^2)^p dx$

**Optimal.** Leaf size=66

$$\frac{(cx)^{m+1} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{c(m+1)}$$

[Out]  $((c*x)^{(1+m)}*(a+b*x^2)^p*Hypergeometric2F1[(1+m)/2, -p, (3+m)/2, -(b*x^2)/a])/((c*(1+m))*(1+(b*x^2)/a)^p)$

**Rubi [A]** time = 0.0507163, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{(cx)^{m+1} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{c(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(c\*x)^m\*(a+b\*x^2)^p,x]

[Out]  $((c*x)^{(1+m)}*(a+b*x^2)^p*Hypergeometric2F1[(1+m)/2, -p, (3+m)/2, -(b*x^2)/a])/((c*(1+m))*(1+(b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 9.44466, size = 51, normalized size = 0.77

$$\frac{(cx)^{m+1} \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, \frac{m}{2} + \frac{1}{2}; \frac{m}{2} + \frac{3}{2}; -\frac{bx^2}{a}\right)}{c(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x)\*\*m\*(b\*x\*\*2+a)\*\*p,x)

[Out]  $(c*x)**(m+1)*(1+b*x**2/a)**(-p)*(a+b*x**2)**p*hyper((-p, m/2+1/2), (m/2+3/2, ), -b*x**2/a)/(c*(m+1))$

**Mathematica [A]** time = 0.0375938, size = 64, normalized size = 0.97

$$\frac{x(cx)^m (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+1}{2} + 1; -\frac{bx^2}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*x)^m\*(a+b\*x^2)^p,x]

[Out]  $(x*(c*x)^m*(a+b*x^2)^p*Hypergeometric2F1[(1+m)/2, -p, 1+(1+m)/2, -(b*x^2)/a])/((1+m)*(1+(b*x^2)/a)^p)$

**Maple [F]** time = 0.099, size = 0, normalized size = 0.

$$\int (cx)^m (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x)^m*(b*x^2+a)^p,x)`

[Out] `int((c*x)^m*(b*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^P (cx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*(c*x)^m,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*(c*x)^m, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^P (cx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*(c*x)^m,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*(c*x)^m, x)`

**Sympy [A]** time = 112.115, size = 54, normalized size = 0.82

$$\frac{a^p c^m x x^m \left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(-p, \frac{m}{2} + \frac{1}{2} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2 \left(\frac{m}{2} + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x)**m*(b*x**2+a)**p,x)`

[Out] `a**p*c**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2, ), b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^P (cx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*(c*x)^m,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*(c*x)^m, x)`

### 3.1060 $\int x^{-8-2p} (a + bx^2)^p dx$

**Optimal.** Leaf size=53

$$\frac{x^{-2p-7} (a + bx^2)^{p+1} {}_2F_1\left(-\frac{5}{2}, 1; \frac{1}{2}(-2p-5); -\frac{bx^2}{a}\right)}{a(2p+7)}$$

[Out]  $-\left((x^{(-7-2*p)}*(a+b*x^2)^{(1+p)}*\text{Hypergeometric2F1}[-5/2, 1, (-5-2*p)/2, -(b*x^2)/a])/(a*(7+2*p))\right)$

**Rubi [A]** time = 0.0530113, antiderivative size = 70, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{x^{-2p-7} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}(-2p-7), -p; \frac{1}{2}(-2p-5); -\frac{bx^2}{a}\right)}{2p+7}$$

Antiderivative was successfully verified.

[In] Int[x^(-8 - 2\*p) \* (a + b\*x^2)^p, x]

[Out]  $-\left((x^{(-7-2*p)}*(a+b*x^2)^p*\text{Hypergeometric2F1}[(-7-2*p)/2, -p, (-5-2*p)/2, -(b*x^2)/a])/(a*(7+2*p)*(1+(b*x^2)/a)^p)\right)$

**Rubi in Sympy [A]** time = 8.67323, size = 54, normalized size = 1.02

$$\frac{x^{-2p-7} \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, -p - \frac{7}{2} \middle| -\frac{bx^2}{a} \right)}{2p+7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(-8-2\*p)\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $-x^{(-2*p-7)}*(1+b*x**2/a)**(-p)*(a+b*x**2)**p*\text{hyper}((-p, -p-7/2), (-p-5/2, ), -b*x**2/a)/(2*p+7)$

**Mathematica [A]** time = 0.0539069, size = 66, normalized size = 1.25

$$\frac{x^{-2p-7} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(-p - \frac{7}{2}, -p; -p - \frac{5}{2}; -\frac{bx^2}{a}\right)}{2p+7}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-8 - 2\*p) \* (a + b\*x^2)^p, x]

[Out]  $-\left((x^{(-7-2*p)}*(a+b*x^2)^p*\text{Hypergeometric2F1}[-7/2 - p, -p, -5/2 - p, -(b*x^2)/a])/(a*(7+2*p)*(1+(b*x^2)/a)^p)\right)$

**Maple [F]** time = 0.083, size = 0, normalized size = 0.

$$\int x^{-8-2p} (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-8-2*p)*(b*x^2+a)^p,x)`

[Out] `int(x^(-8-2*p)*(b*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-2p-8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 8),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^(-2*p - 8), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^p x^{-2p-8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 8),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^(-2*p - 8), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-8-2*p)*(b*x**2+a)**p,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-2p-8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 8),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^(-2*p - 8), x)`

### 3.1061 $\int x^{-7-2p} (a + bx^2)^p dx$

**Optimal.** Leaf size=105

$$-\frac{b^2 x^{-2(p+1)} (a + bx^2)^{p+1}}{a^3 (p+1)(p+2)(p+3)} + \frac{bx^{-2(p+2)} (a + bx^2)^{p+1}}{a^2 (p+2)(p+3)} - \frac{x^{-2(p+3)} (a + bx^2)^{p+1}}{2a(p+3)}$$

[Out]  $-\left(\frac{b^2 x^{2(1+p)} (a + bx^2)^{p+1}}{a^3 (p+1)(p+2)(p+3)} + \frac{bx^{2(2+p)} (a + bx^2)^{p+1}}{a^2 (p+2)(p+3)} - \frac{x^{2(3+p)} (a + bx^2)^{p+1}}{2a(p+3)}\right) + (b^2 x^{2(1+p)} (a + bx^2)^{p+1}) / (a^3 (1+p)^2 (2+p)^2 (3+p)^2 x^{2(1+p)}) - (bx^{2(2+p)} (a + bx^2)^{p+1}) / (a^2 (2+p)^2 (3+p)^2 x^{2(2+p)}) - (x^{2(3+p)} (a + bx^2)^{p+1}) / (2a^3 (3+p)^2 x^{2(3+p)})$

**Rubi [A]** time = 0.132568, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$-\frac{b^2 x^{-2(p+1)} (a + bx^2)^{p+1}}{a^3 (p+1)(p+2)(p+3)} + \frac{bx^{-2(p+2)} (a + bx^2)^{p+1}}{a^2 (p+2)(p+3)} - \frac{x^{-2(p+3)} (a + bx^2)^{p+1}}{2a(p+3)}$$

Antiderivative was successfully verified.

[In] Int[x^(-7 - 2\*p) \* (a + b\*x^2)^p, x]

[Out]  $-\left(\frac{b^2 x^{2(1+p)} (a + bx^2)^{p+1}}{a^3 (p+1)(p+2)(p+3)} + \frac{bx^{2(2+p)} (a + bx^2)^{p+1}}{a^2 (p+2)(p+3)} - \frac{x^{2(3+p)} (a + bx^2)^{p+1}}{2a(p+3)}\right) + (b^2 x^{2(1+p)} (a + bx^2)^{p+1}) / (a^3 (1+p)^2 (2+p)^2 (3+p)^2 x^{2(1+p)}) - (bx^{2(2+p)} (a + bx^2)^{p+1}) / (a^2 (2+p)^2 (3+p)^2 x^{2(2+p)}) - (x^{2(3+p)} (a + bx^2)^{p+1}) / (2a^3 (3+p)^2 x^{2(3+p)})$

**Rubi in Sympy [A]** time = 17.5474, size = 90, normalized size = 0.86

$$-\frac{x^{-2p-6} (a + bx^2)^{p+1}}{2a(p+3)} + \frac{bx^{-2p-4} (a + bx^2)^{p+1}}{a^2 (p+2)(p+3)} - \frac{b^2 x^{-2p-2} (a + bx^2)^{p+1}}{a^3 (p+1)(p+2)(p+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(-7-2\*p) \* (b\*x\*\*2+a)\*\*p, x)

[Out]  $-x^{-(2*p+6)} (a + bx^2)^{p+1} / (2a^3 (p+3)) + bx^{-(2*p+4)} (a + bx^2)^{p+1} / (a^2 (p+2)(p+3)) - b^2 x^{-(2*p+2)} (a + bx^2)^{p+1} / (a^3 (p+1)(p+2)(p+3))$

**Mathematica [A]** time = 0.0920518, size = 71, normalized size = 0.68

$$\frac{x^{-2(p+3)} (a + bx^2)^{p+1} (a^2 (p^2 + 3p + 2) - 2ab(p+1)x^2 + 2b^2 x^4)}{2a^3 (p+1)(p+2)(p+3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-7 - 2\*p) \* (a + b\*x^2)^p, x]

[Out]  $-\frac{(a + bx^2)^{1+p} (a^2 (2 + 3p + p^2) - 2ab(1+p)x^2 + 2b^2 x^4)}{2a^3 (1+p)^2 (2+p)^2 (3+p)^2 x^{2(3+p)}}$

**Maple [A]** time = 0.01, size = 81, normalized size = 0.8

$$\frac{(bx^2 + a)^{1+p} x^{-6-2p} (2b^2 x^4 - 2abpx^2 + a^2 p^2 - 2abx^2 + 3a^2 p + 2a^2)}{(6 + 2p)(2 + p)(1 + p)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-7-2*p)*(b*x^2+a)^p,x)`

[Out] 
$$-1/2*(b*x^2+a)^(1+p)*x^(-6-2*p)*(2*b^2*x^4-2*a*b*p*x^2+a^2*p^2-2*a*b*x^2+3*a^2*p+2*a^2)/(3+p)/(2+p)/(1+p)/a^3$$

**Maxima [A]** time = 1.34294, size = 113, normalized size = 1.08

$$\frac{(2b^3x^6 - 2ab^2px^4 + (p^2 + p)a^2bx^2 + (p^2 + 3p + 2)a^3)e^{(p \log(bx^2+a) - 2p \log(x))}}{2(p^3 + 6p^2 + 11p + 6)a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 7),x, algorithm="maxima")`

[Out] 
$$-1/2*(2*b^3*x^6 - 2*a*b^2*p*x^4 + (p^2 + p)*a^2*b*x^2 + (p^2 + 3*p + 2)*a^3)*e^{(p*\log(b*x^2 + a) - 2*p*\log(x))}/((p^3 + 6*p^2 + 11*p + 6)*a^3*x^6)$$

**Fricas [A]** time = 0.227773, size = 143, normalized size = 1.36

$$\frac{(2b^3x^7 - 2ab^2px^5 + (a^2bp^2 + a^2bp)x^3 + (a^3p^2 + 3a^3p + 2a^3)x)(bx^2 + a)^p x^{-2p-7}}{2(a^3p^3 + 6a^3p^2 + 11a^3p + 6a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 7),x, algorithm="fricas")`

[Out] 
$$-1/2*(2*b^3*x^7 - 2*a*b^2*p*x^5 + (a^2*b*p^2 + a^2*b*p)*x^3 + (a^3*p^2 + 3*a^3*p + 2*a^3)*x)*(b*x^2 + a)^p*x^(-2*p - 7)/(a^3*p^3 + 6*a^3*p^2 + 11*a^3*p + 6*a^3)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-7-2*p)*(b*x**2+a)**p,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-2p-7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 7),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^(-2*p - 7), x)`

### 3.1062 $\int x^{-6-2p} (a + bx^2)^p dx$

**Optimal.** Leaf size=53

$$\frac{x^{-2p-5} (a + bx^2)^{p+1} {}_2F_1\left(-\frac{3}{2}, 1; \frac{1}{2}(-2p-3); -\frac{bx^2}{a}\right)}{a(2p+5)}$$

[Out]  $-\left(\left(x^{(-5-2*p)}\right)\left(a+b*x^2\right)^{(1+p)}\text{Hypergeometric2F1}\left[-\frac{3}{2}, 1, \left(-\frac{3-2*p}{2}, -\left(\frac{b*x^2}{a}\right)\right)\right]\right)/\left(a*(5+2*p)\right)$

**Rubi [A]** time = 0.0541517, antiderivative size = 70, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{x^{-2p-5} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}(-2p-5), -p; \frac{1}{2}(-2p-3); -\frac{bx^2}{a}\right)}{2p+5}$$

Antiderivative was successfully verified.

[In] Int[x^(-6 - 2\*p) \* (a + b\*x^2)^p, x]

[Out]  $-\left(\left(x^{(-5-2*p)}\right)\left(a+b*x^2\right)^p\text{Hypergeometric2F1}\left[\frac{(-5-2*p)}{2}, -p, \left(-\frac{3-2*p}{2}, -\left(\frac{b*x^2}{a}\right)\right)\right]\right)/\left((5+2*p)*\left(1+\left(\frac{b*x^2}{a}\right)^p\right)\right)$

**Rubi in Sympy [A]** time = 9.21745, size = 54, normalized size = 1.02

$$\frac{x^{-2p-5} \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, -p - \frac{5}{2} \middle| -\frac{bx^2}{a} \right)}{2p+5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(-6-2\*p) \* (b\*x\*\*2+a)\*\*p, x)

[Out]  $-x^{(-2*p-5)}\left(1+b*x^2/a\right)^{-p}\left(a+b*x^2\right)^p\text{hyper}\left(\left(-p, -p-\frac{5}{2}\right), \left(-p-\frac{3}{2},\right), -b*x^2/a\right)/\left(2*p+5\right)$

**Mathematica [A]** time = 0.0567384, size = 66, normalized size = 1.25

$$\frac{x^{-2p-5} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(-p - \frac{5}{2}, -p; -p - \frac{3}{2}; -\frac{bx^2}{a}\right)}{2p+5}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-6 - 2\*p) \* (a + b\*x^2)^p, x]

[Out]  $-\left(\left(x^{(-5-2*p)}\right)\left(a+b*x^2\right)^p\text{Hypergeometric2F1}\left[-\frac{5}{2}-p, -p, -\frac{3}{2}-p, -\left(\frac{b*x^2}{a}\right)\right]\right)/\left((5+2*p)*\left(1+\left(\frac{b*x^2}{a}\right)^p\right)\right)$

**Maple [F]** time = 0.084, size = 0, normalized size = 0.

$$\int x^{-6-2p} (bx^2 + a)^p dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-6-2*p)*(b*x^2+a)^p,x)`

[Out] `int(x^(-6-2*p)*(b*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-2p-6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 6),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^(-2*p - 6), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^p x^{-2p-6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 6),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^(-2*p - 6), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-6-2*p)*(b*x**2+a)**p,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-2p-6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 6),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^(-2*p - 6), x)`

### 3.1063 $\int x^{-5-2p} (a + bx^2)^p dx$

**Optimal.** Leaf size=67

$$\frac{bx^{-2(p+1)}(a+bx^2)^{p+1}}{2a^2(p+1)(p+2)} - \frac{x^{-2(p+2)}(a+bx^2)^{p+1}}{2a(p+2)}$$

[Out]  $(b*(a + b*x^2)^(1 + p))/(2*a^2*(1 + p)*(2 + p)*x^(2*(1 + p))) - (a + b*x^2)^(1 + p)/(2*a*(2 + p)*x^(2*(2 + p)))$

**Rubi [A]** time = 0.0587591, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{bx^{-2(p+1)}(a+bx^2)^{p+1}}{2a^2(p+1)(p+2)} - \frac{x^{-2(p+2)}(a+bx^2)^{p+1}}{2a(p+2)}$$

Antiderivative was successfully verified.

[In] Int[x^(-5 - 2\*p)\*(a + b\*x^2)^p, x]

[Out]  $(b*(a + b*x^2)^(1 + p))/(2*a^2*(1 + p)*(2 + p)*x^(2*(1 + p))) - (a + b*x^2)^(1 + p)/(2*a*(2 + p)*x^(2*(2 + p)))$

**Rubi in Sympy [A]** time = 8.7767, size = 56, normalized size = 0.84

$$-\frac{x^{-2p-4}(a+bx^2)^{p+1}}{2a(p+2)} + \frac{bx^{-2p-2}(a+bx^2)^{p+1}}{2a^2(p+1)(p+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(-5-2\*p)\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $-x**(-2*p - 4)*(a + b*x**2)**(p + 1)/(2*a*(p + 2)) + b*x**(-2*p - 2)*(a + b*x**2)**(p + 1)/(2*a**2*(p + 1)*(p + 2))$

**Mathematica [A]** time = 0.0625106, size = 46, normalized size = 0.69

$$-\frac{x^{-2(p+2)}(ap+a-bx^2)(a+bx^2)^{p+1}}{2a^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-5 - 2\*p)\*(a + b\*x^2)^p, x]

[Out]  $-((a + a*p - b*x^2)*(a + b*x^2)^(1 + p))/(2*a^2*(1 + p)*(2 + p)*x^(2*(2 + p)))$

**Maple [A]** time = 0.006, size = 45, normalized size = 0.7

$$-\frac{(bx^2 + a)^{1+p} x^{-4-2p} (-bx^2 + ap + a)}{(4 + 2p)(1 + p)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-5-2*p)*(b*x^2+a)^p,x)`

[Out]  $-1/2*(b*x^2+a)^(1+p)*x^(-4-2*p)*(-b*x^2+a*p+a)/(2+p)/(1+p)/a^2$

**Maxima [A]** time = 1.37147, size = 80, normalized size = 1.19

$$\frac{(b^2x^4 - abpx^2 - a^2(p+1))e^{(p \log(bx^2+a) - 2p \log(x))}}{2(p^2 + 3p + 2)a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 5),x, algorithm="maxima")`

[Out]  $1/2*(b^2*x^4 - a*b*p*x^2 - a^2*(p+1))*e^{(p*\log(b*x^2 + a) - 2*p*\log(x))}/((p^2 + 3*p + 2)*a^2*x^4)$

**Fricas [A]** time = 0.228229, size = 90, normalized size = 1.34

$$\frac{(b^2x^5 - abpx^3 - (a^2p + a^2)x)(bx^2 + a)^p x^{-2p-5}}{2(a^2p^2 + 3a^2p + 2a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 5),x, algorithm="fricas")`

[Out]  $1/2*(b^2*x^5 - a*b*p*x^3 - (a^2*p + a^2)*x)*(b*x^2 + a)^p*x^(-2*p - 5)/(a^2*p^2 + 3*a^2*p + 2*a^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-5-2*p)*(b*x**2+a)**p,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-2p-5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 5),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^(-2*p - 5), x)`

### 3.1064 $\int x^{-4-2p} (a + bx^2)^p dx$

**Optimal.** Leaf size=53

$$\frac{x^{-2p-3} (a + bx^2)^{p+1} {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}(-2p-1); -\frac{bx^2}{a}\right)}{a(2p+3)}$$

[Out]  $-\left((x^{(-3-2*p)}*(a+b*x^2)^{(1+p)}*\text{Hypergeometric2F1}[-1/2, 1, (-1-2*p)/2, -(b*x^2)/a])/(a*(3+2*p))\right)$

**Rubi [A]** time = 0.0540685, antiderivative size = 70, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{x^{-2p-3} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}(-2p-3), -p; \frac{1}{2}(-2p-1); -\frac{bx^2}{a}\right)}{2p+3}$$

Antiderivative was successfully verified.

[In] Int[x^(-4 - 2\*p) \* (a + b\*x^2)^p, x]

[Out]  $-\left((x^{(-3-2*p)}*(a+b*x^2)^p*\text{Hypergeometric2F1}[(-3-2*p)/2, -p, (-1-2*p)/2, -(b*x^2)/a])/(a*(3+2*p)*(1+(b*x^2)/a)^p)\right)$

**Rubi in Sympy [A]** time = 8.58093, size = 54, normalized size = 1.02

$$\frac{x^{-2p-3} \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, -p - \frac{3}{2} \middle| -\frac{bx^2}{a} \right)}{2p+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(-4-2\*p)\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $-x^{(-2*p-3)}*(1+b*x**2/a)**(-p)*(a+b*x**2)**p*\text{hyper}((-p, -p-3/2), (-p-1/2), -b*x**2/a)/(2*p+3)$

**Mathematica [A]** time = 0.0546944, size = 66, normalized size = 1.25

$$\frac{x^{-2p-3} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(-p - \frac{3}{2}, -p; -p - \frac{1}{2}; -\frac{bx^2}{a}\right)}{2p+3}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-4 - 2\*p) \* (a + b\*x^2)^p, x]

[Out]  $-\left((x^{(-3-2*p)}*(a+b*x^2)^p*\text{Hypergeometric2F1}[-3/2-p, -p, -1/2-p, -(b*x^2)/a])/(a*(3+2*p)*(1+(b*x^2)/a)^p)\right)$

**Maple [F]** time = 0.085, size = 0, normalized size = 0.

$$\int x^{-4-2p} (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-4-2*p)*(b*x^2+a)^p,x)`

[Out] `int(x^(-4-2*p)*(b*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-2p-4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 4),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^(-2*p - 4), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^p x^{-2p-4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 4),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^(-2*p - 4), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-4-2*p)*(b*x**2+a)**p,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-2p-4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 4),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^(-2*p - 4), x)`

### 3.1065 $\int x^{-3-2p} (a + bx^2)^p dx$

**Optimal.** Leaf size=30

$$\frac{x^{-2(p+1)} (a + bx^2)^{p+1}}{2a(p+1)}$$

[Out]  $-(a + b*x^2)^{(1 + p)}/(2*a*(1 + p)*x^{(2*(1 + p))})$

**Rubi [A]** time = 0.0213797, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{x^{-2(p+1)} (a + bx^2)^{p+1}}{2a(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^(-3 - 2\*p)\*(a + b\*x^2)^p, x]

[Out]  $-(a + b*x^2)^{(1 + p)}/(2*a*(1 + p)*x^{(2*(1 + p))})$

**Rubi in Sympy [A]** time = 4.03935, size = 26, normalized size = 0.87

$$\frac{x^{-2p-2} (a + bx^2)^{p+1}}{2a(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(-3-2\*p)\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $-x^{(-2*p - 2)}*(a + b*x**2)**(p + 1)/(2*a*(p + 1))$

**Mathematica [A]** time = 0.0448504, size = 30, normalized size = 1.

$$\frac{x^{-2p-2} (a + bx^2)^{p+1}}{2ap + 2a}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-3 - 2\*p)\*(a + b\*x^2)^p, x]

[Out]  $-((x^{(-2 - 2*p)}*(a + b*x^2)^{(1 + p)}))/(2*a + 2*a*p)$

**Maple [A]** time = 0.004, size = 29, normalized size = 1.

$$\frac{x^{-2-2p} (bx^2 + a)^{1+p}}{2a(1+p)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-3-2\*p)\*(b\*x^2+a)^p, x)

[Out]  $-1/2 * x^{(-2-2*p)} * (b * x^2 + a)^{(1+p)} / a / (1+p)$

---

**Maxima [A]** time = 1.36654, size = 50, normalized size = 1.67

$$\frac{(bx^2 + a) e^{(p \log(bx^2 + a) - 2p \log(x))}}{2 a(p + 1)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 3),x, algorithm="maxima")`

[Out]  $-1/2 * (b * x^2 + a) * e^{(p * \log(b * x^2 + a) - 2 * p * \log(x))} / (a * (p + 1) * x^2)$

---

**Fricas [A]** time = 0.227759, size = 46, normalized size = 1.53

$$\frac{(bx^3 + ax)(bx^2 + a)^p x^{-2p-3}}{2(ap + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 3),x, algorithm="fricas")`

[Out]  $-1/2 * (b * x^3 + a * x) * (b * x^2 + a)^p * x^{(-2 * p - 3)} / (a * p + a)$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-3-2*p)*(b*x**2+a)**p,x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-2p-3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 3),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^(-2*p - 3), x)`

### 3.1066 $\int x^{-2-2p} (a + bx^2)^p dx$

**Optimal.** Leaf size=53

$$\frac{x^{-2p-1} (a + bx^2)^{p+1} {}_2F_1\left(\frac{1}{2}, 1; \frac{1}{2}(1-2p); -\frac{bx^2}{a}\right)}{a(2p+1)}$$

[Out]  $-\left(\left(x^{(-1-2*p)}\right)\left(a+b*x^2\right)^{(1+p)}\text{Hypergeometric2F1}\left[\frac{1}{2}, 1, \left(1-2*p\right)/2, -\left(\left(b*x^2\right)/a\right)\right]\right)/\left(a*\left(1+2*p\right)\right)$

**Rubi [A]** time = 0.0533584, antiderivative size = 70, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{x^{-2p-1} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}(-2p-1), -p; \frac{1}{2}(1-2p); -\frac{bx^2}{a}\right)}{2p+1}$$

Antiderivative was successfully verified.

[In] Int[x^(-2 - 2\*p) \* (a + b\*x^2)^p, x]

[Out]  $-\left(\left(x^{(-1-2*p)}\right)\left(a+b*x^2\right)^p\text{Hypergeometric2F1}\left[\left(-1-2*p\right)/2, -p, \left(1-2*p\right)/2, -\left(\left(b*x^2\right)/a\right)\right]\right)/\left(\left(1+2*p\right)*\left(1+\left(b*x^2\right)/a\right)^p\right)$

**Rubi in Sympy [A]** time = 9.20424, size = 53, normalized size = 1.

$$\frac{x^{-2p-1} \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, -p - \frac{1}{2} \middle| -\frac{bx^2}{a}\right)}{2p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(-2-2\*p)\*(b\*x\*\*2+a)\*\*p,x)

[Out]  $-x^{(-2*p-1)}\left(1+b*x**2/a\right)^{-p}\left(a+b*x**2\right)^p\text{hyper}\left(\left(-p,-p-1/2\right),\left(-p+1/2,\right),-b*x**2/a\right)/\left(2*p+1\right)$

**Mathematica [A]** time = 0.0542976, size = 66, normalized size = 1.25

$$\frac{x^{-2p-1} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(-p - \frac{1}{2}, -p; \frac{1}{2} - p; -\frac{bx^2}{a}\right)}{2p+1}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-2 - 2\*p) \* (a + b\*x^2)^p, x]

[Out]  $-\left(\left(x^{(-1-2*p)}\right)\left(a+b*x^2\right)^p\text{Hypergeometric2F1}\left[-1/2-p, -p, 1/2-p, -\left(\left(b*x^2\right)/a\right)\right]\right)/\left(\left(1+2*p\right)*\left(1+\left(b*x^2\right)/a\right)^p\right)$

**Maple [F]** time = 0.083, size = 0, normalized size = 0.

$$\int x^{-2-2p} (bx^2 + a)^p dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-2-2*p)*(b*x^2+a)^p,x)`

[Out] `int(x^(-2-2*p)*(b*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-2p-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 2),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^(-2*p - 2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^p x^{-2p-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 2),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^(-2*p - 2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-2-2*p)*(b*x**2+a)**p,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-2p-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 2),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^(-2*p - 2), x)`

$$3.1067 \quad \int x^{-1-2p} (a + bx^2)^p dx$$

**Optimal.** Leaf size=43

$$\frac{x^{-2p} (a + bx^2)^{p+1} {}_2F_1\left(1, 1; 1 - p; -\frac{bx^2}{a}\right)}{2ap}$$

[Out]  $-\left((a + b*x^2)^{(1 + p)} \text{Hypergeometric2F1}\left[1, 1, 1 - p, -\left(\frac{b*x^2}{a}\right)\right]\right) / \left(2*a*p*x^{(2*p)}\right)$

**Rubi [A]** time = 0.0454165, antiderivative size = 56, normalized size of antiderivative = 1.3, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{x^{-2p} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(-p, -p; 1 - p; -\frac{bx^2}{a}\right)}{2p}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 - 2\*p) \* (a + b\*x^2)^p, x]

[Out]  $-\left((a + b*x^2)^p \text{Hypergeometric2F1}\left[-p, -p, 1 - p, -\left(\frac{b*x^2}{a}\right)\right]\right) / \left(2*p*x^{(2*p)} * \left(1 + \left(\frac{b*x^2}{a}\right)\right)^p\right)$

**Rubi in Sympy [A]** time = 8.49927, size = 42, normalized size = 0.98

$$\frac{x^{-2p} \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, -p; -p + 1; -\frac{bx^2}{a}\right)}{2p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(-1-2\*p) \* (b\*x\*\*2+a)\*\*p, x)

[Out]  $-x^{(-2*p)} * \left(1 + \frac{b*x**2}{a}\right)^{(-p)} * (a + b*x**2)**p * \text{hyper}\left(\left(-p, -p\right), \left(-p + 1\right), -\frac{b*x**2}{a}\right) / (2*p)$

**Mathematica [A]** time = 0.0356083, size = 56, normalized size = 1.3

$$\frac{x^{-2p} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(-p, -p; 1 - p; -\frac{bx^2}{a}\right)}{2p}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 - 2\*p) \* (a + b\*x^2)^p, x]

[Out]  $-\left((a + b*x^2)^p \text{Hypergeometric2F1}\left[-p, -p, 1 - p, -\left(\frac{b*x^2}{a}\right)\right]\right) / \left(2*p*x^{(2*p)} * \left(1 + \left(\frac{b*x^2}{a}\right)\right)^p\right)$

**Maple [F]** time = 0.074, size = 0, normalized size = 0.

$$\int x^{-1-2p} (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(-1-2*p)*(b*x^2+a)^p,x)`

[Out] `int(x^(-1-2*p)*(b*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-2p-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 1),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^(-2*p - 1), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^p x^{-2p-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 1),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^(-2*p - 1), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1-2*p)*(b*x**2+a)**p,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-2p-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p - 1),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^(-2*p - 1), x)`

### 3.1068 $\int x^{-2p} (a + bx^2)^p dx$

**Optimal.** Leaf size=52

$$\frac{x^{1-2p} (a + bx^2)^{p+1} {}_2F_1\left(1, \frac{3}{2}; \frac{1}{2}(3-2p); -\frac{bx^2}{a}\right)}{a(1-2p)}$$

[Out]  $(x^{(1-2p)}*(a+b*x^2)^{(1+p)}*Hypergeometric2F1[1, 3/2, (3-2p)/2, -(b*x^2)/a])/(a*(1-2p))$

**Rubi [A]** time = 0.048455, antiderivative size = 69, normalized size of antiderivative = 1.33, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{x^{1-2p} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}(1-2p), -p; \frac{1}{2}(3-2p); -\frac{bx^2}{a}\right)}{1-2p}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^p/x^{(2*p)}, x]$

[Out]  $(x^{(1-2p)}*(a+b*x^2)^p*Hypergeometric2F1[(1-2p)/2, -p, (3-2p)/2, -(b*x^2)/a])/((1-2p)*(1+(b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 9.07013, size = 48, normalized size = 0.92

$$\frac{x^{-2p+1} \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, -p + \frac{1}{2}; -p + \frac{3}{2}; -\frac{bx^2}{a}\right)}{-2p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x^{**2}+a)**p/(x^{**}(2*p)), x)$

[Out]  $x^{*(-2*p + 1)}*(1 + b*x^{**2}/a)**(-p)*(a + b*x^{**2})**p*\text{hyper}((-p, -p + 1/2), (-p + 3/2, ), -b*x^{**2}/a)/(-2*p + 1)$

**Mathematica [A]** time = 0.0483296, size = 65, normalized size = 1.25

$$\frac{x^{1-2p} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2} - p, -p; \frac{3}{2} - p; -\frac{bx^2}{a}\right)}{1-2p}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b*x^2)^p/x^{(2*p)}, x]$

[Out]  $(x^{(1-2p)}*(a+b*x^2)^p*Hypergeometric2F1[1/2 - p, -p, 3/2 - p, -(b*x^2)/a])/((1-2p)*(1+(b*x^2)/a)^p)$

**Maple [F]** time = 0.082, size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^{2p}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^p/(x^(2*p)),x)`

[Out] `int((b*x^2+a)^p/(x^(2*p)),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-2p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x^(2*p),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^(-2*p), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p}{x^{2p}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x^(2*p),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p/x^(2*p), x)`

**Sympy [A]** time = 91.2144, size = 24, normalized size = 0.46

$$b^p x {}_2F_1\left(-\frac{1}{2}, -p \mid \frac{ae^{i\pi}}{bx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**p/(x**(2*p)),x)`

[Out] `b**p*x*hyper((-1/2, -p), (1/2, ), a*exp_polar(I*pi)/(b*x**2))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^{2p}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/x^(2*p),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p/x^(2*p), x)`

### 3.1069 $\int x^{1-2p} (a + bx^2)^p dx$

**Optimal.** Leaf size=49

$$\frac{x^{2-2p} (a + bx^2)^{p+1} {}_2F_1\left(1, 2; 2 - p; -\frac{bx^2}{a}\right)}{2a(1 - p)}$$

[Out] (x^(2 - 2\*p)\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 2, 2 - p, -(b\*x^2)/a])/(2\*a\*(1 - p))

**Rubi [A]** time = 0.0516875, antiderivative size = 64, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{x^{2-2p} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(1 - p, -p; 2 - p; -\frac{bx^2}{a}\right)}{2(1 - p)}$$

Antiderivative was successfully verified.

[In] Int[x^(1 - 2\*p)\*(a + b\*x^2)^p, x]

[Out] (x^(2 - 2\*p)\*(a + b\*x^2)^p\*Hypergeometric2F1[1 - p, -p, 2 - p, -(b\*x^2)/a])/(2\*(1 - p)\*(1 + (b\*x^2)/a)^p)

**Rubi in Sympy [A]** time = 8.93124, size = 44, normalized size = 0.9

$$\frac{x^{-2p+2} \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, -p + 1; -p + 2; -\frac{bx^2}{a}\right)}{2(-p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(1-2\*p)\*(b\*x\*\*2+a)\*\*p, x)

[Out] x\*\*(-2\*p + 2)\*(1 + b\*x\*\*2/a)\*\*(-p)\*(a + b\*x\*\*2)\*\*p\*hyper((-p, -p + 1), (-p + 2, ), -b\*x\*\*2/a)/(2\*(-p + 1))

**Mathematica [A]** time = 0.0466132, size = 62, normalized size = 1.27

$$\frac{x^{2-2p} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(1 - p, -p; 2 - p; -\frac{bx^2}{a}\right)}{2(p - 1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(1 - 2\*p)\*(a + b\*x^2)^p, x]

[Out] -(x^(2 - 2\*p)\*(a + b\*x^2)^p\*Hypergeometric2F1[1 - p, -p, 2 - p, -(b\*x^2)/a])/(2\*(-1 + p)\*(1 + (b\*x^2)/a)^p)

**Maple [F]** time = 0.083, size = 0, normalized size = 0.

$$\int x^{1-2p} (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1-2*p)*(b*x^2+a)^p,x)`

[Out] `int(x^(1-2*p)*(b*x^2+a)^p,x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-2p+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p + 1),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^(-2*p + 1), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^p x^{-2p+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p + 1),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^(-2*p + 1), x)`

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1-2*p)*(b*x**2+a)**p,x)`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-2p+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p + 1),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^(-2*p + 1), x)`

### 3.1070 $\int x^{2-2p} (a + bx^2)^p dx$

**Optimal.** Leaf size=52

$$\frac{x^{3-2p} (a + bx^2)^{p+1} {}_2F_1\left(1, \frac{5}{2}; \frac{1}{2}(5-2p); -\frac{bx^2}{a}\right)}{a(3-2p)}$$

[Out]  $(x^{3-2p} (a + b*x^2)^{(1+p)} \text{Hypergeometric2F1}[1, 5/2, (5-2p)/2, -(b*x^2)/a]) / (a*(3-2p))$

**Rubi [A]** time = 0.0517598, antiderivative size = 69, normalized size of antiderivative = 1.33, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{x^{3-2p} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}(3-2p), -p; \frac{1}{2}(5-2p); -\frac{bx^2}{a}\right)}{3-2p}$$

Antiderivative was successfully verified.

[In] Int[x^(2 - 2\*p) \* (a + b\*x^2)^p, x]

[Out]  $(x^{3-2p} (a + b*x^2)^p \text{Hypergeometric2F1}[(3-2p)/2, -p, (5-2p)/2, -(b*x^2)/a]) / ((3-2p) * (1 + (b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 8.88666, size = 48, normalized size = 0.92

$$\frac{x^{-2p+3} \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, -p + \frac{3}{2}; -p + \frac{5}{2}; -\frac{bx^2}{a}\right)}{-2p + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(2-2\*p) \* (b\*x\*\*2+a)\*\*p, x)

[Out]  $x^{(-2*p + 3)} * (1 + b*x**2/a)**(-p) * (a + b*x**2)**p * \text{hyper}((-p, -p + 3/2), (-p + 5/2, ), -b*x**2/a) / (-2*p + 3)$

**Mathematica [A]** time = 0.0533408, size = 65, normalized size = 1.25

$$\frac{x^{3-2p} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{2} - p, -p; \frac{5}{2} - p; -\frac{bx^2}{a}\right)}{3-2p}$$

Antiderivative was successfully verified.

[In] Integrate[x^(2 - 2\*p) \* (a + b\*x^2)^p, x]

[Out]  $(x^{3-2p} (a + b*x^2)^p \text{Hypergeometric2F1}[3/2 - p, -p, 5/2 - p, -(b*x^2)/a]) / ((3-2p) * (1 + (b*x^2)/a)^p)$

**Maple [F]** time = 0.083, size = 0, normalized size = 0.

$$\int x^{2-2p} (bx^2 + a)^p dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(2-2*p)*(b*x^2+a)^p,x)`

[Out] `int(x^(2-2*p)*(b*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-2p+2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p + 2),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^(-2*p + 2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^p x^{-2p+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p + 2),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^(-2*p + 2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(2-2*p)*(b*x**2+a)**p,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-2p+2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p + 2),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^(-2*p + 2), x)`

### 3.1071 $\int x^{3-2p} (a + bx^2)^p dx$

**Optimal.** Leaf size=49

$$\frac{x^{4-2p} (a + bx^2)^{p+1} {}_2F_1\left(1, 3; 3 - p; -\frac{bx^2}{a}\right)}{2a(2 - p)}$$

[Out] (x^(4 - 2\*p)\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 3, 3 - p, -(b\*x^2)/a])/(2\*a\*(2 - p))

**Rubi [A]** time = 0.0534336, antiderivative size = 64, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{x^{4-2p} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(2 - p, -p; 3 - p; -\frac{bx^2}{a}\right)}{2(2 - p)}$$

Antiderivative was successfully verified.

[In] Int[x^(3 - 2\*p)\*(a + b\*x^2)^p, x]

[Out] (x^(4 - 2\*p)\*(a + b\*x^2)^p\*Hypergeometric2F1[2 - p, -p, 3 - p, -(b\*x^2)/a])/(2\*(2 - p)\*(1 + (b\*x^2)/a)^p)

**Rubi in Sympy [A]** time = 8.65634, size = 44, normalized size = 0.9

$$\frac{x^{-2p+4} \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, -p + 2; -p + 3; -\frac{bx^2}{a}\right)}{2(-p + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*(3-2\*p)\*(b\*x\*\*2+a)\*\*p, x)

[Out] x\*\*(-2\*p + 4)\*(1 + b\*x\*\*2/a)\*\*(-p)\*(a + b\*x\*\*2)\*\*p\*hyper((-p, -p + 2), (-p + 3, ), -b\*x\*\*2/a)/(2\*(-p + 2))

**Mathematica [A]** time = 0.0504031, size = 62, normalized size = 1.27

$$\frac{x^{4-2p} (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(2 - p, -p; 3 - p; -\frac{bx^2}{a}\right)}{2(p - 2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3 - 2\*p)\*(a + b\*x^2)^p, x]

[Out] -(x^(4 - 2\*p)\*(a + b\*x^2)^p\*Hypergeometric2F1[2 - p, -p, 3 - p, -(b\*x^2)/a])/(2\*(-2 + p)\*(1 + (b\*x^2)/a)^p)

**Maple [F]** time = 0.083, size = 0, normalized size = 0.

$$\int x^{3-2p} (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3-2*p)*(b*x^2+a)^p,x)`

[Out] `int(x^(3-2*p)*(b*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-2p+3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p + 3),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^(-2*p + 3), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)^p x^{-2p+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p + 3),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^(-2*p + 3), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(3-2*p)*(b*x**2+a)**p,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)^p x^{-2p+3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^(-2*p + 3),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^(-2*p + 3), x)`

## 4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result, optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result, optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_, optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result, Complex] || Not[FreeQ[optimal, Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result, Integrate] && FreeQ[result, Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```

ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType, expn]],
  If[Head[expn]==Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]==Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]==Rational, 1,
      Max[ExpnType[expn[[1]]], 2]],
    Max[ExpnType[expn[[1]]], ExpnType[expn[[2]]], 3]],
  If[Head[expn]==Plus || Head[expn]==Times,
    Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3, ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
  If[Head[expn]==RootSum,
    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
  If[Head[expn]==Integrate || Head[expn]==Int,
    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
  9]]]]]]]]]]]]

```

```

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, CsCh,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsCh
  }, func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]
AppellFunctionQ[func_] := MemberQ[{AppellF1}, func]

```

```

# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);
    #This check below actually is not needed, since I only call this grading only for
    #passed integrals. i.e. I check for "F" before calling this.

    if not type(result,freeof('int')) then
        return "F";
    end if;

    if ExpnType_result<=ExpnType_optimal then
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                #both result and optimal complex
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A";
                else
                    return "B";
                end if
            else #result contains complex but optimal is not
                return "C";
            end if
        else # result do not contain complex
            # this assumes optimal do not as well
            if leaf_count_result<=2*leaf_count_optimal then
                return "A";
            else
                return "B";
            end if
        end if
    else #ExpnType(result) > ExpnType(optimal)
        return "C";
    end if
end proc:

```

```

# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,``^``) then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
        max(2,ExpnType(op(1,expn))) end if else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,``+``) or type(expn,``*``) then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

```

```

ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```